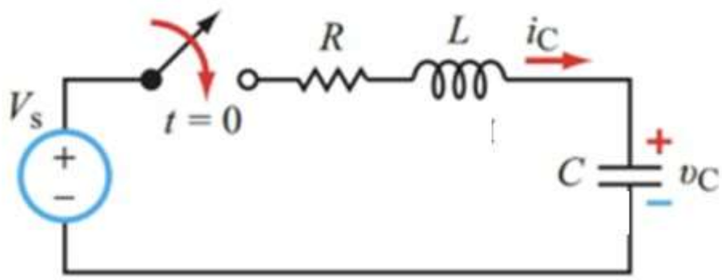
214 Hemmer sections -- Test 3 -- Name JOAQUIN SALAS

214 Test 3

1) For the circuit below, write down the differential equation and any auxiliary equations that are needed to find the current ic for t>0, and find the value of R that makes it critically damped.



For this circuit:

$V_S = R i_c + L\frac{di_c}{dt} + v_c$; $i_c = C\frac{dv_c}{dt}$

When $t > 0$:

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\left(\frac{dv_c}{dt}\right) + \frac{1}{LC} v_c = \frac{V_S}{LC}$$

And:

$$V_c(t) = A_1 e^{s_1 t} + A_o e^{s_2 t} + V_c(\infty)$$

where s_1 and s_2 –

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$\alpha = \frac{R}{2L}$

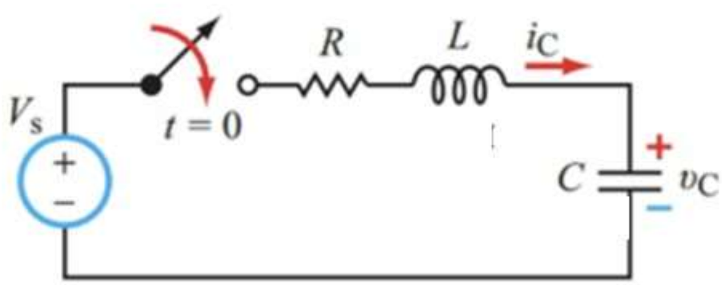
$\omega_0 = \frac{1}{\sqrt{LC}}$

For the circuit to be critically damped $\alpha = \omega_0$.

$$2L \cdot \frac{R}{2L} = \frac{1}{\sqrt{LC}} \cdot 2L$$

$$\boxed{R = 2\sqrt{\frac{L}{C}}}$$

2) For the circuit below, write down all the equations (including any auxiliary ones) needed to find the initial (t = 0+) values of both voltage and current on each element: capacitor, inductor and resistor.



$\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$V_c(t) = \left[A_1 e^{s_1 t} + A_2 e^{s_2 t} + V_c(\infty)\right]$$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$$A_1 = \frac{\frac{1}{C} i_c(0) - s_1\left[V_c(0) - V_c(\infty)\right]}{s_1 - s_2}$$

$$A_2 = -\left[\frac{\frac{1}{C} i_c(0) - s_1\left[V_c(0) - V_c(\infty)\right]}{s_1 - s_2}\right]$$

$$i_c(t) = C\frac{dv_c}{dt}$$

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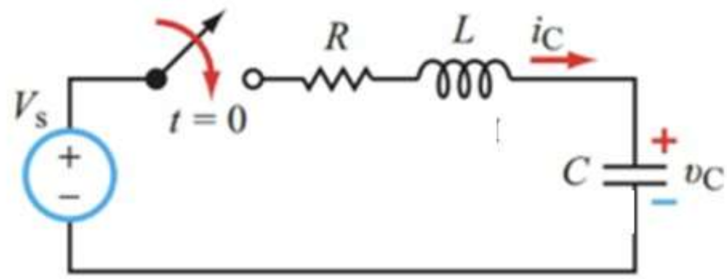
3) For the circuit below, write down all the equations needed to find the current ic when the source voltage is |Vs| / 0° and the switch has been closed for a long time. Then evaluate ic at the frequency where the circuit is critically damped. Hint: Use equations from problem 1 to complete the second part.



