MATH 311, HW 6

Due 11:00pm on Thursday March 9, 2023

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Problem 1. [56pts] Let $\mathbb{R}^{2\times 2}$ be the vector space of 2×2 matrices and consider its standard basis $E = \{e_{11}, e_{12}, e_{21}, e_{22}\}$ (in that order), where

$$e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad e_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad e_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Fix the matrix $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Determine whether the following L are linear operators on $\mathbb{R}^{2\times 2}$. If they are, find the matrix M_L representing L with respect to the standard basis, and find $\ker(L)$ and $range(L)$. (**Note:** in each case the definition of the map L is for every matrix $A \in \mathbb{R}^{2\times 2}$.)

$L(A+B) = L(A)+L(B)$

1. (8pts) $L(A) := A^2C$

Is it a linear transformation?
Yes ☐ No ☒

$L(A+B) = (A+B)^2 C$

$M_L =$		ker =		range =	

CHECK $\underline{R1}$ $L(\alpha A) = \alpha L(A)? = (\alpha A^2 C) = \alpha(A^2C) = \alpha L(A)$ ✓

$A \in M_{2\times 2}$

$\alpha \in \mathbb{R}$

$\underline{R2}$ $L(A+B) = L(A)+L(B)? = (A^2C + B^2C) = L(A^2C) + L(B^2C)$ ✓

$\underline{R2}$ $L(A+B) = L(A)+L(B)? = (A+B)^2C \neq L(A)+L(B)$ X

DOES NOT HOLD R2

$\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}$

2. (8pts) $L(A) := A^T$

Is it a linear transformation?
Yes ■ No □

$M_L =$	$\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$	ker =	$\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\} = \mathbb{R}^4$ $\left\{\begin{pmatrix}0&0\\0&0\end{pmatrix}\right\} = \mathbb{R}^{2\times2}$	range = span	$\boxed{4}$ $\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}$

① CHECK R_1, R_2 $\underline{R1}$ $A \in M_{2\times2}, \alpha \in \mathbb{R}$ $L(\alpha A) = \alpha L(A)$?

$L(\alpha A^T) = \alpha L(A^T)$ ✓

$\underline{R2}$ $L(A+B) = L(A) + L(B)$?

$L(A^T + B^T) = L(A^T) + L(B^T)$ ✓

$\underline{M_L}$:
$\left[L\begin{pmatrix}1&0\\0&0\end{pmatrix}\right]_E = \left[\begin{pmatrix}1&0\\0&0\end{pmatrix}\right]_E = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

$\left[L\begin{pmatrix}0&1\\0&0\end{pmatrix}\right]_E = \left[\begin{pmatrix}0&0\\1&0\end{pmatrix}\right]_E = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$

$\left[L\begin{pmatrix}0&0\\1&0\end{pmatrix}\right]_E = \left[\begin{pmatrix}0&1\\0&0\end{pmatrix}\right]_E = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

$\left[L\begin{pmatrix}0&0\\0&1\end{pmatrix}\right]_E = \left[\begin{pmatrix}0&0\\0&1\end{pmatrix}\right]_E = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

$\underline{M_L}$: $\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$

$\underline{\text{Ker}(L)}$:
IN REF: $\left(\begin{array}{cccc|c}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\end{array}\right) \rightarrow$ So, $\ker(L) = \left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}$

$\text{Range}(L) = \text{span}\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix}\right\} = \mathbb{R}^{2\times2}$

$\underline{\text{So, Range}(L) = 4}$

$L_L(A) = M_L A$ $\qquad \mathbb{R}^{2\times 2} \longrightarrow \mathbb{R}^4$

3. (8pts) $L(A) := CA + AC$

Is it a linear transformation?
Yes ■ No □

RANGE(L) = COLUMN(M_L)
$= \text{span}(c_1, c_2, \dots, c_4)$

$M_L =$	$\begin{pmatrix} 2 & 3 & 2 & 0 \\ 2 & 5 & 0 & 2 \\ 3 & 0 & 5 & 3 \\ 0 & 3 & 2 & 8 \end{pmatrix}$	ker =	$\left\{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right\}$ $= 0$	range =	$\text{Span}\left\{\begin{pmatrix} 2 & 2 \\ 3 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 5 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 5 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 3 & 8 \end{pmatrix}\right\}$ 4

