1.16 Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]\,x[n-2].$$

(a) Is the system memoryless?
Since $y[n]$ depends on previous values of $x[n]$, it is not memoryless

(b) Output: $y[n] = \delta[n]\,\delta[n-2] = 0$

1.17 Consider a continuous-time system w/ input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t))$$

(a) Check $y(\pi) = x(\sin(\pi)) = x(0)$ ✗
$y(2\pi) = x(\sin(2\pi)) = x(0)$ ✗
The system is not casual, $y(t)$ may depend on future values of $x(t)$.

(b) let $y_1(t) = x_1(\sin(t))$ and $y_2(t) = x_2(\sin(t))$

let $x_3(t) = \alpha x_1(t) + \beta x_2(t)$ $\quad \alpha, \beta \in \mathbb{R}$

$$y_3(t) = x_3(\sin(t)) = \alpha x_1(\sin(t)) + \beta x_2(\sin(t)) = \alpha x_1(t) + \beta x_2(t) \checkmark$$

The system is linear.

1.18 (a) $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$ $\quad n_0$ is finite positive number.

$$\left.\begin{aligned} x_1[n] &\rightarrow y_1[n] = \sum_{n-n_0}^{n+n_0} x_1[k] \\ x_2[n] &\rightarrow y_2[n] = \sum_{n-n_0}^{n+n_0} x_2[k] \end{aligned}\right\} \; x_3 = \alpha x_1[n] + \beta x_2[n]$$

$$y_3[n] = \sum_{n-n_0}^{n+n_0} x_3[k] = \sum_{n-n_0}^{n+n_0} (\alpha x_1[k] + \beta x_2[k]) = \alpha \sum_{n-n_0}^{n+n_0} x_1[k] + \beta \sum_{n-n_0}^{n+n_0} x_2[k]$$

$$= \alpha y_1[n] + \beta y_2[n] \checkmark$$

System is linear.

(b) let $x_2[n] = x_1[n+n_1]$,

$$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{n-n_0}^{n+n_0} x_1[k+n_1] = \sum_{n+n_1-n_0}^{n+n_0+n_1} x_1[k]$$

AND

$$y_1[n+n_1] = \sum_{n+n_0-n_1}^{n+n_0+n_1} x_1[k]$$

The system is time-invariant

1.19 Determine if system is linear, time invariant, or both.

(a) $y(t) = t^2 x(t-1)$

linear ?

$$x_1(t) \rightarrow y_1(t) = t^2 x_1(t-1)$$
$$x_2(t) \rightarrow y_2(t) = t^2 x_2(t-1)$$
$$x_3(t) \rightarrow y_3(t) = \alpha x_1(t) + \beta x_2(t)$$

$$\begin{aligned} y_3(t) &= t^2 x_3(t-1) \\ &= t^2(\alpha x_1(t-1) + \beta x_2(t-1)) \\ &= \underline{\alpha y_1(t) + \beta y_2(t)} \checkmark \end{aligned}$$

$\boxed{\text{System is linear}}$

Arbitrary Input $x_1(t) \rightarrow y_1(t) = t^2 x_1(t-1)$

$x_2(t)$ shift $\quad x_2(t) = x_1(t+t_0)$

$$y_2(t) = t^2 x_2(t-1) = t^2 x_1(t-1+t_0)$$
$$y_1(t+t_0) = (t+t_0)^2 x_1(t-1+t_0) \neq y_2(t)$$

$\boxed{\text{Not time-invariant}}$

(b) $y[n] = x^2[n-2]$

linear?

$$x_1[n] \rightarrow y_1[n] = x_1^2[n-2]$$
$$x_2[n] \rightarrow y_2[n] = x_2^2[n-2]$$
$$x_3[n] \rightarrow \alpha x_1[n] + \beta x_2[n]$$

$$y_3[n] = (\alpha x_1[n-2] + \beta x_2[n-2])^2 = \alpha^2 x_1^2[n-2] + \beta^2 x_2^2[n-2] + 2\alpha\beta x_1[n-2]x_2[n-2]$$
$$\neq \underline{\alpha y_1[n] + \beta y_2[n]}$$

$\boxed{\text{System is not-linear}}$

Arbitrary Input $\therefore x_1[n] \rightarrow y_1[n] = x_1^2[n-2]$

$x_2[n] = x_1[n-n_0]$

$$y_2[n] = x_2^2[n-2] = x_1^2[n-2-n_0]$$
$$\rightarrow y_1[n-n_0] = x_1^2[n-2-n_0] \checkmark$$
$$y_2[n] == y_1[n-n_0]$$

$\boxed{\text{Time-Invariant}}$