Validation of a Model Describing Two-Dimensional Heat Transfer During Solid-State Fermentation in Packed Bed Bioreactors

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Abstract: A two-dimensional heat transfer model was validated against two experimental studies from the literature which describe the growth of *Aspergillus niger* during solid-state fermentation in packed bed bioreactors. With the same set of model parameters, the two-dimensional model was able to describe both radial temperature gradients, which dominated in one of the studies, and axial temperature gradients, which dominated in the other study. The sensitivity of the model predictions to the characteristics of the substrate and the microbe were explored. The temperatures reached in the column are most sensitive to parameters which affect the peak heat load, including the substrate packing density, the maximum specific growth rate, and the maximum biomass concentration. Even though the bed is assumed to be aerated with saturated air, the increase in temperature with bed height increases the water-carrying capacity of the air and therefore enables evaporation to contribute significantly to cooling. The model suggests that evaporation can remove as much as 78% of the heat from the bed during times of peak heat generation. Our model provides a tool which can guide the design and operation of packed bed bioreactors. However, further improvements are necessary to do this effectively, the most important of which is the incorporation of a water balance. *Biotechnol Bioeng* **60**: 739–749, 1998.

Keywords: bioreactor design; heat transfer; packed bed bioreactor; mathematical modelling

INTRODUCTION

Solid-state fermentation (SSF) involves the growth of microbes on water insoluble substrates in the absence of free water. SSF has some potential to be used at commercial scale, but applications are limited by the lack of well-founded scale-up criteria (Lonsane et al., 1992). Relatively large-scale processes have been developed for the Koji stage of soy sauce manufacture (Mudgett, 1986), but the details of these processes are not available as they represent important proprietary information. Mathematical models describing heat and mass transfer processes in SSF are essential for the effective design of large-scale bioreactors, but such models are currently lacking (Ramana Murthy et al., 1993).

Bioreactor designs used for SSF include trays, packed beds, rotating drums, stirred reactors, and air–solid fluidized beds. Of these, trays are the simplest, but the size of individual trays is limited. In large trays problems arise with overheating and lack of oxygen in the centre (Ragheva Rao et al., 1993; Rajagopalan and Modak, 1994). The next most simple design is the packed bed, which allows some control of fermentation parameters, through manipulation of the flow rate or temperature of the process air. This design has potential particularly for those fermentations in which agitation is harmful, such as the production of fungal spores. However, overheating is a significant problem, especially at the top of the column (Gowthaman et al., 1993). Under some operating conditions, temperatures in the middle of the bed can be 20°C higher than the temperature of the inlet air, leading to large variations in growth and product formation (Ghildyal et al., 1994).

Sangsurasak and Mitchell (1995) proposed a model describing two-dimensional heat transfer in a packed bed bioreactor, improving on the model of Saucedo-Castaneda et al. (1990) who neglected axial heat transfer, which is only appropriate for thin columns operated at low aeration rates. The two-dimensional heat transfer model can describe heat transfer in bioreactors under a wide range of geometries and operating conditions.

This paper extends the model of Sangsurasak and Mitchell (1995) by including a term describing evaporation, which contributes significantly to the overall heat removal in a packed bed bioreactor (Gutierrez-Rojas et al., 1996). The model predictions are compared with the results of two experimental studies reported in the literature, both of which involve the growth of *Aspergillus niger* in packed bed bioreactors (Ghildyal et al., 1994; Saucedo-Castaneda et al., 1990). The sensitivity of the model predictions to parameter values is also explored.

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MODEL

The system modelled is a cylindrical packed bed bioreactor, aerated from the bottom with moist air (Fig. 1). A moist starchy substrate is inoculated with *A. niger* and placed in the bioreactor at time zero. A water jacket removes heat by convection. During the process the substrate bed remains static. This system was used by both Saucedo-Castaneda et al. (1990) and Ghildyal et al. (1994), whose data we use to validate our model.

The model concentrates on the heat transfer phenomena. Equations for mass transfer have not been incorporated. Gas transfer is not important, since if the aeration satisfies cooling requirements then it will satisfy the need for supply of O_2 and removal of CO_2 (Ghildyal et al., 1994; Gowthaman et al., 1993; Saucedo-Castaneda et al., 1992). Mass transfer of water is the most important mass transfer in this system, since significant evaporation may occur during the fermentation (Gutierrez-Rojas et al., 1996). This could potentially decrease the water activity of the substrate and limit growth, but these effects have been ignored, because the results of Saucedo-Castaneda et al. (1990) and Ghildyal et al. (1994) suggest that growth is limited by the high temperatures achieved in the columns, and not by the availability of water. In practice two strategies might be used to try to avoid limitation of growth by low water availability. Firstly, if the substrate was prepared with a high initial water content, then significant amounts of water could be evaporated before the water activity fell significantly, since the water activities of the solid agricultural materials which are used as substrates typically remain quite high over a wide range of water contents. Secondly, it might be possible to replenish water within the substrate bed, by dripping water through small inlets located at several points in the bed. The change in weight of dry matter in the bed is ignored. The mathematical model simply consists of an equation describing growth kinetics and an equation describing the energy balance over each position within the packed bed bioreactor.

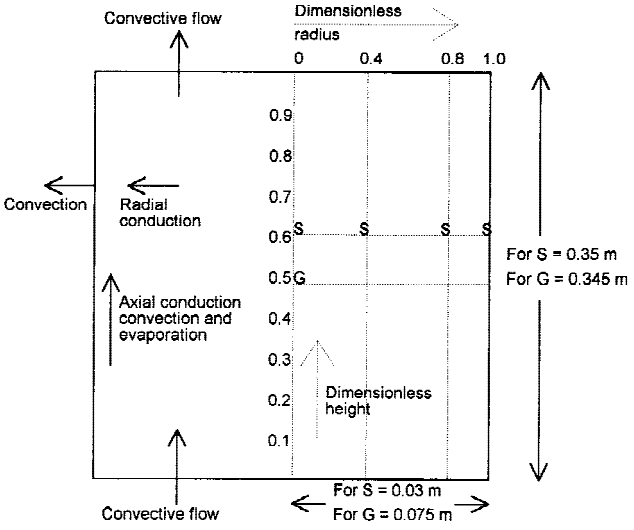

Figure 1. Diagram of the cylindrical packed bed bioreactor modelled in the current study. The left hand side of the diagram shows the heat transfer mechanisms described by the model. The right hand side of the diagram indicates the dimensionless positions of the points for which Saucedo-Castaneda et al. (1990) made measurements (marked ''S'') and for which Ghildyal et al. (1994) made their measurements (marked ''G''). The positions of internal collocation points were adjusted within the numerical technique to coincide with these positions. The heights and radii of the reactors used by these workers are also shown.

