Homework 1

You may either submit this page with your homework or copy the information in this box to your handwritten solutions.

An Aggie does not lie, cheat or steal or tolerate those who do.

Please acknowledge your adherence to the honor code by printing and signing your name below.

This work is the product of my effort and was completed either through my sole effort or with collaborations as permitted by the syllabus. All collaboration, team effort, and sources are disclosed below. I have complied with the requirements of this homework to the best of my knowledge.

JOAQUIN SALAS

Printed Name **Signature**

Disclosure of Sources and Collaborators:

PART 1: Individual Homework (100 Points)

1. **10 Points** In the cartesian coordinate system, a line is described by the equation $x + 2y = 4$. A vector $\bar{A}$ starts at the origin and ends at point S on the line such that $\bar{A}$ is perpendicular to the line.
   a. Sketch the geometry described. Label the intercepts on the x and y axis. (**3 Points**)
   b. If the x coordinate of S is a, use the equation of the line to find the y coordinate. (**1 Point**)
   c. Find a vector $\bar{B}$ that connects the intercept points between line $x + 2y = 4$ and the coordinate axes. (**2.5 Points**)
   d. What is $\bar{A}.\bar{B}$? (**1 Points**)
   e. Find an expression for $\bar{A}$. (2.5 **Points**)

2. **5 Points** Use arrows to sketch the fields. For full credit, label the axes.
   a. $\bar{E} = -ye_x$

3. **15 Points** Three vectors are defined as $\bar{P} = e_x + 2e_y - 3e_z$, $\bar{R} = 2e_x - 4e_y$, $\bar{S} = 2e_y - 4e_z$. Calculate the following:
   a. $\bar{P}.(\bar{R} \times \bar{S})$ (**5 Points**)
   b. $\bar{P} \times (\bar{R} \times \bar{S})$ (**5 Points**)
   c. The component of $\bar{R}$ along $\bar{S}$. (**5 Points**)

4. **5 Points** Evaluate the following (1 point each):
   a. $e_x \cdot e_z$
   b. $e_x \times e_z$
   c. $e_\rho \times e_\rho$
   d. $e_r \times e_\Phi$

e. $e_\phi \times e_\theta$

5. **20 Points** Sketch and calculate the area of the geometries described below (**10 points each –** 3 points for the sketch and 7 for calculating area):
   a. Cylindrical coordinate system: $\rho = 3$, $0 \le \phi \le \pi/3$, $-2 \le z \le 2$
   b. Spherical coordinate system: $0 \le r \le 5$, $\theta = \pi/3$, $0 \le \phi \le 2\pi$

6. **10 Points** Transform the vector $\bar{A} = (x + y)e_x$ into cylindrical coordinates and evaluate it at the point $P_1 = (x = 1, y = 2, z = 3)$. You may use the following steps to do so –
   a. Refer to *Table 2: Relations Between Unit Vectors of Coordinate Systems from Cylindrical to Cartesian* (from the notes) and write the expression for e_x in terms of the unit vectors in the cylindrical coordinates.
   b. Next, refer to *Table 1: Relation Between Coordinates from Cylindrical to Cartesian* and write the expression for x and y in cylindrical coordinates.
   c. Substitute x, y, e_x in $\bar{A}$ with the answers from above and group terms according to unit vectors. (**5 Points**)
   d. Refer to *Table 1: Relation Between Coordinates from Cartesian to Cylindrical* to express P_1 in terms of (ρ, ϕ, z). (**2 Points**)
   e. To evaluate $\bar{A}$ at P_1, substitute the answer in (c) with values from (d). (**3 Points**)

7. **20 Points** Transform the vector $\bar{A} = y^2 e_x + xze_y + 4e_z$ into spherical coordinates and evaluate it at the point $P_1 = (x = 1, y = -1, z = 2)$.

