BASICS

- DOT PRODUCT

$\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}| \cos\theta$

(SCALAR) $= A_x B_x + A_y B_y + A_z B_z$

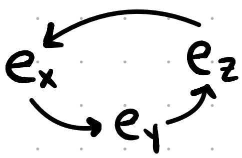
- CROSS PRODUCT

$$\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

(VECTOR)

CARTESIAN COORDINATE SYSTEM

(x, y, z)



DIFFERENTIAL LENGTH:

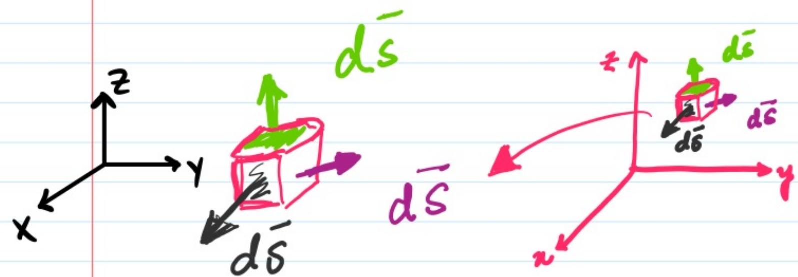
$$d\bar{l} = d_x \hat{e}_x + d_y \hat{e}_y + d_z \hat{e}_z$$

DISTANCE BETWEEN P_1 AND P_2.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

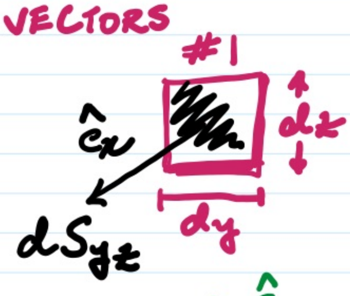
Surfaces and Volume in Cartesian Coordinate System

Tuesday, June 6, 2023 4:51 PM

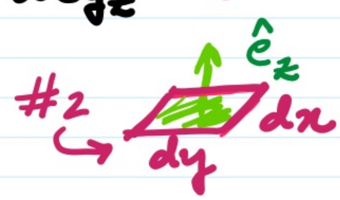


DIFFERENTIAL AREA VECTORS

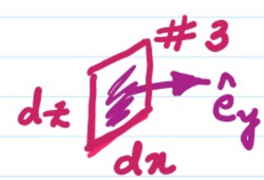
#1 $d\bar{s}_x = d_y d_z \hat{e}_x$



#2 $d\bar{s}_y = dx dz \hat{e}_y$



#3 $d\bar{s}_z = d_x d_y \hat{e}_z$



DIFFERENTIAL VOLUME

(SCALAR) $dV = d_x d_y d_z$

- There is NO VECTOR FOR VOLUME

Made with Goodnotes

Cylindrical Coordinate System (ρ, ϕ, z)

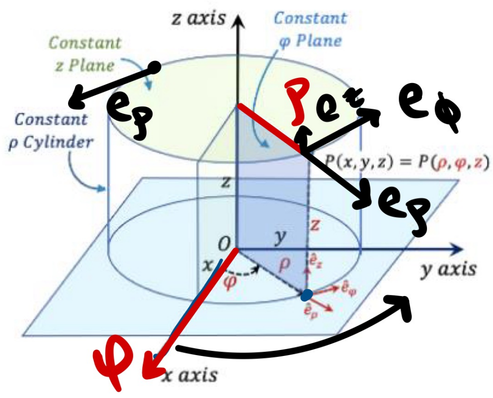


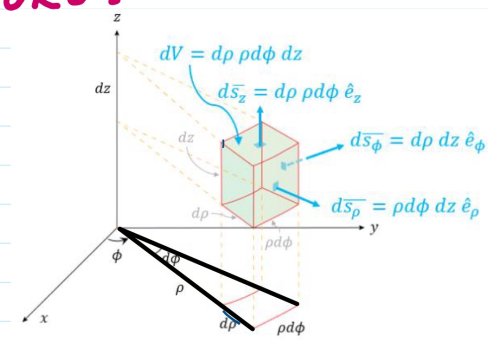
Figure 2: Cylindrical Coordinate System

LENGTH VECTORS:

$$d\bar{l}_z = d_z \hat{e}_z$$

$$d\bar{l}_\rho = d\rho \hat{e}_\rho$$

$$d\bar{l}_\phi = \rho d\phi \hat{e}_\phi$$

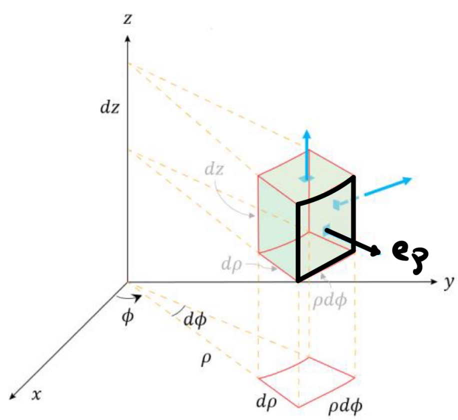


$$d\bar{l} = d\rho \hat{e}_\rho + \rho d\phi \hat{e}_\phi + d_z \hat{e}_z$$

$e_\rho \rightarrow e_\phi \rightarrow e_z \rightarrow e_\rho$

$$e_\rho \times e_\rho = 0$$

$$e_\rho \cdot e_\rho = 1$$



Area Vectors:

$$d\bar{s}_\rho = \rho d\phi dz \hat{e}_\rho$$

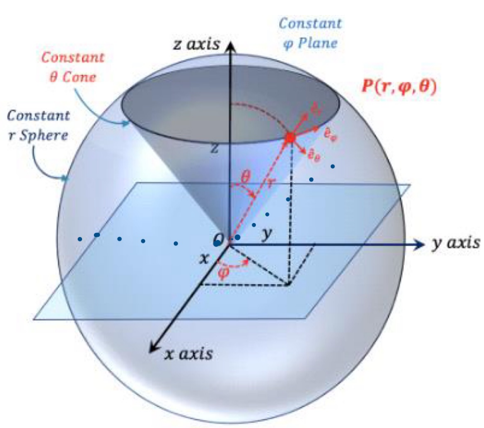
$$d\bar{s}_z = \rho d\phi d\rho \hat{e}_z$$

$$d\bar{s}_\phi = dz d\rho \hat{e}_\phi$$

Differential Volume:

$$dV = d\rho \rho d\phi dz$$

Spherical Coordinate System (r, θ, ϕ)

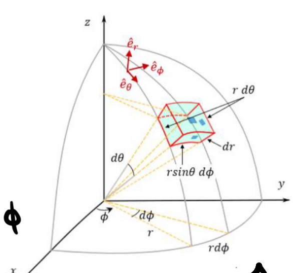


DIFFERENTIAL LENGTHS IN SPHERICAL COORDINATES:

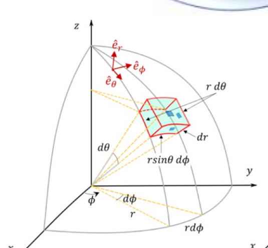
$$d\bar{l}_r = dr \hat{e}_r$$

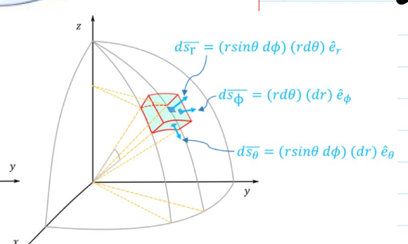
$$d\bar{l}_\theta = r d\theta \hat{e}_\theta$$

$$d\bar{l}_\phi = r d\phi \sin\theta \hat{e}_\phi$$



$$d\bar{l} = dr \hat{e}_r + r d\theta \hat{e}_\theta + r d\phi \sin\theta \hat{e}_\phi$$





