Regulation and Service Provision in Dynamic Oligopoly: Evidence from Mobile Telecommunications

João Granja*

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Abstract

I study the effects of universal service regulation in oligopolistic industries with sunk investment costs, focusing on the Brazilian mobile telecommunications market. To quantify the impact of regulation on service availability, market structure and the speed and cost of roll-out of new technologies, I develop a dynamic game of entry and technology upgrade under regulation and estimate it using new panel data on mobile technology availability in Brazilian municipalities. In counterfactual simulations, I find that the regulation accelerated the introduction of 3G technology by 0.7 years, on average, and reduced firms' profits by 20.04%. Though the regulation may act as a commitment device and deter entry, I find those effects to be small. I show that an alternative subsidization policy similarly accelerates the roll-out of 3G and leads to substantially higher aggregate profits, likely increasing aggregate welfare.

*University College London, Department of Economics, joao.granja@ucl.ac.uk I thank Aviv Nevo, Katja Seim, and Mike Abito for their guidance and support throughout this project. I also thank Gorkem Bostanci, Nick Buchholz, Juan Camilo Castillo, Federica De Stefano, Kathleen Hui, Tomas Larroucau, John Lazarev, Philipp Schmidt-Dengler, Gabrielle Vasey, Sergio Villalvazo, Andrei Zeleneev, and seminar participants at various institutions for suggestions that have substantially improved this paper.

1 Introduction

Firms' inability to appropriate the consumer surplus generated by their new goods and services may lead to underprovision. This possibility is particularly salient in industries featuring large fixed costs and in disadvantaged areas, where the prospects of recouping these costs are dim. Concerns regarding service underprovision have led to regulatory oversight and intervention in many industries, such as postal service, healthcare, airlines, and telecommunications.[1] The purpose of this paper is to evaluate the effects of these interventions and compare their desirability relative to alternative policies.

Concerns of service underprovision have historically been particularly salient in the telecommunications industry (Wu (2010)) because of the substantial investment costs required for network expansion and the many benefits associated with access to telecommunications services.[2] Countries ranging from Nigeria to the United States regulate the roll-out of new mobile telecommunications technologies to ensure their diffusion to low-income, rural, or isolated areas. A common regulatory tool is called coverage requirements. A coverage requirement tasks a single firm (the *regulated firm*) with providing service of a specific technology in a given area by a date set by the regulator, while imposing no constraints on the behavior of its competitors (the *unregulated firms*).[3]

When deciding whether to impose such a requirement, the regulator faces the following trade-off. On the one hand, the regulation presumably generates service in areas that would not be served in its absence and accelerates the introduction of new technologies in other areas, thus increasing the discounted stream of consumer surplus. On the other hand, coverage requirements impose a cost on the regulated firm, for it is required to enter a market or upgrade its technology when it might not have done so in the absence of regulation. Furthermore, a coverage requirement is a credible commitment by the regulated

[1]For example, in the United States the USPS is subject to a Universal Service Obligation; the HRSA runs the Medicare Rural Hospital Flexibility Program; the DOT runs the Essential Air Service and Small Community Air Service Development Program, and The Universal Service Administrative Company spends almost ten billion dollars annually in subsidies for high-speed broadband access (see Q8 here – last accessed May 21, 2021.).

[2]Telecommunications services have been shown to have positive effects on economic growth (Roller and Waverman (2001), Czernich, Falck, Kretschmer, and Woessmann (2011)); labor productivity (Bertschek and Niebel (2016), Akerman, Gaarder, and Mogstad (2015)); market efficiency (Jensen (2007)), and risk-sharing (Jack and Suri (2014)). See Aker and Mbiti (2010) and Hjort and Tian (2021) for reviews of this literature.

[3]This is the implementation of coverage requirements in my empirical setting. Another common implementation is for firms to be obliged to provide service to at least some fraction of the territory covered by their license.

firm to provide service of a relatively advanced technology. This commitment may deter entry by its competitors. This, in turn, may incentivize the regulated firm to delay its own introduction of the new technology to benefit from decreasing costs of adoption. These equilibrium effects may diminish or even reverse the intended effect of the regulation.

To quantify the effects of coverage requirements and alternative policies on service availability and the speed of technology roll-out, I develop and estimate an empirical dynamic game of entry and technology upgrade under regulation. A model of entry and technology upgrade in the mobile telecommunications industry must allow for changes over time in consumers' preferences for and the cost of introducing new technologies. Therefore, I model firms' flow profits as a time-varying function of market structure and local demographics. Similarly, technology introduction costs vary over time and depend on local market characteristics. Moreover, in the model, as in the data, in each market exactly one firm is required to provide 3G service by a date set exogenously by the regulator. I model enforcement by assuming that a noncompliant regulated firm pays a fine every period following the regulation deadline.

The time-varying nature of variable profits and technology adoption costs and the regulation deadline make the environment non-stationary. Existing empirical models of technology adoption make assumptions such as a finite horizon that allow the application of backward induction solution algorithms. I instead assume that structural parameters stabilize before the end of the sample and focus on what I call Quasi-Stationary Markov Perfect Equilibria (QMPE). Essentially, QMPE have a non-stationary phase followed by a stationary phase. This structure allows me to adapt existing estimation techniques used in stationary dynamic games to a non-stationary setting. I also introduce a novel model of flow profits that is estimatable with data on market shares and expenditures but not prices. Moreover, I introduce assumptions that allow quantity data at different levels of geographic granularity to be combined in a GMM estimator that adapts the techniques of S. Berry (1994) and S. Berry, Levinsohn, and Pakes (1995).

I estimate the dynamic parameters of the model using new panel data on mobile technology availability at the municipality level in Brazil from June 2013 to June 2021, focusing on a set of relatively small municipalities. These data show that regulated firms are more likely to enter a market and upgrade their technologies than unregulated firms. Moreover, the latter are less likely to enter a market or upgrade their technologies when the regulated firm is yet to satisfy its coverage requirement, a pattern consistent with the entry deterrence effect outlined above.

I find that the profits and costs associated with 3G are fairly stable over the sample period. The profits associated with 4G rise sharply, and the costs of 4G introduction decrease substantially over time. The cost of non-compliance with the regulation is not directly observed, but it is identified from differences in behavior between regulated and unregulated firms. I estimate it to be sizable: it amounts to 28.35% of the median entry cost.

Counterfactual exercises show that in the absence of regulation, the arrival of 3G technology to relatively small and underdeveloped markets in Brazil would have happened 0.67 years later, on average – and more than 2 years later in some cases. The regulation reduces firms' aggregate expected profits by 321 million BRL, or 20.04% of the profits they obtain in the absence of regulation. I find the entry deterrence effects to be quantitatively small.

I also evaluate alternative policy interventions. I find that a budget neutral subsidy paid to the first firm to introduce 3G technology similarly accelerates its roll-out. Moreover, firms benefit substantially from the subsidy: their aggregate profits increase by 126 million BRL, or 9.83% of their earnings under coverage requirements, after accounting for the financing of the subsidy. These gains stem primarily from a more cost-efficient pattern of technology adoption. The subsidy typically leads an incumbent to introduce the new technology, whereas coverage requirements are imposed on potential entrants in many cases. Incumbents only incur technology installation costs, whereas potential entrants also incur entry costs, which I estimate to be sizable. These gains, however, come at the expense of reduced competition in the market. Nevertheless, I estimate that one more firm in the market has to generate a gain in consumer surplus that exceeds 40% of consumers' average expenditures for coverage requirements to be preferred to the subsidy. These results suggest that subsidization is a more efficient policy than the current form of regulation.

This paper relates to the literature studying how regulation affects market structure and market outcomes in dynamic environments. Ryan (2012) shows that stricter environmental regulation increases entry costs, thus decreasing both the number of firms in the market and consumer surplus. Gowrisankaran, Lucarelli, Schmidt-Dengler, and Town (2011) study the effect of the Medicare Rural Hospital Flexibility Program on health care provision in rural America, and show that the program expanded coverage but had a net adverse effect on consumer welfare due to provisions that limited the size and scope of regulated hospitals. Dunne, Klimek, Roberts, and Xu (2013) study the effects of entry subsidies under the Health Professional Shortage Areas program on local market structure. Most related to this paper, Fan and Xiao (2015) estimate a model of telephone service provider entry into local markets in the US and use the model to evaluate the extent to which different entry subsidies would have reduced monopolization. I extend this literature by modeling not only entry but also the set of products firms offer and by studying the equilibrium effects of asymmetric regulation on market outcomes.

This paper also relates to the empirical literature on technology adoption. Schmidt-Dengler (2006) studies US hospitals' decisions to adopt magnetic resonance imaging (MRI). Igami (2017) studies how cannibalization, preemption, and incumbents' cost advantages shape firms' adoption of new generation hard disk drives. My paper adds to this literature by studying how regulation affects technology adoption. My work also differs from these papers methodologically. Models of technology adoption must allow for time-varying demand and adoption costs. The aforementioned papers apply full solution estimation methods based on backward induction algorithms, feasible in these settings due to a finite horizon assumption (Igami (2017)) or full adoption in finite time (Schmidt-Dengler (2006)). I instead model technology adoption as happening in an infinite horizon and assume that the game has a non-stationary part followed by a stationary part. This allows me to adapt existing iterative estimation methods to this non-stationary setting.

My work also relates to the literature on regulation in telecommunications markets. Björkegren (2019) studied consumer adoption of mobile phones in Rwanda, and in that context evaluated the welfare effect of rural coverage requirements imposed on the dominant mobile network operator. I add to this work by modeling how firms respond to regulation, and moreover by doing so in an oligopoly context. My work also relates to an earlier, mostly theoretical, literature on universal service obligations, such as Armstrong (2001), Choné, Flochel, and Perrot (2002), and Valletti, Hoernig, and Barros (2002). This paper is the first to empirically quantify the effect of such regulation on service provision and the introduction of new technologies. The focus on regulation also distinguishes this paper from recent research that analyzes infrastructure investment by mobile network operators, namely Marcoux (2022) and Lin, Tang, and Xiao (2021).

This paper builds on a long literature on applied dynamic games, going back to Ericson and Pakes (1995). The model I present below is a dynamic game with discrete controls. A number of estimators have been proposed for stationary dynamic games with discrete controls, e.g., Aguirregabiria and Mira (2007), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008). I show that with a cross-section of markets and the notion of Quasi-Stationary Markov Perfect Equilibria, these estimators can be applied to non-stationary settings. Because I am interested in how different markets are affected by regulation, I allow equilibria to depend flexibly on market-level observed heterogeneity. I invoke a standard identification argument for (linear) dynamic games, which goes through if equilibrium selection is continuous, a condition for which I provide supporting evidence.

2 Institutional Setting and Data

Operators of mobile telecommunications networks transmit data through the radio frequency spectrum, a public resource subject to government management in most countries. Spectrum is typically allocated to firms via spectrum license auctions. These licenses typically come with a number of conditions, chief among them the coverage requirements that are the focus of this paper. In Brazil, the first spectrum auction happened in 2007 and since then firms have been suject to coverage requirements. For the purpose of this paper, a coverage requirement is an imposition that a firm provide service in a municipality by a deadline set by the regulator and with a minimum technological requirement (3G or 4G).[4]

The Brazilian mobile telecommunications market is characterized by 7 mobile network operators (MNOs), i.e., carriers that operate their own network infrastructure. Of the 7 MNOs, four provide service in the entire country and have held licenses covering the entire territory since the introduction of mobile telecommunications in the country. The other three MNOs provide more localized service. There is also a handful of very small mobile virtual network operators (MVNOs), which are carriers that do not own infrastructure and instead rent network space from the MNOs. There has been no entry or exit in this market in the past twenty years.

My analysis focuses on municipalities with less than 30,000 inhabitants. The coverage requirements targeting these municipalities are the most likely to in-

[4]The requirement is considered satisfied if service is available in 80% of the municipality's territory.

fluence the availability of service.[5] In these markets, a single firm was required to introduce 3G technology with varying deadlines. All the four large MNOs are subject to a coverage requirement in some of these markets, but not all. Though these coverage requirements target the introduction of 3G, the regulated firm is considered to comply with the regulation if it deploys 4G instead.

The motivation for coverage requirements rests on two premisses. First, mobile telecommunications services generate substantial welfare gains.[6] In the words of the Brazilian telecom regulator:[7]

> [Mobile telecommunications technologies] *create employment opportunities, improve the education system, increase firm productivity, allow access to public digital services, among other benefits.*

Second, for the intervention to be justified it must be that firms do not internalize the entirety of the surplus generated by their entry and new technologies. This seems likely, given how multifaceted these benefits are and firms' limited ability to price discriminate.[8]

The regulator enforces coverage requirements in multiple ways. First, carriers are required to deposit financial guarantees with the regulator, which can be executed in case of noncompliance. Perhaps more importantly, a noncompliant carrier can have its license revoked. In this case, the carrier would also be charged the value paid for its license in proportion to the time used. The regulator can also impose fines on noncompliant carriers.[9]

The main dataset used in this study comes from ANATEL, the Brazilian telecommunications regulator. The data records the technologies (2G, 3G, and 4G) offered by each mobile network operator in all the 5,770 Brazilian municipalities at a monthly frequency. Figure 16 in appendix B illustrates the structure of the data. The second important piece of data coming from ANATEL is the

[5] It is likely that the coverage requirements targeting larger municipalities affect the number of firms in the market, but not the availability of service, which is the primary focus of this paper.

[6] See, e.g., the references in footnote 2.

[7] See, `https://www.anatel.gov.br/setorregulado/telefonia-movel` (last accessed in October 22, 2020).

[8] Recent empirical results in related markets lend support to this hypothesis: using data on the French mobile telecommunications market, Elliott, Houngbonon, Ivaldi, and Scott (2021) estimate that the marginal social value of spectrum is five times firms' willingness to pay for it; studying residential broadband, Nevo, Turner, and Williams (2016) estimate a large gap between social and private incentives to invest in infrastructure.

[9] The final important piece of institutional detail is the process by which the identity of the regulated carrier in each market is determined. I discuss that in appendix C.

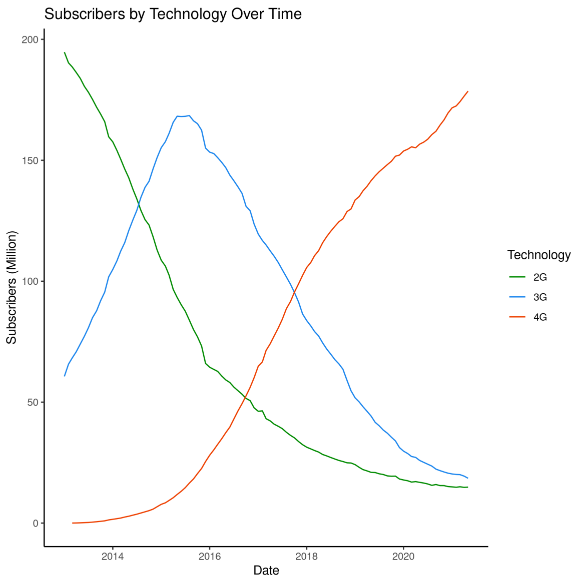

Figure 1: Total number of subscribers in the country, by technology.

Calculated from ANATEL's data on subscription to mobile telecommunications services.

identity of the regulated firm in each municipality and the associated regulation deadline.

ANATEL also provides data on the numbers of subscribers to mobile telecommunications services. These data are available for each carrier-technology combination at a monthly frequency, first at the code-area level from February 2005 to December 2018 and then at the municipality level from January 2019 to May 2021.[10] Figure 1 shows the total number of subscribers in the country by technology from January 2013 to May 2021. The figure shows that 2G has been in decline over the period, initially being overtaken by 3G. Moreover, 3G reaches a peak towards the end of 2015, around the time when the growth of 4G accelerates. To the extent that these patterns are driven by consumer preferences, they shape firms' incentives to introduce new technologies. The empirical model introduced below will account for this pattern in demand by allowing demand-side parameters to vary over time.

I complement the ANATEL data with a number of datasets from IBGE, the Brazilian Census Bureau. First, I utilize municipality demographics and characteristics, such as population, GDP per capita, and area. Summary statistics on these variables are shown in table 1. Second, I use the 2017-2018 Family Budget Survey, which provides information on household income, size and expen-

[10]Code-areas are much coarser than municipalities. There are 67 code-areas in Brazil.

diture on mobile telecommunications services.[11] Summary statistics on these data are provided in table 2. Third, I use the 2010 Population Census to obtain the distribution of individual-level demographics at the municipality level.

Table 1: Summary Statistics – Municipality Characteristics

Variable	N	Mean	Std. Dev.	p10	p90
GDP Per Capita	3,449	16,221.34	20,357.61	5,440.13	30,668.74
Population	3,449	10,907.84	7,564.48	2,917.02	22,655.82
Area	3,449	1,235.85	3,963.64	110.60	2,370.71

Values are averaged over time. GDP per capita is deflated to 2013 BRL. Area is in squared kilometers.

Table 2: Summary Statistics – Mobile Expenses and HH Characteristics

Variable	N	Mean	Std. Dev.	p10	p90
Mobile Spending	80,921	26.64	36.51	7.44	48.39
HH Income PC	80,921	2,159.54	3,962.78	557.54	4,050.98
No. Residents	80,921	2.20	1.04	1.00	4.00
Urban	80,921	0.81	0.39	0.00	1.00

Data from the 2017-2018 Family Budget Survey. The unit of observation is an individual. Mobile spending is the sum of expenditures on voice and data plans, pre-paid expenditure, and SIM cards, in BRL (deflated to 2013). "HH Income PC" is the monthly per capita income in the household. "No. Residents" is the number of residents in the household. "Urban" is a dummy that is equal to 1 if the individual lives in an urban area.

