Application of the null Space:

$Ax = b \Rightarrow$ Fewer equations than unknowns.

$$\Downarrow$$

$\#$ rows $<$ $\#$ columns

- Let u be a general vector in $NULL(A)$
- Let v be any vector that solves $Ax = b$

$x = u + v$ general solution to $Ax = b$

$$Ax = A(u+v) = Au + Av = 0 + b = b$$

Ex:

$$\left.\begin{aligned} 2x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 - 2x_2 - x_3 &= 1 \end{aligned}\right\}$$

$$\begin{pmatrix} 2 & 2 & 1 & 0 \\ 2 & -2 & -1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{4} \end{pmatrix}$$

(I) $NULL(A)$:

$$x_1 = 0$$

$$x_2 = -\frac{1}{2}x_3$$

basis: $\begin{pmatrix} 0 \\ -1/2\, x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 0 \\ -1/2 \\ 1 \end{pmatrix}$

(2) Find V "Particular Solution"

$x_1 = 1/4$

$x_2 + \frac{1}{2}x_3 = -\frac{1}{4}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = a\begin{pmatrix} 0 \\ -1/2 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{4} \\ -1/4 \\ 0 \end{pmatrix}$$