0314 Access Code

① $x(t) = A\cos(Bt + C)$

(a) x(t) is real

(b) $T_0 = 4 \qquad \omega_0 = \frac{2\pi}{4} = \boxed{\frac{\pi}{2} = B}$

(c) $X[1]$ and $X[-1]$

(d) $x(t) = x(-t)$

(e) $\frac{1}{4}\int_{-2}^{2} |x(t)|^2 dx = \underline{2}$

$X[1] = 2$ ← Positive $\underline{\underline{\text{REAL}}}$

$X[-1] = -2$

So,

$$x(t) = \underbrace{2e^{-j\omega_0 t} - 2je^{-j\omega_0 t}}_{-\sin(\omega_0 t)} = -\cos\left(\omega_0 t + \frac{\pi}{2}\right)$$

$$\boxed{A = 1,\ B = \frac{\pi}{2},\ C = \frac{\pi}{2}}$$

Q2 Determine $y(t) = x(t) * h(t)$

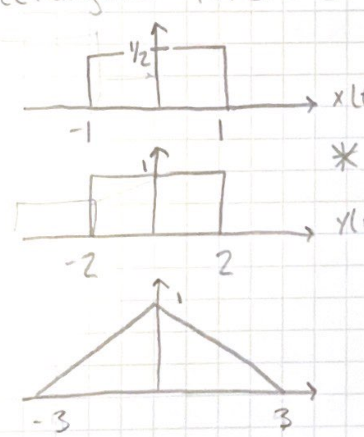
$$x(t) = \frac{\sin(\pi t)}{2\pi t}, \quad h(t) = \frac{2\sin(2\pi t)}{\pi t}$$

Rectangular functions

$x(t) = \frac{\sin(\pi t)}{2\pi t} = \pi\left(\omega/2\omega\right)$

Rectangular function

$X(j\omega) = \begin{cases} 1, & 1 < 2\pi \\ 0, & \end{cases}$



$$\boxed{X(j\omega) = \begin{cases} j/\pi, & -3 \le \omega \le 0 \\ -j/\pi, & 0 \ge \omega \ge 3 \\ 0, & \text{else} \end{cases}}$$

Q3.

$$x(t) = e^{-t}u(t) - 2e^{-4t}u(t)$$
$$y(t) = e^{-t}u(t) - e^{-2t}u(t)$$

$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$$

Fourier Transform

$$x(t) = \frac{1}{1+j\omega} \cdot \frac{2}{4+j\omega} = \frac{2}{(1+j\omega)(4+j\omega)} = X(j\omega)$$
$$y(t) = \frac{1}{(1+j\omega)} - \frac{1}{(2+j\omega)} = \frac{1}{(1+j\omega)(2+j\omega)}$$

$x(t)$:
$$\frac{2}{(1+j\omega)(4+j\omega)} = \frac{A}{1+j\omega} + \frac{B}{4+j\omega}$$
$$2 = A(4+j\omega) + \cancel{B(1+j\omega)}\Big|_{-1} = \frac{2}{3} = \frac{3A}{3} \Rightarrow \underline{A = \frac{2}{3}}$$
$$2 = \cancel{A(4+j\omega)} + B(1+j\omega)\Big|_{-4} = 2 = -3B \Rightarrow \underline{B = -\frac{2}{3}}$$
$$x(t) = \frac{2/3}{(1+j\omega)} - \frac{2/3}{(4+j\omega)} \overset{FT}{\Rightarrow} \underline{\frac{2}{3}e^{-t}u(t) - \frac{2}{3}e^{-4t}u(t)}$$

$y(t)$: $\frac{1}{(1+j\omega)(2+j\omega)} = \frac{A}{1+j\omega} + \frac{B}{2+j\omega} =$

