MIDTERM – 2 SOLUTIONS

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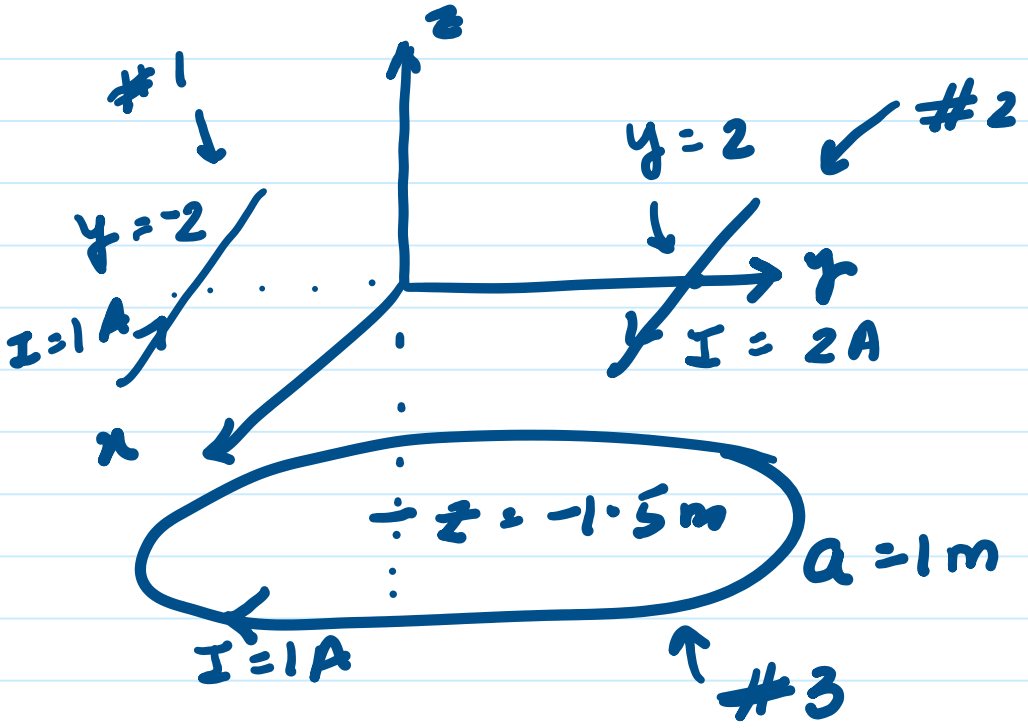
1.



$$\bar{H}_1 = \frac{I}{2\pi\rho}\hat{c}_\phi = \frac{1}{2\pi(2)}(-\hat{c}_z) = -0.0796\,\hat{c}_z$$

$$\bar{H}_2 = \frac{2}{2\pi(2)}(-\hat{c}_z) = -0.159\,\hat{c}_z$$

$$\bar{H}_3 = \frac{1}{2}\frac{I a^2}{(a^2+z^2)^{3/2}}(-\hat{c}_z)$$

$$= \frac{1}{2}\frac{1}{(1+(1.5)^2)^{3/2}}(-\hat{c}_z) = 0.085\,(-\hat{c}_z)$$

$$\Rightarrow \bar{H} = \bar{H}_1 + \bar{H}_2 + \bar{H}_3 = -0.32\,\hat{c}_z \left(\frac{A}{m}\right)$$

2. $\bar{E} = 2e^{-0.126z} \sin(4.3\times10^8 t - 0.126z)\,\hat{e}_x \left(\frac{\mu V}{m}\right)$

(a) $f = \frac{\omega}{2\pi} = \frac{4.3\times10^8}{2\pi} = 0.684\times10^8$ Hz

$= 68.4$ MHz

(b) $\alpha = 0.126$ Np/m

(c) $+z$ DIRECTION

(d) INITIAL AMPLITUDE $= 2\,\mu V/m$
AMPLITUDE $= 2e^{-0.126z}\ \mu V/m$

(e) $\bar{E} = 2e^{-0.126z}\cos\left(4.3\times10^8 t - 0.126z - \frac{\pi}{2}\right)\hat{e}_x$

$\tilde{E} = 2e^{-0.126z}\, e^{j(-0.126z - \pi/2)}\,\hat{e}_x \left(\frac{\mu V}{m}\right)$

(f) $\frac{E_{i0}}{H_{i0}} = \eta_c \Rightarrow H_{i0} = \frac{E_{i0}}{\eta_c} = \frac{2}{0.044\angle 45^\circ}$

$= 45.45\angle -45^\circ$

$$\Rightarrow \vec{H} = 45.45\, e^{-j\pi/4}\, e^{-0.126z}\, e^{j(-0.126z - \pi/2)}\, \hat{e}_y \left(\frac{\mu A}{m}\right)$$

(g) $\vec{H}(z,t)$

$$= 45.45\, e^{-0.126z} \cos\left(4.3\times10^{8}t - 0.126z - \frac{\pi}{2} - \frac{\pi}{4}\right) \hat{e}_y \left(\frac{\mu A}{m}\right)$$

$$= 45.45\, e^{-0.126z} \cos\left(4.3\times10^{8}t - 0.126z - \frac{3\pi}{4}\right) \hat{e}_y \left(\frac{\mu A}{m}\right)$$

