PARAMETRICALLY AMPLIFIED MEMS MAGNETOMETER

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ABSTRACT
This paper demonstrates how parametric amplification can improve the system wide signal to noise ratio (SNR) of MEMS resonant force sensors. Experiments demonstrate how parametric amplification increases the mechanical sensitivity of a Lorentz force based MEMS magnetometer. This increase in mechanical sensitivity amplifies both the Lorentz and Brownian forces. The RMS displacement caused by the 39 nT/√Hz Brownian noise floor is amplified from 13.75 pm to 283 pm. In this study a maximum stable magnetic field sensitivity increase of 51 times was achieved which has an equivalent amplification to a linear resonator with a quality factor (Q) of 40,000.

KEYWORDS
Parametric, Parametric amplification, Lorentz Force, Magnetometer,

INTRODUCTION
State-of-the-art magnetoresistive (MR) magnetometers can achieve noise-limited resolutions on the order of 1 nT/√Hz [1]. In comparison, MEMS-based magnetometers have the advantage that they require no specialized materials, are free from hysteresis, and can be monolithically integrated with high-performance accelerometers to produce a single-chip inertial measurement unit (IMU). However, previous MEMS magnetometers have resolutions that are on the order of ~10 □T/√Hz [2] rendering these devices insufficiently sensitive for navigation applications. Recently Kyynarainen et al produced a magnetometer with a 10nT/√Hz [3]. This work describes an approach to reduce the size and power consumption of such sensors.

Sensor Noise
The noise-limited resolution of MEMS force sensors is determined by a combination of Brownian and electronic noise, as shown in Fig 1. Achieving Brownian noise-limited performance comes at the cost of increasing power consumption since a 50% reduction in electronic noise of the detection electronics requires a 16x increase in the bias current. We are pursuing an alternative route to Brownian noise-limited performance by increasing the force-to-displacement sensitivity of the sensor using parametric amplification [4]. Because the equivalent noise force resulting from electronic noise is inversely proportional to the force sensitivity, increasing this sensitivity allows Brownian noise-limited performance to be achieved using higher noise (and therefore lower power) detection electronics.

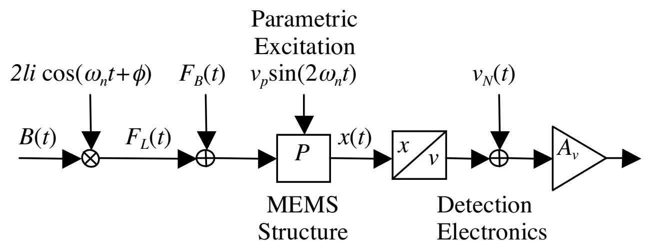


Figure 1: System block diagram. Lorentz force (F_L) and Brownian noise force (F_b) are input to the MEMS structure, resulting in a displacement, x. The input-referred noise of the detection electronics is indicated as v_N

Parametric Amplification
The equation of motion of a parametrically-excited spring-mass-damper system includes a time-varying mass or spring parameter. When the spring constant is modulated at twice the natural frequency the motion is described by the Mathieu equation:

$$m\ddot{x} + \frac{m\omega_n}{Q}\dot{x} + \left[k_{eff} + \Delta k \sin(2\omega t)\right]x = F_L(t)\cos\left(\omega t + \frac{\pi}{2}\right) \qquad (1)$$

Where m, Q, ω and ω_n represent the mass, quality factor, driving frequency and natural frequency of the device, Δk is the spring modulation and k_{eff} is the effective spring stiffness. Parametric amplification is based on decreasing the spring stiffness as the oscillator drives away from its center and increasing the spring stiffness as it drives towards its center, hence the spring is modulated at twice the driving frequency. The Mathieu equation can be solved using perturbations by separating the solution into two time scales; stretched time (ωt) and slow time (η) as shown by [5]:

$$x(t) = A(\eta)\cos(\omega t) + B(\eta)\sin(\omega t) \qquad (2)$$

Where A(η) and B(η) are slow time varying and can be calculated explicitly and converted back into normal time as shown in (3):

$$\begin{aligned} \frac{\partial A}{\partial t} &= \frac{\omega_n^2}{2k_{eff}\omega}\left[-A\frac{k_{eff}}{Q}\left(1-\frac{Q\Delta k}{2k_{eff}}\right) + B\left(k_{eff}-m\omega^2\right) + F_L(t)\right] \\ \frac{\partial B}{\partial t} &= \frac{\omega_n^2}{2k_{eff}\omega}\left[-B\frac{k_{eff}}{Q}\left(1+\frac{Q\Delta k}{2k_{eff}}\right) - A\left(k_{eff}-m\omega^2\right)\right] \end{aligned} \qquad (3)$$

