QUASIOPTICAL TRANSFORMER WHICH TRANSFORMS THE WAVES IN A WAVEGUIDE HAVING A CIRCULAR CROSS SECTION INTO A HIGHLY DIRECTIONAL WAVE BEAM

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UDC 621.372.8.09

The construction is proposed and the calculation is presented of a quasioptical transformer which accomplishes the transformation of waves having the azimuthal indices m = 0, 1 in an ultradimensional waveguide having a circular cross section into a highly directional wave beam or into the principal wave of a rectangular waveguide. The transformation of the H_{14} wave of a circular waveguide into the principal wave of a rectangular waveguide with a power transmission factor of 0.7, or into the natural wave of a quasioptical mirror waveguide with a power transmission factor of 0.75 was accomplished experimentally.

In using cyclotron-resonance maser oscillators from which the high-frequency power is extracted by means of the higher-mode waves propagating in ultradimensional waveguides having a circular cross section, the necessity arises of transforming such waves into lower-mode waves or into quasioptical wave beams. The present paper, which is an expanded exposition of a portion of the report [1], is devoted to the theoretical and experimental investigation of a transformer that transforms the waves having azimuthal indices m = 0, 1 in a waveguide having a circular cross section into a wave beam. A radiator proposed in [2] for symmetrical wave modes is used in the transformer.

1. Theoretical Analysis of the Operation of the Transformer

The principle of operation of the transformer may be clarified from geometric-optics concepts. The natural waves of a waveguide having a circular cross section are represented in the form of a superposition of plane waves whose wave vectors $\mathbf{k}$ form the angles $\theta = \arcsin \varkappa_{mp}/\mathbf{k}$ with the z axis according to the Brillouin concept, where $k = 2\pi/\lambda$, and $\varkappa_{mp}$ is the transverse wave number (for H_{mp}-waves we have $\varkappa_{mp} = \nu'_{mp}/a$, ν'_{mp} is the root of the equation $J'_m(\nu'_{mp}) = 0$; for E_{mn}-waves we have $\varkappa_{mp} = \mu_{mp}/a$, μ_{mp} is the root of the equation $J_m(\mu_{mp}) = 0$; J_m, J'_m are the Bessel function and its derivative, respectively).

The radiation field of waves having the azimuthal indices m = 0, 1 near the open end of an ultradimensional waveguide constitutes a set of diverging waves that are bounded by the waveguide aperture and make an angle θ with the z axis. If the waveguide has a baffle plate, as is shown in Fig. 1, the radiation will occur in the interval of angles $-\pi/2 < \varphi < \pi/2$. The field of the type indicated is transformed into a highly directional wave beam by means of a reflector – a parabolic cylinder whose focal axis coincides with the waveguide axis. In the geometric-optics approximation the beam width in the $\varphi = \pi/2$ plane is equal to 4F, where F is the focal distance of the parabolic cylinder and the field distribution is close to uniform for symmetrical waves and close to cosinusoidal for waves having m = 1; the width of the wave beam in the $\varphi = 0$ plane is equal to $4a \cos\theta$, and the field distribution in it is uniform. The polarization of the radiation beyond the parabolic mirror will be linear. Figure 1 displays the vector $\mathbf{E}$ in the wave beam for the case when a H-mode wave propagates in the waveguide.

The field distribution at the output of the transformer, as found in the geometric-optics approximation, may be used to calculate the radiation field by means of the Kirchhoff integral. In such an approximation diffraction inside the transformer is not taken into account.

Scientific-Research Radio-Physics Institute. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 17, No. 1, pp. 148-154, January, 1974. Original article submitted March 26, 1973.

