IMPEDANCE MATCHING CIRCUIT

FOR PIEZOELECTRIC TRANSDUCERS

by

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ABSTRACT

Ultrasonic systems use quartz and ceramic transducers to produce acoustic power. The major limitation in producing this power is insufficient electrical power input due to low power factor and high impedance of the transducer.

An impedance matching circuit between electrical source and transducer improves the power transfer. Four circuits have been designed which work for 600 KHz, 900 KHz, and 2.7 MHz with quartz transducers, and for 600 KHz to 3 MHz with ceramic transducers.

Special considerations include high voltage and high current components, cables, and connectors, and design simplicity for ease in operation and reliability.

The circuits will be built and a manual describing their operation will be written by September 1978.

Thesis Supervisor
P. P. Lele, Professor of Experimental Medicine

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CHAPTER I

ULTRASONIC SYSTEMS

Piezoelectric transducers form the heart of ultrasonic systems which generate high-intensity sound waves. Given the proper electrical drive, the transducer will vibrate, propagating sound waves into an acoustic medium. The sound waves contain energy, some of which is converted to heat by a property of the medium called absorption. The medium heats in proportion to the intensity of the sound. An ultrasonic system designed to heat an acoustic medium therefore must produce high-intensity sound.

A new medical use of high-intensity ultrasound is in cancer treatment by production of local hyperthermia. This is a recently discovered effect that inhibits cancer growth and increases the tumor's radio-sensitivity, making the tumor more susceptible to radiation therapy. The tumor is first located, then heated by a few degrees Celcius to $42^{o}C$. Ultrasound is ideal for heating deep tumors since it can penetrate deep into the body, can be focussed, and has no known harmful effect on healthy tissue at the intensity levels used, and therefore no harmful side effects in the treatment.

The present limitation in producing high-intensity sound is insufficient electrical drive to the transducer. In MIT's Medical Ultrasonic Laboratory (hereafter called the laboratory), electrical sources exist with up to 1 KW output capability, but due to a low power factor in the transducer, not much electrical power in becomes acoustical power out. Also, impedance levels differ between source and load, further reducing power output. An impedance matching circuit is needed.

Impedance matching circuits transform the load impedance to match that of the source. They are narrowband, operating at the transducer's resonant frequency, but are adjustable to accommodate a range of transducer frequencies and impedances, and therefore allow maximum power output for a variety of transducers.

An ultrasonic system can be broken down into two subsystems: electrical and mechanical. Their only link is the transducer, which is the endpoint of electrical power flow and the beginning of acoustical power flow. It is important to distinguish their properties. For example, travelling waves and standing waves occur in the mechanical subsystem; they do not in the electrical subsystem, since electrical wavelengths are 10^5 larger than acoustical, though both electrical and mechanical components are similarly sized. Thus there are no standing waves in the electrical system. Another example is that only in the transducer will mechanical resonance imply electrical resonance. Thus an impedance matching circuit will resonate with the transducer electrically, causing a power transfer maximum, but the transducer's series resonance, mechanical in origin, need not be at the same frequency.

The impedance matching circuits will maximize the electrical power dissipated in the transducer, and this maximizes acoustic power out. However, it is not necessary, nor was it expedient to consider the mechanical susbystem *per se* in achieving maximum electrical power.

The system of units used throughout is rationalized MKS, and the units for various constants are listed with their definitions.

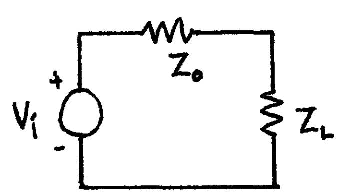
CHAPTER II

ACOUSTIC POWER PRODUCTION

The concern of high-intensity ultrasonic systems is producing acoustic power in an acoustic load. There are many mechanical or acoustical criteria involved, including wave focussing, acoustic coupling, absorption, dispersion, cavitation, and others. These criteria will not be discussed here, as they do not constitute the system's present limitation. Instead, electrical performance is inadequate and electrical criteria of acoustic power production will be presented. First, general source-load considerations are presented, then specific source-transducer considerations applicable to high-intensity ultrasonic systems.

2.1 General Source-Load Considerations

In general, an electrical source can be modelled with a Thevenin equivalent, and the load with a circuit equivalent, as shown below.



Z_o is the output impedance of the source, Z_L is the impedance of the load, and V_i is the unloaded output voltage of the source.

There is only one factor involved in transferring maximum power from the source to the load, but several descriptions of the process. The causal factor is impedance matching, which is described by the maximum power transfer theorem. The various descriptions include power factor, resonance, tuning, and transforming.

All sources have a non-zero output impedance, so the power transferred from source to load depends on the relative values of Z_o and Z_L which are in general complex quantities. By the maximum power transfer theorem, maximum power is transferred from the source to the load when Z_L is a complex conjugate of Z_o.(1) For the case where Z_o is purely resistive, its imaginary part is zero, so maximum power is transferred when the load impedance equals the output impedance. This is the situation encountered in the lab, where Z_o is designed to be resistive to match resistive characteristic impedance of a transmission line.

The power factor description of power transmission is useful when driving the load from a voltage source. Power factor is defined as the ratio of active (dissipated) power to apparent (available) power. It is a measure of how in-phase the voltage and current are in the load, since active power is proportional to the cosine of their phase angle. Apparent power is given by the product of r.m.s. voltage and current and contains no information about their phase angle, hence cannot be used as a measure of the power dissipated. The mathematical description is shown below.

For sinusoidal current (i) and voltage (v) with peak values $\hat{I}$ and $\hat{V}$:

$$i = \hat{I}\sin t$$

$$v = \hat{V}\sin(\omega t + \phi)$$

where ϕ is an arbitrary phase angle

The instantaneous power (p) is:

$$p = vi = \hat{V}\hat{I}\sin(\omega t + \phi)\sin\omega t = \tfrac{1}{2}\hat{V}\hat{I}\,[\cos\phi - \cos(2\omega t+\phi)]$$

which is an oscillating quantity with a mean value called the active power (P):

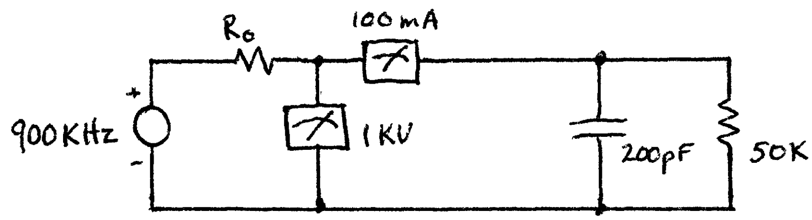
$$P = \tfrac{1}{2}\hat{V}\hat{I}\cos\phi = V_{rms}I_{rms}\cos\phi$$

Active power is that which is converted to either work or heat--it is the dissipated power. Apparent power (P_a) represents an electrical source limitation and so may be thought of as available power, and is given by:

$$P_a = V_{rms}I_{rms}$$

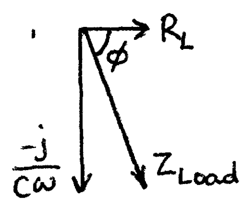
The power factor is then P/P_a, or $\cos\phi$.

An example found in the laboratory is shown here. A high power oscillator (Pulsed Ultrasonic Generator) indicates its output is 1 KV rms and 100 mA rms. The cable and quartz transducer load is modelled by a parallel RC. Find the electrical power dissipated as acoustical power, assuming electromechanical efficiency is 99%. The schematic is shown below.



$$P_a = (1\ \mathrm{KV})(100\mathrm{mA}) = 100\mathrm{W}$$

The phase angle ϕ is defined by this phasor diagram:



where

$$R_L = \frac{50K}{(2\pi \cdot 900\,KHz \cdot 50K \cdot 200pF)^2} = 15.6\,\Omega$$

$$\frac{1}{C\omega} = \frac{1}{2\pi \cdot 900\,KHz \cdot 200pF} = 884\,\Omega$$

thus

$$\phi = \tan^{-1} \frac{Im[Z_L]}{Re[Z_L]} = 1.55\ rad$$

$$\cos\phi = .018$$

so that active power = P = P_a cosϕ = 1.8 W, and

$$P_{acoustic} = \eta P = 1.8\ \mathrm{W}$$

2.2 Specific Source-Load Considerations

Specific source-load considerations in delivering maximum power consist of keeping component losses to a minimum. For the ultrasonic system these components include low-loss cables and high Q impedance matching circuits. In particular, tuning inductors must have a high Q, meaning low wire losses (ohmic losses), low eddy current losses in the core and nearby metallic objects, and high inductance per wire length (closely spaced coil windings).

Generally, lower losses occur at a particular frequency, due to resonances between inherent capacitances and inductances. This behavior means narrowband frequency response for power output. The transducer is a good example of this: high Q transducers are the best ones to use for delivering high acoustic power and these are characterized by narrowband response.

It is important to note, however, that the Q of a transducer is dependent on two loss mechanisms: internal losses from dielectric, backing, and friction, and external losses from the mechanical load. It doesn't matter if the Q of the unloaded transducer is 100 or 10^5 if the acoustic load reduces the working Q to 10. In this case, external losses would be so much greater than internal losses that it wouldn't matter if the internal losses were much smaller. Note, though, that the self-heating of the transducer is lower when the internal losses are lower, an important factor for ceramics, where self-heating causes a power limitation.

Given that a system is low-loss, a high level of power input is still necessary for high acoustic output. Preliminary study indicated an electrical power input of up to 1 KW was needed. The electrical sources used in the laboratory are voltage sources, and driving a 50 ohm load requires 220 V for full power output. The load voltage will be much higher for impedance transducers, and load voltages may reach 10 KV peak. Special components and layout must be used to allow this high voltage without arcing.

As the level of input power increases, limitations from the piezoelectric disc will occur. These limitations are from mechanical strength of the material and from maximum temperature of operation, somewhat below the Curie point. For quartz, the dielectric losses are low, so self-heating is low, and the temperature limitation is unimportant. For ceramics, dielectric losses are significant, and temperature rise forms the primary limitation in high-power applications. Since the piezoelectric disc is expensive and fragile, the electrical system output must not exceed these limitations.

CHAPTER III

TRANSDUCER THEORY

It is necessary to determine the effect of the transducer in the ultrasonic system, specifically its electrical operation. To do this, the relevant piezoelectric constants must be obtained, which requires knowledge of crystal orientation and piezoelectricity. Crystal resonance, which explains frequency of operation, is also discussed.

3.1 Crystal types, Cut, and Orientation

In the laboratory transducers, the piezoelectric disc has alternating voltage applied across its thickness, compressing and extending it in a thickness mode vibration. The electric field and the resulting mechanical strain occur in the same direction. This is not a chance occurrence; the piezoelectric is a crystal or ceramic with definite crystallographic axes which must be properly aligned for a useful thickness mode disc vibrator.

There are many types of piezoelectric materials available, but those found in the laboratory will be of primary interest here. They include X-cut alpha quartz crystals and three types of lead zirconate titanate ceramics called PZT-1, PZT-4, and PZT-5. The primary consideration in selecting which type of ceramic to use is the dielectric loss tangent, which is a measure of how lossy a ceramic material is. A lesser criterion is high Curie point, which allows high temperature rise in the ceramic before degradation in performance (note that performance degrades at a temperature somewhat lower than the Curie point). A large dielectric loss tangent (abbreviated "tan δ") means a large fraction of power stored in the ceramic's capacitance is dissipated in the ceramic as heat. Excessive heating will cause failure of the ceramic. Thus the input power must be limited. Ceramics with the lowest dielectric loss tangents are best suited to high intensity applications. The lowest is PZT-8, then PZT-4 next; the transducers also have the highest unloaded mechanical Qs. PZT-8 has the disadvantage of a reduced electromechanical coupling, and somewhat lower resistance to a depoling field than PZT-4, but its lower loss tangent at high fields makes it the most attractive ceramic. PZT-5A and PZT-5H are five times more lossy than PZT-4 and PZT-8 at low field, and much more lossy at high field. The values for tan δ for low field, and the intensity of field required before the loss tangent increases to .04, are shown below.