Using (2) and (3) and assuming we are forcing the system at its natural frequency, $F_L(t) = F$ and $\omega = \omega_n$, the steady-state solution is given by:

$$x_{ss} = \frac{FQ}{k_{eff}}\left(1 - \frac{Q\Delta k}{2k_{eff}}\right)^{-1}\cos(\omega_n t) \tag{4}$$

In a linear resonator, $\Delta k = 0$ and the sensitivity is Q/k_{eff}. With parametric amplification Δk is arbitrarily set by the designer to obtain a desired additional amplification G_p, shown in (5)

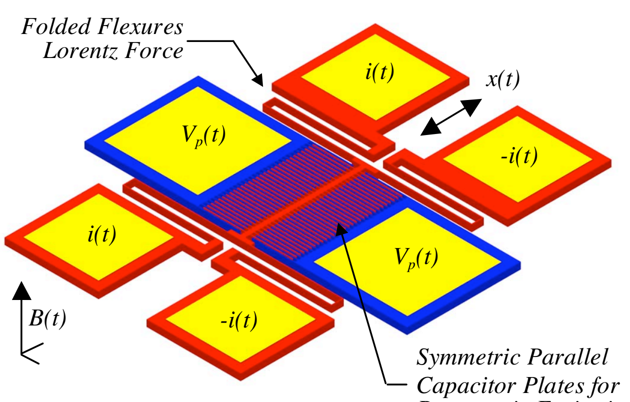
$$G_p = \left(1 - \frac{Q\Delta k}{2k_{eff}}\right)^{-1} \Leftrightarrow \Delta k = \frac{2k_{eff}}{Q}\left(1 - \frac{1}{G_p}\right)$$

$$x_{ss} = \frac{FQ_{eff}}{k_{eff}}\cos(\omega_n t) \qquad Q_{eff} = QG_p \tag{5}$$

The effective quality factor Q_{eff} is defined as the product of the mechanical quality factor and the parametric gain.

MAGNETOMETER DESIGN

The goal of the design is to manufacture a SOI magnetometer that would be suitable for navigation purposes using foundry fabrication techniques allowing for ease of commercialization.



$$i(t) = i\cos\left(\omega_n t + \frac{\pi}{2}\right) \qquad V_p(t) = \sqrt{V_{DC} - V_{AC}\sin(2\omega_n t)}$$

Figure 2: Schematic of the device showing the folded flexures, which generate the Lorentz force, and the symmetrically placed parallel plate electrodes used for parametric excitation of the structure.

The block diagram and layout of the SOI magnetometer are shown in Figs. 1 and 2, respectively. An out-of-plane magnetic field produces an in-plane Lorentz force that is modulated at the device's natural frequency using an ac current passing through the flexures.

$$F_L = Bli \tag{6}$$

Where B is the magnetic field, L is the length of the beam and i is the current. The flexure current is oscillated at the natural frequency of the MEMS magnetometer.

The device is parametrically excited by electrostatically modulating the mechanical spring stiffness at twice the natural frequency using a pump voltage applied to capacitive plates that are disposed symmetrically about the device. The force on the capacitive plates is given by:

$$F(x,V,t) = \frac{e_o A}{2}\left[\frac{1}{(g - x(t))^2} - \frac{1}{(g + x(t))^2}\right]V^2(t) \tag{7}$$

Where e_o is the permittivity, A is the area, g is the nominal gap, V is the voltage and x is the displacement. Expanding (7) and neglecting higher order terms we obtain:

$$F(x,V,t) \approx \frac{2e_o AV^2(t)}{g^3}x(t) \tag{8}$$

Therefore F is a function of displacement hence it acts like a spring where the voltage sets the constant. This allows the modulation of voltage to control the spring constant and if V is chosen as

$$V_p(t) = \sqrt{V_{DC} - V_{AC}\sin(2\omega t)} \qquad V_{DC} \geq V_{AC} \tag{9}$$

The spring terms for the Mathieu equation become.

$$k_{eff} = k - \frac{2e_o AV_{DC}}{g^3} \qquad \Delta k = \frac{2e_o AV_{AC}}{g^3} \tag{10}$$

Where k is the mechanical spring stiffness and $k_{eff} > 0$.

EXPERIMENTAL RESULTS

The fabricated magnetometer has a natural frequency of 4028 Hz, k_{eff} of 2.8 N/m, and Q of 400-1600 for pressures in the range 10mTorr-1Torr. It was tested in a 64mTorr vacuum and in a dc magnetic field of 7 mTesla. A multi-channel data acquisition card was used to synchronize the current drive and parametric inputs and a laser Doppler vibrometer (LDV) with a lock-in-amplifier measured the output.

