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ELEN 444

Texas A&M University
Department of Electrical and Computer Engineering
ELEN 444 - Digital Signal Processing
Fall 2024

Problem Set 1
Assigned: Thursday, Aug. 22, 2024
Due: Thursday, Aug. 29, 2022

Discrete-Time Signals and Systems

1.1: Proakis and Manolakis 2.1

1.2: Proakis and Manolakis 2.2

1.3: Proakis and Manolakis 2.3

1.4: Proakis and Manolakis 2.5

1.5: Proakis and Manolakis 2.6 (a,b)

1.6: Proakis and Manolakis 2.7 (a,c,e,g,i,k,m)

1.7: Proakis and Manolakis 2.17 (a,c)

1.8: Proakis and Manolakis 2.19

1.9: Proakis and Manolakis 2.23

1.10: An LTI system has impulse response $h[n] = u[n]$, find the response of the system to the input $x[n]$ described as follows:

$$x[n] = \begin{cases} 1 - \frac{|n|}{L}, & \text{if } |n| \leq L, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Computer Assignment 1: Discrete-Time Signals and Systems

For the following problems, you should hand in the Matlab scripts and the resulting figures.

L1.1: Use Matlab to reproduce Figs. 2.1.1-2.1.5 in the text book (Proakis and Manolakis).

L1.2: Use “conv” in Matlab to solve Proakis and Manolakis 2.17 again.

L1.3: Use “conv” in Matlab to solve Proakis and Manolakis 2.18.

2.1 A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine its values and sketch the signal $x(n)$.

(b) Sketch the signals that result if we:

1. First fold $x(n)$ and then delay the resulting signal by four samples.
2. First delay $x(n)$ by four samples and then fold the resulting signal.

(c) Sketch the signal $x(-n+4)$.

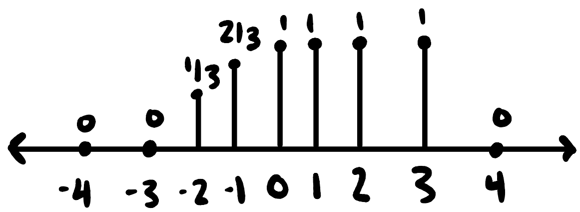
(d) Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

(e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

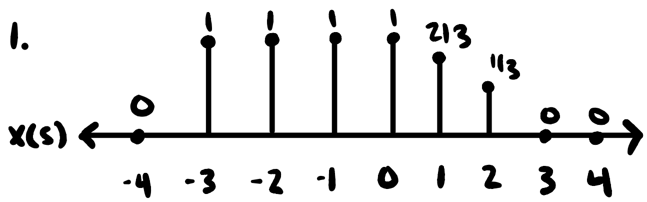
(a)

$1 + \frac{-1}{3}$

n	-4	-3	-2	-1	0	1	2	3	4
$x(n)$	0	0	1/3	2/3	1	1	1	1	0

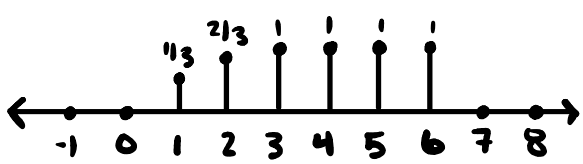


(b) 1.

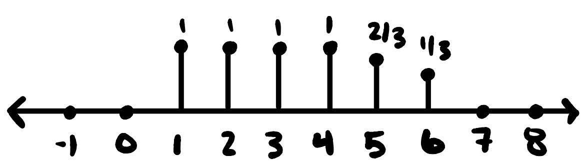


$x(-n) = \{0, 1, 1, 1, 1, 2/3, 1/3, 0, 0\}$

2. DELAY BY 4: $x(s+4)$



(c)



(d) ① Fold $x(n)$ to equal $x(-n)$

② Shift or Delay by k to get $x(-n+k)$

(e) $x(n) = \frac{1}{3} \times \delta(n+2) + \frac{2}{3} \times \delta(n+1) + u(n) - u(n-4)$

