Regulation and Service Provision in Dynamic Oligopoly: Evidence from Mobile Telecommunications

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Abstract

I study the effect of universal service regulation on the speed of roll-out of new mobile telecommunications technologies. I develop a dynamic game of entry and technology upgrade under regulation, which I estimate using new data on mobile technology availability in Brazilian municipalities. Counterfactuals show that the regulation accelerated the introduction of 3G technology to lower-income, rural municipalities by just under 1 year, on average, while reducing firms' aggregate profits by 10%. The regulation can act as a commitment device and marginally deter technology deployment by unregulated firms. A subsidy auction achieves similar acceleration of 3G deployment at 21% of the cost, likely increasing welfare.

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1 Introduction

Firms' inability to appropriate the consumer surplus generated by their new goods and services may lead to underprovision. This possibility is particularly salient in industries featuring large sunk costs and in disadvantaged areas, where the prospects of recouping these costs are dim. Concerns regarding service underprovision have led to regulatory interventions aimed at achieving universal service in the postal, healthcare, airline, and telecommunications industries.[1] The purpose of this paper is to evaluate the effects and design of universal service regulation in the mobile telecommunications industry.

Service underprovision has been a historical concern in the telecommunications industry (Wu (2010)) because of the substantial investment costs required for network expansion and the many benefits associated with access to telecommunications services.[2] Countries ranging from Nigeria to the United States regulate the roll-out of new mobile telecommunications technologies to ensure their diffusion to low-income, rural, or isolated areas. A common regulatory tool is called coverage requirements. A coverage requirement tasks a single firm (the *regulated firm*) with providing service of a specific technology in a given area by a date set by the regulator, while imposing no constraints on the behavior of its competitors (the *unregulated firms*).

When deciding whether to impose such a requirement, the regulator faces the following trade-off. On the one hand, the regulation presumably generates service in areas that would not be served in its absence and accelerates the introduction of new technologies in other areas, thus increasing the discounted stream of consumer surplus. On the other hand, coverage requirements impose a cost on the regulated firm, for it is required to enter a market or upgrade its technology when it might not have done so in the absence of regulation. Furthermore, equilibrium responses to the regulation may imply that the presumption that it increases or accelerates service provision is unwarranted. A

[1]For example, in the United States the USPS is subject to a Universal Service Obligation; the HRSA runs the Medicare Rural Hospital Flexibility Program; the DOT runs the Essential Air Service and Small Community Air Service Development Program, and The Universal Service Administrative Company spent more than seven billion dollars in 2022 in subsidies for high-speed broadband access (see Q8 here – last accessed July 2025.).

[2]Telecommunications services have been shown to have positive effects on economic growth (Roller and Waverman (2001), Czernich, Falck, Kretschmer, and Woessmann (2011)); labor productivity (Bertschek and Niebel (2016), Akerman, Gaarder, and Mogstad (2015)); market efficiency (Jensen (2007)), employment rates (Hjort and Poulsen (2019), Chiplunkar and Goldberg (2022)), health outcomes (Van Parys and Brown (2024)) and risk-sharing (Jack and Suri (2014)). See Aker and Mbiti (2010) and Hjort and Tian (2024) for reviews of this literature.

coverage requirement is a credible commitment by the regulated firm to provide service of a relatively advanced technology. This commitment may deter its competitors from entering or upgrading their technologies, a mechanism I refer to as the *deterrence effect* of regulation. This effect may incentivize the regulated firm to delay its own introduction of the new technology to benefit from decreasing costs of adoption. These equilibrium responses may undermine or even reverse the regulation's intended impact.[3]

To quantify the effects of coverage requirements and alternative policies on the speed of roll-out of third and fourth generation mobile telecommunications technologies, I develop and estimate an empirical dynamic game of entry and technology upgrade under regulation. A model of entry and technology upgrade in the mobile telecommunications industry must allow for changes over time in consumers' preferences for and the cost of introducing new technologies. The time-varying nature of demand and costs, coupled with the regulation deadline, render the environment non-stationary.

Existing empirical models of firm behavior in non-stationary environments typically assume a finite horizon and parameterize the dynamics of time-varying model primitives, thus enabling the application of backward induction solution and estimation routines. I instead do not restrict the dynamics of model primitives during the sample period, assume they stabilize at the end of the sample, and focus on what I call Quasi-Stationary Markov Perfect Equilibria (QS-MPE). Essentially, QS-MPE have a non-stationary phase followed by a stationary phase. This structure allows me to adapt existing techniques used to estimate stationary dynamic games to a non-stationary setting.

I estimate the model using new panel data on mobile technology availability at the municipality level in Brazil from March 2013 to March 2022, focusing on a set of relatively low-income and rural municipalities, i.e., those most susceptible to lack of service provision. In these municipalities, one firm is subject to a requirement to provide 3G service by varying deadlines. Reduced-form analysis of these data in Section 2 shows that the regulation does affect firm behavior. Regulated firms are more likely to enter a market and upgrade their technologies than unregulated firms. The latter are less likely to enter a market or upgrade their technologies when the regulated firm is yet to satisfy its coverage requirement, a pattern consistent with the deterrence effect outlined above.

[3] Appendix A provides a simple example, based on Fudenberg and Tirole (1985), in which a requirement to adopt a new technology leads to delay in its adoption.

Counterfactual exercises based on the estimated model show that in the absence of regulation, the arrival of 3G technology to this set of relatively rural and underdeveloped markets in Brazil would have happened 0.94 years later, on average. There is substantial heterogeneity around this mean, meaning that the regulation has little consequence in some markets, while it is crucial in others: the tenth percentile of the distribution of the acceleration in the roll-out of 3G due to the regulation is 0.08 years, while the ninetieth percentile is 2.43 years. I find that the regulation reduces firms' aggregate expected profits by $3{,}221$ million BRL, or 10% of their profits in the absence of regulation. Moreover, 51% of these costs are borne by regulated firms that were inactive in the relevant market at the start of the data. Many of these markets do have service by other firms at the start of the data. These results suggest substantial inefficiencies in the allocation of coverage requirements.

Motivated by these cost incidence results, I evaluate the reallocation of coverage requirements by means of a subsidy auction. Specifically, for each firm-market pair, I use the estimated model to compute a transfer that would make the firm willing to be regulated in that market. In each market, I assign the requirement to the firm that demands the lowest transfer. I find that the identity of the regulated firm differs from the status-quo in 65% of the markets. The aggregate subsidy amounts to only 21% of the cost of coverage requirements reported above, and the effects on the speed of 3G and 4G roll-out are virtually unchanged. However, subsidy auctions are not without cost. I show that the choice between coverage requirements and subsidy auctions features a trade-off between cost-efficiency and competition. I use the model to compute a lower bound on the gains in consumer surplus from increased competition that would make coverage requirements more efficient than subsidy auctions, in spite of their relative cost-inefficiency. I find that lower bound to be 35% of the mean consumer expenditure in this market. This is substantially larger than the relevant estimates in the literature, thus suggesting that subsidy auctions are more efficient than the status-quo implementation of coverage requirements in this setting.

Related literature. This paper relates to the literature studying how regulation affects market structure and market outcomes in dynamic environments. Ryan (2012) shows that stricter environmental regulation increases entry costs, thus decreasing both the number of firms in the market and consumer surplus. Gowrisankaran, Lucarelli, Schmidt-Dengler, and Town (2011) study the effect of the Medicare Rural Hospital Flexibility Program on health care provision in rural areas of the United States, and show that the program expanded coverage but had an adverse effect on consumer welfare due to provisions that limited the size and scope of regulated hospitals. Dunne, Klimek, Roberts, and Xu (2013) study the effects of entry subsidies under the Health Professional Shortage Areas program on local market structure. Most closely related to this paper, Fan and Xiao (2015) estimate a model of telephone service provider entry into local markets in the US and use the model to evaluate the extent to which different entry subsidies would have reduced monopolization. I extend this literature by modeling not only entry but also the set of technologies firms offer and by studying the equilibrium effects of asymmetric universal service regulation on market outcomes.

This paper also relates to the empirical literature on technology adoption and in particular to the modeling of industry dynamics in non-stationary environments. Schmidt-Dengler (in press) studies US hospitals' decisions to adopt magnetic resonance imaging (MRI). Igami (2017) studies how cannibalization, preemption, and incumbents' cost advantages shape firms' adoption of new generation hard disk drives. This paper adds to this literature by studying how regulation affects technology adoption. Moreover, this paper differs from prior literature in its handling of non-stationarity. Both of the aforementioned papers are instances of the finite-horizon approach previously discussed.[4] I compare the QS-MPE and finite horizon approaches to non-stationarity in greater detail in Section 3.

My work also relates to the literature on the telecommunications industry, particularly on network investment decisions and regulation. Björkegren (2019) studied consumer adoption of mobile phones in Rwanda, and in that context evaluated the welfare effect of rural coverage requirements imposed on the dominant mobile network operator. I add to this work by modeling how firms respond to regulation, and moreover by doing so in an oligopoly context. My work also relates to an earlier, mostly theoretical, literature on universal service obligations, such as Armstrong (2001), Choné, Flochel, and Perrot (2002), and Valletti, Hoernig, and Barros (2002). This paper is the first to empirically quantify the effect of such regulation on service provision and the introduction of new technologies. Recent work by Lin, Tang, and Xiao (2021), Marcoux (2022), and Elliott, Houngbonon, Ivaldi, and Scott (2025) also studies infrastructure in-

[4]The finite horizon is assumed in Igami (2017) and derived from a result of full adoption in finite time in Schmidt-Dengler (in press).

vestment by mobile network operators. This paper differs from these contributions in its modeling of and focus on regulation, and in treating infrastructure investment decisions as the outcome of a dynamic rather than static game.

The rest of the paper is organized as follows. Section 2 introduces the institutional setting, data, and presents preliminary evidence on the effects of coverage requirements on firm behavior. Section 3 introduces a model of entry and technology upgrade with regulation. Section 4 discusses the identification and estimation of the model, and estimation results are presented in Section 5. Section 6 presents counterfactual analyses on the effects of coverage requirements, the equilibrium responses to regulation, and the implications of subsidy auctions. Section 7 concludes.

2 Institutional Setting and Data

2.1 Institutional Setting

To operate, mobile network operators must acquire spectrum licenses, which allow them to transmit radio signals in a given frequency band and geographic area. Coverage requirements are a common condition of spectrum licenses. A coverage requirement, for the purposes of this paper, is an obligation imposed on a firm to provide service in a municipality by a deadline set by the regulator and with a minimum technological requirement (3G or 4G).[5] Mobile network operators have been subject to coverage requirements in Brazil since the first spectrum auction in 2007.[6]

The data used in this paper covers the period from March 2013 to March 2022. During that period, the Brazilian mobile telecommunications market had 7 mobile network operators (MNOs), which are carriers that operate their own network infrastructure. Of the 7 MNOs, the four largest (Claro, Oi, Tim, and Vivo) have held licenses covering the entire national territory since 2010.[7] The other three MNOs have regional footprint.[8] There are also very small mobile

[5] The requirement is considered to be satisfied if service is available in 80% of the municipality's territory.

[6] The 2007 auction granted spectrum licenses for the roll-out of 3G technology. Prior to that auction, spectrum licenses for 2G services were granted by administrative concession.

[7] Oi was the latest of the four to obtain licenses covering the entire territory, by means of its acquisition of Brasil Telecom in 2009. The other three MNOs essentially achieved nationwide coverage with the 2007 auction. *Oi* was sold to a consortium consisting of the other three large MNOs in 2020, and its customers and assets were transferred to the acquiring firms in April 2022. My analysis abstracts from this change in ownership.

[8] These are, in increasing order of size, Sercomtel, Algar, and Nextel. Sercomtel operates in

virtual network operators (MVNOs), which are carriers that do not own infrastructure and instead rent network space from the MNOs.

Every municipality in Brazil is covered by a coverage requirement, meaning that at least one carrier, but in some cases more, is subject to a requirement in that municipality. I focus on municipalities with fewer than 30,000 inhabitants, where a single carrier is required to introduce 3G technology. I focus on this set of municipalities because that is where the regulation is most likely to be binding, as deploying service in larger markets is more likely to be profitable. All the four large MNOs are subject to coverage requirements in some of these markets.[9]

The motivation for coverage requirements rests on two premises. First, mobile telecommunications services generate substantial consumer welfare gains.[10] In the words of the Brazilian telecommunications regulator:[11]

> [Mobile telecommunications technologies] *create employment opportunities, improve the education system, increase firm productivity, allow access to digital public services, among other benefits.*

Second, for the intervention to be justified it must be that firms do not internalize the entirety of the surplus generated by their entry and new technologies. This premise is supported by recent empirical results in related settings.[12]

The regulator enforces coverage requirements in multiple ways. First, carriers are required to deposit financial guarantees with the regulator, which the regulator may draw upon in case of noncompliance. Noncompliant carriers can also have their licenses revoked, in which case they would also be charged the value paid for their license in proportion to the time used. The regulator can also impose additional fines on noncompliant carriers. [13]

only 2 municipalities. Algar operates in 4 states, 7 code-areas, and 139 municipalities. Nextel operates in all states and code-areas, but only 1435 municipalities. Its market share is mostly negligible. Nextel's maximum market share over time exceeds 5% in only 3 code-areas, and 1% in only 8. A code-area is a geographic area collecting large numbers of municipalities that share the same regional code in their phone numbers.

[9]Though these coverage requirements target the introduction of 3G, the regulated firm is considered to comply with the regulation if it deploys 4G instead.

[10]See, e.g., the references in footnote 2.

[11]See `https://web.archive.org/web/20200920182918/https://www.anatel.gov.br/setorregulado/telefonia-movel#expand` (last accessed in May 27, 2025).

[12]Using data on the French mobile telecommunications market, Elliott et al. (2025) estimate that the marginal social value of spectrum is five times firms' willingness to pay for it. Studying residential broadband, Nevo, Turner, and Williams (2016) estimate a large gap between social and private incentives to invest in infrastructure.

[13]The final important piece of institutional detail is the process by which the identity of the regulated carrier in each market is determined. I discuss that process in appendix C.

2.2 Data

The main dataset used in this study comes from ANATEL, the Brazilian telecommunications regulator. The data records the technologies (2G, 3G, and 4G) offered by each mobile network operator in all the 5,770 Brazilian municipalities at a monthly frequency, from March 2013 to March 2022. Figure 18 in appendix B illustrates the data. The second important piece of data coming from ANATEL is the identity of the regulated firm in each municipality and the corresponding deadline for compliance with the regulation.

ANATEL also provides data on the numbers of subscribers to mobile telecommunications services. These data are available for each carrier-technology combination at a monthly frequency, first at the code-area level from February 2009 to December 2018 and then at the municipality level from January 2019 to January 2025.[14] To align the time-series coverage with the deployment data, I use these data up to and including June 2022. Figure 1 shows the evolution of the total number of subscribers in the country by technology. The figure shows that 2G has been in decline over the period, initially being overtaken by 3G. Moreover, 3G reaches a peak towards mid-2015, around the time when the growth of 4G accelerates. To the extent that these patterns are driven by consumer preferences, they shape firms' incentives to introduce new technologies. The empirical model introduced below will account for this pattern in demand by allowing demand-side parameters to vary over time.

I complement the ANATEL data with a number of datasets from IBGE, Brazil's national statistical institute. First, I use municipality characteristics such as GDP per capita, population, and land area. Summary statistics on these variables are shown in Table 1, separately for municipalities in and out of the estimation sample. This illustrates that the focus of the analysis is on a set of municipalities that are relatively low-income, small in terms of population and land area, and more likely to be rural. Second, I use the 2017-2018 Family Budget Survey, which provides information on household income, size and individual expenditure on mobile telecommunications services.[15] Summary statistics on these data are provided in Table 2. Third, I use the 2010 Population Census to obtain the distribution of individual-level demographics at the municipality level.

I will model the behavior of the four large carriers in Brazil, whose aggre-

[14] There are 67 code-areas in Brazil and 5,770 municipalities.

[15] This survey is the *Pesquisa de Orçamentos Familiares* (POF).

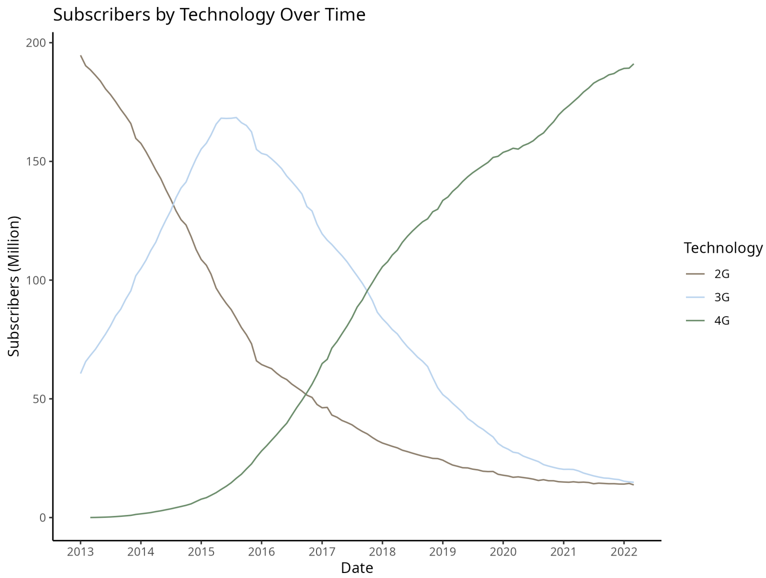


Figure 1: Total number of subscribers, by technology.

Calculated from ANATEL's data on subscription to mobile telecommunications services.

Table 1: Summary Statistics – Municipality Characteristics

Panel A: In-Sample Municipalities

Variable	N	Mean	Std. Dev.	p10	p90
GDP Per Capita	3,461	16,680.91	16,512.45	5,685.95	31,418.20
Population	3,461	10,932.99	7,654.35	2,854.70	22,957.00
Area	3,461	916.15	2,077.03	106.02	1,910.01
Rural	3,461	0.74	0.44	0.00	1.00

Panel B: Out of Sample Municipalities

Variable	N	Mean	Std. Dev.	p10	p90
GDP Per Capita	2,109	19,778.33	19,370.97	6,013.44	36,468.67
Population	2,109	80,375.58	350,684.53	4,374.92	136,152.34
Area	2,109	2,526.92	8,622.76	139.56	4,581.71
Rural	2,109	0.42	0.49	0.00	1.00

Values are averaged over time. GDP per capita is deflated to 2013 BRL. Area is in squared kilometers.

Table 2: Summary Statistics – Mobile Expenses and HH Characteristics

Variable	N	Mean	Std. Dev.	p10	p90
Mobile Spending	75,004	27.69	33.77	3.28	54.59
HH Income PC	75,004	2,065.50	2,106.83	593.19	4,052.78
No. Residents	75,004	2.23	1.04	1.00	4.00
Urban	75,004	0.81	0.39	0.00	1.00

Data from the 2017-2018 Family Budget Survey. The unit of observation is an individual. Mobile spending is the sum of expenditures on voice and data plans, pre-paid expenditure, and SIM cards, in BRL (deflated to 2013). "HH Income PC" is the monthly per capita income in the household. "No. Residents" is the number of residents in the household. "Urban" is a dummy that is equal to 1 if the individual lives in an urban area.

gate market share is 98.88% on average over the sample period. I thus restrict attention to areas where the regional carriers are either absent or have negligible market shares. This is the majority of the country.[16] Moreover, as mentioned above, I focus on municipalities where a single carrier is subject to a 3G coverage requirement. The resulting estimation sample contains 3,461 municipalities. The deadline for compliance with the coverage requirement varies across municipalities in this group, ranging from April 2014 to December 2019. For counterfactual exercises I focus on the subset of municipalities with an April 2016 or December 2019 regulation deadline. There are $1,872$ of those. Because entering a market or upgrading a technology is a non-trivial investment that likely involves some time to build, I use data on a six-month frequency rather than monthly. The unit of observation is thus a municipality-carrier-period, where the length of a period is six months.

