Homework 6

You may either submit this page with your homework or copy the information in this box to your handwritten solutions.

An Aggie does not lie, cheat or steal or tolerate those who do.

Please acknowledge your adherence to the honor code by printing and signing your name below.

This work is the product of my effort and was completed either through my sole effort or with collaborations as permitted by the syllabus. All collaboration, team effort, and sources are disclosed below. I have complied with the requirements of this homework to the best of my knowledge.

JOAQUIN SALAS

Printed Name

Signature

Disclosure of Sources and Collaborators:

OTABEK , HELP ON #5

Note: This homework assignment is worth a total of 100 Points.

PART 1: Individual Homework

1. **20 Points** Two circular loops carrying a current of 40A each are arranged parallel to the x-y plane such that the center of one loop is the origin and the center of the other is at z = 2m. The current through both loops is in the $-e_\phi$ direction. The two loops have the same radius of a = 3m.
   a. For the given problem, determine the expression for the magnetic field intensity at any point z on the z-axis that lies between $0 \le z \le 2$m [5 Points].
   b. What is the magnetic field intensity at $z = 0$ and $z = 2m$? [5 Points]
   c. Plot the magnitude of the magnetic field intensity between $0 \le z \le 2$m using MATLAB. Place a cursor at $z = 1m$. Submit a copy of the code and the plot with cursor. [10 Points]
2. **5 Points** In a certain region, the magnetic field is given in the cylindrical coordinates by
$$\bar{H} = \frac{4}{\rho}[1 - (1 + 3\rho)e^{-3\rho}]\hat{e}_\phi$$
   Find the current density $\bar{J}$.
3. **25 Points** An infinitely long wire carrying a 25 A current in the positive x direction is placed along the x-axis in the vicinity of a 20-turn circular loop located in the x-y plane. The center of the loop is at (0, 2m) and its radius is 1m. If the magnetic field at the center of the loop is zero, what is the direction and magnitude of the current flowing in the loop? *Hint: We derived the expression for the magnetic flux density for a loop with one turn. For a loop with N turns, multiple the expression for $\bar{B}$ with N.*

4. **30 Points** A Current I flows along the positive z-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has a radius a, and the inner and outer radii of the outer conductor are b and c, respectively. Use Ampere's Law to
    a. Determine the magnetic field intensity in the region $0 < \rho < a$. [5 points]
    b. Determine the magnetic field intensity in the region $a < \rho < b$. [5 points]
    c. Determine the magnetic field intensity in the region $b < \rho < c$. [5 points]
    d. Determine the magnetic field intensity in the region $\rho > c$. [5 points]
    e. If I = 10 A, a = 2 cm, b = 4 cm, and c = 5 cm, plot $|\bar{H}|$ as a function of ρ from $0 \leq \rho \leq 8cm$. [10 points]
5. **10 Points** Two infinitely long parallel wires are carrying 6A currents in the opposite directions as shown in Fig. 1. Find the magnetic flux density at point P in Fig. 1.

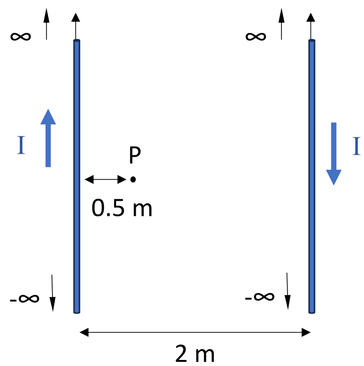


Fig 1: Geometry for Problem 5

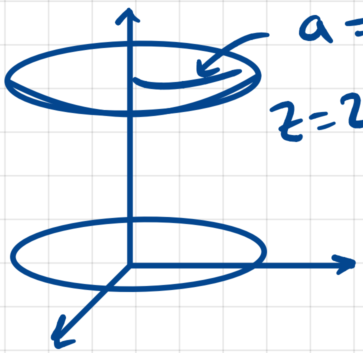
6. **10 Points** Download the file Homework 6 – Reading Assignment from canvas. This file compiles three articles on electromagnetics. Read the articles and write a summary. Your summary should have a minimum page length of ¾ page (font size: 12, single spacing). You may include figures in your summary, but figures are not counted while calculating length of the article.

PART 2: Team Homework (T-HW) (0 Points)

This homework does not have a team component due to fall break.

