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ECEN 325 – 512 Prelab 1
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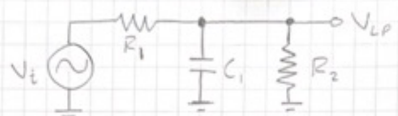
Part 1 – Calculations

JOAQUIN SALAS PRE-LAB 1 ECEN 325

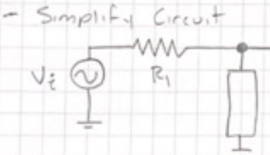
1) Derive. $H_{LP}(s) = \frac{V_{LP}}{V_i}(s) = k_L \frac{1}{1+\frac{s}{\omega_L}}$



- Transfer function is Nothing but Gain

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{V_{LP}(s)}{V_{in}(s)}$$

- Simplify Circuit



$$\frac{1}{C_1 s} \| R_2 = \frac{\frac{1}{C_1 s} \cdot R_2}{\frac{1}{C_1 s} + R_2} = \frac{R_2}{1 + C_1 R_2 s}$$

- Plug in $V_{LP} = \frac{Z}{R_1 + Z} \cdot V_{in} \rightarrow \frac{V_{LP}}{V_{in}} = \frac{Z}{R_1 + Z}$ Plug in

$$\rightarrow H_{LP}(s) = \frac{V_{LP}}{V_i} = \frac{\frac{R_2}{1 + C_1 R_2 s}}{\frac{R_2}{1 + C_1 R_2 s} + R_1} = \frac{R_2}{R_2 + R_1(1 + C_1 R_2 s)}$$

$$= \frac{R_2}{R_2 + R_1 + R_1 R_2 C_1 s}$$

$$= \frac{R_2}{R_1 + R_2}\left[\frac{1}{1 + \frac{R_1 R_2 C_1 s}{R_1 + R_2}}\right]$$

$$k_L = \frac{R_2}{R_1 + R_2}, \quad \omega_L = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

2) $k_L = 0.5$ and $f_L = 5\text{kHz}$ $\quad * \ \omega = 2\pi f$

Find $R_1, R_2, R_3, C_1, C_2, C_3$

$k_L = \frac{R_2}{R_1 + R_2} = 0.5 \Rightarrow 2R_2 = R_1 + R_2 \Rightarrow R_2 = R_1$

$\omega_L = \frac{R_1 + R_2}{R_1 R_2 C_1} = 2\pi \cdot 5k \Rightarrow \frac{2R}{R^2 C_1} = \cancel{2}\pi \times 5 \times 10^3 \Rightarrow RC_1 = \frac{1}{5000\pi}$

$RC_1 = 6.36 \times 10^{-5}$

$R_1 = R_2 = 1k\Omega \quad C_1 = 63.66\,nF$

$$\boxed{\begin{aligned} R_1 &= 1k\Omega \\ R_2 &= 1k\Omega \\ C_1 &= 63.66\,nF \end{aligned}}$$

3) Sketch magnitude and phase Bode plots

For $H_{LP}(S)$: From Lecture, Assume it will look like

- Plug into $H_{LP}(S) = \frac{K_L}{1+\frac{S}{\omega_L}} = \frac{0.5}{1+\frac{S}{10{,}000\pi}}$

$\Rightarrow T(j\omega) = H(s) \Rightarrow \frac{0.5}{1+\frac{j\omega}{10{,}000\pi}} \cdot \frac{10{,}000\pi}{10{,}000\pi} \Rightarrow \frac{5000\pi}{10000\pi + j\omega}$.

