PROBLEM 1

- AMPERE'S LAW RELATES MAGNETIC FIELDS TO THE CURRENTS THAT PRODUCE THEM.
- BOTH AC CURRENT AND DC CURRENT THROUGH A COIL OF WIRE CAN CAUSE A VOLTAGE TO BE INDUCED IN THE COIL. FALSE (ONLY AC)
- FOR A COUPLED MAGNETIC SYSTEM WITH TWO COILS, THE MUTUAL INDUCTANCE M CANNOT BE LARGER THAN $\sqrt{L_1L_2}$ WHERE L_1, L_2 ARE THE SELF INDUCTANCES OF THE TWO COILS. TRUE ($0 \leq k \leq 1$)
- BOTH AC AND DC POWER CAN BE TRANSFERRED THROUGH A POWER TRANSFORMER. FALSE (ONLY AC)
- FOR THE DC MOTOR SYSTEM IN THE FIGURE BELOW, THE COIL SHOULD ROTATE ANTI-CLOCKWISE (IN THE FRONT VIEW) UNDER THE MAGNETIC FIELD. TRUE (LEFT FORCE DOWN, RIGHT FORCE UP)

PROBLEM 2

Magnetic Field Magnitude

how strong or weak a magnetic field is

Magnetic Field Intensity H (unit: A/m) or **Magnetic Flux Density B** (unit: T)

- Consider ideal linear magnetic material:

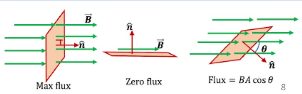
$$B = \mu H$$

where μ is the **permeability** of the material.

- Define **relative permeability** $\mu_r = \dfrac{\mu}{\mu_0}$ $\quad \mu_0 = 4\pi \times 10^{-7} H/m, \quad \mu_r \approx 1$ for air

Magnetic Flux ϕ: measures the total magnetic field passing through a surface A

$$\phi = \int_S \boldsymbol{B} \cdot \boldsymbol{n}\, da \qquad \phi = BAcos\theta \quad [Wb]$$



Procedure for Solving Magnetic Circuits

- Step 1: Establish the **direction** of the coil mmf using **right-hand rule**
- Step 2: Draw equivalent magnetic circuit
- Step 3: Find reluctances
  - $\mathcal{R} = \dfrac{l}{\mu A}$, $A = w \cdot t$

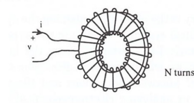
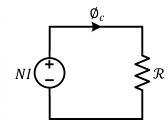


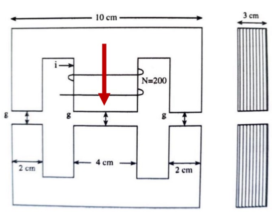
Figure 3.1: Toroid



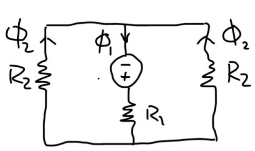
- Step 4: Solve the magnetic circuits
  - $KCL + KVL + Magnetic\ Ohm's\ Law\ \ Ni = \mathcal{R}\phi$
- Step 5: Once flux (∅) is calculated, you can find B and H
  - $\phi = BA \rightarrow B = \dfrac{\phi}{A}$
  - $B = \mu H \rightarrow H = \dfrac{B}{\mu}$

B: magnetic flux density
H: magnetic field intensity
ϕ: magnetic flux

1. In the Magnetic System shown as the figure below, the current i=10A and g=0.25cm. Depth is 3cm. Neglect fringing. μ=∞ in iron.
a) (10 points) Draw the equivalent magnetic circuit.
b) (10 points) Find the flux φ in the centre leg.
c) (10 points) Find the flux density B in each air gap of the outer legs.