2.3 Patterns of Technology Deployment

Figures 2 and 3 summarise the technology deployment data. Figure 2 shows, on the left panel, the average number of firms in a market and, on the right panel, the average number of products, where a product is a firm-technology combination and averages are taken across municipalities. Municipalities are split into those above and below the median population.

Figure 2 illustrates important patterns. First, there is substantial entry and upgrade activity in these markets. Second, firm behavior is heterogeneous across municipalities, as illustrated by the differences in level and slope be-

[16]Specifically, I drop code-areas where the aggregate market share of the smaller carriers ever exceeded 5%. This excludes 6 code-areas out of 67.

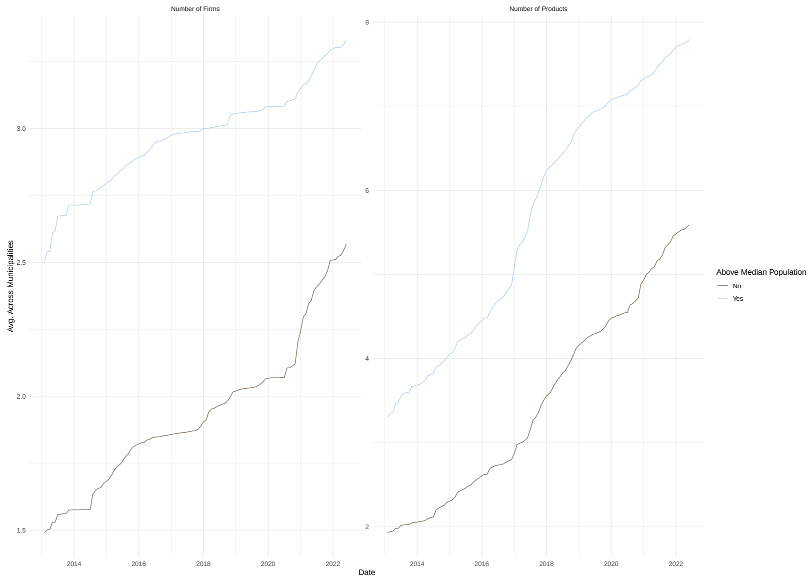


Figure 2: Average numbers of firms and products (firm-technology pairs) over time, separately for municipalities with population above and below the median.

tween markets with population above and below the median. Third, these curves describe rather complex dynamics, being at times concave, at times convex. The patterns in Figure 2 inform the model of section 3 in two important ways. First, a satisfactory model of firm behavior in this industry must account for market heterogeneity. Second, modeling the dynamics illustrated by these figures requires a non-stationary model, as a stationary model would likely generate concave curves for these outcomes.[17]

Figure 3 is a first step in describing the behavior of regulated and unregulated firms. The left panel of figure 3 shows the fraction of markets where the regulated firm has complied with the regulation. The panel on the right shows the fraction of markets where some unregulated firm provides 3G technology or better. The figure shows that regulated firms deploy new technologies at a faster pace than unregulated firms, suggesting that the regulation plays a role in the evolution of service availability. Note, however, that it is not the case that unregulated firms refrain from providing and upgrading service in these markets. Indeed, the slope of the curves in the right panel is far from zero. A second feature of these curves further indicates that the regulation affects firm

[17]Suppose a firm enters a market. If flow profits are decreasing in the degree of competition, this reduces potential entrants' incentives to enter, which leads to a concave curve for the number of firms in the market. The same argument applies to the number of products, with the potential counteracting effects that entry or upgrade by a firm may increase the probability of an upgrade by that very firm and by other incumbents.

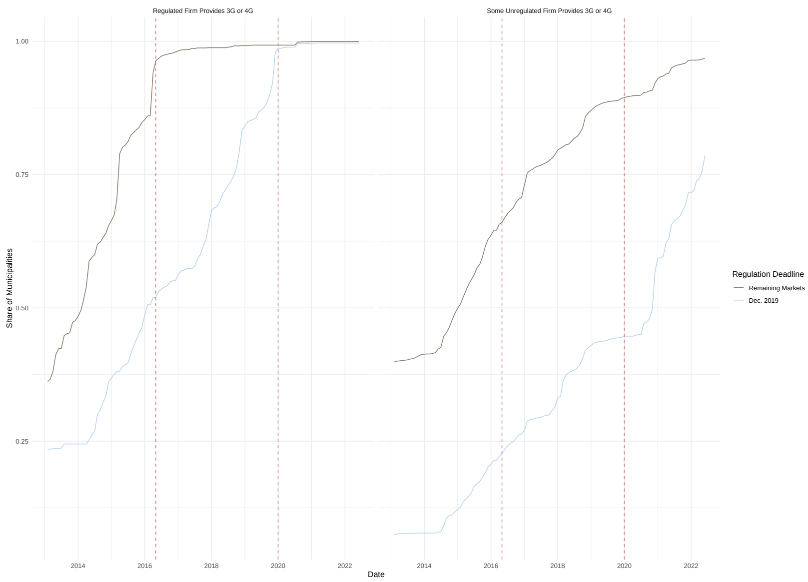


Figure 3: Technology availability over time.

The figure in the top-left shows the fraction of municipalities where 3G or 4G service is available. The one on the top-right shows the same quantity for 4G. The figure in the bottom-left shows the fraction of municipalities where the regulated firm has introduced 3G or 4G technology. The one in the bottom-right shows the fraction of municipalities where some unregulated firm has introduced 3G or 4G technology. Municipalities are split into two groups: those with a coverage requirement with a December 2019 deadline and all the others.

behavior. Namely, regulated firms race to comply with the regulation, as indicated by the steep increase in the share of markets where the regulated firm has complied with the regulation just before the regulation deadlines, which are indicated by the vertical red dashed lines.

The differential rate at which regulated and unregulated firms introduce 3G or 4G technology depicted in Figure 3 can be decomposed into a differential rate of technology introduction in the absence of regulation and two regulation effects: an effect on the behavior of the regulated firm and an effect on the behavior of unregulated firms. The first of these effects is likely positive, and due to the cost of noncompliance with the regulation. The second, however, may be positive or negative. On the one hand, unregulated firms may be less likely to enter new markets or upgrade their technologies because they know that the regulated firm will introduce 3G by the requirement deadline. This knowledge of tougher future competition may reduce their incentives to invest. I call this the *deterrence effect*. Appendix A provides a theoretical example, based on Fudenberg and Tirole (1985), in which the deterrence effect implies that regulation leads to a delay in technology adoption. On the other hand, technology upgrades may be strategic complements, in which case the regulation may increase unregulated firms' incentives to upgrade their technologies. I call this the *strategic complementarity effect*.

Figure 3 establishes a clear difference in behavior between regulated and unregulated firms, but it does not shed light on how the regulation affects firm behavior across these two regulatory contexts. To do so, I leverage variation in firms' regulatory status and the compliance status of regulated firms to estimate linear probability models of firms' technology upgrade decisions, while controlling for market structure and municipality characteristics. Table 3 reports the results. The columns correspond to different subsamples according to firms' best technologies. The dependent variable is a dummy equal to one if and only if firms choose to upgrade their technology. The key explanatory variables are the dummies "Regulated", "Regulated Competitor - Out", and "Regulated Competitor - 2G". The first of these variables is equal to 1 when the firm is regulated, and 0 otherwise. The second is equal to 1 when the firm faces a regulated competitor that is out of the market. The third is equal to 1 when the firm faces a regulated competitor that has 2G technology. The omitted category includes the cases when the regulated firm has complied and when the regulated firm is not one of the four large carriers.[18] The models control for the logarithms of GDP per capita, population, and land area, and include the number of competitors with each technology.[19] [20]

There are two key results in Table 3. First, regulated firms that have not satisfied their coverage requirements are more likely to enter the market and

[18]Because I restrict the sample to regions where the small firms have always had negligible market shares, I interpret both of these cases as no firm being influenced by regulation.

[19]A firm's infrastructure in neighboring municipalities may also be important for their upgrade choices. I test for this mechanism in appendix B. I do find that having service in a neighboring municipality increases the probability of entry and technology upgrade. However, the other coefficients change only slightly, if at all. This suggests that the choice of the regulated firm is uncorrelated with their local network infrastructure. Indeed, in appendix C, I show that service in neighboring municipalities does not change the probability that a firm is regulated in a given market. Firms' presence in neighboring municipalities will not be included as a state variable in the structural model, as doing so would increase the computational burden significantly. The descriptive results discussed here, however, suggest that this omission will not bias the inference regarding the effect of the regulation.

[20]The models reported in Table 3 also control for unobserved municipality-level heterogeneity in a heuristic manner. Specifically, I run a regression of the number of products on municipality and time fixed effects and GDP per capita, population, and area. I then assign municipalities to groups according to evenly-spaced quantiles of the distribution of the residuals of this regression, and include group fixed-effects in the regression specification. This is similar to Collard-Wexler (2013). Appendix B shows the results obtained estimating the model in Table 3 without the group fixed effects. The coefficients on the number of competitors are affected the most by the group fixed effects. The other coefficients change only slightly.

Table 3: Entry/Upgrade Models

	Out	2G	3G
Regulated	0.117	0.167	-0.039
	(0.008)	(0.008)	(0.005)
Regulated Competitor - Out	-0.014	-0.006	-0.040
	(0.003)	(0.007)	(0.009)
Regulated Competitor - 2G	0.001	-0.022	-0.065
	(0.005)	(0.004)	(0.007)
No. Competitors 2G	-0.028	-0.043	-0.029
	(0.006)	(0.004)	(0.004)
No. Competitors 3G	-0.054	-0.086	-0.061
	(0.005)	(0.007)	(0.006)
No. Competitors 4G	-0.084	-0.133	-0.103
	(0.006)	(0.010)	(0.007)
Log GDP PC	-0.002	-0.005	0.006
	(0.002)	(0.003)	(0.003)
Log Pop.	0.086	0.119	0.064
	(0.009)	(0.006)	(0.007)
Log Area	-0.009	-0.015	-0.011
	(0.001)	(0.002)	(0.002)
$\bar{Y}$	0.031	0.074	0.082
Num. obs.	98632	52728	54914

Linear probability models. The dependent variable is a dummy equal to 1 if a technology upgrade is observed. The explanatory variables are, in this order: a dummy that is equal to 1 if the firm is regulated, a dummy that is equal to 1 if the firm faces a regulated competitor that is out of the market, a dummy that is equal to 1 if the firm faces a regulated competitor that has 2G technology, the numbers of competitors with 2G, 3G and 4G technology, and the natural logarithms of GDP per capita, population, and municipality area. Each column corresponds to the subsample where firms' best technology is as indicated in the column heading. Standard errors are clustered at the code-area level.

upgrade their technologies than firms that are not subject to regulation.[21] Second, unregulated firms are less likely to enter and upgrade their technologies when the regulated competitor is either out of the market or has 2G technology. These results show that the regulation indeed accelerates the introduction of the new technology by regulated firms, but also that it delays the introduction of new technologies by unregulated firms. This is consistent with the deterrence effect outlined above. Given these two counteracting effects, it is a priori unclear whether the regulation accelerates the introduction of new technologies. These results further motivate studying the effect of regulation through the lens of a model featuring strategic interactions.

The rest of the paper is concerned with developing tools that allows us to quantify the net effect of regulation on the time to introduction of new mobile telecommunications technologies, as well as to decompose that effect into its components discussed above. This requires a model of entry and technology upgrade decisions under regulation, that I turn to next.

3 Model

This section describes a dynamic game in which firms make entry and technology upgrade decisions in a regulated oligopoly. The model operates at the level of a municipality. I will use municipality and market interchangeably. Each market has four potential firms. Time is discrete and the horizon is infinite. Within a period, the timing of the game is as follows. In the beginning of each period t incumbent firms earn flow profits. Each firm then privately observes action-specific cost shocks, and firms make simultaneous entry and technology upgrade decisions. Potential entrants can enter with any technology and incumbents can choose to upgrade to any technology more advanced than their current best one. The available technologies are 2G, 3G, and 4G. I assume that firms offer every technology less advanced than their best technology.[22] After choosing an action, firms pay the associated sunk costs. Firms start period $t+1$ with the technologies chosen in period t.

I denote firms' technologies by $s \in \mathcal{S} := \{0, 1, 2, 3\}$, where $s = 0$ denotes that

[21]Note that the opposite is true for regulated firms that have complied with the regulation (the third column in Table 3). This is intuitive. Conditional on having 3G technology, regulated firms should be adversely selected, and thus less likely to upgrade to 4G.

[22]This assumption is broadly consistent with the data, as carriers typically keep old technologies in place as a fallback option. It also reduces the cardinality of the state space and thus aids computational tractability.

the firm is out of the market and the other values correspond to 2G, 3G, and 4G, respectively. The market's *technological state* $\boldsymbol{s} \in \mathcal{S}^4$ is a vector recording each firm's technology. Firms' flow profits are given by a time-varying function of the market's technological state $\boldsymbol{s}_t$: $\pi_{ft}(\boldsymbol{s})$. The specification of π_{ft} is deferred to subsection 3.4.

Entry and technology upgrades are costly. If firm f takes action a when its technology is s, it incurs a cost given by $c_{ft}(a, s, \boldsymbol{\varepsilon})$, where $\boldsymbol{\varepsilon}$ is a vector of action-specific cost shocks. As is common in the literature, I assume that the cost shocks are additively separable and independent and identically distributed across firms and over time, i.e., with a slight abuse of notation, $c_{ft}(a, s, \boldsymbol{\varepsilon}) = c_{ft}(a, s) - \varepsilon(a)$, where $\varepsilon(a) \overset{\text{iid}}{\sim} F_{\varepsilon}$. The specification of c_{ft} is deferred to subsection 3.3. Firms have perfect foresight regarding the dynamics of flow profits and entry and technology upgrade costs.

Exactly one firm is required to provide 3G service by a date T exogenously set by the regulator. I will call that firm the *regulated* firm and the other firms the *unregulated* firms. A regulated firm that does not provide 3G technology or better by $T + 1$ pays a fine φ and does so every period until it complies with the regulation.

Firms choose their actions to maximize their discounted expected profits, taking their competitors' behavior as given. I focus on Markov Perfect Equilibria (MPE), as is common in empirical applications of dynamic games.

3.1 Markov Perfect Equilibria

A Markov Perfect Equilibrium (MPE) is a strategy profile $(\sigma_1, \ldots, \sigma_4)$, such that σ_f is a function that maps a firm's state variables into a feasible action and maximizes firm i's expected discounted profits given its competitors' behavior. In this environment, a firm's state consists of the technological state $\boldsymbol{s}$, the date t, and its vector of cost shocks $\boldsymbol{\varepsilon}$. Let a typical state be denoted by $\omega = (\boldsymbol{s}, t, \boldsymbol{\varepsilon})$ and the state space be Ω. A strategy is a function $\sigma_f : \Omega \to \{0, 1, 2, 3\}$ satisfying the restriction that $\sigma_f(\omega) \in A(s_f(\omega)) := \{s_f(\omega), \ldots, 3\}$, where $s_f(\omega)$ is the f-th coordinate of the technological state component of ω, i.e., firm f's current technology.

Let $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_4)$ be a strategy profile and $r \in \{1, \ldots, 4\}$ denote the identity of the regulated firm. Define the implied ex-ante value function

$$V_{f,\boldsymbol{\sigma}}(\boldsymbol{s},t) := \mathbb{E}_{\tilde{\varepsilon},\boldsymbol{\sigma}}\Bigg\{\sum_{\tau=t}^{\infty}\delta^{\tau-t}\Big[\pi_{f\tau}(\boldsymbol{s}_\tau) - c_{f\tau}\left(\sigma_f(\boldsymbol{s}_\tau,\tau,\boldsymbol{\varepsilon}_{f\tau}), s_{f\tau}\right) + \\ + \varepsilon_{f\tau}\left(\sigma_f(\boldsymbol{s}_\tau,\tau,\boldsymbol{\varepsilon}_{f\tau})\right) - \varphi\mathbf{1}\{r=f\}\mathbf{1}\{T<\tau\}\mathbf{1}\{s_{f\tau}<2\}\Big] \;\Big|\; \boldsymbol{s}\Bigg\},$$

where $s_{f\tau} = \sigma_f(\boldsymbol{s}_{\tau-1},\tau-1,\boldsymbol{\varepsilon}_{f,\tau-1})$. The notation $\mathbb{E}_{\tilde{\varepsilon},\boldsymbol{\sigma}}$ indicates that the expectation is taken over the sequence of ε's for all firms, and firms' states evolve according to $\boldsymbol{\sigma}$. The recursive characterization of Markov Perfect Equilibria (e.g., Doraszelski and Escobar (2010)) implies that $\boldsymbol{\sigma}$ is a MPE if and only if

$$\sigma_f(\boldsymbol{s},t,\boldsymbol{\varepsilon}) = \underset{a\in A(s_f)}{\operatorname{argmax}}\Bigg\{\pi_{ft}(\boldsymbol{s}) - c_{ft}(a,s_f) - \varphi\mathbf{1}\{r=f\}\mathbf{1}\{T<t\}\mathbf{1}\{s_f<2\} \\ +\delta\mathbb{E}_{\boldsymbol{\varepsilon}_{-f}}\left[V_{f,\boldsymbol{\sigma}}\left(a,\boldsymbol{s}'_{-f},t+1\right) \mid \boldsymbol{s}\right] + \varepsilon(a)\Bigg\} \tag{1}$$

for all firms f and states $(\boldsymbol{s},t,\boldsymbol{\varepsilon})$. In equation (1), for firms $h \neq f$, $s'_h = \sigma_h(\boldsymbol{s},t,\boldsymbol{\varepsilon}_h)$ and the expectation is with respect to the shocks $\boldsymbol{\varepsilon}_h$, for $h \neq f$.

3.2 Quasi-Stationary Markov Perfect Equilibria

The environment just introduced has two sources of non-stationarity. First, flow profits and entry and technology upgrade costs vary over time. Second, the deadline for compliance with the regulation makes firm behavior vary over time. Indeed, a non-compliant regulated firm should become more likely to comply as the regulation deadline approaches and the threat of enforcement becomes more imminent. Unregulated firms respond to this change in the regulated firm's behavior. Thus, conditional choice probabilities vary over time. It follows that stationary Markov Perfect Equilibrium is not an appropriate solution concept. In this subsection, I introduce assumptions that respect these two sources of non-stationarity while imposing a degree of stationarity that enables the application of tools developed for stationary dynamic games.

I assume that flow profits, entry costs, and technology upgrade costs vary freely within the sample period but remain constant after its end. Formally, letting $\bar{T}$ denote the final period in the sample, $\pi_{ft}(\boldsymbol{s}) = \pi_{f\bar{T}}(\boldsymbol{s})$ for all f, $\boldsymbol{s}$, and $t \geq \bar{T}$ and $c_{ft}(a,s) = c_{f\bar{T}}(a,s)$ for all f, a, s, and $t \geq \bar{T}$. I then assume that firm behavior is stationary starting at $\bar{T}$. Firm behavior is thus allowed to vary over time as long as model primitives do. Because $\bar{T}$ exceeds the latest regulation deadline in the data, this assumption also addresses non-stationarity stemming from the regulation.

Definition 1. Let $\bar{T}$ denote the final period in the sample. A Markov Perfect Equilibrium $\boldsymbol{\sigma}$ is said to be *Quasi-Stationary* if there exist functions $\tilde{\sigma}_f(\boldsymbol{s}, \boldsymbol{\varepsilon})$, $f \in \{1, 2, 3, 4\}$, such that

$$\sigma_f(\boldsymbol{s}, t, \boldsymbol{\varepsilon}_f) = \tilde{\sigma}_f(\boldsymbol{s}, \boldsymbol{\varepsilon}_f) \quad \text{for all } f \in \{1, 2, 3, 4\}, t \geq \bar{T}, \boldsymbol{s}, \text{ and } \boldsymbol{\varepsilon}_f \ .$$

I assume throughout that the data is generated by a Quasi-Stationary Markov Perfect Equilibrium (QS-MPE).

