

# Homework 1

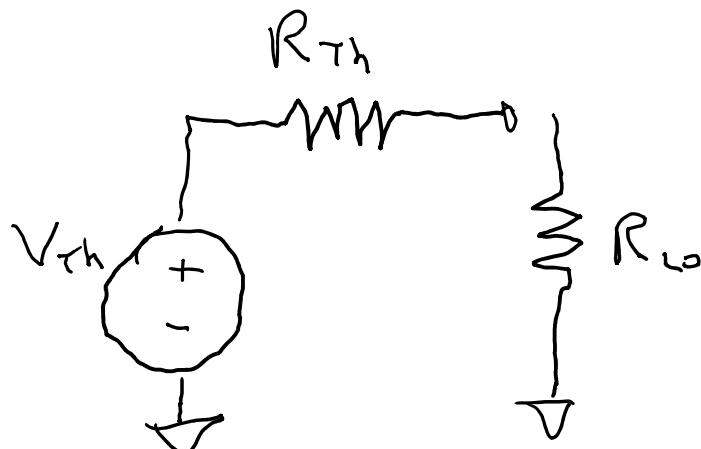
214 2023 Hemmer sections

# Problem 1

$V_o$



- A USB power source has a nominal voltage of 5 V. The allowable voltage range is within 5 % of the nominal. Typical USB cables are in the range AWG 20–28.
- (a) Consider a 10 W USB supply. What is the smallest Thevenin equivalent voltage  $V_{Th}$  and the largest Thevenin equivalent resistance,  $R_{Th}$ , of this supply if it is to meet specs?
- (b) What is the smallest load resistor that can be driven by this supply without overloading?
- (c) What fraction of the supply power is dissipated in the load under this maximal load condition?
- (d) Why is this not optimized for maximum power transfer?
- (e) What is the maximum length of cable that can be used with a 5 W USB load, assuming copper AWG 20?



$$V_L = V_o(1 - \epsilon), \quad V_H = V_o(1 + \epsilon)$$

$$P_o = V_o I_o \quad V_{Th} = V_H, \quad V_L = V_H - R_{Th} I_o$$

$r_{th} \rightarrow \frac{2 e v o^2}{p_0} \quad R_{Th} = 0.25 \Omega \quad (a)$

$$(b) P_{L0} = V_L I_o, \quad R_{L0} = \frac{V_L}{I_o} = 2,375 \Omega$$

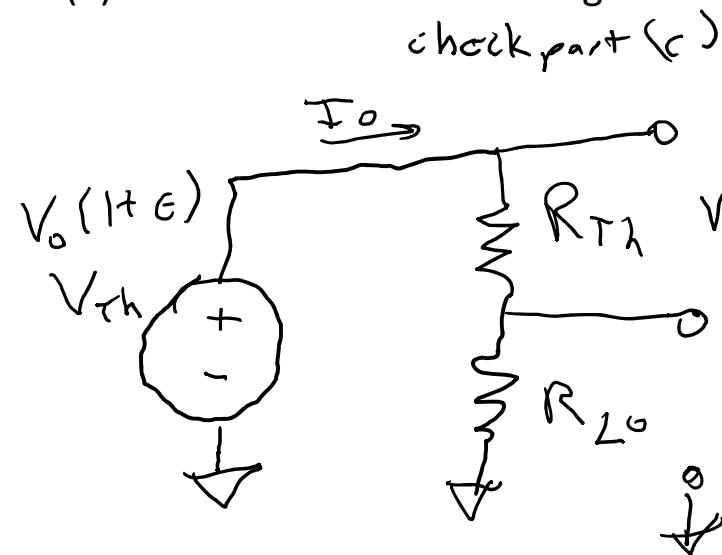
$$(c) P_s = V_{Th} I_o, \quad \text{Fraction} = \frac{P_{L0}}{P_s} = \frac{1 - \epsilon}{1 + \epsilon} = 90.5\%$$

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$$-P_{V_{th}} = \underbrace{I_o V_o (1+\epsilon)}_{P_{R_{th}}} \quad | \quad R_{th} = 26 \frac{V_o^2}{P_o} = 26 \frac{V_o}{I_o}$$

$$P_{R_{th}} = I_o^2 R_{th} \rightarrow \underbrace{2\epsilon V_o I_o}_{P_L} \quad | \quad R_{th} = \frac{V_o(1-\epsilon)}{I_o}$$

$$P_L = I_o^2 R_{th} \rightarrow \underbrace{V_o I_o (1-\epsilon)}_{P_L}$$

$$R_{th} = 0.25 \Omega, R_{th} = 2.375 \Omega$$

(d)

Diagram showing a dependent current source  $K_2$  connected in parallel with a load resistor  $R_{th}$ . The dependent source is controlled by the voltage across  $R_{th}$ , which is  $V_o - V_{th}$ . The current through the dependent source is  $I_o$ .

$$V_o = V_{th} \quad | \quad I_o = 0$$

$$= \frac{1}{2} V_{th} \quad | \quad I_o = I_{max}$$

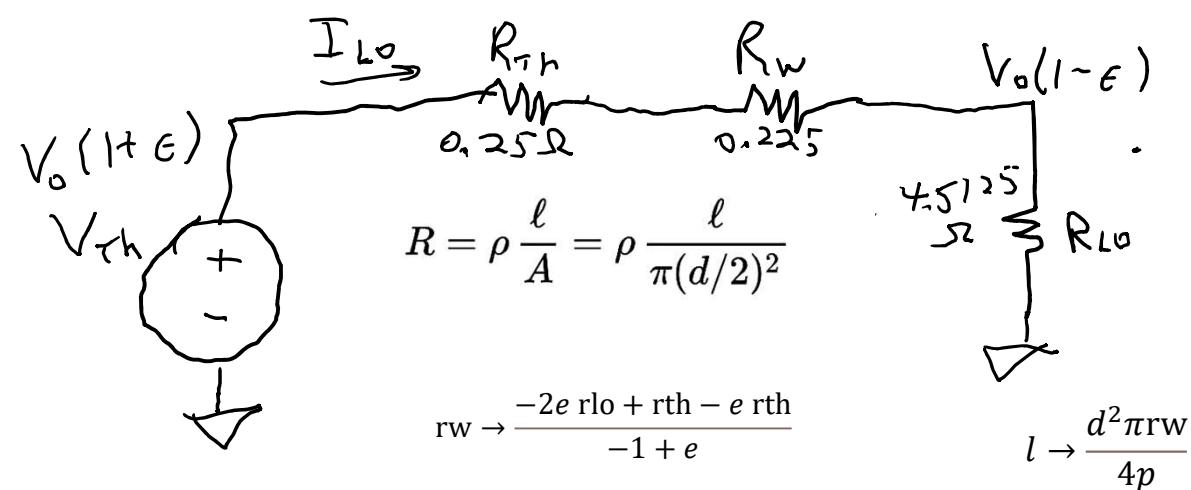
*VSB voltage can only be  $\pm 5\%$  not 50%*

