Question 1

Given:

Mass = 2.25 ± 0.021kg

Radius = 0.180 ± 0.0030m

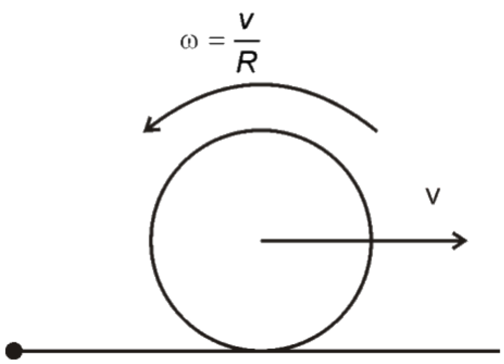
ω = 17.5 ± 0.250 rad/s

L = 1/2MR^2 ω

Find:

The angular momentum L and its uncertainty.

Diagram:



Theory: Propagation of error formula

$$\delta f(x, y, \ldots) = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$$

Assumptions: Uncertainties in mass, radius, and angular velocity are independent.

Solution:

$$L = \sqrt{(0.5 \times 17.5 \times 0.021 \times 0.18^2)^2 + (2.25 \times 0.18 \times 17.5 \times 0.003^2)^2 + (0.5 \times 2.25 \times 0.25 \times 0.18^2)^2}$$

L = 0.64 ± 0.02 kgm^2/s

Question 2

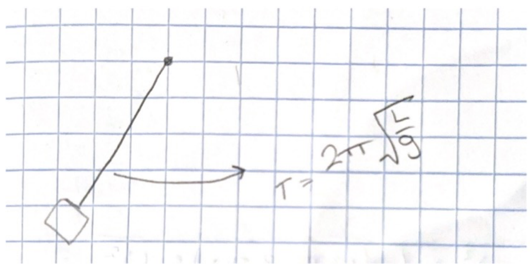
Given:

a) $L = 0.75 \pm 0.011m$
b) $T = 1.75 \pm 0.010s$

Find:

a) What is the predicted value of T?
b) Is a measured value of $T = 1.75 \pm 0.010s$ is consistent with the theoretical prediction from part a.

Diagram:



Theory:

- the period T of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$

Assumptions:

We can assume we will need to get the L_{max} and L_{min} values to compare to the theoretical prediction.

Solution:

Compute max and min values of the lengths

$L_{max} = (0.75 + 0.011)m = 0.761m$
$L_{min} = (0.75 - 0.011)m = 0.739m$

$T \;= 2\pi\sqrt{\frac{0.75}{9.81}} = 1.738$; $T_{maximum} = 2\pi\sqrt{\frac{0.761}{9.81}} = 1.751$; $T_{minimum} = 2\pi\sqrt{\frac{0.739}{9.81}} = 1.725$

$T = 1.75 \pm 0.010s$ -> Max = 1.76, Min = 1.74

Since $1.725 \leq T \leq 1.751$, only the minimum value would fit here and the max value is not consistent.

Question 3

Given:

v1 = 3.54 ± 0.10m/s

v2 = 8.16 ± 0.10m/s

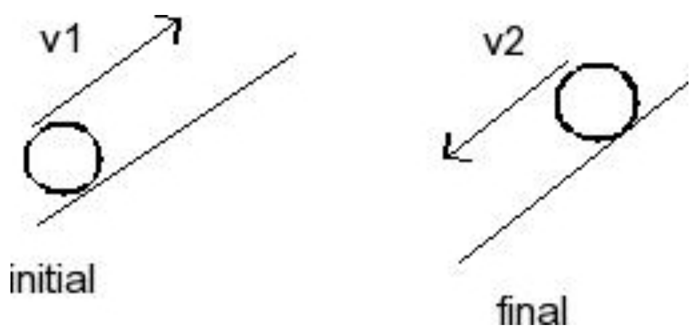
t = 2.79 ± 0.10 s

a = (v2-v1)/t

Find:

a) Acceleration and its uncertainty.
b) Compare the measured acceleration with the predicted.

Diagram:



Theory: Propagation of error formula

$$\delta f(x, y, \ldots) = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$$

Assumptions:

The uncertainties in v1, v2 and t are independent and random.