I drop code-areas where any of the three smaller carriers had a market share of at least 5% at any point in time and then focus on the four major carriers. Moreover, as mentioned above, I focus on municipalities with less than 30,000 inhabitants (in 2006). The resulting estimation sample contains 3,449 municipalities. For counterfactual exercises I focus on the subset of municipalities where the regulation deadline was December 2019. The technology availability data starts in early 2013 and these requirements were imposed in July 2012; requirements in other municipalities were imposed a few years earlier. Therefore, it is only reasonable to consider counterfactual scenarios for the December 2019 group of markets. Finally, because entering a market or upgrading a technology is a non-trivial investment that likely involves some time to build, I use data on a semester frequency rather than monthly. The unit of observation is thus a municipality-carrier-semester.

[11] This survey is the *Pesquisa de Orçamentos Familiares* (POF).

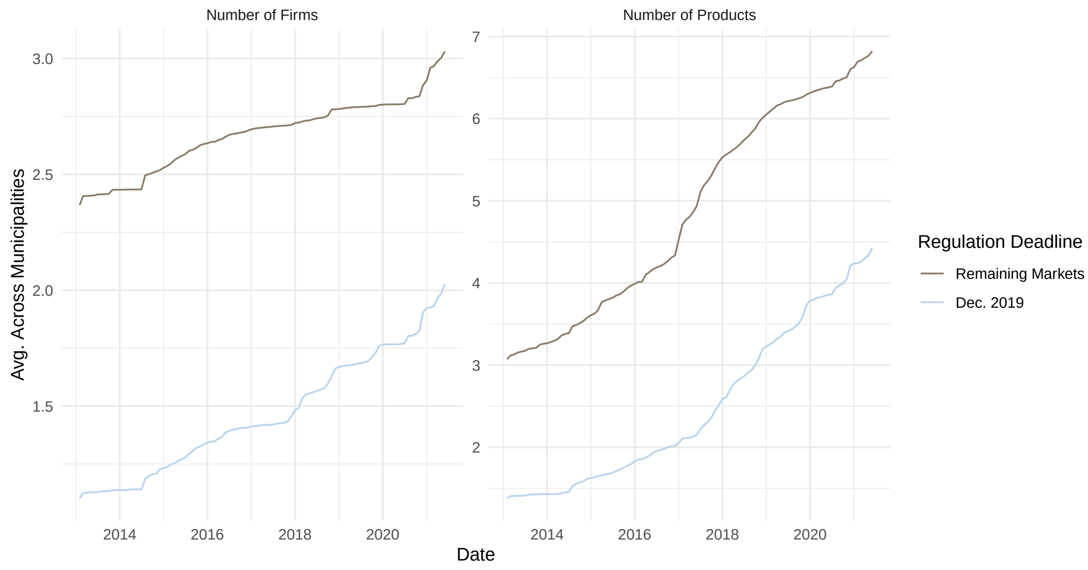

Figure 2: Average numbers of firms and products (firm-technology pairs) over time.

Figures 2 and 3 summarise the technology deployment data. Figure 2 shows, on the left panel, the average number of firms in a market and, on the right panel, the average number of products, where a product is a carrier-technology combination and averages are taken across municipalities. Municipalities are split into two groups: those with a coverage requirement with a December 2019 deadline and all the other ones.[12] The figure shows an increase in the numbers of firms and products, at a slightly faster pace for the December 2019 markets. There is also a clear difference in levels for the two groups.

Figure 3 shows technology availability over time. The figure in the top left shows the fraction of markets where at least 3G is available. The one on the top right show the fraction of markets where 4G is available. The figure on the bottom left shows the fraction of markets where the regulated firm has complied with the regulation. Finally, the figure on the bottom right shows the fraction of markets where some unregulated firm provides 3G technology or better. The availability of 3G technology or better in the December 2019 group grows from just over 25% of markets to 100% over the sample period. The availability of 4G grows from zero to about 96%. The bottom panel shows that regulated firms are more likely to provide advanced technologies since the beginning of the sample and also introduce these technologies at a faster pace than unregulated

[12]This includes cases in which the regulated firm is subject to an earlier deadline – the vast majority of cases – and cases in which the regulated firm is not one of the four large carriers.

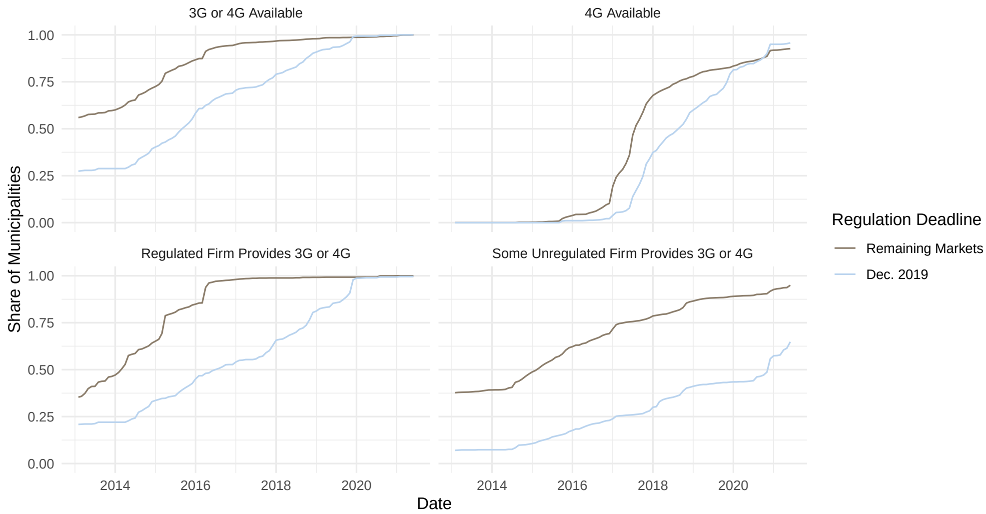

Figure 3: Technology availability over time.

firms.

Figure 3 suggests an important role for coverage requirements in explaining the diffusion of new mobile telecommunications technologies. The difference in behavior between regulated and unregulated firms is potentially composed of two different effects of coverage requirements: a positive effect on regulated firms and a negative effect on unregulated firms. Unregulated firms may be less likely to enter new markets or upgrade their technologies because they know that the regulated firm will introduce 3G by the requirement deadline. This knowledge of tougher competition in the future may reduce the incentives of unregulated firms to invest. See appendix A for a theoretical example, based on Fudenberg and Tirole (1985), in which this mechanism implies that regulation leads to a delay in technology introduction.

The data allow further investigation of these potential effects. In table 3 I report linear probability models of firms' technology upgrade decisions. The dependent variable is a dummy equal to one if and only if firms choose to upgrade their technology (or enter the market if they are not already active). The key explanatory variables are the dummies "Regulated", "Regulated Competitor - Out", and "Regulated Competitor - 2G". The first of these variables is equal to 1 when the firm is regulated, and 0 otherwise. The second is equal to 1 when the firm faces a regulated competitor that is out of the market. The third is equal to 1 when the firm faces a regulated competitor that has 2G technology. The omitted category includes the cases when the regulated firm has complied and when the regulated firm is not one of the four large carriers. [13] The models also control for the logarithms of GDP per capita, population, and area, and include the number of competitors with each technology.[14] Moreover, to account for unobserved municipality-level heterogeneity, these models also include group fixed effects, where the groups are defined by a heuristic pre-estimation step.[15] The columns correspond to different subsamples according to firms' best technologies (out of the market, 2G, or 3G).

There are two key results in Table 3. First, regulated firms that have not satisfied their coverage requirements are more likely to enter the market and upgrade their technologies than firms that are not subject to regulation. Second, unregulated firms are less likely to enter and upgrade their technologies when the regulated competitor is either out of the market or has 2G technology. These results show that the regulation indeed accelerates the introduction of the new technology by regulated firms, but also that it delays the introduction of new technologies by unregulated firms. This is consistent with the entry and technology upgrade deterrence effects outlined above. Given these two counteracting effects, it is a priori unclear whether the regulation accelerates the introduction of new technologies.

The rest of the paper is concerned with developing tools that allows us to quantify the net effect of regulation on the time to introduction of new mobile telecommunications technologies, as well as the entry deterrence effects alluded to above and the costs that the regulation imposes on firms. This requires a model of entry and upgrade decisions.

[13] Because I restrict the sample to regions where the small firms have always had negligible market shares, I interpret both of these cases as no firm being influenced by regulation.

[14] It may also be expected that a firm's infrastructure in neighboring municipalities is important for their choices. I test for that in appendix B. I do find that having service in a neighboring municipality increases the probability of entry and technology upgrade. However, the other coefficients change only slightly, if at all. This suggests that the choice of the regulated firm is uncorrelated with their local network infrastructure. Indeed, in appendix B, I show that service in neighboring municipalities does not increase the probability that a firm is regulated in a given market. Firms' presence in neighboring municipalities will not be included as a state variable in the structural model, as doing so would increase the computational burden significantly. The descriptive results discussed here, however, suggest that this omission will not bias the inference regarding the effect of the regulation.

[15] Specifically, I first run a regression of the number of products on municipality and semester fixed effects. I then project the municipality fixed effects onto (averages over time) of GDP per capita, population, and area. Municipalities are grouped according to quintiles of the residuals of the latter regression. Appendix B shows the results obtained estimating the model in Table 3 without the group fixed effects. The coefficients on the number of competitors are affected the most by the group fixed effects. The other coefficients change only slightly.

Table 3: Entry/Upgrade Models

	Out	2G	3G
Log GDP PC	0.013	0.014	0.021
	(0.001)	(0.003)	(0.002)
Log Pop.	0.026	0.049	0.000
	(0.001)	(0.003)	(0.003)
Log Area	−0.006	−0.014	0.001
	(0.001)	(0.001)	(0.001)
Regulated	0.102	0.150	−0.034
	(0.003)	(0.004)	(0.003)
Regulated Competitor - Out	−0.017	−0.006	−0.035
	(0.002)	(0.006)	(0.009)
Regulated Competitor - 2G	−0.006	−0.051	−0.091
	(0.002)	(0.005)	(0.007)
No. Competitors 2G	−0.012	−0.007	−0.011
	(0.001)	(0.003)	(0.002)
No. Competitors 3G	−0.021	−0.013	−0.002
	(0.001)	(0.003)	(0.003)
No. Competitors 4G	−0.008	−0.027	0.002
	(0.001)	(0.003)	(0.003)
Group FE	Yes	Yes	Yes
$\bar{Y}$	0.026	0.079	0.083
Num. obs.	92088	47074	49245

Linear probability models. The dependent variable is a dummy equal to 1 if a technology upgrade is observed. The explanatory variables are, in this order: the natural logarithms of GDP per capita, population, and municipality area, a dummy that is equal to 1 if the firm is regulated, a dummy that is equal to 1 if the firm faces a regulated competitor that is out of the market, a dummy that is equal to 1 if the firm faces a regulated competitor that has 2G technology, and the numbers of competitors with 2G, 3G and 4G technology. Each column corresponds to the subsample of the data where firms' best technology is as indicated in the column heading.

3 Model

The model operates at the level of a municipality. Firms' flow profits depend on their own technologies, their competitors' technologies, and the local distribution of consumers' demographic characteristics. Inactive firms make entry decisions, and both entrants and incumbents choose what technologies to offer in the market; firms incur sunk costs of entry and technology upgrade. In each market a single firm is required to provide 3G technology by an exogenously specified deadline. If it fails to do so, it pays a fine every period, until it does introduce 3G.

Each market has four potential firms. The available technologies are 2G, 3G, and 4G. I assume that firms offer every technology less advanced than their best technology.[16] Time is discrete and the horizon is infinite. Within a period, the timing of the game is as follows. In the beginning of each period t incumbent firms earn their flow profits. Each firm then privately observes action-specific cost shocks, and firms make simultaneous choices. Potential entrants can enter with any technology and incumbents can choose to upgrade to any technology that is more advanced than their current technology. After choosing an action, firms pay the associated costs. Firms start period $t + 1$ with the technologies chosen in period t.

Let s_{fmt} denote firm f's technology in market m and period t: $s_{fmt} \in \mathcal{S} := \{0, 2, 3, 4\}$, where $s_{fmt} = 0$ denotes that firm f is out of the market and the other values correspond to each of the available technologies (2G, 3G, and 4G). The market's *technological state* $s_{mt} \in \mathcal{S}^4$ is a vector recording each firm's technology. Firms' flow profits are given by a time-varying function of the market's technological state s and the distribution H_x^m of demographics x in market m: $\pi_t(s, H_x^m)$. The specification of π_t is deferred to subsection 3.3.

Entry and technology upgrade are costly. The modeling of these costs reflects the fact that entry requires the installation of passive infrastructure, i.e., cell phone towers. Moreover, service provision requires the installation of technology-specific active infrastructure, the radios or transmitters. The specification of technology upgrade costs also reflects the fact that the costs of introducing new technologies fall over time.[17] Firms know the dynamics of flow profits and en-

[16] This assumption is broadly consistent with the data. Carriers typically keep old technologies in place as a fallback option. This assumption also reduces the dimension of the state space considerably, making the model computationally tractable.

[17] The cost of installing 2G is not allowed to vary over time, reflecting the fact that 2G is an old technology at the beginning of my sample period.

try and technology upgrade costs.

Formally, costs are modeled as

$$c_t\left(a, s_f, z_m, \varepsilon\right) = \begin{cases} -\varepsilon(a) & \text{if } a = s_f \\ \sum_{\{g':g'>s_f\}}^{a} z_m'\theta_{g',t} + \mathbf{1}\left(s_f = 0\right) z_m'\theta_e - \varepsilon(a) & \text{if } a > s_f \end{cases} \tag{1}$$

In equation 1, $a \in \{s_{fmt}, \ldots, 4\}$ is the action chosen by the firm, s_f is firm f's state, and z_m is a vector of observed market characteristics. The term $\varepsilon(a)$ is an action-specific cost shock, ε is a vector collecting all the $\varepsilon(a)$, and the θ's are parameters to be estimated. If $a = s_{fmt}$, the firm pays no costs (other than receiving the cost shock). A potential entrant that decides to enter pays an entry cost $z_m'\theta_e$. Moreover, associated with every technology g there are installation costs $z_m'\theta_{g,t}$. The summation in equation (1) reflects the previous assumption that firms offer all technologies less advanced than their best technology. If, for example, a firm's current best technology is 2G, and that firm upgrades to 4G, equation (1) says that the firm will pay the costs of installing both 3G and 4G.[18] The cost shocks are assumed to be independent across firms, periods, and actions, and they follow a Type 1 Extreme Value distribution with scale parameter λ.

In each market m, exactly one firm is required to provide 3G service by a date T_m exogenously specified by the regulator. I will call that firm the *regulated* firm and the other firms the *unregulated* firms. A regulated firm that does not provide 3G technology or better by $T_m + 1$ pays a fine φ and does so every period until it complies with the regulation.

Firms choose their actions to maximize their discounted expected profits, taking their competitors' behavior as given. I focus on Markov Perfect Equilibria (MPE), as is common in empirical applications of dynamic games. I allow regulated and unregulated firms to behave differently, but beyond that I impose symmetry.

3.1 Symmetric Markov Perfect Equilibria

A Markov Perfect Equilibrium is a strategy profile $(\sigma_1, \ldots, \sigma_4)$, such that σ_i is a function that maps a firm's state variables into a feasible action and maximizes firm i's expected discounted profits given the behavior of its competitors. In

[18] This implies that an entering firm will always offer 2G. Because the cost of installing 2G is only paid by an entering firm, θ_e and θ_{2G} are not separately identified. Therefore, in estimation I drop θ_{2G}. The estimate of θ_e thus includes both entry costs and 2G installation costs.

a symmetric Markov Perfect Equilibrium, strategies do not depend on firms' identities. Instead, I define value and policy functions for regulated and unregulated firms. To simplify the notation, I subsume all the market-specific variables that do not vary over time in a superscript. The state of an unregulated firm f is $(s_f, s_r, s_-, t, \varepsilon)$, where s_f is that firm's technology, s_r is the technology of the regulated firm, and s_- is a vector with the technologies of the other two firms. The state of a regulated firm f is $(s_f, s_-, t, \varepsilon)$ where now s_- denotes the technologies of the three remaining firms. Let Ω_0, Ω_1 denote the state space for unregulated and regulated firms, respectively, with typical element $\omega_r, r \in \{0, 1\}$. A strategy is a function $\sigma_r : \Omega_r \to \{0, 2, 3, 4\}$ satisfying the restriction that $\sigma_r(\omega_r) \in A(s_1(\omega_r)) := \{s_1(\omega_r), \dots, 4\}$, where $s_1(\omega_r)$ is the first coordinate of ω_r, i.e., the firm's current technology.[19]

Let $\sigma^m = (\sigma_0^m, \sigma_1^m)$ be a symmetric strategy profile. Define the implied ex-ante value function

$$V_{r,\sigma^m}^m(s,t) := \mathbb{E}_\varepsilon \Big\{ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \Big[\pi_\tau^m(s_{f\tau}, s_{-f,\tau}) - c_\tau^m(a_{f\tau}, s_{f\tau}) + \\ + \varepsilon_{f\tau}(a_\tau) - \varphi r \mathbf{1}(T_m < \tau, s_{f\tau} < 3) \Big] \Big| r, s, t; \sigma^m \Big\}$$

where m indicates the market, $r \in \{0, 1\}$ is equal to 1 if the firm is regulated and zero otherwise, $\mathbb{E}_\varepsilon$ indicates that the expectation is taken over the sequence of ε's for all firms, and firms' states evolve according to (σ_0^m, σ_1^m). Symmetry implies restrictions on $\sigma_0, \sigma_1, V_0, V_1$. For details, see appendix D.