1. **20 Points** Two circular loops carrying a current of 40A each are arranged parallel to the x-y plane such that the center of one loop is the origin and the center of the other is at z = 2m. The current through both loops is in the $-e_\phi$ direction. The two loops have the same radius of a = 3m.
   a. For the given problem, determine the expression for the magnetic field intensity at any point z on the z-axis that lies between $0 \le z \le 2$m [5 Points].
   b. What is the magnetic field intensity at $z = 0$ and $z = 2m$? [5 Points]
   c. Plot the magnitude of the magnetic field intensity between $0 \le z \le 2$m using MATLAB. Place a cursor at $z = 1m$. Submit a copy of the code and the plot with cursor. [10 Points]

a.



@ Loop 1 - $\bar{H} = \dfrac{40 \cdot 3^2}{2(3^2 + z^2)^{3/2}}$

@ Loop 2 - $\bar{H} = \dfrac{40 \cdot 3^2}{2(3^2 + (2-z)^2)^{3/2}}$

+2 in z component

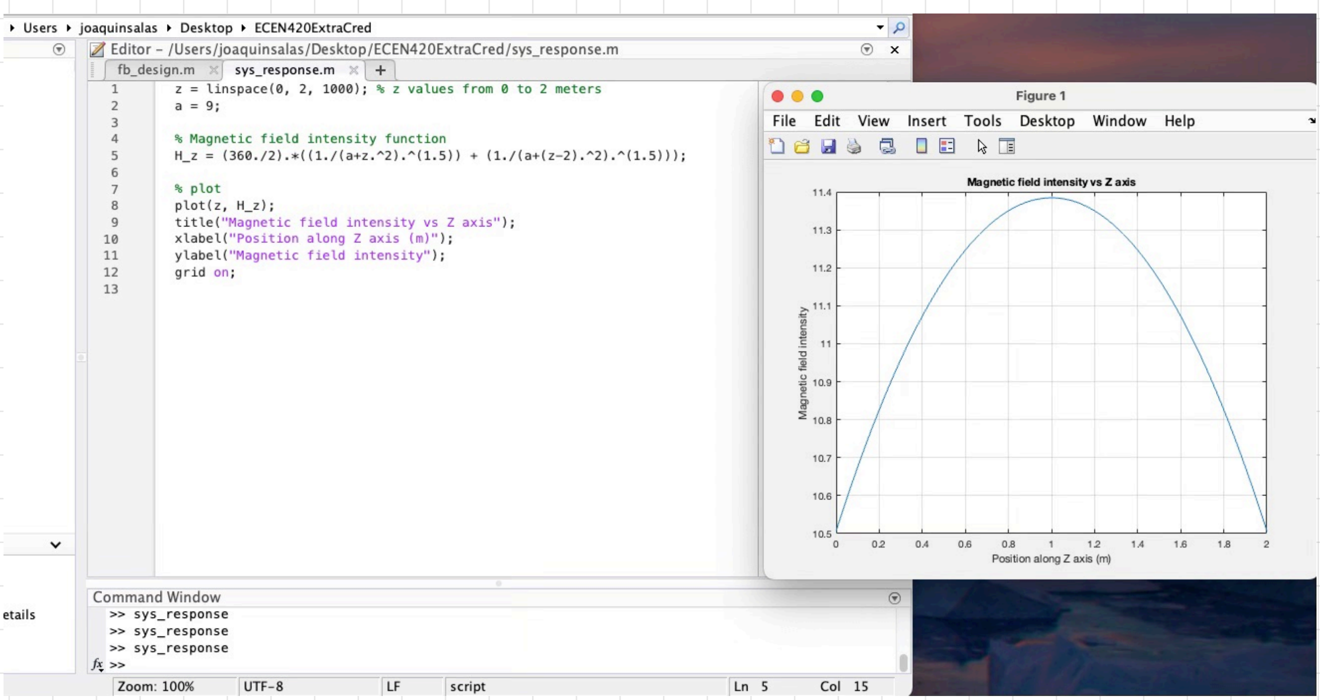
$$\bar{H} = -\frac{I \cdot a^2}{2(a^2 + z^2)^{3/2}}\hat{e}_z - \frac{I \cdot a^2}{2(a^2 + (z-2)^2)^{3/2}}\hat{e}_z$$

b. at z = 0

$$H = -\frac{40 \cdot 3^2}{2(3^2 + 0^2)^{3/2}}\hat{e}_z - \frac{40 \cdot 3^2}{2(3^2 + (-2)^2)^{3/2}}\hat{e}_z = -10.51\,\hat{e}_z \text{ A/m}$$