	PZT-1	PZT-4	PZT-5A	PZT-5H	PZT-8
tan δ	.006	.004	.02	.02	.004
Curie Point ($^{\circ}C$)	350	328	365	193	300
AC Field for tan δ to increase to .04 (10^5 Volt/Meter)	-	3.9	.45	-	>10
Unloaded Mechanical Q	500	500	75	65	1000

To use quartz as a thickness-extension mode vibrator, the crystallographic x axis ("1" axis) must be parallel to the thickness of the disc. The three PZT ceramics also have crystallographic axes, and their z axis ("3" axis) must be parallel to the thickness of the disc.

Although PZT is a ceramic, not a crystal, it still has crystallographic axes because of a manufacturing process called "poling." In general, a ceramic is composed of many microsocpic crystals called domains, bound together in a random orientation. No crystallographic axes are identifiable and no piezoelectric effect occurs. For this to happen, the domains must be aligned. This is simple because the domains have an inherent electrical dipole moment which aligns the domains to an intense external electric field when the ceramic is heated to the Curie point, at which temperature the domains can reorient themselves. After cooling, the orientation remains. This process, called poling, makes most of the domains have a common axis, which produces in the ceramic a crystallographic axis with which to work. The piezoelectric ceramic exhibits a strong piezoelectric effect, though not nearly as strong as if it were a single crystal of the same material.(2)

3.2 Piezoelectricity

The piezoelectric effect relates mechanical and electrical properties of certain anisotropic crystals. The direct piezoelectric effect predicts electrical polarization when the crystal is mechanically strained. This electrical polarization produces charges on crystal faces, which in turn produce a voltage because of crystal capacitance. The converse piezo-electrical effect predicts mechanical stress in the crystal when an electric field is applied.

A complete treatment of piezoelectricity is complex, partly due to several interacting properties of a crystal, including piezoelectricity, thermoelasticity, pyroelectricity, and electrocaloricity. However, many simplifying approximations can be made while incurring little error. Satisfactory results are obtainable using only stress, strain, field, and polarization. The relevant equations are shown below.

Elastic

$x = s*X$	x = strain
$X = c*x$	X = stress
	c = stiffness
	s = compliance

Dielectric

$P = \chi*E$	E = field
$E = \eta*P$	P = polarization
$D = \varepsilon*E$	χ = dielectric stiffness
$E = \beta*D$	η = dielectric susceptibility
	ε = permittivity
	β = dielectric impermiability

<u>Piezoelectric</u>

P = x*e	P = X*d	e = piezoelectric stress constant
x = g*P	X = h*P	g = piezoelectric stress constant
x = d*E	X = e*E	d = piezoelectric strain constant
E = x*h	E = X*g	h = piezoelectric strain constant

Although these relationships are easily defined, they involve the several phenomena mentioned above, and so it is important to note the conditions under which the definitions are made. For example, measuring a stiffness coefficient requires that the electric field in the crystal be constant or zero so that a stress induced piezoelectrically does not appear. There are four conditions: mechanically clamped (constant or zero strain), mechanically free (constant or zero stress), electrically clamped (constant or zero electrical strain or polarization), and electrically free (constant or zero electrical stress or field). The representations of the coefficients defined under these conditions include the following superscripts: "E" for constant field, "D" for constant polarization (displacement), "T" for constant stress, and "S" for constant strain. In some cases a condition is implied without showing a superscript, but an explanatory note usually appears in the text.

The general piezoelectric equations mathematically describe three dimensional effects, and are tensor equations: when a crystal is exposed to a field in one direction, strains occur in other directions also. In general, a given electric field on one crystal axis will produce six responses: a compression stress and a shear stress along each of the three orthogonal axes. The crystal is also sensitive to which axis was electrically stimulated, so three electrical inputs exist. In the most general case the following quantities are needed:

3 components of electric field (volt/meter)	E_x	E_y	E_z
3 components of polarization (coul/m^2)	P_x	P_y	P_z
6 mechanical stress components (newton/m^2)			
3 compressional	X_x	Y_y	Z_z
3 shear	Y_z	Z_x	X_y
6 mechanical strain components (dimensionless)			
3 compressional	x_x	y_y	z_z
3 shear	y_z	z_x	x_y
18 piezoelectric stress constants			
(newton/volt*m and coul/m^2)	e_{ik}	$i = 1$ to 3	
(newton/coul and volt/m)	h_{ik}	$k = 1$ to 6	
18 piezoelectric strain constants			
(m/volt and coul/newton)	d_{ih}	$i = 1$ to 3	
(volt*m/newton and m^2/coul)	g_{ih}	$h = 1$ to 6	

There are four types of piezoelectric constants, e, d, g, and h. Original theories of piezoelectricity used values called e and d as constants. These adequately describe piezoelectrics such as quartz. Later theories needed to describe ferroelectrics such as PZT used values called g and d. It was found that in ferroelectrics the old piezoelectric and dielectric constants varied greatly with temperature. However, the quantity voltage/force, represented by the quotient of piezoelectric to dielectric constants, remained constant. These quotients became the constants in the so-called "displacement" theory, which proposes electrical displacement instead of polarization as a causal quantity. The relation between the various piezoelectric constants is given below. These equations reflect the fact that the crystal's anisotropy leads to tensor qualities.

$$d_{ih} = \sum_{k} s_{hk}{}^{E} * e_{ik} \qquad e_{ik} = \sum_{h} c_{hk}{}^{E} * d_{ih}$$

$$d_{ih} = \sum_{j} \epsilon_{ij}{}^{T} * g_{jh} \qquad g_{jh} = \sum_{i} \beta_{ij}{}^{T} * d_{ih}$$

$$e_{ik} = \sum_{j} \epsilon_{ij}{}^{S} * h_{jk} \qquad h_{jk} = \sum_{i} \beta_{ij}{}^{S} * e_{ik}$$

$$g_{jh} = \sum_{k} s_{hk}{}^{D} * h_{jk} \qquad h_{jk} = \sum_{h} c_{hk}{}^{D} * g_{jh}$$

where h and k = 1 to 6

i and j = 1 to 3

In all crystal classes except triclinic (Class 1), crystallographic symmetries exist which cause certain piezoelectric constants to vanish. For example, alpha-quartz, the type used in transducers, is a trigonal holoaxial (Class 18) crystal, having specialized piezoelectric stress and strain constant matrices below. Also shown are the matrices for PZT, a ceramic composed of tetrogonal crystals.

Piezoelectric and Dielectric Constants for Quartz[3,4]

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_x \\ Y_y \\ Z_z \\ Y_z \\ Z_x \\ X_y \end{bmatrix}$$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} e_{11} & -e_{11} & 0 & e_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -e_{14} & -e_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_x \\ y_y \\ z_z \\ y_z \\ z_x \\ x_y \end{bmatrix}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$d_{11} = 2.3*10^{-12}$ $\qquad$ $e_{11} = .17$

$d_{14} = -5.7*10^{-13}$ $\qquad$ $e_{14} = .40$

$\varepsilon_{11}{}^{T} = 4.6$

Piezoelectric and Dielectric Constants for PZT

$$\begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

PZT-1

$d_{31} = -1.22*10^{-10}$ $g_{31} = -.0125$

$d_{33} = 2.84*10^{-10}$ $g_{33} = .0292$

$\varepsilon_{33}{}^{T} = 1100$ $\varepsilon_{33}{}^{S} = 500$

PZT-4

$d_{31} = -1.23*10^{-10}$ $g_{31} = -.0107$ $e_{31} = -5.2$

$d_{33} = 2.89*10^{-10}$ $g_{33} = .0251$ $e_{33} = 15.1$

$d_{15} = 4.96*10^{-10}$ $g_{15} = .0380$ $e_{15} = 12.7$

$\varepsilon_{11}{}^{T} = 1475$ $\varepsilon_{11}{}^{S} = 730$

$\varepsilon_{33}{}^{T} = 1300$ $\varepsilon_{33}{}^{S} = 635$

PZT-5(H)

$d_{31} = -2.74*10^{-10}$ $g_{31} = -.0091$ $e_{31} = -6.5$

$d_{33} = 5.93*10^{-10}$ $g_{33} = .0197$ $e_{33} = 23.3$

$d_{15} = 7.41*10^{-10}$ $g_{15} = .0268$ $e_{15} = 17.0$

$\varepsilon_{11}{}^{T} = 3130$ $\varepsilon_{11}{}^{S} = 1700$

$\varepsilon_{33}{}^{T} = 3400$ $\varepsilon_{33}{}^{S} = 1470$

Since the piezoelectrics used in the lab are used in thickness-extension mode where the fields and strain are parallel, the crystal axis driven is the one where a peizoelectric constant exists with two identical subscripts. For quartz d_{11} is the only such constant, so quartz is driven along its x-axis. For PZT, d_{33} is the only usable constant, so PZT is driven along the z-axis. These are the relevant constants for use in calculating various quantities relating to the piezoelectrics in the laboratory. Moreover, the relevant dielectric constants, electromagnetic coupling coefficients, etc., have the same subscripts as the piezoelectric constants.

3.3 Resonance

Although piezoelectric crystals will output sound for any alternating electrical input, the amount of acoustic output is frequency dependent. At some frequencies, a constant amplitude input will produce more acoustic output than at other frequencies. This is due to the internal structure of the crystal which causes mechanical resonances to occur. The crystal is a spring-mass system, where the stiffness and mass store elastic and inertial energies respectively. At resonance, the amount of energy stored in each is equal and opposite, so that the reactive part of the mechanical impedance vanishes. The impedance that remains is due to viscous losses from acoustic energy radiated externally and internal frictional losses. Thus at resonance nearly all energy input is radiated as sound, minus the inherent internal crystal losses.

Resonances occur at a fundamental frequency, and at odd harmonics of the fundamental (i.e. third, fifth, seventh, etc.). These frequencies are determined by the thickness of the crystal and the velocity of sound in the crystal. Vibrations produced by the crystal form travelling acoustical waves within the crystal, which originate at a point of symmetrical mechanical impedance within the crystal. For a travelling wave with a sinusoidal amplitude, maximum amplitude occurs at odd multiples of a quarter-wavelength ($\lambda/4$, $3\lambda/4$, $5\lambda/4$, etc.) from the wave's origin. A crystal with a radiating surface at a distance $n\lambda/4$ for n odd from the wave's origin will put out maximum amplitude vibrations. In other words, resonance occurs in a crystal of given thickness for frequencies which cause the radiating surface to be $n\lambda/4$ for n odd from the wave's origin. Using the equation $f\lambda = c$, where c is the velocity of sound in the crystal, the following equation can be written:

$$f_{res} = \frac{nc}{4d}$$

where n = odd integer

d = distance from wave's origin to radiating surface

To find "d" in the above equation, it is necessary to identify the origin of the velocity wave, where displacement is zero. In a crystal clamped at one face, that face becomes the origin of the wave. For crystals which are not clamped, such as those found in the laboratory, both faces will vibrate. Since no net disc translation occurs, forces on the disc must be in equilibrium, and the two disc faces must vibrate out of phase with each other. Thus the plane of zero vibration, the origin of the acoustic wave, lies between the two faces. For a symmetrically loaded disc, the origin is at the midplane.

