Multi-level Facility Location Problems

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Abstract

We conduct a comprehensive review on multi-level facility location problems which extend several classical facility location problems and can be regarded as a subclass within the well-established field of hierarchical facility location. We first present the main characteristics of these problems and discuss some similarities and differences with related areas. Based on the types of decisions involved in the optimization process, we identify three different categories of multi-level facility location problems. We present overviews of formulations, algorithms and applications, and we trace the historical development of the field.

Keywords: Multi-level facility location, Hierarchical, Multi-echelon, Review, Supply chain

1. Introduction

Discrete facility location problems (FLPs) constitute a major area of interest for researchers and practitioners in operations research (OR). The mathematical structure of some FLPs, which has proven fruitful to the development of solution methodologies broadly used today in OR, combined with their applicability to real-life problems, have made FLPs a core topic that has led to a vast number of publications, including several books and surveys (see, for example [35, 38, 69, 85]). A subclass of FLPs called multi-level facility location problems (MLFLPs) has attracted increasing attention in the last two decades. However, to the best of our knowledge, no recent publication consolidates the available material on this particular subject. Thus, we

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felt that the time was adequate to discuss the main aspects of MLFLPs in order to differentiate them from related topics and classify this rapidly emerging area. In this article we review the most representative MLFLPs as well as their historical development, models, solution methods and applications. For this purpose we survey over 60 OR-related studies published since the late 1970s, among which more than 40 have appeared in the last decade.

In an MLFLP we are given a set of customers that have a service or product requirement and a set of potential facilities partitioned into k levels. The goal is to determine which facilities to open simultaneously at each level, so that customers are assigned to one or multiple sequences of opened facilities, while optimizing an objective function. Some of these problems generalize fundamental FLPs such as the uncapacitated facility location problem (UFLP) [29, 67]. For example, in one of the first papers on MLFLPs, Kaufman et al. [61] introduced the so-called warehouse and plant location problem. Later, a slightly different version of that problem was presented and denoted as the two-level uncapacitated facility location problem (TUFLP). A natural extension to more than two levels of facilities corresponds to the multi-level uncapacitated facility location problem (MUFLP).

MLFLPs can also be viewed as a special case of an important class of problems called hierarchical facility location problems (HFLPs), where systems involving different types of interacting facilities that provide services to a set of customers are studied. Applications of HFLPs arise naturally in supply chain management (SCM) [82], where the interactions between plants, warehouses, distribution centers, and retail stores play a major role, and in health care systems [96] in which users must be served from different levels of clinics and hospitals. Other examples arise in hierarchical telecommunication networks [24, 52], freight transportation [46, 47], and solid waste management systems [16]. The two surveys of Şahin and Süral [30] and Zanjirani Farahani et al. [111] provide classifications and overviews of models, applications, and algorithms for HFLPs. Reference [30] covers the literature until 2004. Reference [111] is more recent but does not present most of the papers on MLFLPs in the broader context of HFLPs. Perhaps, one of the reasons for the exclusion of some of these problems is that they are known under different names and can be confused with similar, out-of-scope, problems. When preparing this survey, we have found that the terms *multi-echelon*, *multi-stage*, *multi-level*, *hierarchical*, and *multi-layer* facility location problems have all been used to refer to what we call MLFLPs.

The main contribution of this article is twofold. First, we formally define MLFLPs in order to present a unified framework for this still-growing area of research, and to differentiate it from other related areas within the field of facility location. Second, we consolidate the main contributions in the context of MLFLPs with a comprehensive review dating back to 1977 but with an emphasis on the last two decades. The paper is organized as follows. Section 2 establishes the types of decisions that pertain to MLFLPs and discusses the main characteristics of these problems. It also relates them with well-known areas of research and describes some of the applications that have been most relevant to MLFLPs. In Section 3 we present some of the historical milestones of the area and identify the main categories of MLFLPs that have been studied. We also discuss some variants and summarize the main references. Sections 4 to 6 are divided following the proposed classification scheme for MLFLPs. In each of the latter sections we provide overviews of the corresponding models and algorithms. Conclusions follow in Section 7. To facilitate reading, Table 1 summarizes the main abbreviations used throughout the paper.

Table 1: Summary of the main abbreviations

OR: Operations research	TUFLP: Two-level uncapacitated facility location problem
MILP: Mixed-integer linear programming	TEUFLP: Two-echelon uncapacitated facility location problem
ILP: Integer linear programming	TFLDP: Two-level facility location design problem
LP: Linear programming	TECFLP-S: Two-echelon CFLP with single assignment constraints
PBF: Path-based formulation	TCFLP: Two-level capacitated facility location problem
ABF: Arc-based formulation	TUFLP-S: TUFLP with single assignment constraints
FLP: Facility location problem	TCFLP-E: TCFLP with edge set-up costs
HFLP: Hierarchical facility location problem	MUFLP: Multi-level uncapacitated facility location problem
GNDP: General network design problem	MUFLP-E: MUFLP with edge set-up costs
UFLP: Uncapacitated facility location problem	MUpLP: Multi-level uncapacitated p-location problem
CFLP: Capacitated facility location problem	MUpLP-E : MUpLP with edge set-up costs
p-MP: p-median problem	MFLDP: Multi-level facility location design problem
MLFLP: Multi-level facility location problem	MpMP: Multi-level p-median problem

2. Decisions, related problems and applications

We first discuss the types of decisions that are involved in an MLFLP. For this purpose and for the sake of clarity when referring to these decisions, we introduce some notation that is used to model an MLFLP. Let $G = (V \cup I, E)$ be a graph with vertex set $V \cup I$ and edge set E. The set I corresponds to the customers, and the set V is partitioned into $\{V_1, \cdots, V_k\}$, corresponding to the sets of potential facilities at levels 1 to k. The edges always link two different levels. An MLFLP involves some of the following decisions.

Design decisions: facility location and edge activation The *location decisions* determine where to open the facilities. Given an underlying network G, facilities may be located at both the vertices or the edges of the network. This review focuses on discrete location problems, where it is assumed that facilities can only be located at the vertices of G. We refer to [50, 56, 107] for generalizations of results of Hakimi [55] on node optimality properties for HFLPs. The UFLP [29] and the p-MP [55] are well-known examples where facility location decisions are involved. The *network design decisions* select the edges to be activated. These edges are used to provide transportation services between customers and facilities of the first level, and facilities between different levels. Fixed-charged network design problems [73] are well-known problems involving network design decisions, among others.

Tactical decisions: allocation and routing The *allocation decisions* determine which facilities will be used to serve each customer. In FLPs, two types of allocation strategies have been considered. In single allocation, each customer is assigned to exactly one facility, whereas in multiple allocation each customer is allowed to be assigned to more than one facility, if beneficial. The *routing decisions* indicate the routes (or paths) on G that will be used to satisfy the customer demands. We use the term route to indicate the sequence of edges used to send flows between pairs of vertices. These types of decisions commonly appear in network flow problems which have been widely studied [9]. Since we consider different levels of facilities (V_r), the allocation decisions can also be viewed as the assignment of customers to open facilities of the first level and that of open facilities from one level to the next, sequentially. That is, a path between customers and highest-level facilities is associated with a multi-level allocation structure. Finally, observe that the network design and routing decisions are also interrelated, since the edges that can be used in the paths are determined by the network design decisions.

Both of the above types of decisions are directly related to the fixed and variable costs. For example, when a vertex $j_r \in V_r$ is selected to locate a facility, a set-up cost f_{j_r} is incurred. Analogously, when an edge $\{j_1, j_2\} \in E$ is activated a set-up cost $h_{j_1 j_2}$ must be paid. The tactical decisions are affected by variable costs. A common example is transportation costs which are generally related to the distances between the vertices. Transportation (or distribution) costs $c_{ij_1 \cdots j_k}$ are variable since they also depend on the customer's demands d_i and the sequence of used facilities $j_1, \cdots, j_k$. Some classical problems such as the UFLP involve set-up costs for opening facilities and transportation costs for assigning customers directly to facilities.

In order to better define the scope of this survey, we further discuss the above types of decisions in the context of MLFLPs. First, it is required that non-trivial facility location decisions be taken at every level of the hierarchy, simultaneously. Other problems involve two or more levels of facilities but only in one of them is the selection of facilities considered. We present some examples of this type of problems in Section 2.1.2. Depending on the application, network design and routing decisions may be explicitly considered or not, that is, the activation of edges and flow patterns are not necessarily non-trivial decisions. More importantly, this type of decisions should not be confused with routing decisions commonly encountered in similar problems such as location-routing problems [10, 32], where tours or paths between vertices of the same level in the network are considered. In the case of MLFLPs, there is no direct interaction between customers, and no horizontal interactions between facilities of the same level. This can be seen from the definition of the set E which corresponds to links between facilities and customers of different levels. Typically the edges between facilities of different levels are defined sequentially, i.e., for $r = 1, \cdots, k-1$, let $E_r = \{\{a, b\} \in E : a \in V_r \text{ and } b \in V_{r+1}\}$, and let $E_0 = \{\{i, b\} \in E : i \in I \text{ and } b \in V_1\}$. When this is the case we require a sequence of exactly one open facility at each level. As we will discuss later in this section, this feature corresponds to what is called a single flow pattern in the context of HFLPs. However, some problems with multi-flow patterns are also considered as MLFLPs. These assign customers to sequences of open facilities that can skip levels. Most of these multi-flow pattern problems can be modeled as single-flow-patterns by simply adding dummy vertices in the corresponding missing levels [97, 100], at the expense of increasing the instance size.