2.2 A discrete-time signal $x(n)$ is shown in Fig. P2.2. Sketch and label carefully each of the following signals.

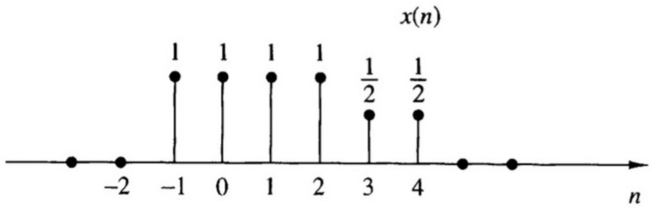
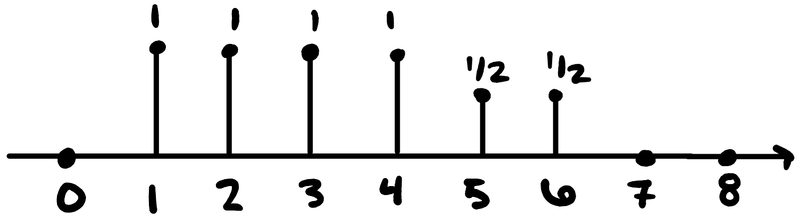


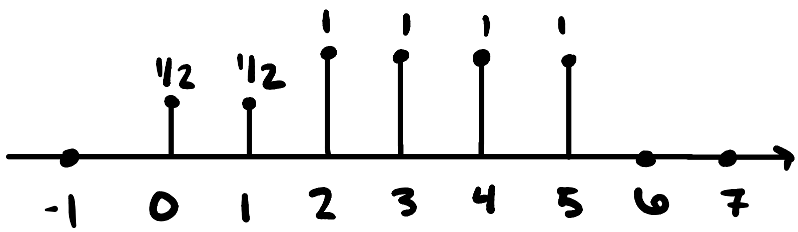
Figure P2.2

(a) $x(n-2)$ **(b)** $x(4-n)$ **(c)** $x(n+2)$ **(d)** $x(n)u(2-n)$ **(e)** $x(n-1)\delta(n-3)$
(f) $x(n^2)$ **(g)** even part of $x(n)$ **(h)** odd part of $x(n)$

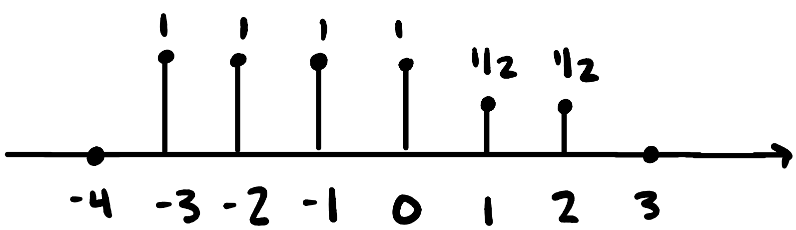
(a)



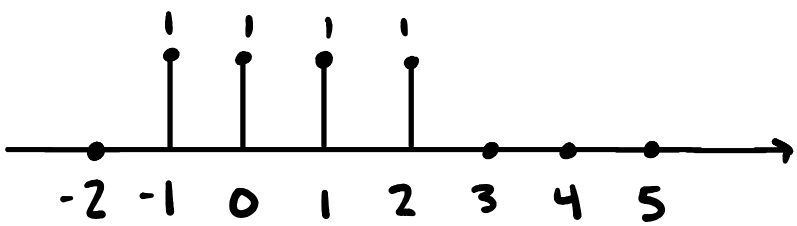
(b)



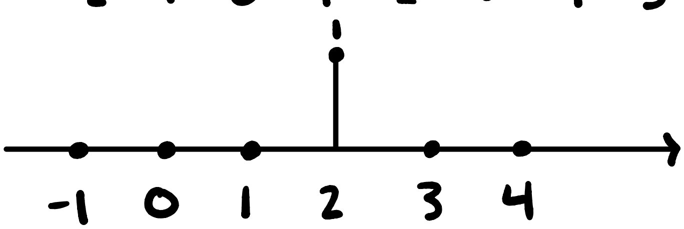
(c)