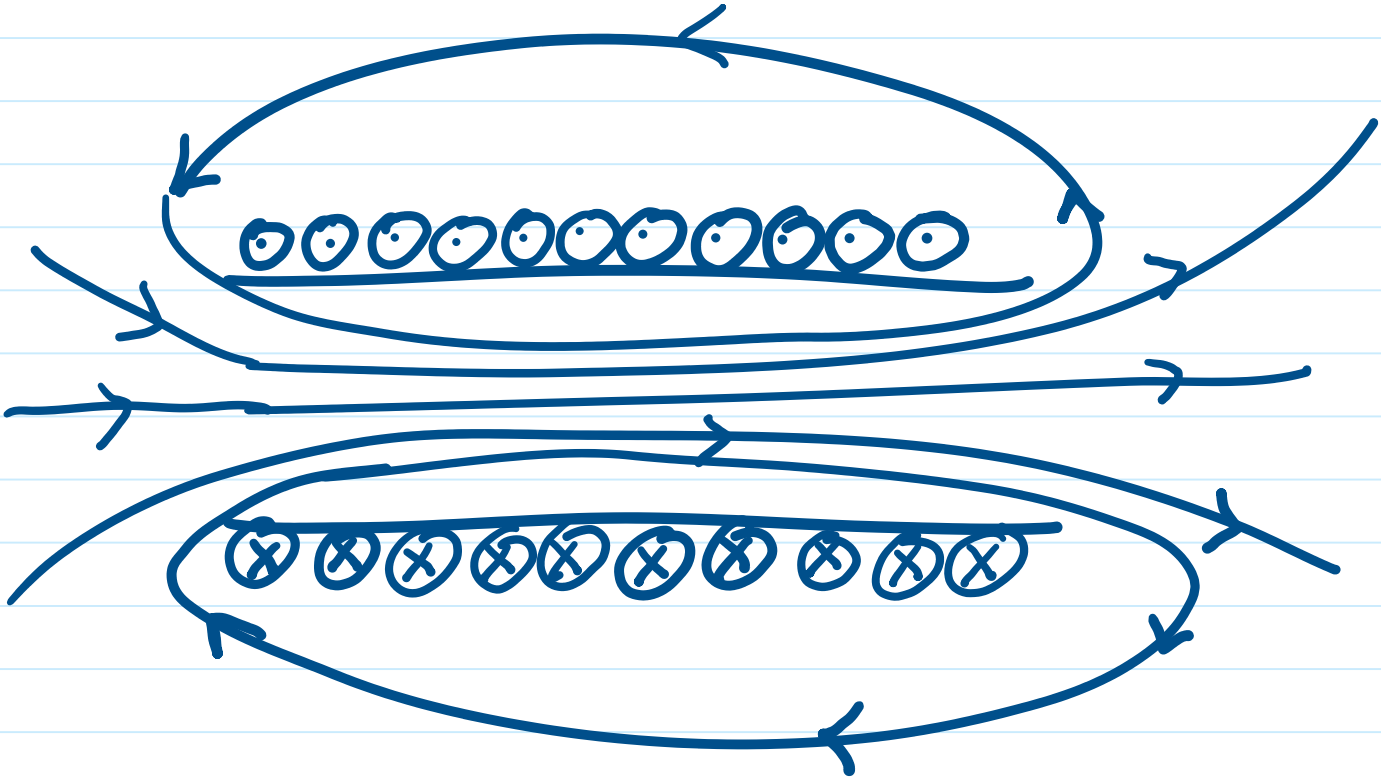
3. $\bar{H}_1 = 7\,\hat{e}_y$ $\qquad \hat{e}_n = \hat{e}_y$

$$B_{1n} = B_{2n}$$

$$\Rightarrow \mu_{1r} H_{1n} = \mu_{2r} H_{2n}$$

$$\Rightarrow \bar{H}_2 = \frac{7}{3}\,\hat{e}_y$$

4. LIKE A PERMANENT MAGNET:



5. $N = 10$

$$V_{emf} = -N \frac{d}{dt} \iint_S \bar{B} \cdot d\bar{s}$$

$$= -10 \frac{d}{dt} \int_{x=-0.5}^{0.5} \int_{y=-0.5}^{0.5} 20 e^t \hat{e}_z \cdot dx\,dy\,\hat{e}_z$$

$$= -10 \times 20 \times e^t \times 1 \times 1$$

$$= -200 e^t \ V$$

6. $\bar{E} = \cos\left(7\pi \times 10^9 t - 176.68 z\right) \hat{c}_x$ (V/m)

$\beta = \omega\sqrt{\mu\varepsilon}$

$\Rightarrow 176.68 = 7\pi \times 10^9 \sqrt{\mu_0 \varepsilon_0 \varepsilon_r}$

$\Rightarrow \sqrt{\varepsilon_r} = 2.41$

$\Rightarrow \varepsilon_r = 5.81$.

7 $a = 1\,m$ $\omega = 2\ rad/s$

$\bar{v} = \omega\rho\,\hat{c}_\phi$ $\bar{B} = \frac{3}{\rho}\hat{c}_\phi$.

$V_{emf} = \int_L (\bar{v} \times \bar{B}) \cdot d\bar{l} = 0$

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8. (a) 1.3 m

9 (b) $-\hat{c}_z$

10 (b) 40 mH

11. (d) 160.75 Ω

12. (c) $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$