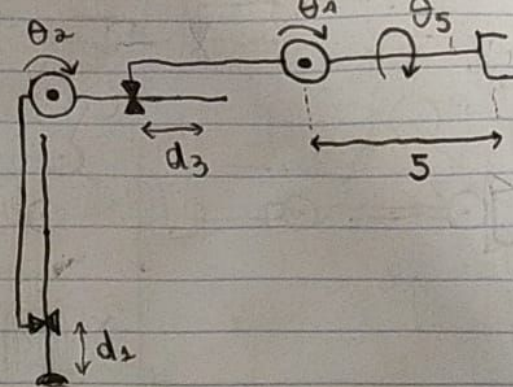
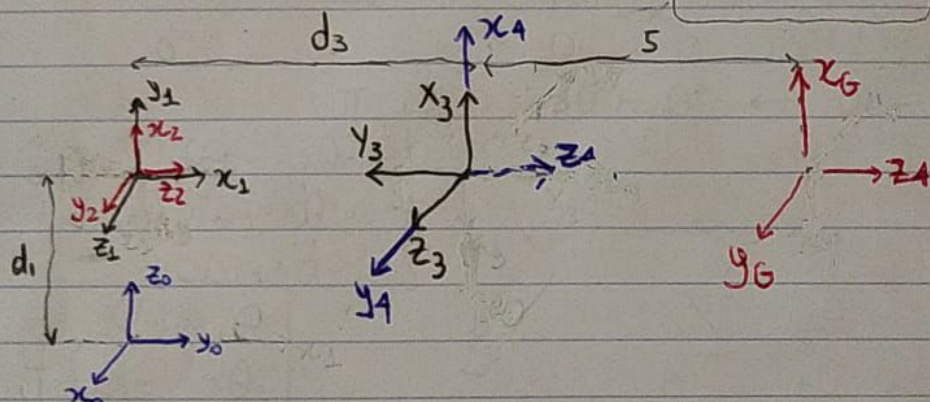
HW_3 (Robótica)

João Vítor Sgotti Veiga, 2017170653
Miguel Mendes Silva, 2017257766

①

	θ_i	d_i	a_i	α_i	off
$0 \to 1$	$\pi/2$	d_1	0	$\pi/2$	0
$1 \to 2$	θ_2	0	0	$\pi/2$	$\pi/2$
$2 \to 3$	0	d_3	0	$-\pi/2$	0
$3 \to 4$	θ_4	0	0	$\pi/2$	0
$4 \to G$	θ_5	5	0	0	0





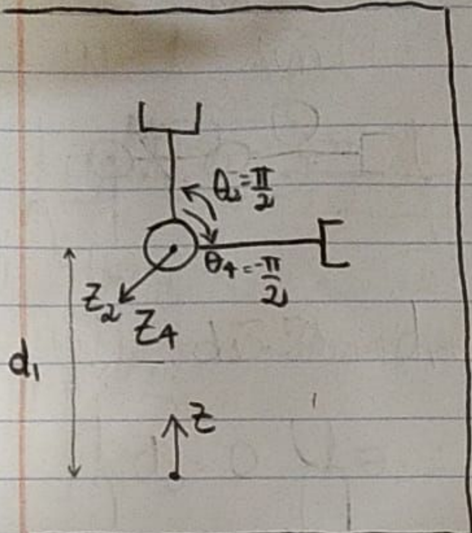
$$
{}^{0}_{G}p = \begin{bmatrix} 0 \\ 5C_{24} + d_3 C_2 \\ 5S_{24} + d_3 S_2 + d_1 \end{bmatrix}
$$

a)

$$
J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -5S_{24} - d_3 S_2 & C_2 & -5S_{24} & 0 \\ 1 & 5C_{24} + d_3 C_2 & S_2 & 5C_{24} & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & C_{24} \\ 0 & 0 & 0 & 0 & S_{24} \end{bmatrix} \rightarrow
$$

$v_z =$

b)



Só podem haver singularidades (quando as componentes das velocidades se anulam) em:

$$\begin{cases} v_y = \dot{\theta}_2(-5 s_{24} - d_3 s_2) + \dot{d}_3 (c_2) + \dot{\theta}_4 (-5 s_{24}) \\ \omega_y = \dot{\theta}_5 (c_{24}) \\ \omega_z = \dot{\theta}_5 (s_{24}) \end{cases}$$

$$v_y = \dot{\theta}_2(-5 s_{24} - d_3 s_2) + \dot{d}_3 c_2 + \dot{\theta}_4(-5 s_{24}) = 0$$

$$\begin{cases} \cos\theta_2 = 0 \rightarrow \theta_2 = \pm \pi/2 \\ -5 \sin(\theta_2 + \theta_4) = 0 \rightarrow \theta_2 + \theta_4 = 0 \text{ ou } \pi \\ -5 s_{24} - d_3 s_2 = 0 \end{cases}$$

