ECEN 248
Introduction to Digital Systems Design

Lecture 2

Boolean Algebra and its Relation to Digital Circuits

- "Traditional" algebra
  - Variables represent real numbers (x, y)
  - Operators operate on variables, return real numbers (2.5*x + y - 3)
- ***Boolean Algebra***
  - Variables represent 0 or 1 only
  - Operators return 0 or 1 only
  - Basic operators
    - AND: *a AND b* returns 1 only when both a=1 and b=1
    - OR: *a OR b* returns 1 if either (or both) a=1 or b=1
    - NOT: *NOT a* returns the opposite of a (1 if a=0, 0 if a=1)

a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

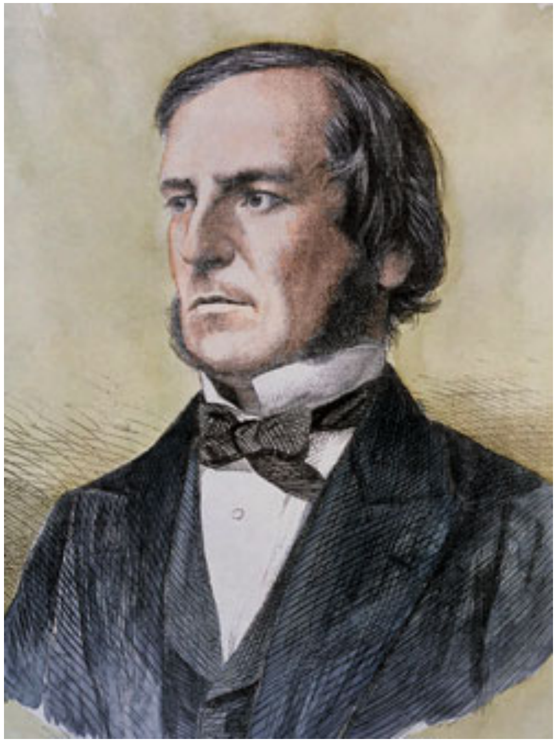
a

a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

a	NOT
0	1
1	0

George Boole

- English mathematician, 1815-1864
- Author of *Laws of Thoughts* (1854)
- Creator of Boolean Algebra



Boolean Algebra and its Relation to Digital Circuits

(Scalable)

- Developed mid-1800's by George Boole to formalize human thought
  - Ex: "I'll go to lunch if Mary goes OR John goes, AND Sally does not go."
    - Let F represent my going to lunch (1 means I go, 0 I don't go)
    - Likewise, m for Mary going, j for John, and s for Sally
    - Then **F = (m OR j) AND NOT(s)**
  - Nice features
    - Formally evaluate
      - m=1, j=0, s=1 --> F = (1 OR 0) AND NOT(1) = 1 AND 0 = **0**
    - Formally transform
      - F = (m and NOT(s)) OR (j and NOT(s))
        - » Looks different, but same function
        - » We'll show transformation techniques soon
    - Formally prove
      - Prove that if Sally goes to lunch (s=1), then I don't go (F=0)
      - F = (m OR j) AND NOT(1) = (m OR j) AND 0 = 0

a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

a	NOT
0	1
1	0

Evaluating Boolean Equations

a • Evaluate the Boolean equation **F = (a AND b) OR (c AND d)** for the given values of variables a, b, c, and d:
  - Q1: a=1, b=1, c=1, d=0.
    - Answer: F = (1 AND 1) OR (1 AND 0) = 1 OR 0 = 1.
  - Q2: a=0, b=1, c=0, d=1.
    - Answer: F = (0 AND 1) OR (0 AND 1) = 0 OR 0 = 0.
  - Q3: a=1, b=1, c=1, d=1.
    - Answer: F = (1 AND 1) OR (1 AND 1) = 1 OR 1 = 1.

a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

a	NOT
0	1
1	0

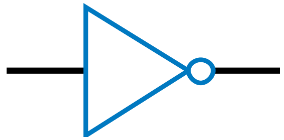
Converting to Boolean Equations

a • Convert the following English statements to a Boolean equation
  - Q1. a is 1 and b is 1.
    - Answer: F = a AND b
  - Q2. either of a or b is 1.
    - Answer: F = a OR b
  - Q3. a is 1 and b is 0.
    - Answer: F = a AND NOT(b) = b is 0
  - Q4. a is not 0.
    - Answer:
      - (a) Option 1: F = NOT(NOT(a))
      - (b) Option 2: F = a

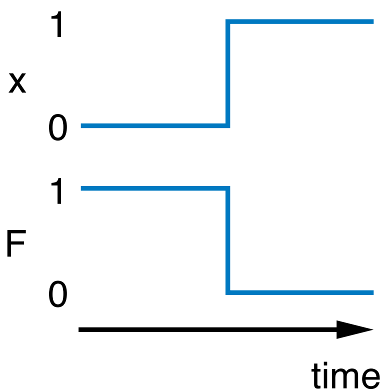
Converting to Boolean Equations

- *a* Q1. A fire sprinkler system should spray water if high heat is sensed and the system is set to enabled.
  - Answer: Let Boolean variable h represent "high heat is sensed," e represent "enabled," and F represent "spraying water." Then an equation is: F = h AND e.
- Q2. A car alarm should sound if the alarm is enabled, and either the car is shaken or the door is opened.
  - Answer: Let a represent "alarm is enabled," s represent "car is shaken," d represent "door is opened," and F represent "alarm sounds." Then an equation is: F = a AND (s OR d).
  - (a) Alternatively, assuming that our door sensor d represents "door is closed" instead of open (meaning d=1 when the door is closed, 0 when open), we obtain the following equation: F = a AND (s OR NOT(d)).

