Universidade Federal do Ceará

Lista de Exercícios - Integrais
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Sumário

1 Integrais - Guidorizzi

Calcule a integral das seguintes equações:

1. $\int xdx$

$$\begin{aligned} \int xdx = \frac{x^{1+1}}{1+1} + C \\ \frac{x^2}{2} + C. \end{aligned} \tag{1}$$

2. $\int 3dx$

$$\int 3dx = 3x + C. \tag{2}$$

3. $\int (3x+1)dx$

$$\begin{aligned} \int (3x+1)dx = \int 3xdx + \int 1dx \\ 3 \times \int xdx + x + C <=> \\ 3 \times \frac{x^{1+1}}{1+1} + C <=> \\ \frac{3x^2}{2} + x + C \end{aligned} \tag{3}$$

4. $\int (x^3+2x+3)dx$

$$\begin{aligned} \int (x^3+2x+3)dx = \int (x^3)dx + 2 \times \int (x)dx + \int (3)dx \\ \frac{x^{3+1}}{3+1} + 2 \times \frac{x^{1+1}}{1+1} + 3x + C \\ \frac{x^4}{4} + \frac{2x^2}{2} + 3x + C \\ \frac{2x^4+8x^2}{8} + 3x + C. \end{aligned} \tag{4}$$

5. $\int x^3 dx$

$$\int x^3 dx = \frac{x^4}{4} + C \tag{5}$$

6. $\int (x^2 + x + 1) dx$

$$\begin{aligned} \int x^2 dx + \int x dx + \int 1 dx <=> \\ \frac{x^3}{3} + \frac{x^2}{2} + x + C \\ \frac{x^3 \times 2 + x^2 \times 3}{6} + x + C \\ \frac{2x^3 + 3x^2 + 6x}{6} + C. \end{aligned} \tag{6}$$

7. $\int \frac{1}{x} dx$

$$\int \frac{1}{x} dx = \ln x + C \tag{7}$$

8. $\int \left(x + \frac{1}{x^3} \right) dx$

$$\begin{aligned} \int x dx + \int \frac{1}{x^3} dx <=> \\ \frac{x^2}{2} + \frac{x^{-3+1}}{-3+1} + C \\ \frac{-2x^2 + 2x^{-2}}{-4} + C \end{aligned} \tag{8}$$

9. $\int \sqrt{x} dx$

$$\begin{aligned} \int \sqrt{x} dx = \int x^{\frac{1}{2}} dx \\ \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2} + 1} + C \\ \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C \\ \int x^{\frac{1}{2}} dx = \frac{2\sqrt{x^3}}{3} + C \end{aligned} \tag{9}$$

10. $\int \sqrt[3]{x}dx$

$$\begin{aligned}\int \sqrt[3]{x}dx &= \int x^{\frac{1}{3}}dx \\ \int x^{\frac{1}{3}}dx &= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} \\ \int x^{\frac{1}{3}}dx &= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} \\ \int x^{\frac{1}{3}}dx &= 3\frac{\sqrt[4]{x^3}}{4}\end{aligned} \tag{10}$$

11. $\int \left(x + \frac{1}{x}\right) dx$

$$\begin{aligned}\int \left(x + \frac{1}{x}\right) dx &= \int xdx + \int \left(\frac{1}{x}\right) dx \\ \int xdx &= \frac{x^2}{2} + C \\ \int \left(\frac{1}{x}\right) dx &= \ln x + C \\ \int \left(x + \frac{1}{x}\right) dx &= \frac{x^2}{2} + \ln x + C.\end{aligned} \tag{11}$$

12. $\int \left(2 + \sqrt[4]{x}\right) dx$

$$\begin{aligned}\int \left(2 + \sqrt[4]{x}\right) dx &= \int 2dx + \int x^{\frac{1}{4}}dx \\ \int 2dx &= 2x + C \\ \int x^{\frac{1}{4}}dx &= \frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C \\ \int x^{\frac{1}{4}}dx &= \frac{4\sqrt[4]{x^5}}{5}\end{aligned} \tag{12}$$

13. $\int (ax+b)dx$, a e b constantes.

$$\begin{aligned}\int axdx + \int bdx <=> \\ \int axdx = a \times \int xdx \\ \int axdx = a \times \frac{x^{1+1}}{1+1} \\ \int axdx = a \times \frac{x^2}{2} \\ \int bdx = bx + C \\ \int (ax+b)dx = a \times \frac{x^2}{2} + bx + C\end{aligned} \tag{13}$$

14. $\int \left(3x^2 + x + \frac{1}{x^3}\right) dx$

$$\begin{aligned}\int \left(3x^2 + x + \frac{1}{x^3}\right) = 3 \times \int x^2dx + \int xdx + \int \frac{1}{x^3}dx <=> \\ 3 \times \frac{x^3}{3} + \frac{x^2}{2} + \frac{x^{-2}}{-2} + C <=> \\ x^3 + \frac{x^2}{2} - \frac{1}{2} \times \frac{1}{x^2} + C <=> \\ x^3 + \frac{x^2}{2} - \frac{1}{2x^2} + C\end{aligned} \tag{14}$$

15. $\int \left(\sqrt{x} + \frac{1}{x^2}\right) dx$

$$\begin{aligned}
\int \left(\sqrt{x} + \frac{1}{x^2}\right) dx \\
\int \left(\sqrt{x} + \frac{1}{x^2}\right) dx = \int x^{\frac{1}{2}} dx + \int \frac{1}{x^2} dx <=> \\
\int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \\
\int x^{\frac{1}{2}} dx = \frac{2\sqrt{x^3}}{3} \\
\int \frac{1}{x^2} dx = \frac{x^{-2+1}}{-1} \\
\int \frac{1}{x^2} dx = \frac{x^{-1}}{-1} = -\frac{1}{x} \\
\int \left(\sqrt{x} + \frac{1}{x^2}\right) dx = \frac{2\sqrt{x^3}}{3} - \frac{1}{x} + C.
\end{aligned} \tag{15}$$

16. $\int \left(\frac{2}{x} + \frac{3}{x^2}\right) dx$

$$\begin{aligned}
2 \times \int \frac{1}{x} dx + 3 \times \int \frac{1}{x^2} dx <=> \\
2 \times \int \frac{1}{x} dx = 2 \times \ln x \\
3 \times \int x^{-2} = 3 \times \frac{x^{-1}}{-1} \\
3 \times \int x^{-2} = -3 \times \frac{1}{x} \\
\int \left(\frac{2}{x} + \frac{3}{x^2}\right) dx = 2 \ln x - 3\frac{1}{x} + C
\end{aligned} \tag{16}$$

17. $\int (3\sqrt[5]{x^2} + 3) dx$

$$\begin{aligned}
\int (3\sqrt[5]{x^2} + 3) dx = 3 \times \int x^{\frac{2}{5}} dx + \int 3 dx <=> \\
3 \times \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 3x + C = \frac{15\sqrt[5]{x^7}}{7} + 3x + C
\end{aligned} \tag{17}$$

18. $\displaystyle\int \left(2x^3 - \frac{1}{x^4}\right) dx$

$$\begin{aligned} 2 \times \int x^3 dx - \int x^{-4} dx <=> \\ 2 \times \frac{x^4}{4} - \frac{x^{-3}}{-3} <=> \\ \frac{x^2}{2} - \frac{1}{-3x^3} = \frac{x^2}{2} + \frac{1}{3x^3} \end{aligned} \tag{18}$$

19. $\displaystyle\int \left(\frac{x^2+1}{x}\right) dx$

$$\begin{aligned} \int \left(\frac{x^2+1}{x}\right) dx = \int \left(x + \frac{1}{x}\right) dx \\ \int \left(x + \frac{1}{x}\right) dx = \int x dx + \int \frac{1}{x} dx <=> \\ \frac{x^2}{2} + \ln x + C \end{aligned} \tag{19}$$

Seja $\alpha \neq 0$ um real fixo. Verifique que:

20. $\displaystyle\int \operatorname{sen} \alpha x dx = -\frac{1}{\alpha} \cos \alpha x + k$

$$\begin{aligned} \cos`\alpha x = -a \times \sin ax \\ \int \sin \alpha x dx = -\frac{1}{\alpha} \int \cos`\alpha x dx \\ -\frac{1}{\alpha} \int \cos`\alpha x dx = -\frac{1}{a} \times \cos \alpha x + k \end{aligned} \tag{20}$$