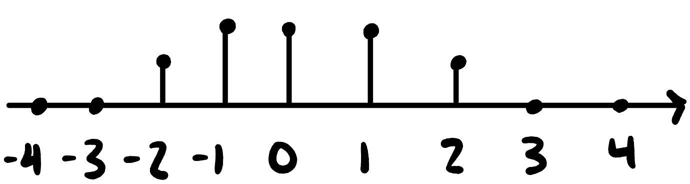
(d)



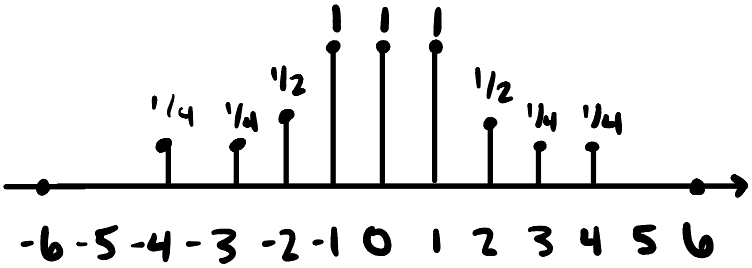
(e)



(f)

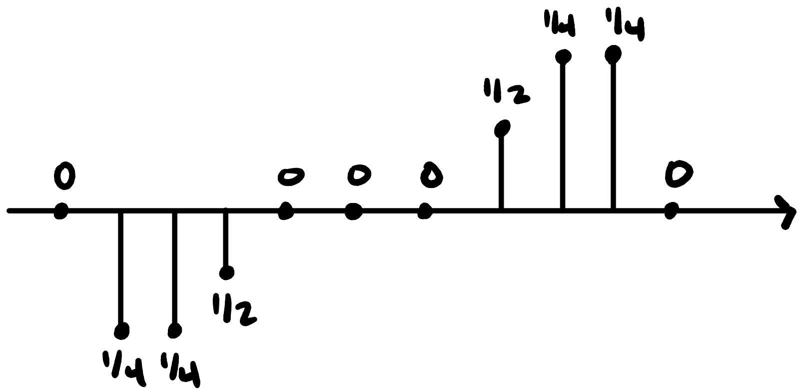


(g)



$x(-n) = \{0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 1, 0\}$

(h)



2.3 Show that

(a) $\delta(n) = u(n) - u(n-1)$

(b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

(a)

$$\delta(n) = \begin{cases} 0, & n > 0 \\ 1, & n = 0 \\ 0, & n < 0 \end{cases}$$

(b)

$$u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 1 \end{cases}$$

$$\sum_{k=0}^{\infty} \quad \begin{matrix} 0 \text{ if } n < 0 \\ 1 \text{ if } n > \text{ or } = 0 \end{matrix}$$

2.5 Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (powers) of its even and odd components.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\rightarrow E = \sum_{n=-\infty}^{\infty} |x_e(n)|^2 + \sum_{n=-\infty}^{\infty} |x_o(n)|^2 = \sum_{n=-\infty}^{\infty} |x_e(n) + x_o(n)|^2$$

$$\rightarrow \sum_{n=-\infty}^{\infty} \left| \frac{x(n) + x(-n)}{2} + \frac{x(n) - x(-n)}{2} \right|^2$$

$$\rightarrow \sum_{n=-\infty}^{\infty} \left| \frac{2x(n)}{2} \right|^2 = \sum_{n=-\infty}^{\infty} |x(n)|^2 = E$$

2.6 Consider the system

$$y(n) = \mathcal{T}[x(n)] = x(n^2)$$

(a) Determine if the system is time invariant.

(b) To clarify the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is applied into the system.

(1) Sketch the signal $x(n)$.

(2) Determine and sketch the signal $y(n) = \mathcal{T}[x(n)]$.

(3) Sketch the signal $y_2'(n) = y(n-2)$.

(4) Determine and sketch the signal $x_2(n) = x(n-2)$.

(5) Determine and sketch the signal $y_2(n) = \mathcal{T}[x_2(n)]$.

(6) Compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion?