Finally, note that the recursive characterization of Markov Perfect Equilibria (e.g., Doraszelski and Escobar (2010)) implies that $\{\sigma_0^m, \sigma_1^m\}$ is a symmetric MPE if and only if

$$\sigma_r^m(s,t,\varepsilon) = \underset{a \in A(s_f)}{\operatorname{argmax}} \Big\{ \pi_t^m(s_{ft}, s_{-f,t}) - c_t^m(a, s_f) + \delta \mathbb{E}_{\varepsilon_{-f}} \big[V_{r,\sigma^m}^m \big(a, s'_{-f}, t+1\big) | r, s, t \big] + \varepsilon(a) \Big\} \tag{2}$$

where, for firms $h \neq f$, $s'_h = \sigma_{r_h}^m(s, t, \varepsilon_h)$ and the expectation is with respect to the shocks ε_h of firms $h \neq f$.

[19] In the absence of the distinction between regulated and unregulated firms and under symmetry, the state variables could be defined to be a firm's own technology and the numbers of competitors with each technology. Here, however, we must keep track of the technology of the regulated firm, as its incentives differ from those of unregulated firms. The symmetry restriction is then imposed via restrictions on policy and value functions.

3.2 Quasi-Stationary Markov Perfect Equilibria

The environment just introduced has two sources of non-stationarity. First, flow profits and entry and technology upgrade costs vary over time. Second, coverage requirements imply that firm behavior depends on the date. Suppose that the regulated firm has not satisfied its commitment and $t < T_m$; as time goes by, the regulated firm gets closer to being fined and therefore should become more likely to comply with the regulation. Unregulated firms respond to this change in behavior. Thus, conditional choice probabilities vary over time. Therefore, stationary Markov Perfect Equilibrium is not an appropriate solution concept. In this subsection, I introduce assumptions that respect these two sources of non-stationarity but beyond that impose that behavior is independent of the date.

I will assume that entry and technology upgrade costs and flow profits vary over time (in a way known to firms) but stabilize at a date known to the firms and the econometrician.[20] I then make two assumptions regarding equilibrium behavior. First, after parameters have stabilized and the regulated firm has complied with the regulation, behavior ceases to depend on the date. Second, the same holds if parameters have stabilized and the regulation deadline has passed. In the latter case, though firms still have to account for the presence of regulation, it affects the environment in a way that does not change over time.

Formally, I focus on *Quasi-stationary Symmetric Markov Perfect Equilibria*, defined below. Let T_θ denote the earliest time period such that flow profits and costs do not vary after T_θ.

Definition 1. A Symmetric Markov Perfect Equilibrium (σ_0, σ_1) is said to be *quasi-stationary* if there exist functions $\tilde{\sigma}_r(s, \varepsilon), r \in \{0, 1\}$, such that, if either

(i) $t \geq \max\{T_m + 1, T_\theta\}$, or

(ii) $t \geq T_\theta$ and $s_r \geq 3$,

then $\sigma_r(s, t, \varepsilon) = \tilde{\sigma}_r(s, \varepsilon)$.

I assume throughout that the data is generated by a Quasi-Stationary Symmetric Markov Perfect Equilibrium. Note that this imposes restrictions on value functions over time. Essentially, the model has a non-stationary phase followed by a stationary phase. Models of technology adoption must somehow contend with the fact that the demand for and costs of adopting a new technology vary

[20]Specifically, costs and flow profits vary yearly until 2018, after which they stabilize.

over time. One way of dealing with the time-varying nature of demand and costs is to assume a finite horizon and solve the game by backward induction; see, e.g., Igami (2017). That method raises the issue of assigning continuation values to different states in the final time period. In Igami (2017), that is done by assuming that the state of the industry does not change after the terminal period, and computing the implied discounted stream of profits. Quasi-stationarity instead assumes that firms will keep playing the entry and technology upgrade game forever, so that firms' continuation values are given by the equilibrium value function in the relevant states.

3.3 Modeling Flow Profits

It is not uncommon in applications of dynamic games for flow profits to be derived from an estimated demand system paired with an assumption on firms' pricing behavior. Following that route would require data on available mobile telecommunications bundles, their prices, and consumers' choices. Unfortunately, such data is not available in my setting. I thus follow a different approach that requires data on the quantities of subscribers to different technologies and consumers' expenditures. Suppose that consumer i in market m with demographic characteristics x_i chooses what carrier to subscribe to, what technology to use, and how much to spend on mobile telecommunications services, e_i. Let $\mu_{fgt}(s, H)$ be the resulting market share of firm-technology pair (f, g) in period t when the industry state is s and the distribution of demographics is H; a model for μ_{fgt} will be specified below. Let M be the size of the market and, as before, let s_f be firm f's state.[21] Finally, denote by $\mathbb{E}_t[e_i|g]$ the expectation of consumers' expenditures e_i, conditional on a consumer choosing technology g

[21] I set the market size to be twice the population of the municipality.

in period t.[22] Assuming zero marginal costs, firms' profits are given by[23]

$$\begin{aligned}\pi_t(s_f, s_{-f}, H) &= M \sum_{g \in s_f} \mu_{fgt}(s, H) \mathbb{E}_t[e_i|g] \\ &= M \sum_{g \in s_f} \mu_{fgt}(s, H) \int \mathbb{E}[e_i|g, x_i] dH_t(x_i|g) \end{aligned} \tag{3}$$

The summation over $g \in s_f$ is over all technologies offered by firm f: $\{g : 2 \leq g \leq s_f\}$. Note that in equation (3), the conditional distribution $H_t(x_i|g)$ is indexed by t. That is because consumer preferences over technologies are allowed to vary over time (as indicated by the t subscript in μ_{fgt}), so that the distribution of demographics conditional on technology choice also varies over time. In contrast, the conditional expectation of expenditures $\mathbb{E}[e_i|g, x_i]$ is assumed to be time-invariant.

I do not observe consumer expenditures together with their technology (and carrier) choices. I will therefore make the following assumption:

Assumption 1. $\mathbb{E}[e_i|g, x_i] = \mathbb{E}[e_i|x_i]$.

This assumption says that conditional on individual characteristics x_i, consumer expenditure is mean independent of the technology chosen by that consumer. This is, admittedly, a strong assumption. It would hold in a world in which consumers pay per usage (a popular model in Brazil) and technology doesn't affect usage. This assumption would fail if better technologies induce consumers to use more data. Assumption 1 would thus be untenable if we were dealing with users of high-bandwith applications. Because we are dealing with small municipalities in Brazil, the assumption is more palatable. Importantly, Assumption 1 does not imply that consumers that subscribe to different technologies spend (on average) the same amount, for individuals with different demographic characteristics are still allowed to sort into different technologies.

[22]Here I condition only on technology, and not on firm identity, because firms are assumed to be symmetric.

[23]The expression on the right hand side of 3 is an approximation. Firms' profits are equal to $\sum_{g \in s_f} \sum_{i \in fg} e_i$, where the second summation is over individuals i subscribing to firm-technology pair (f, g). This approximation holds in the sense that the difference between firms' profits and the right hand side of equation 3 is $O_p(\sqrt{M})$, whereas the included term is $O(M)$. This approximation is analogous to the (implicit) approximation to profit functions used routinely in empirical industrial organization.

Assumption 1 and equation 3 imply that

$$\pi_t(s,H) = M \sum_{g \in s_f} \mu_{fgt}(s,H) \int \mathbb{E}[e_i|x_i] dH_t(x_i|g) \tag{4}$$

I model $\mu_{fgt}(s,H)$ as arising from a nested logit model. Specifically, consumer i's utility of subscribing to firm-technology pair $j = (f,g)$ in market m and year τ is given by[24]

$$u_{ijm\tau} = \underbrace{\gamma_{r(m),p(\tau)} + \mu_{g(j),p(\tau)} + \beta_{g(j),p(\tau)} y_{m\tau} + \theta_{g(j),p(\tau)} d_{m\tau}}_{v_{g(j)m\tau}} + \xi_{jm\tau} + \zeta_{im\tau}(\rho) + (1-\rho)\varepsilon_{ijm\tau} \tag{5}$$

where $r(m)$ is the state of municiality m, $p(\tau) \in \{E, L\}$ groups periods into an early (2015 and earlier) and late (2016 onwards) phase, and $g(j)$ is the technology of the firm-technology pair j. Moreover, $y_{m\tau}$ is GDP per capita, and $d_{m\tau}$ is population density.[25] The term $\xi_{jm\tau}$ is an unobserved product characteristic, $\zeta_{im\tau}(\rho)$ is a disturbance common to all goods other than the outside good, and $\varepsilon_{ijm\tau}$ is a Type 1 Extreme Value shock. The parameter ρ is the nesting parameter, and $\zeta_{im\tau}(\rho)$ has the unique distribution such that $[\zeta_{im\tau}(\rho) + (1-\rho)\varepsilon_{ijm\tau}]$ has an extreme value distribution (see Cardell (1997)).

In equation (5), $\gamma_{r(m),p(\tau)}$ is a state-phase fixed effect meant to capture variation in the share of the outside good and $\mu_{g(j),p(\tau)}$ is a technology-phase fixed effect, which captures changes in technology popularity over time. The effect of income and population density on consumer preferences varies by technology and phase. For example, one might expect the effect of income on preferences for 4G relative to alternative technologies to decrease over time, as the technology becomes more diffused, handsets become more affordable and more consumers desire to join networks that require 4G technology.

The distributional assumptions above imply that market shares are given by

$$\mu_{jm\tau}(s, v_{m\tau}, \xi_{m\tau}) = \frac{e^{(v_{g(j)m\tau} + \xi_{jm\tau})/(1-\rho)}}{D} \times \frac{D^{1-\rho}}{1 + D^{1-\rho}} \tag{6}$$

where $v_{m,\tau}$ is a vector collecting the $v_{gm\tau}$, $\xi_{m\tau}$ is a vector similarly defined, and

[24] I specify equation 5 at the year level because the included demographics are observed with that frequency. A period in the dynamic game is mapped to its corresponding year and the model above is used to compute shares.

[25] Using y_i in equation 5 would add one more integration to the estimation routine and thus add to the computational cost. In what follows, when calculating $H_t(x|g)$, I treat the coefficient on $y_{m\tau}$ as the effect of an individual's income on her utility.

$D := \sum_{j \in s} e^{(v_{g(j)m\tau} + \xi_{jm\tau})/(1-\rho)}$, where the summation is over the products offered in the market. The predicted quantity of subscribers is $M\mu_{jm\tau}(s)$.

I assume that individual i's expenditure, e_i, is given by

$$\log(e_i) = \alpha_{r(i)u} + \alpha_1 \log(y_i) + \alpha_2 n_i + \eta_i \ , \tag{7}$$

where $r(i)$ indicates i's state of residence; u indicates whether the municipality is classified as urban or rural by the Census; y_i is income; n_i is household size; and η_i is an error term that is uncorrelated with the included regressors. The final ingredient needed to compute firms' profits in equation 4 is the distribution $H_t(x_i|g)$. I obtain that distribution using the technology choice model above and Census data on municipality-level demographics; for details, see section 4.

The final aspect of the model is an assumption regarding the distribution of $\xi_{jm\tau}$. I introduce this assumption to deal with the fact that I observe the quantities of subscribers at different levels of geographic granularity over time; see section 4 for details.

Assumption 2. Let $c(m)$ denote the code-area that municipality m belongs to. There exists a distribution F such that the unobserved product characteristic $\xi_{jm\tau}$ satisfies

$$\xi_{jm\tau} = \xi_{jc(m)\tau} + \eta_{jm\tau}$$

where $\eta_{jm\tau} \overset{iid}{\sim} F$ and $\mathbb{E}_F[\eta_{jmt}] = 0$.

Assumption 2 says that $\xi_{jm\tau}$ can be decomposed into a random variable that varies only across code-areas, on which I place no restrictions, and another random variable that varies across municipalities within a code-area, that I assume is iid with some unrestricted distribution F.

Let ω_m be the fraction of the population in area-code c in municipality m. Under Assumption 2, an argument relying on a large number of municipalities within a code-area c implies that

$$\mu_{jc\tau} = \sum_{m \in c} \omega_m \int \mu_{jm\tau}(s_{m\tau}, v_{m\tau}, \xi_{c(m)\tau} + \eta_{m\tau}; \theta) dF(\eta_{m\tau}) \tag{8}$$

holds approximately.[26] I will use equation (8) in estimation; see section 4.

[26]See appendix E for details.

4 Identification and Estimation

I start this section by discussing the estimation of the flow profit function in subsection 4.1. In subsection 4.2 I discuss the estimation of the dynamic parameters of the model, i.e., the entry and upgrade costs and the fine for non-compliance with the regulation.

4.1 Identification and Estimation of the Flow Profit Function

Computing profits requires three objects: $\mu_{fgt}(s, H)$, $\mathbb{E}[e_i|x_i]$, and $H_t(x_i|g)$ – see equation (4). The data on market shares for the 2013-18 period is at the code-area level and for the 2019-20 period at the municipality level. Using the municipality level data, I can proceed as follows. Equation (6) implies (see S. Berry (1994)):

$$\log(s_{jmt}) - \log(s_{0mt}) = v_{g(j)mt} + \rho \log(s_{j|\mathcal{J}_{mt}}) + \xi_{jmt} \tag{9}$$

where $s_{j|\mathcal{J}_{mt}}$ is the share of good j in the total number of subscriptions in the market. This equation yields ξ_{jmt} as a function of data and parameters, $\xi_{jmt}(\theta)$. The term $v_{g(j)mt}$ contains municipality characteristics that are assumed to be uncorrelated with ξ_{jmt}. Nevertheless, $\log(s_{j|\mathcal{J}_{mt}})$ is a function of ξ_{jmt}. I interact $\xi_{jmt}(\theta)$ with instruments Z^1_{jmt} to form moment conditions $\mathbb{E}[\xi_{jmt}(\theta)Z^1_{jmt}] = 0$.

The intuition for the identification of the nesting parameter ρ is similar to that in S. T. Berry and Waldfogel (1999). The nesting parameter determines the extent of business stealing. If we can exogenously vary the number of products in the market, we learn the value of ρ by observing the effect on the aggregate share of the inside goods. Following this intuition, I exploit coverage requirements themselves as a source of exogenous variation in the number of products in the market. The number of regulated firms in a municipality is a function only of population measured in 2006 and 2012, which are credibly uncorrelated with ξ_{jmt}. Therefore, I use the number of firms subject to 3G and 4G coverage requirements as instruments for $\log(s_{j|\mathcal{J}_{mt}})$.

The moments discussed above are informative about the nesting parameter and preference parameters in the later period of the data, but not in the earlier period of the data. To construct additional moments to identify those parameters, I leverage assumption 2 and equation (8). Equation (8), repeated here for convenience, states that market shares at the area-code level are approximately given by

$$\mu_{jct} = \sum_{m \in c} \omega_m \int \mu_{jmt}(s_{mt}, v_{mt}, \xi_{c(m),t} + \eta_{mt}; \theta) dF(\eta_{mt}) \tag{10}$$

Equating observed market shares at the area-code level with their predicted counterparts in equation 10, one can solve for ξ_{jct} as a function of data and parameters. These structural error terms, $\xi_{jct}(\theta)$, can then be interacted with instruments to form moment conditions $\mathbb{E}[\xi_{jct}(\theta) Z^2_{jct}] = 0$. The one hindrance to this approach is the integration with respect to $F(\eta_{jmt})$. Here, again, assumption 2 offers a solution. Given any vector of structural parameters, θ, equation (9) gives $\xi_{jmt}(\theta)$. We can then use assumption 2 to recover $\hat{\eta}_{jmt}(\theta)$, which gives us an empirical distribution of η_{jmt} given θ, $\hat{F}(\eta; \theta)$. The integration in equation (10) can be performed for any guess of θ by sampling from $\hat{F}(\eta; \theta)$.

To summarise the preceding discussion, the steps involved in evaluating the GMM objective function for a given value of θ are as follows. First, use equation (9) to obtain $\xi_{jmt}(\theta)$. Second, use assumption 2 to obtain $\eta_{jmt}(\theta)$. Third, solve for $\xi_{jct}(\theta)$ from

$$s_{jct} = \sum_{m \in c} \omega_m \frac{1}{N_s} \sum_{i=1}^{N_s} \mu_{jmt}(s_{mt}, v_{mt}, \xi_{c(m),t} + \eta_i; \theta) \tag{11}$$

where s_{jct} is the observed market share of firm-technology pair j in area-code c and period t, η_i is a vector of $|\mathcal{J}_{mt}|$ independent draws from $F(\eta; \theta)$ and N_s is the number of simulation draws. Fourth, interact ξ_{jmt} with Z^1_{jmt} and ξ_{jct} with Z^2_{jct} and average, to get sample analogs of the moment conditions discussed above, $\bar{g}^1(\theta)$ and $\bar{g}^2(\theta)$. For a chosen weight matrix W, the GMM objective is then given by

$$J(\theta) := \begin{pmatrix} \bar{g}^1(\theta)' & \bar{g}^2(\theta)' \end{pmatrix} W \begin{pmatrix} \bar{g}^1(\theta) \\ \bar{g}^2(\theta) \end{pmatrix} \tag{12}$$

The GMM estimator is, as usual, $\hat{\theta} := \operatorname{argmin}_\theta J(\theta)$. I have discussed the instruments Z^1_{jct} above. The instruments Z^2_{jct} used in estimation are the population-weighted averages of the demographics included in v_{gmt}. I use the identity matrix as the weighting matrix.

The term $\mathbb{E}[e_i|x_i]$ in equation (4) is calculated from equation (7), which is estimated by ordinaty least squares using the Household Budget Survey. From (7) it follows that $\mathbb{E}[e_{im}|x_i] = \exp(\alpha_{r(m)u} + \alpha_2 n_i) y_i^\alpha \mathbb{E}[\exp(\eta_{im})|x_i]$. I assume that $\exp(\eta_{im})$ is mean independent of x_i and estimate $\mathbb{E}[\exp(\eta_{im})]$ using the residuals from equation (7).