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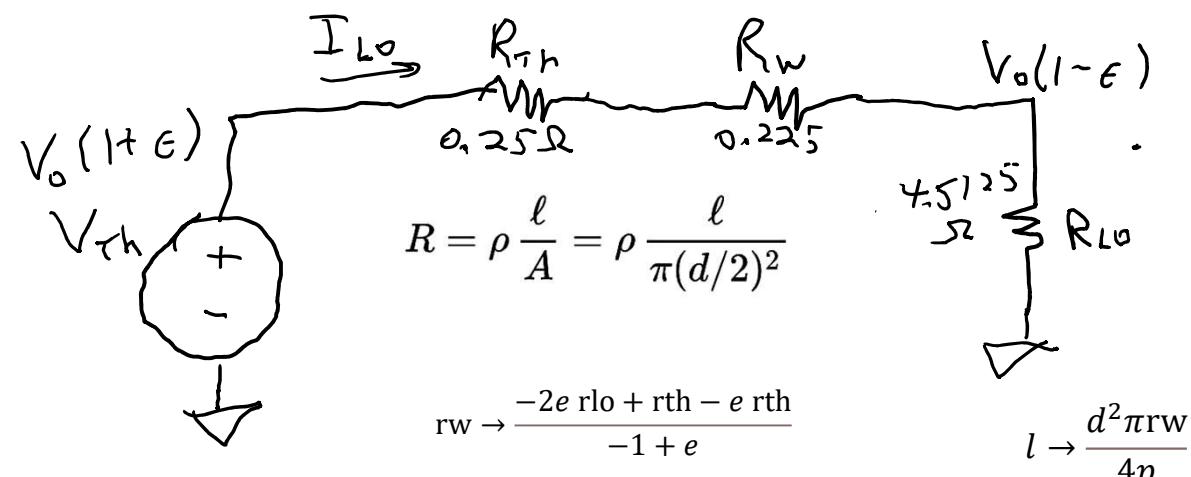
$$\begin{aligned}
 P_{L0} &= I_{L0} V_o (1 - \epsilon) \rightarrow I_{L0} = \frac{P_{L0}}{V_o (1 - \epsilon)} \\
 V_{L0} &= V_o (1 - \epsilon) = \frac{R_{L0} (V_o (1 + \epsilon))}{R_{Th} + R_w + R_{L0}} \\
 V_o (1 - \epsilon) &= I_{L0} R_{L0} \rightarrow R_{L0} = \frac{V_o (1 - \epsilon)}{I_{L0}} \\
 \text{Solve for } R_w &= 0.225 \Omega \quad \left\{ \begin{array}{l} I_{L0} \\ = 4.5125 \Omega \\ = 1.053 A \end{array} \right. \\
 \text{Solve for } l &= 6.575 m
 \end{aligned}$$

