A Two-Stage Approach to Solve Voltage-Stability/Security-Constrained Optimal Power Flow Auction Systems by means of PSO and the Continuation Power Flow

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***Abstract*—Given the recent progresses in designing and implementing electricity market optimization-based tools to support power allocations that bridge the gap between supply and demand, the realistic modeling of optimal power flow-based auction systems provide transparent and higher transaction levels among market players, as well as more accurate locational marginal prices. Nevertheless, actual voltage stability/security representation in these auction systems remains an open question. Thus, this paper proposes a two-stage resolution framework for a nonlinear Voltage-Stability/Security-Constrained Optimal Power Flow (VSCOPF)-based auction system. The first stage of the proposed framework consists in solving a single-period two-sided VSCOPF-based auction system model by a Particle Swarm Optimization (PSO) algorithm, which adjusts active power allocations and generator voltage magnitudes while verifying if the grid's technical-operational constraints are satisfied by power flow calculations. The system voltage stability/security is addressed in the second stage by a maximal loading problem solved by the Continuation Power Flow method. Then, if a minimal voltage stability/security margin condition is not satisfied, active power allocations and generator voltage magnitudes are readjusted by the PSO to comply with stable/secure power allocations. Results with a 6-bus test-system are presented to illustrate this proposal and, therefore, show the effectiveness of proposing a two-stage resolution framework for VSCOPF-based auction systems.**

***Index Terms*—auction system, continuation power flow, optimal power flow, particle swarm optimization, voltage stability/security**

I. Introduction

The power sector has experienced major changes due to its process of deregulation around the world, and these changes are reflected in the design and mathematical modeling of electricity markets [1]–[3]. Auctions have been formally introduced in this sector as a transparent mechanism to attain fair, open, timely and competitive procurement processes as such a deregulation is implemented [4].

In addition to fostering competition in the electricity sector among market players (producers and purchasers), electricity auctions for energy procurement allow for efficient and cost-effective power dispatch [5]. In this context, the use of Optimal Power Flow (OPF)-based tools have become of utmost importance, ensuring power balance and that technical, operational and voltage stability/security limits are fulfilled. However, the proper representation of voltage stability/security in such auction models is still an open question [6].

In [7], voltage stability/security constraints in a single-period OPF-based auction system for short-term energy procurement in hybrid electricity markets are modeled based on a artificial neural network functional approximation of the voltage stability/security boundaries; this approach demands the calculation of maximal loading points in the voltage stability/security boundary for training and validating of the artificial neural network, which demands a high computational effort. [8] and [6] propose voltage stability representation by means of voltage stability indices, namely the minimum singular value of the power system Jacobian and the tangent vector norm of Power Flow (PF) solution manifold, respectively, in a Voltage-Stability/Security-Constrained Optimal Power Flow (VSCOPF) auction system for the short-term energy procurement in hybrid markets; however, such indices are difficult to predict given their nonlinear profile and assigning a general critical value to ensure minimal stability margins is too conservative and potentially lead to inappropriate price signals.

Many optimization techniques have been applied to solve

Optimal Power Flow (OPF) problems and can be divided into two groups: mathematical programming techniques and metaheuristics. Mathematical programming techniques are deterministic and usually use derivatives to determine the search direction to achieve the local optima. However, OPF consists of a class of large nonlinear and non-convex optimization problems in which more than one local optimum may exist. Furthermore, it is pointed out that the non-convex feasible region may still be disconnected [9]. Consequently, the use of stochastic methods that employ probabilistic rules to potentially achieve the global solution of optimization problems, namely metaheuristics, has gained popularity in order to overcome the limitations imposed by the inherent characteristics of OPFs to mathematical programming techniques. Among these metaheuristics, Particle Swarm Optimization (PSO) has shown good performance to solve OPF problems [10], [11], and it is used to address many complex optimization problems that are nonlinear, non-differentiable, and multimodal. One of its main advantages is the noticeable high convergence rate [12]–[14].