(a) Solution:



$$\mathcal{R}_1 = \frac{g}{\mu A_1} = \frac{0.25 \times 10^{-2}}{4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 1.65 \times 10^6 AT/Wb$$

$$\mathcal{R}_2 = \frac{g}{\mu A_2} = \frac{0.25 \times 10^{-2}}{4\pi \times 10^{-7} \times 6 \times 10^{-4}} = 3.31 \times 10^6 AT/Wb$$

(b) Solution:

$$\phi_1 = \frac{Ni}{R_1 + \frac{R_2}{2}} = \frac{2000}{3.31 \times 10^6} = 0.602 \times 10^{-3} Wb$$

(c) $Outer\ leg{:}\ \phi_2 = \dfrac{\phi_1}{2}$ $\quad B_{out} = \dfrac{\phi_2}{A} = 0.5026T$

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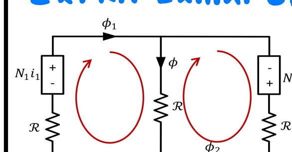
EXTRA EXAMPLE



Reluctance: $\mathcal{R} = 2 \times \frac{l_g}{\mu_0 A} = 9.95 \times 10^6$ At/Wb

Loop Equations:

$$N_1 i_1 = \mathcal{R}\phi_1 + \mathcal{R}(\phi_1 - \phi_2) = 2\mathcal{R}\phi_1 - \mathcal{R}\phi_2$$
$$N_2 i_2 = \mathcal{R}\phi_2 + \mathcal{R}(\phi_2 - \phi_1) = -\mathcal{R}\phi_1 + 2\mathcal{R}\phi_2$$

$$\phi_1 = \frac{(2N_1 + N_2)i}{3\mathcal{R}} = 1.51 \times 10^{-4} \text{ Wb}$$

$$\phi_2 = \frac{(N_1 + 2N_2)i}{3\mathcal{R}} = 1.01 \times 10^{-4} \text{ Wb}$$

$$\phi = \phi_1 - \phi_2 = 5.03 \times 10^{-5} \text{ Wb}$$

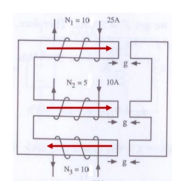
c) Flux Intensity: $B = \phi/_A = 0.0629$,

Magnetic Field Intensity:

$$H = \frac{B}{\mu_0} = 5.0034 \times 10^4$$

Example 3: Magnetic Circuits

For the magnetic circuit shown in the figure: length of the airgap g = 0.1cm and the cross sectional area A = 4 cm². Assume $\mu_r = \infty$ for iron core. Find the flux density B in each airgap.



Step 1 – Establish the direction of the coil mmf (right hand rule)

Step 2 – Draw the equivalent circuit

Step 3 – Solve the magnetic circuit using the principles of electric circuits

Assume $\mu_r = \infty$ for iron core

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A} = 0$$

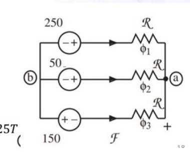
KCL: $\phi_1 + \phi_2 + \phi_3 = 0$

$$\frac{250 - F_{ab}}{\mathcal{R}} + \frac{50 - F_{ab}}{\mathcal{R}} + \frac{-150 - F_{ab}}{\mathcal{R}} = 0 \qquad F_{ab} = 50$$

$$\mathcal{R} = \frac{g}{A\ \mu_0} = 2 \times 10^6 \quad \phi_1 = \frac{250 - F_{ab}}{\mathcal{R}} = 1 \times 10^{-4} \quad B_1 = \frac{\phi_1}{A} = 0.25T$$

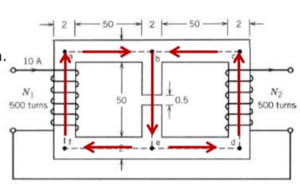
$\phi_2 = 0 \quad B_2 = 0$

$\phi_3 = -1 \times 10^{-4}$ (opposite direction) $B_3 = 0.25\,T$



Example 4: Magnetic Circuits

For the magnetic circuit shown in the figure, all dimensions are in cm. The relative permeability of the core $\mu_r = 1200$.
Find the flux density B in the airgap.