Models of technology adoption must somehow contend with the fact that the demand for and costs of adopting a new technology vary over time. One way of dealing with the time-varying nature of demand and costs is to assume a finite horizon after which model primitives are constant and the industry state does not change. This assumption pins down continuation values in the terminal period, and one can then solve the game by backward induction. See, e.g., Igami (2017). In Igami (2017), the terminal period is the final period in the data. The QS-MPE approach treats the dynamics of the model primitives in a similar way, but it does not restrict continuation values at $\bar{T}$ to be $\pi_{f\bar{T}}(\boldsymbol{s}_{\bar{T}})/(1-\delta)$. Instead, firms' continuation values are given by the equilibrium value function. The assumption that firms cease to invest after the end of the sample may be reasonable in some applications, but restrictive in others, as is the case in the current setting. For the latter applications, QS-MPE can be a useful alternative.

An alternative implementation of the finite-horizon approach sets the terminal period to a date after the end of the sample, e.g., Elliott (2024). This approach has the benefit that, as in the QS-MPE approach, observed decisions are always modeled taking future strategic interactions into account. However, it requires the econometrician to impose parametric restrictions on the dynamics of model primitives. The QS-MPE approach, albeit making the alternative restriction that model primitives are constant after the end of the sample, does not require parametric restrictions on their dynamics during the sample period. Indeed, I estimate year-specific flow profit and cost function parameters, imposing no restrictions on how they vary over time. For this to be feasible, many markets ought to be observed. When that is not the case, or when the assumption that model primitives stabilize after the end of the sample is untenable, the finite-horizon approach may be more appropriate.[23]

[23] Yet another implementation of the finite-horizon approach rests on the assumption that

3.3 Specifying Sunk Investment Costs

The modeling of entry and technology upgrade costs reflects the fact that service provision requires four pieces of infrastructure: cell phone towers, a backhaul connection to the carrier's core network, transmitters, and antennas. Entry into a market requires setting up towers and a backhaul connection. Providing service of a specific technology requires installing transmitters and antennas compatible with that technology.[24]

With these considerations in mind, sunk investment costs are specified as follows:

$$c_{fmt}\left(a,s\right)=\begin{cases}0 & \text{if } a=s\\ \mathbf{1}\left\{s=0\right\}z_{m}^{e}\cdot\theta_{e}+\sum_{\{g':g'>s\}}^{a}z_{fmt}^{g'}\cdot\theta_{g',t} & \text{if } a>s\end{cases}. \tag{2}$$

I have added an m subscript to the definition of c_{fmt} to reflect the fact that the empirical specification allows sunk investment costs to vary across markets and firms as a function of observed firm-market characteristics z_{fmt}. In equation (2), the θ's are parameters to be estimated.

If $a=s$, the firm pays no costs (other than receiving the cost shock). A potential entrant that decides to enter pays the entry cost $z_{m}^{e}\cdot\theta_{e}$. This captures the cost of setting up towers and a backhaul connection. Moreover, associated with every technology g there are installation costs $z_{fmt}^{g}\cdot\theta_{g,t}$.[25] This captures the cost of installing transmitters and antennas compatible with technology

continuation values are known to be zero after the terminal date. See, e.g., Igami and Uetake (2020). This approach is reasonable in some applications. When it is not, the QS-MPE approach is an alternative.

[24]Though multi-technology transmitters were introduced around 2010, and multi-technology and multi-band transmitters around 2015, firms typically had to install new transmitters and antennas when rolling out a new technology. Adding 3G to an existing 2G network required new transmitters because 2G radios were of an older generation and thus technology-specific. Moreover, 3G was rolled out in Brazil using frequency bands different from those used for 2G. The situation was similar with 4G: existing 3G transmitters were typically single-technology or single-band, and 4G was deployed in bands distinct from those used for 2G and 3G. The potential exception is when firms entered a market or introduced 3G after 2015 using multi-technology and multi-band transmitters. In that case, they might have been able to introduce 3G and 4G repurposing existing transmitters – though they still had to acquire software licenses to enable additional technologies. Antennas are technology-agnostic, but are designed for specific (potentially multiple) frequency bands. Given the frequency bands used for different technologies in Brazil, deploying new technologies typically required installing new antennas.

[25]Note that because the cost of installing 2G is only paid by an entering firm, entry costs and 2G installation costs are not separately identified. Therefore, I do not estimate 2G installation cost parameters, so that entry cost estimates include 2G installation costs. Entry costs and 2G installation costs are assumed constant over time.

g.[26] Technology installation costs are allowed to vary over time, to capture upstream changes in the available transmitters and antennas and their prices. The summation in equation (2) reflects the previous assumption that firms offer all technologies less advanced than their best technology. If, for example, a firm's current best technology is 2G, and that firm upgrades to 4G, equation (2) says that the firm will pay the costs of installing both 3G and 4G. Cost shocks are assumed to be independent across firms, periods, and actions, and they follow a Type 1 Extreme Value distribution with scale parameter λ.

The covariates z_m^e include a constant and the municipality's area, reflecting the fact that larger municipalities require more towers to provide coverage. The covariates z_{fmt}^g, for $g \in \{2, 3\}$, include a constant, the municipality's area, and an affine function of the share of out-of-sample nearby markets where firm f newly installs technology g in period t. The latter covariate is meant to capture the geographic interdependence of technology diffusion driven by economies of scale and density.[27] That variable is constructed using out-of-sample municipalities, which are more populous, larger, and wealthier than the municipalities in the estimation sample – see Table 1. It is thus not unreasonable to assume that these variables are invariant to counterfactual policies affecting only the latter set of municipalities. Given the functional form (2), the perfect foresight assumption translates into firms having perfect foresight over the parameters $\theta_{g,t}$ and the covariates z_{fmt}^g. This can be interpreted as firms knowing future transmitter and antenna prices and their technology roll-out plans in out-of-sample markets.[28]

3.4 Specifying Flow Profits

It is not uncommon in applications of dynamic games for flow profits to be derived from an estimated demand system paired with an assumption on firms' pricing behavior. The unavailability of price data precludes that approach in my setting. I thus follow a different approach that instead uses data on the

[26] There are no exits or technology retirements in the data or in the model. Therefore, the estimated entry and technology installation costs also include any fixed costs associated with maintaining existing infrastructure.

[27] In particular, some fixed costs of installing a new technology, such as obtaining construction permits and deploying the necessary capital and specialized labor, can be shared across nearby markets.

[28] The covariates z_{fmt}^g also include a dummy that is equal to 1 if and only if f is the carrier Oi. Oi was going through significant financial distress during the sample period, and that is reflected in its lower rate of 3G and 4G deployment. Including that dummy allows the model to fit Oi's behavior.

quantities of subscribers to different firms and technologies and data on consumers' expenditures.

Suppose that consumer i in market m with demographic characteristics x_i chooses what carrier to subscribe to, what technology to use, and how much to spend on mobile telecommunications services. Denote expenditure by e and let j index firm-technology pairs, or products. Moreover, let j_i denote consumer i's choice of product and M be the size of the market.[29] Then, assuming no marginal costs, firm f's profits are given by [30]

$$\sum_{g \in s_f} \sum_i \mathbf{1}\{j_i = (f,g)\} e_i \approx M \sum_{g \in s_f} \int \mathbb{E}[\mathbf{1}\{j_i = (f,g)\} e_i \mid x_i] \, \mathrm{d}H_t(x_i) \; . \tag{3}$$

In equation (3), the summation over $g \in s_f$ is over all technologies offered by firm f, i.e., $\{g : 1 \leq g \leq s_f\}$, and H_t is the distribution of demographic characteristics in period t.

I do not observe consumer expenditures together with their product choices. I will therefore make the following assumption:

Assumption 1. Given demographic characteristics, consumer expenditures and product choices are conditionally independent, i.e.,

$$j_i \perp\!\!\!\perp e_i \mid x_i \; .$$

This assumption would hold in a world in which consumers pay per usage (a popular model in Brazil) and technology does not affect usage. It would fail if better technologies induce consumers to use more data. Assumption 1 would thus be untenable if we were dealing with users of high-bandwidth applications. Because we are dealing with relatively small, rural, and low-income municipalities in Brazil, the assumption is more palatable. Importantly, Assumption 1 does not imply that consumers that subscribe to different technologies spend the same amount on average, since individuals with different characteristics are allowed to sort into different technologies.

Assumption 1 and equation 3 imply that firms' flow profits can be written as

$$\pi_{ft}(\boldsymbol{s}, H_t) = M \sum_{g \in s_{ft}} \int \mu_{fgt}(\boldsymbol{s}, x_i) \mathbb{E}[e_i \mid x_i] \, \mathrm{d}H_t(x_i) \; , \tag{4}$$

[29] There are more mobile telecommunications subscriptions in Brazil than there are individuals. I set the market size to twice the municipality population.

[30] The approximation in equation 3 is analogous to the (implicit) approximation to profit functions used routinely in empirical industrial organization.

where $\mu_{fgt}(\boldsymbol{s}, x_i)$ is the probability that a consumer with characteristics x_i chooses firm-technology pair (f, g) when the market's technological state is $\boldsymbol{s}$. The distribution $H_t(x_i)$ of demographic characteristics is obtained for each municipality directly from Census data.[31] To fully describe firms' flow profits, it remains to specify choice probabilities $\mu_{fgt}(\boldsymbol{s}, x_i)$ and expected expenditures $\mathbb{E}[e_i \mid x_i]$.

3.4.1 Functional Forms for Market Shares and Expenditures

Market Shares. I model $\mu_{fgt}(\boldsymbol{s}, x_i)$ as arising from a nested logit model. Up to now, I have omitted the dependence of market shares on the market m to ease notation. That dependence must now be introduced. Consumer i's utility of subscribing to firm-technology pair $j = (f, g)$ in market m and year τ is given by[32]

$$u_{ijm\tau} = \overbrace{\gamma_{r(m),f(j)} + \alpha_{g(j),\tau} + \beta_{g(j),\tau} y_{im\tau} + \kappa_{g(j),\tau} d_{m\tau}}^{v_{jm\tau}(x_{im\tau})} + \\ + \xi_{jm\tau} + \zeta_{ig(j)m\tau}(\rho) + (1-\rho)\varepsilon_{ijm\tau} \tag{5}$$

where $r(m)$ is the state of municipality m, $f(j)$ is the firm of the firm-technology pair j, and $g(j)$ is j's technology. Moreover, $y_{im\tau}$ is income, and $d_{m\tau}$ is population density.[33] The term $\xi_{jm\tau}$ is an unobserved product characteristic, $\zeta_{ig(j)m\tau}(\rho)$ is a disturbance common to all goods with the same technology, and $\varepsilon_{ijm\tau}$ is a Type 1 Extreme Value shock.[34] The parameter ρ is the nesting parameter, which determines the extent to which consumers substitute within versus between technologies. The shock $\zeta_{ig(j)m\tau}(\rho)$ has the unique distribution such that $[\zeta_{ig(j)m\tau}(\rho) + (1-\rho)\varepsilon_{ijm\tau}]$ has an extreme value distribution (see Cardell (1997)).

In equation (5), $\gamma_{r(m),f(j)}$ is a state-firm fixed effect meant to capture geographic variation in firms' market shares. The term $\alpha_{g(j),\tau}$ is a technology-year fixed effect, which captures changes in technology popularity over time. The ef-

[31] I use the 2010 Census to obtain the distributions of income and household size. To obtain time-varying distributions, I assume that all individuals sampled in 2010 experience real income growth at the same rate as the municipality's GDP per capita and that household sizes remain unchanged.

[32] I specify equation 5 at the year level because the included demographics are observed with that frequency. I map a period in the dynamic game to the year in which it begins and use the model above to compute shares.

[33] In estimation, I substitute municipality-level GDP per capita for individual income $y_{im\tau}$. See section 4. This economizes on computational cost. When computing the flow profits implied by the model, I use individual-level income as obtained from the Census. See footnote 31.

[34] I trust that using ε for both cost and utility shocks will cause no confusion, as the shocks in Equation (5) will soon be integrated out and will not feature in the rest of the analysis.

fect of income and population density on consumer preferences varies by technology and year. This allows, for instance, for the effect of income on preferences for 4G relative to alternative technologies to decrease over time, as 4G becomes more widely available, handsets become more affordable and more consumers desire to join 4G networks. I label consumer utility net of the error structure as $v_{jm\tau}(x_{im\tau})$, where $x_{im\tau} = (y_{im\tau}, d_{m\tau})$. In what follows, I also refer to $\Delta_{jmt}(x) := v_{jm\tau}(x) + \xi_{jm\tau}$ as the mean-utility of product j in municipality m and year τ.

The distributional assumptions above imply that when the technological state is $\boldsymbol{s}$ and the vector of mean utilities is $\boldsymbol{\Delta}$ market shares are given by

$$\mu_j(\boldsymbol{s}, \boldsymbol{\Delta}) = \begin{cases} 0 & , \text{ if } j \notin \boldsymbol{s} \\ \frac{e^{\Delta_j/(1-\rho)}}{D^{\rho}_{g(j)\cap \boldsymbol{s}} \sum_{g=0}^{3} D^{1-\rho}_{g\cap \boldsymbol{s}}} & , \text{ if } j \in \boldsymbol{s} \end{cases} , \tag{6}$$

where $D_{g\cap \boldsymbol{s}} = D_{g\cap \boldsymbol{s}}(\boldsymbol{\Delta}) = \sum_{k\in g\cap \boldsymbol{s}} e^{\Delta_k/(1-\rho)}$, the summation being over all products with technology g that are available in the market. The choice probability function for municipality m in year τ is then given by

$$\mu_{jm\tau}(\boldsymbol{s}, x) := \mu_j(\boldsymbol{s}, \boldsymbol{\Delta}_{m\tau}(x)) , \tag{7}$$

where $\Delta_{m\tau}(x)$ stacks the $\Delta_{jm\tau}(x)$ for all j.

Expenditures. I assume that individual i's monthly expenditure on mobile telecommunications services, e_i, is given by

$$\log(e_i) = \nu_{r(i)u} + \nu_1 \log(y_i) + \nu_2 n_i + \varepsilon^e_i , \tag{8}$$

where $r(i)$ is i's state of residence, u indicates whether the municipality is classified as urban or rural by the national statistical institute (IBGE), y_i is monthly income, n_i is household size, and ε^e_i is an error term that is independent from the included regressors. Figure 4 shows that log-linearity is a reasonable approximation to the data.

3.4.2 Aggregation of Market Shares to the Code-Area Level

It will be necessary to aggregate market shares up to the code-area level. To do so, I introduce the following assumption.

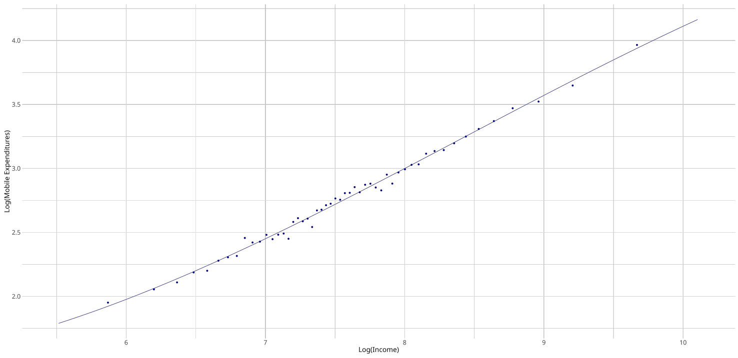


Figure 4: Log of Monthly Expenditures vs. Log of Monthly Income

Assumption 2. Let $c(m)$ denote municipality m's code-area. Municipality-level unobserved product characteristic $\xi_{jm\tau}$ can be decomposed as

$$\xi_{jm\tau} = \xi_{jc(m)\tau} + \eta_{jm\tau}$$

where $\eta_{jm\tau} \overset{iid}{\sim} F(\eta)$ and $\mathbb{E}_F[\eta_{jmt}] = 0$.

Assumption 2 says that $\xi_{jm\tau}$ can be decomposed into a random variable that varies only across code-areas, on which I place no restrictions, and another random variable that varies across municipalities within a code-area, that I assume is independent across products, municipalities, and years and identically distributed with some unrestricted distribution F.

Let ω_m be the fraction of the population in code-area c that resides in municipality m. Under Assumption 2, an argument relying on a large number of municipalities within a code-area implies that

$$\mu_{jc\tau} = \sum_{m \in c} \omega_{m\tau} \int \int \mu_j(\boldsymbol{s}_{m\tau}, \boldsymbol{v}_{m\tau}(x) + \boldsymbol{\xi}_{c\tau} + \boldsymbol{\eta}_{m\tau}; \theta) \, \mathrm{d}H_{m\tau}(x) \, \mathrm{d}F(\boldsymbol{\eta}_{m\tau}) \tag{9}$$

is approximately true, where $\mu_{jc\tau}$ is the predicted market share in code-area c.[35] The exact expression for market shares at the code-area level is similar to (9), simply omitting the outer integral. That expression is not useful, as $\boldsymbol{\eta}_{m\tau}$ is unknown. However, it is possible to recover an empirical counterpart to $F(\eta_{jmt})$, conditional on utility parameters, and thus compute the right-hand side of equation (9). I will use equation (9) in estimation to solve for $\boldsymbol{\xi}_{c\tau}$ and form moment conditions, in a manner analogous to Berry, Levinsohn, and Pakes (1995).

[35]See appendix D for details.

4 Identification and Estimation

I start this section by discussing the estimation of the flow profit function in subsection 4.1. In subsection 4.2, I discuss the estimation of the entry and upgrade costs and the fine for non-compliance with the regulation.

4.1 Identification and Estimation of the Flow Profit Function

Here I discuss the identification and estimation of the product choice and expenditure models, starting with the former. When estimating the product choice model, I substitute municipality-level GDP per capita for individual income, to save on computational cost; see footnote 38 One factor complicating the estimation of the product choice model is that I observe market shares at different levels of geographic granularity over time, as previously discussed in section 2. The data for the 2013-18 period is at the code-area level whereas the data starting in 2019 is at the municipality level.

Consider first the municipality-level data. Equation (6) implies (see Berry (1994)):

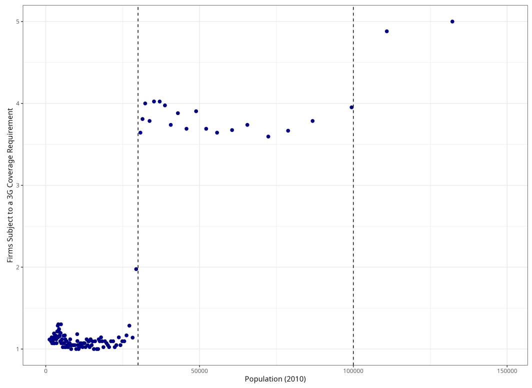
$$\log(s_{jm\tau}) - \log(s_{0m\tau}) = v_{jm\tau}(x_{m\tau}) + \rho \log\left(s_{j|\mathcal{J}_{g(j)m\tau}}\right) + \xi_{jm\tau} \tag{10}$$

where $s_{j|\mathcal{J}_{g(j)m\tau}}$ is the share of good j within its nest, i.e., the group of products with the same technology. This equation can be used to estimate the parameters in $v_{jm\tau}(x_{m\tau})$ and ρ. The municipality characteristics $x_{m\tau}$ are assumed to be uncorrelated with $\xi_{jm\tau}$. The term $s_{j|\mathcal{J}_{g(j)m\tau}}$, however, is a function of $\xi_{jm\tau}$. Therefore, instruments are necessary to identify the nesting parameter ρ.

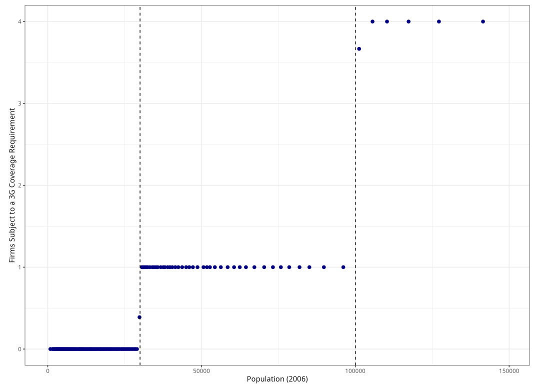
The nesting parameter determines the extent of business stealing within and between nests when new goods are introduced. Therefore, to identify it, we would like to leverage exogenous variation in the number of goods in the market. This intuition is similar to that in Berry and Waldfogel (1999). A common instrument used to identify the nesting parameter is the number of products in the market. In the present setting, the number of products is not a valid instrument, as firms' entry and technology upgrade decisions depend on $\xi_{jm\tau}$'s.

