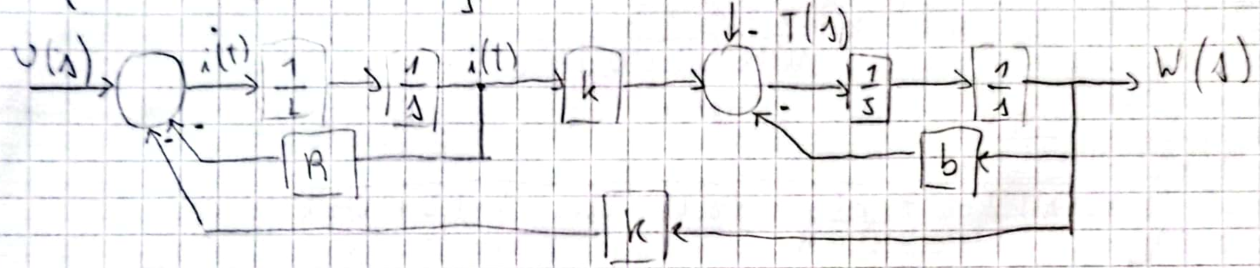
G-57671/3 | Joaquín González Zargón | HOJA N° 1
FECHA

Trabajo práctico 1: Análisis de sistemas lineales

2.2

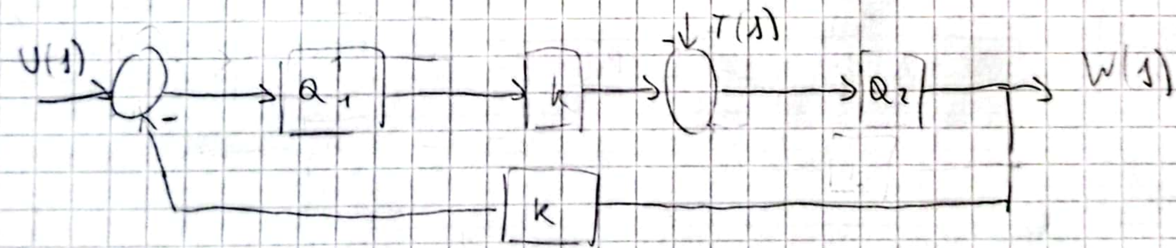
$$\begin{cases} L\,\dot{i}(t) + R\,i(t) + k\,w(t) = U(t) \\ J\,\dot{w}(t) + b\,w(t) + k\,i(t) = -T(t) \end{cases}$$

$$\begin{cases} \dot{i}(t) = \dfrac{U(t) - R\,i(t) - k\,w(t)}{L} \\ \dot{w}(t) = \dfrac{-T(t) - b\,w(t) + k\,i(t)}{J} \end{cases}$$



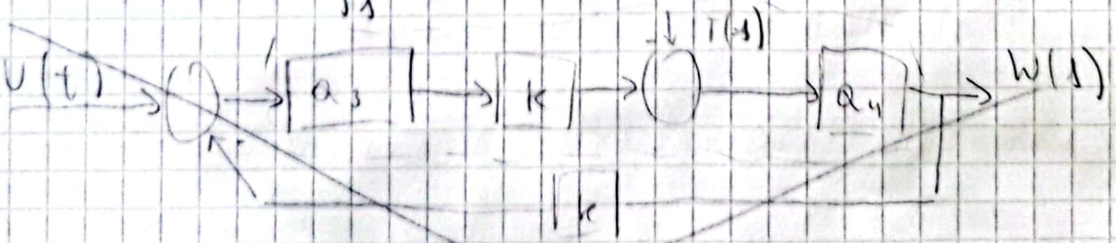
1) Hallar FT $U \to W$ y FT $T \to W$

$U \to W$



$$Q_1 = \frac{\frac{1}{sL}}{1 + \frac{R}{sL}} \Rightarrow Q_1 = \frac{1}{sL + R}$$

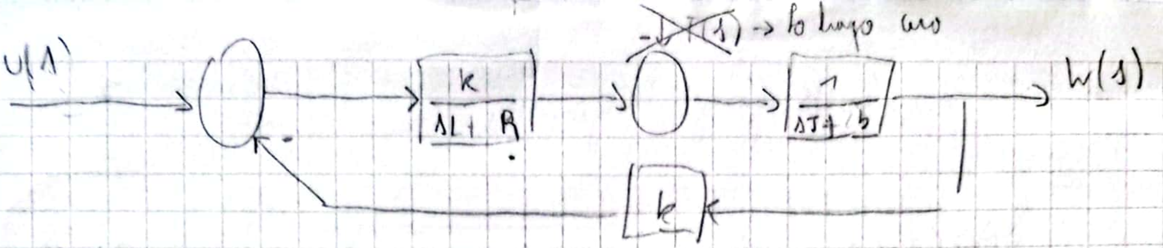
$$Q_2 = \frac{\frac{1}{sJ}}{1 + \frac{b}{Js}} \Rightarrow Q_2 = \frac{1}{Js + b}$$

