MATH 311, HW 2

Due 11:00pm on Thursday February 2, 2023

Instructions: download/print this HW and answer all questions in the space provided, (either in paper or electronically). You must explain your procedure as detailed as possible, but also be concise so that you only use the space provided. Then scan all the pages and upload them through Gradescope. Check the section "Fixed-length submissions (templated)" in the following webpage.

Problem 1. [40 pts] Given $A = \begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & 1 \\ 2 & 5 & 1 \end{pmatrix}$, and $B = \begin{pmatrix} -3 & 1 & 1 \\ -4 & 2 & 1 \\ 3 & 0 & 2 \end{pmatrix}$, compute the following

1. [5pts] $2A =$ $\begin{pmatrix} 2 & 8 & -4 \\ 0 & 6 & 2 \\ 4 & 10 & 2 \end{pmatrix}$

$2\begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & 1 \\ 2 & 5 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 8 & -4 \\ 0 & 6 & 2 \\ 4 & 10 & 2 \end{pmatrix}$

2. [5pts] $A + B =$ $\begin{pmatrix} -2 & 5 & -1 \\ -4 & 5 & 2 \\ 5 & 5 & 3 \end{pmatrix}$

$\begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & 1 \\ 2 & 5 & 1 \end{pmatrix} + \begin{pmatrix} -3 & 1 & 1 \\ -4 & 2 & 1 \\ 3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 5 & -1 \\ -4 & 5 & 2 \\ 5 & 5 & 3 \end{pmatrix}$

3. [5pts] $2A - 3B =$ $\begin{pmatrix} 11 & 5 & -7 \\ 12 & 0 & -1 \\ -5 & 10 & -4 \end{pmatrix}$

$\begin{pmatrix} 2 & 8 & -4 \\ 0 & 6 & 2 \\ 4 & 10 & 2 \end{pmatrix} - \begin{pmatrix} -9 & 3 & 3 \\ -12 & 6 & 3 \\ 9 & 0 & 6 \end{pmatrix} = \begin{pmatrix} 11 & 5 & -7 \\ 12 & 0 & -1 \\ -5 & 10 & -4 \end{pmatrix}$

4. [5pts] $(2A)^T - (3B)^T =$ $\begin{pmatrix} 11 & 12 & -5 \\ 5 & 0 & 10 \\ -7 & -1 & -4 \end{pmatrix}$

$\begin{pmatrix} 2 & 8 & -4 \\ 0 & 6 & 2 \\ 4 & 10 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 0 & 4 \\ 8 & 6 & 10 \\ -4 & 2 & 2 \end{pmatrix}^T = A$ ① Transpose A

$\begin{pmatrix} -9 & 3 & 3 \\ -12 & 6 & 3 \\ 9 & 0 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} -9 & -12 & 9 \\ 3 & 6 & 0 \\ 3 & 3 & 6 \end{pmatrix}^T = B$ ② Transpose B

$\begin{pmatrix} 2 & 0 & 4 \\ 8 & 6 & 10 \\ -4 & 2 & 2 \end{pmatrix} - \begin{pmatrix} -9 & -12 & 9 \\ 3 & 6 & 0 \\ 3 & 3 & 6 \end{pmatrix} = \begin{pmatrix} 11 & 12 & -5 \\ 5 & 0 & 10 \\ -7 & -1 & -4 \end{pmatrix}$ ③ Subtract

$$A = \begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & 1 \\ 2 & 5 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -3 & 1 & 1 \\ -4 & 2 & 1 \\ 3 & 0 & 2 \end{pmatrix}$$

5. [5pts] $AB =$ $\begin{pmatrix} -25 & 9 & 1 \\ -9 & 6 & 5 \\ -23 & 12 & 9 \end{pmatrix}$

A^{row1}

$1(-3) + 4(-4) - 2(3)$ $\quad 1(1) + 4(2) - 2(0)$ $\quad 1(1) + 4(1) - 2(2)$

$0(-3) + 3(-4) + 1(3)$ $\quad 0(1) + 3(2) + 1(0)$ $\quad 0(1) + 3(1) + 1(2)$

$2(-3) + 5(-4) + 1(3)$ $\quad 2(1) + 5(2) + 1(0)$ $\quad 2(1) + 5(1) + 1(2)$

$-3 - 16 - 6 = -25$ $\quad 1 + 8 = 9$ $\quad 1 + 4 - 4 = 1$

$0 - 12 + 3 = -9$ $\quad 6 = 6$ $\quad 0 + 3 + 2 = 5$

$-6 - 20 + 3 = -23$ $\quad 2 + 10 = 12$ $\quad 2 + 5 + 2 = 9$

6. [5pts] $BA =$ $\begin{pmatrix} -1 & -4 & 8 \\ -2 & -5 & 12 \\ 7 & 22 & -4 \end{pmatrix}$

