8/2

T. l → "expresión general"

→ matriz asoc. y las bases → $_B(T)_A = \begin{pmatrix} | \\ \text{coord}_B\, T(v_1) \\ | \end{pmatrix}$

→ $T(v_1) = w_1$

$T(v_2) = w_2$ $\quad \{v_1, v_2, v_3\} \xrightarrow{b} V$

$T(v_3) = w_3$

Teorema

V, W ev. $\quad A \xrightarrow{b} V \quad , \quad B \xrightarrow{b} W$

$T: V \to W$ t. l.

Entonces:

$$\text{coord}_B\, T(v) = {}_B(T)_A\, \text{coord}_A\, v \quad \forall v \in V$$

cod   cod   dom   dom

$v=(1,1,-3,-1) \quad T(v)=?$

$coord_A v=(\alpha,\beta,\gamma,\varepsilon)$

$(1,1,-3,-1)=\alpha(1,1,2,0)+\beta(1,0,0,-1)+\gamma(-2,-2,3,1)+\varepsilon(0,0,1,0)$

$$\begin{cases}\alpha+\beta-2\gamma=1\\ \vdots\end{cases} \sim \left(\begin{array}{cccc|c}1&1&2&0&1\\1&0&-2&0&1\\2&0&3&1&-3\\0&-1&1&0&-1\end{array}\right)\sim\dots$$

$$\boxed{coord_A v=(-1,0,-1,2)}$$

$$B(T)_A\, coord_A v=\boxed{(-2,2,3)}$$

$\downarrow$

$coord_B T(v)$

$$T(v)=-2(1,0,1)+2(0,1,1)+3(1,0,0)$$

$$\boxed{T(v)=(1,2,0)}$$

Ex 2do 2015

VoF ① $\exists$ sist lineales / $m > n$ que son incompatibles (V)

$$\left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{array}\right)$$

② $\{u, v, w\}$ es LI $\Rightarrow \{5u - v, 3w, w - v\}$ es LI (V)

$\Rightarrow \alpha(5u - v) + \beta(3w) + \gamma(w - v) = \vec{0}$

$\Rightarrow \underbrace{(5\alpha)}_{0} u + \underbrace{(-\alpha - \gamma)}_{0} v + \underbrace{(3\beta + \gamma)}_{0} w = \vec{0}$

$$\Rightarrow \begin{cases} 5\alpha = 0 \to \alpha = 0 \\ -\alpha - \gamma = 0 \to \gamma = 0 \\ 3\beta + \gamma = 0 \to \beta = 0 \end{cases} \Rightarrow \text{es LI}$$

③ $T: P_2 \to \mathbb{R}^3 / T(ax^2 + bx + c) = (2a, b, 1)$ (F)

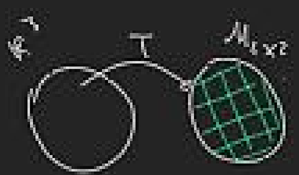
$\Rightarrow$ T es una t. lineal

$T(\alpha u + v) = \alpha T(u) + T(v)$

Si T es lineal $\Rightarrow T(\vec{0}) = \vec{0}$

④ $\exists\, T: \underset{\dim = 3}{\mathbb{R}^3} \to \underset{\dim = 4}{M_{2\times 2}}$ sobreyectiva (F)

$\hookrightarrow \boxed{\dim W = \dim Im(T)}$



Teo dim: $\underset{3}{\dim \mathbb{R}^3} = \underbrace{\dim Im(T)}_{4} + \dim N(T)$ X

⑤ $x + z = 1 \perp y + z = 1$ (F)

$n_{\pi_1} = (1, 0, 1)$ $n_{\pi_2} = (0, 1, 1)$

$\langle (1, 0, 1)(0, 1, 1) \rangle = 0$

$1 = 0$ X

⑥ $A, \underbrace{AB \text{ inv}}$ $\Rightarrow$ B es inv (V)

$|AB| \neq 0 \Rightarrow \underset{\neq 0}{|A|}\, \overset{\neq 0}{|B|} \neq 0 \to$ B es inv

$AB = I$

$\Downarrow$

$\underbrace{A A^{-1}}_{I} B = A^{-1} I$

$B = A^{-1} \Rightarrow$ B es inv.