$\phi = \tan^{-1}\left[\frac{Im(T)}{Re(T)}\right]$ $\quad A(\omega) = \sqrt{T^*T}$ $\quad A(\omega) = \frac{5000\pi}{\sqrt{(10000\pi)^2+\omega^2}}$ L.P

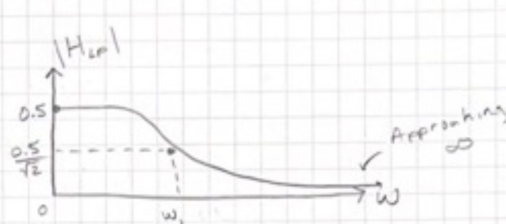
$= -\tan^{-1}\left\{\frac{\omega}{10000\pi}\right\}$

Magnitude Response

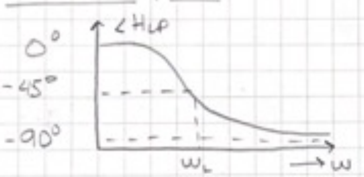
When $\omega = 0 \Rightarrow |H_{LP}| = 0.5$

$\omega = \omega_L \Rightarrow |H_{LP}| = \frac{0.5}{\sqrt{2}}$

$\omega = \infty \Rightarrow |H_{LP}| = 0$



Phase Response



4) Calculate $V_{LP}(t)$ for $V_i(t) = 0.4\sin(2\pi 4000t)$

$V_{LP}(t) = H_{LP}(s)$ plug in $S = j(2\pi 4000)$

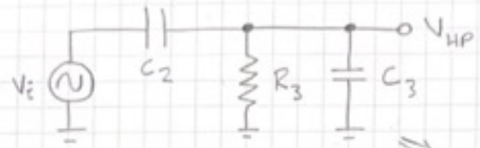
$= \left(\frac{0.5}{1+\frac{8000\pi j}{10000\pi}}\right)(0.4\sin(2\pi 4000t))$ $\quad \frac{4}{5}j$

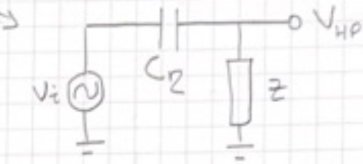
$\Rightarrow \phi = -\tan^{-1}\left(\frac{8000\pi}{10000\pi}\right) = -38.66°$

$\Rightarrow V_{LP}(t) = 0.4\,|H_{LP}(\omega)|\sin(\omega t - \phi)$

$= 0.4(0.468)\sin(10000\pi t - 38.66°) = \underline{0.156\sin(8000\pi t - 38.66°)}$

1) Derive $H_{HP}(S) = \frac{V_{HP}(S)}{V_i} = K_H \frac{S}{S+\omega_H}$





$- \; H(S) = \frac{V_o(S)}{V_{in}(S)} = \frac{V_{HP}(S)}{V_i(S)}$

$- \; Z = R_3 \| \frac{1}{SC_3} = \frac{R_3 \cdot \frac{1}{SC_3} \cdot SC_3}{R_3 + \frac{1}{SC_3} \cdot SC_3} \rightarrow \frac{R_3}{1+SR_3C_3} \rightarrow V_{HP} = \left(\frac{Z}{Z + \frac{1}{SC_2}}\right)\frac{SC_2}{SC_2} V_{in} = \frac{Z\,SC_2}{ZSC_2+1} V_{in}$

Plug in Z $\Rightarrow$ $\frac{V_{HP}(S)}{V_i(S)} = H_{HP}(S) = \frac{\left[\frac{R_3}{1+SR_3C_3}\right][SC_2]}{\frac{R_3}{1+SR_3C_3}(SC_2)+1} = \frac{\frac{SC_2R_3}{1+SR_3C_3}}{\frac{SC_2R_3}{1+SR_3C_3}+1} \Rightarrow \frac{SC_2R_3}{1+SR_3C_3+SC_2R_3} \rightarrow$

Compare to Eq Above $\longrightarrow$

$\frac{SC_2R_2}{1+S(R_3C_3+C_2R_3)} \rightarrow K_H\left[\frac{C_2R_3}{R_3C_3+R_3C_2}\right]\left[\frac{S}{S+\frac{1}{R_3C_3+R_3C_2}}\right]$

$K_H = \frac{C_2}{C_2+C_3} \quad \omega_H = \frac{1}{R_3(C_2+C_3)}$

2) $K_H = \frac{C_2}{C_2+C_3} = 0.5 \Rightarrow \frac{16\times10^{-9}}{16\times10^{-9}+16\times10^{-9}}$ $\quad \omega = 2\pi(5000)$, $\omega = 10000\pi$