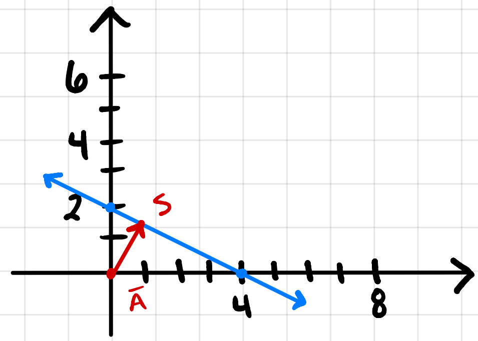
8. **15 Points** Using the MATLAB tutorial (Modules -> MATLAB), download the software on your computer or access the software using VOAL. Avoid using the online version of MATLAB. Familiarize yourself with MATLAB using the same tutorial file.
   You will now learn to define vectors in MATLAB and perform vector multiplication.
   a. To define two vectors $\bar{A} = e_x + e_y + e_z$, and $\bar{B} = e_x - e_z$, type the following in the MATLAB editor.
      ```
      A = [1,1,1];
      B =[1,0,-1];
      ```
      Save the script and run. Vectors are defined in MATLAB as arrays where each element is the coefficient of the corresponding vector component. The order of the elements is important and reflects the order of the unit vectors as defined by right hand rule in the coordinate system. Since $\bar{B}$ does not have a y component, the corresponding element is entered as 0 in MATLAB.
   b. To find the sum of the vectors, type
      ```
      C = A+B;
      ```
      Save the script and run. To display the answer type, C in the command window and hit enter.
   c. Submit a screenshot of your code (script) as well as the sum displayed in the command window. (**2.5 Points**)
   d. Now define another vector $\bar{D} = -e_x + 2e_y + e_z$ and find $\bar{D} + \bar{C}$. Submit a screenshot of your code (script) as well $\bar{D} + \bar{C}$ displayed in the command window. (**2.5 Points**)
   e. MATLAB has an extensive help section which is useful for finding functions and their definitions. You may search for functions by typing key words into the search bar to the top right of the MATLAB window. Alternatively, you may use the command window as discussed below.

f. Type `help dot` in the command window, hit enter and read the description.
g. Next, type `help cross` in the command window, hit enter and read the description.
h. The above two functions calculate the dot and cross products of vectors. Notice that the function can take only two inputs at a time and each input must a column vector (i.e. one row and elements placed in columns like $\bar{A}$ and $\bar{B}$ defined above).
i. Use MATLAB to calculate the solutions to the first two parts of Problem 3 in this homework. You must submit a screenshot of your code as well as the answer displayed in the command window.
    i. Code and solution to $\bar{P}.(\bar{R} \times \bar{S})$ (**5 Points**)
    ii. Code and solution to $\bar{P} \times (\bar{R} \times \bar{S})$ (**5 Points**)

PART 2: Team Homework (0 Points)

This homework assignment does not have a team component.

1. **10 Points** In the cartesian coordinate system, a line is described by the equation $x + 2y = 4$. A vector $\bar{A}$ starts at the origin and ends at point S on the line such that $\bar{A}$ is perpendicular to the line.
   a. Sketch the geometry described. Label the intercepts on the x and y axis. (**3 Points**)
   b. If the x coordinate of S is a, use the equation of the line to find the y coordinate. (**1 Point**)
   c. Find a vector $\bar{B}$ that connects the intercept points between line $x + 2y = 4$ and the coordinate axes. (**2.5 Points**)
   d. What is $\bar{A}.\bar{B}$? (**1 Points**)
   e. Find an expression for $\bar{A}$. (2.5 **Points**)



a. $x + 2y = 4 \rightarrow y = \frac{4-x}{2} \rightarrow y = 2 - \frac{x}{2}$

$y = -\frac{1}{2}x + 2$

· Negative Slope

y-int: $0 = 2 - \frac{x}{2} \rightarrow 2 \cdot \frac{x}{2} = 2$

$x = 4$

$(4,0)$

x-int: $y = 2 - \cancel{0}$

$y = 2$, $(0,2)$

b. $y = -\frac{1}{2}x + 2 \rightarrow y = -\frac{1}{2}a + 2 \leftarrow$ y-coordinate

$x = a$

So, $\left(a, -\frac{1}{2}a + 2\right)$

c. $\vec{B}$ Starts at the x-intercept AND ENDS AT Y-INTERCEPT.

