P4.6 Express FT's of Signals listed below.

(a) $x_1(t) = x(1-t) + x(-1-t)$

$x_1(t) = x(-(t-1)) + x(-(t+1))$

$x(t) \xrightarrow{FT} X(\omega)$

$x(t-t_0) \xrightarrow{FT} e^{-j\omega t_0} X(\omega)$ — ①

$x(-t) \xrightarrow{FT} X(-\omega)$ — ②

for $x(t-1) \xrightarrow{FT} e^{-j\omega} X(\omega)$

for $x(-t-1) \xrightarrow{FT} e^{-j(-\omega)} X(\omega)$

$x(-t-1) \xrightarrow{FT} e^{j\omega} X(-\omega)$

$x(1-t) \xrightarrow{FT} e^{j\omega} X(-\omega)$ — ③

for $x(-1-t)$

$x(t) \xrightarrow{FT} X(\omega)$

$x(t+1) \xrightarrow{FT} e^{-j\omega(-1)} X(\omega)$

$x(t+1) \xrightarrow{FT} e^{j\omega} X(\omega)$

for $x(-(t+1))$

$x(t) \xrightarrow{FT} X(\omega)$

$x(-(t+1)) \xrightarrow{FT} e^{-j\omega} X(-\omega)$ — ④

$FT\{x_1(t)\} = FT\{x(1-t) + x(-1-t)\}$

$FT\{x_1(t)\} = FT\{x(1-t)\} + FT\{x(-1-t)\}$

$FT\{x_1(t)\} = e^{j\omega} X(-\omega) + e^{-j\omega} X(-\omega)$

$\boxed{FT\{x_1(t)\} = X(-\omega)\{e^{j\omega} + e^{-j\omega}\}}$

(b) $x_2(t) = x(3t-6) \rightarrow$ Simplify

$x_2(t) = x(3(t-2))$

Time Shifting

$x(t-t_0) \xrightarrow{FT} e^{-j\omega t_0} X(\omega)$

$FT\{x(t-2)\} = e^{-j\omega 2} X(\omega)$

Scaling Property $x(at) \xrightarrow{FT} \frac{1}{|a|} X\left|\frac{\omega}{a}\right|$

$x(3(t-2)) \xrightarrow{FT} \left(\frac{1}{3}\right) e^{-2j(\omega/3)} X(\omega/3)$

$\boxed{x(3t-6) \xrightarrow{FT} \left(\frac{1}{3}\right) e^{(-2/3)j\omega} X(\omega/3)}$

4.9

$$x(t) = \begin{cases} 0, & |t| > 1 \\ (t+1)/2, & -1 \le t \le 1 \end{cases}$$

(a)
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} \left(\frac{t+1}{2}\right) e^{-j\omega t}\, dt = \frac{1}{2}\left[\int_{-1}^{1} t e^{-j\omega t} dt + \int_{-1}^{1} e^{-j\omega t} dt\right]$$

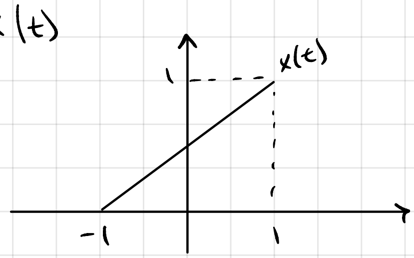
$$\int t e^{-j\omega t} dt = \frac{-t e^{-j\omega t}}{j\omega} + \frac{e^{-j\omega t}}{\omega^2}$$

$$X(j\omega) = \frac{1}{2}\left[\frac{-t e^{-j\omega t}}{j\omega} + \frac{e^{-j\omega t}}{\omega^2}\right]_{-1}^{1} - \left.\frac{e^{-j\omega t}}{j\omega}\right]_{-1}^{1}$$

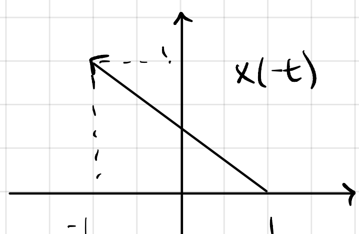
$$X(j\omega) = \frac{1}{2}\left[\frac{-(1)e^{-j\omega(1)}}{j\omega} + \frac{e^{-j\omega}}{\omega^2} + \frac{(-1)e^{-j\omega(-1)}}{j\omega}\right] = \frac{1}{2}\left[-\frac{e^{-j\omega}}{j\omega} + \frac{e^{-j\omega}}{\omega^2} - \frac{e^{-j\omega}}{\omega^2} - \frac{e^{-j\omega}}{j\omega}\right]$$

$$\rightarrow \frac{1}{2}\left[-\frac{2e^{-j\omega}}{j\omega} + \frac{1}{\omega^2}\left(e^{-j\omega} - e^{j\omega}\right)\right.$$