$|V_s| < 0° = V_s$, $X = |V_s| \cos(\theta)$,
$Y = |V_s| \sin(\theta)$

when $t = 0$, $V_c(0) = V_c(0^+) = 0$
AND $V_c(\infty) = V_s$

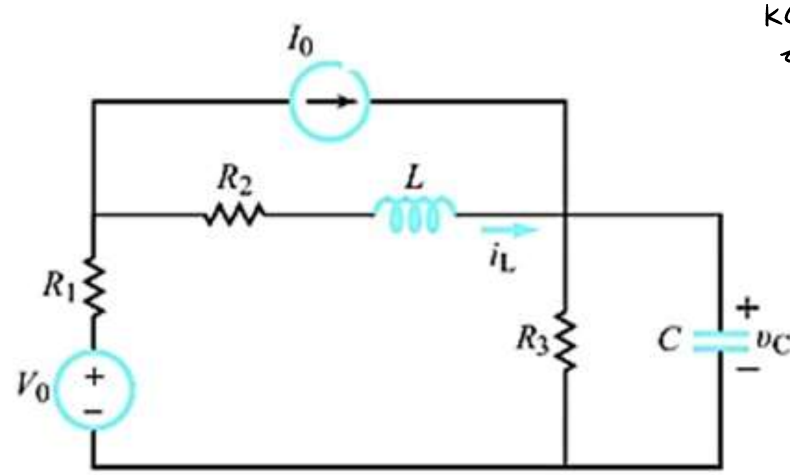
$B_1 = V_c(0) - V_c(\infty)$ $\quad B_2 = \frac{1}{C} i_c(0) + \alpha [V_c(0) - V_c(\infty)]$

$v(t) = (B_1 + B_2 t)e^{-2\alpha t} + V_c(\infty)$

$i_c = C[-\alpha V_s e^{-\alpha t} - \cancel{\alpha}(-V_s - \alpha \cancel{V_s} t)e^{-\alpha t}]$

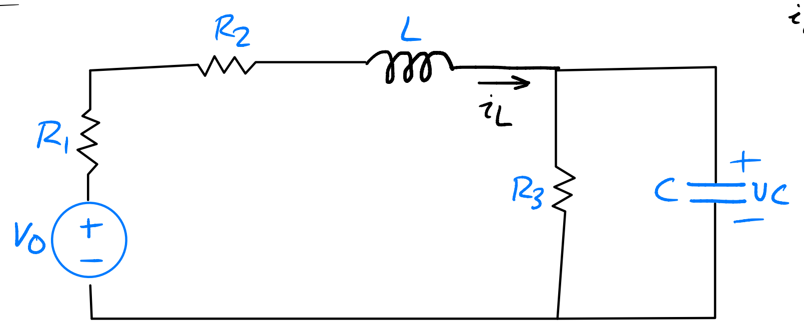
$= C[\alpha^2 V_s t e^{-\alpha t}]$

4) At $t = 0^+$ for the circuit below, write down all the equations needed to find vc and i_L. Hint: Be sure to identify the procedure needed to find a differential equation with only one variable, also write down any auxiliary equations that are needed.



KCL at node 2 leads to
$i_c(0) = I_o + i_L(0) - i_{R_3}(0)$
Determine $V_L(0)$
$i_1(0) = I_o + i_L(0)$
KVL around lower left loop
$V_L(0) = V_o - R_1 i_1 - R_2 i_L - V_{R_3}$

Redraw -



Since $i_c(0^-) = 0$
$i_L(0^-) = \frac{V_o}{R_1 + R_2 + R_3}$
$i_L(0^-) = i_L(0)$
$V_c(0^-) = i_L(0^-) R_3$
$V_c(0) = V_c(0^-)$
Next: $i_{R_3}(0) = \frac{V_c(0)}{R_3}$

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5) At $t = 0^+$ for the circuit below, write down all the equations needed to find the initial currents and voltages on both the inductor and capacitor. Hint: Do not forget any auxiliary equations.

