Texas A&M University
Department of Electrical and Computer Engineering
ELEN 444 - Digital Signal Processing
Fall 2024

Midterm #2, Tuesday, Nov. 5, 2024

---

1. Consider a linear time-invariant system, initially at rest, described by the difference equation

$$y[n] = \frac{1}{16}y[n-2] + x[n]$$

(a) (5%) Determine the system function $H(z)$.

(b) (5%) Determine the impulse response of the system $h[n]$.

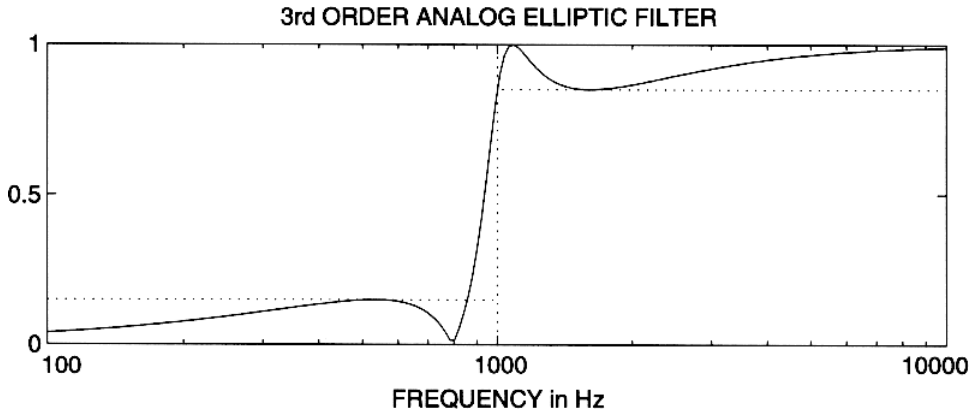
(c) (5%) Determine the direction form realization of this system.

(d) (10%) Determine the cascade form realization of this system.

(e) (10%) Determine the parallel form realization of this system.

(f) (15%) What is the system response to the input signal $x[n] = [(\frac{1}{4})^n + (-\frac{1}{4})^n]u[n]$.

2. Suppose we want to map the analog highpass elliptic filter shown below to a digital filter by using the bilinear transformation (note that the frequency axis is logarithmic in Hertz, so that the stopband edge is at approximately 860 Hz).



(a) (12.5 %) Determine T_d needed in the bilinear transformation so that the passband edge of the digital filter will be at $\omega_p = 0.3\pi$.

(b) (12.5 %) If the edge of the digital highpass filter was placed at $\omega_p = 0.3\pi$ as in part (a), dtermine the width of the transmission band $\Delta\omega = \omega_p - \omega_s$.

3. (12.5%) Prove that an even length, symmetric (or linear phase) FIR filter cannot be a true highpass filter; that is, show that $H(e^{j\pi}) = 0$ for all choice of coefficients.

4. (12.5%) Consider a sequence $x[n]$ whose length is M and another sequence $y[n]$ whose length is N, what is the length of $x[n] * y[n]$?

5 (Extra point question) We start with a prototype lowpass filter $h[n]$ with z-transform $H(z)$ before employing frequency transformation to convert $h[n]$ to other filters.

(a) (5%) If we use the frequency transformation $z^{-1} \to z$ on $H(z)$ to get our first new filter $h_1[n]$, that is $H_1(z) = H(z^{-1})$, what is the relationship between $h_1[n]$ and $h[n]$?

(b) (5%) If we use the frequency transformation $z^{-1} \to -z^{-1}$ on $H(z)$ to get our second new filter $h_2[n]$, that is $H_2(z) = H(-z)$, what is the relationship between $h_2[n]$ and $h[n]$? Is $h_2[n]$ lowpass, highpass, bandpass, or bandstop?