@ z = 2m

$$\bar{H} = -\frac{40 \cdot 3^2}{2(3^2 + 2^2)^{3/2}}\hat{e}_z - \frac{40 \cdot 3^2}{2(3^2 + 0^2)^{3/2}}\hat{e}_z = -10.51\,\hat{e}_z \text{ A/m}$$




```
z = linspace(0, 2, 1000); % z values from 0 to 2 meters
a = 9;

% Magnetic field intensity function
H_z = (360./2).*((1./(a+z.^2).^(1.5)) + (1./(a+(z-2).^2).^(1.5)));

% plot
plot(z, H_z);
title("Magnetic field intensity vs Z axis");
xlabel("Position along Z axis (m)");
ylabel("Magnetic field intensity");
grid on;
```

2. **5 Points** In a certain region, the magnetic field is given in the cylindrical coordinates by

$$\bar{H} = \frac{4}{\rho}[1 - (1 + 3\rho)e^{-3\rho}]\hat{e}_\phi$$

Find the current density $\bar{J}$.

Ampere's Law

$$\bar{\nabla} \times \bar{H} = J$$

Cylindrical Coordinates

$$\bar{A}(\rho, \varphi, z) = e_\rho A_\rho + e_\varphi A_\varphi + e_z A_z$$

$$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)e_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)e_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)e_z$$

$$\left[\cancel{\frac{1}{\rho}\frac{\partial}{\partial \phi}(0)}^{\,0} - \cancel{\frac{\partial}{\partial z}\left(\frac{4}{\rho}[1-(1+3\rho)e^{-3\rho}]\right)}^{\,0}\right]\cancel{\hat{e}_\rho}^{\,0}$$

$$+\left[\cancel{\frac{\partial}{\partial z}\left(\frac{4}{\rho}[1-(1+3\rho)e^{-3\rho}]\right)}^{\,0} - \cancel{\frac{\partial}{\partial \rho}(0)}^{\,0}\right]\hat{e}_\phi$$

$$+\frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\cancel{\rho}\,\frac{4}{\cancel{\rho}}[1-(1+3\rho)e^{-3\rho}]\right) - \cancel{\frac{\partial}{\partial \phi}\left(\frac{4}{\rho}[1-(1+3\rho)e^{-3\rho}]\right)}^{\,0}\right]$$

$\rho \cdot 1 \cdot 1$

$$\frac{4}{\rho}[1-(1+3\rho)e^{-3\rho}]\hat{e}_z$$

Chain Rule

$FS' + F'S$

$= -1 - 3\rho$

$(-1-3\rho)(-3e^{-3\rho})$

$+ (-3)(e^{-3\rho})$

$\cancel{3e^{-3\rho}} + 3\rho e^{-3\rho}$

$+ \cancel{-3e^{-3\rho}}$

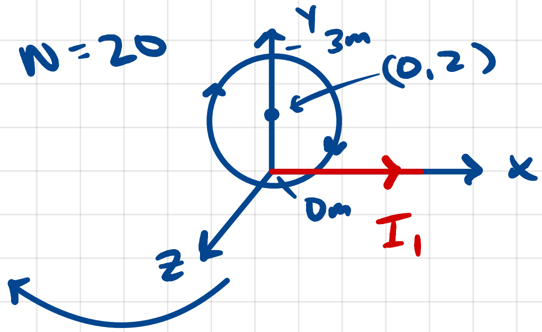
$$= 4[0 + \cancel{3e^{-3\rho}} \cdot 3\rho \cdot -3e^{-3\rho} + \cancel{e^{-3\rho}} \cdot 3\rho]$$

$$= \frac{1}{\rho}4[9\rho e^{-3\rho}]$$

$$\vec{J} = 36e^{-3\rho}\hat{e}_z$$

3. **25 Points** An infinitely long wire carrying a 25 A current in the positive x direction is placed along the x-axis in the vicinity of a 20-turn circular loop located in the x-y plane. The center of the loop is at (0, 2m) and its radius is 1m. If the magnetic field at the center of the loop is zero, what is the direction and magnitude of the current flowing in the loop? *Hint: We derived the expression for the magnetic flux density for a loop with one turn. For a loop with N turns, multiple the expression for $\bar{B}$ with N.*

