Lecture # 13

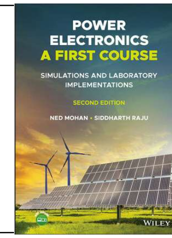
ECEN 438/738 Power Electronics

Spring 2025 Semester

Dr. Prasad Enjeti,
Electrical & Computer Engineering
Texas A&M University
http://enjeti.tamu.edu

1

ECEN 438/738 Power Electronics



Power Electronics
A First Course: 2nd Edition

Free Textbook Online Access Link: https://go.oreilly.com/TAMU/library/view/-/9781119818564/?ar

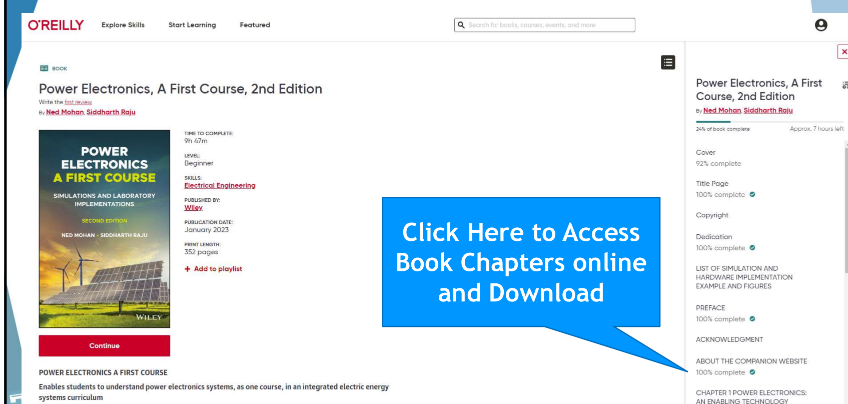
Chapter 7

Magnetic Circuit Concepts

2

ECEN 438/738 Power Electronics

Free Textbook Access Link: https://go.oreilly.com/TAMU/library/view/-/9781119818564/?ar

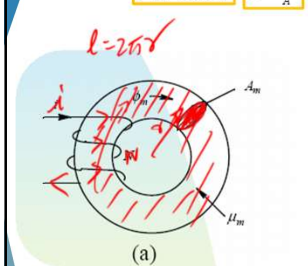


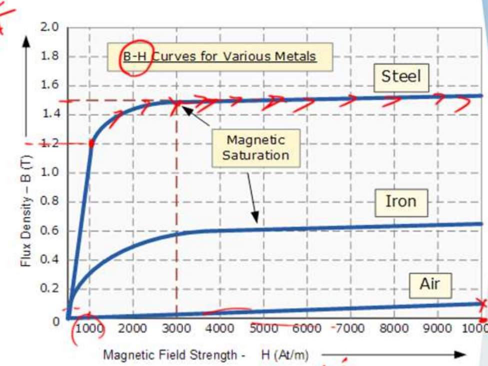
3

Chater # 7 Magnetic Circuit Concepts

https://www.electronics-tutorials.ws/electromagnetism/magnetic-hysteresis.html

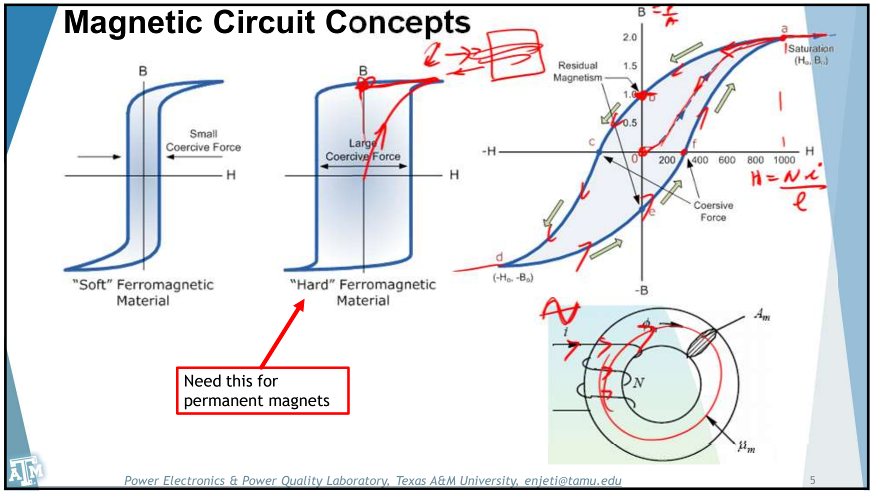
Flux density $B = \frac{\phi}{A}$ $\quad B = \frac{\phi}{A}$