$\vec{B} = [4-0, 0-2] = [4, -2] = 4\hat{x} - 2\hat{y}$

d. $\bar{A} \cdot \bar{B} = 0$. FROM NOTES.

e. $m_1 = -1/2$ $\quad m_2 = \frac{-\frac{1}{2}a + 2}{a} \leftarrow$ Slope

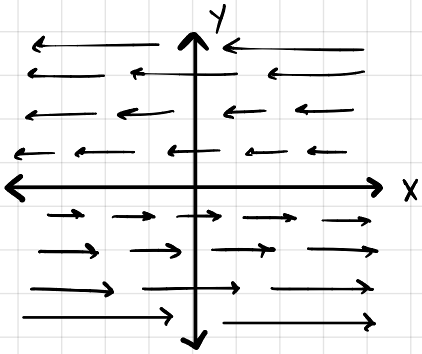
$\left(-\frac{1}{2}\right)\left(\frac{-\frac{1}{2}a + 2}{a}\right) = -1 \rightarrow 2\left(-\frac{1}{2}a + 2\right) = (2a)^{-2} =$

$-a - 4 = 4a$
$+a \qquad +a$
$-4 = -5a$
$a = 4/5$

$\frac{-\frac{1}{2}\left(\frac{4}{5}\right) + 2}{4/5} = \frac{8}{5}$

$\bar{A} = \frac{4}{5}\hat{x} + \frac{8}{5}\hat{y}$

2. **5 Points** Use arrows to sketch the fields. For full credit, label the axes.
   a. $\bar{E} = -ye_x$



· When the y-COORDINATE IS POSITIVE, THE $\vec{E}$ will be

ex. $-1e_x$ or $-2e_x$

· When y-COORDINATE IS NEGATIVE, $\vec{E}$ WILL BE

Example. $-(-1)e_x = 1e_x$, so the field will drift in the $+x$ direction.

3. **15 Points** Three vectors are defined as $\bar{P} = e_x + 2e_y - 3e_z$, $\bar{R} = 2e_x - 4e_y$, $\bar{S} = 2e_y - 4e_z$. Calculate the following:

a. $\bar{P}.(\bar{R} \times \bar{S})$ (**5 Points**)
b. $\bar{P} \times (\bar{R} \times \bar{S})$ (**5 Points**)
c. The component of $\bar{R}$ along $\bar{S}$. (**5 Points**)

a. $\vec{P}\cdot(\vec{R}\times\vec{S})$

1) $\vec{R}\times\vec{S}$ $\quad \det\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -4 & 0 \\ 0 & 2 & -4 \end{vmatrix} = [16-0\,\hat{\imath}] - [-8\hat{\jmath}] + [4\hat{k}]$

$= 16\hat{\imath} + 8\hat{\jmath} + 4\hat{k}$

2) $\vec{P}\cdot(\vec{R}\times\vec{S})$

$(e_x + 2e_y - 3e_z)\cdot(16e_x + 8e_y + 4e_z) = 16 + 16 - 12 = 20$

b. $\vec{P}\times(\vec{R}\times\vec{S})$

$[e_x + 2e_y - 3e_z]\times[16e_x + 8e_y + 4e_z] \Rightarrow \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 1 & 2 & -3 \\ 16 & 8 & 4 \end{vmatrix} =$

$[8-(-24)]\,\hat{e}_x - [4-(-48)]\,\hat{e}_y + [8-32]\,\hat{e}_z$

$= 32\hat{e}_x - 52\hat{e}_y - 24\hat{e}_z$

c. $\text{Proj}_{\bar{R}}\vec{S}$

Proj of $\vec{S} = |\bar{S}|\cos\varphi = \dfrac{\bar{S}\cdot\bar{R}}{|\bar{R}|}$ $\rightarrow \bar{S}\cdot\bar{R} = [2\hat{\jmath} - 4\hat{k}]\cdot[2\hat{\imath} - 4\hat{\jmath}]$

$(\cancel{2\cdot 0}) + (2\cdot -4) + (\cancel{-4\cdot 0})$

$= -8$

$= -\frac{2}{5}[2e_x - 4e_y]$ $\quad \rightarrow |\bar{R}| = 2e_x - 4e_y$

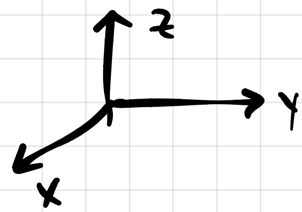
$= \sqrt{2^2 + (-4)^2 + 0^2}$

$= -\frac{4}{5}\hat{e}_x + \frac{8}{5}\hat{e}_y$ $\quad = \underline{4.47}^2 = 20$