21. $\displaystyle\int \cos \alpha x dx = \frac{1}{\alpha} \operatorname{sen} \alpha x + k$

$$\begin{aligned} \sin`\alpha x = \alpha \times \cos \alpha x \\ \int \cos \alpha x dx = \frac{1}{\alpha} \times \int \sin`\alpha x dx \\ \frac{1}{\alpha} \times \int \sin`\alpha x dx = \frac{1}{\alpha} \times \sin \alpha x + k \end{aligned} \tag{21}$$

2 Integrais de funções exponenciais e trigonométricas - Guidorizzi

22. $\int e^{2x}dx$

$$
\begin{aligned}
\frac{d}{dx}e^{2x} &= 2e^{2x} \\
e^{2x} &= \frac{\frac{d}{dx}e^{2x}}{2} \\
\int e^{2x}dx &= \frac{1}{2}\int \frac{d}{dx}e^{2x}dx \\
\frac{1}{2}\int \frac{d}{dx}e^{2x}dx &= \frac{1}{2}e^{2x} + C
\end{aligned}
\tag{22}
$$

23. $\int e^{-x}dx$

$$
\begin{aligned}
\frac{d}{dx}e^{-x} &= -e^{-x} \\
e^{-x} &= -\frac{d}{dx}e^{-x} \\
\int e^{-x}dx &= -\int \frac{d}{dx}e^{-x}dx \\
-\int \frac{d}{dx}e^{-x}dx &= -e^{-x} + C
\end{aligned}
\tag{23}
$$

24. $\int (x + 3e^x)\,dx$

$$
\begin{aligned}
\int (x + 3e^x)\,dx &= \int x + 3 \times \int e^x dx <=> \\
\int (x + 3e^x)\,dx &= \frac{x^2}{2} + 3e^x + C
\end{aligned}
\tag{24}
$$

25. $\int \cos 3x dx$

$$\begin{aligned}
\frac{d}{dx}\sin 3x &= 3\cos 3x \\
\cos 3x &= \frac{\frac{d}{dx}\sin 3x}{3} \\
\int \cos 3x dx &= \frac{1}{3}\int \frac{d}{dx}\sin 3x dx \\
\frac{1}{3}\int \frac{d}{dx}\sin 3x dx &= \frac{1}{3}\sin 3x + C
\end{aligned} \tag{25}$$

26. $\int \sin 5x dx$

$$\begin{aligned}
\frac{d}{dx}\cos 5x &= -5\sin 5x \\
\sin 5x &= -\frac{\frac{d}{dx}\cos 5x}{5} \\
\int \sin 5x dx &= -\frac{1}{5}\int \frac{d}{dx}\cos 5x dx \\
-\frac{1}{5}\int \frac{d}{dx}\cos 5x dx &= -\frac{1}{5}\cos 5x + C
\end{aligned} \tag{26}$$

27. $\int \left(e^{2x} + e^{-2x}\right) dx$

$$\begin{aligned}
\int \left(e^{2x} + e^{-2x}\right) dx &= \int e^{2x} dx + \int e^{-2x} dx \\
\int e^{2x} dx &= \frac{1}{2}e^{2x} + C \\
\frac{d}{dx}e^{-2x} &= -2e^{-2x} \\
e^{-2x} &= -\frac{\frac{d}{dx}e^{-2x}}{2} \\
\int e^{-2x} dx &= -\frac{1}{2}\int \frac{d}{dx}e^{-2x} dx \\
-\frac{1}{2}\int \frac{d}{dx}e^{-2x} dx &= -\frac{1}{2}e^{-2x} + C \\
\int \left(e^{2x} + e^{-2x}\right) dx &= \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + C
\end{aligned} \tag{27}$$

28. $\int \left(x^2 + \sin x\right) dx$

$$\begin{aligned}
\int \left(x^2 + \sin x\right) dx = \int x^2 dx + \int \sin x dx <=> \\
\int x^2 dx = \frac{x^2}{2} \\
\frac{d}{dx} \cos x = -\sin x \\
\sin x = -\frac{d}{dx} \cos x \\
\int \sin x dx = -\int \frac{d}{dx} \cos x dx \\
-\int \frac{d}{dx} \cos x dx = -\cos x + C \\
\int \left(x^2 + \sin x\right) dx = \frac{x^2}{2} - \cos x + C
\end{aligned} \tag{28}$$

29. $\int (3 + \cos x)\, dx$

$$\begin{aligned}
\int (3 + \cos x)\, dx = \int 3dx + \int \cos x dx <=> \\
\frac{d}{dx} \sin x = \cos x \\
\int (3 + \cos x)\, dx = 3x + \sin x + C
\end{aligned} \tag{29}$$

30. $\int \frac{e^x + e^{-x}}{2} dx$

$$\begin{aligned}
\int \frac{e^x + e^{-x}}{2} dx = \frac{1}{2} \int e^x dx + \frac{1}{2} \int e^{-x} dx <=> \\
\int \frac{e^x + e^{-x}}{2} dx = \frac{1}{2} e^x - \frac{1}{2} e^{-x} + C
\end{aligned} \tag{30}$$

31. $\int \frac{1}{e^{3x}} dx$

$$\begin{aligned}
\int \frac{1}{e^{3x}} dx &= \int e^{-3x} dx \\
\frac{d}{dx} e^{-3x} &= -3e^{-3x} \\
e^{-3x} &= -\frac{\frac{d}{dx} e^{-3x}}{3} \\
\int \frac{1}{e^{3x}} dx &= -\frac{1}{3} \int \frac{d}{dx} e^{-3x} dx \\
-\frac{1}{3} \int \frac{d}{dx} e^{-3x} dx &= -\frac{1}{3} e^{-3x} + C <=> \\
\int \frac{1}{e^{3x}} dx &= -\frac{1}{3e^{3x}} + C
\end{aligned} \tag{31}$$

32. $\int (\sin 3x + \cos 5x) dx$

$$\begin{aligned}
\int (\sin 3x + \cos 5x) dx &= \int \sin 3x dx + \int \cos 5x dx \\
\frac{d}{dx} \cos 3x &= -3 \sin 3x \\
\sin 3x &= -\frac{\frac{d}{dx} \cos 3x}{3} \\
\int \sin 3x &= -\frac{1}{3} \int \frac{d}{dx} \cos 3x dx \\
-\frac{1}{3} \int \frac{d}{dx} \cos 3x dx &= -\frac{1}{3} \cos 3x + C \\
\frac{d}{dx} \sin 5x &= 5 \cos 5x \\
\cos 5x &= \frac{\frac{d}{dx} \sin 5x}{5} \\
\int \cos 5x dx &= \frac{1}{5} \int \frac{d}{dx} \sin 5x dx \\
\frac{1}{5} \int \frac{d}{dx} \sin 5x dx &= \frac{1}{5} \sin 5x + C \\
\int (\sin 3x + \cos 5x) dx &= -\frac{1}{3} \cos 3x + \frac{1}{5} \sin 5x + C
\end{aligned} \tag{32}$$

33. $\int \left(\frac{1}{x} + e^x \right) dx,\, x > 0$

$$\begin{aligned} \int \left(\frac{1}{x} + e^x \right) dx = \int \frac{1}{x} dx + \int e^x dx <=> \\ \int \frac{1}{x} dx + \int e^x dx = \ln x + e^x + C \end{aligned} \tag{33}$$

34. $\int \sin \frac{x}{2} dx$

$$\begin{aligned} \frac{d}{dx} \cos \frac{x}{2} = \frac{1}{2} \sin \frac{x}{2} \\ \sin \frac{x}{2} = 2 \frac{d}{dx} \cos \frac{x}{2} \\ \int \sin \frac{x}{2} dx = 2 \int \frac{d}{dx} \cos \frac{x}{2} dx \\ 2 \int \frac{d}{dx} \cos \frac{x}{2} dx = 2 \times \cos \frac{x}{2} + C \end{aligned} \tag{34}$$

35. $\int \cos \frac{x}{3} dx$

$$\begin{aligned} \frac{d}{dx} \sin \frac{x}{3} = \frac{1}{3} \cos \frac{x}{3} \\ \int \cos \frac{x}{3} dx = 3 \int \frac{d}{dx} \sin \frac{x}{3} dx \\ 3 \int \frac{d}{dx} \sin \frac{x}{3} dx = 3 \sin \frac{x}{3} + C \end{aligned} \tag{35}$$