The substrate bed is treated as a pseudo-homogeneous matrix with the mass-weighted average properties of the substrate and the air. With respect to heat transfer, this assumes that the bed acts as though it is a single medium with the weighted thermal conductivity. The effect of a wide range of thermal conductivities on the model predictions was investigated. The other consequence of the pseudo-homogeneous matrix is that the air and the moist solid at any particular location within the bed are assumed to be at the same temperature, with the air saturated with water vapor. This assumption of thermal and moisture equilibrium is reasonable if saturated air is used to aerate the bed, and is supported by the observation by Van Lier et al. (1994) that at bed heights of 0.4–1.2 m in a forcefully aerated compost bed, the air and solids were at the same temperature. The thermal and physical properties of the bed are assumed to be independent of temperature. Air moves only in the axial direction, with a constant velocity profile across the bed. Effects of microbial growth on particle size and pressure drop across the bed are ignored.

Growth Kinetics

The growth kinetics are described empirically by the logistic equation:

$$\frac{dX}{dt} = \mu_g X\left(1 - \frac{X}{X_m}\right), \tag{1}$$

where X is the biomass concentration (kg biomass/kg dry substrate) and X_m is the maximum possible biomass concentration. Since the change in dry matter of the bed is ignored, this equation can be derived from a balance written in terms of mass of biomass by dividing through by the mass of dry substrate. The following empirical equations are used to describe the effect of temperature on the specific growth rate μ_g (s^{-1}):

$$\mu_g = \mu_{gopt},\ T \leqslant T_{opt}, \tag{2a}$$

$$\mu_g = \left(\frac{b + (T_{max} - T_{opt})}{(T_{max} - T_{opt})}\right)\left(\frac{\mu_{gopt}(T_{max} - T)}{b + (T_{max} - T)}\right),$$
$$T_{opt} \leqslant T \leqslant T_{max}, \tag{2b}$$

$$\mu_g = 0,\ T \geqslant T_{max}, \tag{2c}$$

where μ_{gopt} is the specific growth rate (s^{-1}) at the optimal temperature for growth, T_{opt}, and T_{max} is the maximum temperature at which growth can occur. Note that, although specific growth rates are expressed in terms of s^{-1} in the model itself, values are reported as h^{-1} in this paper.

Eq. (2b) is simply an empirical equation which was used to allow different sensitivities to increases in temperature to be described by the adjustment of a single parameter, *b*. The effect of the value of *b* on the change in specific growth rate as the temperature increases is illustrated in Fig. 2. For small values of *b* the specific growth rate only decreases slightly as temperature increases, but then decreases suddenly as the temperature nears T_{max}, reaching zero at T_{max}. For large values of *b* the specific growth rate decreases steadily as temperature increases from T_{opt} to T_{max}. The term within the first set of parentheses is simply a scaling factor which ensures that $\mu_g = \mu_{gopt}$ when $T = T_{opt}$.

This approach to modelling the effects of high temperatures was taken since the effect of temperature variations during growth are not clear. Data are typically obtained from experiments involving a number of cultures held isothermally at a range of temperatures. These experiments typically show a steady decrease in the maximum specific growth rate as the temperature is increased above the optimum temperature for growth. However, Saucedo-Castaneda et al. (1990) and Stuart (1996) suggest that μ_g may remain essentially constant as the temperature increases if the temperature rise occurs after growth has commenced at the optimum temperature for growth. The effect on the model predictions of different sensitivities to temperature increases is explored later.

For *A. niger*, the specific growth rate falls off slowly as the temperature decreases below the optimum temperature. Colony radial extension rates of *A. niger* remained constant over the range of 303–308 K (Smits et al., 1998). In other work the specific growth rate at 303 K was still 90% of the specific growth rate at the optimum temperature of 308 K (Szewczyk and Myszka, 1994). Since the temperature in the system never falls below 303 K if saturated air at 303 K is supplied to the column, for simplicity, at temperatures below the optimum temperature for growth, the specific growth rate was assumed to remain at μ_{gopt}. If air significantly below the temperature optimum for growth was used to aerate the column, it would be simple to incorporate an equation describing a decrease in specific growth rate with temperature into the model.

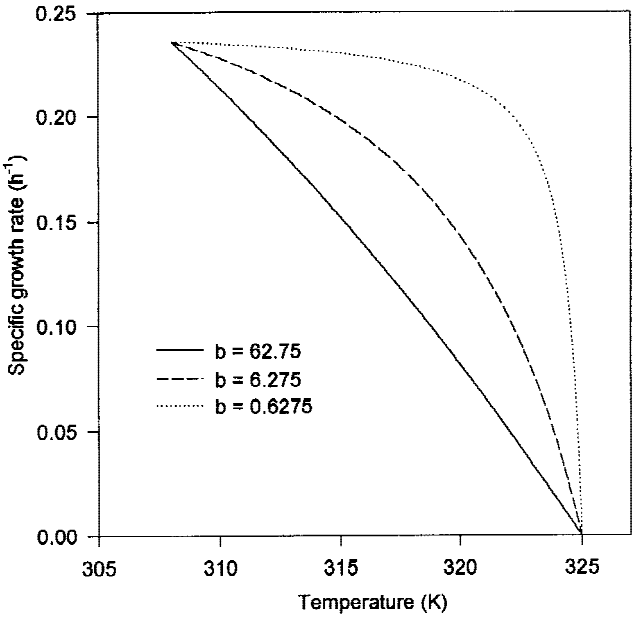

Figure 2. The relationship between the specific growth rate (μ_g) and temperature between the optimum temperature and the maximum temperature for growth, as described by Eq. (2b). The value of the parameter *b* (in K) affects the sensitivity of the specific growth rate to increases in temperature.