$= \dfrac{-8}{20}$

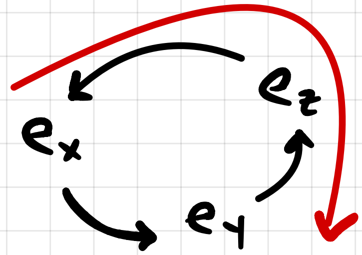
4. **5 Points** Evaluate the following (1 point each):

a. $e_x \,.\, e_z$
b. $e_x \times e_z$
c. $e_\rho \times e_\rho$
d. $e_r \times e_\Phi$

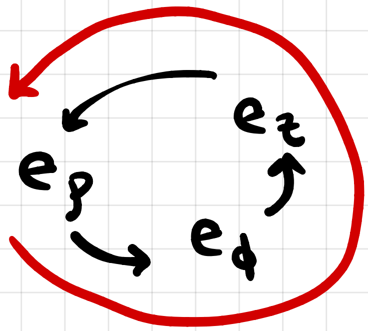


a. $e_x \cdot e_z = 0$

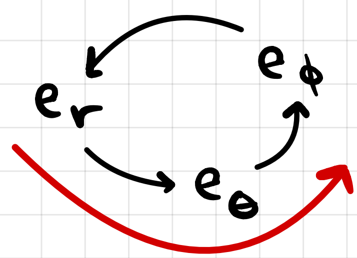
b. $e_x \times e_z = -e_y$



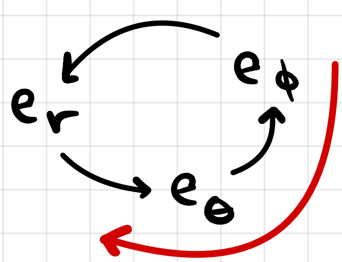
c. $e_\rho \times e_\rho = 0$



d. $e_r \times e_\theta = e_\phi$

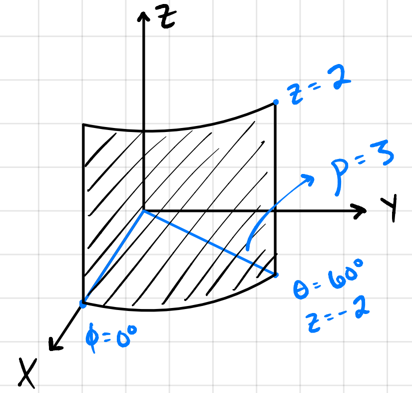


e. $e_\phi \times e_\theta = -e_r$



5. **20 Points** Sketch and calculate the area of the geometries described below (**10 points each** – 3 points for the sketch and 7 for calculating area):
   a. Cylindrical coordinate system: $\rho = 3$, $0 \le \phi \le \pi/3$, $-2 \le z \le 2$
   b. Spherical coordinate system: $0 \le r \le 5$, $\theta = \pi/3$, $0 \le \phi \le 2\pi$

a. $\rho = 3$, $0 \le \phi \le \frac{\pi}{3}$, $-2 \le z < 2$



AREA

$$d\bar{S}_\rho = \rho\, d\phi\, dt\, \hat{e}_\rho$$

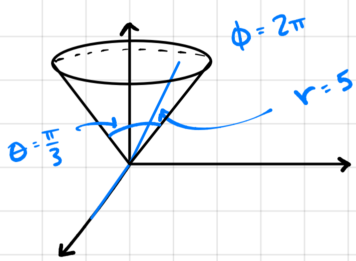
$$\rightarrow |d\bar{S}_\rho| = \rho\, d\phi\, dz$$

$$S = \int_0^{\pi/3}\int_{-2}^{2} \rho\, d\phi\, dz = \rho \int_0^{\pi/3} d\phi \int_{-2}^{2} dz$$

$$= 3\left[\phi\right]_0^{\pi/3} \cdot \left[z\right]_{-2}^{2}$$

$$= 3 \times \left[\pi/3 - 0\right] \times \left[2 - (-2)\right] = \cancel{3} \times \frac{\pi}{\cancel{3}} \times 4 = 4\pi\, [m^2]\, \hat{e}_\rho$$

b. COORDINATES: (r, θ, ϕ)