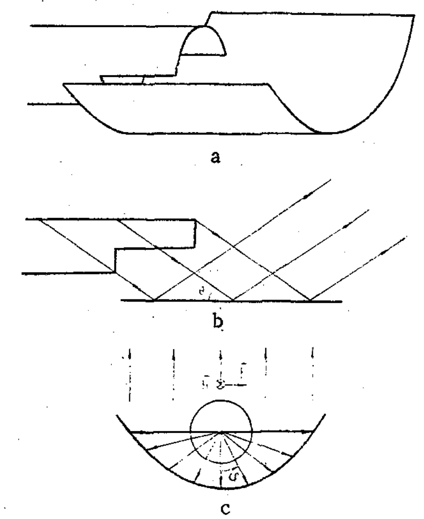

Fig. 1. Quasioptical transformer which transforms H_{0p}, H_{1p} waves into a wave beam.

Corrections to the geometric-optics structure of the field which develops as a consequence of diffraction may be estimated as follows. We shall assume that radiation is incident on the parabolic reflector from the inside surface of the waveguide and baffle plate, which is bounded by the angle values $\pi/2 < |\varphi| < \pi$, while the function describing the dependence of the field on φ inside the waveguide and on the baffle plate has the form

$$g = \begin{cases} \text{const} & \left(\frac{\pi}{2} < |\varphi| < \pi\right) \\ 0 & \left(0 < |\varphi| < \frac{\pi}{2}\right) \end{cases} \tag{1a}$$

for H_{0p}-, E_{0p}-waves and

$$\bar{g} = \begin{cases} \cos\varphi & \left(\frac{\pi}{2} < |\varphi| < \pi\right) \\ 0 & \left(0 < |\varphi| < \frac{\pi}{2}\right) \end{cases} \tag{1b}$$

for H_{1p}-, E_{1p}-waves.

Let us expand the field in a series of φ, and let us apply the following formula to each of the angular harmonics which are proportional to $e^{im\varphi}$ [3]:

$$f_2(z - a\,\mathrm{ctg}\,\theta) = \exp\left\{-ikz\cos\theta - 2i\left[\sqrt{k^2a^2\sin^2\theta - m^2} - m\arcsin d_m - \frac{\pi}{4}\right]\right\}\sqrt{\frac{ik\sin\theta}{4\pi b_m}}\int_{-l_1}^{l_1} f_1(z' + a\,\mathrm{ctg}\,\theta)\exp\left[-\frac{ik\sin^3\theta}{4b_m}(z - z')^2\right]dz', \tag{2}$$

where

$$b_m = a\sqrt{1 - \frac{m^2}{k^2a^2}}, \quad d_m = \sqrt{1 - \frac{m^2}{k^2a^2\sin^2\theta}},$$

while f_1 and f_2 are functions of the coordinate z which describes the field distribution of the m-th harmonic on a cylinder of radius a in waves propagating toward the z axis and from the z axis, respectively, at an angle $\theta = \arccos(h/k)$; h is the propagation constant of the wave in the waveguide. In (2) it is assumed that f_1 is nonvanishing only for $-l_1 < z' + a\cot\theta < l_1$, while f_1 and f_2 vary slightly over dimensions of the order of a wavelength.

If the dependence of the fields on φ can be described with a sufficient degree of accuracy by a superposition of harmonics having $m \ll ka\sin\theta$, so that for each of them the dependence of b_m and d_m on φ may be neglected, then for propagation of the field from the waveguide surface onto the surface near the parabolic mirror no substantial distortions develop. For propagation of fields of the type (1) distortions may be neglected within ≈5-10% for $\varkappa_{mp} \approx 10$, since in this case the first five harmonics ($m = 0, \pm1, \pm2$) already contain more than 95% of the beam energy.

The diffraction corrections to the field distribution with respect to the coordinate z found in the geometric-optics approximation will be small if for each of the harmonics having $m \ll ka$ the condition

$$\frac{4a^2}{\lambda(F + a)}\cos^2\theta\sin\theta \gg 1, \tag{3}$$

is fulfilled; for violation of the conditions (3) the spreading of the beam with respect to z inside the radiator may be taken into account by using Eq. (2) and stipulating the initial field distribution on the surface of the waveguide and the baffle plate rather than on the parabolic mirror.

