7.2 Compute the eight-point circular convolution for the following sequences.

(a) $x_1(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$x_2(n) = \sin \frac{3\pi}{8} n, \quad 0 \le n \le 7$

(b) $x_1(n) = (\frac{1}{4})^n, \quad 0 \le n \le 7$

$x_2(n) = \cos \frac{3\pi}{8} n, \quad 0 \le n \le 7$

(c) Compute the DFT of the two circular convolution sequences using the DFTs of $x_1(n)$ and $x_2(n)$.

(a) CIRCULAR CONVOLUTION

$$y[n] = \sum_{m=0}^{N-1} x_1[m] x_2[(n-m)]_N$$

IF $0 \le l \le N-1$

$\hookrightarrow N-1 \le l+N \le N+N-1$

$= N-1 \le l \le -1$

$$x_2(l) = \sin\left(\frac{3\pi}{8} l + N\right) \quad N-1 \le l \le -1$$

$N = 8$

$\hookrightarrow \sin\left(\frac{3\pi}{8} l + 8\right), \ -7 \le l \le -1$

$$\rightarrow y[n] = \sum_{m=0}^{3} x_1[m] x_2[n-m]_N \rightarrow x_2[n-m] = \sin\left(\frac{3\pi}{8}[n-m]\right)$$

- $n=0$

$y[0] = \sin \frac{3\pi}{8}(0) + \sin \frac{3\pi}{8}(1) + \sin \frac{3\pi}{8}(2) + \sin \frac{3\pi}{8}(3)$

$y[0] = 0 + 0.924 + 0.707 + -0.3826$

$y[0] = 1.248$

- $n=1$

$y[1] = \sin \frac{3\pi}{8}(1) + \sin \frac{3\pi}{8}(0) + \sin \frac{3\pi}{8}(1) + \sin \frac{3\pi}{8}(2)$

$y[1] = 2.555$

- $n=2, \ y[2] = 2.555$
- $n=3, \ y[3] = 1.25$
- $n=4$

$y[4] = x_1[0] \cdot x_2(3) + x_1[1] \cdot x_2(2) + x_1[2] \cdot \bar{x}_2(1) + x_1[3] \cdot x_2[0]$

$= 0.25$

$y[5] = -1.06$

$y[6] = -1.06$

$y[7] = 0.25$

$y[5] = \sin \frac{15\pi}{8} + \sin \frac{12\pi}{8} + \sin \frac{9\pi}{8} + \sin \frac{6\pi}{8}$

Answer:

$y[0]$

$\downarrow$

$\{1.25, 2.56, 2.56, 1.25, 0.25, -1.06, -1.06, 0.25\}$

(b) $x_1(n) = \frac{1}{4}^m,\ 0 \le n \le 7$

$x_2(n) = \cos \frac{3\pi}{8} n,\ 0 \le n \le 7$

$$y[n] = \sum_{m=0}^{7} x_1[m] \cdot x_2(n-m)_N$$

• $n=0$

$$y[0] = \frac{1}{4}^0 \cdot \cos(0) + \frac{1}{4}^1 \cdot \cos\left(-\frac{21\pi}{8}\right) + \frac{1}{4}^2 \cdot \cos\left(\frac{-18\pi}{8}\right) + \frac{1}{4}^7 \cdot \cos\left(-\frac{3\pi}{8}\right)$$

$$y = \frac{1}{4} \cdot 0.9239 + \frac{1}{16} \cdot 0.707$$

$$y[0] = 0.99$$

Follow for all inputs 0-7 of N

ANSWER

$= \{0.99, 0.924, -0.7071, -1.24, -0.48, 0.92, -0.15\}$

(c) DTF of $x_1(n)$

$x_1(n) = \left(\frac{1}{4}\right)^n,\ 0 \le n \le 7$, $\quad X_1[k] = \sum_{n=0}^{7} x_1[n]\, e^{-j\frac{2\pi}{?}kn}$

$x_1[0] = 1,\ x_1[1] = \frac{1}{4},\ x_1[2] = \frac{1}{16}$, etc.