Energy Balance

A macroscopic energy balance over the column, including terms for convective and evaporative heat removal, conduction in the radial and axial directions, and the generation of heat from microbial growth, gives Eq. (3):

$$\rho_b C_{pb}\left(\frac{\partial T}{\partial t}\right) + (\rho_a C_{pa} + \rho_a f\lambda)v_z\left(\frac{\partial T}{\partial z}\right) = \frac{k_b}{r}\left(\frac{\partial T}{\partial r}\right) + k_b\left(\frac{\partial^2 T}{\partial r^2}\right) + k_b\left(\frac{\partial^2 T}{\partial z^2}\right) + \rho_s(1-\varepsilon)Y\frac{dX}{dt}, \quad (3)$$

where each term has the units of W/m^3. The factor $\rho_a f\lambda$ arises since the evaporation of water to keep the air saturated gives the air a higher apparent heat capacity. This apparent heat capacity increases with temperature because the ability of the air to carry water increases. Between T_{opt} and T_{max}, the water content of saturated air can be approximated by a straight line, with slope *f* (kg water vapour/(kg air K)). This is a simplified approach to handling evaporative cooling. The last term of the energy balance assumes that metabolic heat generation is directly proportional to the production of new biomass. Maintenance metabolism is ignored.

Values for density, thermal conductivity and heat capacity of the bed were calculated as mass-weighted averages of the properties of the air and substrate within the bed:

$$\rho_b = (\varepsilon\rho_a) + (1-\varepsilon)\rho_s \quad (4a)$$

$$k_b = (\varepsilon k_a) + (1-\varepsilon)k_s \quad (4b)$$

$$C_{pb} = (\varepsilon C_{pa}) + (1-\varepsilon)C_{ps} \quad (4c)$$

Initial and Boundary Conditions

The boundary conditions are as follows:

$$z = 0, \quad T = T_a, \quad (5a)$$

$$z = H, \quad \frac{\partial T}{\partial z} = 0, \quad (5b)$$

$$r = 0, \quad \frac{\partial T}{\partial r} = 0, \quad (5c)$$

$$r = R, \quad \frac{\partial T}{\partial r} = \frac{\mathrm{Bi}}{R}(T_s - T). \quad (5d)$$

These boundary conditions correspond to the following physical conditions: the temperature at the bottom of the bed being maintained at the inlet air temperature by the inlet air; the absence of external cooling at the top of the bed, with the air leaving the bed simply leaving at the temperature of the top of the bed; the absence of heat transfer through the centre of the bed (due to the bed symmetry there is no temperature gradient at the central axis); and finally, convective cooling at the side walls of the bed. The resistance of the bioreactor wall to heat transfer is neglected.

At the beginning of the fermentation both the temperature (T_o) and the inoculum concentration (X_o) are assumed to be constant over the whole height (H) of the bed:

$$\text{at } t = 0 \qquad T = T_o \qquad 0 \leqslant z \leqslant H \tag{6a}$$

$$\text{at } t = 0 \qquad X = X_o \qquad 0 \leqslant z \leqslant H \tag{6b}$$

COMPUTATIONAL METHODS

Orthogonal collocation, using Jacobi polynomials, was used to discretize the spatial coordinates as a two-dimensional axisymmetric problem (Villadsen and Michelsen, 1978; Finlayson, 1980), leaving a set of ordinary differential equations. The equations were solved using the GEAR package (Hindmash, 1974).

The numbers of internal collocation points were varied to find the minimum number which gave predictions which did not depend on the number of points chosen. As a result of this analysis, fifteen collocation points were used, with five roots in the axial direction and three roots in the radial direction. The positions of these collocation points were manipulated to coincide with the positions for which experimental temperature measurements were available by changing the values of the exponents α and β (Villadson and Michelson, 1978) within the polynomials (Fig. 1). The data of Saucedo-Castaneda et al. (1990) were for a dimensionless height of 0.6, at the central axis and dimensionless radii of 0.4, 0.8, and 1.0. The data of Ghildyal et al. (1994) were taken at the central axis at a dimensionless height of 0.5. Since the numerical method does not actually calculate the temperature at the central axis, a dimensionless radius of 0.05 was used for the central axis.

With five axial roots, values of $\alpha = 1$ and $\beta = 1$ for axial collocation give one of the roots at a dimensionless height of 0.5, whereas values of $\alpha = 5$ and $\beta = 1$ give one of the roots at a dimensionless height of 0.6. With three radial roots, values of $\alpha = 10$ and $\beta = 0.1$ for radial collocation give one of the roots at a dimensionless radius of 0.05 and values of $\alpha = 0.65$ and $\beta = 2$ give roots at dimensionless radii of 0.4, 0.8, and 1.0.

RESULTS

Model predictions were compared with the experimental results of Saucedo-Castaneda et al. (1990) and Ghildyal et al. (1994) for growth of *A. niger* on cassava and wheat bran, respectively. Saucedo-Castaneda et al. (1990) gives temporal temperature profiles at different radial positions within the bed, while Ghildyal et al. (1994) gives temporal temperature profiles at the centre of the bed for different superficial air velocities.

Parameter Estimation

Table I lists parameters estimated for the systems of Ghildyal et al. (1994) and Saucedo-Castaneda et al. (1990). Their design and operating conditions are given in Table II. The same microbial and substrate parameters were used for the two simulations, with only the parameters describing the operating conditions and the bioreactor configuration being varied.

The optimum temperature for growth of *A. niger* was estimated as 35°C (308 K) (Szewczyk and Myszka, 1994). Unfortunately the growth rate data of Szewczyk and Myszka (1994) only go up to 40°C, for which the specific growth rate is still quite high. However, since Ghildyal et al. (1994) observed temperatures up to 52°C, this was taken as the maximum temperature for growth. A value of b was chosen which gave a moderate effect of temperature on growth (see Fig. 2), because there is conflicting evidence of the effect of temperature. Raimbault and Alazard (1980) showed that at 45°C the growth rate of *A. niger* in SSF was the same as the specific growth rate at the optimum temperature, whereas the incomplete data of Szewczyk and Myszka (1994) suggests a response closer to that obtained for a value of *b* of 62.75.

In the absence of direct biomass measurements by

Table I. Parameter values used in validation of the 2-dimensional model against the data of Saucedo-Castaneda et al. (1990) and Ghildyal et al. (1994).