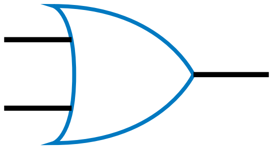
NOT gate



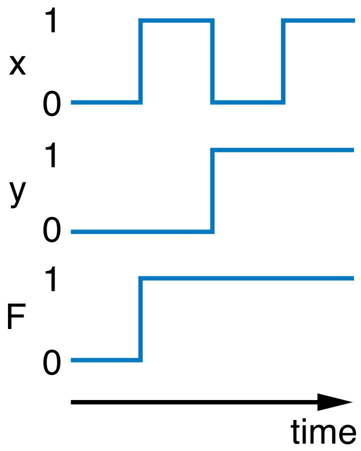
x	F
0	1
1	0



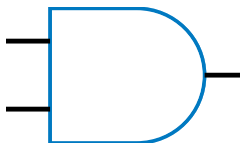
OR gate



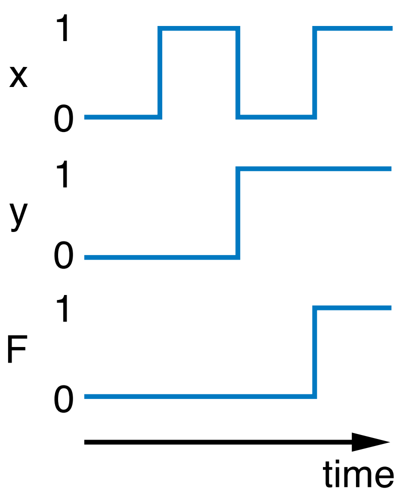
x	y	F
0	0	0
0	1	1
1	0	1
1	1	1



AND gate

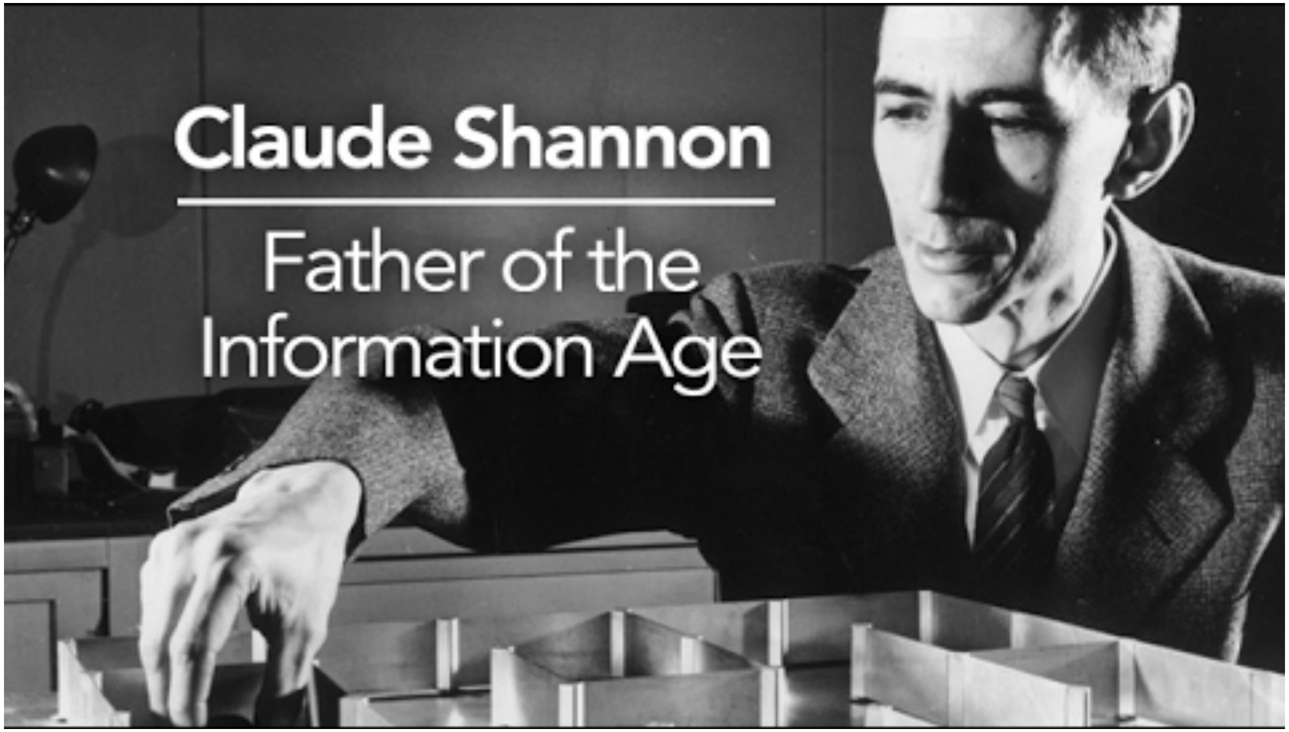


x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

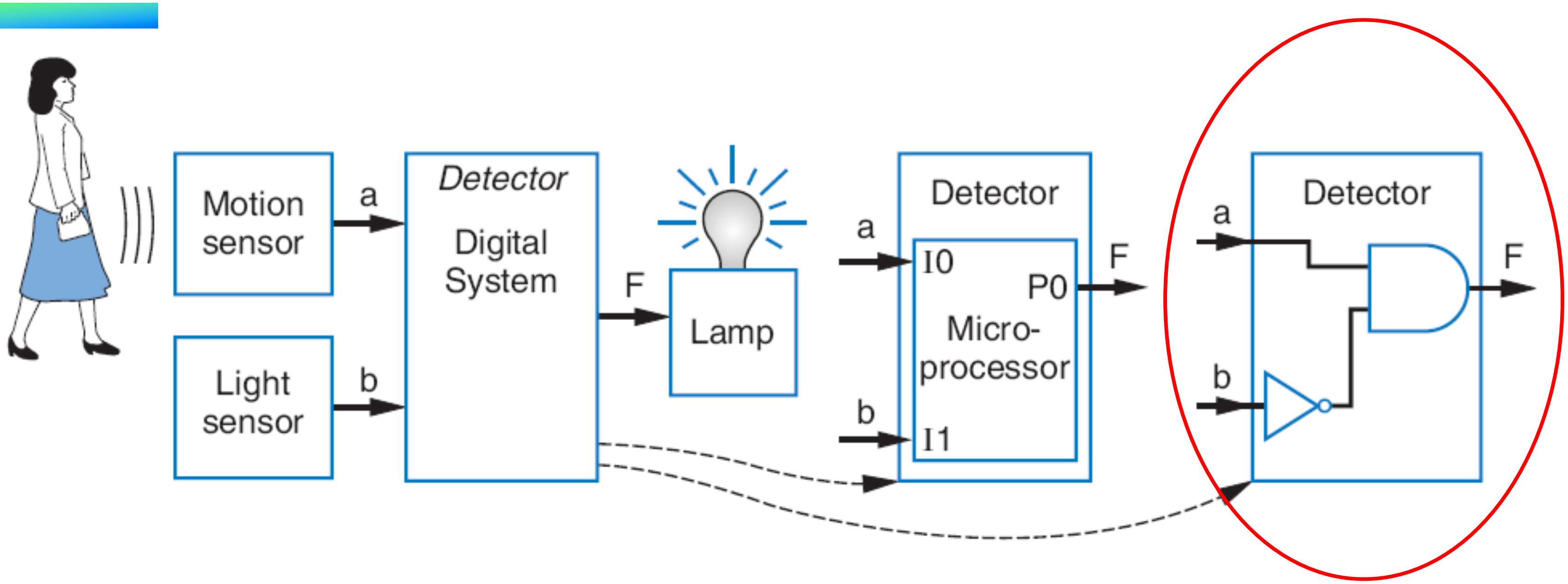


Claude Shannon

- American mathematician & electrical engineer (1916-2001)
- *A symbolic analysis of switching and delay circuits*, Master's thesis, MIT , 1937.



Building Circuits Using Gates



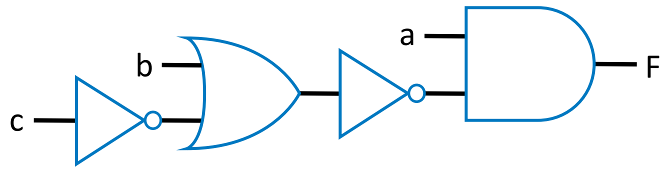
- Turn on lamp (F=1) when motion sensed (a=1) and no light (b=0)
- F = a AND NOT(b)
- Build using logic gates, AND and NOT, as shown
- We just built our first digital circuit!

Example: Converting a Boolean Equation to a Circuit of Logic Gates

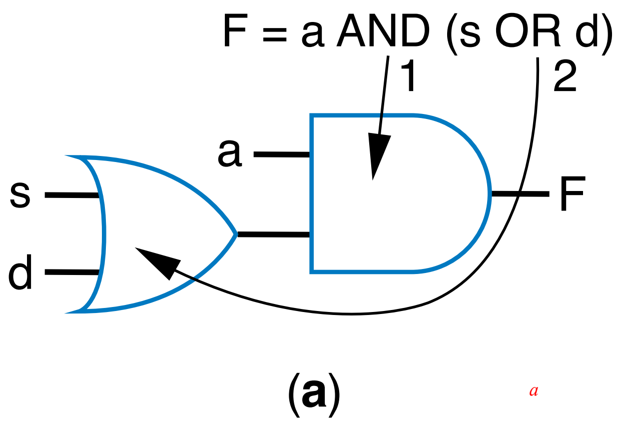
Start from the output, work back towards the inputs

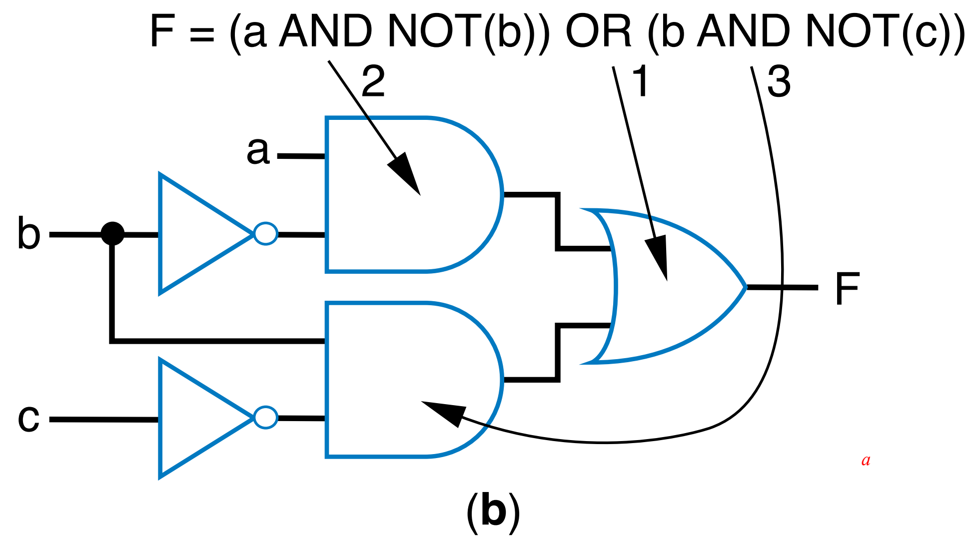
- Q: Convert the following equation to logic gates:
  F = a AND NOT(b OR NOT(c))