N=20



BY USING RHR - B IS GOING OUT OF THE PAGE

$I = 25A \quad n = 20\text{-turns}$

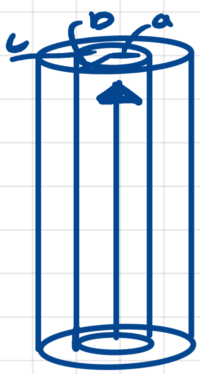
$$\bar{B} = \mu\bar{H} = \frac{\mu I_1}{2\pi\rho}$$

AROUND THE LOOP $\quad B = \dfrac{\mu I_2}{2\rho} \longrightarrow \dfrac{\cancel{\mu} I_1}{\cancel{2}\pi\rho} = \dfrac{\cancel{\mu} N I_2}{\cancel{2}\rho}$

$$I_2 = \frac{25\times 1}{20\pi(2)} \rightarrow I_2 = 0.2A$$

4. **30 Points** A Current I flows along the positive z-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has a radius a, and the inner and outer radii of the outer conductor are b and c, respectively. Use Ampere's Law to

   a. Determine the magnetic field intensity in the region $0 < \rho < a$. [5 points]
   b. Determine the magnetic field intensity in the region $a < \rho < b$. [5 points]
   c. Determine the magnetic field intensity in the region $b < \rho < c$. [5 points]
   d. Determine the magnetic field intensity in the region $\rho > c$. [5 points]
   e. If $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm, plot $|\bar{H}|$ as a function of ρ from $0 \leq \rho \leq 8cm$. [10 points]



$$\oint \bar{H} \cdot d\bar{l} = I_{enc} \qquad (\rho, \phi, z)$$

(a) $0 < \rho < a$

$$= \int_0^{2\pi} \bar{H} \cdot \rho \, d\phi = (2\pi\rho \bar{H} \hat{e}_\phi) = I_{enc} = I \cdot \left(\frac{\rho^2}{a^2}\right)$$

$$\bar{H} = \frac{I}{2\pi a^2} \cdot \rho \ \hat{e}_\phi \ (A/m)$$

(b) $a < \rho < b$

CURRENT IS ENCLOSED BY LOOP.

$$d\bar{l} = 2\pi\rho \, d\phi \rightarrow \bar{H} \cdot 2\pi\rho \hat{e}_\phi = I$$

$$\bar{H} = \frac{I}{2\pi\rho} \hat{e}_\phi \ (A/m)$$

(c) $b < \rho < c$

$$\oint_0^{2\pi} \bar{H} \cdot \rho \hat{e}_\phi = I_{enc} \rightarrow I_{enc} = \frac{\pi(\rho^2 - b^2)}{\pi(c^2 - b^2)}$$

$$\rightarrow \bar{H} 2\pi\rho \hat{e}_\phi = \left[\frac{\cancel{\pi}(\rho^2 - b^2)}{\cancel{\pi}(c^2 - b^2)}\right] I$$

