

Row space, null space and rank:

$$A = m \times n$$

$$\left(\begin{pmatrix} | & & | \end{pmatrix} \right) \dots \left(\begin{pmatrix} | & & | \end{pmatrix} \right)$$

$\text{NULL}(A)$ = subspace of all $n \times 1$ ~~column~~ matrices.

$\text{col}(A)$ = subspace of all $(m \times 1)$ matrices.

$\text{Row}(A) = \text{col}(A^T) \Rightarrow$ subspace of all $m \times 1$ matrices

$\text{Left null}(A) = \text{NULL}(A^T) \Rightarrow$ subspace of $m \times 1$ matrices.

$AX = 0 \rightarrow$ Vectors in row space are orthogonal to vectors in the null space.

* $\text{row} \rightarrow$ reduced Echelon form.

$$\text{rank}(A)$$

$$\left(\begin{pmatrix} | & & | \end{pmatrix} \right) \dots \left(\begin{pmatrix} | & & | \end{pmatrix} \right)$$

$\dim(\text{NULL}(A)) = \# \text{ no pivot columns}$

$\dim(\text{row}(A)) = \# \text{ of PIVOT columns}$

\rightarrow The sum of these 2 ~~is~~ is going to be the total of columns of the matrix.

$\text{NULL}(A) \ \& \ \text{Row}(A)$ are orthogonal complements.

$\dim \text{col}(A) = \# \text{ of pivot columns.}$

$$\dim(\text{Row}(A)) = \dim(\text{col}(A))$$



$$\dim(\text{Row}(A)) = \dim(\text{col}(A)) = \text{rank} (= \# \text{ linear}$$

independent columns of the matrix A .)