Compute $X_1[k]$

$k=0: \quad X_1[0] = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \cdots$

$k=1: \quad X_1[1] = \sum_{n=0}^{7} \left(\frac{1}{4}\right)^1 e^{-j\frac{\pi}{4}n}$

$k=2: \quad X_1[2] = \sum_{n=0}^{7} \left(\frac{1}{4}\right)^2 e^{-j\frac{\pi}{16}n}$

$k=3: \quad X_1[3] = \sum_{n=0}^{7} \left(\frac{1}{4}\right)^3 e^{-j\frac{\pi}{64}n}$

$k=4: \quad X_1[4] = \sum_{n=0}^{7} \left(\frac{1}{4}\right)^4 e^{-j\frac{\pi}{256}n}$

$k=5: \quad X_1[5] = \sum_{n=0}^{7} \left(\frac{1}{4}\right)^5 e^{-j\frac{\pi}{4096}n}$

$k=6: \quad X_1[6] = \sum_{n=0}^{7} \left(\frac{1}{4}\right)^6 e^{-j\frac{\pi}{16384}n}$

$k=7: \quad X_1[7] = \sum_{n=0}^{7} \left(\frac{1}{4}\right)^7 e^{-j\frac{\pi}{32884}n}$

Next compute $X_2(n)$

$$X_2[k] = \sum_{n=0}^{7} x_2[n] e^{-j\frac{2\pi}{8}kn}$$

$x_2[n]$

$x_2[0] = 1,\ x_2[1] = \cos\frac{3\pi}{8},\ x_2[2] = \cos\frac{6\pi}{8},\ x_2[3] = \frac{9\pi}{8}, \ldots$

Compute $X_2[k]$ $k = 0, 1, 2.$

$k=0:\ X_1[0] = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \cdots$

$k=1:\ X_1[1] = \sum_{n=0}^{7} \cos\frac{3\pi}{8} e^{-j\frac{\pi}{4}n}$

$k=2:\ X_1[2] = \sum_{n=0}^{7} \frac{\cos 3\pi}{8} e^{-j\frac{\pi}{16}n}$

$k=3:\ X_1[3] = \sum_{n=0}^{7} \frac{\cos 3\pi}{8} e^{-j\frac{\pi}{64}n}$

$k=4:\ X_1[4] = \sum_{n=0}^{7} \cos\frac{3\pi}{8} e^{-j\frac{\pi}{256}n}$

$k=5:\ X_1[5] = \sum_{n=0}^{7} \cos\frac{3\pi}{8} e^{-j\frac{\pi}{4096}n}$

$k=6:\ X_1[6] = \sum_{n=0}^{7} \cos\frac{3\pi}{8} e^{-j\frac{\pi}{16384}n}$

$k=7:\ X_1[7] = \sum_{n=0}^{7} \cos\frac{3\pi}{8} e^{-j\frac{\pi}{32889}n}$

multiply both together.

7.7 If $X(k)$ is the DFT of the sequence $x(n)$, determine the N-point DFTs of the sequences

$$x_c(n) = x(n)\cos\frac{2\pi k_0 n}{N}, \qquad 0 \le n \le N-1$$

and

$$x_s(n) = x(n)\sin\frac{2\pi k_0 n}{N}, \qquad 0 \le n \le N-1$$

in terms of $X(k)$.

$$X_c(k) = \sum_{n=0}^{N-1} x_c(n) e^{-j\frac{2\pi}{N}kn}$$ Substitute $x_c(n) = x(n)\cos\frac{2\pi k_0 n}{N}$

$$X_c(k) = \sum_{n=0}^{N-1} x(n)\cos\left(\frac{2\pi k_0 n}{N}\right) e^{-j\frac{2\pi}{N}kn}$$

Euler's Formula

$$X_c(k) = \frac{1}{2}\sum_{n=0}^{N-1} x(n)\left(e^{j\frac{2\pi k_0 n}{N}} + e^{-j\frac{2\pi k_0 n}{N}}\right) e^{-j\frac{2\pi}{N}kn}$$

Split in 2 terms

$$X_c(k) = \frac{1}{2}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n(k-k_0)} + \frac{1}{2}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n(k+k_0)}$$

Fits

$$X_c(k) = \frac{1}{2}\left(X(k-k_0) + X(k+k_0)\right)$$

Part 2

$$X_c(k) = \sum_{n=0}^{N-1} x(n)\sin\left(\frac{2\pi k_0 n}{N}\right) e^{-j\frac{2\pi}{N}kn}$$

Euler's Formula

$$X_c(k) = \frac{1}{2j}\sum_{n=0}^{N-1} x(n)\left(e^{j\frac{2\pi k_0 n}{N}} - e^{-j\frac{2\pi k_0 n}{N}}\right) e^{-j\frac{2\pi}{N}kn}$$

Split in 2 terms

$$X_c(k) = \frac{1}{2j}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n(k-k_0)} - \frac{1}{2j}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n(k+k_0)}$$

