Ej ①

$\mathcal{B} = \{v_1, v_2, v_3\} \xrightarrow{b} V$

$\{v_1, v_2+v_3, v_3\} \xrightarrow{b} V$

$T(v_1) = v_1 + v_2 - v_3$

$T(v_2+v_3) = -v_1 + 5v_2$

$T(v_3) = v_1 + 2v_2 + 3v_3$

$${}_B(T)_B = \begin{pmatrix} | & | & \\ \text{coord}_B T(v_1) & \text{coord}_B T(v_2) & \text{coord}_B T(v_3) \\ | & | & \end{pmatrix}$$

$${}_B(T)_B = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 3 & 2 \\ -1 & -3 & 3 \end{pmatrix}$$

$T(v_2+v_3) = T(v_2) + T(v_3)$

$\boxed{T(v_2) = -2v_1 + 3v_2 - 3v_3}$

Teorema:

$\dim N(T) = \dim N({}_B(T)_B)$

$\rightarrow v \in N({}_B(T)_B) \Longleftrightarrow {}_B(T)_B \cdot v = \vec{0}$

$\rightarrow v \in N(T)_B \ldots [T]_B \cdot v = 0$

$$v = (x, y, z) \quad \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & 3 & 2 & 0 \\ -1 & -3 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & -5 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\dim N(T)$

gr de libertad: $\underset{3}{n} - \overset{2}{p} = 1$

SÍ FUERA LD

$5y + z = 0$

$\boxed{z = -5y}$

$x - 2y - 5y = 0$

$\boxed{x = 7y}$

$\boxed{y \in \mathbb{R}}$

$$\left.\right\} \quad (7y, y, -5y)$$

$$y(7, 1, -5)$$

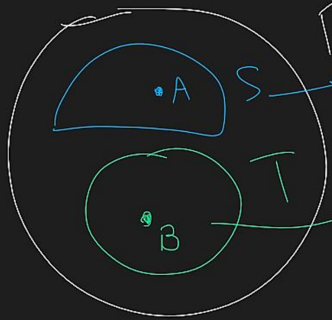
$$N(T) = [(7, 1, -5)]$$

$\hookrightarrow \dim N(T) = 1$

$S+T = \{ v \in V : \exists s \in S \text{ y } \exists t \in T / v = s+t \}$

Ej (4)

$\mathbb{R}^4 \Rightarrow \dim \mathbb{R}^4 = 4$



$S \to \dim S = 2$

$T \to \dim T = 2$

$\dim S+T = \dim S + \dim T - \dim S \cap T$

$$\boxed{\begin{array}{l}\textbf{Teorema:} \\ \left.\begin{array}{l} A \xrightarrow{g} S \\ B \xrightarrow{g} T \end{array}\right\} \Rightarrow A \cup B \xrightarrow{g} S+T \end{array}}$$

$A \downarrow g\ S$ $B \downarrow g\ T$

→ la unión de las bases de los sev es generador de la suma

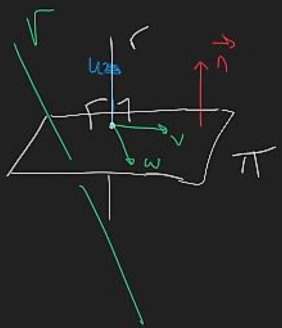
$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\boxed{\dim S+T = 3}$$

$$n - p = 2$$

$n = 4$, $p = 3$

⑥



$$r)\begin{cases} y-az=2 \\ ax+z=1 \end{cases} \xrightarrow{\text{paramétrica}} \boxed{z=t} \qquad ax+t=1 \qquad \boxed{x=\frac{1-t}{a}} \qquad \boxed{y=2+at}$$

$$\pi)\begin{cases} x = 2+2\mu-\lambda \\ y=-1+\mu \\ z=3+\lambda \end{cases}$$

pto de paso: $(2,-1,3)$
$\vec{v}=(2,1,0)$
$\vec{w}=(-1,0,1)$

$$r=\begin{cases} x=1/a-1/a\,t \\ y=2+at \\ z=t \end{cases}$$

$$\boxed{u=(-1/a,a,1)}$$

Paso π a
paramétri
resoluci

$$\pi:\begin{cases} x \\ y \\ z \end{cases}$$

n_π

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = (1,-2,1)$$

$$\| \downarrow$$

$$(-1/a, a, 1)$$

$$\boxed{a=-1} \neq \boxed{a=-2}$$

$$r \perp \pi \iff \langle u,v\rangle = 0 \text{ y } \langle u,w\rangle=0$$

$$\langle(-1/a, a, 1),(2,1,0)\rangle = 0$$

$$\hookrightarrow -\frac{2}{a}+a=0 \iff a=\frac{2}{a}$$

$$\Downarrow$$

$$a^2=2 \to \boxed{a=\sqrt{2}},\ \boxed{a=-\sqrt{2}}$$

$$\langle(-1/a, a, 1)(-1,0,1)\rangle=0$$

$$\hookrightarrow \frac{1}{a}+1=0 \Rightarrow \frac{1}{a}=-1$$

$$\hookrightarrow \boxed{a=-1}$$

Paso π de paramétrica a reducida:

$$\pi : \begin{cases} x = 2 + 2\mu - \lambda \\ y = -1 + \mu \\ z = 3 + \lambda \end{cases}$$

$$\pi : \begin{vmatrix} \overbrace{x-2 \quad y+1 \quad z-3}^{X - \text{pto de paso}} \\ 2 \quad\quad 1 \quad\quad 0 \\ -1 \quad\quad 0 \quad\quad 1 \end{vmatrix} = 0$$

$$= (x-2) - 2(y+1) + (z-3) = 0$$

$$\boxed{\pi : \; x - 2y + z - 7 = 0}$$

$\rightarrow n_{\pi} = (1, -2, 1)$

$\lambda = \sqrt{2}$

⑦ $T: V \to V \;/\; T \circ T = I$

I) T es biyectiva V

A, B matrices asoc.

$$T \circ T = Id$$
$$\Downarrow$$
$$AB = Id \Rightarrow T \text{ es invertible}$$
$$\Downarrow$$
$$T \text{ es biyec}$$

II) $\overbrace{N(T + Id)}^{S}$

$Id(v) = v$

III) A Mat. asoc

$$T \circ T = I$$
$$\boxed{A^2 = I}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\boxed{A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$