Therefore, the objective of this work is to propose a two-stage resolution framework for a nonlinear Voltage-Stability/Security-Constrained Optimal Power Flow (VSCOPF)-based auction system. The first stage of the proposed framework consists in solving a single-period two-sided VSCOPF-based auction system model by a Particle Swarm Optimization (PSO) algorithm, which adjusts active power allocations and generator voltage magnitudes while verifying if the grid's technical -operational constraints are satisfied by power flow calculations. The system voltage stability/security is addressed in the second stage by a maximal loading problem solved by the Continuation Power Flow (CPF) method. Then, if a minimal voltage stability/security margin condition is not satisfied, active power allocations and generator voltage magnitudes are readjusted by the PSO to comply with stable/secure power allocations. Results with a 6-bus test-system are presented to illustrate this proposal and, therefore, show the effectiveness of proposing a two-stage resolution framework for VSCOPF-based auction systems.

The remainder of this work is organized as follows. Section II presents the modeling of the VSCOPF auction system and the system of algebraic equations and inequations from which the maximal loading is determined by the CPF. Section III features the two-stage proposed framework to solve such a VSCOPF. In Section IV, the numerical results for a 6-bus test-system are presented to validate our proposal. At last, the main conclusions of this work are highlighted in Section V.

II. VSCOPF Auction System Modeling

The modeling of the VSCOPF auction system and the system of algebraic equations and inequations from which the maximal loading is determined are presented below.

A. VSCOPF-Based Auction System

The VSCOPF-based auction system considered in this paper aims at maximizing the social welfare, so that producers maximize their income from the allocation of supply bid blocks while purchasers minimize the prices paid for their allocated demand bid blocks, subject to power balance, supply and demand bid limits and technical, operational and voltage stability/security constraints. Power system voltage stability/security is ensured by a constraint based on an implicit function that represents the maximal loading problem and takes as its argument the allocated bid blocks and voltage magnitudes in generation buses. Thus, considering a transmission grid with n_B buses and n_T transmission branches, the single-period two-sided VSCOPF auction system is formulated as:

$$\max_{P_S, P_D, Q_G, \delta, V} \quad C_D^T P_D - C_S^T P_S \tag{1a}$$

$$\text{s.t:} \quad \Delta P(P_S, P_D, \delta, V) = 0, \tag{1b}$$

$$\Delta Q(P_D, Q_G, \delta, V) = 0, \tag{1c}$$

$$P_S^{\min} \leq P_S \leq P_S^{\max}, \tag{1d}$$

$$P_D^{\min} \leq P_D \leq P_D^{\max}, \tag{1e}$$

$$Q_G^{\min} \leq Q_G \leq Q_G^{\max}, \tag{1f}$$

$$V^{\min} \leq V \leq V^{\max}, \tag{1g}$$

$$\sigma(P_S, P_D, V_G) > 1, \tag{1h}$$

where $P_S \in \mathcal{R}_+^{n_B}$ is the nonnegative vector of supply power bid blocks in per unit (p.u.), with $P_S^{\min} \in \mathcal{R}_+^{n_B}$ and $P_S^{\max} \in \mathcal{R}_+^{n_B}$ representing the lower and upper supply bid blocks limits, respectively; $P_D \in \mathcal{R}_+^{n_B}$ is the nonnegative vector of demand power bid blocks in p.u., with lower and upper limits given, respectively, by $P_D^{\min} \in \mathcal{R}_+^{n_B}$ and $P_D^{\max} \in \mathcal{R}_+^{n_B}$; $Q_G \in \mathcal{R}^{n_B}$ is the vector of reactive power generation in p.u., with limits given by $Q_G^{\min} \in \mathcal{R}^{n_B}$ and $Q_G^{\max} \in \mathcal{R}^{n_B}$; $\delta \in \mathcal{R}^{n_B}$ is the vector of voltage phase angles in radians (for the slack bus is set to 0 rad.); $V \in \mathcal{R}_+^{n_B}$ is the vector of voltage magnitudes in p.u. with limits given by $V^{\min} \in \mathcal{R}_+^{n_B}$ and $V^{\max} \in \mathcal{R}_+^{n_B}$, where $V_G \subset V$ is the vector of voltage magnitudes on generation buses and $V_L \subset V$ is the vector of voltage magnitudes on load buses; $C_S \in \mathcal{R}_+^{n_B}$ and $C_D \in \mathcal{R}_+^{n_B}$ are, respectively, the vectors of supply and demand price bids in \$/MWh; $\Delta P : \mathcal{R}_+^{4n_B} \to \mathcal{R}_+^{n_B}$ and $\Delta Q : \mathcal{R}_+^{4n_B} \to \mathcal{R}_+^{n_B}$ are the vectors of active and reactive power balance, respectively; $\sigma : \mathcal{R}^{3n_B} \to \mathcal{R}_+$ is an adimensional implicit function defined in terms of P_S, P_D and V_G that represents the power system critical loading associated with voltage stability/security boundaries and imposes that supply and demand acceptance blocks allow minimum levels of security.

