Homework 4 | Page 1
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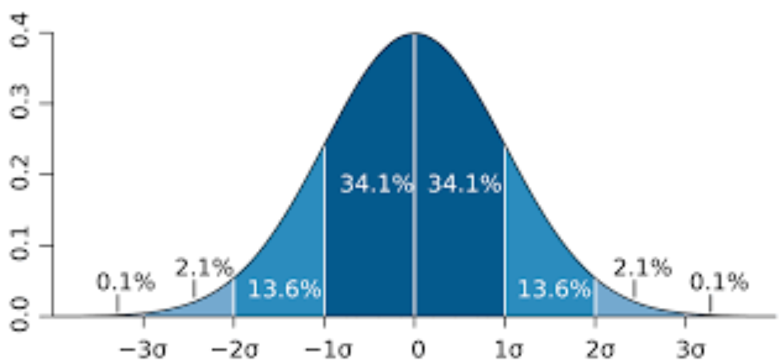
Question 1

Given:
- Standard normal distribution curve.

Find:
a. Area between $z = 0$ and $z = 1.3$.
b. Area to the right of $z = 1.87$.
c. Area to the left of $z = -0.7$.
d. Area between $z = -1.47$ and $z = 2.39$.

Diagram:



Theory:
- The standard normal distribution curve is a probability density function with a mean (μ) of 0 and a standard deviation (σ) of 1.

Assumptions:
- Assuming we are working with the standard normal distribution.

Solution:

a. For area between $z = 0$ and $z = 1.3$, the area is about 0.4032.

b. For the area to the right of $z = 1.87$, the area is about 0.0307.

c. For the area to the left of $z = -0.7$, the area is about 0.2420.

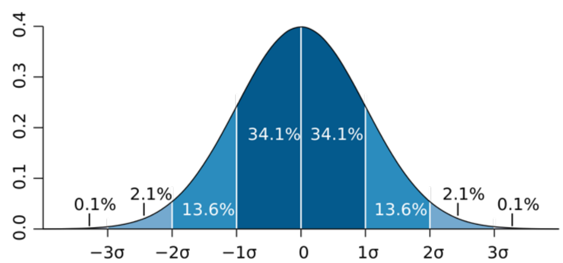
d. For the area between $z = -1.47$ and $z = 2.39$, find the area to the left of $z = 2.39$ and subtract the area to the left of $z = -1.47$. Using a standard normal distribution table, the area to the left of $z = 2.39$ is about 0.9933, and the area to the left of $z = -1.47$ is about 0.0708. Area between these two values is approximately 0.9933 - 0.0708 = 0.9208.

Question 2

Given:
- Mean (μ) = 80
- Standard Deviation (σ) = 6.0

Find:
- Probability of a student having a quiz grade of 95 or greater.

Diagram:



Theory:
To find probability, use the standard normal distribution and the z-score formula:

$$Z = \frac{X - \mu}{\sigma}$$

- Z is the z-score.
- X is the value we want to find the probability for.
- μ is the mean.
- σ is the standard deviation.

Find the z-score for 95 and then use the z-score to find the probability using the z-table

Assumptions:
Quiz grades follow a normal distribution.

Solution:
Calculate the z-score for a quiz grade of 95:

$$Z = \frac{95 - 80}{6.0} = 2.5$$

from scipy.stats import norm

```
# Find the cumulative probability for Z = 2.5 (probability of grade ≥ 95)
cum_prob_grade_95_or_greater = 1 - norm.cdf(2.5)
# Print the result
print(cum_prob_grade_95_or_greater)
```

When I ran this code, it will output the probability:

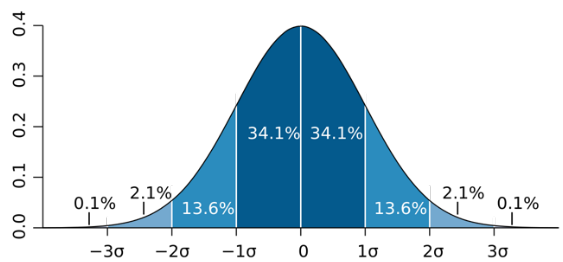
Probability of a student having a quiz grade of 95 or greater: 0.0062097 or .62%

Question 3

Given:
- Mean (μ) of Dr. Pepper dispensed = 63.0 oz
- Standard Deviation (σ) of Dr. Pepper dispensed = 1.75 oz
- Cup capacity = 64.0 oz

Find:
- Probability that a randomly selected cup will be overfilled.