Figs. 3 and 4, show the parametrically amplified RMS displacements of the Lorentz and Brownian forces. In Fig. 3 the displacement magnitudes have been normalized relative to linear resonator (V_p=0) leaving only the parametric amplification (G_p). In this experiment the pump frequency was fixed at twice the drive frequency. A maximum stable amplification G_p = 28 was achieved at a voltage of 0.22V. The noise floor of the LDV is sufficiently low that measurement of Brownian motion is possible when the flexure current is zero. Fig. 4 shows the displacement spectrum $|X(f)|$ at various parametric gains. The experimental measurements show excellent agreement with theoretical predictions for a 2nd order system with effective quality factor $Q_{eff} = G_{para}Q$ excited by thermomechanical noise. The measured Brownian noise floor of 13.75 pm/√Hz corresponds to a noise-limited magnetic field resolution of 39 nT/√Hz at a 1 mA flexure bias current. Note that parametric amplification does not improve the Brownian noise-limited performance; instead, parametric amplification merely makes it possible to achieve this performance with a lower resolution measurement system than our LDV.

During standard operation the Lorentz force input is modulated at the device's natural frequency and electrostatically pumped at twice the natural frequency, $\omega_p = 2\omega_n$. Fig. 4 shows the parametric amplification as a function of the pump voltage in standard operating mode. It can be seen from Fig. 4 that the stability limit is 0.23V. Voltages above 0.23V represent an unstable region where the amplification becomes independent of the input forces. As the pump voltage is driven closer to the stability limit the amplification increases, however, the likelihood of instability also increases. Three marginally stable parametric gains of 33, 47 and 52 were observed for a short duration. It is believe that the instabilities were caused by nonlinear behavior occurring at large deflections.

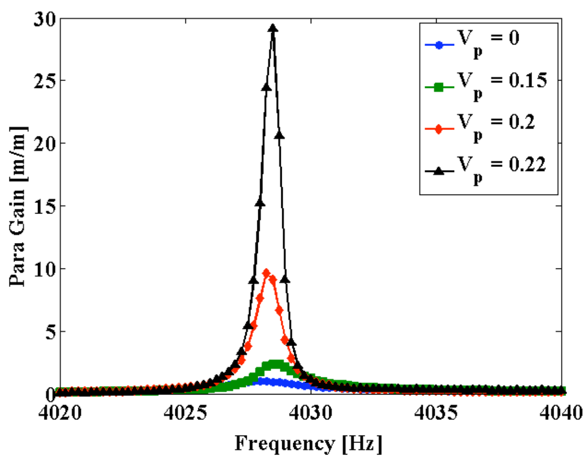


Figure 3: Magnetometer frequency response measured at various pump amplitudes. The y-axis magnitude scale is normalized relative to the peak magnitude at $v_p = 0$ to show the increase in sensitivity relative to a linear resonator. Device has a Q of 1600

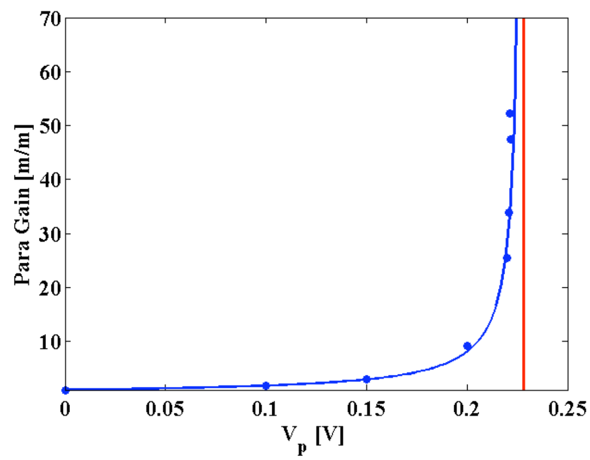


Figure 5: Experimental (points) and the theoretical (line) parametric gain versus pump voltage driven at $\omega_p = 2\omega_n$. Device has a Q of 1600. Stability boundary at $V_{pump} = 0.23V$

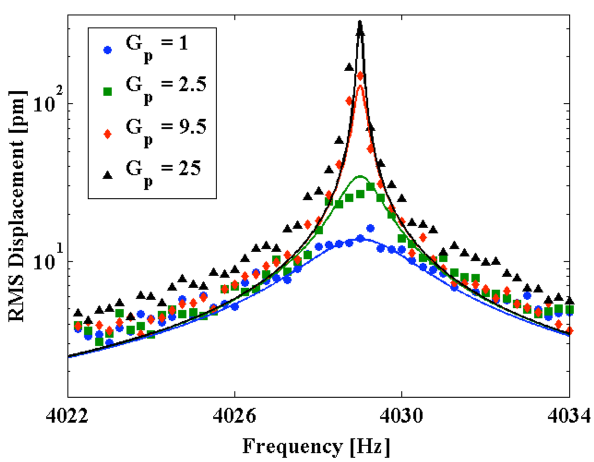


Figure 4: Brownian motion spectrum measured at various parametric gains. Device has a Q of 1600