$\omega_H = \frac{1}{R_3(C_2+C_3)} = \frac{1}{1000(16\times10^{-9}+16\times10^{-9})} = 10000\pi$

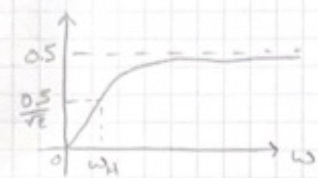
So, $R_3 = 1\,k\Omega$
$C_2 = 16\times10^{-9}\,F$ or $16\,nF$
$C_3 = 16\,nF$

3) $H_{HP} = \frac{0.5S}{S+\omega_H} \Rightarrow H_{HP} = \frac{0.5(j\omega)}{j\omega+\omega_H} \Rightarrow A(j\omega) = \frac{0.5\omega}{\sqrt{\omega^2+(\omega_H)^2}}$

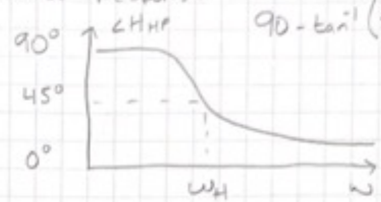
when $\omega = 0 = |H_{HP}| = \frac{\frac{0.5\,\omega_H}{10000\pi}}{\sqrt{2}}$ $\quad \omega_H = 0$ $\quad = \boxed{0}$

$\omega = \omega_H \Rightarrow |H_{HP}| = \frac{\frac{0.5\,\omega_H}{10000\pi\,\omega_H}}{\sqrt{2}} = \frac{0.5}{\sqrt{2}}$

$\omega = \infty \Rightarrow |H_{HP}| = \underline{0.5}$



Phase Response $\quad 90 - \tan^{-1}\left(\frac{\omega}{\omega_H}\right)$



4) For H_{HP}

$$= \frac{0.5s}{s + 10000\pi}[0.4]\sin(8000\pi t), \quad s = j(8000\pi)$$

$$= \frac{0.5(j8000\pi)}{j8000\pi + j10000\pi}[0.4]\sin(8000\pi t)$$

$$\Rightarrow \cancel{0.24} \; \frac{4000\pi}{\sqrt{(10000\pi)^2 + (8000)^2}} = 0.312$$

$$= \phi = \tan^{-1}\left(\frac{8000\pi}{10000\pi}\right) = 38.66^\circ \qquad \underline{V_{HP} = 0.125\sin(8000\pi t + 38.66^\circ)}$$

5) For H_{HP} and H_{LP}

$V_i(t) = 0.3\sin(2\pi 6000t)$. $\quad$ Low Pass

$$\phi = \tan^{-1}\left(\frac{12000\pi}{10000\pi}\right) = -50.19^\circ$$

$$V_{LP}(t) = \cancel{0.3} \; 0.3\,|H_{LP}(\omega)|\sin(\omega t + \phi)$$

$$= \left(\frac{5000\pi}{\sqrt{(10000\pi)^2 + (12000\pi)^2}}\right) \Rightarrow 0.320$$

Low Pass

$$= 0.3 \times 0.320 \sin(\omega t - 50.19^\circ) \Rightarrow \boxed{0.096\sin(\omega t - 50.19^\circ)}$$