$-3(1) + 1(0) + 1(2)$ $\quad -3(4) + 1(3) + 1(5)$ $\quad -3(-2) + 1(1) + 1(1)$

$-4(1) + 2(0) + 1(2)$ $\quad -4(4) + 2(3) + 1(5)$ $\quad -4(-2) + 2(1) + 2(1)$

$3(1) + 0(0) + 2(2)$ $\quad 3(4) + 0(3) + 2(5)$ $\quad 3(-2) + 0(1) + 2(1)$

$-3 + 2 = -1$ $\quad -12 + 3 + 5 = -4$ $\quad 6 + 1 + 1 = 8$

$-4 + 2 = -2$ $\quad -16 + 6 + 5 = -5$ $\quad 8 + 2 + 2 = 12$

$3 + 4 = 7$ $\quad 12 + 0 + 10 = 22$ $\quad -6 + 0 + 2 = -4$

7. [5pts] $A^T B^T =$ $\begin{pmatrix} -1 & -2 & 7 \\ -4 & -5 & 22 \\ -8 & 11 & -4 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & 1 \\ 2 & 5 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} -3 & 1 & 1 \\ -4 & 2 & 1 \\ 3 & 0 & 2 \end{pmatrix} \longrightarrow$ $A^T = \begin{pmatrix} 1 & 0 & 2 \\ 4 & 3 & 5 \\ -2 & 1 & 1 \end{pmatrix}$ $\quad B^T = \begin{pmatrix} -3 & -4 & 3 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

$-3(1) + 1(0) + 1(2)$ $\quad -4(1) + 2(0) + 1(2)$ $\quad 3(1) + 0(0) + 2(2)$

$-3(4) + 1(3) + 1(5)$ $\quad -4(4) + 2(3) + 1(5)$ $\quad 3(4) + 0(3) + 2(5)$

$-3(-2) + 1(1) + 1(1)$ $\quad -4(-2) + 2(1) + 1(1)$ $\quad 3(-2) + 0(1) + 2(1)$

$-3 + 0 + 2 = -1$ $\quad -4 + 0 + 2 = -2$ $\quad 3 + 0 + 4 = 7$

$-12 + 3 + 5 = -4$ $\quad -16 + 6 + 5 = -5$ $\quad 12 + 0 + 10 = 22$

$6 + 1 + 1 = 8$ $\quad 8 + 2 + 1 = 11$ $\quad -6 + 0 + 2 = -4$

8. [5pts] $(BA)^T =$ $\begin{pmatrix} -1 & -2 & 7 \\ -4 & -5 & 22 \\ 8 & 12 & -4 \end{pmatrix}$

$BA = \begin{pmatrix} -1 & -4 & 8 \\ -2 & -5 & 12 \\ 7 & 22 & -4 \end{pmatrix}$, $BA^T = \begin{pmatrix} -1 & -2 & 7 \\ -4 & -5 & 22 \\ 8 & 12 & -4 \end{pmatrix}$

Problem 2. [20 pts] Given $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$, compute the following powers

$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

1. [5pts] $A^2 =$ $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

$A^2 = A \cdot A$ $\quad \begin{matrix} 0(0) + 1(1) & 0(1) + 1(1) \\ 1(0) + 1(1) & 1(1) + 1(1) \end{matrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

