Métodos numéricos - Lista 3

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a)

Here I used the same function from the first problem set. Following are displayed the grid of the shock and the markov chain matrix.

$$\begin{bmatrix} -0.07 \\ -0.05 \\ -0.03 \\ -0.02 \\ 0.0 \\ 0.02 \\ 0.03 \\ 0.05 \\ 0.07 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} 0.57 & 0.4 & 0.03 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.08 & 0.55 & 0.35 & 0.02 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.11 & 0.58 & 0.29 & 0.01 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.15 & 0.6 & 0.24 & 0.01 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.19 & 0.61 & 0.19 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.01 & 0.24 & 0.6 & 0.15 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.01 & 0.29 & 0.58 & 0.11 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.02 & 0.35 & 0.55 & 0.08 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.03 & 0.4 & 0.57 \end{bmatrix} \tag{2}$$

b)

In the Hugget model we have an *ad hoc* debt limit. So I opted to use it as large as possible, so I used the result of complete markets, that $r = \frac{1}{\beta} - 1$, and the Aiyagari natural debt limit I have $\phi = \exp(z_1)\frac{\beta}{1-\beta} + \epsilon$, where $\epsilon = 0.0001$ and is used to the natural limit is not available. z_1 is the smallest shock of our grid of shocks. The grid of capital is linearly spaced between $a_1 = -\phi$ and $a_{500} = \phi$.

The way I constructed the function to solve the model it answers both items *b*) and *c*). The function *higget_vfi* solves the value function iteration and calculate the capital police function and the stationary distribution, naturally the function depends on an interest rate. The function *final_f* computes the excess of demand and the function *solve_hugget* finds the equilibrium interest rate and calculate the right value function, the capital and consumption police functions and the stationary distribution. In 1 it is posted a graph of excess demand for interest rates between 3.5% and the complete markets yield (4.16%), in the next item I will run the algorithm to find the one that makes it equal to zero and we can see in the graph this yield is in the posted interval. I am also posting the value function (2), capital (3) and consumption (4) police for the 4% yield.

c)

The function I've made is similar to the one used in problem set 2. Since the Cobb-Douglas production function and the CRRA utility function are both concave and monotone we have that our value function is concave and monotone. I opted to also use the accelerator, so the function maximizes in the first 100 iterations and after that it maximizes each 10 iterations. I am also using the Ben Moll's initial guess for the value function, where $a = a'$. With a guess for the interest rate in hands this function solves the problem in 5 seconds.
For the stationary distribution we know that a probability matrix has at least one eigenvalue that equals one, so to use this result I constructed the markov matrix of transitions as in Ljungqvist and Sargent (2004). This is a $4500x4500$ matrix where the entry (a_i, s_j) is $P(a_{t+1} = a', z_{t+1} = z'|a_t = a, z_t = z) = I(a', a, z)P(s_{t+1} = s'|s_t = s)$, where $I(a', a, z)$ is an indicator function, where is equal one when $g(a, z) = a'$. This is a matrix full of zeros, so I transformed it in a sparse matrix. After that I needed to solve a linear system: $Qx = x$, where x is the eigenvector associated with the eigenvalue that equals one and Q is the transition matrix. The problem is to exclude the feasible solution that $x = 0$, so I normalized the first entry of the vector to one, to exclude this possibility. This way of finding the invariant distribution is faster than the iteration, that I also tried. I used the function *find_zero* from the package *Roots.jl* to find the desired yield. It took 3.5 minutes to find the right rate and calculate the police functions. The equilibrium rate is 4.15% and in 5 we can see the value function for this rate, in 6 the capital police function, 7 the consumption police function, 8 the stationary distribution and in 10 the Euler Equation Errors (EEE). The mean EEE was -2.2 what means that for each hundred dollar spent in the model, there is a 1 dollar error. In 9 is the pdf of the stationary distribution, in other words, this is the sum of the density of all states for each level of capital, we can see there is a bigger concentration on both extremes, meaning there are a meaningful quantity of rich and poor people in this economy.

d)

Since we changed the ρ parameter I had to create a new set for the shocks and hence also for the capital. Again it took three minutes and a half to run. The new equilibrium rate is 4.14. We have a smaller rate because now the shocks are more permanents and since the agents are risk averse they will save more. Since the supply of savings is bigger, the rate will drop to discourage the agents to save and make the total amount equals zero. The mean EEE is -2.2 for those parameters. In 11 is displayed the value function, 12 the capital police, 13 the consumption police, 14 the stationary distribution, 15 the pdf and in 16 the EEE.

e)

Now I am using the ρ and the σ of the first item, so the same capital and shock grids too. Now the agents are much more risk averse, so naturally they will save more to defend themselves against consumption variations. As the total amount of savings must equal zero the rate will drop even more, now we have 4% as que equilibrium rate. The mean EEE is now -2.2. In 17 is displayed the value function, 18 the capital police, 19 the consumption police, 20

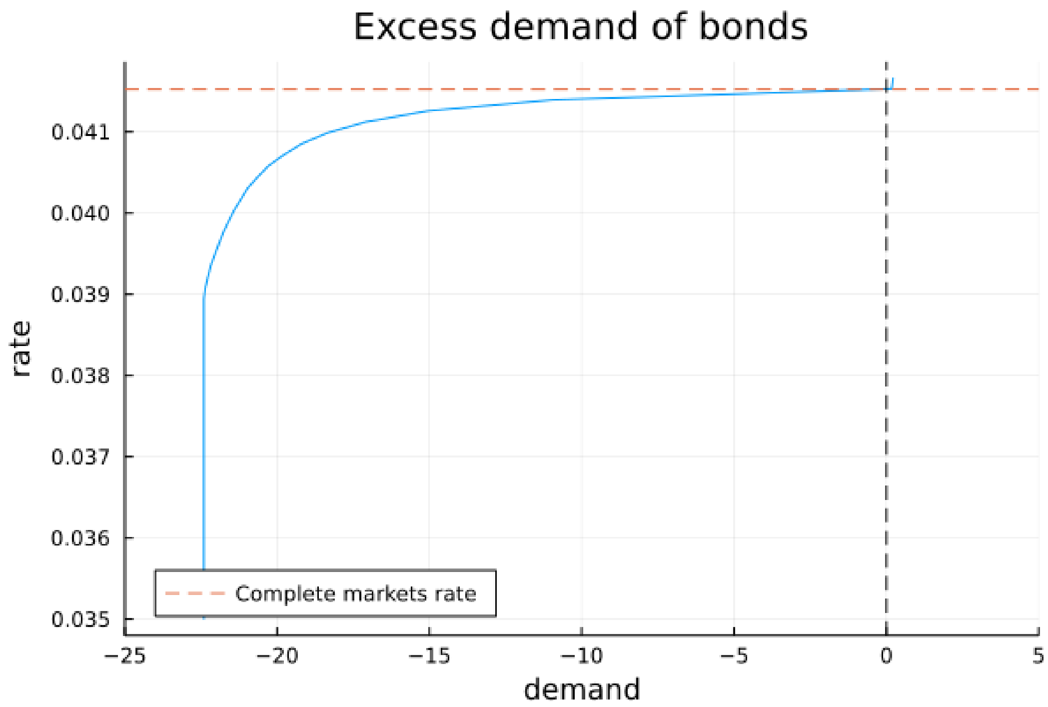

Figure 1: Excess demand of bonds

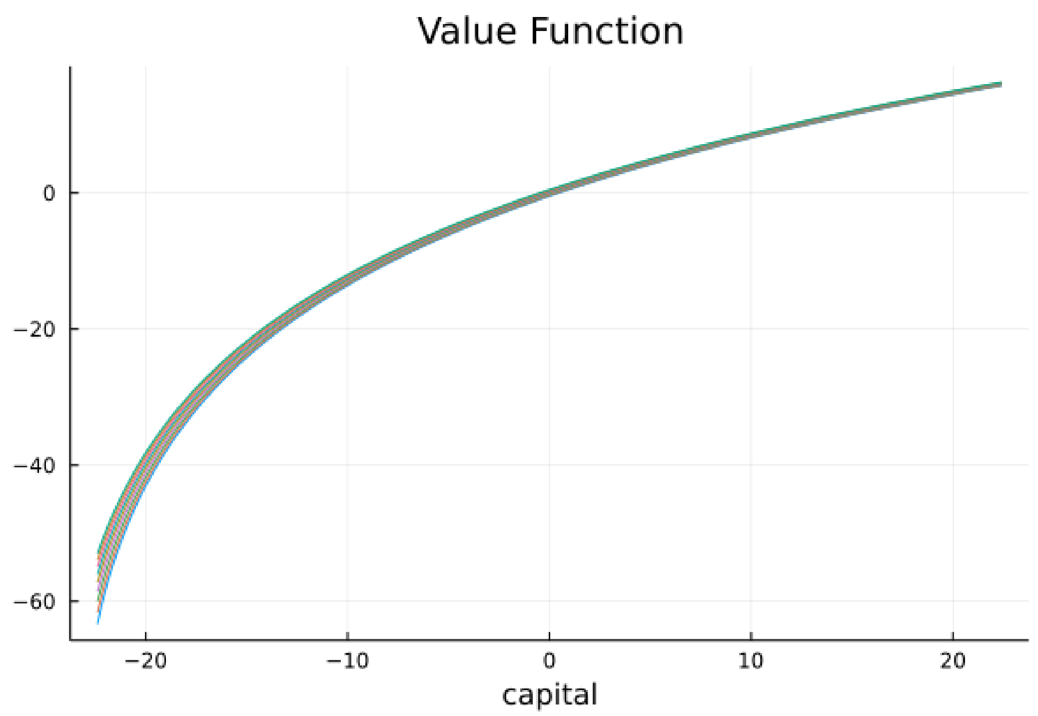

Figure 2: Value function (r = 4%)

