11 September 2023 731000141
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Question 1

Given:

Clock time	0:00:00	0:59:12	2:01:46	2:58:55	3:47:01	4:13:00	5:36:17
Odometer reading	102	157.8	217.6	264.1	315.2	341.7	420.3

Find:

Time interval in hours
Distance in miles
Average speed in miles per hour

Theory:

$\Delta t = t2\text{-}t1$
$\Delta x = x2\text{-}x1$
$\text{Average speed} = \text{Distance}/\ t2\text{-}t1$

Assumptions:

Moving in positive direction

Solution:

Time interval (hr) = 0:59:12 == .99 hours
Distance (miles) = 157.8 – 102 = 55.8 miles
Average speed (mph) = 55.8/.99 = 56.55 mph
Total average speed (mph) = [(56.55+57.35+48.82+63.74+61.19+56.63)] / (6) = 57.8 mph

Question 2

Given:

Time t, seconds: [0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0]
Position x, meters: [0.00, 4.10, 7.53, 10.92, 12.31, 12.35, 11.83, 10.49, 7.95]

Find:

Velocity and acceleration of a moving ball at t = 2, 3, 4, 5, and 6 seconds using finite difference methods (forward, backward, and centered).

Theory:

Velocity is rate of change of position with respect to time, v(t) = dx/dt (first derivative).
Acceleration is rate of change of velocity with respect to time, a(t) = dv/dt (second derivative).

Assumptions:

Position vs. time data is accurate.
Use finite difference methods to approximate velocity and acceleration at specified time intervals.

Solution:

Time	Velocity - Forward	Velocity - Backward	Velocity - Centered	Acceleration - Forward	Acceleration - Backward	Acceleration - Centered
2	3.39	3.43	3.41	-2	-0.67	-0.04
3	1.39	3.39	2.39	-1.35	-0.04	-2
4	0.04	1.39	0.715	-0.56	-2	-1.35
5	-0.52	0.04	-0.24	-0.82	-1.35	-0.56
6	-1.34	-0.52	-0.93	-1.2	-0.56	-0.82
7	-2.54	-1.34	-1.94	-5.41	-0.82	-1.2

2 FORWARD FINITE

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(3) = \frac{f(3+1) - f(3)}{1} = \frac{12.31 - 10.92}{1}$$

$$= \underline{1.39}$$

BACKWARDS FINITE

$$f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x} = \underline{3.39}$$

CENTERED FINITE

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = 2.39$$

FORWARD FINITE ACCELERATION

$$f''(t) = \frac{f(t) - 2f(\cancel{x}+\Delta x) + f(x+2\Delta x)}{\Delta x^2}$$

$$= \underline{-0.82}$$

BACKWARD FINITE Acceleration

$$f''(x) = \frac{f(x) - 2f(x-\Delta x) + f(x+\Delta x)}{\Delta x^2} = -0.56$$

CENTERED FINITE Acceleration

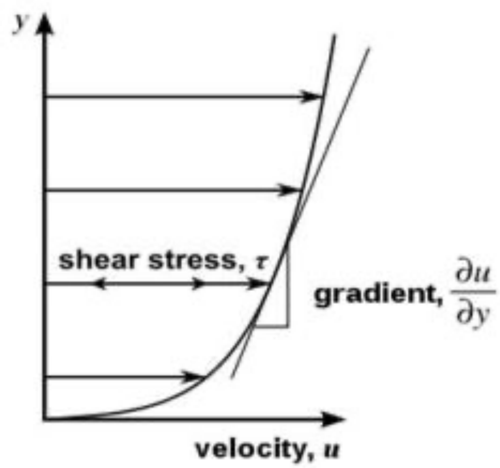
$$f''(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{(\Delta x)^2} = -0.56$$

Question 3

Given:
Dynamic viscosity (μ) = 1.8 × 10^-5 Ns/m^2
Distances from the surface (y) in meters: [0.000, 0.002, 0.006, 0.012, 0.018, 0.024]
Air velocity (u) in m/s: [0.000, 0.067, 0.572, 2.291, 5.047, 9.041]

Find:
Shear stress τ at distances y = 0.006 m, 0.012 m, and 0.018 m using the second-order centered first finite difference method.

Diagram:



Viscosity, $\tau = \mu \frac{du}{dy}$

Where,

μ = Dynamic viscosity

τ = Shear stress= F/A

$\frac{du}{dy}$ = Rate of shear deformation

Theory:
Newton's viscosity law relates the shear stress τ to the dynamic viscosity μ and the velocity gradient as τ = μ * (du/dy).

Assumptions:
Assume the airflow near the surface is sufficiently steady and incompressible.
Use the provided values for dynamic viscosity and air velocity.

Solution:

Using higher-order methods would likely give you more accurate results, especially if the velocity data is noisy or irregular. These methods provide better approximations for derivatives and could better capture subtle changes in velocity, resulting in more precise estimates of shear stress. However, it may also require more data points for accurate calculations.

