

ANSI/ASAE EP433 DEC01
Loads Exerted by Free-Flowing Grain on Bins



American Society of Agricultural Engineers

**S
T
A
N
D
A
R
D**

ASAE is a professional and technical organization, of members worldwide, who are dedicated to advancement of engineering applicable to agricultural, food, and biological systems. ASAE Standards are consensus documents developed and adopted by the American Society of Agricultural Engineers to meet standardization needs within the scope of the Society; principally agricultural field equipment, farmstead equipment, structures, soil and water resource management, turf and landscape equipment, forest engineering, food and process engineering, electric power applications, plant and animal environment, and waste management.

NOTE: ASAE Standards, Engineering Practices, and Data are informational and advisory only. Their use by anyone engaged in industry or trade is entirely voluntary. The ASAE assumes no responsibility for results attributable to the application of these ASAE Standards, Engineering Practices, and Data. Conformity does not ensure compliance with applicable ordinances, laws and regulations. Prospective users are responsible for protecting themselves against liability for infringement of patents.

This standard may be designated ANSI/ASAE. If so, this standard is an American National Standard. Approval of an American National Standard requires verification by ANSI that the requirements for due process, consensus, and other criteria for approval have been met by the standards developer.

Consensus is established when, in the judgment of the ANSI Board of Standards Review, substantial agreement has been reached by directly and materially affected interests. Substantial agreement means much more than a simple majority, but not necessarily unanimity. Consensus requires that all views and objections be considered, and that a concerted effort be made toward their resolution.

CAUTION NOTICE: In the case that this standard is an ANSI/ASAE standard, this American National Standard may be revised or withdrawn at any time. The procedures of the American National Standards Institute require that action be taken periodically to reaffirm, revise, or withdraw this standard. Purchasers of American National Standards may receive current information on all standards by calling or writing the American National Standards Institute.

Copyright American Society of Agricultural Engineers. All rights reserved.

ASAE-The Society for engineering in agricultural, food, and biological systems
2950 Niles Rd., St. Joseph, MI 49085-9659, USA ph. 269-429-0300, fax 269-429-3852,
hq@asae.org

Loads Exerted by Free-Flowing Grain on Bins

Developed by the ASAE Loads Due to Bulk Grains and Fertilizers Subcommittee of the Structures Group; approved by the Structures and Environment Division Standards Committee; adopted by ASAE December 1988; revised editorially February 1991; revised editorially June 1991; approved as an American National Standard September 1991; reaffirmed by ASAE December 1993, December 1994, December 1995, December 1996, December 1997; reaffirmed by ASAE for two years December 1999; revised editorially March 2000; reaffirmed by ANSI June 2000; reaffirmed for five years December 2001.

1 Purpose

1.1 This Engineering Practice, presents methods of estimating the grain pressures within centrally loaded and unloaded bins used to store free-flowing, agricultural whole grain.

2 Terminology

2.1 Terms used in this Engineering Practice are defined as follows:

2.1.1 antidynamic tube: A vertical conduit, generally at the center of a bin, with the bottom of the tube placed directly over an orifice through which grain can be unloaded from the bin.

2.1.2 bin: A container with a height to diameter (or shortest side) ratio greater than 0.5.

2.1.3 flume: A vertical tube attached to the wall of a bin through which grain can flow. Discharge outlets may be placed in the bin wall at any location along the vertical rise of the conduit.

2.1.4 funnel flow: Flow from a bin in which all grain movement occurs through a central core with no movement occurring along the bin wall (see Fig. 1).

2.1.5 funnel flow hopper: A hopper in which a flow channel is formed within the stagnant grain (see Fig. 2).

2.1.6 hopper: The sloped portion of a bin which is used to aid gravity discharge through an orifice.

2.1.7 mass flow hopper: A hopper in which all of the grain in the hopper is in motion whenever any grain is withdrawn through the hopper outlet (see Fig. 2).

2.1.8 moisture induced or hygroscopic pressures: Pressures induced by expansion of grain resulting from increases in moisture content.