$$= 0.096\sin(12000\pi t - 50.19^\circ)$$

H_{HP} -

$$\phi = \tan^{-1}\left(\frac{12000\pi}{10000\pi}\right) = 50.19^\circ$$

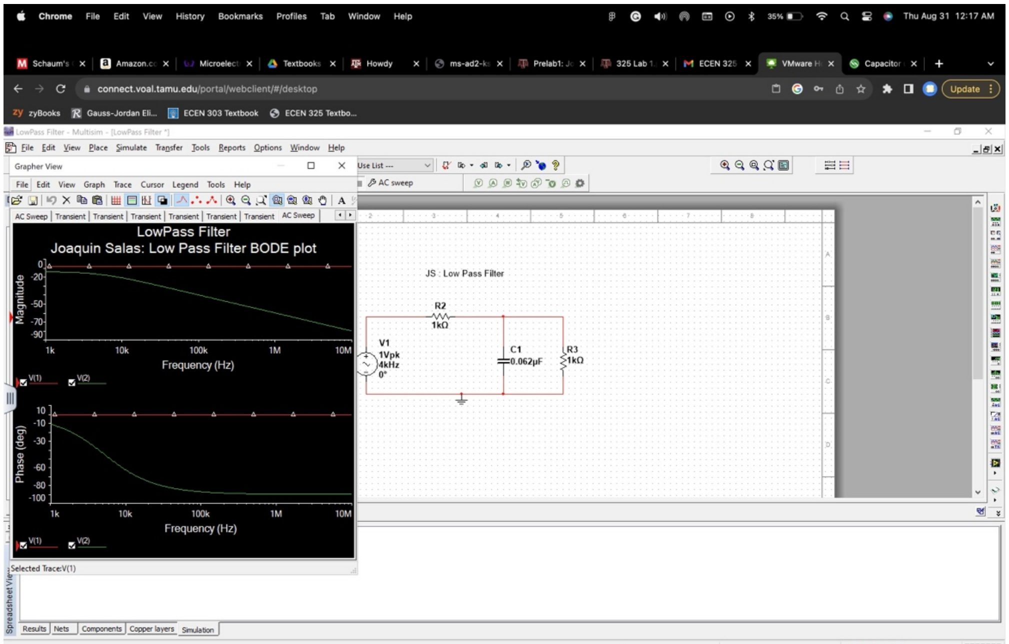
$$\Rightarrow \frac{6000\pi}{\sqrt{(10000\pi)^2 + (12000\pi)^2}} = 0.384$$

$$0.3\,|H_{HP}(\omega)|\sin(\omega t + \phi)$$

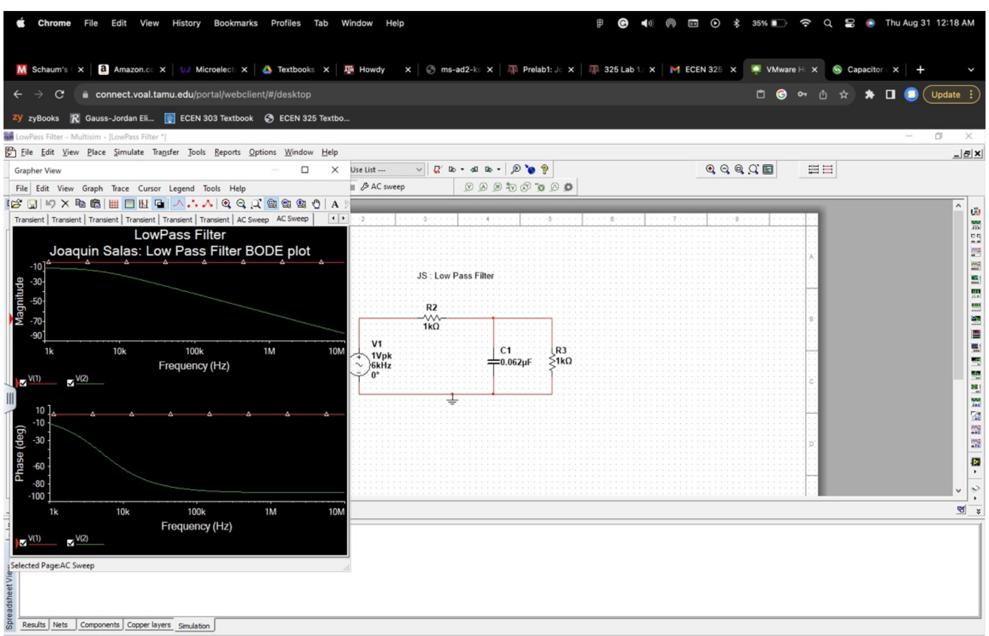
$$= 0.115\sin(12000\pi t + 50.19^\circ)$$