a



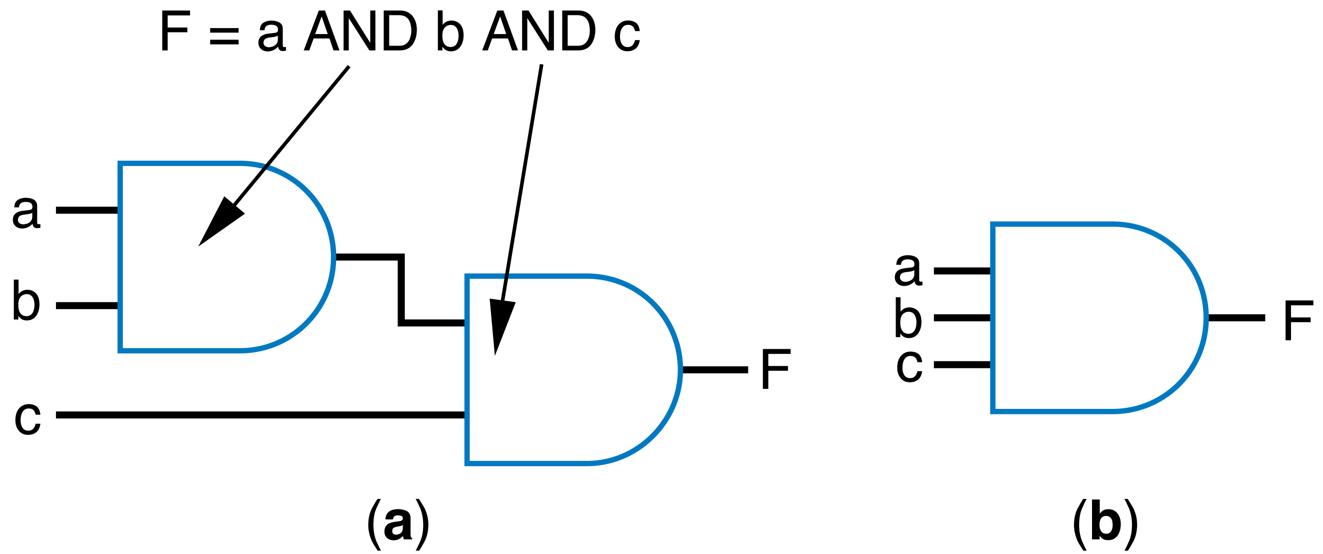
More examples





Start from the output, work back towards the inputs

Using gates with more than 2 inputs

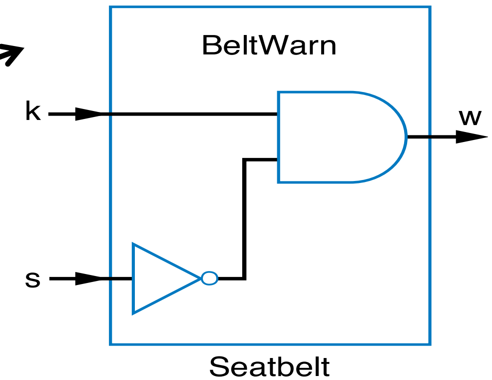


Can think of as AND(a,b,c)

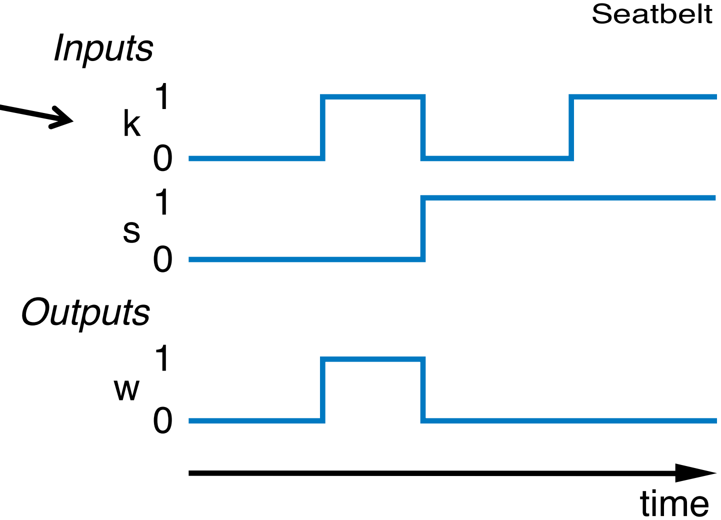
Example: Seat Belt Warning Light System

- Design circuit for warning light
- Sensors
  - s=1: seat belt fastened
  - k=1: key inserted
- Capture Boolean equation
  - seat belt not fastened, and key inserted
- Convert equation to circuit

w = NOT(s) AND k

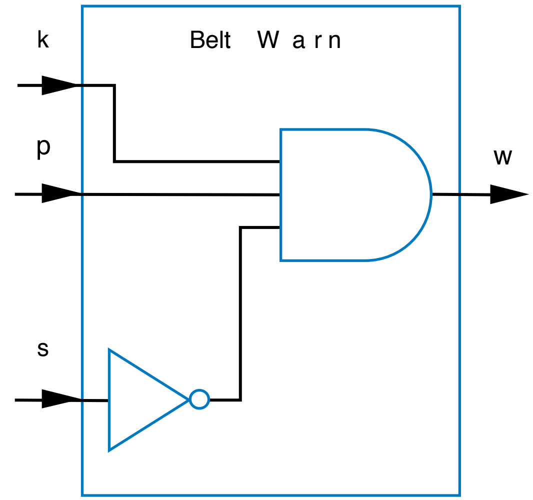


- *Timing diagram* illustrates circuit behavior
  - We set inputs to any values
  - Output set according to circuit

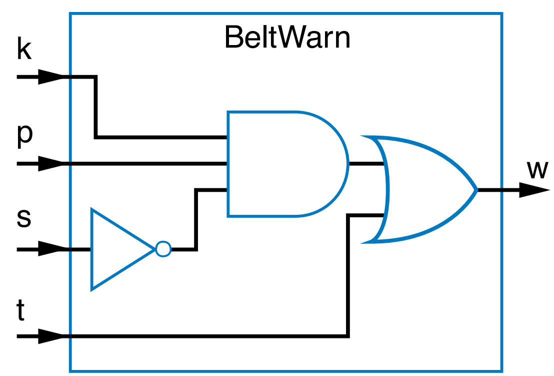


More examples: Seat belt warning light extensions

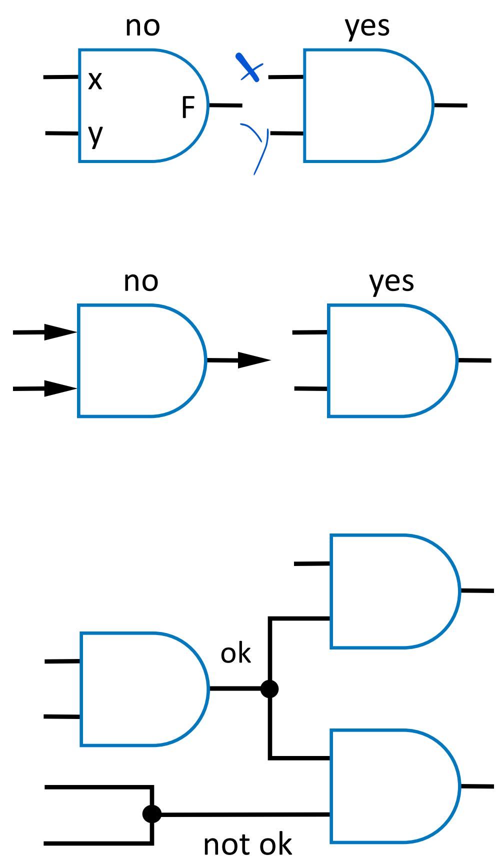
- Only illuminate warning light if person is in the seat (p=1), and seat belt not fastened and key inserted
- w = p AND NOT(s) AND k



- Given t=1 for 5 seconds after key inserted. Turn on warning light when t=1 (to check that warning lights are working)
- w = (p AND NOT(s) AND k) OR t



Some Gate-Based Circuit Drawing Conventions



Boolean Algebra

- By defining logic gates based on Boolean algebra, we can *use algebraic methods to manipulate circuits*
- Notation: Writing a AND b, a OR b, NOT(a) is cumbersome
  - Use symbols: a * b (or just ab), a + b, and a'
    - Original: w = (p AND NOT(s) AND k) OR t
    - New: w = ps'k + t
      - Spoken as "w equals p and s prime and k, or t"
      - Or just "w equals p s prime k, or t"
      - s' known as "complement of s"
    - While symbols come from regular algebra, ***don't*** say "times" or "plus"
      - "product" and "sum" are OK and commonly used

Boolean algebra precedence, highest precedence first.

Symbol	Name	Description
( )	Parentheses	Evaluate expressions nested in parentheses first
'	NOT	Evaluate from left to right
*	AND	Evaluate from left to right
+	OR	Evaluate from left to right

Boolean Algebra Operator Precedence

- Evaluate the following Boolean equations, assuming a=1, b=1, c=0, d=1.
  - Q1. F = a * b + c.
    - Answer: * has precedence over +, so we evaluate the equation as F = (1 *1) + 0 = (1) + 0 = 1 + 0 = 1.
  - Q2. F = ab + c.
    - Answer: the problem is identical to the previous problem, using the shorthand notation for *.
  - Q3. F = ab’.
    - Answer: we first evaluate b’ because NOT has precedence over AND, resulting in F = 1 * (1’) = 1 * (0) = 1 * 0 = 0.
  - Q4. F = (ac)’.
    - Answer: we first evaluate what is inside the parentheses, then we NOT the result, yielding (1*0)’ = (0)’ = 0’ = 1.
  - Q5. F = (a + b’) * c + d’.
    - Answer: Inside left parentheses: (1 + (1’)) = (1 + (0)) = (1 + 0) = 1. Next, * has precedence over +, yielding (1 * 0) + 1’ = (0) + 1’. The NOT has precedence over the OR, giving (0) + (1’) = (0) + (0) = 0 + 0 = 0.

Boolean algebra precedence, highest precedence first.

Symbol	Name	Description
( )	Parentheses	Evaluate expressions nested in parentheses first
’	NOT	Evaluate from left to right
*	AND	Evaluate from left to right
+	OR	Evaluate from left to right

Boolean Algebra Terminology

- Example equation: **F(a,b,c) = a' bc + abc' + ab + c**
- ***Variable***
  - Represents a value (0 or 1)
  - Three variables: a, b, and c

x^2+2x+3

- ***Literal***
  - Appearance of a variable, in true or complemented form
  - Nine literals: a' , b, c, a, b, c' , a, b, and c
- ***Product term***
  - Product of literals
  - Four product terms: a' bc, abc' , ab, c

F=ac+bc+d

- ***Sum-of-products***
  - Equation written as OR of product terms only
  - Above equation is in sum-of-products form. "F = (a+b)c + d" is not.