Constant	Value	Source
b	6.275	See text
Bi	10	Saucedo-Castaneda et al. (1990)
C_{pa}	1180 J/(kg K)	Himmelblau (1982)
C_{ps}	2500 J/(kg K)	Sweat (1986)
f	0.00246 kg water/(kg air K)	Himmelblau (1982)
k_a	0.0206 W/(m K)	Perry et al. (1984)
k_s	0.03 W/(m K)	Saucedo-Castaneda et al. (1990)
T_{opt}	35°C = 308 K	Szewczyk and Myszka (1994)
T_{max}	52°C = 325 K	Ghildyal et al. (1994)
X_0	0.001 kg biomass/kg substrate	Saucedo-Castaneda et al. (1990)
X_m	0.125 kg biomass/kg substrate	Gumbira-Sa'id et al. (1992)
Y	8.366×10^6 J/kg biomass	Saucedo-Castaneda et al. (1990, 1992)
ε	0.35	Terzic and Todorovic (1992)
λ	2414300 J/kg water	Himmelblau (1982)
μ_{gopt}	0.236 h^{-1} = 6.56×10^{-5} s^{-1}	Szewczyk and Myszka (1994)
ρ_2	700 kg/m^3	Saucedo-Castaneda et al. (1990)
ρ_a	1.14 kg/m^3	Weast (1974)

Table II. Values of design and operating variables used in validation of the 2-dimensional model against the data of Saucedo-Castaneda et al. (1990) and Ghildyal et al. (1994).

Variable	Saucedo-Castaneda et al. (1990)	Ghildyal et al. (1994)
R	0.03 m	0.075 m
H	0.35 m	0.345 m
T_a	35°C = 308 K	30°C = 303 K
T_s	35°C = 308 K	25°C = 298 K
T_o	25°C = 298 K	30°C = 303 K
v_z	0.01 m/s	0.0047, 0.0141, 0.0235 m/s

Saucedo-Castaneda et al. (1990) and Ghildyal et al. (1994), a maximum biomass concentration of 0.125 kg biomass/kg substrate was assumed (Gumbira-Sa'id et al., 1992). The value for initial biomass was chosen as 0.001 kg biomass/kg substrate although the actual value cannot be estimated from the available data.

Saucedo-Castaneda et al. (1990) estimated many of the necessary physical parameters. They did not determine the thermal conductivity, but this was estimated indirectly from their Peclet number of 2551, assuming a substrate heat capacity of 2500 J/(kg K).

Validating the Model against the Data of Saucedo-Castaneda et al. (1990)

Saucedo-Castaneda et al. (1990) studied the growth of *A. niger* on cassava meal in a packed bed bioreactor. The values of X_o, X_m, and μ_{gopt} used in the present simulations are different from those of Saucedo-Castaneda et al. (1990), who did not determine these parameters independently but rather used least-squares optimization to fit the model predictions to the temperature profiles. In the final version of their model, they kept μ_g constant at 0.3 and expressed the maximum possible biomass concentration X_m as a function of temperature.

Figure 3 shows that reasonable agreement was obtained between the predicted temperature profiles and the experimental data of Saucedo-Castaneda et al. (1990). Our two-dimensional model describes higher temperatures during the heating and cooling phases, and it predicts the height and timing of the temperature peak reasonably closely.

There are some interesting differences between the predictions of the two-dimensional model and the model of Saucedo-Castaneda et al. (1990) which describes only radial heat removal (hereafter referred to as the ''radial model''). The two-dimensional model is able to describe phenomena over the whole column, while the radial model can describe the bioreactor only after the first 5 cm of the bed, in which there were significant axial temperature gradients.

The two-dimensional model predicts more rapid heating of the substrate during the early lag period (within the first 10 h) than does the radial model (Saucedo-Castaneda et al., 1990), with the discrepancy being largest near the central axis of the column. During this lag period metabolic heat production is negligible and the radial model only allows

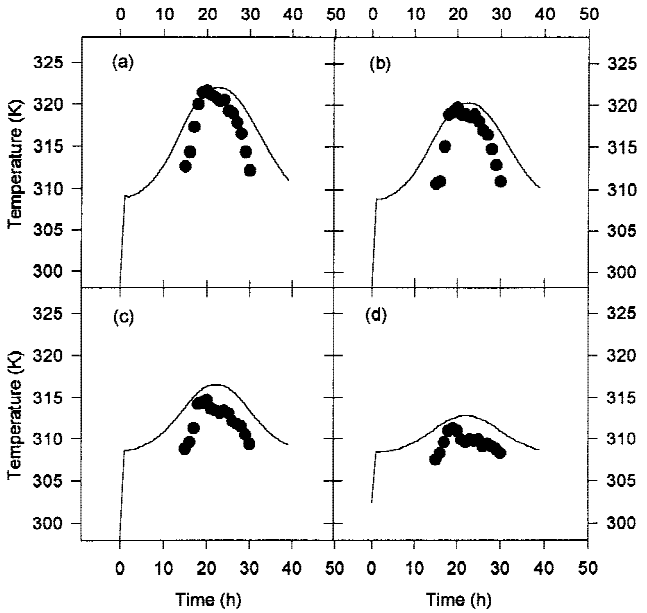

Figure 3. Comparison of the predictions of the two-dimensional model (lines) with the experimental data of Saucedo-Castaneda et al. (1990) (symbols). The data are plotted for a dimensionless height of 0.6 and dimensionless radii of (a) 0.05, (b) 0.4, (c) 0.8, and (d) 1.0.

the substrate temperature to increase by conduction of heat radially inwards from the column wall. The further towards the inside of the column, the longer the radial model predicts that it takes for the substrate temperature to reach the temperature of the surroundings. On the other hand, with the two-dimensional model the substrate heats up to the aeration air temperature by axial convection, which is a much faster process. In fact, the two-dimensional model suggests that radial temperature profiles during this period will be negligible, whereas the radial model suggests there will be a significant gradient until the interior heats up. Saucedo-Castaneda et al. (1990) did not collect temperature data during this early period, so it is not possible to say with certainty that the early profile predicted by the two-dimensional model is more realistic.

