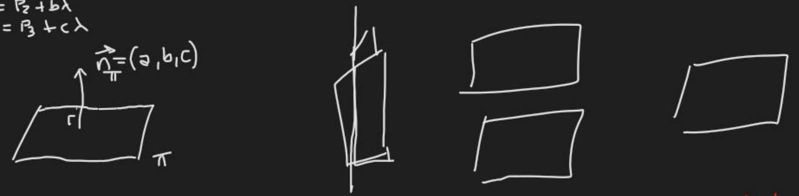
Rectas:

- Ec. vectorial: $r: (x,y,z) = (P_1,P_2,P_3) + \lambda(a,b,c)$
- Ec. reducida: $r = \begin{cases} ax+by=c \\ cy+dz=f \end{cases}$
- Ec. paramétrica: $r = \begin{cases} x = P_1 + a\lambda \\ y = P_2 + b\lambda \\ z = P_3 + c\lambda \end{cases}$

Plano: $\boxed{ax+by+cz+d=0}$ red.



Ej ③ $r_1 \cap r_2 = \{P\}$

$r_1 = (x,y,z) = (0,0,0) + \lambda(-1,1,0)$

$r_2 = (x,y,z) = (1,2,0) + \mu(1,0,0)$

$r_1: \begin{cases} x = -\lambda \\ y = \lambda \\ z = 0 \end{cases}$ | $r_2: \begin{cases} x = 1+\mu \\ y = 2 \\ z = 0 \end{cases}$

Igualo:

$x: \ -\lambda = 1 + \mu \rightarrow \boxed{\mu = -3}$

$y: \ \boxed{\lambda = 2}$

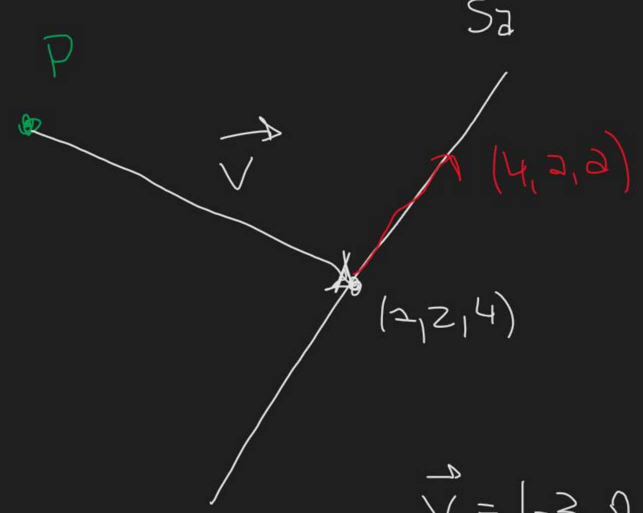
$z: \ 0 = 0$

sustituyo en r_1: $\boxed{P = (-2, 2, 0)}$

$S_a = (x,y,z) = (1,2,4) + t(4,a,a)$

$$\boxed{d(P, \underbrace{(1,2,4)}_{\text{pto de paso de } S_a}) = d(S_a, P)}$$

Hallar a



$\vec{v} = (-3, 0, -4)$

$\langle \vec{v}, (4,a,a) \rangle = 0$

$\langle (-3,0,-4), (4,a,a) \rangle = 0$

$-12 - 4a = 0 \Rightarrow \boxed{a = -3}$

⑥ Teorema:

Sean V, W ev. $T: V \to W$ t.l.

$A = \{v_1, \dots, v_n\} \xrightarrow{b} V$, $w_1, \dots, w_n$ arbitrarios.

Entonces:

Existe una única t.l tal que $T(v_i) = w_i \quad \forall i = 1, \dots, n$.

Obs: La transf. está bien definida (def en una base)

Ej.

