First assignment

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Let F define the map

$$F: \begin{array}{ccc} [-\pi,\pi]\times[-\pi,\pi] & \longrightarrow & [-\pi,\pi]\times[-\pi,\pi] \\ \begin{pmatrix} x \\ y \end{pmatrix} & \longmapsto & \begin{pmatrix} x + a\sin(x+y) \\ x+y \end{pmatrix} \end{array},$$

where a is a constant, and we take modulo 2π to ensure the image lies in the square again (this means that the function maps the torus to itself). Study the dynamical system defined by this map.

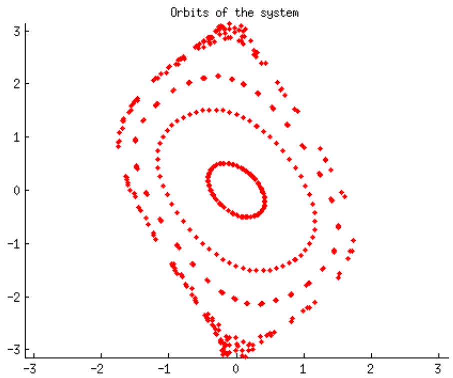
1. Simulate the dynamics taking 100 iterates for each initial condition. Try different initial conditions. Then, modify the code such that the program takes N initial conditions on the x-axis. Plot the results.

We have a MATLAB function *map.m* that computes the map for us anytime we need. We use it at the script *test1.m*, that generates any number of orbits that the user wishes, with the initial conditions that one gives explicitly. Observe that the code asks for values of a and the number of orbits, but has default values. Anytime this scripts ask for a parameter that takes a default value, the user can just press Enter and the default value will be used. We always take $a = -0.7$ and number of iterates $= 100$.

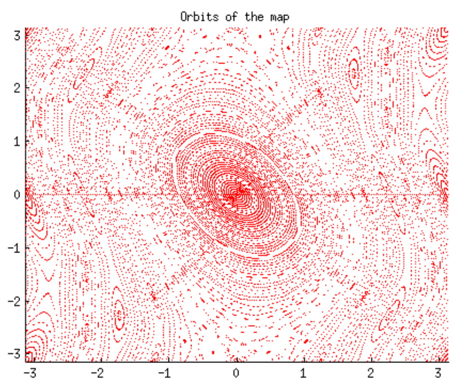
We runned *test1.m*, and took the following five initial conditions (for no particular reason):

$$x_1 = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}, x_2 = \begin{pmatrix} -0.3 \\ 3 \end{pmatrix}, x_3 = \begin{pmatrix} 0.1 \\ 2 \end{pmatrix}, x_4 = \begin{pmatrix} 0.1 \\ -0.5 \end{pmatrix}, x_5 = \begin{pmatrix} 0.1 \\ 3 \end{pmatrix},$$

and this gave us the plot



We also have the script *test2.m*, that plots the number of orbits we wish starting at N equidistributed points at the x-axis between $-\pi$ and π. The result taking 200 initial points, for instance, is



2- Find exact initial condition for a 2-periodic orbit

We can see that $x_0 = \begin{pmatrix} \pi \\ 0 \end{pmatrix}$ gives a 2-periodic point. This is because

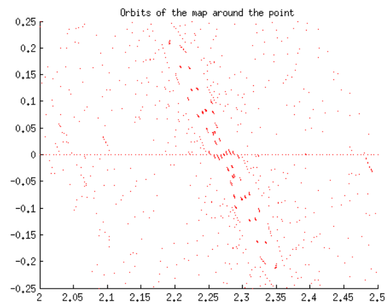
$$F\begin{pmatrix} \pi \\ 0 \end{pmatrix} = \begin{pmatrix} \pi - 0.7\sin(\pi) \\ \pi \end{pmatrix} = \begin{pmatrix} \pi \\ \pi \end{pmatrix},$$

$$F\begin{pmatrix} \pi \\ \pi \end{pmatrix} = \begin{pmatrix} \pi - 0.7\sin(2\pi) \\ 2\pi \end{pmatrix} = \begin{pmatrix} \pi \\ 0 \end{pmatrix}.$$

3. Find approximate initial condition for a 3-periodic orbit.

We prepared the script *zoom.m* to make more detailed plots of the system around certain plots. Using this, and the *test2.m*, we can get a detailed picture of the system around the "periodic islands".

For instance, if we take 100 orbits around the point $\begin{pmatrix} 2.25 \\ 0 \end{pmatrix}$ and a square of size 0.5×0.5, we get the plot



To have a more refined approximations, we prepared the function *mapk.m* and the script *findPeriods.m*, that allow us to find periodic points via solving the equation

$$F^k(x) - x = 0,$$

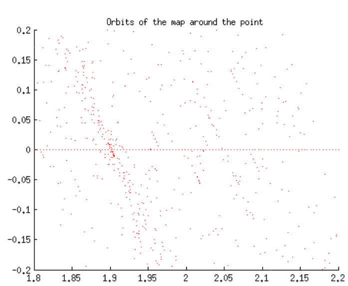
taking $k = 3$ in this case. We need a good initial approximation to ensure that our code converges to a real 3-periodic point, and here is where *zoom.m* proves to be useful.

In this case, taking the initial point $x_0 = \begin{pmatrix} 2.3 \\ 0 \end{pmatrix}$ as suggested by the plot, we find the approximate 3-periodic orbit

$$x_1 = \begin{pmatrix} 2.272589 \\ 0 \end{pmatrix}, x_2 = F(x_1) = \begin{pmatrix} 1.738008 \\ 2.272589 \end{pmatrix}, x_3 = F^2(x_1) = \begin{pmatrix} 2.272589 \\ -2.272589 \end{pmatrix}.$$

4- Find approximate initial condition for a 4-periodic point.

Using the same process and codes as before, we get a plot near the 4-periodic island, around the point $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$



And *findPeriods.m* gives us the 4-periodic orbit

$$x_1 = \begin{pmatrix} 1.901797 \\ 0 \end{pmatrix}, x_2 = F(x_1) = \begin{pmatrix} 1.239795 \\ 1.901797 \end{pmatrix},$$

$$x_3 = F^2(x_1) = \begin{pmatrix} 1.239795 \\ -\pi \end{pmatrix}, x_4 = F^3(x_1) = \begin{pmatrix} 1.901797 \\ -1.901797 \end{pmatrix}.$$