Inverse matrices

$$AA^{-1} = I = A^{-1}A$$

A has to be $n \times n$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}}_{A^{-1}} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{I} \Rightarrow \begin{cases} ax_1 + by_1 = 1 \\ ax_2 + by_2 = 0 \\ cx_1 + dy_1 = 0 \\ cx_2 + dy_2 = 1 \end{cases} \Rightarrow \begin{cases} y_1 = -(c/d)x_1 \\ y_2 = -(a/b)x_2 \end{cases}$$

$$A^{-1} = \begin{cases} ax_1 - \frac{bc}{d}x_1 = 1 \Rightarrow x_1 = \frac{d}{ad-bc} \\ \Downarrow \\ y_1 = \frac{-c}{ad-bc} \\ \\ cx_2 - \frac{ad}{b}x_2 = 1 \Rightarrow x_2 = \frac{b}{bc-ad} \\ \Downarrow \\ y_2 = \frac{a}{ad-bc} \end{cases}$$

$$\Downarrow$$

$$A^{-1} = \frac{1}{\underbrace{ad-bc}_{\det A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$