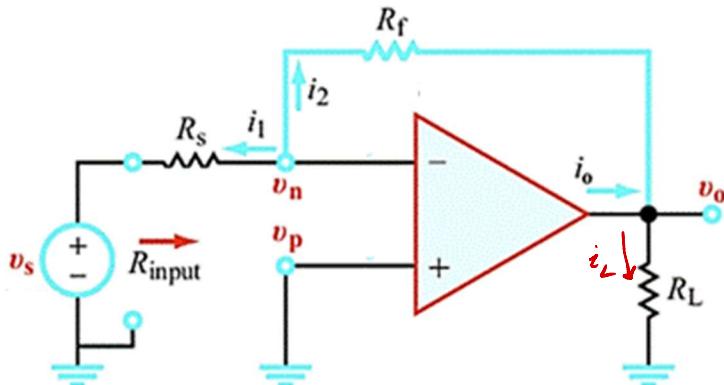


Test 2

Special instructions: When equations are asked to be labeled. Use labels like: node, mesh, auxiliary, initial or final condition, KCL, KVL, divider, etc.

- 1) For the op-amp below, what are the values for the currents i_1 , i_2 and i_o ?



For i_1 and i_2 use KCL

$$i_1 + i_2 = 0$$

$$\text{or } \frac{v_n - v_s}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

For i_o use KCL AND OHM'S LAW

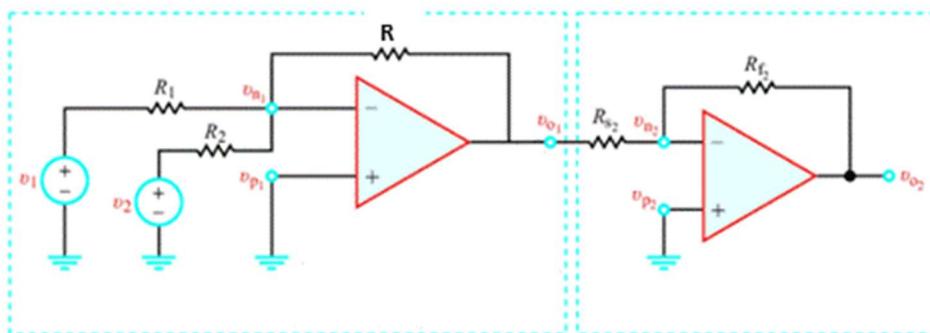
$$i_o = i_L - i_2 = \frac{v_o}{R_L} - \frac{v_n - v_o}{R_s}$$

$$i_1 = \frac{v_n - v_s}{R_s}$$

$$i_2 = \frac{v_n - v_o}{R_s}$$

$$i_o = i_L - i_2 = \frac{v_o}{R_L} - \frac{v_n - v_o}{R_s}$$

- 2) What values for R_1 , R_2 , R_{f2} , R_{s2} are needed below to have $v_{o1} = a v_1 + b v_2$.



① for v_{o1} ,

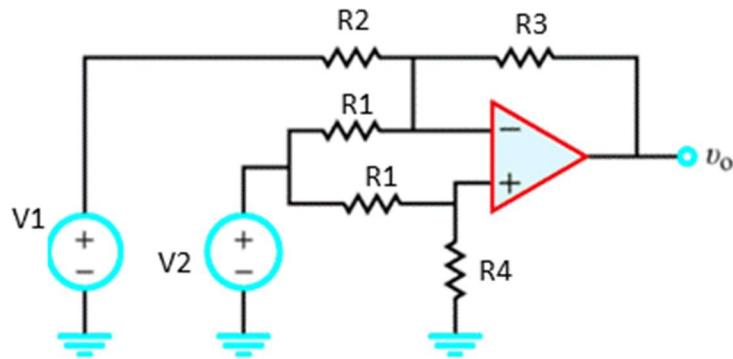
$$-\frac{R}{R_1} = a \text{ and } -\frac{R}{R_2} = b$$

② for v_{o2}

$$v_{o2} = \left(-\frac{R_{s2}}{R_{s2}} \right) v_{o1}$$

$$R_1 = -\frac{R}{a} \text{ and } R_2 = -\frac{R}{b}$$

- 3) Write down and label all the equations needed to find v_o for the circuit below.