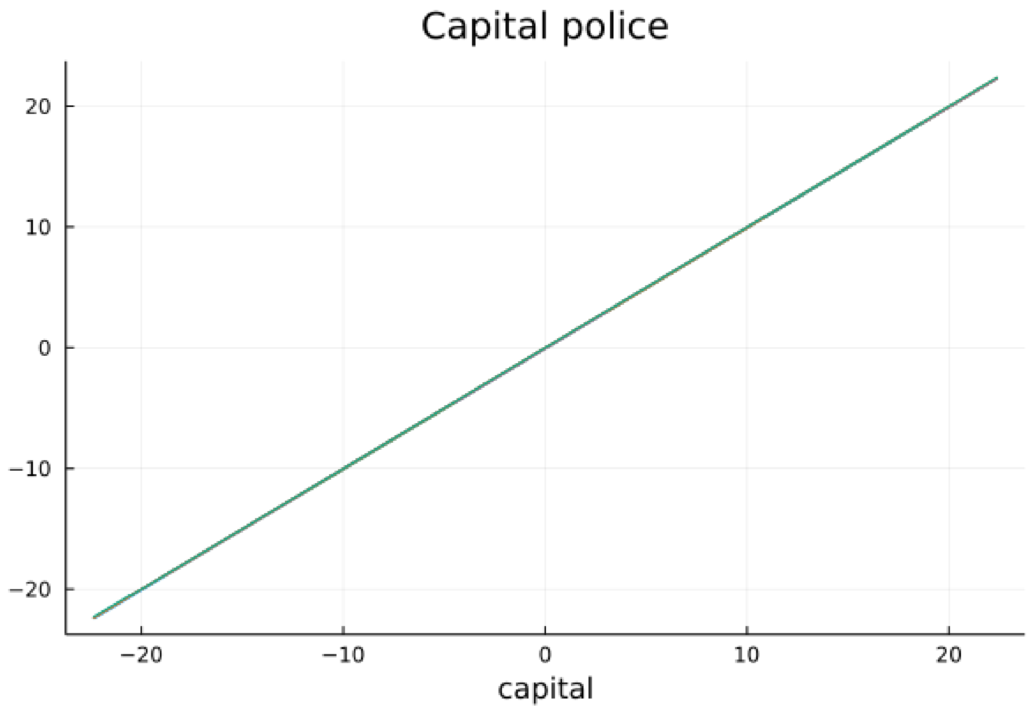

Figure 3: Capital police function (r = 4%)

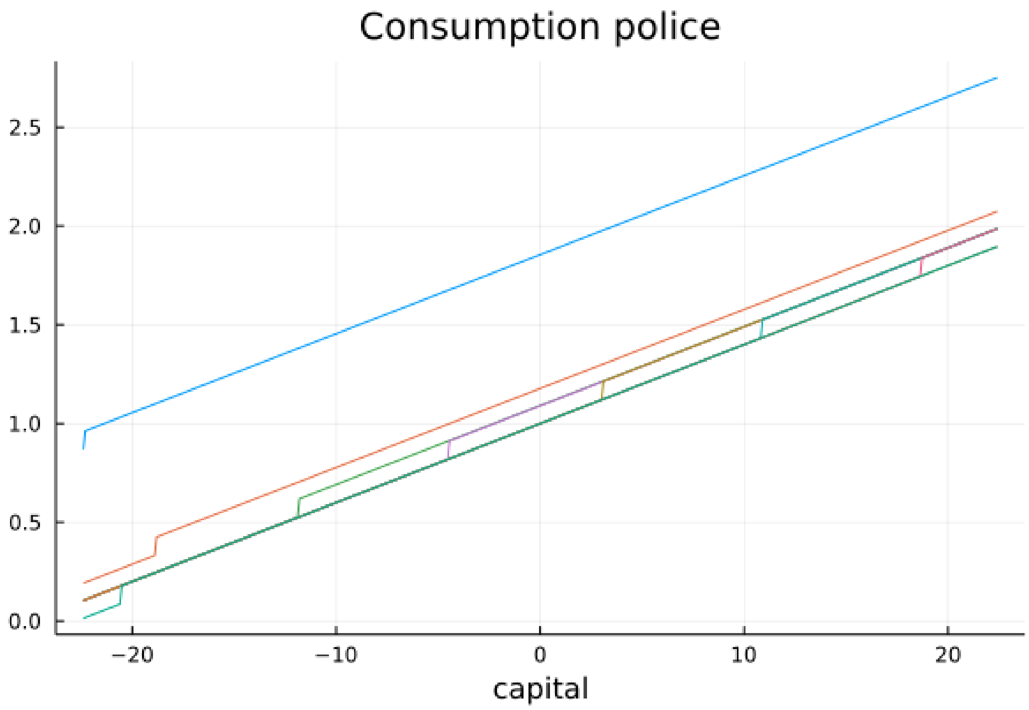

Figure 4: Consumption police function (r = 4%)

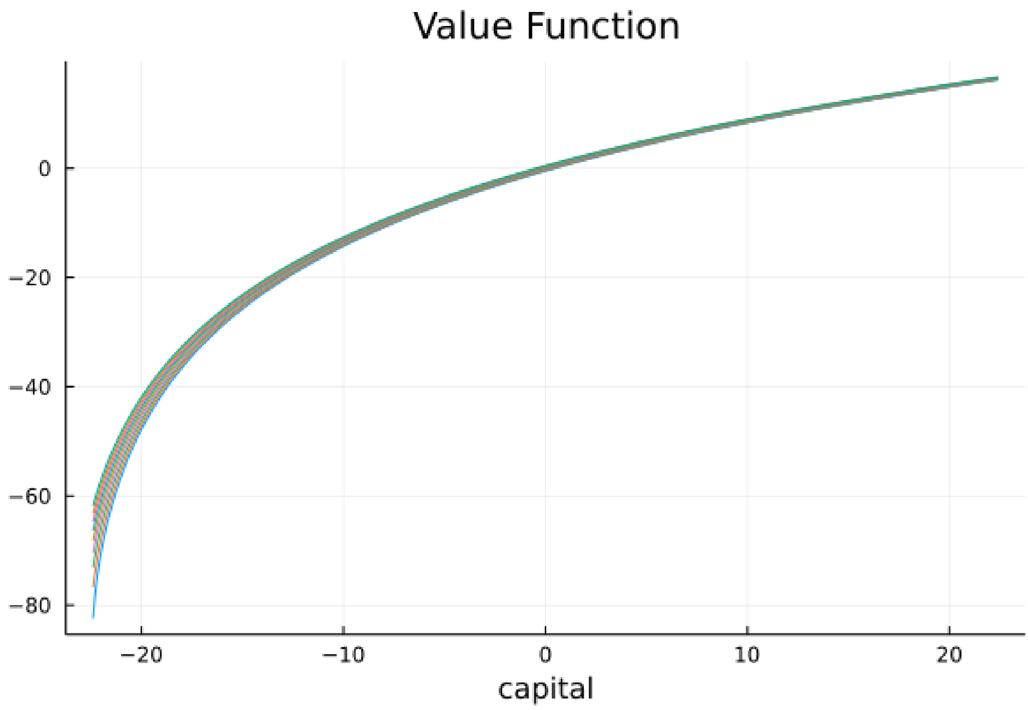

Figure 5: Value function ($\mathrm{r}^* = 4.15$, $\rho = 0.9, \sigma = 0.01$)

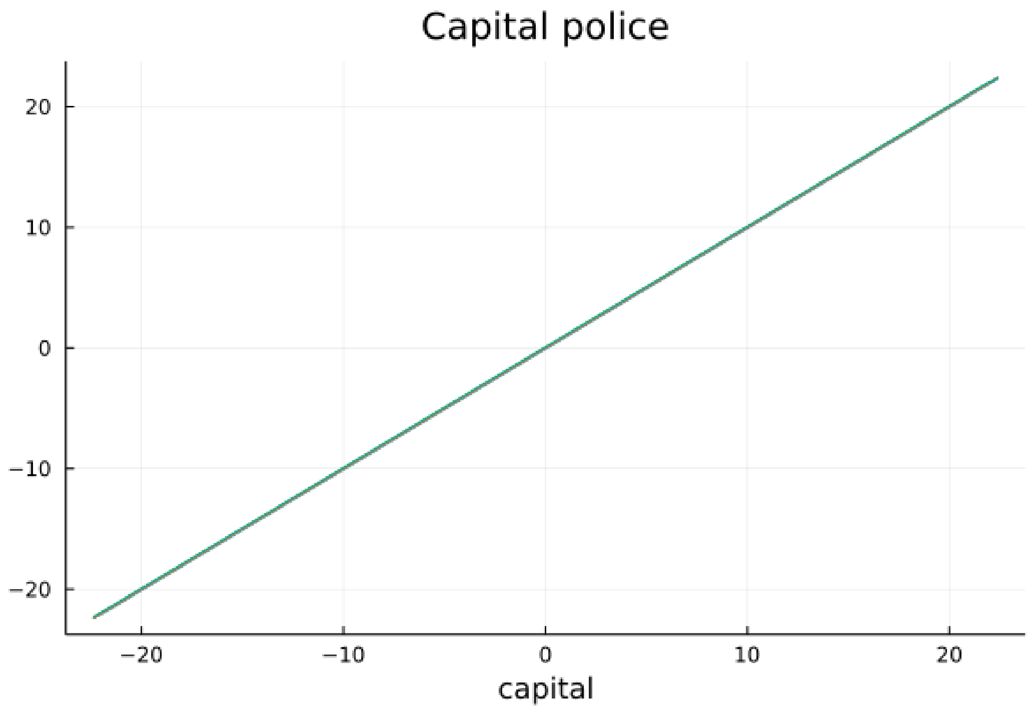

Figure 6: Capital police function ($\mathrm{r}^* = 4.15$, $\rho = 0.9, \sigma = 0.01$)