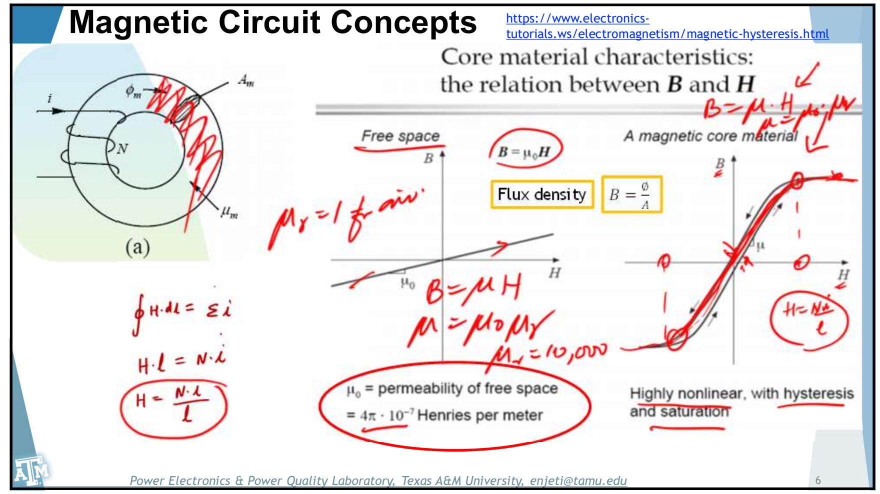


$H = \frac{N \cdot i}{l}$

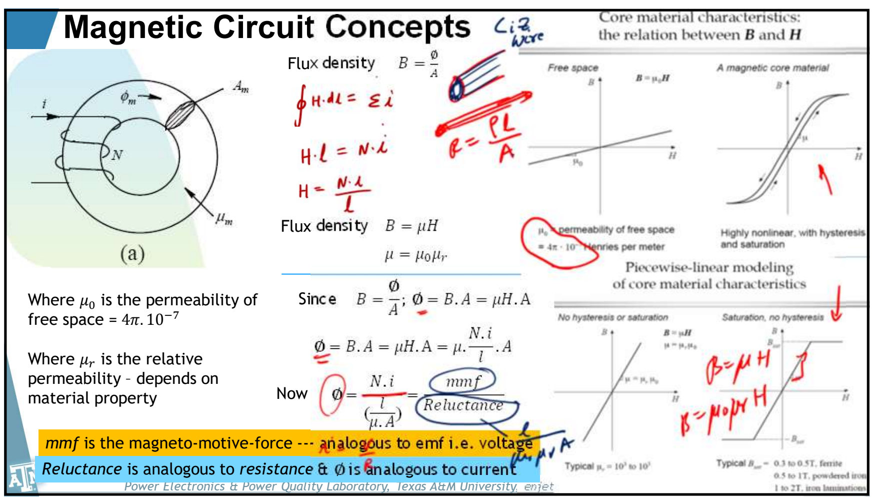
4


Magnetic Circuit Concepts
B
H
Small
Coercive Force
"Soft" Ferromagnetic
Material
B
H
Large
Coercive Force
"Hard" Ferromagnetic
Material
Need this for
permanent magnets
B
Residual
Magnetism
2.0
1.5
1.0
0.5
-H
200
400
600
800
1000
H
Saturation
(Hₛ, Bₛ)
Coersive
Force
(-Hₛ, -Bₛ)
-B
$H = \frac{N \cdot i}{l}$
A_m
N
μ_m
Power Electronics & Power Quality Laboratory, Texas A&M University, enjeti@tamu.edu