A common requirement in MLFLPs is that every served customer must be allocated to an open facility of the k^{th} level either directly or through a sequence of open facilities, and every open facility of level r must be connected to an open facility of level $r + 1$, except those of level k. When flow patterns are considered, the flow between levels must go in one direction and there ought to be only one type of arc available. Some HFLPs, especially those that arise in the framework of waste management systems, consider flows in two directions or more than one type of arc (see, for instance[16, 88]). These types of problems lie outside the scope of this paper.

Another important feature that differentiates MLFLPs from similar problems is that the set of vertices $V \cup I$ consisting of potential sites and customers is partitioned from the input into $k + 1$ levels. This means that the set V is also partitioned into k subsets, one for each level of facilities. Notably, in early works the partitioning of the set V did not necessarily consist of pairwise disjoint sets [61, 97]. However, most of the more recent papers assume pairwise disjoint sets. In any case, in contrast to some HFLPs where one can open different facilities at any vertex of the network, including those that model customer zones, in MLFLPs the sets V_r differ from V_{r+1} for all r. This also means that in MLFLPs the number of levels is not part of the decision process and facilities of type r can only be located in V_r, i.e. the hierarchy is given as an input of the problem. Note also that the hierarchy is imposed only on the vertices and not on the edges, in contrast for instance to multi-level network design problems where usually the network design decisions are predominant [15, 51]. Finally, in terms of the objective function we restrict this review to those MLFLPs with median and fixed charge objective (minisum) functions. We note that in recent years variations of some MLFLPs allow the planner to have the option of incurring a penalty instead of serving all customers. Such penalties are included in the objective function and take into account the benefit of deciding which customers to serve. Therefore, we do not restrict MLFLPs to require each customer to be allocated to a sequence of open facilities.

2.1. Related problems

Different classes of FLPs are related to MLFLPs. We next discuss some of the areas that we consider to be most relevant to this review and we point out the main differences and similarities with MLFLPs.

2.1.1. Hierarchical facility location problems

We have already discussed some applications, definitions and references [30, 111] for this class of FLPs. In particular, since we consider MLFLPs as a special case of HFLPs, we have mentioned some of the differences between the two types of problems. We now emphasize other relevant differences between them. Hence we use the classification scheme and terminology of HFLPs given in [30] in order to categorize MLFLPs in that context. It is based on four criteria: *flow pattern*, *service availability*, *spatial configuration* and *objective*. A flow pattern refers to the way in which a facility at a given level receives or offers services or products to another facility at a different level and is either single-flow or multi-flow. In a network with single-flow patterns, the flow from or to the customers must pass through all higher levels until it reaches its point of origin or destination, whereas in an multi-flow pattern, facilities of some level may receive or send flow directly from or to any higher level. Service availability specifies whether a higher-level facility provides all services offered by its lower-level facilities plus another one (nested), or whether facilities at each level provide different services (non-nested). In the spatial configuration category a network can be coherent or non-coherent. In a coherent network, an open facility of a lower-level must receive or send service from or to exactly one higher-level facility. Non-coherent systems allow more than one higher-level facility serving a given lower-level facility. Median, covering and fixed charge objectives are considered in HFLPs. Therefore, for an MLFLP we have noted that the single-flow pattern is more common and in principle a non-nested structure is considered. In terms of the coherency criterion some papers have included certain assumptions while others simply impose single assignment constraints which in both cases imply a coherent structure [for example, 24, 46, 93].

Three main differences thus arise between MLFLPs and HFLPs apart from those mentioned above, namely the type of objective function, the type of demands and the service availability criterion. First, we note that other HFLPs that consider covering or pure median objectives typically appear in the context of having the same set V as potential sites for all types of facilities. In MLFLPs it is common to observe fixed-charge-type objective functions. On the other hand, the service availability criterion which was first discussed by Narula [89], is strictly interrelated with the presence of different types of demand. In some HFLPs the requirements from the customers are services, and the same customer can demand different types of service offered by certain types of facilities in the hierarchy. These problems are generally motivated by health care applications where geographical zones require service from regional hospitals, local hospitals or clinics. Examples of this feature are provided in [84, 101]. In contrast, in MLFLPs there is only one type of demand, more in the spirit of a production-distribution system where for instance, plants serve warehouses which in turn serve customers. Therefore, we assume a non-nested configuration for MLFLPs since we refer to different types of facilities instead of services, although this is also application-dependent.

2.1.2. Multi-echelon location-routing problems

The term multi-level is not the only one used in the context of MLFLPs. For example, we found *multi-echelon*, *multi-stage*, *multi-layer* and *multi-tier* among the more common terminologies. Typically the terms *layer* and *level* are used as synonyms, referring to the sets V_r or types of facilities as we did above. On the other hand, the term *echelon* is generally associated with distribution networks where products are transported between each pair of levels. Such pairs are called echelons [17, 43, 71]. Multi-echelon FLPs are thus very similar to MLFLPs. In Section 3 we highlight the main steps in the evolution of both terms. In fact, some of the papers that we review as MLFLPs denote their problems as multi-echelon. There are two main characteristics that we can use to differentiate the terminology in this case. The first one is that although all of the multi-echelon problems involve a multi-level environment, not all of them require facility location decisions at every level. For example, Geoffrion [48] Klose [62, 63] and Li et al. [71] study two-echelon FLPs in which facilities to be opened are only selected at one of the levels. This is partially because the predominant decisions are made at the echelons, and these typically involve routing variables. Indeed, the second differentiating feature lies precisely in the routing patterns. In MLFLPs we are concerned with problems where facility, and sometimes network design decisions, are predominant with no routing decisions between vertices of the same level involved. Cuda et al. [32] recently reviewed two-echelon routing problems. Another term that is generally related to echelons is the word *tier*, which has mainly been used in the context of freight transportation systems and city logistics [74, 75]. These problems also involve vehicle routing decisions and are therefore out of the scope of this paper. The term *stage* has also been used in the MLFLPs context. This is probably the most elusive one when trying to associate it to something in particular. To mention a few MLFLPs references, in [68, 79, 87] the term stage is used when referring to what we denote as levels. However, in other papers it has been used in the sense of what we identified as echelons [e.g. 62, 63, 71]. Finally, the term stages may also apply for dynamic FLPs and stochastic programs. In this review we attempt to select those that are concerned with MLFLPs.

2.1.3. General network design problems

We note that the types of decisions involved in MLFLPs mentioned above are very much in the spirit of those identified by Contreras and Fernández [27], who classified a broader class of optimization problems referred to as general network design problems (GNDPs), where both the facility location and network design decisions are predominant and non-trivial. Thus, MLFLPs can also be seen as a special case of the more general class of GNDPs. However, these authors concentrated on single-level problems, excluding MLFLPs from their study. Nonetheless, their classification of GNDPs based on the type of demand can be useful for our study of MLFPLs. Contreras and Fernández [27] present two main categories of GNDPs: problems that involve User-Facility demands (UF), and those with User-User demands (UU). In UF, facilities are the service providers to users and typically there are no interactions between facilities. Therefore, demands are routed from facilities to users. On the other hand in UU, facilities consolidate commodities that are routed from origins to destinations and thus, they are used as intermediate locations. The network design and routing decisions influence the optimal solution structure by deciding how to connect users to facilities and facilities to each other. This means that in most UU cases facilities interact with each other. An example of GNDPs with UU demands in which there is a multi-level environment are the so-called hierarchical hub location problems [11, 110]. In MLFLPs, from the perspective of GNDPs, we restrict our attention to those problems that have a UF demand and incorporate non-trivial network design decisions.

2.1.4. Supply chain network design problems

MLFLPs also relate to the well-studied area of supply chain management [13, 99]. There has been a great effort to establish the importance of location problems in SCM [33, 76, 82]. For instance, Melo et al. [82] review facility location models in the context of SCM and identify features that such models must capture in order to be consistent with the strategic decisions involved in SCM. In particular, the authors discuss the importance of having a different types of facilities, very much like an MLFLP, where the strategic decisions of the SCM system are considered. However, SCM usually involve decisions on the inventory, procurement, production, routing, etc, and thus, reviewing such a general class of problems is beyond the scope of this paper. Nevertheless, MLFLPs can be considered as a simple version of a supply chain network design problem where most of the tactical and operational decisions are not involved.

2.2. Applications

Two types of applications arise frequently in MLFLPs-related papers. The first one is concerned with production-distribution systems where customers require a product that must be provided from first-level facilities (warehouses) which in turn is sent from production plants. This line of research naturally evolved from some early works where the warehouse and plant location problem was introduced [61, 97]. Some variations include additional levels in the distribution network such as retail stores or distribution centers and more sophisticated models in freight transportation [26, 46, 47]. Also note that the applications that motivate this type of MLFLP generally do not exceed more than three levels of facilities. Some examples where a production-distribution system is studied are [40, 43, 54, 68, 79, 80, 87, 94, 95, 100, 103, 108, 109]. However, other papers more involved in the development of approximation algorithms for the general k-level case also make reference to distribution systems [7, 22, 25]. Moreover, an interesting variant of the problem arises when decision maker determines whether to provide the service to each customer or to pay a penalty for those that are not served [12, 23].