By Voltage Division

Ohm's Law

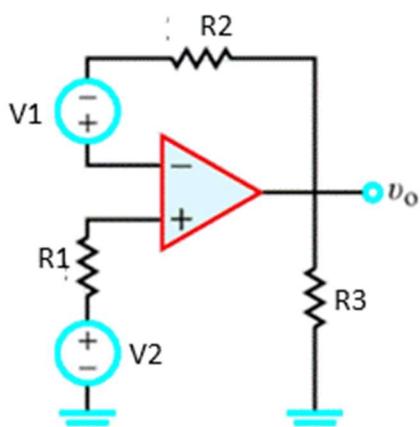
$$V_p = V_2 \times \frac{R_4}{R_1 + R_4}$$

$V_n = V_p$ Ideal op-amp constraint

KCL

$$\frac{V_n - V_1}{R_2} + \frac{V_n - V_2}{R_1} + \frac{V_n - V_o}{R_3} = 0$$

- 4) Write down and label all the equations needed to find v_o for the circuit below.



Here, $i_p = 0$.

There's no voltage drop across R_1 .

$$i_n = 0$$

There is no voltage drop across R_2 .

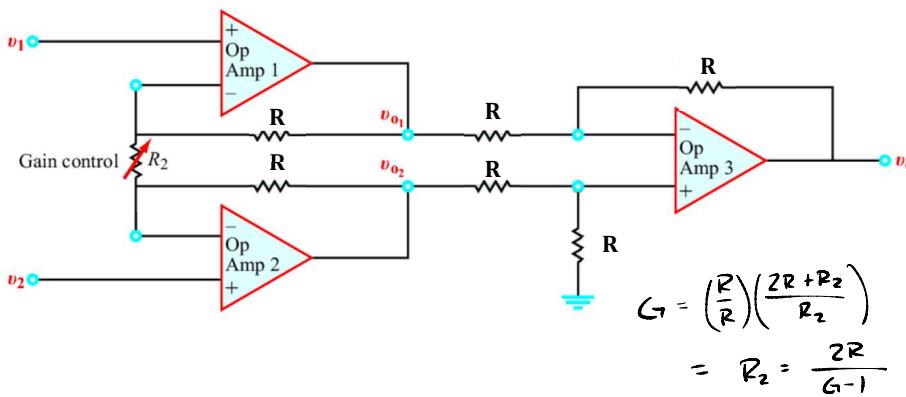
So,

$$V_p = V_2$$

Subtracting Voltages Ohm's Law
 $V_o = V_n - V_1$

$$V_p = V_n \quad \text{IDEAL OP-AMP CONSTRAINT}$$

- 5) For the amplifier below, what must be the value of R_2 to produce a gain of G ?



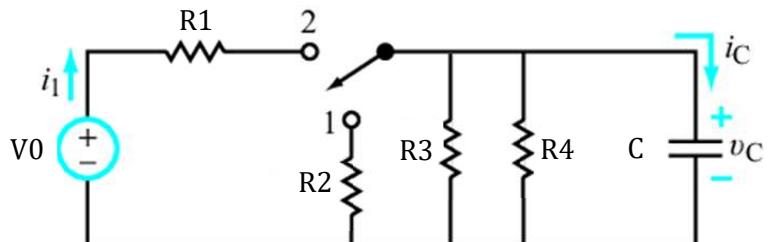
GAIN CONTROL

R_2 controls the overall gain of the amp.

$$G = \left(\frac{R}{R} \right) \left(\frac{2R + R_2}{R_2} \right)$$

$$= R_2 = \frac{2R}{G-1}$$

- 6) Having been in position 2 for a long time, the switch below moves to position 1 at $t = 0$. Find the initial capacitor current $i_C(0)$ and the time constant τ .