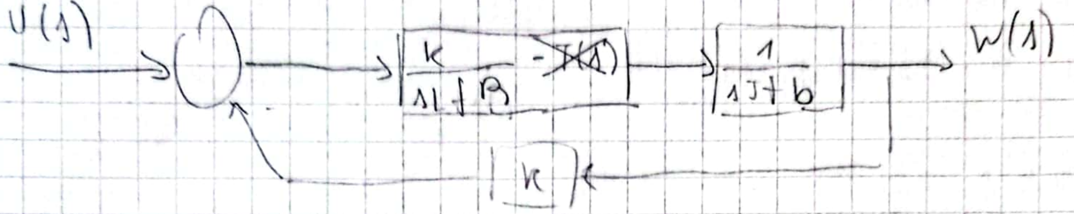


$$\cancel{Q_3 = \frac{\frac{1}{sL+R}}{1 + \frac{R}{sL+R}} \Rightarrow Q_3 = \frac{1}{sL + 2R}}$$

$$\cancel{Q_4 = \frac{\frac{1}{sJ+b}}{1 + \frac{b}{sJ+b}} \Rightarrow Q_4 = \frac{1}{sJ + 2b}}$$

NOTA





Resuelvo el lazo simple

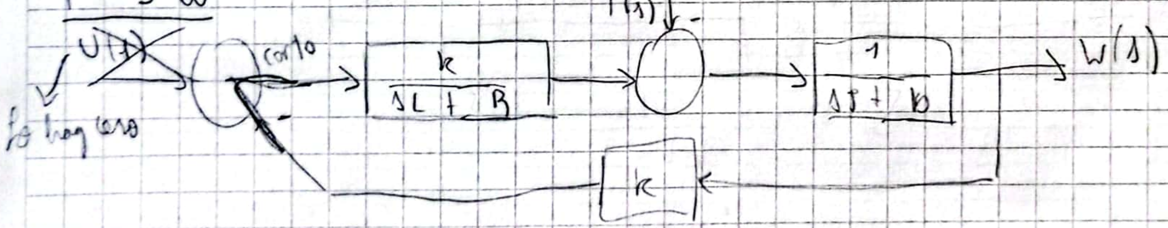
$$\frac{W(s)}{U(s)} = \frac{\left(\frac{k}{sL+R}\right)\left(\frac{1}{sJ+b}\right)}{1 + k\left(\frac{k}{sL+R}\right)\left(\frac{1}{sJ+b}\right)}$$

$$\frac{\frac{k}{sL+R}}{sJ+b+\frac{k^2}{sL+R}} = \frac{k}{s^2LJ + sLb + sRJ + bR + k^2}$$

$$\frac{W(s)}{U(s)} = \frac{k}{s^2(LJ) + s(Lb+RJ) + (Rb+k^2)}$$

$$\frac{W(s)}{U(s)} = \frac{\frac{k}{LJ}}{s^2 + s\left(\frac{b}{J}+\frac{R}{L}\right) + \left(\frac{Rb+k^2}{LJ}\right)}$$

$T \to W$



$$\frac{W(s)}{T(s)} = \frac{\frac{1}{sJ+b}}{1+\frac{1}{sJ+b}\,\frac{k}{sL+R}\,k} = \frac{1}{sJ+b+\frac{k^2}{sL+R}} =$$

$$\frac{sL+R}{s^2LJ + sRJ + sbL + Rb + k^2} = \frac{sL+R}{s^2LJ + s(RJ+bL) + (Rb+k^2)}$$

$$\frac{W(s)}{T(s)} = \frac{\frac{1}{J}\left(s+\frac{R}{L}\right)}{s^2 + s\left(\frac{R}{L}+\frac{b}{J}\right) + \left(\frac{Rb+k^2}{LJ}\right)}$$