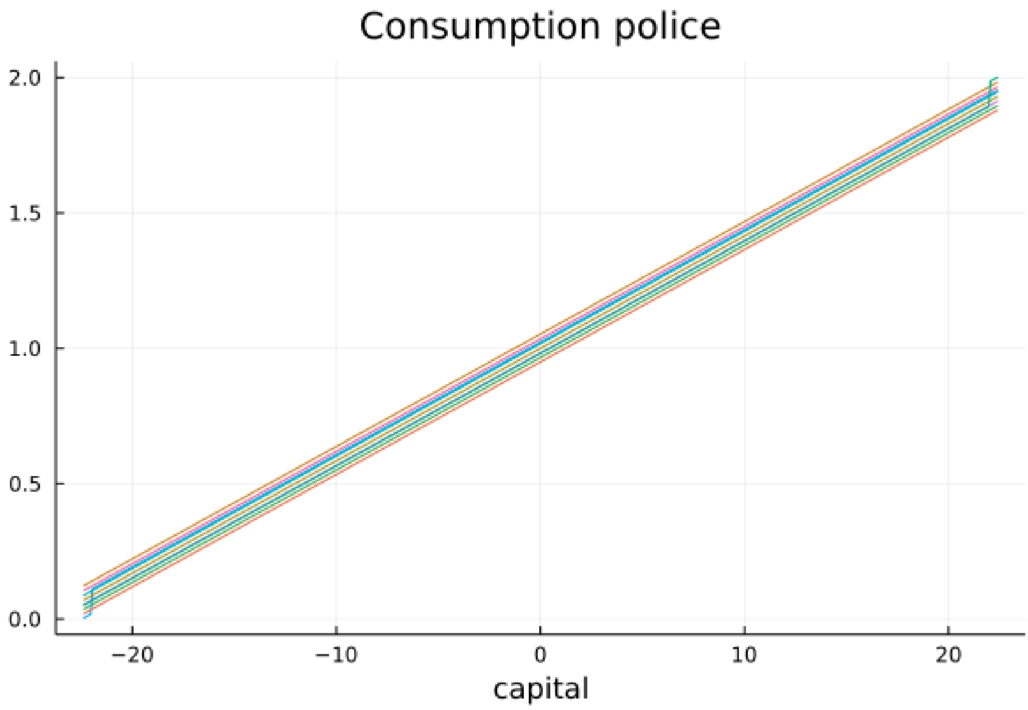

Figure 7: Consumption police function (r* $= 4.15$, $\rho = 0.9, \sigma = 0.01$)

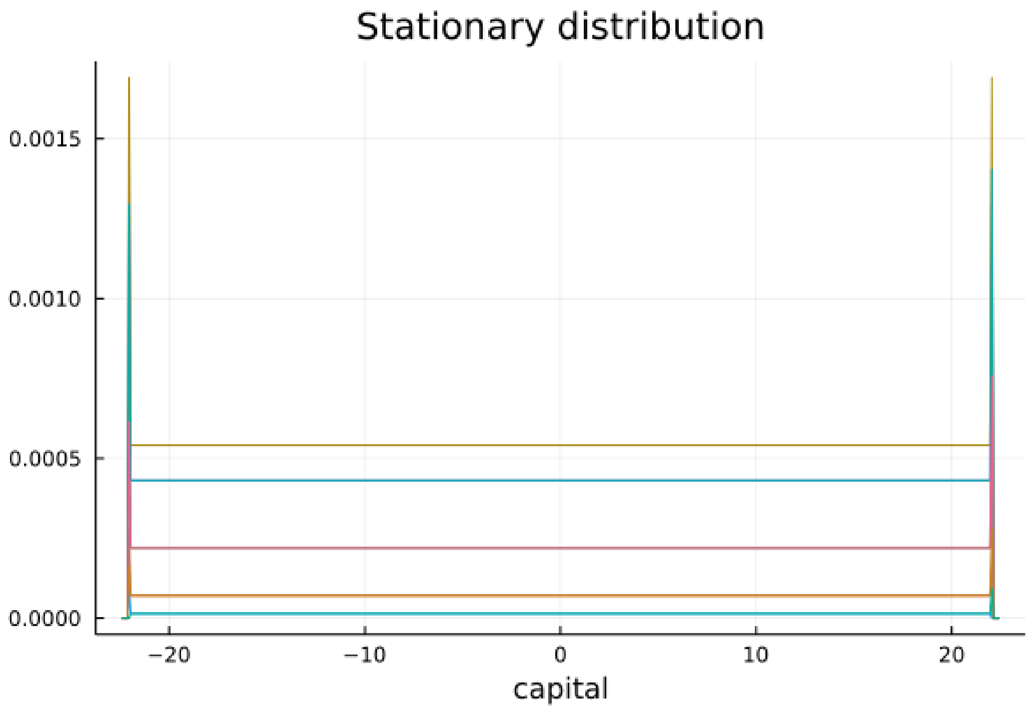

Figure 8: Stationary distribution (r* $= 4.15$, $\rho = 0.9, \sigma = 0.01$)

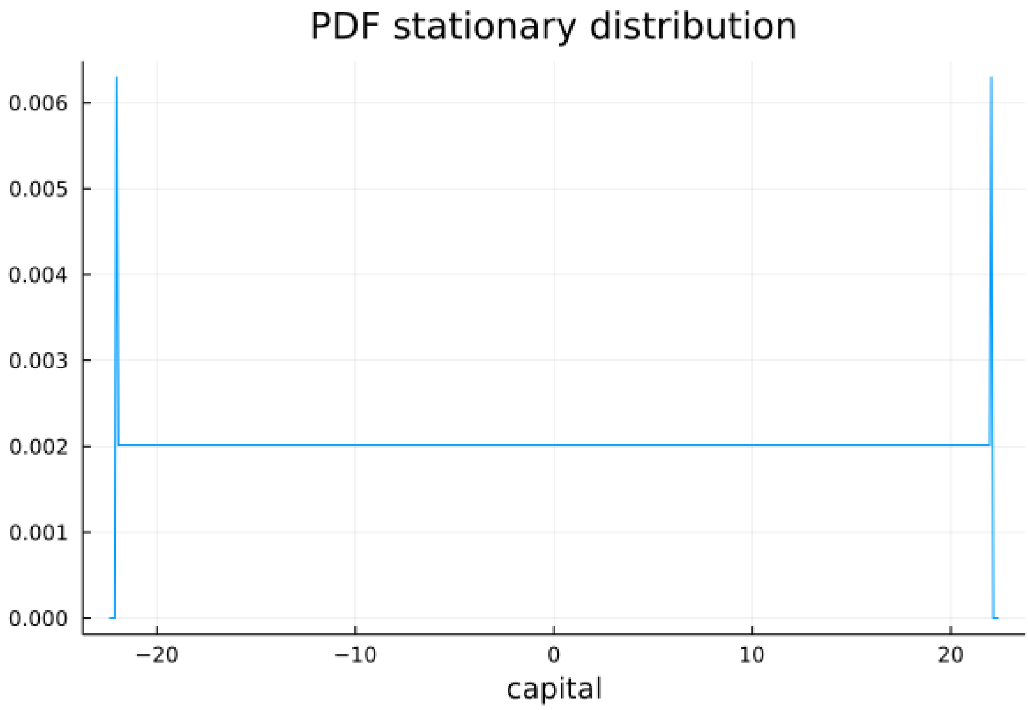

Figure 9: pdf stationary distribution (r* = 4.15, $\rho = 0.9, \sigma = 0.01$)