$$i_{R_2} = \frac{V_C(0)}{R_2}$$

$$i_{R_{34}} = \frac{V_C(0)}{R_{34}}$$

$$i_C(0) = -i_{R_2} - i_{R_{34}}$$

At $t = \infty$

$$R_{34} = \frac{R_3 R_4}{R_3 + R_4}$$

$$\tau = \left(\frac{R_2 R_{34}}{R_2 + R_{34}} \right) C$$

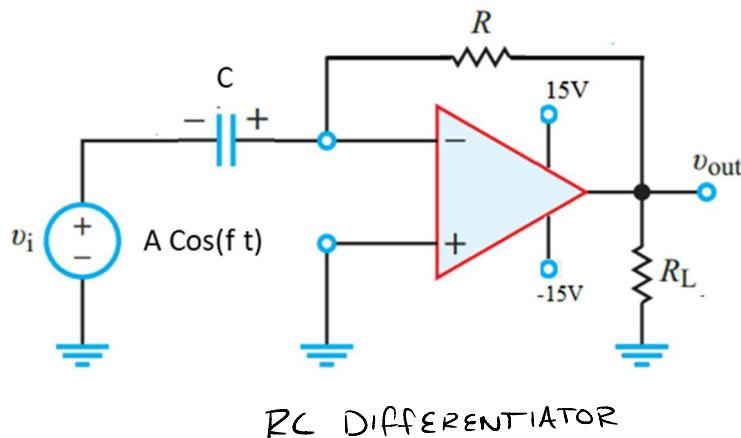
$$V_c(0) = i_C R_{34}$$

$$= \frac{V_0 R_{34}}{R_1 + R_{34}}$$

Initial Capacitor Current

$$i_C(0) = C \frac{d}{dt} V_C$$

- 7) For the circuit below, find $v_{out}(t)$.



find $V_{out}(t)$

$$V_{out} = -RC \frac{d}{dt} V_i$$

$$\rightarrow V_{out} = -RC \times \frac{d}{dt} [A \cos(\omega t)]$$

$$\underline{V_{out} = R C A \omega \sin(\omega t)}$$

- 8) If the waveform below is input to an inverting op amp differentiator with resistor R and capacitor C, what is the output waveform?

For time segment $t = t_1$ and $t = t_2$, the slope

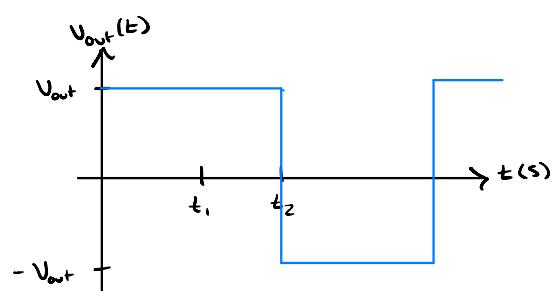
of the Input Signal is $(\frac{a+b}{t_2})$.

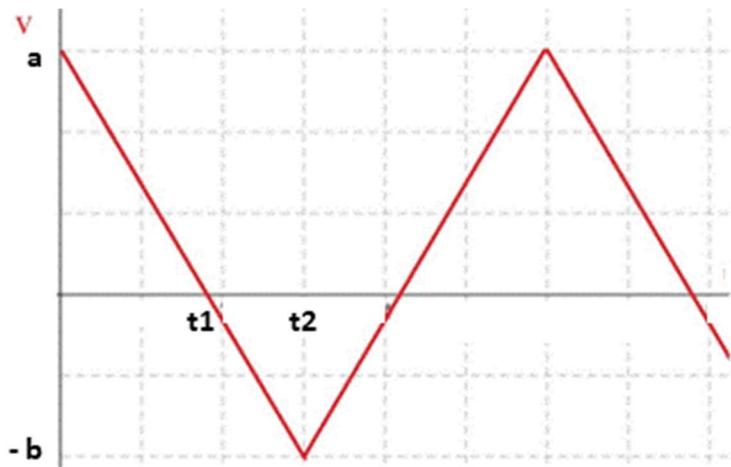
Output Voltage is given by :