7)

$$\begin{vmatrix} d & e & 3f \\ 2a & 2b & 6c \\ g & h & 3i \end{vmatrix} = ? \qquad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5$$

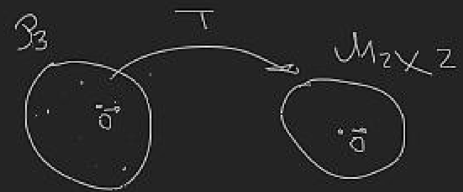
$$= 2.3 \begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -6 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -30$$

F

8) $T: P_3 \to M_{2\times 2}$ $\boxed{\text{inyectivas}}$ y otras que no

V

$\dim N(T) = 0$



Teo dim

$$\underset{4}{\underset{\shortparallel}{\dim P_3}} = \underset{0}{\underset{\shortparallel}{\dim N(T)}} + \underset{4}{\underset{\shortparallel}{\dim Im(T)}}$$

✓

9) $B \xrightarrow{b} V$

$B' \xrightarrow{b} W$

$T: V \to W$ t.l $\boxed{\text{INVERTIBLE}} \Rightarrow {}_{B'}(T)_B$ es INVERTIBLE

↓

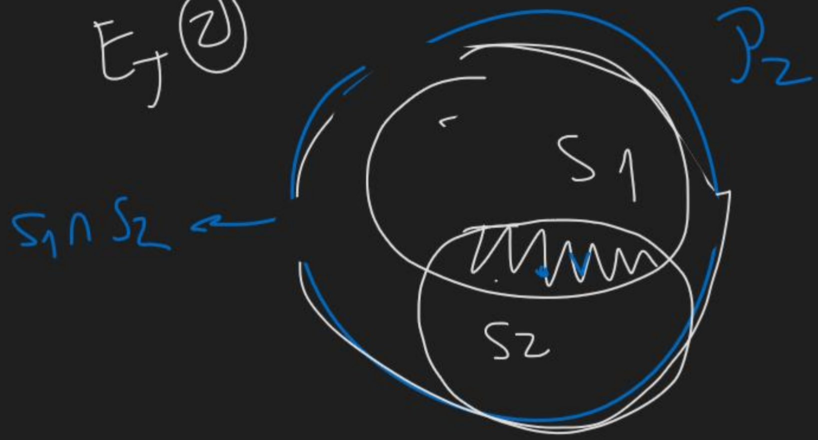
biyectiva

↓

"me llevan de base en base"

$$B \xrightarrow{b} V \Rightarrow T(B) \xrightarrow{b} W$$

Ej ②



$S_1+S_2 = \{ \exists s_1 \in S_1$ y $\exists s_2 \in S \,/\, v = s_1+s_2 \}$

$S_1 \oplus S_2$ es cuando esos s_1 y s_2 son únicos.

ⓐ $S_1 \cap S_2 = \{\vec{0}\} \iff \underline{S_1 \oplus S_2 = P_2}$

Teo: Si la suma es directa $\Rightarrow$ la unión de las bases de S_1 y S_2 es base de P_2

$\rightarrow \dim S_2 = 2$

$\rightarrow \dim S_1 = 3$

$\rightarrow \dim P_2 = 3$

$v \in S_1 \cap S_2 \iff v \in S_1$ y $v \in S_2$

③ ortogonal: $A^tA = I$

① A, B ortogonal $\Rightarrow$ $A+B$ ortogonal $\boxed{F}$

② $\quad$ " $\quad \Rightarrow AB$ " $\quad$ V

③ A, AB " $\Rightarrow$ B " $\quad$ V

$A^tA = I$

$B^tB = I$

$(A+B)^t(A+B) = I$?

$(A^t + B^t)(A+B) = I$.

$$\underbrace{A^tA}_{I} + A^tB + B^tA + \underbrace{B^tB}_{I} = I$$

$$A^tB + B^tA \overset{?}{=} I$$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$A+B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$$\underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}_{(A+B)^t}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \neq I$$

② AB ortogonal

$\Updownarrow$

$$\underbrace{(AB)^t}AB = I \iff B^t\underbrace{A^tA}_{I}B = I \iff \underbrace{B^tB}_{I} = I \quad ✓$$