The Metacircular Evaluator

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Abstract

This document provides a comprehensive explanation of a metacircular evaluator implemented for Scheme. Developing a metacircular evaluator is a fundamental step toward mastering the creation of domain-specific languages.

The implementation, detailed in the metacircular-evaluator.scm file, closely follows the evaluator described in *Structure and Interpretation of Computer Programs*[2]. It incorporates several enhancements and simplifications to improve both clarity and functionality.

Additionally, the discussion in Section 1.4 draws significant inspiration from the lecture "Metacircular Evaluator" [5], delivered by Professor Gerald Jay Sussman at MIT.

For a deeper understanding, it is strongly recommended to carefully read both the accompanying quotes and the text.

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Chapter 1

The Evaluator

1.1 Motivation

[...] It's in words that the magic is — Abracadabra, Open Sesame, and the rest — but the magic words in one story aren't magical in the next. The real magic is to understand which words work, and when, and for what; the trick is to learn the trick. [...] And those words are made from the letters of our alphabet: a couple-dozen squiggles we can draw with the pen. This is the key! And the treasure, too, if we can only get our hands on it! It's as if — as if the key to the treasure is the treasure! [3]

John Barth, Chimera

A programmer may want to use different programming languages according to his desired-program needs. In programming a scheduler — for example — one may want to use a considerably amount of pointers and access to low-level instructions. In programming a machine learning algorithm, for instance, the same programmer may want to use a language that allows a huge load of abstraction, since it is not of his desire to be worried about memory or small details. By the same argument, the definition of low-level is arguable. According to Alan Jay Perlis, "A programming language is low level when its programs require attention to the irrelevant.[4]".

The point is that a language that is designed for a certain purpose is generally more appropriate for solving a certain issue. Singular, for example, may be a spectacular language for Computer Algebra, but may not be appropriate for simply plotting a sin function. Python, on the other hand, can be used to plot a sin function in no more than 3 lines of code, but is not the best for Computer Algebra. By the same argument, this document was made in LaTeX instead of C or some other general purpose language.

In conclusion, domain-specific languages prove to be particularly effective in scenarios where

specialized tasks cannot be efficiently addressed by general-purpose languages. To allow a programer to develop its own language to address a program is to allow the creation of an instrument capable of shaping the complexity of the problem to the specific needs of the domain.

1.2 The Evaluator as a Program

It is no exaggeration to regard this as the most fundamental idea in programming: The evaluator, which determines the meaning of expressions in a programming language, is just another program. To appreciate this point is to change our images of ourselves as programmers. We come to see ourselves as designers of languages, rather than only users of languages designed by others. In fact, we can regard almost any program as the evaluator for some language. [2]

Gerald Jay Sussman & Harold Abelson, Structure and Interpretation of Computer Programs

By implementing an interpreter for solving a specific problem, the developer is allowed to make decisions about the semantics and syntax of *his* programming language.

Note that most — maybe all — of the programs that are written can be easily seen as a tiny extension of a existing programming language. For example, imagine the following factorial procedure:

```
In Scheme:
                                                          In C:
                                In Python:
(define (factorial n)
                                def factorial(n):
                                                           int factorial(int n) {
  (if (<= n 1)
                                   if n <= 1:
                                                             if (n \le 1) {
                                     return n
      n
                                                               return n;
      (* n
                                                             }
                                   else:
         (factorial (- n
                                     return n *
                                                             else {
                        1)))))
                                         factorial(n-1)
                                                               return n *
                                                                  factorial(n-1);
                                                             }
                                                           }
```

The factorial program defined above is in fact a tiny programming language with its own semantics and syntax. All the three versions of factorial have the same semantics, they compute the factorial of a given number following the same mathematical definition. On the other hand, the three of them have different syntax. Note that the Scheme implementation require a lot of parentheses and the C implementation require some parentheses, curly brackets and semicolons. while the Python implementation relies mostly of its syntax on the indentation¹.

¹Note that in C and Scheme, indentation is only used for better visualization and understanding of the defini-

The factorial procedure is a simple program considering that it has a very strict input and a very strict output, exactly like a mathematical function $f: \mathbb{N} \to \mathbb{N}$ such that:

$$\begin{cases} f(n) = n & \text{if } n \le 1 \\ f(n) = n \times f(n-1) & \text{if } n > 1 \end{cases}$$

The interpreter, on the other hand, will have far more expressiveness than the factorial procedure. In fact, the factorial procedure will be able to be passed as an argument to the interpreter, in what is known as a universal machine. This flexibility highlights the core concept of computation: the ability to evaluate any function, define new functions, and even create new languages and paradigms for computation. The interpreter serves as the bridge between the abstract, high-level description of a program and its execution on a physical machine.

