$S = \{p \in \mathbb{R}[x] : p(\alpha) = 0\}$ con $\alpha \in \mathbb{R}$, α valor fijo

son sev de $\mathbb{R}[x]$

SEV de $\mathbb{R}^3$:
- $\vec{0}$, $\mathbb{R}^3$ } triviales
- planos x el origen
- rectas x el origen

dominio, codominio

③ $T : M_{3\times3} \to \mathbb{R} \,/\, T(A) = \det(A)$ es una $\boxed{\text{t. lineal}}$ F

$\forall u, v \in V$
- $T(u+v) = T(u) + T(v)$
- $T(\alpha v) = \alpha T(v)$;

$\to |A+B| = |A| + |B|$ ✗

$|AB| = |A|\,|B|$ ✓

$$T\begin{pmatrix} \alpha a & \alpha b & \alpha c \\ \alpha d & \alpha e & \alpha f \\ \alpha g & \alpha h & \alpha i \end{pmatrix} \neq \alpha^3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

④ Base $\boxed{\text{conj. LI + generador}}$

dim = # base

$\to$ bases canónicas de $\mathbb{R}^n$, P_n, $M_{n\times n}$

dim = n, dim = n+1

$$\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix}\right\} \xrightarrow{b} M_{2\times2}$$

dim = 4

⑤ $A, B \in M_{\boxed{5\times5}} \,/\, AB$ es inv $\Rightarrow$ A es inv

A es inv si $\exists B \,/\, AB = I$

A es inv $\Leftrightarrow |A| \neq 0$

$|A.B| \neq 0$

$|A|.|B| \neq 0$

$\neq 0 \quad \neq 0$

⑥ $W = \{ p \in \mathbb{R}[x] \mid p(1) = 0 \}$ es un sev de $\mathbb{R}[x]$

- $W \subset \mathbb{R}[x]$ ✓
- $\vec{0} \in W$ ✓
- $u + v \in W$ ✓
- $\alpha u \in W$ ✓

vector nulo $\mathbb{R}[x]$

$p(x) = ax^n + bx^{n-1} + \dots + cx + d$

¿ $p(x) = 0 \in W$?

como $p(1) = 0 \Rightarrow \underline{\vec{0} \in W}$ ✓

- $p(x) = ax^n + bx^{n-1} + \dots + cx + d$ / $q = ex^n + fx^{n-1} + \dots + gx + h$

$p(1) = 0$ / $q(1) = 0$

$p(1) = a + b + \dots + c + d = 0$ / $q = e + f + \dots + g + h = 0$

$\rightarrow \underline{(p+q)(x) \in W}$?

$(p+q)(x) = (a+e)x^n + \dots + (c+g)x + d + h$

$(p+q)(1) = a + e + \dots + c + g + d + h$

$(p+q)(1) = \underbrace{a + c + d + \dots}_{0} + \underbrace{f + g + h}_{0} = 0$

$\Big\}$ $\underline{(p+q) \in W}$

$\underline{(\alpha p)(x) \in W}$?

$(\alpha p)(x) = \alpha a x^n + \dots + \alpha c x + \alpha d$

$(\alpha p)(1) = \alpha a + \dots + \alpha c + \alpha d$

$= \alpha \underbrace{(a + b + \dots + c + d)}_{0} = 0$

$\Big\}$ $\underline{\alpha p \in W}$

Ex. Dic 2019

Ej ①. SCD $\longrightarrow$ 1 única sol.

SCI $\longrightarrow$ ∞ sol.

SI $\longrightarrow$ $\nexists$ sol.

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 1 & 2 & k & 9 \\ 4 & k & 10 & 6 \end{array}\right) \sim \dots \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & k-8 & 6 & -2 \\ 0 & 0 & k-1 & 7 \end{array}\right)$$

Discuto según k: $k \in \mathbb{R}$

si $k=8$:

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 6 & -2 \\ 0 & 0 & 7 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 6 & -2 \\ 0 & 0 & 0 & \boxed{56} \end{array}\right)$$

S. Incomp.

$k=1$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -7 & 6 & -2 \\ 0 & 0 & 0 & \boxed{7} \end{array}\right)$$ S. Incomp.

$k=2$:

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -6 & 6 & -2 \\ 0 & 0 & 1 & 7 \end{array}\right)$$ SCD

②

$$A = \begin{pmatrix} \alpha_1 & & \\ 0 & \ddots & \\ 0 & \cdots\ 0 & \alpha_n \end{pmatrix} \qquad \det(A) = \alpha_1 \cdots \alpha_n.$$

$$|A+B| \neq |A| + |B|$$

Prop:

$$D = \left|\begin{array}{c|c} A & B \\ \hline 0 & C \end{array}\right| \qquad A, B, C \text{ matrices.}$$

$$|D| = |A|\,|C|.$$

(9) $\{P_1, P_2, P_3, P_4\}$ es LI o LD?

Planteo $\lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3 + \lambda_4 P_4 = \vec{0}$

$\boxed{SCD}$

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$

$\Rightarrow$ LI

$\boxed{SCI}$

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$

pero hay más sol

$\Rightarrow$ LD.

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 6 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$6\lambda_3 + 6\lambda_4 = 0 \rightarrow \boxed{-\lambda_3 = \lambda_4}$

$2\lambda_2 + 2\lambda_3 = 0$

$\boxed{\lambda_2 = -\lambda_3}$

$\lambda_1 + \lambda_3 = 0$

$\boxed{\lambda_1 = -\lambda_3}$

SCI $\Rightarrow$ LD

grados de libertad: $n - P = 1$

n: incógnitas

P: escalones

el vector asociado a λ_3 es comb. lineal de los demás $\Leftarrow$ $\lambda_3 \in \mathbb{R}$