Fundamental Noise Limit of Piezoelectric Accelerometer

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Abstract—**Since significant progress is achieved in the development of piezoelectric (PE) accelerometers for small signal applications nowadays (for example, piezoelectric seismic vibration sensors), the question about the fundamental noise limit of these sensors becomes vital. The noise of the PE transducer is the fundamental noise limit of the PE accelerometer and should be taken into account if the noise of the electronics is small enough. The two noise sources of PE transducer, the mechanical-thermal noise of the damped mechanical harmonic oscillator and the electrical-thermal noise of the PE element's material, are analyzed in this paper. The equation of the total fundamental noise limit of the PE accelerometer is presented. This equation can be used for the calculation of the fundamental noise limit of PE accelerometers if their parameters are known or can be obtained by measurement.**

Index Terms—**Accelerometer, low-noise accelerometer, noise, piezoelectric, transducer.**

I. Introduction

LOW noise PE accelerometers head the list of vibration sensors used for the measurement of small vibration signals at a frequency range from about 0.001 Hz to 10 kHz at present time [1]–[5]. Since significant progress is achieved in the development of high sensitivity PE transducers and low-noise electronics, the noise floor of these accelerometers decreased greatly [3]–[8]. Some modern low-noise seismic accelerometers, for example, have a noise floor estimated at a few dozen $\mathrm{ng}/\sqrt{\mathrm{Hz}}$ at frequency 1 Hz and a few $\mathrm{ng}/\sqrt{\mathrm{Hz}}$ at frequency 100 Hz [3], [5]. Designers try to decrease the noise floor more and more. In this connection, this question becomes vital: what is the fundamental noise limit of PE accelerometer?

The typical PE accelerometer incorporates the PE transducer and low-noise electrical amplifier used for the amplification of signals going from transducer. The fundamental noise limit of the PE accelerometer is determined by the noise of the PE transducer. Usually, the noise of the amplifier dominates the noise of the transducer. Therefore, the noise of the PE transducer is often neglected, but some modern very low-noise PE accelerometers using very low-noise amplifiers exhibit noise comparable with the noise of the transducer itself. Therefore, the last one should be taken into account for these sensors.

It is known that every PE transducer has two functional parts: a mechanical damped harmonic oscillator and an electrical PE element, which produces a charge on its terminals that is proportional to the mechanical motion. Each of these parts makes their noise contributions in total noise of the PE transducer. Mechanical-thermal noise of a damped harmonic oscillator is associated with a mechanical resistance noise and is well known [9]–[16]. With reference to the sensors, usually this noise is observed at high frequencies in MEMS small signal accelerometers, miniature signal acoustic sensors, and small pressure sensors [9]–[14]. Electrical-thermal noise of the PE element is determined by the loss factor of the PE element's material [1], [17], and is usually neglected in design and analysis. Most of the literature sources about the noise limit of vibration and acoustic sensors describe mainly the mechanical-thermal noise of a damped harmonic oscillator as a basic, and the only, component of the sensor's noise limit [9]–[14]. Perhaps, therefore, the electrical-thermal noise is shown inadequately in the literature, but it can play a significant role in many sensor types, particularly, in the low-noise, low frequency seismic sensors, and should be considered as a part of the fundamental noise limit of the accelerometer. In this paper, both of these noise sources are presented for determination of the total fundamental noise limit of the PE accelerometer.

Manuscript received February 28, 2003; revised July 21, 2003. This work was supported by Endevco Corporation. The associate editor coordinating the review of this paper and approving it for publication was Dr. Ewald Benes.

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Digital Object Identifier 10.1109/JSEN.2003.820366

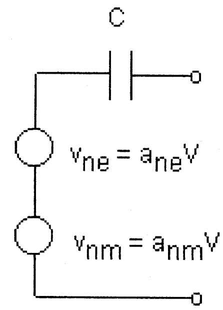

Fig. 1. Equivalent electrical schematic of the transducer's noise.