36. $\int \left(\sqrt[3]{x} + \cos 3x \right) dx$

$$\begin{aligned} \int \left(\sqrt[3]{x} + \cos 3x \right) dx = \int \sqrt[3]{x} dx + \int \cos 3x dx \\ \int \sqrt[3]{x} dx + \int \cos 3x dx = x^{\frac{1}{3}} dx + \int \cos 3x dx \\ \int x^{\frac{1}{3}} dx + \int \cos 3x dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - \frac{1}{3} \sin 3x \\ \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - \frac{1}{3} \sin 3x = \frac{3 \sqrt[3]{x^4}}{4} - \frac{1}{3} \sin 3x \end{aligned} \tag{36}$$

37. $\int \left(x + e^{3x}\right) dx$

$$\begin{aligned} \int \left(x + e^{3x}\right) dx &= \int x dx + \int e^{3x} dx \\ \int x dx + \int e^{3x} dx &= \tfrac{x^2}{2} + \tfrac{1}{3} e^{3x} + C \end{aligned} \tag{37}$$

38. $\int \left(3 + e^{-x}\right) dx$

$$\begin{aligned} \int \left(3 + e^{-x}\right) dx =& 3dx + \int e^{-x} dx \\ \int 3dx + \int e^{-x} dx &= 3x + \left(-e^{-x}\right) + C \\ &= 3x - e^{-x} + C \end{aligned} \tag{38}$$

39. $\int 5e^{7x}$

$$\begin{aligned} \int 5e^{7x} &= 5 \int e^{7x} dx \frac{d}{dx} e^{7x} = 7e^{7x} \\ e^{7x} &= \frac{\frac{d}{dx} e^{7x}}{7} \\ 5 \int e^{7x} dx &= \frac{5}{7} \int \frac{d}{dx} e^{7x} dx \\ \frac{5}{7} \int \frac{d}{dx} e^{7x} dx &= \frac{5}{7} e^{7x} + C \end{aligned} \tag{39}$$

40. $\int (1 - \cos 4x) \, dx$

$$\begin{aligned} \int (1 - \cos 4x) \, dx &= \int 1dx - \int \cos 4x dx \\ \frac{d}{dx} \sin 4x &= 4 \cos 4x \\ \cos 4x &= \frac{\frac{d}{dx} \sin 4x}{4} \\ \int 1dx - \int \cos 4x dx &= \int 1dx - \frac{1}{4} \int \frac{d}{dx} \sin 4x dx \\ \int 1dx - \frac{1}{4} \int \frac{d}{dx} \sin 4x dx &= x - \frac{1}{4} \sin 4x + C \end{aligned} \tag{40}$$

41. $\displaystyle\int \left(2+\sin\frac{x}{3}\right)dx$

$$\begin{aligned}\int \left(2+\sin\frac{x}{3}\right)dx = \int 2dx + \int \sin\frac{x}{3}dx \\ \frac{d}{dx}\cos\frac{x}{3} = -\frac{1}{3}\sin\frac{x}{3} \\ \sin\frac{x}{3} = \frac{d}{dx}\cos\frac{x}{3}\times(-3) \\ \int \sin\frac{x}{3}dx = -3\int \frac{d}{dx}\cos\frac{x}{3}dx \\ \int 2dx - 3\int \frac{d}{dx}\cos\frac{x}{3}dx = 2x - 3\cos\frac{x}{3} + C\end{aligned} \tag{41}$$

Verifique que:

42. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + k$

$$\begin{aligned}&\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \\ &\int \frac{d}{dx}\arcsin x dx = \arcsin x + k\end{aligned} \tag{42}$$

43. $\displaystyle\int \frac{1}{1+x^2}dx = \arctan + k$

$$\begin{aligned}&\textstyle\int \frac{1}{1+x^2}dx = \operatorname{arctg} x + k \\ &\frac{d}{dx}\operatorname{arctg} x = \frac{1}{1+x^2} \\ &\textstyle\int \frac{d}{dx}\operatorname{arctg} x dx = \arctan x + k\end{aligned} \tag{43}$$

Determine a função y = y(x), $x \in \mathbb{R}$

44. $\dfrac{dy}{dx} = 3x - 1$ e y(0) = 2

$$\begin{aligned} y(x) &= \int (3x - 1)\, dx \\ y(x) &= 3\int x - \int 1dx \\ y(x) &= 3\frac{x^2}{2} - x + C \\ 2 &= 3\frac{0^2}{2} - 0 + C \\ C &= 2 \\ y(x) &= \frac{3x^2}{2} - x + 2 \end{aligned} \tag{44}$$

45. $\dfrac{dy}{dx} = x^3 - x + 1$ e $y(1) = 1$

$$\begin{aligned} y(x) &= \int \left(x^3 - x + 1\right) dx \\ y(x) &= \int \frac{x^4}{4} dx - \int xdx + \int 1dx \\ y(x) &= \frac{x^4}{4} - \frac{x^2}{2} + x + C \\ 1 &= \frac{1^4}{4} - \frac{1^2}{2} + 1 + C \\ C &= 1 - \frac{1}{4} + \frac{1}{2} \\ C &= 1 - \frac{6}{8} \\ C &= \frac{1}{4} \\ y(x) &= \frac{x^4}{4} - \frac{x^2}{2} + x + \frac{1}{4} \end{aligned} \tag{45}$$

46. $\dfrac{dy}{dx} = \dfrac{1}{2}x + 3$ e $y(-1) = 0$.

$$\begin{aligned}
y(x) &= \int \left(\frac{1}{2}x + 3\right) dx \\
y(x) &= \frac{1}{2}\int x dx + \int 3dx \\
y(x) &= \frac{1}{2}\frac{x^2}{2} + 3x + C \\
0 &= \frac{1}{2}\frac{(-1)^2}{2} + 3(-1) + C \\
\frac{1}{2}\frac{1}{2} - 3 &= -C \\
1 - 3 &= -C \\
C &= 2 \\
y(x) &= \frac{x^2}{4} + 3x + 2
\end{aligned} \tag{46}$$

47. $\dfrac{dy}{dx} = \sin 3x$ e $y(0) = 1$

$$\begin{aligned}
y(x) &= \int \sin 3x dx \\
\frac{d}{dx}\cos 3x &= -3\sin 3x \\
\sin 3x &= -\frac{\frac{d}{dx}\cos 3x}{3} \\
y(x) &= \frac{1}{3}\int \frac{d}{dx}\cos 3x dx \\
y(0) &= -\frac{1}{3} \times \cos 0 + C \\
C &= 1 + \frac{1}{3} \\
C &= \frac{4}{3} \\
y(x) &= -\frac{\cos 3x}{3} + \frac{4}{3} \\
y(x) &= -\frac{\cos 3x + 4}{3}
\end{aligned} \tag{47}$$

48. $\dfrac{dy}{dx} = \cos x$ e $y(0) = 0$

$$\begin{aligned} y(x) &= \int \cos x dx \\ y(x) &= \int \frac{d}{dx} \sin x dx \\ y(x) &= \sin x + C \\ 0 &= \sin 0 + C \\ C &= 0 \\ y(x) &= \sin x \end{aligned} \tag{48}$$

49. $\dfrac{dy}{dx} = e^{-x}$ e $y(0) = 1$

$$\begin{aligned} y(x) &= \int e^{-x} dx \\ y(x) &= -\int \frac{d}{dx} e^{-x} dx \\ y(x) &= -e^{-x} + C \\ 1 &= -e^{-1} + C \\ C &= 1 + e^{-1} \\ y(x) &= -e^{-x} + 1 + e^{-1} \end{aligned} \tag{49}$$

50. $\dfrac{dy}{dx} = \dfrac{1}{x^2}$ e $y(1) = 1$

$$\begin{aligned} y(x) &= \int \frac{1}{x^2} dx \\ y(x) &= \frac{x^{-2}}{2} + C \\ y(1) &= \frac{1^{-2}}{2} + C \\ 1 - \frac{1}{2} &= C \\ C &= \frac{1}{2} \\ y(x) &= \frac{1}{2x^2} + \frac{1}{2} \end{aligned} \tag{50}$$

51. $\dfrac{dy}{dx} = 3 + \dfrac{1}{x}$ e $y(1) = 2$

$$\begin{aligned} y(x) &= \int 3 + \frac{1}{x} dx \\ y(x) &= \int 3dx + \int \frac{1}{x} dx \\ y(x) &= 3x + \ln x + C \\ y(1) &= 3 \times 1 + \ln 1 + C \\ C &= 2 - 3 \\ C &= -1 \\ y(x) &= 3x + \ln x - 1 \end{aligned} \tag{51}$$

52. $\dfrac{dy}{dx} = x + \dfrac{1}{\sqrt{x}}$ e y(1) = 0

$$\begin{aligned} y(x) &= \int x dx + \int \frac{1}{\sqrt{x}} dx \\ y(x) &= \frac{x^2}{2} + \frac{\sqrt{x}}{2} + C \\ y(1) &= \frac{1^2}{2} + \frac{\sqrt{1}}{2} + C \\ C &= 0 - 1 \\ C &= -1 \\ y(x) &= \frac{x^2}{2} + \frac{\sqrt{x}}{2} - 1 \end{aligned} \tag{52}$$

53. $\dfrac{dy}{dx} = \dfrac{1}{x} + \dfrac{1}{x^2}$ e y(1) = 1

$$\begin{aligned} y(x) &= \int \frac{1}{x} dx + \int \frac{1}{x^2} dx \\ y(x) &= \ln x + \frac{1}{2x} + C \\ y(1) &= \ln 1 + \frac{1}{1^2} + C \\ 1 &= 0 + 1 + C \\ C &= 0 \\ y(x) &= \ln x + \frac{1}{2x} \end{aligned} \tag{53}$$

Pergunta para as questões 54, 55, 56. Uma partícula desloca-se sobre o eixo x com velocidade $v(t) = t + 3$, $t \geq 0$. Sabe-se que, no instante t = 0, a partícula encontra-se na posição x = 2.

