Texas A&M University
Department of Electrical and Computer Engineering
ELEN 444 - Digital Signal Processing
Fall 2024

Problem Set 6
Assigned: Thursday, Oct. 10, 2024
Due: Thursday, Oct. 17, 2024

Structures for discrete-time systems

6.1: Proakis and Manolakis 9.3

6.2: Proakis and Manolakis 9.6

6.3: Proakis and Manolakis 9.7

6.4: Proakis and Manolakis 9.8

6.5: Proakis and Manolakis 9.9 (b,d,f)

Computer Assignment 6: Transform analysis of LTI systems

L5.1: Two type I FIR linear phase filters $h'[n]$ and $\tilde{h}'[n]$ are given as follows:

$$h'[0] = 0.037829, h'[1] = -0.023849, h'[2] = -0.110624, h'[3] = 0.37740, h'[4] = 0.852699;$$

$h'[n] = h'[8-n]$, for $5 \leq n \leq 8$;

$$\tilde{h}'[0] = -0.064539, \tilde{h}'[1] = -0.04069, \tilde{h}'[2] = 0.418092, \tilde{h}'[3] = 0.788485;$$

$\tilde{h}'[n] = \tilde{h}'[6-n]$, for $4 \leq n \leq 6$; and $h'(n) = \tilde{h}'(n) = 0$ for other n.

(a) Use "freqz" in Matlab to plot the log magnitude and phase of $h'[n]$ and $\tilde{h}'[n]$.

Since both $h'[n]$ and $\tilde{h}'[n]$ have nonzero phase response (or group delay), we define two new FIR linear phase filters $h[n]$ and $\tilde{h}[n]$ by shifting $h'[n]$ and $\tilde{h}'[n]$, repectively. That is: we set $h[n] = h'[n+4]$ and $\tilde{h}[n] = \tilde{h}'[n+3]$.

(b) Plot both $h[n]$ and $\tilde{h}[n]$ using "stem" in Matlab.

$h[n]$ and $\tilde{h}[n]$ are now symmetric with repect to zero, thus they both have zero phase response, i.e., both $H(e^{j\omega})$ and $\tilde{H}(e^{j\omega})$ are real functions. But $H(e^{j\omega}) = H'(e^{j\omega})e^{j4\omega}$ and $\tilde{H}(e^{j\omega}) = \tilde{H}'(e^{j\omega})e^{j3\omega}$ by definition, so $H'(e^{j\omega})e^{j4\omega}$ and $\tilde{H}'(e^{j\omega})e^{j3\omega}$ are real functions.

(c) Plot $H(e^{j\omega})$ (or $H'(e^{j\omega})e^{j4\omega}$) and $\tilde{H}(e^{j\omega})$ (or $\tilde{H}'(e^{j\omega})e^{j3\omega}$).

It can be shown that $H(e^{j\omega})\tilde{H}(e^{j\omega}) + H(e^{j(\omega+\pi)})\tilde{H}(e^{j(\omega+\pi)}) = 2$. This is the condition that $h[n]$ and $\tilde{h}[n]$ have to satisfy in order to be called a pair of biorthogonal wavelet filters.

(d) Plot $H(e^{j\omega})\tilde{H}(e^{j\omega}) + H(e^{j(\omega+\pi)})\tilde{H}(e^{j(\omega+\pi)})$ and check if it is indeed a constant that equals to 2.

9.3 Determine the system function and the impulse response of the system shown in Fig. P9.3.

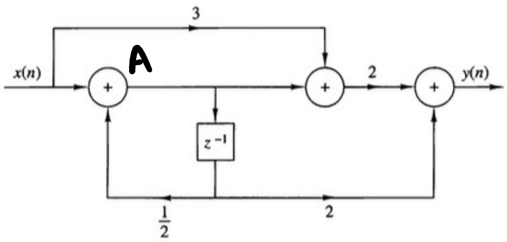


Figure P9.3

$$A = x(n) + \frac{1}{2} z^{-1} \cdot A$$

$$A - \frac{1}{2} z^{-1} \cdot A = x(n)$$

$$A\left(1 - \frac{1}{2} z^{-1}\right) = x(n)$$

$$\rightarrow A = \frac{x}{1 - \frac{1}{2} z^{-1}}$$

$$Y(n) = 2(3x(n)) + 2A(n) + 2z^{-1}A(n)$$

$$Y(z) = 2(3X(z)) + 2A(z) + 2z^{-1}(A(z))$$

Sub in $A(z)$

$$Y(z) = 6X(z) + 2\left[\frac{X(z)}{1 - \frac{1}{2} z^{-1}}\right] + 2z^{-1}\left[\frac{X(z)}{1 - \frac{1}{2} z^{-1}}\right]$$

