

## Problem 1.

A USB power source has a nominal voltage of 5 V. The allowable voltage range is within 5 % of the nominal. Typical USB cables are in the range AWG 20–28.

(a) Consider a 10 W USB supply. What is the smallest Thevenin equivalent voltage  $V_{th}$  and the largest Thevenin equivalent resistance,  $R_{th}$ , of this supply if it is to meet specs?

(b) What is the smallest load resistor that can be driven by this supply without overloading?

(c) What fraction of the supply power is dissipated in the load under this maximal load condition?

(d) Why is this not optimized for maximum power transfer?

(e) What is the maximum length of cable that can be used with a 5 W USB load, assuming copper AWG 20?

(A)  $5\% = 0.25$

min Voltage = 4.75 V

max Voltage = 5.25 V

$$R = \frac{E^2}{P} \rightarrow R = \frac{V_{th}^2}{P} = \frac{(4.75)^2}{10} = 2.26 \Omega$$

SMALLEST THEVENIN EQUIVALENT VOLTAGE = 4.75

LARGEST = 2.26  $\Omega$  RESISTOR

(B)

POWER = 10 W  
NOMINAL VOLTAGE = 5 V

$$P = IE \rightarrow I = \frac{P}{E} \rightarrow \frac{10}{5} = 2 A$$

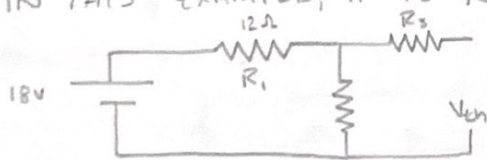
$$R = \frac{V}{I} = \frac{4.75}{2} = 2.38 \Omega$$

(C)

$P = 10 W$   $P = \frac{E^2}{R} = \frac{(4.75)^2}{2.38} = 9.5 W$

$$\frac{9.5 W}{10 W} = 0.95 = 95\%$$

(D) THE LOAD RESISTOR HAS TO EQUAL THE THEVENIN EQUIVALENT RESISTANCE. IN THIS EXAMPLE, IT IS NOT OPTIMIZED FOR POWER SUPPLY.



$$18 \cdot \left( \frac{24}{24+12} \right) \rightarrow 18 \cdot \left( \frac{24}{36} \right) \rightarrow 18 \cdot \frac{2}{3} \rightarrow 12 V$$

$$V_L = 12 V - V_A \quad P_{max} = \frac{\frac{1}{4} V_{th}^2}{R_{th}} = \frac{\frac{1}{4} (12)^2}{25} = 1.5 W$$

(E) AWG 20 = 0.8 mm  $\rightarrow A = \pi R^2 = A = \pi (0.4)^2 = 0.5$

$$R = P \frac{L}{A} \rightarrow L = \frac{P L}{A} \rightarrow \frac{R_{th} K A}{0.05 A_p} \rightarrow \frac{2.27 \cdot (1.68 \times 10^{-8}) (0.518)}{0.05 (1.68 \times 10^{-8})} =$$

Copper Resistance =  $(1.68 \times 10^{-8})$

$$= 39.087 m$$

\* 10W POWER SUPPLY

→ 5V

→ 2.1A

## Problem 2.

Consider the USB-powered mobile-battery charger circuit shown below. Assume the USB power supply is the newer 10 W version but the mobile battery is the older 2.5 W version. Assume the transistor beta is 100 and  $V_{BE} = 0.7$  V. Take  $R_1 = 470 \Omega$ .

Bipolar Junction Transistor

$\beta$  = current gain or amplification factor of the transistor

(a) Sketch the equivalent circuit.

(b) What is the maximum voltage to which the mobile battery that can be charged with this circuit.

(c) What is the charging current as a function of mobile battery voltage?

(d) What percent of the power generated by the USB supply is being stored in the mobile battery, as a function of mobile battery voltage? Note that energy lost due to the mobile battery resistance does not count toward power stored in the battery.

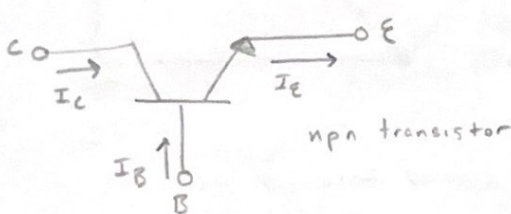
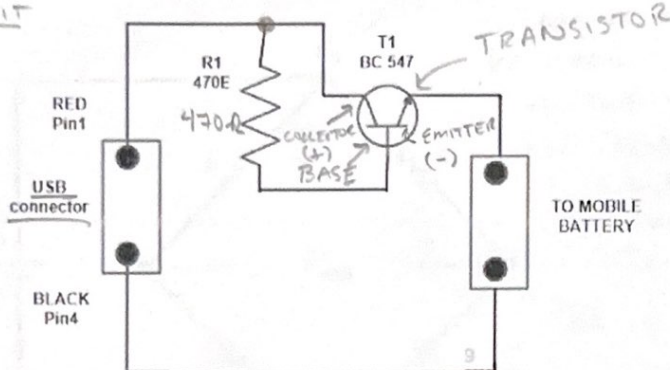
TRANSISTOR BETA

\* RATIO OF COLLECTOR CURRENT TO BASE CURRENT IS 100.  
\* Beta affects Amplification and Switching characteristics of the transistor.