II. Equivalent Electrical Schematic of the Transducer's Noise

Fig. 1 shows the equivalent electrical schematic of the transducer's noise in terms of equivalent noise voltage spectral density. Here v_{nm} is a transducer's equivalent voltage noise spectral density, caused by the mechanical-thermal noise, $v_{nm} = a_{nm}V$. v_{ne} is a transducer's equivalent noise voltage spectral density, caused by the electrical-thermal noise, $v_{ne} = a_{ne}V$. a_{nm} is a transducer's equivalent acceleration noise spectral density, caused by the mechanical-thermal noise. a_{ne} is a transducer's equivalent acceleration noise spectral density, caused by electrical-thermal noise. The C is a transducer's electrical capacitance, V is a transducer's voltage sensitivity, $V = Q_T/C$,

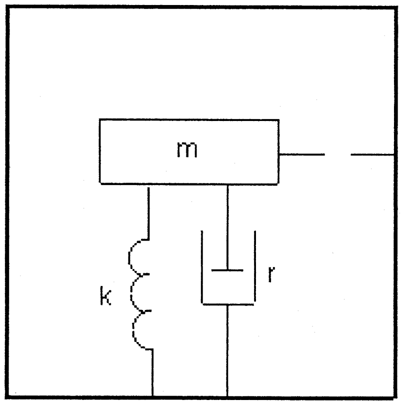

Fig. 2. Schematic diagram of a simple accelerometer.

where Q_T is a transducer's charge sensitivity. v_{nm} is determined by the mechanical resistance of the damped mechanical harmonic oscillator v_{ne} is determined by the loss factor of the PE material of the PE element. We can suppose that these noise sources are not correlated. Then, the total noise of the PE transducer in terms of equivalent voltage noise spectral density v_{ntr} at the output of the transducer, or equivalent acceleration noise spectral density a_{ntr} can be found as a square root of the sum of these noise sources squares

$$\begin{aligned} v_{ntr} &= a_{ntr}V = \sqrt{v_{nm}^2 + v_{ne}^2} = V\sqrt{a_{nm}^2 + a_{ne}^2}, \\ a_{ntr} &= \sqrt{a_{nm}^2 + a_{ne}^2}. \end{aligned} \tag{1}$$

a_{nm}, a_{ne}, v_{nm}, and v_{ne} are completely determined by the transducer's parameters and their expressions will be found below.

III. Mechanical–Thermal Noise of the Transducer

The formula of mechanical–thermal noise for a simple PE accelerometer can be obtained from analysis of this type of noise based on the well-known mechanism of Brownian Motion [9], [12]. For a simple accelerometer containing a damped harmonic oscillator with mass m, spring constant k, and mechanical resistance r (see Fig. 2), signal-to-noise ratio expressed in terms of spectral density at any frequency $f \ll f_0$ (that is typical for accelerometer) equals [9]

$$\frac{Z_s^2}{Z_n^2} = \frac{a_s^2 mQ}{4k_B T\omega_0}. \tag{2}$$

Here, Z_s is a signal response spectral density, $Z_s = a_s/\omega_0^2$, a_s is an input acceleration spectral density, ω_0 is the resonance frequency, $\omega_0^2 = 4\pi^2 f_0^2 = k/m$, Q is the quality factor of the harmonic oscillator, $Q = \omega_0 m/r$, k_B is the Boltzmann's constant (1.38×10^{-23} J/K), T is the absolute temperature (°K), Z_n is a noise response spectral density, $Z_n = \sqrt{4k_B T/\omega_0^3 mQ}$. From (1), we can find the equivalent acceleration spectral density $a_s = a_{nm}$ that corresponds to the thermal mechanical noise spectral density of accelerometer if we suppose $Z_s = Z_n$

$$a_{nm} = \sqrt{\frac{4k_B T\omega_0}{mQ}}. \tag{3}$$

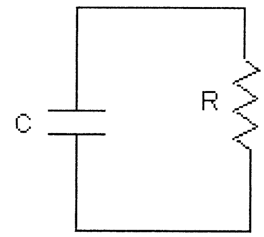

Fig. 3. Equivalent schematic of the transducer's electrical capacitance.

