Homework 10
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Page 1
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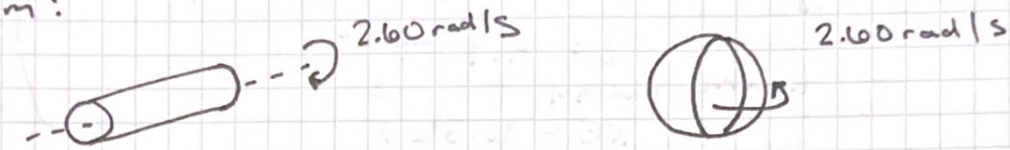
PHYS 216-510

Problem 1

Given: Angular velocity (ω) = 2.60 rad/s
Mass(m) = 7.50 kg
Radius (r) = 0.501 m

Find: Angular momentum and kinetic Energy of:
- Solid Cylinder
- Hollow Cylinder
- Solid Sphere
- Hollow Sphere.

Diagram:



Theory: Angular Momentum of a rotating object is given by the formula: $L = I\omega$
I - moment of Inertia
ω - Angular velocity
I depends on Geometry of the object
$KE = \frac{1}{2} I \omega^2$

Assumptions - Objects are ~~rotational~~ rotating about their central axis.

Solution -
Solid Cylinder - $L = I\omega = \left(\frac{1}{2} MR^2\right)\omega$
Rotational KE - $KE = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2} MR^2\right)\omega^2$

$\rightarrow L = \left(\frac{1}{2}(7.50\,kg)(0.501m)^2\right)(2.60\,m/s)$

$L = 2.447\ kgm^2s^{-1}$

$\rightarrow KE = \frac{1}{4}(7.50\,kg)(0.501m)^2(2.60\,rad/s)^2$

$KE = 3.18\,J$

Angular Momentum of a solid cylinder
$L = 2.45\ \frac{kgm^2}{s}$
Kinetic Energy = 3.18 J

For Hollow Cylinder - $L = (mr^2)\omega$

$$L = (7.5kg)(0.501m)^2(2.60\ rad/s)$$
$$= 4.9\ \frac{kgm^2}{s}$$

$$KE = \left[\frac{1}{2}mr^2\right]\omega$$
$$\rightarrow = \left[\frac{1}{2}(7.5kg)(0.501m)^2\right](2.60\ rad/s)$$
$$= 6.36J$$

Angular Momentum of Hollow Cylinder
$$L = 4.9\ \frac{kgm^2}{s}$$
Kinetic Energy
$$KE = 6.36J$$

Solid Sphere. $L = I\omega$, $I = \frac{2}{5}mr^2$
$KE = \frac{1}{2}I\omega^2$

$$L = \left[\frac{2}{5}mr^2\right]\omega \rightarrow \frac{2}{5}(7.5kg)(0.501m^2)(2.60\ rad/s)$$
$$= 1.96\ \frac{kgm^2}{s}$$

$$KE = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{5}mr^2\right]\omega \rightarrow \frac{1}{5}(7.5kg)(0.501m^2)(2.60\ rad/s)^2$$
$$\rightarrow = 2.55J$$

Angular Momentum of Solid Sphere
$= 1.96\ kgm^2/s$
Kinetic Energy $= 2.55J$

Hollow Sphere - $L = I\omega = \left[\frac{2}{3}mr^2\right]\omega$
$KE = \frac{1}{2}\left[\frac{2}{3}mr^2\right]\omega^2$

$$L = \frac{2}{3}(7.5kg)(0.501m^2)(2.60\ rad/s)$$
$$= 3.26\ kgm^2/s$$
$$KE = \frac{1}{3}(7.5kg)(0.501m^2)(2.60\ rad/s)^2$$
$$= 4.24J$$

$L = 3.26\ kgm^2/s$
$KE = 4.24J$

~~Homework~~
Problem 2

Given: mass = 26g
mass of rotating disk : 350g
Radius : 12 cm
Distance fallen : 55cm

Find : Speed of the block :

Diagram :

Theory : $\Delta PE = mgd$, $E_{initial} = \Delta PE$

For Solid disk
$I = \frac{1}{2} Mr^2$
$\tau = r \times F$
$\rightarrow r \times F_T \times \sin 90$

Linear acceleration
$a = \alpha \times R$
$T\cancel{R} = \frac{1}{2} m_2 \cancel{r^2} \times \frac{a}{\cancel{r}}$
$\rightarrow T = \frac{1}{2} m_2 a$ on disk

From the drawing,
$F = mg - T \rightarrow ma = mg - \frac{1}{2} m_2 a$
$\rightarrow a = mg \left(\frac{1}{m}\right)\left(\frac{2}{m_2}\right)$
$\rightarrow a = m_1 g \left(\frac{2}{m_2} \times \frac{1}{m_1}\right)$

Assumptions : No Extra External forces.