$$d\bar{s}_r = r^2 \sin\theta d\theta d\phi \hat{e}_r$$

$$d\bar{s}_\phi = r dr d\theta \hat{e}_\phi$$

$$d\bar{s}_\theta = r\sin\theta d\phi dr \hat{e}_\theta$$

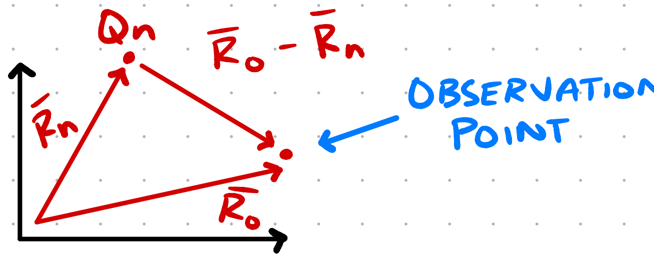
Differential Volume

(Scalar) $dV = r^2 \sin\theta d\theta d\phi dr$

$e_r \rightarrow e_\theta \rightarrow e_\phi \rightarrow e_r$

OPERATORS:

OPERATOR	NAME	INPUT	OUTPUT
$\bar{\nabla}\psi$	GRADIENT	SCALAR	VECTOR
$\bar{\nabla}\cdot\bar{A}$	DIVERGENCE	VECTOR	SCALAR
$\bar{\nabla}\times\bar{A}$	CURL	VECTOR	VECTOR
$\bar{\nabla}^2\psi$	LAPLACIAN	SCALAR	SCALAR



VECTOR ADDITION

$$\bar{R}_1 + \bar{R}_{12} - \bar{R}_2 = 0$$

$$= \bar{R}_{12} = \bar{R}_2 - \bar{R}_1$$

OBSERVATION

GRADIENT: GIVES MAX RATE OF CHANGE AND POINTS IN THE DIRECTION OF MAX INCREASE

Table 2: The gradient $\bar{\nabla}\Psi$ in different coordinate systems

Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
$\Psi(x,y,z)$	$\Psi(\rho,\varphi,z)$	$\Psi(r,\varphi,\theta)$
$\bar{\nabla}\Psi = e_x\frac{\partial\Psi}{\partial x} + e_y\frac{\partial\Psi}{\partial y} + e_z\frac{\partial\Psi}{\partial z}$	$\bar{\nabla}\Psi = e_\rho\frac{\partial\Psi}{\partial\rho} + e_\varphi\frac{1}{\rho}\frac{\partial\Psi}{\partial\varphi} + e_z\frac{\partial\Psi}{\partial z}$	$\bar{\nabla}\Psi = e_r\frac{\partial\Psi}{\partial r} + e_\theta\frac{1}{r}\frac{\partial\Psi}{\partial\theta} + e_\varphi\frac{1}{rsin\theta}\frac{\partial\Psi}{\partial\varphi}$

DIVERGENCE: $\bar{\nabla}\cdot\bar{A} > 0 \rightarrow$ SOURCE
$\bar{\nabla}\cdot\bar{A} < 0 \rightarrow$ SINK
$\bar{\nabla}\cdot\bar{A} = 0 \rightarrow$ PARALLEL FIELD LINES

Table 3: $\bar{\nabla}\cdot\bar{A}$

Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
$\bar{A}(x,y,z) = e_xA_x + e_yA_y + e_zA_z$	$\bar{A}(\rho,\varphi,z) = e_\rho A_\rho + e_\varphi A_\varphi + e_zA_z$	$\bar{A}(r,\varphi,\theta) = e_rA_r + e_\theta A_\theta + e_\varphi A_\varphi$
$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{rsin\theta}\frac{\partial(A_\theta sin\theta)}{\partial\theta} + \frac{1}{rsin\theta}\frac{\partial A_\varphi}{\partial\varphi}$

CURL: INDICATES WHETHER A VECTOR FIELD HAS MICROSCOPIC ROTATION.