The role of the interpreter becomes clear when it is considered that it can evaluate any expression or program written in a language. In a sense, an interpreter for a language is itself a program that reads another program, interprets its meaning, and produces an output. Thus, the interpreter is both a tool for executing programs and a means of creating new languages.

This perspective encourages us to view programming not just as writing instructions for a computer but as the creation of languages through which computations are described. By designing and implementing interpreters, we become language designers, defining the syntax and semantics that govern the execution of our programs. The ability to define new languages is powerful, enabling the creation of more expressive, efficient, or domain-specific solutions.

All these programming languages have different ways of parsing through the user's definitions and transcribing them to machine code language (if the language is compiled) or evaluating them (if the language is interpreted). Note that Scheme, despite having the most unusual syntax of the three languages shown above for the average programmer, actually makes parsing significantly easier for several reasons:

1. Homoiconicity: the fact that Scheme code is in fact a huge Scheme list makes it easier to manipulate its values.

For an average program — such as the factorial function or a scheduler — the manipulated data typically consists of numbers, characters, arrays and user-defined types. However, in programs like interpreters or compilers, the data being manipulated is code itself. Notice that Scheme code is itself a Scheme list. Manipulating this list (which is, in fact, code) isn't much different from manipulating other lists.

See, for example, a variation of the symbolic differentiation procedure made by Gerald Jay Sussman and Harold Abelson in Structure and Interpretation of Computer Programs[2]:

tions. However, these are not required.

```
((product? exp)
                        (product-rule exp var))
((division? exp)
                        (quotient-rule exp var))
((exponentiation? exp) (power-rule
                                        exp var))
((composite? exp)
                        (chain-rule
                                        exp var))
((ln? exp)
                        (ln-rule
                                        exp var))
((exp? exp)
                        (expt-rule
                                        exp var))
((sin? exp)
                        (sin-rule
                                        exp var))
((cos? exp)
                        (cos-rule
                                        exp var))
(else
  (error "case not found!"))))
```

The procedure deriv takes as input an expression exp and a variable var with respect to which the expression will be differentiated. The argument exp is typically a Scheme list such as (+ (* a (* x x)) (+ (* b x) c)). If the variables a, b, c, and x were defined, this expression could be evaluated to yield a numeric result. In this sense, deriv operates on exp in a way analogous to how the evaluator processes expressions. Furthermore, both deriv and the evaluator accept structurally similar arguments. The procedures deriv and eval (the procedure responsible for evaluating expressions) both manipulate lists in a very similar way. Since Scheme code is itself a Scheme list, it is extremely simple for eval to interpret and manipulate expressions directly, as the problem can be reduced to simply manipulating Scheme lists (which, in fact, are not any different from Scheme code).

2. S-expressions: In Scheme, all function calls have the function as the first argument of the expression.

For most non-lisp users, Scheme code can be seen as pretty strange, "why does one prefer to write (+ 1 2) instead of (1 + 2)?".

A convenience of having + as the first argument of the expression is to facilitate the parsing. This makes it very easily to apply a function to its arguments. In most cases, all the Scheme evaluator does is apply the first element of the list (the function) to the remaining evaluated elements of the list (the arguments). This is, in fact, the eval-apply cycle that will be better discussed in section 1.3.

Try to imagine how are these following codes parsed in their respective languages:

```
In Scheme:
                              In Python:
                                                            In C:
(define x 1)
                              x = 1
                                                            int main() {
                              2 + x * 10 / 2 - 3
(-(+2)
                                                              int x = 1;
      (* x
                                                              2 + x * 10 / 2 - 3;
          (/ 10
                                                              return 0;
                                                            }
             2)))
   3)
```

Most programmers struggle to form an intuitive understanding or make good guesses about how C or Python code is parsed. "How is the order of operations determined? How are the operations precedence implemented? How exactly does the parser work?".

In contrast, the Scheme version of the code is remarkably simple and intuitive.

Scheme code doesn't need to be heavily parsed and manipulated in order for it to be evaluated. Scheme code undergoes minimal parsing, allowing for straightforward traversal, interpretation,

and transformation of expressions without the need for complex intermediate representations (similar to the deriv procedure).