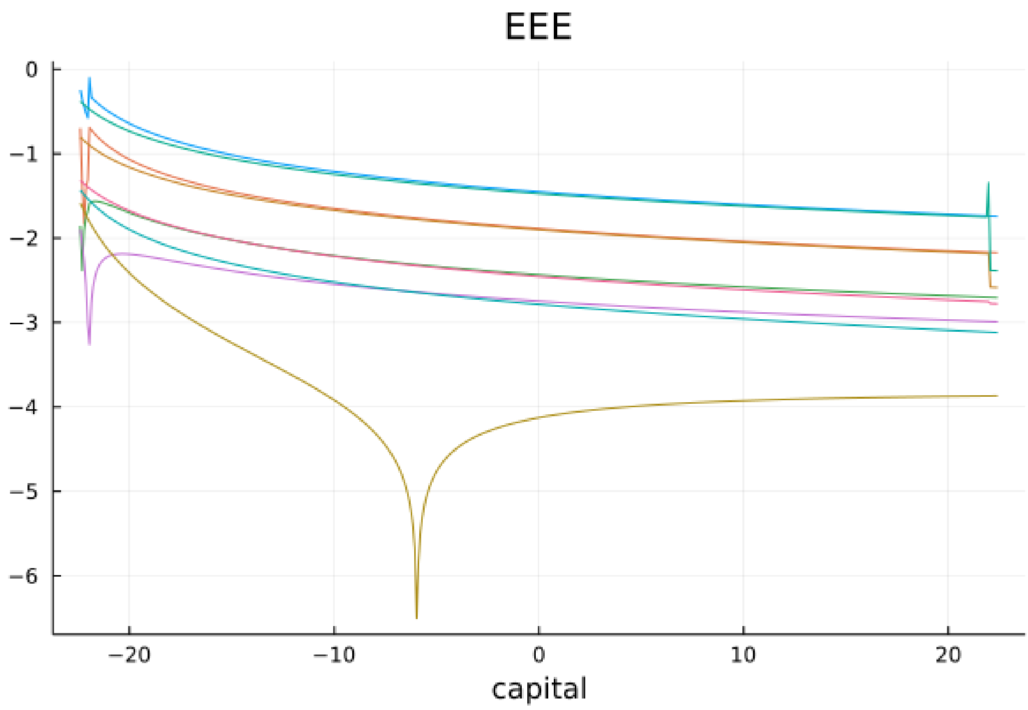

Figure 10: Euler Equation Errors (r* = 4.15, $\rho = 0.9, \sigma = 0.01$)

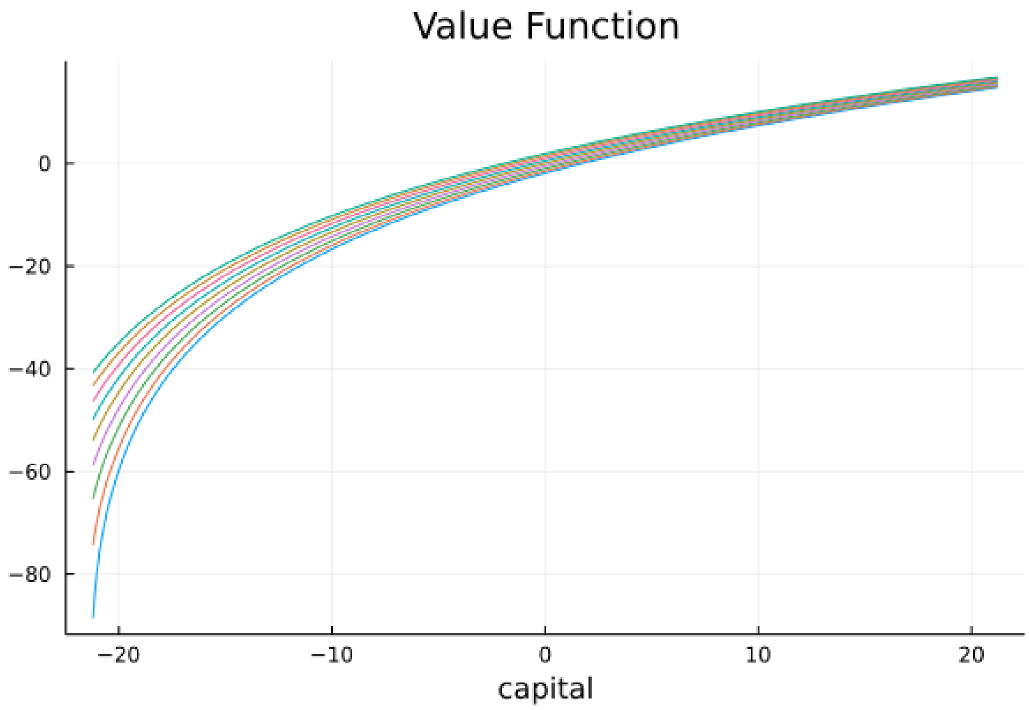

Figure 11: Value function ($r^* = 4.14,\ \rho = 0.97, \sigma = 0.01$)

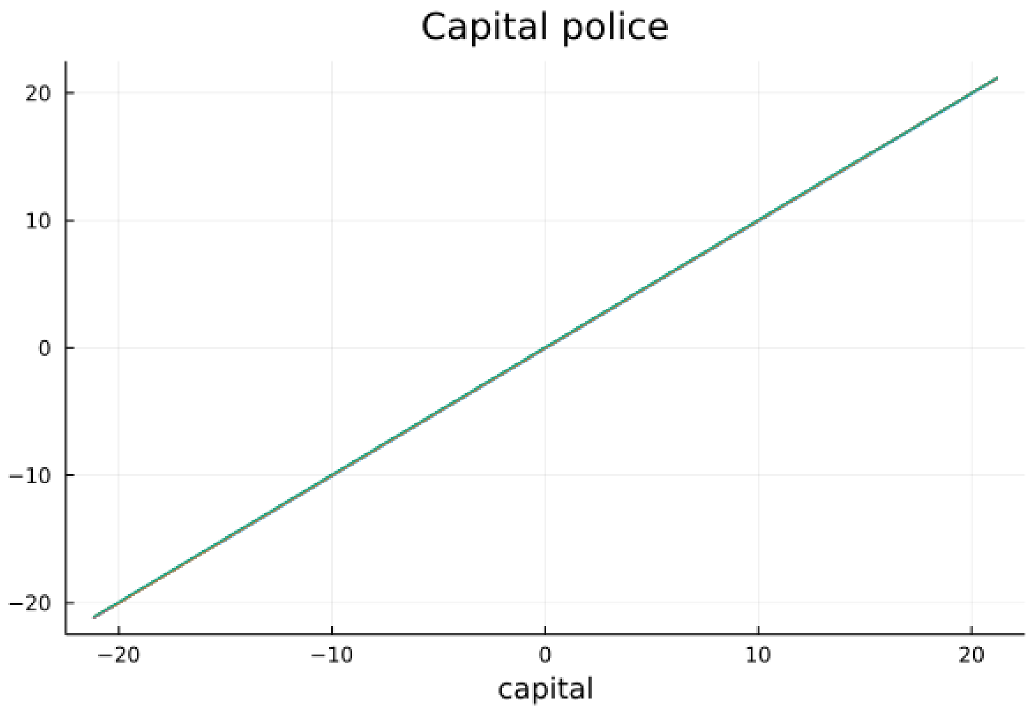

Figure 12: Capital police function ($r^* = 4.14,\ \rho = 0.97, \sigma = 0.01$)

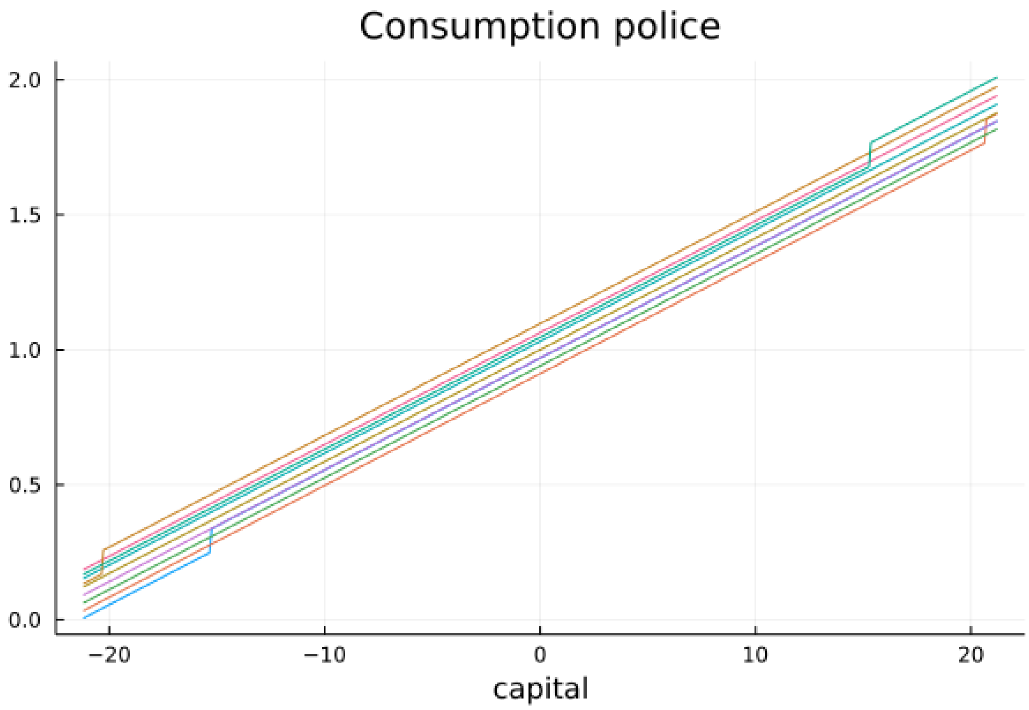

Figure 13: Consumption police function (r* = 4.14, $\rho = 0.97, \sigma = 0.01$)