Table 4: $\bar{\nabla}\times\bar{A}$

Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
$\bar{A}(x,y,z) = e_xA_x + e_yA_y + e_zA_z$	$\bar{A}(\rho,\varphi,z) = e_\rho A_\rho + e_\varphi A_\varphi + e_zA_z$	$\bar{A}(r,\varphi,\theta) = e_rA_r + e_\theta A_\theta + e_\varphi A_\varphi$
$(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})e_x + (\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})e_y + (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})e_z$	$(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z})e_\rho + (\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho})e_\varphi + \frac{1}{\rho}(\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\varphi})e_z$	$\frac{1}{rsin\theta}(\frac{\partial}{\partial\theta}(A_\varphi sin\theta) - \frac{\partial A_\theta}{\partial\varphi})e_r + \frac{1}{r}(\frac{1}{sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi))e_\theta + \frac{1}{r}(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta})e_\varphi$

CALCULUS.

$$\frac{d}{dx}(\sin x) = \cos x \qquad \int \sin x = -\cos x$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \int \cos x = \sin x$$

BIOT SAVART' LAW (FINDING $\bar{H}$)

$\bar{B}$: MAGNETIC FLUX DENSITY

$\bar{H}$: MAGNETIC FIELD INTENSITY

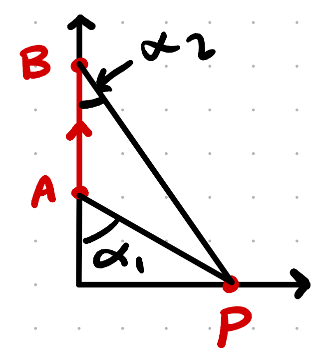
LINE CURRENT: $\bar{H} = \int_l \frac{I\, d\bar{l} \times \bar{R}}{4\pi |\bar{R}|^3} \left(\frac{A}{m}\right)$

SURFACE CURRENT: $\bar{H} = \iint_s \frac{\bar{J}_s ds \times \bar{R}}{4\pi |\bar{R}|^3} \left(\frac{A}{m}\right)$ J_s: SURFACE CURRENT DENSITY

VOLUME CURRENT: $\bar{H} = \iiint_v \frac{\bar{J}_v dv \times \bar{R}}{4\pi |\bar{R}|^3} \left(\frac{A}{m}\right)$ J_s: VOLUME CURRENT DENSITY

MAGNETIC FIELD INTENSITY DUE TO A FINITE CURRENT CARRYING CONDUCTOR.

$$\bar{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right] \hat{e}_\phi$$



INFINITE WIRE: ρ: DISTANCE FROM WIRE

$\bar{H} = \frac{I}{2\pi\rho} \hat{e}_\phi$ $\hat{e}_\phi$: RIGHT HAND RULE.

MAGNETIC FLUX DENSITY

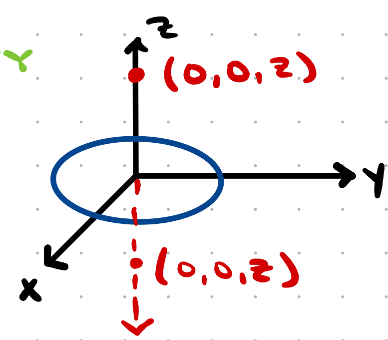
$\bar{B} = \mu \bar{H} \leftarrow$ MAGNETIC FIELD INTENSITY (T).