AREA $d\bar{S}_r = r^2 \sin\theta\, d\theta\, d\phi\, \hat{e}_r$

$$= r^2 \int_0^{\pi/3} \sin\theta\, d\theta \int_0^{2\pi} d\phi\ \hat{e}_r$$

$$= 25\left[-\cos\theta\right]_0^{\pi/3} \times \left[\phi\right]_0^{2\pi}\ \hat{e}_r$$

$$= \frac{25}{\cancel{2}} \times \cancel{2}\pi = 25\pi\, [m^2]\, \hat{e}_r$$

6. **10 Points** Transform the vector $\bar{A} = (x+y)e_x$ into cylindrical coordinates and evaluate it at the point $P_1 = (x=1, y=2, z=3)$. You may use the following steps to do so –

a. Refer to *Table 2: Relations Between Unit Vectors of Coordinate Systems from Cylindrical to Cartesian* (from the notes) and write the expression for e_x in terms of the unit vectors in the cylindrical coordinates.

b. Next, refer to *Table 1: Relation Between Coordinates from Cylindrical to Cartesian* and write the expression for x and y in cylindrical coordinates.

c. Substitute x, y, e_x in $\bar{A}$ with the answers from above and group terms according to unit vectors. (**5 Points**)

d. Refer to *Table 1: Relation Between Coordinates from Cartesian to Cylindrical* to express P_1 in terms of (ρ, ϕ, z). (**2 Points**)

e. To evaluate $\bar{A}$ at P_1, substitute the answer in (c) with values from (d). (**3 Points**)

a. $\bar{A} = (x+y)\hat{e}_x$ CARTESIAN → CYLINDRICAL.

$$\hat{e}_x = \hat{e}_\rho \cos\phi - \hat{e}_\phi \sin\phi$$

b. $(x+y)$ → CYLINDRICAL.

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$$\rightarrow [(\rho\cos\phi) + (\rho\sin\phi)]$$

c. $[(\rho\cos\phi) + (\rho\sin\phi)]\ \hat{e}_\rho\cos\phi - \hat{e}_\phi\sin\phi$

$= \rho\cos\phi\,\hat{e}_\rho\cos\phi - \rho\cos\phi\,\hat{e}_\phi\sin\phi + \rho\sin\phi\,\hat{e}_\rho\cos\phi - \rho\sin\phi\,\hat{e}_\phi\sin\phi$

$= \hat{e}_\rho[\rho\cos\phi\cos\phi + \rho\sin\phi\cos\phi] - \hat{e}_\phi[\rho\cos\phi\sin\phi + \rho\sin\phi\sin\phi]$

$= \rho\hat{e}_\rho[\cos^2\phi + \sin\phi\cos\phi] - \rho\hat{e}_\phi[\cos\phi\sin\phi + \sin^2\phi]$

d. $P_1 = \overset{x,y,z}{(1,2,3)}$

$\rho = \sqrt{1^2 + 2^2} = 2.24$

$\phi = \arctan\left(\frac{2}{1}\right) = 1.107$ rad

$z = 3$

$\Rightarrow P_1 = (2.24, 1.107, 3)$

e. $= \rho\hat{e}_\rho[\cos^2\phi + \sin\phi\cos\phi] - \rho\hat{e}_\phi[\cos\phi\sin\phi + \sin^2\phi]$

$= 2.24\,\hat{e}_\rho[\cos^2(1.107) + \sin(1.107)\cos(1.107)] - 2.24\,\hat{e}_\phi[\cos(1.107)\sin(1.107) + \sin^2(1.107)]$

$\bar{A} = 1.34\,\hat{e}_\rho - 2.69\,\hat{e}_\phi$

7. **20 Points** Transform the vector $\bar{A} = y^2 e_x + xze_y + 4e_z$ into spherical coordinates and evaluate it at the point $P_1 = (x = 1, y = -1, z = 2)$.

