Exercícios robótica:


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1. ${}^{A}_{B}T = I \rightarrow$

a) Deslocar sist. de coordenadas A em 2 und segundo o eixo de rotação ${}^{B}\vec{r} = [0,0,1]^T$.

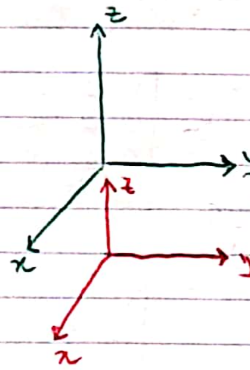
1º

deslocado

$${}^{A'}_{B}T = {}^{A'}_{A}T \cdot {}^{A}_{B}T$$

A

B



$$D_z(2) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$${}^{A'}_{B}T = {}^{A}_{B}T \cdot \left[D_z(2) \cdot \left({}^{B}_{A}T\right)\right]^{-1}$$

$${}^{A'}_{B}T = I_{4\times4} \cdot \left[D_z(2) \cdot I_{4\times4}\right]^{-1}$$

$${}^{A'}_{B}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$- \; {}^{A}_{B}T = \boxed{ATB} \cdot D$$

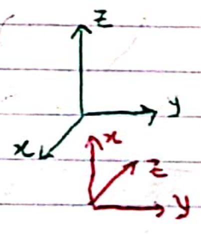
2º Rodar sist. de coordenadas B de um angulo $-\pi/2$ sobre o eixo de rotacão ${}^{A_{ini}}\vec{r} = [0,-1,0]^T$ inicial.

$${}^{A'}_{B'}T = {}^{A'}_{B}T \cdot {}^{B}_{B'}T$$

$${}^{B}_{B'}T = {}^{B}_{A}T \cdot {}^{B}_{A}T = \left({}^{B}_{A}T\right)^{-1} T_{\vec{r}} \; {}^{A}_{B}T$$

$${}^{A'}_{B'}T = {}^{A'}_{B}T \cdot {}^{B}_{A}T \cdot R_{-y}(-\pi/2) \cdot {}^{A}_{B}T$$

${}^{A'}_{A}T$



$$R_{-y}\left(-\frac{\pi}{2}\right) = \begin{bmatrix} 0 & 0 & +1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{A'}_{B'}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & +1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3º Rodar o sist. de coordenadas B de um ângulo igual a $+\pi/2$ sobre o eixo ${}^{A}\vec{r} = [1, 0, 0]$

$$ {}^{A'}_{B''}T = {}^{A'}_{B'}T \cdot \underbrace{R_x(\pi/2)}_{{}^{B'}_{B''}T} $$

$$ {}^{A'}_{B''}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

$$ {}^{A'}_{B''}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

b) $\quad {}^{A'}P = {}^{A'}_{B''}T \cdot {}^{B''}P$

$$ {}^{A'}_{B''}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

c) $\quad {}^{B}_{B''}T = {}^{B}_{A'}T \cdot {}^{A'}_{B''}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$ {}^{B}_{B'''}T = T_{\vec{r}} \cdot {}^{B}_{B''}T = \begin{bmatrix} 0 & \sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2} \\ 1 & 0 & 0 & 2 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

d) ${}^{A}_{B'''}R = {}^{A}_{B}R \cdot {}^{B}_{B'''}R = I_{3\times3} = \begin{bmatrix} 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}$

$$q_1 = \frac{r_{32} \cdot r_{23}}{4q_0} = 0{,}653$$

$$q_2 = \frac{r_{13} - r_{31}}{4 \cdot q_0} = 0{,}653$$

$$q_3 = \frac{r_{21} - r_{12}}{4q_0} = 0{,}271$$

$$q_0 = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}}$$

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$1 \rightarrow R \Rightarrow P = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right]^T$

$2 \rightarrow$ deslocamento $t = [10, 30, 0]^T$

a) $\|p\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 0} = 1 \rightarrow$ 1ª ação

$\|p\| = tg(\theta) \Leftrightarrow \theta = \pi/2$

$$K = \frac{P}{\|p\|} = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right]$$

EIXO ARBITRÁRIO

$$R = \begin{bmatrix} 1/2 & 1/2 & \sqrt{2}/2 \\ 1/2 & 1/2 & -\sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \end{bmatrix}$$

2ª AÇÃO ⇒ $T = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$${}^{W}_{R}T = \begin{bmatrix} 1/2 & 1/2 & \sqrt{2}/2 & 10 \\ 1/2 & 1/2 & -\sqrt{2}/2 & 30 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) ${}^{w}_{obj}T = ?$

$${}^{w}_{obj}T = {}^{w}_{R}T \cdot {}^{R}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{b}T \cdot {}^{b}_{c}T \cdot {}^{c}_{obj}T$$

$$= \begin{bmatrix} 1/2 & 1/2 & \sqrt{2}/2 & 10 \\ 1/2 & 1/2 & -\sqrt{2}/2 & 30 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 30 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -20 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{c}_{cN}T = {}^{c}_{1}T \cdot T_{\vec{r}} \cdot {}^{1}_{c}T$$

$${}^{3}_{c}T = {}^{1}_{2}T \cdot {}^{2}_{p}T \cdot {}^{p}_{c}T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -25 \\ 0 & 0 & -1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow {}^{c}_{1}T = \begin{bmatrix} 0 & -1 & 0 & -25 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{\vec{r}} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 10\sqrt{2} \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 10\sqrt{2} \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}_{c}T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -25 \\ 0 & 0 & -1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{C}_{CU}T = \begin{bmatrix} 0 & -1 & 0 & -25 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 10\sqrt{2} \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 10\sqrt{2} \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -25 \\ 0 & 0 & -1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{C}_{CM}T = \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 & 0 & -6307/111 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 1393/394 \\ 0 & 0 & -1 & 40 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$