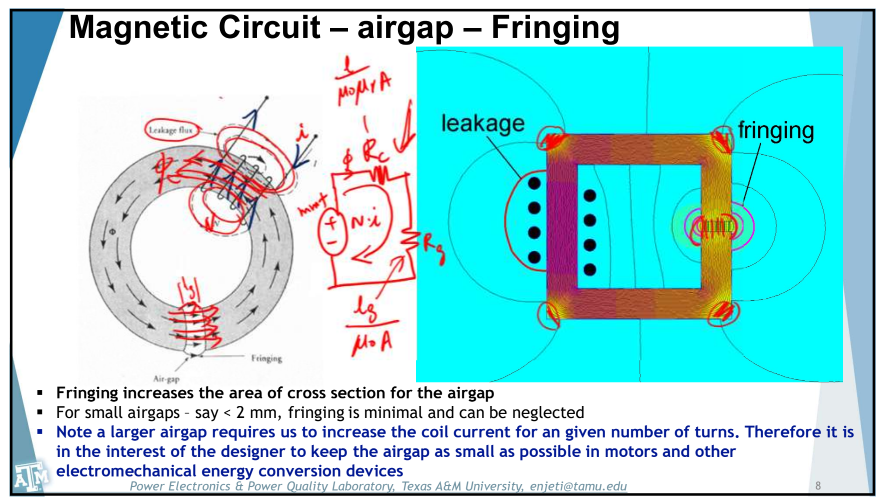
5


Magnetic Circuit Concepts
https://www.electronics-tutorials.ws/electromagnetism/magnetic-hysteresis.html
Core material characteristics:
the relation between B and H
Free space
$B = \mu_0 H$
Flux density
$B = \frac{\emptyset}{A}$
A magnetic core material
$\mu_r = 1$ for air
$B = \mu H$
$\mu = \mu_0 \mu_r$
$\mu_r = 10{,}000$
$\oint H \cdot dl = \Sigma i$
$H \cdot l = N \cdot i$
$H = \frac{N \cdot i}{l}$
μ_0 = permeability of free space
$= 4\pi \cdot 10^{-7}$ Henries per meter
Highly nonlinear, with hysteresis
and saturation
(a)
Power Electronics & Power Quality Laboratory, Texas A&M University, enjeti@tamu.edu

6

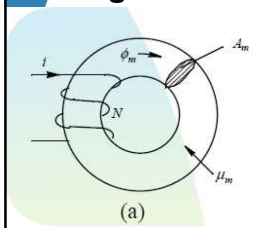

Magnetic Circuit Concepts
Core material characteristics:
the relation between B and H
Flux density $B = \frac{\emptyset}{A}$
$\oint H \cdot dl = \Sigma i$
$H \cdot l = N \cdot i$
$H = \frac{N \cdot i}{l}$
$R = \frac{\rho l}{A}$
Flux density $B = \mu H$
$\mu = \mu_0 \mu_r$
Where μ_0 is the permeability of free space = $4\pi . 10^{-7}$
Where μ_r is the relative permeability - depends on material property
Since $B = \frac{\emptyset}{A}$; $\emptyset = B.A = \mu H.A$
$\emptyset = B.A = \mu H.A = \mu.\frac{N.i}{l}.A$
Now $\emptyset = \frac{N.i}{\left(\frac{l}{\mu.A}\right)} = \frac{mmf}{Reluctance}$
Piecewise-linear modeling
of core material characteristics
No hysteresis or saturation
Saturation, no hysteresis
$B = \mu H$
$B = \mu_0 \mu_r H$
mmf is the magneto-motive-force --- analogous to emf i.e. voltage
Reluctance is analogous to resistance & $\emptyset$ is analogous to current
Power Electronics & Power Quality Laboratory, Texas A&M University, enjeti