(a)

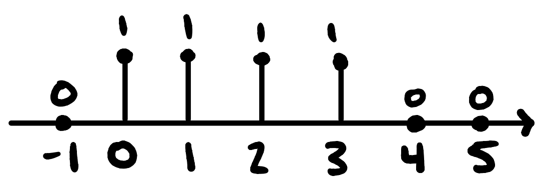
DOES $x(n-k) = x(n^2)$

$x(n-k) = x|(n-k)^2|$

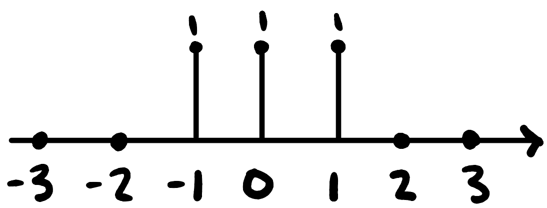
$= x|(n^2 + k^2 + 2kn)|$

DOES NOT $= y(n-k)$ TIME VARIANT

(b) 1.



2.



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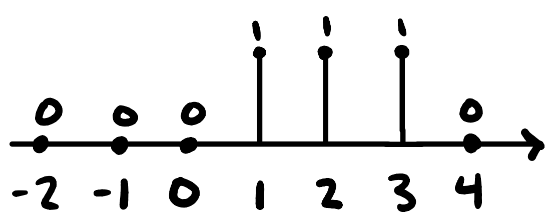

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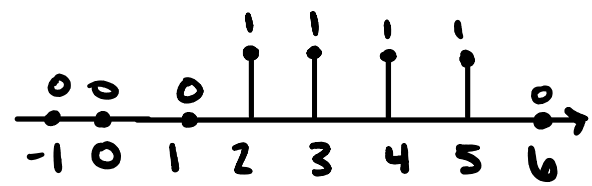

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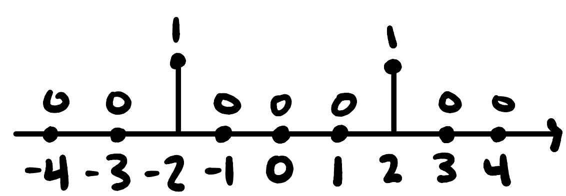


6. DOES NOT EQUAL

SYSTEM IS TIME-VARIANT.

2.7 A discrete-time system can be

(1) Static or dynamic
(2) Linear or nonlinear
(3) Time invariant or time varying
(4) Causal or noncausal
(5) Stable or unstable

Examine the following systems with respect to the properties above.

(a) $y(n) = \cos[x(n)]$

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

(c) $y(n) = x(n)\cos(\omega_0 n)$

(d) $y(n) = x(-n+2)$

(e) $y(n) = \text{Trun}[x(n)]$, where $\text{Trun}[x(n)]$ denotes the integer part of $x(n)$, obtained by truncation

(f) $y(n) = \text{Round}[x(n)]$, where $\text{Round}[x(n)]$ denotes the integer part of $x(n)$ obtained by rounding

Remark: The systems in parts (e) and (f) are quantizers that perform truncation and rounding, respectively.

(g) $y(n) = |x(n)|$

(h) $y(n) = x(n)u(n)$

(i) $y(n) = x(n) + nx(n+1)$

(j) $y(n) = x(2n)$

(k) $y(n) = \begin{cases} x(n), & \text{if } x(n) \geq 0 \\ 0, & \text{if } x(n) < 0 \end{cases}$

(l) $y(n) = x(-n)$

(m) $y(n) = \text{sign}[x(n)]$

(n) The ideal sampling system with input $x_a(t)$ and output $x(n) = x_a(nT)$, $-\infty < n < \infty$

(a) STATIC, NONLINEAR, TIME INVARIANT, CASUAL, STABLE.

(C) STATIC, LINEAR, TIME VARIANT, CASUAL, STABLE.

(e) STATIC, NON-LINEAR, TIME VARIANT, CASUAL, STABLE.