STEP 1 CARTESIAN $\rightarrow$ SPHERICAL

$$\hat{e}_x = \hat{e}_r \sin\theta\cos\phi + \hat{e}_\theta \cos\theta\cos\phi - \hat{e}_\phi \sin\phi$$
$$\hat{e}_y = \hat{e}_r \sin\theta\sin\phi + \hat{e}_\theta \cos\theta\sin\phi + \hat{e}_\phi \cos\phi$$
$$\hat{e}_z = \hat{e}_r \cos\theta - \hat{e}_\theta \sin\theta$$

STEP 2

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi \rightarrow y^2 = (r\sin\theta\sin\phi)^2 = r^2\sin^2\theta\sin^2\phi$$
$$z = r\cos\theta$$

STEP 3

$P_1(1,-1,2)$

$$r = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}$$
$$\theta = \arccos\frac{z}{\sqrt{6}} \rightarrow \operatorname{Arccos}\left(\frac{2}{\sqrt{6}}\right) = 0.615$$
$$\phi = \operatorname{Arctan}\left(\frac{y}{x}\right) \rightarrow \operatorname{Arctan}\left(\frac{-1}{1}\right) = -0.79$$

$$r^2\sin^2\theta\sin^2\phi\left[\hat{e}_r\sin\theta\cos\phi + \hat{e}_\theta\cos\theta\cos\phi - \hat{e}_\phi\sin\phi\right] + \left[r\sin\theta\cos\phi\right]\left[r\cos\theta\right]\hat{e}_r\sin\theta\sin\phi + \hat{e}_\theta\cos\theta\sin\phi + \hat{e}_\phi\cos\phi +$$
$$4\left[\hat{e}_r\cos\theta - \hat{e}_\theta\sin\theta\right]$$

$$= \hat{e}_r\sin\theta\cos\phi\, r^2\sin^2\theta\sin^2\phi + \hat{e}_\theta\cos\theta\cos\phi\, r^2\sin^2\theta\sin^2\phi - \hat{e}_\phi\sin\phi\, r^2\sin^2\theta\sin^2\phi$$
$$= .409\hat{e}_r + .579\hat{e}_\theta + .716\hat{e}_\phi$$

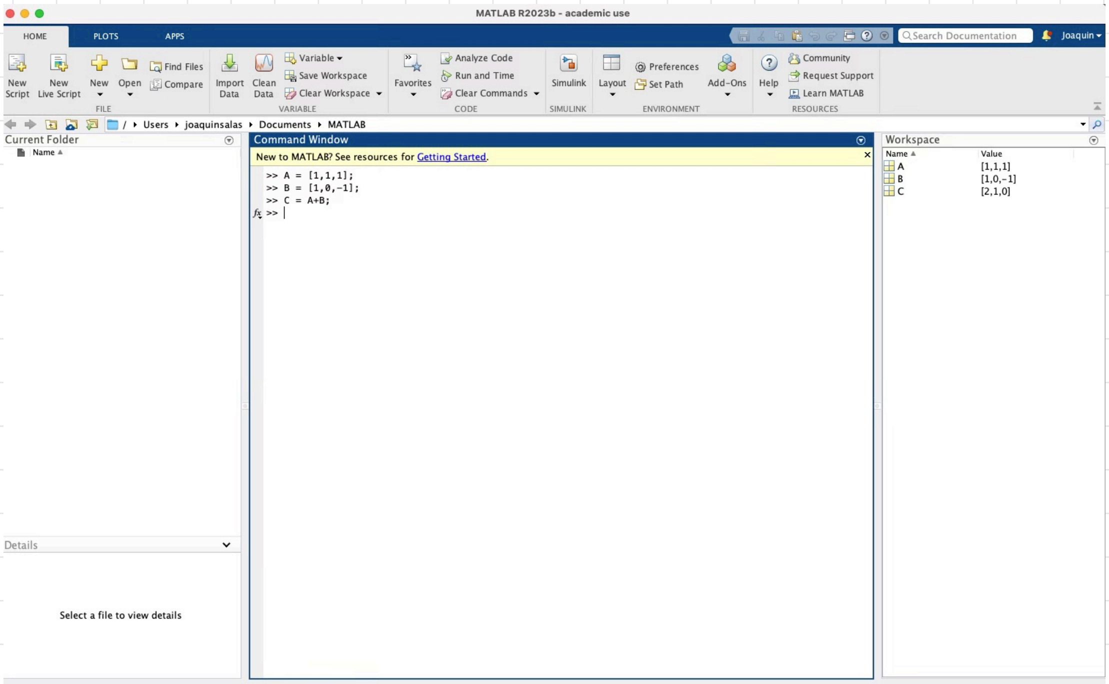
$$= \left[\left[r\sin\theta\cos\phi\right]\left[r\cos\theta\right]\right]\hat{e}_r\sin\theta\sin\phi + \hat{e}_\theta\cos\theta\sin\phi + \hat{e}_\phi\cos\phi$$
$$r^2\sin\theta\cos\phi\cos\theta\left[\hat{e}_r\sin\theta\sin\phi + \hat{e}_\theta\cos\theta\sin\phi + \hat{e}_\phi\cos\phi\right]$$