Let us estimate the coefficients of transformation of the obtained distributions into the principal mode of a rectangular waveguide and into a Gaussian beam while restricting the analysis to transformers with one intermediate phase corrector [5, 6]. The transformation coefficient in this case could be equal to unity only for an infinite corrector for the transformation of fields satisfying the relationships derived in [5]. In our case these relationships are not fulfilled, and therefore the transformation coefficient is less than unity. For obtaining the lower bound of the transformation efficiency and choosing the correctors it

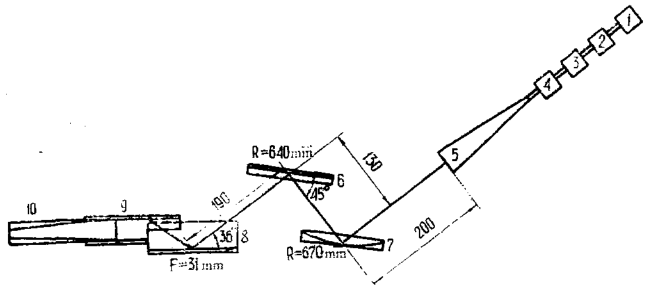

Fig. 2. Diagram of the excitation of the H_{14} wave of a circular waveguide by the principal wave of a horn: 1) oscillator; 2) wavemeter; 3) attenuator; 4) measurement line; 5) horn; 6, 7) correcting mirrors; 8) parabolic cylinder; 9) waveguide with baffle plate; 10) conical bushing.

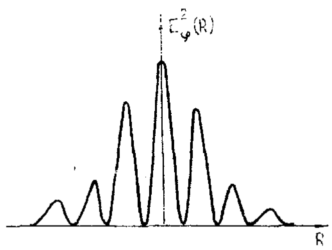

Fig. 3. Distribution of the field intensity of the H_{14} wave in a circular waveguide excited by a quasioptical transformer.

is expedient to apply a simplified procedure which is based on approximating the transverse field distribution in the wave beam traveling from the parabolic mirror by Gaussian functions $\exp(-r^2/\omega^2)$; the field distribution of the П-type having a width 2d corresponds to $\omega = 0.94d$, while the cosinusoidal distribution $\cos(\pi x/2d)$ having the width 2d corresponds to $\omega = 0.70d$ [10] For such an approximation the greatest transformation factor is attained when quadratic correctors (lenses or mirrors) are used, while the difference of the transformation coefficient from unity is due to the difference of the real field structure from a Gaussian structure. In accordance with such a calculation the maximum transformation coefficient in the case of optimal selection of the corrector may amount to 0.7 and 0.8 for transformation of H_{0p}- and H_{1p}-waves into the natural wave of a rectangular horn. The coefficients for transformation into a Gaussian beam amount to 0.8 and 0.9 for H_{0p}- and H_{1p}-waves, respectively. Note that the magnitude of the transformation losses is comparable with the accuracy with which the fields are approximated on the parabolic mirror by the П and cosinusoidal distributions. In order to increase the transformation coefficient it is necessary not only to use nonquadratic correctors but also to take account of the diffraction inside the transformer more correctly.

2. Experimental Investigation of the Transformers

Quasioptical transformers of the type described above were used a) for matching the principal waves of a horn that goes over into a one-mode waveguide to the H_{14} wave of a waveguide having a circular cross section; b) for transforming the H_{03} wave of a circular waveguide into the natural wave of a mirror quasioptical transmission line. The experiment was carried out at the wavelength $\lambda = 5.1$ mm. The waveguide diameter was equal to 32 mm.

a) Matching of the principal wave of the horn to the H_{14} wave was accomplished according to the scheme depicted in Fig. 2. The horn having an aperture 42×36 mm had a length of 250 mm and made the transition into a standard one-mode waveguide 3.6×1.8 mm. The focal distance of the parabolic cylinder was equal to 31 mm. Auxiliary correcting mirrors were used for matching.