Step 1 – Establish the direction of the coil mmf

Step 2 – Draw the equivalent circuit

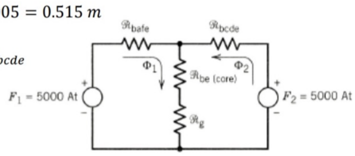
Step 3 – Solve the magnetic circuit using the principles of electric circuits

$$l_{bafe} = 3 \times 0.52 = 1.56\, m = l_{bcde} \qquad l_{be} = 0.52 - 0.005 = 0.515\, m$$

$$\mathcal{R}_{bafe} = \frac{l_{bafe}}{\mu_r \mu_0 A} = \frac{1.56}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}} = 2.58 \times 10^6 = \mathcal{R}_{bcde}$$

$$\mathcal{R}_{be} = \frac{l_{be}}{\mu_r \mu_0 A} = \frac{0.515}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}} = 0.82 \times 10^6$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A} = \frac{0.005}{4\pi 10^{-7} \times 4 \times 10^{-4}} = 9.94 \times 10^6$$

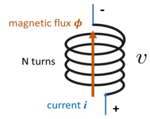


Apply magnetic version of "KVL" and "Ohm's Law"

PROBLEM 3 - SELF INDUCTANCE

Self-Inductance Time-varying current of a coil produces a time-varying flux linkage, which **induces a voltage on itself**

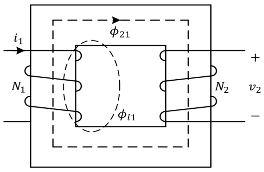
$$v_1 = \frac{d\lambda_1}{dt} = L\frac{di_1}{dt}$$



Mutual Inductance Time-varying current of a coil produces a time-varying flux linkage, which **induces a voltage on another coil**

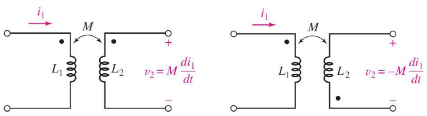
$$v_2 = \frac{d\lambda_{21}}{dt} = M\frac{di_1}{dt}$$

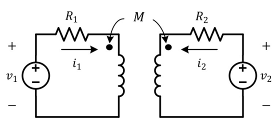
- In both cases, the **induced voltage** equals to the **changing rate of the flux linkage**.
- **Inductance** is the **linear coefficient** between voltage and the changing rate of the current.
- Mutual Inductance is the basis of **Transformer**.



Equations with Mutually Coupled Coils

- If a reference current **enters a dotted** terminal of a coil, the induced voltage on the other coil will be **positive at the dotted** terminal





$$v_1 = i_1R_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$v_2 = i_2R_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

EXAMPLE # 2

Solution. (a) Time domain

$$v_1 = i_1R_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \Leftrightarrow 2\sqrt{2}\cos t = i_1 + \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$0 = i_2R_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \Leftrightarrow 0 = i_2 + \frac{di_2}{dt} + \frac{di_1}{dt}$$

Phasor: $V_1 = R_1I_1 + jwL_1I_1 + jwMI_2 \iff 2 = I_1 + jI_1 + jI_2$

$$0 = R_2I_2 + jwL_2I_2 + jwMI_1 \Leftrightarrow 0 = I_2 + jI_2 + jI_1 \Leftrightarrow I_1 = j(1+j)I_2$$

(b) $2 = (1+j)I_1 + jI_2 = ((1+j)^2 j + j)I_2$

$$\Leftrightarrow \quad I_2 = -0.8 - 0.4j\ A = 0.8944\angle-153.4349^o\ A$$

$$I_1 = j(1+j)I_2 = 1.2 - 0.4i\ A = 1.2649\angle-18.4349^o A$$

$$i_1(t) = \sqrt{2}\times 1.2649\cos(t - 18.4349^o) = 1.7888\cos(t-18.4349^o)\ A$$

$$i_2(t) = \sqrt{2}\times 0.8944\cos(t-153.4349^o) = 1.2649\cos(t-153.4349^o)\ A$$