$$V_{out,t} = -RC \frac{d}{dt} V_i$$

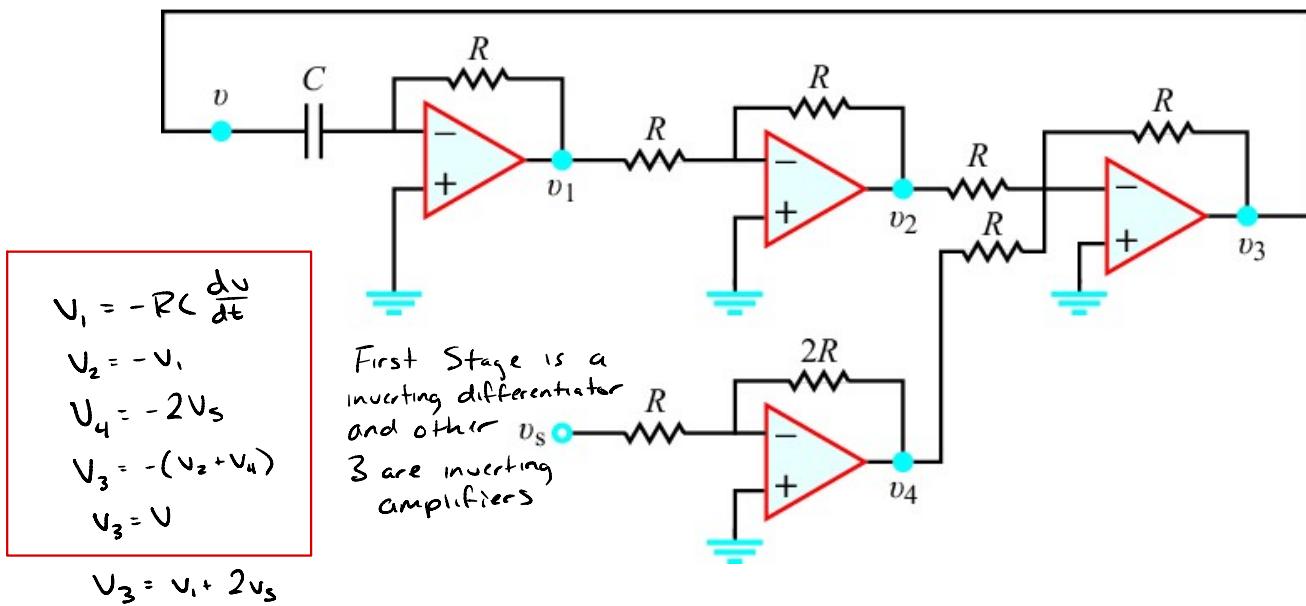
$$= -RC \times \left(\frac{a+b}{t_2} \right)$$

Slope, Op amp differentiator



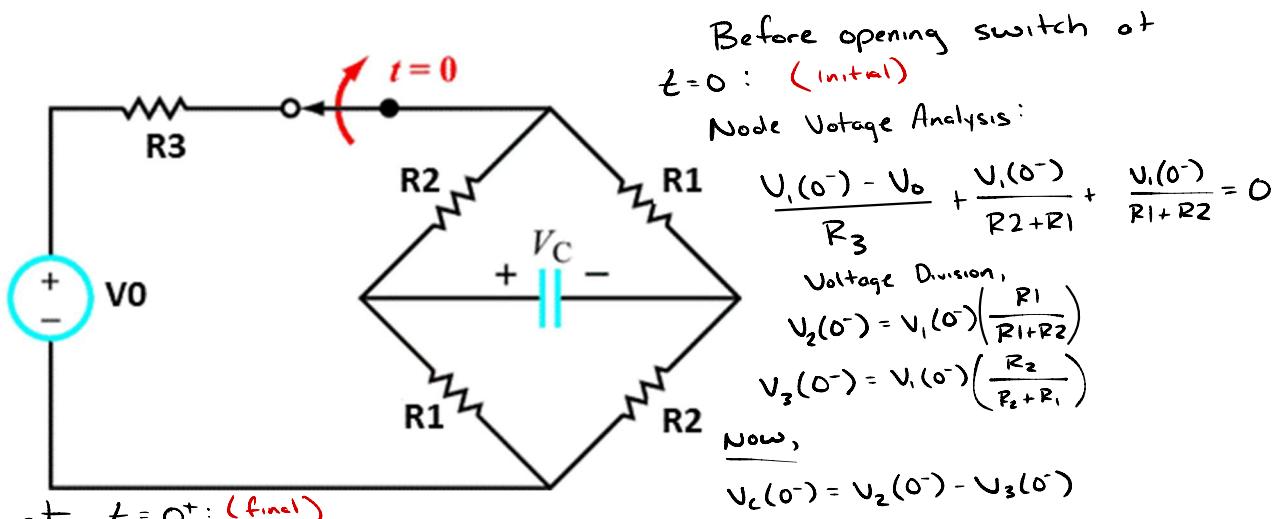


9) Write down and label all the equations needed to find v_o for the circuit below.



10) Write down and label all the equations needed to determine $v_C(t)$ in the circuit of the figure below for $t \geq 0$, given that the switch had been closed for a long time prior to $t=0$.

Hint: You will need equations for the initial and final conditions, the time constant, and then combine them to make the final equation for $v_C(t)$.