(b) $x(t)$

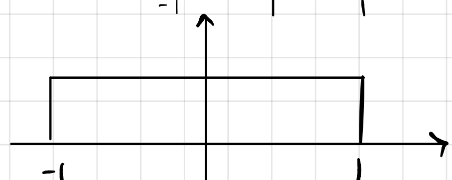


$\rightarrow$

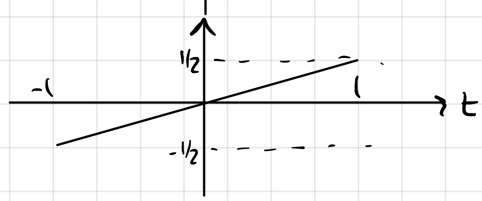


$x(-t)$

$$\text{Even}\{x(t)\} = \frac{x(t) + x(-t)}{2}$$



$$\text{Odd}\{x(t)\} = \frac{x(t) - x(-t)}{2}$$



$$F\{\text{Even}(x(t))\} = \frac{1}{2}\int_{-1}^{1} e^{-j\omega t} dt$$

$$= \frac{1}{2}\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^{1} = -\frac{1}{2j\omega}\left[e^{-j\omega} - e^{j\omega}\right]$$

$$= \frac{1}{\omega}\left[\frac{e^{j\omega} - e^{-j\omega}}{2j}\right]$$

$$= \underline{\frac{1}{\omega}\sin(\omega)}$$

$$F\{\text{Odd}(x(t))\} = \frac{1}{2}\int_{-1}^{1} t e^{-j\omega t} dt$$

$$= \frac{1}{2}\left[\frac{-t e^{-j\omega t}}{j\omega} + \frac{e^{-j\omega t}}{\omega^2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{-e^{-j\omega}}{j\omega} + \frac{e^{-j\omega}}{\omega^2} - \frac{e^{j\omega}}{j\omega} - \frac{e^{j\omega}}{\omega^2}\right]$$

$$= \frac{1}{2}\left[-\frac{1}{\omega}\left[\frac{e^{j\omega} + e^{-j\omega}}{j}\right] + \frac{1}{\omega^2}\left[e^{-j\omega} - e^{j\omega}\right]\right]$$

$$= \frac{-1}{\omega}\left[\frac{e^{j\omega} + e^{-j\omega}}{2j}\right] + \frac{j}{2\omega^2}\left[\frac{e^{-j\omega} - e^{j\omega}}{2j}\right]$$

$$= j\left[\frac{\cos(\omega)}{\omega} - \frac{1}{\omega^2}\sin(\omega)\right]$$

$$= \frac{j}{\omega}\left[\cos(\omega) - j\sin(\omega)\right] + \frac{j}{\omega^2}\sin(\omega)$$

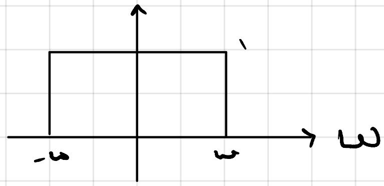
$$X(j\omega) = \frac{j}{\omega}\cos(\omega) - \frac{j^2}{\omega}\sin(\omega) - \frac{j}{\omega^2}\sin(\omega)$$

$$= \frac{1}{\omega}\sin(\omega) + j\left(\frac{1}{\omega}\cos(\omega) - \frac{1}{\omega^2}\sin(\omega)\right)$$

$F\{\text{Even}(x(t))\}$ is the Real Part of $X(j\omega)$

4.10 Determine FT

(a) $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$



$= \pi\left(\frac{\omega}{2w}\right)$

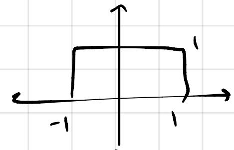
$x(t) * y(t) \xrightarrow{FT} X(j\omega) * Y(j\omega)$

