

1 Null space of a matrix:

$$A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$$

$$\text{REF}(A) = \begin{pmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Reduced Echelon Form

$\text{NULL}(A)$ is a vector space of all column vectors X such that $AX=0$. Here the nullspace $\text{NULL}(A)$ is a subspace of 5×1 matrices.



$$X_1 = 2X_2 + X_4 - 3X_5$$

$$X_3 = -2X_4 + 2X_5$$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} = \begin{pmatrix} 2X_2 + X_4 - 3X_5 \\ X_2 \\ -2X_4 + 2X_5 \\ X_4 \\ X_5 \end{pmatrix}$$

→ $\text{NULL}(A)$ is a linear combination of these 3.

$$= X_2 \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + X_4 \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{pmatrix} + X_5 \begin{pmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

Basis for $\text{NULL}(A)$

$x_1, x_3 \rightarrow$ basic Voronobles

$x_2, x_4, x_5 \rightarrow$ free Voronobles