The origin also lies at the midplane for a half-wavelength disc which is mounted with an air backing. The air presents low mechanical impedance, but sound passing from that boundary to the midplane travels a quarter-wavelength, and a quarter-wavelength of mechanical transmission line transforms the impedance. The low impedance is transformed to a high impedance at the origin, thus acting as a rigid backing for the other half-thickness of the disc. The net effect is then a quarter-wavelength resonator.

The half-wavelength disc describes the operation of discs in the laboratory. There the piezoelectric discs within a transducer are loaded with a waterbath on the front face, but loosely coupled to a massive electrode, lightly spring loaded, on the back face. The spring-mass electrode arrangement is driven by the transducer beyond its resonant frequency, and the loose coupling between it and the disc ensures a low mechanical impedance load. Thus the back face is lightly loaded, making the entire disc exhibit the half-wavelength effects described.

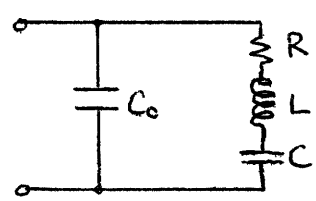
CHAPTER IV

TRANSDUCER MODELS

Understanding the effect of a transducer on the electrical subsystem requires an electrical characterization of the transducer. Since the frequency of operation of the transducer is less than 3 MHz, a lumped-parameter equivalent circuit is a valid modelling method, except for predicting harmonic resonances, which will be explained later. The complexity of the model depends on how many transducer effects are accounted for. Since the only concern here is impedance matching, the model need only describe the electrical impedance of the transducer. Further, the transducer operates at a mechanical resonance frequency, so that its driving voltage will be a single frequency. The model thus consists of an impedance equivalent over a narrowband frequency range.

4.1 Piezoelectric Disc Model

The model is obtained by examining a more complex model valid over a large frequency range, then simplifying it to frequencies of interest. The general model is shown below.



There are two circuit branches. The one with C_o represents the capacitance inherent in a transducer where metal electrodes are separated by an insulating crystal. This arrangement produces a parallel-plate capacitor whose value is given by the equation:

$$C_o = \frac{\varepsilon S}{\ell}$$

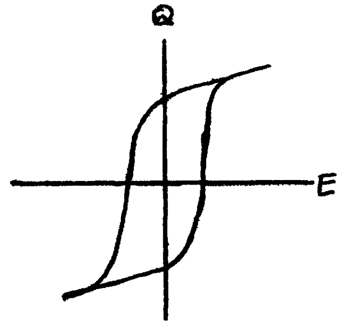
where ε = permittivity of the crystal in the direction of applied field

S = area of the electrode

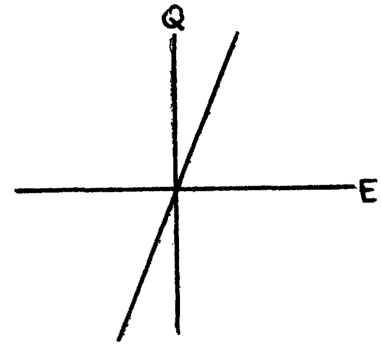
ℓ = distance between electrodes

Since C_o arises from a purely electrical source, independent of crystal motion, it is sometimes called the "static" capacitance.

C_o is a constant capacitance only if the crystal permittivity (ε) us constant. For quartz, ε is constant with temperature and applied field, so C_o is a constant, linear ($Q = CV$) circuit element. For PZT, and ceramics in general, permittivity is a strong function of temperature; even more important is the change in capacitance with applied field. A ceramic is composed of many independent domains, each of which has an electric dipole moment, and they reverse their polarity under varying intensities of field. This causes a change in the net charge observed on the ceramic faces, not predicted by the linear $Q = C*V$ relationship. The sudden reversal of dipole domains under external stimulus is called "ferroelectricity" and gives rise to a hysteresis effect of Q vs. E., shown below.



ferroelectric



linear

The incremental capacitance is proportional to the slope of the Q-E curve, and it can be seen that for PZT, the value of capacitance changes significantly with field. The effective permittivity is also proportional to the slope of the Q-E curve, and it can be seen that the low field value is much less than the maximum value which is specified by the ceramic manufacturer.

The changing capacitance is difficult to model, and for purposes of impedance matching, impossible to completely correct with fixed components. Therefore an approximation will be used which considers the capacitance to be constant, with a value determined by a constant permittivity with a value for low field, equal to ε^S, specified by the manufacturer. The effect of this in impedance matching is that under varying intensity field, the capacitance will be matched exactly only part of the time, but fairly closely the rest of the time.

The other branch of the transducer circuit model is mechanical in origin and arises from an electrical-mechanical equivalence intorduced by the disc's piezoelectric effect.

For the piezoelectric disc to radiate acoustic energy, electrical energy must be input, since energy is conserved. The amount of acoustic energy determines the amount of electric energy, and the acoustic load on the transducer directly causes the electric load on the electrical source. Also, the mechanical impedance of the disc itself, arising from its friction, stiffness, and mass, is reflected as a load on the electrical source. The electrical equivalent is determined by an electrical-mechanical circuit analog which is based on the piezoelectric effect; electric field causes stress, strain causes polarization. An electrical-mechanical circuit equivalent requires equations that relate voltage to force, displacement to charge, and thus velocity to current by taking time derivatives. The step between piezoelectric effect and equivalent circuits involves a transformation factor. It is derived from normalizing factors of length (thickness) and area, and definitions of various quantities: stress is force per area, field is voltage per length, polarization is charge per area, and strain is displacement per length. The equations and derivations are shown below.

piezoelectric effect: $X = e\,E \qquad P = e\,x$

definitions: $X = F/S \qquad E - V/\ell \qquad P = Q/S \qquad x = d/\ell$

thus: $F/S = X = e\,E = e\,V/\ell$ or simply $F = \frac{Se}{\ell}V$

and: $Q/S = P = e\,x - e\,d/\ell$ or simply $Q = \frac{Se}{\ell}d$

differentiating: $\frac{dQ}{dt} = \frac{Se}{\ell}\frac{dd}{dt}$ or simply $i = \frac{Se}{\ell}u$

Relating the mechanical state variables to electrical ones thus requires the factor S•e/ℓ (hereafter called α) where S is the radiating area, e is the relevant piezoelectric stress constant, and ℓ is the thickness of the disc. Relating mechanical circuit elements to electrical ones requires the factor α^2. For example, Hooke's Law states F = K d where K is the spring constant. The electrical equivalent is derived below.

$$F = K\ d$$

$$K = F/d = \frac{\alpha V}{Q/\alpha} = \alpha^2\ \frac{V}{Q}$$

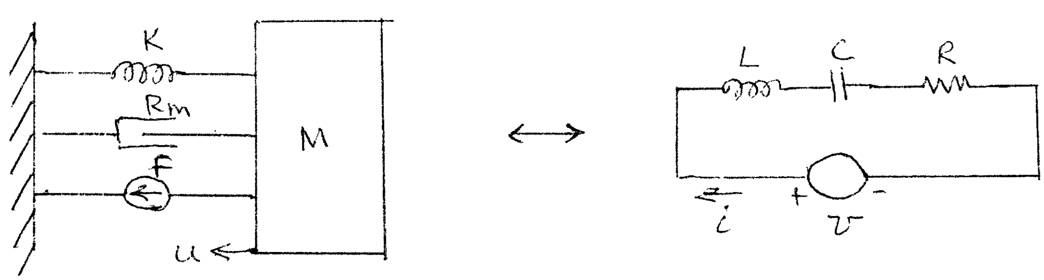
$$Q = \frac{\alpha^2}{K}\ V$$

For a capacitor: $Q = C^* V$

thus the spring is modelled as a capacitor of value $C = \alpha^2/K$.

The electrical model of the piezoelectric disc can thus be obtained from the mechanical circuit. This is shown below.[6]

$$f = M\frac{du}{dt} + R_m u + K\int u\,dt \longleftrightarrow v = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt$$



$$f,\ u,\ R_m,\ M,\ K \longleftrightarrow v,\ i,\ \alpha^2 R,\ \alpha^2 L,\ \frac{\alpha^2}{C}$$

The disc's stiffness is modelled as C, its mass as L, and its frictional and viscous damping as R.

Although R is composed of two parts, the frictional losses of the vibrating disc are much less significant than the viscous losses. This is observed by the difference between a water loaded and an air loaded transducer. The mechanical impedance of air is virtually zero, so the mechanical losses in an air loaded transducer arise from internal frictional losses. The mechanical losses determine the mechanical Q of the transducer at mechanical resonance, and Q of quartz unloaded can be above 10^5 whereas loaded the Q drops to several hundred. The viscous effect of loading is therefore the dominant loss mechanism.

Assuming the acoustic load is well coupled to the vibrating disc (no air pockets or bubbles) and that the load absorbs all acoustic energy put out (no reflections or standing waves), the load can be completely characterized by its characteristic acoustic impedance. The value is given by $Z = \rho c$ where ρ is density and c is sound velocity in the medium. For transducers used in biological applications the load consists of a water bath and biological tissues. Tissues which are mostly water have an acoustic impedance close to that of water, $1.5*10^6$ Kg/m^2* sec. The tissues which are different are bone and lung, which have higher and lower densities than water, respectively. The acoustic impedance, ρc, has units of pressure/velocity. To convert to a mechanical impedance, with units of force/velocity, such as R_m in the mechanical circuit, ρc must be multiplied by the radiating surface area.

The value of R in the electrical circuit can be derived from the mechanical impedance R_m, shown below.

$$R = \frac{R_m}{\alpha^2} = \frac{\rho c S}{\alpha^2} = \frac{\rho c (\ell^*)^2}{e_{ii}^2 S}$$

For transducers in the laboratory, ℓ^* will be half the disc's thickness, or $\lambda/4$. For the conventional ℓ which is $\lambda/2$, the equation becomes:

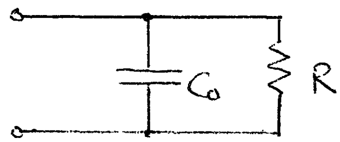
$$R = \frac{\rho c \ell^2}{4 e_{ii}^2 S}$$

and

$$\alpha = \frac{2 e_{ii} S}{\ell}$$

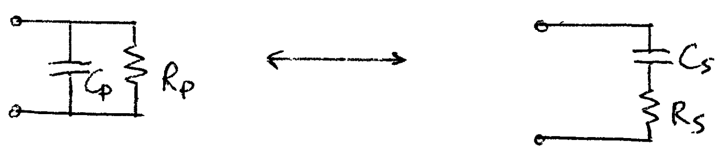
The L and C of the electrical model are reactive elements which vanish at resonance frequency, leaving R alone for that branch. Note that the basic model shows two distinct resonances, one involving C_o, C, and L, called the "parallel" resonance, the other involving C and L, called the "series" resonance. The series resonance is the significant one, because the mechanical impedance is then at a minimum and maximum power may be delivered to the load, R, for a constant voltage drive.

Driving the transducer at its resonance means the following circuit is a valid model.



$$C_o = \frac{\epsilon S}{\ell} \qquad R = \frac{\rho c \ell^2}{4 e_{ii}^2 S}$$

An alternative circuit topology exhibiting equivalent impedance can be made though the circuit elements have no physical significance. This series RC model is useful for identifying measureemnts made on an impedance bridge and for calculating resistive load impedance when impedance matching with a series coil. Transforming the values of the parallel circuit can be done in several ways, the one presented below being the one used for dissipative capacitors, involving the calculation of the dissipation factor, D.[7]



$$C_s = C_p\,(1 + D^2)$$
$$R_s = R_p\left(\frac{D^2}{1 + D^2}\right)$$
$$D = \frac{1}{\omega R_p C_p} = \omega R_s C_s$$

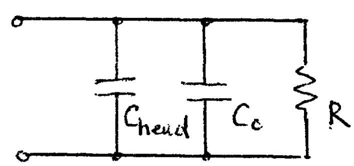
When the parallel resistance is very large, as for many quartz transducers, so that $D \leq .01$, the following approximations can be made, introducing less than 1% error: $C_s = C_p$ and

$R_s = D^2 R_p$.