$$\rightarrow Y(z) = 6X(z) + \frac{2X(z)}{1 - \frac{1}{2} z^{-1}} + 2z^{-1} \cdot \frac{X(z)}{1 - \frac{1}{2} z^{-1}}$$

$$\rightarrow \frac{Y(z)}{X(z)} = 6 + \frac{2}{1 - \frac{1}{2} z^{-1}} + \frac{2z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$H(z) = \frac{8 \cdot z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

z - transform

$$H(z) = \frac{8}{1 - \frac{1}{2} z^{-1}} - \frac{z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$h(n) = 8\left(\frac{1}{2}\right)^{n} u(n) - \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

9.6 Determine a_1, a_2 and c_1, and c_0 in terms of b_1 and b_2 so that the two systems in Fig. P9.6 are equivalent.

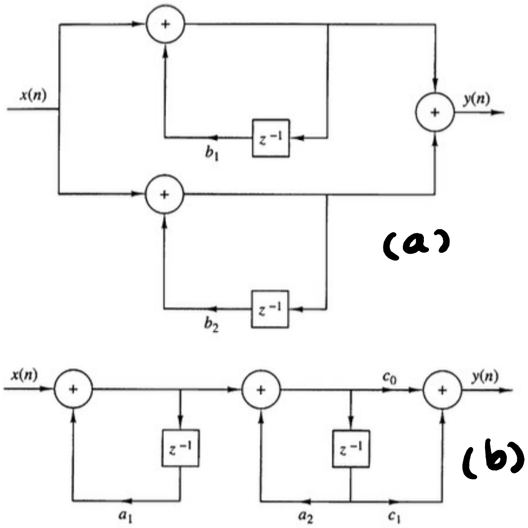


Figure P9.6

$$y(n) = c_0 x(n) + c_1 z^{-1} y(n) + a_1 z^{-1} x(n) + a_2 z^{-2} x(n)$$

(a) $y(n) = x(n) + b_1 z^{-1} y(n) + b_2 z^{-1} x(n)$

$\rightarrow y(n) - b_1 z^{-1} y(n) = x(n) + b_2 z^{-1} x(n)$

$$\rightarrow y(n) = \frac{x(n) + b_2 z^{-1} x(n)}{1 - b_1 z^{-1}}$$

(b) $$y(n) = \frac{c_0 + (a_1 + a_2 z^{-1}) z^{-1}}{1 - c_1 z^{-1}} x(n)$$

Set Equal.

$1 - b_1 z^{-1} = 1 - c_1 z^{-1} \rightarrow b_1 = c_1$

$1 + b_2 z^{-1} = c_0 + a_1 z^{-1} + a_2 z^{-2}$

$c_0 = 1$

$a_1 = -(b_1 + b_2)$

$a_2 = b_2$

9.7 Consider the filter shown in Fig. P9.7.

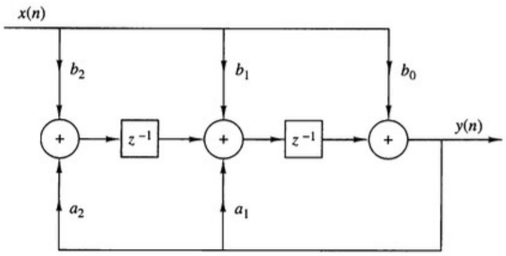


Figure P9.7

(a) Determine its system function.

(b) Sketch the pole–zero plot and check for stability if

1. $b_0 = b_2 = 1, \quad b_1 = 2, \quad a_1 = 1.5, a_2 = -0.9$
2. $b_0 = b_2 = 1, \quad b_1 = 2, \quad a_1 = 1, a_2 = -2$

(c) Determine the response to $x(n) = \cos(\pi n/3)$ if $b_0 = 1$, $b_1 = b_2 = 0$, $a_1 = 1$, and $a_2 = -0.99$.