The process of evaluation of the Scheme code shown above goes as followed:

- 1. the special form define is applied to the arguments x and 1. What define does is assign the value of the third element of the list (1) to the second element of the list (x).
- 2. The evaluation of '(- (+ 2 (* x (/ 10 2))) 3) involves applying the procedure to the results of evaluating '(+ 2 (* x (/ 10 2))) and '3. The evaluation of '(+ 2 (* x (/ 10 2))) is done by applying the procedure + to the results of evaluating '2 and '(* x (/ 10 2)). The evaluation of '(* x (/ 10 2)) applies the procedure * to the evaluation of 'x and '(/ 10 2). The evaluation of 'x yields the number 1, while the evaluation of '(/ 10 2) applies the function / to the evaluation of '10 and '2. As expected, the evaluations of '3, '2, '10, and '2 are simply the values themselves.

It is easily imaginable that the Scheme interpreter does something similar to²:

```
(eval '(define x 1))
...
(eval '(- (+ 2 (* x (/ 10 2))) 3))
(apply (eval '-) (eval '(+ 2 (* x (/ 10 2)))) (eval '3))
(apply - (apply (eval '+) (eval '2) (eval '(* x (/ 10 2)))) 3)
(apply - (apply + 2 (apply (eval '*) (eval 'x) (eval '(/ 10 2)))) 3)
(apply - (apply + 2 (apply * 1 (apply (eval '/) (eval '10) (eval '2)))) 3)
(apply - (apply + 2 (apply * 1 (apply / 10 2))) 3)
(apply - (apply + 2 (apply * 1 5)) 3)
(apply - (apply + 2 5) 3)
(apply - 7 3)
```

Questions about how code is evaluated are resolved through an understanding of the evalapply cycle.

²This eval-apply cycle expansion is somewhat similar to what the interpreter defined in the metacircular -evaluator.scm file does. However, observe that the second argument of apply should have been a list since eval invokes apply with the second argument begin a map of eval throughout the arguments of the function passed to apply.

1.3 Implementation

We're going to understand what we mean by a program a little bit more profoundly than we have up till now. We've been thinking of programs as describing machines. [...] There's something very remarkable that can happen in the computational world which is that you can have something called a universal machine. [...] We'll see that among other things, it's extremely simple. Now, we are getting very close to the real spirit in the computer at this point. [...] There's a certain amount of mysticism that will appear here. [...] I wish to write for you the evaluator for Lisp. The evaluator isn't very complicated, it's very much like all the programs we've seen already: that's the amazing part of it. [5]

```
Gerald Jay Sussman,
Lecture 7A: Metacircular Evaluator. Timestamp: 0:18
```

The interpreter for Scheme is commonly referred to as the *Metacircular* Evaluator. It is called Meta because it is written in the language itself, meaning Scheme code is interpreting Scheme. It is called Circular because it — for the most part — consists of a recursive loop between the functions eval and apply known as the eval-apply cycle.

The main function of the evaluator is eval itself, which definition is:

```
(define (EVAL exp env)
  (cond
    ((self-value? exp)
    ((variable? exp)
                         (lookup-variable-value exp env))
    ((quoted? exp)
                         (text-of-quotation exp))
    ((assignment? exp)
                         (eval-assignment exp env))
    ((definition? exp)
                         (eval-definition exp env))
    ((if? exp)
                         (eval-if exp env))
                         (make-procedure (lambda-ps exp) (lambda-body exp) env))
    ((lambda? exp)
    ((begin? exp)
                         (eval-sequence (begin-actions exp) env))
                         (eval-and exp env))
    ((and? exp)
    ((or? exp)
                         (eval-or exp env))
    ((cond? exp)
                         (EVAL (cond->if exp) env))
    ((let? exp)
                         (EVAL (let->combination exp) env))
    ((let*? exp)
                         (EVAL (let*->nested-lets exp) env))
    ((application? exp) (APPLY (EVAL (operator exp) env)
                                (map (lambda (exp) (EVAL exp env))
                                     (operands exp))))))
```

Something noticeable is that eval is simply a case analysis which determines what should be done to the given expression. Notice that there are 3 kinds of procedures on the eval definition:

- 1. Special Forms: These constructs cannot be implemented as simple functions because their evaluation behavior differs from that of regular functions. Take, for example, the if special form. When evaluating (if x y z), it first checks whether x is true. If it is, y is evaluated and returned; otherwise, z is evaluated and returned. The distinction lies in how the expressions y and z are handled: if if were implemented as a regular function, both y and z would be evaluated before the function is called, even though only one of them is actually needed. This difference is particularly important when y or z involve side effects, as unnecessary evaluations could lead to unintended behavior.
- 2. Derived Expressions: These constructs can easily be implemented as a list manipulation procedure that converts an expression to a special form. Take, for example, the cond expression. Cond can easily be implemented as a syntactic-sugar to nested if expressions. The same happens to let, which can be implemented as a lambda expression and let*, which can be implemented as nested let expressions.
- 3. Applications: These constructs are the remaining building blocks in the evaluation process. In an application, a function is applied to a sequence of arguments. The function is first evaluated to determine its value (a procedure), and then its arguments are evaluated. The procedure is then invoked with the results of the arguments' evaluations. This process forms the backbone of computation in functional programming, as nearly every computation reduces to applying functions to arguments.

