Prod. interno → devuelve un n°

→ $\langle \; , \; \rangle = 2$

→ Props: $\langle \alpha u, v\rangle = \alpha \langle u, v\rangle$

$\langle u, v\rangle = \langle v, u\rangle$

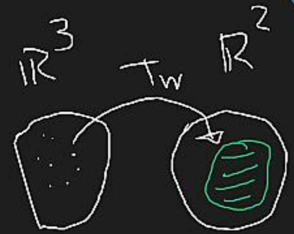
$\langle u+v, w\rangle = \langle u, w\rangle + \langle v, w\rangle$

(2) $T_w : \mathbb{R}^3 \to \mathbb{R}^2 \,/\, T_w(v) = \underbrace{\big(\underbrace{\langle v, 2w\rangle}_{\in \mathbb{R}}, \underbrace{\langle v, -w\rangle}_{\in \mathbb{R}} \big)}$

$\boxed{\begin{matrix} w \in \mathbb{R}^3 \\ w \neq 0 \end{matrix}}$

$$T_w(v) = \big(2\langle v, w\rangle, -\langle v, w\rangle\big)$$

$$\boxed{T_w(v) = \underbrace{\langle v, w\rangle}_{\in \mathbb{R}} (2, -1)}$$



$$\operatorname{Im}(T_w) = [(2, -1)] = [(-2, 1)]$$

$$\dim \operatorname{Im}(T_w) = 1$$

Teo dim:

$$\underset{\underset{3}{\shortparallel}}{\dim \mathbb{R}^3} = \underset{\underset{1}{\shortparallel}}{\dim \operatorname{Im}(T_w)} + \underset{\underset{2}{\Downarrow}}{\dim N(T_w)}$$

$$v \in N(T_w) \Leftrightarrow T_w(v) = (0, 0)$$

$$\langle v, w\rangle (2, -1) = (0, 0)$$

$$\Updownarrow$$

$$\boxed{\langle v, w\rangle = (0, 0)}$$ condición

$$\dim N(T_w) = \underset{\underset{3}{\shortparallel}}{\dim \mathbb{R}^3} - \underset{\underset{1}{\shortparallel}}{\# \text{cond}}$$

$$\Rightarrow \dim N(T_w) = 2$$

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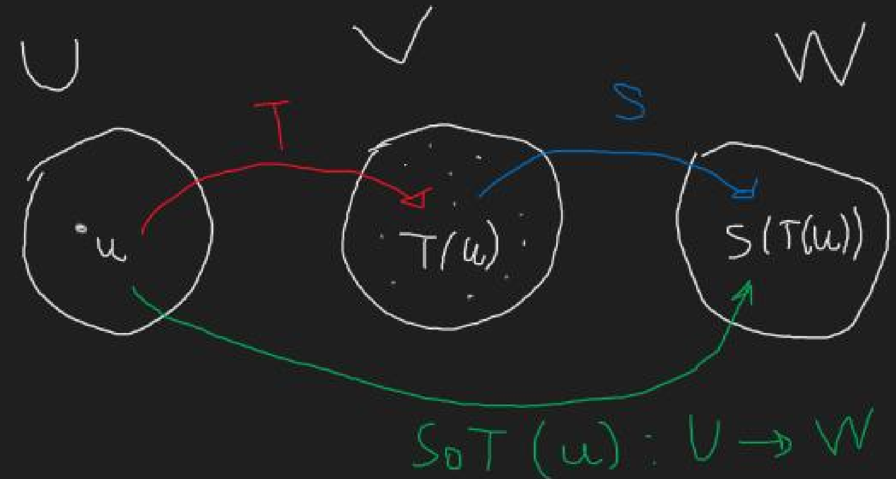
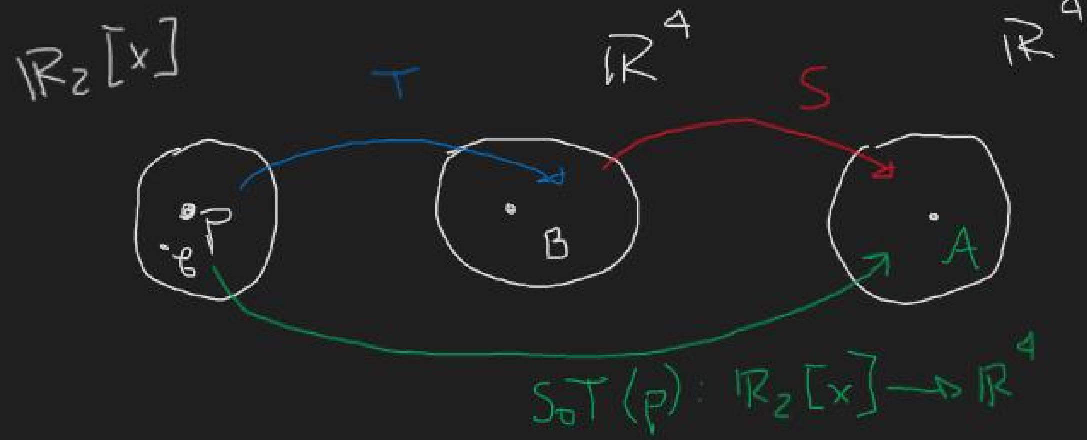