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) - a_1 y(n-1) - a_2 y(n-2)$$

(c) $$H(z) = \frac{1}{1 + z^{-1} - 0.99 z^{-2}}$$

$$= \frac{1}{1 + e^{-j\pi/3} - 0.99 e^{-j2\pi/3}}$$

(a) $$y(n) = b_0 x(n) + b_1 z^{-1} x(n) + b_2 z^{-2} x(n) - a_1 y(n) z^{-1} - a_2 y(n) z^{-2}$$

$$y(n)\left(1 + a_1 z^{-1} + a_2 z^{-2}\right) = x(n)\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

(b) $$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 + z^{-1} - 2z^{-2}}$$

ZEROS $\rightarrow$ $z_1 = -1, z_2 = -1$

Poles

$x^2 + x - 2 \rightarrow (x-1)(x+2)$

$x^2 + 2x + 1$

Poles : $p_1 = 1$

$p_2 = -2$

$(x+1)(x+1) \rightarrow x = -1, -1$

Not Within unit circle, System is unstable

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 + 1.5z^{-1} + 0.9z^{-2}}$$

Zeros: $z_1 = -1, z_2 = -1$

Poles

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1.5 \pm \sqrt{(1.5)^2 - 4(0.9)}}{2(1)}$$

System is stable

Poles $= -\frac{3}{4} \pm j\,\frac{29}{50}$

9.8 Consider an LTI system, initially at rest, described by the difference equation

$$y(n) = \frac{1}{4}y(n-2) + x(n)$$

(a) Determine the impulse response, $h(n)$, of the system.

(b) What is the response of the system to the input signal

$$x(n) = [(\tfrac{1}{2})^n + (-\tfrac{1}{2})^n]u(n)$$

(c) Determine the direct form II, parallel-form, and cascade-form realizations for this system.

(d) Sketch roughly the magnitude response $|H(\omega)|$ of this system.

(a) $$Y(z) = \frac{1}{4}z^{-2}Y(z) + X(z)$$

$$\rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{4}z^{-2}} \rightarrow \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}}$$

$$1 = A(1-\tfrac{1}{2}z^{-1}) + B(1-\tfrac{1}{2}z^{-1}) \qquad A+B=1$$

$$A = 1/2,\ B = 1/2 \qquad \tfrac{1}{2}A - \tfrac{1}{2}B = 0$$

$$H(z) = \frac{1/2}{1-\frac{1}{2}z^{-1}} + \frac{1/2}{1-\frac{1}{2}z^{-1}}$$

$$\rightarrow h(n) = \tfrac{1}{2}(\tfrac{1}{2})^n u(n) + \tfrac{1}{2}(\tfrac{1}{2})^n u(n)$$

(b)

$$x(n) \rightarrow X(z)$$

$$x(n) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{2}z^{-1}} = \frac{2}{1-\frac{1}{4}z^{-2}} \cdot \frac{1}{1-\frac{1}{4}z^{-2}} \quad (\text{x by } H(z))$$

$$Y(z) = \frac{2}{1-\frac{1}{4}z^{-2}} \cdot \left(\frac{1/2}{1-1/2z^{-1}} + \frac{1/2}{1-\frac{1}{2}z^{-1}}\right)$$

$$= \frac{2}{1-\frac{1}{4}z^{-2}} \cdot \left(\frac{1}{2(1-1/2z^{-1})} + \frac{1}{2(1-\frac{1}{2}z^{-1})}\right)$$

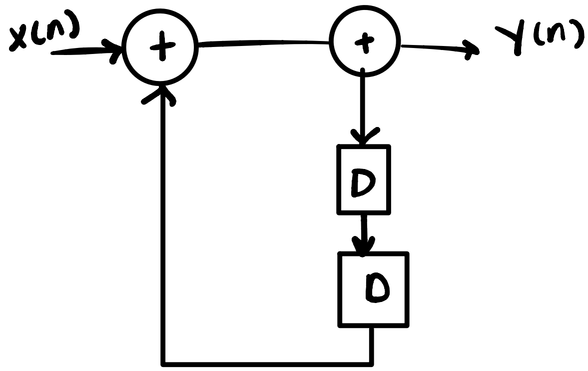
$$Y(z) = \underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}_{(\frac{1}{2})^n u(n)} + \underbrace{\frac{1}{1+\frac{1}{2}z^{-1}}}_{(-\frac{1}{2})^n u(n)} - \underbrace{\frac{z^{-1}}{(1-\frac{1}{2}z^{-1})^2}}_{(\frac{1}{2})^n u(n)} + \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$$