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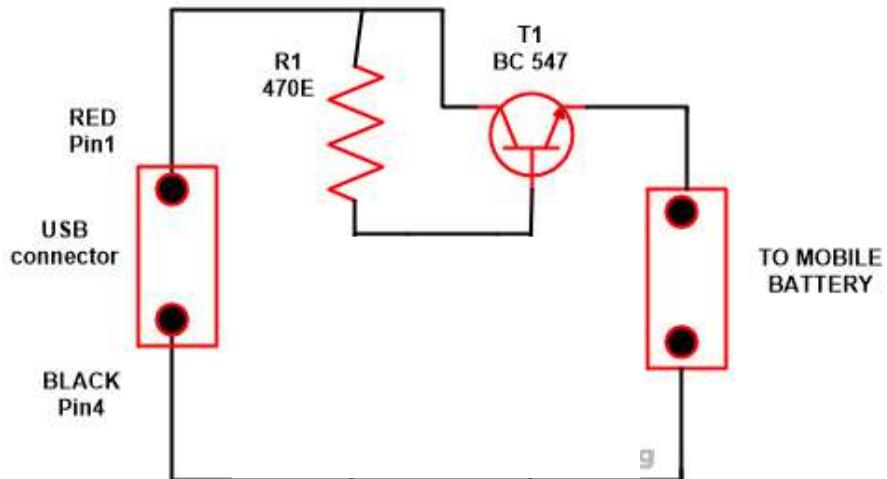
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"problem 1"
"part a"
values={v0→5,e→0.05,p0→10,ilo→5}
assign={vl→v0(1-e),vh→v0(1+e),i0→p0/v0}
as2={vth→vh}
s1=Solve[vl==vh-rth i0,rth]/.as2
Factor[s1/.assign]
s1/.assign/.values
"part b"
rlo→vl/i0/.assign/.values
"part c"
fraction→(vl i0)/(vth i0)/.as2/.assign
fraction→(vl i0)/(vth i0)/.as2/.assign/.values
"part e"
as3=rlo→v0 (1-e)/ilo/.ilo→p0/(v0 (1-e))
s2=Solve[v0(1-e)==v0(1+e)(rlo/(rlo+rw+rth)),rw]
s2=s2/.as3/.s1/.as2/.assign
Expand[Factor[s2]]
s2/.values
values2={p→1.72 10^-8,d→0.8 10^-3}
s4=Solve[rw=p 1/(Pi (d/2)^2),1]

```

## Problem 2

- Consider the USB-powered mobile-battery charger circuit shown below. Assume the USB power supply is the newer 10 W version but the mobile battery is the older 2.5 W version. Assume the transistor beta is 100 and  $V_{BE} = 0.7$  V. Take  $R_1 = 470 \Omega$ .
  - (a) Sketch the equivalent circuit.
  - (b) What is the maximum voltage to which the mobile battery can be charged with this circuit?
  - (c) What is the charging current as a function of mobile battery voltage?
  - (d) What percent of the power generated by the USB supply is being stored in the mobile battery, as a function of mobile battery voltage? Note that energy lost due to the mobile battery resistance does not count toward power stored in the battery.



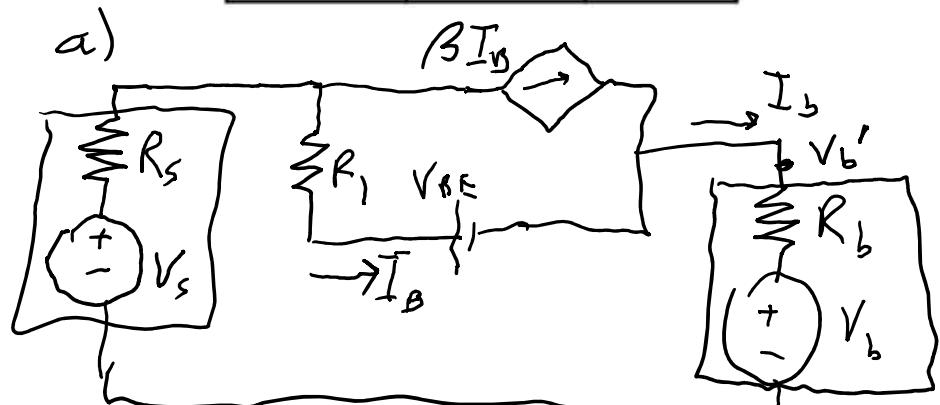
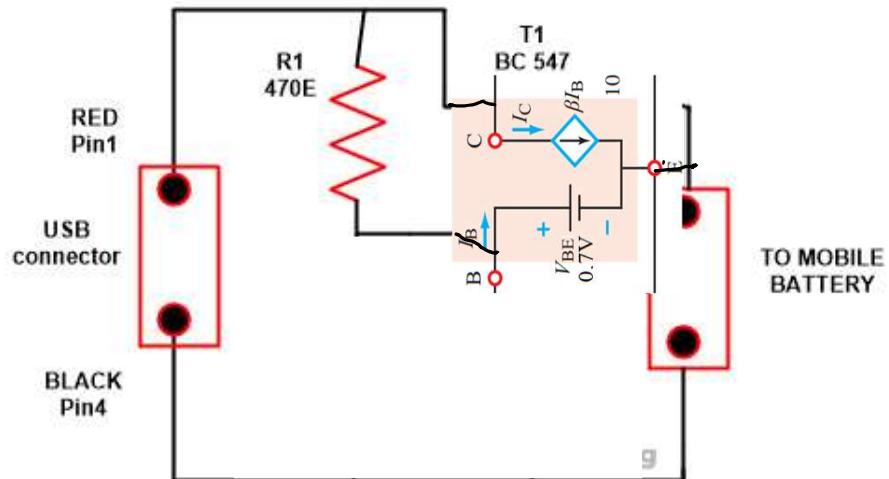
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$$b) V_s - V_{BE} = V_b^{\max} = 5.25 - 0.7 = 4.55$$

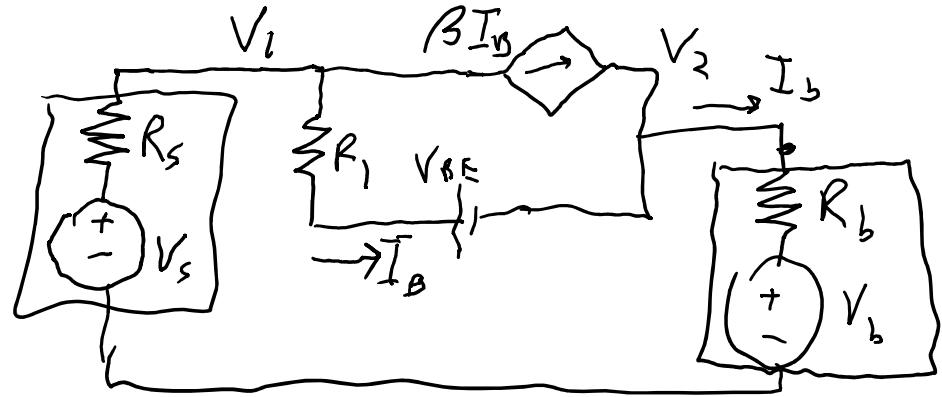
