Decision Support Methods - Assignment 2

Diogo Cordeiro João Varelas

Introduction

This problem consist in locating one or more facilities (DC) given a set of candidates towns. The available dataset [1] contains population of each city and coordinates allowing us to calculate the distances between them. Distance will reflect in delivery costs for towns that are far away from distribution center.

The goal is to decide how many DCs along the country to locate (and where) constrained by a certain budget. Our model is generic and works for both problems. The only difference is the amount company can afford (budget) to open one or several DCs.

Question 1

Optimization Model

Data

Name	Description
$City$	set of cities
$latitude_c,\ c \in City$	cities' latitudes
$longitude_c,\ c \in City$	cities' longitudes
$population_c,\ c \in City$	cities' population
R	earth radius (km)
$deliveries_c = \lceil 3\frac{population_c}{1000} \rceil$	avg. anual deliveries of c
$distance_{c_ic_j}, c_i, c_j \in City$	distance between c_i and c_j
$yearly\_cost = 25000€$	yearly cost of opening a DC
$cost_{c_ic_j} = distance_{c_ic_j} \times cost_{c_j}$	cost of deliveries from c_i to c_j
$max\_dc = \frac{budget}{yearly\_cost}$	maximum no. of DCs to open given a budget (afford limit)

Distance is given by the following formula: $distance_{c_ic_j} = 2\pi R \times \frac{|latitude_{c_j} - latitude_{c_i}| + |longitude_{c_j} - longitudec_i|}{360}$

For this question, we assume: $budget = 25000€$ and therefore, $max\_dc = \frac{25000}{25000} = 1$.

Company can afford to open 1 DC only.

Variables

Name	Description
$dc_c \in \{0, 1\}$	DC is placed at city c
$deliver_{c_ic_j} \in \{0, 1\}$	DC at city c_i delivers to city c_j

Formulation

$$\text{minimize} \quad z = \sum_{c \in City} dc_c \times yearly\_cost + \sum_{\substack{c_i \in City \\ c_i \in City}} deliver_{c_i c_j} \times cost_{c_i c_j} \tag{1}$$

$$\text{subject to:} \quad \sum_{c \in City} dc_c \leq max\_dc, \tag{2}$$

$$\sum_{i \in City} deliver_{c_i c_j} = 1, \qquad \forall c_j \in City \tag{3}$$

$$deliver_{c_i c_j} \leq dc_i \qquad \forall c_i, c_j \in City \tag{4}$$

Solution

Our model was implemented in GLPK. The solution can be obtained by running `$ ./solve_problem.sh 1`.

Optimum cost z is `3831575`€ and the location of DC is `Santarém`. The town with largest delivery costs `167036`€ is `Lisbon` (from `Santarém`).

Question 2

Our solution to question 2 use the same model for question 1. The only difference is the maximum amount of money (budget) that the company is willing spend to open one or several DCs (max. 5 DCs equivalent to 125000€).

Optimization Model

The model is analogue of the previous, except $budget = 125000$€. Therefore, $max\_dc = \frac{125000}{25000} = 5$. Company is able to open 5 DCs.

Solution

The solution can be obtained by running `$ ./solve_problem.sh 2`.

Optimum cost z is:

- `255836`€ in `Lisbon` (DC)
- `67782`€ in `Loulé` (DC)
- `156246`€ in `Ourém` (DC)
- `265507`€ in `Pedrouços` (DC)
- `119155`€ in `Sernancelhe` (DC)

The towns with largest delivery costs for each DC listed above are:

- `255836`€ is `Évora` (from `Lisbon DC`)
- `12070`€ is `Beja` (from `Loulé DC`)
- `26200`€ is `Coimbra` (from `Ourém DC`)
- `21418`€ is `Braga` (from `Pedrouços DC`)
- `19004`€ is `Bragança` (from `Sernancelhe DC`)

We conclude that `Évora` is the town with largest delivery costs.

References

[1] https://www.dcc.fc.up.pt/~jpp/mad1920/PopulationContPT-2020.csv

Appendix

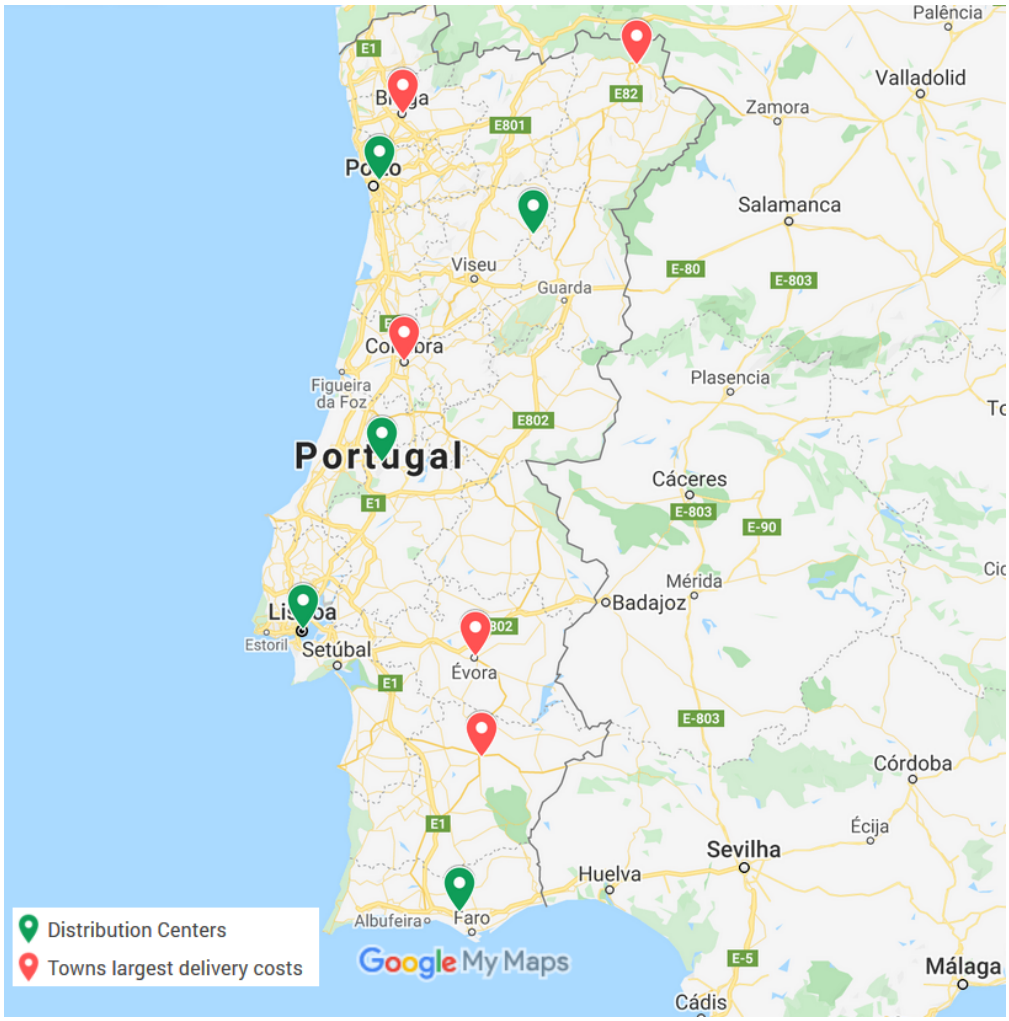

Figure 1: Map of Portugal - Visualization of solution for question 2