$t\,x(t) \longleftrightarrow j\frac{d}{d\omega}\left[X(j\omega)\right]$

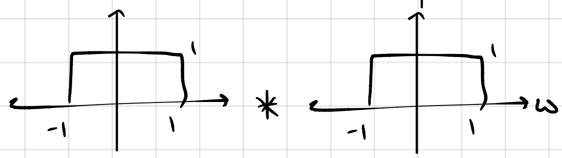
Given

$x(t) = t\cdot\left(\frac{\sin t}{\pi t}\right)\cdot\left(\frac{\sin t}{\pi t}\right)$

$\frac{\sin t}{\pi t} \longleftrightarrow$

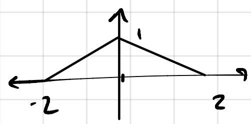


$x_1(t) = \frac{\sin(t)}{\pi t}\cdot\frac{\sin(t)}{\pi t} \longleftrightarrow$

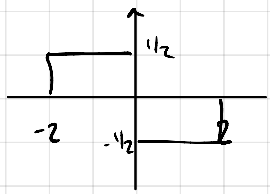


$x(t) = t\cdot x_1(t)$

$X(\omega) = j\frac{d}{d\omega}\left[X_1(j\omega)\right]$



$X(\omega) = \begin{cases} j/2 & -2 < \omega < 0 \\ -j/2 & 0 < \omega < 2 \end{cases}$



4.13 let $x(t)$

$$X(j\omega) = \delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)$$

and

$$h(t) = u(t) - u(t-2)$$

(a) Periodic?

$$x(t) = \frac{1}{2\pi} + \frac{e^{j\pi t}}{2\pi} + \frac{e^{j5t}}{2\pi}$$

$$= \frac{1}{2\pi}\left[1 + e^{j\pi t} + e^{j5t}\right]$$

$e^{j\pi t}$ is periodic w/ $\omega_0 = \pi$ and $T_0 = 2$

e^{j5t} periodic, $T_0 = \frac{2\pi}{5}$

No LCM for 2 and $\frac{2\pi}{5}$ so,

$x(t)$ is not periodic

(b) $h(t) = u(t) - u(t-2)$

$$= \text{rect}\left(\frac{t-1}{2}\right) = H(j\omega) = e^{-j\omega}\left(\frac{2\sin(\omega)}{\omega}\right)$$

Output:

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$= \delta(\omega) + \delta(\omega-\pi) + \delta(t-2)\, H(j\omega)$$

$$Y(j\omega) = \delta(\omega)H(0) + \delta(\omega-\pi)H(j\pi) + \delta(\omega-5)H(j5)$$

$$= 2\delta(\omega) + 0 - e^{-j5}(j\pi)\,\delta(\omega-5)$$

$$Y(t) = \frac{1}{2\pi}\left[2 - e^{-j5}(j\pi)\, e^{-j5t}\right]$$

$Y(t)$ is periodic

(c) The convolution of the two signals is periodic

4.14

① Fourier Series

$$(1+j\omega)X(j\omega) = \frac{A}{2+j\omega}$$

$$X(j\omega) = \frac{A}{(2+j\omega)(1+j\omega)}, \quad X(j\omega) = \frac{a}{2+j\omega} + \frac{b}{1+j\omega}$$

$$X(j\omega) = \frac{a(1+j\omega)+b(2+j\omega)}{(2+j\omega)(1+j\omega)}$$

Partal fraction

$$A = a(1+j\omega)+b(2+j\omega)$$

$$j\omega = 1, \quad b = A$$

$$j\omega = -2, \quad -A = a$$

$$X(j\omega) = \frac{-A}{2+j\omega} + \frac{A}{1+j\omega}$$

② Parsevals'

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = 1$$

③ Integrate FT

$$\int_{-\infty}^{\infty} |Ae^{-t}u(t) - Ae^{-2t}u(t)|^2 dt = 1$$

$$\int_{-\infty}^{\infty} A^2e^{-2t}dt + \int_{0}^{\infty} A^2e^{-4t}dt - \int_{0}^{\infty} 2A^2e^{-3t}dt = 1$$

$$\frac{A^2}{2} + \frac{A^2}{4} - \frac{2A^2}{3} = 1, \quad A = \sqrt{12}$$

Equation

$$x(t) = \sqrt{12}\left(e^{-t} - e^{-2t}\right)u(t)$$

$$A = \sqrt{12}$$