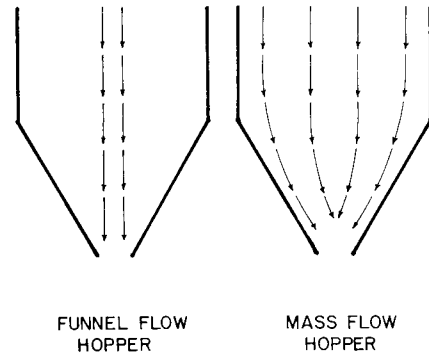


Figure 2 – Hopper flow types

2.1.9 plug flow: Flow from a bin in which the grain moves out of the bin in a manner such that movement occurs along all or part of the bin wall (see Fig. 1).

2.1.10 thermally induced pressures: Pressures induced in a filled bin when subjected to a decline in ambient temperature.

2.1.11 vibration induced pressures: Pressures induced by ground or machinery vibrations.

3 Nomenclature

a = length of the short side of a rectangular bin, m (ft)

b = length of the long side of a rectangular bin, m (ft)

c = equivalent bin wall length, m (ft)

k = ratio of lateral to vertical pressure, dimensionless

u = integration variable for equivalent material depth, m (ft), see Fig. 3.

D = bin diameter, m (ft), see Fig. 3

F = overpressure factor, dimensionless

G = gravity constant, 9.8×10^{-3} kN/kg (1.0 lbf/lb)

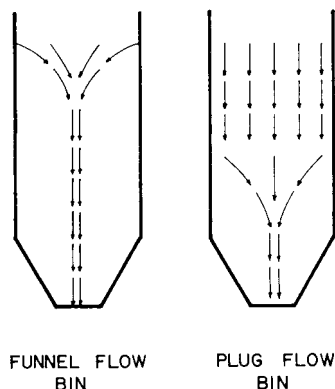


Figure 1 – Bin flow patterns

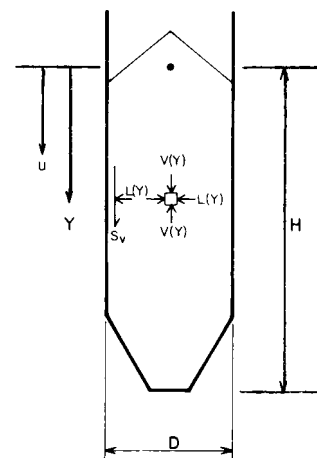


Figure 3 – Bin dimensions

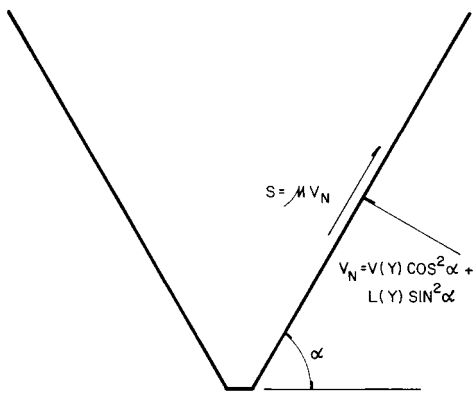


Figure 4 – Hopper stresses

- H = height of material from the lowest point of discharge to 1/3 of the height of the surcharge, if present, m (ft), see Fig. 3
- R = hydraulic radius of the bin (cross section area divided by perimeter), m (ft)
- S = maximum shear stress between inclined surface and grain, kPa (lbf/ft²), see Fig. 4
- W = bulk density of stored grain, kg/m³ (lb/ft³)
- Y = equivalent grain depth, m (ft), see Fig. 3
- P_v = vertical wall load per unit length of bin wall, kN/m (lbf/ft)
- S_v = shear stress between vertical wall and grain, kPa (lbf/ft²)
- V_n = normal pressure on a surface inclined at an angle, α , to horizontal, kPa (lbf/ft²), see Fig. 4
- $L(Y)$ = lateral pressure of grain at depth, Y , kPa (lbf/ft²)
- $V(Y)$ = vertical pressure of grain at depth, Y , kPa (lbf/ft²)
- α = angle from horizontal to inclined surface, deg, see Fig. 4
- μ = coefficient of friction of grain on structural surfaces, dimensionless