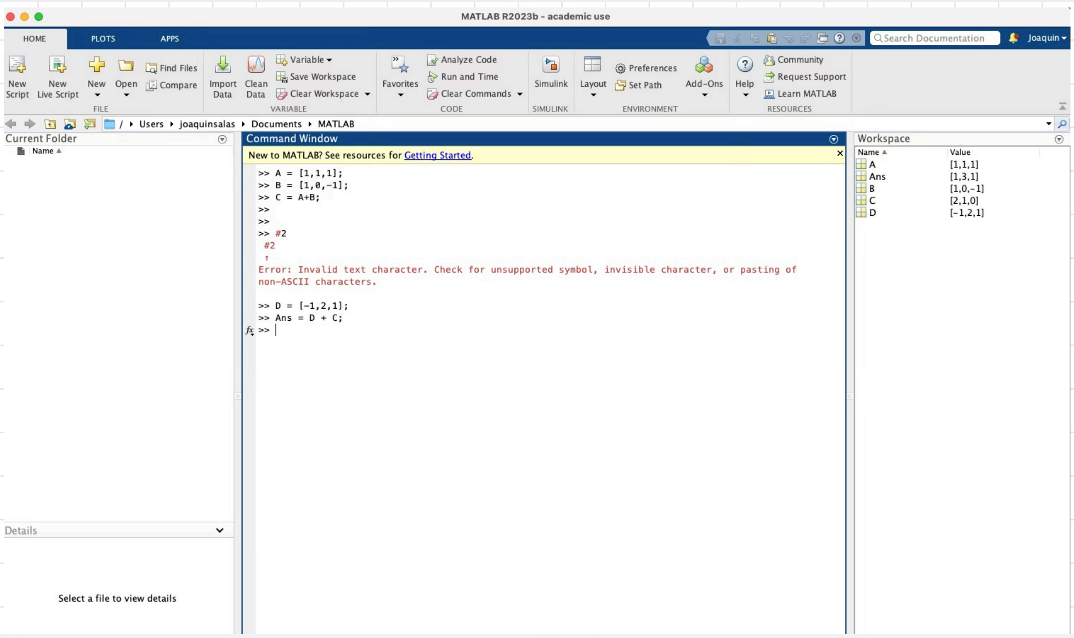
$$\rightarrow \hat{e}_r\sin\theta\sin\phi\, r^2\sin\theta\cos\phi\cos\theta + \hat{e}_\theta\cos\theta\sin\phi\, r^2\sin\theta\cos\phi\cos\theta + \hat{e}_\phi\cos\phi\, r^2\sin\theta\cos\phi\cos\theta$$
$$= -.816\hat{e}_r - 1.15\hat{e}_\theta + 1.4\hat{e}_\phi$$

