Lecture # 9

ECEN 438/738 Power Electronics

Spring 2025 Semester

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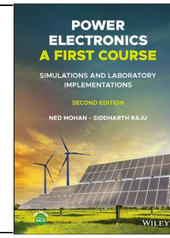
ECEN 438/738 Power Electronics



Power Electronics
A First Course: 2nd Edition

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Chapter 5
RECTIFICATION OF UTILITY INPUT USING DIODE RECTIFIERS

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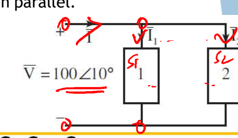
Conservation of Complex Power - ECEN 340

Suppose the two loads in Example # 1 and Example # 2 are connected in parallel.

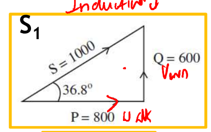
$\overline{S}_1 = 800 + j600 \text{ VA}$ $\qquad \overline{S}_2 = 433 - j250 \text{ VA}$



The Total Power S is: $\overline{S} = \overline{S}_1 + \overline{S}_2 = (800 + 433) + j(600 - 250)$

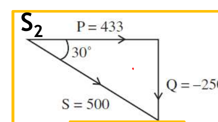
$= 1233 + j350 \text{ VA}$

Inductive



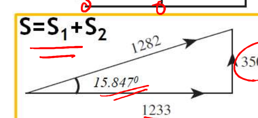
Power triangle

+



Power triangle

=



Combined Power Triangle

The Total Input current I : $\overline{I} = \dfrac{\overline{S}^*}{\overline{V}^*} = \dfrac{1233 - j350}{100\angle -10^\circ} = \dfrac{1282\angle -15.847^\circ}{100\angle -10^\circ} = 12.82\angle -5.847^\circ \text{ A}$

Same

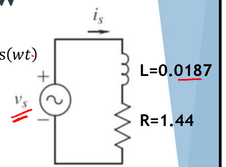
From KVL, Input current: $\overline{I} = \overline{I}_1 + \overline{I}_2 = 10\angle -26.8^0 + 5\angle 30^0 = 12.82\angle -5.847^0$

This proves the conservation of complex power

Power Factor / RMS - Fundamentals - Review

$$I = \frac{V}{Z} = \frac{120\angle 0}{1.44 + j7.054} = 16.67\angle -78.46^\circ$$

$v_s = \sqrt{2} * 120 * \cos(wt)$
$w = 2 * \pi * f$ $= 2\pi \times 60$, $V = 377$

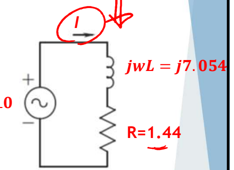


Complex Power

V I^*

$$S = V \cdot I^* = (120\angle 0)(16.67\angle +78.46^\circ)$$
$$= 2000\angle 78.46$$
$$\boxed{S = 400 + j1959.6}$$

78.46° I (lagging) voltage



Real Power P = 400 W; Reactive Power Q = 1959.6 VAR

$$Power\ Factor: \quad PF = \frac{real\ power}{apparent\ power} = \frac{P}{V_{rms} * I_{rms}} = \frac{P}{|S|} = \frac{400}{2000} = 0.2$$

$$PF = \frac{V * I * \cos(\emptyset)}{V * I} = \cos(\emptyset) = \cos(78.46^0) = 0.2$$

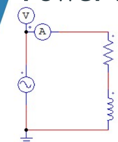
Power Factor / RMS - Fundamentals - PSIM Simulations

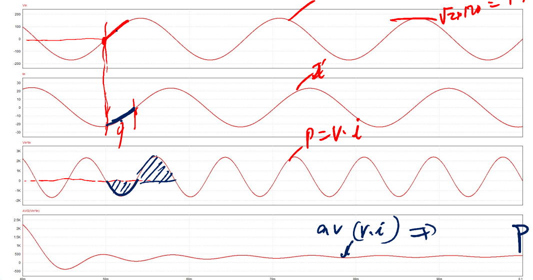
$P = V * I * \cos(\emptyset) = 400\ W$

$I = 16.67\ A$

$v_s = \sqrt{2} * 120 * \cos(wt)$
$w = 2 * \pi * f$

If PF = 1, i.e. ∅=0, then for V=120, we needs only I = 3.33 A for P = 400 Watts
Q - How can accomplish this??