PERMEABILITY: $\mu = \mu_0 \mu_r$

$\mu_0 = 4\pi \times 10^{-7} (H/m)$

MAGNETIC FLUX DENSITY DUE TO A LOOP.

$$B(z) = \frac{1}{2} \frac{\mu I a^2}{(a^2 + z^2)^{3/2}} \hat{e}_z$$



AT THE CENTER.

$$B(z=0) = \frac{\mu I}{2a} \hat{e}_z$$

AMPERE'S LAW (1st MAXWELLS EQUATION)

$$\oint \bar{H} \cdot d\bar{l} = I_{enc} \rightarrow \text{INTERGAL FORM}$$

$$\bar{\nabla} \times \bar{H} = J \longrightarrow \text{DIFFERENTIAL FORM}$$

THE LINE INTERGAL OF $\bar{H}$ AROUND A CLOSED PATH IS THE SAME AS THE TOTAL CURRENT ENCLOSED BY THE PATH.

2nd MAXWELLS EQUATION

MAGNETIC MONOPOLES DO NOT EXIST

FIELD LINES GOING IN
= FIELD LINES GOING OUT.

$$\oiint \bar{B} \cdot d\bar{s} = \iiint_V (\bar{\nabla} \cdot \bar{B})\, dv = 0$$

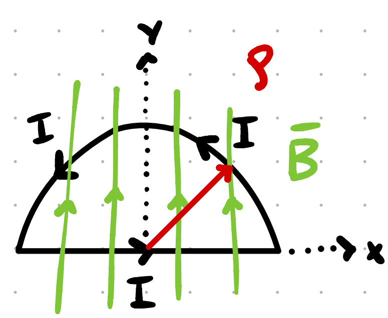
$$\longrightarrow \bar{\nabla} \cdot \bar{B} = 0$$ NO SOURCE OR SINK.

FORCE ON A CURRENT CARRYING CONDUCTOR.

EQUATION: $\bar{F}_m = \int_L I\, d\bar{l} \times \bar{B}$ (N)

STRAIGHT CONDUCTOR SECTION:

ALONG x AXIS $\rightarrow 2IB_0\rho\,\hat{e}_z$ (N)



CURVED CONDUCTOR:

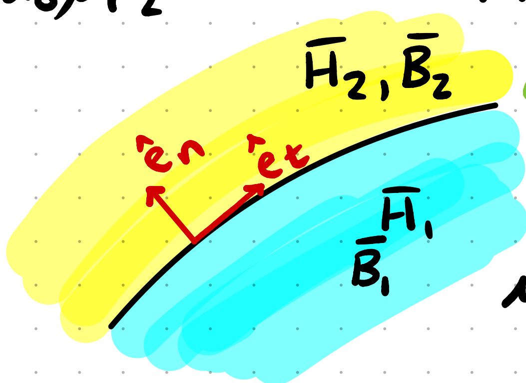
$\bar{F} = -2I\rho B_0\hat{e}_z$ (N)

MAGNETOSTATICS BOUNDARY CONDITION

$\mu_2 = \mu_0 \mu_{r2}$

MAG. MAT. #1

NO CURRENT AT BOUNDARY



MAG. MAT #2

$\mu_1 = \mu_0 \mu_{r1}$

1 NORMAL COMPONENT IS CONTINUOUS

$$\bar{B}_{1n} = \bar{B}_{2n} \rightarrow \mu_1 \bar{H}_{1n} = \mu_2 \bar{H}_{2n}$$

2 TANGENTAL IS CONTINUOUS.

$$\bar{H}_{1t} = \bar{H}_{2t}$$

PERFECT CONDUCTOR. MATERIAL 1 IS CONDUCTOR
2 IS MAGNETIC

$$\bar{B}_{2n} = 0 = \bar{H}_{2t}$$

$$\bar{B}_{1n} = 0, \quad \bar{H}_{1t} = J_s \leftarrow \text{SURFACE CURRENT DENSITY}$$