7


Magnetic Circuit – airgap – Fringing
$\frac{l}{\mu_0 \mu_r A}$
R_c
$N \cdot i$
R_g
$\frac{l_g}{\mu_0 A}$
leakage
fringing
Leakage flux
Fringing
Air-gap
▪ Fringing increases the area of cross section for the airgap
▪ For small airgaps - say < 2 mm, fringing is minimal and can be neglected
▪ Note a larger airgap requires us to increase the coil current for an given number of turns. Therefore it is in the interest of the designer to keep the airgap as small as possible in motors and other electromechanical energy conversion devices
Power Electronics & Power Quality Laboratory, Texas A&M University, enjeti@tamu.edu

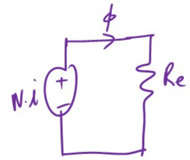
8

Magnetic Circuit Concepts

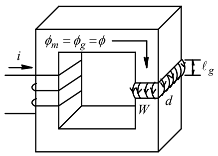


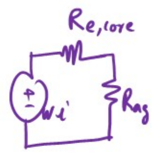
Since $\emptyset = \frac{N.i}{(\frac{l}{\mu.A})} = \frac{mmf}{Reluctance}$

The magnetic equivalent circuit is

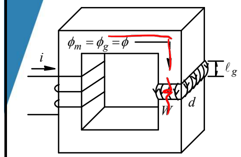


Magnetic equivalent circuit with an airgap





Magnetic Circuit – with an airgap

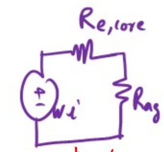


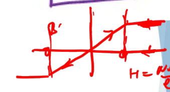
$\emptyset = \frac{mmf}{Reluctance}$

$\emptyset = \frac{N.i}{\left(\frac{l_m}{\mu_0\mu_r.A}\right)+\left(\frac{l_g}{\mu_0.A}\right)}$

Neglect fringing and assume A is same for the core and for the airgap

l_m is the length of the core
l_g is the length of the airgap





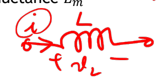
Inductance *L*

Is defined as flux linkages per ampere of current

At any given instant of time, the flux linkages of the coil λ_m, due to the flux lines entirely in the core, is equal to the flux $\emptyset_m$ times the number of turns N that are linked.

This flux linkages is related to the current i by a parameter defined as the inductance L_m

$v_L = N\frac{d\phi}{dt}$



$\lambda_m = N.\emptyset_m = L_m.i;$ $Therefore$ $L_m = \frac{\lambda_m}{i} = \frac{N.\emptyset_m}{i}$

$Therefore$ $L_m = \frac{N^2}{\frac{l_m}{\mu_0\mu_r.A}+\frac{l_g}{\mu_0.A}} = \frac{N^2}{Relucatnce\ of\ the\ core + Reluctance\ of\ the\ airgap}$

$v_L = L\frac{di}{dt}$

Magnetic Circuit Concepts – Inductance L – Energy Storage

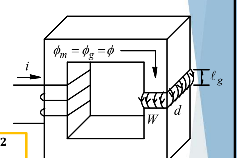
Energy stored in an inductor is: $W = \frac{1}{2}L_m i^2 \ [J]$