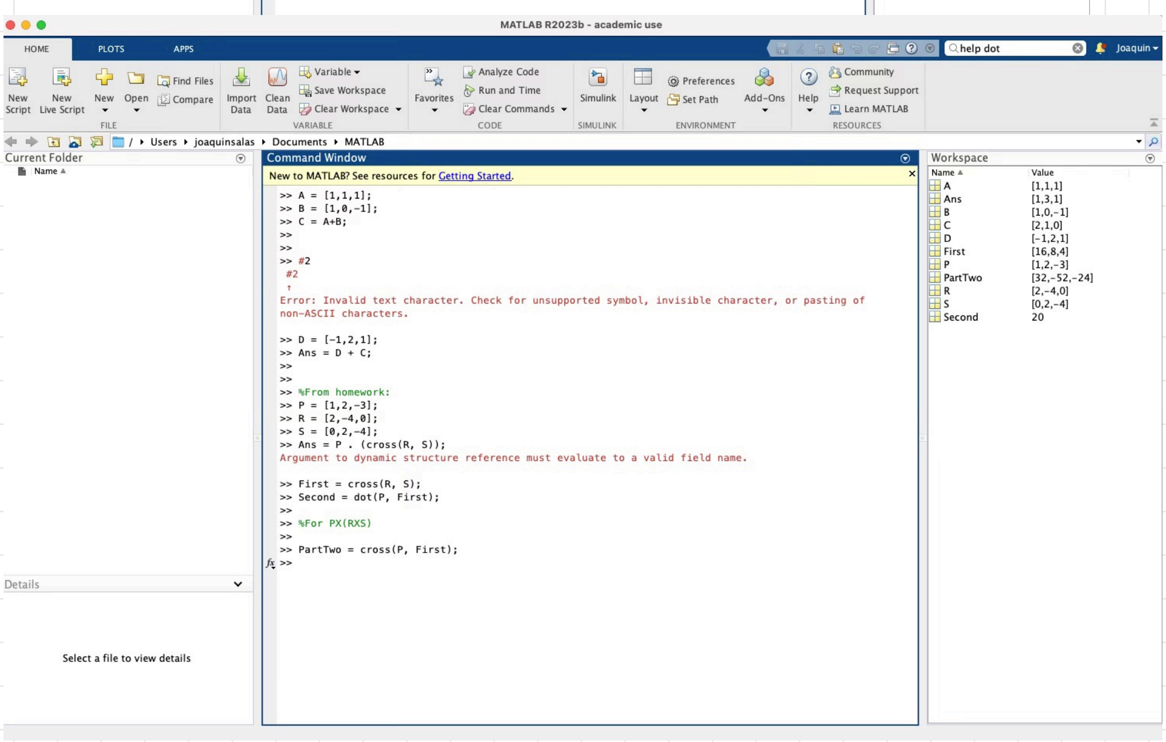
$$\rightarrow 4\left[\hat{e}_r\cos\theta - \hat{e}_\theta\sin\theta\right] = 4\hat{e}_r\cos\theta - 4\hat{e}_\theta\sin\theta$$
$$= 3.27\hat{e}_r - 2.3\hat{e}_\theta$$

$$\bar{A} = 2.86\hat{e}_r - 2.87\hat{e}_\theta + 2.12\hat{e}_\phi$$

8.


MATLAB R2023b - academic use
Command Window
New to MATLAB? See resources for Getting Started.
>> A = [1,1,1];
>> B = [1,0,-1];
>> C = A+B;
>>
Workspace
Name Value
A [1,1,1]
B [1,0,-1]
C [2,1,0]
Select a file to view details


MATLAB R2023b - academic use
Command Window
New to MATLAB? See resources for Getting Started.
>> A = [1,1,1];
>> B = [1,0,-1];
>> C = A+B;
>>
>>
>> #2
#2
↑
Error: Invalid text character. Check for unsupported symbol, invisible character, or pasting of non-ASCII characters.
>> D = [-1,2,1];
>> Ans = D + C;
>>
Workspace
Name Value
A [1,1,1]
Ans [1,3,1]
B [1,0,-1]
C [2,1,0]
D [-1,2,1]
Select a file to view details


MATLAB R2023b - academic use
help dot
Command Window
New to MATLAB? See resources for Getting Started.
>> A = [1,1,1];
>> B = [1,0,-1];
>> C = A+B;
>>
>>
>> #2
#2
↑
Error: Invalid text character. Check for unsupported symbol, invisible character, or pasting of non-ASCII characters.
>> D = [-1,2,1];
>> Ans = D + C;
>>
>>
>> %From homework:
>> P = [1,2,-3];
>> R = [2,-4,0];
>> S = [0,2,-4];
>> Ans = P . (cross(R, S));
Argument to dynamic structure reference must evaluate to a valid field name.
>> First = cross(R, S);
>> Second = dot(P, First);
>>
>> %For PX(RXS)
>>
>> PartTwo = cross(P, First);
>>
Workspace
Name Value
A [1,1,1]
Ans [1,3,1]
B [1,0,-1]
C [2,1,0]
D [-1,2,1]
First [16,8,4]
P [1,2,-3]
PartTwo [32,-52,-24]
R [2,-4,0]
S [0,2,-4]
Second 20
Select a file to view details