To circumvent this difficulty, I exploit the fact that the number of firms subject to coverage requirements is a function only of population measured, at the latest, in 2010.[36] Figure 5 shows that the number of firms subject to 3G and

[36] When estimating the product choice model, I use data on the entire country, in contrast with the estimation of the dynamic parameters, which restricts attention to municipalities where a



(a) 3G Requirements



(b) 4G Requirements

Figure 5: Number of Regulated Firms against 2010 Population

Binned scatterplot of the number of firms subject to 3G (left panel) and 4G (right panel) coverage requirements against municipality population measured in July 2010.

4G coverage requirements is indeed close to a step function of 2010 population. These variables are, therefore, credibly uncorrelated with the unobserved product characteristics $\xi_{jm\tau}$. I use equation (10) to obtain unobserved product characteristics as a function of data and parameters, $\xi_{jm\tau}(\Phi)$, where Φ denotes the vector of parameters in the product choice model. I interact $\xi_{jm\tau}(\Phi)$ with the number of firms subject to 3G and 4G requirements and the exogenous variables included in $v_{jm\tau}$ to form moment conditions $\mathbb{E}[\xi_{jm\tau}(\Phi)Z^1_{jm\tau}] = 0$.

The moments just discussed are informative about the nesting parameter and preference parameters in the later period of the data, but not about those governing preferences in 2013-2018. To construct additional moments to identify those parameters, I leverage assumption 2 and equation (9). Predicted market shares at the code-area level are given by[37]

$$\mu_{jc\tau} = \sum_{m \in c} \omega_{m\tau} \int \mu_j(\boldsymbol{s}_{m\tau}, \boldsymbol{v}_{m\tau}(x_{m\tau}) + \boldsymbol{\xi}_{c\tau} + \boldsymbol{\eta}_{m\tau}; \Phi) \, \mathrm{d}F(\boldsymbol{\eta}_{m\tau}) \,. \tag{11}$$

Equating observed market shares at the code-area level with their predicted counterparts from equation (11), one can solve for $\boldsymbol{\xi}_{c\tau}$ as a function of data and parameters. Because the kernel μ_j in equation (11) is given by a nested logit model, I do so using the Brenkers and Verboven (2006) and Grigolon and Verboven (2014) modification of the Berry et al. (1995) contraction mapping. One

single firm is subject to a 3G requirement. See section 2.

[37] This is equation (9) with the simplification that municipality-level GDP per capita is used instead of individual income, as stated above. See also footnote 38.

can then form additional moment conditions of the form $\mathbb{E}[\xi_{jc\tau}(\Phi)Z^2_{jc\tau}] = 0$.

The one hindrance to this approach is the integration with respect to $F(\boldsymbol{\eta}_{m\tau})$. Here, again, assumption 2 offers a solution. Given any vector of structural parameters, Φ, equation (10) returns $\xi_{jm\tau}(\Phi)$. Given the independence condition in assumption 2, we can then project $\xi_{jm\tau}(\Phi)$ onto product by area-code by time fixed effects and obtain residuals $\hat{\eta}_{jm\tau}(\Phi)$. This gives an empirical distribution of $\eta_{jm\tau}$ given Φ, $\hat{F}(\eta;\Phi)$. The integration in equation (9) can be performed for any guess of Φ by sampling from $\hat{F}(\eta;\Phi)$.

To summarize the preceding discussion, the steps involved in evaluating the GMM objective function for a given value of Φ are as follows. First, use equation (10) to obtain $\xi_{jm\tau}(\Phi)$. Second, use assumption 2 to obtain $\eta_{jm\tau}(\Phi)$. Third, solve for $\xi_{jc\tau}(\Phi)$, for the years up to and including 2018, from

$$s_{jc\tau} = \sum_{m\in c} \omega_{m\tau} \frac{1}{N_s} \sum_{i=1}^{N_s} \mu_j(\boldsymbol{s}_{m\tau}, \boldsymbol{v}_{m\tau}(x_{m\tau}) + \boldsymbol{\xi}_{c\tau} + \boldsymbol{\eta}_{m\tau i}(\Phi); \Phi) \tag{12}$$

where $s_{jc\tau}$ is the observed market share of firm-technology pair j in code-area c and year τ, $\boldsymbol{\eta}_{m\tau i}(\Phi)$ is a vector of $|\mathcal{J}_{m\tau}|$ independent draws from $F(\eta;\Phi)$ and N_s is the number of simulation draws. Fourth, form sample analogs of the moments $\mathbb{E}[\xi_{jm\tau}(\Phi)Z^1_{jm\tau}]$ and $\mathbb{E}[\xi_{jc\tau}(\Phi)Z^2_{jc\tau}]$, denoted $\bar{g}^1(\Phi)$ and $\bar{g}^2(\Phi)$, respectively. For a chosen weight matrix W, the GMM objective is then

$$J(\Phi) := \begin{pmatrix}\bar{g}^1(\Phi)' & \bar{g}^2(\Phi)'\end{pmatrix} W \begin{pmatrix}\bar{g}^1(\Phi) \\ \bar{g}^2(\Phi)\end{pmatrix} \tag{13}$$

The GMM estimator is, as usual, $\hat{\Phi} := \operatorname{argmin}_{\Phi} J(\Phi)$.[38] I have discussed the instruments $Z^1_{jc\tau}$ above. The instruments $Z^2_{jc\tau}$ are the exogenous covariates in equation (5), computed using the population-weighted average of the characteristics $x_{m\tau}$ of the municipalities in code-area c. I use a block-diagonal weighting matrix, where the blocks are given by sample analogs of the variance-covariance matrices of $Z^1_{jm\tau}$ and $Z^2_{jc\tau}$.

[38] The substantial computational savings obtained by using municipality-level GDP per capita instead of individual income are now clear. They are threefold. First, using individual income would add one more system of nonlinear equations to be solved numerically, namely those to obtain $\xi_{jm\tau}(\theta)$. Second, the right-hand side of equation (12) would include a second integral. Third, the derivatives of the $\eta_{m\tau i}(\theta)$ draws would cease to be independent from the parameters.

Expenditure Model. I estimate equation (8) by ordinary least squares using the Household Budget Survey data. From (8) it follows that $\mathbb{E}[e_i \mid x_i] = \exp(\nu_{r(i)u} + \nu_2 n_i) y_i^{\nu_1} \mathbb{E}[\exp(\varepsilon_i^e) \mid x_i]$. Because ν_i is assumed independent from the included regressors, I substitute $\mathbb{E}[\exp(\varepsilon_i^e)]$ for $\mathbb{E}[\exp(\varepsilon_i^e) \mid x_i]$. I use the residuals from equation (8) to estimate $\mathbb{E}[\exp(\varepsilon_i^e)]$ by $N^{-1}\sum_{i=1}^{N}\exp(\hat{\varepsilon}_i^e)$.

4.2 Identification and Estimation of Dynamic Parameters

Identification. The flow payoffs of the dynamic game are linear in the structural parameters. It is possible to show that the structural parameters of dynamic games with linear flow payoffs are identified if conditional choice probabilities are identified.[39] In Appendix E, I show that there exist known functions $\Upsilon_{fm,P_m}(a, \boldsymbol{s}, t)$ and $\chi_{fm,P_m}(a, \boldsymbol{s}, t)$ such that

$$\Upsilon_{fm,P_m}(a, \boldsymbol{s}, t) = \chi_{fm,P_m}(a, \boldsymbol{s}, t)' \frac{\Psi}{\lambda} ,$$

where $\Psi = (1, \theta', \varphi)'$, and θ is a vector collecting the investment cost parameters. Therefore, if the matrix stacking $\chi_{fm,P_m}(a, \boldsymbol{s}, t)'$ across firms, markets, feasible actions, and states has full column rank, $\lambda^{-1}\Psi$ is identified.[40]

The intuition for identification is that structural parameters are identified by exogenous variation in market characteristics and firms' regulatory status. For instance, comparing municipalities that differ in land area, holding other variables constant, identifies the effect of land area on investment costs. This simple argument is complicated by the fact that the cost specification features technology-specific parameters. Thus, varying a covariate such as land area changes actions' payoff differences, and thus CCPs, through the cost of more than one action. The solution to this difficulty is to first condition on firms that have 3G technology, so that the previous thought experiment identifies the effect of land area on the cost of deploying 4G. One can then condition on firms that have 2G technology, and again compare markets with different land area to identify its effect on the cost of deploying 3G. One can then move on to potential entrants to identify the effect of land area on the cost of entry and 2G deployment.[41]

[39] This is a known result, see, e.g., Aguirregabiria and Nevo (2013). Appendix E provides details specific to the present setting.

[40] It follows from this that λ and Ψ are separately identified. However, estimation of λ led to implausibly large estimates for sunk investment costs. I fix $\lambda = 1$ in estimation.

[41] Another difficulty with this intuitive argument is the fact that comparing municipalities with different characteristics leads to changes not only in firms' flow costs of technology de-

The fine parameter φ is identified by the difference in behavior between regulated and unregulated firms. Time variation also helps its identification. Intuitively, for small φ the behavior of regulated firms will change only slightly as the regulation deadline approaches. Large φ, on the other hand, will lead to larger changes in behavior over time.

The identification argument above requires municipality-specific CCPs P_m to be estimable from the data, and indeed in estimation and counterfactuals I allow for CCPs to vary freely across municipalities. Doing so is at the service of the economic and policy questions that motivate this paper.[42] For municipality-specific CCPs P_m to be recoverable from the data, it is necessary that the map from market observables to QS-MPE be continuous. Under these conditions, one can use nearby (in the space of market characteristics) markets to estimate a market's CCPs.[43] Computational experimentation supports uniqueness.[44] Figure 19 in appendix E provides evidence for QS-MPE continuity in market characteristics. Note that this identification argument is applicable only in settings with a large cross-section of markets, due not only to the foregoing discussion but also to the need to recover period-specific CCPs. A large cross-section of markets is a feature of the setting in this paper.

Estimation. I apply Aguirregabiria and Mira (2007)'s Nested Pseudo Likelihood (NPL) algorithm to estimate the dynamic parameters. In light of the results of Pesendorfer and Schmidt-Dengler (2010), this choice of estimator requires some justification. A popular alternative is to use a two-step estimator, e.g. Bajari, Benkard, and Levin (2007), Pakes, Ostrovsky, and Berry (2007) or Pesendorfer and Schmidt-Dengler (2008). These estimators all proceed by flexibly estimating policy functions in a first stage and then using those policy functions to construct a second-stage objective function that is minimized to yield structural estimates. In the present setting that would require estimating CCPs

ployment, but also in continuation values. Parametric restrictions are important here, as they pin down those changes in continuation values as functions of the parameter of interest and known conditional choice probabilities – see Appendix E.

[42]An alternative but coarser approach would be to group markets with similar observables and posit that the data comes from a unique equilibrium within each group, as in Dunne et al. (2013).

[43]See De Paula (2013) for a related discussion.

[44]The dynamic game of section 3 is a *directional* game, as defined in Iskhakov, Rust, and Schjerning (2016). It may be possible to use their techniques to compute all equilibria of the game in this paper, and thus potentially establish uniqueness for a given value of the structural parameters. Note that even if uniqueness fails, municipality-specific CCPs remain estimable as long as the map from market characteristics to the QS-MPE is continuous, i.e., the data are generated by a continuous selection of the QS-MPE correspondence.

that vary flexibly with municipality characteristics and over time. Though I do observe a large cross-section of markets, allowing for such flexibility continues to ask a lot of the data. I thus opt to use an estimator that makes fuller use of the already imposed structural assumptions.

There are alternatives to the NPL estimator that also make full use of the model structure. The most obvious one is the maximum likelihood estimator, but its computational cost is prohibitive in the case of dynamic games. Another alternative is the k-EPL estimator proposed by Dearing and Blevins (2025).[45] Their estimator enjoys good theoretical properties. However, this estimator requires the econometrician to solve systems of linear equations for choice-specific value functions. NPL, instead, requires the solution to linear equations for the ex-ante value functions. A NPL iteration is thus cheaper than a k-EPL iteration. Because I allow for municipality-specific choice probabilities, the added computational cost of EPL relative to NPL is multiplied by the number of municipalities. I thus choose to employ the NPL estimator.

A Nested Pseudo Likelihood (NPL) fixed point is a pair $((\tilde{\theta}, \tilde{\varphi}), \{\tilde{P}_m\}_m)$ that satisfies

(i) $(\tilde{\theta}, \tilde{\varphi}) = \text{argmax}_{\theta,\varphi} \sum_{m,t,f} \ln \mathcal{P}_{fm}(a_{fmt} \mid \boldsymbol{s}_{mt}, t; \theta, \varphi, \tilde{P}_m)$.

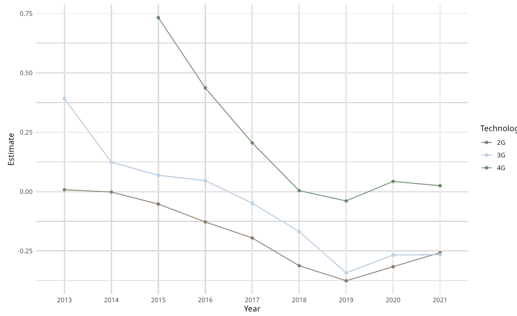
(ii) $\tilde{P}_m = \mathcal{P}_m(\tilde{\theta}, \tilde{\varphi}, \tilde{P}_m)$ for all m.

where $\mathcal{P}_{fm}(a_{fmt} \mid \boldsymbol{s}_{mt}, t; \theta, \varphi, \tilde{P}_m)$ is the probability that the observed action a_{fmt} is optimal for firm f in state $(\boldsymbol{s}_{mt}, t)$ in market m, when the firm believes that its competitors' and its own future behavior will follow the CCPs $\tilde{P}_m$ and the structural parameters are (θ, φ). The object $\mathcal{P}_m(\tilde{\theta}, \tilde{\varphi}, \tilde{P}_m)$ is an array collecting such probabilities for all firms, states, and feasible actions. The NPL *estimator* is the NPL fixed point with the maximum value of the pseudo-likelihood. The set of NPL fixed points is known to be non-empty but need not be a singleton.

In practice, one finds NPL fixed points via an iterative algorithm. Starting with a guess for CCPs, $\{\tilde{P}_m\}_m$, the implied pseudo likelihood is maximized. One then uses the resulting guess for $(\tilde{\theta}, \tilde{\varphi})$ to update firms' CCPs. These two steps are repeated until the CCPs and the structural parameters converge. Due to the potential multiplicity of NPL fixed points, it is important to experiment with different starting values for CCPs.

[45] See also Aguirregabiria and Marcoux (2021) for a related contribution.

(a) Technology-Year FEs



(b) Income

(c) Population Density

Figure 6: Market Share Model Estimates

The three panels show, respectively, estimates of technology-year fixed effects, income coefficients by technology, and population density coefficients by technology over time.

5 Estimation Results

Static Parameters. Figure 6 presents estimates of some of the main parameters of the market-share model introduced in section 3. It shows estimates of technology-year fixed effects, income coefficients by technology, and population density coefficients by technology, from 2013 to 2021. The most striking pattern in the figure is that the 4G estimates change much more rapidly than the equivalent 2G and 3G estimates. This is consistent with 4G being a new technology during this period. The effect of income on a technology's utility is larger the more advanced the technology. Therefore, as income grows, market shares of more advanced technologies grow relative to those of less advanced technologies. The effect of population density on utility is almost always positive. This is consistent with individuals in more densely populated areas having more social connections and thus higher demand for mobile telecommunications services. That effect is similar for 2G and 3G products at the beginning of the sample, but those effects decouple over time. The effects of population

Table 4: Expenditure Equation

Dependent Variable:	$\log(e_i)$
Model:	(1)
Variables	
$\log(y_i)$	0.5238***
	(0.0054)
HH Size	0.0537***
	(0.0038)
Fixed-effects	
State-Rural/Urban	Yes
Fit statistics	
Observations	75,004
R^2	0.16516

IID standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

density on the utility from 3G and 4G products are similar towards the end of the sample. In light of the quasi-stationarity assumption of section 3, it is important to observe that all parameters are fairly stable towards the end of the sample. Table 4 reports estimates of the expenditure equation (8). It shows that mobile telecommunications services are a necessity good, and expenditures in those services are increasing in household size.

More important than the parameter values are the implications of these estimates for firms' incentives to enter markets and upgrade their technologies. Figure 7 shows the distribution of the relative changes in firms' marginal profits with respect to the technology of the regulated firm. Let $\Delta_{fm}(s', s, s_r, t) := \pi_{fm}((s', s_r, \mathbf{0}), t) - \pi_{fm}((s, s_r, \mathbf{0}), t)$, i.e., the marginal profit for firm f in market m from a change to its technology from s to s' when the regulated firm has technology s_r in period t, while keeping other competitors out of the market. Figure 7 shows the distribution of relative changes in $\Delta_{fm}(s', s, s_r, t)$ when the regulated firm's technology changes, i.e.,

$$\rho_{fm}(s', s, s'_r, s_r, t) := \frac{\Delta_{fm}(s', s, s'_r, t) - \Delta_{fm}(s', s, s_r, t)}{\Delta_{fm}(s', s, s_r, t)} . \tag{14}$$

Focusing on the case where $s' > s$ and $s'_r > s_r$, the quantity $\rho_{fm}(s', s, s'_r, s_r, t)$ indicates whether technology upgrades are strategic complements or substitutes.

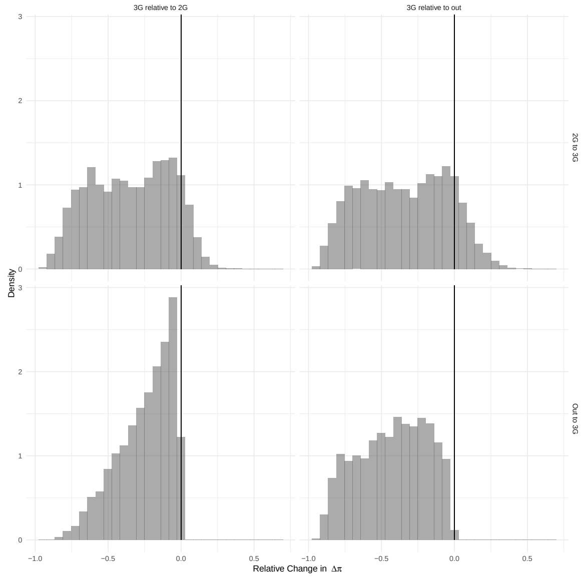


Figure 7: Second differences of firms' flow profits.

Distribution of relative changes in firms' marginal profits for a given change in the regulated firm's technology. Column labels indicate the technologies of the regulated firm, i.e., s_r and s_r' in the definition of ρ_{fm} in equation (14). Row labels indicate the technologies of the firm whose marginal profits are being evaluated, i.e, s and s' in the definition of ρ_{fm} in equation (14). See the main text for further details.

When $\rho_{fm} > 0$, technology upgrades are strategic complements. When $\rho_{fm} < 0$, technology upgrades are strategic substitutes.

The panels in Figure 7 vary the values of s, s', s_r, and s_r'. The first column sets $s_r = 1$ and $s_r' = 2$, while the second column uses $s_r = 0$ and $s_r' = 2$. The first row sets $s = 1$ and $s' = 2$, while the second row uses $s = 0$ and $s' = 2$. Figure 7 shows that technology upgrades are almost always strategic substitutes, though they can be strategic complements when the focal firm is already active. Going back to the deterrence and strategic complementarity effects discussed in the introduction, this result indicates that the former will be the more prevalent one.

Dynamic Parameters. I estimate the median entry cost (which accounts for the cost of setting up 2G service) to be equal to 8.07 million BRL, the fifth and ninety-fifth percentiles being 6.25 million and 10.93 million BRL. This variation is due to differences in municipality area. The cost of non-compliance with the regulation, φ, is estimated to be 1.14 million BRL, or 14.1% of the median entry cost.