The fact that all expressions start with the function itself makes it extremely easy to implement the checking procedures since they are simply verifying if the first element of the expression is equal to something.

```
(define (tagged-list? exp tag)
  (if (pair? exp)
      (eq? (car exp) tag)
      false))
(define self-value?
                                  number?)
(define variable?
                                  symbol?)
(define (quoted? exp)
                                  (tagged-list? exp 'quote))
(define (assignment? exp)
                                  (tagged-list? exp 'set!))
(define (definition? exp)
                                  (tagged-list? exp 'define))
(define (if? exp)
                                  (tagged-list? exp 'if))
(define (lambda? exp)
                                  (tagged-list? exp 'lambda))
(define (begin? exp)
                                  (tagged-list? exp 'begin))
(define (and? exp)
                                  (tagged-list? exp 'and))
                                  (tagged-list? exp 'or))
(define (or? exp)
(define (cond? exp)
                                  (tagged-list? exp 'cond))
(define (let? exp)
                                  (tagged-list? exp 'let))
                                  (tagged-list? exp 'let*))
(define (let*? exp)
(define application?
                                  pair?)
(define (compound-procedure? p)
                                  (tagged-list? p 'procedure))
(define (primitive-procedure? p) (tagged-list? p 'primitive))
```

Apply is the procedure responsible for searching the procedure in the environment and applying it to the evaluated remaining arguments.

These are the most important definitions that are needed for understanding the evaluator.

It's definitely uneven to think that a huge program can be evaluated by a program this small³, but that's precisely the elegance of a metacircular evaluator: it leverages the power of the language it interprets to reflect its own semantics, demonstrating that even the most complex systems can be built upon simple, recursive principles.

If you examine the implementation of the metacircular evaluator in detail, you will notice that many definitions are simply aliases for basic list operations like car and cdr. This design choice creates a clear separation between syntax and semantics, making it easier to modify the language's behavior without changing its underlying semantics. For instance, consider the evaluator's final check:

Here, the operator and operands procedures are defined as followed:

```
(define operator car) ; first element of the expression
(define operands cdr) ; all elements of the expression except for the first one
```

To modify Scheme for educational purposes — such as placing the operator as the second element of the list — it is possible to simply redefine these procedures:

```
(define operator cadr) ; second element of the expression
(define (operands x) ; all elements of the expression
  (cons (car x) (cddr x))) ; except for the second one
```

This change would allow expressions like (4 + 2) and (4 factorial) to evaluate correctly while preserving the original semantics⁴. The ability to adapt the syntax so easily highlights the power and flexibility of this approach.

³The program includes more definitions than just eval and apply that, for the sake of simplicity and conciseness, will not be included here.

 $^{^4}$ With this modification some strange syntax would appear, like evaluating (2 * 3 5 7) in order to compute the product of the 4 first prime numbers.

Notice the similarities between eval and deriv. Both are recursive⁵ procedures that traverse⁶ a Scheme list and, based on the specific case encountered, invoke an appropriate procedure to operate on the given expression.

1.4 Lisp as a Fixed Point

There's an awful lot of strange nonsense here. After all, he purported to explain to me Lisp, and he wrote me a Lisp program on the blackboard. The Lisp program was intended to be an interpreter for Lisp, but you need a Lisp interpreter in order to understand that program. How could that program have told me anything there is to be known about Lisp? [...] The whole thing is sort of like these Escher's hands. [5]

Gerald Jay Sussman,

Lecture 7A: Metacicular Evaluator. Timestamp: 56:19

Given the set of equations

$$\begin{cases} x = 3 - y \\ y = -1 + x, \end{cases}$$

notice that x is defined in terms of y and y is defined in terms of x. This system has not only a solution but a unique one in x and y.

Given the set of equations

$$\begin{cases} 2x = 6 - 2y \\ y = 3 - x \end{cases}$$

notice that x is once again defined in terms of y and y is defined in terms of x. Strangely enough, this system has no solution in x and y.

Given the set of equations

$$\begin{cases} x = 1 + y \\ y = x - 2, \end{cases}$$

notice that the pattern of recursive definitions once again appears. However, this system has no solutions in x and y.