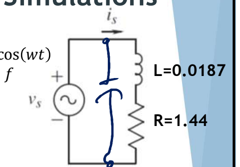


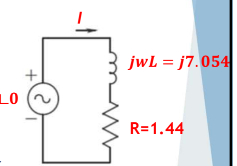


$\sqrt{2} \times 120 = 170\ V$

$p = v \cdot i$

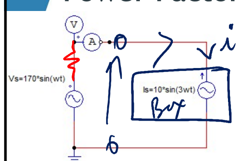
av $(v \cdot i) \Rightarrow$ $P = \frac{1}{T}\int_0^T v \cdot i \cdot dt$

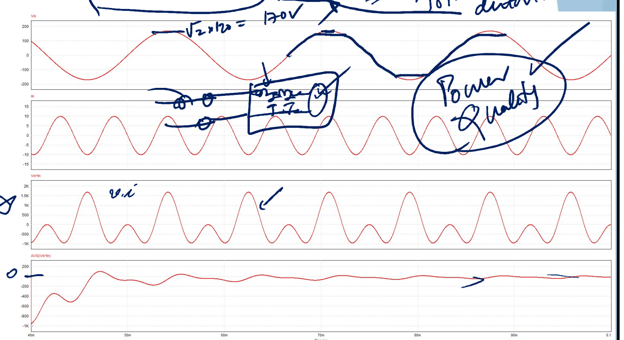




Power Factor / RMS - Harmonics - THD - DF

Total harmonic distortion





$\sqrt{2} \times 120 = 170\ V$

Power Quality

$v \cdot i$

v(t) = 170 * cos(wt) V

i(t) = 10 * cos(3wt) V

Instantaneous Power
p(t) = v(t) * i(t)

Average Power

$$P = \frac{1}{2\pi}\int_0^{2\pi} 170 * \cos(wt) * 10 * \cos(3wt)\, d(wt) \quad = 0 \text{ (from calculus)}$$

Therefore voltage and current of different frequencies do not contribute to real power W

Definition of rms

if $v(t) = V_1 * \sin(\omega_1 t) + V_2 * \sin(\omega_2 t)$ + - - - -

$$v(t) = V_1 \sin\omega t + V_2 \sin 3\omega t$$

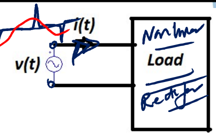
Then V_{rms} is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (v_1 + v_2)^2\, dt}$$
$$= \sqrt{\frac{1}{T}\int_0^T (v_1^2 + 2v_1v_2 + v_2^2)\, dt}$$
$$= \sqrt{\frac{1}{T}\int_0^T V_1^2 \sin^2\omega t + \frac{1}{T}\int 2V_1V_2 \sin\omega t \cdot \sin 3\omega t\, dt + \frac{1}{T}\int_0^T V_2^2 \sin^2 3\omega t\, dt}$$
$$V_{rms} = \sqrt{\left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2}$$

Therefore V_{rms} is

$$V_{rms} = \sqrt{\left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2}$$

Total Harmonics - THD - PF - DF



Power Factor: $PF = \dfrac{Real\ Power\ (watts)}{Apparent\ Power\ (volt.amps)}$

$v(t) = \sqrt{2}V * \cos(wt)$ Volts

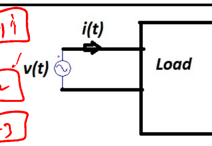
$i(t) = \sqrt{2}I_1 \cos(wt - \emptyset_1) + \sqrt{2}I_3 \cos(3wt + \emptyset_3) + \sqrt{2}I_5 \cos(5wt + \emptyset_5)$ Amps

Real Power "P" =

odd

even

Total Harmonics - THD - PF - DF



Power Factor: $PF = \dfrac{Real\ Power\ (watts)}{Apparent\ Power\ (volt.amps)}$

$v(t) = \sqrt{2}V * \cos(wt)$ Volts

$i(t) = \sqrt{2}I_1 \cos(wt - \emptyset_1) + \sqrt{2}I_3 \cos(3wt + \emptyset_3) + \sqrt{2}I_5 \cos(5wt + \emptyset_5)$ Amps