IS L LINEAR? R1 $A \in M_{2\times 2}, \alpha \in \mathbb{R}$ $\qquad L(\alpha A) \stackrel{?}{=} \alpha L(A)$

$L(\alpha A) = (\alpha CA + \alpha AC) = \alpha(CA + AC)$
$= \alpha L(A)$ ✓

R2 $A, B \in M_{2\times 2}$ $\qquad L(A+B) = L(A) + L(B)$?

$L(A+B) = C(A+B) + (A+B)C$
$= CA + CB + CA + CB$
$= C(A+A) + C(B+B)$
$= L(A) + L(B)$ ✓

M_L: $\quad L(A) = CA + AC \qquad \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\left[L\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 2 & 2 \\ 3 & 0 \end{pmatrix}\right]_E = \begin{pmatrix} 2 \\ 2 \\ 3 \\ 0 \end{pmatrix}$

$\left[L\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 3 & 5 \\ 0 & 3 \end{pmatrix}\right]_E = \begin{pmatrix} 3 \\ 5 \\ 0 \\ 3 \end{pmatrix}$

$\left[L\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 2 & 0 \\ 5 & 2 \end{pmatrix}\right]_E = \begin{pmatrix} 2 \\ 0 \\ 5 \\ 2 \end{pmatrix}$

$\left[L\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 0 & 2 \\ 3 & 8 \end{pmatrix}\right]_E = \begin{pmatrix} 0 \\ 2 \\ 3 \\ 8 \end{pmatrix}$

$L\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right]$
$= \begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}$

$L\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right]$

$L\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right]$

$L\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right]$

So, $M_L = \begin{pmatrix} 2 & 3 & 2 & 0 \\ 2 & 5 & 0 & 2 \\ 3 & 0 & 5 & 3 \\ 0 & 3 & 2 & 8 \end{pmatrix}$

$\ker(L) = N(M_L)$

$\begin{pmatrix} 2 & 3 & 2 & 0 \\ 2 & 5 & 0 & 2 \\ 3 & 0 & 5 & 3 \\ 0 & 3 & 2 & 8 \end{pmatrix} \xrightarrow{\text{REF}} \left(\begin{array}{cccc|c} 2 & 3 & 2 & 0 & 0 \\ 0 & 2 & -2 & 2 & 0 \\ 0 & 0 & -5/2 & 15/2 & 0 \\ 0 & 0 & 0 & 20 & 0 \end{array}\right)$ So, $\ker(L) = 0$

$\text{col}(M_L) = \text{Span}\left\{\begin{pmatrix} 2 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 5 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \\ 8 \end{pmatrix}\right\} = \mathbb{R}^4$

$\text{Range}(L) = \text{span}\left\{\begin{pmatrix} 2 & 2 \\ 3 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 5 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 5 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 3 & 8 \end{pmatrix}\right\} = \mathbb{R}^{2\times 2}$

4. (8pts) $L(A) := 3A$

Is it a linear transformation?
Yes ☒ No ☐

$M_L =$	$\begin{pmatrix} 3&0&0&0\\0&3&0&0\\0&0&3&0\\0&0&0&3 \end{pmatrix}$	ker $=$	$\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}$ $\left\{\begin{pmatrix}0&0\\0&0\end{pmatrix}\right\} = 0$	range $=$	Span$\left\{\begin{pmatrix}3&0\\0&0\end{pmatrix}, \begin{pmatrix}0&3\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\3&0\end{pmatrix}, \begin{pmatrix}0&0\\0&3\end{pmatrix}\right\}$ $\boxed{4}$

Check R1 $A \in M_{2\times2}, \alpha \in \mathbb{R}$

$L(\alpha A) = \alpha L(A)$?