NOTA

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + 2\xi W_n s + W_n^2}$$

Hoja 2

2) A partir de la respuesta temporal de la Fig. 3, identifique la FT $U \to W$

T.V.F. : $\lim_{t\to\infty} W(t) = 440 \lim_{s\to 0} G(s) = \frac{b_0 \, 440}{W_n^2} = 65 \Rightarrow \frac{b_0}{W_n^2} = 0{,}15$

T.V.I. : $\lim_{t\to 0^+} W(t) = \lim_{s\to\infty} G(s) = 0$

T.V.I. : $\lim_{t\to 0^+} \dot{W}(t) = \lim_{s\to\infty} s\,G(s) = 0$

Sobrevalor: $\frac{|h(t+T_a/2) - h_f|}{h(t) - h_f} = 0{,}4 = e^{-\sigma T_a/2} = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$

$\ln 0{,}4 = \frac{-\pi \xi}{\sqrt{1-\xi^2}} \Rightarrow \sqrt{1-\xi^2}\,\ln 0{,}4 = -\pi \xi \Rightarrow 1-\xi^2 = \frac{\pi^2 \xi^2}{\ln^2 0{,}4}$

$1 = \xi^2 \left(\frac{\pi^2}{\ln^2 0{,}4} + 1 \right) \Rightarrow \xi = \sqrt{\frac{1}{\left(\frac{\pi^2}{\ln^2 0{,}4} + 1\right)}} = 0{,}28$

$T_a = 0{,}2\,seg = \frac{2\pi}{W_a}$

$W_a = 31{,}42\ rad/seg \Rightarrow W_a = W_n \sqrt{1-\xi^2}$

$W_n = \frac{W_a}{\sqrt{1-\xi^2}} = 32{,}73\ rad$, $\sigma = \xi W_n = 9{,}16$

$\frac{b_0}{W_n^2} = 0{,}15 \Rightarrow b_0 = 160$

$2\xi W_n = 18{,}32$

$W_n^2 = 1071{,}25$

$$G(s) = \frac{160}{s^2 + 18{,}32 s + 1071{,}25}$$

3) Calcular k, R y L asumiendo $J = 15\ kg\,m^2$ y $b = 1{,}1\ \frac{Nms}{rad}$

$x = L,\ y = k,\ z = R$

$$\begin{cases} \dfrac{k}{LJ} = 160 & (1) \\ \dfrac{b}{J} + \dfrac{R}{L} = 18{,}32 \Rightarrow \dfrac{bL + RJ}{JL} = 9{,}16 & (2) \\ \dfrac{Rb + k^2}{LJ} = 1071{,}25 & (3) \end{cases}$$

$$\cancel{(1)\quad k = 31911{,}75 \cdot L}$$

$$\cancel{(3)\quad \frac{R \cdot 1{,}1 + 31911{,}75^2\, L^2}{15\,L} = 1071{,}25}$$

$$\cancel{\frac{R}{L} \cdot 1{,}1 + 31911{,}75^2\, L = 16068{,}75 \quad (2)}$$

$$\cancel{R \cdot 1{,}1 + k^2 = 16068{,}78 \cdot L}$$

Wolfram Alpha

$L = 2{,}79\ Hy$

$k = 6{,}69$

$R = 0{,}051\ \Omega$

$\boxed{B.2}$

1) Identificamos FTD de $U_k \to W_k$ a partir de la gráfica de la resp. al escalón

$$G(z) = \frac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0}$$

Como nos concentramos solo en los polos complejos conjugados

$$G(z) = \frac{b_2 z^2 + b_1 z + b_0}{z^2 - z2a\cos(W_0) + a^2}$$

- TVI: $h(0) = 440 \lim_{z \to \infty} G(z) = \boxed{b_2 = 0}$
- TVI: $h(1) = 440 \lim_{z \to \infty} z\left[G(z) - \underbrace{h(0)}_{b_2}\right] + \underbrace{h(0)}_{b_2}$

$$h(1) = 440 \lim_{z \to \infty} \left[\frac{b_2 z^3 + b_1 z^2 + b_0 z}{z^2 - 2a\cos(W_0)z + a^2} - b_2 z\right] + b_2$$

$$h(1) = 440 \lim_{z \to \infty} \left[\frac{\cancel{b_2 z^3} + b_1 z^2 + b_0 z - \cancel{b_2 z^3} + z^2 2a\cos(W_0) b_2 - a^2 b_2 z}{z^2 - 2a\cos(W_0) z + a^2}\right] + b_2$$

L'Hopital

$$h(1) = \left[b_1 + 2a\cos(W_0) b_2 + b_2\right] 440 \approx 3$$

$b_2 = 0$

$$h(1) = b_1 . 440 \Rightarrow b_1 = \boxed{6,82 . 10^{-3}}$$

- TVF: $h(\infty) = G(1) = \dfrac{b_2 + b_1 + b_0}{1 - 2a\cos(W_0) + a^2} = 65$

(*) Hoja siguiente

Debo obtener "a" y "W_0"

- W_0 ...