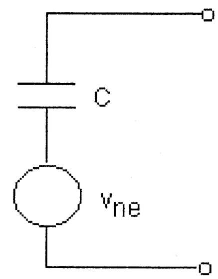

Fig. 4. Equivalent noise schematic of the transducer's electrical capacitance.

This expression coincides with a similar equation for equivalent input acceleration of a vibration sensor represented in [10], [11]. Acceleration noise spectral density a_{nm} creates a voltage noise spectral density at the output of the transducer v_{nm}, which apparently equals

$$v_{nm} = a_{nm}V = \sqrt{\frac{4k_B T\omega_0 V^2}{mQ}} = \sqrt{\frac{4k_B T\omega_0 Q_T^2}{mQC^2}}. \tag{4}$$

It is easy to see from (3) that a_{nm} can be decreased by increasing the mass m and the quality factor Q, or by decreasing the resonance frequency ω_0. Besides, a_{nm} does not depend on frequency f at all $f \ll f_0$.

IV. Electrical–Thermal Noise of the Transducer

The mechanism of this type of noise is related with losses in the PE material and depends on its loss factor or dissipation factor η, which is inverse of the PE material's quality factor [1], [17]. η is a property of any electrical capacitor, which has some active losses. The equivalent schematic of the transducer's electrical capacitance C is shown in Fig. 3. Here, R reflects active losses in this capacitor and can change with the frequency change. By definition [17], η equals

$$\eta = \frac{1}{\omega CR}, \quad \omega = 2\pi f. \tag{5}$$

To obtain an expression for the equivalent noise generator v_{ne} in Fig. 1, we need to transform the schematic in Fig. 3 into the equivalent noise schematic shown in Fig. 4. It is easy to find v_{ne}

$$v_{ne} = \sqrt{4k_B T R_n}. \tag{6}$$

Here, R_n is an equivalent noise resistor

$$R_n = \frac{R}{1 + (\omega RC)^2}, \quad R_n = \frac{1}{\omega C\left(\eta + \frac{1}{\eta}\right)}. \tag{7}$$

Usually for PE material $\eta \ll 1$, in practice, it can be in range from 0.001 to 0.05. Then, (7) becomes simpler

$$R_n = \frac{\eta}{\omega C}. \tag{8}$$

By substituting R_n into (6) with (8), we will finally obtain

$$v_{ne} = \sqrt{\frac{4k_B T\eta}{\omega C}},\ a_{ne} = \frac{v_{ne}}{V} = \sqrt{\frac{4k_B T\eta}{\omega C V^2}} = \sqrt{\frac{4k_B T\eta C}{\omega Q_T^2}}. \tag{9}$$

As we can see from (9), a_{ne} decreases with the decreasing of η and the increasing of V. Besides, a_{ne} and v_{ne} are functions of frequency f and these functions depend on frequency variation of η. Measurements of η, C, and R of the PE element made of PZT (lead zirconate titanate) material typically used for piezoelectric accelerometers were performed at frequency range from 100 Hz to 100 kHz using the HP Impedance Analyzer 4194A. Measurement results are the following: η and C are practically constant at this frequency range. Indeed, the values of η were in the range from 0.014 to 0.017, values of C were in the range from 1176 pF to 1230 pF, but R changes inversely proportionally to the frequency at this frequency range that corresponds to (5) if η and C are constant. Unfortunately, typical impedance analyzers or impedance bridges do not allow to measure η at frequencies below 100 Hz. We can expect change η with the frequency at low frequencies below 100 Hz [17], [18].

Note should be taken, expression (5) and Fig. 3 represent loss mechanisms of the PE element at frequency range restricted by the some high frequencies $f \geq 10^7$ Hz. At these frequencies, behavior of loss factor can be characterized by multiple relaxation and resonance modes and, therefore, equivalent circuit representing this behavior will be more complicated [17]. At very low frequencies, $f \leq 10$ Hz additional component of low frequency noise associated with noise of $1/f$ type can be expected [19]. This noise contribution depends on conductivity, impurities, and defects of PE material and usually increases with increasing of each of factors mentioned above [19].

