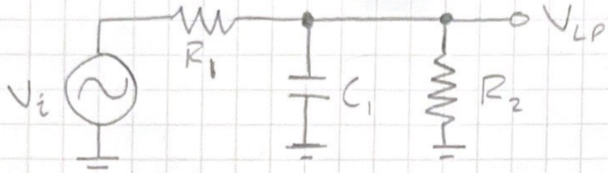
JOAQUIN SALAS PRE-LAB 1 ECEN 325

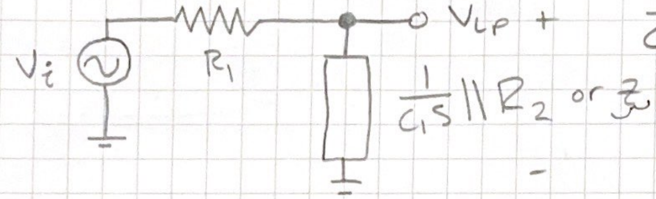
1) Derive. $H_{LP}(s) = \frac{V_{LP}}{V_1}(s) = k_L \frac{1}{1+\frac{s}{\omega_L}}$



- Transfer function is Nothing but Gain

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{V_{LP}(s)}{V_{in}(s)}$$

- Simplify Circuit



$$\frac{1}{C_1 s} \| R_2 = \frac{\frac{1}{C_1 s} \cdot R_2}{\frac{1}{C_1 s} + R_2} = \frac{R_2}{1 + C_1 R_2 s}$$

- Plug in $V_{LP} = \frac{Z}{R_1 + Z} \cdot V_{in} \rightarrow \frac{V_{LP}}{V_{in}} = \frac{Z}{R_1 + Z}$ Plug in

$$\rightarrow H_{LP}(s) = \frac{V_{LP}}{V_i} = \frac{\frac{R_2}{1 + C_1 R_2 s}}{\frac{R_2}{1 + C_1 R_2 s} + R_1} = \frac{R_2}{R_2 + R_1(1 + C_1 R_2 s)}$$

$$= \frac{R_2}{R_1 + R_2}\left[\frac{1}{1 + \frac{R_1 R_2 C_1 s}{R_1 + R_2}}\right] = \frac{R_2}{R_2 + R_1 + R_1 R_2 C_1 s}$$

$$K_L = \frac{R_2}{R_1 + R_2}, \quad \omega_L = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

2) $K_L = 0.5$ and $f_L = 5\,kHz$

$* \; \omega = 2\pi f$

Find $R_1, R_2, R_3, C_1, C_2, C_3$

$$K_L = \frac{R_2}{R_1 + R_2} = 0.5 \Rightarrow 2R_2 = R_1 + R_2 \Rightarrow R_2 = R_1$$

$$\omega_L = \frac{R_1 + R_2}{R_1 R_2 C_1} = 2\pi \cdot 5k \Rightarrow \frac{2R}{R^2 C_1} = \cancel{2}\pi \times 5 \times 10^3 \Rightarrow RC_1 = \frac{1}{5000\pi}$$

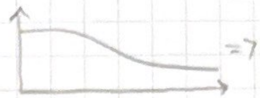
$RC_1 = 6.36 \times 10^{-5}$

$R_1 = R_2 = 1k\Omega$ $C_1 = 63.66\,nF$

$$\boxed{\begin{aligned} R_1 &= 1k\Omega \\ R_2 &= 1k\Omega \\ C_1 &= 63.66\,nF \end{aligned}}$$

3) Sketch magnitude and phase Bode plots

For $H_{LP}(s)$: From Lecture, Assume it will look like



- Plug into $H_{LP}(s) = \frac{K_L}{1+\frac{s}{\omega_L}} = \frac{0.5}{1+\frac{s}{10{,}000\pi}}$

$\Rightarrow T(j\omega) = H(s) \Rightarrow \frac{0.5}{1+\frac{j\omega}{10{,}000\pi}} \cdot \frac{10{,}000\pi}{10{,}000\pi} \Rightarrow \frac{5000\pi}{10000\pi + j\omega}$.