At $t = \infty$:

The mesh Equation for loop 1 is

$$-V_0 + R_1 i_1 + R_2(i_1 - i_2) + R_3 i_1 = 0$$

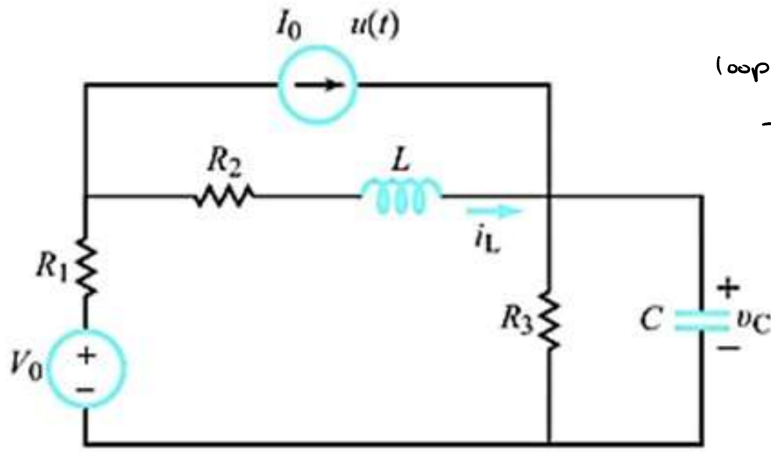
for loop 2,

$$i_2 = I_0$$

This leads to...

$$i_L(\infty) = i_1 - I_0$$

$$V_c(\infty) = i_1 R_3$$



6) For the circuit below, write down all the equations needed to find the current vc and i_L when the voltage is a cosine function |Vs| /_ 0° and current source is |Io| /_ 90° and the switch has been closed for a long time.

Polar form

Given $|V_s| \angle 0^\circ = V_s$, $|I_0| \angle 90^\circ = I_0 j = I_0\sqrt{-1}$

KVL for loop 1:

$$-V_s + R_i I_i + R_2(i_1 - i_2) + R_3 i_1 = 0$$

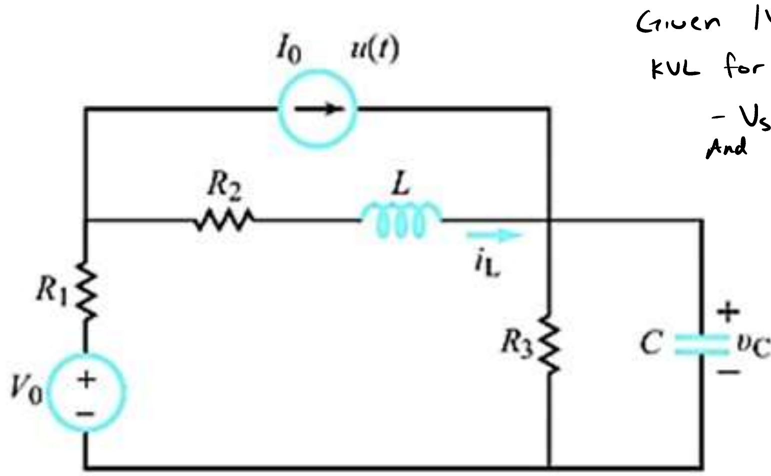
And at loop 2

$$i_2 = I_0\sqrt{-1}$$

$$\frac{V_s}{i_1} = R_1 + R_2 - \frac{R_2 i_2}{i_1} + R_3$$

$$\frac{R_2 i_2}{i_1} + \frac{V_s}{i_1} = R_1 + R_2 + R_3$$

$$i_1 = \frac{V_s + R_2 i_2}{R_1 + R_2 + R_3}$$



for $t = \infty$.

$$i_L(\infty) = i_1 - I_0\sqrt{-1}$$

$$V_c(\infty) = i_1 R_3$$

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7) For the circuit below use superposition to find the contribution to Vx from both the current and the voltage sources. Hint: Be sure to write down separate equations for each source.

