

Question 1

Given:

Weekly expenses for dinners with five friends:

- Week 1: Blake paid \$25, Jamie paid \$50.
- Week 2: Amari paid \$50, Emerson paid \$25, Kai paid \$25.
- Week 3: Amari paid \$15, Blake paid \$30, Jamie paid \$35, Kai paid \$30.
- Week 4: Emerson and Kai both paid \$35.

Find:

- Who ends up owing money and how much does each person owe?
- Write out the fewest transactions needed to get even (make it so everyone paid the same amount).

Diagram:

Theory:

Calculate the total expenses incurred by each friend over the four weeks.

Calculate the average expenses per person.

Determine who owes money, how much, or if everyone has already paid their fair share.

Assumptions:

That everybody paid their fair share.

Solution:

- Finding how much each person has paid, add up all the above information to get a total.

Total expenses / Number of friends = $\$355 / 5 = \71

All added up from the givens-

Amari - \$65

Blake - \$55

Emerson - \$60

Jamie - \$85

Kai - \$90

Anyone who has paid less than \$71 owes money. So, Emerson, Amari, and Blake owe money.

- Who owes who money –

Emerson owes Kai \$11.

Amari owes Kai \$6.

Blake owes Jamie \$14.

Blake owes Kai \$2.

Question 2

Given:

Height = 85.0 cm

Diameter = 57.0 cm

Water flows into the bucket at a rate of 1.34 L/s

The bucket dumps its contents when the water level reaches 78.9 cm

Find:

How long does it take for the bucket to fill up and dump water over the playground?

Diagram:

Theory:

Calculate the volume of the bucket using its dimensions.

Calculate the time it takes to fill the bucket with water at a rate of 1.34 L/s.

Assumptions:

The bucket is going to spill when it reaches the level of 78.9 cm.

Solution:

a. The time it takes for the bucket to fill and empty is equal to Volume of full bucket divided by inflow Rate

$$\text{Eq. } T_0 = \frac{\pi r^2 h}{R_i} = \frac{\pi \times (0.285)^2 \times 0.789}{1.34 \times 10^{-3}}$$
$$= \underline{150.25 \text{ seconds}}$$

b. Net rate is change in Rate in - out

$$T_E = \frac{\pi r^2 h}{R_i - R_o} = \frac{\pi (0.285)^2 \times 0.789}{(1.34 \times 10^{-3}) - (0.162 \times 10^{-3})}$$
$$= \underline{170.9 \text{ seconds}}$$
$$\Delta T = 170.9 - 150.25 = \underline{20.7 \text{ seconds}}$$

(a) So, time it takes to fill and empty water bucket = 150.25 seconds

(b) And time added is 20.7 seconds extra

Question 3

Given:

Maximum height of the playground slide, $h = 2.25 \text{ m}$

Radius of the slide, $r = 8.75 \text{ m}$

Mass of the toddler, $m = 18.6 \text{ kg}$

Initial velocity of the toddler at the top, $u = 0 \text{ m/s}$

Final velocity of the toddler at the bottom, $v = 3.52 \text{ m/s}$

Gravitational acceleration, $g = 9.81 \text{ m/s}^2$

Find:

- a. Length of the slide (arc length)
- b. Average frictional force acting on the toddler over this distance.

Diagram:

Theory:

Arc Length (s) = Radius (r) \times Angle (θ) (in radians)

Gravitational Potential Energy (at the top) = Kinetic Energy (at the bottom)

$$mgh = \frac{1}{2} * m * v^2$$

$$v = \sqrt{2gh}$$

$$\theta = \text{Arc Length } (s) / \text{Radius } (r)$$

- b. To find the average frictional force, use the work-energy principle.

$$\Delta KE = KE_{\text{final}} - KE_{\text{initial}}$$

$$\Delta KE = \left(\frac{1}{2}\right) * m * v^2 - \left(\frac{1}{2}\right) * m * u^2$$

Assumptions:

There is no air resistance.

The slide is a perfect circle, not taking any irregularities into account.

Solution:

Diagram

$m = 18.6 \text{ kg}$
Height: 2.25 m
Radius: 8.75 m
Velocity: 3.52 m/s

Solution

(a) To find length of slide, find radians first.

$$\phi = \cos^{-1} \left(\frac{\text{Radius} - \text{Height}}{\text{Radius}} \right)$$
$$= \cos^{-1} \left(\frac{8.75 - 2.25}{8.75} \right)$$
$$= 0.733 \text{ radians}$$

So, the length can be calculated by

$$L = R\phi$$
$$L = 0.733 \times 8.75 = 6.42$$

(b) Frictional force on baby

Equation, work-energy.

$$MgH = \frac{1}{2}mv^2 + Lf$$
$$\rightarrow 18.6 \times 9.8 \times 2.25 = \frac{1}{2} \times 18.6 \times (3.52)^2 + 6.42 \times$$
$$\rightarrow 45.93 \text{ N}$$

Since going down

$$\rightarrow -45.93 \text{ N}$$

Question 4

Given:

Cost of plain chocolate per pound = \$3.50

Cost of durian per pound = \$15.75

Max allowable cost per pound to produce durian flavored chocolate = \$4.50

Total pounds of durian-flavored chocolate bars to produce = 216 pounds

Find:

The number of pounds of durian needed to produce 216 pounds of durian-flavored chocolate bars.

Diagram:

- Not applicable.

Theory:

To find the number of pounds of durian needed to produce 216 pounds of durian-flavored chocolate bars, find that the total cost of the durian and plain chocolate used does not exceed the maximum allowable cost per pound, which is \$4.50.

Cost per pound = (Cost of durian per pound + Cost of plain chocolate per pound)

Assumptions:

The cost of producing durian-flavored chocolate bars is a linear combination of the cost of durian and plain chocolate.

Solution:

4.
Solution
make 2 equations
 $15.75x + 3.50y = 972$
And
 $y = 216 - x$
Plug in
 $15.75x + 3.50(216 - x) = 972$
 $15.75x - 3.50x = 972 - 756$
 $\rightarrow 12.25x = 216$
 $x = 17.63$
It requires 17.63 pounds of durian to make 216 durian cookies.