Polyhedral model

What is Polyhedral model?

The polyhedral model is a compilation technique which

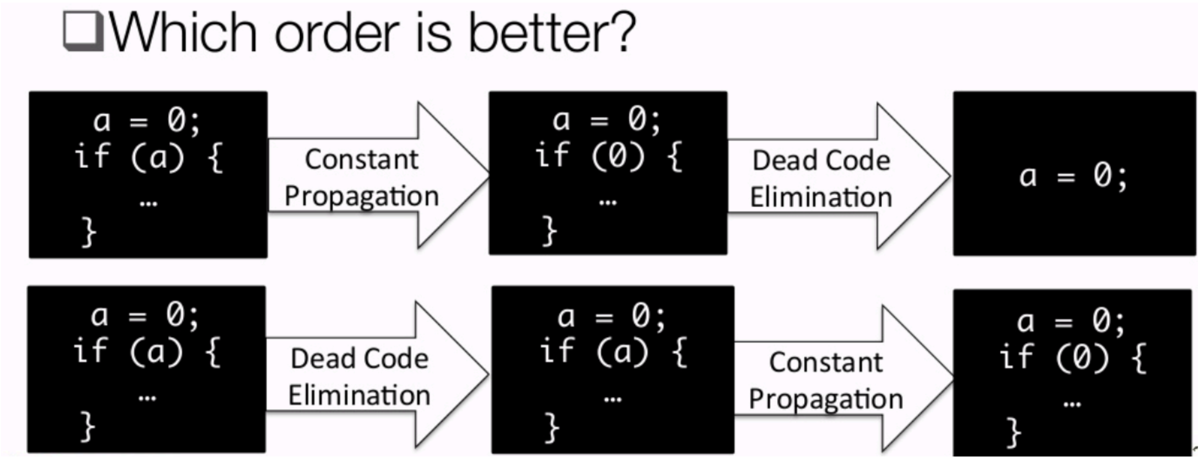
- treats each loop iteration within nested loops as lattice points inside mathematical objects called polyhedra
- performs affine transformations or more general non-affine transformations such as tiling on the polytopes
- converts the transformed polytopes into equivalent, but optimized (depending on targeted optimization goal), loop nests through polyhedra scanning.
- Nested loop programs are the typical, but not the only example, and the most common use of the model is for loop nest optimization in program optimization

Introduction to Polyhedral Compilation

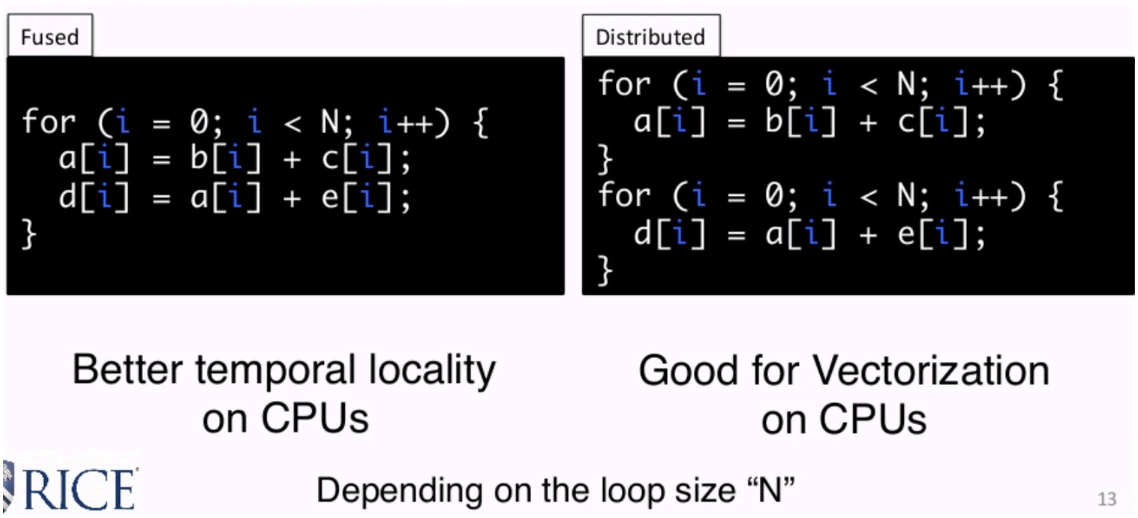
Why Polyhedral Model

> The Polyhedral Model is a convenient alternative representation which combines analysis power, expressiveness and high flexibility" - OpenScop Specification and Library