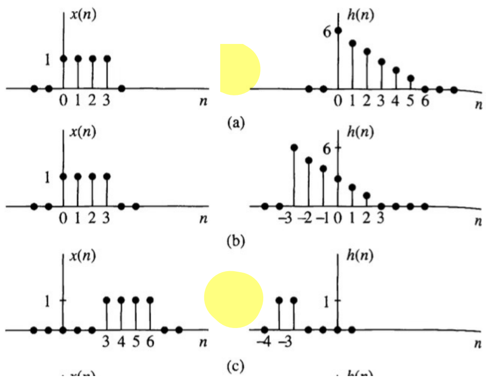
(g) STATIC, NON-LINEAR, TIME VARIANT, CASUAL, STABLE.

(i) DYNAMIC, LINEAR, TIME-VARIANT, NON-CASUAL, UNSTABLE

(K) STATIC, NON-LINEAR, TIME-VARIANT, CASUAL, STABLE.

(M) STATIC, NON-LINEAR, TIME-VARIANT, CASUAL, STABLE.

2.17 Compute and plot the convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown in Fig. P2.17.



(a) $x(n) = \{1, 1, 1, 1\}$ (indices 0 1 2 3) $\quad h(n) = \{6, 5, 4, 3, 2, 1\}$ (indices 0 1 2 3 4 5)

FORMULA:

$$\sum_{k=0}^{n} x(k)\,h(n-k)$$

$y(0) = 6$

$y(1) = 5 + 6 = 11$

$y(2) = 4 + 5 + 6 = 15$

$y(3) = 3 + 4 + 5 + 6 = 18$

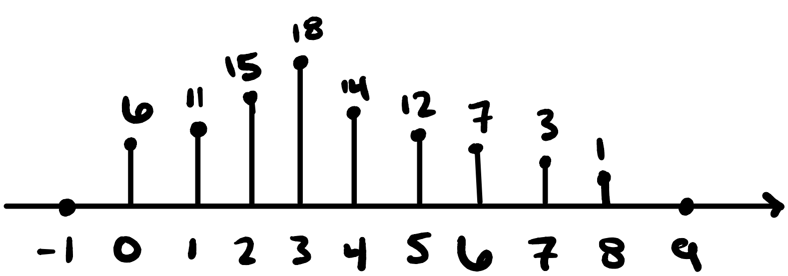
$y(4) = 2 + 3 + 4 + 5 + 0 = 14$

$y(5) = 1 + 2 + 3 + 4 + 2 + 1 = 12$

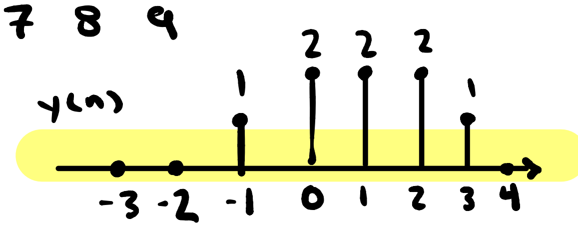
$y(6) = 1 + 2 + 4 = 7$

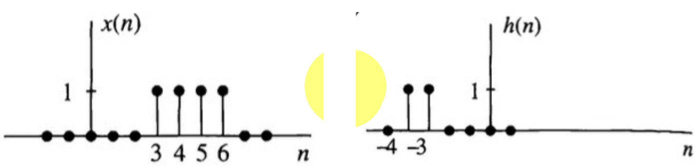
$y(7) = 1 + 2 = 3$

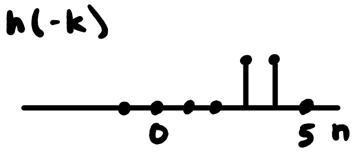
$y(8) = 1$

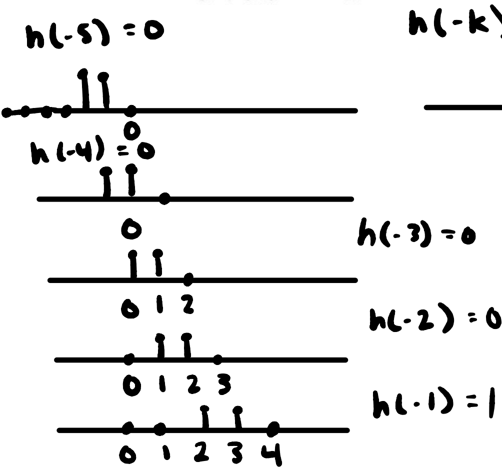


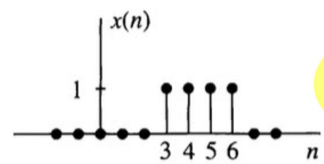
(b)