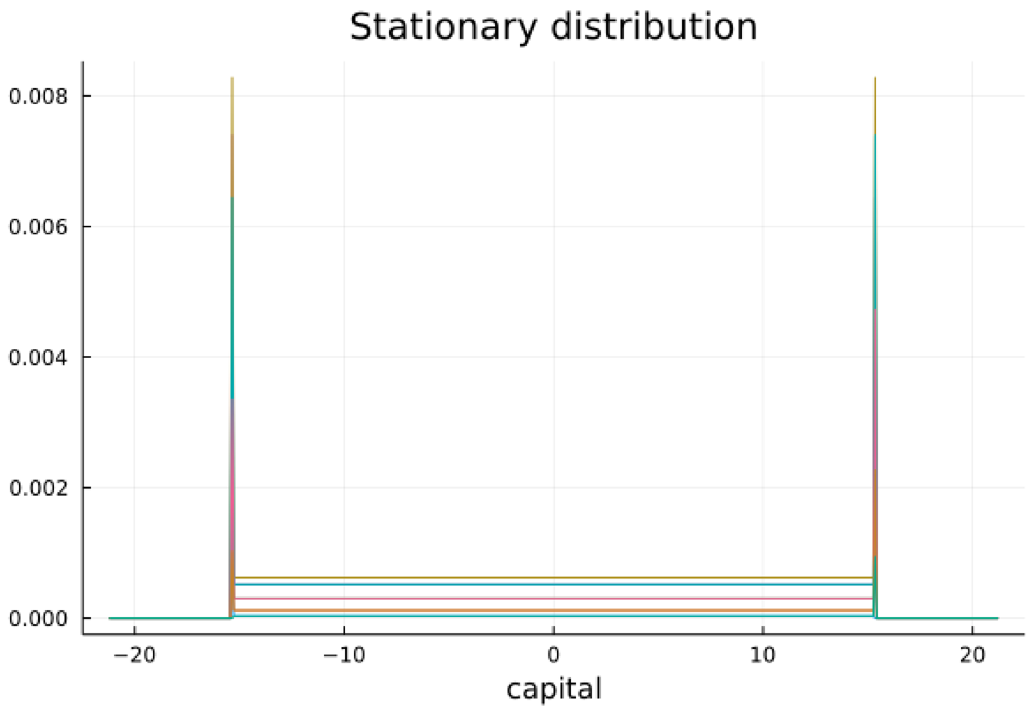

Figure 14: Stationary distribution (r* = 4.14, $\rho = 0.97, \sigma = 0.01$)

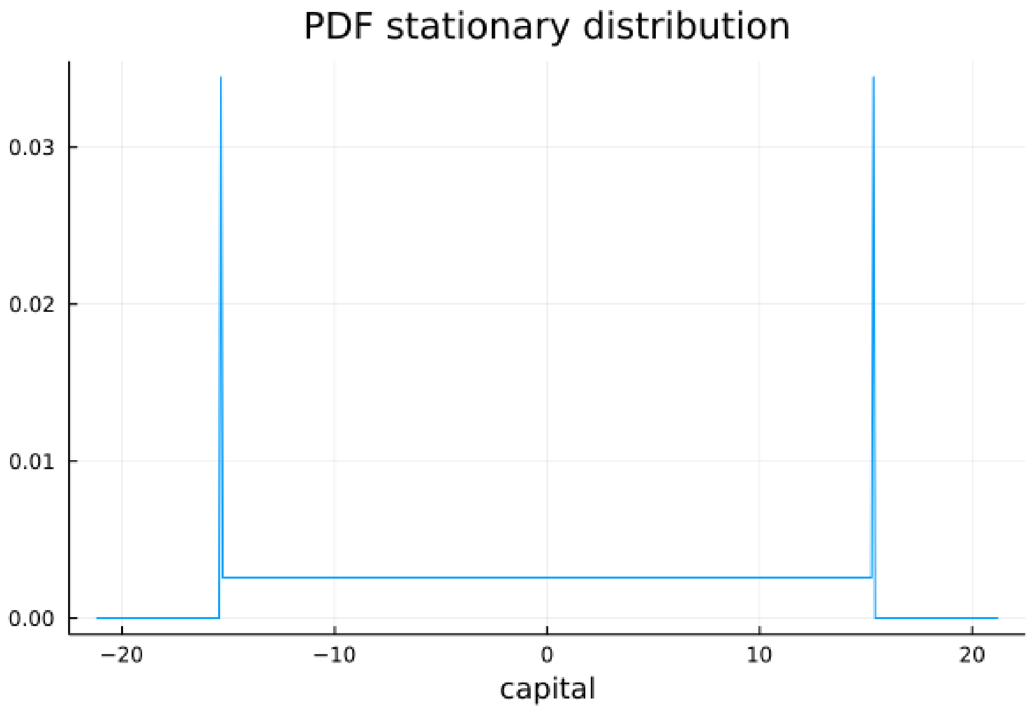

Figure 15: pdf stationary distribution (r* $= 4.14,\ \rho = 0.99, \sigma = 0.01$)

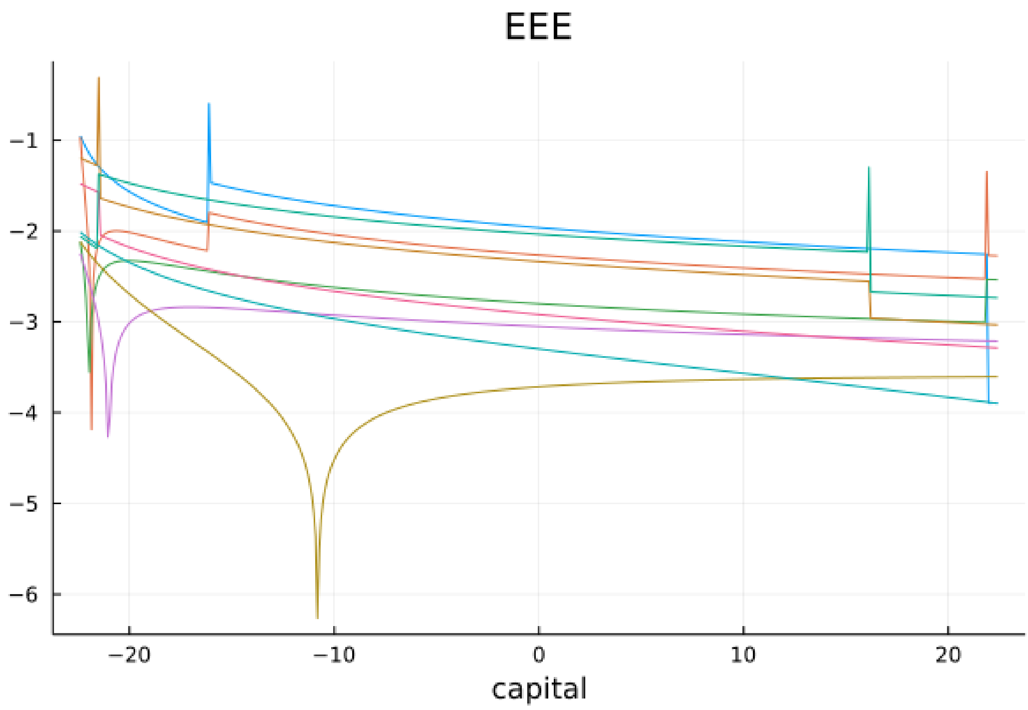

Figure 16: Euler Equation Errors (r* $= 4.14,\ \rho = 0.97, \sigma = 0.01$)

the stationary distribution, 21 the pdf and in 22 the EEE. We have a bigger concentration in two points more in the middle of the grid of capital and not in the extremes any more.

f)

Again I recalculated the grid and capital shocks, beside the sigma, I am using the others parameters as in item a). The algorithm took 4 minutes to find the 4% equilibrium rate. The rate dropped comparing to the first equilibrium rate we found for the same reason from the last items. Now the volatility is higher and the risk averse agents want to save more, so to make the net savings of the economy to equal zero the rate must drop. The mean EEE here is -2.4. For all items we got a higher error than the last problems sets. Once again in 23 is displayed the value function, 24 the capital police, 25 the consumption police, 26 the stationary distribution, 27 the pdf and in 28 the EEE. The pdf now is not concentrated in two points as before, we have a bigger mass in the middle of the assets.

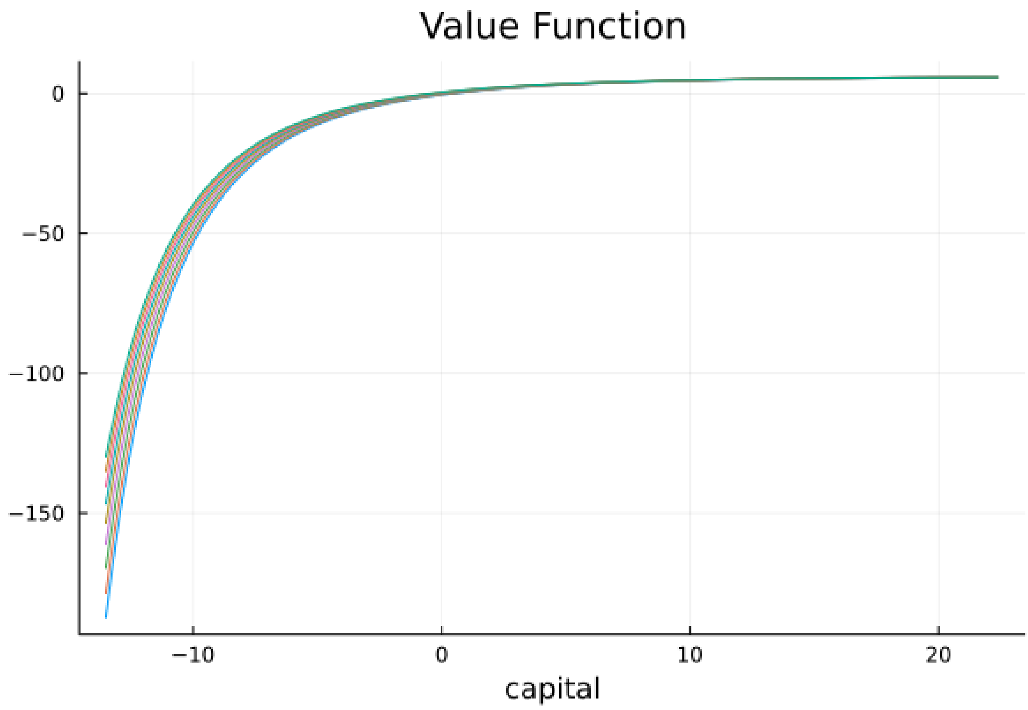

Figure 17: Value function (r* = 4.14, $\rho = 0.97, \sigma = 0.01$)

