Question 1

Given:

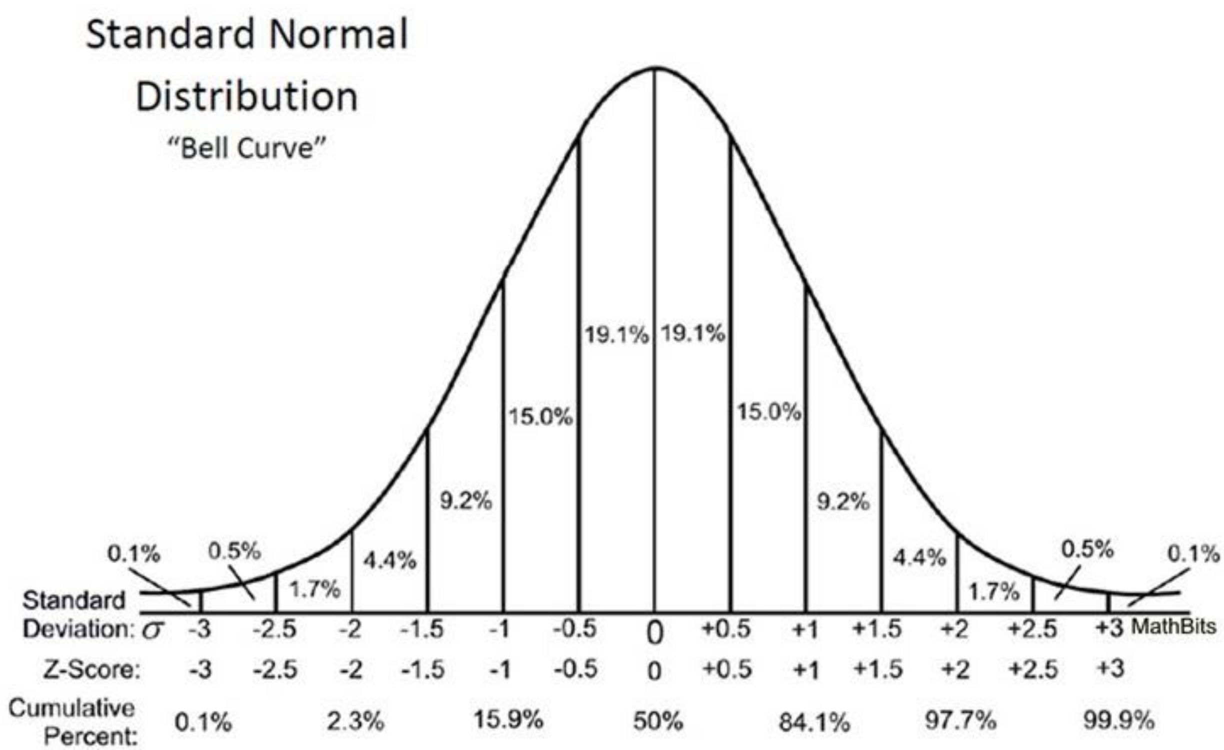
A normal population with a known population standard deviation σ.

Confidence interval: $\bar{x} - 1.17\sigma/\sqrt{n} \le \mu \le \bar{x} + 1.17\sigma/\sqrt{n}$.

Find:

The confidence level associated with this confidence interval.

Diagram:



Theory:

The confidence level represents the probability that the calculated confidence interval contains the true population parameter, μ.

$$(1 - \alpha) \times 100\%$$

- α is the probability of making a Type I error.

Equations:

Assumptions:
The population follows a normal distribution.
The population standard deviation σ is known.
The sample mean $\bar{x}$ follows a normal distribution.

