Question 1

$x(t) = 2e^{-2t}u(t)$ $\quad h(t) = 4e^{4t}u(-t)$

Fourier Transform

$$e^{-at}u(t) \longleftrightarrow \frac{1}{(a+j\omega)}$$

$$e^{-at}u(-t) \qquad \frac{1}{(a-j\omega)}$$

$$X(\omega) = \frac{2}{2+j\omega} + \frac{4}{4-j\omega} \qquad Y(\omega) = X(\omega)H(\omega)$$

Partial Fractions

$$= \frac{A}{2+j\omega} + \frac{B}{4-j\omega} = \frac{1}{(2+j\omega)(4-j\omega)}$$

$$A = (2+j\omega)Y(\omega)\Big|_{-2} = \boxed{\frac{1}{6}} = A$$

$$B = (4-j\omega)Y(\omega)\Big|_{4} = \frac{1}{2+4} = \boxed{\frac{1}{6}} = B$$

$$Y(\omega) = \frac{1}{6}\left[\frac{2}{(2+j\omega)} - \frac{4}{4-j\omega}\right]$$

$$e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a+j\omega}$$

$$\boxed{Y(\omega) = \frac{1}{6}\left[2e^{-2t}u(t) - 4e^{4t}u(-t)\right]}$$