

Row space, null space and rank: 1 (M. Mat)

$A = m \times n$

$$((A)_{m \times n}) \text{ and } ((I_m)_n)$$

$\text{NULL}(A) = \text{subspace of all } m \times 1 \text{ } \cancel{\text{matrices}}$

$\text{col}(A) = \text{subspace of all } (m \times 1) \text{ matrices}$

$\text{Row}(A) = \text{col}(A^T) \Rightarrow \text{subspace of all } (m \times 1) \text{ matrices}$

$\text{left null}(A) = \text{NULL}(A^T) \Rightarrow \text{subspace of } m \times 1 \text{ matrices}$

$Ax = 0 \rightarrow$ Vectors in row space are orthogonal
to vectors in the null space.

* Mat \rightarrow reduced row echelon form.

$\text{null}(A)$

$\dim(\text{NULL}(A)) = \# \text{ no pivot columns}$

$\dim(\text{row}(A)) = \# \text{ of pivot columns}$

The sum of these 2 ~~is~~ is going to be the total of columns of the matrix.

$\text{NULL}(A) \& \text{Row}(A)$ are orthogonal complements.

$\dim(\text{col}(A)) = \# \text{ of pivot columns.}$

$$\dim(\text{Row}(A)) = \dim(\text{col}(A))$$

$\dim(\text{Row}(A)) = \dim(\text{col}(A)) = \text{rank} (= \boxed{\# \text{ of linearly independent columns of } A})$