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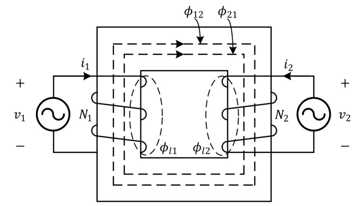
Coupling Coefficient k

- $k \triangleq \frac{M}{\sqrt{L_1L_2}}$ (the amount of coupling)
- $0 \le k \le 1$ and $0 \le M \le \sqrt{L_1L_2}$



1) When $k = 0$: No coupling: $M = 0$
2) When $k = 1$: Perfect coupling: ideal coupling with zero leakage. Then, $M = \sqrt{L_1L_2}$

$$v_1 = \frac{d\lambda_1}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$v_2 = \frac{d\lambda_2}{dt} = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

Example

2. (20 points) The circuit below shows a set of coupled coils.

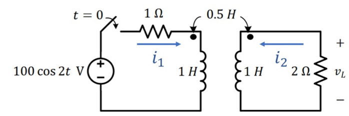


Figure 2 Question 2 Circuits

$$V_L = \frac{100\sqrt{2}}{-6-j}V = -22.93 + 3.82j = 23.25\angle 170.54^o\ V$$

$$v_L(t) = 32.87\cos(2t + 170.54^o)V$$

(a) (10 points) Write the loop equations in both time domain and phasor domain for $t \ge 0$.
(b) (10 points) Solve for $v_L(t)$ using phasors.

Solution (a) Time domain:

$$v_1 = i_1R_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad 100\cos(2t) = i_1 + \frac{di_1}{dt} + 0.5\frac{di_2}{dt}$$

$$v_2 = i_2R_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \qquad 0 = 2i_2 + \frac{di_2}{dt} + 0.5\frac{di_1}{dt}$$

Phasor domain: $w = 2$

$$V_1 = I_1R_1 + jwL_1I_1 + jwMI_2 \qquad 50\sqrt{2} = I_1 + j2I_1 + jI_2 = (1+j2)I_1 + jI_2$$

$$V_2 = I_2R_2 + jwL_2I_2 + jwMI_1 \qquad 0 = 2I_2 + j2I_2 + jI_1 = (2+j2)I_2 + jI_1$$

Solution (b) $I_1 = -\frac{(2+j2)}{j}I_2$

$$50\sqrt{2} = -\frac{(1+j2)(2+j2)}{j}I_2 + jI_2$$

$$I_2 = \frac{50\sqrt{2}}{-6-j}A$$

$$V_L = I_2R_2 = \frac{100\sqrt{2}}{-6-j}V$$

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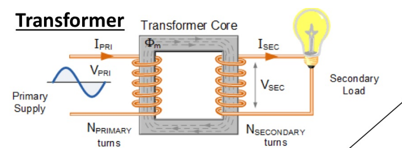
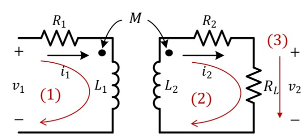
PROBLEM 4

Transformer



Practical Transformer



- R_1 & R_2: winding resistances,
- L_1, L_2, M: self and mutual inductances
- $a = N_1/N_2$: turn ratio

Original Circuit Equations

(1) $v_1 = i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$

(2) $0 = R_2i_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} + i_2R_L$

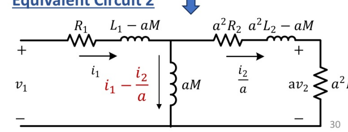
(3) $v_2 = i_2R_L$

Equivalent Circuit 2 Equations

(1) $v_1 = i_1R_1 + (L_1 - aM)\frac{di_1}{dt} + aM\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right)$ (3) $av_2 = \left(\frac{i_2}{a}\right)a^2R_l$

(2) $0 = \left(\frac{i_2}{a}\right)a^2R_2 + (a^2L_2 - aM)\frac{d}{dt}\left(\frac{i_2}{a}\right) + \left(\frac{i_2}{a}\right)a^2R_L - aM\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right)$

