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ECEN-314 HW 1

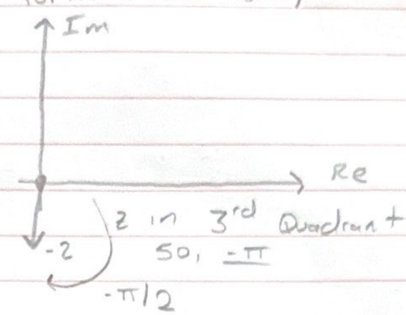
1. (a) $-2j$

Complex # → Exponential form (Polar form)
(Rectangular)

$0-2j$

$z = a + jb$

- $a = Re(z)$: Real #
- $b = Im(z)$: real #
- $j = \sqrt{-1}$: Imaginary



- $r = \sqrt{a^2+b^2}$
  - $r = \sqrt{0^2 + (-2)^2} = \pm\underline{2}$
- $\theta = \tan^{-1}(b/a) - \pi$
  - $\theta = \cancel{\tan^{-1}(-2/0)} - \pi$ (→ 0)
  - $\theta = -\pi/2$

$$\boxed{\begin{array}{l} \cdot\ z = 2\cos(-\pi/2) + 2j\sin(-\pi/2) = 2e^{j\frac{-\pi}{2}} \\ \cdot\ (2, -\pi/2) \end{array}}$$

(b) $(1-j)^2 = (1-j)(1-j)$ $\quad -j^2 = (\sqrt{-1})^2 = -1$

$= \cancel{1} - j - j - \cancel{1}$

$= -2j$

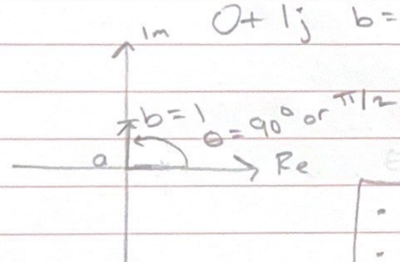
$$\boxed{\begin{array}{l} \cdot\ z = 2\cos(-\pi/2) + 2j\sin(-\pi/2) = 2e^{j\frac{-\pi}{2}} \\ \cdot\ \text{Polar Coordinates } (2, -\pi/2) \end{array}}$$

(c) $\dfrac{1+j}{1-j}$ • multiply by 1 (complex conjugate)

- $\dfrac{1+j}{1-j} \cdot \dfrac{1+j}{1+j} = \dfrac{(1+j)(1+j)}{(1-j)(1+j)}$ $\quad -j^2 = (-\sqrt{-1})^2$

$z = a + jb =$ $\quad = 1 + 2j + (-1) / 1 - (-1) = 2$

Im $0 + 1j$ $\quad b = 1, a = 0$ $\quad = \underline{2j/2} = \underline{j}$



- $r = \sqrt{0^2+1^2} = \underline{1}$
- $\theta = (\pi/2)$

$$\boxed{\begin{array}{l} \cdot\ z = \cos(\pi/2) + j\sin(\pi/2) = e^{j\pi/2} \\ \cdot\ \text{Polar Coordinates } (1, \pi/2) \end{array}}$$

2. $\cos^2\theta = 1/2(1+\cos 2\theta)$

① $e^{j\theta} = \cos\theta + j\sin\theta$ $\quad j=\sqrt{-1},\ j^2=-1$

$e^{j\theta} = \cos\theta + j\sin\theta$
$e^{-j\theta} = \cos\theta - j\sin\theta$

② $e^{j\theta} + e^{-j\theta} = \cos\theta + \cancel{j\sin\theta} + \cos\theta - \cancel{j\sin\theta} = 2\cos\theta$

$\cos\theta = \frac{1}{2}(e^{j\theta}+e^{-j\theta})$

$(e^{j\theta}+e^{-j\theta})^2 = (2\cos\theta)^2 \rightarrow \frac{(e^{j\theta}+e^{-j\theta})^2}{4} = \frac{\cancel{4}\cos^2\theta}{\cancel{4}}$

$\rightarrow \frac{1}{4}(e^{j\theta}+e^{-j\theta})(e^{j\theta}+e^{-j\theta}) = \cos^2\theta$

$\rightarrow \frac{1}{4}(e^{2j\theta} + 2\cancel{e^{j\theta}e^{-j\theta}}^{1} + e^{-2j\theta}) = \cos^2\theta$

$2\cos 2\theta = (e^{2j\theta}+e^{-2j\theta})$

$\rightarrow \frac{1}{4}(2\cos 2\theta + 2) = \cos^2\theta$

$\rightarrow \boxed{\frac{1}{2}(1+\cos 2\theta) = \cos^2\theta}$

3. $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)$ when $n=0$

- a: first term
- r: common ratio

$0 + 0 + -1\left(\frac{1}{4}\right) + 0 + \frac{1}{16} + 0 + \frac{1}{32} + 0 + \frac{1}{128}$

$r \neq 1,\quad \frac{a - ar^N}{1-r}$

① Convert $\left(\frac{1}{2}\right)^n\left(\frac{e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}}{2}\right)$