$$c) V_s = V_o(1 + \epsilon) = V_b , I_s R_s = V_o(1 + \epsilon) - V_o(1 - \epsilon) = 2\epsilon V_o \quad V_o = 5V, \epsilon = 5\%$$

$$I_s^{\max} = \frac{P_o}{V_o} , R_s = \frac{2\epsilon V_o}{P_o/V_b} = 2\epsilon \frac{V_o^2}{P_o} , P_{os} \approx 10W, P_{ob} \approx 2.5W$$



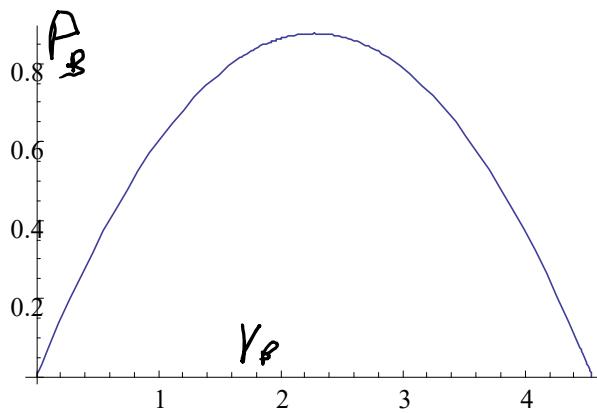
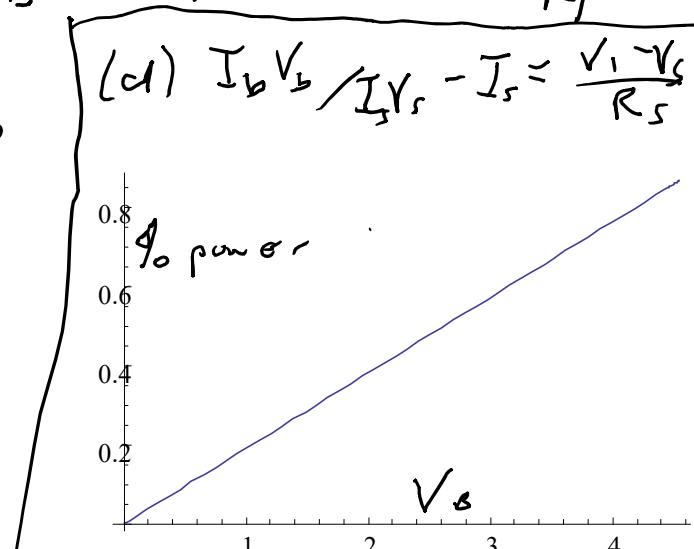
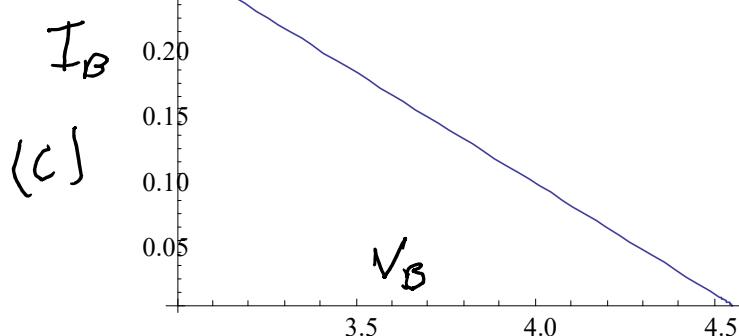
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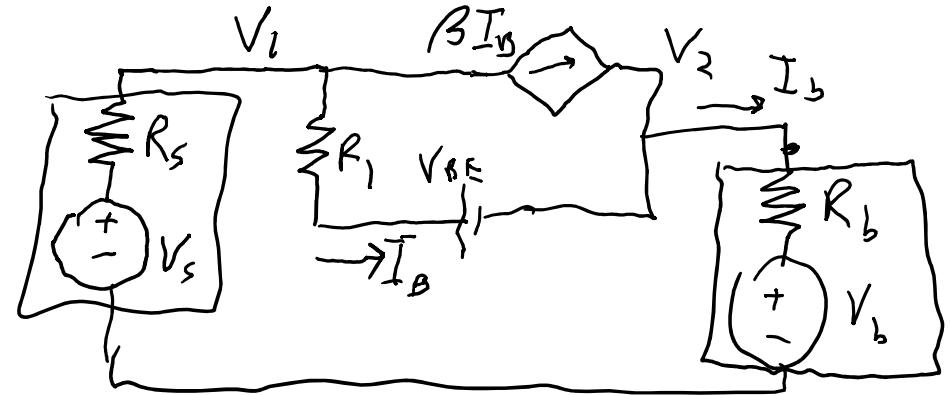
$$\frac{V_1 - V_s}{R_s} + \frac{V_1 - (V_{BE} + V_2)}{R_1} + \beta I_B = 0, \quad I_B = \frac{V_1 - (V_{BE} + V_2)}{R_1}, \quad I_b = \frac{V_2 - V_b}{R_b}$$

$$-\beta I_B - I_B + \frac{V_2 - V_b}{R_b} = 0$$



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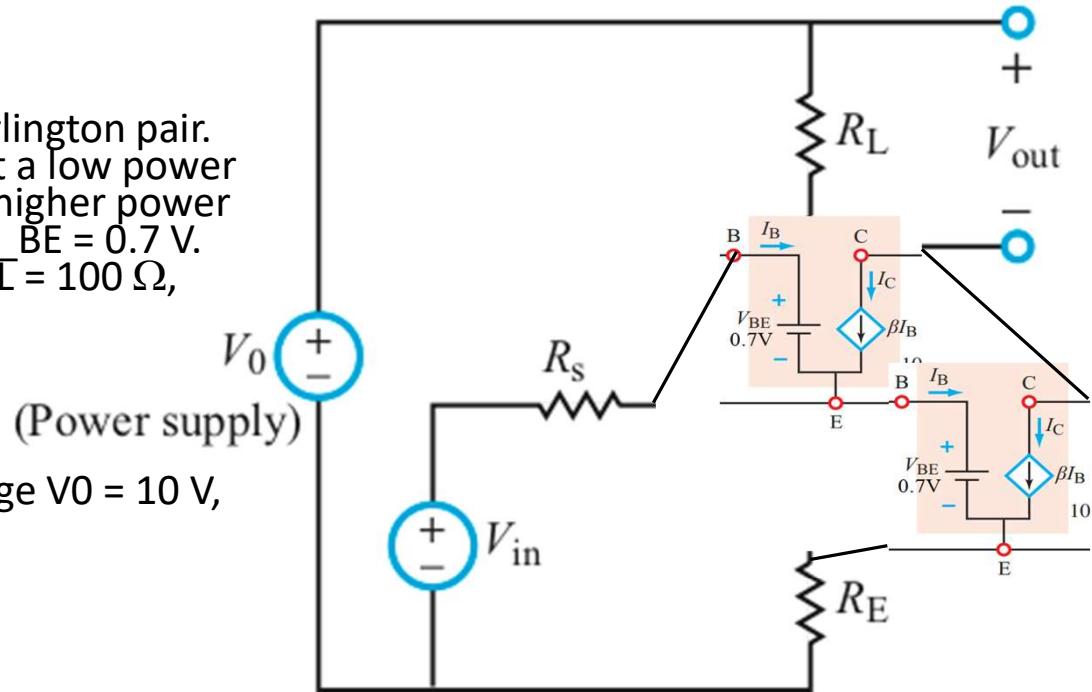
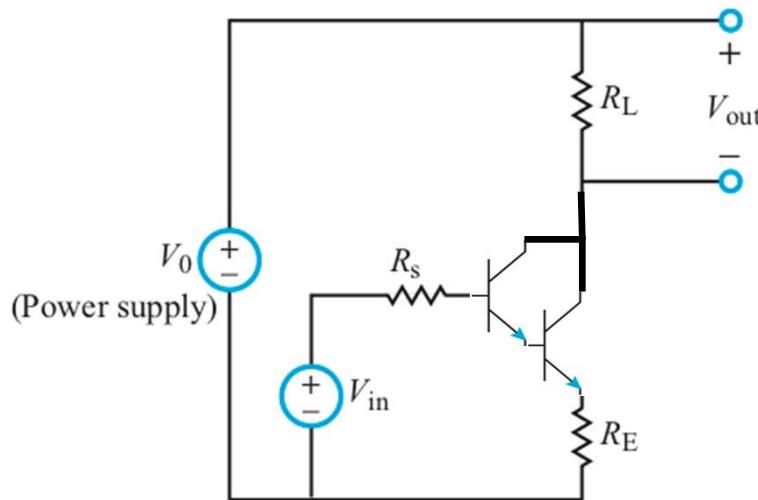
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"problem 2"
values={v0→5,e→0.05,ps→10,pb→2.5,b→100,vbe→0.7,r1→470}
assign={rs→2 e v0^2/ps,rb→2 e v0^2/pb,vs→v0(1+e)}
resistances→{rs,rb}/.assign/.values
e1=(v1-vs)/rs+ibb+b ibb=0
e2=ibb=(v1-(vbe+v2))/r1
e3=(v2-vb)/rb-ibb-b ibb=0
eqn={e1,e2,e3}
vec={v1,v2,ibb}
s1=Solve[eqn,vec]
e5=ibb=(v2-vb)/rb/.s1
e5=ibb=(v2-vb)/rb/.s1/.assign
s2=Solve[e5,ib]
s2=s2/.values
Plot[ib/.s2,{vb,3,4.55}]
"power check"
c1={prs→(v1-vs)^2/rs,prb→(v2-vb)^2/rb,pr1→(v1-(vbe+v2))^2/r1}
c2={pvs→vs -(vs-v1)/rs,pvb→vb -(v2-vb)/rb,pt→-(v2-v1)b ibb}
c3={c1,c2}/.s1/.assign/.values
-(pvb/pvs)/.Flatten[c3]
Plot[-(pvb/pvs)/.Flatten[c3],{vb,0,4.55}]
Plot[pvb/.Flatten[c3],{vb,0,4.55}]
Manipulate[{c1,c2}/.s1/.assign/.values/.vb→vbm,{{vbm,4},0,4.55}]
Manipulate[{(prs+prb+pr1),(pvs+pvb+pt)}/.Flatten[{c1,c2}]/.s1/.assign/.values/.vb→vbm,{{vbm,4},0,4.55}]