Equivalent Circuit 2



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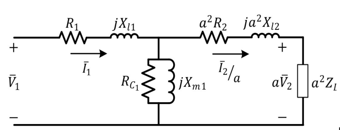
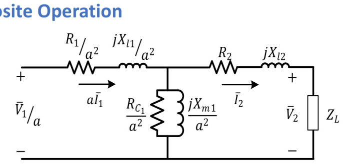
- Equivalent circuit with all quantities referred to **winding 1**:



- **Impedances** are **divided by a^2**
- **Voltage** is **divided by a**
- **Current** is **multiplied by a**

Opposite Operation

- Equivalent circuit with all quantities referred to **winding 2**:



Example (Transformer)

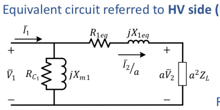
A 10kVA, 2400/240V transformer has $R_c = 340\Omega$ and $X_m = 250\Omega$ referred to the LV side. And the transformer has $R_{eq} = 9\Omega$ and $X_{eq} = 15\Omega$ referred to the HV side. The transformer is supplying a load of 8kVA on the LV side at 0.94PF lagging at 230V.

(a) Draw the approximate equivalent circuit referred to the HV side (mark the components and parameters).
(b) Find the transformer efficiency.
(c) Find the voltage regulation of the transformer.

Solution: (a) 2400/240 → step-down transformer and turn ratio a =2400/240 = 10

Equivalent circuit referred to **HV side (Primary)** $\quad R_{c1} = a^2R_c = 34000\ \Omega \quad X_{m1} = a^2X_m = 25000\ \Omega$

$R_{1eq} = 9\Omega, \quad X_{1eq} = 15\Omega$



(b) Power factor angle $\theta = \cos^{-1}0.94 = 19.95^o$

$\bar{V}_2 = 230\angle 0^oV \quad$ Apparent power $S = 8000VA$

RMS Current $I_2 = \frac{8000}{230} = 34.78A \quad \bar{I}_2 = 34.78\angle-19.95^o\ A$

$$KVL:\ \bar{V}_1 = (R_{1eq} + jX_{1eq})\frac{\bar{I}_2}{a} + a\bar{V}_2 = (9+15j)(3.478\angle-19.95^o) + 2300\angle 0^o = 2347.2 + 38.36j\ V = 2347.5\angle 0.9363^o$$

$$P_c = \left(\frac{I_2}{a}\right)^2 R_{1eq} = 3.478^2\times 9 = 108.9W \quad P_i = \frac{V_1^2}{R_{C1}} = \frac{2347.5^2}{34000} = 162.1W \quad \eta = \frac{P_{out}}{P_{out}+P_c+P_i} = \frac{7520}{7791} = 96.52\%$$

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(c) With load, $V_{load} = V_2 = 230V$ $\quad$ With no load, the secondary is open circuit, and no current namely $I_2 = 0A$

$V_{no\ load} = V_1/a = 234.75V \quad$ Voltage Regulation $= \frac{V_{no\ load} - V_{load}}{V_{load}} = \frac{234.75-230}{230} = 2.07\%$

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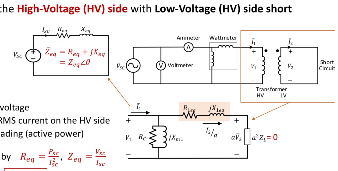
Transformer Short-Circuit (SC) Test

- **Performed on** the **High-Voltage (HV) side** with **Low-Voltage (HV) side short**



- Measurements:
  - V_{sc}: applied RMS voltage
  - I_{sc}: short-circuit RMS current on the HV side
  - P_{sc}: wattmeter reading (active power)
- Obtain R_{eq} and X_{eq} by $R_{eq} = \frac{P_{sc}}{I_{sc}^2}$, $Z_{eq} = \frac{V_{sc}}{I_{sc}}$