Solution : $a = (26g \times 9.81\, m/s)\left(\frac{1}{26}\right)\left(\frac{2}{350g}\right)$
$= 1.27\, m/s^2$

~~Displacement~~
Distance fallen = 0.55m
$V = \sqrt{2a\Delta d} = \sqrt{2 \times 1.27\, m/s \times 0.55\, m} = 1.18\, m/s$

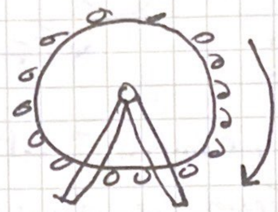
Speed of block once dropped = 1.2 m/s

Problem 3

Given:
- diameter = 150 m
- Time for one complete Revolution = 30 mins × 60 s = 1800 s
- Mass of Rim = 7×10^{5} kg
- Mass of each capsule = 1×10^{4} kg
- # of capsules = 28
- Mass of Avg person = 70 kg
- # of passengers = 784

Find: (a) Magnitude of Angular Momentum at full capacity.
(b) Average Net external force applied to stop wheel in 15 mins.

Diagram:



time to stop.

Theory: Angular Momentum – $L = I\omega$

~~Di~~ Moment of Inertia of Rim, Capsules, Passengers to find total moment of Inertia.

Angular velocity = $2\pi/T$

Time to stop wheel – $\tau = \Delta L/\Delta t$.

Assumption – Wheel rotates at constant velocity.

Solution –

1. Angular Velocity – $\omega = \frac{2\pi}{1800} = 0.0035$ rad/s
2. mr^2 – Inertia for Rim
3. I for Capsule – nmr^2
4. I for people – nmr^2

$$\Sigma I = mr^2 + nmr^2 + nmr^2 =$$

$$= (7\times10^5\ \text{kg})(75\text{m})^2 + (28)(1\times10^4\text{kg})(75\text{m}^2) + 784(70\text{kg})(75\text{m}^2)$$

$$= 5.82\times10^9\ \text{kgm}^2$$

Angular Momentum – $L = I\omega = 5.82\times10^9\ \text{kgm}^2 \times 0.0035\ \frac{\text{rad}}{\text{s}}$

$$= \underline{2.04\times10^7\ \text{kgm}^2/\text{s}}$$

Angular acceleration –

$$\alpha = \frac{\Delta\omega}{900\text{s}} = -3.89\times10^{-6}\ \text{rad/s}^2$$

~~Dz~~ $\tau = I\alpha = (5.82\times10^9\ \text{kgm}^2)(3.89\times10^{-6}\ \text{rad/s})$

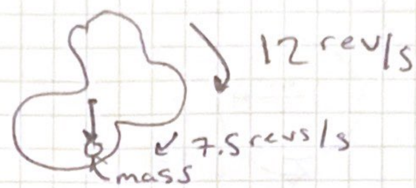
$$= 2.26\times10^4\ \text{Nm}$$

Problem 4

Given - $\omega_1 = 12 \text{ rev/s}$ $r = 0.032m$
$m = 0.025kg$ $\omega_f = 7.5 \text{ rev/s}$
$I_m = 2.5 \times 10^{-6} \text{ kgm}^2$

Find - Moment of Inertia of the odd shaped object

Diagram -



Theory - $L_i = L_f$

Assumptions - No unknown external forces

Solutions -

$$12I_1 = \left(2.5 \cdot 10^{-6}\right) + \left(I_0\right) + \left((0.025kg)(0.032^2)\right) \cdot 7.5$$

$$12I - 7.5I = 4.5I$$

$$4.5I = 28.1 \times 10^{-6} \text{ kgm}^2 (7.5 \text{ rad/s})$$

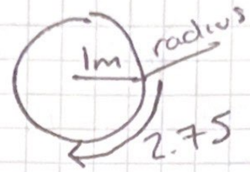
$$\boxed{I = 4.68 \times 10^{-5} \ \frac{kgm^2}{s}}$$

Problem 5

Given - $r = 1m$, $I = 135\ kgm^2$ $m = 21.5\ kg$ $v = 2.75\ m/s$

Find - (a) Magnitude of Angular Momentum
(b) Angular Velocity

Diagram -



Theory - $\vec{L} = \vec{r} \times \vec{P}$ $L_i = L_F$

Assumptions - No extra externall forces will be included in the calculation.

Solutions - (a) $\vec{L} = r \times mv$

$= 1m \times 21.5kg \times 2.75\ m/s$

$\boxed{\vec{L} = 59.13\ kgm^2/s}$

(b) $(1m)(21.5kg)(2.75\ m/s) = (135\ kgm^2 + (21.5kg)(2.75)^2)$

$\frac{59.125}{156.5} = \frac{156.5\ \omega_F}{156.5}$

$\boxed{\omega_F = 0.377\ rad/s}$