⁵The recursive nature of deriv may not be immediately evident from its definition. However, consider the sum rule for differentiation, for instance, and the recursive structure of the procedure becomes apparent.

 $^{^6}$ Notice that not all functions within an expression passed to eval are evaluated. For instance, consider the if special form.

Given these three sets of equations, note that the number of solutions is not a consequence of their format, but of their content. The equation of interest is the one that has a unique solution.

$$\begin{cases} x = 3 - y \\ y = -1 + x. \end{cases}$$

A way of seeing this set of equations is as a transformation T such that

$$\begin{bmatrix} x \\ y \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix}.$$

Note that the solution of this equation is a fixed point of the transformation T.

Take a closer look at the factorial procedure implemented in section 1.2. Observe that factorial is a kind of recursive equation somewhat similar to the set of equations in x and y.

To find the fixed point of factorial, it is first needed to rewrite it in such a way that the transformation T becomes apparent.

Observe that f is a procedure such that, if the solution g were provided, the result would be the factorial procedure. Assuming g is the factorial procedure, f would produce precisely the same factorial procedure described in section 1.2.

It's strange that the g procedure is needed for g to be the result of this function, by the same way that somehow the value of x is needed to compute x in x = 4 - x. As shown in linear algebra, a way to approximate the solution x of this equation in simply by having a start guess x_0 and apply the transformation T to x multiple times.

$$x_1 = Tx_0$$

$$x_2 = Tx_1$$

$$x_3 = Tx_2$$

$$\vdots$$

$$x_k = Tx_{k-1}$$

Such that $x_i, \forall i \in \{1, ..., k\}$, is an approximation of x. A similar approximation can be made to the factorial procedure:

```
; computes 0! and 1! with no errors
```

```
(define factorial-0 (lambda (n) 1)); initial guess is 0
; computes 2! with no errors
(define factorial-1 (f factorial-0))
; computes 3! with no errors
(define factorial-2 (f factorial-1))
; computes 4! with no errors
(define factorial-3 (f factorial-2))
```

Factorial-3 is an approximation of factorial that computes factorials up to 4 with no errors. It can be concluded that the procedure factorial is equal to $\lim_{n\to\infty}$ factorial-n. It can also be said that factorial is equal to (f (f (...(f factorial-0)...))) or that factorial is a fixed-point of the function f.

A way of making an infinite loop is by the Curry's Paradoxical Combinator of Y:

We can conclude that (Y F) = (F (Y F)) which is exactly what we wanted (An infinite loop).

What Lisp is, is the fixed point of the process which says "If I knew what Lisp was and substituted it in for eval and apply, and so on, on the right hand side of all those recursive equations, [...] Then the left hand side would also be Lisp". [5]

Gerald Jay Sussman, Lecture 7A: Metacircular Evaluator. Timestamp: 1:16:37

In the same way that the f procedure, when given a factorial procedure, returns factorial, the procedures eval can be defined similarly as being the fixed point eval = L eval.

```
(define L
  (lambda (ev)
  <...>
```

By this definition, L is a procedure that takes ev as argument and returns an approximation to the eval procedure defined in section 1.3. By the same argument used in factorial, we could say that eval (or even Scheme) is equal to (L (...(L ev0)...)), with ev0 being in fact any initial guess.

1.5 Conclusion

There are many languages that have made a mess of themselves by adding huge numbers of features. [...] Many systems suffer from what is called "creeping featurism". [...] After a while, the thing has a manual 500 pages long that no one can understand. [...] In computer languages, I think it's a disaster to have too much stuff in them. [6]

Gerald Jay Sussman, Lecture 7B: Metacircular Evaluator. Timestamp: 2:48

The fact that **Scheme** code is inherently represented as lists, with functions appearing as the first element of these lists, makes it exceptionally simple, convenient, and elegant to implement a metacircular evaluator. **Scheme**'s minimalistic design, combined with its remarkable abstraction capabilities, allows it to serve as an ideal foundation for creating domain-specific languages.

Understanding how to implement such an easy evaluator for a language like Scheme, is the first step towards learning how to create programming languages ourselves.

Once you have the interpreter in your hands, you have all this power to start playing with the language. [...] There's this notion of metalinguistic abstraction, which says [...] that you can gain control of complexity by inventing new languages, sometimes. One way to think about computer programming is that it only incidentally has to do with getting a computer to do something. Primarily, what a computer program has to do with is a way of expressing ideas, of communicating ideas. Sometimes, when you want to communicate new kinds of ideas, you'd like to invent new modes of expressing them. [1]

Lecture 8A: Logic Programming. Timestamp: 1:45, Harold Abelson

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