Use of Weibull Distribution for Describing Outdoor Multipath Fading

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Abstract

In this work we introduce the Weibull Distribution as a less complex and accurate description for the outdoor multipath fading channel than some of the existing models. Using the method of moments we compare the Weibull Distribution and other statistical models, such as the Rayleigh, Rice and Nakagami Distributions with experimental results taken in urban and suburban environment. Apart from introducing the Weibull Distribution as a useful tool for describing outdoor multipath fading, we verify that the Rayleigh distribution is inadequate in many cases for describing the modern urban environment.

Index Terms- Multipath fading channel, Weibull Distribution, Rice Distribution, Rayleigh Distribution, Nakagami Distribution, method of moments.

I. Introduction

The most important problem that appears in cellular wireless communication systems is that of multipath fading. This effect is causing a fluctuation in the received signal as we can see in Figure 1. To reduce or solve this problem we need to be able to predict this fading effect. In order to do so there are a lot of statistical models used to describe the outdoor multipath fading [1]. Some of these models produce very accurate results especially the Rice and Nakagami distributions [2],[3]. The drawback of using these statistical models is that they include very complex computations.

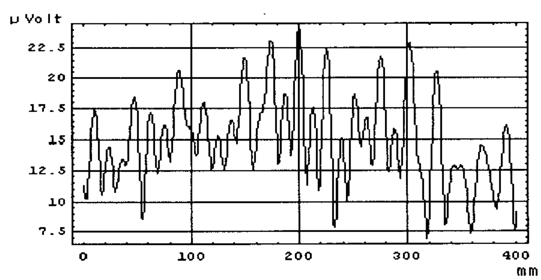

Figure 1 : The fluctuation (due to multipath fading) of the received signal across a 40 cm sector.

II. Weibull Distribution

The Weibull distribution is often used to model the time until failure of many different physicals systems. During the last two years it was used for describing the indoor radio channel fading. The PDF and the CDF of Weibull is:

$$P_{Weibull}(r) = ab\, x^{b-1} e^{-ax^{b}} \quad (1)$$

$$P_{Weibull}(r) = 1 - e^{-ax^{b}} \quad (2)$$

For b =1 the Weibull distribution is identical to the exponential distribution, while for b=2 the Weibull distribution is identical to the Rayleigh distribution. This is also the reason the Weibull distribution is always better than Rayleigh distribution.

Using the method of moments we end up with the following system:

$$E\{x\} = \int_{-\infty}^{+\infty} x\; ab\, x^{b-1} e^{-ax^{b}}\, dx = a^{-\frac{1}{b}}\, \Gamma(1+\frac{1}{b}) \quad (3)$$

$$\text{Variance} = E\{x^2\} - E\{x\}^2 = a^{-\frac{2}{b}}\, [\, \Gamma(1+\frac{2}{b}) - \Gamma^2(1+\frac{1}{b})] \quad (4)$$

Since we know the mean and the variance from the experimental data we solve for a and b (the Weibull parameters) and we come up with:

$$\frac{Variance}{E\{x\}^2} = \frac{2b * \Gamma(\frac{2}{b})}{\Gamma^2(\frac{1}{b})} - 1 \quad (5)$$

So we can use formula 14 to calculate b and then from formula 12 we get a as:

$$\alpha = \left(\frac{\Gamma(1+\frac{1}{b})}{E\{x\}}\right)^{b} \quad (6)$$

III. Processing of the experimental results

Measurements were taken in urban and suburban environments around 900 MHz for a GSM network. Each set of measurements was taken in a sector of specific length, since there was a need for a constant mean value of the transmitted signal. In general, this sector must be around 20 λ (where λ the wavelength) or a few centimeters. To get a general idea about the propagation of the signal, one must combine all these measurements. In order to do so from all sectors, one can not just add all the data, since they do not cover the need for a constant mean for our signal. The way to do that is by scanning all our sectors, pick up the maximum (preferably one of the largest values) for the mean of the signal and normalize all measurements accordingly to the mean of our "control" sector. To do that the following formula is used:

$d'_i = d_i + dm$ (7)

where d'_i is our new sample,

d_i is our old sample,

and dm is the difference between the mean of the sector where our sample is and the mean of the control sector:

$$dm = E\{x\}_{\text{sector of } i} - E\{x\}_{\text{control sector}} \quad (8)$$

That way all sectors have the same mean, so now they can be added without damaging the generality of the problem.

Another important note is that for our processing we use the CDF (cumulative density function). The reason we choose to use the CDFs and not the PDF's when we are comparing the experimental data with the theoretical distributions is that, as we can see in Figures 2 (a and b), that the CDF is more continuous (for the experimental data).