Finally, the conditional distribution $H_t(x_i|g)$ is obtained by Bayes' rule:

$$h_t(x_i|g) = \frac{\mu_t(g|x_i)h(x_i)}{\int \mu_t(g|x_i')h(x_i')dx_i'} \tag{13}$$

The term $\mu_t(g|x_i)$ is derived from the technology choice model; the unconditional distribution of x_i comes from the Census data. I obtain $h_t(x_i|g)$ by drawing a uniform random sample from the municipality-level Census data, computing $\mu_t(g|x_i)$ for each draw x_i, and calculating $\mu_t(g|x_i)/\sum_j \mu_t(g|x_j)$.

4.2 Identification and Estimation of Dynamic Parameters

The flow payoffs of the dynamic game are linear in the structural parameters. For dynamic games with linear flow payoffs, it is possible to show that structural parameters are identified if conditional choice probabilities are identified.[27]

The conditional value functions inherit the linearity from the flow payoffs: there exist functions $f_{rt,P^m}(a,s)$ and $g_{rt,P^m}(a,s,z)$ such that

$$\lambda^{-1}v_{r,t}^m(a,s) = f_{rt,P^m}(a,s) + g_{rt,P^m}(a,s,z)'\lambda^{-1}\Psi$$

where Ψ is a vector collecting all structural parameters (see appendix F for details). Since the idiosyncratic errors follow a Type 1 Extreme Value distribution, the conditional choice probabilities have the logit form:

$$P^m(a|s,r,t) = \frac{\exp(v_{r,t}^m(a,s)/\lambda)}{\sum_{a'\in A(s_f)}\exp(v_{r,t}^m(a',s)/\lambda)}$$

We can apply the usual logit inversion to this equation to obtain:

$$\ln(P^m(a|s,r,t)) - \ln(P^m(s_f|s,r,t)) = \frac{v_{r,t}^m(a,s)}{\lambda} - \frac{v_{r,t}^m(s_f,s)}{\lambda}$$

Using the linear representation of the conditional value functions we can then write

$$h^m(a,s,r,t) = \left[g_{rt,P^m}(a,s,z) - g_{rt,P^m}(s_f,s,z)\right]'\frac{\Psi}{\lambda} \tag{14}$$

where $h^m(a,s,r,t) := \ln(P^m(a|s,r,t)) - \ln(P^m(s_f|s,r,t)) - f_{rt,P^m}(a,s) + f_{rt,P^m}(s_f,s)$. Equation (14) leads to an OLS-like formula for Ψ/λ. Moreover, as shown in ap-

[27] This is a known result, see, e.g., Aguirregabiria and Nevo (2013). I review the argument for completeness.

pendix F, the first coordinate of Ψ is 1, so that λ is also identified.

The intuition for identification is that the structural parameters are identified by exogenous variation in (π_m, z_m, s, r, t) and the fact that we observe how firms respond to this variation. For example, varying a municipality's area while holding constant variables that affect firms' flow profits identifies how area affects investment costs. Note that we can do so for firms that have 3G technology, so we identify the effect of area on the cost of 4G. Knowing that we can move to firms that have 2G, so we identify the effect of area on the cost of 3G, and then we move to potential entrants. The fine parameter φ is identified by the difference in behavior between regulated and unregulated firms. Time variation also helps to identify φ. Intuitively, for small φ the behavior of regulated firms will change only slightly as the regulation deadline approaches; large φ, on the other hand, should lead to larger changes in behavior as time goes by.

Note that the argument above refers to municipality-specific CCPs P^m, and indeed in estimation and counterfactuals I allow for municipality-specific CCPs. Doing so fully exploits observed market-level heterogeneity and is at the service of the economic and policy questions that motivate this paper.[28] Regardless, CCPs P^m have to be recoverable from the data. For this to be the case, it is sufficient that there be a unique (quasi-stationary, symmetric) MPE for each market and that the map from market-level observables to MPE be continuous. Under these conditions, one can use nearby markets (in the space of observables) to estimate a market's CCPs.[29] Figure 17 in appendix F provides evidence in favor of continuity. Computational experimentation supports uniqueness. Finally, observe that the CCPs also vary with t due to non-stationarity. In both cases, it is important to observe a large cross-section of markets, as is the case in the present paper.

I apply the Nested Pseudo Likelihood (NPL) algorithm of Aguirregabiria and Mira (2007) to estimate the dynamic parameters. In light of the results of Pesendorfer and Schmidt-Dengler (2010), my choice of estimator requires some justification. A popular alternative is to use a two-step estimator, e.g. Bajari, Benkard, and Levin (2007), Pakes et al. (2007) or Pesendorfer and Schmidt-Dengler (2008). These estimators all proceed by flexibly estimating policy functions in a first stage and then using those policy functions to construct a second-

[28] An alternative but coarser approach would be to group markets with similar observables and posit that the data comes from a unique equilibrium within each group, as in Dunne et al. (2013).

[29] See De Paula (2013) for a related discussion.

stage objective function that is minimized to yield structural estimates. Because of the high degree of flexibility I require of CCPs, I opt to use an estimator that makes fuller use of the already imposed structural assumptions.

The maximum likelihood estimator is a natural option, but its computational cost is prohibitive in the case of dynamic games. I thus adopt Aguirregabiria and Mira (2007). An alternative that was recently proposed is Dearing and Blevins (2019).[30] The estimator proposed by Dearing and Blevins (2019) enjoys good theoretical properties. In particular, it is guaranteed to converge, thus overcoming the main issue of NPL raised by Pesendorfer and Schmidt-Dengler (2010). However, that algorithm requires solving large systems of linear equations, which renders its application to the empirical setting in this paper substantially costlier than Aguirregabiria and Mira (2007).

A Nested Pseudo Likelihood (NPL) fixed point is a pair $(\tilde{\theta}, \{\tilde{P}^m\}_m)$ that satisfies

(i) $\tilde{\theta} = \text{argmax}_{\theta} \sum_{m,t,f} \ln \Psi(a_{fmt}|s_{mt}, r_{fm}, t, m; \theta, \tilde{P}^m)$

(ii) $\tilde{P}^m = \Psi(\tilde{P}^m; \tilde{\theta})$ for all m

where $\Psi(a_{fmt}|s_{mt}, r_{fm}, t, m; \theta, \tilde{P}^m)$ is the probability that action a is optimal at state (s_{mt}, r_{fm}, t) in market m, when the firm believes that its competitors' and its own behavior will follow the CCPs $\tilde{P}^m$ from $t+1$ onwards, and $\Psi(\tilde{P}^m; \tilde{\theta})$ is an array collecting all such probabilities. The NPL *estimator* is the NPL fixed point with the maximum value of the pseudo-likelihood. The set of NPL fixed points is known to be non-empty but need not be a singleton. Therefore, the researcher must explore the parameter space to ensure that the pseudo-likelihood is being maximized in the set of NPL fixed points.

In practice, one finds NPL fixed points via an iterative algorithm. Starting with a guess for CCPs, $\{\tilde{P}^m\}_m$, the implied pseudo likelihood is maximized. One then uses the resulting guess for θ to update firms' CCPs. These two steps are repeated until the CCPs and the structural parameters converge.

5 Estimation Results

In table 4 I present estimates of the parameters in the market-share model. In table 5 I present estimates of the expenditure equation. The effect of income on a technology's utility is stronger the more advanced the technology, both in

[30]See also Aguirregabiria and Marcoux (2021) for a related recent contribution.

the early and late periods. This suggests that individuals with higher earnings make more frequent use of high-bandwidth applications of mobile telecommunications. Population density also increases the demand for mobile telecommunications services, and to a similar degree for all technologies. This is consistent with individuals in more densely populated areas having more social connections and thus higher demand for mobile telecommunications services. Table 5 shows that expenditures in mobile telecommunications are increasing in income and household size, perhaps because individuals in larger households have more reasons to communicate.

Table 4: Parameter Estimates – Market Shares

Variable	Phase	Technology	Estimate	2.5 Quantile	97.5 Quantile
GDP	early	2G	0.121	-0.186	0.598
GDP	early	3G	0.322	-0.072	0.827
GDP	early	4G	0.869	0.457	1.466
GDP	late	2G	-0.425	-0.665	0.121
GDP	late	3G	-0.304	-0.600	0.230
GDP	late	4G	0.184	-0.086	0.806
Intercept	early	3G	-2.032	-3.192	-0.704
Intercept	early	4G	-9.226	-11.727	-7.100
Intercept	late	3G	-0.334	-1.447	1.108
Intercept	late	4G	-4.150	-5.618	-2.159
Pop Density	early	2G	0.268	0.060	0.490
Pop Density	early	3G	0.297	0.159	0.541
Pop Density	early	4G	0.302	0.104	0.539
Pop Density	late	2G	0.442	0.026	0.557
Pop Density	late	3G	0.420	0.105	0.502
Pop Density	late	4G	0.434	0.031	0.463
Nesting Parameter			0.262	0.126	0.392

This table displays GMM estimates of the market share model. A combination of the Variable, Phase, and Technology columns defines a parameter in the model. GDP and Pop Density are the logarithms of GDP per capita and population density. The Estimate column shows the point estimate and the final two columns define a 95% confidence interval for the respective parameter. The confidence interval is calculated by bootstrap, which is performed at the area-code level. The estimated models also include state-phase fixed effects.

Regarding the dynamic parameters, I estimate the median entry cost (which accounts for the cost of setting up 2G service) to be equal to 5.33 million BRL, the fifth and ninety-fifth percentiles being 4.99 million and 5.53 million BRL. This variation is due to differences in municipality area. The cost of non-compliance with the regulation, φ, is estimated to be 1.51 million BRL, or 28.35% of the

Table 5: Expenditure Equation

	Log of Expenditures
Log Income	0.360
	(0.004)
Number of Residents	0.029
	(0.002)
State * Rural/Urban FE	Yes
Num. obs.	72775
R^2	0.201
Adj. R^2	0.200

This table displays OLS estimates of equation 7.

median entry cost.

In figure 4, I plot the estimated dynamics of the cost of introducing 3G technology. I show, for the years 2014-2018, the average cost together with the tenth and the ninetieth percentiles of the distribution – where, as before, the variation is driven by municipality area. The average cost of introducing 3G declined from just below 4 million BRL in 2014 to about 2.5 million BRL in 2018. Figure 5 is an analogous figure for 4G technology. I estimate a much more marked decrease in the cost of introducing 4G. This is consistent with the notion that 4G was a relatively new technology in 2014. Estimates of the dynamic parameters and confidence intervals can be found in appendix G.

6 Counterfactual Analysis

The counterfactual exercises in this section directly address the questions posed in the beginning. In subsection 6.1, I analyze the effect of coverage requirements on the time to introduction of 3G technology, quantify the cost that the regulation imposes on firms, and measure equilibrium effects. In subsection 6.2, I evaluate an alternative regulation in which the first firm to introduce 3G technology receives a subsidy.[31]

[31] As noted in section 2, the counterfactual exercises focus on markets with a December 31, 2019 regulation deadline. There are 941 such markets in my sample. I perform the simulations for the subset of these markets in which the regulated firm does not offer 3G technology at the start of the data. There are 743 such markets in my sample. Moreover, when comparing the time to 3G introduction under alternative scenarios, I only consider markets that start without 3G, of which there are 679.

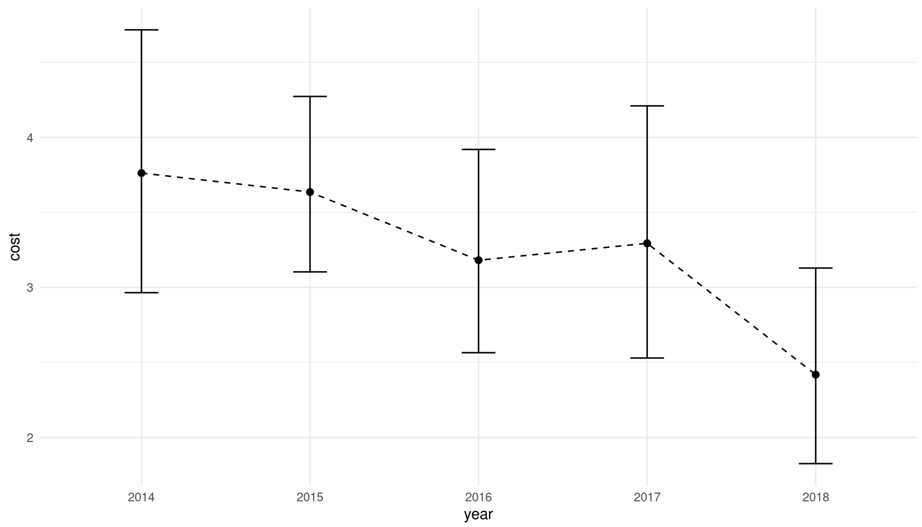

Figure 4: Estimated dynamics of the cost of 3G introduction.

The dot shows the average cost across municipalities. The whiskers indicate the tenth and ninetieth percentiles of the distribution, where the variation is driven by municipality area.

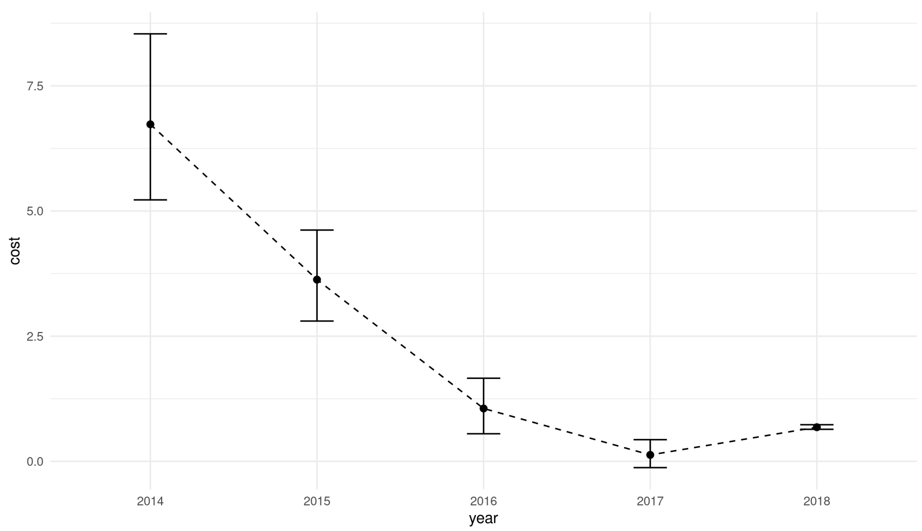

Figure 5: Estimated dynamics of the cost of 4G introduction.

The dot shows the average cost across municipalities. The whiskers indicate the tenth and ninetieth percentiles of the distribution, where the variation is driven by municipality area.

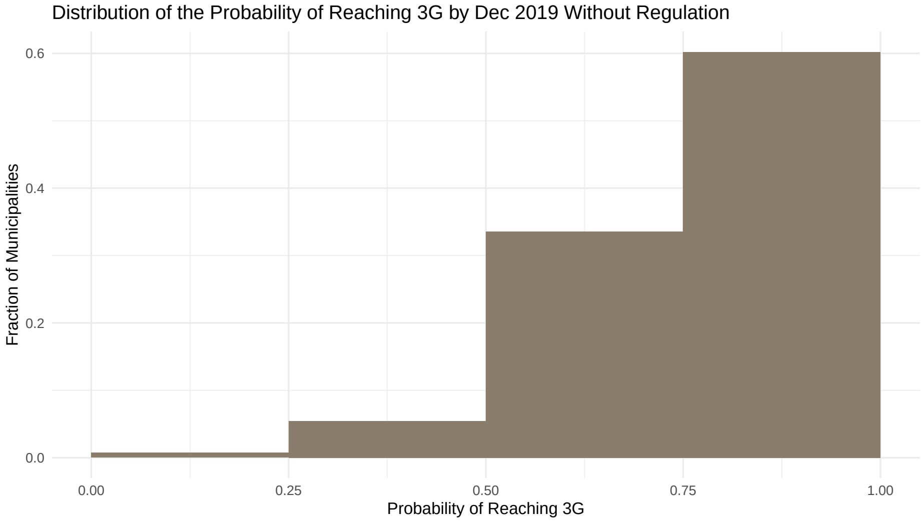

Figure 6: Distribution of the Probability of Reaching 3G by December 2019 Without Regulation.

6.1 The Effect of Coverage Requirements

To quantify the effect of coverage requirements on the time to introduction of 3G technology and firms' ex-ante expected profits, I solve the game and simulate data for each municipality under two regulatory regimes: under the estimated fine $\hat{\varphi}$ and setting $\varphi = 0$, i.e., with no regulation. In each case, I simulate 250 paths of play for each municipality.

The first question we can ask the model is whether coverage requirements are really necessary. More precisely, without regulation, would 3G technology be introduced within a reasonable amount of time? To answer this question, I compute the share of simulations in which some firm introduced 3G technology by December 2019. In figure 6, I show the distribution of these probabilities across municipalities. The figure shows that just over 60% of the municipalities in the sample would have had access to 3G technology by December 2019 with at least 75% probability. For 33.72% of the municipalities, the probability of having 3G access is between 50% and 75%. Therefore, for most municipalities, market forces would most likely than not be sufficient to guarantee provision of 3G service. Nevertheless, for just over 6% of municipalities, the probability of having 3G service by December 2019 is less than 50%. In these municipalities, market forces are most likely insufficient to guarantee service provision.