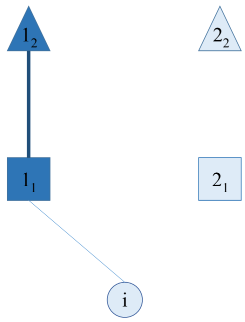
The second major application area emerges from the telecommunications industry and the design of computer networks. In this case, one must decide where to locate devices such as routers and multiplexers and how to allocate customers (terminals) to a sequence of devices. Examples of references where this type of application is discussed are [5, 6, 24, 34, 52, 53, 59, 60, 86]. Finally, other studies in the context of MLFLPs have been motivated by applications in different fields. For instance, waste disposal systems [17, 19, 20, 104], supply chain of disaster relief system [49], and health care systems [31].

3. A classification scheme and overview of the related literature

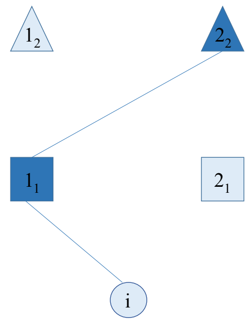
We now present a classification scheme for MLFLPs based on the types of decisions involved and on the different possible combinations of them. On the one hand, design decisions correspond to (*i*) opening of facilities and (*ii*) activating edges, while on the other hand, (*iii*) allocation and routing decisions are made to satisfy customer demands. Since selecting which facilities are opened at each level is a requirement of every MLFLP, we are left with the three possible combinations of (*ii*) and (*iii*) to define our categories. Given that non-trivial decisions are closely related to the types of costs (or profits) considered in the definition of each problem, we could also refer to the corresponding category by type of cost. We have selected one fundamental problem from each category in order to identify them more easily as follows. When there are only design decisions (*i*) and (*ii*) involved, we refer to them as multi-level facility location design problems (MFLDPs). When there are facility location and tactical decisions (*i*) and (*iii*), we refer to the MUFLP. Finally, when all three types of decisions are present, we refer to the MUFLP with edge set-up costs (MUFLP-E). The latter is clearly a more general version combining the former two. In the following example we sketch an instance of the three problems in a two-level environment in order to illustrate this categorization. We summarize the main notation used throughout the paper in Table 2.

Example 1. Consider an underlying network consisting of $I = \{i\}$, $V_1 = \{1_1, 2_1\}$ and $V_2 = \{1_2, 2_2\}$ and all edges between I and V_1 as well as those between V_1 and V_2 exist. The fixed costs for opening facilities are $f_{1_1} = 5$, $f_{2_1} = 10$, $f_{1_2} = 20$, and $f_{2_2} = 25$ and we assume that there are no fixed costs for opening edges between I and V_1. We analyze three scenarios, one for each of the aforementioned representative problems in this two-level context (TFLDP, TUFLP, TUFLP-E). For the TFLDP and the TUFLP-E, consider edge set-up cost $h^1_{1_1 1_2} = 5$, $h^1_{1_1 2_2} = 10$, $h^1_{2_1 1_2} = 3$, and $h^1_{2_1 2_2} = 5$. For the TUFLP and the TUFLP-E let $c_{i1_1 1_2} = 10$, $c_{i1_1 2_2} = 1$, $c_{i2_1 1_2} = 5$, and $c_{i2_1 2_2} = 5$ be the corresponding transportation costs. Therefore, we obtain three different optimal solutions, one for each problem. For the TFLDP the optimal value is 30, opening facilities 1_1 and 1_2 as well as the edge $\{1_1, 1_2\}$. This solution is depicted in Figure 1a. We have represented with darker colors open facilities and links. Similarly, for the TUFLP and the TUFLP-E we have optimal values equals to 31 and 38, respectively. The corresponding solutions are shown in Figures 1b and 1c.

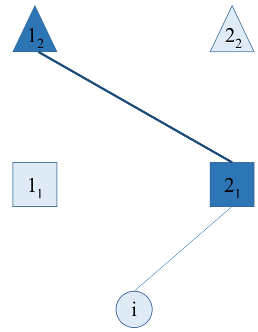
In Sections 4 to 6 we present formal definitions of the problems and discuss the related variants and references in more detail for each category of MLFLPs. We also present what we consider to be milestones of the field, and the trends that they have



(a) TFLDP solution



(b) TUFLP solution



(c) TUFLP-E solution

Figure 1: Three examples of two-level FLPs

defined. On our historical path towards defining those most representative MLFLPs, we introduce some commonly used MILP formulations within each category, and thus we illustrate the differences and relationships with each other as well as their variations. Table 3 summarizes the main MLFLP publications and includes side criteria such as capacitated/uncapacitated and the solution approach (exact or approximate) that was applied in the corresponding reference. Some references may therefore appear in more than one box of the table. We also include papers containing polyhedral studies or introducing MILP formulations only in the "exact" columns.

From Table 3 we can observe that certain areas have received considerably more attention than others. For example, in the uncapacitated cases an important number of publications are concerned with the development of approximation algorithms, except for the MUFLP-E variant. Thus, more research must be carried out to further investigate whether adding fixed costs on the edges changes the problem drastically from an approximation perspective. Also, in the uncapacitated case, we see that models and exact algorithms have been proposed for almost all categories listed in the table. However, only recently was an exact solution method designed for large-scale instances of the general MUFLP-E with $k > 2$ [92]. On the other hand, in the capacitated versions, the effort appears to have focused on the two-level variants. This leaves aside only a few references where approximation algorithms have been designed for the general case where $k > 2$.

4. MLFLPs with tactical decisions

Perhaps the simplest version of an MLFLP, yet the most studied, is the TUFLP which can be defined as follows. Assuming that all facilities are uncapacitated and

Table 2: Summary of main notation

	Notation	Definition
Sets	$G = (V \cup I, E)$	graph with vertices $V \cup I$ and edges E
	V	set of potential sites
	I	set of customers
	V_r	set of potential sites of level r, for $r = 1, \cdots, k$
	E_r	set of edges between V_r and V_{r+1}, for $r = 1, \cdots, k-1$
	E_0	set of edges between I and V_1
	Q	set of possible paths of facilities, exactly one from each level e.g. $q = j_1, \cdots, j_k \in Q$
Parameters	k	number of levels
	f_{j_r}	fixed cost for opening facility $j_r \in V_r$, for $r = 1, \cdots, k$
	$c_{ij_1 \cdots j_k}$	variable cost (or profit) for serving customer i through the sequence $j_1 \cdots j_k$
	$h^1_{j_1 j_2}$	fixed costs for opening edge $\{j_1, j_2\} \in E_1$
	$h^0_{ij_1}$	fixed costs for opening edge $\{i, j_1\} \in E_0$
	p_r	maximum number of facilities to open at level r, for $r = 1, \cdots, k$
	d_i	demand of customer $i \in I$
	β_{j_1}, α_{j_2}	capacities at facilities $j_1 \in V_1$ and $j_2 \in V_2$, respectively
Variables	y_{j_r}	binary decision for opening facility $j_r \in V_r$, for $r = 1, \cdots, k$
	$x_{ij_1 \cdots j_k}$	binary (continuous) decision for assigning customer $i \in I$ to sequence $j_1 \cdots j_k$ with $j_r \in V_r$ for $r = 1, \cdots, k$
	$t^1_{j_1 j_2}$	binary decision for opening edge $\{j_1, j_2\} \in E_1$
	$t^0_{ij_1}$	binary decision for opening edge $\{ij_1\} \in E_0$
	z^r_{iab}	binary decision determining whether the edge $\{a, b\} \in E_r$ is used to serve customer i, for $r = 1, \cdots, k$
	v_{ij_1}	binary decision if customer i is assigned to $j_1 \in V_1$. Also used as amount of flow between $i \in I$ and $j_1 \in V_1$
	$w_{j_r j_{r+1}}$	(fraction of) flow between $j_r \in V_r$ and $j_{r+1} \in V_{r+1}$, for $r = 0, \cdots, k-1$, with $V_0 = I$
	η^i	continuous variable for the profit of serving customer $i \in I$
	x_q	binary decision for opening path $q \in Q$

given fixed costs f_{j_r} for setting up facility j_r, for $r = 1, 2$, as well as distribution costs $c_{ij_1 j_2}$ for serving customer i through the pair j_1, j_2, the problem consists of determining which facilities to open at each level so that every customer is served via a pair of open facilities (j_1, j_2), while minimizing the total cost. Consider the binary decision variables y_{j_r} equal to 1 if and only if facility $j_r \in V_r$ is open, and the continuous variable $x_{ij_1 j_2}$ equal to the fraction of the demand of customer i satisfied by second-level facility j_2 through first-level facility j_1. The TUFLP can then be

