Linear Independence:

The set of vectors $\{v_1, v_2, \ldots, v_m\}$ are linear independent if $c_1 v_1 + c_2 v_2 + \cdots + c_m v_m = 0$ has the only solution $c_1 = c_2 = c_m = 0$.

Meaning: No vector in the set can be written as linear combination of other vectors.

Example:

$U = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $V = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $W = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$, Linear independent?

$\Downarrow$

NO, $W = 0V + 3V$

$\Downarrow$

$W - 0V - 3V = 0$

$\Downarrow$

$c_1 \neq c_2 \neq c_3 \neq 0$

Example:

U, V above and $W = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$?

Yes, linearly independent.

$a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow a = b = c = 0$