$y(t) = e^{-t}u(t) - e^{-2t}u(t)$

$$1 = A(2+j\omega) + \cancel{B(1+j\omega)}\Big|_{-1} = A = 1$$
$$1 = \cancel{A(2+j\omega)} + B(1+j\omega)\Big|_{-2} \quad B = -1 \quad \underline{\frac{1}{(j\omega+1)} - \frac{1}{(2+j\omega)}}$$

$$\frac{2}{(1+j\omega)^2(2+j\omega)(4+j\omega)} = \frac{A}{(1+j\omega)} + \frac{B}{(1+j\omega)^2} + \frac{C}{(2+j\omega)} + \frac{D}{(4+j\omega)}$$

$A(1+j\omega)^2(2+j\omega)(4+j\omega) + B(1+j\omega)(2+j\omega)(4+j\omega) +$
$C(1+j\omega)(1+j\omega)^2(4+j\omega) + D(1+j\omega)(1+j\omega^2)(2+j\omega)$

~~At~~

$$\boxed{H(j\omega) = \frac{\frac{1}{(1+j\omega)} - \frac{1}{(2+j\omega)}}{\frac{2/3}{(1+j\omega)} - \frac{2/3}{(4+j\omega)}}}$$

$$\boxed{h(t) = \frac{e^{-t}u(t) - e^{-2t}u(t)}{\frac{2}{3}e^{-t}u(t) - \frac{2}{3}e^{-4t}u(t)}}$$

Q4 Mathematical and Pictorial

$$x(t) = \cos\left(\frac{\pi}{2}t\right) - j\sin\left(\frac{\pi}{2}t\right)$$

FT's

$a_1 = a_{-1} = \frac{1}{2}$

① $\cos(\omega_0 t) \xleftrightarrow{FT} \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$ $a_k = 0$, else

$\cos\left(\frac{\pi}{2}t\right) \longleftrightarrow \pi\left[\delta\left(\omega-\frac{\pi}{2}\right) + \delta\left(\omega+\frac{\pi}{2}\right)\right]$ $a_1 = 1/2$

② $\sin(\omega_0 t) \xleftrightarrow{FT} \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$ $a_1 = -a_{-1} = \frac{1}{2j}$
$a_k = 0$, else

$\sin\left(\frac{\pi}{2}t\right) \xleftrightarrow{FT} \frac{\pi}{j}\left[\delta\left(\omega-\frac{\pi}{2}\right) - \delta\left(\omega+\frac{\pi}{2}\right)\right]$

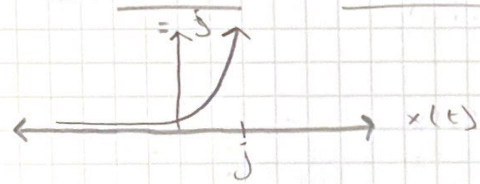
$$x(t) = \left[\pi\left[\delta\left(\omega-\frac{\pi}{2}\right) + \delta\left(\omega+\frac{\pi}{2}\right)\right]\right] - j\left[\frac{\pi}{j}\left[\delta\left(\omega-\frac{\pi}{2}\right) - \delta\left(\omega+\frac{\pi}{2}\right)\right]\right]$$

$$\cancel{\pi\delta\left(\omega-\frac{\pi}{2}\right)} + \pi\delta\left(\omega+\frac{\pi}{2}\right) - \cancel{\pi\delta\left(\omega-\frac{\pi}{2}\right)} + \pi\delta\left(\omega+\frac{\pi}{2}\right)$$

$$x(t) = 2\pi\delta\left(\omega+\frac{\pi}{2}\right) \xleftrightarrow{FT} e^{j\omega_0 t}$$

$\rightarrow \underline{x(t) = e^{-j\frac{\pi}{2}t}}$ <u>Mathematical Description</u>

<u>Pictorial Description</u>



Q5 DTFT $X(e^{j\omega})$

Plug into:

$x[n] = \left(\frac{1}{3}\right)^n u[n]$ $X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$

$\sum_{-\infty}^{\infty}\left(\frac{1}{3}\right)^n u[n]e^{-j\omega n} \rightarrow \sum_{0}^{\infty}\left(\frac{1}{3}\right)^n e^{-j\omega n}$

$$\boxed{X(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}} \quad = \sum_{0}^{\infty}\left(\frac{1}{3}e^{-j\omega}\right)^n$$

$$= \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$