V. Complete Formulae for the Fundamental Noise Limits

Now, we can write a complete expression for the fundamental noise limit by substituting a_{nm}, a_{ne}, v_{nm}, v_{ne} in (1) with (3), (4), and (9). In terms of the acceleration noise spectral density, this noise limit equals

$$a_{ntr} = \sqrt{4k_B T\left(\frac{\omega_0}{mQ} + \frac{\eta C}{\omega Q_T^2}\right)}. \tag{10}$$

The noise limit in terms of the voltage noise spectral density at the output of the transducer equals

$$v_{ntr} = \sqrt{4k_B T\left(\frac{\omega_0 Q_T^2}{C^2 mQ} + \frac{\eta}{\omega C}\right)}. \tag{11}$$

According to (10), (11), a_{ntr}, and v_{ntr} can be calculated for any PE accelerometer at any frequency f if such parameters as ω_0, m, Q, Q_T, C, and η are known or found as a result of measurement. Note should be taken, generally speaking, such parameters of the transducer as Q, C, and η are functions of f. Therefore for calculation a_{ntr} and v_{ntr} at some particular frequency it is needed to use the value of these parameters corresponding to this frequency.

Apparently, for the determination of the total noise of the accelerometer including the noise of the electronic amplifier, it is needed to add the electronic amplifier's noise to the (10) and (11)

$$a_{ta} = \sqrt{4k_B T\left(\frac{\omega_0}{mQ} + \frac{\eta C}{\omega Q_T^2}\right) + \left(\frac{e_n C}{Q_T}\right)^2} \tag{12}$$

$$v_{ta} = \sqrt{4k_B T\left(\frac{\omega_0 Q_T^2}{C^2 mQ} + \frac{\eta}{wC}\right) + e_n^2}. \tag{13}$$

In (12) and (13), a_{ta} is a total equivalent acceleration noise spectral density of the accelerometer, v_{ta} is a total equivalent voltage noise spectral density of the accelerometer, and e_n is an equivalent voltage noise spectral density of the electronic amplifier referred to the amplifier's input or to the transducer's output. We supposed in (12) and (13) that e_n is not correlated with v_{ntr} that corresponds to practice.

VI. Some Practical Examples

Let us calculate a_{nm}, a_{ne}, and a_{ntr} for the seismic accelerometer having the following parameters: $f_0 = 400$ Hz, $m = 0.5$ kg, $Q = 50$, $Q_T = 20{,}000$ pC/g $= 2 \cdot 10^{-9}$ C/m/s^2, $C = 3000$ pF $= 3 \cdot 10^{-9}$ F, $\eta = 0.02$, T $= 300°$ K, the operating frequency range is from 0.005 Hz to 100 Hz. Using (3) and (9) we will obtain: $a_{nm} \approx 13 \cdot 10^{-10}$ m/s$^2/\sqrt{\text{Hz}} \approx 0.13$ ng$/\sqrt{\text{Hz}}$ at any frequency $f \lll f_0$, $a_{ne} \approx 2 \cdot 10^{-7}$ m/s$^2/\sqrt{\text{Hz}} \approx 20$ ng$/\sqrt{\text{Hz}}$ at frequency 1 Hz, $a_{ne} \approx 6.3$ ng$/\sqrt{\text{Hz}}$ at frequency 10 Hz, and $a_{ne} \approx 2$ ng$/\sqrt{\text{Hz}}$ at frequency 100 Hz. At this calculation, we supposed that η is constant at all frequencies. For this accelerometer a_{ne} prevails over a_{nm} at all frequency range. Therefore, total fundamental noise limit is determined basically by the electrical-thermal noise of the transducer: $a_{nt} \approx a_{ne}$ at all accelerometer frequency range. Noise floor of 20 ng$/\sqrt{\text{Hz}}$ at frequency 1 Hz and 2 ng$/\sqrt{\text{Hz}}$ at frequency 100 Hz will create noise at the output of the transducer of this accelerometer about 130 nV$/\sqrt{\text{Hz}}$ at frequency 1 Hz and about 13 nV$/\sqrt{\text{Hz}}$ at frequency 100 Hz. For some low-noise electronic amplifiers this level of noise can be comparable with the noise contribution of the electronic amplifier e_n at these frequencies [6]–[8] and therefore should be considered in design and analysis.