4.2 Mounting Head Capacitance

The piezoelectric is the dominant electrical element in the transducer, but other effects exist which must be included. These are mounting head capacitance and cable capacitance.

Mounting head capacitance arises primarily from the proximity of the electrode to the shell of the mounting head. A value of capacitance for the 8 cm disc mounting head was obtained experimentally. A mock piezoelectric disc made from cardboard was mounted in the transducer, and the input impedance was measured on an impedance bridge at various frequencies. In all cases the impedance was purely capacitive, equal to 14 pF. This capacitance occurs between the electrode and ground, so its effect would be in parallel with the piezoelectric capacitance, C_o, as shown below.



4.3 Cable Capacitance

Cable capacitance occurs from the use of coaxial cables, which are useful for providing electromagnetic shielding, convenience, and safety, at the expense of relatively high interlead capacitance. Cable capacitance is undesirable because it further reduces the power factor of the combined cable and transducer load.

The effect of the cable in the electrical system is determined by using a lumped circuit equivalent. The validity of this method will be demonstrated by calculating the impedance of a typical cable and transducer combination using general transmission line theory and then calculating the impedance using a lumped equivalent for the cable. The lumped model is expected to be valid because for lengths of line short compared to the electrical wavelength, no distributed effect is apparent, and all cables in the laboratory are much shorter than the electrical wavelengths. Even the longest cables, about two meters, are 3% of the shortest wavelength, 66 meters (for 3 MHz operation in RG-11/U cable with v_p = 66%).

The general transmission line equation of interest defines the input impedance of an arbitrary length of an arbitrary line terminated in an arbitrary impedance:(8)

$$Z_{in} = \frac{Z_T + Z_o \tanh(\alpha + j\beta)\ell}{1 + \frac{Z_T}{Z_o}\tanh(\alpha + j\beta)\ell}$$

where z_{in} = input impedance of the line

z_o = characteristic impedance of the line

z_t = terminating impedance

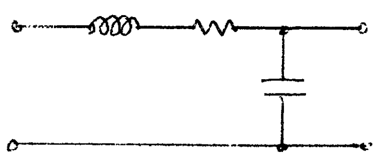
α = attenuation constant (nepers/m)

β = phase constant (radians/m)

ℓ = length of line (m)

The constants in the equation α and ℓ are not the electromechanical transformation factor and disc thickness as previously defined. All of the constants are available from tables of cable data, except for β, which can be easuly calculated by using the high-frequency approximations of cable parameters, valid for frequencies above tens of KHz. Thus $\beta = \omega/v_p$ where v_p is the velocity of propagation, a parameter of the cable.

The lumped model of the line consists of a series resistance, series inductance, and shunt capacitance, shown below.



The values of L and C are usually supplied in normalized form as cable specifications; R must be calculated from Z_o using high-frequency approximations:

$$R_o = Z_o$$

$$R = 2\alpha R_o$$

The dominant effect of the cable is capacitive, so the lumped model is frequently simplified to include only the shunt capacitor.

The example to which these two analysis techniques are applied is this: three feet of RG-11/U cable is connected to a ceramic transducer, 847 pF in series with 20 ohms. First the constants are calculated:

$$\omega = (1\ \text{MHz})(2\pi) = 6.28\ \text{MRad/sec}$$

$$ZT = 20 - j188\ \Omega$$

$$v_p = (66\%)(3*10^8\ \text{m/sec}) = 1.99*10^8\ \text{m/sec}$$

$$\beta = \omega/v_p = .0317\ \text{rad/m}$$

$$\alpha = .27\ \text{dB/100 ft} = .00102\ \text{nepers/m}$$

$$\ell = 1\ \text{m}$$

Then the input impedance of the cable is calculated using the transmission line equation:

$$\tanh\,(\alpha + j\beta)\ell = .00100 + j0.0320$$

$$z_{in} = \frac{20 - j188 + 75(.001+j\ .032)}{1 + (.267 - j2.51)(.001 + j.032)} = 17.6 - j172$$

The lumped model elements are then calculated:

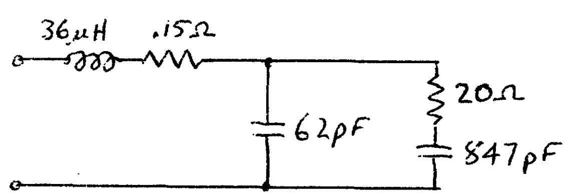
$$C = (3\ \text{ft})(20.5\ \text{pF/ft}) = 62\ \text{pF}$$

$$R = 2\alpha Z_o = .15\,\Omega$$

$$L_o = CZ_o^2 = .12\ \text{uH/ft}$$

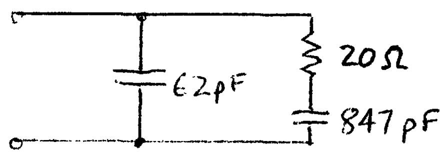
$$L = (3\ \text{ft})(.12\ \text{uH/ft}) = .36\text{uH}$$

The lumped circuit equivalent is shown below.



$$z_{in} = j2.26 + .15 + \frac{(-j2590)(20 - j188)}{20 - j2780} = 17.5 - j173$$

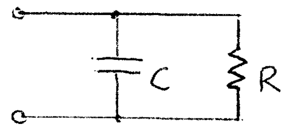
The percentage error in the lumped model impedance is 0.6% in each the real and imaginary parts. The simplification of the lumped model ignores the inductance and resistance, so that the cable is modelled by a shunt capacitance. For this model, shown below, the percentage error is 1% in the real part, and 2% in the imaginary.



$$Z_{in} = 17.4 - j175$$

Since the error that this model introduces is less than 2%, a negligible fraction, the shunt capacitor model will be the one used for a cable capacitance. Note, though, that this usage is based in short lengths of cable similar to RG-8A/U, such as RG-11/U. If the cable used has a much different inductance, capacitance, or resistance (attenuation), the circuit equivalent model for the cable may need the additional circuit elements, and if the cable has a much lower velocity of propagation or much longer length, the lumped model may not be valid at all.

The complete model for the combined cable and transducer load at resinance, which is the total electrical load on the electrical source, is shown below.



where

$$C = C_{cable} + C_{head} + C_{static}$$

$$C_{cable} = \text{(capacitance/length)(cable length)}$$

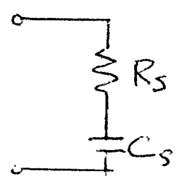
$$C_{head} = 14\text{pF (for the 8 cm disc head)}$$

$$C_{static} = \frac{\epsilon S}{\ell} = \frac{\epsilon_0 \epsilon_r S}{\ell}$$

and

$$R = \frac{\rho_i c_0 S}{4\alpha^2} = \frac{1.5 \cdot 10^6 \ell^2}{4 e_{ii}^2 S}$$

The series model for the combined cable and transducer uses the values of the parallel model, and is shown below.



where

$$R_s = R\left(\frac{D^2}{1+D^2}\right)$$

$$C_s = C\,\frac{1+D^2}{D^2}$$

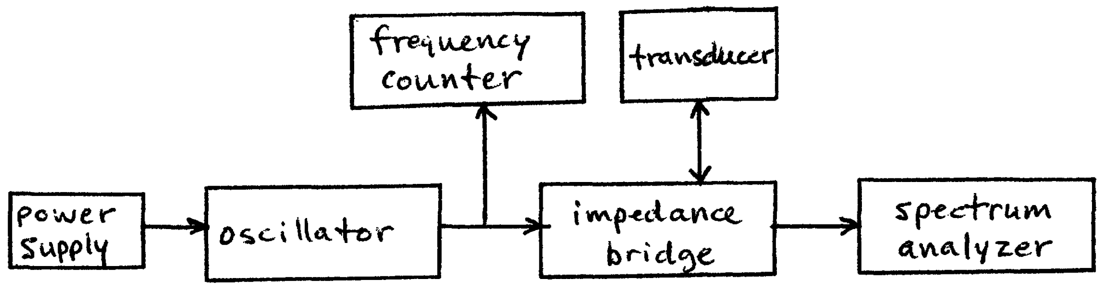
$$D = \frac{1}{\omega RC} = \omega R_s C_s$$

CHAPTER V

TRANSDUCER MEASUREMENTS

Two methods of measuring transducers were investigated. The electrical method uses an impedance bridge with suitable frequency range. The electrical impedance of the transducer can be obtained directly using this method, and impedance measurements of both quartz and ceramic transducers have been obtained. Mechanical measurements involve taking dimensions of the disc, obtaining elastic constants for the material, and calculating electrical parameters from equations governing electrical-mechanical equivalence. Mechanical measurements are much easier and quicker than electrical ones, but are at best approximations. They can have up to 40% error for ceramic transducers where manufacturing tolerances only ensure 20% tolerance on piezoelectric constants, 5% on elastic constants, and 10% on dielectric constants. Still, this rough estimate may be useful, especially since transducer impedance measurements will be used only for rough setting of controls on the impedance matching circuits.

Electrical measurements were made with a General Radio 916-AL impedance bridge (with high frequency transformer in place), General Radio 1211-C oscillator, Data Precision frequency counter, and Hewlett-Packard 121-T spectrum analyzer. The set-up is shown below.



The procedure is roughly this: for each frequency setting, the output of the bridge was nulled, observing the spectrum analyzer waveform. The initial null (zero impedance reference) of the bridge was checked every few frequency changes. Details of the bridge's operation are given in its instruction manual.

Measuring quartz transducers without a coaxial cable required the use of a shunt capacitance to reduce the impedance to a value on the bridge's scale. The shunt needed was about 100 pF. The exact value was obtained by first measuring a 300 pF capacitor, then using it to shunt the 100 pF capacitor for a measurement.

Noise was not such a problem as was suggested by the bridge's instruction manual, so shielding and groundplane techniques were not necessary. A reasonable large signal was required, which meant keeping the oscillator's output from .25 to .75 of the maximum.

The transducer could be measured alone or with its coaxial cable. For resonant frequency determination, it is more accurate to measure the transducer alone.

The bridge measures impedance, $Z = R + jX$, R and X corresponding to a series RC where $X = 1/C\omega$. Thus at resonance, R_s has a maximum. With a parallel RC model, at resonance, R_p would reach a minimum due to the inverse relationship of R_{series} and $R_{parallel}$, shown by the series-parallel transformation.

$$R_p = \frac{R_s}{D^2} = \frac{X_s^{\,2}}{R_s}$$

Therefore at resonance, R_p reaches a minimum as predicted by the basic transducer model.