		Uncapacitated		Capacitated	
		Heuristics	Exact	Heuristics	Exact
MUFLP	$k = 2$	[44, 66, 86, 98, 112]	[4, 19, 31, 61, 68, 79, 97]	[6, 40, 54, 72, 87, 94, 95, 109]	[1, 2, 5, 20, 72, 80, 94, 95, 104, 108, 109]
	$k > 2$	[3, 7, 8, 12, 21–23, 37, 41, 42, 53, 64, 66, 70, 77, 78, 81, 83, 93, 100, 105, 106, 113]	[42, 65, 93, 100]	[7, 22, 25, 36]	
MUFLP-E	$k = 2$	[17, 43]	[17, 24, 43, 45, 46]	[49, 59, 102]	[49, 59, 103]
	$k > 2$		[92]		
MFLDP	$k = 2$	[86]	[14, 24]	[86]	[24]
	$k > 2$	[34, 41, 60]			

Table 3: Summary of MLFLPs references

formulated as

$$
\begin{aligned}
\text{(F1-TUFLP)}\quad \text{minimize}\quad & \sum_{i \in I} \sum_{j_1 \in V_1} \sum_{j_2 \in V_2} c_{ij_1j_2} x_{ij_1j_2} + \sum_{r=1}^{2} \sum_{j_r \in V_r} f_{j_r} y_{j_r} & & \\
\text{subject to}\quad & \sum_{j_2 \in V_2} \sum_{j_1 \in V_1} x_{ij_1j_2} = 1 & i \in I & \qquad (1) \\
& \sum_{j_1 \in V_1} x_{ij_1j_2} \le y_{j_2} & i \in I,\ j_2 \in V_2 & \qquad (2) \\
& \sum_{j_2 \in V_2} x_{ij_1j_2} \le y_{j_1} & i \in I,\ j_1 \in V_1 & \qquad (3) \\
& x_{ij_1j_2} \ge 0 & i \in I,\ j_1 \in V_1,\ j_2 \in V_2 & \qquad (4) \\
& y_{j_r} \in \{0, 1\} & j_r \in V_r, \quad r = 1, 2. & \qquad (5)
\end{aligned}
$$

Note that the variables $x_{ij_1j_2}$ are allowed to be declared as continuous since in the uncapacitated case they will take integer values in any case [4]. However, the earliest version of the problem that we were able to identify is a slightly different variant which was denoted as the warehouse and plant location problem in the seminal work of Kaufman et al. [61]. Assuming that $V_2 \subseteq V_1$, the authors imposed the additional constraint that with each open plant there must be an open warehouse in the same location:

$$y_{j_2} \le y_{j_1} \qquad j_1 \in V_1, j_2 \in V_2.$$

A few years later, Ro and Tcha [97] introduced a modified version of this constraint by including a set of "adjunct" warehouses to each plant, thus enforcing the constraint that if a plant is opened the associated warehouses are opened, but not vice versa. When the sets of adjunct warehouses are empty, the problem corresponds to what we call the TUFLP. The same year, Tcha and Lee [100] presented a problem without this additional constraint, which is then a TUFLP. These authors also generalized the problem to k levels and denoted it as the MUFLP. They introduced an MILP for the MUFLP which is nowadays referred to as *path-based formulation* (PBF), where each sequence of facilities $j_1, j_2, \cdots, j_k$, with $j_r \in V_r$, is called a path, and every customer must be allocated to a path of open facilities. It is straightforward to derive the corresponding MILP formulation for the MUFLP by extending the decision variables y_{j_r} and $x_{ij_1 \cdots j_k}$ for $r = 1, \cdots, k$ from those of the F1-TUFLP. We thus select the MUFLP as the representative problem for those MLFLPs encompassed in this category. We divide this section into the uncapacitated and the capacitated cases. The former is in turn divided into three parts namely, formulations, exact algorithms and heuristics.

4.1. Uncapacitated case

As for the single-level case, we follow the uncapacitated/capacitated criterion for MLFLPs. This distinction is important since capacity constraints usually play a major role in models and algorithms.

4.1.1. Formulations

Two main families of MILP formulations are commonly used for MLFLPs. The first is related to the PBF explained before that extends from the F1-TUFLP using variables $x_{ij_1 \cdots j_k}$ for the allocation of customers, and y_{j_r} variables for selecting facilities. The second type of formulation is the so-called *arc-based formulation* (ABF). In contrast with PBF, in an ABF the decision variables are in a sense split between levels, that is, the variables are associated with arcs instead of paths. At this point it is important to note that a path in k levels (a sequence of k facilities, one from each level), actually coincides with arcs in the two-level case. Also, we refer to arcs and edges indistinctly unless otherwise needed. Therefore, the initial formulation F1-TUFLP can be viewed as a PBF or as an ABF, from which the extended versions are derived. For example, Gabor and van Ommeren [42] introduced the following MILP for the MUFLP in which decision variables are associated to arcs instead of paths. Assuming that the sets E_r contain all possible edges between levels r and $r + 1$, for $r = 1, \cdots, k - 1$ and that $c_{ij_1 \cdots j_k} = c_{ij_1} + \cdots + c_{j_{k-1} j_k}$, let $z^r_{iab} = 1$ if customer i uses the edge $\{a, b\} \in E_r$, y_{j_r} as defined before and $v_{ij_1} = 1$ if customer i

is assigned to $j_1 \in V_1$. An ABF for the MUFLP is then (F2-MUFLP)

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i\in I}\sum_{j_1\in V_1} c_{ij_1} v_{ij_1} + \sum_{i\in I} \sum_{r=1}^{k-1} \sum_{\{a,b\}\in E_r} c_{ab} z^r_{iab} + \sum_{r=1}^{k}\sum_{j_r\in V_r} f_{j_r} y_{j_r} \\
\text{subject to} \quad & \sum_{j_1\in V_1} v_{ij_1} = 1 && i \in I && (6)\\
& \sum_{b\in V_2} z^1_{ij_1b} = v_{ij_1} && \{i, j_1\} \in E_0 && (7)\\
& \sum_{b\in V_{r+1}} z^r_{iab} = \sum_{b'\in V_{r-1}} z^{r-1}_{ib'a} && i \in I,\ a \in V_r,\ r = 2, \cdots, k-1 && (8)\\
& v_{ij_1} \le y_{j_1} && \{i, j_1\} \in E_0 && (9)\\
& \sum_{a\in V_{r-1}} z^{r-1}_{iab} \le y_b && i \in I,\ b \in V_r,\ r = 2, \cdots, k && (10)\\
& v_{ij_1} \ge 0 && \{i, j_1\} \in E_0 && (11)\\
& z^r_{iab} \ge 0 && i \in I,\ \{a, b\} \in E_r,\ r = 1, \cdots, k-1 && (12)\\
& y_{j_r} \in \{0, 1\} && j_r \in V_r,\ r = 1, \cdots, k. && (13)
\end{aligned}
$$

Note that the variables v_{ij_1} can be eliminated from the model either by consolidating the sets of constraints (6) and (7) or by setting $z_{iij_1} = v_{ij_1}$, in which case (7) can be embedded in (8). However, we have opted to reproduce the model as presented in [42]. When $k = 2$ we obtain the initial formulation F1-TUFLP described above.

Other ABFs have been studied for the problem, in particular those that consider variables $w_{j_rj_{r+1}}$ representing the flow from facility $j_r \in V_r$ to facility $j_{r+1} \in V_{r+1}$, with $V_0 = I$. As we will discuss, this type of formulation is common in the capacitated cases. One example in the uncapacitated variant arises in the seminal work of Aardal et al. [4], which introduces an ABF for the TUFLP by defining $w_{ij_1} = \sum_{j_2\in V_2} x_{ij_1j_2}$ and $w_{j_1j_2} = \sum_{i\in I} x_{ij_1j_2}$. The authors compared the LP relaxations of the two formulations concluding that the bound of the F1-TUFLP formulation is always better than that obtained with their ABF. This result typically generalizes to k levels, but the size of a PBF grows much faster than that of an ABF. Aardal et al. [4] also conducted a polyhedral study of the associated polytope of F1-TUFLP. In particular, they developed a characterization of the extreme points of its LP relaxation as well as results extending all nontrivial facets of the single-level UFLP to the TUFLP. They proved, among other results, that (2)–(4) define facets of the convex hull of the associated polyhedral set of F1-TUFLP. Moreover, they introduced two classes of facet-defining inequalities for a modified version of the F1-TUFLP and stated conditions under which these inequalities also induce facets for the single-level case. However, these results have never been extended to the case $k > 2$.

Another example of an ABF using the variables $w_{j_r j_{r+1}}$ for the TUFLP was studied by Marín [79]. Landete and Marín [68] also used the disaggregated version of constraints (2) and (3) and introduced a reformulation of the TUFLP as a set packing problem for which the corresponding polyhedral study was developed, along with facet-defining inequalities and an algorithm. More recently, Kratica et al. [65] and Marić et al. [78] independently introduced a new ABF for the MUFLP, very much in the spirit of the ABF introduced in [4] for the two-level case. Kratica et al. [65] provided computational results comparing on general purpose solvers the performance of the new formulation with those of the PBF and of the F2-MUFLP.