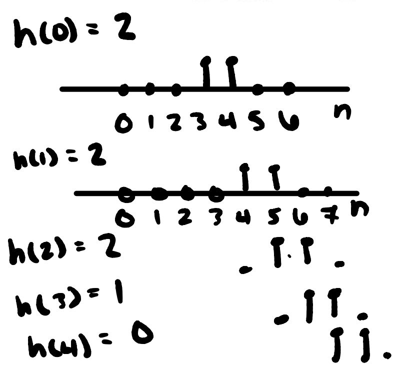










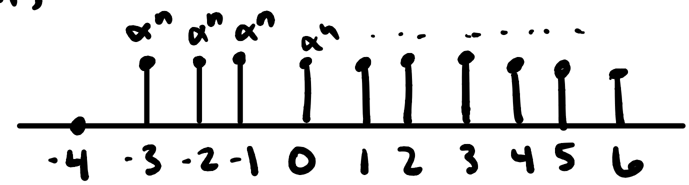


2.19 Compute the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \alpha^n, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

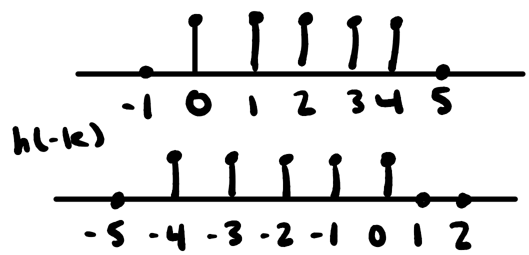
$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