Following is measured data for three cases: 900 KHz quartz transducer operated at fundamental frequency with a cable, 900 KHz quartz transducer operated at third harmonic without a cable, and 1.02 MHz ceramic transducer operated at fundamental without a cable. In the tables, R_d and X_d are direct reading from the bridge at final null, R_d and X_m are the impedance at the bridge's terminals including shunt capacitance, and R and X are the resistive and reactive parts of the transducer (and cable) load alone. The relationship between these quantities is shown below.

$$X_m = \frac{X(\text{initial}) - X_d}{.01\ f(\text{KHz})} \qquad R = \frac{R_m\, X_{shunt}^{\,2}}{R_m^{\,2} + (X_m - X_{shunt})^2}$$

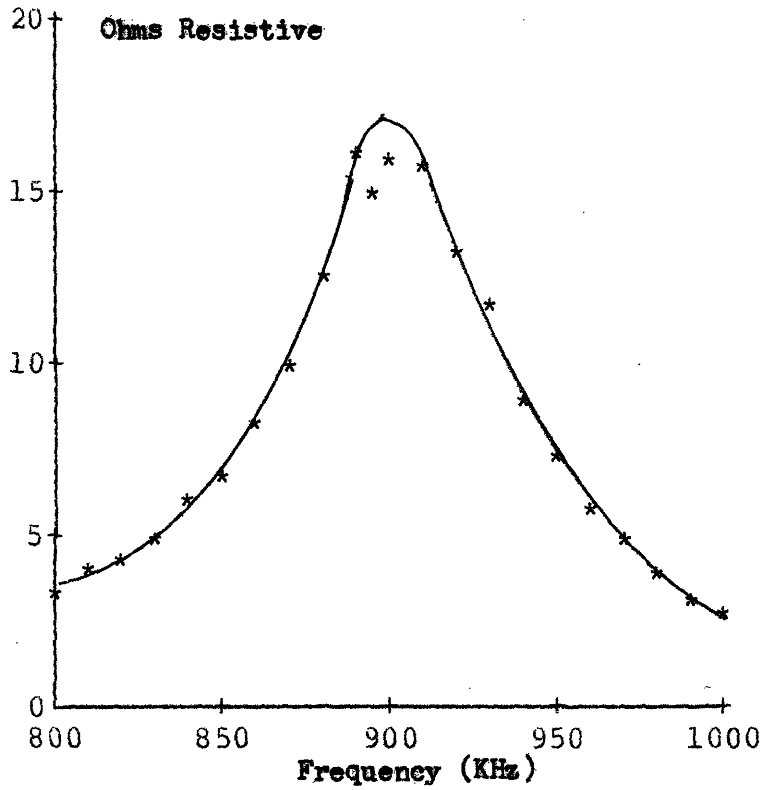
$$X = \frac{-X_{shunt}\,(R_m^{\,2} + X_m\,(X_m - X_{shunt}))}{R_m^{\,2} + (X_m - X_{shunt})^2}$$

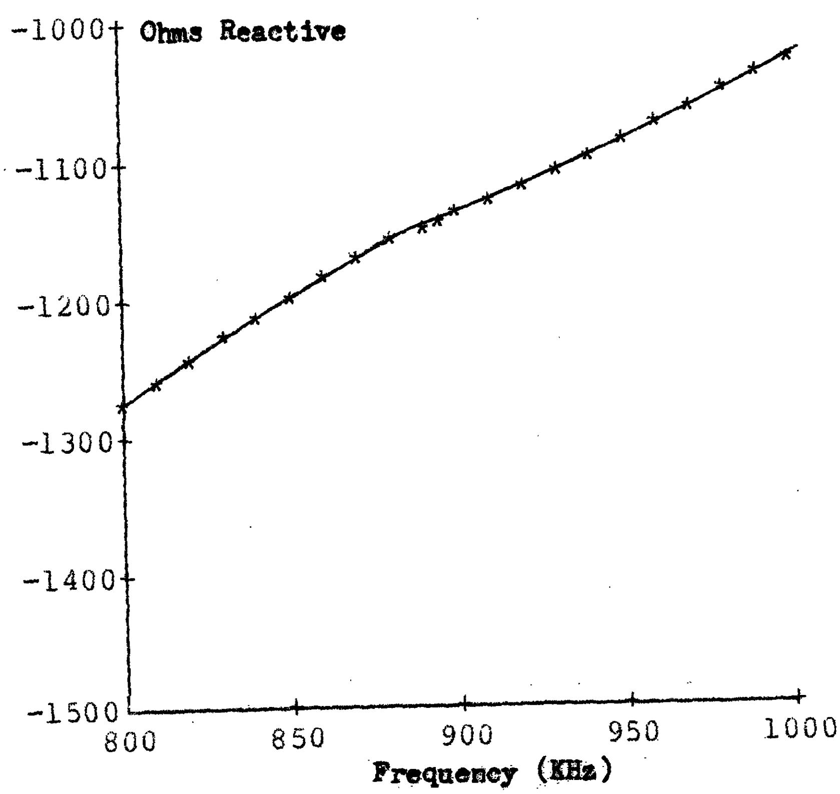
Unknown: quartz, 900 KHz resonance, with 1m RG-11/U cable
Shunt: none
Initial Null: R = 0.0, X = 11000

freq (KHz)	R_d	X_d	X_m	R	X
800	3.2	760	-1280	3.2	-1280
810	3.9	760	-1264	3.9	-1264
820	4.2	770	-1248	4.2	-1248
830	4.8	780	-1231	4.8	-1231
840	5.9	780	-1217	5.9	-1217
850	6.6	790	-1201	6.6	-1201
860	8.1	795	-1187	8.1	-1187
870	9.8	795	-1173	9.8	-1173
880	12.4	795	-1160	12.4	-1160
890	16.0	750	-1152	16.0	-1152
895	14.8	726	-1148	14.8	-1148
900	15.8	727	-1141	15.8	-1141
910	15.6	690	-1133	15.6	-1133
920	13.1	668	-1123	13.1	-1123
930	11.6	652	-1113	11.6	-1113
940	8.8	650	-1102	8.8	-1102
950	7.2	645	-1090	7.2	-1090
960	5.6	647	-1078	5.6	-1078
970	4.8	653	-1067	4.8	-1067
980	3.8	657	-1055	3.8	-1055
990	3.0	662	-1044	3.0	-1044
1000	2.6	668	-1033	2.6	-1033
1100	0.8	694	- 694	0.8	- 694

Note: Resonance was determined by the frequency of maximum resistive impedance

Impedance for Quartz Transducer



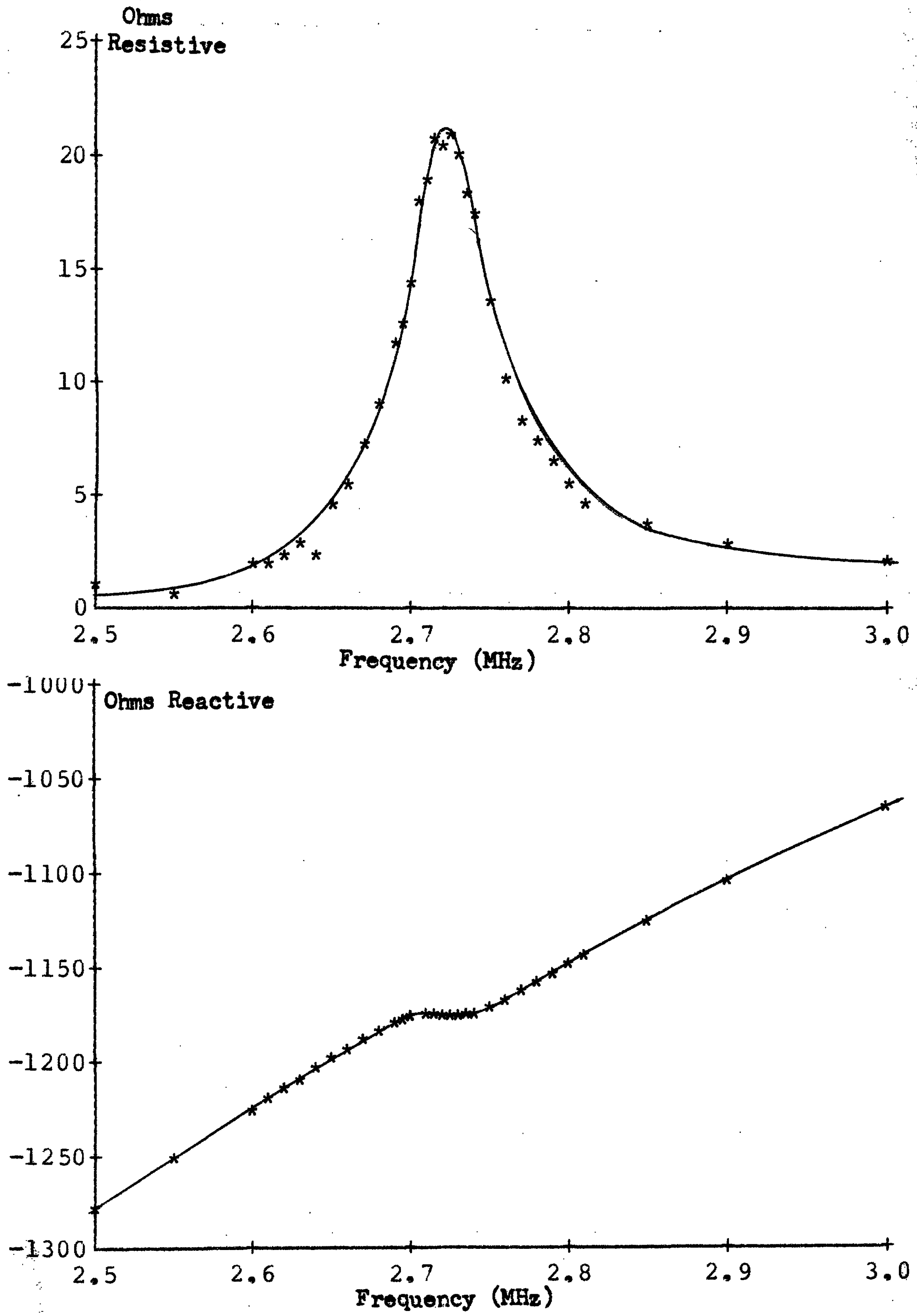


Unknown: quartz, 900 KHz resonance, without cable
Shunt: 99.3 pF (measured)
Initial Null: R = 0.0, X = 11000

freq (MHz)	R_d	X_d	X_m	R	X
2.500	.1	322	-427	0.90	-1280
2.550	.05	329	-418	0.45	-1252
2.600	.2	333	-410	1.79	-1227
2.610	.2	336	-409	1.79	-1221
2.620	.25	337	-407	2.23	-1216
2.630	.3	339	-405	2.68	-1211
2.640	.35	341	-404	2.23	-1205
2.650	.5	343	-402	4.45	-1200
2.660	.6	345	-401	5.34	-1195
2.670	.8	347	-399	7.11	-1190
2.680	1.0	349	-397	8.89	-1185
2.690	1.3	348	-396	11.56	-1181
2.695	1.4	347	-395	12.45	-1179
2.700	1.6	345	-395	14.24	-1177
2.705	2.0	340	-394	17.83	-1177
2.710	2.1	335	-394	18.76	-1176
2.715	2.3	328	-393	20.60	-1176
2.720	2.25	320	-393	20.21	-1177
2.725	2.3	313	-392	20.71	-1177
2.730	2.2	306	-392	19.86	-1177
2.735	2.0	302	-391	18.09	-1176
2.740	1.9	296	-391	17.22	-1176
2.750	1.48	293	-389	13.43	-1173
2.760	1.1	291	-388	9.99	-1169
2.770	.9	295	-386	8.16	-1164
2.780	.8	296	-385	7.25	-1159
2.790	.7	297	-384	6.34	-1155
2.800	.6	298	-382	5.43	-1150
2.810	.5	301	-381	4.52	-1145
2.850	.4	308	-375	3.61	-1127
2.900	.3	313	-369	2.70	-1106
3.000	.22	320	-356	1.98	-1067