Prop:

$$_{A}(S\circ T)_{\mathcal{E}} = {}_{A}(S)_{B}\;{}_{B}(T)_{\mathcal{E}}$$

quiero

A es base canónica de $\mathbb{R}^4$

$_A(S)_B = \begin{pmatrix} \text{coord}_A S(\ldots) \end{pmatrix}$

$$_A(S)_B = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 2 & -1 & -1 \\ -1 & 0 & 2 & -2 \end{pmatrix}$$

Multiplico y obtengo:

$$_A(S \circ T)_{\mathcal{B}} = \left(\begin{array}{ccc|c} 3 & 2 & 3 & 5 \\ 9 & 2 & 10 & 11 \\ 5 & -2 & 2 & 3 \\ -9 & -3 & -19 & -12 \end{array}\right) \sim \ldots$$

$$\left(\begin{array}{ccc|c} 3 & 2 & 3 & 5 \\ 0 & -4 & 1 & -4 \\ 0 & 0 & 13 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\to 3a + 2 = 5 \to \boxed{a = 1}$

$\to -4b = -4 \quad \boxed{b = 1}$

$\to -13c = 0 \quad \boxed{c = 0}$

$p \in \mathbb{R}_2[x] \;/\; \overbrace{S \circ T(p) = (5, 11, 3, -12)}^{\checkmark}$

$p(1) = ?$

$$\overset{\checkmark}{_A(S \circ T)_{\mathcal{B}}} \cdot \overset{(a,\,b,\,c)}{\text{coord}_{\mathcal{B}}\, p} = \overbrace{\text{coord}_A\, S \circ T(p)} \checkmark$$

$\mathcal{B} = \{1, x, x^2\}$

$p(x) = cx^2 + bx + a$

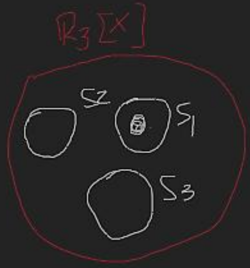
$\Big\}\ \text{coord}_{\mathcal{B}}\, p = (a, b, c)$

$\text{coord}_{\mathcal{B}}\, p = (1, 1, 0)$

$p(x) = x + 1$

$$\boxed{p(1) = 2}$$

(5) $S_1=\{p\in\mathbb{R}_3[x]: p(0)+p'(0)+p''(0)=0\}$



Vamos hallar base de S_1:

$$p(x)=ax^3+bx^2+cx+d \rightarrow p(0)=d$$
$$p'(x)=3ax^2+2bx+c \rightarrow p'(0)=c$$
$$p''(x)=6ax+2b \rightarrow p''(0)=2b$$

$$S_1=\{p\in\mathbb{R}_3[x] \,/\, \underbrace{d+c+2b=0}_{d=-c-2b}\}$$

$$S_1=\{ax^3+bx^2+cx-c-2b \,,\; a,b,c\in\mathbb{R}\}$$

$$S_1=\{a(x^3)+b(x^2-2)+c(x-1)\}$$

$$S_1=[x^3,\, x^2-2,\, x-1]$$

$$\{x^3, x^2-2, x-1\}\xrightarrow{b} S_1 \Rightarrow \boxed{\dim S_1=3}$$

$$S_2=[x^3,\, x^2,\, x-1] \quad \dim S_2=3$$

$$S_3=[x^3,\, x,\, 1] \quad \dim S_3=3$$

$$\boxed{\dim \mathbb{R}_3[x]=4}$$

$$S_2+S_3=\{x^3, x^2, x, 1\}\xrightarrow{b} S_2+S_3$$

$$\dim S_2+S_3=4$$

$$\mathbb{R}_3[x]=S_2\oplus S_3$$

P/ verificar:

$$\{x^3+1, x^3, x^2-2, x-1\}\xrightarrow{b}\mathbb{R}_3[x]$$

$$S_1\cap[x^3+1]=\{\vec{0}\}\Leftrightarrow S_1\oplus[x^3+1]=\mathbb{R}_3[x]$$