4 General design information

4.1 Static pressures and dynamic pressures on bin walls and flat floors.

4.1.1 Static pressures. Estimate static pressures at depth, Y , by Janssen's equation

$$V(Y) = \frac{WRG}{\mu k} \left[1 - e^{-\frac{\mu k Y}{R}} \right] \quad (1)$$

$$L(Y) = kV(Y) \quad (2)$$

4.1.1.1 Estimate the shear stress between the vertical wall and grain using equation 3.

$$S_v = \mu L(Y) \quad (3)$$

4.1.1.2 Rectangular bins. To estimate the pressure next to the short side of rectangular bins, use $R = a/4$ and for pressures next to the long side use $R = c/4$

where

$$c = \frac{2ab}{a+b} \quad (4)$$

4.1.1.3 Conical surcharge. If a conical surcharge of grain is present at the top of the material mass, increase the grain depth, Y , by 1/3 of the conical surcharge height.

4.1.1.4 Bulk density. A maximum of 834 kg/m³ (52 lb/ft³) is recommended for the bulk density of any free-flowing grain. For pressures imposed by a specific commodity other than wheat, use bulk

densities determined by the Winchester Bushel Test (USDA, 1980) or those listed in ASAE Data D241, Density, Specific Gravity and Weight-Moisture Relationships of Grain for Storage, increased by a compaction factor of 1.08. Other material properties are those listed in Table 1.

4.1.2 Dynamic pressures

4.1.2.1 Funnel flow. Funnel flow bins have lateral wall pressures predictable by equation 2. Funnel flow will normally occur in bins which have H/D ratios less than 2.0. H is measured from the lowest point of discharge to the top of the grain surface, or if a surcharge is present, to 1/3 of the surcharge height (see Fig. 3).

4.1.2.2 Plug flow. Dynamic lateral wall pressures during plug flow are larger than those predicted by equation 2. Bins with an H/D ratio greater than 2.0 may unload by plug flow. Estimate lateral wall pressure in bins which unload by plug flow by the static pressure determined using equation 2 multiplied by an overpressure factor. Values of the overpressure factor, F , are given in Table 1. For flat bottom plug flow bins apply this factor from the grain surface to within a distance of $D/4$ from the bottom.

4.1.2.3 Reductions in overpressure factor in bins which unload by plug flow. A reduction in the overpressure factor is allowed within a distance of $D/4$ from the base of flat bottom bins. Interpolate the overpressure factor between the value obtained from Table 1 at a height of $D/4$ to 1.0 at the bottom of the bin.

4.1.3 Calculation of vertical wall loads. Calculate vertical wall loads at depth, Y , from the following expression.

$$P_v = [WGY - V(Y)]R \quad (5)$$

4.1.4 Pressures on floors of flat bottom bins. Estimate vertical floor pressures on flat bottom bins using equation 1.

4.2 Hopper pressures

4.2.1 Exclusions. This Engineering Practice does not apply to pressures in mass flow hoppers.

4.2.2 Load estimation techniques

4.2.2.1 Normal pressures. For pressure normal to an inclined hopper surface (see Fig. 4).

$$V_n = V(Y) \cos^2 \alpha + L(Y) \sin^2 \alpha \quad (6)$$

To determine V_n at a discrete location within a hopper, determine $V(Y)$ and $L(Y)$ using equations 1 and 2 with equivalent grain depth, Y . Use the bin geometry at the intersection of the hopper and the bin wall to calculate hydraulic radius. Apply overpressure factors at the top of the hopper. Overpressure factors may be linearly reduced from F at the top of the hopper to 1.0 at the point of hopper discharge.

4.2.2.2 Tangential stresses. For frictional stresses tangential to inclined hopper surface (see Fig. 4).

$$S = \mu V_n \quad (7)$$

4.3 Pressures on antidynamic tubes and flumes

4.3.1 Lateral pressures on antidynamic tubes and flumes. Lateral pressures are exerted both internally and externally in a direction normal to the wall surface on an antidynamic tube or a flume.