The contribution of convection is relatively small with the thin column design and the low superficial velocity used by Saucedo-Castaneda et al. (1990), and therefore both the radial model and the two-dimensional model predict significant radial temperature gradients. However, the two-dimensional model also predicts significant axial temperature profiles, with temperature differences of 10°C between heights of 3 and 21 cm, and differences of up to 5°C between heights of 10 and 21 cm. Experimentally, after the first 5 cm of the 35 cm column, where the bed temperature rose sharply, the temperature differences along the remainder of the column were no more than 3°C and were often much less (Saucedo-Castaneda et al., 1990).

Validating the Model against the Data of Ghildyal et al. (1994)

Ghildyal et al. (1994) measured temporal temperature profiles at the centre of a packed bed during the SSF of wheat

bran by *A. niger*. Since they did not determine many of the parameters necessary for the two-dimensional model, many of the parameters of Saucedo-Castaneda et al. (1990) were used, although the different substrates used in the two studies might affect both heat transfer properties and growth kinetics. Despite this, the two-dimensional model reasonably closely predicts the magnitude and timing of the temperature peaks for the three different air flow rates that Ghildyal et al. (1994) used (Fig. 4). The good agreement with the experimental results over such a wide range of superficial velocities suggests that the model describes the contributions of convective and evaporative cooling reasonably well, since these are the heat removal mechanisms which are most directly affected by the superficial velocity.

Sensitivity of the Two-Dimensional Model to Parameter Values

The comparisons above suggest that the two-dimensional model adequately describes heat transfer in a packed bed bioreactor. The current work takes a different perspective from our previous exploration of the effect of design and operational parameters (Sangsurasak and Mitchell, 1995). It identifies the substrate and microbial parameters which have the greatest influence on the model predictions. Future experimental studies of the performance of SSF bioreactors, including packed beds, will involve a wide range of substrates and microbes. Most effort should be put into accurate determination of those parameters to which the model predictions are most sensitive. It may be sufficient to obtain estimates from the literature for parameters with less influence on the model predictions.

The various substrates used in SSF will have different physical and heat transfer properties, while microbes used in SSF will vary widely in growth characteristics such as maximum specific growth rate, maximum biomass density, and the sensitivity of the specific growth rate to increases in temperature. The effects of various substrate and microbial characteristics on the temperature reached in the centre of the bed were explored by varying them one at a time from the base case represented by the simulation of the superficial velocity of 0.0141 m/s of Ghildyal et al. (1994), illustrated in Fig. 4b.

Packing Density of Substrate

The substrate packing density, defined as $\rho_s(1 - \varepsilon)$, was adjusted by varying the void fraction (ε). As void fraction decreases the amount of substrate per unit volume of the bioreactor increases. Void fractions were varied from 0.2 to 0.7. The lowest value represents tight packing that might be expected with regularly shaped cylindrical particles. The void fraction of 0.7 represents a substrate with a fluffy consistency.

Since the model assumes that the growth of biomass per unit volume depends directly on the substrate packing density, a decreasing void fraction significantly affects the pre-

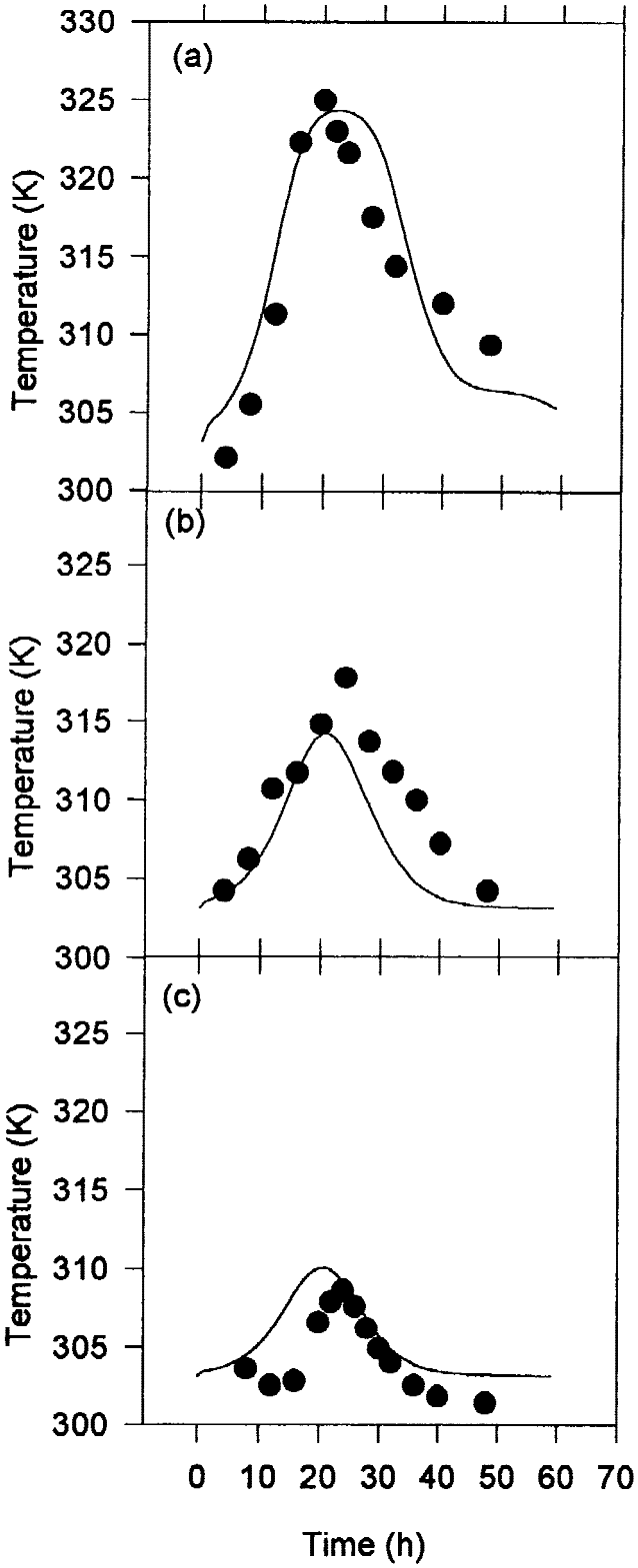

Figure 4. Comparison of the predictions of the two-dimensional model (lines) with the experimental data of Ghildyal et al. (1994) (symbols). The data are plotted for a position at a dimensionless radius of 0.05 and a dimensionless height of 0.5, and for superficial velocities of (a) 0.0047, (b) 0.0141, and (c) 0.0235 m/s.