Solution:

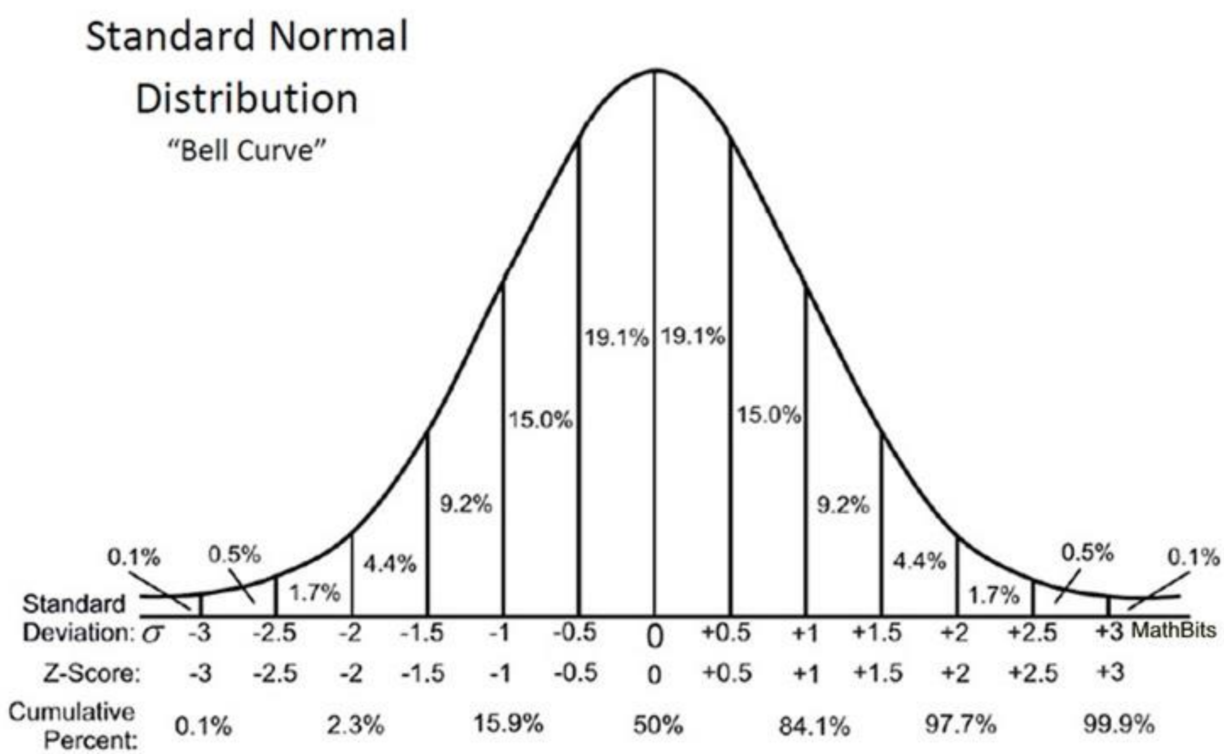
Plug into a confidence level calculator the Z value of 1.17, the answer is 75.79%

Question 2a)

Given:
Sample size, n = 91
Sample mean, $\bar{x}$ = 216
Confidence interval: 90%
Confidence interval range: 210 to 222

Find:
The population standard deviation σ

Diagram:



Theory:
I need to determine σ using this formula.

$$ME = Z * (\frac{\sigma}{\sqrt{n}})$$

Z - z-score corresponding to the desired confidence level.
σ - population standard deviation.
n - sample size.

Assumptions:
The sample is random and representative of the population.
The population follows the Central Limit Theorem.

Solution:

Find the margin of error using the given confidence interval:

ME = (Upper Limit - Lower Limit) / 2
ME = (222 - 210) / 2
ME = 6

For a 90% confidence level, the z-score is approximately 1.645.

Now, we can use the formula for σ:

σ = ME * (√n / Z)
σ = 3 * (6.54 / 1.645)
σ ≈ 34.79

The population standard deviation is 34.79.

Question 2b)

Given:
Sample size is doubled to 2 * 91 = 182.
Calculate the new confidence interval while keeping the same confidence level

Find:
The new confidence interval with the doubled sample size.

Theory:
New ME = (Upper Limit + Lower Limit) / 2
New SE = (σ / √(2n))

Assumptions:
The sample is random and representative of the population.
2. The population follows the Central Limit Theorem.