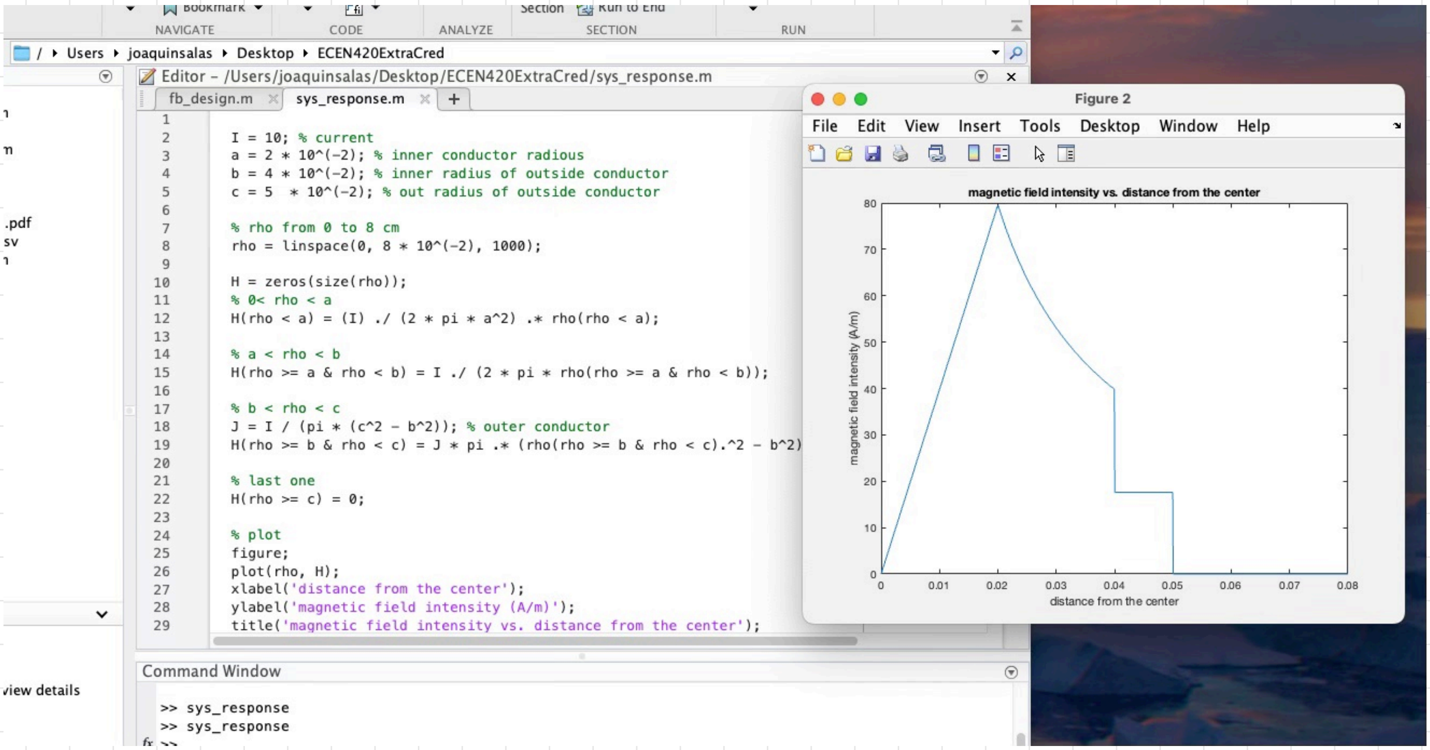
$$\rightarrow \bar{H} = \frac{I}{2\pi\rho}\left[\frac{\rho^2 - b^2}{c^2 - b^2}\right] \hat{e}_\phi \ (A/m)$$

(d) Outside of c, the farthest point out. INTENSITY is 0

$\rho > c$

$$\boxed{\bar{H} = 0}$$

(e)



5. **10 Points** Two infinitely long parallel wires are carrying 6A currents in the opposite directions as shown in Fig. 1. Find the magnetic flux density at point P in Fig. 1.

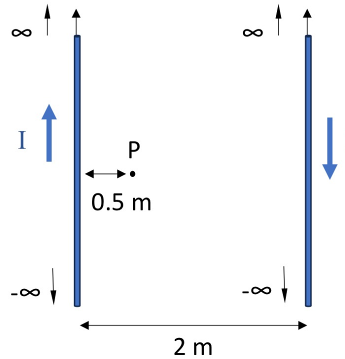


Fig 1: Geometry for Problem 5

CURRENT$_1$ = 6A

C2 = −6A

$$\bar{H} = \frac{I}{2\pi\rho}\,\hat{e}_\phi \text{ A/m}$$

$$\bar{H}_1 = \frac{6A\,\mu_0}{2\pi(0.5)}\,\hat{e}_\phi = -2.4\times10^{-6}\,T\,\hat{e}_\phi$$ INTO PAGE

$$\bar{H}_2 = \frac{-6A\,\mu_0}{2\pi(0.5-2)}\,\hat{e}_\phi = -0.8\times10^{-6}\,T\,\hat{e}_\phi$$ OUT OF PAGE

$$\text{Net} = \bar{H}_1 + \bar{H}_2 = -2.4\times10^{-6}T - 0.8\times10^{-6} = -3.2\times10^{-6}T$$

$$= -3.2\times10^{-6}\,\hat{e}_\phi\ (T)$$ In to page

Since symmetrical and opposing at P.

$$\text{ANS} = 3.2\times10^{-6}\,\hat{e}_\phi\ (T)$$

Out of the Page