The circuit at $t=0^+$: (final)

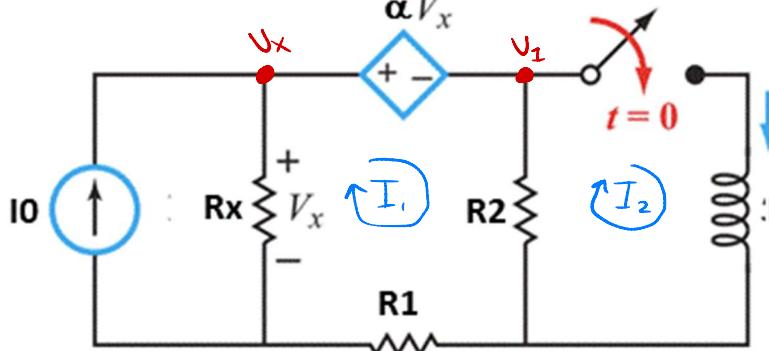
$$R_{eq} = (R_1+R_2) \parallel (R_1+R_2)$$

$$\tau = \text{time constant} = R_{eq} \times C$$

$$V_c(\infty) = 0$$

$$\text{Final Equation: } V_c(t) = V_c(\infty) + [V_c(0^-) - V_c(\infty)] e^{-\frac{t}{\tau}}$$

- 11) Write down and label all the equations needed to determine $i_L(t)$ in the circuit below for $t \geq 0$. Hint: You will need equations for the initial and final conditions, the time constant, and then combine them to make the final equation for $i_L(t)$. Note that to find time constant you will need to find the Thevenin equivalent of a circuit with a dependent source.



$$\text{Final Equation: } i_L(t) = i_L(\infty) + [i_L(0^-) - i_L(\infty)] e^{-\frac{t}{\tau}} \text{ AND } \tau = \frac{L}{R_{TH}}$$

① Simplify into Equivalent RL Circuit using Thevenin Equivalent to the left of the switch:

At supernodes U_x and U_1 :

$$-I_o + \frac{U_x}{R_x} + \frac{U_1}{R_1+R_2} = 0$$

$$I_2 = \alpha U_x$$

$$V_{oc} = U_2 \left(\frac{R_2}{R_1+R_2} \right) \rightarrow V_{TH} = V_{oc}$$

Mesh Analysis

$$R_x I_1 + \alpha U_x + R_2(I_1 - I_2) + R_1(I_1) = 0$$

$$R_{TH} = \frac{U_{ex}}{I_{ex}} = \frac{U_{ex}}{-I_2} \text{ AND } I_N = \frac{V_{TH}}{R_{TH}}$$

- 12) Write down and label all the equations needed to determine $i_L(t)$ in the circuit below for $t \geq 0$ when it is driven with the voltage waveform shown. Hint: You will have a driven and transient solution.

Using the triangular-source excitation

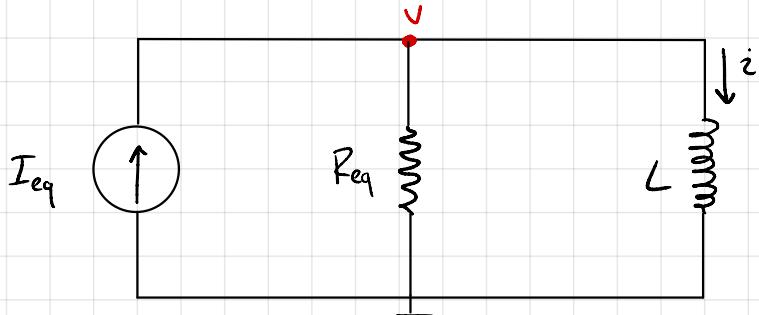
$$\text{Equation: } \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

12)

① Find R_{eq}

$$R_{eq} = R_3 + \frac{R_1 \times R_2}{R_1 + R_2} \quad \text{AND} \quad Z = \frac{L}{R_{eq}} = Z^{-1}$$