```

# Problem 3

- The transistor circuit shown below is known as a Darlington pair. In this device the input transistor has a high beta but a low power rating. The second transistor has a lower beta but a higher power rating. For simplicity assume both transistors have  $V_{BE} = 0.7$  V. Take  $\beta_1 = 100$  and  $\beta_2 = 10$ . Choose  $R_s = 1 \text{ k}\Omega$  and  $R_L = 100 \Omega$ ,  $R_E = 10 \Omega$ . Assume  $V_0 > V_{in}$  and  $V_{in} > V_{BE}$
- (a) What is the voltage gain of the circuit?
- (b) What is the limiting gain for large beta.
- (c) For an input voltage of  $V_{in} = 5$  V and supply voltage  $V_0 = 10$  V, what is the power dissipated in each resistor?
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$$\frac{V_{out}}{R_L} = \beta_1 I_{B1} + \beta_2 I_{B2}$$

$$\left\{ -\frac{v_{out}}{r_l} = b_1 i_{b1} + b_2 i_{b2}, (1 + b_2) i_{b2} r_e + i_{b1} r_{in} + 2v_{be} - v_{in} = 0, i_{b2} = (1 + b_1) i_{b1} \right\}$$

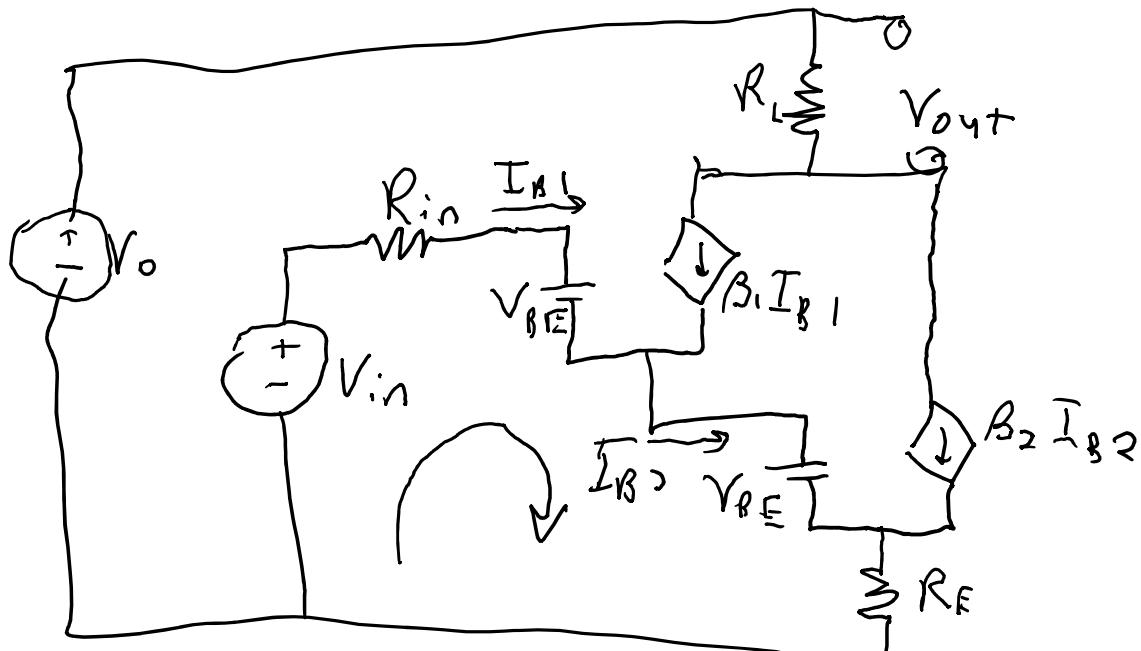
$$-V_{in} + R_{in} I_{B1} + V_{BE} + R_E (1 + \beta_2) I_{B2}$$

$$v_{out} \rightarrow \frac{(b_1 r_l + b_2 r_l + b_1 b_2 r_l)(2v_{be} - v_{in})}{r_e + b_1 r_e + b_2 r_e + b_1 b_2 r_e + r_{in}}$$

$$I_{B2} = (1 + \beta_1) I_{B1}$$

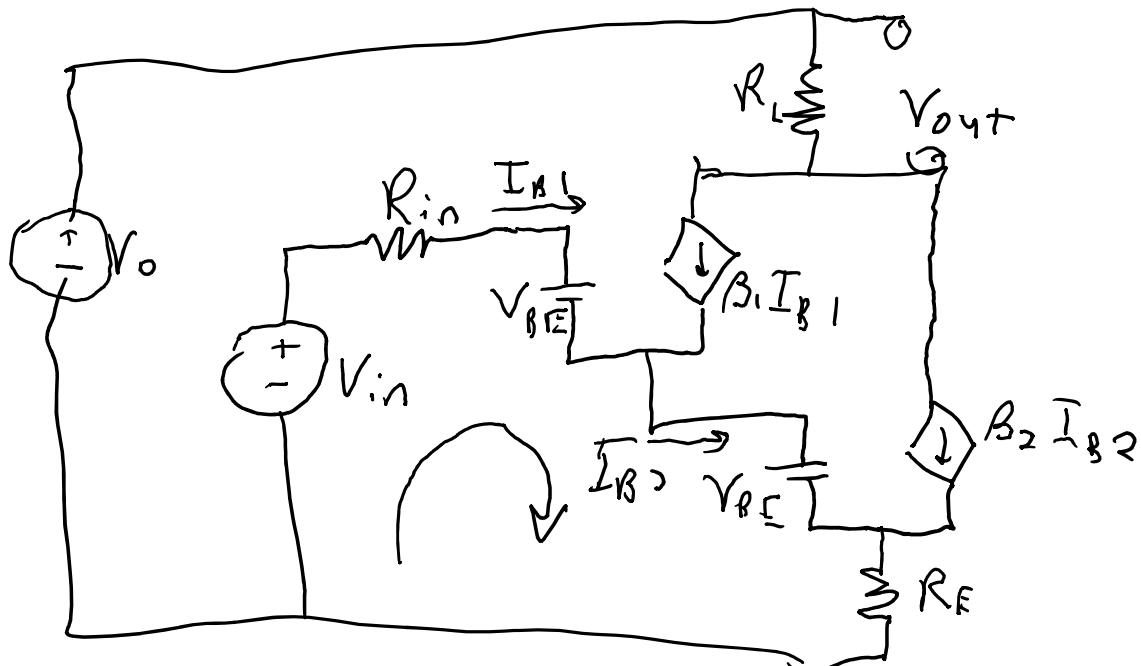
$$(a) V_{out} = \underline{9.1566(V_{in} - 1.4)}$$

$$(b) V_{out} = \frac{-R_L (\beta_1 + \beta_2 + \beta_1 \beta_2) (V_{in} - 2V_{BE})}{R_E (1 + \beta_1 + \beta_2 + \beta_1 \beta_2) + R_{in}} \xrightarrow{\beta \text{ large}} -\frac{R_L}{R_E} \quad \begin{cases} V_{in} \gg V_{BE} \\ R_E > \frac{R_{in}}{\beta_1 \beta_2} \end{cases}$$



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$$\left\{ \left\{ \begin{array}{l} p_{rin} \rightarrow 0.00008837241335105328, \\ prl \rightarrow 10.888365048983275, \\ pre \rightarrow 1.0907992561988544 \end{array} \right\}, \left\{ \begin{array}{l} p_{vin} \rightarrow -0.0014863748967795212, \\ pv0 \rightarrow -3.299752270850537, \\ pt1c \rightarrow -0.8026498086287207, \\ pt2c \rightarrow -7.896589656745457 \end{array} \right\}, \dots \right\}$$