In figure 8, I plot the estimated dynamics of the cost of introducing 3G and

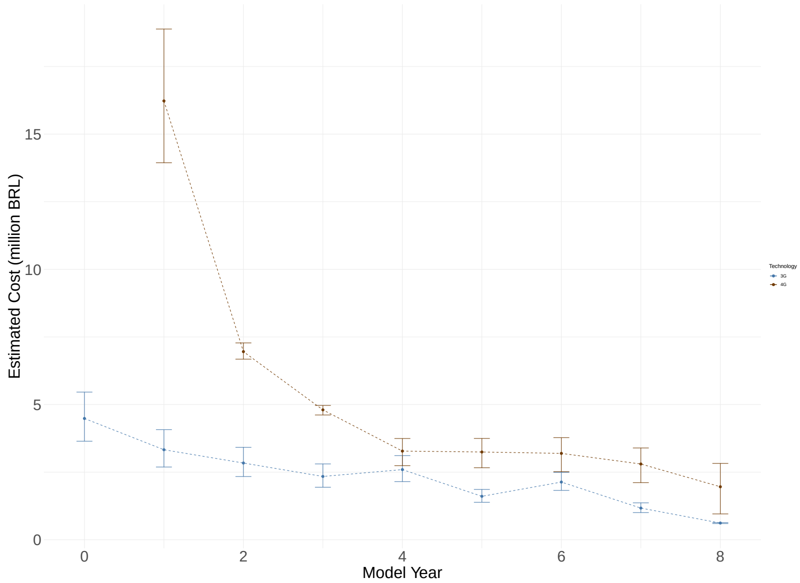


Figure 8: Estimated Dynamics of the Cost of 3G and 4G Introduction

4G technology. I show, for the years 2013-2021, the average cost together with the tenth and the ninetieth percentiles of the distribution – where, as before, the variation is driven by municipality area.[46] [47] The costs of introducing 3G and 4G technology are decreasing over time. These costs fall at a much faster rate for 4G than for 3G. These patterns are consistent with 3G being a relatively mature technology by the beginning of my sample period, while 4G was a new technology at that time. Though the costs of introducing 3G and 4G fall over time, they are fairly stable in the final years of the sample. This suggests that the assumption that deep parameters stabilize after the end of the sample, required for the use of QS-MPE, is not unreasonable. Estimates of the dynamic parameters and their bootstrap confidence intervals are reported in Appendix F.2.

[46]When computing these costs, I set variables pertaining to firms' deployment in out-of-sample markets to zero. See section 3 for a description of those variables.

[47] The costs of introducing 4G reported in Figure 8 start from year 1 rather than 0 because I assume that the cost of introducing 4G technology in 2013 and 2014 is the same. Moreover, note that estimates of the cost function in equation (2) account for the reduction in the number of cost shock draws available to firms when they upgrade their technologies. To make the levels of the cost functions more interpretable, in Figure 8 I adjust those estimates by adding the term $\sum_{\tilde{a}\geq a-1} P_{a-1}(\tilde{a}) \ln P_{a-1}(\tilde{a}) - \sum_{\tilde{a}\geq a} P_a(\tilde{a}) \ln P_a(\tilde{a})$, where P_a is the vector of action probabilities conditional on the firm's technology being a, which is estimated from the data unconditionally, and a is the technology whose cost is being computed.

6 Counterfactual Analysis

The counterfactual exercises in this section directly address the questions posed in the Introduction. In subsection 6.1, I use the estimated model to measure the effect of coverage requirements on the time to introduction of 3G technology. In subsection 6.2, I quantify the cost that the regulation imposes on firms. In subsection 6.3, I use the model to quantify how equilibrium responses to the regulation shape its effect on the roll-out of 3G. In subsection 6.4, I consider the reallocation of coverage requirements across firms by means of a subsidy auction, and compute its effects on 3G roll-out and the aggregate cost to the government.[48]

6.1 The Effect of Coverage Requirements on 3G Roll-Out

To quantify the effect of coverage requirements on the time to introduction of 3G technology, I solve the game and simulate data for each municipality under the estimated fine $\hat{\varphi}$ and under $\varphi = 0$, i.e., with no regulation. In each case, I simulate 200 paths of play for each municipality.

Before tackling the question of the effect of coverage requirements per se, I use the computed equilibria with $\varphi = 0$ to ask the model whether 3G technology would be introduced within a reasonable amount of time in the absence of regulation. Figure 9 sheds light on this question. It plots, over time, the share of municipalities whose probability of 3G availability is at least 90% in the absence of regulation. The figure shows the familiar S-shape for technology diffusion. For the purpose of universal service regulation, Figure 9 has two important implications. First, there is a group of markets that do not attain a high probability of 3G availability, even after 2020. If we take the latest regulation deadline, December 2019, as a reasonable benchmark, we find that just below 50% of markets fail to achieve a 90% probability of 3G availability. Regulation may be necessary to ensure service provision in these markets.[49] Second, many markets do achieve a high probability of 3G availability reasonably quickly. In this group of markets, some firm finds it privately optimal to introduce 3G, and

[48] As noted in section 2, the counterfactual exercises focus on markets with a December 2019 and April 2016 regulation deadline. I perform the simulations for the subset of these markets where the regulated firm does not offer 3G technology at the start of the data. There are 1,872 such markets. When comparing the time to 3G introduction under alternative scenarios, I only consider markets that start without 3G, of which there are 1,347.

[49] The 90% threshold is, of course, arbitrary. However, the qualitative message from Figure 9 persists if that threshold is changed.

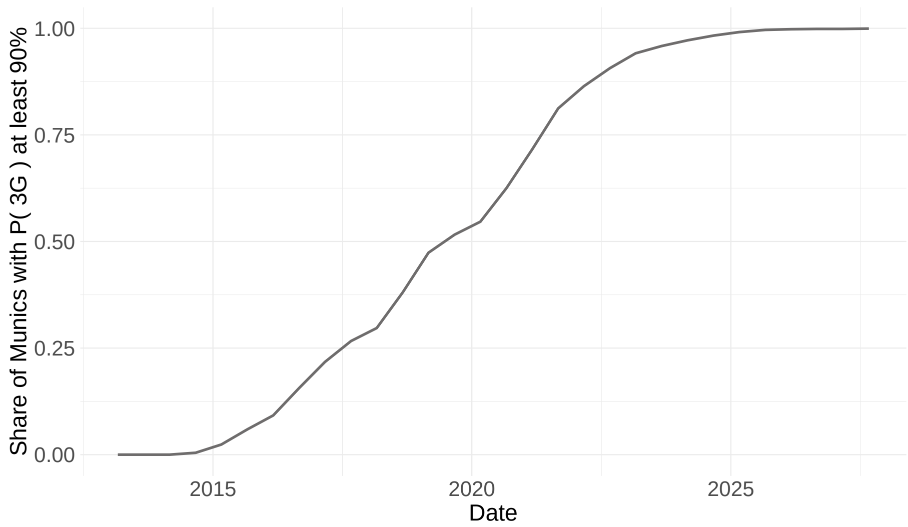


Figure 9: Share of municipalities with at least 90% probability of 3G availability over time, without regulation.

need not be further incentivized to do so. Coverage requirements that are imposed on some firm other than that profitable firm generate unnecessary costs if the goal is to achieve service provision.

Next, I use the model to quantify the effect of coverage requirements on the time to the introduction of 3G. Let the *regulation effect* to be the difference in the time to 3G introduction between the no regulation ($\varphi = 0$) and status quo ($\varphi = \hat{\varphi}$) scenarios. Figure 10 shows the distribution of the regulation effect across municipalities, separately for municipalities with April 2016 and December 2019 regulation deadlines. The effects are sizable. The average effect for those municipalities with an April 2016 deadline is 1.25 years, with the tenth percentile equal to 0.06 years and the ninetieth percentile equal to 3.4 years. For those municipalities with a December 2019 deadline, the average effect is 0.59 years, with the tenth percentile equal to 0.1 years and the ninetieth percentile equal to 1.33 years. The regulation accelerates the introduction of 3G technology in all but 5 municipalities. Though the deterrence effect discussed in the introduction is present – see subsection 6.3 – it is dominated by the regulation effect on the behavior of the regulated firm.

Though the regulation targets 3G technology, there are spillover effects on the introduction of 4G. Figure 11 shows the distribution of the regulation effect on the time to 4G introduction. The effects are smaller than those for 3G, but non-negligible. The average effect for those municipalities with a December 2019 (respectively, April 2016) deadline is 0.44 years (resp., 0.28 years), with the

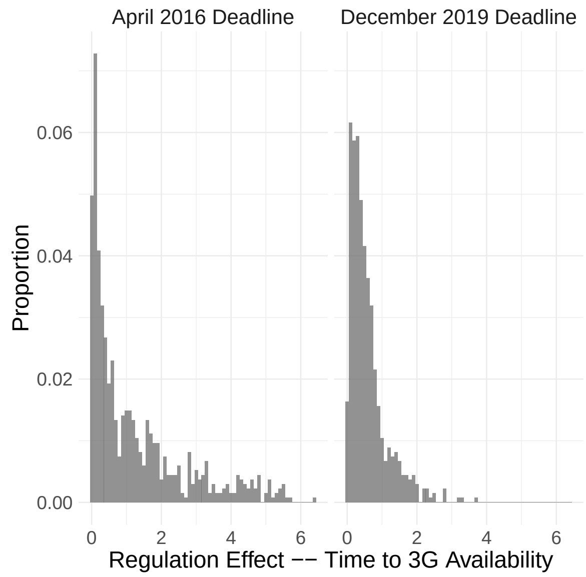


Figure 10: Distribution of the Regulation Effect on the Time to 3G Introduction

tenth percentile equal to 0.05 years (resp., 0 years) and the ninetieth percentile equal to 1.04 years (resp., 0.78 years). Interestingly, the regulation does cause delays in the introduction of 4G in 3.69 % of municipalities, though those delays are small.

6.2 The Cost of Regulation

Next, I use the model to calculate the cost imposed on firms by the regulation.[50] This cost is equal to

$$\text{Regulation Cost} = \sum_m \sum_f \left(V^0_{fm}(s_{fm0}, \boldsymbol{s}_{-fm0}, t=0) - V^{\hat{\varphi}}_{fm}(s_{fm0}, \boldsymbol{s}_{-fm0}, t=0) \right)$$

where $V^{\varphi}_{fm}(\omega)$ is firm f's ex-ante expected profit in municipality m and state ω when the fine is equal to φ. I calculate that the regulation cost amounts to $3{,}221$ million 2013 BRL, or $1{,}498$ million 2013 USD.[51] This amounts to 9.73% of firms' aggregate ex-ante expected profits without regulation.

In table 5 I report the incidence of these costs by the firm's regulatory status and its initial state in the data. The table shows the total cost, the average cost and the fraction of the total cost borne by the respective group of firms. Average costs are positive for all groups. Most of the costs are borne by regulated

[50] Part of this cost is borne by the government via reduced revenue in spectrum auctions.

[51] This conversion uses the average exchange rate in 2013 of 0.4651 USD per BRL.

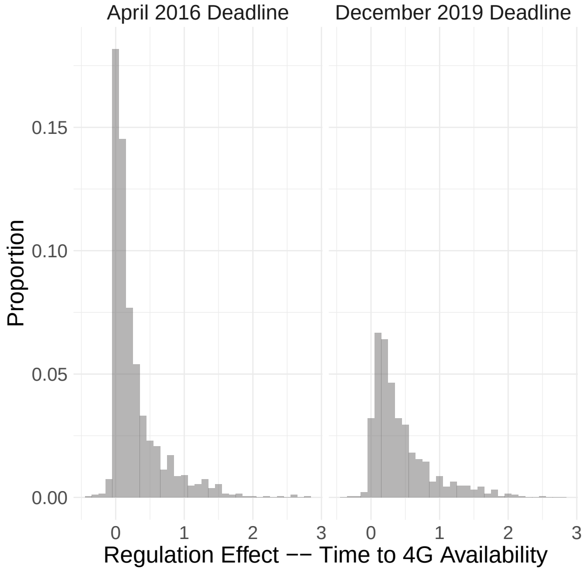


Figure 11: Distribution of the Regulation Effect on the Time to 4G Introduction

firms. These firms are forced to take costly actions they would not have taken in the absence of regulation. Costs are highest when the regulated firm is not active in the beginning of the data, as in that case compliance requires paying both entry and technology installation costs. Unregulated firms bear a non-negligible share of total costs. These are due to the regulation exposing them to tougher competition. This effect is more pronounced for firms that are active in the market in the beginning of the data because they necessarily face tougher competition, whereas potential entrants are only hurt conditional on entry.

Table 5: Incidence of Regulation Cost

Regulated	Firm State	Total Cost	Average Cost	Percentage of Total Cost
No	Out	149.66	0.04	4.65
No	2G	217.11	0.15	6.74
No	3G	141.48	0.25	4.39
Yes	Out	1,630.47	2.86	50.62
Yes	2G	1,082.32	0.83	33.60

This table shows the total, average, and fraction of total costs borne by firms as a function of their regulatory status and their initial technology in the data. Total and average costs are in million BRL.

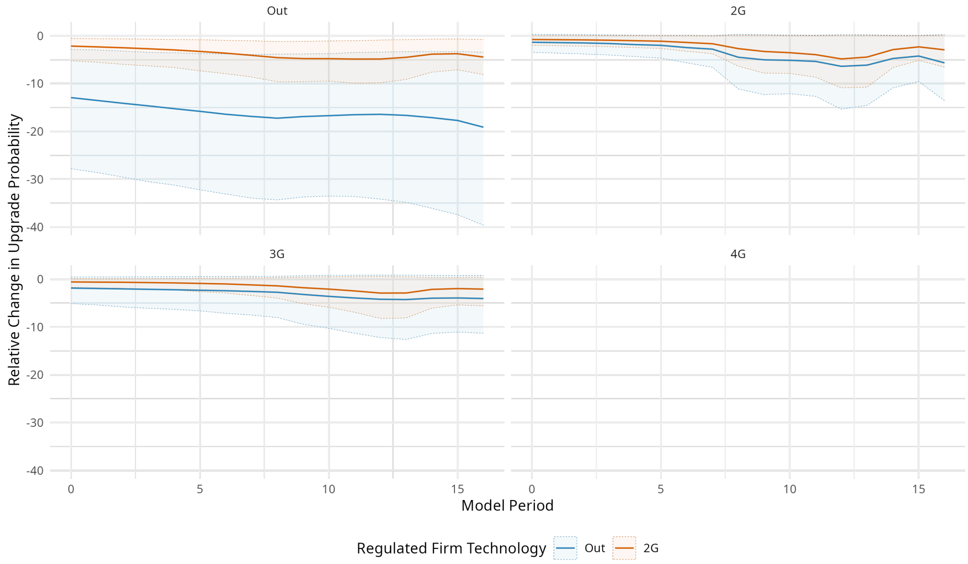


Figure 12: Regulation Effect on Probabilities of Upgrades by Unregulated Firms

Relative changes in the probability of upgrade by unregulated firms due to regulation, by firm technology and technology of the regulated firm. For every municipality, model period, and unregulated carrier, I compute its upgrade probability given its own technology and the technology of the regulated firm. The other two firms are assumed to be inactive. The solid lines show average relative changes across municipalities and carriers in a given period. The shaded regions are delimited by the 5th and 95th percentiles of the distribution of relative changes.

6.3 Quantifying Equilibrium Effects

In this section I use the model to quantify the relevance of equilibrium responses to the regulation. Consider a change from a no-regulation scenario to a situation with $\varphi > 0$. Let the *direct effect* of the regulation be the change in behavior by the regulated firm holding the behavior of unregulated firms constant at the no-regulation equilibrium. The *equilibrium responses* to the regulation are twofold. First, the change in behavior by the unregulated firms in response to the changed behavior of the regulated firm. Second, the change in behavior by the regulated firm, relative to its behavior when accounting only for the direct effect, in response to the changed behavior of the unregulated firms.

Figure 12 illustrates how the first of these effects operates. It plots the relative changes in the probabilities of upgrades by unregulated firms due to the regulation, by their technology and the technology of the regulated firm. The solid lines show average relative changes across municipalities and carriers, whereas the shaded regions are delimited by the 5th and 95th percentiles of the distribution of relative changes. Relative changes in unregulated firms' upgrade probabilities are almost always negative: in anticipation of tougher competition in the future, these firms upgrade their technologies less often. In a few cases, the strategic complementarity effect dominates the preemption effect, and unregulated firms are more likely to upgrade their technologies. This occurs only when the unregulated firm is already active.[52] The magnitude of the regulation effect on the behavior of unregulated firms is increasing over time. As time goes by, a non-compliant regulated firm becomes more likely to upgrade its technology. The more imminent toughening of competition further reduces the probability of upgrades by unregulated firms. The reduction in unregulated firms' upgrade probabilities due to the regulation can be substantial: the relative change in entry probabilities can be as large as 40% and its conditional mean can be close to 20%.

To quantify how equilibrium responses to the regulation affect the time to 3G introduction, I simulate firms' decisions according to the equilibrium policies obtained with the estimated fine $\hat{\varphi}$ and accounting only for the direct effect of the regulation as defined above. To compute the latter, I first solve for QS-MPE setting $\varphi = 0$. Then, I set the fine to $\hat{\varphi}$ and solve for the regulated firm's optimal policy, holding the unregulated firms' policy functions constant. The desired equilibrium effects are the difference in the time to 3G introduction between these two scenarios.

Figure 13 shows the distribution of equilibrium effects across municipalities. Most of the values are positive: equilibrium responses to the regulation lead to a longer time to 3G introduction. This reflects the reduced incentives to invest faced by unregulated firms, in light of the anticipated tougher competition. Quantitatively, however, these effects are small relative to the overall effects reported in subsection 6.1.

6.4 Subsidy Auctions

The cost incidence figures above show that coverage requirements impose a large cost on firms, especially when the regulated firm is not active in the market. That result, coupled with the fact that the majority of markets does have active firms in the beginning of the data, suggests sizable inefficiencies in the implementation of coverage requirements. In particular, substantial cost savings might be achieved by reallocating coverage requirements from potential entrants to incumbents. Other dimensions of firm heterogeneity can further

[52] This is in line with the results in Figure 7.

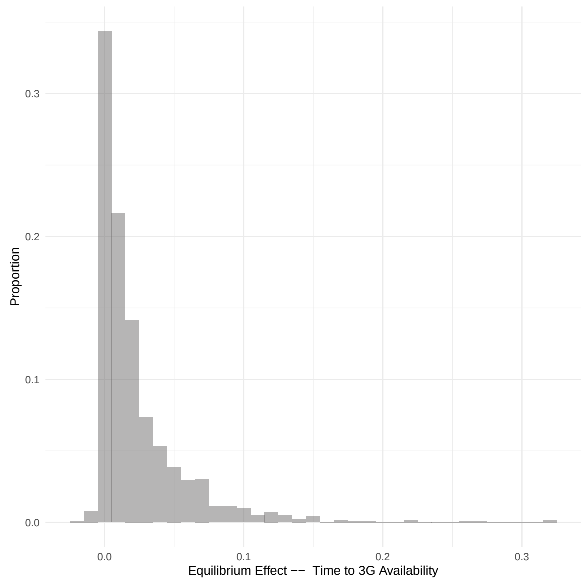


Figure 13: Equilibrium Effects.

This figure shows the difference between the time to the introduction of 3G (or 4G) under the equilibrium with regulation and when only the regulated firm responds to the regulation.

contribute to potential cost savings from such reallocations. There are two reasons why the selection of the firm subject to a requirement in a given market may be inefficient. First, when a firm chooses a municipality where it will be subject to a coverage requirement according to the mechanism described in Appendix C, it seeks to minimize its own costs of being regulated, not internalizing the fact that its cost savings from being regulated in that market relative to some alternative market may be smaller than those of another firm. Second, constraints on the number of markets where firms are regulated prevent an allocation in which an efficient firm is subject to coverage requirements in a large number of markets.