OVERALL:

$$\bar{B}_{1n} = \bar{B}_{2n} \rightarrow \mu_1 \bar{H}_{1n} = \mu_2 \bar{H}_{2n} \text{ (NORMAL)}$$

$$(\bar{H}_1 - \bar{H}_2) \times \hat{e}_n = \bar{J} \text{ (TANGENTIAL)}$$

IF $\bar{J} = 0 \rightarrow \quad \bar{H}_{1t} = \bar{H}_{2t}$

INDUCTANCE

MAGNETIC FLUX THRU A LOOP

$$\Psi_{MAG} = \iint \bar{B} \cdot d\bar{s}$$

INDUCTANCE OF A LOOP

$$L = \frac{\Psi_{MAG}}{I}$$

FOR N LOOPS: $L = \frac{N \Psi_{MAG}}{I}$

INDUCTANCE OF A LONG SOLENOID: $\ell \gg a$ [a: RADIUS]

$$L = \frac{\mu N^2 S}{\ell} \ (H)$$

N: TURNS S: AREA
ℓ: LENGTH.

$S = \pi a^2$ $N = n\ell$

TIME VARYING FIELDS:

FARADAY'S LAW:

$$V_{emf} = -N \frac{d}{dt} \Psi_{MAG} \leftrightarrow V_{emf} = -N \frac{d}{dt}\left[\iint_S \bar{B} \cdot d\bar{s}\right]$$

STATIONARY LOOP IN TIME VARYING $\bar{B}$ FIELD.
(TRANSFORMER EMF)

$$V^{tr}_{emf} = -N \iint \frac{\partial}{\partial t} \bar{B} \cdot d\bar{s}$$

MOTIONAL EMF REFERENCE.

$$V^{m}_{emf} = \oint_C{}_A^B (\bar{v} \times \bar{B}) \cdot d\bar{\ell}$$

MAXWELL'S EQUATIONS

GAUSS LAW

$$\bar{\nabla} \cdot \bar{D} = q_0 \quad \text{OR} \quad \iint_S \bar{D} \cdot d\bar{s} = \iiint_v q_v \, dv$$

FARADAY'S LAW

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad \text{OR} \quad \oint \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \iint \bar{B} \cdot d\bar{s}$$

CHANGE IN MAGNETIC FIELD WILL GIVE RISE TO INDUCED CURRENT.

NON-EXISTENCE OF MAGNETIC MONOPOLES

$$\bar{\nabla} \cdot \bar{B} = 0 \quad \text{OR} \quad \oiint \bar{B} \cdot d\bar{s} = 0$$

AMPERE'S LAW

$$\bar{\nabla} \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t} \quad \text{OR} \quad \oint \bar{H} \cdot d\bar{l} = I + \underbrace{\iint_S \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s}}_{\text{DISPLACEMENT CURRENT.}}$$

WAVES

PROPAGATING IN +x DIRECTION

$$y(x,t) = A\cos(\omega t - \beta x + \phi)$$

A : Amplitude

ω : ANGULAR FREQUENCY

$$\omega = 2\pi f = \frac{2\pi}{T} \left(\frac{rad}{s}\right)$$

β : PHASE CONSTANT / ANGULAR WAVENUMBER.

$$\beta = \frac{2\pi}{\lambda} \left(\frac{rad}{m}\right)$$

v_p : PHASE VELOCITY

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda \left(\frac{m}{s}\right)$$

λ : WAVELENGTH

$$\lambda = \frac{v_p}{f} = \frac{2\pi}{\beta} \, (m)$$

PHASOR

$$\tilde{Y} = A e^{(-j\beta x + j\phi_0)} \cdot e^{j\omega t}$$

COMPLEX PERMITTIVITY

$$\tilde{D} = \underbrace{(\varepsilon' - j\varepsilon'')}_{\text{COMPLEX PERMITTIVITY}} \tilde{E} = \overbrace{\varepsilon_0 \varepsilon_r}^{\text{PERMITTIVITY}} - j \overbrace{\frac{\sigma}{\omega}}^{\text{CONDUCTIVITY}}$$

IN MOST MATERIALS:

$\varepsilon' = \varepsilon = \varepsilon_0 \varepsilon_r$ [REAL VALUED PERMITTIVITY]

$\varepsilon'' = \frac{\sigma}{\omega}$ [HIGHER ORDER TERMS]