$L(3\alpha A) = L(\alpha 3A) = \alpha L(A)$ ✓

R2

$L(A+B) = L(A) + L(B)$?

$L(3A+3B) = L(3A) + L(3B) = 3(L(A)+L(B))$ ✓

$L(3(A+B)) = 3(L(A+B)) = 3(L(A)+L(B))$

M_L: $L(A) = 3A$

$\left[L\begin{pmatrix}1&0\\0&0\end{pmatrix}\right]_E = \left[\begin{pmatrix}3&0\\0&0\end{pmatrix}\right]_E$

$\left[L\begin{pmatrix}0&1\\0&0\end{pmatrix}\right]_E = \left[\begin{pmatrix}0&3\\0&0\end{pmatrix}\right]_E$

$\left[L\begin{pmatrix}0&0\\1&0\end{pmatrix}\right]_E = \left[\begin{pmatrix}0&0\\3&0\end{pmatrix}\right]_E$

$\left[L\begin{pmatrix}0&0\\0&1\end{pmatrix}\right]_E = \left[\begin{pmatrix}0&0\\0&3\end{pmatrix}\right]_E$

$M_L = \begin{pmatrix} 3&0&0&0\\0&3&0&0\\0&0&3&0\\0&0&0&3 \end{pmatrix}$

$\ker(L) = N(M_L)$

$\left(\begin{array}{cccc|c} 3&0&0&0&0\\0&3&0&0&0\\0&0&3&0&0\\0&0&0&3&0 \end{array}\right) = \ker(L) = \left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}$

$\text{Range}(L) = \text{Span}\left\{\begin{pmatrix}3&0\\0&0\end{pmatrix}, \begin{pmatrix}0&3\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\3&0\end{pmatrix}, \begin{pmatrix}0&0\\0&3\end{pmatrix}\right\}$

$\text{Range}(L) = 4$

5. (8pts) $L(A) := A + I$

Is it a linear transformation?
Yes ☐ No ■

$M_L =$		ker =		range =	

$L(\alpha A) = \alpha A + I \overset{?}{=} \alpha L(A)$

When $(\alpha A + I)$, cant pull a "α" out of I so does not pass.

$L(A+B) = (A+B) + I$

$\neq$

$L(A) + L(B) = (A+I)(+)(B+I) = A + B + 2I$

$A+B+2I$ X Does NOT WORK

$(A+B)^T = A^T + B^T$

6. (8pts) $L(A) := A^T - A$

Is it a linear transformation?
Yes ☑ No ☐

$M_L =$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	ker =	$\left\{\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}\right\}$ Ker(L) = 1	range =	Span $\left\{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right\}$ Ran(L) = 3

R1 $A \in M_{2\times 2}, \alpha \in \mathbb{R}$ $\quad L(\alpha A) = \alpha L(A)$?

$L(\alpha A^T - \alpha A) = L(\alpha(A^T - A)) = \alpha L(A^T - A)$ ✓

R2 if $(A+B)^T = A^T + B^T$ $\quad L(A+B) = L(A) + L(B)$?

$\hookrightarrow L((A+B)^T - (A+B)) = L(A^T - A) + (B^T - B))$ ✓

$\underline{M_L}$:

$\left[L\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\right]_E$

$\left[L\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\right]_E$

$\left[L\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right]_E$

$\left[L\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\right]_E$

M_L: $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\text{Ker}(L) = \begin{pmatrix} 0 & 0 \\ -1 & 1 \\ 1 & -1 \\ 0 & 0 \end{pmatrix} \rightarrow$ $\quad c - b = 0, \quad b - c = 0$

$c = b \quad b = k \quad c = k$

$\text{Ker}(L) = \left\{\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}\right\} = 1$

