

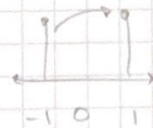
1204

① $y[n] = 2x[1-n] = y[n] = 2x[-n+1]$

Memory because of (+1)

Time Invariant?

$$\begin{aligned} y[0] &= 2x[1] \\ y[1] &= 2x[0] \\ y[2] &= 2x[-1] \end{aligned}$$



$y_1[n] = 2x[-n+1]$

Shift + n let $x[n-n_0] = x[n]$ $2x[1-n-n_0]$

$y_2[n] = 2x[-n-n_0+1] = 2x[-(n-n_0)+1]$

2 $y_1[n-n_0] = 2x[-n-n_0+1] = 2x[-(n-n_0)+1]$ Time Invariant

Linear?

$x_1(t) \rightarrow y_1[n] = 2x_1[1-n]$

$x_2(t) \rightarrow y_2[n] = 2x_2[1-n]$

$x_3 = ax_1 + bx_2 \rightarrow y_3[n] = a(2x_1[1-n]) + b(2x_2[1-n])$
 $= 2ax_1[1-n] + 2bx_2[1-n]$

Causal? Yes

$= a(2x_1[1-n]) + b(2x_2[1-n])$
 $= ay_1[n] + by_2[n] \checkmark$
Linear

$y[2] = 2x[1-2]$

$= 2x[-1]$ Past

$y[3] = 2x[1-3]$

$= 2x[-2]$ Past

Stable? Yes



Invertible?

Yes.

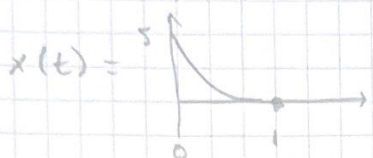
Answers

- Time Invariant
- Linear
- Causal
- Stable
- Invertible

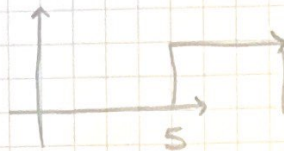
Q2 $y(t) = x(t) * h(t)$

CT convolution

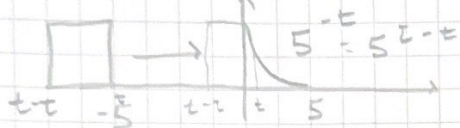
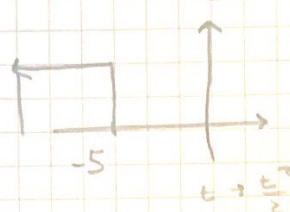
$$\int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$



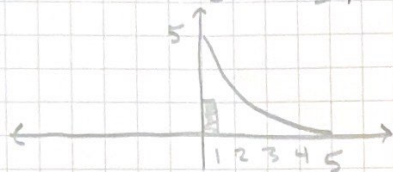
$h(t) = \delta(t-5)$



$h(-\tau)$



- $t < 0 \Rightarrow \tau = 0$
- $0 \leq t < 5$

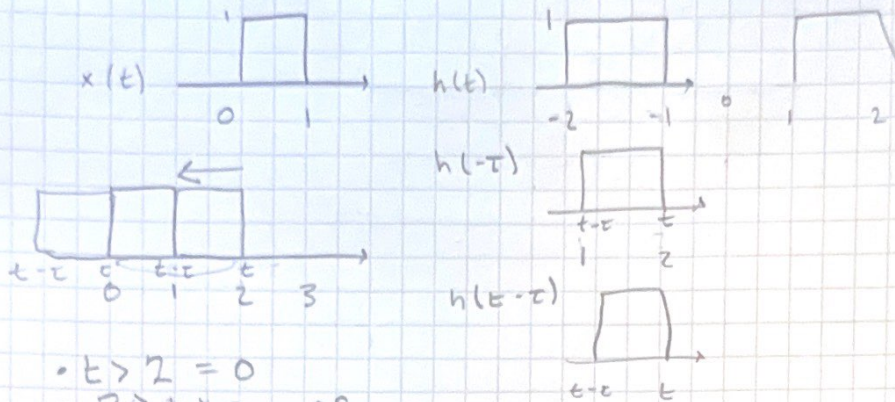


$$\int_0^5 1 \cdot 5^{\tau-t+\tau} d\tau = \int_0^5 5^{-t} d\tau = 5^{-t} \tau \Big|_0^5 = 5^{-t} \cdot 5$$

$\int 5^x = \ln(x)$

$$x(t) = \begin{cases} 0, & t < 0 \\ 5, & 0 \leq t < 5 \\ 0, & t \geq 5 \end{cases}$$

Q3 $y(t) = x(t) * h(t)$



$\bullet t > 2 = 0$

$\bullet 2 \geq t > 0$

$$\int_2^0 h(t-\tau) d\tau = \int_2^0 \tau d\tau = \left[\frac{\tau^2}{2} \right]_2^0 = (0 - 2t) - [-2] = -2t + 2$$

