P.3.1 A continuous-time periodic signal, fundamental period $T = 8$. Non-zero Fourier series coefficients are:

$$a_1 = a_{-1} = 2,\ a_3 = a^*_{-3} = 4j$$

Express in form $$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

$$x(t) = \sum_{k=-\infty}^{\infty} A_k e^{jk\omega_0 t} = 2e^{-j\omega_0 t} + 2e^{j\omega_0 t} - 4e^{-j3\omega_0 t} + 4je^{j3\omega_0 t}$$

Eulers - $e^{i\theta} = \cos\theta + i\sin\theta$

$$x(t) = 2\left[2\cos(\omega_0 t) + 4j(2j\sin(3\omega_0 t)\right]$$

$$\rightarrow T = \frac{2\pi}{\omega_0} \rightarrow 8 = \frac{2\pi}{\omega_0} \rightarrow \omega_0 = \frac{2\pi}{8} = \boxed{\frac{\pi}{4}}$$

$$\rightarrow 2\left[2\cos\left(\frac{\pi t}{4}\right)\right] + 4j\left[2j\sin\left(\frac{3\pi t}{4}\right)\right]$$

$$\rightarrow \boxed{4\cos\left(\frac{\pi t}{4}\right) - 8\sin\left(\frac{3\pi t}{4} - \frac{\pi}{2}\right)}$$

P3.2 $N = 5$, $a_0 = 1$, $a_2 = e^{j\pi/4}$, $a^*_{-2} = e^{j\pi/4}$, $a_4 = a^*_4 = 2e^{j\pi/3}$

$$a_2 = e^{j\pi/4} = \cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} = \frac{1+j}{\sqrt{2}}$$

$$a_4 = 2e^{j\pi/3} = 2\cos\left(\frac{\pi}{3}\right) + 2j\sin\left(\frac{\pi}{3}\right) = 1 + j\sqrt{3}$$

$$\omega_0 = \frac{2\pi}{N} = \frac{2\pi}{5}$$

$$x[n] = \sum_{k=\langle N\rangle} A_k e^{jk\omega_0 n} = \sum_{k=-4}^{4} A_{-4} e^{-j4\omega_0 n} + A_{-2} e^{-j2\omega_0 n} + A_2 e^{j2\omega_0 n} + A_4 e^{j4\omega_0 n}$$

$$\rightarrow 2e^{-j\frac{\pi}{3}} \cdot e^{-j4\omega_0 n} + e^{-j\frac{\pi}{4}} e^{-j2\omega_0 n} + 1 + 2e^{j\frac{\pi}{3}} \cdot e^{j4\omega_0 n} + e^{j2\omega_0 n} \cdot e^{j2\omega_0 n}$$

$$\rightarrow (1 - j\sqrt{3})e^{-j\frac{8\pi}{5}} + \left(\frac{1-j}{\sqrt{2}}\right)e^{-j\frac{4\pi}{5}} + 1 + \left(\frac{1+j}{\sqrt{2}}\right)e^{j\frac{\pi 4}{5}} + (1 + j\sqrt{3})e^{j\frac{8\pi}{5}}$$

$$\rightarrow 1 + \left(e^{j\frac{8\pi}{5}} + e^{-j\frac{8\pi}{5}}\right) + \sqrt{3}j\left(e^{j\frac{8\pi}{5}} - e^{-j\frac{8\pi}{5}}\right) + \frac{1}{\sqrt{2}}\left(e^{j\frac{4\pi}{5}n} + e^{-j\frac{4\pi}{5}n}\right) + \frac{j}{\sqrt{2}}\left(e^{j\frac{4\pi}{5}} - e^{-j\frac{4\pi}{5}}\right)$$

$$\rightarrow 1 + 2\cos\left(\frac{8\pi}{5}n\right) + j\sqrt{3} \cdot j2\sin\left(\frac{8\pi}{5}n\right) + \frac{2}{\sqrt{2}}\cos\left(\frac{4\pi}{5}n\right) + j/\sqrt{2} \cdot j2\sin\left(\frac{4\pi}{5}n\right)\Big]$$

$$\rightarrow 1 + 4\left[\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{8\pi}{5}n\right) - 4\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{8\pi}{5}n\right) + 2\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{4\pi}{5}n\right) - 2\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{4\pi}{5}n\right)\right]$$

$$x(n) = 1 + 2\cos\left(\frac{4\pi}{5}n + \frac{\pi}{4}\right)^{-\left(\frac{\pi}{2}\right)} + 4\cos\left(\frac{8\pi}{5} + \frac{\pi}{3}\right) \quad * \sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\boxed{x(n) = 1 + 2\sin\left(\frac{4\pi}{5}n + \frac{3\pi}{4}\right) + 4\sin\left(\frac{8\pi}{5} + \frac{5\pi}{6}\right)}$$

$A_0 = 1,\ A_2 = 2,\ A_4 = 4$

P3.3

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$$

$$\rightarrow x(t) = 2 + \left(\frac{e^{j\frac{2\pi}{3}t} + e^{-j\frac{2\pi}{3}t}}{2}\right) + 4\left(\frac{e^{j\frac{5\pi}{3}t} - e^{-j\frac{5\pi}{3}t}}{2j}\right)$$

$$= 2 + \frac{1}{2}e^{j2\frac{\pi}{3}t} + \frac{1}{2}e^{-j2\frac{\pi}{3}t} - 2je^{j5\frac{\pi}{3}t} + 2je^{-j5\frac{\pi}{3}t}$$