② Equivalent Circuit for $0 \leq t \leq t_1$



$$U_s(t) = 4t(V)$$

$$I_{eq} = \frac{V}{R_{eq}} + i$$

$$V = L \frac{di}{dt} \text{ So,}$$

$$\frac{L}{R_{eq}} \frac{di}{dt} + i = I_{eq}$$

Multiply both sides by e^{at} , integrate $t=0$ to t . $\rightarrow \frac{di}{dt} + \frac{R_{eq}}{L} i = \frac{R_{eq} I_{eq}}{L}$

$$\rightarrow \int_0^t \left(\frac{di}{dt} + \frac{R_{eq}}{L} i \right) e^{at} dt = \int_0^t \frac{R_{eq}}{L} e^{at} dt$$

$$\rightarrow \int_0^t \frac{di}{dt} (e^{at}) dt + \frac{R_{eq}}{L} \int_0^t i e^{at} dt = L \int_0^t t e^{at} dt$$

$$\rightarrow \int_0^t \frac{d}{dt} (i e^{at}) dt = \int_0^t \frac{1}{a} t e^{at} dt$$

$$\rightarrow i e^{at} \Big|_0^t = \int_0^t \frac{1}{a} t e^{at} dt$$

Apply $\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$, $i(t)e^{at} - i(0)$ for $0 \leq t \leq t_1$.

Because U_s was zero before $t=0$

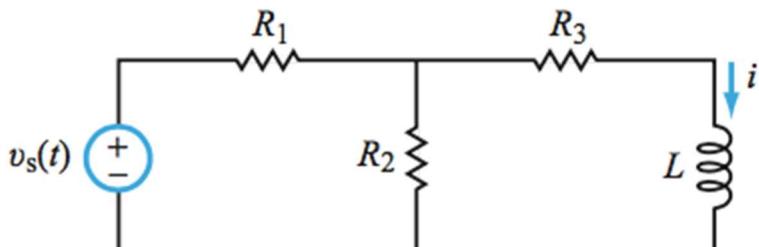
$$i(0) = i(0^-) = 0$$

2. Time Segment $t \geq t_1$

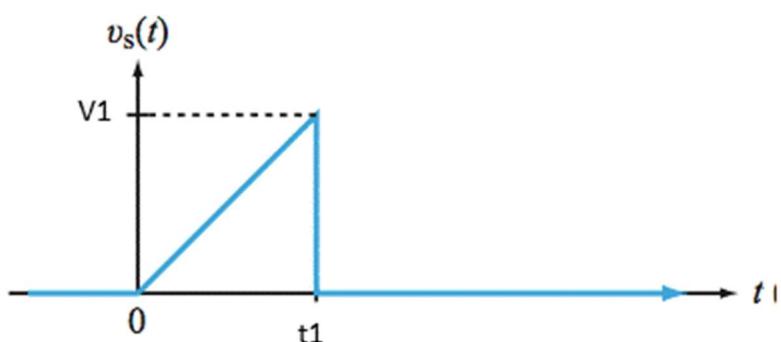
At $t = t_1$, $i(t = t_1)$ Plug into Equation Above

