

Inverse matrix with Gaussian elimination:

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{pmatrix}$$

$$\left\{ \begin{array}{l} 1 \ 1 \ 1 \ 0 \ 0 \ 0 \\ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \end{array} \right\} \quad \left\{ \begin{array}{l} 1 \ 1 \ 1 \ 0 \ 0 \ 0 \\ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \end{array} \right\}$$

Definitions:

$$AA^{-1} = I$$

$$A\vec{a}_i = e_i$$

Steps:

I - $\left(\begin{array}{ccc|ccc} -3 & 2 & -1 & 1 & 0 & 0 \\ 6 & -6 & 7 & 0 & 1 & 0 \\ 3 & -4 & 4 & 0 & 0 & 1 \end{array} \right) \xrightarrow[1]{2} \xrightarrow[1]{3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$

II - $\left(\begin{array}{ccc|ccc} -3 & 2 & -1 & 1 & 0 & 0 \\ 0 & -2 & 5 & 2 & 1 & 0 \\ 0 & -2 & 3 & 1 & 0 & 1 \end{array} \right) \xrightarrow[1]{1} \xrightarrow[1]{3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$

III - $\left[\begin{array}{ccccc|c} -3 & 0 & 4 & 3 & 1 & 0 \\ 0 & -2 & 5 & 2 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array} \right] \xrightarrow[2]{1} \left[\begin{array}{ccccc|c} 1 & 0 & -2 & -1 & -1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$

$$\text{IV} - \left(\begin{array}{cccccc} -3 & 0 & 0 & 1 & -1 & 2 \\ 0 & -2 & 0 & -1/2 & -3/2 & 5/2 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array} \right) \div -2$$

$$\left(\begin{array}{cccccc} 1 & 0 & 0 & 1/2 & -1/2 & -1/2 \\ 0 & 1 & 0 & 1/4 & 3/4 & -5/4 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array} \right)$$

$$\text{V} - \left(\begin{array}{cccccc} 1 & 0 & 0 & -1/3 & 1/3 & -2/3 \\ 0 & 1 & 0 & 1/4 & 3/4 & -5/4 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array} \right)$$

$$\boxed{\quad}$$

$$A^{-1}$$

$$A^{-1} = \frac{1}{6} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$V = \left(\begin{array}{cccccc} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{array} \right)$$

$$\left(\begin{array}{cccccc} 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{array} \right)$$

$$\left(\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right)$$