A cylindrical transfer cavity resonator with the H_{141} oscillation mode was used to tune to maximum excitation of the H_{14} wave. The indicator of maximum excitation of oscillations in the resonator was a detector coupled to the resonator by a one-mode waveguide excited through a coupling aperture on the cylindrical surface of the resonator. After tuning to maximum excitation of the wave, the resonator was cut out of the path.

Fig. 3 displays the distribution $E^2_\varphi(R)$ on the aperture of the waveguide having a circular cross section in the plane perpendicular to the electric field lines, as obtained by a probe consisting of a one-mode waveguide without a flange. The high purity of the excitation of such a complex wave as the H_{14} wave via a quasioptical transformer should be noted.

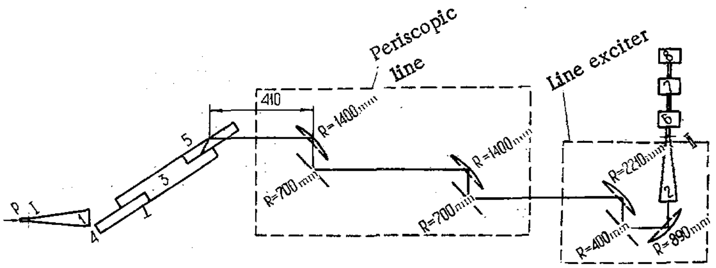

Fig. 4. Diagram of the excitation of a mirror quasioptical transmission line: 1, 2) horn; 3) circular waveguide; 4, 5) parabolic cylinders; 6) attenuator; 7) detectors; 8) amplifier.

In order to determine the magnitude of the coefficient of transformation of the principal wave of the horn into the H_{14} wave of a waveguide having a circular cross section the method of finding the scattering matrix of a fourport [5] by measuring the complex reflection coefficient at its output was used. In the system considered a measurement line connected directly to the horn was used to determine the complex reflection coefficient for movement of a reflecting plunger in the circular waveguide. The proposed method of finding the transformation coefficient as the power transmission factor of a fourport is valid if the plunger reflects the measured wave completely, does not impede the radiation of parasitic waves from the waveguide, and does not reradiate the principal wave into parasitic waves. In the experiment a bushing consisting of a conical transition from a diameter of 32 mm to a diameter of 16 mm and having a length of 250 mm was used as the plunger. Under these conditions the transformation of the H_{14} wave into parasitic waves on the cone was estimated to be several percent.

The conical transition completely reflected not only the measured wave but also those parasitic waves whose propagation constants were less than the propagation constant of the H_{14} wave; this would substantially complicate the interpretation of the results of the experiment if the excitation of parasitic waves in the circular waveguide were to be significant. However, the points which map the complex reflection coefficient in a polar coordinate system fell on a closed curve which was close to an ellipse having a small eccentricity and a center coinciding with the origin.

A curve having such a shape is evidence of the fact that only one parasitic wave is reflected from the plunger or develops on it with a noticeable amplitude: the modulus of the reflection coefficient is maximal when the fields of the reflected waves (the principal wave and the parasitic wave) in the one-mode waveguide measurement line turn out to be in phase; it is minimal if they are in phase opposition. The coincidence of the center of the ellipse with the origin is quite natural, since for the dimensions used for the horn and the waveguide having the circular cross section the coefficients s_{11} and s_{22} of the scattering matrix of the equivalent fourport are small. In the described experiment the major half-axis of the ellipse was equal to 0.78, while the minor half-axis was equal to 0.62; this corresponds to a voltage reflection coefficient of the H_{14} wave of approximately 0.70. The desired power transformation coefficient is also equal to this value as a consequence of the smallness of the coefficients s_{11} and s_{22}.