In Equation (1b) and Equation (1c), the active and reactive power balances in bus $k \in B$, where B is the set of all system buses, are:

$$\Delta P_k = P_{G_k} + P_{S_k} - (P_{L_k} + P_{D_k}) - \sum_{m \in \mathcal{K}} P_{km}(\delta, V), \tag{2}$$

$$\Delta Q_k = Q_{G_k} + (Q_{L_k} + K_{L_k} P_{D_k}) - \sum_{m \in \mathcal{K}} Q_{km}(\delta, V), \tag{3}$$

where $P_G \in \mathcal{R}_+^{n_B}$ represents the active power outputs of must-run generators in p.u. and, therefore, does not participate in market bidding; $P_L \in \mathcal{R}_+^{n_B}$ and $Q_L \in \mathcal{R}_+^{n_B}$ are the inelastic active and reactive loads in p.u. and do not take part in the market bidding; $K_L \in \mathcal{R}_+^{n_B}$ is a vector of adimensional parameters used to model the increase in reactive load in terms of P_D to maintain a constant power factor; $\mathcal{K}$ is the set of buses indexes connected to the $k \in B$ bus plus the k bus itself; and P_{km} and Q_{km} are the active and reactive power flows through transmission branch $\{k, m\} \in T$, where T is the ordered set of bus index pairs representing a transmission line or a transformer connecting one bus to another, given by:

$$P_{km} = V_k V_m (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km}), \tag{4}$$

$$Q_{km} = V_k V_m (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}), \tag{5}$$

where G_{km} and B_{km}, are respectively the real and imaginary parts of the element Y_{km} in the bus admittance matrix; and $\delta_{km} = \delta_k - \delta_m$.

In (1), the objective function (1a) is the social welfare. Constraints (1b) and (1c) are the PF equations. Constraints (1d) and (1e) limits the accepted power supply and demand bid blocks. Constraint (1f) denotes the limits of reactive power generation. Constraint (1g) limits the voltage magnitudes in all buses. At last, constraint (1h) is the voltage stability/security constraint defined by an implicit function of P_S, P_D and V_G. This function represents the system maximal loading level with allocated supply and demand bid blocks and ensure that the accepted bid blocks allow minimal stability/security margins. The problem formulation to determine such a loading level is presented in the following.

B. Power System Maximal Loading Problem

The voltage-stability/security constraint (1h) is an implicit function in therms of P_S, P_D and V_G, which associates the system loading level with the state of the system at that point. It claims that the maximal (critical) loading σ must be bigger than 1, imposing that supply and demand blocks allow minimum levels of stability/security. The maximal loading is, thus, determined by the CPF, which consists in successively solving the following algebraic system of equations and inequations in a parameterized fashion (since, for n equalities, there are $n+1$ variables) [15]:

$$\Delta P^c(P_S, P_D, \sigma, K_G, \delta^c, V^c) = 0, \tag{6a}$$
$$\Delta Q^c(P_D, \sigma, Q_G^c, \delta^c, V^c) = 0, \tag{6b}$$
$$Q_G^{\text{min}} \leq Q_G^c \leq Q_G^{\text{max}}, \tag{6c}$$
$$V^{\text{min}} \leq V^c \leq V^{\text{max}}, \tag{6d}$$
$$I^c(\delta^c, V^c) \leq I^{\text{max}}, \tag{6e}$$

