Tarea 9
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1 Exercise 1, page 247

A card is drawn at random from a deck consisting of cards numbered 2 through 10. A player wins 1 dollar if the number on the card is odd and losses 1 dollar if the number is even. ¿What is the expected value of his winnings?
$E = \sum x * P(X = x)$.
$E = (-1)P(x = -1) + 1(1)P(x = 1)$.
$P(X = 1) = \frac{4}{9}$.
$P(X = -1) = \frac{4}{9}$.
$E = (-1)(\frac{4}{9}) + (1)(\frac{4}{9})$.
$E = -\frac{1}{9}$.

2 Exercise 6, page 247

A die is rolled twice. Let X denote the sum of the two numbers that turn up, and Y the difference of the numbers (specifically, the number on the first roll minus the number on the second). Show that $E(XY) = E(X)E(Y)$. ¿Are X and Y independent?
$A =$ result of the first die.
$B =$ result of the second die.
$X = A + B$.
$Y = A - B$.
$E(A) = E(B)$.
$E(A^2) = E(B^2)$.
We will now show $E(XY) = E(X)E(Y)$.
$E(XY) = E(X)E(Y)$.
$E(XY) = E((A + B)(A - B))$.
$E(XY) = (A^2 - B^2 + BA - AB)$.
$E(XY) = (A^2 - B^2)$.
$E(XY) = E(A^2) - E(B^2)$.
$E(XY) = 0$.
$E(X)E(Y) = E(A + B)E(A - B)$.
$E(X)E(Y) = (E(A) + E(B))(E(A) - E(B))$.
$E(X)E(Y) = (E(A))^2 - (E(B))^2$.
$E(X)E(Y) = 0$.
then we have $E(XY) = E(X)E(Y)$.
We will show now that X and Y are independent. By definition we have $P(X|Z) = P(x)$ if they are independent. We set $X = 2$ if we calculate the probability $P(X = 2) = \frac{1}{36}$ And now we calculate $P(X = 2|Y = 5) = 0$. Since $P(X = 2) \neq P(X = 2|Y = 5)$,then, X and Y are not independent.

3 Exercise 18, page 249

Exactly one of six similar keys opens a certain door. If you try the keys, one after another, ¿what is the expected number of keys that you will have to try before success?
$E(X) = \sum_{x=1}^{6} x * P(X = x)$.
$E(X) = \sum_{x=1}^{6} x * \frac{1}{6}$.
$E(X) = (1 + 2 + 3 + 4 + 5 + 6) * \frac{1}{6}$.

$E(X) = 3.5.$

4 Exercise 1, page 263

A number is chosen at random from the set $S = \{-1, 0, 1\}$. Let X be the number chosen. Find the expected value, variance, and standard deviation of X.
$E(X) = \sum_{x=1}^{3} x * P(X = x).$
$E(X) = \sum_{x=1}^{3} x * \frac{1}{3}.$
$E(X) = (-1, 0, 1) * \frac{1}{3}.$
$E(X) = 0.$
Now for the variance: $\sigma^2 = V(X) = E((x - E(x))^2).$
$\sigma^2 = E((x - 0)^2).$
$\sigma^2 = E(x^2).$
$\sigma^2 = \sum x^2 * P(X^2 = x^2).$
$\sigma^2 = (0 * \frac{1}{3}) + (1 * \frac{2}{3}).$
$\sigma^2 = \frac{2}{3}.$
Now for standar deviation: $\sigma = \sqrt{\frac{2}{3}}.$