Fits

$$X_c(k) = \frac{-j}{2}\left(X(k-k_0) - X(k+k_0)\right)$$

7.8 Determine the circular convolution of the sequences

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{4}, 3, 2, 2\}$$

using the time-domain formula in (7.2.39).

$$y(n) = \sum_{k=0}^{N-1} x_1(k)\, x_2(n-k)_N \quad , n = 0, 1, 2, N-1$$

$N = 4$

. $n = 0$

$y(0) = x_1(0)x_2(0) + x_1(1)x_2(3) + x_1(2)x_2(2) + x_1(3)x_2(1)$

$y(0) = 1\cdot4 + 2\cdot2 + 3\cdot2 + 1\cdot3 = 4 + 4 + 6 + 3 = 17$

$y(1) = x_1(0)x_2(1) + x_1(1)x_2(0) + x_1(2)x_2(3) + x_1(3)x_2(2)$

$y(1) = 1\cdot3 + 2\cdot4 + 3\cdot2 \cdot 1\cdot2 = 3 + 8 + 6 + 2 = 19$

$y(2) = x_1(0)x_2(2) + x_1(1)x_2(1) + x_1(2)x_2(0) + x_1(3)x_2(3)$

$y(2) = 1\cdot2 + 2\cdot3 + 3\cdot4 + 1\cdot2 = 2 + 6 + 12 + 2 = 22$

$y(3) = x_1(0)x_2(3) + x_1(1)x_2(2) + x_1(2)x_2(1) + x_1(3)x_2(0)$

$y(3) = 1\cdot2 + 2\cdot2 + 3\cdot3 + 1\cdot4 = 2 + 4 + 9 + 4 = 19$

$y(n) = \{17, 19, 22, 19\}$

7.10 Compute the energy of the N-point sequence

$$x(n) = \cos \frac{2\pi k_0 n}{N}, \qquad 0 \le n \le N-1$$

$$\mathcal{E} = \sum_{n=0}^{N-1} \cos^2\left(\frac{2\pi k_0 n}{N}\right) \qquad \cos^2\theta = \frac{1 + \cos\left(\frac{4\pi k_0 n}{N}\right)}{2}$$

$$\mathcal{E} = \frac{1}{2}\sum_{n=0}^{N-1} 1 + \frac{1}{2}\sum_{n=0}^{N-1} \cos \frac{4\pi k_0 n}{N}$$

FIRST TERM

$$\frac{1}{2}\sum_{n=0}^{N-1} 1 = \frac{1}{2} \cdot N = \frac{N}{2}$$

SECOND TERM

$$\sum_{n=0}^{N-1} \cos\left(\frac{4\pi k_0 n}{N}\right) = 0$$

Answer

$$E = \frac{N}{2}$$

7.17 Determine the eight-point DFT of the signal

$$x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$$

and sketch its magnitude and phase.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$k=0$

$$X(0) = 1+1+1+1+1+1+1 = 7$$

$k=1$

$$X(1) = \sum_{n=0}^{6} e^{-j\frac{2\pi}{8}n}$$

$k=2$

$$X\,2 = \sum_{n=0}^{5} e^{-j\frac{2\pi}{8}n}$$

$k=3$

$$X(3) = \sum_{n=0}^{4} e^{-j\frac{2\pi}{8}n}$$

$k=4$

$$X(4) = \sum_{n=0}^{3} e^{-j\frac{2\pi}{8}n}$$

$k=5$

$$X(5) = \sum_{n=0}^{2} e^{-j\frac{2\pi}{8}n}$$

$k=6$

$$X(6) = \sum_{n=0}^{2} e^{-j\frac{2\pi}{8}n}$$

$k=7$

$$X(7) = \sum_{n=0}^{1} e^{-j\frac{2\pi}{8}n}$$

$$|X(k)| = \sqrt{X(k)^2 + X(k)^2}$$

$$\angle X(k) = \tan^{-1}\left(\frac{Im(X(k))}{Re(X(k))}\right)$$

9.6

FIRST STEP IS TO FIND CONJUGATE OF $X[k]$

Such as $X^*[k]$ AND NORMALIZE THE OUTPUT

Such as $x[n] = \frac{1}{N} \cdot x^*[n]$

Next, find symmetry by MAKING A SEQUENCE.

$$X'[k] = X[k]\, e^{-j\frac{2\pi}{N}kn}$$

And FIND the DFT by plugging it into the function.