$$(-\tfrac{1}{2})^n u(n)$$

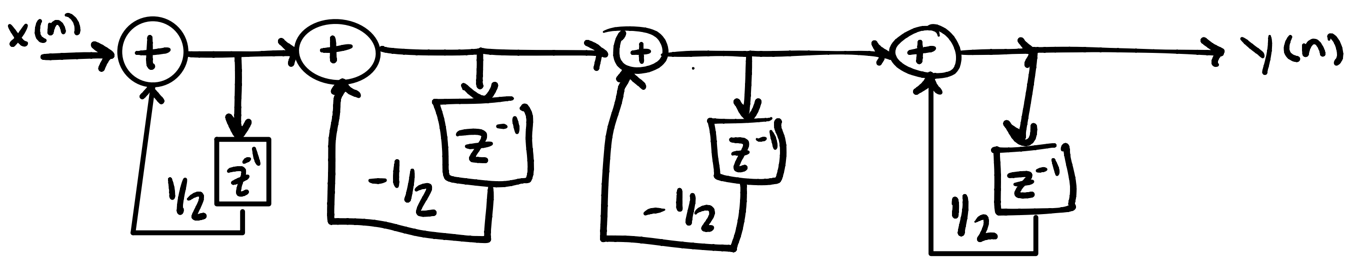
$$y(n) = (\tfrac{1}{2})^n u(n) + (-\tfrac{1}{2})^n u(n) - n\left[(\tfrac{1}{2})^n + (-\tfrac{1}{2})^n\right]$$

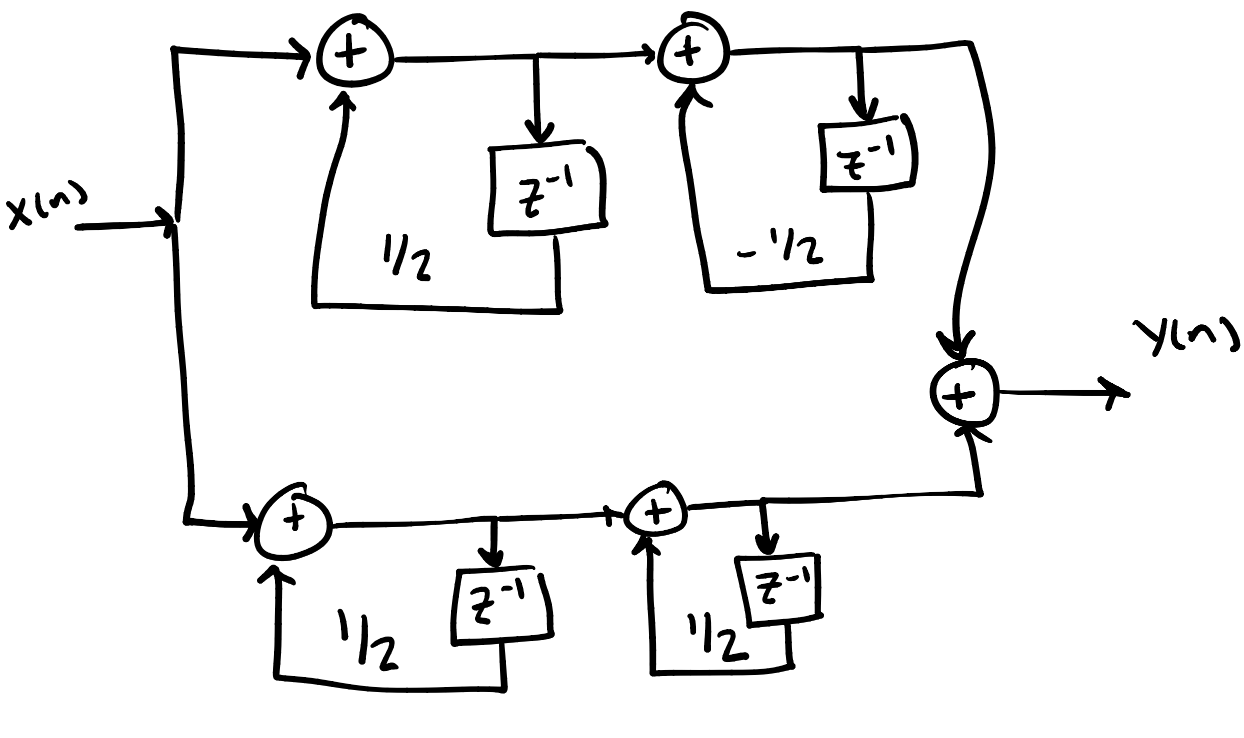
(c.)