These results may suggest that the regulation has a limited effect. However, it may affect not only the ultimate availability of 3G but also how quickly it

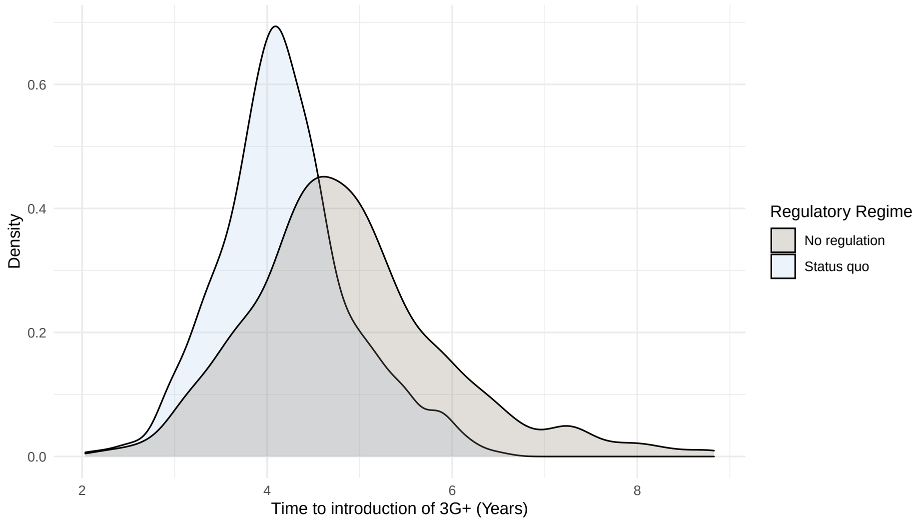

Figure 7: Distribution of the time to introduction of 3G technology or better under alternative regulatory regimes.

is introduced. To evaluate effects on the speed of roll-out, I calculate the time to 3G introduction under both regulatory regimes.[32] In figure 7 I show the resulting distributions. The label "Status quo" refers to the case $\varphi = \hat{\varphi}$ whereas "No regulation" corresponds to $\varphi = 0$. Coverage requirements reduce the average time to 3G introduction by 0.67 years, on average. The regulation also considerably reduces the dispersion in the time to introduction of 3G, mostly by eliminating a long right tail present in the absence of regulation. Figure 8 shows the distribution of the acceleration in the introduction of 3G due to the regulation. The effects are concentrated between 0 and 1 year, though there is a long right tail, consisting of the most vulnerable markets.

To further understand the determinants of the effects of the regulation, in table 6 I project the time to introduction of 3G with no regulation and the acceleration induced by coverage requirements onto observable market characteristics and variables that capture the initial market structure. The dependent variable in column 1 of table 6 is the time to 3G introduction without regulation, in years. The time to 3G introduction without regulation is decreasing in a municipality's population and increasing in its area. The effect of GDP per capita is not statistically significant.[33] Moreover, the time to 3G introduction

[32] I simulate data until 2023. In those instances in which 3G is not introduced by the end of the simulation, I set the time to 3G introduction equal to the length of the simulated sample. Therefore, the numbers I present on the effect of the regulation are, in some cases, a lower bound.

[33] GDP per capita does have a negative and statistically significat effect on the time to 4G

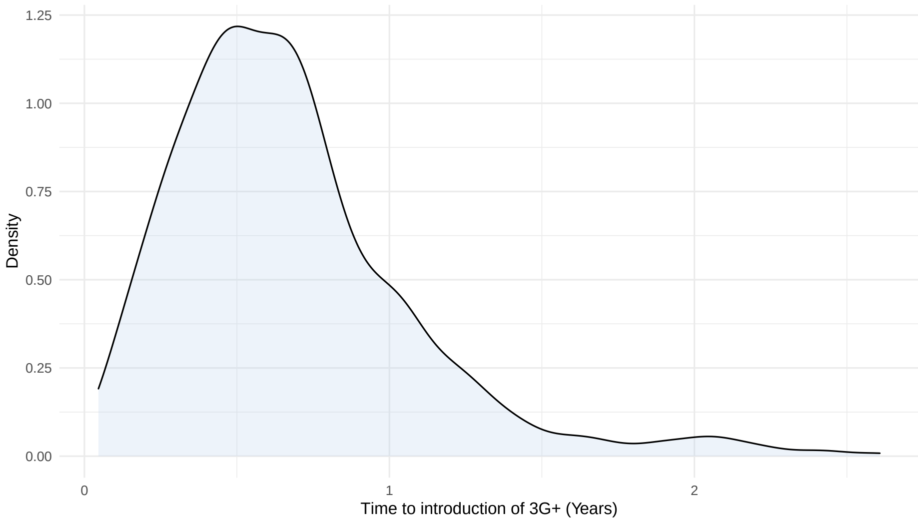

Figure 8: How much faster is the introduction of 3G+ under regulation?

Table 6: Explaining Time to Adoption and the Effect of Regulation

	Time to 3G	Reg. Effect
Log Area	0.604	0.208
	(0.019)	(0.007)
Log Population	−0.651	−0.254
	(0.053)	(0.020)
Log GDP Per Capita	0.049	−0.008
	(0.041)	(0.015)
Number of Firms in t = 0	−1.594	−0.357
	(0.058)	(0.025)
Regulated Firm Active in t = 0		0.090
		(0.017)
Intercept	7.941	1.923
	(0.675)	(0.249)
R^2	0.728	0.629
Adj. R^2	0.727	0.626
Num. obs.	679	655

The dependent variable in column (1) is the time to 3G introduction without regulation, measured in years. The dependent variable in column (2) is the acceleration of 3G introduction due to the regulation. Municipality characteristics are averaged over time.

is decreasing in the number of firms in the market in $t = 0$. These results are intuitive: firms are more likely to enter and upgrade their technologies in more populous markets with a smaller area to be covered; since incumbents have a lower cost of introducing 3G than potential entrants, a larger initial number of firms leads to faster 3G introduction.

The coefficients in the second column of table 6 show the same pattern as in column 1, i.e., markets where, in the absence of regulation, 3G would be introduced faster also experience a smaller acceleration, as one might expect. Additionally, the estimates imply that regulating an incumbent leads to a slightly larger acceleration than regulating a potential entrant. [34]

Next, I use the model to calculate the cost that the regulation imposes on firms.[35] Solving the dynamic game under the estimated fine and under no regulation, I obtain, for each municipality, firms' ex-ante expected profits under those two regimes. The cost of the regulation is the aggregate difference in firms' ex-ante expected profits in the no-regulation and status-quo regimes:

$$\text{Regulation Cost} = \sum_m \sum_f \left(V^m_{\varphi=0}(s_{f0}, s_{-f0}, t=0) - V^m_{\varphi=\hat{\varphi}}(r_f, s_{f0}, s_{-f0}, t=0) \right)$$

where $V^m_\varphi(\omega)$ is the firm's ex-ante expected profit in municipality m and state ω when the fine is set to φ.[36] I calculate that the cost of the regulation amounts to 321 million 2010 BRL, or 183 million 2010 USD.[37] This amounts to 20.04% of firms' aggregate ex-ante expected profits without regulation.

In table 7 I show the incidence of these costs. The first column indicates whether or not the firm is regulated and the second column indicates its initial state in the data. The next three columns show, respectively, the total cost, the average cost and the fraction of the total cost borne by the respective group of firms. Costs are shown in millions of BRL. Average costs are positive for all groups. The regulation imposes costs on unregulated firms because it makes competition tougher. This effect is more pronounced for firms that are active in the market in the beginning of the data because they necessarily face tougher

introduction.

[34]To aid in the interpretability of the coefficient on the dummy, this regression further restrict attention to those municipalities that had at least one active firm in the beginning of the data.

[35]Note that in the real world part of this cost is borne by the government via reduced revenue in spectrum auctions.

[36]Note that in the first term, $V^m_{\varphi=0}(s_{f0}, s_{-f0}, t=0)$, I do not include the regulation indicator r_f as an argument because there is no regulation in that case; r_f does appear as an argument in the second term.

[37]This conversion uses the average exchange rate in 2010 of 0.5685 USD per BRL.

competition, whereas potential entrants are only hurt in case they enter the market. The costs imposed on regulated firms are larger, especially when the regulated firm is not active in the beginning of the data. Regulated firms are forced to take actions they might not have taken in the absence of regulation. If they are not active in the beginning of the data, they have to pay not only technology installation costs, but also entry costs. The last column shows that most of the costs fall on regulated firms, in particular those that have to enter the market to comply with the regulation.

Table 7: Incidence of Regulation Cost

Regulated	Initial State	Total Cost	Average Cost	Fraction of Total Cost
0	Out	6.027	0.003	0.019
0	2G	11.781	0.033	0.037
0	3G	2.020	0.032	0.006
1	Out	267.335	0.747	0.831
1	2G	34.646	0.090	0.108

This table shows the total, average, and fraction of total costs borne by firms as a function of their regulated status and their initial technology in the data. Total and average costs are in millions of BRL.

Lastly, I quantify the importance of equilibrium effects. I proceed in three steps. First, I solve the game and simulate data in the absence of regulation. I then set the fine to its estimated value $\hat{\varphi}$ and solve for the regulated firm's optimal policy, holding the policy functions of the unregulated firms fixed. Next, I solve for the Markov Perfect Equilibrium under regulation. The difference between the time to adoption under the regulation equilibrium and the time to adoption when only the regulated firm responds to the regulation gives the desired equilibrium effects.

Figure 9 shows the distribution, across municipalities, of the equilibrium effects. Most of the values are positive: the equilibrium adjustment leads to a longer time to 3G introduction, relative to the case when only the regulated firm responds to the policy. This reflects the reduced incentives to invest faced by unregulated firms, resulting from the increased future competition induced by the regulation. Quantitatively, however, these effects are small. The total effects of the policy are therefore almost entirely explained by the direct effects on the regulated firm. Appendix H provides further detail on the equilibrium effects by looking at changes in policy functions in the two regimes.

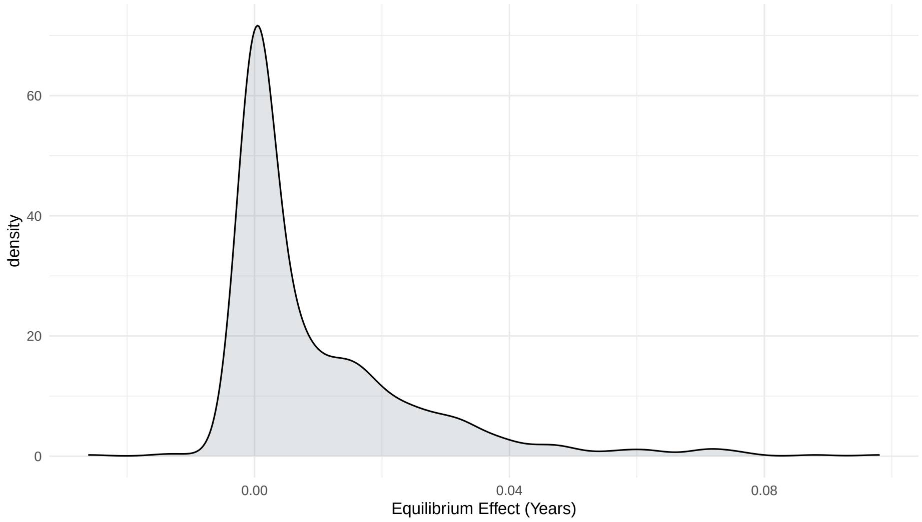

Figure 9: Equilibrium Effects.

This figure shows the difference between the time to the introduction of 3G (or 4G) under the equilibrium with regulation and when only the regulated firm responds to the regulation.

6.2 Alternative Regulatory Interventions

The final question posed in the beginning of this paper was whether we can design regulation that is more effective than coverage requirements. As before, I am mostly concerned with two dimensions of a policy's effect: to what extent it accelerates the introduction of new technologies and their cost of adoption. I will also highlight the effect of different policies on market structure.

6.2.1 Subsidizing the Introduction of 3G

The large estimated cost of non-compliance and the counterfactual results above show that coverage requirements provide strong incentives for 3G introduction, ensuring service provision. However, this comes at a substantial cost for firms, especially when regulated and not active in the market.

A policy that treats firms symmetrically, instead of targeting a single firm, may save on these costs. The intuition is simple. By providing the same incentive to all firms, the firm that will eventually choose to introduce the new technology will tend to be the most cost-efficient one.

Motivated by this reasoning, I evalute a regulation that subsidizes the first firm to introduce 3G technology or better. I denote the subsidy by β. If more than one firm introduces the new technology, those firms split the subsidy equally. Therefore, I add the following term to firms' flow profits for each state of the game and each possible action a_f:

$$\beta \times \underbrace{\mathbf{1}\left\{\left(\max_{f'} s_{f'}\right) < 3 \leq a_f\right\}}_{\text{Subsidy is paid}} \times \underbrace{\sum_{n=0}^{3} \mathbb{P}\left(\left(\sum_{f' \neq f} \mathbf{1}\{a_{f'} \geq 3\}\right) = n\right) \times \frac{1}{1+n}}_{\text{Expected fraction of the subsidy}}$$

where the probabilities in this expression are are given by the ensuing equilibrium behavior.

I experiment with two subsidy designs. I start with a budget given by

$$\text{Budget} = \sum_m \sum_f \left(V^m_{\varphi=0}(s_{f0}, s_{-f0}, t=0) - V^m_{\varphi=\hat{\varphi}}(r_f, s_{f0}, s_{-f0}, t=0)\right), \quad (15)$$

which is simply the aggregate cost of the regulation. Note that firms would be willing to pay this amount to move from the status quo world to a world with a subsidy. In that sense, the subsidies considered below are self-financed.[38]

I start by splitting the budget in equation (15) equally across municipalities. Figure 10 shows the acceleration in the introduction of 3G technology under coverage requirements (labeled "status quo") and the subsidy. The average effect is very similar; the subsidy accelerates the introduction of 3G by 0.62 years on average, relative to 0.68 years under coverage requirements. As figure 10 shows, relative to coverage requirements, the subsidy eliminates some small effects, but also loses some large ones. The large effects lost by the subsidy come from municipalities that would experience relatively late introduction of 3G without regulation. Consider, for example, those municipalities where coverage requirements accelerate the introduction of 3G by one year or more. The average time to introduction of 3G without regulation in these municipalities is more than two years larger than in the remaining municipalities. These are relatively unprofitable markets, and the homogeneous subsidy provides less incentives for 3G introduction in these markets than coverage requirements. For this group of markets, the subsidy delays the introduction of 3G by 0.7 years relative to coverage requirements, on average.

The municipalities where coverage requirements generate small accelerations (less than 6 months) are relatively competitive. The average number of firms in $t = 0$ in those municipalities is 1.24, relative to 0.96 in the remaining

[38] The practical application of this idea is based on the observation that the alternative policies discussed here would increase the value of spectrum licenses. Therefore, spectrum auction revenues would increase and these increased revenues could be used to finance the subsidies proposed here.

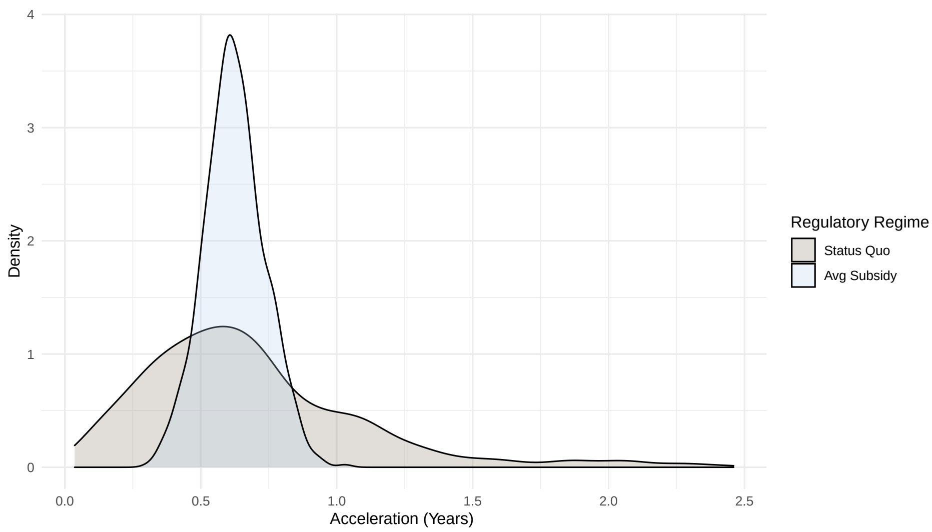

Figure 10: Acceleration of 3G Introduction Under Coverage Requirements and Subsidy

municipalities. The introduction of 3G in these municipalities in the absence of regulation is relatively fast: 3.89 years, on average, compared to 5.44 years in the other municipalities. In these markets, the effect of the subsidy is very close to the mean effect, so that they are moved from the left tail of the "Status Quo" distribution in figure 10 to the middle of the subsidy distribution. In summary, relative to coverage requirements, a flat subsidy increases the acceleration of 3G introduction in some localities where there seems to be little need for regulation, and has smaller effects in some municipalities where regulation seems to be particularly important.

This point is shown clearly in figure 11, where the time to introduction of 3G in the absence of regulation is plotted against the effects of coverage requirements and the flat subsidy. Each dot in the figure is a municipality. For the case of coverage requirements, we see a positive correlation: the regulation has stronger effects in those markets where, in the absence of intervention, it would take longer for 3G to be introduced. The flat subsidy does not display the same correlation. In fact, for the most vulnerable municipalities the correlation seems to be slightly negative.