$$P_{R_{in}} \sim 0 \quad P_{V_{in}} \sim 0 \quad \underline{(c) \& (d)}$$

$$P_{R_L} = 10.9 \text{ W} \quad P_{V_0} = -3.3 \text{ W}$$

$$P_{R_E} = 1.1 \text{ W} \quad P_{T1} = -0.8 \text{ W}$$

$$P_{T2} = -7.9 \text{ W}$$

Total power

Resists  $12 \text{ W}$ , Sources  $-12 \text{ W}$

$\{11.97925267759548, -12.000061926150396\}$

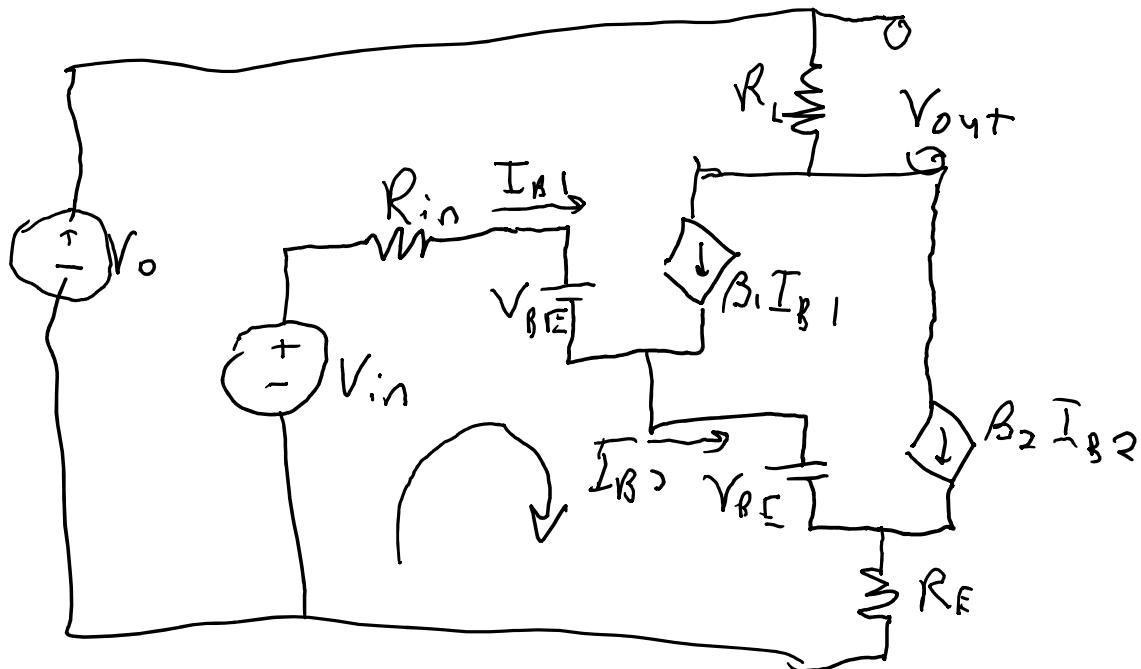
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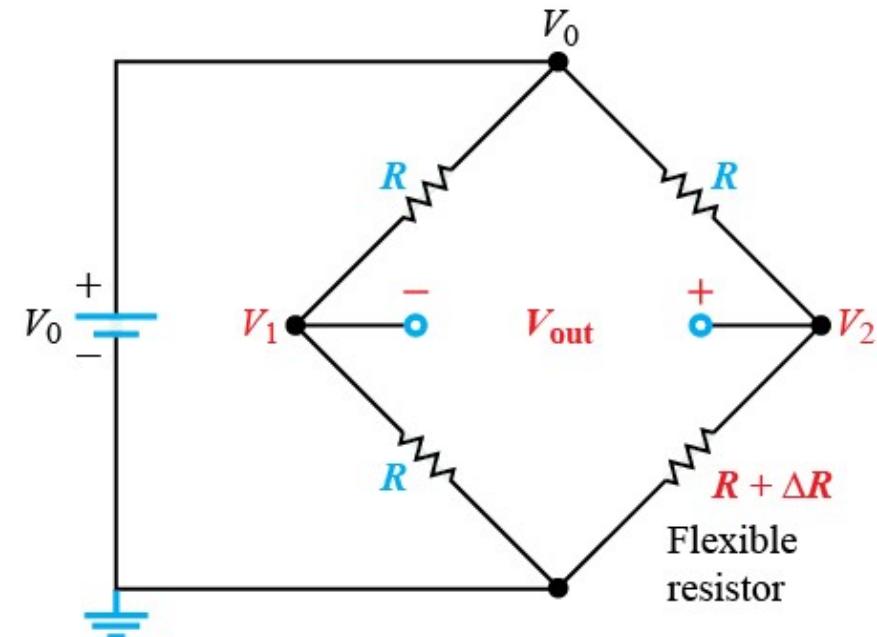
"problem 3"
values={b1→100,b2→10,vbe→0.7,rin→1000,r1→100,re→10}
values2={vin→5,v0→10}
e1=vout /r1=b1 ib1+b2 ib2
e2=-vin+rin ib1+vbe+vbe+re ib2(1+b2)==0
e3=ib2==ib1(1+b1)
eqn={e1,e2,e3}
vec={ib1,ib2,vout}
s1=Solve[eqn, vec]
s1/.values
N[vout/.s1/.values]
p1={{prin→rin ib1^2,pr1→r1 (b1 ib1 +b2 ib2)^2,pre→re ((1+b2) ib2)^2},
 {pvin→-vin ib1,pv0→-v0 (b1 ib1+b2 ib2),pt1c→((v0-vout)-(vin-rin ib1-vbe))b1 ib1,pt2c→((v0-vout)-(vin-rin ib1-vbe-vbe))b2
 ib2},pt1v→vbe ib1,pt2v→vbe ib2}
p2=p1/.s1/.values2
{prin+pr1+pre,pvin+pv0+pt1c+pt2c+pt1v+pt2v}/.Flatten[p2]

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# Problem 4

- Consider the Wheatstone bridge circuit shown below. Suppose you have a voltmeter that can measure in the range of 1 mV to 100 V, with an accuracy of 1 % from 10 mV to 100 V and 10 % from 1 to 10 mV.
- (a) What is the smallest percentage change you can measure in the flexible resistor?
- (b) Assuming you can measure the resistance directly with the voltmeter to 1 %, how much improvement do you get for  $V_0 = 100$  V.
- (c) If you treat the  $V_{out}$  terminal as a power supply, what is its Thevenin equivalent voltage and resistance as a function of the fractional change in the flexible resistor?
- (d) What is the power consumed in each resistor, the Thevenin equivalent resistor, and in the source when  $R = 100 \Omega$  and the fractional resistor change is 1 %?



$$\frac{(V_1 - V_0)}{R} + \frac{V_1}{R} = 0$$

$$\frac{dV_0}{2(2+d)} = V_{0,V+} = \frac{V_0}{2} \left( \frac{\Delta}{2+\Delta} \right)$$

Equates to  $V_{min}$

$$\frac{(V_2 - V_0)}{R} + \frac{V_2}{R(1+\Delta)} = 0$$

$$V_{min} = 0.1 \text{ mV} (1 - 10 \text{ mV}) \\ 0.1 \text{ mV} (10 \text{ mV} - 100 \text{ V})$$

$$(b) \frac{\Delta_d}{\Delta} = \frac{0.01}{4V_{min}/V_0} \approx 2500$$

$$d \rightarrow \frac{4V_{min}}{V_0 - 2V_{min}}$$

(a)

$$\Delta = \frac{4V_{min}}{V_0 - 2V_{min}} \rightarrow 4 \frac{V_{min}}{V_0}$$

# Problem 4

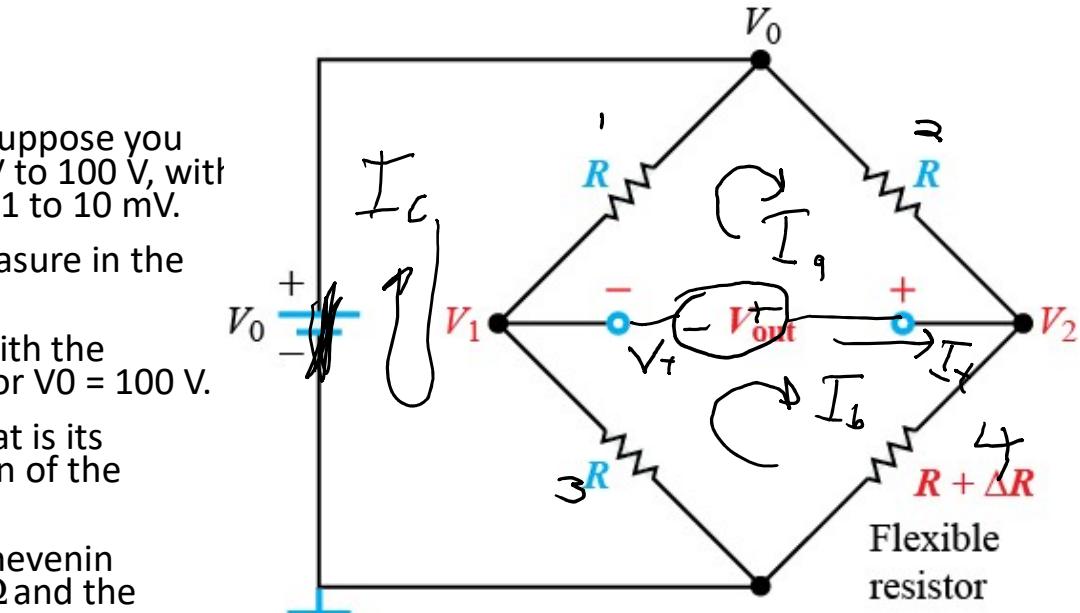
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$$\frac{dV_0}{2(2+d)} = V_{0,V^+} = \frac{V_0}{2} \left( \frac{\Delta}{2+\Delta} \right) > V_{Th} \quad - R_{Th} = \frac{V_t}{I^+} \quad \text{rth} \rightarrow \frac{(4+3d)r}{2(2+d)} \quad R_{Th} = R \frac{1 + \frac{3}{4}\Delta}{1 + \frac{1}{2}\Delta}$$

