Inner and Outer Products

- Inner Product: (Dot Product)

$$U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}, \quad V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$U^T V = (U_1 \ U_2 \ U_3) \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = U_1 V_1 + U_2 V_2 + U_3 V_3 = \sum_{i=1}^{n} U_i V_i$$

$U^T V = 0 \Rightarrow U, V$ are orthogonal (perpendicular)

$$\Downarrow$$

<u>norm</u>: $\|U\| = (U^T U)^{1/2} = \left(\sum_{i=1}^{n} U_i U_i\right)^{1/2} = \sqrt{(U_1^2 + \ldots + U_n^2)}$

U is a normalized if $\|U\| = 1$

If a vector is orthogonal + normalized $\Rightarrow$ orthonormal

Outer Product:

$$uv^T = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix} = \begin{pmatrix} u_1v_1 & u_1v_2 & u_1v_3 \\ u_2v_1 & u_2v_2 & u_2v_3 \\ u_3v_1 & u_3v_2 & u_3v_3 \end{pmatrix}$$