$\hookrightarrow \theta_2 + \theta_4 = 0 \Rightarrow \theta_4 = -\theta_2$

$\hookrightarrow \theta_2 + \theta_4 = \pi \Rightarrow \theta_4 = \pi - \theta_2$

$\hookrightarrow \theta_2 = \pi/2$

$\hookrightarrow \theta_4 = -\pi/2$

$-5 \underbrace{s(0)}_{0} - d_3 \underbrace{s(\pi/2)}_{1} = 0$

$d_3 = 0$

$\hookrightarrow \theta_2 = -\pi/2$

$\hookrightarrow \theta_4 = 3\pi/4$

$-5 \underbrace{s(\pi)}_{0} - d_3 \underbrace{s(-\pi/2)}_{-1} = 0$

$$\boxed{\begin{matrix} \theta_2 = \pm \pi/2 \\ \theta_4 = -\theta_2 \\ d_3 = 0 \end{matrix}}$$

$$\omega_y = \cos(\theta_2 + \theta_4) = 0 \rightsquigarrow \theta_2 + \theta_4 = \pm\pi/2 \rightsquigarrow \boxed{\theta_2 = \pm\frac{\pi}{2} - \theta_4}$$

$$\omega_z = \sin(\theta_2 + \theta_4) = 0 \rightsquigarrow \theta_2 + \theta_4 = 0 / \pi \rightsquigarrow \boxed{\begin{matrix} \theta_2 = -\theta_4 \\ \theta_2 = \pi - \theta_4 \end{matrix}}$$

$v_x = 0$

c) $v_y = \dot{\theta}_2(-5s_{21} - d_3 s_2) + \dot{d}_3(C_2) + \dot{\theta}_1(-5s_{21}) = 0$

$v_z = d_1$

$\rightarrow v_y = 0 \Rightarrow d_3 = 0,\ \theta_2 = \pm\pi/2,\ \theta_1 = -\theta_2$

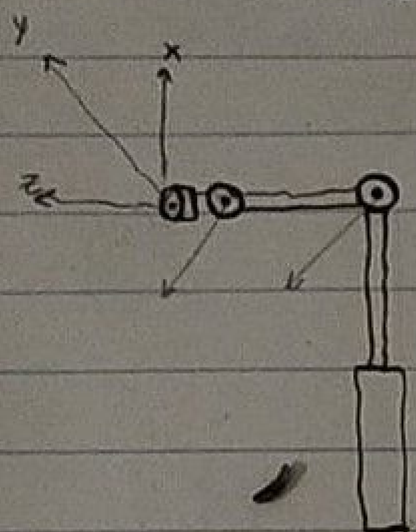
$\omega_y = \dot{\theta}_5(C_{21}) \rightsquigarrow \omega_y = 0 \Rightarrow \theta_2 + \theta_1 = \pm\pi/2$

$\omega_z = \dot{\theta}_5(S_{21}) \rightsquigarrow \omega_z = 0 \Rightarrow \theta_2 + \theta_1 = \pm\pi$

$\theta_5(S_{21}) \rightsquigarrow$

1d) $$\begin{array}{cccccccc} & & v_x & v_y & v_z & \omega_x & \omega_y & \omega_z \\ {}^4F_{1,apli} = & \Big[& 2 & -1 & 3 & 0 & 10 & 0 & \Big]^T \\ & & \square & \boxtimes & \square & \boxtimes & \square & \square \end{array}$$

estas forças estão contra as juntas. (logo, só atuam contra a resistência do material. Portanto não é necessário que exista uma compensação do motor p/ a estática)



Melhor visualização através da simulação do MATLAB: "HW3_ExTeorico_1"

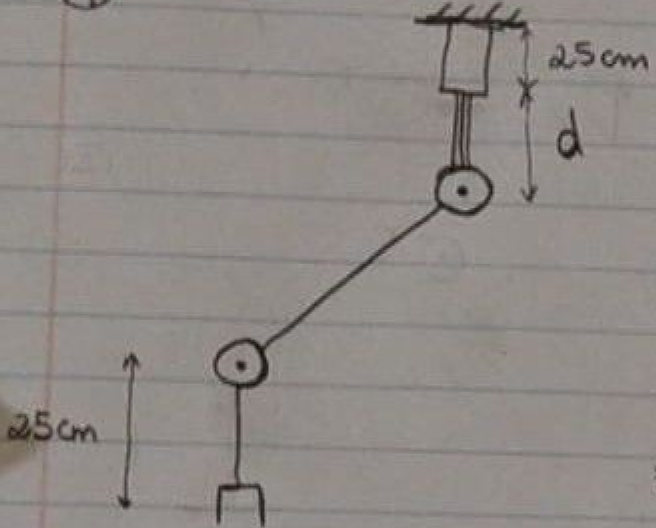
$${}^4F_{1,apli} = [2\ 0\ 3\ |\ 0\ 10\ 0]^T$$

