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Valero

Subject: 731000141

Date:

By:

1. CASUAL LTI SYSTEM DESCRIBED BY:

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n]$$

(a) Find impulse response $h[n]$.

$$y[n] = Y(z),\ \frac{1}{4}y[n-1] = \frac{z^{-1}}{4}Y(z),\ y[n-2] = -\frac{z^{-2}}{8}Y(z)$$

$$x[n] = X(z)$$

$$Y(z) - \frac{1}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) = X(z)$$

$$Y(z)\left[1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right] = X(z)$$

$$H(z) = \frac{1}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} = \frac{1}{(z-\frac{3}{8})(z-\frac{3}{8})}$$

$$x^2 - \frac{1}{4}x - \frac{1}{8}$$

$$(z+\frac{1}{4})(z-\frac{1}{2}) = z^2 - \frac{1}{2}z + \frac{1}{4}z - \frac{1}{8}$$

$$H(z) = \frac{1}{(z+\frac{1}{4})(z-\frac{1}{2})} = \frac{A}{z+\frac{1}{4}} + \frac{B}{z-1/2}$$

$$1 = A(z-1/2) + B(z+1/4)$$

$z = 1/2$ $\quad 1 = \frac{3}{4}B \rightarrow B = 4/3$

$z = -1/4$ $\quad 1 = -\frac{3}{4}A \rightarrow A = -4/3$

$$H(z) = \frac{-4/3}{z+\frac{1}{4}} + \frac{4/3}{z-1/2}$$

$$\boxed{h[n] = -\frac{4}{3}\left(-\frac{1}{4}\right)^n u[n] + \frac{4}{3}\left(\frac{1}{2}\right)^n u[n]}$$

1.

(b) Output $= y[n] = 3\delta(n)$

Input $\rightarrow$ $x[n] = \delta(n) - \frac{2}{3}\delta[n-1]$

$y[n] = x(n) * h(n)$

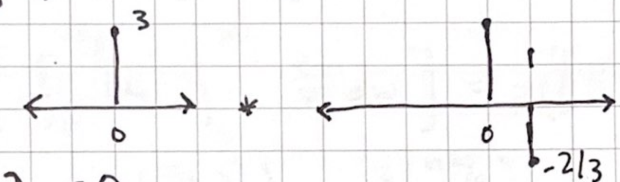
$3 = 1 - \frac{2}{3} e^{-j\omega} * h(n)$

Find $h[n]$

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

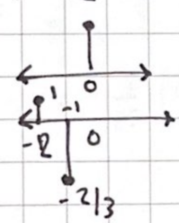
~~n=0 → 2~~

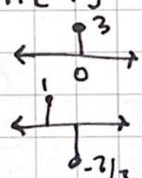
~~y[1] →~~



$h[n] = \cancel{2} -2 = 0$

$h[-1] = \cancel{0/3} -2$

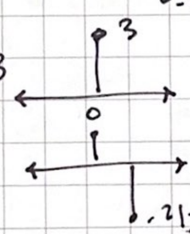




Convolution

$\boxed{\text{~~Impulse response~~} \quad \cancel{h[n] = \delta[n] - \frac{2}{3}\delta[n+1]}}$

$h(0) = \cancel{3}\,3$



$\{0, \underset{\uparrow 0}{3}, -2, 0\} = h[n] = 3\delta[n] - 2\delta[n-1]$

$$\boxed{\begin{array}{l}\text{Impulse Respone} \\ h[n] = 3\delta[n] - 2\delta[n-1]\end{array}}$$

2. Consider x[n], FT is

$$X(e^{j\omega}) = \begin{cases} 1-\left|\frac{3\omega}{\pi}\right|, & \text{if } |\omega| \leq \pi/3 \\ 0, & \pi/3 < |\omega| \leq \pi \end{cases}$$

~~(a)~~ Compute x[0]

$$x[0] = \frac{1}{2\pi}\int_{-\omega}^{\omega} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$|\omega| \leq \pi/3$ so, ω to $\pi/3$ AND $|\omega|$ to $\pi/3$