Boolean Algebra Properties

- Commutative
  - a + b = b + a
  - a * b = b * a
- Distributive
  - a * (b + c) = a * b + a * c
    - Can write as: a(b+c) = ab + ac
  - a + (b * c) = (a + b) * (a + c)
    - (This second one is tricky!)
    - Can write as: a+(bc) = (a+b)(a+c)
- Associative
  - (a + b) + c = a + (b + c)
  - (a * b) * c = a * (b * c)
- Identity
  - 0 + a = a + 0 = a
  - 1 * a = a * 1 = a
- Complement
  - a + a' = 1
  - a * a' = 0
- To prove, just evaluate all possibilities

Example uses of the properties

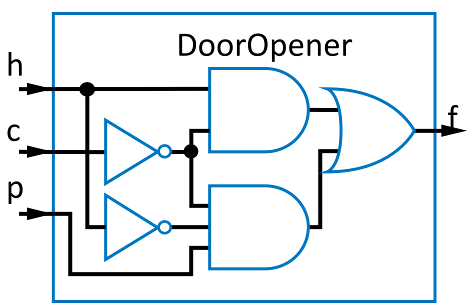
- Show abc' equivalent to c'ba.
  - Use commutative property:
    - a*b*c' = a*c'*b = c'*a*b = c'*b*a
- Show abc + abc' = ab.
  - Use first distributive property
    - abc + abc' = ab(c+c').
  - Complement property
    - Replace c+c' by 1: ab(c+c') = ab(1).
  - Identity property
    - ab(1) = ab*1 = ab.
- Show x + x'z equivalent to x + z.
  - Second distributive property
    - Replace x+x'z by (x+x')*(x+z).
  - Complement property
    - Replace (x+x') by 1,
  - Identity property
    - replace 1*(x+z) by x+z.

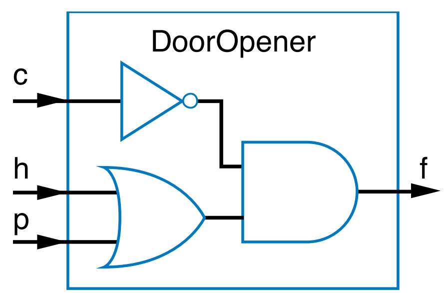
Example that Applies Boolean Algebra Properties

- Want automatic door opener circuit (e.g., for grocery store)
  - Output: f=1 opens door
  - Inputs:
    - p=1: person detected
    - h=1: switch forcing hold open
    - c=1: key forcing closed
  - Want open door when
    - h=1 and c=0, or
    - h=0 and p=1 and c=0
  - Equation: f = hc' + h'pc'

- Can the circuit be simplified?

f = hc' + h'pc'	
f = c'h + c'h'p	(by the commutative property)
f = c'(h + h'p)	(by the first distrib. property)
f = c'((h+h')*(h+p))	(2nd distrib. prop.; tricky one)
f = c'((1)*(h + p))	(by the complement property)
f = c'(h+p)	(by the identity property)



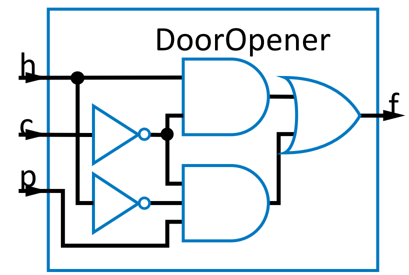


Simplified circuit

Simplification of circuits is covered in Sec. 2.11 / Sec 6.2.

Example that Applies Boolean Algebra Properties

- Found inexpensive chip that computes:
    - $f = c'hp + c'hp' + c'h'p$
  - Can we use it for the door opener?
    - Is it the same as $f = hc' + h'pc'$?



- Apply Boolean algebra:

$f = c'hp + c'hp' + c'h'p$

$f = c'h(p + p') + c'h'p$ (by the distributive property)

$f = c'h(1) + c'h'p$ (by the complement property)

$f = c'h + c'h'p$ (by the identity property)

$f = hc' + h'pc'$ (by the commutative property)

Same! Yes, we can use it.

a

- Commutative
  - $a + b = b + a$
  - $a * b = b * a$
- Distributive
  - $a * (b + c) = a * b + a * c$
  - $a + (b * c) = (a + b) * (a + c)$
- Associative
  - $(a + b) + c = a + (b + c)$
  - $(a * b) * c = a * (b * c)$
- Identity
  - $0 + a = a + 0 = a$
  - $1 * a = a * 1 = a$
- Complement
  - $a + a' = 1$
  - $a * a' = 0$

Boolean Algebra: Additional Properties

- Null elements
  - $a + 1 = 1$
  - $a * 0 = 0$
- Idempotent Law
  - $a + a = a$
  - $a * a = a$
- Involution Law
  - $(a')' = a$
- DeMorgan's Law
  - $(a + b)' = a'b'$
  - $(ab)' = a' + b'$
  - *Very useful!*
- To prove, just evaluate all possibilities

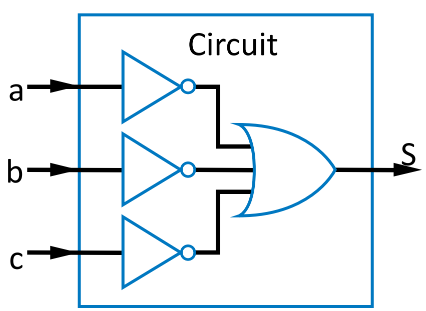
Example Applying DeMorgan's Law

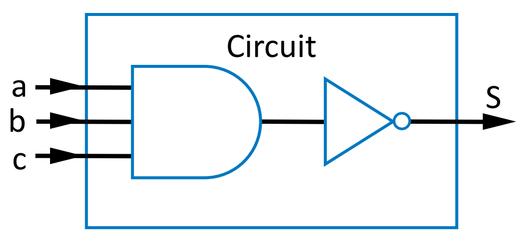
$(a + b)' = a'b'$

$(ab)' = a' + b'$

Aircraft lavatory sign example

- Behavior
  - Three lavatories, each with sensor (a, b, c), equals 1 if door locked
  - Light "Available" sign (S) if any lavatory available
- Equation and circuit
  - $S = a' + b' + c'$
- Transform
  - $(abc)' = a'+b'+c'$ (by DeMorgan's Law)
  - $S = (abc)'$
- New circuit





- Alternative: Instead of lighting "Available," light "Occupied"
  - Opposite of "Available" function $S = a' + b' + c'$
  - So $S' = (a' + b' + c')'$
    - $S' = (a')' * (b')' * (c')'$ (by DeMorgan's Law)
    - $S' = a * b * c$ (by Involution Law)
  - Makes intuitive sense
    - Occupied if all doors are locked

Example Applying Properties

- For door opener f = c'(h+p) , *prove* door stays closed (f=0) when c=1
  - f = c'(h+p)
  - *Let c = 1* (door forced closed)
  - f = 1'(h+p)
  - f = 0(h+p)
  - f = 0h + 0p (by the distributive property)
  - f = 0 + 0 (by the null elements property)
  - f = 0

- Commutative
  - a + b = b + a
  - a * b = b * a
- Distributive
  - a * (b + c) = a * b + a * c
  - a + (b * c) = (a + b) * (a + c)
- Associative
  - (a + b) + c = a + (b + c)
  - (a * b) * c = a * (b * c)
- Identity
  - 0 + a = a + 0 = a
  - 1 * a = a * 1 = a
- Complement
  - a + a' = 1
  - a * a' = 0
- Null elements
  - a + 1 = 1
  - a * 0 = 0
- Idempotent Law
  - a + a = a
  - a * a = a
- Involution Law
  - (a')' = a
- DeMorgan's Law
  - (a + b)' = a' b'
  - (ab)' = a' + b'

Complement of a Function

- Commonly want to find complement (inverse) of function F
  - 0 when F is 1; 1 when F is 0
- Use DeMorgan's Law repeatedly
  - Note: DeMorgan's Law defined for more than two variables, e.g.:
    - (a + b + c)' = (abc)'
    - (abc)' = (a' + b' + c')
- Complement of f = w'xy + wx'y'z'
  - f ' = (w'xy + wx'y'z')'
  - f ' = (w'xy)'(wx'y'z')' (by DeMorgan's Law)
  - f ' = (w+x'+y')(w'+x+y+z) (by DeMorgan's Law)
- Can then expand into sum-of-products form

Representations of Boolean Functions

English 1: F outputs 1 when a is 0 and b is 0, or when a is 0 and b is 1.