The ohmic power losses in the horn, on the surfaces of the mirrors, and in the circular waveguide were estimated to be several percent. The measured value of the magnitudes of the transformation coefficient is in good agreement with the calculated result.

b) Matching of the ultradimensional and quasioptical waveguides was studied in a system consisting of a circular waveguide, a transformer which transforms a H_{03} wave into a wave beam, and a mirror periscopic transmission line (Fig. 4). The line was confocal with a distance between identical mirrors equal to 100 cm and a distance between the centers of mirrors in a pair equal to 13 cm; the mirror dimensions corresponded to a Fresnel parameter $C = 4$. The focal distance of the cylinder was equal to 16 mm in this experiment. The line was excited without intermediate matching mirrors, since for a mutual arrangement of the transformer and the line depicted in Fig. 4 the beam had dimensions and a phase-front curvature close to the corresponding values of the natural wave of the quasioptical line as the first pair of mirrors was approached. The H_{03} wave in the waveguide having the circular cross section was excited by the horn and the transformer, as depicted in Fig. 4. Tuning to the maximum excitation of the H_{03} wave was accomplished by means of an auxiliary cavity resonator with the H_{031} oscillation mode, which was coupled to a detector through an aperture in the cylindrical wall by a one-mode waveguide. The auxiliary resonator was cut out of the path after the system had been tuned. At the output the mirror line was matched to the horn of the system consisting of three mirrors with losses of 1 dB. The total losses in the system were equal to 5.3 dB. The losses for excitation of the H_{03} wave were determined by measuring the passage of power through the symmetrical system: horn–transformer–secular waveguide–transformer –horn, and amounted to 3 dB per exciter. Under these conditions the losses for excitation of the mirror line by the quasioptical transformer amounted to 1.3 dB, which corresponds to a magnitude ≈0.75 of the transformation coefficient.

Thus, in the investigated wave transformers the power wave transformation coefficient is only 0.7-0.75. It may be increased for a more exact choice of the transformer parameters based on consideration of diffraction inside the radiator. Such a calculation may be performed only by numerical methods. However, transformers that have already been developed may compete successfully in magnitude of the transformation coefficient with wave transformers of other types in those cases when the necessity arises to transform higher wave modes; under these conditions the advantage of the devices described here increases with a shortening of the wavelength to $\lambda \doteq 1$ mm, since the characteristics of other transformers become worse.

The authors are indebted to M. I. Petelin for his discussion, to V. S. Averbakh and N. M. Sheronova for providing the elements of the quasioptical waveguide, and to V. M. Perepelitsa for help in performing the measurements.

LITERATURE CITED

1. S. N. Vlasov, L. I. Zagryadskaya, N. F. Kovalev, I. M. Orlova, and M. I. Petelin, Report to the Fifth All-Union Symposium on the Diffraction and Propagation of Radio Waves [in Russian], Leningrad (1970).
2. L. I. Pangonis and M. V. Persikov, Radiotekhnika i Élektronika, 15, 2300 (1970).
3. N. A. Zinichev, Diplomate Paper, GGU [in Russian], Gor'kii (1970).
4. S. N. Vlasov, I. M. Orlova, and M. I. Petelin, Izv. VUZ, Radiofiz., 15, No. 12, 1913 (1972).
5. N. G. Bondarenko and V. I. Talanov, ibid., 7, No. 2, 313 (1964).
6. R. B. Vaganov, Radiotekhnika i Élektronika, 18, No. 1, 187 (1973).
7. V. S. Averbakh, S. N. Vlasov, É. M. Popova, and N. M. Sheronova, ibid., 11, No. 4, 750 (1966).
8. H. Kogelnick and T. Li, Trans. IEEE, 54, No. 10, 95 (1966).
9. E. Ginston, Measurements at Centimeter Wavelengths [Russian translation], IL, Moscow (1960).
10. R. B. Vaganov, Quadratic Phase Correction in Waveguide Quasioptics, Preprint of the Institute of Radio Electronics, Academy of Sciences of the USSR [in Russian], Moscow (1972).