Second Order Centered First Finite Difference

For Velocity -

$$V_1 = \frac{d(t+\Delta t) - d(t-\Delta t)}{2\Delta t} = \frac{d_2 - d_0}{2(t_1 - t_0)}$$

Newtons Viscosity Law

$$\tau = \mu \frac{du}{dy}$$

At 0.0120 $\rightarrow$ $f(x-\Delta x)$

$$\frac{5.047 - 0.572}{2 \cdot 0.006} = 372.9166$$

$f(x+\Delta x)$ $\quad$ Δx

$$\frac{du}{dy} = 372.9166$$

$$\mu = 1.8\times10^{-5}\left(\frac{du}{dy}\right) = 6.7125$$

@ 0.0120 $\rightarrow$ SHEAR STRESS = 6.7125×10^{-3} N/m²

@ 0.006 $\rightarrow$

$$\rightarrow \frac{2.291 - 0.067}{2 \times \Delta x\ 0.002} = [556] \quad \cancel{\Delta x = 0.01}$$

$$1.8\times10^{-5}\left(\frac{du}{dy}\right) = 1.0008\times10^{-5}$$

@ 0.018

$$= 1.0008\times10^{-5}\ \text{N/m}^2$$

$\longrightarrow$ 281.25

$$1.8\times10^{-5}(281.25) = 0.0050625$$

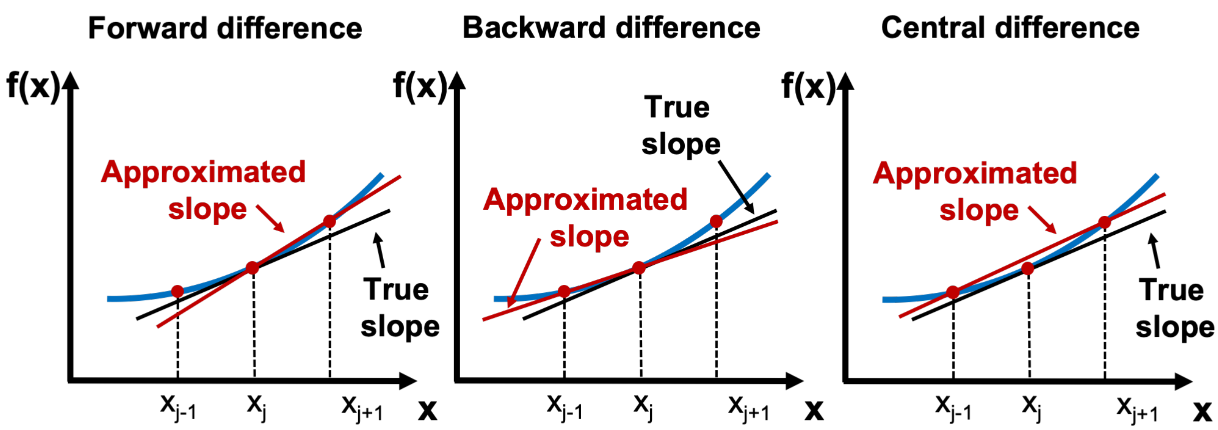
$$\Rightarrow 5.06\times10^{-4}\ \text{N/m}^2$$

Question 4

Given:
The function: $f(x)$ = ln(x) sinh(x) / e^x
Derivative value at x = 1.5: $f'(1.5)$ = 0.336925
Step size: Δx = 0.25

Find:
Numerical estimates of the derivative using forward, backward, and centered finite differences.
Percent error between the true value and each estimated value.

Diagram:



Theory:
- The forward finite difference formula for approximating the derivative is: $f'(x) \approx [f(x + \Delta x) - f(x)] / \Delta x$
- The backward finite difference formula for approximating the derivative is: $f'(x) \approx [f(x) - f(x - \Delta x)] / \Delta x$
- The centered finite difference formula for approximating the derivative is: $f'(x) \approx [f(x + \Delta x) - f(x - \Delta x)] / (2\Delta x)$
- Percent error formula: $\varepsilon = |true\ value - estimated\ value| / |true\ value| \times 100\%$

Assumptions:
The function $f(x)$ can be accurately approximated using finite differences.

Solution:

First Order Forward

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x} \Rightarrow \frac{\frac{\ln(1.5+0.25)\sinh(1.5+0.25)}{e^{(1.5+0.25)}} - 0.192639}{0.25}$$

$$f'(x) = 0.3149$$

Backwards

$$f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x} \Rightarrow 0.3609$$

0.102413

$$f'(x) = 0.3609$$

Centered

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = 0.337974$$

$$f'(x) = 0.337974$$

% Error Forward

$$\varepsilon = \left| \frac{0.336925 - 0.314877}{0.336925} \right| \times 100\% = 6.54\%$$

% Error Centered

$$\varepsilon = .3113\%$$

% Error Backwards

$$\varepsilon = 7.12\%$$