4.3.1.1 External lateral pressures. The pressure at any given level on an antidynamic tube or flume is estimated as equal to the lateral pressure at the wall at the same level using the techniques described in paragraph 4.1.

4.3.1.2 Internal lateral pressure. The pressures on the wall at any given level in an antidynamic tube or a flume may be neglected or estimated using the techniques described in paragraph 4.1 with a bin diameter equal to the equivalent internal diameter of the antidynamic tube or the flume.

4.3.2 Vertical stresses on antidynamic tubes or flumes. Vertical stresses act on both internal and external surfaces of antidynamic tubes and flumes.

Table 1 – Overpressure factors and material properties

Wall material	μ	k	F
Steel	0.30	0.5	1.4
Concrete	0.40	0.5	1.4
Corrugated steel	0.37	0.5	1.4

4.3.2.1 External vertical stresses. Estimate external stresses by multiplying the external lateral pressure at a given level on the antidynamic tubes and flume as estimated by the method described in paragraph 4.3.1.1 by an appropriate coefficient of friction presented in Table 1.

4.3.2.2 Internal vertical stresses. Estimate internal stresses by multiplying the internal lateral pressure by the appropriate coefficient of friction from Table 1.

4.4 Special load considerations

4.4.1 Thermally induced pressures. Estimate thermal pressures for circular steel bins as 8% of the static load for temperature declines of 10 °C per hour and as 15% of the static load for temperature declines of 20 °C per hour.

4.4.2 Moisture induced or hygroscopic pressures

4.4.2.1 Magnitude. Moisture content increases during storage of 4% or more can cause lateral pressures to increase several times static load conditions.

4.4.2.2 Precautions. Precautions should be taken in the design, location and management of bins to prevent the occurrence of grain moisture content increases.

4.4.3 Vibration induced pressures. There are insufficient data available to predict the magnitude or significance of vibration induced pressures.

5 Commentary

5.1 This section includes the basis for the design methods suggested in Section 1—Purpose, Section 2—Terminology, Section 3—Nomenclature, and Section 4—General Design Information. Further discussion of the provisions of the Engineering Practice may be found in Bokhoven *et al.* (1986), Britton and Moysey (1986), Bucklin *et al.* (1986), Manbeck *et al.* (1986), and Ross *et al.* (1986). The methods described in this Engineering Practice apply only to bins which are centrally loaded and emptied.

5.1.1 Static pressures. An accepted method of predicting static loads on bin walls and floors is that proposed by Janssen (1895). Janssen assumed that the bulk density, lateral to vertical pressure ratio, and coefficient of friction between the grain and bin wall were constants for any given configuration. Janssen's technique assumes that the grain pressure does not vary across the bin cross section. Values of k and μ listed in Table 1 and values of W listed in ASAE Data D241, Density, Specific Gravity, and Weight-Moisture Relationships of Grain for Storage, are values that will produce estimates of the upper bound grain pressures.

5.1.2 Dynamic pressures. Janssen's equation was derived for static conditions. Under dynamic or plug flow conditions, forces are generated which are larger than those predicted using Janssen's technique.

5.1.2.1 Funnel flow. Pressures can be predicted by Janssen's equation in bins which empty by funnel flow. Material movement occurs in a center core of the mass, and overpressures are not generated. Studies of flow patterns in bins under 3 m (10 ft) in diameter indicate that the transition between funnel and plug flow may be at a point as low as H/D equal to 1.3 for small bins (Nguyen, 1980). However, field observation of bins over 3 m (10 ft) in diameter indicate that the transition point in large bins occurs at a point near H/D equal to 2.0 (Usry and Thompson, 1986). For

any specific situation when it can be shown or it is suspected that plug flow will exist in a bin, lateral wall pressures should be estimated by the method described in paragraph 4.1.2.2.