54. Qual a posição da partícula no instante t?

$$\begin{aligned} \int t dt + \int 3dt = \frac{t^2}{2} + 3t + C \\ \frac{0^2}{2} + 3 \times 0 + C = 2 \\ C = 2 \\ x(t) = \frac{t^2}{2} + 3t + 2 \end{aligned} \tag{54}$$

55. Determine a posição da partícula no instante t = 2.

$$\begin{aligned} x(2) = \frac{2^2}{2} + 3 \times 2 + 2 \\ x(2) = 10 \end{aligned} \tag{55}$$

56. Determine a aceleração.

$$\frac{d^2x}{dt^2} = 1 \tag{56}$$

57. Uma partícula desloca-se sobre o eixo x com velocidade $v(t) = 2t - 3$, $t \geq 0$. Sabe-se que no instante t = 0 a partícula encontra-se na posição x = 5. Determine o instante em que a partícula estará mais próxima da origem.

$$f \tag{57}$$

58. $\dfrac{dx}{dt} = 2t + 3$ e $x(0) = 2$

$$\begin{aligned} x(t) = \int (2t + 3)\, dt \\ x(t) = 2\int tdt + \int 3dt \\ x(t) = t^2 + 3t + C \\ 2 = 0^2 + 3 \times 0 + C \\ C = 2 \\ x(t) = t^2 + 3t + 2 \end{aligned} \tag{58}$$

59. $v(t) = t^2 - 1$ e $x(0) = -1$

$$\begin{aligned}
x(t) &= \int \left(t^2 - 1\right) dt \\
x(t) &= \int t^2 dt - \int 1dt \\
x(t) &= \frac{t^2}{2} - t + C \\
-1 &= \frac{0^2}{2} - 0 + C \\
C &= -1 \\
x(t) &= \frac{t^2}{2} - t - 1
\end{aligned} \tag{59}$$

60. $\dfrac{d^2x}{dt^2} = 3$, $v(0) = 1$ e $x(0) = 1$

$$\begin{aligned}
v(t) &= \int 3dt \\
v(t) &= \frac{3t^2}{2} + C \\
v(0) &= 3 \times 0^2 + C \\
1 &= C \\
v(t) &= 3t^2 + 1 \\
x(t) &= 3\int t^2 dt + \int 1dt \\
x(t) &= \frac{3t^2}{2} + t + C \\
1 &= \frac{3 \times 0^2}{2} + C \\
C &= 1 \\
x(t) &= \frac{3t^2}{2} + t + 1
\end{aligned} \tag{60}$$

61. $\dfrac{d^2x}{dt^2} = e^{-t}$, $v(0) = 0$ e $x(0) = 1$

$$\begin{aligned}
v(t) &= \int e^{-t} dt \\
v(t) &= -e^{-t} + C \\
v(0) &= -e^{0} + C \\
C &= 1 \\
v(t) &= -e^{-t} + 1 \\
x(t) &= -\int e^{-t} dt + \int 1 dt \\
x(t) &= e^{-t} + t + C \\
x(0) &= e^{0} + 0 + C \\
C &= 0 \\
x(t) &= e^{-t} + t
\end{aligned} \tag{61}$$

62. $\dfrac{d^2x}{dt^2} = \cos 2t,\ v(0) = 1$ e $x(0) = 0$

$$\begin{aligned}
v(t) &= \int \cos 2t dt \\
\frac{d}{dt}\sin 2t &= 2\cos 2t \\
\cos 2t &= \frac{\frac{d}{dt}\sin 2t}{2} \\
v(t) &= \frac{1}{2}\int \frac{d}{dt}\sin 2t dt \\
v(t) &= \frac{1}{2}\sin 2t + C \\
v(0) &= \frac{1}{2}\sin 2\times 0 + C \\
C &= 1 \\
v(t) &= \frac{\sin 2t}{2} + 1 \\
\frac{d}{dt}\cos 2t &= -2\sin 2t \\
\sin 2t &= -\frac{\frac{d}{dt}\cos 2t}{2} \\
x(t) &= -\frac{1}{2}\int \sin 2t dt + \int 1 dt \\
x(t) &= -\frac{1}{2}\cos 2t + t + C \\
x(0) &= -\frac{1}{2}\cos 2\times 0 + 0 + C \\
C &= \frac{1}{2} \\
x(t) &= -\frac{\cos 2t + 1}{2} + t
\end{aligned} \tag{62}$$

63. $\dfrac{d^2x}{dt^2} = \sin 3t$, $v(0) = 0$ e $x(0) = 0$

$$\begin{aligned}
v(t) &= \int \sin 3t dt \\
\frac{d}{dt}\cos 3t &= -3 \times \sin 3t \\
\sin 3t &= -\frac{\frac{d}{dt}\cos 3t}{3} \\
v(t) &= -\frac{1}{3}\int \cos 3t dt \\
v(t) &= -\frac{\cos 3t}{3} + C \\
v(0) &= -\frac{1}{3}\cos 3 \times 0 + C \\
C &= -\frac{1}{3} \\
v(t) &= \frac{\cos 3t - 1}{3} \\
x(t) = -\frac{1}{9}\int \frac{d}{dt}\sin 3t dt &- \frac{1}{3}\int 1 dt \\
x(t) &= -\frac{1}{9}\sin 3t - \frac{1}{3} \times \frac{t}{2} + C \\
x(0) &= -\frac{1}{9}\sin 3 \times 0 - \frac{0}{6} + C \\
C &= 0 \\
x(t) &= -\frac{1}{9}\sin 3t - \frac{t}{6}
\end{aligned} \tag{63}$$

64. $\dfrac{dx}{dt} = \dfrac{1}{1+t^2}$ e $x(0) = 0$

$$\begin{aligned}
x(t) &= \int \left(\frac{1}{1+t^2}\right) dt \\
x(t) &= \int \frac{d}{dt}\arctan t dt \\
x(t) &= \arctan t
\end{aligned} \tag{64}$$

3 Integrais de Riemann

65. $\int_0^1 (x+3)\,dx$

$$\begin{aligned}\int_0^1 (x+3)\,dx &= \left[\frac{x^2}{2}+3x\right]_0^1\\ \left[\frac{x^2}{2}+3x\right]_0^1 &= \frac{1^2}{2}+3\times 1-\frac{0^2}{2}+3\times 0\\ \int_0^1 (x+3)\,dx &= \frac{7}{2}\end{aligned} \tag{65}$$

66. $\int_0^4 (2x+1)\,dx$

$$\begin{aligned}\left[x^2+x\right]_{-1}^1 &= 1^2+1-((-1)^2-1)\\ \int_0^4 (2x+1)\,dx &= 2-0=2\end{aligned} \tag{66}$$

67. $\int_0^4 \frac{1}{2}dx$

$$\begin{aligned}\int_0^4 \frac{1}{2}dx &= \left[\frac{x}{2}\right]_0^4\\ \int_0^4 \frac{1}{2}dx &= \frac{4}{2}-\frac{0}{2}\\ \int_0^4 \frac{1}{2}dx &= 2\end{aligned} \tag{67}$$