$L_m = \frac{N^2}{\Re_m} = \frac{N^2}{\frac{\ell_m}{\mu_m A_m}}$ $i = \frac{H_m \ell_m}{N}$

$$\therefore W = \frac{1}{2}\left(\frac{N^2}{\frac{\ell_m}{\mu_m A_m}}\right)\left(\frac{H_m^2\ell_m^2}{N^2}\right)$$

$$= \frac{1}{2}(A_m\ell_m)(\mu_m H_m^2)$$

$$= \frac{1}{2}(A_m\ell_m)\frac{B_m^2}{\mu_m}$$



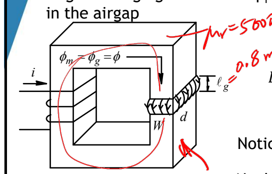
Energy Stored in the core is: $W_{core} = \frac{1}{2}\frac{B^2}{\mu_0\mu_r}$

$$w = \frac{1}{2}\frac{B^2}{\mu} \quad [J/m^3]$$

Energy Stored in the airgap is: $W_{airgap} = \frac{1}{2}\frac{B^2}{\mu_0}$

Magnetic Circuit Concepts – Inductance L

Problem # 1: The magnetic circuit shown in the figure, has the following dimensions: Area of cross section A_c = 1cm², mean length of the core l_m = 15cm, length of the airgap l_g = 0.8mm, number of turns N = 480. Neglect fringing and assume μ_r=5000 for the core, a) Calculate the inductance, b) energy stored in the core and in the airgap



Note $\mu_0 = 4\pi.10^{-7}$ for MKS units

$L_m = \frac{N^2}{\frac{l_m}{\mu_0\mu_r.A}+\frac{l_g}{\mu_0.A}} = \frac{480^2}{(0.24 * 10^6)+(6.37 * 10^6)} = 34.85\ milli - H$

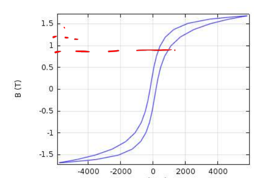
Notice the core reluctance is only 3.7% of the reluctance of the airgap

Neglecting the core reluctance the inductance is $L_m \approx 36.17\ milli - H$

Now lets calculate the flux density if the current i = 1A

$B = \frac{\emptyset}{A};$ $and\ \emptyset = \frac{N.i}{\frac{l_m}{\mu_0\mu_r.A}+\frac{l_g}{\mu_0.A}}$

$B = 0.73\ T$

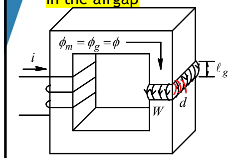


Since the iron core saturates at B ~ 1.4 T, we can conclude the above inductors Current rating should not exceed 1.5 A or so

Magnetic Circuit Concepts – Inductance L

Problem # 1: The magnetic circuit shown in the figure, has the following dimensions: Area of cross section A_c = 1cm², mean length of the core l_m = 15cm, length of the airgap l_g = 0.8mm, number of turns N = 480. Neglect fringing and assume μ_r=5000 for the core, a) Calculate the inductance, b) energy stored in the core and in the airgap

Note $\mu_0 = 4\pi.10^{-7}$ for MKS units



$$Energy\ Stored\ in\ the\ core\ is:\ W_{core} = \frac{1}{2}\frac{B^2}{\mu_0\mu_r} = \frac{1}{2}\frac{(0.78)^2}{\mu_0\mu_r} = 48\,J/m^3$$

$$Energy\ Stored\ in\ the\ airgap\ is: W_{airgap} = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{(0.78)^2}{\mu_0} = 240{,}000\,J/m^3$$

Energy stored in the airgap is much greater than the energy stored in the core

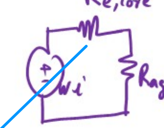
13

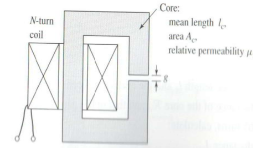
Magnetic Circuit Concepts – Inductance L