5.1.2.2 Plug flow. Pressures in plug flow bins are greater than those predicted by Janssen's equation. These pressures can be predicted by using Janssen's equation combined with overpressure factors, F . Recommended values of F are presented in Table 1 along with representative values of k and μ for different wall surfaces. The grain bin wall coefficient of friction for corrugated bins is the grain-on-grain coefficient of friction (Moore *et al.*, 1983). Plug flow is defined as flow from a bin in which all or part of the material moves as a unit, with material movement along the bin walls. The overpressure factors presented are based on an analysis of the results reported by Platanov and Kovtun (1959) on full scale bins filled with wheat. Wheat is considered to exert the highest pressures on bins. The results of Platanov and Kovtun (1959) serve as the basis of the Russian (Soviet Code, 1965), German (DIN, 1964) and American Concrete Institute (ACI, 1983) recommendations for the calculation of loads exerted by granular materials.

5.1.2.3 Overpressure factor. Overpressure factors can be reduced in the lower portions of plug flow, flat bottom bins. This reduction is based on the effect of the stationary grain along the bottom of the bin wall. No reduction is allowed if material movement is present along the entire bin wall.

5.1.3 Calculation of wall loads. The vertical wall load per unit of wall length in a bin at depth, Y , is:

$$P_v = \int_0^Y \mu L(u) du$$

Settling of the material may produce smaller forces on the walls and greater forces on the floor.

5.2 Hopper pressures. Hoppers are commonly classified as funnel flow or mass flow. For free-flowing agricultural grains, funnel flow hoppers are the most common. When grain flows from the hopper outlet, an expanding channel is formed within the stagnant material until the channel either intersects the bin wall or intersects the top surface of the material. Flow along the hopper walls is nonexistent until a major portion of the bin has been emptied. A second, less common hopper for free-flowing agricultural grains is a mass flow hopper. Mass flow hoppers are sufficiently steep and smooth to cause all of the grain in the bin and hopper to be in motion whenever any of it is withdrawn through the hopper outlet. This Engineering Practice only presents methods for predicting pressures within funnel flow hoppers. For pressure in mass flow hoppers, see discussions by Jenike (1980), Walker (1966), Walters (1973), and Wilms (1985).

5.3 Loads on antidynamic tubes and flumes. Antidynamic tubes and flumes are generally placed in bins containing free-flowing grains to promote top unloading of the material contained in the bin by funnel flow. Antidynamic tubes are typically supported and stabilized by their connection to the bin bottom, while flumes are typically attached to the wall. Both devices are normally equipped with multiple openings in the wall of the tube or flume along the vertical axis to allow grain to flow into an enclosed flow channel formed by the device. The grain will then flow through this channel to an outlet from the bin. Normally the flow channel will be filled through the uppermost opening at which the grain is present with little or no material entering the flow channel at lower levels. Once the level of the grain falls below the uppermost opening, the grain will begin to flow through the next lower opening in an antidynamic tube or flume. If the flow of the grain is restricted at the uppermost opening, flow into the channel will occur at the next lower opening. If complete blockage of the flow channel occurs for any reason in an antidynamic tube or flume, inflow to the flow channel will occur at the next opening below the blockage. Likewise, if the flow channel is partially blocked so that the flow from the bin is greater than the flow through the partial blockage, grain will flow into the channel through the next opening below

the partial blockage. These situations can create unloading patterns which may result in lateral and vertical loads which are normally associated with plug flow.

5.3.1 Antidynamic tubes. Antidynamic tubes have been shown to be effective devices for reducing pressures in bins which have height to diameter ratios or hopper configurations which cause plug flow from the bin. Antidynamic tubes facilitate and encourage funnel flow throughout the entire height of a bin. Antidynamic tubes are normally installed in the center of bins over an outlet orifice. Off-center loading or unloading of a bin in which a center located antidynamic tube is installed may create lateral forces or overturning moments on the antidynamic tube which are greater than those expected with center loading and unloading. Center unloading through an antidynamic tube should result in symmetric loading of the bin walls and the antidynamic tube. Blockage of the upper inlets or the lower part of the central channel of an antidynamic tube could result in a bin unloading by plug flow.