68. $\int_{-2}^{1}\left(x^2-1\right)dx$

$$\begin{aligned}\int_{-2}^{1}\left(x^2-1\right)dx &= \left[\frac{x^3}{3}-x\right]_{-2}^{1}\\ \int_{-2}^{1}\left(x^2-1\right)dx &= \left[\frac{1^3}{3}-1\right]-\left[\frac{(-2)^3}{3}-(-2)\right]\\ \int_{-2}^{1}\left(x^2-1\right)dx &= -\frac{2}{3}-\left[-\frac{8}{3}+2\right]\\ \int_{-2}^{1}\left(x^2-1\right)dx &= -\frac{2}{3}+\frac{14}{3}\\ \int_{-2}^{1}\left(x^2-1\right)dx &= -4\end{aligned}\tag{68}$$

69. $\int_{1}^{3} x^3 dx$

$$\begin{aligned}\int_{1}^{3} dx_1^3 &= [x]_1^3\\ \int_{1}^{3} dx_1^3 &= 3-1\\ \int_{1}^{3} dx_1^3 &= 2.\end{aligned}\tag{69}$$

70. $\int_{-1}^{2} 4dx$

$$\begin{aligned}\int_{-1}^{2} 4dx &= [4x]_{-1}^{2}\\ \int_{-1}^{2} 4dx &= 4\times 2-4\times(-1)\\ \int_{-1}^{2} 4dx &= 8+4\\ \int_{-1}^{2} 4dx &= 12\end{aligned}\tag{70}$$

71. $\int_1^3 \frac{1}{x^3} dx$

$$\begin{aligned}
\int_1^3 \frac{1}{x^3} dx &= \left[\frac{x^{-2}}{-2}\right]_1^3 \\
\left[\frac{x^{-2}}{-2}\right]_1^3 &= \left[\frac{3^{-2}}{-2}\right] - \left[\frac{1^{-2}}{-2}\right] \\
\left[\frac{3^{-2}}{-2}\right] - \left[\frac{1^{-2}}{-2}\right] &= \frac{1}{-18} - \frac{1}{-2} = \frac{-2+18}{36} \\
\frac{-2+18}{-36} &= \frac{-16}{-36} \\
\frac{16}{36} &= \frac{4}{9}
\end{aligned} \tag{71}$$

72. $\int_{-1}^1 5dx$

$$\begin{aligned}
\int_{-1}^1 5dx &= |5x|_{-1}^1 \\
|5x|_{-1}^1 &= 5 \cdot 1 - (5 \cdot (-1)) \\
5 \cdot 1 - (5 \cdot (-1)) &= 5 + 5 = 10
\end{aligned} \tag{72}$$

73. $\int_0^2 \left(x^2 + 3x - 3\right) dx$

$$\begin{aligned}
\int_0^2 \left(x^2 + 3x - 3\right) dx &= \left[\frac{x^3}{3} + \frac{3x^2}{2} - 3x\right]_0^2 \\
\int_0^2 \left(x^2 + 3x - 3\right) dx &= \left[\frac{2^3}{3} + \frac{3 \cdot 2^2}{2} - 3 \cdot 2\right] - \left[\frac{0^3}{2} + \frac{3 \cdot 0^2}{2} - 3 \cdot 0\right] \\
\int_0^2 \left(x^2 + 3x - 3\right) dx &= \frac{8}{3}
\end{aligned} \tag{73}$$

74. $\int_0^1 \left(5x^3 - \frac{1}{2}\right) dx$

$$\begin{aligned}\int_0^1 \left(5x^3 - \frac{1}{2}\right) dx &= \left[\frac{5x^4}{4} - \frac{x}{2}\right]_0^1 \\ \int_0^1 \left(5x^3 - \frac{1}{2}\right) dx &= \left[\frac{5\cdot 1^4}{4} - \frac{1}{2}\right] - \left[\frac{5\cdot 0^4}{4} - \frac{0}{2}\right] \\ \int_0^1 \left(5x^3 - \frac{1}{2}\right) dx &= \frac{3}{4}\end{aligned} \tag{74}$$

75. $\int_1^1 (2x+3)\,dx$

$$\begin{aligned}\int_1^1 (2x+3)\,dx &= \left[\frac{2x^2}{2} + 3x\right]_1^1 \\ \left[\frac{2x^2}{2} + 3x\right]_1^1 &= [1+3] - [1+3] \\ [1+3] - [1+3] &= 0\end{aligned} \tag{75}$$

76. $\int_1^0 (2x+3)$

$$\begin{aligned}\int_1^0 (2x+3) &= \left[x^2 + 3x\right]_1^0 \\ \left[x^2 + 3x\right]_1^0 &= \left[0^2 + 3\cdot 0\right] - \left[1^2 + 3\cdot 1\right] \\ \left[x^2 + 3x\right]_1^0 &= -4\end{aligned} \tag{76}$$

77. $\int_{-2}^{-1} \left(\frac{1}{x^2} + x\right) dx$

$$\begin{aligned}\int_{-2}^{-1} \left(\frac{1}{x^2} + x\right) dx &= \left[\frac{x^{-1}}{-1} + \frac{x^2}{2}\right]_{-2}^{-1} \\ \left[\frac{x^{-1}}{-1} + \frac{x^2}{2}\right]_{-2}^{-1} &= \left[\frac{-1}{-1} + \frac{(-1)^2}{2}\right] - \left[\frac{-1}{-2} + \frac{(-2)^2}{2}\right] \\ \left[\frac{x^{-1}}{-1} + \frac{x^2}{2}\right]_{-2}^{-1} &= \left[1 + \frac{1}{2}\right] - \left[\frac{1}{2} + 2\right] \\ \left[\frac{x^{-1}}{-1} + \frac{x^2}{2}\right]_{-2}^{-1} &= \frac{3}{2} - \frac{5}{2} \\ \frac{-2}{2} &= -1\end{aligned} \tag{77}$$

78. $\int_0^4 \sqrt{x}dx$

$$\begin{aligned}
\int_0^4 \sqrt{x}dx &= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^4 \\
\int_0^4 \sqrt{x}dx &= \left[\frac{2\sqrt{x^3}}{3}\right]_0^4 \\
\int_0^4 \sqrt{x}dx &= \left[\frac{2\sqrt{4^3}}{3}\right] - \left[\frac{2\sqrt{0^3}}{3}\right] \\
\int_0^4 \sqrt{x}dx &= \frac{16}{3}
\end{aligned} \tag{78}$$

79. $\int_1^4 \frac{1}{\sqrt{x}}dx$

$$\begin{aligned}
\int_1^4 \frac{1}{\sqrt{x}}dx &= [2 \cdot \sqrt{x}]_1^4 \\
\int_1^4 \frac{1}{\sqrt{x}}dx &= 2\sqrt{4} - 2\sqrt{1} \\
2\sqrt{4} - 2\sqrt{1} &= 2 \\
2 \cdot 2 - 2 \cdot 1 &= 2
\end{aligned} \tag{79}$$

80. $\int_0^8 \sqrt[3]{x}dx$

$$\begin{aligned}
\int_0^8 \sqrt[3]{x}dx &= \left[\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_0^8 \\
\left[\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right] = \left(\frac{3\sqrt[3]{x^4}}{4}\right)\left(\frac{3\sqrt[3]{x^4}}{4}\right)_0^8 = \left[\frac{3\sqrt[3]{8^4}}{4}\right] - \left[\frac{3\sqrt[3]{x^4}}{4}\right] &<=> \\
\frac{3 \cdot 16}{4} - \frac{3}{4} &= 12
\end{aligned} \tag{80}$$

81. $\int_{-1}^{0} \left(x^3 - 2x + 3\right) dx$

$$\int_{-1}^{0} \left(x^3 - 2x + 3\right) dx = \left(\frac{0^4}{4} - 0^2 + 3 \cdot 0\right) - \left(\frac{(-1)^4}{4} - (-1)^2 + 3 \cdot (-1)\right) \tag{81}$$

82. $\int_{0}^{8} \sqrt[8]{x} dx$

$$\begin{aligned} \int_{0}^{8} \sqrt[8]{x} dx &= \left[\frac{x^{\frac{1}{8}+1}}{\frac{1}{8}+1}\right]_0^8 \\ \int_{0}^{8} \sqrt[8]{x} dx &= \left[\frac{8\sqrt[8]{x^9}}{9}\right] \\ \int_{0}^{8} \sqrt[8]{x} dx &= \left[\frac{8\sqrt[8]{8^9}}{9}\right] - \left[\frac{8\sqrt[8]{0^9}}{9}\right] \\ \left[\frac{8\sqrt[8]{1^9}}{9}\right] - \left[\frac{8\sqrt[8]{0^9}}{9}\right] &= \frac{8\sqrt[8]{1}}{9} \\ \frac{8\sqrt[8]{1}}{9} &= \frac{8}{9} \end{aligned} \tag{82}$$