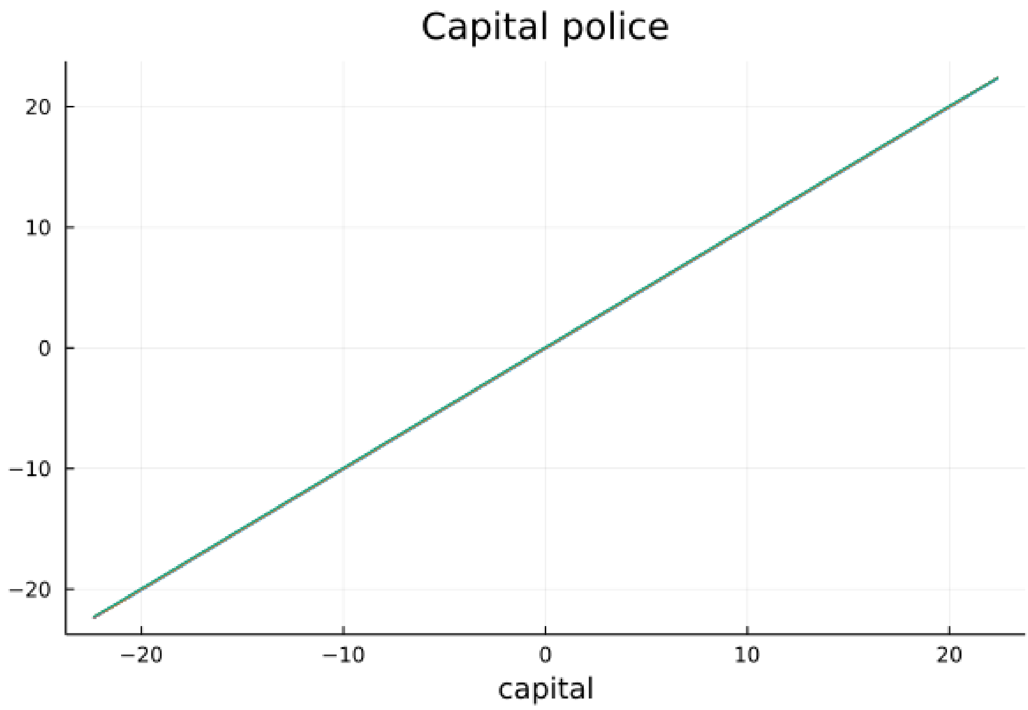

Figure 18: Capital police function (r* = 4.14, $\rho = 0.97, \sigma = 0.01$)

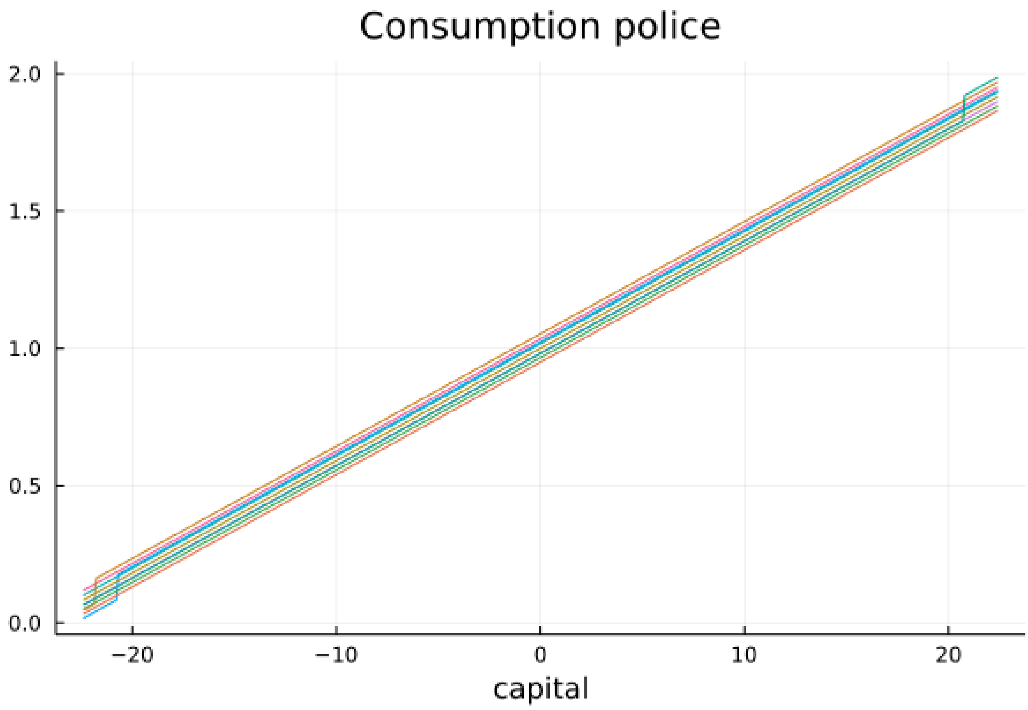

Figure 19: Consumption police function (r* = 4.14, $\rho = 0.97, \sigma = 0.01$)

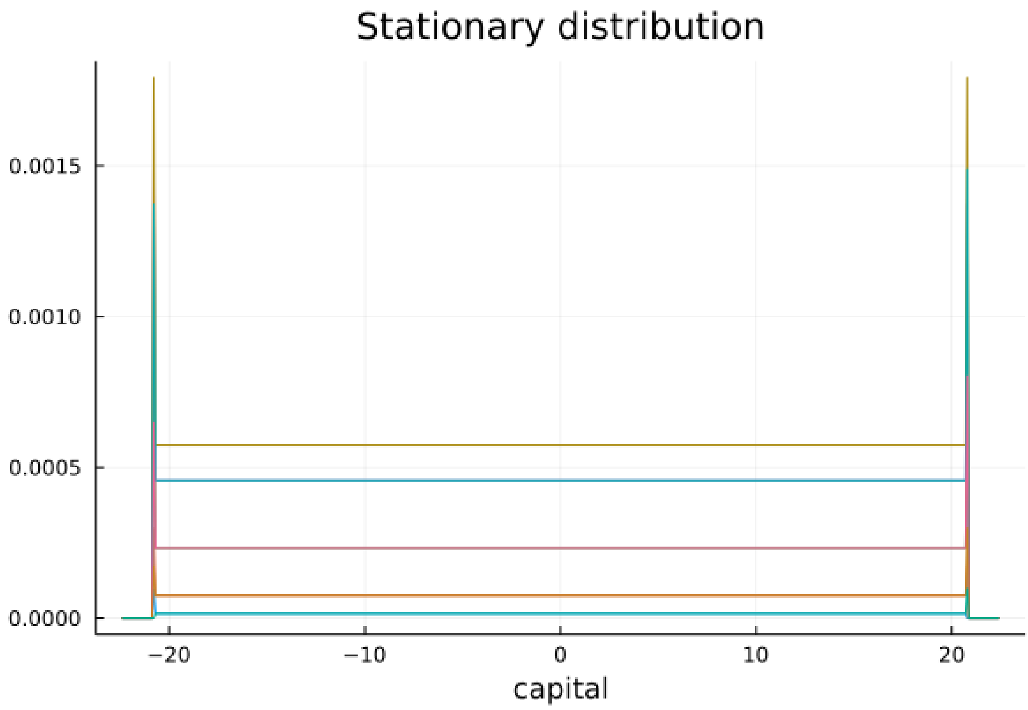

Figure 20: Stationary distribution (r* = 4.14, $\rho = 0.97, \sigma = 0.01$)

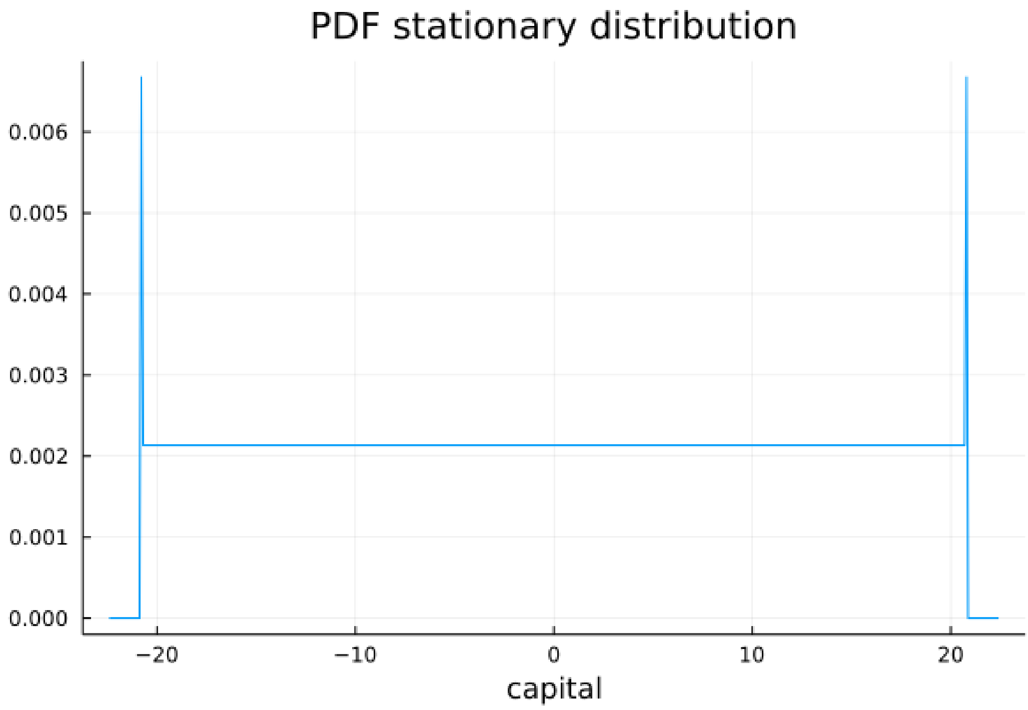

Figure 21: pdf stationary distribution (r* $= 4.14$, $\rho = 0.97, \sigma = 0.01$)