Las muestras en la gráfica están en segundos, período de muestreo $T_s = 0,01$ seg

$$T_a = 0,2 \text{ seg} \Rightarrow k = \frac{T_a}{T_s} = \frac{F_a}{F_s} = 20 \text{ muestras}$$

$$k = \frac{2\pi}{W_0} \Rightarrow W_0 = \frac{2\pi}{k} = \boxed{0,31 \text{ rad/seg} = W_0}$$

- a ...

$$\frac{|h(k + k/2) - h_f|}{|h(k) - h_f|} = a^{\pi/W_0} = \frac{52-65}{92-65} \Rightarrow a = (0,48)^{W_0/\pi} = \boxed{a = 0,93}$$

$$G(1) = \frac{\overset{\nearrow 0}{(b_2)} + \overset{\nearrow 1350}{(b_1)} + b_0 \cdot 440}{1 - 2a\cos(w_0) + a^2} = 65$$

$$b_0 = 65/440 \left(1 - 2a\cos(W_0) + a^2\right) - b_1$$

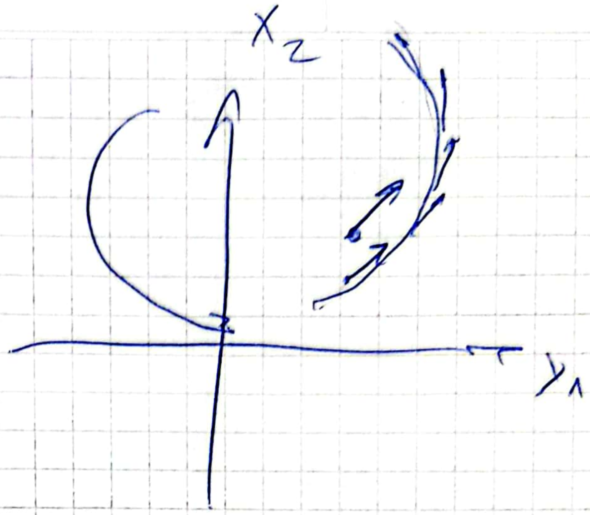
$$b_0 = 6{,}07$$

$$G(1) = \frac{6{,}82 \cdot 10^{-3} z + 0{,}007354}{z^2 - 1{,}77z + 0{,}86}$$

Polos: $z_{1,2} = 0{,}885 \pm j\,0{,}28$

$\dot{x} = Ax + Bu$

$y = Cx + Du$



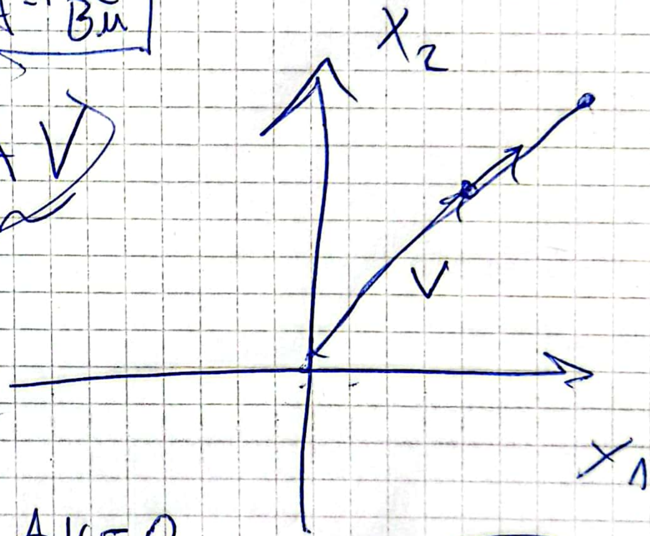
$\dot{x} = Ax + B\bar{u} = 0$

$A^{-1}Ax = A^{-1}(-B\bar{u})$

$Ix = -A^{-1}B\bar{u}$

$\boxed{\bar{x} = -A^{-1}B\bar{u}}$

$\lambda V = AV$



$\dot{v} = A.v$

$\lambda V - AV = 0$

$(\lambda I - A)\; V = 0$

$\boxed{\dot{w} = \Lambda w}$