VoF

① $T: V \to V$ t.l. y $\exists v \neq 0 / T^3(v) = 0 \Rightarrow T$ no invertible

no biyectiva

no inyect ⇔ $\dim N(T) \neq 0$

no sobreyect

$$T \circ T \circ T(v) = T^3(v)$$

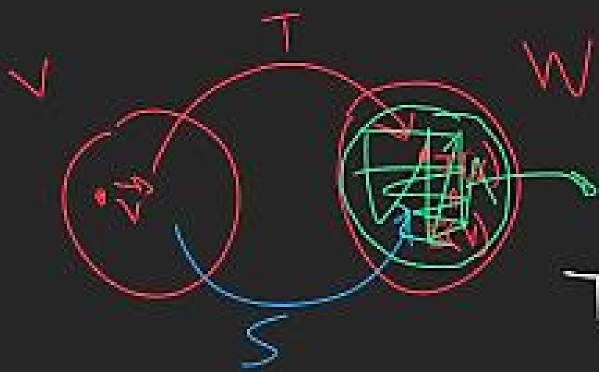
$$T^3(v) = T^2(\underbrace{T(v)}_{w}) = T(\underbrace{T(w)}_{u}) = T(u) = 0$$

• si $u \neq 0$
$\Rightarrow \dim N(T) \neq 0$ ✓

• si $u = 0$: $T(w) = 0$
& $w \in N(T) \Rightarrow \dim N(T) \neq 0$ ✓

② $\begin{matrix} T: V \to W \\ S: V \to W \end{matrix} \Big/ Im(T) = Im(S)$ ← **F**

$\Rightarrow \exists v \neq 0 \in V / T(v) = S(v)$



$\rightarrow Im(T) = Im(S)$

$T(x, y) = (x, y) \rightarrow Im(T) = \mathbb{R}^2$

$S(x, y) = (-x, -y) \rightarrow Im(S) = \mathbb{R}^2$

$\exists \boxed{v \neq 0} / (x, y) = (-x, -y)$?

③ $V = S_1 \oplus S_2$ y $S \subset V$ es un sev. de V

Entonces $S \subset S_1$ ó $S \subset S_2$.

$V = \mathbb{R}^3$

$S_1 = [(1,0,0)]$

$S_2 = [(0,1,0)(0,0,1)]$

$S = [(1,0,0)(0,1,0)]$

④ $A, B \xrightarrow{b} V$, $T: V \to V$ t.l /

F

$_B(T)_A = I_n \Rightarrow A = B$

$$_B(T)_A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\boxed{\text{coord}_B T(v_1)} = (1,0)$

$T(x,y) = (2x, 2y)$

$A = \{(1/2, 0)(0, 1/2)\}$

$B = \{(1,0)(0,1)\}$

$T(1/2, 0) = (1,0)$

$T(0, 1/2) = (0,1)$

$$_B(T)_A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\{(1,8,-2)\,(2,1,5)\,(1,-2,4)\}$ es base de $\mathbb{R}^3$?

$\lambda_1(1,8,-2)+\lambda_2(2,1,5)+\lambda_3(1,-2,4)=(0,0,0)$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 8 & 1 & -2 & 0 \\ -2 & 5 & 4 & 0 \end{array}\right) \sim \dots \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -15 & -10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow \lambda_1 - \frac{4}{3}\lambda_3 + \lambda_3 = 0 \quad \boxed{\lambda_1 = \frac{1}{3}\lambda_3}$$

SCI.

$-15\lambda_2 - 10\lambda_3 = 0 \quad \boxed{\lambda_2 = -\frac{2}{3}\lambda_3} \quad \lambda_3 \in \mathbb{R}$

Tomo $\lambda_3 = 1 \Rightarrow \lambda_2 = -2/3$, $\lambda_1 = 1/3$

$1/3(1,8,-2) - 2/3(2,1,5) + (1,-2,4) = (0,0,0)$

Aplico T

$T\left(1/3(1,8,-2) - 2/3(2,1,5)\right) = T\left(-1(1,-2,4)\right)$

$1/3\,T(1,8,-2) - 2/3\,T(2,1,5) = -T(1,-2,4)$

$1/3(-3,1,1,4) - 2/3(0,-1,2,5) = -(1,-1,-1,2)$

$(-1,1,-1,-2) = (-1,1,-1,-2)$ ✓

$T(\alpha u + \beta v) = \alpha T(u) + \beta$

¿$(2,0,3)$ es cl de $\{(1,8,-2),(2,1,5)\}$?

$(2,0,3) = \lambda_1(1,8,-2) + \lambda_2(2,1,5)$

$\Rightarrow (2,0,3)$ <u>no</u> es c.l. !!!

$\{(2,0,3)\,(1,8,-2)\,(2,1,5)\} \rightarrow B\,\mathbb{R}^3$