Part 2: Simulations

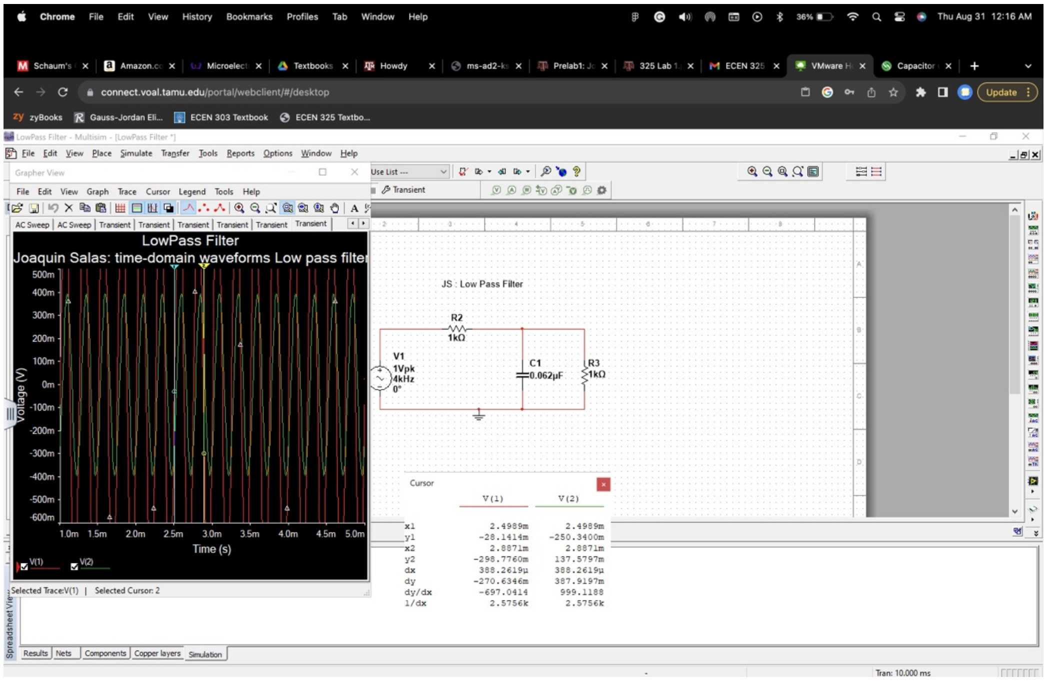
a) Low Pass Filter Bode Plot at 4kHz and 4mV.