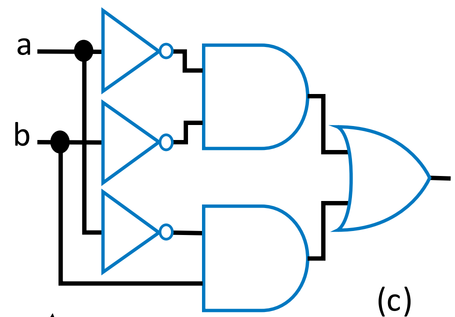
English 2: F outputs 1 when a is 0, regardless of b's value

(a)

Equation 1: F(a,b) = a' b' + a' b

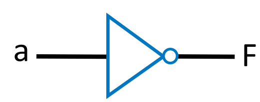
Equation 2: F(a,b) = a'

(b)



Circuit 1

(c)



Circuit 2

a	b	F
0	0	1
0	1	1
1	0	0
1	1	0

Truth table

(d)

The function F

- A function can be represented in different ways
  - Above shows seven representations of the same functions F(a,b), using four different methods: English, Equation, Circuit, and Truth Table

Truth Table Representation of Boolean Functions

- Define value of F for each possible combination of input values
  - 2-input function: 4 rows
  - 3-input function: 8 rows
  - 4-input function: 16 rows
- Q: Use truth table to define function F(a,b,c) that is 1 when abc is 5 or greater in binary

a	b	F
0	0	
0	1	
1	0	
1	1	

(a)

a	b	c	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(b)

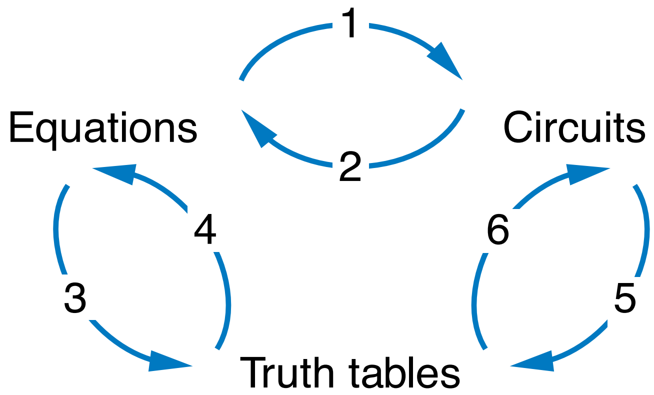
a	b	c	d	F
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

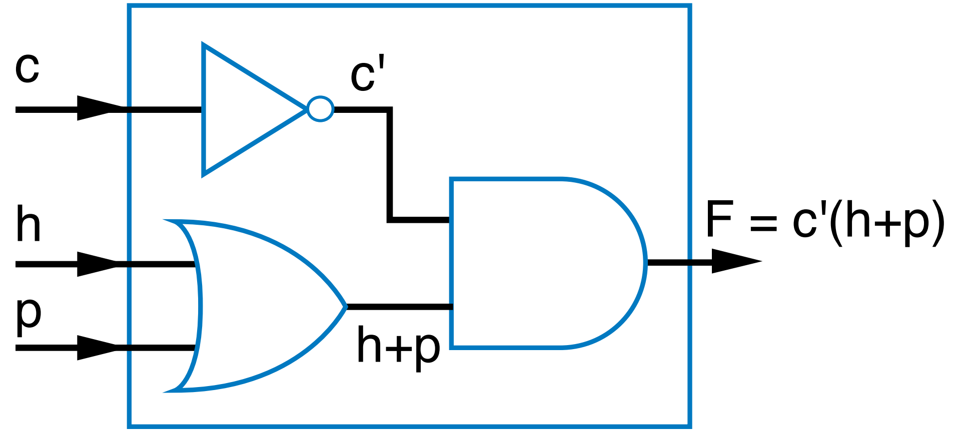
(c)

a	b	c	F
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

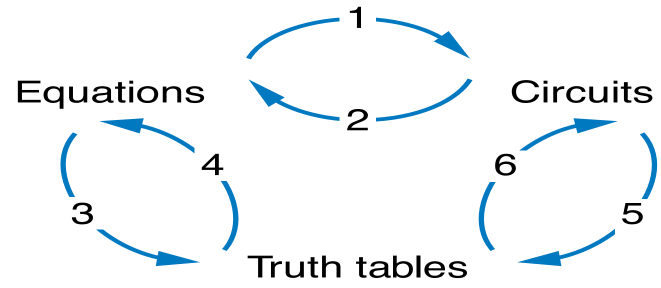
Converting among Representations

- Can convert from any representation to another
- Common conversions
  - Equation to circuit (we did this earlier)
  - Circuit to equation
    - Start at inputs, write expression of each gate output





Converting among Representations



- Additional common conversions
  - Truth table to equation (which we can then convert to circuit)
    - Easy–just OR each input term that should output 1
  - Equation to truth table
    - Easy—just evaluate equation for each input combination (row)
    - Creating intermediate columns helps

Inputs		Outputs	Term
a	b	F	F = sum of
0	0	1	a' b'
0	1	1	a' b
1	0	0	
1	1	0	

$F = a'b' + a'b$

Q: Convert to equation

a	b	c	F	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	ab' c
1	1	0	1	abc'
1	1	1	1	abc

$F = ab'c + abc' + abc$

Q: Convert to truth table: $F = a'b' + a'b$

Inputs				Output
a	b	a' b'	a' b	F
0	0	1	0	**1**
0	1	0	1	**1**
1	0	0	0	**0**
1	1	0	0	**0**

Example: Converting from Truth Table to Equation

- Parity bit
  - Extra bit added to data, intended to enable detection of error (a bit changed unintentionally)
  - e.g., errors can occur on wires due to electrical interference
- Even parity
  - Set parity bit so total number of 1s (data + parity) is even
  - e.g., if data is 001, parity bit is 1 → 0011 has even number of 1s
- Want equation, but easiest to start from truth table for this example

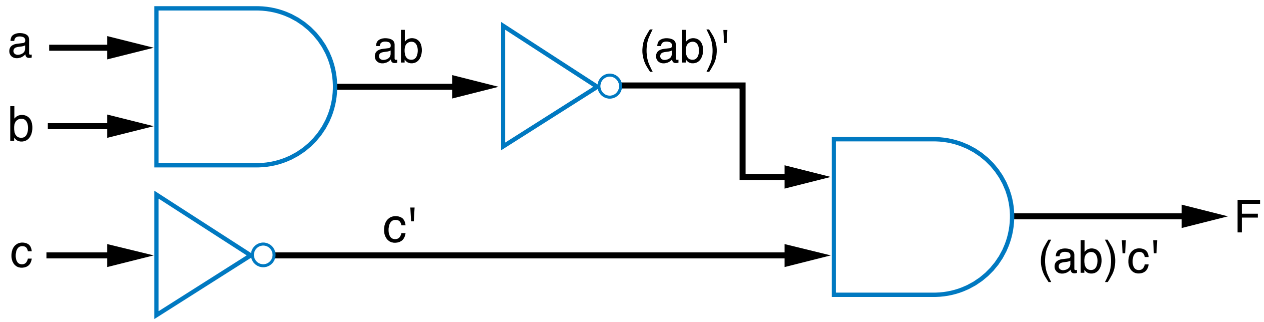
a	b	c	P	
0	0	0	0	(0 1s: even)
0	0	1	1	(2 1s: even)
0	1	0	1	
0	1	1	0	
1	0	0	1	
1	0	1	0	
1	1	0	0	
1	1	1	1	

Convert to equation

$$P = a'b'c + a'bc' + ab'c' + abc$$

Example: Converting from Circuit to Truth Table

- First convert to circuit to equation, then equation to table



Inputs						Outputs
a	b	c	ab	(ab)'	c'	F
0	0	0	0	1	1	1
0	0	1	0	1	0	0
0	1	0	0	1	1	1
0	1	1	0	1	0	0
1	0	0	0	1	1	1
1	0	1	0	1	0	0
1	1	0	1	0	1	0
1	1	1	1	0	0	0

Standard Representation: Truth Table

- How can we determine if two functions are the same?
  - Recall automatic door example
    - Same as $f = hc' + h'pc'$? $= ?$
    - Used algebraic methods
    - But if we failed, does that prove *not* equal? No.
- Solution: Convert to truth tables
  - Only ONE truth table representation of a given function
    - ***Standard*** representation—for given function, only one version in standard form exists

$f = c'hp + c'hp' + c'h'$

$f = c'h(p + p') + c'h'p$

$f = c'h(1) + c'h'p$

$f = c'h + c'h'p$ (what if we stopped here?)

