

1 Exercise 1, page 247

A card is drawn at random from a deck consisting of cards numbered 2 through 10. A player wins 1 dollar if the number on the card is odd and losses 1 dollar if the number is even. ¿What is the expected value of his winnings?

$$E = \sum x * P(X = x)$$

$$E = (-1)P(x = -1) + 1(1)P(x = 1)$$

$$P(X = 1) = \frac{4}{9}$$

$$P(X = -1) = \frac{4}{9}$$

$$E = (-1)(\frac{4}{9}) + (1)(\frac{4}{9})$$

$$E = -\frac{1}{9}$$

2 Exercise 6, page 247

A die is rolled twice. Let X denote the sum of the two numbers that turn up, and Y the difference of the numbers (specifically, the number on the first roll minus the number on the second). Show that $E(XY) = E(X)E(Y)$. ¿Are X and Y independent?

A = result of the first die

B = result of the second die

$$X = A + B$$

$$Y = A - B$$

$$E(A) = E(B)$$

$$E(A^2) = E(B^2)$$

we will now show $E(XY) = E(X)E(Y)$

$$E(XY) = E(X)E(Y)$$

$$E(XY) = E((A + B)(A - B))$$

$$E(XY) = (A^2 - B^2 + BA - AB)$$

$$E(XY) = (A^2 - B^2)$$

$$E(XY) = E(A^2) - E(B^2)$$

$$E(XY) = 0$$

$$E(X)E(Y) = E(A + B)E(A - B)$$

$$E(X)E(Y) = (E(A) + E(B))(E(A) - E(B))$$

$$E(X)E(Y) = (E(A))^2 - (E(B))^2$$

$$E(X)E(Y) = 0$$

then we have $E(XY) = E(X)E(Y)$

we will show now that X and Y are independent. By definition we have $P(X|Z) = P(x)$ if they are independent. We set $X = 2$ if we calculate the probability $P(X = 2) = \frac{1}{36}$ and now we calculate $P(X = 2|Y = 5) = 0$. Since $P(X = 2) \neq P(X = 2|Y = 5)$, then, X and Y are not independent.

3 Exercise 18, page 249

Exactly one of six similar keys opens a certain door. If you try the keys, one after another, ¿what is the expected number of keys that you will have to try before success?

$$E(X) = \sum_{x=1}^6 x * P(X = x)$$

$$E(X) = \sum_{x=1}^6 x * \frac{1}{6}$$

$$E(X) = (1 + 2 + 3 + 4 + 5 + 6) * \frac{1}{6}$$

$$E(X) = 3.5$$

4 Exercise 1, page 263

A number is chosen at random from the set $S = \{-1, 0, 1\}$. Let X be the number chosen. Find the expected value, variance, and standard deviation of X .

$$E(X) = \sum_{x=-1}^1 x * P(X = x)$$

$$E(X) = \sum_{x=-1}^1 x * \frac{1}{3}$$

$$E(X) = (-1, 0, 1) * \frac{1}{3}$$

$$E(X) = 0$$

now for the variance $\sigma^2 = V(X) = E((x - E(x))^2)$

$$\sigma^2 = E((x - 0)^2)$$

$$\sigma^2 = E(x^2)$$

$$\sigma^2 = \sum x^2 * P(X^2 = x^2)$$

$$\sigma^2 = (0 * \frac{1}{3}) + (1 * \frac{2}{3})$$

$$\sigma^2 = \frac{2}{3}$$

now for standar deviation $\sigma = \sqrt{\frac{2}{3}}$

5 Exercise 9, page 264

A die is loaded so that the probability of a face coming up is proportional to the number on that face. The die is rolled with outcome X . Find $V(X)$ and $D(X)$.

6 Exercise 12, page 264

Let X be a random variable with $\mu = E(X)$ and $\sigma^2 = V(X)$. Define $X^* = (X - \mu)/\sigma$. The random variable X^* is called the *standardized random variable associated* with X . Show that this standardized random variable has expected value 0 and variance 1.

7 Exercise 3, page 278

The lifetime, measure in hours, of the ACME super light bulb is a random variable T with density function $f_T(t) = \lambda^2 t e^{-\lambda t}$, where $\lambda = 0.05$. ¿What is the expected litetime of this light bulb? ¿What is its variance?

8 Exercise 15, page 249

A box contains two gold balls and three silver balls. You are allowed to choose successively balls from the box at random. You win 1 dollar each time you draw a gold ball and lose 1 dollar each time you draw a silver ball. After a draw, the ball is replaced. Show that, if you draw until you are ahead by 1 dollar or until there are no more gold balls, this is a favorable game.