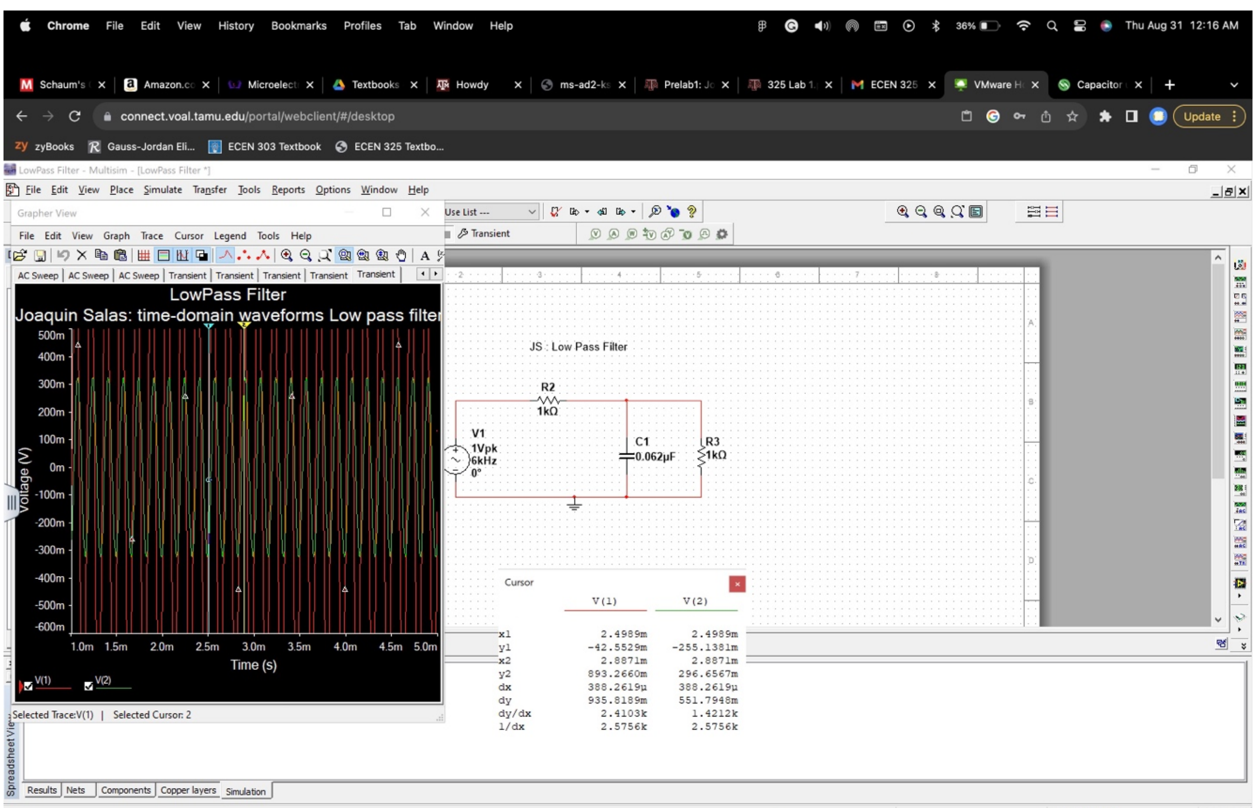
b) Low Pass Filter Bode Plot at 6kHz and 3mV.



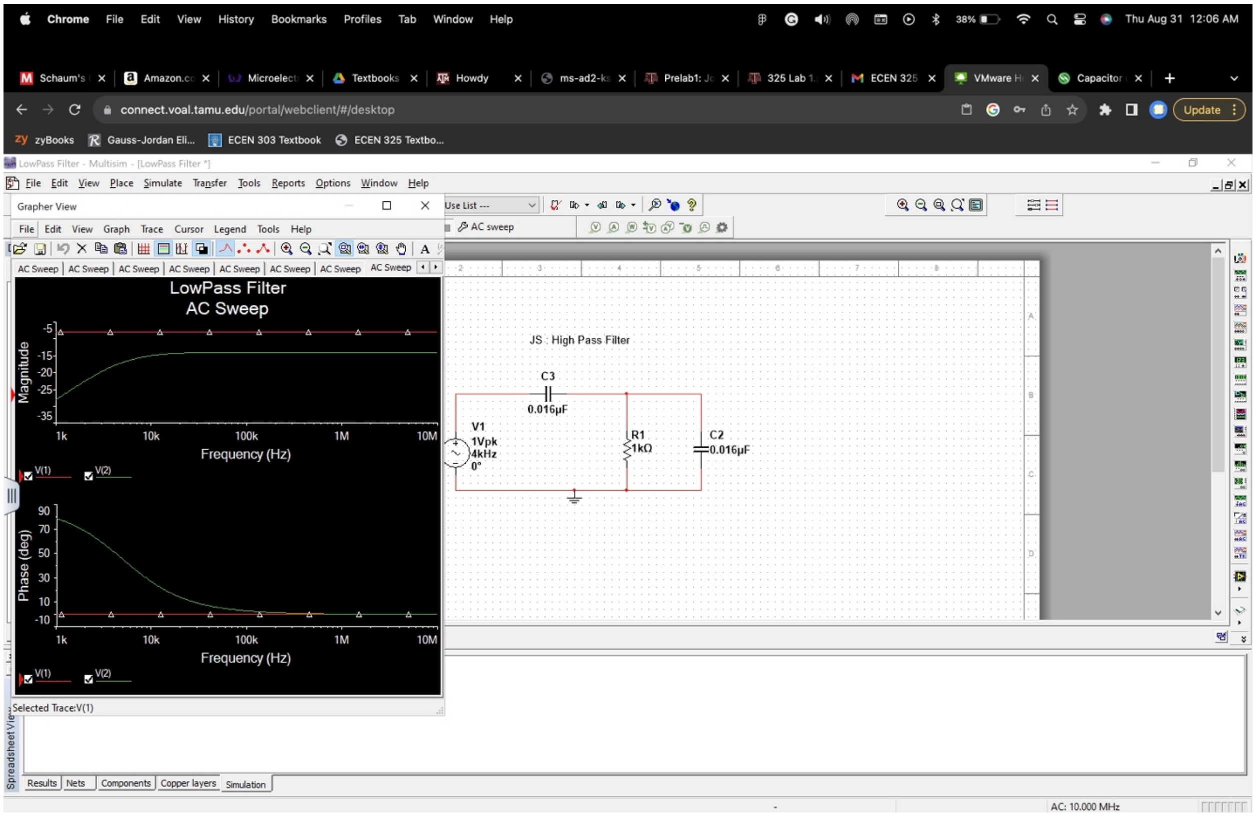
c) Low Pass Filter Waveform Graph at 4kHz and 4mV.