83. $\int_{1}^{2} \left(x^3\right) dx$

$$\begin{aligned} \int_{1}^{2} \left(x^3\right) dx &= \left|\frac{x^4}{4}\right|_1^2 \\ \int_{1}^{2} \left(x^3\right) dx &= \left[\frac{2^2}{2}\right] - \left[\frac{1^2}{2}\right] \\ \int_{1}^{2} \left(x^3\right) dx &= \frac{3}{2} \end{aligned} \tag{83}$$

84. $\int_1^2 \left(x^3 + x + \frac{1}{x^3}\right) dx$

$$\begin{aligned}
\int_1^2 \left(x^3 + x + \frac{1}{x^3}\right) dx &= \left[\frac{x^4}{4} + \frac{x^2}{2} - \frac{1}{2x^2}\right]_1^2 \\
\int_1^2 \left(x^3 + x + \frac{1}{x^3}\right) dx &= \left[\frac{2^4}{4} + \frac{2^2}{2} - \frac{1}{2 \cdot 2^2}\right] - \left[\frac{1^4}{4} + \frac{1^2}{2} - \frac{1}{2 \cdot 1^2}\right] \\
\int_1^2 \left(x^3 + x + \frac{1}{x^3}\right) dx &= \frac{48}{8} - \frac{1}{4} \\
\int_1^2 \left(x^3 + x + \frac{1}{x^3}\right) dx &= \frac{45}{8}
\end{aligned} \tag{84}$$

85. $\int_0^1 \left(x + \sqrt[4]{x}\right) dx$

$$\begin{aligned}
\int_0^1 \left(x + \sqrt[4]{x}\right) dx &= \left[\frac{x^2}{2} + \frac{4\sqrt[4]{x^5}}{5}\right]_0^1 \\
\int_0^1 \left(x + \sqrt[4]{x}\right) dx &= \left[\frac{4\sqrt[4]{(-1)^2}}{5}\right] \\
\int_0^1 \left(x + \sqrt[4]{x}\right) dx &= \frac{13}{10}
\end{aligned} \tag{85}$$

86. $\int_1^3 \left(5 + \frac{1}{x^2}\right) dx$

$$\begin{aligned}
\int_1^3 \left(5 + \frac{1}{x^2}\right) dx &= \left[5x - \frac{1}{x}\right]_1^3 \\
\int_1^3 \left(5 + \frac{1}{x^2}\right) dx &= \left[5 \cdot 3 - \frac{1}{3}\right] - \left[5 \cdot 1 - \frac{1}{1}\right] \\
\int_1^3 \left(5 + \frac{1}{x^2}\right) dx &= \frac{32}{3}
\end{aligned} \tag{86}$$

87. $\int_{-3}^{3} x^3 dx$

$$\begin{aligned}
\int_{-3}^{3} x^3 dx &= \left[\frac{x^4}{4}\right]_{-3}^{3} \\
\int_{-3}^{3} x^3 dx &= \left[\frac{3^4}{4}\right] - \left[\frac{(-3)^4}{4}\right] \\
\int_{-3}^{3} x^3 dx &= \frac{81}{4} - \frac{81}{4} \\
\int_{-3}^{3} x^3 dx &= 0.
\end{aligned} \tag{87}$$

88. $\int_{-1}^{1} \left(x^7 + x^3 + x\right) dx$

$$\begin{aligned}
\int_{-1}^{1} \left(x^7 + x^3 + x\right) dx &= \left[\frac{x^8}{8} + \frac{x^4}{4} + \frac{x^2}{2}\right]_{-1}^{1} \\
\int_{-1}^{1} \left(x^7 + x^3 + x\right) dx &= \left[\frac{1^8}{8} + \frac{1^4}{4} + \frac{1^2}{2}\right] - \left[\frac{(-1)^8}{8} + \frac{(-1)^4}{4} + \frac{(-1)^2}{2}\right] \\
\int_{-1}^{1} \left(x^7 + x^3 + x\right) dx &= \frac{7}{8} - \frac{7}{8} \\
\int_{-1}^{1} \left(x^7 + x^3 + x\right) dx &= 0
\end{aligned} \tag{88}$$

89. $\int_{\frac{1}{2}}^{1} (x+3)\, dx$

$$\begin{aligned}
\int_{\frac{1}{2}}^{1} (x+3)\, dx &= \left[\frac{x^2}{2} + 3x\right]_{\frac{1}{2}}^{1} \\
\left[\frac{x^2}{2} + 3x\right]_{\frac{1}{2}}^{1} &= \left[\frac{1^2}{2} + 3 \cdot 1\right] - \left[\frac{\left(\frac{1}{2}\right)^2}{2} + 3 \cdot \frac{1}{2}\right] \\
\left[\frac{x^2}{2} + 3x\right]_{\frac{1}{2}}^{1} &= \frac{1}{2} + 3 - \left(\frac{1}{8} + \frac{3}{2}\right) \\
\left[\frac{x^2}{2} + 3x\right]_{\frac{1}{2}}^{1} &= \frac{15}{8}
\end{aligned} \tag{89}$$

90. $\int_1^4 \left(5x + \sqrt{x}\right) dx$

$$
\begin{aligned}
\int_1^4 \left(5x + \sqrt{x}\right) dx &= \left[\frac{5x^2}{2} + \frac{2\sqrt{x^3}}{2}\right]_1^4 \\
\int_1^4 \left(5x + \sqrt{x}\right) dx &= \left[\frac{5 \cdot 4^2}{2} + \frac{2\sqrt{4^3}}{2}\right] - \left[\frac{5 \cdot 1^2}{2} + \frac{2\sqrt{1^3}}{2}\right] \\
\left[\frac{5x^2}{2} + \frac{2\sqrt{x^3}}{2}\right]_1^4 &= \left(5 \cdot 8 + \frac{2}{3} \cdot 8\right) - \left(\frac{5}{2} + \frac{2}{3}\right) \\
\left[\frac{5x^2}{2} + \frac{2\sqrt{x^3}}{2}\right]_1^4 &= \left(40 + \frac{16}{3}\right) - \left(\frac{19}{6}\right) \\
\left[\frac{5x^2}{2} + \frac{2\sqrt{x^3}}{2}\right]_1^4 &= \frac{136}{3} - \frac{19}{6} \\
\left[\frac{5x^2}{2} + \frac{2\sqrt{x^3}}{2}\right]_1^4 &= \frac{253}{6}
\end{aligned}
\tag{90}
$$

91. $\int_1^0 \left(x^7 - x + 3\right) dx$

$$
\begin{aligned}
\int_1^0 \left(x^7 - x + 3\right) dx &= \left[\frac{x^8}{8} - \frac{x^2}{2} + 3x\right]_1^0 \\
\left[\frac{x^8}{8} - \frac{x^2}{2} + 3x\right]_1^0 &= \left[\frac{0^8}{8} - \frac{0^2}{2} + 3 \cdot 0\right] - \left[\frac{1^8}{8} - \frac{1^2}{2} + 3 \cdot 1\right] \\
\left[\frac{x^8}{8} - \frac{x^2}{2} + 3x\right]_1^0 &= -\left(\frac{1}{8} - \frac{1}{2} + 3\right) \\
\left[\frac{x^8}{8} - \frac{x^2}{2} + 3x\right]_1^0 &= -\frac{21}{8}
\end{aligned}
\tag{91}
$$

92. $\int_1^2 \left(\frac{1+x}{x^3}\right) dx$

$$\begin{aligned}
\int_1^2 \left(\frac{1+x}{x^3}\right) dx &= \int_1^2 \frac{1}{x^3} dx + \int_1^2 \frac{1}{2} dx \\
\int_1^2 \frac{1}{x^3} dx + \int_1^2 \frac{1}{2} dx &= \left[-\frac{1}{2x^2} + \left(\frac{-1}{x}\right)\right]_1^2 \\
\left[-\frac{1}{2x^2} - \left(\frac{1}{x}\right)\right]_1^2 &= \left[-\frac{1}{2\cdot 2^2} + \left(\frac{-1}{2}\right)\right] - \left[-\frac{1}{2\cdot 1^2} - \left(\frac{1}{1}\right)\right] \\
\left[-\frac{1}{2\cdot 2^2} - \left(\frac{1}{2}\right)\right] - \left[-\frac{1}{2\cdot 1^2} - \left(\frac{1}{1}\right)\right] &= \frac{7}{8}
\end{aligned} \tag{92}$$

