

Null space of a matrix:

$$A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$$

$$\text{null}(A) = \begin{pmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Reduced Echelon Form

$\text{null}(A)$ is a vector space of all column vectors X such that $AX=0$. Here the nullspace $\mathcal{N}(A)$ is a subspace of 5×1 matrices.



$$x_1 = 2x_2 + x_4 - 3x_5$$

$$x_3 = -2x_4 + 2x_5$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 2x_2 + x_4 - 3x_5 \\ x_2 \\ -2x_4 + 2x_5 \\ x_4 \\ x_5 \end{pmatrix}$$

$$= x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

$\Rightarrow \text{nullspace}(A)$ is a linear combination of these 3.

\Rightarrow Basis for $\text{null}(A)$

$x_1, x_3 \rightarrow$ basic variables

$x_2, x_4, x_5 \rightarrow$ free variables