Note: Resonance was determined by the frequency of maximum resistive impedance

Impedance for Quartz Transducer



Unknown: ceramic, 1.02 MHz resonance, without cable
Shunt: none
Initial Null: R = 0.0, X = 2500 up to 1037 KHz; X = 3500 thereafter

freq (KHz)	R_d	X_d	X_m	R	X
900	1.7	360	-238	1.7	-238
902.5	2.5	360	-237	2.5	-237
906.6	2.2	340	-238	2.2	-238
910	2.2	350	-236	2.2	-236
920	3.2	370	-232	3.2	-232
940	5.2	390	-224	5.2	-224
950	5.0	440	-217	5.0	-217
955	6.2	435	-216	6.2	-216
957.5	7.7	450	-214	7.2	-214
960	8.0	440	-215	8.0	-215
965	7.6	455	-212	7.6	-212
967.5	6.8	475	-209	6.8	-209
970	9.2	485	-208	9.2	-208
972.5	9.6	485	-208	9.6	-208
975	12.7	510	-204	12.7	-204
977.5	12.9	470	-208	12.9	-208
980	11.4	485	-206	11.4	-206
985	13.0	520	-201	13.0	-201
990	19.8	600	-192	19.8	-192
992.5	22.4	560	-195	22.4	-195
995	26.2	550	-196	26.2	-196
997.5	22.8	515	-199	22.8	-199
1000	19.2	600	-190	19.2	-190
1002.5	29.0	670	-183	29.0	-183
1005	33.8	620	-187	33.8	-187
1007.5	45	660	-183	45	-183
1010	49	520	-196	49	-196
1015.5	46	670	-180	46	-180
1025	69	350	-210	69	-210
1030	75	95	-233	75	-233
1031	72	30	-240	72	-240
1032	65	10	-241	65	-241
1033	60	30	-239	60	-239
1034	60	85	-234	60	-234
(note 1)					
1034	56	30	-239	56	0239
1035	60	100	-232	60	-232
1036	67	100	-232	67	-232
1037	78	940	-247	78	-247
1038	69	800	-260	69	-260
1039	62	780	-262	62	-262
1040	54	740	-265	54	-265

(data continued)

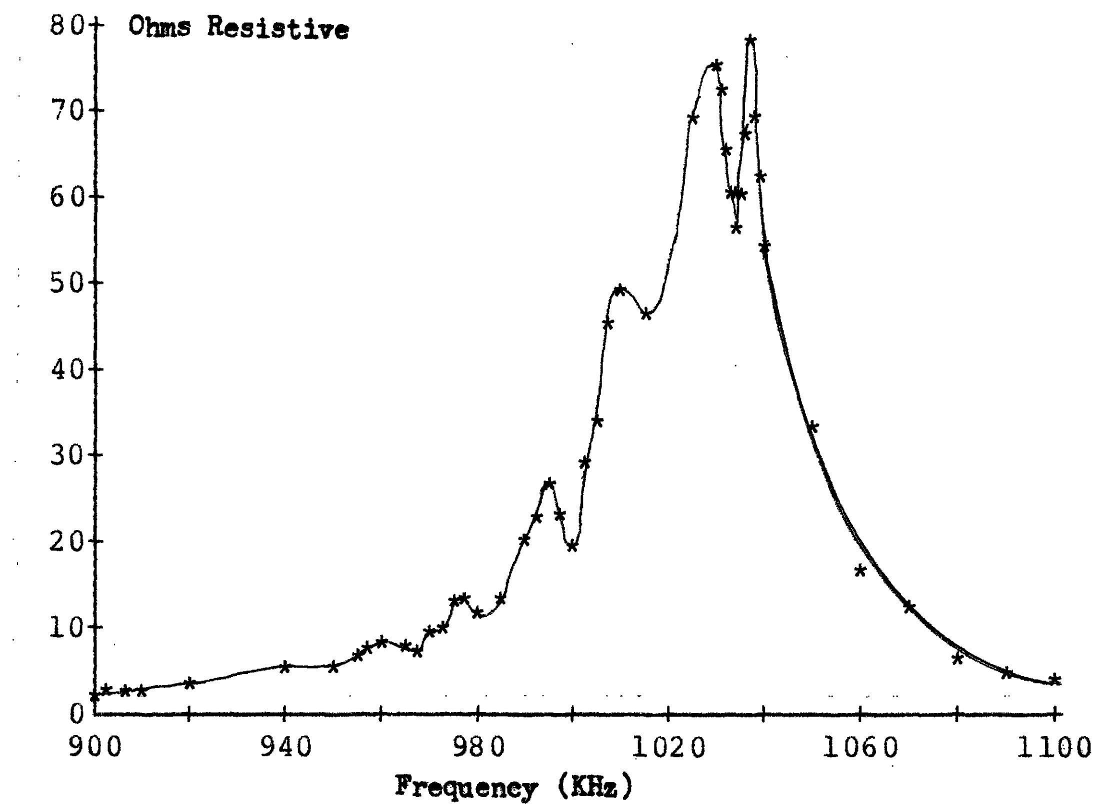
1050	33	760	-261	33	-261
1060	16.3	820	-253	16.3	-253
1070	12	920	-241	12	-241
1080	6	960	-235	6	-235
1090	4.5	1000	-229	4.5	-229
1100	3.6	1010	-226	3.6	-226

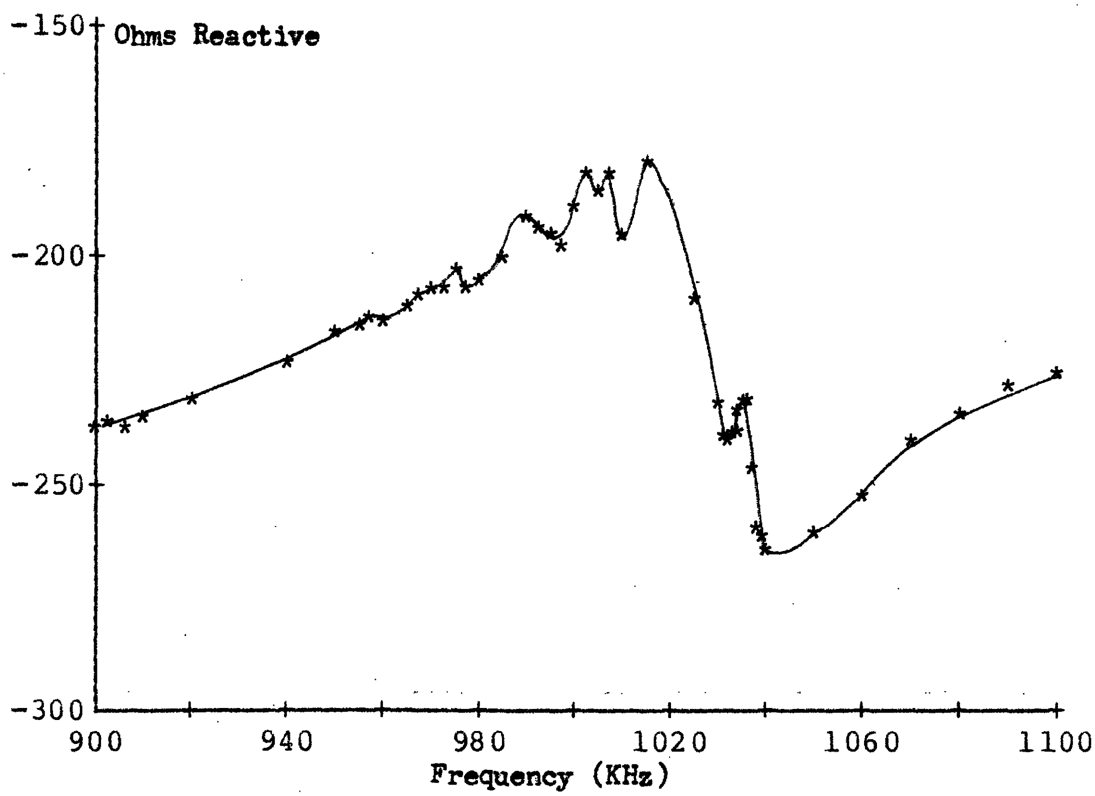
Note 1: Between measurements at 1034 KHz there was a one-hour delay. The reason for the discontinuity in impedance is unexplainable. Factors which could be ruled out are self-heating, since the input power was so low, and ferroelectric domain re-orientation, since the field intensity was so low.

Note 2: From 950 KHz to 1040 KHz there were many peaks and dips in the impedance curve, not all of which were recorded. Their amplitudes were generally less than five ohms.

Note 3: Resonance is determined by the frequency at which reactive impedance equals impedance due to static capacitance alone. A maximum in resistive impedance coincides.

Impedance of Ceramic Transducer





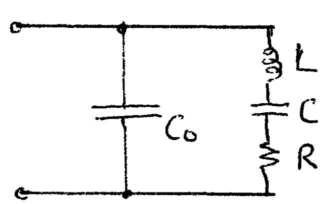
It is desirable to obtain values for the transducer model R and C in the simplest way possible. Electrical measurements are easily misunderstood if only a few measureemnts are made, since away from resonance the RC model is invalid. A whole curve should be reproduced to verify resonance frequency and resonance RC values, and this is time consuming. An easier though less accurate method is mechanical measurement and calculation of electrical values. The equations shown below use the electro-mechanical transformation factor, α, described in Chapter IV,[7] mechanical quantities:

motional mass $M = \frac{\rho S \ell}{2}$

motional stiffness $K = \frac{\pi^2 c_{ii} S}{2\ell}$

radiation resistance $Z_R = \rho_o c_o S$

electrical model:



Transformation factor:

Eletrical quantities:

$L = \frac{M}{4\alpha^2}$ $R = \frac{Z_R}{4\alpha^2}$

$C = \frac{4\alpha^2}{K}$ $C_o = \frac{\epsilon_o \epsilon_r S}{\ell}$

Again, of interest is the operation at resonant frequency so that L and C cancel and a parallel RC_o remains.

A range of transducer impedances found in the laboratory was obtained by selecting large and small piezoelectric discs of both quartz and ceramic, measuring them physically, and calculating their static capacitance and radiation resistance. There are two disc areas that are important. The static capacitance depends on the area of the spring-loaded electrode which is one plate of the parallel-plate capacitor. The radiation resistance depends on the total disc area which is acoustically coupled to the acoustic load. The piezoelectric disc's diameter is measured and the electrode is assumed to be of radius 1 cm smaller, to prevent arcing between electrod and mounting head which may have up to 10 KV potential difference. The measurements are shown below.

<u>Range of Transducers in the Laboratory</u>

<u>quartz:</u>

(1)	15.0 cm diameter,	.318 cm thickness,	C = 170pF,	R = 9.9K ohms
(2)	12.7 cm diameter,	.477 cm thickness,	C = 77pF,	R = 3 K ohms
(3)	8.25 cm diameter,	1.91 cm thickness,	C = 7pF,	R = 1 M ohms
(4)	8.25 cm diameter,	.312 cm thickness,	C = 40pF,	R = 50 K ohms

<u>ceramic:</u>

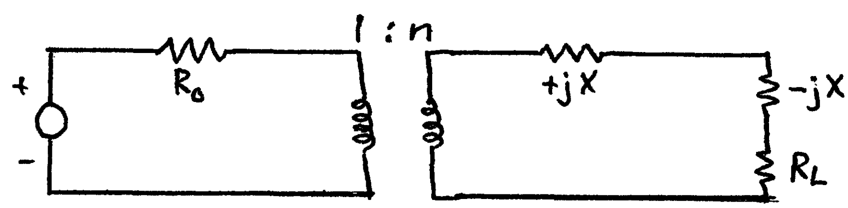
(1)	14.5 cm diameter,	.280 cm thickness,	C = 20 nF,	R = 210 ohms
(2)	8.25 cm diameter,	.278 cm thickness,	C = 1 nF,	R = 830 ohms

Typical transducers are the quartz #4 and ceramic #2. In deciding the range of the impedance matching circuits, it may be necessary to forego extreme transducer loads, but in all cases typical transducers must be matched.