where the superscript c is used to differentiate functions and dependent variables in the auction system VSCOPF (1) from those in maximal loading problem (6); K_G is an adimensional scalar used to represent a distributed slack bus (it is assumed that the active power losses in the load flow calculation are distributed among all generators in proportion to their respective power injections); $I : \mathcal{R}^{2n_B} \to \mathcal{R}_+^{n_B}$ is the vector of current line magnitude in p.u. in transmission branches, whose limits are given by $I^{\text{max}} \in \mathcal{R}_+^{n_T}$ in p.u. (thermal limits); and P_S, P_D and V_G are the parameters in (6) determined in the VSCOPF auction system (1).

The maximal (critical) loading point in (6) is associated to the voltage magnitude, thermal and voltage stability limits.

In Equation (6a) and Equation (6b), respectively, the active and reactive power balances for the $k \in B$ bus are:

$$\begin{aligned}\Delta P_k^c =& (\sigma + K_G)(P_{G_k} + P_{S_k}) - \sigma(P_{L_k} + P_{D_k}) - \\ & \sum_{m \in \mathcal{K}} P_{km}(\delta^c, V^c)\end{aligned} \tag{7}$$

$$\Delta Q_k^c = Q_{G_k}^c - \sigma(Q_{L_k} + K_{L_k} P_{D_k}) - \sum_{m \in \mathcal{K}} Q_{km}(\delta^c, V^c) \tag{8}$$

III. Proposed Approach

This section presents the proposed two-stage framework to solve the proposed VSCOPF-based auction system by a PSO algorithm and the maximal loading problem by the CPF method. Furthermore, the proposed approach flowchart is summarized in Figs. 1 and 2.

A. Particle Swarm Optimization

The PSO is a population-based search method, in which there is a simulation of the social behavior of birds within a flock [16]. In it, individuals designated as particles follow a very simple behavior: imitating the success of neighboring individuals and their own successes, where each particle represents a potential solution [17]. It is initialized with a random group of particles and they move to seek the ideal position with the best solution for the objective function.

In the solution space, each particle occupies a given position and moves at a certain speed, using its own experience, in addition to the experience of all particles. This new position is a random combination of its previous speed and the current position, where, in each iteration, the position of each particle is updated by the best fitness value, namely P_{best}. Among these P_{best}, each particle knows the best value in the group, namely G_{best} [18], [19]. After a certain number of iterations, search points are expected to reach the global optimum.

The updated speed and position of the particles are determined according to (9) and (10):

$$\begin{aligned}V_i^{k+1} =& W * V_i^k + c_1 * rand_1 * (P_{best_i} - X_i^k) \\ & + c_2 * rand_2 * (G_{best_i} - X_i^k)\end{aligned} \tag{9}$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{10}$$

where V_i^k is the current velocity of particle i in iteration k; V_i^{k+1} is the updated particle speed in iteration $k+1$; W is the weight of inertia and it is used to control the impact of the previous speed story, $rand_1$ and $rand_2$ are random numbers between 0 and 1; P_{best_i} is the best value found by particle i up to iteration k, G_{best_i} is the best particle i found in the group up to iteration k; X_i^k is the current position of particle i in iteration $k+1$; X_i^{k+1} is the current position (modified search point) of particle i in iteration $k+1$; c_1 and c_2 are acceleration constants and represent the weighting factors of the acceleration terms that pull each particle to positions P_{best} and G_{best}.

B. Continuation Power Flow

The CPF is an algorithm based on the successive resolution of the PF problem reformulated to include the loading parameter σ as in (2) and (3). With the addition of this parameter, the PF equations are solved in a parameterized fashion as a function of the increase of the loading parameter [15].

Thus, it is possible to find the maximal (critical) loading point (σ), which characterizes the voltage collapse point. At this point, the stable and unstable equilibrium points merge, and this merging is mathematically represented by a bifurcation (either a saddle-node or a limit-induced bifurcation). From that point on, the set of the PF equations no longer has solutions [15].