$f = hc' + h'pc'$

Q: Determine if $F=ab+a'$ is same function as $F=a'b'+a'b+ab$, by converting each to truth table first

$F = ab + a'$

a	b	F
0	0	1
0	1	1
1	0	0
1	1	1

$F = a'b' + a'b + ab$

a	b	F
0	0	1
0	1	1
1	0	0
1	1	1

Same

a

Truth Table Canonical Form

- Q: Determine via truth tables whether ab+a' and (a+b)' are equivalent

$F = ab + a'$

a	b	F
0	0	1
0	1	1
1	0	0
1	1	1

$F = (a+b)'$

a	b	F
0	0	1
0	1	0
1	0	0
1	1	0

Not equivalent

a

Canonical Form – Sum of Minterms

- Truth tables too big for numerous inputs
- Use standard form of equation instead
  - Known as ***canonical form***
  - Regular algebra: group terms of polynomial by power
    - $ax^2 + bx + c$ $(3x^2 + 4x + 2x^2 + 3 + 1 \rightarrow 5x^2 + 4x + 4)$
  - Boolean algebra: create sum of minterms
    - ***Minterm***: product term with every function literal appearing exactly once, in true or complemented form
    - Just multiply-out equation until sum of product terms
    - Then expand each term until all terms are minterms

Q: Determine if F(a,b)=ab+a’ is equivalent to F(a,b)=a’b’+a’b+ab, by converting first equation to canonical form (second already is)

F = ab+a’ (already sum of products)
F = ab + a’(b+b’) (expanding term)
F = ab + a’b + a’b’ (Equivalent – same three terms as other equation)

Canonical Form – Sum of Minterms

- Q: Determine whether the functions G(a,b,c,d,e) = abcd + a'bcde and H(a,b,c,d,e) = abcde + abcde' + a'bcde + a'bcde(a' + c) are equivalent.

G = abcd + a'bcde

G = abcd(e+e') + a'bcde

G = abcde + abcde' + a'bcde

G = a'bcde + abcde' + abcde *(sum of minterms form)*

Equivalent

a

H = abcde + abcde' + a'bcde + a'bcde(a' + c)

H = abcde + abcde' + a'bcde + a'bcdea' + a'bcdec

H = abcde + abcde' + a'bcde + a'bcde + a'bcde

H = abcde + abcde' + a'bcde (last two terms were redundant)

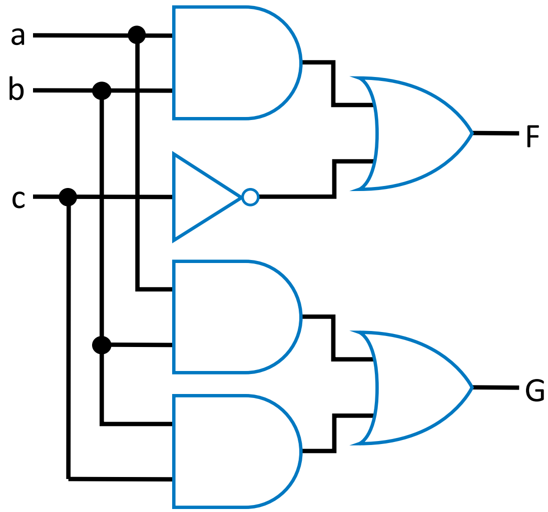
H = a'bcde + abcde' + abcde

Compact Sum of Minterms Representation

- List each minterm as a number
- Number determined from the binary representation of its variables' values
  - a'bcde corresponds to 01111, or 15
  - abcde' corresponds to 11110, or 30
  - abcde corresponds to 11111, or 31
- Thus, H = a'bcde + abcde' + abcde can be written as:
  - H = $\sum$m(15,30,31)
  - "H is the sum of minterms 15, 30, and 31"

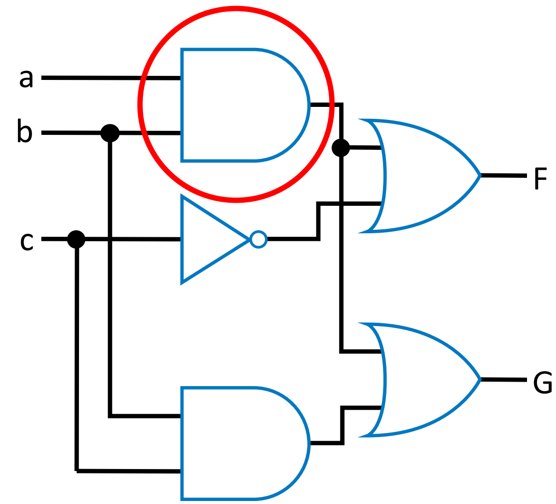
Multiple-Output Circuits

- Many circuits have more than one output
- Can give each a separate circuit, or can share gates
- Ex: F = ab + c' , G = ab + bc



(a)

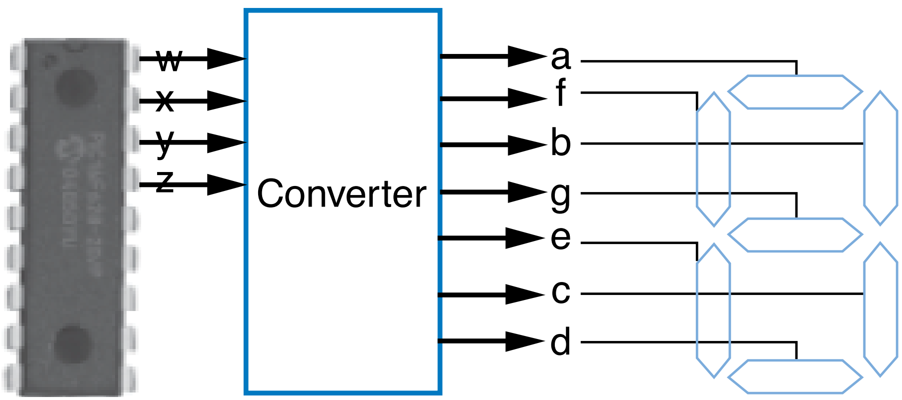
Option 1: Separate circuits

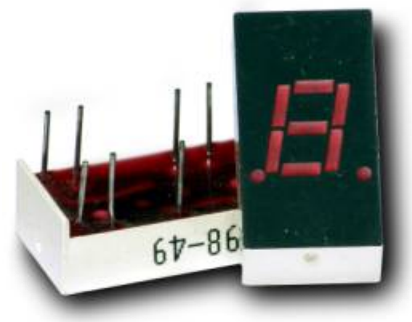


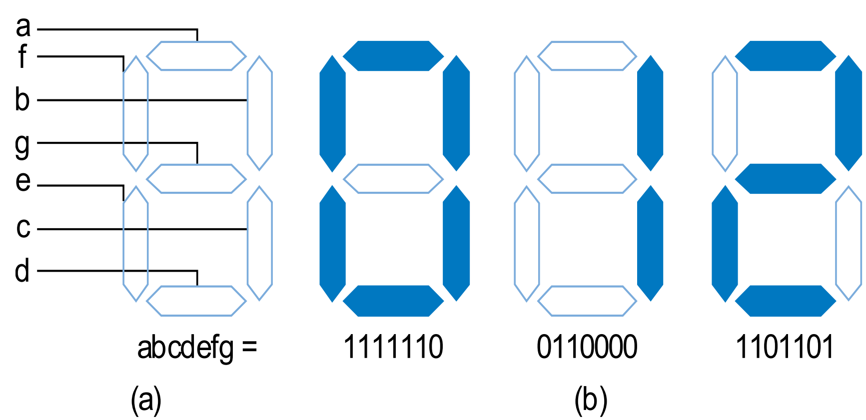
(b)

Option 2: Shared gates

Multiple-Output Example:
BCD to 7-Segment Converter



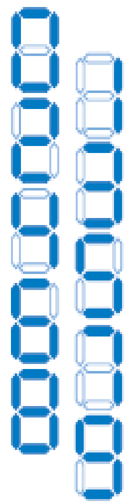


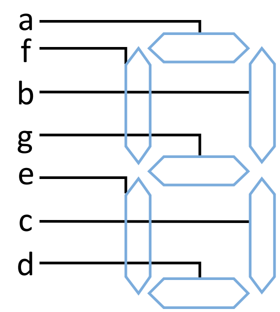


Multiple-Output Example: BCD to 7-Segment Converter

TABLE 2-4 4-bit binary number to seven-segment display truth table

w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0





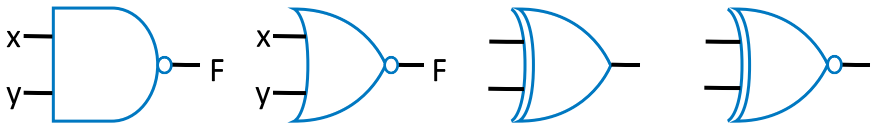
$a = w'x'y'z' + w'x'yz' + w'x'yz + w'xy'z + w'xyz' + w'xyz + wx'y'z' + wx'y'z$

$b = w'x'y'z' + w'x'y'z + w'x'yz' + w'x'yz + w'xy'z' + w'xyz + wx'y'z' + wx'y'z$

•••

More Gates

NAND NOR XOR XNOR



x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

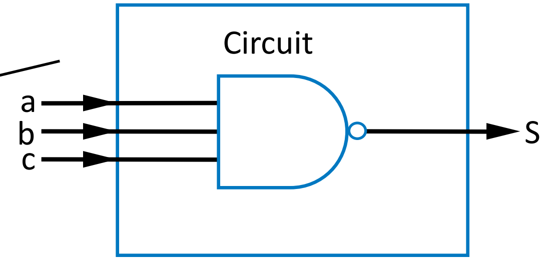
x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

x	y	F
0	0	1
0	1	0
1	0	0
1	1	1

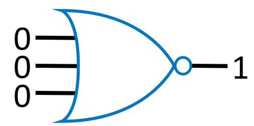
- NAND: Opposite of AND (“NOT AND”)
- NOR: Opposite of OR (“NOT OR”)
- XOR: Exactly 1 input is 1, for 2-input XOR. (For more inputs -- odd number of 1s)
- XNOR: Opposite of XOR (“NOT XOR”)