Condiciones → es una base del dominio?
- → Sí → $\exists!$ t.l. (teorema)
- → NO
  - → $\nexists$ t.l (probar que no se cumple $T(\alpha u + v) = \alpha T(u) + T(v)$)
  - → $\exists\, \infty$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$-2\lambda_3 - \lambda_4 = 0$

$\to \boxed{\lambda_4 = -2\lambda_3}$

$\boxed{\lambda_2 = -\lambda_3}$

$\boxed{\lambda_1 = -\lambda_3}$

$\lambda_3 \in \mathbb{R}$

grados de libertad: $\underset{4}{n} - \underset{3}{p} = 1$

- Tomo $\boxed{\lambda_3 = 1}$

$\lambda_1 = -1$, $\lambda_2 = -1$, $\lambda_4 = -2$

$$(-1)(x^2+2) + (-1)(x^3+x^2+x+1) + (1)\boxed{(x^3-x+1)} + (-2)(x^3) = \vec{0}$$

$$T\big((-1)(x^2+2) + (-1)(x^3+x^2+x+1) + (-2)(x^3)\big) = T\big((-1)(x^3-x+1)\big)$$

$$(-1)\underbrace{T(x^2+2)}_{(1,1,2,0)} + (-1)\underbrace{T(x^3+x^2+x+1)}_{(3,2,1,0)} + (-2)\underbrace{T(x^3)}_{(3,2,1,0)} = (-1)\underbrace{T(x^3-x+1)}_{(1,1,2,0)}$$

$(-10, \quad) = (-1, \quad \dots, \quad)$ X

⇒ T no es lineal

Ej 4

$A = (a_{ij}) \in M_{5\times 5}$

fila   columna

$a_{i,j} = 0$ si $i > j$

A es tri diag sup.

$B = ($

$B|$

$$A = \begin{pmatrix} a_{11} & & \\ 0 & \ddots & \\ \boxed{0 \cdots 0 \; a_{55}} & & \end{pmatrix}$$

A es inv.

$\exists$ $B = (b_{i,j})$ es su inv.

$\Rightarrow AB = I_{5\times 5}$

$\longrightarrow b_{i,i} = -a_{i,i}$

$\longrightarrow \boxed{b_{i,i} = 1/a_{i,i}}$

$$\underbrace{a_{44} b_{44}}_{=1} + \underbrace{a_{45} b_{54}}_{=0} = 1$$

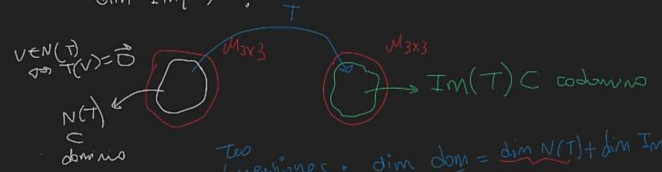
$$\longrightarrow \underbrace{a_{45}}_{=0} \; \underbrace{b_{54}}_{=-1} = 0$$

$$B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

(2) $T: M_{3x3} \to M_{3x3} \,/\, \boxed{T(A) = A + A^t}$

$\dim Im(T) = ?$



Teo dimensiones: $\underbrace{\dim dom}_{9} = \underbrace{\dim N(T)}_{3} + \dim Im(T)$ $\to \dim Im(T) = 6$

Hallar $N(T)$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in N(T) \Leftrightarrow T(A) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Leftrightarrow$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} + \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2a & b+d & c+g \\ d+b & 2e & f+h \\ g+c & h+f & 2i \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} a = 0 \\ b = -d \\ c = -g \\ e = 0 \\ f = -h \\ i = 0 \end{cases} \qquad \boxed{d, g, h \in \mathbb{R}}$$

PROP:

$$\underbrace{\dim dom}_{9} - \underbrace{\# cond}_{6} = \underbrace{\dim N(T)}_{3}$$

$$N(T) = \begin{pmatrix} 0 & -d & -g \\ d & 0 & -h \\ g & h & 0 \end{pmatrix}$$

$$N(T) = \left[\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \right]$$

$$\boxed{\dim N(T) = 3}$$