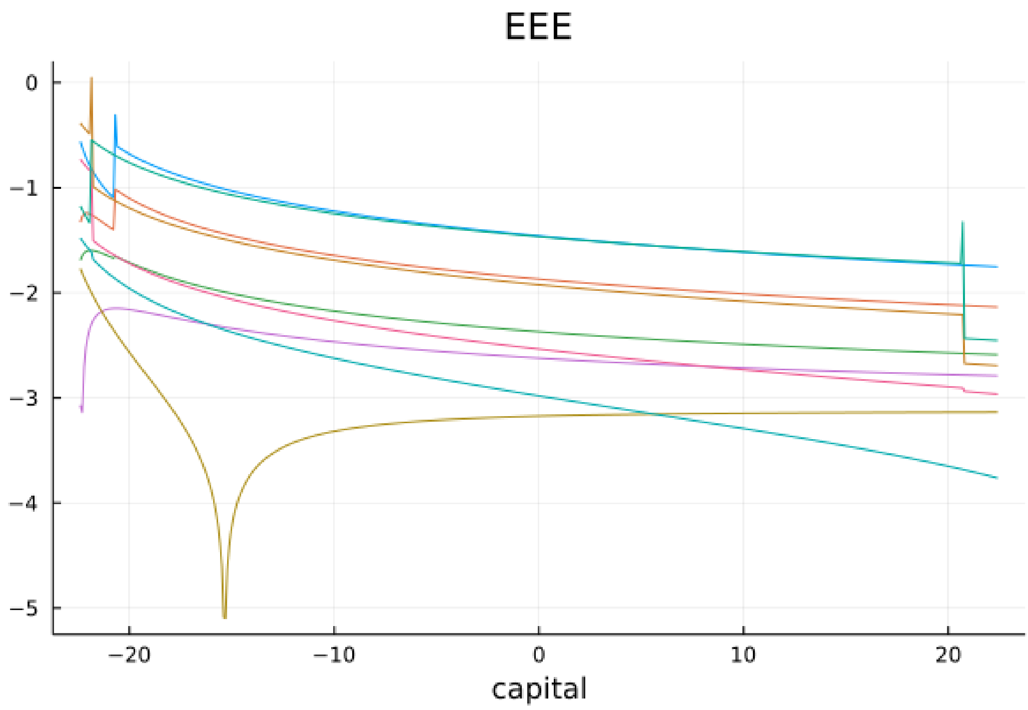

Figure 22: Euler Equation Errors (r* $= 4.14$, $\rho = 0.97, \sigma = 0.01$)

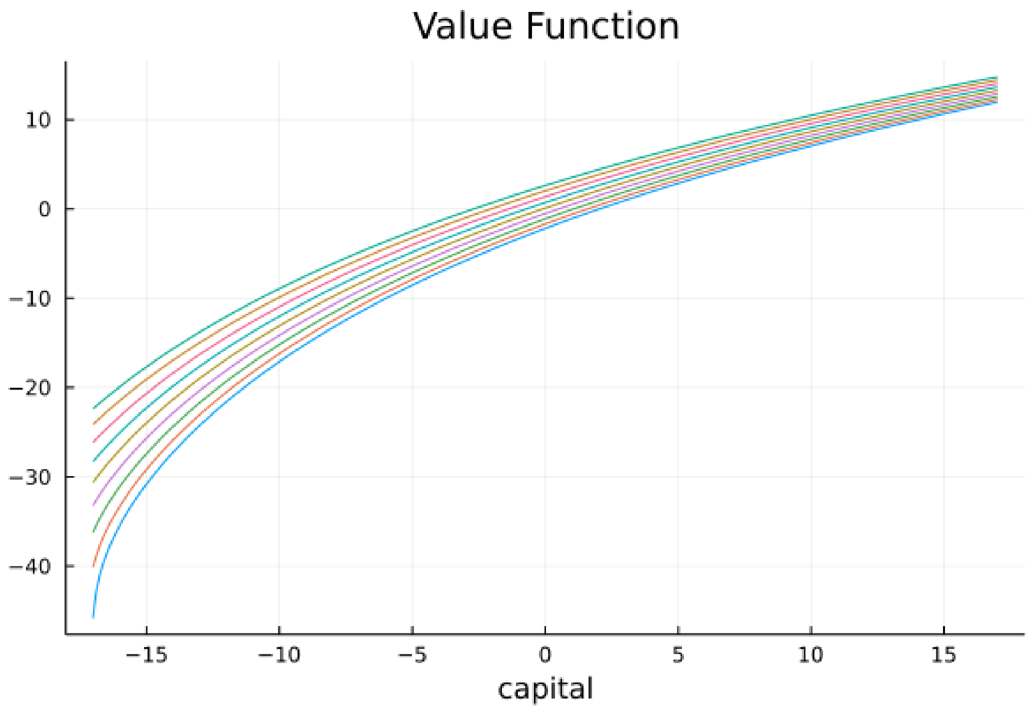

Figure 23: Value function (r* = 4.0, $\rho = 0.9, \sigma = 0.05$)

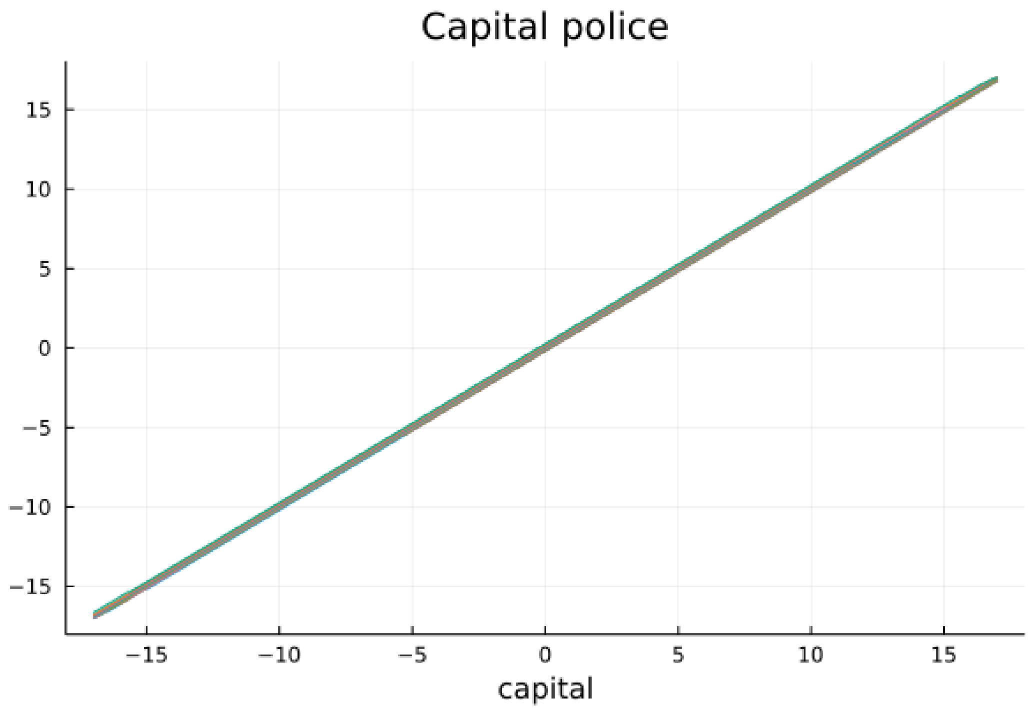

Figure 24: Capital police function (r* = 4.0, $\rho = 0.9, \sigma = 0.05$)

