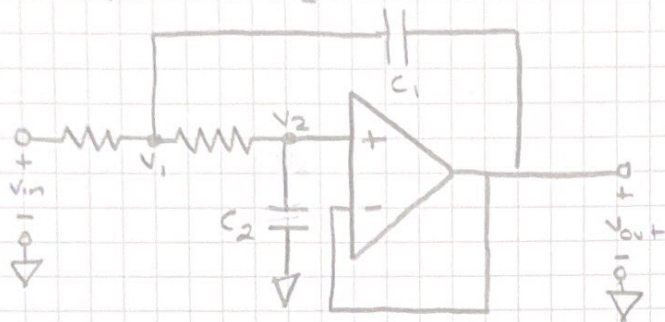


Step 1 $V_2 = V_{out}$

KCL at NODE 1: FOR Sallen-Key 2nd ORDER CIRCUIT

$$\frac{V_{in} - V_1}{R_1} + \frac{V_{out} - V_1}{R_2} + C_1 \frac{d}{dt}(V_{out} - V_1) = 0$$



NODE 2:

$$\frac{V_1 - V_{out}}{R_2} = C_2 \frac{dV_{out}}{dt}$$

Plugging all in:

$$R_1 C_1 R_2 C_2 \frac{d^2 V_{out}(t)}{dt^2} + (R_1 + R_2) C_2 \frac{dV_{out}(t)}{dt} + V_{out}(t) = V_{in}(t)$$

$$\frac{d^2 V_{out}(t)}{dt^2} + \frac{(R_1 + R_2) C_2}{R_1 C_1 R_2 C_2} \cdot \frac{dV_{out}(t)}{dt} + \frac{1}{R_1 C_1 R_2 C_2} V_{out}(t) = \frac{V_{in}(t)}{R_1 C_1 R_2 C_2}$$

In form, $\frac{d^2 y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = f(x)$

Use formula,

Assume Solution in form: $V_{out}(t) = e^{st}$

differentiating twice: $\frac{dV_{out}(t)}{dt} = s e^{st}$, $\frac{d^2 V_{out}(t)}{dt^2} = s^2 e^{st}$

Plug in: $R_1 C_1 R_2 C_2 (s^2 e^{st}) + (R_1 + R_2) C_2 (s e^{st}) + V_{out}(t) \cdot e^{st} = V_{in}(t)$

Divide by e^{st} :

① $R_1 C_1 R_2 C_2 s^2 + (R_1 + R_2) C_2 \cdot s + V_{out} = \frac{V_{in}(t)}{e^{st}}$

② $\frac{V_{out}}{e^{st}} = \frac{s^2 + \frac{(R_1 + R_2) C_2}{R_1 C_1 R_2 C_2} \cdot s + 1}{R_1 C_1 R_2 C_2} \cdot \frac{V_{in}(t)}{e^{st}}$

Multiply by $R_1 C_1 R_2 C_2$

Char. eq $R_1 C_1 R_2 C_2 s^2 + (R_1 + R_2) C_2 \cdot s + 1 = 0$