$\text{Range}(L) = 4 - 1 = 3$

7. (8pts) $L(A) := C^2A$

Is it a linear transformation?
Yes ■ No ☐

$M_L =$	$\begin{matrix} 7 & 0 & 10 & 0 \\ 0 & 7 & 0 & 10 \\ 15 & 0 & 22 & 0 \\ 0 & 15 & 0 & 2 \end{matrix}$	ker =	$\{(0)\}$	range =	$\underline{4}$ $\text{Span}\left\{\begin{pmatrix} 7 & 0 \\ 15 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 7 \\ 0 & 15 \end{pmatrix}, \begin{pmatrix} 10 & 0 \\ 22 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 10 \\ 0 & 22 \end{pmatrix}\right\}$

$\underline{R1}$ $A \in M_{2\times 2}$, $\alpha \in \mathbb{R}$ $\quad L(\alpha A) = \alpha L(A)$?

$L(C^2 \alpha A) = L(\alpha C^2 A) = \alpha L(C^2 A)$ ✓

$\underline{R2}$ $A, B \in M_{2\times 2}$

$L(C^2(A+B)) = L(C^2A) + L(C^2B)$ ✓

$\underline{M_L}$: $\left[L\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^2 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 7 & 0 \\ 15 & 0 \end{pmatrix}\right]_E$

$\left[L\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 0 & 7 \\ 0 & 15 \end{pmatrix}\right]$

$\left[L\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 10 & 0 \\ 22 & 0 \end{pmatrix}\right]$

$\left[L\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right]_E = \left[\begin{pmatrix} 0 & 10 \\ 0 & 22 \end{pmatrix}\right]$

$\underline{M_L} = \begin{pmatrix} 7 & 0 & 10 & 0 \\ 0 & 7 & 0 & 10 \\ 15 & 0 & 22 & 0 \\ 0 & 15 & 0 & 22 \end{pmatrix}$

$$\text{Ker}(L) = \left(\begin{array}{cccc|c} 1 & 0 & 132/90 & 0 & 0 \\ 0 & 1 & 0 & 132/90 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) = \{(0)\}$$

$$\text{Range}(L) = \text{Span}\left\{\begin{pmatrix} 7 & 0 \\ 15 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 7 \\ 0 & 15 \end{pmatrix}, \begin{pmatrix} 10 & 0 \\ 22 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 10 \\ 0 & 22 \end{pmatrix}\right\} = \mathbb{R}^{2\times 2}$$

Problem 2. [35pts] Let P_n be the vector space of polynomials of degree n or less, that has standard basis $E = \{1, x, x^2, \dots x^n\}$ (in that order).

Determine whether the following maps $L : P_2 \to P_3$ are linear transformations. If they are, find the matrix representing L with respect to the standard basis, and find $\ker(L)$ and $range(L)$. (**Note:** in each case the definition of the map L is for every polynomial $p \in P_2$. Also, p' denotes the derivative of p.) You do not need to explain the procedure here, just provide your answers.

1. (5pts) $L(p(x)) = xp(x)$

Is it a linear transformation?
Yes ■ No ☐

$M_L =$	$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	ker =	$\{(0)\}$	range =	Span $= \{x, x^2, x^3\}$

$p, q \in P_2$, $\alpha \in \mathbb{R}$, $L(\alpha p(x) + q(x)) = \alpha L(p(x)) + L(q(x))$

$P_2 = \{1, x, x^2\}$, $P_3 = \{1, x, x^2, x^3\}$.