σ : CONDUCTIVITY

LOSS TANGENT - $\tan\delta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}$

IN AN UNBOUNDED HOMOGENEOUS MEDIUM

$$\bar{\nabla}^2 \tilde{E} - \gamma^2 \tilde{E} = 0 \qquad \gamma = \sqrt{-\omega^2 \mu \varepsilon_c}$$

$$\bar{\nabla}^2 \tilde{H} - \gamma^2 \tilde{H} = 0 \qquad \text{IN FORM } \alpha + j\beta$$

α : ATTENUATION CONSTANT $\left(\frac{Np}{m}\right)$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

β : PHASE CONSTANT (rad/m)

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

REAL PART

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon'$$

IMAGINARY PART

$$2\alpha\beta = \omega^2 \mu \varepsilon''$$

LOSSY WAVE

$$\bar{E}(z,t) = E_0^+ e^{-\alpha z} \cos(\omega t - \beta z + \phi_0)\, \hat{e}_E \ (V/m)$$

E_0^+ : INITIAL AMPLITUDE (V/m)

α : ATTENUATION CONSTANT (Np/m)

ω : ANGULAR FREQUENCY $= 2\pi f$

β : PHASE CONSTANT (rad/m)

$-\beta x$: DIRECTION OF PROPAGATION.

η : INTRTINSIC IMPEDANCE OF MATERIAL (Ω)

$\eta : E_0^+ / H_0^+$

COMPLEX NUMBER

$$\eta = |\eta| \angle \theta_\eta$$

$$\downarrow$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega \varepsilon}$$

MAGNITUDE $|\eta| = \dfrac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$

IMAGINARY PART $\theta_\eta = \dfrac{\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}{2}$

IN PHASOR FORM

$$\tilde{E} = E_0^+ e^{-\alpha z} e^{j[-\beta x + \phi_0]}\, \hat{e}_E \ (V/m)$$

WHEN SOLVING FOR ε' AND ε''.

MAGNITUDE $|\eta| = \dfrac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]^{1/4}}$

IMAGINARY PART $\theta_\eta = \dfrac{\tan^{-1}\left(\frac{\varepsilon''}{\varepsilon'}\right)}{2}$

TO FIND ε' AND ε''

REAL PART

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon'$$

IMAGINARY PART

$$2\alpha\beta = \omega^2 \mu \varepsilon''$$

LOSSLESS PERFECT DIELECTRIC [NON-CONDUCTING]

$\sigma = 0$ CONDUCTIVITY

$\alpha = 0$ AND $\beta = \omega\sqrt{\mu\varepsilon}$ (rad/m) PHASE CONSTANT

$\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$ [η IS A REAL NUMBER FOR LOSSLESS MATERIALS]

PHASE VELOCITY $v_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{\mu\varepsilon}} = 3\times10^{8}$ m/s

IN FREE SPACE

$\varepsilon = \varepsilon_0 = 8.854\times10^{-12}$ F/m $\quad \mu = \mu_0 = 4\pi\times10^{-7}$ H/m

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ (\Omega)$$

FOR ANY LOSSLESS DIELECTRIC.

$$\eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}\ (\Omega) \qquad v_p = \frac{c}{\sqrt{\mu_r\varepsilon_r}}\ (\text{m/s})$$

WAVELENGTH

$$\lambda = \frac{c}{f\sqrt{\mu_r\varepsilon_r}}$$

IN FREE SPACE

$$\lambda_0 = \frac{c}{f}$$

PHASE CONSTANT

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

[IN LOSSLESS MATERIALS]

RELATIONSHIP BETWEEN $\bar{E}$ AND $\bar{H}$.

$$\eta = \frac{E_0^+}{H_0^+} \qquad (\hat{e}_E \times \hat{e}_H) = \hat{e}_K$$

RADIAN TO DEGREE

$$\text{RADIAN} \times \frac{180}{\pi}$$