This was confirmed by measurements of noise contribution of the low-noise JFET amplifier to the overall noise floor of seismic accelerometer that have been made with the help of HP Dynamic Analyzer 3562A. The accelerometer comprised the PE transducer having parameters mentioned above and low-noise JFET amplifier. The measured results were the following. Noise contribution of JFET amplifier was about 20 ng$/\sqrt{\text{Hz}}$ at frequency 1 Hz and about 2.5 ng$/\sqrt{\text{Hz}}$ at frequency100 Hz that was comparable with the noise floor contribution of PE transducer. So, total noise floor of accelerometer (PE transducer plus amplifier) should be about 28 ng$/\sqrt{\text{Hz}}$ at frequency 1 Hz and about 3.2 ng$/\sqrt{\text{Hz}}$ at frequency 100 Hz. Measurement of noise contribution of the PE transducer to the overall noise floor is a challenging task since it is very difficult to eliminate the influence of vibration environmental noise on PE transducer.

Now, we will calculate the fundamental noise limit for a small PE accelerometer having the following parameters: $f_0 = 30$ kHz, $m = 3 \cdot 10^{-4}$ kg, $Q = 70$, $Q_T = 1$ pC/g $= 10^{-13}$ C/m/s^2, $C = 100$ pF $= 10^{-10}$ F, $\eta = 0.02$, T $= 300°$ K, the operating frequency range is from 0.5 Hz to 10 kHz. Using (3) and (9), we will obtain $a_{nm} \approx 40$ ng/$\sqrt{\text{Hz}}$ at all frequency range, $a_{ne} \approx 73$ μg/$\sqrt{\text{Hz}}$ at frequency 1 Hz, $a_{ne} \approx 7.3$ μg/$\sqrt{\text{Hz}}$ at frequency 100 Hz, $a_{ne} \approx 0.73$ μg/$\sqrt{\text{Hz}}$ at frequency 10 kHz. For this accelerometer, the total fundamental noise limit is also determined basically by the electrical-thermal noise of the transducer $a_{ntr} \approx a_{ne}$ at all frequency range. The fundamental noise limit of this accelerometer mentioned above will create the noise at the output of the accelerometer's transducer about 730 nV/$\sqrt{\text{Hz}}$ at frequency 1 Hz and about 7.3 nV/$\sqrt{\text{Hz}}$ at frequency 10 kHz. These values of noise are also comparable with e_n of some very low-noise amplifiers [7], [8] and should be considered in design and analysis.

The examples mentioned above illustrate that the transducer's electrical-thermal noise a_{ne} usually predominates over its mechanical-thermal noise a_{nm} at frequencies up to 10 kHz if η is constant at all this frequency range or its dependence on frequency is small. We can expect that for miniature small signal accelerometers, a_{ne} will be masked by a_{nm} at some high frequencies $f > 10$ kHz, because a_{ne} is proportional to $1/\sqrt{f}$, and a_{nm} is frequency independent. At these high frequencies, a_{ne} can be neglected and only a_{nm} has to be taken into account.

VII. Conclusion

Engineering formulae for the fundamental noise limits of a PE accelerometer in terms of the equivalent noise acceleration spectral density and the equivalent noise voltage spectral density at the output of the PE transducer are obtained. The fundamental noise limit of the accelerometer is considered as a noise of the PE transducer of this accelerometer. Analysis of the fundamental noise limit is carried out on the base of two noise sources of the PE transducer: the mechanical – thermal noise of the damped mechanical harmonic oscillator and the electrical – thermal noise of the PE element's material. The fundamental noise limit of low-noise PE accelerometers can be comparable with the noise contribution of some modern low-noise electronic amplifiers and therefore should be considered in design and analysis.

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