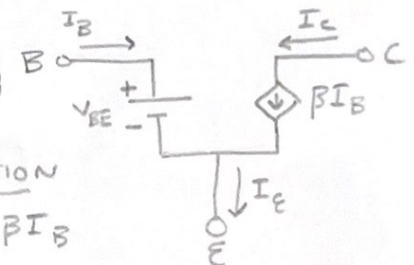
$V_{BE}$

VOLTAGE DROP ACROSS BASE-EMITTER JUNCTION OF A BJT when it is forward-biased

IN THIS CIRCUIT

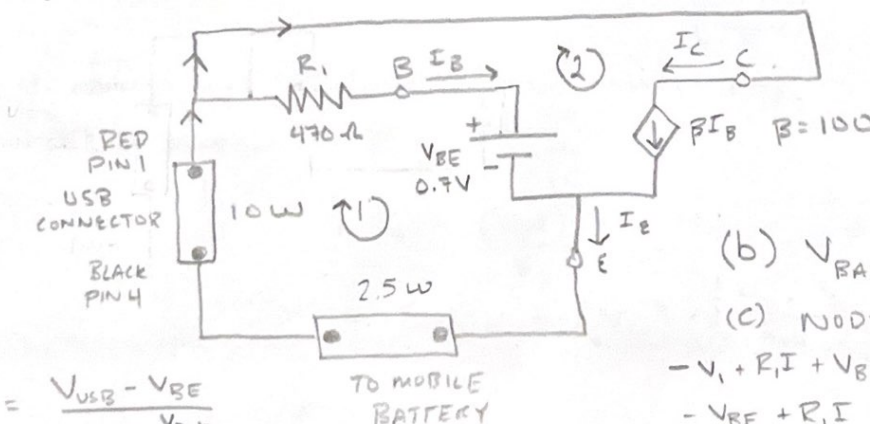


Equivalent T-model for npn transistor with  $V_{BE} = 0.7$  V



EQUATION  
 $I_C = \beta I_B$

(a) SKETCH EQUIVALENT CIRCUIT.



$P = VI \rightarrow V = \frac{P}{I}$   
MAX VOLTAGE  
 $\frac{P}{I} = V_{max}$

(b)  $V_{BAT} + V_{BE} = 5.7$  V

(c) NODE EQUATION

$-V_1 + R_1 I + V_{BE} + R_2 I + V_2 = 0$   
 $-V_{BE} + R_1 I + I_B = 0$

(d)  $\frac{P_{ch}}{P_{USB}} = \frac{V_{USB} - V_{BE}}{V_{USB} + \frac{V_{BAT}}{R}}$   
 $= \frac{3.7 - 0.7}{5 + \frac{3.7}{470}} = 60\%$

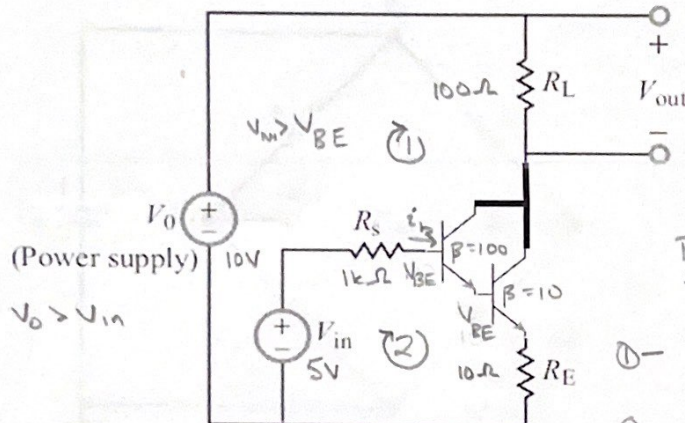
$I_C = \frac{V_{BAT} + V_{BE}}{(R_1 + R_2)}$   $I_{ch} = \frac{\beta}{R_1} \left[ (V_{USB} - V_{BE}) - \frac{V_{BAT}}{R} \right]$



### Problem 3.

The transistor circuit shown below is known as a Darlington pair. In this device the input transistor has a high beta but a low power rating. The second transistor has a lower beta but a higher power rating. For simplicity assume both transistors have  $V_{BE} = 0.7$  V. Take  $\beta_1 = 100$  and  $\beta_2 = 10$ . Choose  $R_s = 1$  k $\Omega$  and  $R_L = 100$   $\Omega$ ,  $R_E = 10$   $\Omega$ . Assume  $V_0 > V_{in}$  and  $V_{in} > V_{BE}$

- (a) What is the voltage gain of the circuit? 1000
- (b) What is the limiting gain for large beta. 0.009  $\Omega$
- (c) For an input voltage of  $V_{in} = 5$  V and supply voltage  $V_0 = 10$  V, what is the power dissipated in each resistor?
- (d) What is the power supplied by each source?