2 integrals

$$x[0] = \frac{1}{\cancel{2}\pi}\left[\cancel{2}\int_0^{\pi/3} 1-\left|\frac{3\omega}{\pi}\right| d\omega\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/3} 1\,d\omega - \int_0^{\pi/3}\frac{3\omega}{\pi}\,d\omega\right] = \frac{1}{\pi}\left(\left[\frac{\pi}{3}-0\right]-\left[\frac{3\omega^2}{2\pi}-0\right]_0^{\pi/3}\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi}{3} - \frac{3(\pi/3)^2}{2\pi}\right) \qquad \frac{3(\pi^2/9)}{2\pi} = \frac{\cancel{\pi}^2/3}{2\cancel{\pi}} \cdot \frac{\frac{1}{\pi}}{\frac{1}{\pi}}$$

$$= \frac{1}{\pi}\left(\frac{2\pi}{6} - \frac{\pi}{6}\right) \qquad = \frac{\pi/3}{2} \cdot \frac{1/2}{\frac{1}{2}} = \frac{\pi}{6}$$

$$= \frac{1}{\cancel{\pi}}\left(\frac{\cancel{\pi}}{6}\right) = \boxed{\frac{1}{6}} \qquad \boxed{x[0] = 1/6}$$

(b) Compute $\sum_{n=-\infty}^{\infty} x[n]$

At $-\infty, \omega = 0$

$$\boxed{\sum_{n=-\infty}^{\infty} x[n] = 1} \quad \rightarrow \sum_{n=-\infty}^{\infty} 1-\left|\frac{3\omega)}{\pi}\right| = 1-\left|\frac{3(0)}{\pi}\right|$$

(c) Compute $\sum_{n=-\infty}^{\infty} (-1)^n \left(1-\left|\frac{3\omega}{\pi}\right|\right)$

Here, $\frac{\pi}{3} < \pi$ AND $> -\pi$

$$X(e^{j\pi}) = 0$$

$$\boxed{\sum_{n=-\infty}^{\infty} (-1)^n x[n] = 0}$$

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(d) DRAW FT of $y[n] = x[n]\cos\left(\frac{\pi n}{3}\right)$

* multiply by $x[n]$ by cosine = shift.

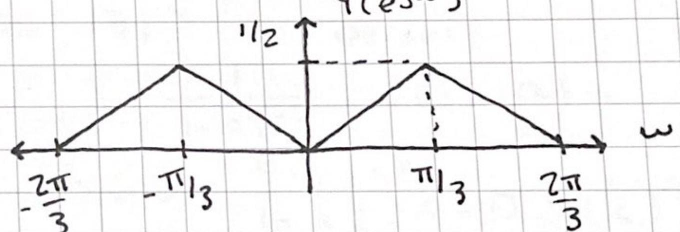
$$\cos(\omega_0 n) \overset{F.T.}{\longleftrightarrow} \frac{1}{2}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$\cos\left(\frac{\pi n}{3}\right) = \frac{1}{2}\left[\delta\left(\omega-\frac{\pi}{3}\right) + \delta\left(\omega+\frac{\pi}{3}\right)\right]$$

$$y[n] \leftrightarrow Y(e^{j\omega})$$

$$Y(e^{j\omega}) = \frac{1}{2}\left[X(e^{j(\omega-\pi/3)}) + X(e^{j(\omega+\pi/3)})\right]$$

$X(e^{j\omega})$ Shift $\pi/3$ Left AND RIGHT

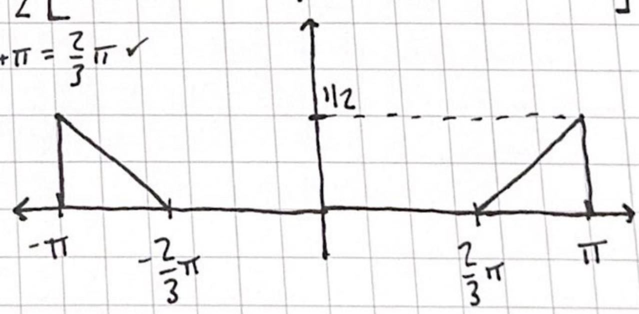


(e) DRAW FT OF $z[n] = x[n]\cos(\pi n)$

$$\cos(\pi n) = \frac{1}{2}\left[\delta(\omega-\pi) + \delta(\omega+\pi)\right]$$

$$Z(e^{j\omega}) = \frac{1}{2}\left[X(e^{j(\omega-\pi)}) + X(e^{j(\omega+\pi)})\right]$$