93. $\int_0^1 (x+1)^2 \, dx$

$$\begin{aligned}
\int_0^1 (x+1)^2 \, dx &= \int_0^1 u^2 du \\
\int_0^1 u^2 du &= \left[\frac{u^3}{3}\right]_0^1 \\
\left[\frac{u^3}{3}\right]_0^1 &= \left[\frac{(x+1)^3}{3}\right]_0^1 \\
\left[\frac{(x+1)^3}{3}\right]_0^1 &= \left[\frac{2^3}{3}\right] - \left[\frac{1^3}{3}\right] \\
\left[\frac{2^3}{3}\right] - \left[\frac{1^3}{3}\right] &= \frac{8}{3} - \frac{1}{3} \\
\int_0^1 (x+1)^2 \, dx &= \frac{7}{3}
\end{aligned} \tag{93}$$

94. $\int_1^4 \left(\frac{1+x}{\sqrt{x}}\right)dx$

$$\begin{aligned}
\int_1^4 \left(\frac{1+x}{\sqrt{x}}\right)dx &= \int_1^4 \frac{1}{\sqrt{x}}dx + \int_1^4 \sqrt{x}dx \\
\int_1^4 \frac{1}{\sqrt{x}}dx + \int_1^4 \sqrt{x}dx &= \left[2\sqrt{x}dx + \frac{2\sqrt{x}}{3}\right]_1^4 \\
\left[2\sqrt{x}dx + \frac{2\sqrt{x}}{3}\right]_1^4 &= \left[2\sqrt{4} + \frac{2\sqrt{4^3}}{3}\right] - \left[2\sqrt{1} + \frac{2\sqrt{1^3}}{3}\right] \\
\left[2\sqrt{4} + \frac{2\sqrt{4^3}}{3}\right] - \left[2\sqrt{1} + \frac{2\sqrt{1^3}}{3}\right] &= 2\cdot 2 + \frac{2\cdot 8}{3} - \left[2\sqrt{1} + \frac{2\sqrt{1^3}}{3}\right] \\
\int_1^4 \left(\frac{1+x}{\sqrt{x}}\right)dx &= \frac{28}{3} + \frac{8}{3} \\
\int_1^4 \left(\frac{1+x}{\sqrt{x}}\right)dx &= \frac{20}{3}
\end{aligned} \tag{94}$$

95. $\int_0^1 (x-3)^2 dx$

$$\begin{aligned}
\int_0^1 (x-3)^2 dx &= \int_0^1 \left(x^2 - 6x + 9\right) dx \\
\int_0^1 \left(x^2 - 6x + 9\right) dx &= \left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^1 \\
\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^1 &= \left[\frac{1^3}{3} - 3\cdot 1^2 + 9\right] \\
\left[\frac{1^3}{3} - 3\cdot 1^2 + 9\right] &= -\frac{16}{9} + \frac{9}{1} \\
\frac{16}{9} + \frac{9}{1} &= -\frac{-16+54}{6} \\
-\frac{38}{6} &= \frac{19}{3}
\end{aligned} \tag{95}$$

96. $\int_0^2 \left(t^2 + 3t - 1\right) dt$

$$
\begin{aligned}
\int_0^2 \left(t^2 + 3t - 1\right) dt &= \left[\frac{t^3}{3} + \frac{3t^2}{2} - t\right]_0^2 \\
\left[\frac{t^3}{3} + \frac{3t^2}{2} - t\right]_0^2 &= \frac{8}{3} + 4 \\
\frac{8}{3} + 4 &= \frac{20}{3}
\end{aligned}
\tag{96}
$$

97. $\int_1^2 \left(\frac{1+t^2}{t^4}\right) dt$

$$
\begin{aligned}
\int_1^2 \left(\frac{1+t^2}{t^4}\right) dt &= \int_1^2 \frac{1}{t^4} dt + \int_1^2 \frac{1}{t^2} dt \\
\int_1^2 \frac{1}{t^4} dt + \int_1^2 \frac{1}{t^2} dt &= \left[\frac{2^{-3}}{-3} - 2^{-1}\right] - \left[\frac{1^{-3}}{-3} - 1^{-1}\right] \\
\left[\frac{2^{-3}}{-3} - 2^{-1}\right] - \left[\frac{1^{-3}}{-3} - 1^{-1}\right] &= -\frac{1}{24} - \frac{1}{2} - \left[-\frac{1}{3} - 1\right] \\
-\frac{1}{24} - \frac{1}{2} - \left[-\frac{1}{3} - 1\right] &= -\frac{26}{48} + \frac{2}{3} \\
-\frac{26}{48} + \frac{2}{3} &= -\frac{78+96}{144} \\
-\frac{78+96}{144} &= \frac{18}{144} \\
\frac{18}{144} &= \frac{1}{8}
\end{aligned}
\tag{97}
$$

98. $\int_{\frac{1}{2}}^{1} (s+2)\, ds$

$$\begin{aligned}
\int_{\frac{1}{2}}^{1} (s+2)\, ds &= \left[\frac{s^2}{2} + 2s\right]_{\frac{1}{2}}^{1} \\
\left[\frac{s^2}{2} + 2s\right]_{\frac{1}{2}}^{1} &= \frac{s}{2} - \left[\frac{\frac{1}{4}}{2} + 1\right] \\
\frac{s}{2} - \left[\frac{\frac{1}{4}}{2} + 1\right] &= \frac{5}{2} - \frac{9}{8} \\
\frac{5}{2} - \frac{9}{8} &= \frac{22}{16} \\
\int_{\frac{1}{2}}^{1} (s+2)\, ds &= \frac{11}{8s}
\end{aligned} \tag{98}$$

99. $\int_{0}^{3} \left(u^2 - 2u + 3\right) du$

$$\begin{aligned}
\int_{0}^{3} \left(u^2 - 2u + 3\right) du = \left[\frac{u^3}{3} - u^2 + 3u\right]_{0}^{3} &= \left[\frac{3^3}{3} - 3^2 + 9\right] - \left[\frac{0^3}{3} - 0^2 + 3 \cdot 0\right] \\
\int_{0}^{3} \left(u^2 - 2u + 3\right) du &= 3^2 - 3^2 + 9 \\
\int_{0}^{3} \left(u^2 - 2u + 3\right) du &= 9
\end{aligned} \tag{99}$$

100. $\int_1^2 \left(s^2+3s+1\right) ds$

$$\begin{aligned}
\int_1^2 \left(s^2+3s+1\right) ds &= \left[\frac{s^3}{3}+\frac{3s^2}{2}+s\right]_1^2 \\
\left[\frac{s^3}{3}+\frac{3s^2}{2}+s\right]_1^2 &= \left[\frac{2^3}{3}+\frac{3\cdot 2^2}{2}+2\right]-\left[\frac{1^3}{3}+\frac{3\cdot 1^2}{2}+1\right] \\
\left[\frac{2^3}{3}+\frac{3\cdot 2^2}{2}+2\right]-\left[\frac{1^3}{3}+\frac{3\cdot 1^2}{2}+1\right] &= \left[\frac{8}{3}+8\right]-\left[\frac{1}{3}+\frac{5}{2}\right] \\
\left[\frac{8}{3}+8\right] &= \frac{32}{3}-\frac{17}{6} \\
\int_1^2 \left(s^2+3s+1\right) ds &= \frac{47}{6}
\end{aligned} \tag{100}$$

101. $\int_{-1}^1 \sqrt[3]{t}dt$

$$\begin{aligned}
\int_{-1}^1 \sqrt[3]{t}dt &= \left[\frac{t^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_{-1}^1 \\
\left[\frac{t^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_{-1}^1 &= \left[\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]_{-1}^1 \\
\left[\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]_{-1}^1 &= \left[\frac{1^{\frac{4}{3}}}{\frac{4}{3}}\right]-\left[\frac{(-1)^{\frac{4}{3}}}{\frac{4}{3}}\right] \\
\left[\frac{1^{\frac{4}{3}}}{\frac{4}{3}}\right]-\left[\frac{(-1)^{\frac{4}{3}}}{\frac{4}{3}}\right] &= \frac{3}{4}-\frac{3}{4} \\
\int_{-1}^1 \sqrt[3]{t}dt &= 0
\end{aligned} \tag{101}$$