Superposition

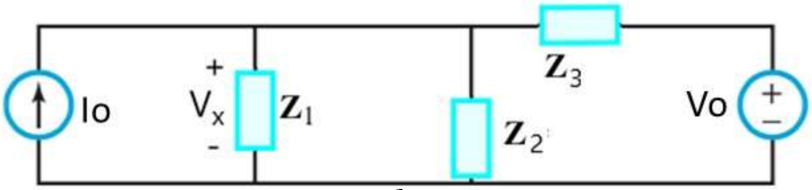
$(Z_1+Z_2)I_1' - Z_2 I_2' = V_s$

$(Z_2+Z_3)I_2' - Z_2 I_1' = 0$

$(Z_1+Z_2)I_1'' - Z_2 I_2 = 0$

$(Z_2+Z_3)I_2'' - Z_2 I_1' = V_o$

$V = (I_1'' + I_1')Z_1$



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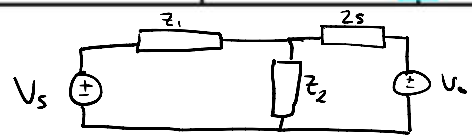
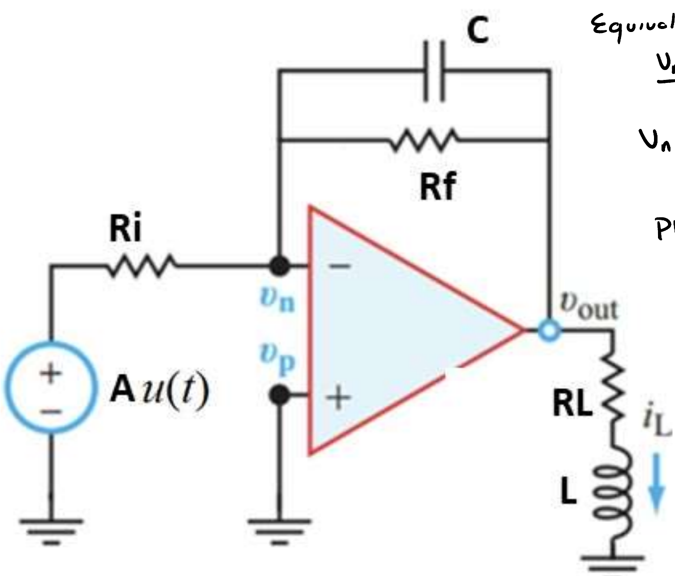


$V_s = I_s \cdot Z_1$

6.8.3

8) At $t = 0^+$ for the circuit below, write down all the equations (differential and auxiliary) needed to find i_L. Hint: Be sure to identify the procedure needed to find a differential equation with only one variable.



At node V_n: $i_1 + i_n + i_2 + i_3 = 0$

Equivalently:

$$\frac{V_n - A}{R_i} + i_n + \frac{V_n - V_{out}}{R_f} + C\frac{d}{dt}(V_n - V_{out}) = 0$$

$V_n = V_p = 0,\ i_n = 0$

$$V_{out} = R_L i_L + L\frac{di_L}{dt}$$

Plug into first Eq.

$$\frac{R_L}{R_f} i_L + \left(\frac{L}{R_f} + R_L C\right)\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = -\frac{A}{R_i}$$