$\frac{\pi}{3} - \pi = -\frac{2}{3}\pi \qquad \frac{\pi}{3} + \pi = \frac{2}{3}\pi$ ✓



3. Signal $x[n]$, z-transform $X(z)$, 2 poles $z = \frac{1}{2}$ AND $z = 2$

(a) FIND $X(z)$

$$a^n u(n) = \frac{1}{1-az^{-1}} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$1 - \frac{1}{2}z^{-1} = 0$$
$$1 = \frac{1}{2}z^{-1}$$
$$z = 1/2 \checkmark$$

$$a^n u(n) = \frac{1}{1-az^{-1}} = \frac{1}{1-2z^{-1}} = 1-2z^{-1} = 0$$

$$\frac{1}{z^{-1}} = \frac{2z^{-1}}{z^{-1}} \quad z = 2 \checkmark$$

$$X(z) = \frac{1z^{-1}}{(1-\frac{1}{2}z^{-1})^2} + \frac{1z^{-1}}{(1-2z^{-1})^2}$$

$$\boxed{X(z) = \frac{5/6\, z^{-1}}{(1-1/2\, z^{-1})^2} + \frac{1}{1-2z^{-1}}}$$

(b) ROC: $|z| > 5/6$ because

$$\frac{1}{1-az^{-1}} \quad |z| > |a|$$

$$x[n] = \frac{5}{6}\left(\frac{1}{2}\right)^n u[n] + (2)^n u[n]$$

(c) IF $x[n]$ IS RIGHT SIDED SIGNAL

ROC: ~~|z| > 1/2 AND [illegible]~~

$|z| > 2$, LARGEST VALUE OF Z

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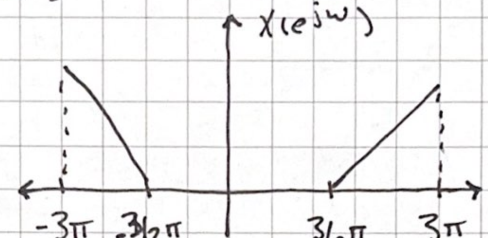
4. BONUS SAMPLED WITH $T = \frac{2\pi}{3\Omega_0}$ to form $x[n] = x_c(nT)$

(a) SKETCH

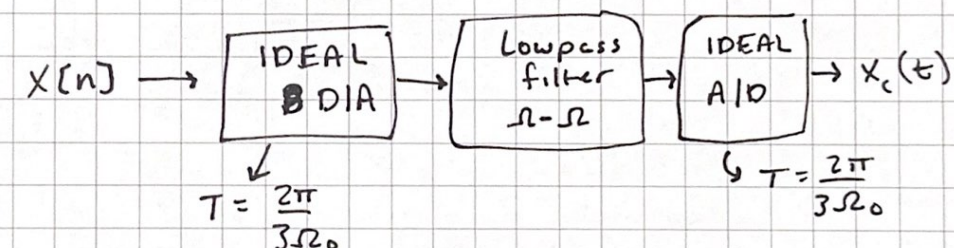
$\Omega_s = 2\pi f$ AND $f = \frac{1}{T}$

$\frac{1}{f} = \frac{2\pi}{\Omega_s} \rightarrow T = \frac{2\pi}{\Omega_s} = \Omega_s = \frac{2\pi}{T} = \frac{\cancel{2\pi}}{2\pi/3\Omega_0} = \underline{3\Omega_0}$



(b) Recover $x_c(t)$ from $x[n]$

$x[n]$ IS Digital so, D/A conv.



(c) Sample freq. $\Omega_s = 3\Omega_0 = f = \frac{1}{T}$

$\frac{1}{T} = 3\Omega_0 \rightarrow \boxed{T = \frac{1}{3}}$