Solution:

$$a = \sqrt{(2.79^{-1} \times .1)^2 + (-2.79^{-1} \times .1)^2 + \left(-\left(\frac{8.16 - 3.54}{2.79^2}\right) \times .1\right)^2}$$

L = 1.656 ± 0.08 m/s^2

Question 4

Given:
- $m1$ = 102 ± 1.0 grams
- $m2$ = 86 ± 0.90 grams

Find:
- Equation for the uncertainty in the expected acceleration (δa)
- Calculate a ± δa using the given values

Theory:
- Formula for acceleration is:
  - $a = \frac{g*(m1-m2)}{(m1+m2)}$
- Formula for propagation of uncertainties:
  - $\frac{\delta \mathrm{a}}{a} = \sqrt{(\left(\frac{\delta \mathrm{m1}}{m1}\right)^2 + \left(\frac{\delta \mathrm{m2}}{m2}\right)^2)}$

Where:
- δa is uncertainty in the acceleration.
- $\delta m1$ is uncertainty in mass $m1$.
- $\delta m2$ is uncertainty in mass $m2$.

Assumptions: Masses m1 and m2 are independent measurements, there is no correlation between them.

Solution:

Solve for acceleration:

$$a = \frac{g*(m1-m2)}{(m1+m2)} \rightarrow a = \frac{9.81*(0.102-0.086)}{(0.102+0.086)} = 0.835 \text{ m/s}^2$$

Rearrange the propagation of uncertainties:

$$\delta \mathrm{a} = \frac{g*(m1-m2)}{(m1+m2)} \times \sqrt{(\left(\frac{\delta \mathrm{m1}}{m1}\right)^2 + \left(\frac{\delta \mathrm{m2}}{m2}\right)^2)}$$

$$\delta \mathrm{a} = 0.835 \times \sqrt{(\left(\frac{1.0}{102}\right)^2 + \left(\frac{0.90}{86}\right)^2)} = 0.014339 \times 0.83489 = 0.012$$

a ± δa is approximately 0.835 ± 0.012 m/s².

Question 5

Given:

H = 3.20 ± 0.15

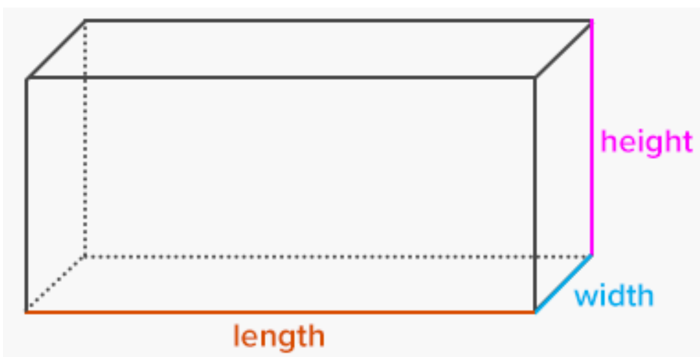
W = 5.00 ± 0.05

L = 4.25 ± 0.13

Find:

a) The volume of the rectangular prism and its associated uncertainty.

Diagram:



Theory: Propagation of error formula

$$\delta f(x, y, \ldots) = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$$

Volume = Base x Height

Base = Length x Width

Assumptions:

Uncertainties in height, length, width are independent and random

Solution:

1. Base

$B = (5.00\,cm \pm 0.05\,cm) \cdot (4.25\,cm \pm 0.13\,cm)$

$B = 21.25\,cm^2 \pm (0.05\,cm \cdot 4.25 + 5.00\,cm \cdot 0.13\,cm)$

$B = 21.25\,cm^2 \pm 0.21375\,cm^2$

2. Volume

$V = (21.25\,cm^2 \pm 0.21375\,cm^2) \cdot (3.20\,cm \pm 0.15\,cm)$

$V = 68\,cm^3 \pm (0.684\,cm^3 + 3.1875\,cm^3)$

$V = 68\,cm^3 \pm 3.8715\,cm^3$

$\boxed{V = 68\,cm^3 \pm 3.8715\,cm^3}$