Note: $I_{rms} = \sqrt{I_1^2 + I_3^2 + I_5^2}$

Watts

Real Power "P" $= VI_1 * Cos(\emptyset_1)$

Apparent Power $= V * I_{rms}$

Power Factor: $PF = \dfrac{Real\ Power\ (watts)}{Apparent\ Power\ (volt.amps)} = \dfrac{VI_1 * Cos(\emptyset_1)}{V * I_{rms}} = \dfrac{I_1 * Cos(\emptyset_1)}{I_{rms}}$

$I_{rms} > I_1$

Power Factor: $PF = \dfrac{I_1}{I_{rms}} * Cos(\emptyset_1)$

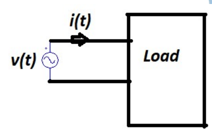
Distortion Factor (DF) $= \dfrac{I_1}{I_{rms}}$

$0 \le DF \le 1$

Displacement Factor (DPF) = $Cos(\emptyset_1)$

Power Factor: $PF = DF * DPF$

Total Harmonics - THD - PF - DF



$v(t) = \sqrt{2}V * \sin(wt)$ Volts

$i(t) = \sqrt{2}I_1 \sin(wt - \emptyset_1) + \sqrt{2}I_3 \sin(3wt + \emptyset_3) + \sqrt{2}I_5 \sin(5wt + \emptyset_5)$ Amps

$THD = \dfrac{rms\ of\ the\ harmonic\ components}{rms\ of\ the\ fundamental\ component}$

$DF = \dfrac{I_1}{I_{rms}}$

Note: $I_{rms} = \sqrt{I_1^2 + I_3^2 + I_5^2}$

Therefore rms of the harmonic components $= \sqrt{I_3^2 + I_5^2}$

Not: $\sqrt{I_3^2 + I_5^2} = \sqrt{I_{rms}^2 - I_1^2}$

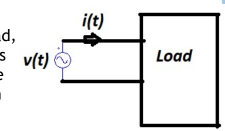
But THD $\equiv 0$

Therefore $THD = \dfrac{\sqrt{I_3^2 + I_5^2}}{I_1} = \dfrac{\sqrt{I_{rms}^2 - I_1^2}}{I_1} = \sqrt{\dfrac{I_{rms}^2}{I_1^2} - 1} = \sqrt{\dfrac{1}{DF^2} - 1}$

$$\mathbf{THD = \sqrt{\frac{1}{DF^2} - 1}}$$

Example Power Computations

A sinusoidal voltage of $v(t) = 170 * \sin(wt)\ V$ is applied to a nonlinear load, resulting in a non-sinusoidal current that is expressed in Fourier series as $i(t) = 10\sin(wt - 30^0) + 6\sin(3wt + 45^0) + 3\sin(5wt + 20^0)\ A$. Determine a) the power (W) absorbed by the load, b) the power factor, c) distortion factor DF, and d) THD of the load current.

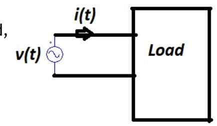


$P = V_{rms} * I_{rms} * \cos(\emptyset)\ W$

Where $\emptyset$ is the angle between the voltage and the current of the same frequency

Example Power Computations

A sinusoidal voltage of $v(t) = 170 * \sin(wt)\ V$ is applied to a nonlinear load, resulting in a non-sinusoidal current that is expressed in Fourier series as $i(t) = 10\sin(wt - 30^0) + 6\sin(3wt + 45^0) + 3\sin(5wt + 20^0)\ A$. Determine a) the power (W) absorbed by the load, b) the power factor, c) distortion factor DF, and d) THD of the load current.



$P = V_{rms} * I_{rms} * \cos(\emptyset)\ W$ Where $\emptyset$ is the angle between the voltage and the current of the same frequency

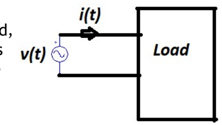
$$Therefore:\ P = \frac{170}{\sqrt{2}} * \frac{10}{\sqrt{2}}\cos(30^0) = 736.12\ \ W$$

$$I_{rms} = \sqrt{\left(\frac{10}{\sqrt{2}}^2\right) + \left(\frac{6}{\sqrt{2}}^2\right) + \left(\frac{3}{\sqrt{2}}^2\right)} = 8.515\ A$$

$$PF = \frac{real\ power}{apparent\ power} = \frac{736.12}{V_{rms} * I_{rms}} = \frac{736.12}{\frac{170}{\sqrt{2}} * 8.515} = 0.72$$