$L(1) = x = 0 \cdot 1 + 1 \cdot x + 0 \cdot x^2 + 0 \cdot x^3$

$L(x) = x^2 = 0 \cdot 1 + 0 \cdot x + 1 \cdot x^2 + 0 \cdot x^3$

$L(x^2) = x^3 = 0 \cdot 1 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3$

$[M_L] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$Ker(L) = \{p = a + bx + cx^2 \in P_2 : L(p) = 0\} = \{(0)\}$, Range$(L)$ = Span$\{L(1), L(x), L(x^2)\}$ = Span$\{x, x^2, x^3\}$

2. (5pts) $L(p(x)) = x^2 + p(x)$

Is it a linear transformation?
Yes ☐ No ■

$M_L =$		ker =		range =	

$L(\alpha p(x)) = x^2 + \alpha p(x) \neq \alpha L\left(\frac{x^2}{\alpha} + p(x)\right)$

DOES NOT WORK

3. (5pts) $L(p(x)) = p(x) + xp(x) + x^2p'(x)$

Is it a linear transformation?
Yes ■ No ☐

$M_L =$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$	ker =	$\{(0)\}$	range =	span$\{1+x,\ x+2x^2,\ x^2+3x^3\}$

So, $p(x), q(x) \in \mathbb{R}^2;\ \alpha \in \mathbb{R}$

$L(1) = 1 + x + 0\cdot x^2 + 0\cdot x^3$

$L(x) = 0\cdot 1 + x + 2\cdot x + 0\cdot x^3$

$L(x^2) = 0\cdot 1 + 0\cdot x + 1\cdot x^2 + 3x^3$

$\ker(L) = \{L(p) = 0 = 0\} = \{0\}$

$L(\alpha p(x) + q(x)) = \alpha p(x) + q(x) + x(\alpha p(x) + q(x)) + x^2(\alpha p'(x) + q'(x))$

$= \alpha(p(x) + xp(x) + x^2p'(x)) + q(x) + xq(x) + x^2q'(x)$

$= \alpha L(p(x)) + L(q(x))$ R1 & R2 ✓

4. (5pts) $L(p(x)) = p(0)x + p(1)$

Is it a linear transformation?
Yes ■ No ☐

$M_L =$	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	ker =	$\{x - x^2\}$	range =	span$\{1, 1+x\}$

5. (5pts) $L(p(x)) = p(x)p'(x)$

Is it a linear transformation?
Yes ☐ No ■

$M_L =$		ker =		range =	

R1 $L(\alpha p(x)) = L(\alpha p(x)\, p'(x)) = \alpha L(p(x)p'(x))$ ✓

R2 $L(p(x)+q(x)) = L(p(x)) + L(q(x)) = L((p(x)+q(x))p'(x))$

$p(x)p'(x) + q(x)q'(x)$

$\neq$

$(p(1)+q(1))(p'(1)+q'(1))$

R2 DOES NOT HOLD BECAUSE
$p(x)p'(x) + q(x)q'(x)$
$\neq$
$(p(x)+q(x))(p'(x)+q'(x))$

6. (5pts) $L(p(x)) = p(x) - p'(x)$

Is it a linear transformation?
Yes ■ No ☐

$M_L =$	$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	ker =	$\{(0)\}$	range =	$\text{Span}\{1, x-1, x^2-2x\}$

R1 $p(x) \in \mathbb{P}_2$, $\alpha \in \mathbb{R}$ $\quad L(\alpha p) = \alpha L(p)$?

$L(\alpha p(x) - \alpha p'(x)) = \alpha L(p(x) - p'(x))$

$L(A+B) = L(A) + L(B) = L(A+B) - (A+B) = L(A+B) = L(A+A)(B-B)$ ✓

7. (5pts) $L(p(x)) = xp'(x) + p''(x)$

Is it a linear transformation?
Yes ■ No ☐

$M_L =$	$\begin{pmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$	ker =	All real numbers $\{a \in \mathbb{R}\}$	range =	$\text{Span}\{x, 2+2x^2\}$

Problem 3. [9pts] If A and B be are similar $n \times n$ matrices, explain why $det(A) = det(B)$.

IF THESE ARE SIMILAR THAN THERE IS AN INVERTIBLE MATRIX P. $\quad B = P^{-1}AP$

$\det(B) = \det(P^{-1})\det(A)\det(P)$ and

$\det(B) = \det(P^{-1})\det(P)\det(A)$ and P and P^{-1} cancel out

$\underline{\det(B) = \det(A)}$ ✓