

MATH 311, HW 1

Due 11:00pm on Thursday January 26, 2023

Instructions: download/print this HW and answer all questions in the space provided, (either in paper or electronically). You must explain your procedure as detailed as possible, but also be concise so that you only use the space provided. Then scan all the pages and upload them through Gradescope. Check the section “Fixed-length submissions (templated)” in the following [webpage](#).

Question 1. [10 pts] Answer the following questions (there are no wrong answers here):

- (3pts) What do I plan to do after graduation? How does this course will help me with that?

After I graduate, i'd like to use my degree in Electrical Engineering to work as a substation Engineer and put my knowledge in Applied Mathematics to use. My ultimate goal is to one day start my own Engineering Consulting Firm in the field of Power Engineering.

- (3pts) What topic of the course I am more interested in learning? Why?

I am most interested in learning about Eigenvalues and Eigenvectors, I believe they are interesting and fun to solve from my brief introduction to them in Differential Equations from last semester.

- (4pts) Which of the following proposed office hours work for me?

- ☐ Just Tuesdays 3:10 pm-4:10 pm
- ☒ Just Thursdays 12:30 pm-1:30 pm
- ☐ Both work
- ☐ None. If none of them work,
provide some interval times on TR that may work: _____

Problem 2. [16 pts] Are the following matrices in **reduced row echelon form**? Fill the corresponding square.

Yes No Matrix

☐ ☒ $\begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

☒ ☐ $\begin{pmatrix} 1 & 4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

☐ ☒ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

☒ ☐ $\begin{pmatrix} 1 & 0 & 0 & 3 & 5 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 2 & 5 \end{pmatrix}$

Yes No Matrix

☐ ☒ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

☒ ☐ $\begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

☐ ☒ $\begin{pmatrix} 1 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 6 \end{pmatrix}$

☐ ☒ $\begin{pmatrix} 0 & 1 & 11 & 111 \\ 0 & 0 & 1 & 11 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Problem 3. [24 pts] For each of the following augmented matrices, find how many solutions does the corresponding linear system of equations has. Fill the corresponding square.

0 1 2 ∞ Matrix

☐ ☐ ☐ ☒ $\begin{pmatrix} 1 & 2 & 3 & | & 0 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$
 $\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ x_3 = 2 \end{cases} \rightarrow x_1 + 2x_2 = -6$

☐ ☒ ☐ ☐ $\begin{pmatrix} 1 & 3 & | & 2 \\ 0 & 1 & | & -2 \\ 0 & 0 & | & 0 \end{pmatrix} \begin{matrix} x_1 - 6 = 2 \\ x_2 = -2 \end{matrix}$

☐ ☐ ☐ ☒ $\begin{pmatrix} \boxed{1} & 3 & 2 & 5 & | & 6 \\ 0 & 0 & \boxed{2} & 1 & | & -1 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$

☐ ☐ ☐ ☒ $\begin{pmatrix} 1 & 2 & 2 & 2 & | & 2 \\ 0 & 3 & -2 & 1 & | & 1 \\ 0 & 0 & 2 & 2 & | & 5 \end{pmatrix}$

0 1 2 ∞ Matrix

☒ ☐ ☐ ☐ $\begin{pmatrix} 1 & 3 & 4 & | & 5 \\ 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$

☐ ☒ ☐ ☐ $\begin{pmatrix} 1 & 0 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & 0 & | & 1 \\ 0 & 0 & 1 & 0 & | & -3 \\ 0 & 0 & 0 & 1 & | & 5 \end{pmatrix}$

☒ ☐ ☐ ☐ $\begin{pmatrix} 1 & 2 & 0 & | & 3 \\ 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & | & 1 \end{pmatrix}$

☒ ☐ ☐ ☐ $\begin{pmatrix} 1 & 5 & 2 & -1 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 2 \end{pmatrix}$

Problem 4. [10 pts] Solve the following system of linear equations.

$$\begin{cases} 4x_1 - x_2 - x_3 + 2x_4 = 3 \\ 2x_2 - 4x_3 - 4x_4 = -4 \\ -x_3 + x_4 = -3 \end{cases}$$

$\rightarrow -x_3 + 2 = -3 \rightarrow -x_3 = -5 \rightarrow x_3 = 5$
 $x_4 = 2$
 $2x_2 - 20 - 8 = -4 \rightarrow 2x_2 = 24 \rightarrow x_2 = 12$
 $4x_1 - 12 - 5 + 4 = 3 \rightarrow 4x_1 - 13 = 3 \rightarrow 4x_1 = 16 \rightarrow x_1 = 4$

Answer: $x_1 = 4, x_2 = 12, x_3 = 5, x_4 = 2$

Problem 5. [20 pts] Solve the following system of linear equations.

$$\begin{cases} x_1 + 2x_2 + 6x_3 + 4x_4 = -4 \\ 2x_1 + 4x_2 + 12x_3 + 9x_4 = -4 \\ 3x_1 + 6x_2 + 18x_3 + 9x_4 = -23 \end{cases}$$

Answer: NO SOLUTION

$$\begin{pmatrix} 1 & 2 & 6 & 4 & -4 \\ 2 & 4 & 12 & 9 & -4 \\ 3 & 6 & 18 & 9 & -23 \end{pmatrix} \xrightarrow{R_2 - 2R_1, R_3 - 3R_1} \begin{pmatrix} 1 & 2 & 6 & 4 & -4 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & -1 & -23 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 6 & 4 & -4 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & -1 & -23 \end{pmatrix} \xrightarrow{R_3 + R_2} \begin{pmatrix} 1 & 2 & 6 & 4 & -4 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & -2 & -19 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 6 & 4 & -4 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & -2 & -19 \end{pmatrix} \xrightarrow{R_3 + 2R_2} \begin{pmatrix} 1 & 2 & 6 & 4 & -4 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

$$R_3 \rightarrow R_3 + 3R_2$$

$$\begin{pmatrix} 1 & 2 & 6 & 4 & -4 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & -23 \end{pmatrix} \rightarrow \text{NO SOLUTIONS}$$

$-23 + 3(4) = -23 + 12 = -11$
 $-23 \neq 0$

Problem 6. [20 pts] Solve the following system of linear equations.

$$\begin{cases} x_1 - 2x_2 + 4x_3 = 6 \\ -x_1 + 2x_2 - x_3 = -1 \\ x_1 - 2x_2 + 10x_3 = 16 \\ -2x_1 + 4x_2 + x_3 = 3 \end{cases}$$

Answer:	$2\alpha - \frac{2}{3}, \alpha, \frac{5}{3}$
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$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ -1 & 2 & -1 & -1 \\ 1 & -2 & 10 & 16 \\ -2 & 4 & 1 & 3 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 + R_1} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 0 & 3 & 5 \\ 1 & -2 & 10 & 16 \\ -2 & 4 & 1 & 3 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - R_1} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 6 & 10 \\ -2 & 4 & 1 & 3 \end{array} \right) \xrightarrow{R_4 \rightarrow R_4 + 2R_1}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 6 & 10 \\ 0 & 0 & 9 & 15 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 15 \end{array} \right) \xrightarrow{R_4 \rightarrow R_4 - 3R_2} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\text{REF}} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 0 & 3 & 5/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{aligned} x_1 - 2x_2 + 4x_3 &= 6 \\ x_3 &= 5/3 \end{aligned}$$

$\bullet x_2$ is a free variable
 $x_2 = 2\alpha$

$$x_1 - 2\alpha + \frac{20}{3} = 6$$

$$x_1 = 2\alpha - \frac{2}{3}$$

$$x_2 = -2\alpha$$

$$x_3 = 5/3$$