2. [5pts] $A^3 =$ $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

$0(1)+1(1)=1 \quad 1(1)+1(1)=2$

$0(1)+1(2)=2 \quad 1(1)+1(2)=3$

3. [5pts] $A^4 =$ $\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$

$A^4 = A^2 \cdot A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} =$

$1(1)+1(1)=2 \quad 1(1)+2(1)=3$

$1(1)+1(2)=3 \quad 1(1)+2(2)=5$

4. [5pts] $A^5 =$ $\begin{pmatrix} 3 & 5 \\ 5 & 8 \end{pmatrix}$

$A^5 = A^4 \cdot A = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

$0(2)+1(3)=3 \quad 1(2)+1(3)=5$

$0(3)+1(5)=5 \quad 1(3)+1(5)=8$

Problem 3. [20 pts] Find the inverse of the following matrices

if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

1. [5pts] $\begin{pmatrix} 1 & 0 \\ 3 & 3 \end{pmatrix}$

Ans $\begin{pmatrix} 1 & 0 \\ -1 & 1/3 \end{pmatrix}$

① find $|A| = 1(3) - 0(3) = 3 - 0 = 3$ $\quad \begin{pmatrix} 3 & 0 \\ -3 & 1 \end{pmatrix}$

$\rightarrow \frac{1}{3}\begin{pmatrix} 3 & 0 \\ -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -1 & 1/3 \end{pmatrix}$

2. [5pts] $\begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Ans $\begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$

$\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{3(5)-7(2)} = \frac{1}{15-14} = \frac{1}{1}$

$\frac{1}{1}\begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$

3. [5pts] $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Ans $\begin{pmatrix} -1/2 & 3/2 \\ 1 & -2 \end{pmatrix}$

$\frac{1}{4(1)-6} = -\frac{1}{2}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}$

$= \begin{pmatrix} -1/2 & 3/2 \\ 1 & -2 \end{pmatrix}$

4. [5pts] $\begin{pmatrix} 3 & 5 \\ 4 & 8 \end{pmatrix}$ $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Ans	$\begin{pmatrix} 2 & -5/4 \\ -1 & 3/4 \end{pmatrix}$

$\frac{1}{24-20} = \frac{1}{4}\begin{pmatrix} 8 & -5 \\ -4 & 3 \end{pmatrix}$
$= \begin{pmatrix} 2 & -5/4 \\ -1 & 3/4 \end{pmatrix}$

Problem 4. [20 pts] Find the inverse of the following matrix:

$$A = \begin{pmatrix} -5 & 3 & 4 \\ 2 & -1 & 0 \\ 3 & -1 & 3 \end{pmatrix}$$

$A^{-1} =$	$\begin{pmatrix} -3 & -13 & 4 \\ -6 & -27 & 8 \\ 1 & 4 & -1 \end{pmatrix}$

Determinate $= -5\begin{vmatrix} -1 & 0 \\ -1 & 3 \end{vmatrix} - 3\begin{vmatrix} 2 & 0 \\ 3 & 3 \end{vmatrix} + 4\begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix}$

$= -5(-3+0) - 3(6-0) + 4(-2+3)$

$= +15 - 18 + 4 = 1 \leftarrow$ $\det(A) = 1$

$$\begin{pmatrix} + \begin{vmatrix} -1 & 0 \\ -1 & 3 \end{vmatrix} & - \begin{vmatrix} 2 & 0 \\ 3 & 3 \end{vmatrix} & + \begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix} \\ - \begin{vmatrix} 3 & 4 \\ -1 & 3 \end{vmatrix} & + \begin{vmatrix} -5 & 4 \\ 3 & 3 \end{vmatrix} & - \begin{vmatrix} -5 & 3 \\ 3 & -1 \end{vmatrix} \\ + \begin{vmatrix} 3 & 4 \\ -1 & 0 \end{vmatrix} & - \begin{vmatrix} -5 & 4 \\ 2 & 0 \end{vmatrix} & + \begin{vmatrix} -5 & 3 \\ 2 & -1 \end{vmatrix} \end{pmatrix} \rightarrow \begin{pmatrix} [(-3-0)] - (6-0) + (-2+3) \\ -(9+4) + (-15-12) - (5-9) \\ +(0+4) - (0-8) + (5-6) \end{pmatrix}$$

$$\begin{pmatrix} -3 & -6 & 1 \\ -13 & -27 & 4 \\ 4 & 8 & -1 \end{pmatrix}^T = \begin{pmatrix} -3 & -13 & 4 \\ -6 & -27 & 8 \\ 1 & 4 & -1 \end{pmatrix} \rightarrow \frac{1}{1}\begin{pmatrix} -3 & -13 & 4 \\ -6 & -27 & 8 \\ 1 & 4 & -1 \end{pmatrix} = \begin{pmatrix} -3 & -13 & 4 \\ -6 & -27 & 8 \\ 1 & 4 & -1 \end{pmatrix}$$