(a) Current gain would be roughly the product of the current gains of each transistor.

$$\beta_1 \cdot \beta_2 = \text{Current gains}$$

$$10 \cdot 100 = \boxed{1000}$$

(b) Ratio of load resistance parallel with emitter resistance to input resistance

$$\textcircled{1} - R_L \parallel R_E = \frac{100 \Omega \times 10 \Omega}{100 \Omega + 10 \Omega} = 9.09 \Omega$$

$$\textcircled{2} - \frac{R_L \parallel R_E}{R_s} = \frac{9.09 \Omega}{1 \text{ k}\Omega} = \boxed{0.009 \Omega}$$

where  $V_{in} = 5$  V,  $V_0 = 10$  V

(c) (1) Current

$$I = \frac{V_0 - V_{BE}}{R_s + (\beta_1 + 1) R_E} \rightarrow \frac{10 - 0.7}{1000 + (100 + 1)10} = \underline{0.0047 \text{ A}}$$

(2) Across  $R_s$

$$V_{in} - V_{BE} = 5 \text{ V} - 0.7 \text{ V} = \underline{4.3 \text{ V}}$$

$$P = (4.3 \text{ V})(0.0047 \text{ A}) = \underline{0.021 \text{ W}}$$

$$\textcircled{3} - R_E = \frac{(V_0 - V_{BE})^2}{R_E} = \frac{(9.3)^2}{10 \Omega} = \underline{8.65 \text{ mW}}$$

$$\textcircled{4} - R_L = \frac{(V_0 - V_{BE})^2}{R_L} = \frac{(9.3)^2}{100 \Omega} = \underline{0.865 \text{ W}}$$

(d)  $I = 0.0047 \text{ A}$

$$V \times I = P, \text{ Power supplied by } V_{in} = 5 \text{ V} \times 0.0047 \text{ A}$$

$$\boxed{P = 0.0235 \text{ W}}$$

$$\text{For Power supply } V_0 = 10 \text{ V} \times 0.0047 \text{ A}$$

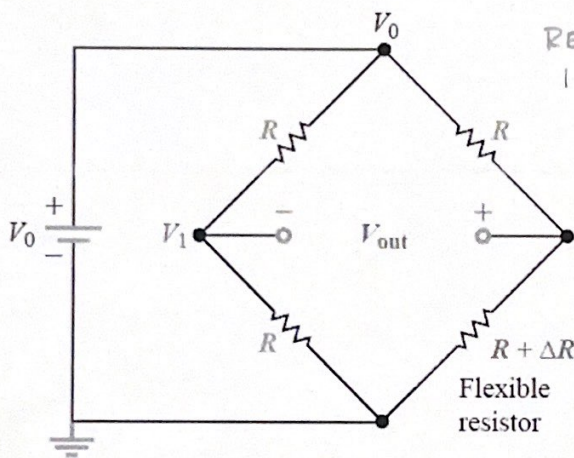
$$\boxed{P = 0.047 \text{ W}}$$



#### Problem 4

Consider the Wheatstone bridge circuit shown below. Suppose you have a voltmeter that can measure in the range of 1 mV to 100 V, with an accuracy of 1 % from 10 mV to 100 V and 10 % from 1 to 10 mV.

- What is the smallest percentage change you can measure in the flexible resistor? 10%
- Assuming you can measure the resistance directly with the voltmeter to 1 %, how much improvement do you get for  $V_0 = 100$  V. 9%
- If you treat the  $V_{out}$  terminal as a power supply, what is its Thevenin equivalent voltage and resistance as a function of the fractional change in the flexible resistor?
- What is the power consumed in each resistor, the Thevenin equivalent resistor, and in the source when  $R = 100$  W, and the fractional resistor change is 1 %?



(a) IF THE FLEXIBLE RESISTOR HAS A RESISTANCE OF  $1k\Omega$  and a CHANGE OF  $1m\Omega$ , THE SMALLEST PERCENT CHANGE WE CAN MEASURE IS 10%. IF RESISTOR  $= 1k\Omega$  AND PERCENT CHANGE OF 100 mV, SMALLEST CHANGE MEASURED IS 1%.

(b)  $\% \text{ Change in Resistance} = \left( \frac{0.1R}{100} \right) \cdot 100 = 10 \cdot 1 = \underline{9\%}$

(c) Thevenin Equivalent Voltage ( $V_{th}$ ) =  $V_{out} \cdot \left( \frac{1 - \Delta R}{R} \right)$   
 Thevenin Equivalent Resistance ( $R_{th}$ ) =  $\frac{R}{\left( \frac{1 - \Delta R}{R} \right)}$

(d)  $R = 100W$  with fractional resistor change 10%.

Power consumed in  $R_1 = \frac{(V_{out})^2}{R_1}$

In  $R_2 = \frac{(V_{out})^2}{R_2}$

In Source =  $(V_{out}) \cdot \left( \frac{V_{out}}{R_{th}} \right)$