Direct Form II



Cascade





9.9 Obtain the direct form I, direct form II, cascade, and parallel structures for the following systems.

(a) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$

(b) $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$

(c) $y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

(d) $H(z) = \dfrac{2(1-z^{-1})(1+\sqrt{2}z^{-1}+z^{-2})}{(1+0.5z^{-1})(1-0.9z^{-1}+0.81z^{-2})}$

(e) $y(n) = \frac{1}{2}y(n-1) + \frac{1}{4}y(n-2) + x(n) + x(n-1)$

(f) $y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2)$

Which of the systems above are stable?

$$y(n) = y(n)\left(-0.1z^{-1} + 0.72z^{-2}\right) + x(n)\left[0.7 - 0.252z^{-2}\right] \qquad (1-0.6z^{-1})^2$$

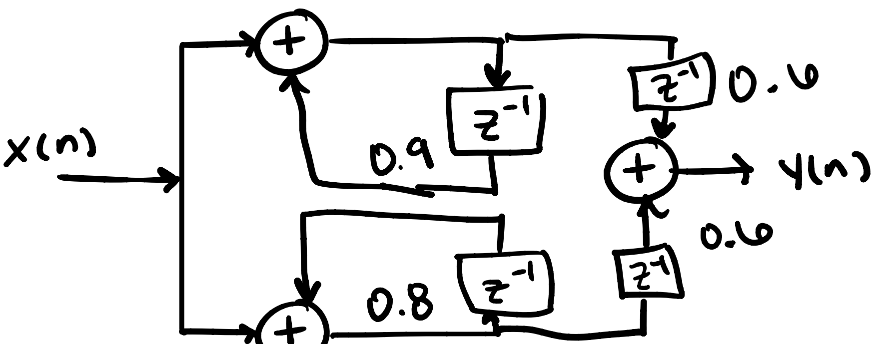
$$\rightarrow \quad H(n) = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} = \frac{0.7\left[1 - 0.36z^{-2}\right]}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$\hookrightarrow (1 + 0.9z^{-1})(1 - 0.8z^{-1})$

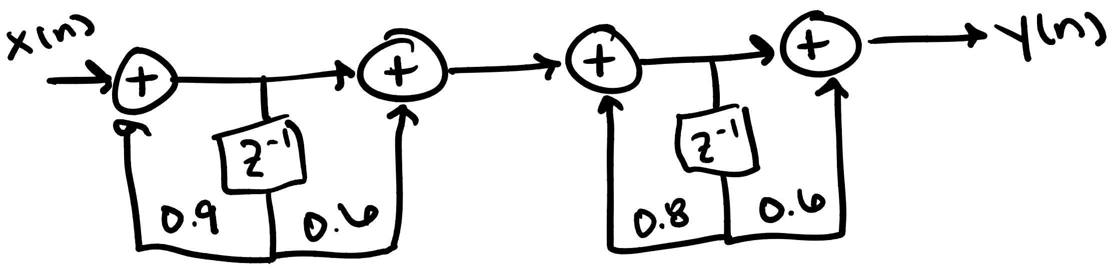
$x^2 + 0.1x - 0.72$

$(x + 0.9)(x - 0.8)$

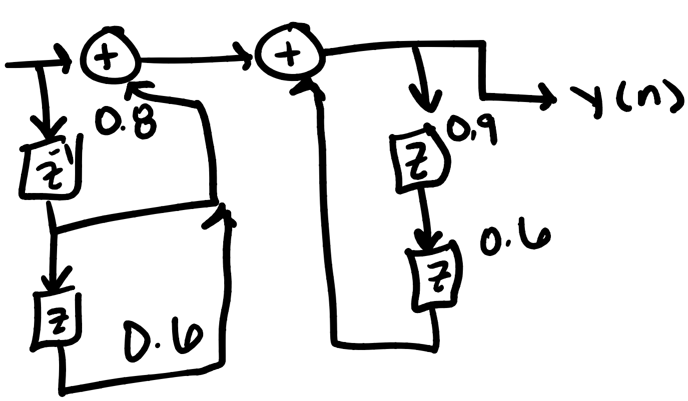
(b) PARALLEL:



CASCADE



DIRECT I :



Direct II :