Rearrange,

$$i_L'' + a i_L' + b i_L = c$$

In which,

$$a = \frac{L + R_f R_L C}{R_f L C}$$

$$b = \frac{R_L}{R_f L C}$$

$$c = \frac{-A}{R_i L C}$$

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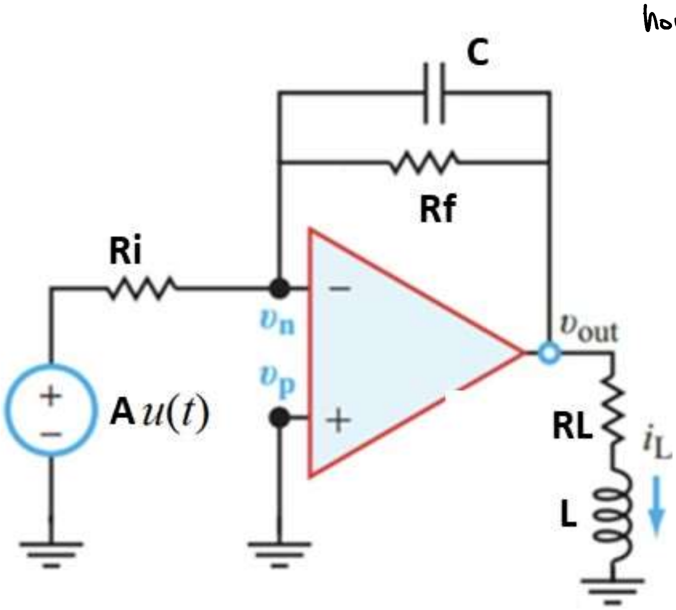
9) For the circuit below, write down all the equations (including any auxiliary ones) needed to find the initial (t = 0+) values of both voltage and current on the capacitor and the inductor.



The damping behavior of i_L is determined by how the magnitude of α compares with ω_0

$$\alpha = \frac{a}{2} \quad \text{and} \quad \omega_0 = \sqrt{b}$$

$$i_L(t) = \left[A_1 e^{s_1 t} + A_2 e^{s_2 t} + i_L(\infty)\right] u(t).$$

where, $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

At $t = \infty$. $V_{out}(\infty) = -\frac{R_f}{R_1} A.$

And $i_L(\infty) = \frac{V_{out}(\infty)}{R_L} = -\frac{R_f A}{R_1 R_L}$

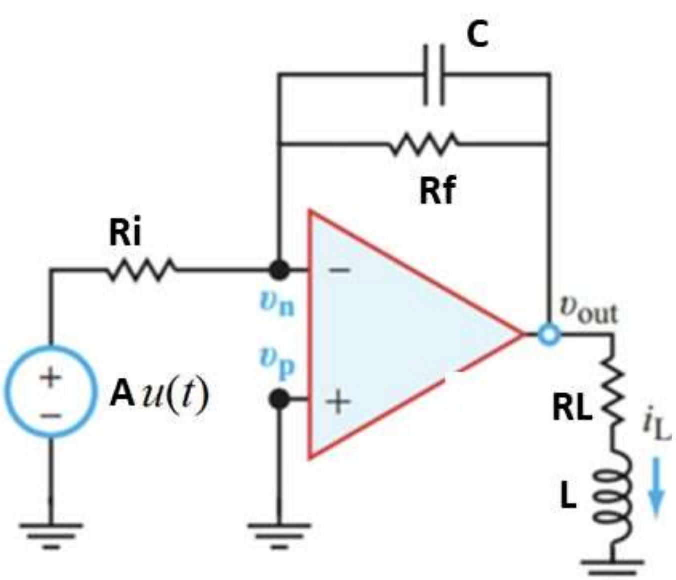
The charge can not change immediately

Inductor acts as an open circuit at $t = 0$.

$V_{out}(0) = 0$, $V_L(0) = 0$ and $i_L'(0) = \frac{1}{L} V_L(0) = 0$

$$A_1 = \frac{i_L'(0) - s_2[i_L(0) - i_L(\infty)]}{s_1 - s_2} \quad \text{AND} \quad A_1 = -\left[\frac{i_L'(0) - s_2[i_L(0) - i_L(\infty)]}{s_1 - s_2}\right]$$

10) For the circuit below, write down all the equations needed to find the current iL when the source voltage is a cosine with |A| $\underline{/}$ 0° and has been on for a long time (t >> 0).