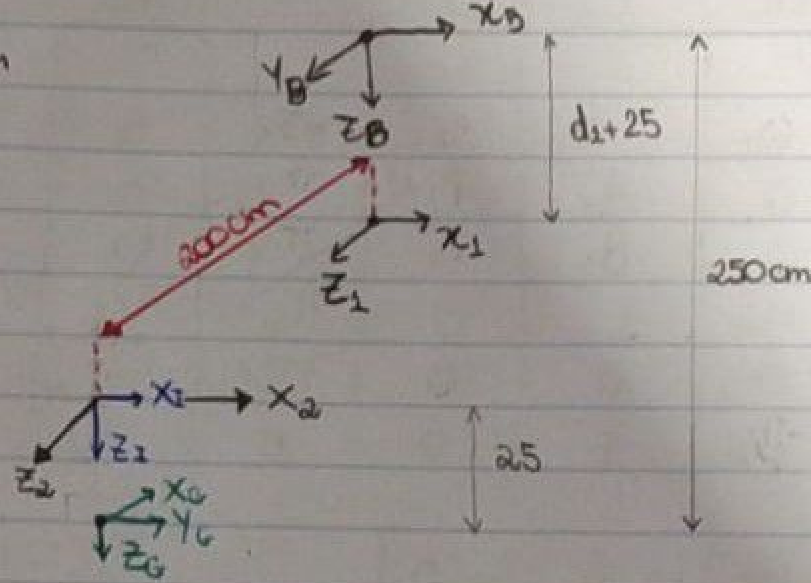
$${}^0_4R = \begin{bmatrix} 0 & 1 & 0 \\ -S_{24} & 0 & C_{24} \\ C_{24} & 0 & S_{24} \end{bmatrix} \qquad {}^0_4F = \begin{bmatrix} {}^0_4R & 0_{3\times3} \\ 0_{3\times3} & {}^0_4R \end{bmatrix}$$

$${}^GF = \begin{bmatrix} -1 \\ 3C_{24} - 2S_{24} \\ 2C_{24} + 3S_{24} \\ 10 \\ 0 \\ 0 \end{bmatrix} \qquad \mathcal{J} = J_{ae}^T \cdot {}^0_4F = \begin{bmatrix} 2C_{24} + 3S_{24} \\ 2d_3C_4 + 3d_3S_4 + 20 \\ 3C_4 - 2S_4 \\ 20 \\ 0 \end{bmatrix}$$

$$q^T = [\underset{\substack{\uparrow \\ d_1}}{10}\ \ \underset{\substack{\uparrow \\ \theta_2}}{0}\ \ \underset{\substack{\uparrow \\ d_3}}{10}\ \ \underset{\substack{\uparrow \\ \theta_4}}{0}\ \ \underset{\substack{\uparrow \\ \theta_5}}{0}]$$

④





✱	θ_i	d_i	a_i	α_i	off
B→1	0	d_1	0	$-\pi/2$	25
1→2	θ_2	0	200	0	0
2→I	θ_3	0	0	$\pi/2$	0
I→G	$-\pi/2$	25	0	0	0

a) $$Tf = \begin{bmatrix} \sin(25) & \cos(25).C_{23} & \cos(25).S_{23} & 25\cos(25)S_{23}+8C_2 \\ -\cos(25) & \sin(25)C_{23} & \sin(25)S_{23} & 25\sin(25)S_{23}+8C_2 \\ 0 & -S_{23} & C_{23} & d_1+25C_{23}-200S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Se utilizando do script de matlab (anexo), obtivemos o seguinte resultado p/ o ${}^{B}J_{6\times3}$:

$$
{}^{B}J = \begin{bmatrix} 0 & 25\cos(25)C_{23} - 8S_2 & 25\cos(25)C_{23} \\ 0 & 25\cos(25)C_{23} - 8S_2 & 25\sin(25)C_{23} \\ 1 & -25S_{23} - 200C_2 & -25S_{23} \\ 0 & -\sin(25) & -\sin(25) \\ 0 & \cos(25) & \cos(25) \\ 0 & 0 & 0 \end{bmatrix}
$$

c) $|v_{Bx}| = 10\ m/s$

$$
\begin{bmatrix} \dot{d}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = {}^{B}Jac^{-1} \begin{bmatrix} v_x \\ v_z \\ \omega_y \end{bmatrix} = \text{pin}(Jac) \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\dot{q} = \begin{bmatrix} \dot{d}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -10C_2/S_2 \\ -1/20S_2 \\ 1/20S_2 \end{bmatrix}
$$