- In contrast to Abstract Syntax Tree
- One solution for tackling the phase-ordering problem

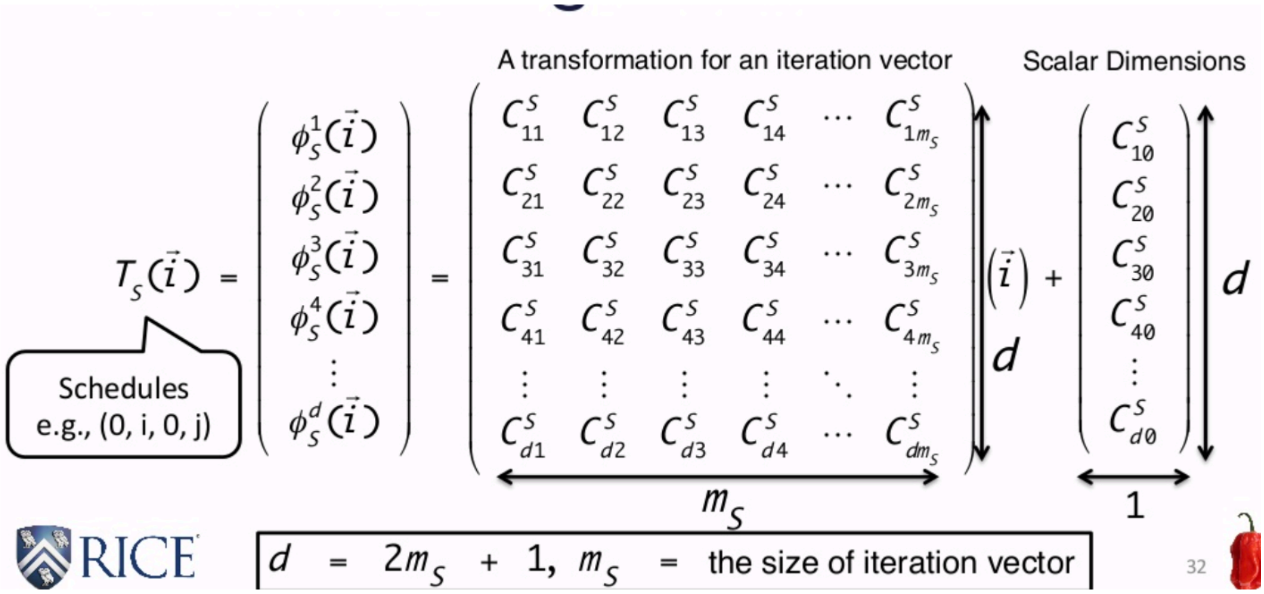


- Good for performing a set of loop transformations
  - Loop permutation (interchange) : stride access or offset access
  - Loop fusion/distribution:



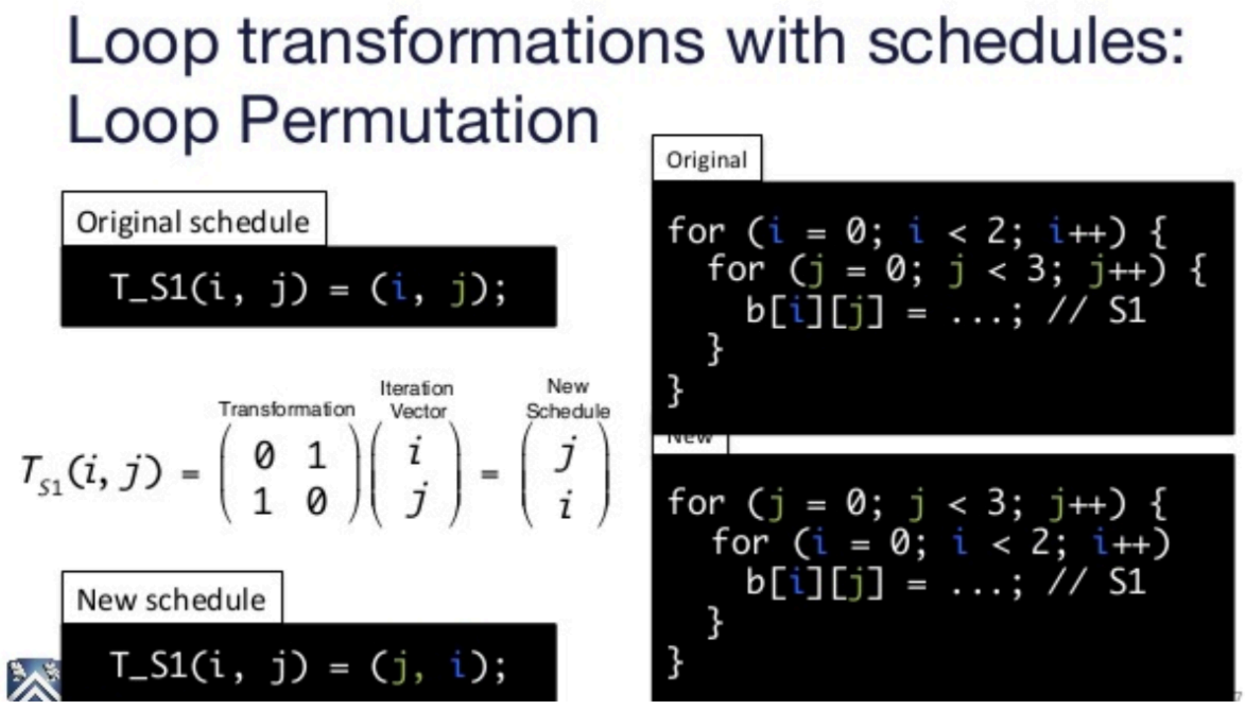
- Loop tiling

Schedules



- s denotes a statement, $\vec{i}$ denotes the iteration vector
- Function T: return the logical state of each statement

An Example: Loop permutation



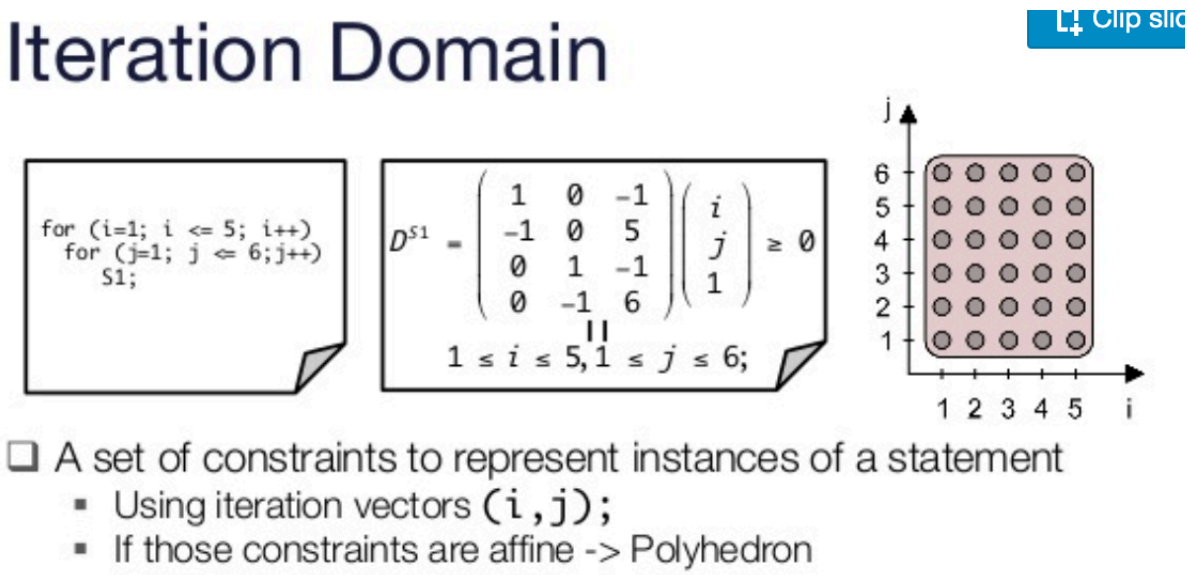
3 important things

- Domain: a set of instances for a statement
- Scattering (Scheduling): an instance -> time stamp (function T)
- Access: an instance -> array elements

Limitation

- Only applicable for Static Control Part (SCoP) in general -> Loop bounds and conditions are affine function of the surroundings the loop iterators.

Iteration Domain



- If those constraints are affine -> Polyhedron (limitation)

Legality