The advantage of using the CPF over direct methods (optimization-based) as a way to determined the maximal loading is, besides being based on standard PF algorithms, to obtain additional information regarding the PF solution manifold accounting for technical and operational limits towards the maximal point.

C. Approach Operation

The proposed framework is introduced in this subsection. First, the PSO is deployed to solve the VSCOPF and obtain the values for its decision variables that will, in the next stage, be used as parameters in the maximum loading problem. Fig. 1 describes the steps for solving the VSCOPF-based auction system.

In the first four steps, the PSO parameters are provided to solve the auction system and obtain the supply and demand bid allocations and the voltage magnitude in generation buses. Then, the PF is calculated to determine the social welfare. Thus, it is obtained the vectors of all dependent and independent variables and, therefore, the state associated with the best fitness is determined.

The Fig. 2 describes the maximal loading problem solved by the CPF method.

After obtaining the results from the VSCOPF-based auction system by the PSO algorithm, the vector with the best fitness is used as parameter in the maximal loading problem. Then, the critical loading value is determined by the CPF method. If a minimal voltage stability/security margin condition is not satisfied, active power allocations and generator voltage magnitudes are readjusted in another round of execution of the first stage to comply with stable/secure power allocations. This process continues while any constraint in the VSCOPF-based auction system is not met.

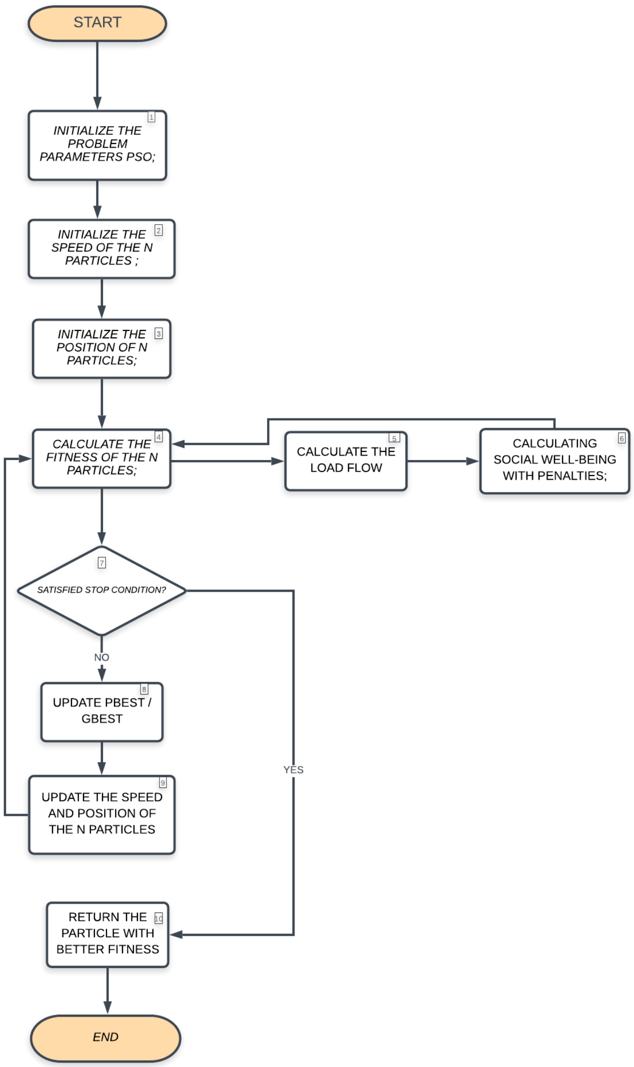

Fig. 1. Operation of the Proposed Approach of the PSO Auction System.

IV. EXPERIMENTS AND RESULTS

This section presents numerical results and discussions to validate the proposed VSCOPF-based auction system and the CPF by means of a 6-bus test-system, where the bus, branch and security data are presented in [5].