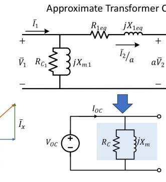
$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$

Open-Circuit (OC) Test

- To obtain **1) magnetizing reactance** X_m and **2) resistive core loss** R_C

- **Performed on** the **Low-Voltage (LV) side** with **High-Voltage (HV) side open**
- Measurements:
  - ✓ **(RMS) Voltage** measurement V_{oc}
  - ✓ **(RMS) Current** measurement I_{oc}
  - ✓ **Active Power** measurement P_{oc}
- Parameters Calculation: $R_c = \frac{V_{oc}^2}{P_{oc}}$

Approximate Transformer Circuit



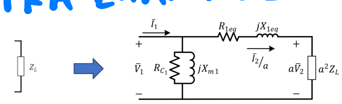
$$\bar{I}_{oc} = \bar{I}_R + \bar{I}_x = \frac{V_{oc}}{R_c} + \frac{V_{oc}}{jX_m} = \frac{V_{oc}}{R_c} - j\frac{V_{oc}}{X_m}$$

Since $\bar{I}_R$ and $\bar{I}_x$ are orthogonal (phase difference of 90°)

$\rightarrow I_x = \sqrt{I_{oc}^2 - I_R^2} \rightarrow X_m = \frac{V_{oc}}{I_x}$

X_m and R_c are values referred to the LV side (Use turn ratio to calculate their values referred to the HV side)

EXTRA EXAMPLE.



the HV side. Mark all
e numerical values, but

$R_{1eq} = 0.1778\Omega, \quad X_{1eq} = 0.9162\Omega$

$R_{c1} = 25*115.2 = 2880\Omega,\ X_{m1} = 25*15.1288 = 378.22\Omega$

er supplies load at
f 0.95 lead.

3. Solution: (c)

power factor angle $\theta = -\cos^{-1}0.95 = -18.2^o$ (*lea*

valent transformer

Turn ratio $a = 5, \quad V_2 = 240\angle 0^oV,$

$I_2 = \frac{150\times 1000}{240}\angle 18.2^oA = 625\angle 18.2^oA = 593.73 + j195.$

Z_L

$$V_1 = \left(\frac{I_2}{a}\right)(R_{1eq} + jX_{1eq}) + aV_2$$
$$= 125\angle 18.2^o(0.1778 + j0.9162) + 1200\angle 0^o$$
$$= 1185.3 + 115.74j = 1191\angle 5.5767^oV$$

HV side. Mark all
merical values, but

$$P_i = \frac{1191^2}{2880} = 492.5281\ W,$$

$$P_c = \left(\frac{I_2}{a}\right)^2 R_{1eq} = 125^2\times 0.1778 = 2778.1W$$

$$\eta = \frac{P_{out}}{P_{out} + P_i + P_c} = \frac{150000*0.95}{150000*0.95 + 492.5281 + 2778.1} = 97.76\%$$

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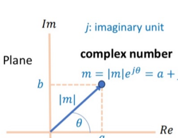
Solution: (d)

$V_{load} = 240V, \quad V_{no\ load} = \frac{1191}{5} = 238.2V,$

voltage regulartion $= \frac{238.2-240}{240} = -0.75\%$

CONVERTING



Relation:

$m = a + jb \qquad m = |m|e^{j\theta} = |m|\angle\theta$

$a = |m|(\cos(\theta)) \qquad |m| = \sqrt{a^2+b^2}$

$b = |m|(\sin(\theta)) \qquad \theta = tan^{-1}(b/a)$

Calculation: (work for phasor!)

$m_1 + m_2 = (a_1 + a_2) + j(b_1 + b_2)$

$m_1 - m_2 = (a_1 - a_2) + j(b_1 - b_2)$

$m_1 \times m_2 = |m_1||m_2|\angle\theta_1 + \theta_2$

$m_1/m_2 = |m_1|/|m_2|\angle\theta_1 - \theta_2$