$x(n)$



$h(n)$

$h(-k)$



$h(-4) = 0$

$h(-3) = \alpha^{-3}$

$h(-2) = \alpha^{-5}$

$h(-1) = \alpha^{-6}$

$h(0) = 1 + \alpha^{-6}$

$h(1) = 1 + \alpha^{-5}$

$h(2) =$

$h(3)$

$h(4)$

$h(5)$

$h(6)$

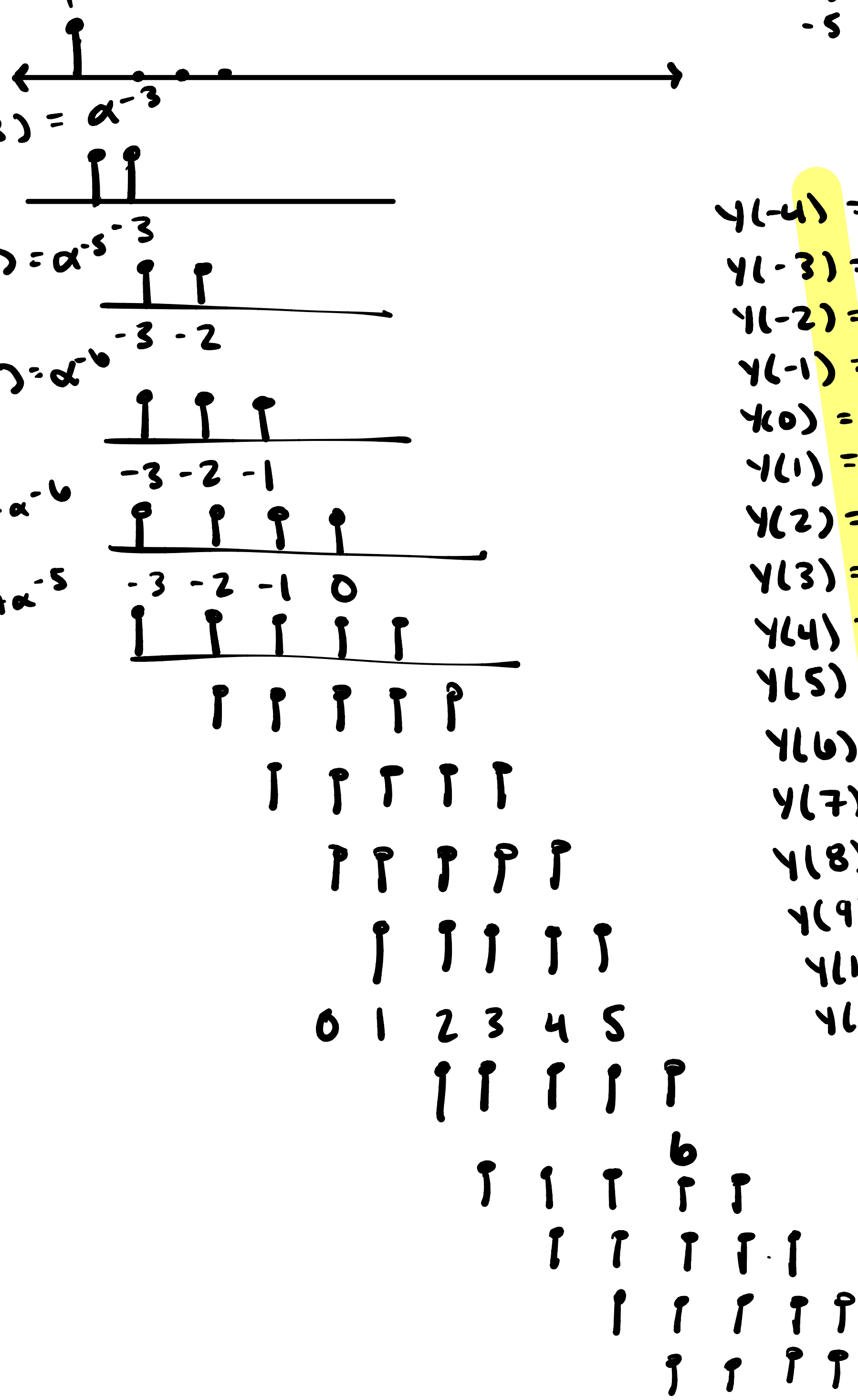
$h(7)$

$h(8)$

$h(9)$

$h(10)$

$h(11)$



$y(-4) = 0$

$y(-3) = \alpha^{-3}$

$y(-2) = \alpha^{-5}$

$y(-1) = \alpha^{-6}$

$y(0) = \alpha^{-6} + 1$

$y(1) = \alpha^{-5} + 1 + \alpha$

$y(2) = \alpha^{-6} + 1 + \alpha + \alpha^2$

$y(3) = \alpha^{-1} + 1 + \alpha + \alpha^5$

$y(4) = \alpha^{10} + 1$

$y(5) = \alpha^{15}$

$y(6) = \alpha^{14}$

$y(7) = \alpha^{12}$

$y(8) = \alpha^{9}$

$y(9) = \alpha^{5}$

$y(10) = 0$

$y(11) = 0$

2.23 Express the output $y(n)$ of a linear time-invariant system with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$.

IMPULSE:

$h(n) = s(n) - s(n-1)$

$y(n) = x(n) * h(n)$

Substitute:

$y(n) = x(n) * [s(n) - s(n-1)]$

$= x(n) * s(n) - x(n) * s(n-1)$

$\rightarrow y(n) = s(n) * x(n) - s(n-1) * x(n)$

1.10: An LTI system has impulse response $h[n] = u[n]$, find the response of the system to the input $x[n]$ described as follows:

$$x[n] = \begin{cases} 1 - \frac{|n|}{L}, & \text{if } |n| \leq L, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

$h(n) = u(n)$

AND $y(n) = x(n) * h(n)$

$= \sum_{k=-\infty}^{n} x(k)$

For $n < L$, $y(n) = 0$

For $-L \leq n < 0$

$y(n) = \sum_{k=-L}^{n} \left(1 + \frac{k}{L}\right)$

$0 \leq n \leq L$

$y(n) = \sum_{k=-L}^{0} \left(1 + \frac{k}{L}\right) + \sum_{k=1}^{n} \left(1 - \frac{k}{L}\right)$

$n > L$

$y(n) = \sum_{k=-L}^{L} \left(1 - \frac{|k|}{L}\right)$