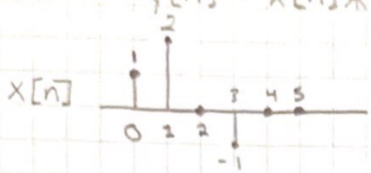
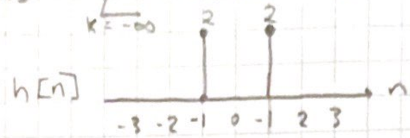
P.2.1 $y[n] = x[n] * h[n]$ let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$

$h[n] = 2\delta[n+1] - 2\delta[n-1]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$





a) $y[n] = x[n] * h[n]$

$$(\delta[n] + 2\delta[n-1] - \delta[n-3]) * (2\delta[n+1] - 2\delta[n-1])$$

$$\rightarrow 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-2] - 2\delta[n-4]$$

$$\boxed{y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]}$$

b) $y[n] = x[n+2] * h[n-1]$

$$\rightarrow \delta[n+2]\left[2\delta[n] + 2\delta[n-2]\right] + 2\delta[n+1]\left[2\delta[n] + 2\delta[n-2]\right] - \delta[n-1]\left[2\delta[n] + 2\delta[n-2]\right] +$$

$$2\delta[n+2] + 2\delta[n] + 4\delta[n+1] + 2\delta[n-1] - 2\delta[n-3]$$

$$\boxed{y_1[n] = 2\delta[n+2] + 2\delta[n] + 4\delta[n+1] + 2\delta[n-1] - 2\delta[n-3]}$$

P2.3 Consider input $x[n]$ and a unit impulse response $h[n]$ given by

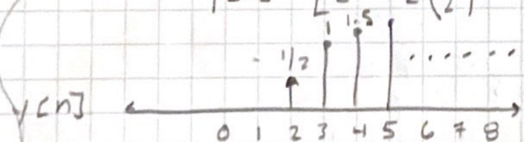
$$x[n] = \left(\tfrac{1}{2}\right)^{n-2} u[n-2], \quad h[n] = u[n-2]$$

Determine and plot $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{k-2} u[k-2]\, u[n-k-2]$$

$$y[n] = \sum_{k=0}^{n+2} \left(\tfrac{1}{2}\right)^{k+2} = \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^3 + \cdots + n = \frac{1 + \left(\tfrac{1}{2}\right)^{n+1}}{1 - 1/2}$$

$$\boxed{y[n] = \left[2 - 2\left(\tfrac{1}{2}\right)^{n-1}\right] u[n]}$$



P2.4 Compute AND Plot $y[n] = x[n] * h[n]$

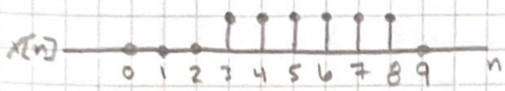
$$x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & \text{otherwise} \end{cases} \qquad h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & \text{Otherwise} \end{cases}$$

① $y[n] * h[n] = \sum_{-\infty}^{\infty} x[k]\, h[n-k]$ 3 4 ~~5~~ 6 7 8 9 10 11 12 13 14 15

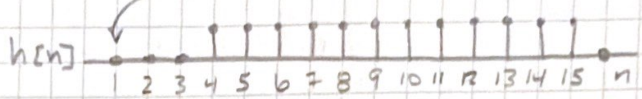
$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

$$y[n] = \begin{cases} n-6 & 7 \le n \le 11 \\ 6 & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{else} \end{cases}$$

$x[n] =$



0, otherwise

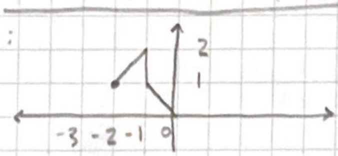


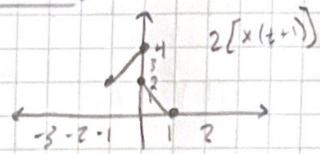
2.8 $x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ -t, & 1 < t \le 2 \\ 0, & \text{else} \end{cases}$ $h(t) = \delta(t+2) + 2\delta(t+1)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, dt$$

$y(t) = x(t) * [\delta(t+2) + 2\delta(t+1)]$
$y(t) = [x(t) * \delta(t+2)] + 2[x(t) * \delta(t+1)]$.
$y(t) = [x(t+2)] + 2[x(t+1)]$

$x(t+2)$:





$2[x(t+1)]$

$$y(t) = \begin{cases} t+3 & -2 < t \le -1 \\ t+4, & -1 < t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & \text{else} \end{cases}$$

2.9 $h(t) = e^{2t} u(-t+4) + e^{-2t} u(t-5)$

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)}, & \tau < A \\ 0, & A < \tau < B \\ e^{2(t-\tau)}, & B < \tau \end{cases}$$

Rewrite

$$h(\tau) = e^{2\tau} u(-r+4) + e^{-2\tau} u(r-5) = \begin{cases} e^{-2\tau}, & r > 5 \\ 0, & 4 < r < 5 \\ e^{2\tau}, & r < 4 \end{cases}$$

$$h(t-r) = \begin{cases} e^{-2(t-r)}, & r < t-5 \\ 0, & t-5 < r < t-4 \\ e^{2(t-r)}, & r > t-4 \end{cases} \qquad h(-r) = \begin{cases} e^{+2r}, & r < -5 \\ e^{-2r}, & r > -4 \\ 0, & -5 < r < -4 \end{cases}$$

Plug in $(t-r)$