$\rightarrow \frac{1}{2}\left(\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n e^{j\frac{\pi n}{2}} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n e^{-j\frac{\pi n}{2}}\right)$ PLUG IN FOR $e^{j\frac{\pi n}{2}}$

$\rightarrow \frac{1}{2}\left(\left(\frac{1}{2}\right)^n\left(\cos\left(\frac{\pi n}{2}\right) + j\sin\left(\frac{\pi n}{2}\right)\right) + \left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right) - j\sin\left(\frac{\pi n}{2}\right)\right)$

$a_1 = 1(1+0) = 1$
$a_2 = \frac{1}{2}(0+j) = \frac{1}{2}j$
$\frac{a_1}{1-r} = \frac{1}{1-j/2}\cdot\frac{2}{2} = \frac{2}{2-j}$
$\frac{2}{2-j}\cdot\frac{(2+j)}{(2+j)} = \frac{4+2j}{4+1} = \frac{4+2j}{5}$

$a_1 = 1$
$a_2 = -j/2$
$= \frac{1}{1+j/2}$
$\frac{2}{2+j}\cdot\frac{2-j}{2-j} = \frac{4-2j}{5}$

$\frac{1}{2}\left(\frac{4}{5} + \frac{2j}{5} - \frac{2j}{5} + \frac{4}{5}\right) = \frac{1}{\cancel{2}}\left(\frac{8}{5}\right) = \boxed{\frac{4}{5}}$

4. 5th Roots of $z = 2e^{j\frac{\pi}{4}}$

$$z = 2e^{j\frac{\pi}{4}} = 2e^{j\left(\frac{\pi}{4}+2\pi\right)} = 2e^{j\left(\frac{\pi}{4}+4\pi\right)} = 2e^{j\left(\frac{\pi}{4}+6\pi\right)} = 2e^{j\left(\frac{\pi}{4}+8\pi\right)}$$

- 1st root: $\sqrt[5]{2e^{j\frac{\pi}{4}}} = \sqrt[5]{2}\, e^{j\frac{\pi}{20}}$
- 2nd root: $\sqrt[5]{2e^{j\left(\frac{\pi}{4}+2\pi\right)}} = \sqrt[5]{2}\, e^{j\left(\frac{\pi}{20}+\frac{2\pi}{5}\right)}$
- 3rd root: $\sqrt[5]{2e^{j\left(\frac{\pi}{4}+4\pi\right)}} = \sqrt[5]{2}\, e^{j\left(\frac{\pi}{20}+\frac{4\pi}{5}\right)}$
- 4th root: $\sqrt[5]{2e^{j\left(\frac{\pi}{4}+6\pi\right)}} = \sqrt[5]{2}\, e^{j\left(\frac{\pi}{20}+\frac{6\pi}{5}\right)}$
- 5th root: $\sqrt[5]{2e^{j\left(\frac{\pi}{4}+8\pi\right)}} = \sqrt[5]{2}\, e^{j\left(\frac{\pi}{20}+\frac{8\pi}{5}\right)}$

1. Express each of the following complex numbers in exponential form.

(a) $-2j$
(b) $(1-j)^2$
(c) $\frac{1+j}{1-j}$

2. Using Euler's relation, derive the following relationship.

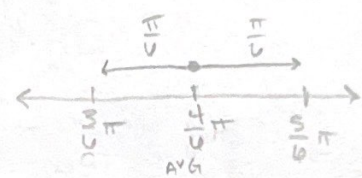
$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

3. Using Euler's and geometric sum formulas, evaluate the following sum. The final answer is a numerical value.

$$\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)$$

4. Find all the fifth roots of $z = 2e^{j\frac{\pi}{4}}$.

5. Compute the magnitude and the angle of a complex number below.



$$z = e^{j\frac{5\pi}{6}} + e^{j\frac{1\pi}{2}} \quad \frac{?}{3} = \frac{3\pi}{6}$$

$z = e^{j\frac{5\pi}{6}} + e^{j\frac{\pi}{2}} =$

- $\frac{5\pi}{6} = \frac{4\pi}{6} + \frac{\pi}{6}$
- $\frac{\pi}{2} = \frac{4\pi}{6} - \frac{\pi}{6}$
- $z = e^{j\frac{5\pi}{6}} + e^{j\frac{\pi}{2}} = e^{j\frac{4\pi}{6}}\left(e^{j\frac{\pi}{6}} + e^{-j\frac{\pi}{6}}\right) = e^{j\frac{4\pi}{6}} \times 2\cos\frac{\pi}{6}$
  $= 2e^{j\frac{4\pi}{6}}\cos\left(\frac{\pi}{6}\right)$
- $\boxed{\text{Magnitude } |z| = \cancel{2}\cdot 1\cdot \frac{\sqrt{3}}{\cancel{2}} = \boxed{\sqrt{3}}}$
- $\boxed{\text{Angle } \angle z = \frac{4\pi}{6}}$