5.3.2 Flumes. Center loading and unloading of bins is desirable because it normally maintains symmetrical loading of the bin walls and bottom. Off-center loading and unloading of bins will cause nonsymmetric loading of bin walls, flat floors, and hopper bottoms. Flumes are used to encourage funnel flow of material into the uppermost opening in the flume at which the grain is present. Side unloading of bins through a flume will create an uneven top surface of the granular mass. The top most surface assumes the shape of a conic section with the apex at the flume inlet and the surface radiating from the apex at the angle-of-repose of the grain. Blockage of the upper inlets or the lower central channel of a flume could result in side unloading at a location near the bottom of the flume. This type of unloading will cause unsymmetrical loading patterns.

5.4 Special load-considerations. Special load considerations are effects which are imposed on bin walls by events that are not related to proper bin operation under normal environmental conditions. They can be accounted for by selection of the appropriate factor of safety.

5.4.1 Thermally induced pressures. Rapid decreases in ambient temperature can increase wall stresses because the bin wall does not undergo free contraction. Laboratory studies with steel model circular bins indicate that design lateral pressures may vary with static pressure levels and rates of air temperature decline (Manbeck and Muzzelo, 1985; Britton, 1973; Zhang *et al.*, 1987). The recommendations given in this Engineering Practice are based on these laboratory studies. Qualitative results collected from full size bins (Blight, 1985) indicate that this effect occurs, but quantitative results needed for design purposes are not available from large bins.

5.4.2 Moisture induced or hygroscopic loads. Stored grains are hygroscopic; that is, they absorb moisture from liquid sources and from the atmosphere. When grains absorb moisture, they expand. When grains are confined within a structure, the expansion is restrained. The consequence is an increase in bin wall pressure, herein defined as the moisture induced or hygroscopic load. Data relating to increases in bin loads caused by increases in grain moisture content are limited in both number and in scope. Research reports which deal with the subject are Dale and Robinson (1954), Risch and Herum (1982), and Blight (1986). Dale and Robinson (1954) reported that lateral pressure increased six times as grain moisture content increased 4%, and increased by a factor of 10 for a 10% increase in grain moisture content. Because of the potential for high loads, it is recommended that grain bins be designed, located and managed to prevent grain moisture contents from increasing more than one or two percent during storage.

5.4.3 Vibration induced pressures. Vibration can lead to increased bulk density that causes increases in grain pressures. Sources of vibration include earthquakes, moving equipment and vehicles traveling on nearby roads or railroads. The effects of vibration are not well understood.

5.5 Grain properties. The grain properties recommended in paragraph 4.1 are based on values suitable for design of bins used for storage of wheat. The grain-bin wall coefficient of friction varies with the bin wall material. The value for corrugated bins is for grain-on-grain. If a bin is to be used to store a variety of grains over its lifetime, it is recommended

that it be designed for the storage of wheat. Values of bulk density for other grains are given in ASAE Data D241, Density, Specific Gravity, and Weight-Moisture Relationships of Grains for Storage. These values are based on standard tests and should be multiplied by a factor of 1.08 to account for the effects of compaction in an actual bin. The use of values of bulk density determined by the Winchester Bushel Test (USDA, 1980) in lieu of the values listed in ASAE Data D241 is acceptable.

Cited Standard:

ASAE D241, Density, Specific Gravity, and Weight-Moisture Relationships of Grain for Storage

