GEOPH 431-531 - Inverse Problems

Assignment:	4
Assigned	March 27, 2023
Due:	April 10, 2023
Topic:	Fourier Reconstruction using Fourier Matrices and Operators

See Chapter 7 of my notes and ISTA notes

1) Write a program to show that you can replace the DFT matrix and its Hermitian transpose matrix by `fft` and `ifft`, respectively. Show that the operators pass the dot product test. Be careful with the normalization of the `fft` and `ifft`. It would be best to multiply them by scaling factors to make them behave like the DFT matrix and its hermitian transpose (`help ifft` and `help ifft` will make things clear for you). I am asking you to convince yourself via the dot product test that the implicit-form operators work as if you have explicit matrices.

2) The data `data_to_reconstruct.mat` contains the time and values of a complex time series. The original data contained 512 samples acquired every 1 unit. The data contain two erasures that you need to reconstruct. For this purpose, use Fourier sparse reconstruction. You can use a DFT (FFT) of 512 Fourier coefficients. It would help if you designed the sampling matrix first. The latter is quite simple. Please, look at the times I have provided, and you will see what is missing. The standard error of the noise is $\sigma = 0.05$.

a Write a program to reconstruct the data using the IRLS ($l_2 - l_1$) algorithm. First, write a code that uses DFT matrices. Use the program to estimate the Fourier coefficients and then use the Fourier coefficients to reconstruct the complex time series. Plot the absolute value of the Fourier coefficients (Spectrum) versus discrete frequency indices k. Also, plot the real part of the signal versus time. I suggest you plot the original signal versus time (to visualize the erasures) and the reconstructed signal versus time. I have also provided `sigma` (the standard error of the noise) to tune the trade-off parameter μ if needed.

File in eclass: `data_to_reconstruct.txt` (three columns `.txt` file)
File in eclass: `data_to_reconstruct.mat` (use `load data_to_reconstruct.mat`)

b Write a new code using an IRLS algorithm that calls a CGLS solver. The CGLS solver replaces the DFT and IDFT matrices with `fft` and `ifft`, respectively. Plot the absolute value of the Fourier coefficients (Spectrum) versus discrete frequency indices k and the real part versus time. I suggest that you plot the original signal versus time (to visualize the erasures) and the reconstructed signal versus time.

c Repeat the inversion and reconstruction using ISTA. Careful of how you define the threshold operator for complex coefficients.