In light of these results, I consider a policy that allocates a larger share of the budget towards the most vulnerable municipalities. Specifically, let τ_m be the time for 3G introduction in municipality m in the absence of regulation and let f be a positive and increasing real function. Allocate to municipality m the fraction $f(\tau_m)/\sum_{m'} f(\tau_{m'})$ of the budget in equation (15). The more convex f,

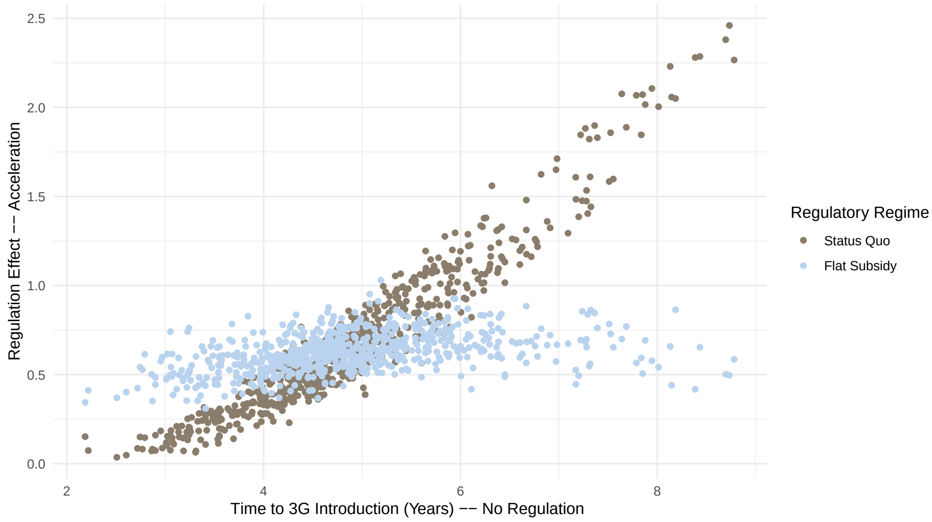

Figure 11: Targeting Properties of Coverage Requirements and a Flat Subsidy

the stronger the targeting towards the most vulnerable municipalities. I set $f(\tau) = \tau^{3/2}$.[39] Figure 12 shows the results. This subsidy leads to an acceleration in the roll-out of 3G of 0.65 years relative to 0.68 for coverage requirements and 0.62 for the flat subsidy. The municipality-specific subsidy restores the desired positive correlation between the effect of the regulation and the time to 3G introduction without regulation. In fact, this subsidy leads to larger accelerations in the most vulnerable municipalities than coverage requirements. This comes at the expense of slightly smaller effects in those municipalities that even in the absence of regulation obtain access to 3G technology relatively quickly. The optimal way to navigate this trade-off (e.g., the optimal choice of exponent in $f(\tau)$) depends on the relative changes in consumer surplus in those two groups of municipalities, which cannot be quantified with the limited data available for this study.

Firms substantially benefit from the municipality-specific subsidy relative to coverage requirements.[40] Firms' ex-ante aggregate expected profits grow by 126 million BRL, after accounting for their financing of the subsidy; this amounts to 9.83% of firms' aggregate profits without regulation. These gains essentially come from reallocating the introduction of the new technology from inactive and regulated firms, who have to pay entry costs, to incumbents, who only pay

[39] This subsidy design relies on τ_m, and one may thus be concerned that its informational requirements are substantial. However, note that the results in table 6 show that a substantial portion of the variation in τ_m is explained by observables. Therefore, it might be possible to design a subsidy with similar properties that relies only on data that is available to regulators.

[40] Similar results hold for the flat subsidy.

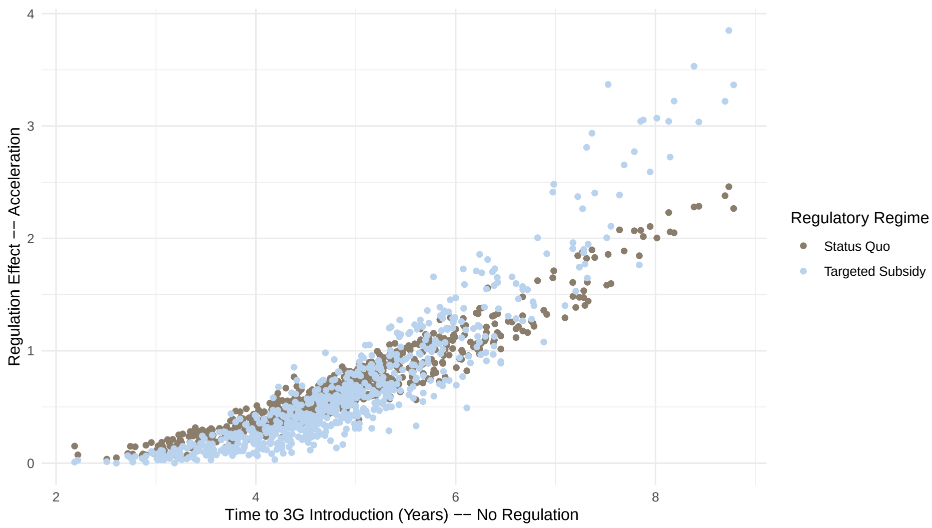

Figure 12: Targeting Properties of Coverage Requirements and a Municipality-Specific Subsidy

technology installation costs.

This reallocation leads to a more cost-efficient technology roll-out, but at the expense of reduced competition in the market. The subsidy leads to entry of 0.93 firms, on average, by the end of 2022. In contrast, coverage requirements lead to entry of 1.17 firms. This difference is entirely driven by those markets where the regulated firm is a potential entrant. The average number of entrants in these markets, under coverage requirements, is 1.60, whereas it is equal to 1.00 under the subsidy. In the remaining markets, coverage requirements result in entry of 0.83 firms, on average; the subsidy results in entry of 0.87 firms.

The model can be used to perform a heuristic calculation that is informative of where we stand in this cost-competition trade-off. For each simulation s, I compute the discounted number of firms present in each market: $\bar{n}_s = \sum_{t=0}^{T} \delta^t \sum_f \mathbf{1}(s_{ft} > 0)$. I then average this quantity across simulations for each regulatory regime to obtain an average discounted number of firms for each regulatory regime and municipality. Combining this with municipality population, I find that 243,303 consumers are exposed to an additional firm under coverage requirements, relative to the targeted subsidy.[41] For this added competition to overturn the cost efficiency results discussed above, the average gain in consumer surplus from one more firm has to be at least 8.21 BRL.[42] This is

[41] The aggregate population (averaged over time) of the municipalities considered in these counterfactuals is 3.2 million.

[42] This is a conservative estimate, as it assumes that the gain in consumer surplus from addi-

equal to 44.99% of the mean predicted expenditure for these markets, thus suggesting that the targeted subsidy is more efficient than coverage requirements.

7 Conclusion

Concerns regarding lack of service provision are present in many industries and so is regulatory intervention. This paper studies the effect of coverage requirements, a common form of regulation in the mobile telecommunications industry, on the speed of roll-out of new technologies, market structure, and firms' profits. To do so, I use new data on mobile technology availability in Brazilian municipalities to estimate a dynamic game of entry and technology upgrade under regulation.

I show that the regulation accelerated the arrival of third generation mobile telecommunications technology to relatively small and underdeveloped markets in Brazil by 0.7 years on average – and by more than 2 years in some cases. This, however, comes at a high cost: the regulation reduces firms' ex-ante expected profits by 20.04%. I also show that a policy that subsidizes the first firm to introduce 3G technology, by an amount that the firms themselves would be willing to finance, achieves a similar acceleration of 3G introduction, better targets the most vulnerable markets, and leads to a more cost-efficient roll-out. These cost-efficiency gains are accompanied by reduced competition in some markets, but the losses in consumer surplus needed to overturn the efficiency gains are implausibly large. These findings have immediate implications for the design of regulation in mobile telecommunications markets, and potentially to other markets where universal service is also a concern.

Some interesting and related questions are not addressed in this paper. First, though my results are informative for the design of regulation, data limitations preclude me from conducting a complete welfare analysis. It would be interesting to combine data such as the one used in this paper with detailed price and quantity data to compare the gains in consumer surplus from earlier access to new technologies and the regulatory costs imposed on firms. Second, my analysis abstracted away from geographic interdependencies in firms' costs. It would be interesting, though challenging, to extend the model to allow for such interdependencies. These topics, however, are left for future research.

tional entrants is constant.

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Online Appendix

Appendix A Regulation and Delay in the Fudenberg-Tirole Model

A.1 The Model

There are two firms. Firm 1 is an incumbent and firm 2 a potential entrant. Time is continuous and the discount rate is r. Firm 1 initially operates as a monopolist with constant marginal cost $\bar{c}$. At any point in time $t \geq 0$, firms can adopt a technology with constant marginal cost $\underline{c}$. Adopting this technology at time t costs $C(t)$, where $C(t) > 0$, $C'(t) < 0$ and $C''(t) > 0$, for all $t \geq 0$.

Let $p^m(c)$ and $\pi^m(c)$ be, respectively, the monopoly price and profit when marginal cost is c. I focus on the case in which the innovation is *non-drastic*, i.e., $p^m(\underline{c}) \geq \bar{c}$. If both firms are in the market, they compete à la Bertrand. Let $\pi^d(c, c')$ be a firm's profit when its cost is c and its competitor's cost is c'. Under the assumption of a non-drastic innovation and Bertrand competition, π^d satisfies

$$\pi^d(\underline{c}, \bar{c}) = (\bar{c} - \underline{c})D(\bar{c}), \quad \pi^d(\bar{c}, \underline{c}) = 0 \quad \text{and} \quad \pi^d(c, c) = 0 \quad \forall c$$

Firms' strategies specify their decisions to adopt or not the new technology as a function of t and their competitor's technology.[43] Note that due to the Bertrand assumption, a firm will never adopt the new technology after its competitor has adopted, as they would incur the positive adoption cost but their flow profits would stay at zero.

If the incumbent is first to adopt at date t_1, its overall profit is

$$L_1(t_1) = \int_0^{t_1} \pi^m(\bar{c})e^{-rt}dt + \int_{t_1}^{\infty} \pi^m(\underline{c})e^{-rt}dt - C(t_1)e^{-rt_1} \tag{16}$$

If the incumbent is preempted at date t_2, its present discounted profit is

$$F_1(t_2) = \int_0^{t_2} \pi^m(\bar{c})e^{-rt}dt \tag{17}$$

[43] The discussion here is somewhat informal. Fudenberg and Tirole (1985) provide a careful description of appropriate strategies for this game.

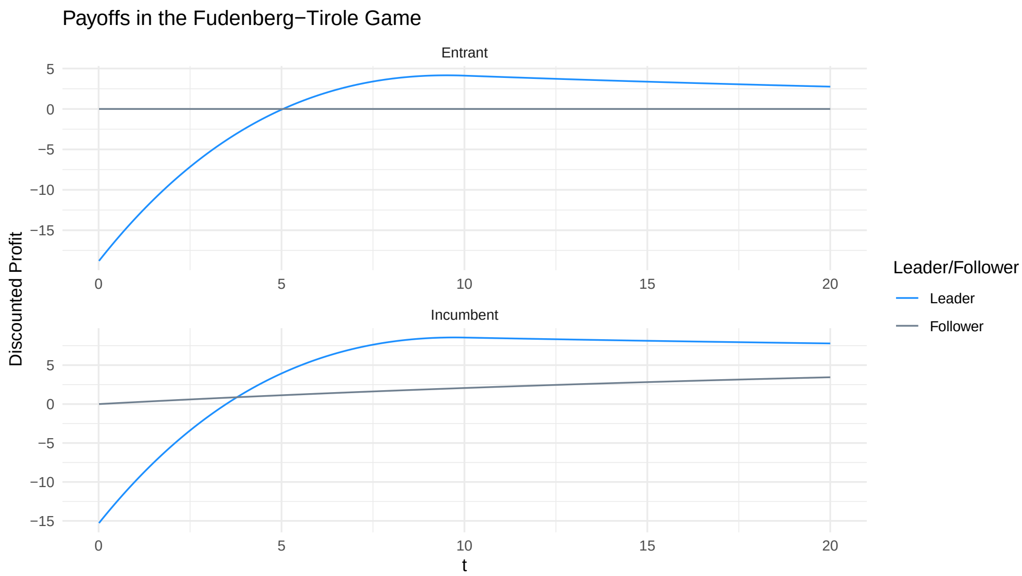

Figure 13: Payoffs in the Fudenberg-Tirole Model.

If the entrant is first to adopt at date t_2, its overall profit is

$$L_2(t_2) = \int_{t_2}^{\infty} \pi^d(\underline{c}, \bar{c}) e^{-rt} dt - C(t_2) e^{-rt_2} \tag{18}$$

Finally, it the entrant is preempted at time t_1, its profit is given by $F_2(t_1) = 0$.

Figure 13 plots the functions L_1, F_1, L_2, F_2.[44] That figure is sufficient to determine the equilibrium outcome of the game.[45] Let t_2^* be defined by $F_2(t_2) = L_2(t_2)$. In Figure 13, $t_2^* \approx 5$. Firm 2 will not adopt before t_2^*, as it would prefer to be preempted by firm 1. Knowing this, firm 1 will wait to adopt, as $L_1(t_1)$ is increasing over $t_1 < t_2^*$. Now suppose firm 2 is first to adopt at some $t_2 > t_2^*$. Since $L_1(t_2) > F_1(t_2)$, firm 1 prefers to adopt at $t_2 - \varepsilon$. In equilibrium, firm 1 adopts at $t_1 = t_2^*$, and firm 2 never adopts.

A.2 Incorporating Regulation

Now suppose that the incumbent is regulated: is must adopt by some exogenously set deadline τ, lest it pay an exorbitant fine. The L_i and F_i functions are

[44] The specification is as follows. $D(p) = 2 - p$, $\bar{c} = 1$, $\underline{c} = 3/4$, $C(t) = \mathbf{1}\{t <= 10\}\left(\frac{t^2}{4} - 5 * t + 25\right) + 0.1$.

[45] But not the equilibrium itself.

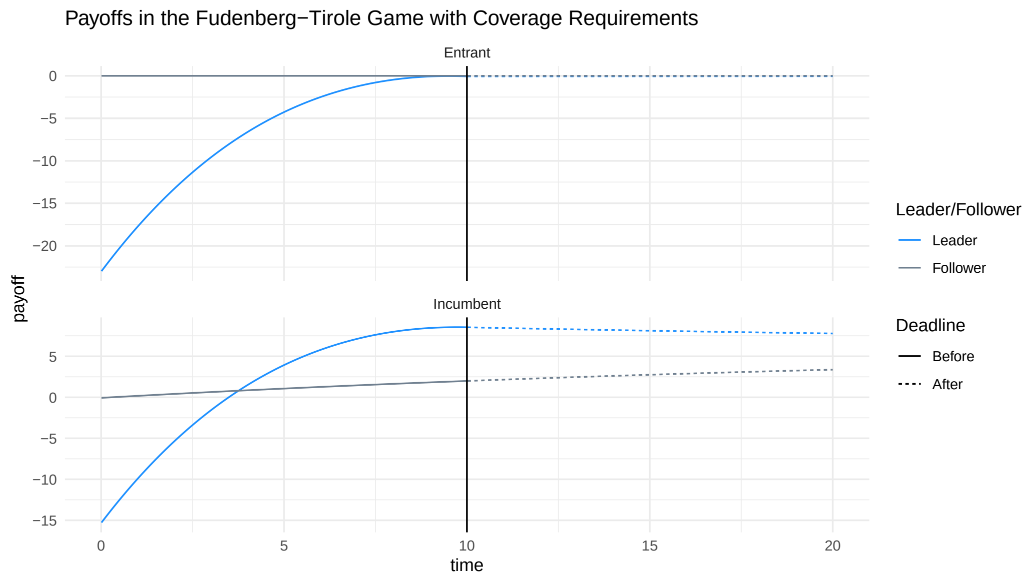

Figure 14: Payoffs in the Fudenberg-Tirole Model with Regulation.

now defined (for $t_i \leq \tau$) as follows:

$$\begin{aligned} L_1(t_1) &= \int_0^{t_1} \pi^m(\bar{c})e^{-rt}dt + \int_{t_1}^{\infty} \pi^m(\underline{c})e^{-rt}dt - C(t_1)e^{-rt_1} \\ F_1(t_2) &= \int_0^{t_2} \pi^m(\bar{c})e^{-rt}dt - C(\tau)e^{-r\tau} \\ L_2(t_2) &= \int_{t_2}^{\tau} \pi^d(\underline{c}, \bar{c})e^{-rt}dt - C(t_2)e^{-rt_2} \\ F_2(t_1) &= 0 \end{aligned} \tag{19}$$

Figure 14 plots these payoffs for the same parametrization underlying Figure 13, and $\tau = 10$. As can be seen from the figure, the fact that the incumbent will adopt the technology at time τ, at the latest, eliminates all incentive for the entrant to adopt the new technology. With no need to preempt the entrant, the incumbent is free to delay its own adoption to its most preferred time, which in this example is $t_1^* \approx 9.7$. Therefore, the regulation delays the adoption of the new technology from $t \approx 5$ to $t \approx 9.7$. Of course, if $\tau < 5$, the regulation speeds up the adoption of the new technology.

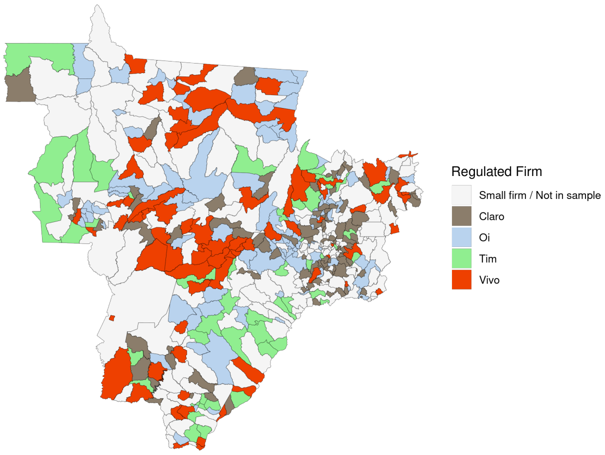

Figure 15: Regulated Carriers – Midwest.