More Gates: Example Uses

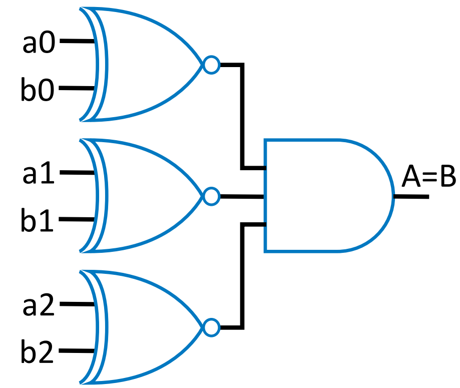
- Aircraft lavatory sign example
  - S = (abc)'



- Detecting all 0s
  - Use NOR



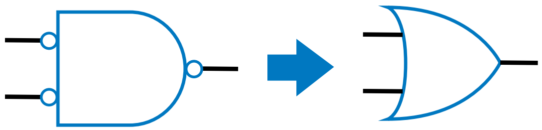
- Detecting equality
  - Use XNOR



- Detecting odd # of 1s
  - Use XOR
  - Useful for generating "parity" bit common for detecting errors

Completeness of NAND

- Any Boolean function can be implemented *using just NAND gates.* Why?
  - Need AND, OR, and NOT
  - NOT: 1-input NAND (or 2-input NAND with inputs tied together)
  - AND: NAND followed by NOT
  - OR: NAND preceded by NOTs
  - Thus, NAND is a universal gate
    - Can implement any circuit using just NAND gates



- Likewise for NOR

Number of Possible Boolean Functions

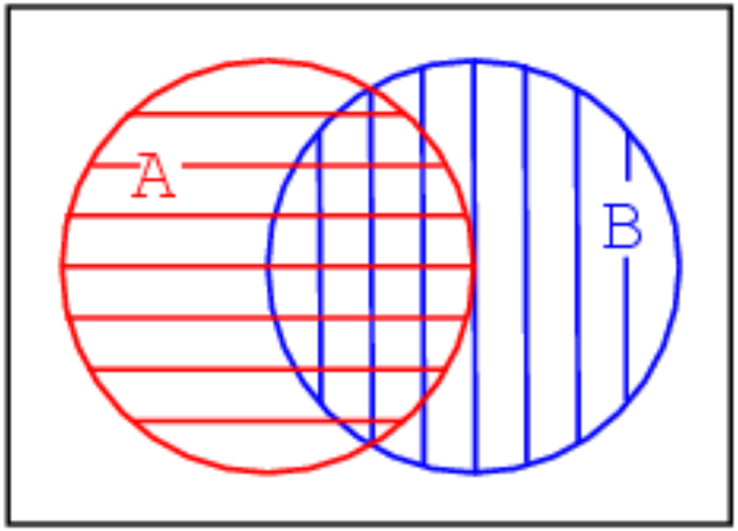
- How many possible functions of 2 variables?
  - 2^2 rows in truth table, 2 choices for each
  - $2^{(2^2)} = 2^4 = 16$ possible functions
- N variables
  - 2^N rows
  - $2^{(2^N)}$ possible functions

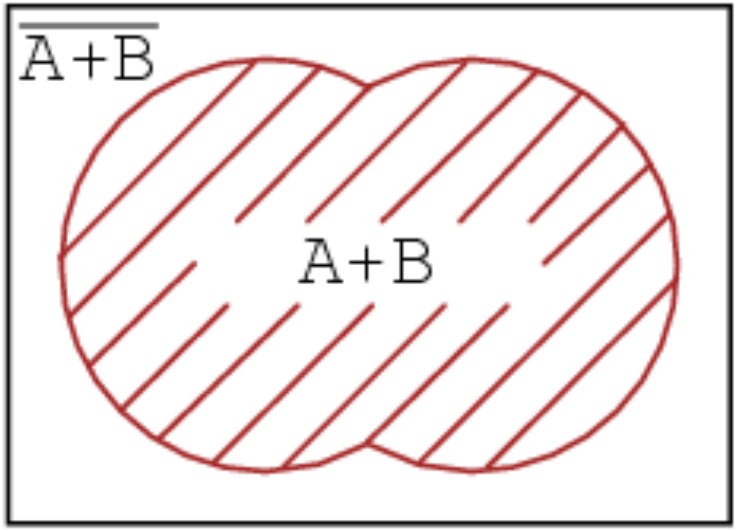
a	b	F	
0	0	0 or 1	2 choices
0	1	0 or 1	2 choices
1	0	0 or 1	2 choices
1	1	0 or 1	2 choices

$2^4 = 16$
possible functions

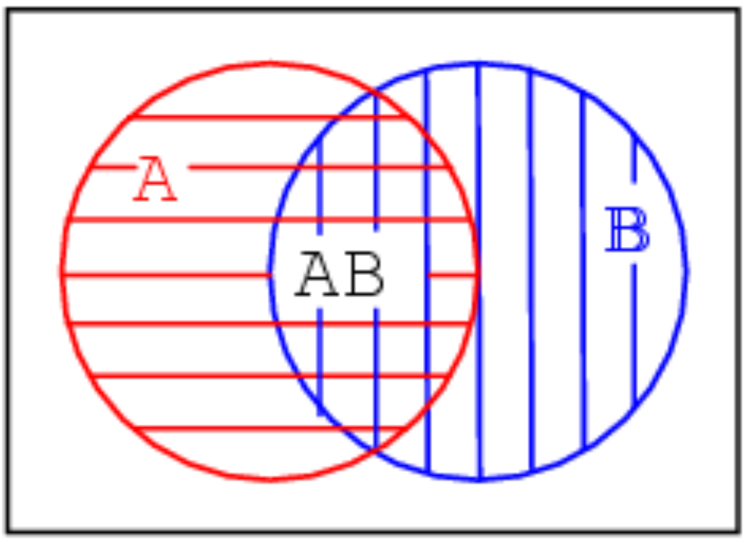
a	b	f0	f1	f2	f3	f4	f5	f6	f7	f8	f9	f10	f11	f12	f13	f14	f15
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
0	**1**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**
1	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**
1	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**1**
		0	a AND b		a		b	a XOR b	a OR b	a NOR b	a XNOR b	b’		a’		a NAND b	1