Example Power Computations

A sinusoidal voltage of $v(t) = 170 * \sin(wt)\ V$ is applied to a nonlinear load, resulting in a non-sinusoidal current that is expressed in Fourier series as $i(t) = 10\sin(wt - 30^0) + 6\sin(3wt + 45^0) + 3\sin(5wt + 20^0)\ A$. Determine a) the power (W) absorbed by the load, b) the power factor, c) distortion factor DF, and d) THD of the load current.



(c) $DF = \frac{I_1}{I_{rms}} = \frac{(10/\sqrt{2})}{8.515} = 0.83$

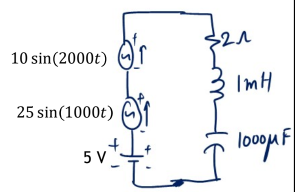
(d) $THD = \frac{\sqrt{\left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2}}{\left(10/\sqrt{2}\right)} = 0.672$ or $\underline{67.2\%}$

$$THD = \sqrt{\frac{1}{DF^2} - 1}$$

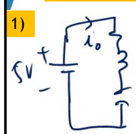
Example Power Computations

A voltage source of $v(t) = 5 + 25\sin(1000t) + 10\sin(2000t)\ V$ is series combination of 2-ohm resistor, a 1 milli-H inductor and a 1000 micro-F capacitor. Determine the rms current in the circuit, and determine the power absorbed by each component.

Apply Superposition

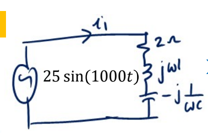


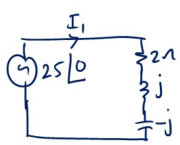
1)



$i_0 = 0$ (Cap is open cct)

2)





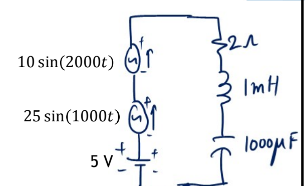
$\underline{I_1 = 12.5\angle 0}$

$i_1(t) = 12.5\sin(1000t)$

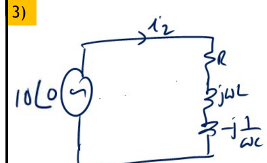
Example Power Computations

A voltage source of $v(t) = 5 + 25\sin(1000t) + 10\sin(2000t)\ V$ is series combination of 2-ohm resistor, a 1 milli-H inductor and a 1000 micro-F capacitor. Determine the rms current in the circuit, and determine the power absorbed by each component.

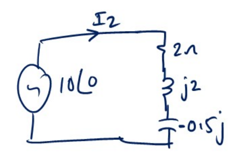
Apply Superposition



3)



$\Rightarrow$



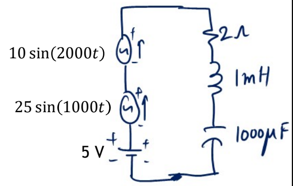
$$I_2 = \frac{10\angle 0^\circ}{2 + j1.5} = 4\angle -36.86^\circ$$

$i_2(t) = 4\sin(2000t - 36.86^0)$

Example Power Computations

A voltage source of $v(t) = 5 + 25\sin(1000t) + 10\sin(2000t)\ V$ is series combination of 2-ohm resistor, a 1 milli-H inductor and a 1000 micro-F capacitor. Determine the rms current in the circuit, and determine the power absorbed by each component.

Apply Superposition



$i(t) = 0 + 12.5\sin(1000t) + 4\sin(2000t - 36.86^0)\ A$

$$I_{rms} = \sqrt{\left(\frac{12.5}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = 9.28\ A$$

$$P = I_{rms}^2 \cdot (2) = \underline{172.25\ \text{Watts}}$$

Note: Inductors / Capacitors DO NOT consume real power

$$P = \left(\frac{25}{\sqrt{2}}\right)\left(\frac{12.5}{\sqrt{2}}\right)\cos 0 + \left(\frac{10}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right)\cos(36.86^\circ) = \underline{172.25\ \text{Watts}}$$

Thank you!!