CHAPTER VI

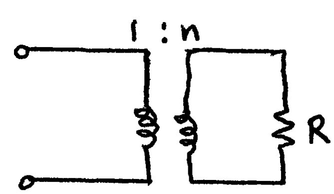
IMPEDANCE MATCHING CIRCUITS

With knowledge of what electrical load the transducer and cable comprise, an impedance match can be designed. To reiterate, the goal is maximum power transfer from electrical source to load, which means the load impedance must be a complex conjugate of the source impedance. The transducer load impedance is that of a parallel RC with a large reactive component while the source impedance is 50 ohms resistive. The impedance matching circuit would then consist of two parts: a tuning element to cancel the load reactance, and a transforming element to change the load resistance. Symbolically this is represented below.



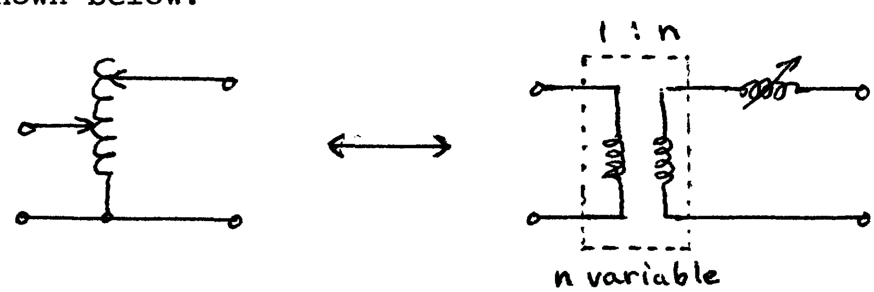
The reactive elements, jX and -jX, sum to zero, and the transformer multiplies the impedance according to its turns ratio squared, as shown below.

$$R_{in} = \frac{R}{n^2}$$



Note the transformation by $1/n^2$ is valid only for the ideal transformer where there is perfect coupling between primary and secondary and no core losses. These two requirements cannot be met simultaneously in practice, though for low frequencies and low power levels they can be closely approximated. Perfect coupling requires all magnetic flux produced by one winding to link the other winding. A ferromagnetic core (high magnetic susceptibility) common to both coils helps by containing the flux within itself, but since ferromagnetic materials are also conductors, eddy currents will circulate in the core causing substantial core heating, hence power loss. Air cores have virtually zero losses, but are paramagnetic and will not confine magnetic flux, leading to loose coupling.

A reduction in wire losses and physical size results from reusing the primary winding in an autotransformer configuration. Furthermore, the loose coupling between primary and secondary (in their entirety, not the common winding) causes substantial leakage inductance in the secondary so that the reactive impedance matching element is achieved. The model, using an ideal transformer, is shown below.



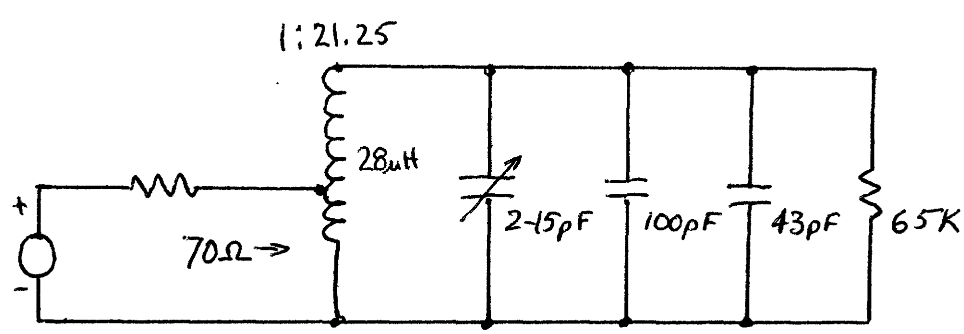
A primary winding is adjustable to achieve a range of transformation ratios to match a variety of load resistances, and the secondary is adjustable to provide a range for cancelling a variety of load reactances.

6.1 Prototype Circuits

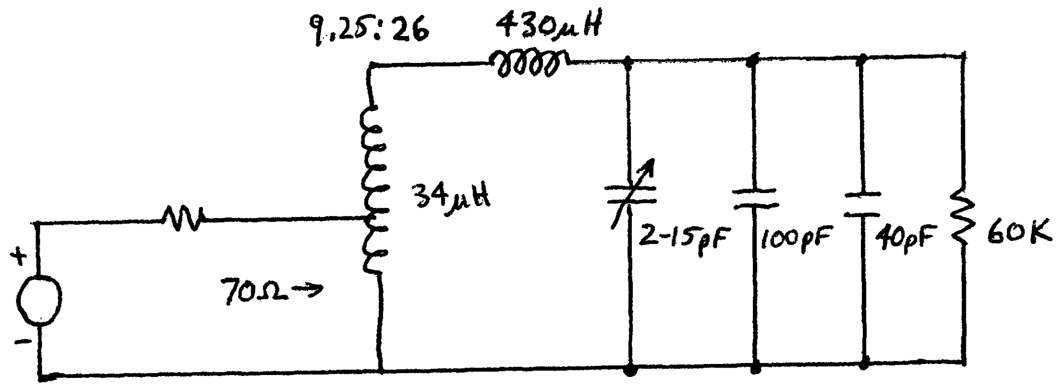
Several prototype systems were built using an autotransformer. Every attempt to maximize power was successful. The procedure is briefly as follows. Transducer and cable load were measured at resonance. Values for tuning inductance and turns ratio were determined. Using an inductor design nomograph, suitable inductors were built or obtained. Input impedance of the matching circuit with load attached was measured on the bridge, and inductor taps were adjusted for optimal impedance (70 ohms resistive). The matching circuit and load were driven by a low-power amplifier and taps were again adjusted to provide maximum acoustic output. An adjustable capacitor maximized acoustic output which was observed as water bath displacement and "cold steam."

Various circuits and component values for the test cases are shown below.

First test: quartz transducer at 2.72 MHz

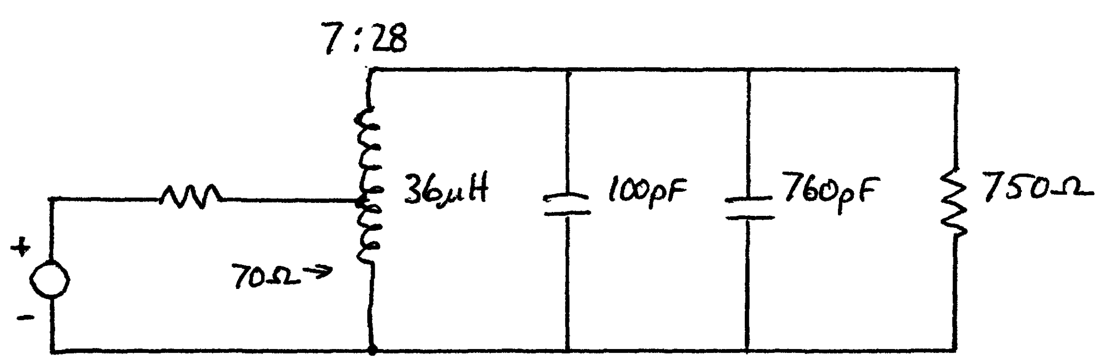


Second test: quartz transducer at 600 KHz

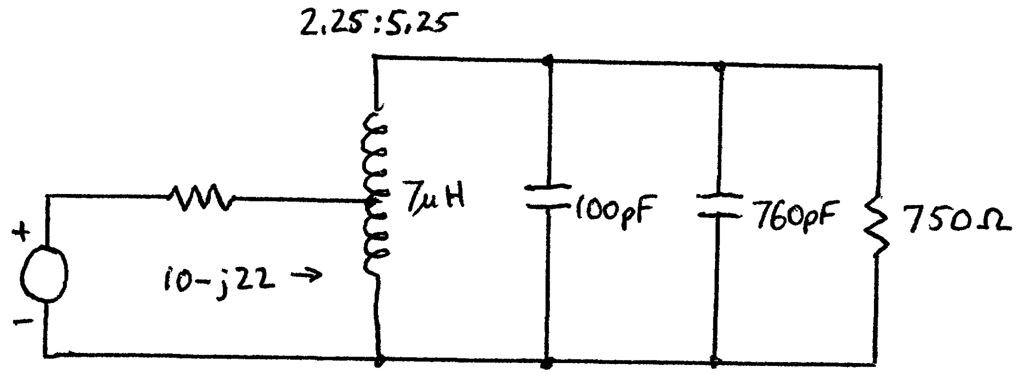


Third test: ceramic at 1.02 MHz

Note: Good acoustic output could be achieved over a frequency range of 970 KHz to 1.5 MHz and coil settings ±1 turn

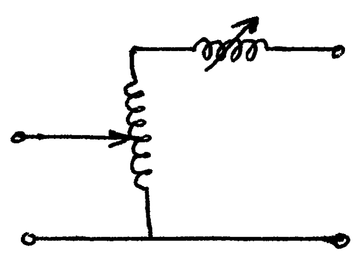


Fourth test: ceramic at 3.07 MHz



6.2 Two-Inductor Circuits

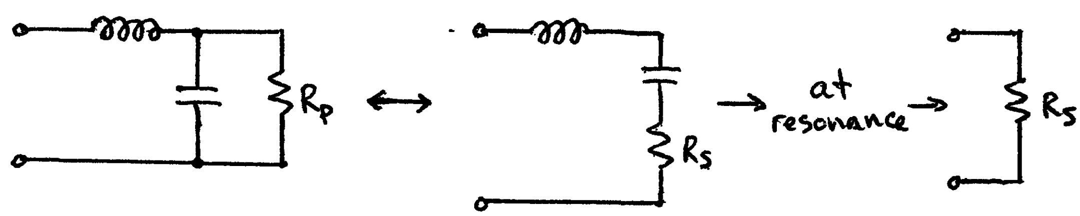
Unfortunately, autotransformers of sufficient size are not available commercially, so an alternative method was developed. The alternative method, which was implemented, uses a combination of autotransformer and inductor, both of which are singly-adjustable inductors, as shown below.



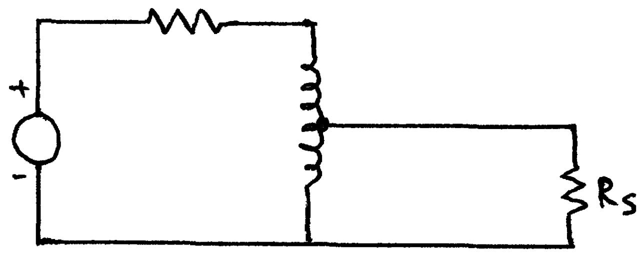
The two-terminal inductor performs load reactance cancellation while the three-terminal autotransformer performs impedance transformation as well as load reactance cancellation because of its leakage inductance.

The tap settings are difficult to analyze. The primary is difficult since the impedance is not transformed by the turns ratio of the autotransformer. Moreover, the setting of the primary affects the inductance of the secondary. It will be assumed that as the primary tap is increased, the input impedance approaches a value between R_{series} and $R_{parallel}$ of the load. It is not reasonable to assume that the variable inductor completely cancels the load capacitance and therefore increasing the tap causes the impedance to approach R_{series}. This false assumption makes some theoretical sense, but is not observed in the prototype circuits. Its rationale is presented below.

If the variable inductor cancelled load capacitance, the following circuits would be equivalent, and at resonance, R_{series} would result.



