Decision Support Methods - Assignment 2

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Introduction

We were given a problem to locate a facility given a set of candidates towns. The available dataset [1] contains the population of each city and its coordinates allowing us to calculate the distances between them. The goal is to decide how many distribution centres along the country to locate constrained by a certain budget. Our model is generic and works for both problems. The only difference is the amount company has to open one or several DCs (budget).

Question 1

Optimization Model

Data

Question data:

Name	Description
$City$	set of cities
$latitude_c, c \in City$	cities' latitudes
$longitude_c, c \in City$	cities' longitudes
$population_c, c \in City$	cities' population
R	earth radius (km)
$deliveries_c = \lceil 3\frac{population_c}{1000} \rceil$	delivery cost between
$distance_{c_i c_j}, c_i, c_j \in City$	distance between c_i and c_j
$yearly\_cost = 25000€$	yearly cost of opening a DC
$cost_{c_i c_j} = distance_{c_i c_j} \times cost_{c_j}$	cost of deliveries from c_i to c_j
$max\_dc = \frac{budget}{yearly\_cost}$	maximum no. of DCs to open given a budget (afford limit)

Distance is given by the following formula: $distance_{c_i c_j} = 2 \times \pi \times R \times \frac{|latitude_{c_j} - latitude_{c_i}| + |longitude_{c_j} - longitude c_i|}{360}$

For this question, we assume: $budget = 25000€$

Variables

Name	Description
$dc_c \in \{0, 1\}$	DC is placed at city c
$deliver_{c_i c_j} \in \{0, 1\}$	City c_i delivers to c_j

Formulation

$$\text{minimize } z = \sum_{c \in City} dc_c \times yearly\_cost + \sum_{\substack{c_i \in City \\ c_i \in City}} deliver_{c_i c_j} \times cost_{c_i c_j} \tag{1}$$

$$\text{subject to:}$$
$$\sum_{c \in City} dc_c \leq max\_dc, \tag{2}$$
$$\sum_{\substack{i \in City \\ j \in City}} deliver_{c_i c_j} = 1, \tag{3}$$
$$deliver_{c_i c_j} \leq dc_i \tag{4}$$

Solution

Our model was implemented in GLPK. The solution can be obtained by running `$ ./solve_problem.sh 1`. Optimum cost z is 3831575€ and the location of the DC is `Santarém`. The town with largest delivery costs 167036€ is `Lisbon` (from `Santarém`).

Question 2

Our solution to question 2 is analogue of the previous. The only difference is the maximum amount of money (budget) that the company is willing spend to open one or several DCs (max. 5 DCs equivalent to 125000€).

Optimization Model

The model is analogue, except $budget = 125000€$.

Solution

The solution can be obtained by running `$ ./solve_problem.sh 2`. Optimum cost z is:

- 255836€ in `Lisbon` (DC)
- 67782€ in `Loulé` (DC)
- 156246€ in `Ourém` (DC)
- 265507€ in `Pedrouços` (DC)
- 119155€ in `Sernancelhe` (DC)

The towns with largest delivery costs for each DC listed above are:

- 255836€ is Évora (from `Lisbon DC`)
- 12070€ is Beja (from `Loulé DC`)
- 26200€ is Coimbra (from `Ourém DC`)
- 21418€ is Braga (from `Pedrouços DC`)
- 19004€ is Bragança (from `Sernancelhe DC`)

We also conclude that Évora is the town with largest delivery costs.

References

[1] https://www.dcc.fc.up.pt/~jpp/mad1920/PopulationContPT-2020.csv