This subsection quantifies these inefficiencies by reallocating coverage requirements across firms by means of a subsidy auction. Specifically, for each municipality, I solve for QS-MPE with no regulation and assigning the coverage requirement to each of the four firms in turn. These QS-MPE return value functions $V^0_{fm}(\boldsymbol{s}_{m0}, t = 0)$, as defined in section 6.2, and $V^{\hat{\varphi}}_{fm}(\boldsymbol{s}_{m0}, t = 0; r)$, respectively, where the additional argument $r \in \{1, 2, 3, 4\}$ indicates the identity of the regulated firm. For each firm and market, I compute the following transfer:

$$T_{fm} := V^0_{fm}(\boldsymbol{s}_{m0}, t = 0) - V^{\hat{\varphi}}_{fm}(\boldsymbol{s}_{m0}, t = 0; f) \ .$$

If firm f were to receive this transfer, it would be willing to be subject to a cov-

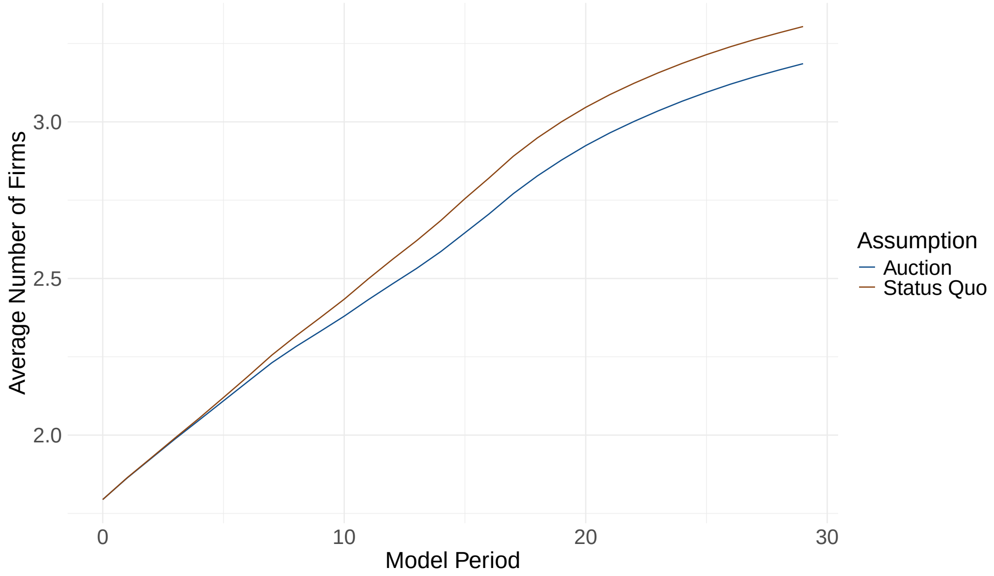


Figure 14: Number of Firms over Time, under Coverage Requirements and Subsidy Auction

erage requirement in municipality m.[53] I then assign the coverage requirement to the firm with the smallest T_{fm} in each municipality, and that firm receives the transfer T_{fm}.

Subsidy Auction Results. I find that the status-quo implementation of coverage requirements fails to identify the least-costly firm to regulate in 65% of municipalities. The aggregate subsidy, $\sum_m \min_f T_{fm}$, amounts to 661 million BRL, only 21% of the cost of the status-quo regulation. These cost savings come at no loss in terms of the speed of 3G roll-out: 3G is introduced 0.04 years earlier on average under the auction. The subsidy auction generates a small acceleration in the introduction of 4G technology, relative to the status quo, of 0.14 years on average.

The cost savings from the subsidy auction are obtained in part by reallocating coverage requirements from potential entrants to incumbents. This leads to a reduction in competition, as illustrated by Figure 14. Could the loss in consumer surplus from this reduction in competition offset the cost savings from the subsidy auction? It is possible to use the estimated model to shed light on

[53]These transfers are larger than necessary. If firm f is not regulated itself, some other firm will be. Therefore, the appropriate transfer is $V^{\hat{\varphi}}_{fm}(s_{m0}; r) - V^{\hat{\varphi}}_{fm}(s_{m0}; f)$ for some $r \neq f$. Because regulation hurts unregulated firms, as shown in subsection 6.2, the approach in the text is conservative.

this question. Let us make the assumption that the gain in monthly consumer surplus due to the marginal entrant is constant and equal to Δ_{CS}. Then, the aggregate gain in consumer surplus due to the increased competition in the status-quo is larger than the cost savings under the subsidy auction if[54]

$$\sum_{m} \text{Pop}_m \mathbb{E}\left[\sum_{t\geq 0} \delta^t (N_{mt}^{SQ} - N_{mt}^{Auction}) 6\Delta_{CS}\right] \geq \Delta\text{Cost} ,$$

where N_{mt}^{SQ} and $N_{mt}^{Auction}$ are the number of firms in municipality m and period t under the status-quo and subsidy auction scenarios, respectively, and the factor 6 is the length of a period in the model in months. We can simulate the equilibria under the status-quo and the subsidy auction to compute the left-hand side of this inequality. The right-hand side comes from subsection 6.2. I find that for this inequality to hold, it must be that $\Delta_{CS} \geq 7.65$ BRL. This is a large number, and it is not credible that the marginal entrant generates this much consumer surplus. Indeed, this number is equal to 35% of the estimated average monthly expenditure on mobile telecommunications services in this set of municipalities.[55] Elliott et al. (2025) estimate that the gain in consumer surplus from moving from 2 to 3 firms is slightly more than 10% of the cheapest plan prices.[56] Bourreau, Sun, and Verboven (2021), using a different framework, estimate consumer surplus gains from the entry of a new firm in the French mobile telecommunications industry of 7.7% of industry sales. Both of these estimates are substantially smaller than the threshold of 35% estimated above. This suggests that subsidy auctions are more efficient than the status-quo implementation of coverage requirements.

7 Conclusion

Concerns of service underprovision are common in network industries, and so is regulatory intervention. This paper studies the effect of coverage requirements, a common form of universal service regulation in the mobile telecommunications industry, on the speed of roll-out of third and fourth generation mobile telecommunications technologies, quantifies the cost of such regulation, and analyzes alternative regulatory designs. To do so, I leverage new panel

[54]This calculation uses the average municipality population over the sample period.
[55]This estimate is obtained from the expenditure model estimates of section 4.
[56]See their Figures 7 and 9.

data on mobile telecommunications technology availability in Brazilian municipalities to estimate a dynamic game of entry and technology upgrade under regulation.

I show that the regulation accelerated the arrival of third generation technology to relatively low-income and rural municipalities in Brazil by 0.94 years, on average. The regulation effect can be even more substantial: the ninetieth percentile of its distribution is 2.43 years. This acceleration in the availability of 3G technology comes at a substantial cost. Firms' ex-ante expected profits decrease by 10%. Reallocating coverage requirements across firms by means of a subsidy auction leads to a very similar acceleration in the availability of 3G and 4G technology, but at only 21% of the cost. These cost-efficiency gains are accompanied by a small reduction in competition, but the losses in consumer surplus needed to overturn the cost-efficiency gains are implausibly large. These findings have immediate implications for the design of regulation in the mobile telecommunications industry, and potentially to others where universal service is a concern.

Some related questions are not addressed in this paper. First, though my results can inform the design of regulation, data limitations preclude me from conducting a complete welfare analysis. Combining data such as the one used in this paper with detailed data on consumer behavior to precisely characterize the trade-off between the regulatory cost imposed on firms and the benefits to consumers would be a valuable contribution. Second, the framework in this paper accommodates geographic interdependencies in firms' costs in a reduced-form manner. A more complete treatment of such interdependencies and their effects on firm behavior would be another valuable, albeit challenging, addition to the literature. These topics, however, are left for future research.

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Regulation and Service Provision in Dynamic Oligopoly: Evidence from Mobile Telecommunications

Online Appendix

Online Appendix

Appendix A Regulation and Delay in the Fudenberg-Tirole Model

A.1 The Model

There are two firms. Firm 1 is an incumbent and firm 2 a potential entrant. Time is continuous and the discount rate is r. Firm 1 initially operates as a monopolist with constant marginal cost $\bar{c}$. At any point in time $t \geq 0$, firms can adopt a technology with constant marginal cost $\underline{c}$. Adopting this technology at time t costs $C(t)$, where $C(t) > 0$, $C'(t) < 0$ and $C''(t) > 0$, for all $t \geq 0$.

Let $p^m(c)$ and $\pi^m(c)$ be, respectively, the monopoly price and profit when marginal cost is c. I focus on the case in which the innovation is *non-drastic*, i.e., $p^m(\underline{c}) \geq \bar{c}$. If both firms are in the market, they compete à la Bertrand. Let $\pi^d(c, c')$ be a firm's profit when its cost is c and its competitor's cost is c'. Under the assumption of a non-drastic innovation and Bertrand competition, π^d satisfies

$$\pi^d(\underline{c}, \bar{c}) = (\bar{c} - \underline{c})D(\bar{c}), \quad \pi^d(\bar{c}, \underline{c}) = 0 \quad \text{and} \quad \pi^d(c, c) = 0 \quad \forall c$$

Firms' strategies specify their decisions to adopt or not the new technology as a function of t and their competitor's technology.[57] Note that due to the Bertrand assumption, a firm will never adopt the new technology after its competitor has adopted, as they would incur the positive adoption cost but their flow profits would stay at zero.

If the incumbent is first to adopt at date t_1, its overall profit is

$$L_1(t_1) = \int_0^{t_1} \pi^m(\bar{c})e^{-rt}dt + \int_{t_1}^{\infty} \pi^m(\underline{c})e^{-rt}dt - C(t_1)e^{-rt_1} \tag{15}$$

If the incumbent is preempted at date t_2, its present discounted profit is

$$F_1(t_2) = \int_0^{t_2} \pi^m(\bar{c})e^{-rt}dt \tag{16}$$

[57] The discussion here is somewhat informal. Fudenberg and Tirole (1985) provide a careful description of appropriate strategies for this game.

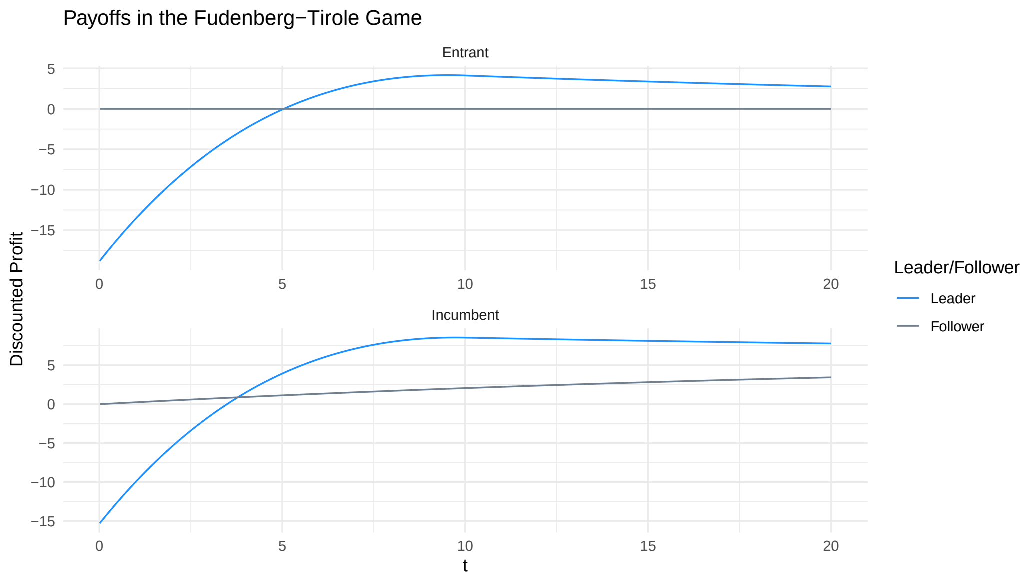

Figure 15: Payoffs in the Fudenberg-Tirole Model.

If the entrant is first to adopt at date t_2, its overall profit is

$$L_2(t_2) = \int_{t_2}^{\infty} \pi^d(\underline{c}, \bar{c})e^{-rt}dt - C(t_2)e^{-rt_2} \tag{17}$$

Finally, it the entrant is preempted at time t_1, its profit is given by $F_2(t_1) = 0$.

Figure 15 plots the functions L_1, F_1, L_2, F_2.[58] That figure is sufficient to determine the equilibrium outcome of the game.[59] Let t_2^* be defined by $F_2(t_2) = L_2(t_2)$. In Figure 15, $t_2^* \approx 5$. Firm 2 will not adopt before t_2^*, as it would prefer to be preempted by firm 1. Knowing this, firm 1 will wait to adopt, as $L_1(t_1)$ is increasing over $t_1 < t_2^*$. Now suppose firm 2 is first to adopt at some $t_2 > t_2^*$. Since $L_1(t_2) > F_1(t_2)$, firm 1 prefers to adopt at $t_2 - \varepsilon$. In equilibrium, firm 1 adopts at $t_1 = t_2^*$, and firm 2 never adopts.

A.2 Incorporating Regulation

Now suppose that the incumbent is regulated: is must adopt by some exogenously set deadline τ, lest it pay an exorbitant fine. The L_i and F_i functions are

[58] The specification is as follows. $D(p) = 2 - p$, $\bar{c} = 1$, $\underline{c} = 3/4$, $C(t) = \mathbf{1}\{t <= 10\}\left(\frac{t^2}{4} - 5 * t + 25\right) + 0.1$.

[59] But not the equilibrium itself.

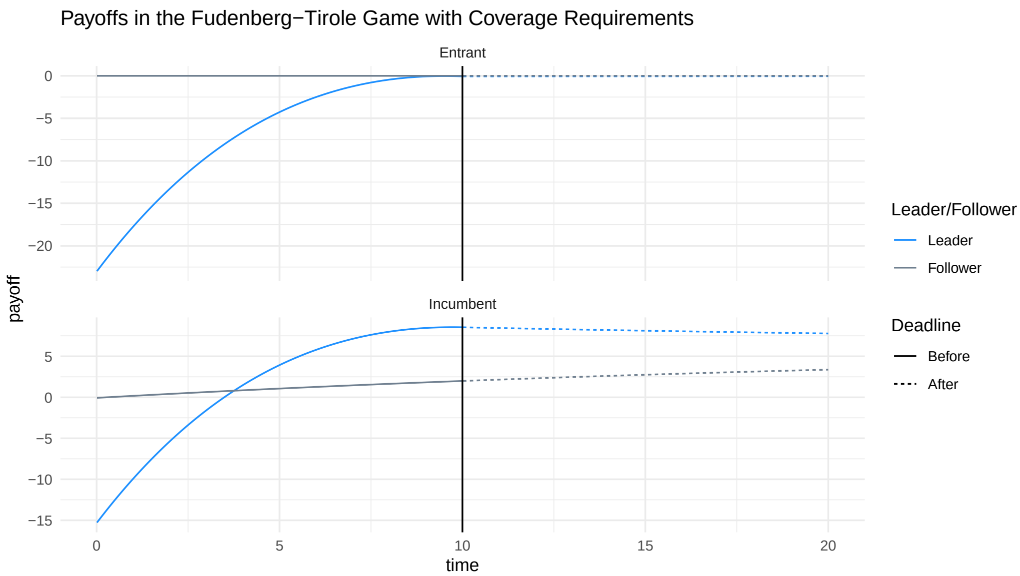

Figure 16: Payoffs in the Fudenberg-Tirole Model with Regulation.

now defined (for $t_i \leq \tau$) as follows:

$$L_1(t_1) = \int_0^{t_1} \pi^m(\bar{c})e^{-rt}dt + \int_{t_1}^{\infty} \pi^m(\underline{c})e^{-rt}dt - C(t_1)e^{-rt_1} \tag{18}$$
$$F_1(t_2) = \int_0^{t_2} \pi^m(\bar{c})e^{-rt}dt - C(\tau)e^{-r\tau}$$
$$L_2(t_2) = \int_{t_2}^{\tau} \pi^d(\underline{c}, \bar{c})e^{-rt}dt - C(t_2)e^{-rt_2}$$
$$F_2(t_1) = 0$$

Figure 16 plots these payoffs for the same parametrization underlying Figure 15, and $\tau = 10$. As can be seen from the figure, the fact that the incumbent will adopt the technology at time τ, at the latest, eliminates all incentive for the entrant to adopt the new technology. With no need to preempt the entrant, the incumbent is free to delay its own adoption to its most preferred time, which in this example is $t_1^* \approx 9.7$. Therefore, the regulation delays the adoption of the new technology from $t \approx 5$ to $t \approx 9.7$. Of course, if $\tau < 5$, the regulation speeds up the adoption of the new technology.

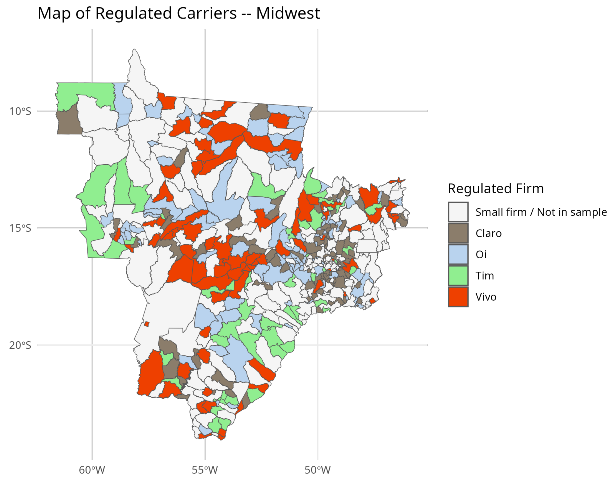

Figure 17: Regulated Carriers – Midwest.

This figure shows a map of the Brazilian Midwest, color-coded according to the identity of the single regulated firm in each market. The subdivisions in the map are municipalities. The municipalities without a color are either not in the estimation sample (municipalities with more than 30,000 inhabitants in 2006) or the regulated firm is one of the small carriers. Claro, Oi, Tim, and Vivo are the four large carriers in Brazil.

Appendix B Supplementary Figures and Alternative Specifications of Descriptive Models

Figure 17 illustrates the result of carriers' iterative choices of municipalities where they would be suject to a coverage requirement. The figure shows a map of the Brazilian Midwest, color-coded according to the identity of the regulated carrier. Figure 18 illustrates the data on technology availability. Each cell contains a map of the state of Pará, in the north of Brazil. The subdivision within each map are the municipalities in that state. Each row shows data for one of the four large carriers, and columns indicate the year for which the data is plotted. Municipalities are color-coded according to the best technology provided by the respective carrier in that municipality at the end of the year indicated in the column.

The tables below report alternative specifications of the descriptive models in table 3 in the main text. In particular, table 6 reports models without group fixed effects, and table 7 reports models that include characteristics of firms' networks in neighboring states. Specifically, the models include dummies for

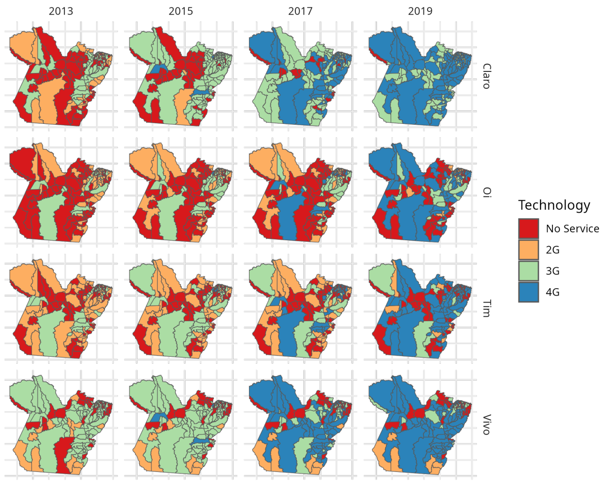

Figure 18: Technology availability in the state of Pará.

Each cell in this matrix contains a map of the state of Pará, in the north of Brazil. The subdivision within the state are municipalities. Rows correspond to the four large carriers in Brazil: Claro, Oi, Tim, and Vivo. Columns correspond to calendar years. Municipalities are color-coded according to the best technology offered in that municipality by the relevant firm in the end of the year.

whether or not the firm provides 2G, 3G, and 4G service in any neighboring municipality. Comparing table 6 and table 3 shows the importance of the group fixed effects. Without them in table 6, the competition coefficients are mostly small in absolute value and sometimes positive. That is in contrast with the results in table 3, where the competition coefficients are almost all negative and larger in absolute value. This suggests that the group fixed effects capture important unobserved factors related to how desirable it is to provide service in a given market.