$\phi = \tan^{-1}\left[\frac{Im(T)}{Re(T)}\right]$

$A(\omega) = \sqrt{T^*T}$ $A(\omega) = \frac{5000\pi}{\sqrt{(10000\pi)^2+\omega^2}}$ L.P

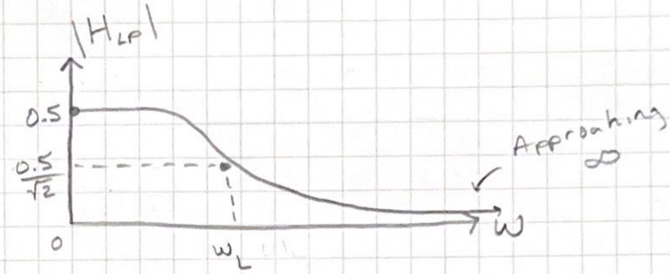
$= -\tan^{-1}\left\{\frac{\omega}{10000\pi}\right\}$

Magnitude Response

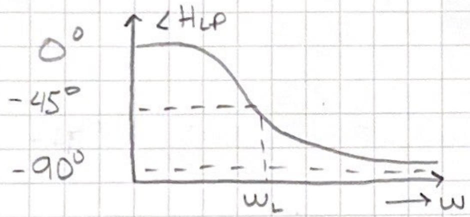
when $\omega = 0 \Rightarrow |H_{LP}| = 0.5$

$\omega = \omega_L \Rightarrow |H_{LP}| = \frac{0.5}{\sqrt{2}}$

$\omega = \infty \Rightarrow |H_{LP}| = 0$



Phase Response



4) Calculate $V_{LP}(t)$ for $V_i(t) = 0.4\sin(2\pi 4000t)$

$V_{LP}(t) = H_{LP}(s)$ plug in $s = j(2\pi 4000)$

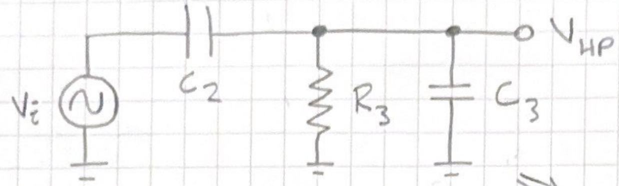
$= \left(\frac{0.5}{1+\frac{8000\pi j}{10000\pi}}\right)(0.4\sin(2\pi 4000t))$ $\frac{4}{5}j$

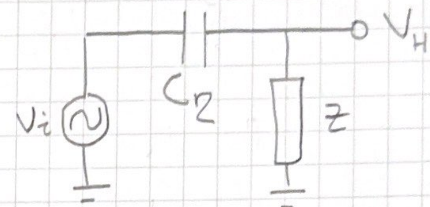
$\Rightarrow \phi = -\tan^{-1}\left(\frac{8000\pi}{10000\pi}\right) = -38.66^\circ$

$\Rightarrow V_{LP}(t) = 0.4\,|H_{LP}(\omega)|\sin(\omega t - \phi)$

$= 0.4(0.468)\sin(10000\pi t - 38.66^\circ) = 0.156\sin(8000\pi t - 38.66^\circ)$

1) Derive $H_{HP}(S) = \frac{V_{HP}(S)}{V_i} = K_H \frac{S}{S+\omega_H}$





- $H(S) = \frac{V_o(S)}{V_{in}(S)} = \frac{V_{HP}(S)}{V_i(S)}$

- $Z = R_3 \| \frac{1}{SC_3} = \frac{R_3 \cdot \frac{1}{SC_3} \cdot SC_3}{R_3 + \frac{1}{SC_3} \cdot SC_3} \rightarrow \frac{R_3}{1+SR_3C_3} \rightarrow V_{HP} = \left(\frac{Z}{Z+\frac{1}{SC_2}}\right)\frac{SC_2}{SC_2} V_{in} = \frac{ZSC_2}{ZSC_2+1} V_{in}$

Plug in Z $\Longrightarrow$ $\frac{V_{HP}(S)}{V_i(S)} = H_{HP}(S) = \frac{\left[\frac{R_3}{1+SR_3C_3}\right][SC_2]}{\frac{R_3}{1+SR_3C_3}(SC_2)+1} = \frac{\frac{SC_2R_3}{1+SR_3C_3}}{\frac{SC_2R_3}{1+SR_3C_3}+1} \Rightarrow \frac{SC_2R_3}{1+SR_3C_3+SC_2R_3} \rightarrow$

Compare to Eq Above $\longrightarrow$ $\frac{SC_2R_2}{1+S(R_3C_3+C_2R_3)} \longrightarrow K_H\left[\frac{C_2R_3}{R_3C_3+R_3C_2}\right]\left[\frac{S}{S+\frac{1}{R_3C_3+R_3C_2}}\right]$