102. $\int_1^3 \left(1+\frac{1}{x}\right) dx$

$$\begin{aligned}
\int_1^3 \left(1+\frac{1}{x}\right) dx &= [x+\ln x]_1^3 \\
[x+\ln x]_1^3 &= 3+\ln 3-[1+\ln 1] \qquad (102)\\
\int_1^3 \left(1+\frac{1}{x}\right) dx &= 2+\ln 3
\end{aligned}$$

103. $\int_1^2 \left(\frac{1+3x^2}{x}\right) dx$

$$\begin{aligned}
\int_1^2 \left(\frac{1+3x^2}{x}\right) dx &= \int_1^2 \frac{1}{x}dx+3\int_1^2 xdx \\
&\int_1^2 \frac{1}{x}dx+3\int_1^2 xdx \\
\left[\ln x+\frac{3x^2}{2}\right]_1^2 &= \left[\ln 2+3\cdot\frac{2^2}{2}\right]-\left[\ln 1-3\cdot\frac{1^2}{2}\right] \qquad (103)\\
\left[\ln 2+3\cdot\frac{2^2}{2}\right]-\left[\ln 1-3\cdot\frac{1^2}{2}\right] &= \ln 2+6-\frac{3}{2} \\
\ln 2+6-\frac{3}{2} &= \frac{9}{2}+\ln 2
\end{aligned}$$

104. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2x dx$

$$
\begin{aligned}
\frac{d}{dx}\sin 2x &= 2\cos 2x \\
\cos 2x &= \frac{\frac{d}{dx}\sin x}{2} \\
\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{d}{dx}\sin 2x dx &= \left[\frac{\sin 2x}{2}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \\
\left[\frac{\sin 2x}{2}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{2}} &= \left[\frac{\sin\frac{2\pi}{2}}{2}\right] - \left[-\frac{\sin\frac{2\pi}{3}}{2}\right] \\
\left[\frac{\sin\frac{2\pi}{2}}{2}\right] - \left[-\frac{\sin\frac{2\pi}{3}}{2}\right] &= \frac{0}{2} + \frac{\frac{\sqrt{3}}{2}}{2} \\
\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2x dx &= \frac{\sqrt{3}}{4}
\end{aligned} \tag{104}
$$

105. $\int_{-\pi}^{0} \sin 3x dx$

$$
\begin{aligned}
\frac{d}{dx}\cos 3x &= -3\sin 3x \\
\sin 3x &= -\frac{\frac{d}{dx}\cos 3x}{3} \\
-\frac{1}{3}\int_{-\pi}^{0} \frac{d}{dx}\cos 3x dx &= -\left[\frac{\cos 3x}{3}\right]_{-\pi}^{0} \\
-\left[\frac{\cos 3x}{3}\right]_{-\pi}^{0} &= -\frac{\cos 3\cdot 0}{3} + \frac{\cos(-3\pi)}{3} \\
-\frac{\cos 3\cdot 0}{3} + \frac{\cos(-3\pi)}{3} &= -\frac{2}{3}
\end{aligned} \tag{105}
$$

106. $\int_{-1}^{1} e^{2x} dx$

$$
\begin{aligned}
\frac{d}{dx} e^{2x} &= 2e^{2x} \\
e^{2x} &= \frac{\frac{d}{dx} e^{2x}}{2} \\
\int_{-1}^{1} e^{2x} dx &= \frac{1}{2} \int_{-1}^{1} \frac{\frac{d}{dx} e^{2x}}{2} dx \\
\int_{-1}^{1} e^{2x} dx &= \left[\frac{e^{2x}}{2} \right]_{-1}^{1} \\
\int_{-1}^{1} e^{2x} dx &= \left[\frac{e^{2 \cdot 1}}{2} \right] - \left[\frac{e^{2 \cdot 0}}{2} \right] \\
\int_{-1}^{1} e^{2x} dx &= \frac{e^{2}}{2} - \frac{e^{-2}}{2} \\
\int_{-1}^{1} e^{2x} dx &= \frac{e^{2} - e^{-2}}{2}
\end{aligned}
\tag{106}
$$

107. $\int_{0}^{1} \left(\frac{1}{1+t^2} \right) dt$

$$
\begin{aligned}
\frac{d}{dt} \arctan t &= \frac{1}{1+t^2} \\
\int_{0}^{1} \left(\frac{1}{1+t^2} \right) dt &= \int_{0}^{1} \frac{d}{dt} \arctan t dt \\
\int_{0}^{1} \left(\frac{1}{1+t^2} \right) dt &= \arctan 1 - \arctan 0 \\
\int_{0}^{1} \left(\frac{1}{1+t^2} \right) dt &= \frac{\pi}{4}
\end{aligned}
\tag{107}
$$

108. $\int_0^{\frac{\pi}{4}} \sin x dx$

$$\begin{aligned}
\frac{d}{dx}\cos x &= -\sin x \\
\int_0^{\frac{\pi}{4}} \sin x dx &= |-\cos x|_0^{\frac{\pi}{4}} \\
\int_0^{\frac{\pi}{4}} \sin x dx &= -\cos\frac{\pi}{4} - (-\cos 0) \\
\int_0^{\frac{\pi}{4}} \sin x dx &= -\frac{\sqrt{2}}{2} + 1 \\
\int_0^{\frac{\pi}{4}} \sin x dx &= -\frac{\sqrt{2}+2}{2}
\end{aligned} \tag{108}$$

109. $\int_{-1}^{0} e^{-2x} dx$

$$\begin{aligned}
\frac{d}{dx}e^{-2x} &= -2e^{-2x} \\
e^{-2x} &= -\frac{\frac{d}{dx}e^{-2x}}{2} \\
\int_{-1}^{0} e^{-2x} dx &= -\frac{1}{2}\int_{-1}^{0} \frac{d}{dx}e^{-2x} dx \\
\int_{-1}^{0} e^{-2x} dx &= \left[-\frac{e^{-2x}}{2}\right]_{-1}^{1} \\
\int_{-1}^{0} e^{-2x} dx &= \frac{-e^{-2\cdot 0}}{2} - \left(\frac{-e^{2\cdot(-1)}}{2}\right) \\
\int_{-1}^{0} e^{-2x} dx &= -\frac{1}{2} + \frac{e^2}{2} \\
\int_{-1}^{0} e^{-2x} dx &= \frac{-1+e^2}{2}
\end{aligned} \tag{109}$$

110. $\int_0^{\frac{\pi}{3}} (3+\cos 3x)\,dx$

$$\begin{aligned}
\int_0^{\frac{\pi}{3}} (3+\cos 3x)\,dx &= \int_0^{\frac{\pi}{3}} 3dx + \int_0^{\frac{\pi}{3}} \cos 3x dx \\
\int_0^{\frac{\pi}{3}} 3dx + \int_0^{\frac{\pi}{3}} \cos 3x dx &= \left[3x + \frac{1}{3}\sin 3x\right]_0^{\frac{\pi}{3}} \\
\int_0^{\frac{\pi}{3}} (3+\cos 3x)\,dx &= \frac{3\frac{\pi}{3}}{3} + \sin\frac{\frac{3\pi}{3}}{3} - \left(3\cdot 0 + \frac{\sin 3\cdot 0}{3}\right) \\
\int_0^{\frac{\pi}{3}} (3+\cos 3x)\,dx &= \pi
\end{aligned} \tag{110}$$

111. $\int_0^1 \sin 5x dx$

$$\begin{aligned}
\int_0^1 \sin 5x dx &= \left[-\frac{1}{5}\cos 5x\right]_0^1 \\
\int_0^1 \sin 5x dx &= -\frac{\cos 5\cdot 1}{5} - \left(\frac{\cos 5\cdot 0}{5}\right) \\
\int_0^1 \sin 5x dx &= \frac{1-\cos 5}{5}
\end{aligned} \tag{111}$$

112. $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$

$$\begin{aligned}
\frac{d}{dx}\arcsin x &= \frac{1}{\sqrt{1-x^2}} \\
\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx &= \int_0^{\frac{1}{2}} \frac{d}{dx}\arcsin x dx \\
\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx &= \arcsin\frac{1}{2} - \arcsin 0 \\
\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx &= \frac{\pi}{6}
\end{aligned} \tag{112}$$

113. $\int_0^2 2^x dx$

$$\begin{aligned}\int_0^2 2^x dx &= \left|\frac{2^x}{\ln 2}\right|_0^2 \\ \int_0^2 2^x dx &= \frac{2^2}{\ln 2} - \frac{2^0}{\ln 2} \\ \int_0^2 2^x dx &= \frac{3}{\ln 2}\end{aligned} \tag{113}$$

114. $\int_0^1 2xe^{x^2} dx$

$$d \tag{114}$$