Turning to table 7, the first thing to note is that service in neighboring municipalities is important. The estimated coefficients on 3G service and 4G service are sizeable and precisely estimated. Interestingly, the coefficients on 2G service in neighboring municipalities are negative. This is surprising because these coefficients are relative to not having service in the neighboring municipality. The next thing to observe is the effect of the network variables on the competition coefficients. These effects are mostly small, except perhaps for the number of competitors with 4G technology. Albeit small, the effects are always in the direction of increasing (in absolute value) the estimated competition coefficients.

Table 6: Entry/Upgrade Models – Without group fixed effects

	Out	2G	3G
Log GDP PC	0.010	0.000	0.008
	(0.003)	(0.003)	(0.002)
Log Pop.	0.020	0.030	−0.014
	(0.004)	(0.003)	(0.003)
Log Area	−0.004	−0.010	0.002
	(0.001)	(0.002)	(0.002)
Regulated	0.123	0.142	−0.039
	(0.011)	(0.008)	(0.005)
Regulated Competitor - Out	−0.009	−0.003	−0.035
	(0.004)	(0.006)	(0.008)
Regulated Competitor - 2G	−0.002	−0.044	−0.086
	(0.005)	(0.004)	(0.007)
No. Competitors 2G	0.004	0.007	−0.001
	(0.003)	(0.003)	(0.003)
No. Competitors 3G	−0.004	0.008	0.014
	(0.003)	(0.002)	(0.003)
No. Competitors 4G	0.003	−0.011	0.016
	(0.003)	(0.002)	(0.003)
Group FE	No	No	No
$\bar{Y}$	0.031	0.075	0.083
Num. obs.	108221	55708	57679

Linear probability models. The dependent variable is a dummy equal to 1 if a technology upgrade is observed. The explanatory variables are, in this order: the natural logarithms of GDP per capita, population, and municipality area, a dummy that is equal to 1 if the firm is regulated, a dummy that is equal to 1 if the firm faces a regulated competitor that is out of the market, a dummy that is equal to 1 if the firm faces a regulated competitor that has 2G technology, and the numbers of competitors with 2G, 3G and 4G technology. Each column corresponds to the subsample of the data where firms' best technology is as indicated in the column heading.

This may suggest that there are unobservable factors that are geographically correlated. Finally, and most importantly for the analysis in this paper, note that the effect of the network variables on the regulation variables is very minor, if present at all. This suggests that the regulation variables (in particular, whether or not a firm is regulated) are not correlated with the surrounding network infrastructure. Appendix C delves deeper into this.

Table 7: Entry/Upgrade Models – With Neighboring Network Info

	Out	2G	3G
Log GDP PC	0.010	−0.000	0.011
	(0.003)	(0.003)	(0.003)
Log Pop.	0.020	0.034	−0.009
	(0.004)	(0.003)	(0.003)
Log Area	−0.004	−0.013	−0.003
	(0.001)	(0.002)	(0.002)
Regulated	0.126	0.155	−0.026
	(0.011)	(0.008)	(0.005)
Regulated Competitor - Out	−0.006	0.008	−0.011
	(0.004)	(0.006)	(0.007)
Regulated Competitor - 2G	0.002	−0.031	−0.064
	(0.005)	(0.004)	(0.006)
No. Competitors 2G	0.004	0.008	0.006
	(0.003)	(0.003)	(0.003)
No. Competitors 3G	−0.007	0.005	0.015
	(0.003)	(0.002)	(0.003)
No. Competitors 4G	−0.006	−0.032	−0.014
	(0.003)	(0.003)	(0.004)
Nb. Service 2G	−0.022	−0.021	−0.033
	(0.006)	(0.009)	(0.010)
Nb. Service 3G	0.015	0.031	0.021
	(0.002)	(0.004)	(0.005)
Nb. Service 4G	0.024	0.084	0.118
	(0.003)	(0.008)	(0.008)
Group FE	Yes	Yes	Yes
$\bar{Y}$	0.031	0.075	0.083
Num. obs.	108221	55708	57679

Linear probability models. The dependent variable is a dummy equal to 1 if a technology upgrade is observed. The explanatory variables are as in table 6, with the addition of "Nb. Service 2G", "Nb. Service 3G", and "Nb. Service 4G". These are dummies indicating whether the carrier has service of the respective technology in a neighboring municipality. Each column corresponds to the subsample of the data where firms' best technology is as indicated in the column heading.

Appendix C Identity of Regulated Firms

The identity of the regulated firm in a given municipality was determined as follows. First, the country was divided into 131 *service areas*. These varied substantially in size, from a single municipality to an entire code area, which include on average 83 municipalities. A service area contains, on average, 42.52 municipalities. Within each of these service areas, the four large carriers would take turns selecting small numbers of municipalities where they would be subject to a coverage requirement, until all munics in the service area had been chosen. This process occurs immediately after each spectrum auction. The total number of municipalities to be selected in each service area by each firm and the number of municipalities chosen per turn were defined in the spectrum licenses.

The analysis in the main text, and in particular the identification of the fine parameter, rests on comparing the behavior of regulated and unregulated firms. Therefore, we must consider which factors make firms choose some markets to be regulated in and not others. First, demand for the products of the regulated firm may be higher than demand for the products of unregulated firms. Second, firms may select markets where they already have service, which lowers the cost of complying with the regulation. Third, they may select markets that are close to markets they serve. The first mechanism is accounted for in the structural model, as demand is allowed to vary flexibly across firms and municipalities. The second mechanism is also accounted for, as the model includes the firm's existing infrastructure in the municipality as a state variable. The model does not account for service in nearby markets, as doing so would be computationally prohibitive.

That choice is supported by the evidence in Table 8. Table 8 tests the hypothesis of no correlation between a firm's status as the regulated firm in a municipality and that firm's infrastructure in neighboring markets. The unit of analysis in Table 8 is a firm-market pair, and it uses information for the first period in the data only (March 2013). The table reports a linear probability model where the dependent variable is a dummy that takes the value 1 if the firm is regulated, and 0 otherwise. The explanatory variables are a constant and a set of dummies. The variable "2G Service" is equal to 1 if the firm provides 2G service in that market; "3G service" is analogously defined. "2G Service Nb." is equal to 1 if the firm provides 2G service in some neighboring market, and "3G Service Nb." is defined similarly. The results show that, conditional on

Table 8: Testing for Selection on Infrastructure in Neighboring Municipalities

	Regulated
2G Service	0.307
	(0.069)
3G Service	0.194
	(0.078)
2G Service Nb.	0.005
	(0.020)
3G Service Nb.	−0.017
	(0.012)
Num. obs.	4020
R^2	0.512
Adj. R^2	0.488

Linear probability model. The unit of observation is a firm-municipality pair. The dependent variable is a dummy indicating whether or not the respective firm is regulated in the market. "2G Service" and "3G Service" are dummies indicating whether the firm has service of the respective technology in that market. "2G Service Nb." and "3G Service Nb." are dummies indicating whether the firm has service of the respective technology in a neighboring municipality.

the technologies offered by a firm in the market, its infrastructure in neighboring municipalities has no effect on the probability that the firm is regulated. This finding assuages any potential concerns that the difference in behavior between regulated and unregulated firms is driven not by the regulation itself but by omitted differences in firms' neighboring infrastructure.

Appendix D Market Shares at the Code Area Level

This appendix justifies equation 9. As discussed in the main text, in estimation I substitute municipality GDP per capita for individual-level income, to save on computational cost. With that modification, equation 9 becomes[60]

$$\mu_{jc\tau} = \sum_{m \in c} \omega_m \int \mu_{j\tau}(s_{m\tau}, v_{m\tau}, \xi_{c(m)\tau} + \eta_{m\tau}; \theta) \, \mathrm{d}F(\eta_{m\tau}) \,. \tag{19}$$

Let h_i denote the alternative chosen by a consumer i. Within a given code-

[60]This simplification is completely inconsequential for the argument in this section.

area c, we have, by the Law of Total Probability

$$\mu_{jc} = \mathbb{P}(h_i = j) = \sum_{m \in c} \omega_m \mathbb{P}(h_i = j \mid m) = \sum_{m \in c} \omega_m \mu_j(s_m, v_m, \xi_{c(m)} + \eta_m; \theta) \; , \tag{20}$$

where I have dropped time subscripts, ω_m is the probability that the consumer comes from municipality m in code-area c, and $\mathbb{P}(h_i = j|m)$ is the probability that consumer i chooses j given that she comes from market m (and thus her demographic attributes come from a market-specific distribution). I will simplify the notation further and write simply $\mu_{jm}(\eta_m)$ instead of $\mu_j(s_m, v_m, \xi_{c(m)} + \eta_m; \theta)$.

I will show that $\operatorname{plim}_{n\to\infty} \left(\sum_{m=1}^{n} \omega_{nm}\mu_m(\eta_m) - \sum_{m=1}^{n} \omega_{nm}\mathbb{E}[\mu_m(\eta_m)]\right) = 0$. When considering $n \to \infty$, I am considering an infinite sequence of markets with known μ_m (i.e., known characteristics (s_m, v_m)) and a triangular array of weights ω_{nm}

$$\begin{array}{cccc} \omega_{1,1} & & & \\ \omega_{2,1} & \omega_{2,2} & & \\ \omega_{3,1} & \omega_{3,2} & \omega_{3,3} & \\ \vdots & \vdots & \vdots & \ddots \end{array}$$

such that $\omega_{nm} > 0$ for all n, m, $\lim_{n\to\infty} \omega_{nm} = 0$ for all m, and $\sum_{m=1}^{n} \omega_{nm} = 1$ for all n. Moreover, I assume that the η_m's are independent – see assumption 2.

By Chebyshev's Inequality

$$\begin{aligned} \mathbb{P}\left(\left|\sum_{m=1}^{n} \omega_{nm}\{\mu_m(\eta_m) - \mathbb{E}[\mu_m(\eta_m)]\}\right| > \varepsilon\right) &\leq \frac{1}{\varepsilon^2}\sum_{m=1}^{n} \omega_{nm}^2 \mathrm{Var}(\mu_m(\eta_m)) \\ &\leq \frac{1}{\varepsilon^2}\sum_{m=1}^{n} \omega_{nm}^2 \end{aligned}$$

where the second inequality follows because $\mu_m(\eta) \in [0, 1]$ for all η.

Now it suffices to show that $\sum_{m=1}^{n} \omega_{nm}^2 \to_n 0$. Assume without loss of generality that $w_{n1} = \max_{1\leq m\leq n} w_{nm}$. Then

$$\sum_{m=1}^{n} \omega_{nm}^2 \leq w_{n1}^2 + v(w_{n1}) \tag{21}$$

where

$$
\begin{aligned}
v(w_{n1}) := \max_{w_{n2},\ldots,w_{nn}} & \sum_{m=2}^{n} w_{nm}^2 \\
\text{s.t.} \quad & 0 < w_{nm} \leq w_{n1} \\
& \sum_{m=2}^{n} \omega_{nm} = 1 - w_{n1}
\end{aligned}
$$

Let k_n be the largest integer such that $k_n w_{n1} \leq 1 - w_{n1}$, i.e., $k_n = \left\lfloor \frac{1-\omega_{n1}}{\omega_{n1}} \right\rfloor$. Then

$$
\begin{aligned}
v(\omega_{n1}) &\leq k_n \omega_{n1}^2 + [1 - (k_n + 1)\omega_{n1}]^2 \\
&\leq \frac{1 - \omega_{n1}}{\omega_{n1}} \omega_{n1}^2 + \left(1 - \frac{1 - \omega_{n1}}{\omega_{n1}} \omega_{n1}\right)^2 \\
&= (1 - \omega_{n1})\omega_{n1} + \omega_{n1}^2
\end{aligned}
$$

It follows that $\lim_{n\to\infty} v(\omega_{n1}) = 0$ and, by 21, $\lim_{n\to\infty} \sum_{m=1}^{n} \omega_{nm}^2 = 0$.

Appendix E Identification of Dynamic Parameters

Let ω denote a generic state of the form $\omega = (\boldsymbol{s}, t)$.[61] Firm f's flow payoffs in market m, net of the idiosyncratic shocks, are given by

$$
\pi_{fm}(\omega) - c_{fm}(a, s_f(\omega); \theta) - \varphi \mathbf{1}\{r = f\}\mathbf{1}\{T_m < t\}\mathbf{1}\{s_f(\omega) < 2\}
$$

where $c_{fm}(a, s; \theta)$ is the deterministic part of costs as defined in equation (2). The cost function is linear in parameters. Abusing notation slightly, write $c_{fm}(a, s; \theta) = c_{fm}(a, s)\theta$.

Define

$$
g_{fm}(\omega, a) := (\pi_{fm}(\omega), c_{fm}(a, s), \mathbf{1}\{r = f\}\mathbf{1}\{T_m < t\}\mathbf{1}\{s_f(\omega) < 2\})
$$

and

$$
\Psi := (1, \theta', \varphi)' \,.
$$

With this notation, flow payoffs can be written as

$$
\pi_{fm}(\omega) - c_{fm}(a, s_f(\omega); \theta) - \varphi \mathbf{1}\{r = f\}\mathbf{1}\{T_m < t\}\mathbf{1}\{s_f(\omega) < 2\} = g_{fm}(\omega, a)\Psi \,.
$$

[61] In the main text, I included the firm's vector of action-specific cost shocks ε_f in the definition of ω. The definition given here is not used elsewhere, so I trust it will cause no confusion.

Firm f's value function in market m satisfies the Bellman equation

$$V_{fm}(\omega, \boldsymbol{\varepsilon}) = \max_{a \in A(s_f(\omega))} g_{fm}(\omega, a)\Psi + \boldsymbol{\varepsilon}(a) + \delta \sum_{\omega'} V_{fm}(\omega') F_{P_m}(\omega' \mid \omega, a)$$

where F_{P_m} denotes the state transitions induced by the equilibrium conditional choice probabilities P_m and

$$V_{fm}(\omega') := \int V_{fm}(\omega', \boldsymbol{\varepsilon}) \, \mathrm{d}G(\boldsymbol{\varepsilon})$$

Denote firm f's equilibrium policy in market m by $\sigma^*_{fm}(\omega, \boldsymbol{\varepsilon})$. Then (using σ^*_{fm} as shorthand for $\sigma^*_{fm}(\omega, \boldsymbol{\varepsilon})$)

$$V_{fm}(\omega, \boldsymbol{\varepsilon}) = g_{fm}(\omega, \sigma^*_{fm})\Psi + \boldsymbol{\varepsilon}(\sigma^*_{fm}) + \delta \sum_{\omega'} V_{fm}(\omega') F_{P_m}(\omega' \mid \omega, \sigma^*_{fm})$$

Integrating both sides of this equation yields

$$\begin{aligned} V_{fm}(\omega) = &\left(\int g_{fm}(\omega, \sigma^*_{fm}) \, \mathrm{d}G(\boldsymbol{\varepsilon}) \right) \Psi \\ &+ \int \boldsymbol{\varepsilon}(\sigma^*_{fm}) \, \mathrm{d}G(\boldsymbol{\varepsilon}) + \delta \sum_{\omega'} V_{fm}(\omega') \int F_{P_m}(\omega' \mid \omega, \sigma^*_{fm}) \, \mathrm{d}G(\boldsymbol{\varepsilon}) \end{aligned}$$

Let $\mathcal{C}^*_{fm}(a, \omega)$ be the set of shocks $\boldsymbol{\varepsilon} \in \mathbb{R}^{|A(s_f)|}$ such that $\sigma^*_{fm}(\omega, \boldsymbol{\varepsilon}) = a$. Then

$$\begin{aligned} \int g_{fm}(\omega, \sigma^*_{fm}) \, \mathrm{d}G(\boldsymbol{\varepsilon}) &= \sum_{a \in A(s_f)} \int_{\mathcal{C}^*_{fm}(a,\omega)} g_{fm}(\omega, \sigma^*_{fm}) \, \mathrm{d}G(\boldsymbol{\varepsilon}) \\ &= \sum_{a \in A(s_f)} g_{fm}(\omega, a) \int_{\mathcal{C}^*_{fm}(a,\omega)} \mathrm{d}G(\boldsymbol{\varepsilon}) \\ &= \sum_{a \in A(s_f)} g_{fm}(\omega, a) P_{fm}(a \mid \omega) \end{aligned}$$

where $P_{fm}(a \mid \omega)$ is the equilibrium probability that firm f chooses action a in state ω.

Similarly,

$$\int F_{P_m}(\omega' \mid \omega, \sigma^*_{fm}) \, \mathrm{d}G(\boldsymbol{\varepsilon}) = \underbrace{\sum_{a \in A(s_f)} F_{P_m}(\omega' \mid \omega, a) P_{fm}(a \mid \omega)}_{F_{P_m}(\omega' \mid \omega)}$$

The term on the right hand side of this equation is the probability that the state moves from ω to ω' under the equilibrium conditional choice probabilities.

Finally, observe that

$$\begin{aligned}\int \boldsymbol{\varepsilon}(\sigma^*_{fm})\,\mathrm{d}G(\boldsymbol{\varepsilon}) &= \sum_{a\in A(s_f)} \int_{\mathcal{C}^*_{fm}(a,\omega)} \boldsymbol{\varepsilon}(a)\,\mathrm{d}G(\boldsymbol{\varepsilon}) \\ &= \sum_{a\in A(s_f)} P_{fm}(a \mid \omega)\mathbb{E}[\boldsymbol{\varepsilon}(a) \mid a = \sigma^*_{fm}(\omega,\boldsymbol{\varepsilon})]\end{aligned}$$

It is well known that for the Type I Extreme Value distribution, $\mathbb{E}[\boldsymbol{\varepsilon}(a) \mid a = \sigma^*_{fm}(\omega,\varepsilon)] = \lambda(\gamma - \ln P_{fm}(a \mid \omega))$, where γ is the Euler-Mascheroni constant. Therefore

$$\int \boldsymbol{\varepsilon}(\sigma^*_{fm})\,\mathrm{d}G(\boldsymbol{\varepsilon}) = \lambda \sum_{a\in A(s_f)} P_{fm}(a \mid \omega)(\gamma - \ln P_{fm}(a \mid \omega))$$

Putting these pieces together, we have

$$\begin{aligned}V_{fm}(\omega) = &\left(\sum_{a\in A(s_f)} g_{fm}(\omega,a)P_{fm}(a \mid \omega)\right)\Psi + \lambda \sum_{a\in A(s_f)} P_{fm}(a \mid \omega)(\gamma - \ln P_{fm}(a \mid \omega)) \\ &+ \delta \sum_{\omega'} V_{fm}(\omega')F_{P_m}(\omega' \mid \omega)\end{aligned}$$

or

$$V_{fm}(\omega) = \mathbb{E}_{P_{fm}(\omega)}[g_{fm}(\omega,a)]\Psi + \lambda\gamma - \lambda\mathbb{E}_{P_{fm}(\omega)}[\ln P_{fm}(a \mid \omega)] + \delta F_{P_m(\omega)}V$$

where $\mathbb{E}_{P_{fm}(\omega)}$ denotes an expectation with respect to a using the distribution defined by $P_{fm}(\omega)$, $F_{P(\omega)}$ is a row vector with the transition probabilities in state ω, and V is a vector with the value function in each state ω.