Given

$|A| \angle 0° = A$

$V_{out}(\infty) = -\frac{R_f}{R_i} A$

$t = \infty$

$i_L(\infty) = \frac{V_{out}(\infty)}{RL} = -\frac{R_f A}{R_i RL}$

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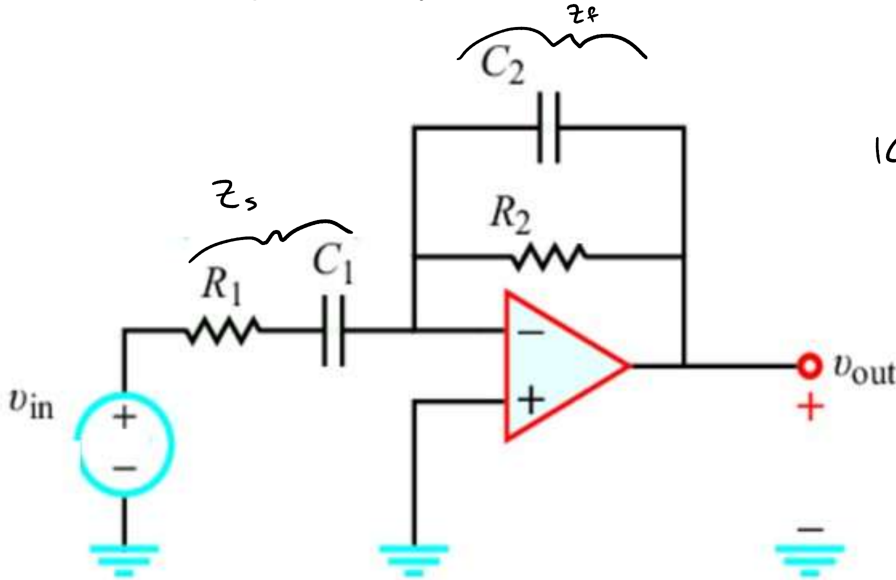
11) For the circuit below, write down the differential and auxiliary equations that are needed to find the output voltage, Vout. Hint: Be sure to outline how to find a differential equation of only one variable.



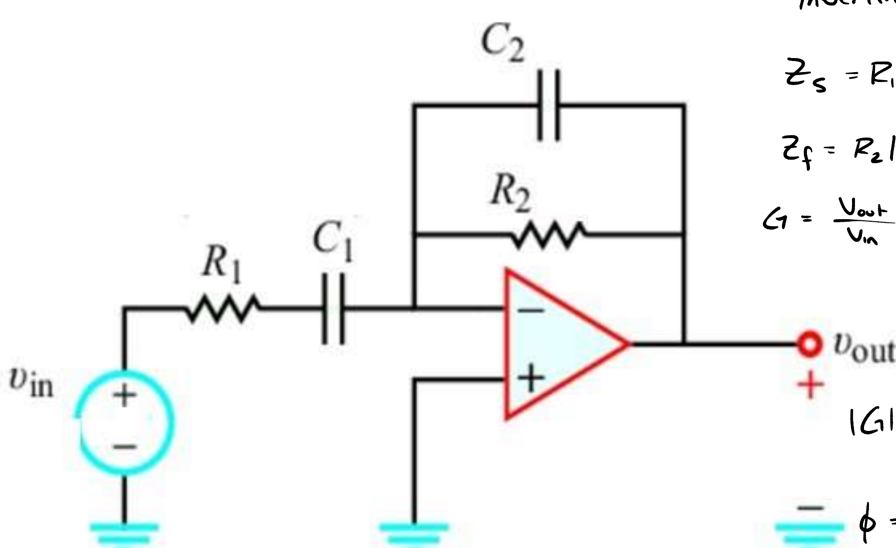
$\omega_s = 2\pi \times V_{in}$ (frequency)

$$|G| = \frac{\omega R_2 C_1}{\left[(1+\omega^2 R_1^2 C_1^2)(1+\omega^2 R_2^2 C_2^2)\right]^{1/2}}$$

$|V_{out}| = |G| V_{in}$

12) For the circuit below, write down all the equations needed to find the output voltage Vout, when the source voltage is |Vin| $\angle$ 0°.