Ortiz-Astorquiza et al. [93] presented a new type of MILP for a slightly more general MUFLP in which for given values of p_r, $r = 1, \cdots, k$, cardinality constraints ($\sum_{j_r \in V_r} y_{j_r} \leq p_r$) are imposed at each level. They called this problem the multi-level uncapacitated p-location problem (MUpLP) since it generalizes the well-known UpLP presented by Cornuéjols et al. [28], which in turn subsumes the UFLP and the p-median problem (p-MP) [55]. The multi-level version of the p-median problem is denoted by MpMP. In [93], the authors developed the new formulation of the maximization version of the problem based on an alternative combinatorial representation given in [91], in which the objective function satisfies the submodularity property. Thus, considering the variables η^i representing the profit (or cost) of serving customer $i \in I$, and Q the set of all possible paths $q = j_1, \cdots j_k$ having exactly one facility at each level, one can project out the variables x_{iq} to x_q from the PBF and obtain the submodular formulation

$$
\begin{aligned}
\text{(SF-MUpLP) maximize } & \sum_{i \in I} \eta^i \\
\text{subject to } & \eta^i \leq c_{iq_t} + \sum_{q \in Q} (c_{iq} - c_{iq_t})^+ x_q \quad i \in I, \quad t = 0, \cdots, |Q| - 1, && (14) \\
& \sum_{q \in Q : j_r \in q} x_q \leq M_r y_{j_r} \qquad j_r \in V_r, \quad r = 1, \cdots, k, && (15) \\
& \sum_{j_r \in V_r} y_{j_r} \leq p_r \qquad r = 1, \cdots, k, && (16) \\
& x_q \in \{0, 1\} \qquad q \in Q, && (17) \\
& y_{j_r} \in \{0, 1\} \qquad j_r \in V_r, \quad r = 1, \cdots, k, && (18)
\end{aligned}
$$

where for each $i \in I$, and $r = 1, \cdots, k$, $0 = c_{iq_0} \leq c_{iq_1} \leq \cdots \leq c_{iq_{|Q|}}$ and M_r are sufficiently large numbers. Note that the disaggregated version of constraints (16) may also be used. In [93] a computational comparison of formulations for the MUpLP was carried out. Because of the large number of constraints (14) the authors embedded the SF-MUpLP in a branch-and-cut framework exploiting an efficient way of solving the separation problem. For comparison purposes, they considered the PBF extension of the F1-TUFLP, the F2-MUFLP, the branch-and-cut SF-MUpLP and the ABF of [65]. Their results showed that when the cardinality constraints are predominant, the SF-MUpLP dominates the other three formulations in terms of CPU time spent to obtain the optimal solution, while in the case of $p_r = |V_r|$, i.e. for the MUFLP, there is no clear dominance of one model over the others. While the PBF grows considerably faster when $k > 2$, it is the one that yields the best LP bound, and therefore a smaller enumeration tree. The F2-MUFLP modified for the general problem seems to take much longer when the cardinality constraints are active, but it is rather efficient in the MUFLP case. The ABF of [65] and the submodular formulation achieve a balance between memory usage and computing time spent when the problem is more general. However, the experiments pointed to a better average performance for the submodular formulation. Instances with up to 2,000 customers, 200 potential facilities, and four levels of hierarchy were solved to optimality.

4.1.2. Exact algorithms

All of the early works on MLFLPs introduced exact algorithms for different versions of the problem. For example, the three ground-setting papers [61, 97, 100] presented branch-and-bound methods that extended those known for the single-level case. However, the algorithms of [97, 100] were based on the assumption that the submodular property extends directly from the single-level version. The correctness of such methods was later discussed by Barros and Labbé [18] who showed that the objective function of the representation of the corresponding problems does not satisfy this property. More recently, Ortiz-Astorquiza et al. [91] introduced an alternative combinatorial representation of the (maximization version of the problem) whose objective function does satisfies submodularity.

Tcha and Lee [100] presented a modified version for the MUFLP of the dual ascent procedure of Erlenkotter [39] known for the single-level UFLP. However, ever since these solution methods were proposed, only a few papers have dealt with the development of specialized exact solution algorithms for variants of the MUFLP. The papers of Marín [79], Landete and Marín [68], Gendron et al. [46] and Ortiz-Astorquiza et al. [92] are perhaps the exceptions. However, [46, 92] study the more general case where fixed costs on the edges are considered, so these contributions will be discussed in the following sections.

As already mentioned, an ABF for the TUFLP was proposed in [79] from which the authors studied several Lagrangian relaxations. The authors showed when the so-called Lagrangian bound satisfied the integrality property, that is, the case when the optimal value obtained from the Lagrangian dual coincides with that of the LP relaxation. Moreover, they presented several results in which dominance relationships between bounds of the different relaxations are given, and developed a bounding procedure based on the lower bounds obtained by applying a subgradient optimization procedure for one of the Lagrangian relaxations. They argued for the selection of a relaxation based on a balance between dominance and ease of solution. Landete and Marín [68] reformulated the TUFLP as a set packing problem and presented different classes of facet-defining inequalities for the reformulation. Based on these inequalities, they developed a branch-and-cut algorithm and compared its performance with that of a general purpose solver.

4.1.3. Heuristics

Most research efforts towards the development of algorithms for MUFLP-related problems have focused on heuristics. We can start by differentiating two main research streams: heuristics without a performance guarantee, and ρ-approximation algorithms i.e., polynomial-time heuristics that yield a feasible solution with an objective function value lying within a factor of ρ of the optimal value. Most of the work on heuristics has focused on the latter stream.

In the area of heuristics without a bounded worst-case ratio, Korać et al. [64], Marić [77], Marić et al. [78] presented algorithms for the MUFLP considering that the costs c are metric (i.e. nonnegative, symmetric and satisfying the triangle inequality) and additive with respect to the k levels. In [77] a genetic algorithm is presented including an implementation with a dynamic programming scheme to find the sequences of open facilities to satisfy customers demands. According to the authors, the dynamic programming component is the main ingredient that enables the genetic algorithm to solve large-scale instances within a short amount of time. Later, in [64, 78] improvements on the genetic algorithm were introduced. For instance, improving the implementation of the dynamic programming approach and incorporating local search procedures designed for the MUFLP, which are denoted as memetic algorithms. Another memetic algorithm was designed by Mišković and Stanimirović [86] to obtain solutions of the TUFLP using the formulation introduced in [68]. Gendron et al. [44] developed a metaheuristic for the two-level uncapacitated facility location problem with single assignment constraints, denoted TUFLP-S. In the TUFLP-S the restrictions that each open first-level facility can be connected to at most one open second-level facility are required. The authors developed what they called a multi-layer variable neighborhood search metaheuristic for the TUFLP-S and a similar variant with modular costs. The term multi-layer comes from partitioning the neighborhood structures into several layers, where for each layer a variable neighborhood search scheme is applied. They compared the performance of their algorithm with that of a MILP solved using a general purpose solver, and with that of a slope-scaling heuristic based on the same formulation.

There also exist approximation algorithms with performance guarantee. However, since there are two versions of the MUFLP and its variants namely, a maximization and a minimization version, we must review them separately. The reason for this additional classification comes from the fact that from an approximation perspective, the maximization and minimization versions of an optimization problem are not necessarily comparable. This was discussed in [58, 98] for the single-level case and in [93, 113] for the multi-level case.

a) *Maximization version*

We note that in the maximization version of the MUFLP, the values of $c_{ij_1\cdots j_k}$ correspond to the profit of serving customer i through path $j_1\cdots j_k$. This can be thought as $c_{iq} = b_i - D_{iq}$, where b_i is the price that client $i \in I$ pays for the service, and D_{iq} is the total operational cost of serving client i through path $q = j_1\cdots j_k$. Observe also that adding to c_{iq} a constant γ_i for every possible path q does not change the optimal solution. This is because in the MUFLP one must serve every client and thus, having new values of c defined as $c'_{iq} = c_{iq} + \gamma_i$ changes the cost of every feasible solution by the same amount. This property is well known for the single-level case [29]. The price b_i can thus be seen as the corresponding constant γ_i and therefore, only the costs are relevant for the decision, yielding the minimization version of the problem. This is why from an optimization point of view, it seems to be more common to work with the minimization version than with its maximization counterpart. Moreover, note that the objective function

$$z = \sum_{i\in I}\sum_{r=1}^{k}\sum_{j_r\in V_r} c_{ij_1\cdots j_k}x_{ij_1\cdots j_k} - \sum_{r=1}^{k}\sum_{j_r\in V_r} f_{j_r}y_{j_r}$$

can take positive or negative values and thus, a correcting factor in the definition of measure of relative deviation for approximate solutions must be added [28].