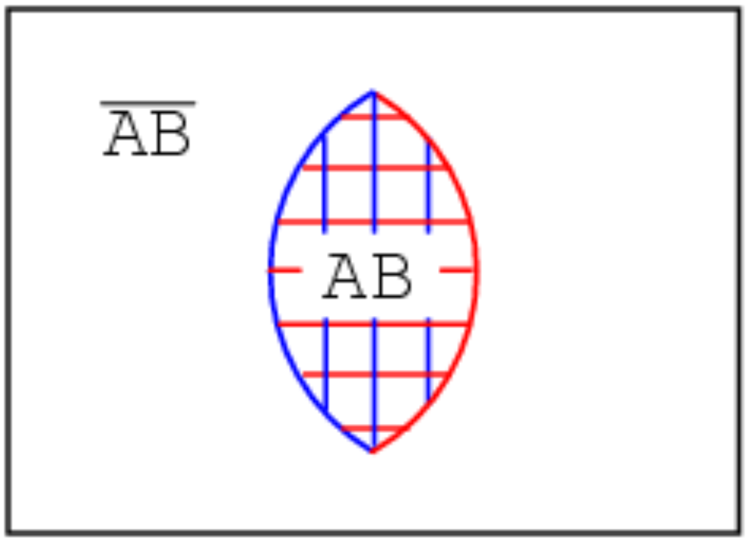
Venn Diagram: A+B



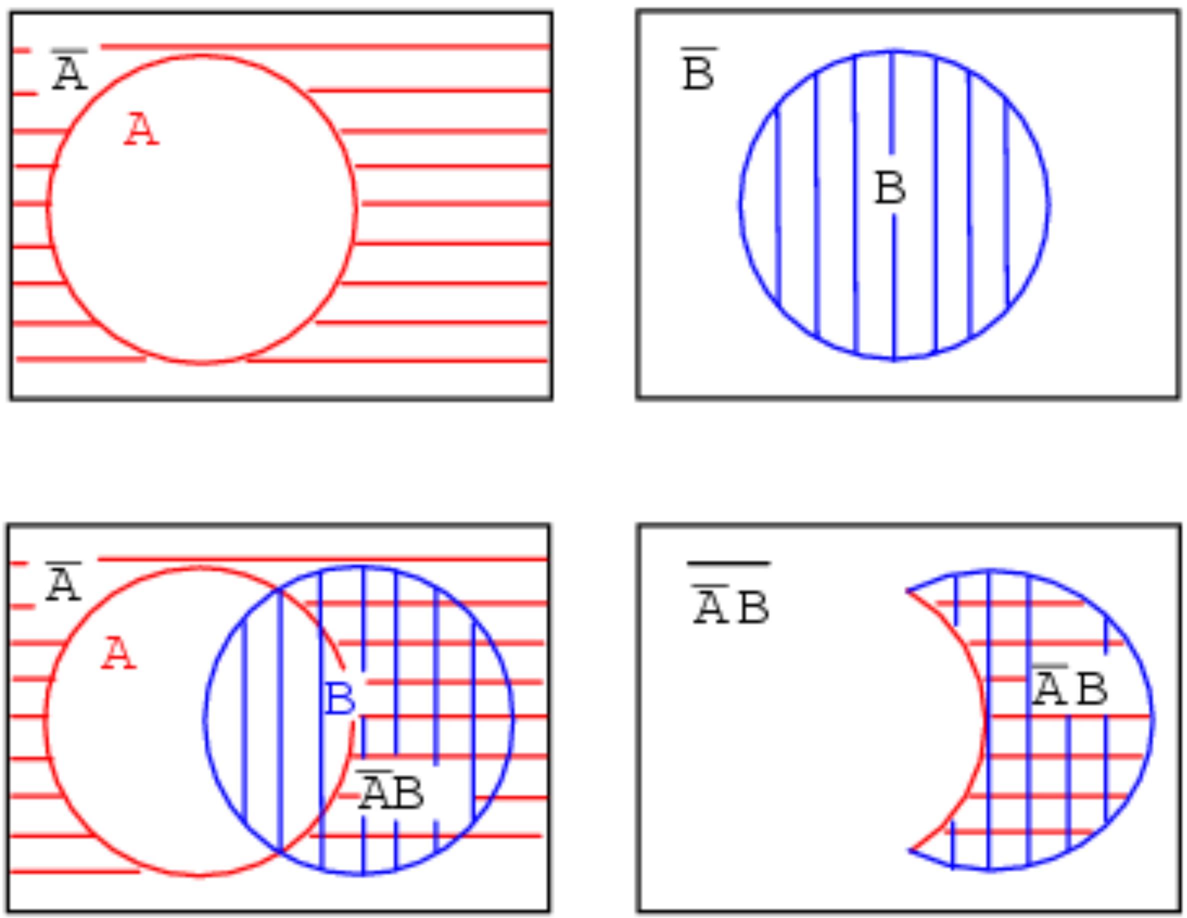


Venn Diagram: A•B

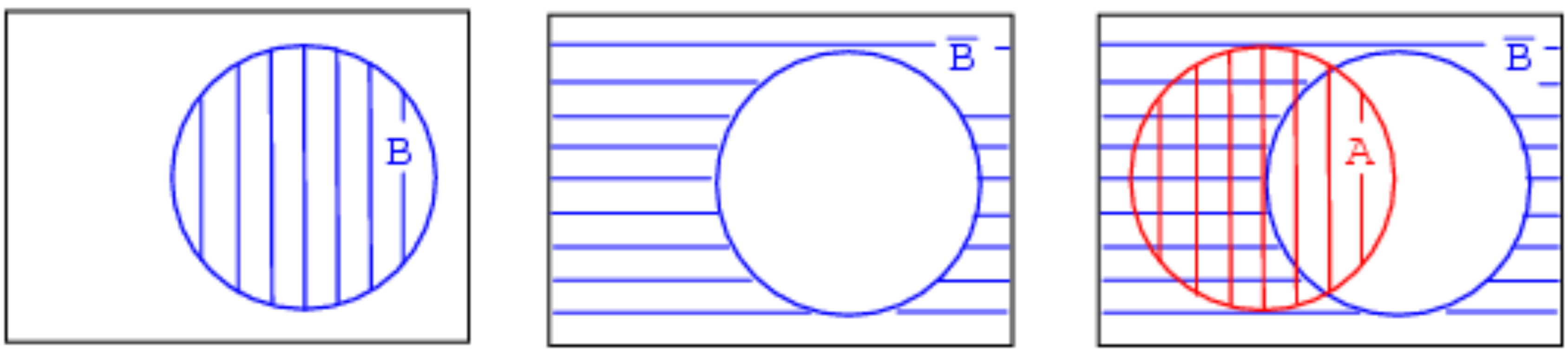




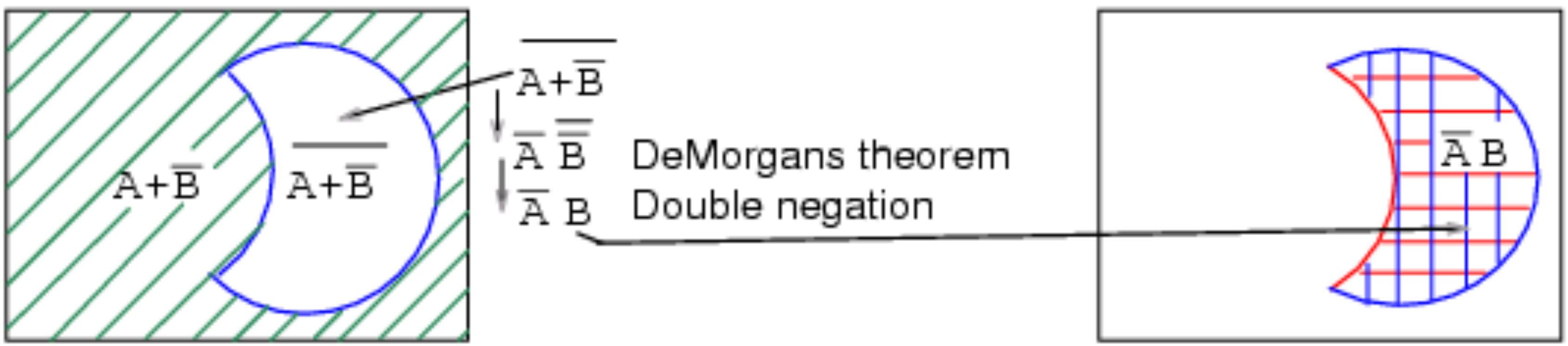
Venn Diagram: A’ B



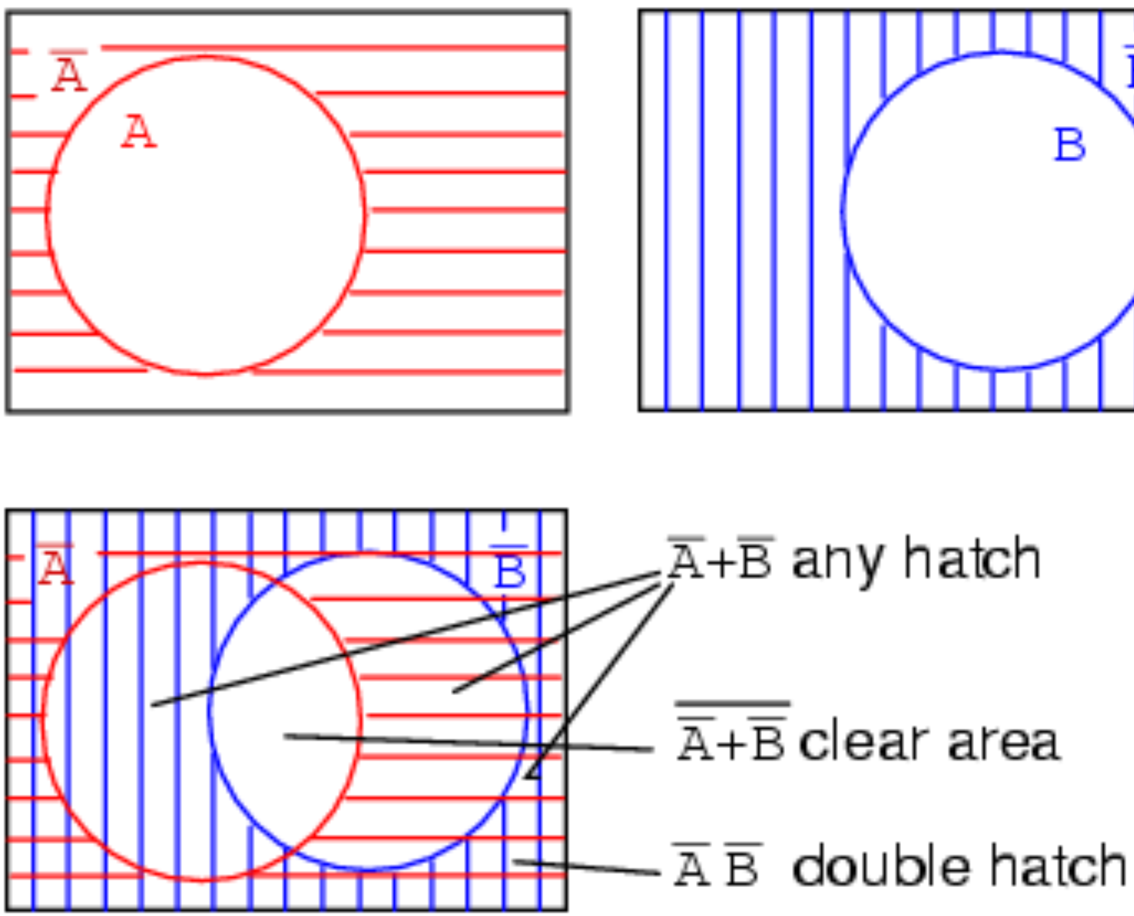
Venn Diagram: A+B'



Venn Diagram: (A+B’)’ = A’ •B



Venn Diagram: A’ +B’ =?



Venn Diagram: A’ +B’ = (AB)’