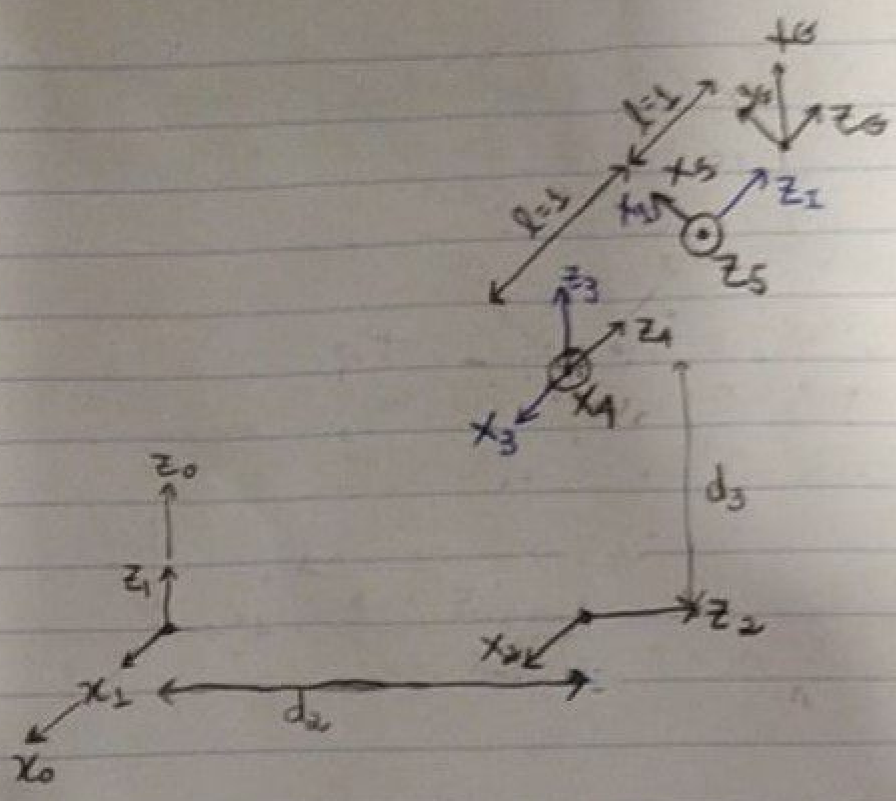
d)

$$
\tau = Jac(Q)^T \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 255 & 0 & 0 & 0 & 1 & 0 \\ 25 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2500 \\ 250 \end{bmatrix}
$$

#7

12



	θ_i	d_i	a_i	α_i	off
1→2	θ_1	0	0	$-\pi/2$	0
2→3	0	d_2	0	$\pi/2$	0
3→4	0	d_3	0	$-\pi/4$	0
4→5	θ_4	1	0	$\pi/2$	$-\pi/2$
5→I	θ_5	0	0	$+\pi/2$	0
I→6	$-\pi/2$	1	0	0	0

$$T_i(:,:,1) = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & -1 & C_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(:,:,4) = \begin{bmatrix} S_4 & 0 & -C_4 \\ -C_4 & 0 & -S_4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T(:,:,2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(:,:,5) = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(:,:,3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(:,:,6) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12b)

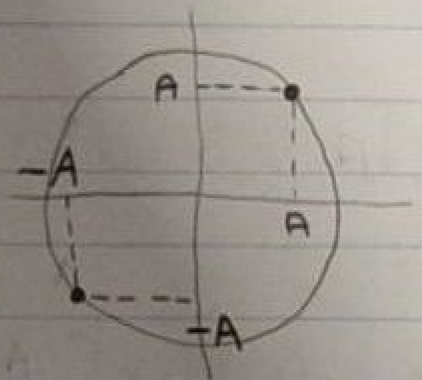
$$ {}^{0}J\_v = \begin{bmatrix} -C_1\left(d_2+\sqrt{2}/2\right) & -S_1 & 0 \\ -S_1\left(d_2+\sqrt{2}/2\right) & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ -C_1^2\left(d_2+\sqrt{2}/2\right) + S_1^2\left(d_2+\sqrt{2}/2\right) = 0 $$

$$ \left(d_2+\sqrt{2}/2\right)\left(S_1^2 - C_1^2\right) = 0 $$

$$ \begin{cases} d_2 = -\sqrt{2}/2 \\ \sin^2(t_1) = \cos^2(t_1) \end{cases} $$

$$ \hookrightarrow t_1 = \pm\frac{\pi}{4} $$



$A = \dfrac{\pi}{4}$