This figure shows a map of the Brazilian Midwest, color-coded according to the identity of the single regulated firm in each market. The subdivisions in the map are municipalities. The municipalities without a color are either not in the estimation sample (municipalities with more than 30,000 inhabitants in 2006) or the regulated firm is one of the small carriers. Claro, Oi, Tim, and Vivo are the four large carriers in Brazil.

Appendix B Supplementary Figures and Alternative Specifications of Descriptive Models

Figure 15 illustrates the result of carriers' iterative choices of municipalities where they would be suject to a coverage requirement. The figure shows a map of the Brazilian Midwest, color-coded according to the identity of the regulated carrier. Figure 16 illustrates the data on technology availability. Each cell contains a map of the state of Pará, in the north of Brazil. The subdivision within each map are the municipalities in that state. Each row shows data for one of the four large carriers, and columns indicate the year for which the data is plotted. Municipalities are color-coded according to the best technology provided by the respective carrier in that municipality at the end of the year indicated in the column.

The tables below report alternative specifications of the descriptive models in table 3 in the main text. In particular, table 8 reports models without group fixed effects, and table 9 reports models that include characteristics of firms' networks in neighboring states. Specifically, the models include dummies for

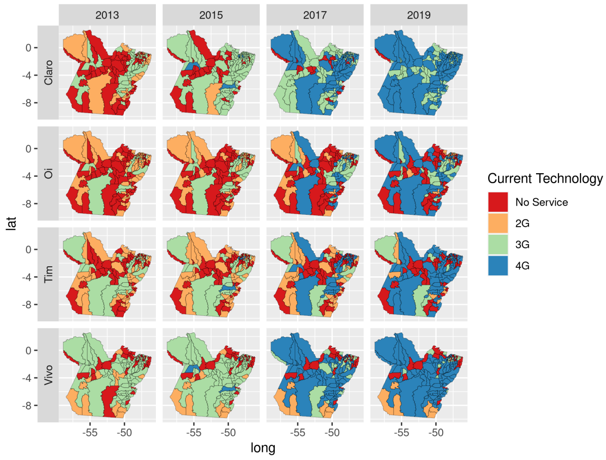

Figure 16: Technology availability in the state of Pará.

Each cell in this matrix contains a map of the state of Pará, in the north of Brazil. The subdivision within the state are municipalities. Rows correspond to the four large carriers in Brazil: Claro, Oi, Tim, and Vivo. Columns correspond to calendar years. Municipalities are color-coded according to the best technology offered in that municipality by the relevant firm in the end of the year.

whether or not the firm provides 2G, 3G, and 4G service in any neighboring municipality. Comparing table 8 and table 3 shows the importance of the group fixed effects. Without them in table 8, the competition coefficients are mostly small in absolute value and sometimes positive. That is in contrast with the results in table 3, where the competition coefficients are almost all negative and larger in absolute value. This suggests that the group fixed effects capture important unobserved factors related to how desirable it is to provide service in a given market.

Turning to table 9, the first thing to note is that service in neighboring municipalities is important. The estimated coefficients on 3G service and 4G service are sizeable and precisely estimated. Interestingly, the coefficients on 2G service in neighboring municipalities are negative. This is surprising because these coefficients are relative to not having service in the neighboring municipality. The next thing to observe is the effect of the network variables on the competition coefficients. These effects are mostly small, except perhaps for the number of competitors with 4G technology. Albeit small, the effects are always in the direction of increasing (in absolute value) the estimated competition coefficients.

Table 8: Entry/Upgrade Models – Without group fixed effects

	Out	2G	3G
Log GDP PC	0.002	0.001	0.015
	(0.001)	(0.002)	(0.002)
Log Pop.	0.011	0.034	-0.008
	(0.001)	(0.002)	(0.002)
Log Area	-0.002	-0.011	0.003
	(0.000)	(0.001)	(0.001)
Regulated	0.105	0.155	-0.035
	(0.003)	(0.004)	(0.003)
Regulated Competitor - Out	-0.015	0.001	-0.028
	(0.002)	(0.006)	(0.009)
Regulated Competitor - 2G	-0.007	-0.045	-0.089
	(0.002)	(0.005)	(0.007)
No. Competitors 2G	-0.001	0.005	-0.001
	(0.001)	(0.002)	(0.002)
No. Competitors 3G	-0.011	0.006	0.008
	(0.001)	(0.002)	(0.002)
No. Competitors 4G	0.004	-0.011	0.013
	(0.001)	(0.002)	(0.002)
Group FE	No	No	No
$\bar{Y}$	0.026	0.079	0.083
Num. obs.	92088	47074	49245

Linear probability models. The dependent variable is a dummy equal to 1 if a technology upgrade is observed. The explanatory variables are, in this order: the natural logarithms of GDP per capita, population, and municipality area, a dummy that is equal to 1 if the firm is regulated, a dummy that is equal to 1 if the firm faces a regulated competitor that is out of the market, a dummy that is equal to 1 if the firm faces a regulated competitor that has 2G technology, and the numbers of competitors with 2G, 3G and 4G technology. Each column corresponds to the subsample of the data where firms' best technology is as indicated in the column heading.

This may suggest that there are unobservable factors that are geographically correlated. Finally, and most importantly for the analysis in this paper, note that the effect of the network variables on the regulation variables is very minor, if present at all. This suggests that the regulation variables (in particular, whether or not a firm is regulated) are not correlated with the surrounding network infrastructure. Appendix C delves deeper into this.

Table 9: Entry/Upgrade Models – With Neighboring Network Info

	Out	2G	3G
Log GDP PC	0.016	0.020	0.032
	(0.001)	(0.003)	(0.002)
Log Pop.	0.031	0.059	0.017
	(0.001)	(0.003)	(0.003)
Log Area	−0.007	−0.019	−0.006
	(0.001)	(0.001)	(0.001)
Regulated	0.104	0.163	−0.021
	(0.003)	(0.004)	(0.003)
Regulated Competitor - Out	−0.014	0.003	−0.024
	(0.002)	(0.006)	(0.009)
Regulated Competitor - 2G	−0.001	−0.038	−0.072
	(0.002)	(0.005)	(0.006)
No. Competitors 2G	−0.014	−0.013	−0.017
	(0.001)	(0.003)	(0.002)
No. Competitors 3G	−0.026	−0.023	−0.015
	(0.001)	(0.003)	(0.003)
No. Competitors 4G	−0.022	−0.059	−0.043
	(0.001)	(0.003)	(0.003)
Nb. Service 2G	−0.012	−0.025	−0.017
	(0.002)	(0.012)	(0.009)
Nb. Service 3G	0.018	0.033	0.014
	(0.001)	(0.003)	(0.006)
Nb. Service 4G	0.025	0.094	0.125
	(0.001)	(0.003)	(0.003)
Group FE	Yes	Yes	Yes
$\bar{Y}$	0.026	0.079	0.083
Num. obs.	92088	47074	49245

Linear probability models. The dependent variable is a dummy equal to 1 if a technology upgrade is observed. The explanatory variables are as in table 8, with the addition of "Nb. Service 2G", "Nb. Service 3G", and "Nb. Service 4G". These are dummies indicating whether the carrier has service of the respective technology in a neighboring municipality. Each column corresponds to the subsample of the data where firms' best technology is as indicated in the column heading.

Appendix C Identity of Regulated Firms

The choice of the regulated firm in a given municipality occurred as follows. First, the country was divided into 131 "service areas". These varied substantially in size, from a single municipality to an entire code area, which include on average 83 municipalities. Within each of these service areas, the four large carriers would take turns selecting small numbers of municipalities where they would be subject to a requirement. This process occurs immediately after each spectrum auction and both the total number of municipalities to be selected and the number of municipalities chosen per turn are determined by the acquired license. Figure 15 in Appendix B shows the result of this process. This selection procedure may raise worries of selection into being regulated. Firms may select markets where they already have service, as that implies lower costs of compliance. Similarly, they may select markets that are close to markets they serve. The structural model fully accounts for service in a given market, but not for service in nearby markets.

Table 10 tests the hypothesis of no correlation between a firm's status as the regulated firm and that firm's infrastructure in neighboring markets. The unit of analysis for the models in table 10 is a firm-market pair, and only data from the June 2013 (the first period in the data) is used. The table reports estimation results of a linear probability model (included for the sake of interpretability) where the dependent variable is a dummy that takes the value 1 if the firm is regulated, and 0 otherwise. The explanatory variables are a constant and a set of dummies. The variable "2G Service" is equal to 1 if the firm provides 2G service in that market; "3G service" is analogously defined. "2G Service Nb." is equal to 1 if the firm provides 2G service in some neighboring market, and "3G Service Nb." is defined similarly. The results show that, conditional on the technologies offered by a firm in the market, which are included in the structural model, its infrastructure in neighboring municipalities has a small effect on the probability that the firm is regulated. The point estimates are in fact negative. These results suggest that there is no cause for concern that the difference in behavior between regulated and unregulated firms, which identifies the fine parameter φ in the structural model, is driven not by the regulation itself but by omitted differences in firms' neighboring infrastructure. Therefore, despite the importance of neighboring infrastructure shown in table 9, I omit these variables from the structural model, as doing so would likely not bias the inference regarding the effects of regulation and would increase the computational bur-

Table 10: Testing for Selection on Infrastructure in Neighboring Municipalities

	Regulated
Intercept	0.111
	(0.015)
2G Service	0.226
	(0.007)
3G Service	0.203
	(0.010)
2G Service Nb.	-0.016
	(0.016)
3G Service Nb.	-0.047
	(0.007)
R^2	0.137
Adj. R^2	0.137
Num. obs.	13796

Linear probability model. The unit of observation is a firm-municipality pair. The dependent variable is a dummy indicating whether or not the respective firm is regulated in the market. "2G Service" and "3G Service" are dummies indicating whether the firm has service of the respective technology in that market. "2G Service Nb." and "3G Service Nb." are dummies indicating whether the firm has service of the respective technology in a neighboring municipality.

den by several orders of magnitude.

Appendix D Restrictions on Value and Policy Functions

As in the main text, let T_θ be the first date after which parameters do not vary anymore and let T_m be the regulation deadline in market m. The assumptions of symmetry and quasi-stationarity imply the following restrictions on value functions (and policy functions):

- $V_0(s_1, s_r, s_-, t) = V_0(s_1, s_r, P(s_-), t)$ for any permutation P.
- $V_1(s_1, s_{-1}, t) = V_1(s_1, P(s_{-1}), t)$, for any permutation P.
- In $V_0(s_1, s_r, s_-)$, write $s_- = (s_-^1, s_-^2)$. If $s_r \geq 3$ and $\exists\ j \in \{1, 2\}$ such that $s_-^j \geq 3$, then $V_0(s_1, s_r, s_-) = V_0(s_1, s_-^j, s_r, s_-^{-j})$.
- If $s_1, s_r \geq 3$, then $V_1(s_1, P(s_r, s_-)) = V_0(s_1, s_r, s_-)$ for any permutation P.

- If $t > T_\theta > T_m$, $V_r(s_1, s_-, t) = V_r(s_1, s_-, T_\theta)$.
- If $T_\theta < T_m$, $T_\theta < t$ and $s_r \geq 3$, then $V_0(s_1, s_r, s_-, t) = V_0(s_1, s_r, s_-, T_\theta)$ and $V_1(s_r, s_-, t) = V_1(s_r, s_-, T_\theta)$.

Starting from the state space

$$\Omega := \{0, \ldots, \max\{T_m + 1, T_\theta\}\} \times \{0, 1\} \times \{0, 2, 3, 4\}^4 \ ,$$

we can reduce its cardinality using the restrictions above by mapping each state to an element of the equivalence class induced by these restrictions. I do so by using a variable-base number system to order the states in Ω and then mapping each state to the minimal (in this order) state in its equivalence class.

Appendix E Market Shares at the Code Area Level

This appendix justifies equation 8 in the main text. That equation, repeated here for convenience, states that market shares at the code-area level are given by

$$\mu_{jc\tau} = \sum_{m \in c} \omega_m \int \mu_{j\tau}(s_{m\tau}, v_{m\tau}, \xi_{c(m)\tau} + \eta_{m\tau}; \theta) dF(\eta_{m\tau}) \tag{8}$$

Let h_i denote the alternative chosen by a consumer i. Within a given code-area c, we have, by the Law of Total Probability

$$\mu_{jc} = \mathbb{P}(h_i = j) = \sum_{m \in c} \omega_m \mathbb{P}(h_i = j | m) = \sum_{m \in c} \omega_m \mu_j(s_m, v_m, \xi_{c(m)} + \eta_m; \theta) \ , \tag{20}$$

where I have dropped time subscripts, ω_m is the probability that the consumer comes from municipality m in code-area c, and $\mathbb{P}(h_i = j|m)$ is the probability that consumer i chooses j given that she comes from market m (and thus her demographic attributes come from a market-specific distribution). I will simplify the notation further and write simply $\mu_{jm}(\eta_m)$ instead of $\mu_j(s_m, v_m, \xi_{c(m)} + \eta_m; \theta)$.

I will show that $\text{plim}_{n \to \infty} \left(\sum_{m=1}^{n} \omega_{nm} \mu_m(\eta_m) - \sum_{m=1}^{n} \omega_{nm} \mathbb{E}[\mu_m(\eta_m)] \right) = 0$. When considering $n \to \infty$, I am considering an infinite sequence of markets with known μ_m (i.e., known characteristics (s_m, v_m)) and a triangular array of weights

ω_{nm}

$$\begin{array}{cccc} \omega_{1,1} & & & \\ \omega_{2,1} & \omega_{2,2} & & \\ \omega_{3,1} & \omega_{3,2} & \omega_{3,3} & \\ \vdots & \vdots & \vdots & \ddots \end{array}$$

such that $\omega_{nm} > 0$ for all n, m, $\lim_{n\to\infty} \omega_{nm} = 0$ for all m, and $\sum_{m=1}^{n} \omega_{nm} = 1$ for all n. Moreover, I assume that the η_m's are independent – see assumption 2.

By Chebyshev's Inequality

$$\begin{aligned} \mathbb{P}\left(\left|\sum_{m=1}^{n} \omega_{nm}\{\mu_m(\eta_m) - \mathbb{E}[\mu_m(\eta_m)]\}\right| > \varepsilon\right) &\leq \frac{1}{\varepsilon^2} \sum_{m=1}^{n} \omega_{nm}^2 \mathrm{Var}(\mu_m(\eta_m)) \\ &\leq \frac{1}{\varepsilon^2} \sum_{m=1}^{n} \omega_{nm}^2 \end{aligned}$$

where the second inequality follows because $\mu_m(\eta) \in [0, 1]$ for all η.

Now it suffices to show that $\sum_{m=1}^{n} \omega_{nm}^2 \to_n 0$. Assume without loss of generality that $w_{n1} = \max_{1\leq m\leq n} w_{nm}$. Then

$$\sum_{m=1}^{n} \omega_{nm}^2 \leq w_{n1}^2 + v(w_{n1}) \tag{21}$$

where

$$\begin{aligned} v(w_{n1}) := \max_{w_{n2},\ldots,w_{nn}} & \sum_{m=2}^{n} w_{nm}^2 \\ \text{s.t.} \quad & 0 < w_{nm} \leq w_{n1} \\ & \sum_{m=2}^{n} \omega_{nm} = 1 - w_{n1} \end{aligned}$$

Let k_n be the largest integer such that $k_n w_{n1} \leq 1 - w_{n1}$, i.e., $k_n = \left\lfloor \frac{1-\omega_{n1}}{\omega_{n1}} \right\rfloor$. Then

$$\begin{aligned} v(\omega_{n1}) &\leq k_n \omega_{n1}^2 + [1 - (k_n + 1)\omega_{n1}]^2 \\ &\leq \frac{1-\omega_{n1}}{\omega_{n1}} \omega_{n1}^2 + \left(1 - \frac{1-\omega_{n1}}{\omega_{n1}}\,\omega_{n1}\right)^2 \\ &= (1-\omega_{n1})\omega_{n1} + \omega_{n1}^2 \end{aligned}$$

It follows that $\lim_{n\to\infty} v(\omega_{n1}) = 0$ and, by 21, $\lim_{n\to\infty} \sum_{m=1}^{n} \omega_{nm}^2 = 0$.

Appendix F Conditional Value Functions are Linear in Parameters

In this section I will simplify notation by letting ω denote a generic state of the form $\omega = (t, r, s_f, s_{-f})$. Flow payoffs, net of the idiosyncratic shock, are given by

$$\pi(\omega) - \varphi r \mathbf{1}\{s_f < 3, T < t\} - c(a, s_f, z; \theta)$$

where $c(a, s_f, z; \theta)$ is the deterministic part of costs: $c(a, s_f, z; \theta) = \sum_{\{g' : g' > s_f\}}^{a} z_m' \theta_{g',t} + \mathbf{1}\left(s_f = 0\right) z_m' \theta_e$ if $a > s_f$ and zero otherwise. This is a linear function of parameters. Abusing notation slightly, write $c(a, s_f, z; \theta) = c(a, s_f, z)\theta$.