We can now stack these equations. Define the following objects:

$$\mathbb{E}_{P_{fm}}[g_{fm}(\Omega,a)] := \begin{bmatrix} \mathbb{E}_{P_{fm}(\omega_1)}[g_{fm}(\omega_1,a)] \\ \vdots \\ \mathbb{E}_{P_{fm}(\omega_{|\Omega|})}[g_{fm}(\omega_{|\Omega|},a)] \end{bmatrix},$$

$$\mathbb{E}_{P_{fm}}[\ln P_{fm}(a \mid \Omega)] := \begin{bmatrix} \mathbb{E}_{P_{fm}(\omega_1)}[\ln P_{fm}(a \mid \omega_1)] \\ \vdots \\ \mathbb{E}_{P_{fm}(\omega_{|\Omega|})}[\ln P_{fm}(a \mid \omega_{|\Omega|})] \end{bmatrix},$$

and

$$M_{P_m} = \begin{bmatrix} F_{P_m(\omega_1)} \\ \vdots \\ F_{P_m(\omega_{|\Omega|})} \end{bmatrix}$$

Then[62]

$$V_{fm} = \mathbb{E}_{P_{fm}}[g_{fm}(\Omega, a)]\Psi + \lambda\gamma - \lambda\mathbb{E}_{P_{fm}}[\ln P_{fm}(a \mid \Omega)] + \delta M_{P_m} V_{fm}$$

From this equation we obtain

$$\begin{aligned} V_{fm} &= (I - \delta M_{P_m})^{-1}\Big\{\mathbb{E}_{P_{fm}}[g_{fm}(\Omega, a)]\Psi + \lambda\gamma - \lambda\mathbb{E}_{P_{fm}}[\ln P_{fm}(a \mid \Omega)]\Big\} \\ &= \lambda K(P_m) + (I - \delta M_{P_m})^{-1}\mathbb{E}_{P_{fm}}[g_{fm}(\Omega, a)]\Psi \end{aligned}$$

where $K(P_m) := (I - \delta M_{P_m})^{-1}(\gamma - \mathbb{E}_{P_{fm}}[\ln P_{fm}(a \mid \Omega)])$

The conditional value function is, by definition,

$$v_{fm}(a, \omega) = g_{fm}(\omega, a)\Psi + \delta \sum_{\omega'} V_{fm}(\omega') F_{P_m}(\omega' | \omega, a) = g_{fm}(\omega, a)\Psi + \delta F_{P_m}(\omega, a) V_{fm}$$

where $F_{P_m}(\omega, a)$ is the distribution over ω' induced by taking action a in state ω when competitors follow P_m. Using the result above for V yields

$$\begin{aligned} \frac{v_{fm}(a, \omega)}{\lambda} &= \delta F_{P_m}(\omega, a) K(P_m) + \\ &+ \Big\{g_{fm}(\omega, a) + \delta F_{P_m}(\omega, a)(I - \delta M_{P_m})^{-1}\mathbb{E}_{P_{fm}}[g_{fm}(\Omega, a)]\Big\}\lambda^{-1}\Psi \\ &= f_{P_m}(\omega, a) + h_{fm,P_m}(\omega, a)\lambda^{-1}\Psi \ . \end{aligned}$$

Since the idiosyncratic errors follow a Type 1 Extreme Value distribution, the conditional choice probabilities have the logit form:

$$P_{fm}(a \mid \omega) = \frac{\exp(v_{fm}(a, \omega)/\lambda)}{\sum_{a' \in A(s_f)} \exp(v_{fm}(a', \omega)/\lambda)} \ .$$

The usual logit inversion gives

$$\ln(P_{fm}(a \mid \omega)) - \ln(P_{fm}(s_f \mid \omega)) = \frac{v_{fm}(a, \omega)}{\lambda} - \frac{v_{fm}(s_f, \omega)}{\lambda}$$

Using the linear representation of the conditional value functions it follows

[62] In this equation, it is to be understood that the scalar $\lambda\gamma$ is added to all coordinates.

that

$$\Upsilon_{fm,P_m}(a,\omega) = \chi_{fm,P_m}(a,\omega)' \frac{\Psi}{\lambda} \tag{22}$$

where

$$\Upsilon_{fm,P_m}(a,\omega) := [\ln(P_{fm}(a \mid \omega)) - \ln(P_{fm}(s_f \mid \omega))] - [f_{P_m}(\omega,a) - f_{P_m}(\omega,s_f)]$$

and

$$\chi_{fm,P_m}(a,\omega) := h_{fm,P_m}(\omega,a) - h_{fm,P_m}(\omega,s_f)\ .$$

If P_m is identified, then so are $\Upsilon_{fm,P_m}(a,\omega)$ and $\chi_{fm,P_m}(a,\omega)$. In that case, as long as the matrix χ stacking $\chi_{fm,P_m}(a,\omega)$ for all f, m, $a \in A(s_f) \setminus \{s_f\}$ and $\omega \in \Omega$ has full column rank, λ^{-1}/Ψ is identified. Moreover, the first coordinate of Ψ is 1, so that λ and Ψ are separately identified.

This argument allows for municipality-specific equilibria P^m. For municipality-specific CCPs to be estimable from the data, it is necessary that the map from market-level observables to the Quasi-Stationary Markov Perfect Equilibrium be continuous. Figure 19 provides evidence in favor of that. Each panel shows data for one of four randomly sampled municipalities. Each dot in a scatterplot corresponds to one of the municipalities with the same regulation deadline and regulated carrier as the focal municipality. The x-axis is the Euclidean distance of municipality characteristics – the estimated flow profits and covariates entering the cost model. The y-axis is the mean absolute difference of equilibrium conditional choice probabilities in both municipalities. These conditional choice probabilities are computed at the estimated parameters and under the status-quo regulatory regime. These scatterplots show that as the distance of demographics approahces zero so does the distance of CCPs, as desired.

Appendix F Parameter Estimates

F.1 Estimates of Static Parameters

Tables 9 and 10 show estimates of the parameters of the product choice model, i.e. those in equation (5). The parameter labels are as in that equation.

F.2 Estimates of Dynamic Parameters

Table 11 shows estimates of the dynamic parameters of the model, i.e. the entry and technology upgrade cost parameters and the fine for non-compliance with

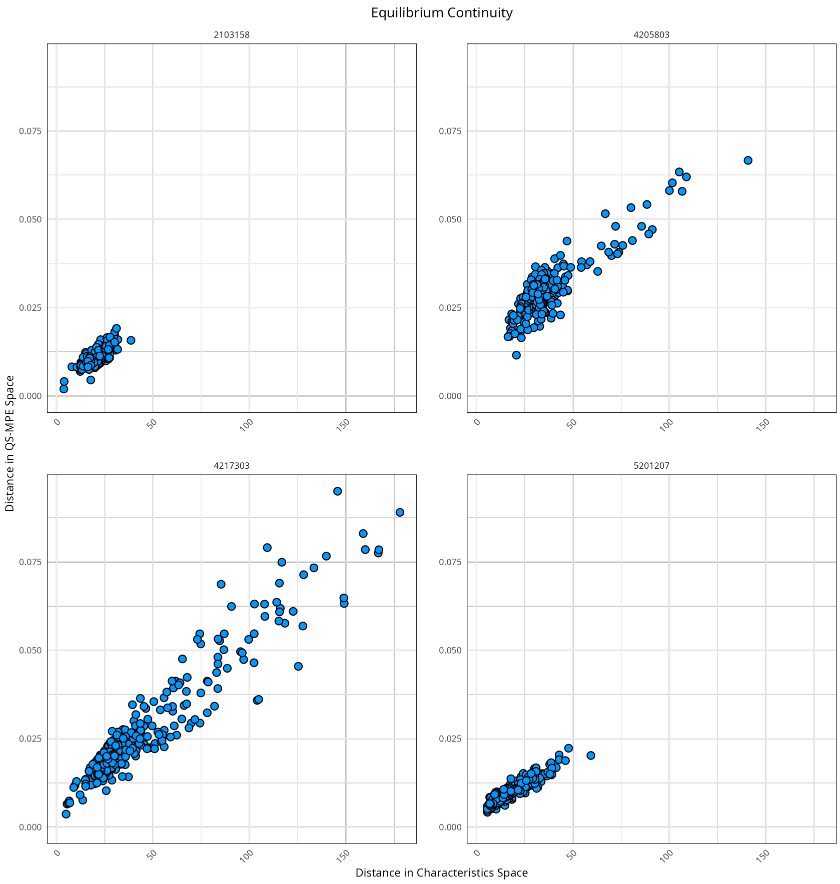

Figure 19: Equilibrium continuity.

Each panel shows data for one of four randomly sampled municipalities. Each dot in a scatterplot corresponds to one of the municipalities with the same regulation deadline and carrier as the focal municipality. The x-axis shows the Euclidean distance of municipality characteristics – the estimated flow profits and covariates entering the cost model. The y-axis shows the mean absolute difference of equilibrium conditional choice probabilities in both municipalities.

Parameter	Estimate	Std. Error	2.5-th Perc.	97.5-th Perc.
$\alpha_{2G,2013}$	-1.4056	0.183366	-2.0271	-1.2269
$\alpha_{2G,2014}$	-1.4918	0.230877	-2.2578	-1.3357
$\alpha_{2G,2015}$	-1.4487	0.100363	-1.7896	-1.3917
$\alpha_{2G,2016}$	-1.5296	0.0491533	-1.6233	-1.4092
$\alpha_{2G,2017}$	-1.0409	0.0341098	-1.1353	-0.9950
$\alpha_{2G,2018}$	-0.3020	0.0336517	-0.3881	-0.2528
$\alpha_{2G,2019}$	-0.0514	0.0445251	-0.1028	0.0514
$\alpha_{2G,2020}$	-0.8554	0.0713141	-1.0467	-0.7676
$\alpha_{2G,2021}$	-1.3162	0.0741907	-1.5254	-1.2150
$\alpha_{3G,2013}$	-6.0324	0.0364207	-6.1197	-5.9795
$\alpha_{3G,2014}$	-2.9444	0.0344033	-3.0152	-2.8910
$\alpha_{3G,2015}$	-1.9359	0.0435661	-2.0220	-1.8571
$\alpha_{3G,2016}$	-2.0256	0.0553112	-2.1829	-1.9575
$\alpha_{3G,2017}$	-1.5625	0.0901411	-1.8977	-1.5242
$\alpha_{3G,2018}$	-0.9537	0.0328903	-1.0423	-0.9007
$\alpha_{3G,2019}$	-0.2787	0.0830337	-0.5167	-0.1391
$\alpha_{3G,2020}$	-1.5807	0.074082	-1.8054	-1.4894
$\alpha_{3G,2021}$	-1.5345	0.109833	-1.8634	-1.4198
$\alpha_{4G,2015}$	-9.8833	0.0314885	-9.9677	-9.8331
$\alpha_{4G,2016}$	-5.4236	0.0349428	-5.5060	-5.3485
$\alpha_{4G,2017}$	-3.8686	0.0357715	-3.9361	-3.7960
$\alpha_{4G,2018}$	-1.9024	0.233497	-2.6186	-1.6311
$\alpha_{4G,2019}$	-2.2121	0.171531	-2.8009	-2.1242
$\alpha_{4G,2020}$	-2.9098	0.0773721	-3.1101	-2.8202
$\alpha_{4G,2021}$	-2.3535	0.194228	-3.0355	-2.3055
$\beta_{2G,2013}$	0.0079	0.00949428	-0.0079	0.0157
$\beta_{2G,2014}$	-0.0041	0.0051768	-0.0082	0.0041
$\beta_{2G,2015}$	-0.0560	0.0306907	-0.1120	-0.0099
$\beta_{2G,2016}$	-0.1371	0.0398212	-0.2265	-0.0734
$\beta_{2G,2017}$	-0.1996	0.0377373	-0.2959	-0.1547
$\beta_{2G,2018}$	-0.3055	0.0323921	-0.4046	-0.2862
$\beta_{2G,2019}$	-0.3785	0.0349853	-0.4887	-0.3601
$\beta_{2G,2020}$	-0.3193	0.0350039	-0.4259	-0.3006
$\beta_{2G,2021}$	-0.2594	0.034057	-0.3630	-0.2352
$\beta_{3G,2013}$	0.3900	0.0389403	0.2893	0.4479
$\beta_{3G,2014}$	0.1237	0.0413491	0.0163	0.1835
$\beta_{3G,2015}$	0.0680	0.0413195	-0.0386	0.1186
$\beta_{3G,2016}$	0.0405	0.0337802	-0.0399	0.0810
$\beta_{3G,2017}$	-0.0539	0.0286894	-0.1079	-0.0084
$\beta_{3G,2018}$	-0.1698	0.03596	-0.2719	-0.1374
$\beta_{3G,2019}$	-0.3446	0.0323734	-0.4372	-0.3161
$\beta_{3G,2020}$	-0.2696	0.0318478	-0.3596	-0.2419
$\beta_{3G,2021}$	-0.2669	0.0334297	-0.3595	-0.2339
$\beta_{4G,2015}$	0.7289	0.0316155	0.6369	0.7564
$\beta_{4G,2016}$	0.4367	0.0503934	0.3058	0.4941
$\beta_{4G,2017}$	0.2054	0.0380264	0.1115	0.2551
$\beta_{4G,2018}$	0.0040	0.00508896	-0.0040	0.0080
$\beta_{4G,2019}$	-0.0414	0.018145	-0.0828	-0.0194
$\beta_{4G,2020}$	0.0457	0.0284917	-0.0417	0.0670
$\beta_{4G,2021}$	0.0211	0.0194445	-0.0211	0.0422

The first column identifies parameters in Equation (5), the second reports point estimates, and the third displays standard errors. The final two columns define a 95% confidence interval for each parameter. Standard errors and confidence intervals are computed by block-bootstrap at the municipality level, using 300 bootstrap replications.

Table 9: Static Parameter Estimates: Fixed Effects and Income Coefficients

Parameter	Estimate	Std. Error	2.5-th Perc.	97.5-th Perc.
$\kappa_{2G,2013}$	0.1095	0.0415805	-0.0089	0.1467
$\kappa_{2G,2014}$	0.0877	0.0457186	-0.0328	0.1455
$\kappa_{2G,2015}$	0.0843	0.0488529	-0.0668	0.1558
$\kappa_{2G,2016}$	0.1157	0.0486669	-0.0263	0.1618
$\kappa_{2G,2017}$	0.0527	0.0406099	-0.0527	0.1054
$\kappa_{2G,2018}$	0.0132	0.017168	-0.0132	0.0264
$\kappa_{2G,2019}$	0.0394	0.0249909	-0.0236	0.0771
$\kappa_{2G,2020}$	0.0153	0.0168505	-0.0153	0.0306
$\kappa_{2G,2021}$	0.0262	0.022188	-0.0262	0.0524
$\kappa_{3G,2013}$	0.1366	0.0509983	0.0109	0.2080
$\kappa_{3G,2014}$	0.1432	0.0542589	0.0021	0.2269
$\kappa_{3G,2015}$	0.1041	0.0557637	-0.0254	0.1946
$\kappa_{3G,2016}$	0.0847	0.0441259	-0.0144	0.1489
$\kappa_{3G,2017}$	0.0862	0.0406294	-0.0292	0.1337
$\kappa_{3G,2018}$	0.0881	0.0342024	-0.0136	0.1296
$\kappa_{3G,2019}$	0.1523	0.0237425	0.0791	0.1769
$\kappa_{3G,2020}$	0.1701	0.0219939	0.1098	0.1905
$\kappa_{3G,2021}$	0.1225	0.0255287	0.0537	0.1536
$\kappa_{4G,2015}$	0.0107	0.0149953	-0.0107	0.0214
$\kappa_{4G,2016}$	-0.0824	0.0648898	-0.1648	0.0585
$\kappa_{4G,2017}$	0.0460	0.0425537	-0.0460	0.0921
$\kappa_{4G,2018}$	0.0837	0.0420462	-0.0306	0.1270
$\kappa_{4G,2019}$	0.1681	0.0258985	0.0956	0.1962
$\kappa_{4G,2020}$	0.1499	0.0213974	0.0902	0.1692
$\kappa_{4G,2021}$	0.1775	0.0255952	0.1096	0.2135
ρ	0.6799	0.0905152	0.3415	0.6849

The first column identifies parameters in Equation (5), the second reports point estimates, and the third displays standard errors. The final two columns define a 95% confidence interval for each parameter. Standard errors and confidence intervals are computed by block-bootstrap at the municipality level, using 300 bootstrap replications.

Table 10: Static Parameter Estimates: Population Density Coefficients and Nesting Parameter

the regulation. In the table, superscripts 0 refer to intercept terms, whereas superscripts A refer to parameters that interact with the logarithm of a municipality's area. The subscripts indicate the technology and year a parameter refers to. Thus, if z_m is the logarithm of the area of municipality m, then the cost of introducing 3G in that municipality in 2013, net of spillover effects, is $\theta^0_{3G,2013} + \theta^A_{3G,2013} z_m$. The parameters with an s subscript capture spillovers from out-of-sample markets. Finally, φ is the fine for non-compliance with the regulation.

Parameter	Estimate	2.5-th Perc.	97.5-th Perc.
θ^0_{entry}	0.8006	0.7388	1.3353
θ^A_{entry}	1.2433	1.1552	1.2784
$\theta^0_{3G,2013}$	0.1661	0.1339	0.2506
$\theta^A_{3G,2013}$	0.6266	0.5898	0.6755
$\theta^0_{3G,2014}$	-0.0902	-0.2789	-0.0186
$\theta^A_{3G,2014}$	0.4769	0.4380	0.5510
$\theta^0_{3G,2015}$	0.0432	-0.0510	0.1494
$\theta^A_{3G,2015}$	0.3729	0.3306	0.4337
$\theta^0_{3G,2016}$	0.0004	-0.0819	0.0958
$\theta^A_{3G,2016}$	0.2973	0.2568	0.3672
$\theta^0_{3G,2017}$	0.0468	-0.0127	0.1205
$\theta^A_{3G,2017}$	0.3318	0.2873	0.4077
$\theta^0_{3G,2018}$	0.0675	0.0068	0.1571
$\theta^A_{3G,2018}$	0.1638	0.1183	0.2450
$\theta^0_{3G,2019}$	0.1963	0.1648	0.3731
$\theta^A_{3G,2019}$	0.2302	0.1894	0.3149
$\theta^0_{3G,2020}$	-0.1302	-0.2133	-0.0520
$\theta^A_{3G,2020}$	0.1242	0.0765	0.2352
$\theta^0_{3G,2021}$	0.1356	0.0665	0.3094
$\theta^A_{3G,2021}$	-0.0122	-0.0479	0.0938
$\theta^0_{4G,2014}$	0.2923	0.1717	0.3307
$\theta^A_{4G,2014}$	1.7066	1.1464	1.8880
$\theta^0_{4G,2015}$	0.0261	-0.0100	0.0480
$\theta^A_{4G,2015}$	0.2071	0.0985	0.2539
$\theta^0_{4G,2016}$	-0.1518	-0.3355	-0.1062
$\theta^A_{4G,2016}$	-0.1218	-0.2334	-0.0873
$\theta^0_{4G,2017}$	-0.3235	-0.5396	-0.2406
$\theta^A_{4G,2017}$	-0.3471	-0.4625	-0.3040
$\theta^0_{4G,2018}$	-0.1946	-0.3455	-0.1254
$\theta^A_{4G,2018}$	-0.3742	-0.5181	-0.3314
$\theta^0_{4G,2019}$	0.1167	0.0859	0.3213
$\theta^A_{4G,2019}$	-0.4346	-0.6252	-0.3858
$\theta^0_{4G,2020}$	-0.2288	-0.3704	-0.1623
$\theta^A_{4G,2020}$	-0.4425	-0.6131	-0.3877
$\theta^0_{4G,2021}$	0.1430	0.0755	0.4014
$\theta^A_{4G,2021}$	-0.6445	-0.8818	-0.5802
$\theta^0_{s,3G}$	-0.8534	-0.9265	-0.5641
$\theta_{s,3G}$	-0.3582	-0.4162	-0.1860
$\theta^0_{s,4G}$	-0.8066	-0.9304	-0.7389
$\theta_{s,4G}$	-0.1615	-0.2378	0.0610
θ_{Oi}	4.9747	4.8331	6.6451
φ	1.1388	1.1007	1.4355

This table displays estimates of the dynamic parameters – the entry and technology upgrade cost parameters and the fine for non-compliance with the regulation – together with 95% confidence intervals. The first column identifies parameters. The second column shows the point estimates. The final two columns define a 95% confidence interval for each parameter, computed by block-bootstrap at the municipality level, using 300 bootstrap replications.

Table 11: Dynamic Parameter Estimates