Solution:

Mean x = (Upper Limit + Lower Limit) / 2
Mean = (222 + 210) / 2
Mean = 216

From part a:
σ ≈ 34.79
n ≈ 91
New MOE ≈ (1.645) * (34.79 / √(2 * 2 * 91))

New Confidence Interval = (x̄ - New MOE, x̄ + New M)E)
New Confidence Interval = (216 – 3, 216 + 3)
New Confidence Interval = (212, 219)

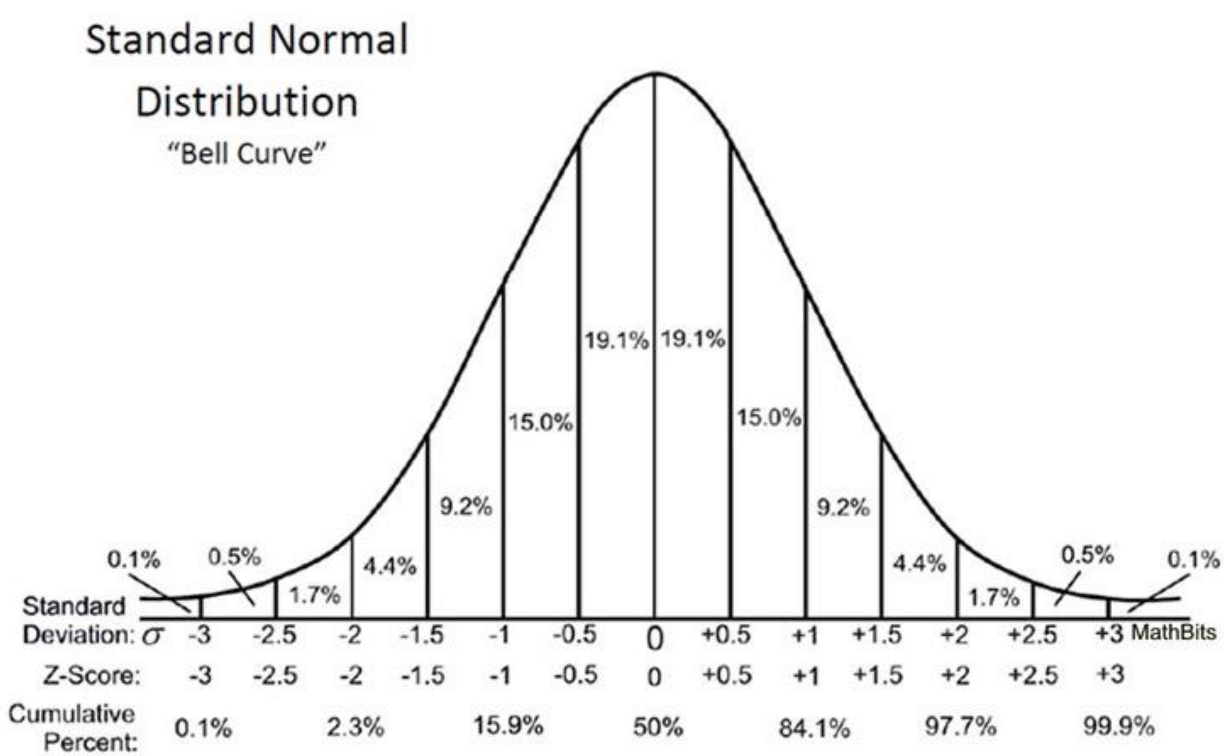
So, with the doubled sample size, the new confidence interval (keeping the same confidence level) is 212 to 219.

Question 3

Given:
- Calculate a 95% confidence level estimate of the average length of stay within +/- 2.0 days, 102 visitors' check-in/check-out records need to be looked at.
- The goal is to obtain a 95% confidence level estimate within +/- 1.0 days.

Find:
- Number of records that should be looked at to achieve new margin of error of +/- 1.0 days and keep a 95% confidence level.

Diagram:



Theory:

$$n = [\frac{(Z * \sigma)}{E}]^2$$

n - required sample size
Z - z-score corresponding to the confidence level
σ - population standard deviation
E - margin of error

Assumptions:
Sample is representative of the population.

Solution:

Solution:

Use Formula $n = \left(\frac{Z * \sigma}{E}\right)^2$

$E = \pm 2.0$
$n = 102$

$E = Z_{\alpha/2} \frac{\sigma}{n}$ Compare Old vs. New

$$E_1 \sqrt{n_1} = E_2 \sqrt{n_2}$$

$$2.0 \sqrt{102} = 1.0 \sqrt{n_2}$$

$$n_2 = \left(\frac{1.0}{2.0} \times \sqrt{102}\right)^2$$

$$\boxed{n_2 = 408}$$