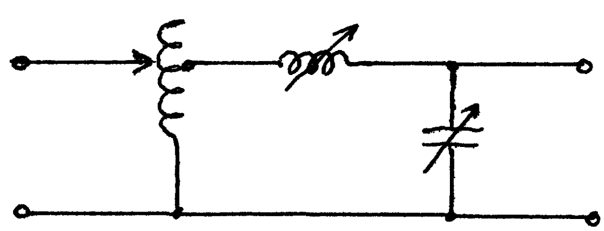
Since R_{series} is a small fraction of $R_{parallel}$, an impedance set-up would be necessary to match it to a 50 ohm source impedance, as shown below.



In actual tests, the prototype worked with a step-down transformer to the source. This is due to a leakage inductance which is distributed throughout the autotransformer so that changing tap position leads to a progressive loading effect at the primary.

It is possible that an exact analysis of the autotransformer could be obtained by using Maxwell's formulas for concentric solenoids or integrating the mutual inductance of many single turn loops, but this degree of analysis proved unnecessary. Instead, the design process was as follows. The total inductance required to cancel the load capacitance was determined for the range of transducers at a given frequency. If the inductance range was too large, a variable capacitor was used in parallel with the load. Since the coils are continuously adjustable, the turns ratio is not a specific design criterion.

The design circuit topology is shown below.

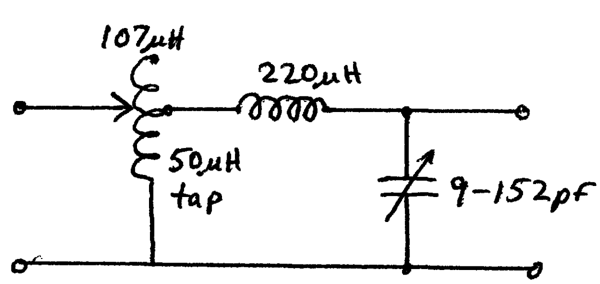


The addition of the variable capacitor lowers the circuit Q, though the efficiency remains unchanged since no power is dissipated in a capacitor, except for dielectric losses which are virtually zero for air or vacuum capacitors.

The circuit values were determined as follows. Three quartz transducers and two ceramic transducers have capacitances of: (1) 210 pF, (2) 95 pF, (3) 8 pF, (4) 700 pF, (5) 5 nF. Mounting head and cable capacitances add 100 pF. The following table is useful for determining rough inductance values for matching the load capacitance directly.

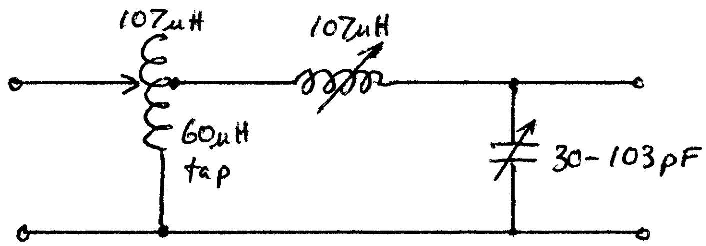
	(1)	(2)	(3)	(4)	(5)
600 KHz	230 uH	360 uH	650 uH	100 uH	14 uH
950 KHz	90 uH	140 uH	260 uH	40 uH	6 uH
3 MHz	9 uH	14 uH	26 uH	4 uH	.6 uH

<u>The 600 KHz circuit for quartz:</u> Using a variable capacitor of 9 to 152 pF the capacitance range is 260 pF (= 108 pF + 152 pF) to 320 pF (= 310 pF + 9 pF), thus needing an inductance of 220 uH to 270 uH. Choosing a fixed inductor of 270 uH allows good matching except for the largest quartz disc. The load resistance ranges from 10K to 1M so a variable autotransformer is needed. The best compromise is shown below.



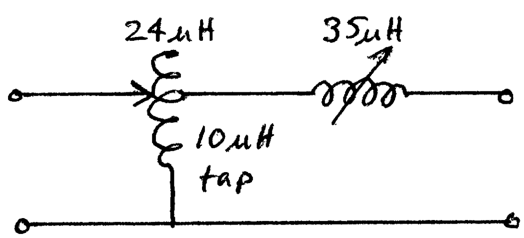
The circuit matches capacitive load from 110 pF to 250 pF.

<u>The 900 KHz circuit for quartz:</u> Using a 30-103 pF adjustable capacitor, the capacitance range is 210 pF (= 108 pF + 103 pF) to 340 pF (= 310 pF + 30 pF), needing an inductance range of 90 uH to 150 uH.



The circuit matches capacitive load from 62 pF to 440 pF.

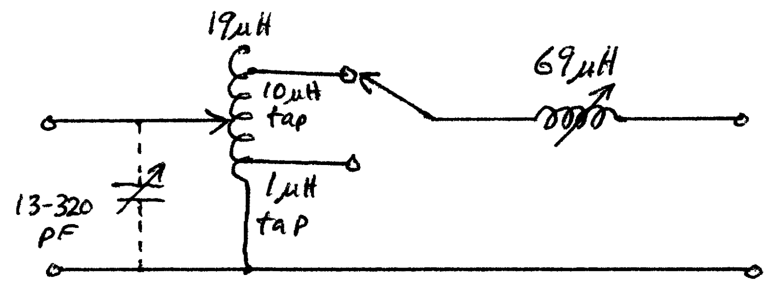
<u>The 2.7 MHz circuit for quartz:</u> Since the inductance range is small, no adjustable capacitor is necessary. The load capacitance range is 110 pF to 310 pF, needing an inductance range of 11 uH to 32 uH.



The circuit matches capacitive loads from 77 pF to 350 pF.

<u>The 600 KHz, 1 MHz, and 3 MHz circuit for ceramics:</u> A wide range of inductance is needed, down to 1 uH and lower. A high power switch achieves the necessary range as shown below, along with the load capacitance ranges for various

frequencies. The adjustable capacitor may be needed for canceling inductive reactance at the primary when the setting produces excessive leakage inductance.



Ranges for C_{load}

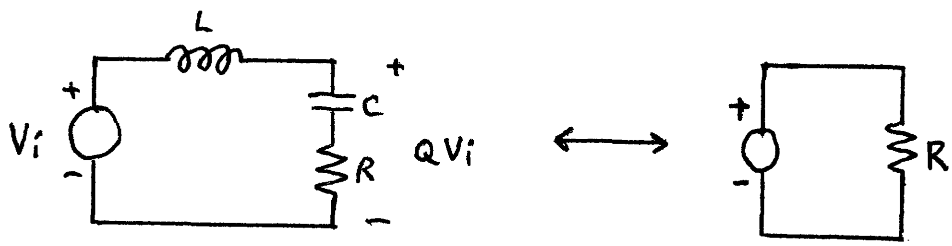
	10 uH tap	1 uH tap
600 KHz	890 pF - 7 nF	8.9 nF - 70 nF
1 MHz	320 pF - 2.5 nF	3.2 nF - 25 nF
3 MHz	37 pF - 280 pF	370 pF - 2.6 nF

These four circuits have overlapping ranges to allow simultaneous operation of similar set-ups. They also gain flexibility due to possible use outside their frequency ranges. To determine the load capacitance they will completely cancel, use the formula $C = 1/\omega^2 L$.

6.3 Maximum Current and Voltage

Other design criteria include sizing the coils and capacitors for maximum current and voltage. These ratings are determined by analyzing the circuits as a mixture between series and parallel RLC circuits. The topology of the circuit is nearly a series RLC, but measurements made on the prototype circuits indicate a high impedance at electrical resonance which is the behavior of a parallel RLC. First the analysis of series and parallel RLC circuits is given, then the practical combination of the two.

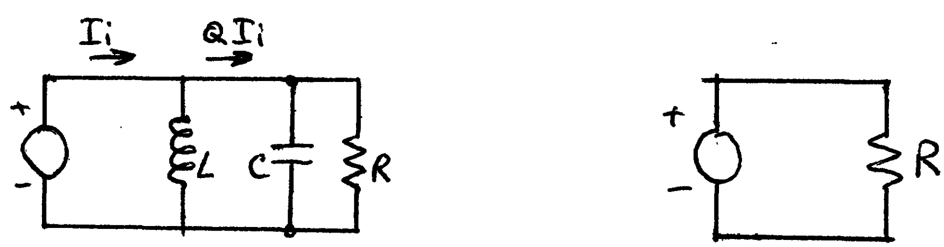
For series RLC, current through each element is equal, but load voltage is Q times the source voltage, where Q is the electrical quality factor.(10) This is shown below.



where

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

For parallel RLC, the voltage across the coil and load is the same, but the current through them is Q time higher, as shown below.(11)



where

$$Q = \frac{1}{R_s}\sqrt{\frac{L}{C}}$$

$$R_s \approx \frac{1}{R\,C^2\omega^2}$$

Where R_p is very large, as for quartz transducers, the matching circuit behaves more like the series RLC with an impedance of R_s, much lower than the parallel load resistance. This makes sense because the quartz transducer has high impedance so voltage tends to be high and current low for a given power level. When R_p is low, as for ceramic transducers, the matchint circuit behaves like a parallel RLC, evidenced by an observed impedance similar to R_p. Thus current is high and voltage low for a given power level. Below is a table of transducer values, matching coil values, and electrical Qs. Note that R_s is used to calculate Q.

	freq	C	L	R_s	Q
ceramic	600 KHz	700 pF	100 uH	72	5
	1 MHz	700 pF	36 uH	26	9
	3 MHz	700 pF	4 uH	3	25
quartz	600 KHz	260 pF	270 uH	13	80
	900 KHz	260 pF	120 uH	6	110
	2.7 MHz	150 pF	23 uH	2	200

Assuming 1 KW is input to the impedance matching circuit, the input voltage is $V = PR^2 = 220\ V_{rms}$. The maximum load voltage would then be 50 to 100 times that, or 10 KV to 20 KV. Due to the poor availability and high cost of voltage components, a limit of 10 KV peak was chosen, which also ensures a reasonable voltage gradient across the piezoelectric.

Assuming maximum source current is output (about 5 Amps for laboratory sources) into 5 ohms, again giving a power of 1 KW, the maximum load current would be 5 to 10 times that.

A maximum coil current of 30 A was chosen for the ceramic matching circuit.

The maximum load current of the quartz matching circuit is nearly the same as the maximum source output current since the circuit behaves in a series RLC manner. Thus 15 A coils are sufficient.

6.4 Using the Impedance Matching Circuits

The electrical connections consist of a cable from low impedance or 50 ohm output of the electrical source to the input of the impedance matching circuit, and a cable from output of the circuit to the transducer. The first cable and its connectors should be low voltage (250 V rms) and high power (up to 1 KW) but the second cable must be high voltage (10 KV peak) as well as high power (up to 1 KW) and high current (15 A). High voltage is needed for the quartz transducers, high current for the ceramic transducers.

The control settings are achieved by successive adjustment of inductor and autotransformer from predetermined approximate settings. The approximate setting for the inductor is given by $L \approx 1/C\omega^2 - L_x$, where C is the load capacitance and L_x is the inductance of the autotransformer. The approximate setting for the autotransformer will be given by an equation of turns ratio to resistance transformation which must be determined empirically with the actual coils used. After presetting the controls, some optimization may be necessary.

Due to the interaction between inductor setting and input impedance, the autotransformer setting and output inductive impedance, successive adjustment of the controls may be necessary. Since the transducer load is the basis for setting the controls and the transducer may heat up under use and change its electrical properties, readjustment of controls after a period of time may be necessary.

A complete instruction manual with specific operating procedure and examples will be written when the circuits are completed. A FORTRAN computer program may also be written if the calculations are involved or difficult.

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