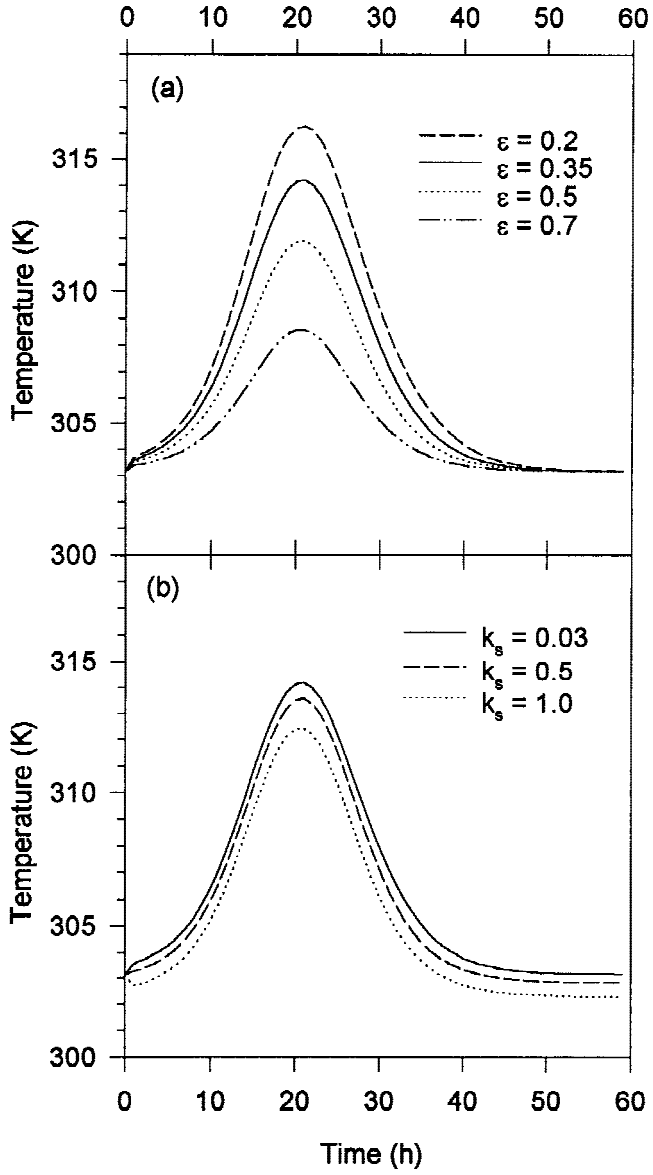

Figure 5. Sensitivity of the temperature profiles predicted by the two-dimensional model to the characteristics of the substrate. (a) Effect of void fraction (ε). (b) Effect of thermal conductivity (k_s in W/m K) of the substrate. All other parameters are the same as for Fig. 4b. The predictions are made for the centre of the bed (point G in Figure 1).

dictions (Fig. 5a) because the production of heat per unit volume increases. The same inoculation density, in kg biomass per kg of substrate, was assumed for the different packing densities, since typically SSF processes are inoculated simply on a weight percent basis. Therefore, for higher substrate packing densities, the inoculation density is higher on a volumetric basis (kg biomass per m^3), enabling faster early growth and heat production. However, since there are no changes to the growth kinetics per mass of substrate, the void fraction only affects the height reached by the temperature profile, not the timing of the increase.

These predictions are reasonable as long as the higher packing density does not in itself limit growth. For example, at high packing densities the pressure drop through the bed will be greater, affecting air flow. Also, if particles are packed so tightly so that they deform and their surfaces are pushed into close contact, then air will be excluded. Close packing of substrate particles might enable the biomass to reach its maximum packing density in the substrate voids (Laukevics et al., 1985) before the nutrients in the substrate are exhausted. These considerations raise interesting questions into the role of particle size, shape, and packing in determining the available surface area for growth.

The decrease in the severity of the overheating problem at lower packing densities suggests that overheating problems could be minimized by ''bulking'' the substrate mass with an inert material. For example, for a starch-utilizing organism the starchy substrate could be mixed with a cellulosic material such as bagasse, lowering the volumetric heat production rate and therefore decreasing the peak temperature. This strategy does reduce one advantage of solid state fermentation, namely the high volumetric productivity which is often achievable. However, this strategy might be essential for a fast growing microbe in situations where the substrate bed cannot afford to be stirred.

The sensitivity to the substrate density itself (ρ_s) was not explored because it is not clear how the density of the substrate would affect the growth of the microbe. For example, if growth is limited by available surface area, then substrate density will have little effect. Also, the nutrient content of a substrate might not relate directly to the density. Higher densities might mean more water and less nutrient. However, any changes to the substrate which increase the amount of biomass per unit volume in the bioreactor will increase volumetric heat production, causing a response similar to the effect of increasing substrate packing density.

Thermal Properties of the Substrate

The thermal conductivity was varied from 0.03 W/(m K) for the base case up to 1.0 W/(m K), which might be expected for very moist substrates (Van Lier et al., 1994). However, since convection and evaporation are the dominant heat transfer mechanisms, the thermal conductivity of the substrate is predicted to have relatively little effect on performance (Fig. 5b). At values below 0.1 W/(m K), there is no discernible difference in the predicted temperature profile, suggesting that at these values conduction is such a minor contributor to heat removal that the conduction terms could be removed from the model. The slightly lower temperature profiles for thermal conductivities of 0.5–1.0 W/(m K) indicate that at such values conduction begins to make a noticeable contribution to heat removal; however, it is still only a small contributor compared to convection and evaporation.

The substrate heat capacity (C_{p_s}), when varied from 1000 to 4000 J/kg K, has almost no effect (results not shown), indicating that the amount of energy stored in the substrate bed is a negligible fraction of the heat released by the microbe.

Specific Growth Rate

Specific growth rate is a key kinetic parameter, since in the model heat generation is directly growth associated. A wide range of specific growth rates can occur in SSF. The highest specific growth rate (μ_{gopt}) used in the simulations was 0.5 h^{-1}, representing growth of *Schwanniomyces castelli* in SSF (Saucedo-Castaneda et al., 1992). The lowest specific growth rate used was 0.1 h^{-1}, representing growth of *Rhizopus arrhizus* in SSF (Soccol et al., 1993). The predicted temperature profiles are very sensitive to the specific growth rate (Fig. 6a). The higher the specific growth rate, the higher the peak heat generation rate and therefore the higher the temperature reached. The peak temperature is also reached sooner.