$K_H = \frac{C_2}{C_2+C_3} \qquad \omega_H = \frac{1}{R_3(C_2+C_3)}$

2) $K_H = \frac{C_2}{C_2+C_3} = 0.5 \Rightarrow \frac{16\times10^{-9}}{16\times10^{-9}+16\times10^{-9}}$ $\qquad \omega = 2\pi(5000)$, $\omega = 10000\pi$

$\omega_H = \frac{1}{R_3(C_2+C_3)} = \frac{1}{1000(16\times10^{-9}+16\times10^{-9})} = 10000\pi$

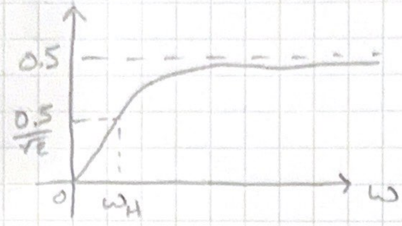
So, $R_3 = 1k\Omega$
$C_2 = 16\times10^{-9}F$ or $16nF$
$C_3 = 16nF$

3) $H_{HP} = \frac{0.5S}{S+\omega_H} \Rightarrow H_{HP} = \frac{0.5(j\omega)}{j\omega+\omega_H} \Rightarrow A(j\omega) = \frac{0.5\omega}{\sqrt{\omega^2+(\omega_H)^2}}$

when $\omega = 0 = |H_{HP}| = \frac{0.5\,\omega_H}{\frac{10000\pi}{\sqrt{2}}}$ $\quad \omega_H = 0$ $\quad = 0$

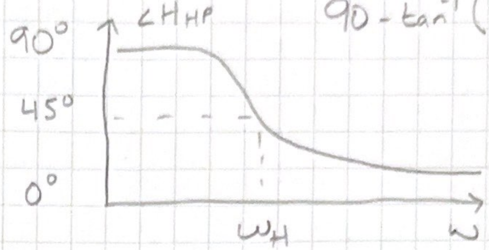
$\omega = \omega_H \Rightarrow |H_{HP}| = \frac{0.5\,\omega_H}{\frac{10000\pi}{\sqrt{2}}} = \frac{0.5}{\sqrt{2}}$

$\omega = \infty \Rightarrow |H_{HP}| = 0.5$



Phase Response

$90 - \tan^{-1}\left(\frac{\omega}{\omega_H}\right)$



4) For H_{HP}

$$= \frac{0.5s}{s + 10000\pi}[0.4]\sin(8000\pi t), \quad s = j(8000\pi)$$

$$= \frac{0.5(j8000\pi)}{j8000\pi + j10000\pi}[0.4]\sin(8000\pi t)$$

$$\Rightarrow \frac{4000\pi}{\sqrt{(10000\pi)^2 + (8000)^2}} = 0.312$$

$$= \phi = \tan^{-1}\left(\frac{8000\pi}{10000\pi}\right) = 38.66^\circ \qquad \underline{V_{HP} = 0.125\sin(8000\pi t + 38.66^\circ)}$$

5) For H_{HP} and H_{LP}

$V_i(t) = 0.3\sin(2\pi 6000t)$. Low Pass

$$\phi = \tan^{-1}\left(\frac{12000\pi}{10000\pi}\right) = -50.19^\circ$$

$$V_{LP}(t) = 0.3\,|H_{LP}(\omega)|\sin(\omega t + \phi)$$

$$= \left(\frac{5000\pi}{\sqrt{(10000\pi)^2 + (12000\pi)^2}}\right) \Rightarrow 0.320$$

$$= \underline{0.3 \times 0.320 \sin(\omega t - 50.19^\circ)} \Rightarrow \boxed{0.096\sin(\omega t - 50.19^\circ)}$$

Low Pass

$$= \underline{0.096\sin(12000\pi t - 50.19^\circ)}$$

H_{HP} -

$$\phi = \tan^{-1}\left(\frac{12000\pi}{10000\pi}\right) = 50.19^\circ$$

$$\Rightarrow \frac{6000\pi}{\sqrt{(10000\pi)^2 + (12000\pi)^2}} = 0.384$$

$$0.3\,|H_{HP}(\omega)|\sin(\omega t + \phi)$$

$$= 0.115\sin(12000\pi t + 50.19^\circ)$$