Problem # 2: Design an inductor using the core shown in the figure. The dimensions are as follows: A_c = 5cm², l_c = 25cm. Calculate the air-gap length g and the number of turns N such that the inductance is 1.4mH and that the inductor can operate at peak currents of 6A without saturating. Assume that saturation occurs when the peak flux density in the core exceeds 1.7T and that, below saturation the core has a μ_r=3200

$$L = \frac{N^2}{R_c + R_{ag}}$$

$L = 0.03$ H
$= 30$ mH



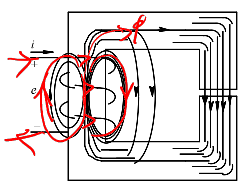


$$R_c = \frac{l_c}{\mu_0\mu_r A} = \frac{15\times10^{-2}}{4\pi\times10^{-7}\times1000\times1\times10^{-4}}$$
$= 1193662 \Rightarrow 1.2\times10^6$

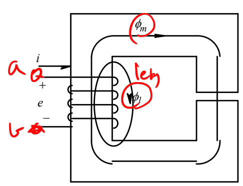
$$R_{ag} = \frac{l_g}{\mu_0 A} = \frac{0.8\times10^{-3}}{4\pi\times10^{-7}\times1\times10^{-4}}$$
$= 6.4\times10^6$

14

Magnetizing Inductance & Leakage



(a)



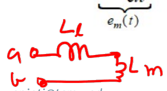
(b)

Flux linkages: $\lambda = N\phi = \underbrace{N\phi_m}_{\lambda_m} + \underbrace{N\phi_\ell}_{\lambda_\ell} = \lambda_m + \lambda_\ell$

Therefore: $L_{self} = L_m + L_\ell$

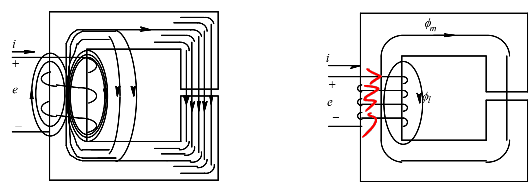
$$\underbrace{\frac{\lambda}{i}}_{L_{self}} = \underbrace{\frac{\lambda_m}{i}}_{L_m} + \underbrace{\frac{\lambda_\ell}{i}}_{L_\ell}$$

$$e(t) = \underbrace{L_m\frac{di}{dt}}_{e_m(t)} + L_\ell\frac{di}{dt}$$

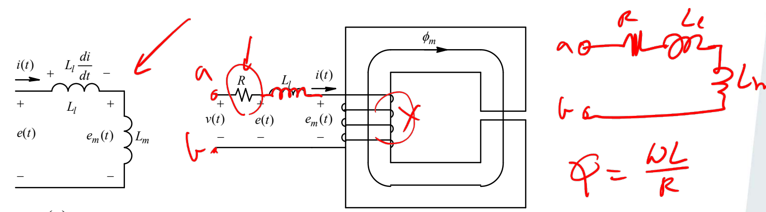


15

Magnetizing Inductance & Leakage



$$e(t) = \underbrace{L_m\frac{di}{dt}}_{e_m(t)} + L_\ell\frac{di}{dt}$$



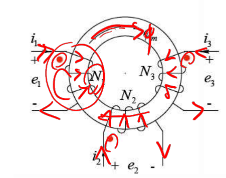
$$\varphi = \frac{\omega L}{R}$$

Where R is the resistance of the coil & Rc is the core loss representation for hysteresis and eddy current losses

16

Transformers – Basics

Faraday's Law – a rate of change in flux results in induced voltage



Transformer with three windings

Understand the dot convention

$$e_1 = N_1 \frac{d\phi_m}{dt}$$

$$e_2 = N_2 \frac{d\phi_m}{dt}$$

$$e_3 = N_3 \frac{d\phi_m}{dt}$$

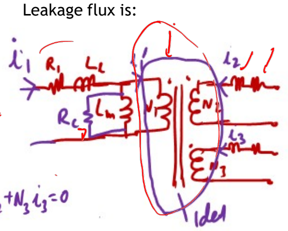
Therefore: $\frac{d\phi_m}{dt} = \frac{e_1}{N_1} = \frac{e_2}{N_2} = \frac{e_3}{N_3}$

$$\Rightarrow \quad \phi_m = \frac{1}{N_1}\int e_1 dt = \frac{1}{N_2}\int e_2 dt = \frac{1}{N_3}\int e_3 dt$$

$N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

The equivalent circuit considering the Leakage flux is:



In many instances we neglect leakage inductance – assuming the coil is tightly wound – i.e. tightly coupled – hence no leakage

Choosing a core with high μ_r will result in high L_m therefore can be neglected.