A. Case Study

In this case study, the proposed framework is deployed to analyze active power allocations by VSCOPF-based auction system when the voltage stability/security boundary is defined by the binding lower voltage magnitude limit in bus 5. In this analysis, the power base S^{base} is 100 MVA, the voltage base V^{base} is 500 kV and the upper and lower limits of voltage magnitudes are 0.90 p.u. and 1.10 p.u., respectively.

The parameters of the PSO in this analysis are: number of particles ($n = 70$), weight of linearly decreasing inertia ($w0 = 1.0$ and $wf = 0.4$), cognitive acceleration component and social acceleration component ($c1 = 2.0$ and $c2 = 2.0$),

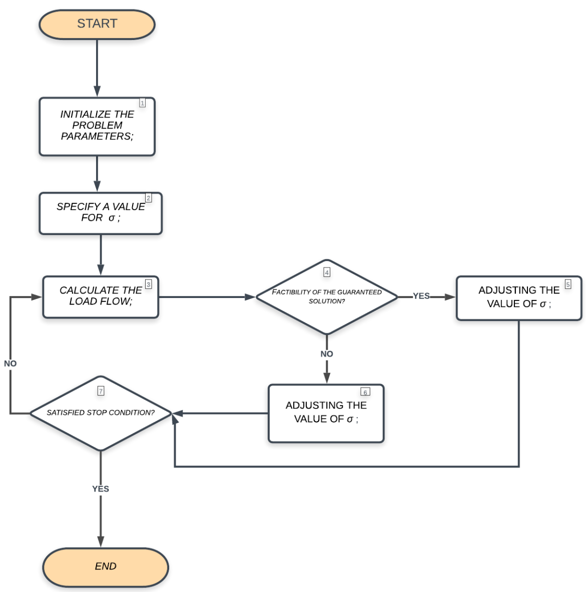

Fig. 2. Operation of the Proposed Continuation Power Flow Approach.

number of executions (30), error tolerance for the fitness function (10^4), number of iterations and network star topology. The parameters of the CPF in this analysis are: error tolerance for PF convergence (10^{-6}).

The statistical analyses that are performed are: calculation of the average of the executions for the objective function and decision variables, calculation of the mode for the values of the objective function and calculation of the standard deviation of the values of the objective function.

The results below are compared to the ones in [8]. Table I shows the results of the best solution for the applied meta-heuristic and the above mentioned parameter settings.

TABLE I
RESULT OF THE BEST SOLUTION FOR THE STUDY CASE

Participant[k]	P_S[MW]	P_D[MW]	V[p.u.]	Q_G[MVar]
1	0.000000	-	1.100000	44.7849
2	25.000000	-	1.100000	77.2048
3	20.000000	-	1.100000	73.9445
4	-	25.000000	1.021081	-
5	-	10.000000	1.012586	-
6	-	8.124160	1.038778	-
Social Welfare [$]:122.1795240452				

Fig. 3 shows how random start particles converge towards the best solution, reaching a point of stagnation where all particles remain at that point.

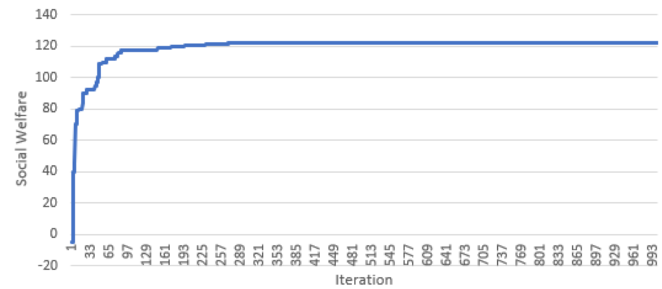

Fig. 3. Best Solution with PSO.

Fig. 4 presents the evaluation of executions. Notice that the standard deviation reaches zero when the particles converge to the same solution.