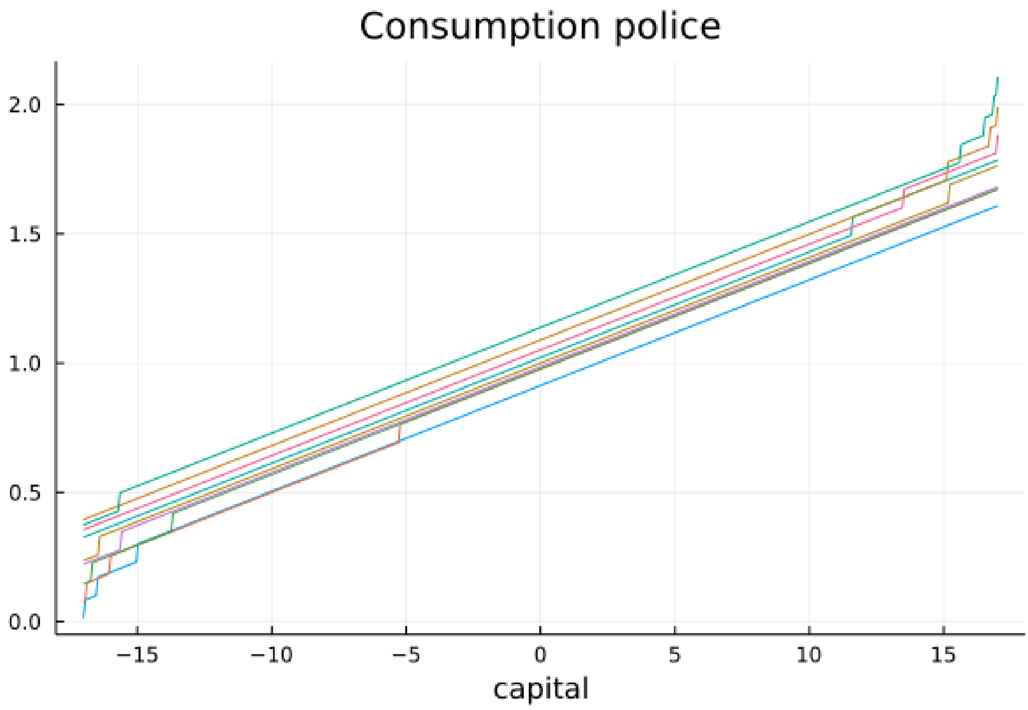

Figure 25: Consumption police function (r* = 4.0, $\rho = 0.9, \sigma = 0.05$)

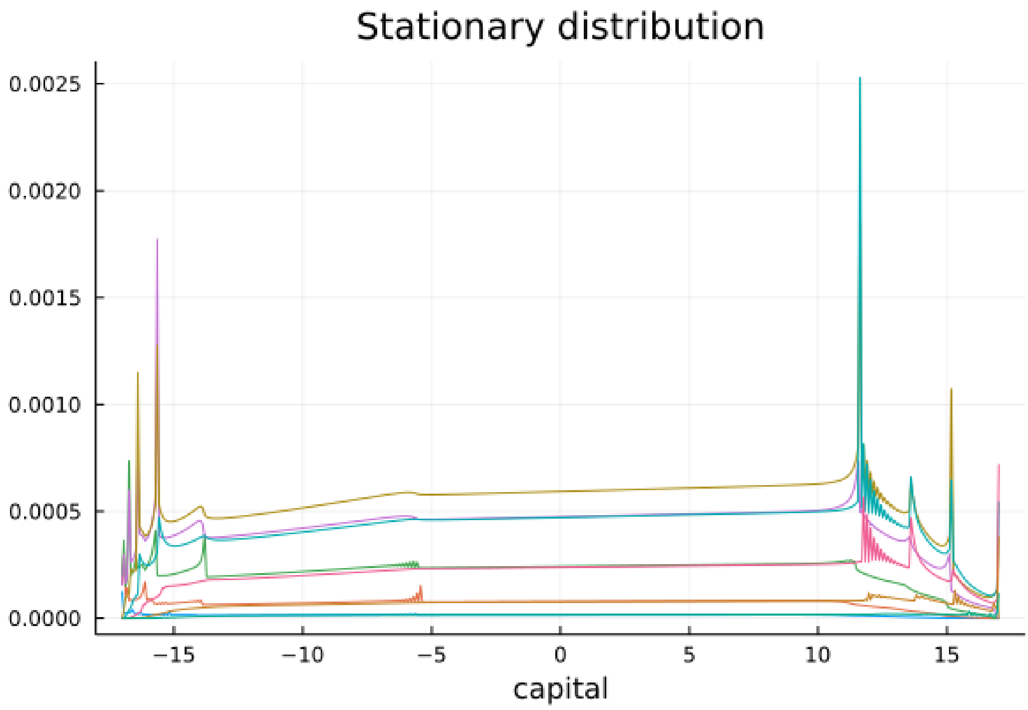

Figure 26: Stationary distribution (r* = 4.0, $\rho = 0.9, \sigma = 0.05$)

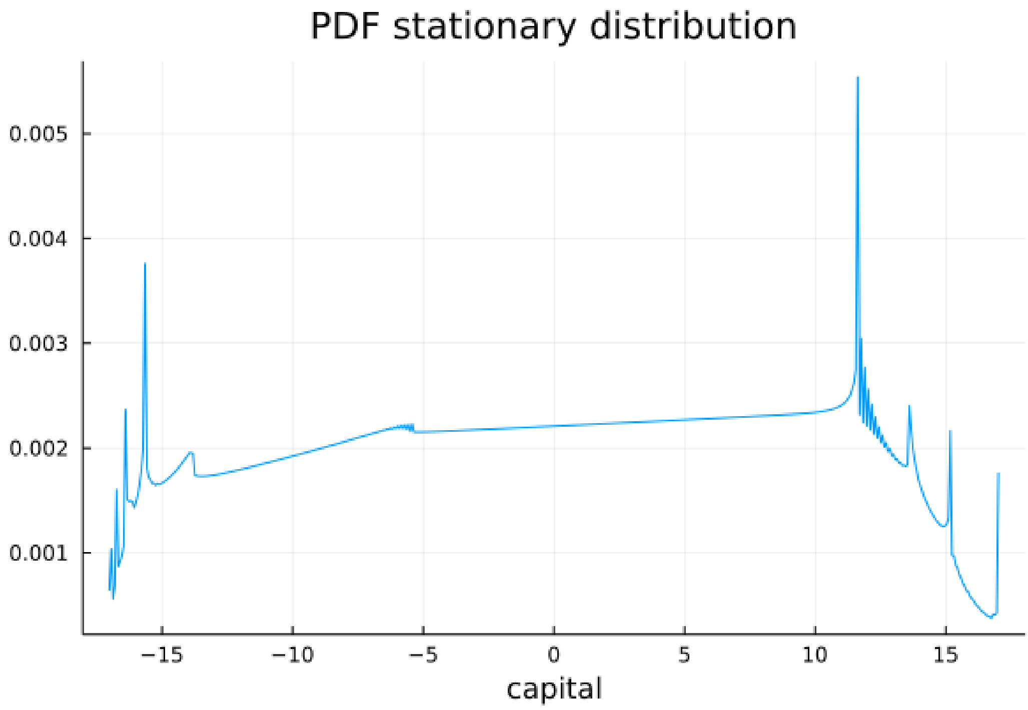

Figure 27: pdf stationary distribution ($r^* = 4.0,\ \rho = 0.9, \sigma = 0.05$)

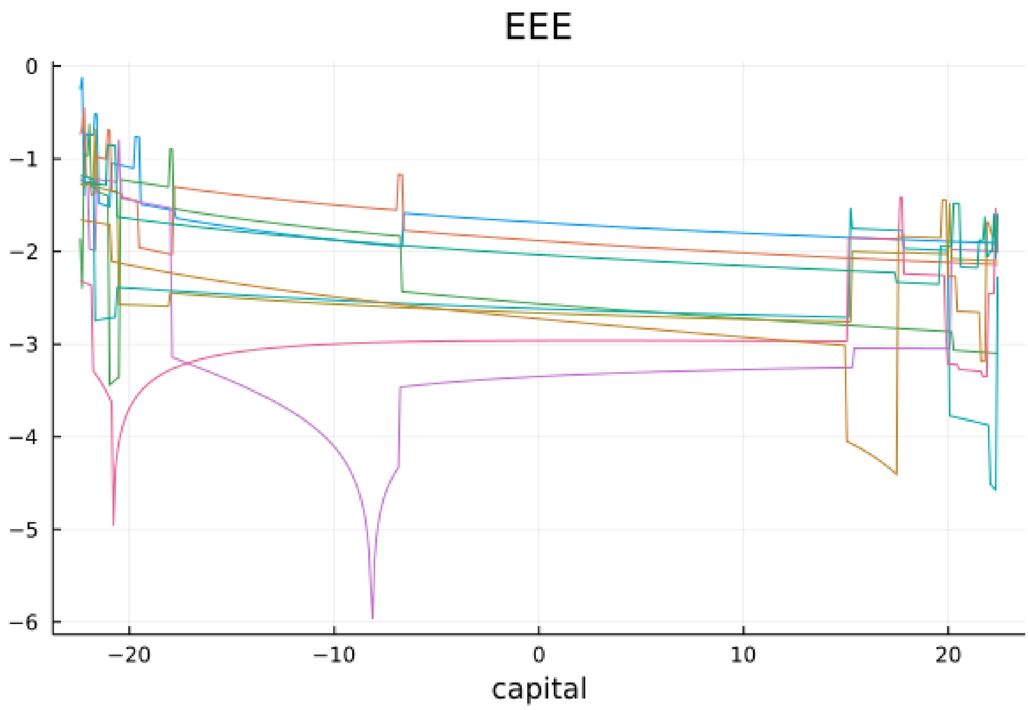

Figure 28: Euler Equation Errors ($r^* = 4.0,\ \rho = 0.9, \sigma = 0.05$)

g)

In the table we can see the same as in the other items. For a bigger risk aversion parameter we have a smaller rate so the savings market can clear. For a larger persistence parameter the same happens, as described in item *d*). The last parameter in analysis is the volatility of the shock, as in our model, a higher volatility implies a smaller rate. We do not have the same ratio as in the table, because for the Hugget model the aggregate saving equals zero.