Let z^* be the optimal value of the problem and let z_R be a sufficiently small number, typically defined depending on the input of the problem, such that $z^* \leq$

$z \leq z_R$, where z is the value of any feasible solution. Bumb [21] assumed that all costs and profits are non-negative and presented an approximation algorithm based on the technique of independently randomized rounding which yields a solution Z satisfying

$$\frac{Z - z_R}{z^* - z_R} \geq 0.47.$$

This worst-case bound was soon improved to 0.5 by Zhang and Ye [113]. Recently, based on an alternative representation of the MUFLP [91] in which the objective function satisfies the submodularity property, Ortiz-Astorquiza et al. [93] were able to extend to the k-level case the constant-performance guarantees of Cornuéjols et al. [28] and Nemhauser et al. [90] derived for the single-level case. In [93], they studied the MUpLP which includes as special cases the MUFLP and the Mp-MP. The authors showed that when the profits c are additive, a polynomial time greedy algorithm always yields a solution satisfying

$$\frac{Z - z_R}{z^* - z_R} \geq 1 - \frac{1}{e} \approx 0.63.$$

Based on the foreseen difficulties of extending their algorithm to the general case of k levels, Bumb [21] questioned whether there exists an approximation algorithm with performance guarantee independent on the number of levels for the maximization version, as was the case at the time for the minimization counterpart. The recent result of [93] answers this question in a positively manner.

b) *Minimization version*

Since almost all the related papers assumed that the costs c are induced by a metric on $V \cup I$ and are additive with respect to the levels as already mentioned, in the remainder of this section we retain these assumptions unless otherwise stated. We observe that Shmoys et al. [98] and Aardal et al. [3] were the first to present approximation algorithms with constant-performance guarantees for the two-level and multi-level cases, respectively. These papers set the ground for a rich line of research. In [98] a 3.16-approximation algorithm was introduced which was soon improved in [3] to a 3-approximation algorithm for the general k-level case. However, the drawback of these algorithms seems to be that they are based on randomized rounding of the optimal solution of an LP relaxation. Even if the algorithms have polynomially bounded running times, the LP relaxation contains an exponential number of variables and thus, solving it may be difficult in practice. Guha et al.

[53], Meyerson et al. [83] were the first to design efficient combinatorial algorithms capable of finding a solution within a factor of $O(\log|I|)$ and 9.2 of the optimal value, respectively. They presented these results for the MUFLP as a special case of more general network design problems. These worst-case bounds were improved by Bumb and Kern [22] who developed a dual ascent algorithm for the MUFLP with a performance guarantee of 6, and by Ageev [7] and Ageev et al. [8], using the result of Edwards [37] who proved that any ρ-approximation algorithm for the UFLP leads to a 3ρ-approximation algorithm for the MUFLP. This yielded combinatorial 4.83- and 3.27-approximation algorithms for any $k \geq 2$ with worst-case bounds of 2.8446 and 3.1678 for $k = 2$ and $k = 3$, respectively. Zhang [112] later combined techniques such as randomized rounding, dual fitting and a greedy procedure to obtain the best-to-date 1.77-approximation algorithm for the TUFLP. Moreover, the author also obtained an $O(\ln|I|)$-approximation algorithm for the non-metric TUFLP. In the same year, Fleischer et al. [41] published their results which consisted of an $O(\ln^k|I|)$-approximation algorithm for the non-metric MUFLP. A few years later, Gabor and van Ommeren [42] described a 3-approximation algorithm for the MUFLP based on LP-rounding using a new MILP formulation. The importance of this model lies in its polynomial number of variables and constraints in contrast with the previous formulation by [3]. Finally, since many techniques used for the development of such algorithms extend from those applied to the single-level versions, a natural question is whether the MUFLP is computationally harder than the UFLP. This question remained open until recently when Krishnaswamy and Sviridenko [66] proved inapproximability results which showed that there exists no approximation algorithm with performance guarantee better than 1.539 for the TUFLP unless $P = NP$. They also showed that for the general case of $k > 2$, when k tends to infinity, the hardness factor is 1.61.

Similarly, approximation algorithms with performance guarantee were developed for variants of the MUFLP. For example, Wang et al. [105, 106] proposed a 4-approximation algorithm based on LP-rounding techniques for the stochastic MUFLP, that is, when demands are uncertain. Melo et al. [81] improved the performance guarantee to $4 - o(1)$. Another variant of the MUFLP that has received attention in the last few years is the so-called MUFLP with penalties [12, 23, 70], in which the decision maker determines whether to provide the service to each customer or to pay a penalty for those that are not served. In particular, Byrka et al. [23] presented the MUFLP as a special case and provides the best known constant-performance guarantee for $k > 2$ which tends to 3 when k is sufficiently large.

4.2. Capacitated case

Several features of single-level FLPs, including applications, solution methods and variants of the problems have been extended to MLFLPs. However, since the number of publications suggests that the uncapacitated cases have attracted more attention, defining streams of research for the capacitated case seems more challenging. Here we discuss the main contributions corresponding to the capacitated variant of the TUFLP, called the two-level capacitated facility location problem (TCFLP). Analogously to the uncapacitated case, it seems that the TCFLP is the one that has been the most studied among capacitated MLFLPs. In this problem, capacities in one or both levels of facilities are imposed, denoted by α_{j_2} and β_{j_1}. From the early works on the TCFLP we note that of Aardal [1], who presented MILP formulation for the problem and a polyhedral study. The same author [2] later introduced a reformulation along with computational results. In contrast with the TUFLP, in this case a more common formulation involves ABF or also called two-index formulations. Let d_i be the demand value for each $i \in I$, and let v_{ij_1} and $w_{j_1j_2}$ be continuous variables representing the flow to customer i from j_1 and that of the plant j_2 to warehouse j_1, respectively. Denoting by c_{ij_1} and $c_{j_1j_2}$ the unit transportation costs from i to j_1 and from j_1 to j_2, respectively, the TCFLP can be formulated as

$$
\begin{aligned}
\text{(TCFLP)} \quad \text{minimize} \quad & \sum_{i\in I}\sum_{j_1\in V_1} c_{ij_1}v_{ij_1} + \sum_{j_1\in V_1}\sum_{j_2\in V_2} c_{j_1j_2}w_{j_1j_2} + \sum_{r=1}^{2}\sum_{j_r\in V_r} f_{j_r}y_{j_r} \\
\text{subject to} \quad & \sum_{j_1\in V_1} v_{ij_1} \geq d_i && i \in I && (19)\\
& \sum_{j_2\in V_2} w_{j_1j_2} \geq \sum_{i\in I} v_{ij_1} && j_1 \in V_1 && (20)\\
& \sum_{j_1\in V_1} w_{j_1j_2} \leq \alpha_{j_2}y_{j_2} && j_2 \in V_2 && (21)\\
& \sum_{j_2\in V_2} w_{j_1j_2} \leq \beta_{j_1}y_{j_1} && j_1 \in V_1 && (22)\\
& y_{j_r} \in \{0,1\} && j_r \in V_r,\ r = 1,2 && (23)\\
& v_{ij_1} \geq 0,\ w_{j_1,j_2} \geq 0 && i \in I,\ j_1 \in V_1,\ j_2 \in V_2. && (24)
\end{aligned}
$$

Marín and Pelegrín [80] compared a two-index and a three-index formulation for the development of an exact algorithm for the TCFLP based on Lagrangian relaxations. More recently, Litvinchev and Ozuna Espinosa [72], Wildbore [108] developed exact and approximate algorithms mainly based on Lagrangian relaxations along with the corresponding computational results obtained for the TCFLP. Fernandes et al. [40] introduced a genetic algorithm, while Guo et al. [54] proposed a hybrid evolutionary algorithm for the same version of the problem. Chen and Wang [25] designed an approximation algorithm for the general k-level version. As in the uncapacitated case, assuming that the values of c are induced by a metric, for k levels, Ageev [7], Bumb and Kern [22], and Du et al. [36] developed ρ-approximation algorithms with values of ρ equal to 12, 9 and $k + 2 + \sqrt{k^2 + 2k + 5} + \epsilon$, respectively.

Other authors have studied some variations of the TCFLP. For example, Bloemhof-Ruwaard et al. [20] solved a slightly different version of the problem in the context of a waste disposal system, while Pirkul and Jayaraman [94] presented a MILP formulation and heuristic methods for a multi-commodity, single-source TCFLP in which cardinality constraints are imposed at both levels. Pirkul and Jayaraman [95] considered the case without the single-source requirements. Yet another variant introduced by Addis et al. [5, 6] is the TCFLP with single source constraints at both levels and dimensioning of the facilities, that is, with modular capacities. The authors provided exact and heuristic algorithms to solve instances with up to 200 customers and 50 potential sites of facilities. A similar version of the problem was studied by Wu et al. [109] who developed a Lagrangian relaxation-based procedure. Finally, Wang and Yang [104] and [87] considered variations of the TCFLP under uncertainty.