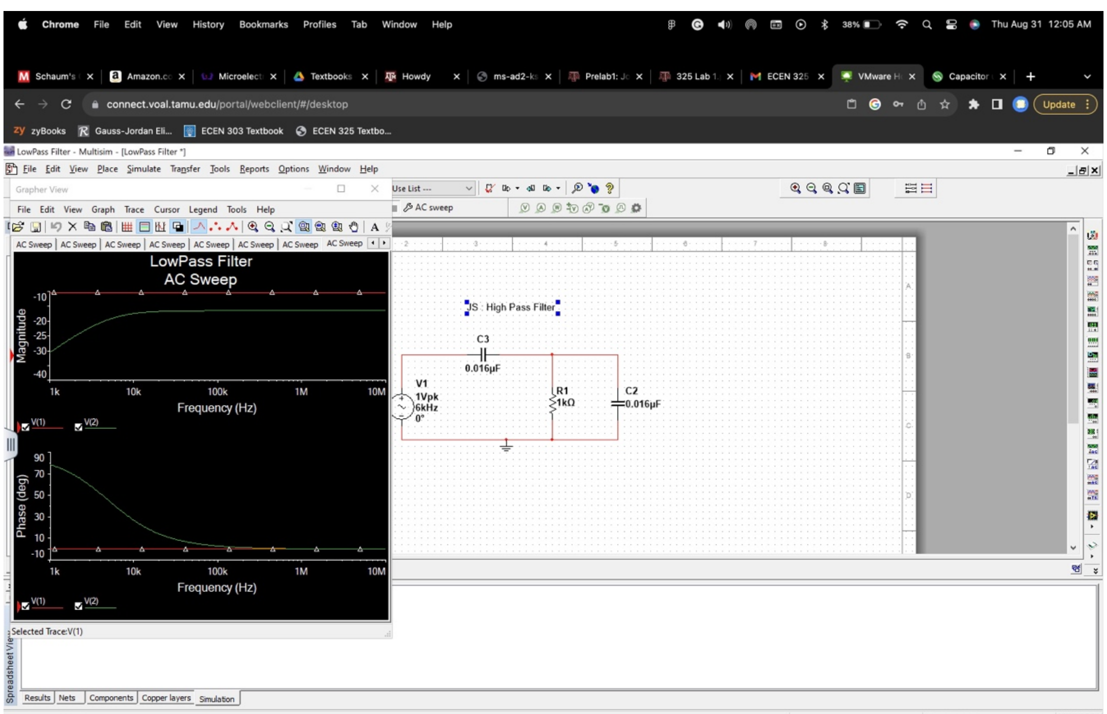
d) Low Pass Filter Waveform Graph at 6kHz and 3mV.



e) High Pass Filter Bode Plot at 4kHz and 4mV.



f) High Pass Filter Bode Plot at 6kHz and 3mV.



g) High Pass Filter Waveform Graph at 6kHz and 3mV.