Define

$$g(a, \omega, z) := (\pi(\omega), c(a, s_f, z), r\mathbf{1}\{s_f < 3, T < t\})$$

and

$$\Psi := (1, \theta', \varphi)'$$

Then we have

$$\pi(\omega) - \varphi r \mathbf{1}\{s_f < 2, T < t\} - c(a, s_f) = g(a, \omega, z)\Psi$$

The value function satisfies the Bellman equation

$$V(\omega, \varepsilon_f) = \max_{a \in A(s_f)} g(a, \omega, z)\Psi + \varepsilon_f(a) + \delta \sum_{\omega'} V(\omega') F_P(\omega' | \omega, a)$$

where F_P denotes the state transitions induced by the equilibrium conditional choice probabilities P and

$$V(\omega') := \int V(\omega, \varepsilon_f) dG(\varepsilon_f)$$

Denote the equilibrium policy by $\sigma^*(s, \varepsilon_f)$. Then (using σ^* as shorthand for $\sigma^*(s, \varepsilon_f)$)

$$V(\omega, \varepsilon_f) = g(\sigma^*, \omega, z)\Psi + \varepsilon_f(\sigma^*) + \delta \sum_{\omega'} V(\omega') F_P(\omega' | \omega, \sigma^*)$$

Integrating both sides of this equation yields

$$\begin{aligned} V(\omega) = &\left(\int g(\sigma^*, \omega, z) dG(\varepsilon_f) \right) \Psi \\ &+ \int \varepsilon_f(\sigma^*) dG(\varepsilon_f) + \delta \sum_{\omega'} V(\omega') \int F_P(\omega'|\omega, \sigma^*) dG(\varepsilon_f) \end{aligned}$$

Let $\mathcal{C}(a, \omega)$ be the set of shocks $\varepsilon_f \in \mathbb{R}^{|A(s_f)|}$ such that $a = \sigma^*(\omega, \varepsilon_f)$. Then

$$\begin{aligned} \int g(\sigma^*, \omega, z) dG(\varepsilon_f) &= \sum_{a \in A(s_f)} \int_{\mathcal{C}(a,\omega)} g(\sigma^*, \omega, z) dG(\varepsilon_f) \\ &= \sum_{a \in A(s_f)} g(a, \omega, z) \int_{\mathcal{C}(a,\omega)} dG(\varepsilon_f) \\ &= \sum_{a \in A(s_f)} g(a, \omega, z) P(a|\omega) \end{aligned}$$

where here $P(a|\omega)$ are the equilibrium conditional choice probabilities.

Similarly,

$$\int F(\omega'|\omega, \sigma^*) dG(\varepsilon_f) = \underbrace{\sum_{a \in A(s_f)} F_P(\omega'|\omega, a) P(a|\omega)}_{F_P(\omega'|\omega)}$$

The term on the right hand side of this equation is simply the probability that the state moves from ω to ω', induced by the equilibrium conditional choice probabilities. I will denote that term by $F_P(\omega'|\omega)$.

Finally, observe that

$$\int \varepsilon_f(\sigma^*) dG(\varepsilon_f) = \sum_{a \in A(s_f)} \int_{\mathcal{C}(a,\omega)} \varepsilon_f(a) dG(\varepsilon) = \sum_{a \in A(s_f)} P(a|\omega) \mathbb{E}[\varepsilon_f(a) | a = \sigma(\omega, \varepsilon_f)]$$

It is well known that for the Type I Extreme Value distribution, $\mathbb{E}[\varepsilon_f(a)|a = \sigma(\omega, \varepsilon)] = \lambda(\gamma - \ln P(a|\omega))$, where γ is the Euler-Mascheroni constant. Therefore

$$\int \varepsilon_f(\sigma^*) dG(\varepsilon_f) = \lambda \sum_{a \in A(s_f)} P(a|\omega)(\gamma - \ln P(a|\omega))$$

Putting these pieces together, we have

$$V(\omega) = \left(\sum_a g(a,\omega,z)P(a|\omega)\right)\Psi + \lambda \sum_{a\in A(s_f)} P(a|\omega)(\gamma - \ln P(a|\omega))$$
$$+ \delta \sum_{\omega'} V(\omega')F_P(\omega'|\omega)$$

or

$$V(\omega) = \mathbb{E}_P[g(a,\omega,z)]\Psi + \lambda\gamma - \lambda\mathbb{E}_P[\ln P(a|\omega)] + \delta F_P(\omega)V$$

where $\mathbb{E}_P$ denotes an expectation with respect to a using the distribution over a defined by P, $F_P(\omega)$ is a row vector with the transition probabilities in state ω, and V a vector with the value function in each state ω.

We can now stack these equations. Let M_P denote the transition matrix induced by P, $M = [F_P(\omega'|\omega)]_{\omega,\omega'}$. Then[46]

$$V = \mathbb{E}_P[g(a,z)]\Psi + \lambda\gamma - \lambda\mathbb{E}_P[\ln P(a)] + \delta M_P V$$

From this equation we obtain

$$\begin{aligned} V &= (I-\delta M_P)^{-1}\Big\{\mathbb{E}_P[g(a,z)]\Psi + \lambda\gamma - \lambda\mathbb{E}_P[\ln P(a)]\Big\} \\ &= \lambda K(P) + (I-\delta M_P)^{-1}\mathbb{E}_P[g(a,z)]\Psi \end{aligned}$$

where $K(P) := (I-\delta M_P)^{-1}(\gamma - \mathbb{E}_P[\ln P(a)])$

The conditional value function is, by definition,

$$v(a,\omega) = g(a,\omega,z)\Psi + \delta\sum_{\omega'} V(\omega')F_P(\omega'|\omega,a) = g(a,\omega,z)\Psi + \delta F_P(\omega,a)V$$

where $F_P(\omega,a)$ is the distribution over ω' induced by taking action a in state ω when competitors follow P. Using the result above for V yields

$$\frac{v(a,\omega)}{\lambda} = \delta F_P(\omega,a)K(P) + \Big\{g(a,\omega,z) + \delta F_P(\omega,a)(I-\delta M_P)^{-1}\mathbb{E}_P[g(a,z)]\Big\}\lambda^{-1}\Psi$$

Finally, the analysis in the main text allows for municipality-specific equilibria P^m. As noted there, for the usual identification argument, based on the derivation above, to go through, it is sufficient that the map from market-level observables to the quasi-stationary symmetric Markov Perfect Equilibrium be

[46]In this equation, it is to be understood that the scalar $\sigma\gamma$ is added to all coordinates. The ω-th coordinate of $\mathbb{E}_P[g(a,z)]$ is equal to $\sum_{a\in A(s_f)} g(a,\omega,z)P(a|\omega)$. Similarly for $\mathbb{E}_P[\ln P(a)]$.

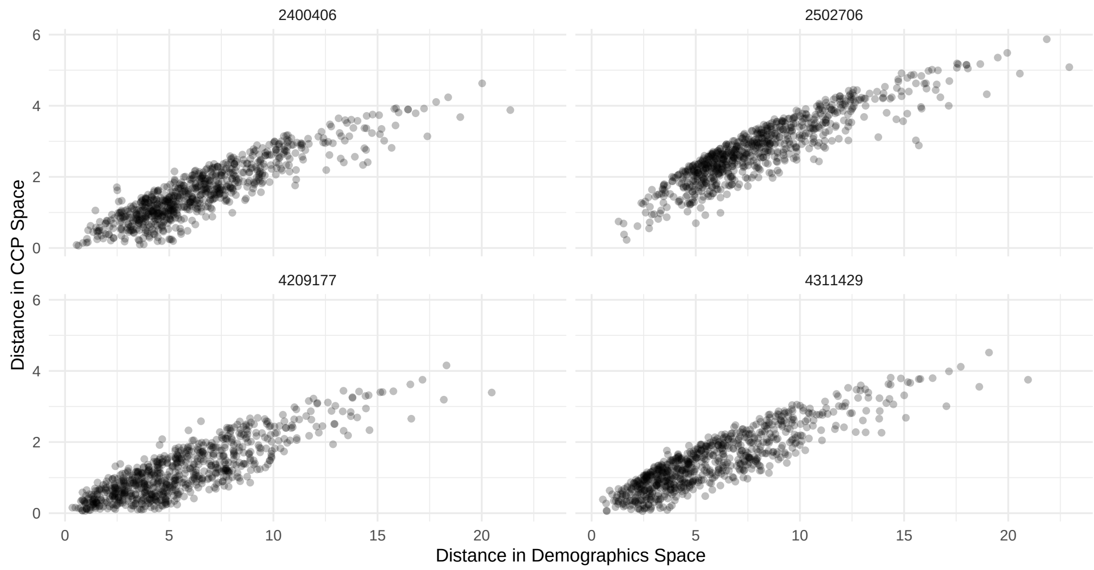

Figure 17: Equilibrium continuity.

Each panel shows data for one of four randomly sampled municipalities. Each dot in a scatterplot corresponds to one of the other 742 municipalities that are part of the 3G counterfactuals. The x-axis is the Euclidean distance of the demographics of the two municipalities (a vector containing the time series of GDP per capita, population, and area). On the y-axis is the Frobenius distance of the conditional choice probability matrices of the two municipalities.

continuous. Figure 17 provides evidence in favor of that. Each panel shows data for one of four randomly sampled municipalities. Each dot in a scatterplot corresponds to one of the other 742 municipalities that are part of the 3G counterfactuals. The x-axis is the Euclidean distance of the demographics of the two municipalities (a vector containing the time series of GDP per capita, population, and area). On the y-axis is the Frobenius distance of the conditional choice probability matrices of the two municipalities.[47] These scatterplots show that as the distance of demographics goes to zero so does the distance of CCPs, as desired.

Appendix G Estimates of Dynamic Parameters

Table 11 shows estimates of the dynamic parameters of the model, i.e. the entry and technology upgrade cost parameters and the fine for non-compliance with the regulation.

[47] Conditional choice probabilities are represented in a matrix of as many rows as (minimal, see Appendix D) states and one column per technology/action.

Table 11: Dynamic Parameter Estimates

Technology	Variable	Year	Estimate	2.5 Quantile	97.5 Quantile
Entry	Area	All	-0.138	-0.183	-0.068
Entry	Intercept	All	6.142	5.669	6.424
3G	Area	2014	0.572	0.498	0.650
3G	Area	2015	0.382	0.271	0.463
3G	Area	2016	0.442	0.337	0.525
3G	Area	2017	0.548	0.439	0.638
3G	Area	2018	0.425	0.301	0.522
3G	Intercept	2014	0.274	-0.203	0.742
3G	Intercept	2015	1.307	0.773	1.974
3G	Intercept	2016	0.484	-0.006	1.132
3G	Intercept	2017	-0.052	-0.650	0.638
3G	Intercept	2018	-0.177	-0.816	0.546
4G	Area	2014	1.082	0.993	1.222
4G	Area	2015	0.593	0.544	0.699
4G	Area	2016	0.362	0.282	0.426
4G	Area	2017	0.182	0.110	0.239
4G	Area	2018	0.030	-0.066	0.107
4G	Intercept	2014	0.130	-0.218	0.270
4G	Intercept	2015	0.012	-0.697	0.292
4G	Intercept	2016	-1.152	-1.530	-0.684
4G	Intercept	2017	-0.983	-1.329	-0.502
4G	Intercept	2018	0.499	0.033	1.092
	Fine	All	1.510	1.358	1.740

This table displays estimates of the dynamic parameters – the entry and technology upgrade cost parameters and the fine for non-compliance with the regulation. A combination of the Technology, Variable, and Year columns defines a parameter in the model. For example, the row identified by 4G, Area, and 2016 indicates to what extent the logarithm of a municipality's area increases the cost of introducing 4G technology. The Estimate column shows the point estimate and the final two columns together define a 95% confidence interval for the respective parameter. The confidence interval is calculated by bootstrap, which is performed at the municipality level.

Appendix H Equilibrium Effects of Coverage Requirements: Policy Function Adjustments

As in the main text, here I consider moving from the situation in which only the regulated firm responds to the regulation to a scenario with regulation. For each municipality considered in the counterfactuals and each state of the game, I compute the difference in upgrade probabilities. Then, focusing on unregulated firms first, I average those differences across states of the game conditioning on firm technology, model period, and the technology of the regulated firm. Figure 18 shows the results.

We see that the move to the equilibrium with regulation leads to reductions in the upgrade probabilities of unregulated firms. This is due to the the anticipated tougher competition in the market following the entry or technology upgrade by the regulated firm. These effects are more pronounced as time passes. As the regulation deadline approaches, the upgrade by the regulated firm and the consequent reduction in flow profits become imminent, implying a stronger reduction in unregulated firms' incentives to enter or upgrade. The effect of the move to equilibrium is also stronger in those states where the regulated firm is out of the market. In those situations, compliance with the regulation will lead to a larger decrease in unregulated firms' flow profits relative to the case where the regulated firm is already active. Finally, the largest effects are observed in the behavior of "weak" incumbents – i.e., those with 2G technology. Those are the ones that experience the largest reductions in flow profits from the added competition, as inactive firms are only affected if they enter the market and incumbents with advanced technologies are in a stronger position to compete with the regulated firm.[48]

Figure 19 shows analogous results for the regulated firms, conditioning on firm technology, model period, and the number of competitors in the market. Upgrade probabilities of regulated firms are lower in the equilibrium with regulation than after their unilateral response to the regulation. Entry and technology upgrade have value as deterrents of upgrades by competitors. As the regulation decreases the probability of such upgrades, it reduces this deterrence incentive, leading to the results in figure 19. These effects are (mostly) increasing over time due to the similar pattern shown in 18. Finally, note that the

[48]The figure also includes a plot for when the unregulated firms have 4G technology as a sanity check. The changes in probability in that case are zero by construction – firms have no decision to make once they have 4G technology.

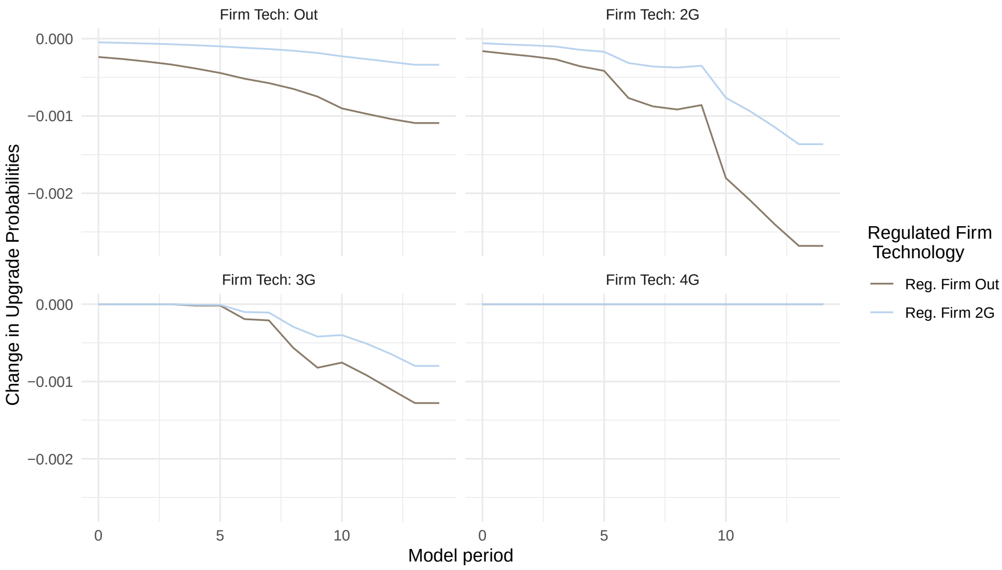

Figure 18: Equilibrium Effects: Changes in the Policy Functions of Unregulated Firms.

Difference between upgrade probabilities in the equilibrium with regulation and when only the regulated firm responds to the regulation.

reductions in regulated firms' upgrade probabilities are decreasing in competition. This is in line with the previous discussion, as deterrence incentives are less relevant the more competition there is in the market.

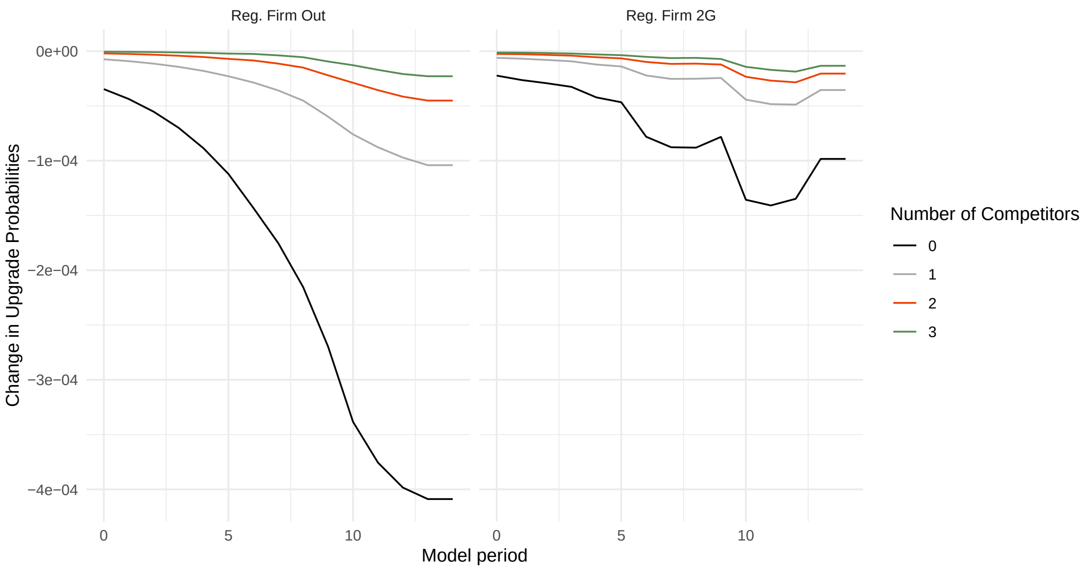

Figure 19: Equilibrium Effects: Changes in the Policy Functions of Regulated Firms.

Difference between upgrade probabilities in the equilibrium with regulation and when only the regulated firm responds to the regulation.