ECEN-314 { Homework 2

① Evaluate, in Cartesian Form

$$\int_0^6 e^{\frac{j\pi t}{2}}\,dt = e^{\frac{j\pi t}{2}} \Big/ \frac{\pi j}{2}\Bigg|_0^6 = e^{\frac{j\pi t}{2}} \cdot \frac{2}{\pi j}\Bigg|_0^6 = \left(e^{\frac{j\pi 6}{2}} \cdot \frac{2}{\pi j}\right) - \left(\frac{2}{\pi j}\right)$$

$$= \frac{2e^{3j\pi}}{\pi j} - \frac{2}{\pi j}$$

Put into Cartesian form.

$* \; e^{3j\pi} = \cos(3\pi) + j\sin(3\pi)$

$\rightarrow \frac{2}{\pi j}\left(e^{3j\pi} - 1\right)$

$\rightarrow \frac{2}{\pi j}\left(\cos(3\pi) + j\sin(3\pi) - 1\right)$

$\rightarrow \frac{2}{\pi j}(-2) = \frac{-4}{\pi j} = \frac{4j}{\pi}$

Answer

$\frac{-4}{\pi j}$ or $\frac{4j}{\pi}$

② Determine P_∞ and E_∞

(b) $x_2(t) = e^{j(2t + \frac{\pi}{4})}$

$P_\infty \rightarrow |x(t)| = \left|e^{j(2t+\frac{\pi}{4})}\right| = 1 \rightarrow P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 1\,dt$

$= \lim_{T\to\infty}\left(\frac{1}{2T}\right)(2T) = \boxed{1}$

$E_\infty \rightarrow \int_{-\infty}^{\infty} |x(t)|^2\,dt$

$\rightarrow \int_{-\infty}^{\infty} 1\,dt = T\Big|_{-\infty}^{\infty} = \boxed{\infty}$

(f) $x_3[n] = \cos\left(\frac{\pi}{4}n\right) \rightarrow |x_3[n]|^2 = \left|\cos\left(\frac{\pi}{4}n\right)\right|^2 \rightarrow \cos^2\left(\frac{\pi}{4}n\right)$

$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4}n\right)$ IDENTITY $\cos^2\theta = \frac{1+\cos(\theta)}{2}$

$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1+\cos\left(\frac{\pi}{4}n\right)}{2}$ Evaluate

$P_\infty = \lim_{N\to\infty} \frac{1}{4N+2}\left[\sum_{n=-N}^{N} 1 + \cancel{\cos\left(\frac{\pi}{4}n\right)}^{\,0}\right] = \lim_{N\to\infty} \frac{2N}{4N+2} = \lim_{N\to\infty}\left(\frac{2}{4}\right) = \boxed{\frac{1}{2}}$

$E_\infty = \sum_{-\infty}^{\infty} |x_3[n]|^2$

$\rightarrow = \sum_{-\infty}^{\infty} \frac{1+\cos\left(\frac{\pi}{2}n\right)}{2} = \boxed{\infty}$

③ Determine value of n for which its garunteed to be 0.

(a) $x[n-3]$ $\quad$ $x[n] \neq 0$ for $n < -2$; $n > 4$

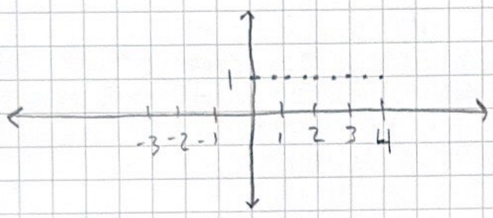
$N_0 = 3$ $\quad$ $n < -2 + 3 = \underline{1}$, $\quad$ $n > 4 + 3 = 7$

$\boxed{n = 0 \text{ at } n < 1 \text{ ; } n > 7}$

(d) $x[-n-2]$ $\quad$ $N_0 = -2$, $\quad$ $n < -2 - 2 = \underline{-4}$, $n > 4 - 2 = \underline{2}$

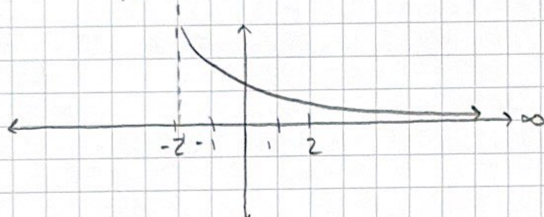
$\boxed{n = 0 \text{ when } n < -4 \text{ ; } n > 2}$

④ (a) $x_1[n] = u[n] - u[n-4]$



@ $n > 4$ the signal will be $\underline{0}$ garunteed.

(d) $x_4[t] = e^{-5t} u[t+2]$



Signal Garunteed to be 0 @ $t < -2$ AND $t > \infty$.

⑤ Determine if Signal Periodic

(a) $x_1(t) = je^{j10t} = je^{j10t} \cdot je^{2\pi j} = je^{2j(5t+\pi)} = x(5t+\pi)$

$\boxed{T_0 = \frac{\pi}{5}}$

(b) $x_2(t) = e^{(-1+j)t}$ $\quad$ it is a decaying exponential signal

decaying
$a = -1 + j < 0$ AND $\underline{\text{NOT PERIODIC}}$

(e) $x_5[n] = 3e^{j\frac{3}{5}(n+\frac{1}{2})}$ $\quad$ $\underline{\text{NOT PERIODIC}}$

⑥ $x[n] = \underbrace{1}_{1} + \underbrace{e^{j 4\pi \frac{n}{7}}}_{b} - \underbrace{e^{j 2\pi \frac{n}{5}}}_{c}$

(a) Period = $\boxed{1}$

(b) Period of $e^{j\frac{4\pi n}{7}} \longrightarrow \frac{4\pi n}{7} \rightarrow N = \frac{\cancel{14\pi m}}{\cancel{2\pi}} = \frac{7m}{2}$

$\boxed{\begin{matrix} N_0 = 7 \\ m = 2 \end{matrix}}$

(c) $e^{j\frac{2\pi n}{5}}$ $\quad \frac{2\pi n}{5} = 2\pi m$

$N = \frac{\cancel{10\pi m}}{\cancel{2\pi}} = 5m$ $\quad \boxed{\begin{matrix} N_0 = 5 \\ m = 1 \end{matrix}}$

Fundamental Period of Signal is LCM
of 1, 7 & 5 $\quad$ <u>35</u>