$$R(I_a - I_c) + RI_a + V_t = 0$$

$$R(I_b - I_c) - V_t + I_b R (1+\Delta) = 0$$

$$(I_c - I_a) R + (I_c - I_b) R = 0$$



$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} \text{pr1} \rightarrow 0.024876084453447772, \\ \text{pr2} \rightarrow 0.025124223411264152, \\ \text{pr3} \rightarrow 0.024876084453447772, \\ \text{pr4} \rightarrow 0.024875468724023908 \\ \text{"psum"} \rightarrow 0.0997518610421836, \\ \text{"prth"} \rightarrow 0.09975186104218361, \\ \{pt \rightarrow 0.09975186104218361} \end{array} \right\}, \\ (d) \end{array} \right\}$$

$P_1 = 24.9 \text{ mW}$   
 $P_2 = 25.1$   
 $P_3 = 24.9$   
 $P_4 = 24.9$   
 $P_{sum} = P_{Th} = 10 \text{ mV}$   
 $P_t = 10 \text{ mW}$

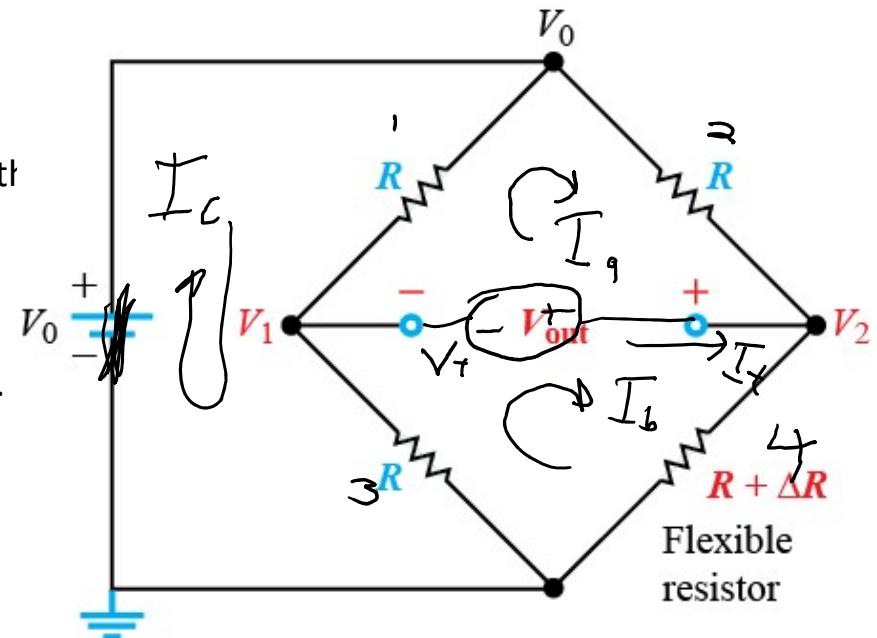
# Problem 4

- Consider the Wheatstone bridge circuit shown below. Suppose you have a voltmeter that can measure in the range of 1 mV to 100 V, with an accuracy of 1 % from 10 mV to 100 V and 10 % from 1 to 10 mV.
  - (a) What is the smallest percentage change you can measure in the flexible resistor?
  - (b) Assuming you can measure the resistance directly with the voltmeter to 1 %, how much improvement do you get for  $V_0 = 100$  V.
  - (c) If you treat the  $V_{out}$  terminal as a power supply, what is its Thevenin equivalent voltage and resistance as a function of the fractional change in the flexible resistor?
  - (d) What is the power consumed in each resistor, the Thevenin equivalent resistor, and in the source when  $R = 100 \Omega$  and the fractional resistor change is 1 %?

```

"problem 4"
values={vmin→0.1/1000,v0→100,dd→0.01}
e1=(v1-v0)/r+v1/r==0
e2=(v2-v0)/r+v2/(r(1+d))==0
eqn={e1,e2}
vec={v1,v2}
s1=Solve[eqn,vec]
s2=(v2-v1)/.s1
Factor[s2]
s3=Solve[s2==vmin,d]
dd/d/.s3/.values
values2={r→10,vt→1,v0→100,d→0.01}
e3=(ia-ic)r+ia r+vt==0
e4=(ib-ic)r-vt+ib r (1+d)==0
e5=(ic-ia)r+(ic-ib) r==0
eqn={e3,e4,e5}
vec={ia,ib,ic}
s3=Solve[eqn,vec]

```





# Useful tables

Material	Conductivity $\sigma$ (S/m)	Resistivity $\rho$ ( $\Omega\text{-m}$ )
<b>Conductors:</b>		
Silver	$6.17 \times 10^7$	$1.62 \times 10^{-8}$
Copper	$5.81 \times 10^7$	$1.72 \times 10^{-8}$
Gold	$4.10 \times 10^7$	$2.44 \times 10^{-8}$
Aluminum	$3.82 \times 10^7$	$2.62 \times 10^{-8}$
Iron	$1.03 \times 10^7$	$9.71 \times 10^{-8}$
Mercury (liquid)	$1.04 \times 10^6$	$9.58 \times 10^{-7}$
<b>Semiconductors:</b>		
Carbon (graphite)	$7.14 \times 10^4$	$1.40 \times 10^{-5}$
Pure germanium	2.13	0.47
Pure silicon	$4.35 \times 10^{-4}$	$2.30 \times 10^3$
<b>Insulators:</b>		
Paper	$\sim 10^{-10}$	$\sim 10^{10}$
Glass	$\sim 10^{-12}$	$\sim 10^{12}$

