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W4415: Problem Set 5

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Problem 1

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Part (a):

Substitutes Game:

	H	M	L
H	(5, 5)	(0, 8)	(0, 6)
M	(8, 0)	(4, 4)	(0, 6)
L	(6, 0)	(6, 0)	(3, 3)

Complements Game:

	H	M	L
H	(5, 5)	(6, 3)	(10, 1)
M	(3, 6)	(4, 4)	(5, 2)
L	(1, 10)	(2, 5)	(3, 3)

In the substitutes case, H is a strictly dominated strategy for Colin. Therefore, Colin will never play H since he knows what state the game is in. On the other hand, H is a strictly DOMINANT strategy in the complements case, therefore Colin will ALWAYS play H since he has full knowledge of the current state of the game.

Part (b):

We find the expected payoff for Row Boat player given that Colin will play Medium, without knowing the current state of the game and assigning a probability of each state as ρ, $1-\rho$. We find that the cutoff values for each BR are given by:

1) Play L iff $\rho \geq 1/2$
2) Play H iff $\rho \leq 1/3$
3) Play M iff $1/3 < \rho < 1/2$

Part (c):

We follow the same operation as before, finding the BR for Row Boat given Colin's strategy

Profile and find:

1) Play L iff $\rho \leq 2/5$
2) Play H iff $\rho > 2/5$

While Colin plays H for complements and M for substitutes.
Above is the Pure Strategey Bayesian Nash Equilibrium for different values of ρ

Problem 2

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Part (a):
As we saw in class, this game has an equilibrium of (1/3 BB + 2/3 BF, 2/3 Call + 1/3 Fold), with an expected payoff for the professor of 1/3 and for the student of -1/3. We of course intuit that a higher number of queens will help bring the game to fairness under equilibrium. First, we derive the payoffs at mixed-strategy equilibrium in terms of p, the probability of getting a king, q_p mixing probability for the professor (bluff), and q_s mixing probability of the student (call). For this to be a fair game, we need:

$$(4p-2)q_s + (1-q_s) = (3p-1)q_s + (2p-1)(1-q_s)$$

From this we get q_s is always equal to 2/3. Solving for optimal strategy for Professor, we need to solve :

$$(2-4p)q_p + (1-3p)(1-q_p) = -q_p + (1-2p)(1-q_p)$$

Solving the above, we get:

$$q_p = \frac{p}{3-3p}$$

To solve for the optimal p value, we need to plug in the values of q_p and q_s in the expected utility equations given below and equating them to each other (Note, one will be the negative of the other) :

$$q_p q_s(4p-2) + q_p(1-q_s) + (1-q_p)q_s(3p-1) + (1-q_p)(1-q_s)(1-2p)$$
$$=$$
$$q_p q_s(2-4p) - q_p(1-q_s) + (1-q_p)q_s(-3p+1) + (1-q_p)(1-q_s)(-2p+1)$$

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Solving the above we get: p=3/8
Which is also that we have 3 kings and 5 queens.
Part (b):

Figure 1: Game Tree

B	C	F
BBB	0,0	1, -1
BBF	1/3, -1/3	1/3, -1/3
BFF	0, 0	-1/3, 1/3
BFB	-1/3, 1/3	1/3, -1/3
FFB	-4/3, 4/3	-1/3, 1/3
FBF	-2/3,2/3	-1/3, 1/3
FBB	-1, 1	1/3, -1/3
FFF	-1, 1	-1, 1

The pure strategy Nash equilibrium will be at (BBF, C) with expected payouts 1/3, -1/3 in equilibrium.

Thus the Professor's payoff will be 1/3.

Problem 3

Part (a):

Suspending belief of common rationality, we suppose that Nash would want to make sure he has positive utility, i.e. $\frac{3}{2}\theta - p \geq 0$. Since Nash knows that $\theta \sim U[0,1]$, he will make sure he has positive utility in expectation. Therefore,

$$3/2\mathbb{E}[\theta] \geq p$$
$$p \leq \frac{3}{4}$$

Therefore the highest price Nash would bid is 3/4. This offer will also be accepted, so he will secure a non-negative utility.

Part (b):

Reinstating rationality, so that Nash wants to maximize his utility, he will make the LOWEST offer that he knows Akerlof will accept. Since both players face uncertainty, Nash will make an offer that gives Akerlof non-negative utility in expectation. Since the expectation of θ will be 1/2, then a price of 1/2 will give Akerlof 0 utility in expectation. Akerlof therefore accepts and Nash enjoys a utility of 1/4.

Part (c):

Now that Akerlof KNOWS the true quality of the car, we can immediately see he will accept prices that are greater than or equal to θ. From Nash's perspective, Akerlof's acceptance of his offer implies the quality of the car must be less than or equal to what he offered in the first place! Nash then infers that $\theta|\text{accept} \sim [0, p]$, which will have expectation $p/2$, taking Nash's expected utility to $\frac{3}{4}p - p$, which will always be negative. Any accepted offer would give Nash a negative expected utility, so he is better off offering 0 (which will never be accepted if we assume that the probability of θ being exactly 0 is 0).

Part (d):

The smog test works as a lower bound on θ, while the accepted offer, continues to be the upper bound. This results in $\theta|\text{accept} \sim U[1/4, p]$. In expectation, Nash's utility will be

$$\frac{3}{2}\frac{(p - 1/4)}{2} - p = \frac{3p}{4} - p - \frac{3}{16}$$
$$\frac{p}{2} - \frac{3}{16} = \mathbb{E}\left[U(p)|\text{accept}\right]$$

Therefore, Nash will still have a negative payoff for ANY accepted offer. Indeed, he will still optimally offer 0.

Problem 4

In the world of online dating, everything is a signal. Each picture chosen, funny quote or opening line– they are all meant to convey something about an individual's potential as a mate.

In this particular case we will consider a world where we expect all players to have a desire to be matched with someone of "high attractiveness" (type H, or hot), over those of "low attractiveness" (type N, or not). Attractiveness, of course, expands beyond physical attributes, so conveying it through an online dating profile is not immediate. Therefore, each player has uncertainty about the other player's type, but not about their own. At the same time, "putting yourself out there" has non-zero cost, which we shall denote c_L, and c_R is the cost of actually meeting someone in real life ($c_R > c_L$). In most dating applications, the game is simply to choose to match or not (swipe left or right). Nature plays first and assigns both player their type, Player 1 and 2 choose simultaneously to swipe left or right.

We denote the player type under uncertainty be a random variable θ which can take the values 1 and 0 for High and Low types respectively. The payoff is then for both players (if they both swipe right):

$$U_i(\theta_i|RR) = 2\theta_{-i} - 1 - \alpha$$

$$U_i(\theta_i|LR, LL, RL) = -\beta$$

While the utility of not matching is simply the cost β.

Initially, we consider $\theta \sim$ Bernoulli(0.5) random variable. This is our prior distribution. In this case, it would never make sense to swipe right since we have a condition on $\alpha > \beta$. This would be true of an environment where getting either is 50/50 odds.

We now consider the utility of a player who faces $\theta \sim$ Bernoulli(0.9) but is N type. $0.8 - \alpha \geq \beta$ would give him a positive utility in expectation conditional on the other player swiping right. But what if the player is insecure, and considers that his signals are counterproductive, so $\theta_i \sim$ Bernoulli(0.1). Then his point estimate of the other player's type will also shift towards 0, since it would only be profitable for N type player to swipe right on him.