$N_1 i_1 + N_2 i_2 + N_3 i_3 = 0$

17

Transformers – Continued

A three-winding transformer with N1=10 turns, N2=5 turns, and N3=5 turns uses a magnetic core that has the following properties: Am=0.639 cm², the magnetic path length l_m = 3.12 cm, and the relative permeability of the material μ_r=5000

A square-wave voltage of ±30V amplitude at a frequency of 100kHz is applied to winding 1. Windings 2 and 3 are open. Ignore the leakage inductance. Calculate and draw the magnetizing current waveforms and the voltages induced in the open windings 2 and 3

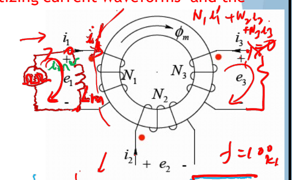
$R_m = \frac{l_m}{\mu_0 \mu_r A} = \frac{3.12 \times 10^{-2}}{4\pi \times 10^{-7} \times 5000 \times 0.639 \times 10^{-4}} = 777.09$

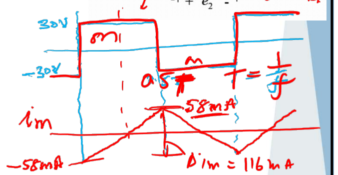
$L_1 = \frac{N_1^2}{R_m} = \frac{(10)^2}{777.09} = 1.29\,\mu H$ (L_m)

$L \frac{di}{dt} = v \Rightarrow L_1 \frac{\Delta i_m}{DT} = 30$

$\Delta i_m = \frac{30 \times 0.5\,T}{L_1} = \frac{30 \times 0.5}{1.29 \times 10^{-6} \times 100 \times 10^3}$

$\boxed{\Delta i_m = 116\ mA}$





20

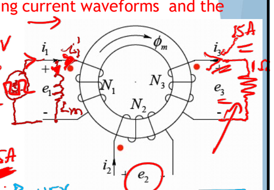
Transformers – Continued

A three-winding transformer with N1=10 turns, N2=5 turns, and N3=5 turns uses a magnetic core that has the following properties: Am=0.639 cm², the magnetic path length l_m = 3.12 cm, and the relative permeability of the material μ_r=5000

A square-wave voltage of ±30V amplitude at a frequency of 100kHz is applied to winding 1. Windings 2 and 3 are open. Ignore the leakage inductance. Calculate and draw the magnetizing current waveforms and the voltages induced in the open windings 2 and 3

$\frac{e_3}{e_1} = \frac{N_3}{N_1} \Rightarrow e_3 = e_1 \cdot \frac{N_3}{N_1} = (\pm 30V)\left(\frac{5}{10}\right) = \pm 15V$

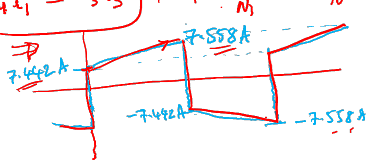
$e_2 = \pm 15V$

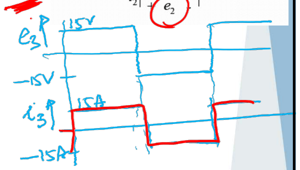


Now a $1\,\Omega$ resistance is connected to e_3

Now $N_1 i_1' = N_3 i_3 \Rightarrow i_1' = \frac{N_3}{N_1} i_3 = \frac{5}{10} \times 15 = 7.5A$

$i_1 = i_m + i_1' \Rightarrow$





23

Thank you!!

24