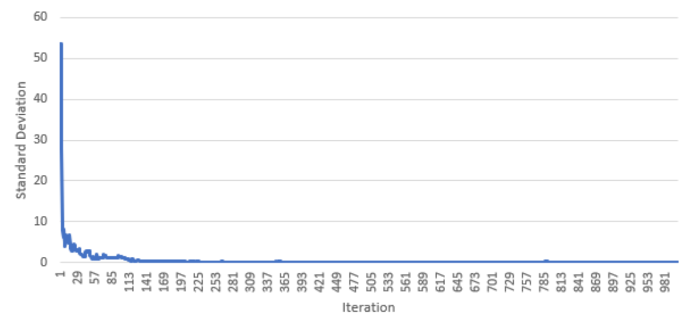

Fig. 4. Statistical Analysis of the Standard Deviation of Executions.

Fig. 5 features the average of the evaluations, showing that the particles always tend to converge to the best solution, following the behavior of their neighbors.

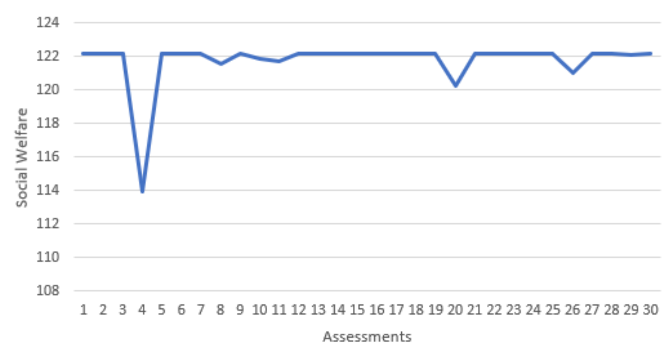

Fig. 5. Statistical Analysis of Average Executions.

In the best solution, $\sigma = 1.66885$, and it is possible to observe that bus 5 reached its lower voltage magnitude limit (0.90 p.u.), characterizing the maximal loading. The system state at the maximal loading is shown in Table II, and the PV curves determined by the CPF are presented in Fig. 6.

TABLE II
SYSTEM STATE AT THE MAXIMAL LOADING ($\sigma = 1.66885$)

Bus	V (p.u.)	δ (deg.)	P_G (MW)	Q_G (Mvar)
1	1.1	0	156.22	140.17
2	1.0708	-0.79437	286.4	150
3	1.0686	-3.3798	138.86	150
4	0.9264	-6.4267	0	0
5	0.9000	-8.7959	0	0
6	0.9507	-7.9688	0	0
K_G = 0.0669				

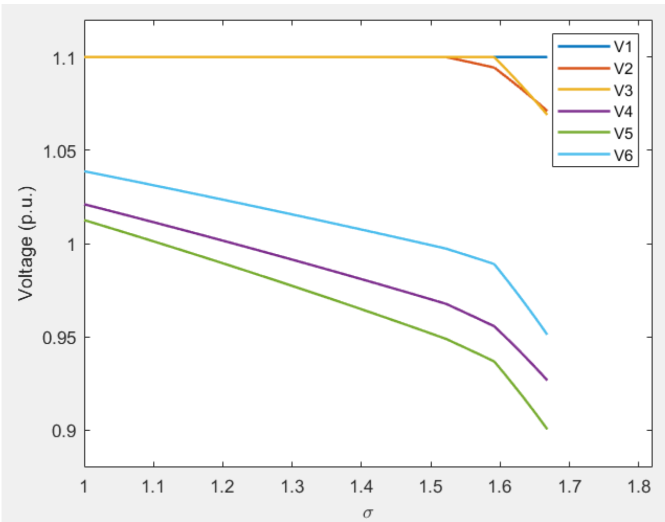

Fig. 6. PV curves with stable/secure equilibrium points for the 6-bus test-system.

V. CONCLUSIONS

This work presented a two-stage framework to account for voltage stability/security in OPF-based auction systems for the short-term energy procurement in hybrid electricity markets. The featured framework is based on the application of the PSO algorithm and the CPF method, which, in turn, are entirely based on PF calculations. As a result, the proposed framework provided stable, secure and accurate power allocations for supply and demand when compared to results in the correlate literature, and for the presented case study, this framework may be deemed effective and robust. Future work will consider different case studies in which voltage stability/security is associated with thermal limits and saddle-node and limit-induced bifurcations, as well as larger power systems.

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