$\bullet t < 0 = 0$

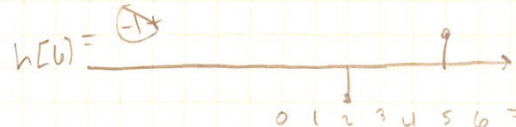
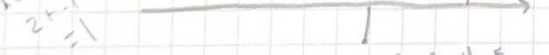
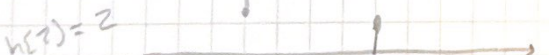
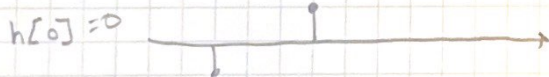
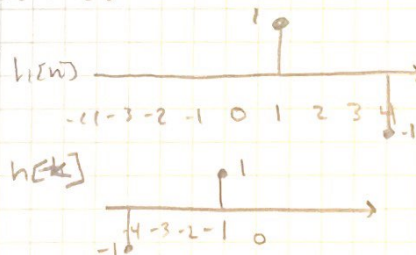
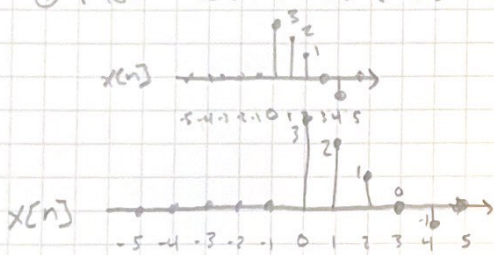
$$y(t) = \begin{cases} 0, & t > 2 \\ -2t + 2, & 2 \geq t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$t\tau \Big|_2^0 - \frac{\tau^2}{2} \Big|_2^0 = (0 - 2t) - [-2] = -2t + 2$$

Q4 Provide pictorial description $y[n] = x[n] * h[n]$

$$x[n] = \begin{cases} -n+3 & 0 \leq n \leq 4 \\ 0 & \text{other} \end{cases} \quad h[n] = u[n-1] - u[n-4]$$

① Plot $x[0]=3, x[1]=2, x[2]=1, x[3]=0, x[4]=-1$



$$h[0] = 0$$

$$1 =$$

$$h[1] = 3$$

$$h[2] = 2$$

$$h[3] = 1$$

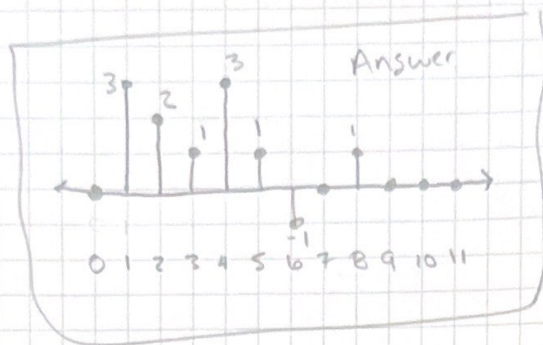
$$h[4] = 3$$

$$h[5] = 1$$

$$h[6] = -1$$

$$h[7] = 0$$

$$h[8] = 1$$



Q5. Determine the Fourier Series for the periodic signal $x(t)$

$$x(t) = \begin{cases} 2, & t = 0 \\ 0, & 0 < t < \pi \\ 1/3, & t > 2 \end{cases}$$

$$x(t) = \begin{cases} 2, & t = 0 \\ 0, & 0 < t < \pi \\ 1/3, & t > 2 \end{cases}$$

$$x(t)$$

$$a_0 = 2$$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}$$

$$S_n =$$

$$S_n(\pi n) = (e^{j\pi n} - e^{-j\pi n})$$

$$x(t) = \frac{1}{6}(e^{j\pi t} + e^{-j\pi t}) = \frac{1}{6}(e^{j\pi t} - e^{-j\pi t})^2 = \left(\frac{5}{8}\right) 0$$

time shift

$$x(t-t_0) = \frac{5\pi}{N} = 3$$

$$x(t) = \frac{1}{6}e^{\frac{1}{3}\pi n} + \frac{1}{6}e^{-\frac{1}{3}\pi n} + \frac{1}{3} = \frac{1}{6}e^{\frac{1}{3}\pi n} + \frac{1}{6}e^{-\frac{1}{3}\pi n} = \left(\frac{1}{6}\right) 3$$

$$x(t) = \begin{cases} 2, & t = 0 \\ 0, & 0 < t < \pi \\ 1/3, & t > 2 \end{cases}$$

Q6

- $x(t)$ is real
- $x(t)$ $T_0 = 6$ and is the fundamental T.P.

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}$$

- $x[k] = 0$ for $k = 0$ and $|k| > 1$

Possible values $x[\pm 1]$

- $x(t) = -x(-t)$ Reflection

$$-e^{-j\omega_0 k} x[k]$$

- $x[1] = -(-1)^{-1}, x[1] = 1$

- $x[-1] = -(-1) x[1] = x[1]$

$$\frac{1}{6} \int_{-3}^3 |x(t)|^2 dt = \frac{1}{2} \quad |x(t)|^2 = |x[-1]|^2 = \frac{1}{2}$$

$$x[1]^2 = -\frac{1}{2}$$

$$x[1]^2 = -\frac{1}{2}j \rightarrow x[1] = \pm \frac{1}{\sqrt{2}}j$$

$x[1]$ Positive imaginary

$$x(t) = x[1] e^{j\omega_0 t} + x[-1] e^{-j\omega_0 t} = \frac{1}{\sqrt{2}} (e^{j\frac{\pi}{3}t} + e^{-j\frac{\pi}{3}t}) \cos\left(\frac{\pi}{3}t\right)$$