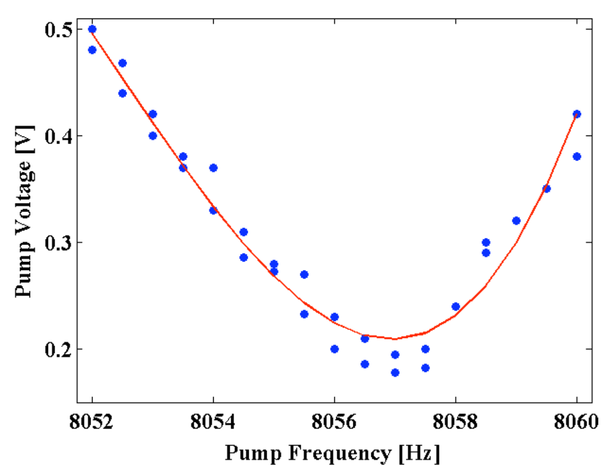


Figure 6: Stability boundary limit versus parametric frequency. Theory (line), Experimental (points). Device has a Q of 1600

Fig 6 shows the instability boundary when the parametric input frequency is slightly offset from twice the natural frequency, $\omega_p = 2\omega_n(1{+}\Delta)$. The theoretical stability limit, evaluated using the perturbation method described above, is indicated with a solid line on the plot and shows good agreement with measured data. When the fractional mismatch $\Delta = 0.05\%$, the instability threshold is approximately doubled. As a result, a pump amplitude that achieves $G_p = 50$ with $\Delta = 0$ will result in $G_p \sim 2$ with $\Delta = 0.05\%$, underlining the sensitivity of the parametric gain to the pump frequency.

Although parametric resonance is a nonlinear phenomenon, for small displacements the force-displacement relationship for the Lorentz force input is linear. For a second device with $Q = 400$, the displacement at a parametric gain at $G_p = 51$ was measured by placing the device in a fixed magnetic field $B = 7.8$ mT and varying the amplitude of the AC flexure current, as shown in Fig. 7. This measurement shows a linear response with a current sensitivity of 68.2 nm/□A for displacements below ~100 nm. At larger displacements, nonlinearity is caused by the electrostatic force arising from the pump electrodes.

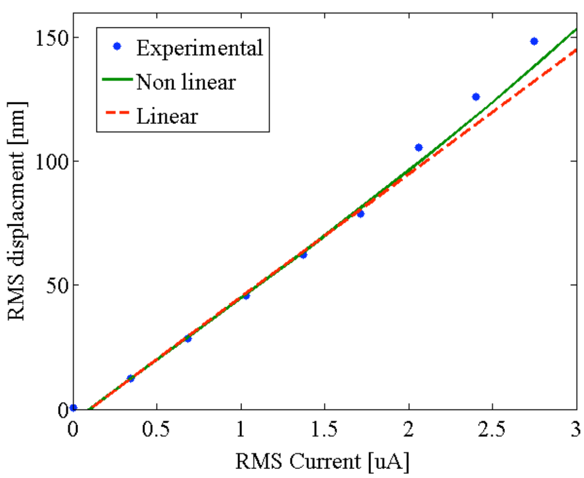


Figure 7: Magnetometer displacement versus flexure current amplitude. Measurements were performed in a constant field B = 7.8 mT with the pump amplitude adjusted to achieve G_p = 51. Device has a Q of 400.

Discussion

The significance of parametric amplification lies in the ability to arbitrarily increase the mechanical sensitivity of a MEMS sensor. In the situation where electronic noise dominates over Brownian noise, parametric amplification will improve the overall signal to noise ratio, since increases in the mechanical sensitivity attenuate the equivalent force noise arising from electronic noise. Electronic noise dominates when the detection electronics are operated at low bias currents or when the displacement-to-voltage sensitivity of the transducer is poor. The latter case arises in capacitive sensors having a relatively low sense capacitance in comparison to the surrounding parasitics. Increasing the sense capacitance requires more device area and comes at the cost of lower Q and therefore increased Brownian noise. Parametric amplification is therefore an attractive solution for reducing transducer size and power consumption. Parametric amplification does have its drawbacks as it requires additional circuitry for driving the pump and may require active stability control for large parametric gains.

CONCLUSION

In conclusion, parametric amplification has been theoretically and experimentally shown to arbitrarily increase the mechanical sensitivity amplifying both the Lorentz and Brownian forces. In the situation where Brownian noise dominates over electronic noise, parametric amplification will improve the system wide SNR. Parametric amplification is most advantageous when the Brownian noise floor is designed to be lower than the electronic noise floor. Parametric amplification does have it drawbacks as it requires additional circuitry to drive the pump and active control may be required for large gains.

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