Diagram:



Theory:
To find the probability, use the standard normal distribution and the z-score formula:

$$Z = \frac{X - \mu}{\sigma}$$

- Z is the z-score.
- X is the value we want to find the probability for (cup being overfilled at 64.0 oz).
- μ is the mean.
- σ is the standard deviation.

Assumptions:
The amount of Dr. Pepper dispensed follows a normal distribution.

Solution:
Let's calculate the z-score for a cup being filled to 64.0 oz:

$$Z = \frac{64 - 63}{1.75} = 0.5714$$

from scipy.stats import norm

# Find the cumulative probability for Z ≈ 0.5714 (probability of overfilling)
cum_prob_overfill = 1 - norm.cdf(0.5714)

```
# Print the result
print(cum_prob_overfill)
```

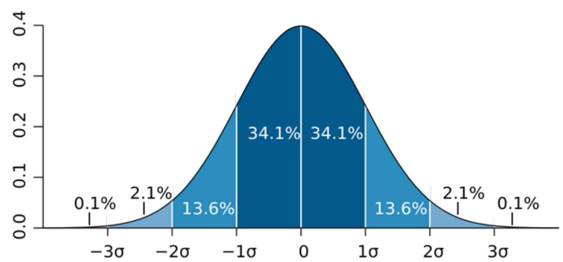
Probability of a randomly selected cup being overfilled: 28.57%

Question 4:

Given:
- Mean lifetime (μ) of LED light bulbs = 4,600 hours
- Standard deviation (σ) of lifetime = 250 hours
- Desired proportion of bulbs burning out before claimed lifetime = 4.0%

Find:
- The lifetime the company should promote for these bulbs.

Diagram:



Theory:
To find the promoted lifetime, find the value of lifetime, X, such that only 4.0% of bulbs burn out before this claimed lifetime.

$$Z = \frac{X - \mu}{\sigma}$$

- Z is the z-score corresponding to the desired proportion.
- X is the claimed lifetime we want to find.
- μ is the mean lifetime.
- σ is the standard deviation.

Assumptions:
Assume that the lifetime of the LED light bulbs follows a normal distribution.

Solution:
(0.04) = -1.7507. Use the z-score formula to find the claimed lifetime (X):

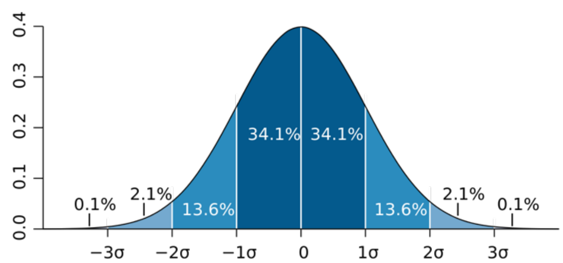
$$-1.7507 = \frac{X - 4{,}600}{250}$$

The company should promote a lifetime of about 4,162.325 hours for these LED light bulbs to ensure that only 4.0% of them burn out before the claimed lifetime.

Given:
- Mean lifetime μ of electric scooters = 15 years
- Standard deviation σ of lifetime = 1.75 years
- Percentage of scooters to be replaced p = 5.0%

Find:
- The length of the guarantee (in years) that the manufacturer should offer.

Diagram:



Theory:

Find a lifetime, X, such that the manufacturer is willing to replace 5.0% of the scooters that fail before this time. This involves finding value (z) from the standard normal distribution corresponding to the percentage 5.0%, and then using it to calculate the lifetime.

The z-score formula is:

$$Z = \frac{X - \mu}{\sigma}$$

- Z is the z-score.
- X is the desired lifetime.
- μ is the mean lifetime.
- σ is the standard deviation.

Assumptions:
Assume the lifetimes of electric scooters follow a normal distribution.

Solution:

$$-1.645 = \frac{X - 15}{1.75}$$

The manufacturer should offer a guarantee of approximately 12.11 years to be willing to replace 5.0% of the scooters that fail before that time.