For $t > t_1$,

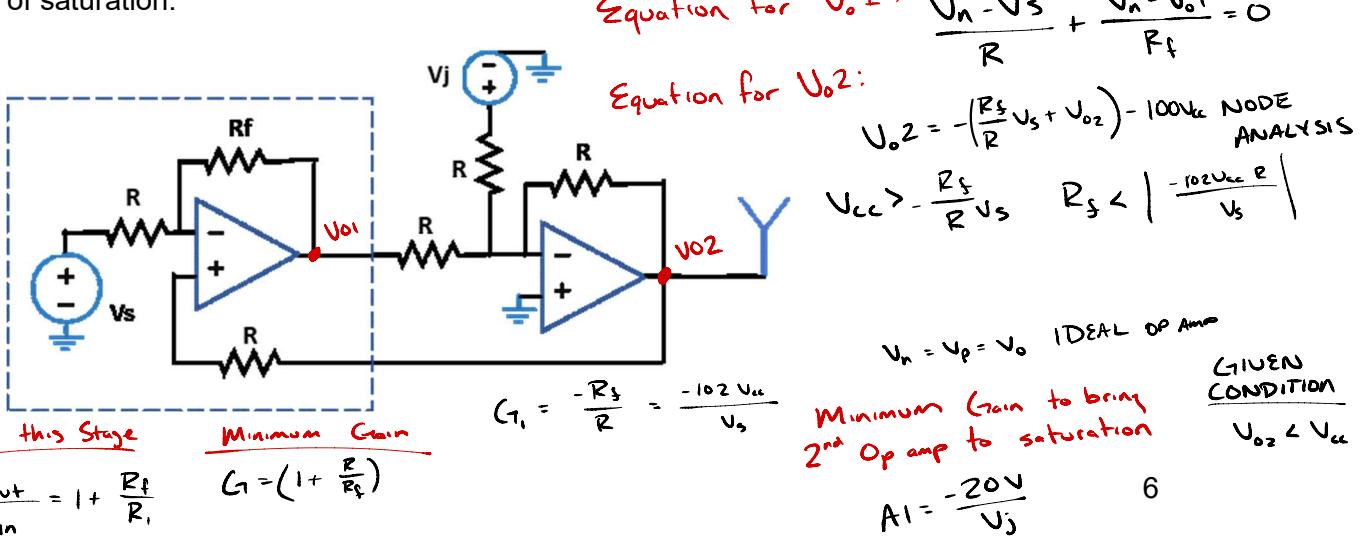
$$\underline{Z = \frac{L}{R_{eq}}}$$



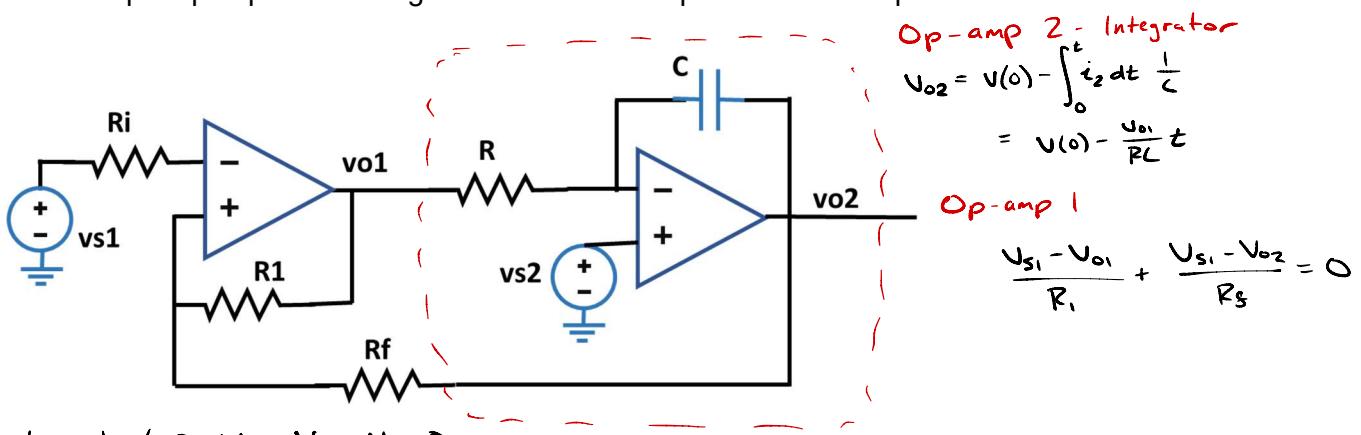
(a) Circuit



Extra credit 1) Consider a dual stage op amp amplifier, shown below that drives a transmitter antenna. The first amplifier is high gain and operates inside a protective enclosure. The second op amp is lower gain but can produce a higher output power and is placed outside the enclosure. As shown, feedback is applied by sampling the second op amp output and applying it to the input of the first op amp. Consider the case when a jamming signal is applied to the outside amplifier. For simplicity model this as a voltage v_j applied through a resistor R_j , as shown below. Assume the jammer voltage is 100 times larger than V_{cc} for the op amps. Write down and label all the equations needed to find the minimum gain of the first op amp that is needed to bring the second op amp out of saturation.



Extra credit 2) The dual op amp circuit below has two outputs v_{o1} and v_{o2} . Note that the first op amp has positive feedback and so is saturated to $+V_{cc}$ or $-V_{cc}$. Because of this the two input voltages, v_{s1} and v_{p1} , are not necessarily equal. Begin with $V_{s1}=0=V_{s2}$. Write down and label all the equations to determine the ratio of R_f to R_1 needed for the second op amp to produce a signal of maximum amplitude almost equal to V_{cc} .



Starting at $t=0$, $v_{o1} = V_{cc}$, $v_{o2} = 0$

$$v_{o2} = -V_{cc} \frac{t}{T}, T = RC$$

Now,

$$v_{s1} = 0 = v_{s2} \quad v_{o1} = -\frac{R_1}{R_s} v_{o2}$$

$$\frac{v_{o1}}{R} = v_{o2} \quad v_{o1} = V_{cc}$$

$$\boxed{\frac{-V_{cc}}{v_{o2}} = \frac{R_1}{R_s}}$$