VARIÁVEIS

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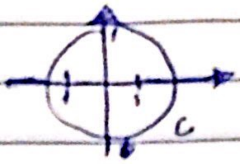
$$\int_{|z|=2} \frac{dz}{z^2-1}$$



$$\frac{1}{(z-1)(z+1)} = \frac{A(z+1)+B(z-1)}{(z-1)(z+1)}$$

Para $z=1 \rightarrow 2A=1 \quad A=\frac{1}{2}$

Para $z=-1 \rightarrow 1=-2B \quad B=\frac{1}{2}$

$$\frac{1}{2}\int_{|z|=2} \frac{dz}{z-1} - \frac{1}{2}\int_{|z|=2} \frac{dz}{z+1} = \int_{|z|=2} \frac{dz}{z^2-1}$$

$$\frac{1}{2}\int_0^{\pi} \frac{dz}{2e^{it}-1}\,dz - \frac{1}{2}\int_0^{\pi} \frac{1}{2e^{it}+1}\,dz = \int \frac{1}{z^2-1}\,dz$$

Usando $\int \frac{1}{z-1}\,dz$ | Usando $\int \frac{1}{z+1}\,dz$

Por Cauchy

$f(z)=1$, $z_0=1$ $\Rightarrow \int \frac{1}{z-1}\,dz = 2\pi i$

$\Rightarrow f(z)=1$, $z_0=-1$ $\Rightarrow \int \frac{1}{z-1} = 2\pi i$

$$\int_c \frac{1}{z^2-1}\,dz = \frac{1}{2}\left[\int_c \frac{1}{z-1}\,dz - \int_c \frac{1}{z+1}\,dz\right]$$

$$\int_c \frac{1}{z^2-1}\,dz = \frac{1}{2}\left[\overbrace{2\pi i - 2\pi i}^{0}\right]$$

$$\boxed{\int_c \frac{1}{z^2-1}\,dz = 0}$$