5. MLFLPs with network design and tactical decisions

In the early 1990s, Gao and Robinson [43] introduced the term *echelon* in the context of MLFLPs by presenting a new MILP for a variant of the TUFLP, denoted as two-echelon uncapacitated facility location problem (TEUFLP). This formulation was mainly motivated by the desire to extend the dual adjustment procedures of [39] for the TUFLP, which Tcha and Lee [100] had previously been unable to achieve. This modification of the problem can be viewed as if the fixed costs for opening warehouses also depend on the plants from which they are served, that is, there is a fixed cost associated with each pair of facilities from levels one and two, i.e. operating together. Equivalently, this variant can be seen as having fixed costs for opening edges between facilities of different levels associated with the selection of facilities at the first level. Thus, the authors consider $h^1_{j_1 j_2}$ to be the fixed costs for opening warehouse j_1 and supplying it from plant j_2, and the binary variables $t^1_{j_1 j_2} = 1$ if j_1 is opened and simultaneously served from j_2. The TEUFLP can be formulated as

$$
\begin{aligned}
\text{(F1-TEUFLP)}\quad \text{minimize}\quad & \sum_{i \in I} \sum_{j_1 \in V_1} \sum_{j_2 \in V_2} c_{ij_1j_2} x_{ij_1j_2} + \sum_{j_2 \in V_2} f_{j_2} y_{j_2} + \sum_{j_2 \in V_2} \sum_{j_1 \in V_1} h_{j_1j_2} t^1_{j_1j_2} \\
& (1), (4) \\
& t^1_{j_1j_2} \le y_{j_2} \qquad j_1 \in V_1,\ j_2 \in V_2, && (25) \\
& x_{ij_1j_2} \le t^1_{j_1j_2} \qquad i \in I,\ j_1 \in V_1,\ j_2 \in V_2, && (26) \\
& y_{j_2} \in \{0,1\} \qquad j_2 \in V_2, && (27) \\
& t^1_{j_1j_2} \in \{0,1\} \qquad j_1 \in V_1,\ j_2 \in V_2. && (28)
\end{aligned}
$$

Soon after, Barros and Labbé [17] introduced a general version of the problem that subsumes both the TUFLP and the TEUFLP. The authors considered fixed costs associated with opening facilities at both levels as well as those for activating edges between facilities of different levels. Fixed costs for opening edges between customers and first-level facilities are not considered in the problem. We denote this variant as the TUFLP-E. The authors discussed three variants of a MILP formulation for the problem and studied the relationships between the corresponding LP relaxations. The TUFLP-E of [17] is formulated as

$$
\begin{aligned}
\text{(F1-TUFLP-E)}\ \text{maximize}\quad & \sum_{i \in I} \sum_{j_1 \in V_1} \sum_{j_2 \in V_2} c_{ij_1j_2} x_{ij_1j_2} - \sum_{r=1}^{2} \sum_{j_r \in V_r} f_{j_r} y_{j_r} - \sum_{j_2 \in V_2} \sum_{j_1 \in V_1} h^1_{j_1j_2} t^1_{j_1j_2} \\
\text{subject to}\quad & (1), (4), (5), (25), (26), (28) \\
& t^1_{j_1j_2} \le y_{j_1} \qquad j_1 \in V_1,\ j_2 \in V_2, && (29)
\end{aligned}
$$

or equivalently, exchanging constraints (25) and (29) by (2) and (3) when the fixed costs are non-negative. They proved in [17] that using the sets of constraints (2) and (3) yields a better LP bound. We also note that the authors formulated the problem in the maximization form, perhaps for the first time for MLFLPs. The values of $c_{ij_1j_2}$ in this case actually correspond to profits, instead of transportation costs. Indeed, as discussed by Cornuéjols et al. [29] for the single-level and by Ortiz-Astorquiza et al. [93] for the multi-level case, the maximization and minimization versions of the UFLP are equivalent from an optimization point of view. However, from an algorithmic perspective, especially for approximation algorithms, this is not the case as mentioned in Section 4.1.3. This version of the problem can be generalized to the case of k levels, even including link activation costs between customers and the set V_1. We denote it as MUFLP-E, which is the representative problem of this category.

We next review the main contributions to MLFLPs that involve non-trivial network design and tactical decisions. We divide this section into the uncapacitated and capacitated cases.

5.1. Uncapacitated case

Barros and Labbé [17] seem to have been the first to study the general version of the problem. They developed a branch-and-bound procedure using the corresponding upper and lower bounds obtained from different Lagrangian relaxations of two of the formulations discussed, and those obtained from an extension of the greedy heuristic proposed for the UFLP. This method also benefits from the efficient solution of a particular Lagrangian relaxation which coincides with a min-cut problem. The authors presented comparative computational results with the previous special cases of [43] and [100]. They observed that solving the proposed Lagrangian relaxations provides an easier way to obtain better bounds than those yielded by the modified dual ascent method for MLFLPs. However, this category of MLFLPs was put aside for some years until very recently when Gendron et al. [45, 46] studied a more general version of the problem of Chardaire et al. [24]. The MLFLP studied in [24] considers only design costs in a two-level FLP where single assignment constraints between levels of facilities are imposed. The problem addressed in [46] additionally includes transportation costs c since it appears as a subproblem in a more sophisticated MLFLP in the context of freight transportation [47]. We refer to this variant as the TUFLP with edge costs and single assignment constraints (TUFLP-E-S) which can be formulated as

$$\text{(F1-TUFLP-E-S) minimize } \sum_{i\in I}\sum_{j_1\in V_1}\sum_{j_2\in V_2} c_{ij_1j_2}x_{ij_1j_2} + \sum_{r=1}^{2}\sum_{j_r\in V_r} f_{j_r}y_{j_r} + \sum_{j_2\in V_2}\sum_{j_1\in V_1} h^1_{j_1j_2}t^1_{j_1j_2}$$

$$\begin{aligned}
\text{subject to } & \sum_{j_2\in V_2}\sum_{j_1\in V_1} x_{ij_1j_2} = 1 && i\in I && (30)\\
& \sum_{j_1\in V_1} x_{ij_1j_2} \le y_{j_2} && i\in I,\ j_2\in V_2, && (31)\\
& \sum_{j_2\in V_2} x_{ij_1j_2} \le y_{j_1} && i\in I,\ j_1\in V_1, && (32)\\
& x_{ij_1j_2} \le t^1_{j_1j_2} && i\in I,\ j_1\in V_1,\ j_2\in V_2, && (33)\\
& \sum_{j_2\in V_2} t^1_{j_1j_2} \le 1 && j_1\in V_1, && (34)\\
& t^1_{j_1j_2} \le y_{j_1} && j_1\in V_1,\ j_2\in V_2 && (35)
\end{aligned}$$

$$x_{ij_1j_2} \geq 0 \qquad i \in I,\ j_1 \in V_1,\ j_2 \in V_2, \tag{36}$$

$$y_{j_r} \in \{0,1\} \qquad j_r \in V_r, \quad r = 1,2. \tag{37}$$

As noted in [46], if the fixed costs are non-negative, one can project out the variables y_{j_1} for $j_1 \in V_1$ based on the set of constraints (34). Thus, $y_{j_1} = \sum_{j_2 \in V_2} t^1_{j_1j_2}$ and the fixed costs f_{j_1} can be embedded within the new edge costs $l_{j_1j_2} = h^1_{j_1j_2} + f_{j_2}$ for each $j_1 \in V_1$. Constraints (35) are actually redundant but allow relaxing the integrality conditions on the y_{j_2} variables in addition to improving the LP bound. After projecting out the variables y_{j_1} the objective function becomes

$$\text{minimize} \quad \sum_{i \in I} \sum_{j_1 \in V_1} \sum_{j_2 \in V_2} c_{ij_1j_2} x_{ij_1j_2} + \sum_{j_2 \in V_2} f_{j_2} y_{j_2} + \sum_{j_2 \in V_2} \sum_{j_1 \in V_1} l_{j_1j_2} t^1_{j_1j_2},$$

which coincides with that of the TEUFLP described in [43]. Also, when the set-up costs on the edges $h^1_{j_1j_2}$ are zero, we obtain the TUFLP-S version of the problem. However, in this case the single assignment constraints can be dropped under some conditions on the costs $c_{ij_1j_2}$ [46, 93], yielding a class of instances for which the TUFLP and the TUFLP-E are equivalent. In [46] a branch-and-bound procedure is also developed based on specialized branching rules and a Lagrangian relaxation that was not previously studied in [17, 24].

All the papers relating to MUFLP-E mentioned so far consider the two-level version of the problem. Ortiz-Astorquiza et al. [92] recently introduced a general k-level setting where all three types of costs are considered and cardinality constraints are imposed at each level. This problem is denoted as MUpLP-E. In comparison with the other two categories of MLFLP, little research has been carried out in this category, especially in what regards the development of exact solution methods. These authors developed an exact Benders-based algorithm decomposition scheme for the solution of large-scale instances. The algorithm exploits the structure of the extended F2-MUFLP formulation for the MUpLP-E in which the subproblems can be efficiently solved. The authors conducted an extensive computational study on the impact of different variations of the Benders decomposition procedure, such as implementing Pareto-optimal cuts or using alternative feasibility cuts.