Due to the effect of temperature on μ_g the growth profiles (not shown) are slightly more linear during the growth phase than would normally be predicted by logistic growth kinetics. During early growth the logistic model predicts an acceleration of growth, but this tends to be counteracted by a decrease in μ_g due to the increase in temperature above the optimum. Likewise, after the temperature maximum is reached the logistic model predicts a deceleration of growth, but this tends to be counteracted by an increase in μ_g due to the decreasing of the temperature back towards the optimum temperature. Linear growth kinetics have been observed in SSF (Huang et al., 1986; Auria et al., 1990), and these considerations provide one possible explanation for how such kinetics might arise.

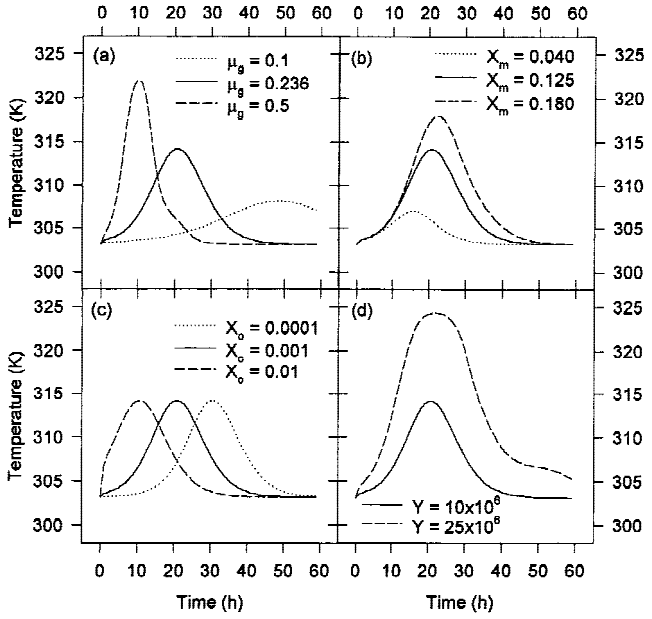

Figure 6. Sensitivity of the temperature and growth profiles predicted by the two-dimensional model to the characteristics of the microorganism. (a) Effect of maximum specific growth rate (μ_{gopt} in h^{-1}). (b) Effect of maximum biomass density (X_m in kg/kg substrate). (c) Effect of initial biomass density (X_o in kg/kg substrate). (d) Effect of the heat yield from growth (Y in J/kg biomass). All other parameters are the same as for Fig. 4b. The predictions are made for the centre of the bed (point G in Fig. 1).

Maximum Biomass Density

The maximum biomass density obtained can vary widely between SSF systems. Sargantanis et al. (1993) obtained biomass densities of 0.45 kg per kg dry matter, whereas Auria et al. (1995) obtained biomass densities of 0.10 kg per kg dry matter. Both had water contents of approximately 60% (w/w) and therefore these values correspond to maximum biomass densities of 0.18 and 0.04 kg biomass per kg moist fermented material, respectively. Simulations were run with these maximum biomass densities (Fig. 6b).

For the logistic equation, the maximum growth rate occurs when the biomass density is equal to half of the maximum density. Therefore the peak growth rate and peak heat production rate increase as the maximum biomass density increases, significantly increasing the maximum temperature reached (Fig. 6b). In addition, with the same value of μ_g, it takes longer to reach higher maximum biomass densities and therefore the temperature peak is broader.

Initial Biomass Density

Inoculation strategies can be quite variable in SSF, and the effects of inoculation strategy are not well understood. Figure 6c shows that the initial inoculation density potentially has a significant effect on the timing of the temperature peak, although it does not affect the maximum temperature reached. However, much more experimental work is required to understand inoculum effects and to allow these effects to be modelled more realistically. High inoculation densities may adversely affect growth (Trevelyan, 1974). In addition, spores are commonly used as the inoculum in SSF processes involving fungi, and the model does not describe the germination period for these spores.

Heat Yield from Growth

Heat yields on different energy sources increase with the degree of reduction of the compound. Unfortunately there are very few reliable estimates of stoichiometric heat yields in SSF systems. In liquid culture heat yields range from approximately 10×10^6 J/kg biomass for growth on carbohydrates to approximately 25×10^6 J/kg biomass for growth on ethanol and hydrocarbons (Bailey and Ollis, 1986). As Fig. 6d shows, heat yield has a significant effect on the model predictions. In fact, for the higher heat yield, the temperature is predicted to increase to just below the maximum temperature for growth. However, the true effects of varying the energy source are not clear, because the energy source will also affect the specific growth rate.

Sensitivity of the Growth Rate to Increasing Temperature

As mentioned previously, the literature is not clear on the effects of increasing temperatures on the growth rates of fungi. Therefore the parameter b was varied to simulate

different sensitivities to temperature. The highest sensitivity to temperature was modelled with a value for b of 62.75, which gives an almost linear decrease in specific growth rate between T_{opt} and T_{max} (Fig. 2). The lowest sensitivity to temperature was modelled with a value of b of 0.6275, for which the specific growth rate is still above 0.2 h^{-1} at 320 K, and then decreases rapidly to zero at 325 K. This variation had relatively little effect on the predictions (results not shown). The maximum temperature reached did increase slightly the lower the sensitivity of growth to high temperatures (i.e. the lower the value of b). However, the maximum temperature with $b = 0.6275$ was only 1°C above that for $b = 62.75$. Therefore both these temperature profiles were very similar to the predicted temperature profile for the base case in Fig. 4b, which had a value for b of 6.275.

Fungal death might occur at high temperatures, but it is not clear how death kinetics should be incorporated into a model to describe the response to temperatures above the maximum and below the optimum. To approximate experimental profiles for the effect of temperature on the observed specific growth rate, Szewczyk and Myszka (1994) used a combination of growth and death kinetics, with the rates of both increasing with temperature. However, their model predicts significant death would occur at the optimum temperature for growth, which seems unrealistic. In any case, it is not simple to investigate the death of fungi, due to their mycelial mode of growth, and therefore the necessary data are not available.