References

1. ACI. 1983. Recommended practice for design and construction of concrete bins, silos, and bunkers for storage of granular materials. Revised 1983, and Commentary. ACI 313R-77. American Concrete Institute, Detroit, MI.
2. Blight, G. E. 1985. Temperature changes affect pressures in steel bins. *International Journal of Bulk Solids Storage in Silos* 1(3):1-7.
3. Blight, G. E. 1986. Swelling, pressures of wetted grain. *Bulk Solids Handling* 6(6):1135-1140.
4. Bokhoven, W. H., I. J. Ross, and P. C. Hammar. 1986. Flow considerations and loads on hoppers. ASAE Paper No. 86-4505. ASAE, St. Joseph, MI 49085.
5. Britton, M. G. 1973. Strain in deep bin walls due to ambient temperature decrease. Ph.D. Thesis. Texas A&M University.
6. Britton, M. G. and E. B. Moysey. 1986. Grain properties in the proposed new engineering practice on bin loads. ASAE Paper No. 86-4502. ASAE, St. Joseph, MI 49085.
7. Bucklin, R. A., S. A. Thompson, and C. D. Fankhauser. 1986. Methods of predicting static and dynamic loads in bins. ASAE Paper No. 86-4503. ASAE, St. Joseph, MI 49085.
8. Dale, A. C. and R. N. Robinson. 1954. Pressures in deep storage structures. *Agricultural Engineering* 35(8):570-573.
9. DIN. 1964. Design loads for buildings—loads in silos. DIN 1055, Blatt 6, Deutsche Normen, Berlin; also Beton und Stahlbetonbau. 1965, 5:126-128.
10. Janssen, H. A. 1895. Versuche uber getreidedruck in silozellen. *VDI Zeitschrift* 39:1045-1049.
11. Jenike, A. W. 1980. Effect of solids flow properties and hopper configuration on silo loads. *Unit and Bulk Materials Handling*. F. J. Loeffle and C. R. Procter (ed) pp. 97-106. American Society of Mechanical Engineers, New York, NY.
12. Manbeck, H. B. and L. M. Muzzelo. 1985. Thermally induced pressure in a model grain bin. *Transactions of the ASAE* 28(4):1253-1258.
13. Manbeck, H. B., V. M. Puri, and D. L. Wambeke. 1986. Special load considerations in bins. ASAE Paper No. 86-4504. ASAE, St. Joseph, MI 49085.
14. Moore, D. W., G. M. White, and I. J. Ross. 1983. Coefficient of friction of wheat on smooth and corrugated metal surfaces. ASAE Paper No. 83-2111. ASAE, St. Joseph, MI 49085.
15. Nguyen, T. V., C. E. Brennen, and R. H. Sabersky. 1980. Funnel flow in hoppers. *Transactions of the ASME Series E*, 102:729-735.
16. Platanov, P. N. and A. P. Kovtun. 1959. Davlenic zerna na stenki silosov elevatorov (The pressure of grain on silo walls). *Mukomolno Elevatornaia Promyshlennost* 25(12):22-24.
17. Risch, E. and F. L. Herum. 1982. Bin wall stresses due to aeration of stored shelled corn. ASAE Paper No. 82-4072. ASAE St. Joseph, MI 49085.
18. Ross, I. J., W. H. Bokhoven, and P. C. Hammar. 1986. Loads on antidynamic tubes and flumes. ASAE Paper No. 86-4506. ASAE, St. Joseph, MI 49085.
19. Soviet Code. 1965. Ukanzia po proectirovaniu silosov dlia siputshich materialov. (Instructions for design of silos for granular materials). Soviet Code CH-302-65 Gosstroy, Moscow, USSR.
20. Usry, J. L. and S. A. Thompson. 1986. Loads on vertical stiffeners on corrugated grain bins. *Transactions of the ASAE* 29(5):1355-1363.
21. USDA. 1980. Grain Inspection Handbook. U.S. Federal Grain Inspection Service, Washington, DC.
22. Walker, D. M. 1966. An approximate theory for pressures and arching in hoppers. *Chemical Engineering Science* 21(11):975-997.
23. Walters, J. K. 1973. A theoretical analysis of stresses in axially-symmetric hoppers and bunkers. *Chemical Engineering Science* 28:779-789.
24. Wilms, H. 1985. Calculation of stresses in silos by the method of characteristics. *Bulk Solids Handling* 5(2):425-429.
25. Zhang, Q., V. M. Puri, H. B. Manbeck, and M. C. Wang. 1987. Finite element model for predicting static and thermally induced bin wall pressures. *Transactions of the ASAE* 30(6):1797-1806.