③ $T: \mathcal{P}_2 \to \mathbb{R}^3$

$$\boxed{T(ax^2+bx+c) = (\lambda a,\ a+\lambda b+c,\ a+b+\lambda c)}$$

T es invertible $\iff$ T es biyectiva — Equivalencias.

$\Updownarrow$ $\qquad\qquad$ $\Updownarrow$

$\forall B \xrightarrow{b} V \Rightarrow T(B) \xrightarrow{b} W \iff B(T)_A$ es invertible

$$B = \{1, x, x^2\} \longrightarrow \begin{array}{l} T(1) = (0, 1, \lambda) \\ T(x) = (0, \lambda, 1) \\ T(x^2) = (\lambda, 1, 1) \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 0 & \lambda & 0 \\ 1 & \lambda & 1 & 0 \\ \lambda & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & \lambda & 1 & 0 \\ 0 & 1-\lambda^2 & 1-\lambda & 0 \\ 0 & 0 & \lambda & 0 \end{array}\right)$$

Si $\boxed{\lambda = 0} \Rightarrow$ T no invertible.

Si $1-\lambda^2 = 0 \Rightarrow \boxed{\lambda = 1}$, $\boxed{\lambda = -1}$ $\Big\} \Rightarrow$ T no invertible.

⑦ $A, B \in M_{7\times 7}$

(I) $tr(AB) = tr(A)\,tr(B)$ **F**

$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $tr(A) = 1$

$B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $tr(B) = 1$

$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $tr(AB) = 0$

(II) AB es inv $\Rightarrow$ A y B inv **V**

(III) Si A inv $\Rightarrow$ cualquier vector de $\mathbb{R}^7$ se escribe como cl de las filas de A **V**

$\boxed{A \text{ es inv} \iff rg(A) = n}$

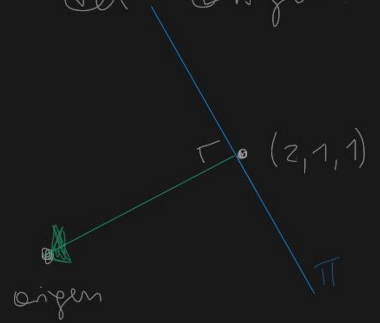
(8) $r = (x,y,z) = (1,1,1) + \lambda(2,2,-2)$

π por $(2,1,1)$ y que está más lejos del origen.

$r \cap \pi = ?$

$ax + by + cz + d = 0$

Vista superior:



$n_\pi : (2,1,1)$

$P = (2,1,1)$

$\Rightarrow 2x + y + z + d = 0$

$2(2) + 1 + 1 + d = 0$

$\boxed{d = -6}$

$\boxed{\pi : 2x + y + z - 6 = 0}$

$r \cap \pi$:

$2(1 + 2\lambda) + 1 + \cancel{2\lambda} + 1 - \cancel{2\lambda} - 6 = 0$

$\boxed{\lambda = 1/2}$

$(x,y,z) = (1,1,1) + (1/2)(2,2,-2)$

$(x,y,z) = (2,2,0)$

⑨ $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$

sol gral → $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

tercera columna de A?

$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix} \Leftrightarrow$

col 1   col 3

$\begin{cases} \boxed{a}x + by + \boxed{c}z = 6 \\ \boxed{d}x + ey + \boxed{f}z = 3 \\ \boxed{g}x + hy + \boxed{i}z = 3 \end{cases}$

$\Leftrightarrow x.\,\text{col}1 + y\,\text{col}2 + z\,\text{col}3 = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$

De la sol gral: $\begin{cases} x = 1 + \lambda + 2\mu \\ y = 1 - 2\lambda \\ z = \mu \end{cases}$

$\underbrace{(1+\lambda+2\mu)}_{=0}\text{col}\,1 + \underbrace{(1-2\lambda)}_{=0}\text{col}\,2 + \mu\,\text{col}3 = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$ $\forall \lambda, \mu \in \mathbb{R}$

$\begin{cases} 1 + \lambda + 2\mu = 0 \\ 1 - 2\lambda = 0 \rightarrow \lambda = 1/2 \end{cases}$ $\longrightarrow \boxed{\mu = -3/4}$

$(-3/4)\begin{pmatrix} c \\ f \\ i \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$

$\begin{cases} -3/4\, c = 6 \longrightarrow \boxed{c = -8} \\ -3/4\, f = 3 \longrightarrow \boxed{f = -4} \\ -3/4\, i = 3 \longrightarrow \boxed{i = -4} \end{cases}$

$\text{col}(3) = \begin{pmatrix} -8 \\ -4 \\ -4 \end{pmatrix}$

(10) $A, B \in \mathcal{M}_{n\times n}$

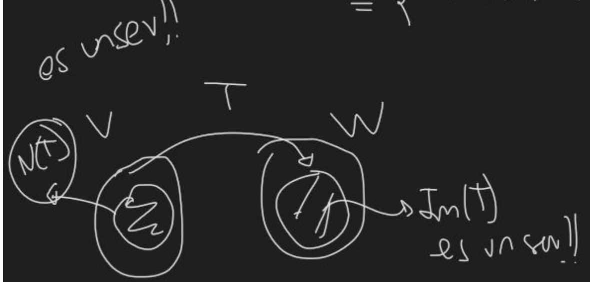
$S_1 = \{x \in \mathbb{R}^n : Ax = Bx\}$ es un sev?

$= \{x \in \mathbb{R}^n : Ax - Bx = 0\}$

$= \{x \in \mathbb{R}^n : \boxed{(A-B)}\,x = 0\}$

sev

- $\vec{0} \in S_1$
- $s_1 + s_2 \in S_1$
- $\alpha s_1 \in S_1$.



Si $A - B = C$

$\rightarrow$ S_1 es el $N(C)$

$\Longrightarrow$ S_1 es un sev!

$S_2 = \{x \in \mathbb{R}^n : A^2 x = B^2 x\}$ es un sev?

$S_2 = \{\ '' : \boxed{(A^2 - B^2)}\,x = 0\}$

$\Longrightarrow$ S_2 es un sev.

$S_3 = \{x \in \mathbb{R}^n : Ax = Bx - x\}$

$= \{\ '' : Ax = (B - I)x\}$

$= \{\ '' : \boxed{(A - B + I)}\,x = 0\}$

$\Longrightarrow$ S_3 es un sev!