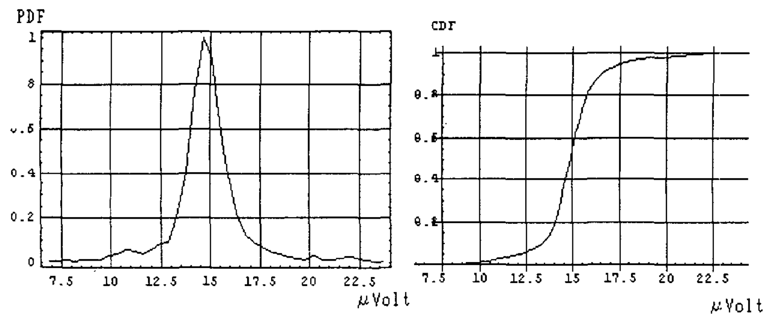

Figure 2 (a,b): The PDF and the CDF of the same experimental data. Notice the discontinuity at the edges of the PDF compared to the clear shape of the CDF. Both plots are originated from the same data as the plot in Figure 1.

The general information obtained from the two above plots is that the level of the signal is around 15 μVolt and the dispersion looks like a Gausssian bell. Most of the signal is between 12.5 and 17.5 μVolts, which gives a typical spread of 30%.

Figure 3a shows that in two of the sections shown how all theoretical distributions match the our experimental results very well. In Fugure 3b, we notice that the Rayleigh distribution does not follow the experimental results which indicates no line of sight. In all cases Weibull distribution fits perfectly the theoretical distribution

IV. Conclusions

In this paper we introduced the Weibull distribution for outdoor multipath fading, using the method of moments to calculate it's parameters. Making a direct comparison between the theoretical statistical models and the experimental results it is obvious (from the graphs shown below) that Weibull, Rice and Nakagami are describing the multipath fading adequately [4], with Weibull being the simplest to calculate. On the other hand Rayleigh does not follow the experimental results in all cases.

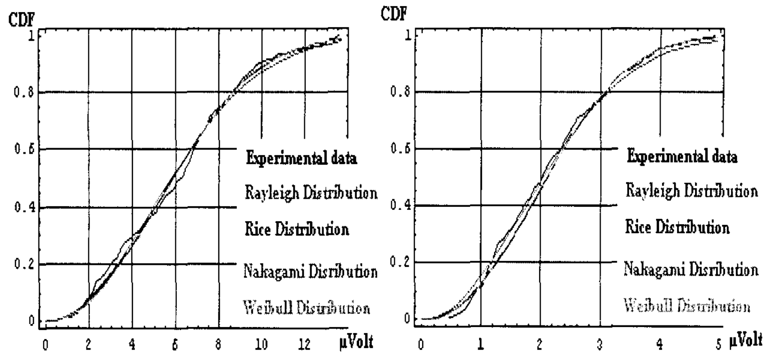

Figure 3(a,b): Figure 3a comes from sector # 0027 and Figure 3b comes from sector # 0002.

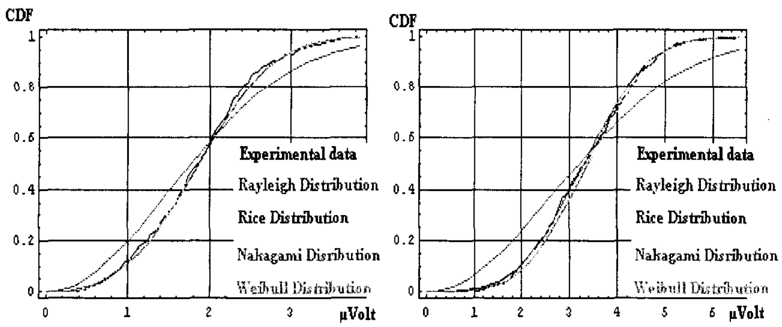

Figure 4(a,b): Figure 4a comes from sector #0003 and Figure 4b comes from sector # 0030..

V. Acknowledgement

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VI. References

[1] H.Suzuki: "A Statistical Model for Urban Radio Propagation" IEEG Trans. Comm,Vol.25. July 1977.
[2] W.C.Y. Lee and Y.S Yeh, "On the Education Of the Second - Order Statistics of Log Norman Fading in the mobile Radio Environment." IEEE Trans. Comm. Vol.22, June 1974, Pg 869-870
[3] M. Yacoub, "Fading Distributions and Cochannel Interference in Wireless Communications", IEEE Antennas and Propagation Magazine, Vol 42/1 February 2000.
[4] Ali Abli, Kyle Wills, H. Allen Barger, Mohamed Slim Alouini, Mostafa Kaveh "Comparison of the Level Crossing Rate and Average Fade Duration of Rayleigh Rice and Nakagami Fading Models" IEEE, VTC 2000