DISCUSSION

The two-dimensional model predicts packed bed operation when either radial or axial heat transfer dominates, or when they both make significant contributions. It described the data of Saucedo-Castaneda et al. (1990) in which the narrow column and low superficial velocity allowed radial heat transfer to dominate, and the data of Ghildyal et al. (1994) where the system allowed axial heat transfer to dominate. The radial heat transfer model of Saucedo-Castaneda et al. (1990) cannot describe the data of Ghildyal et al. (1994).

Packed beds will remain the most important bioreactor for static SSFs since the environment can be controlled better than in trays. Static SSFs will be required for spore production, since agitation tends to disrupt the reproductive aerial hyphae (Silman, 1980). However, our simulations show that, depending on the rate of heat generation by the microorganism, overheating is difficult to avoid, especially at the top of the bed. Previously we explored the effect of operational variables such as inlet air temperature and flow rate in order to overcome the overheating problem (Sangsurasak and Mitchell, 1995). The current work has suggested another strategy, namely bulking the substrate bed with an inert material to decrease the volumetric rate of heat production. Alternatively, modifications to the basic packed bed design may be required, such as the provision of closely spaced internal cooling surfaces (Roussos et al., 1993). Combinations of these strategies may be required to cultivate fast growing fungi in packed bed bioreactors.

The model allows analysis of the contribution of the various heat transfer mechanisms to overall heat removal. For the base case used in the sensitivity analysis, the heat production rate at the peak temperature, at which time heat production equals heat removal, is 6609 W/m^3. The contribution of convective cooling is 1027 W/m^3 and the contribution of evaporative cooling is 5174 W/m^3, representing percentage contributions of 16% and 78%, respectively. These are reasonably similar to the contributions to heat removal calculated by Gutierrez-Rojas et al. (1996), which were 27% for convection and 65% for evaporation. This analysis shows that even when saturated air is used to aerate a packed bed, due to the increase in temperature and therefore water-carrying capacity as the air travels up the bed, evaporation still makes very significant contributions to heat removal.

Desirable Improvements of the Model

The two-dimensional model involves many assumptions and simplifications. For example, the bed voidage is assumed not to change during the fermentation. However, the growth of the fungus into the interparticle spaces can significantly decrease the bed voidage, increasing the pressure drop through the bed (Auria et al., 1995). To maintain the same superficial velocity, and therefore the effectiveness of convective and evaporative heat removal, the upline pressure must be increased during the fermentation, otherwise the back-pressure built up during the fermentation will decrease the flow through the column. The current model does not describe the dependence of air flow on pressure and voidage. The Ergun equation, which is typically used to describe the pressure drop in packed beds will not apply for the situation where the interparticle spaces are filled by the fibrous biomass, although it can be used to estimate apparent void fractions from pressure drop data (Auria et al., 1995). Modification of the model to describe these pressure drop effects might be appropriate.

More attention is required into the effect of temperature on the growth and death kinetics of fungi in SSF. Unfortunately, this is difficult due to problems in making direct biomass measurements in SSF. Techniques must be developed to enable investigations of how growth is affected when the temperature varies during growth. It is unlikely that the approach taken to date of incubating cultures isothermally at a range of temperatures adequately describes what happens. For example, Raimbault and Alazard (1980) noted that the growth rate of *A. niger* remained high when the temperature increased to 45°C during SSF, even though it will not germinate and grow if incubated at this temperature from the start of the fermentation.

The most pressing need is to incorporate a water balance into the model. Significant amounts of water are predicted to be evaporated in the column. Since the air is assumed to be saturated at the column temperature, an approximate cal-

culation can be done for the system of Ghildyal et al. (1994). The amount of water evaporated in the bottom half of the column is proportional to the area under the temperature profile for the mid point of the column. For the superficial velocity of 0.014 m/s, it is predicted that approximately 600 g of water evaporates over the 40 h of growth. However, this section of column only contains around 700 g of substrate. In practice it would be difficult to replenish this water continuously and evenly in the bed without mixing. Water would have to be distributed by drips located throughout the column. A model modified to include a water balance could predict the rate at which water needed to be added as a function of height within the column.

CONCLUSIONS

The present work has clearly demonstrated that the overheating problem in packed bed bioreactors is not only affected by operating variables as has been previously reported (Sangsurasak and Mitchell, 1995), but is also strongly influenced by the microbial and substrate properties, especially those that affect the volumetric heat production rate, such as substrate packing density, the specific growth rate and the maximum biomass concentration.

We have developed a two-dimensional heat transfer model of packed bed operation which can describe both radial and axial heat transfer, either of which can dominate depending on the geometry and superficial air velocity. Our model provides a tool which can guide the design and operation of packed bed bioreactors. However, to do this effectively further improvements are desirable, the most important of which is the incorporation of a water balance.

NOMENCLATURE

b	sensitivity of growth kinetics to increase in temperature (K)
Bi	Dimensionless Biot number (h R/k_b)
C_{p_a}	heat capacity of moist air (J/kg K)
C_{p_b}	heat capacity of bed (J/kg K)
C_{p_s}	heat capacity of substrate (J/kg K)
f	water-carrying capacity of air (kg water/kg air K)
h	convective heat transfer coefficient (W/m^2 K)
H	bed height (m)
k_a	thermal conductivity of moist air (W/m K)
k_b	thermal conductivity of the bed (W/m K)
k_s	thermal conductivity of the substrate (W/m K)
r	radial position (m)
R	reactor radius (m)
t	fermentation time (s)
T	bed temperature (K)
T_a	temperature of inlet air (K)
T_o	initial bed temperature (K)
T_s	temperature of the surroundings (K)
T_{max}	maximum temperature for growth (K)
T_{opt}	optimum temperature for growth (K)
v_z	superficial velocity (m/s)
X	biomass concentration (kg biomass/kg substrate)
X_m	maximum biomass concentration (kg biomass/kg substrate)
X_o	initial biomass (kg biomass/kg substrate)
Y	metabolic heat yield coefficient (J/kg biomass)
z	axial position (m)
ε	void fraction
λ	enthalpy of vaporization of water (J/kg)
μ_g	specific growth rate (s^{-1})
μ_{gopt}	specific growth rate at the optimum temperature (s^{-1})
ρ_a	density of moist air (kg/m^3)
ρ_b	density of bed (kg/m^3)
ρ_s	density of substrate (kg/m^3)

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