5 Exercise 9, page 264

A die is loaded so that the probability of a face coming up is proportional to the number on that face. The die is rolled with outcome X. Find $V(X)$ and $D(X)$.
The probability of each face is $P(X = x) = \frac{x}{21}.$
With this we can get our expected value.
By definition we have
$E(X) = \sum x * P(X = x).$
$E(X) = \sum x * \frac{x}{21}.$
$E(X) = \frac{1}{21} + \frac{4}{21} + \frac{9}{21} + \frac{16}{21} + \frac{25}{21} + \frac{36}{21}.$
$E(X) = \frac{91}{21}.$
Using the definition we have $V(cX) = E((cX)^2) - E((cX))^2$,
then, we need to get $E(X^2)$.
$E(X^2) = \sum X^2 * P(X^2 = X^2).$
However since it will be the same probability. $E(X^2) = \sum X^2 * P(X = X).$
$E(X^2) = \sum X^2 * \frac{x}{21}.$
$E(X^2) = \sum X^3 * \frac{x}{21}.$
$E(X^2) = \frac{1}{21} + \frac{8}{21} + \frac{27}{21} + \frac{64}{21} + \frac{125}{21} + \frac{216}{21} = 21.$
With this we can get our variance. $V(X) = E(x^2) - (E(x))^2.$
$V(X) = 21 - (\frac{91}{21})^2.$
$V(X) = \frac{981}{441}.$
$V(X) = 2.22.$
Finally the standar deviation.
$D(X) = \sqrt{2.2}$

6 Exercise 12, page 264

Let X be a random variable with $\mu = E(X)$ and $\sigma^2 = V(X)$. Define $X^* = (X - \mu)/\sigma$. The random variable X^* is called the *standardized random variable associated* with X. Show that this standardized random variable has expected value 0 and variance 1.
First we will get our expected value.

$E(X) = E(\frac{x-\mu}{\sigma})$.
$E(X) = E(\frac{1}{\sigma}(x-\mu))$.
$E(X) = \frac{1}{\sigma}E(x-\mu)$.
$E(X) = \frac{1}{\sigma}(E(x)-E(\mu))$.
$E(X) = \frac{1}{\sigma}E(\mu-\mu)$.
$E(X) = 0$.
Next the variance.
$V(X) = V(\frac{x-\mu}{\sigma})$.
$V(X) = V(\frac{1}{\sigma}(x-\mu))$.
$V(X) = (\frac{1}{\sigma})^2 V(x-\mu)$.
$V(X) = (\frac{1}{\sigma^2})V(x)$.
$V(X) = (\frac{1}{\sigma})(\sigma^2)$.
$V(X) = 1$.

7 Exercise 3, page 278

The lifetime, measure in hours, of the ACME super light bulb is a random variable T with density function $f_T(t) = \lambda^2 t \exp^{-\lambda t}$, where $\lambda = 0.05$. ¿What is the expected litetime of this light bulb? ¿What is its variance?
By definition we have $E(T) = \int_0^\infty (t)(f_T(t)dt)$ to get our expected value of a continuous variable.
$E(T) = \int_0^\infty (t)(\lambda^2 t \exp^{-\lambda t})dt)$.
$E(T) = \int_0^\infty t^2 \lambda^2 \exp^{-\lambda t}\, dt$.
$E(T) = 40$.
By definition we have $V(cX) = E((cX)^2) - E((cX))^2$,
then, we need to get $E(T^2)$.
$E(T^2) = \int_0^\infty (t^2)(\lambda^2 t \exp^{-\lambda t})dt$.
$E(T^2) = \int_0^\infty t^3 \lambda^2 \exp^{-\lambda t}\, dt$.
$E(T^2) = 2400$.
With this we can get the variance.
$V(X) = E((X)^2) - E((X))^2$.
$V(X) = 2400 - 1600\dot{}\ V(X) = 800$.

8 Exercise 15, page 249

A box contains two gold balls and three silver balls. You are allowed to choose successively balls from the box at random. You win 1 dollar each time you draw a gold ball and lose 1 dollar each time you draw a silver ball. After a draw, the ball is replaced. Show that, if you draw until you are ahead by 1 dollar or until there are no more gold balls, this is a favorable game.
We need the probability of each case.
$P(W=1) = \frac{1}{2}$.
$P(W=0) = \frac{1}{5}$.
$P(W=-1) = \frac{3}{10}$.
By definition we have:
$E(W) = \sum(g)P(G=g)$.
$E(W) = (-1)\frac{3}{10} + (0)\frac{1}{5} + (1)(\frac{1}{2})$.
$E(W) = -\frac{3}{10} + (1)(\frac{2}{10})$.
$E(W) = \frac{1}{5}$.
with this our strategy gives a expected value greater that 0, which means is favorable.