5.2. Capacitated case

The first articles to consider a TCFLP with fixed costs for opening edges are [102, 103]. Tragantalerngsak et al. [102] developed several Lagrangian heuristics for a two-level CFLP with single source constraints (single assignment in the TUFLP) and capacities at the first-level facilities only. The MILP formulation described by the authors resembles that of the TEUFLP of [43], where fixed costs on the edges between facilities of different levels replace those of opening facilities in one level. In a sequel paper [103], the same authors presented an exact algorithm for the problem based on the previous Lagrangian relaxations. We refer to this problem as the TECFLP-S. It can be formulated as

$$\begin{aligned}
\text{(TECFLP-S) minimize } & \sum_{i\in I}\sum_{j_1\in V_1}\sum_{j_2\in V_2} c_{ij_1j_2}x_{ij_1j_2} + \sum_{j_1\in V_1}\sum_{j_2\in V_2} h_{j_1j_2}t_{j_1j_2} \sum_{j_2\in V_2} f_{j_2}y_{j_2} && \\
\text{subject to } & \sum_{j_1\in V_1}\sum_{j_2\in V_2} x_{ij_1j_2} = 1 && i\in I && (38)\\
& \sum_{i\in I} d_i x_{ij_1j_2} \le \beta_{j_1} && j_1\in V_1,\ j_2\in V_2 && (39)\\
& \sum_{j_2\in V_2} t_{j_1j_2} \le 1 && j_1\in V_1 && (40)\\
& x_{ij_1j_2} \le t_{j_1j_2} && j_1\in V_1,\quad j_2\in V_2 && (41)\\
& t_{j_1j_2} \le y_{j_2} && && (42)\\
& x_{ij_1j_2},\ t_{j_1,j_2},\ y_{j_2} \in \{0,1\} && i\in I,\quad j_1\in V_1,\ j_2\in V_2. && (43)
\end{aligned}$$

Other related problems have also been studied. These are more general and typically include additional requirements. For example, Ignacio et al. [59] presented a two-level capacitated facility location problem with edge costs (in E_0 and E_1) and single-source constraints (TCFLP-E-S) in a computer network environment. There, both levels of facilities, routers and concentrators, have capacities and fixed costs for opening facilities at the two levels are considered. The authors designed an exact solution method based on a Lagrangian relaxation an a tabu search heuristic. Another example of a TCFLP-E arises in the context of a disaster relief facility location system in [49]. Ghezavati et al. [49] considered a more general version of the problem where capacities are also imposed on the edges and studied the problem under some uncertain parameters. Finally, some related problems were addressed in [26, 47, 57]. Çinar and Yaman [26] introduced two variants of the so-called vendor location problem as special cases of capacitated MLFLPs. The work of Gendron and Semet [47] was motivated by a freight transportation problem. It set the ground for the study of different variants of the TUFLP which can be seen as a capacitated MLFLP. The authors considered a multi-commodity two-level facility location problem with single-source constraints, capacities in the arcs and modular transportation costs. Hinojosa et al. [57] studied a multi-period TCFLP.

6. MLFLPs with network design decisions

The last category of MLFLPs that we review is concerned with non-trivial network design decisions, but in which no tactical decisions are explicitly considered. We call the MFLDP this "design-only" version of the problem, which can also be viewed as a special case of the MUFLP-E. This problem is relevant to strategic supply chain management. In such a scenario, only the design decisions are involved through the fixed cost on facilities and edges and the allocation of customers is implicitly given by the opening of the corresponding edge. MLFLPs belonging to this category, either for two or k levels, have been studied by several authors [14, 24, 34, 41, 60]. Remarkably, with the exception of [24], none of the above references presents an exact algorithm and all date from the last decade; three of them develop approximation algorithms with performance guarantee, while [14] presents a polyhedral study for a ILP formulation. In particular, the latter paper provides three families of valid inequalities and extends to the TFLDP non-trivial facet defining inequalities for the single-level UFLP. Moreover, the authors study integrality conditions of the polytope associated with the TFLDP, that is, they introduce conditions on the graph $G = (V \cup I, E)$ so that the LP relaxation of the problem outputs an integral solution. They also show how to determine whether a given graph G satisfies such conditions using a polynomial time algorithm developed for the single-level case. For the two-level case, we reproduce the version presented in [14] which uses the sets of arcs $A_r \subseteq V_r \times V_{r+1}$ between levels of facilities, considering $A_0 = I$, to introduce their ILP formulation:

$$
\begin{aligned}
\text{(F1-TFLDP) minimize} \quad & \sum_{(i,j_1)\in A_0} h^0_{ij_1} t^0_{ij_1} + \sum_{(j_1,j_2)\in A_1} h^1_{j_1j_2} t^1_{j_1j_2} + \sum_{r=1}^{2} \sum_{j_r\in V_r} f_{j_r} y_{j_r} \\
& \sum_{(i,j_1)\in A_0} t^0_{ij_1} = 1 && i \in I && (44)\\
& t^0_{ij_1} \le y_{j_1} && (i,j_1) \in A_0, && (45)\\
& \sum_{(j_1,j_2)\in A_1} t^1_{j_1j_2} = y_{j_1} && j_1 \in V_1, && (46)\\
& t^1_{j_1j_2} \le y_{j_2} && (j_1,j_2) \in A_1, && (47)\\
& t^0_{ij_1}, t^1_{j_1j_2} \in \{0,1\} && (i,j_1) \in A_0,\ (j_1,j_2) \in A_1, && (48)\\
& y_{j_r} \in \{0,1\} && r = 1,2,\ j_r \in V_r, && (49)
\end{aligned}
$$

where $t^0_{ij_1}$ and $h^0_{ij_1}$ are the decision variables and costs for opening a link between customer i and first-level facility j_1, respectively. We make two remarks on the above formulation. First, it corresponds to a more general version in which arcs are considered between levels of facilities instead of taking the sets $V_r \times V_{r+1}$. This slightly more general version of the problem could also be reproduced for the MUFLP-E. Second, constraints (46) ensure the allocation of open facilities of the first level to those of the second one, and also enforce single assignment for open facilities of the first level. This is important because in this case the number of edges adjacent to an open facility yields a capacitated version of the problem. This follows from the fact that there are no flow or transportation variables, but only design-type variables. For this reason we exclude the uncapacitated/capacitated subdivision from this category.

Fleischer et al. [41] developed a $\ln^k |I|$-approximation algorithm for the k-level extension of the problem. The authors use general costs h, that is, they also consider the non-metric case. Later, Drexl [34] presented a $3/2(3^k - 1)$-approximation algorithm. They assumed that the costs h are induced by a metric and that the values of f_{j_r} are non-negative. Kantor and Peleg [60] studied a similar version of the problem in which edges need to be opened as well as facilities at each of the k levels, but only in one level is there an associated fixed cost for opening facilities f_{j_k}. The authors developed a $(1 + 3^\beta)(3^{\beta+1})^{k-1}$-approximation algorithm, where $\beta \geq 1$ is a parameter used to define the values of the costs h from the distances between vertices j_r, j_{r+1} in the graph.

Finally, Chardaire et al. [24], mainly motivated from a telecommunications application, studied a variant of the two-level problem in which single-assignment constraints between levels of facilities are enforced. That is, each open facility of the first level can be assigned to at most one open facility of the second level (coherent structure in the HFLP classification). Following the notation of this article,

$$\sum_{j_2 \in V_2} t^1_{j_1 j_2} \leq 1 \qquad j_1 \in V_1.$$

The authors presented two MILP formulations and obtained lower bounds via a Lagrangian relaxation, thus improving one of the formulations with a family of facet-defining inequalities. They also developed a simulated annealing algorithm to improve the upper bounds returned by the Lagrangian relaxation. Mišković and Stanimirović [86] used the model of [24] to design a metaheuristic.

7. Conclusions

We have identified the main characteristics of MLFLPs, an important class of discrete location problems that has received increasing attention in the last two decades. We have pointed out the main differences and similarities with well-known related areas in an attempt to delimit the borders of this class of problems and thus the scope of the survey. In the context of MLFLPs, we have identified three main categories based on the types of decisions involved in the optimization process: MLFLPs with tactical decisions, MLFLPs with network design and tactical decisions, and MLFLPs with network design decisions only. These decisions are closely related to the types of input costs to the problem. Using this classification scheme we have presented a comprehensive review of the most relevant publications and we have discussed the variations between problems along with formulations and algorithms. We have also considered the uncapacitated/capacitated distinction to further identify where most of the efforts in the area have been expended. We first observed that with one exception [92], all papers concerned with the development of exact algorithms (or polyhedral studies) refer to the special case where $k = 2$. Thus, all contributions related to the most general versions of the problems arise from the approximate algorithms context. In particular, a large number of papers have been published on the development of approximation algorithms with performance guarantee for the MUFLP. Notably, this same category of MLFLPs is the one that has received the most attention in comparison with the other two.

Some recent publications have concentrated on different variants and extensions of the main MLFLPs. For instance, we have mentioned some articles in which uncertain parameters are included, as well as dynamic facility location problem where facilities can be opened and closed at each time period, and MLFLPs with service penalties where customers may not to be serviced. In many cases, fundamental MLFLPs arise as subproblems of these more general versions. Other sophisticated models in SCM and HFLPs also present MLFLPs as subproblems. Therefore, efficient algorithms for MLFLPs may help solve related problems. The fact that MLFLPs generalize well-known single-level FLPs while retaining several of their mathematical properties can be further exploited in the development of such algorithms. One important step towards a more systematic growth of the field is the incorporation of a common set of MLFLP instances, which would allow fairer algorithmic comparisons. Finally, we consider that MLFLPs constitute a very promising research area, not only from a theoretical and modeling point of view, but also in terms of devising efficient algorithms.

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