Angular freq. $= \underline{\omega_0 = \frac{\pi}{3}}$

$a_0 = 2$ $\quad a_5 = -2j$
$a_2 = 1/2$ $\quad a_{-5} = 2j$
$a_{-2} = 1/2$

$$\boxed{a_0 = 2,\ a_{-2} = a_2 = \frac{1}{2},\ a_5 = a_{-5}^* = -2j}$$

P3.4

A_k for periodic signal $x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$

fundamental freq. $\omega_0 = \pi$ $\quad T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$

$$a_0 = \frac{1}{2}\int_0^2 x(t)\,dt = \frac{1}{2}\left[\int_0^1 1.5\,dt - \int_1^2 1.5\,dt\right] = \frac{1}{2}\left[\cancel{1.5t}\Big|_0^1 - \cancel{1.5t}\Big|_1^2\right] = \underline{0}$$

$$a_k = \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}\,dt = \frac{1}{2}\left[\int_0^1 1.5e^{-jk\pi t}\,dt - \int_1^2 1.5e^{-jk\pi t}\,dt\right]$$

$$= \frac{1}{2}\left[1.5\times\left(\frac{e^{-jk\pi t}}{-jk\pi}\Big|_0^1\right) - 1.5\left(\frac{e^{-jk\pi t}}{-jk\pi}\Big|_1^2\right)\right]$$

$$a_k = \frac{3}{4(-jk\pi)}\left[e^{-jk\pi} - 1 - (e^{-j2k\pi} - e^{-jk\pi})\right]$$

$$\rightarrow \frac{3}{4jk\pi}\left[1 + e^{-j2k\pi} - 2e^{-jk\pi}\right] \qquad * \ e^{-j2k\pi} = \cos(2k\pi) - \cancel{j\sin(2k\pi)}^{0} = 1$$

$$\rightarrow \frac{3}{4jk\pi}\left[2 - 2e^{-jk\pi}\right] = \frac{3}{2jk\pi}\left[e^{-j0} - e^{-jk\pi}\right] = \frac{3}{2jk\pi}\left[e^{j\left(\frac{\pi}{2}-\frac{\pi}{2}\right)} - e^{-j\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}\right]$$

$$= \frac{3}{2jk\pi}\left[e^{j\frac{\pi}{2}k}e^{-j\frac{\pi}{2}k} - e^{-j\frac{\pi}{2}k}e^{-j\frac{\pi}{2}k}\right]$$

$$\rightarrow \frac{3e^{-j\frac{\pi}{2}k}}{k\pi}\left[\frac{e^{jk\frac{\pi}{2}} - e^{-j\frac{\pi}{2}k}}{2j}\right]$$

$$= \frac{3e^{-j\frac{\pi}{2}k}}{k\pi}\left[\sin\left(\frac{k\pi}{2}\right)\right]$$

$$\boxed{a_k = \begin{cases} 0, & k = 0 \\ \frac{3e^{-j\frac{\pi}{2}k}}{k\pi}\sin\left(\frac{k\pi}{2}\right), & k \ne 0 \end{cases}}$$

Q5 $T = 4,\ x[2] = x[-2] = 2,\ x[3] = x^*[-3] = 4j$

$$x(t) = \sum_{n=0}^{\infty} A_k \cos(\omega_k t + \phi) = \sum_{-\infty}^{\infty} x[k] e^{jk\omega_0 t}$$

$$x(t) = a_2 e^{2j\omega_0 t} + a_{-2} e^{-2j\omega_0 t} + a_3 e^{j\omega_0 t} + a_{-3} e^{-j4\omega_0 t}$$

$$= e^{j\frac{2\pi}{4}t} + e^{-j\frac{2\pi}{4}t} + 4j e^{j3\frac{2\pi}{4}t} - 4j e^{-j3\frac{2\pi}{4}t}$$

$$= e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t} + 4e^{j\frac{\pi}{2}} e^{j\frac{3\pi}{2}} + 4e^{-j\frac{\pi}{2}} e^{j\frac{3\pi}{2}t}$$

$$= 2\cos(2 \times \tfrac{\pi}{2}t + 0^\circ) + 4\cos(3 \times \tfrac{\pi}{2}t + 90^\circ)$$

$$\boxed{x(t) = 2\cos(\pi t) + 4\cos\left(\tfrac{3}{2}\pi t + \tfrac{\pi}{2}\right)}$$

Q6 $y(t) = x(t) + x(t-1)$ and $x(t) = e^{-2jt}$

$y(t) = H(s)e^{st}$ What is $H(s)$?

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, dt = \int_{-\infty}^{\infty} h(\tau)\, e^{-2j(t-\tau)}\, d\tau$$

$$y(t) = e^{-2jt} \int_{-\infty}^{\infty} h(\tau)\, e^{2j\tau}\, d\tau \qquad s = -2j$$

$$\rightarrow e^{st} \int_{-\infty}^{\infty} h(\tau)\, e^{-st}\, d\tau \quad - ①$$

$$y(t) = H(s)\, e^{st} \quad - ②$$

$$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$