Inverting amp w/ complex source



$$Z_s = R_1 + \frac{1}{j\omega C_1} = \frac{j\omega R_1 C_1 + 1}{j\omega C_1}$$

$$Z_f = R_2 \,||\, \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$G = \frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_s} = \frac{-j\omega R_2 C_1}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$$

$$G = Ge^{j\phi}$$

$$|G| = \frac{\omega R_2 C_1}{\left[(1+\omega^2 R_1^2 C_1^2)(1+\omega^2 R_2^2 C_2^2)\right]^{1/2}}$$

$$\phi = 0° - \tan^{-1}(\omega R_1 C_1) - \tan^{-1}(\omega R_2 C_2)$$

$$V_{out}(t) = |G| V_1 \cos(\omega t + \phi)$$

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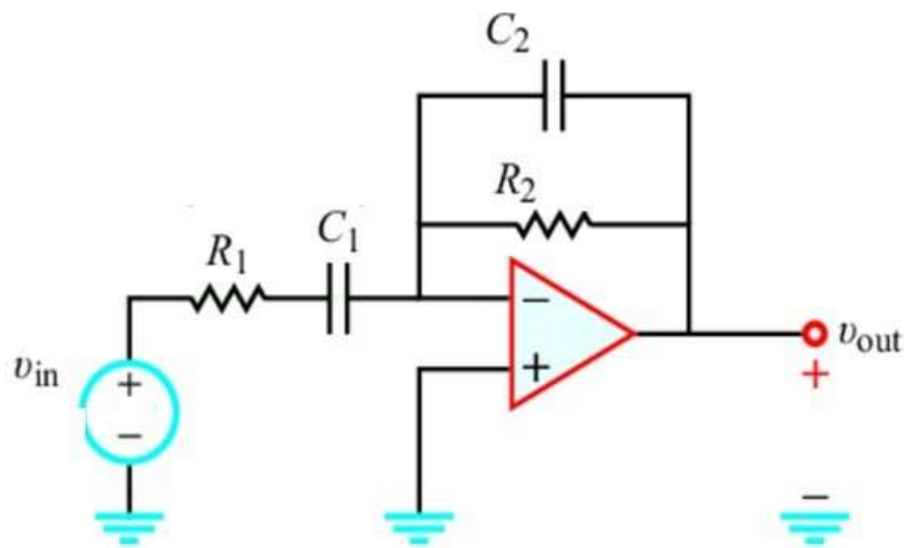
Extra credit section

EC1) For the circuit below, make a log-log plot of the gain magnitude |Vout/Vin| vs w (use can use angular frequency). Also make a semi-log plot of the phase shift vs log w. Label x and y values at locations where there are noticeable changes in slope (for magnitude plots) or phase. Choose two cases: 1) R1 C1 >> R2 C2 (at least 100 X) and 2) R1 C1 << R2 C2. Hint: there will be 4 plots total, 2 magnitude and 2 phase.



EC2) For the two circuits below, make a log-log plot of the gain magnitude |Vout/Vin| vs w (use can use angular frequency). Also make a semi-log plot of the phase shift vs log w. Label x and y values at locations where there are noticeable changes in slope (for magnitude plots) or phase. Choose two cases: 1) R2 >> R1 and C2 << C1 (at least 100 X) but R1 C1 = R2 C2, and 2) R1 = R2 and C1 = C2. Hint: there will be 4 plots total, 2 magnitude and 2 phase for each of the two circuits.