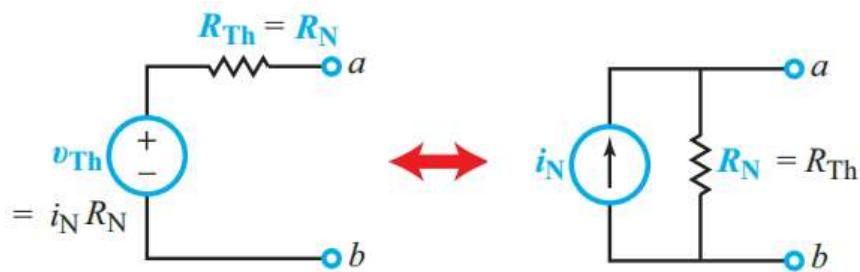
Teflon	$\sim 3.3 \times 10^{-13}$	$\sim 3 \times 10^{12}$
Porcelain	$\sim 10^{-14}$	$\sim 10^{14}$
Mica	$\sim 10^{-15}$	$\sim 10^{15}$
Polystyrene	$\sim 10^{-16}$	$\sim 10^{16}$
Fused quartz	$\sim 10^{-17}$	$\sim 10^{17}$
<b>Common materials:</b>		
Distilled water	$5.5 \times 10^{-6}$	$1.8 \times 10^5$
Drinking water	$\sim 5 \times 10^{-3}$	$\sim 200$
Sea water	4.8	0.2
Graphite	$1.4 \times 10^{-5}$	$71.4 \times 10^3$
Rubber	$1 \times 10^{-13}$	$1 \times 10^{13}$
<b>Biological tissues:</b>		
Blood	$\sim 1.5$	$\sim 0.67$
Muscle	$\sim 1.5$	$\sim 0.67$
Fat	$\sim 0.1$	$\sim 10$

AWG size designation	Diameter $d$ (mm)
0	8.3
2	6.5
4	5.2
6	4.1
10	2.6
14	1.6
18	1.0
20	0.8

# Useful equations – chapter 3

$$I = \frac{V}{R} = GV. \quad RI = V_t,$$

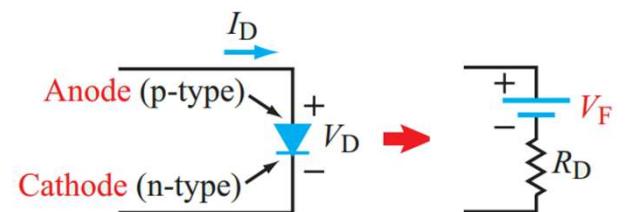
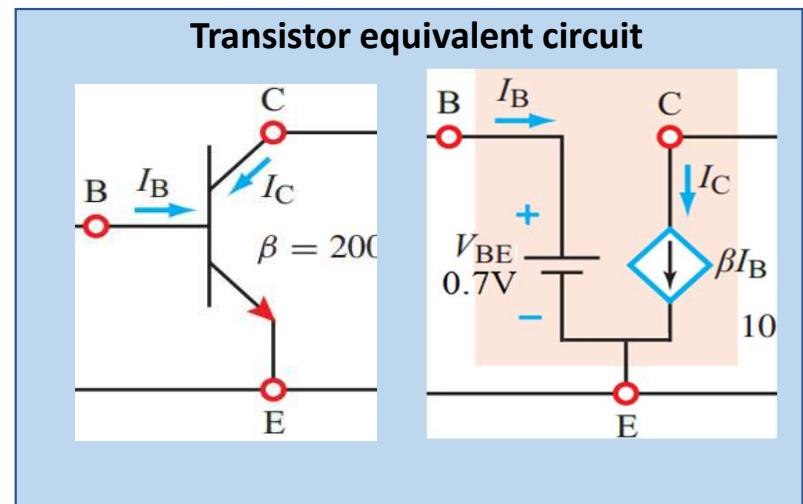
$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} \quad i_N = \frac{v_{\text{Th}}}{R_{\text{Th}}} \quad R_N = R_{\text{Th}}$$



Thévenin  
equivalent  
circuit

Norton equivalent  
circuit

$R_L = R_s$  (maximum power transfer).



# Useful equations – chapter 2 part 2

$$R_{eq} = \sum_{i=1}^N R_i \quad (\text{resistors in series}),$$

$$\frac{1}{R_{eq}} = \sum_{i=1}^N \frac{1}{R_i} \quad (\text{resistors in parallel}).$$

$$G_{eq} = \sum_{i=1}^N G_i \quad (\text{conductances in parallel}).$$

$$v_i = \left( \frac{R_i}{R_{eq}} \right) v_s \quad (\text{voltage division})$$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

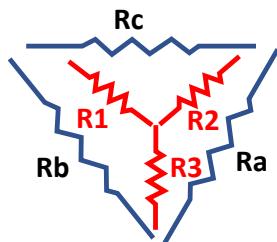
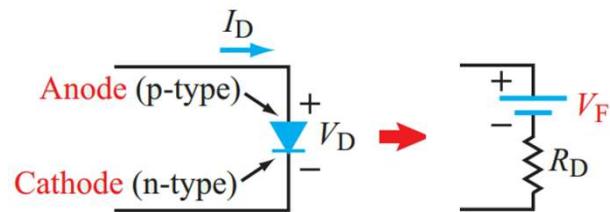
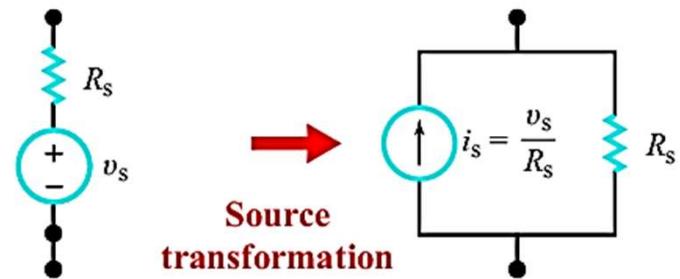
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$



$$R_1 = R_2 = R_3 = \frac{R_a}{3} \quad (\text{if } R_a = R_b = R_c),$$

$$R_a = R_b = R_c = 3R_1 \quad (\text{if } R_1 = R_2 = R_3).$$

# Useful equations – chapter 2 part 1

$$i = \frac{v_a - v_b}{R} \quad v = iR, \quad G = \frac{1}{R} \quad (\text{S}), \quad i = \frac{v}{R} = Gv,$$

$$R = \frac{\ell}{\sigma A} \quad R = \rho \frac{\ell}{A} = \rho \frac{\ell}{\pi(d/2)^2}$$

$$p = iv = i^2 R = \frac{v^2}{R} \quad (\text{W}). \quad p = iv = Gv^2 \quad (\text{W}).$$

$$R = R_0(1 + \alpha T), \quad R_D \approx \Delta V_D / \Delta I. \quad R = \frac{dv}{di}$$

$$\sum_{n=1}^N i_n = 0 \quad (\text{KCL}) \quad \sum_{n=1}^N v_n = 0 \quad (\text{KVL}),$$

# Useful equations– chapter 1

$$p(t) = v(t) \cdot i(t)$$

$$i = \frac{dq}{dt} \text{ (A).}$$

$$W = \int_0^{\infty} p(t) dt$$

$$dq = i \ dt.$$

$$\sum_{k=1}^n p_k = 0,$$

$$q(t) = \int_{-\infty}^t i \ d\tau \text{ (C).}$$

$$i(t) = \frac{v(t)}{R} :$$

$$W = \int_0^{\infty} \left( \frac{v^2}{R} \right) dt$$

Dependent Sources	
<b>Voltage-Controlled Voltage Source (VCVS)</b>	<b>Voltage-Controlled Current Source (VCCS)</b>
 $v_s = \alpha v_x$	 $i_s = g v_x$
<b>Current-Controlled Voltage Source (CCVS)</b>	<b>Current-Controlled Current Source (CCCS)</b>
 $v_s = r i_x$	 $i_s = \beta i_x$