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Final Exam Ecen 444

1.

$$x[n] = \delta[n-2] + \delta[n-4] + \delta[n-6] + \delta[n-8] + \delta[n-10]$$

a) find the phase of $X(e^{jw})$, which is the Fourier Transform of $x[n]$.

FT of $\delta[n-k] = e^{-jwk}$

$$\downarrow$$

$$X(e^{jw}) = e^{-jw2} + e^{-jw4} + e^{-jw6} + e^{-jw8} + e^{-jw10}$$

b) Define $Y(e^{jw}) = e^{jw} X^*(e^{jw})$, where $X^*(e^{jw})$ is the complex conjugate of $X(e^{jw})$. Compute & sketch the inverse FT of $Y(e^{jw})$

$$X^*(e^{jw}) = e^{jw2} + e^{jw4} + e^{jw6} + e^{jw8} + e^{jw10}$$

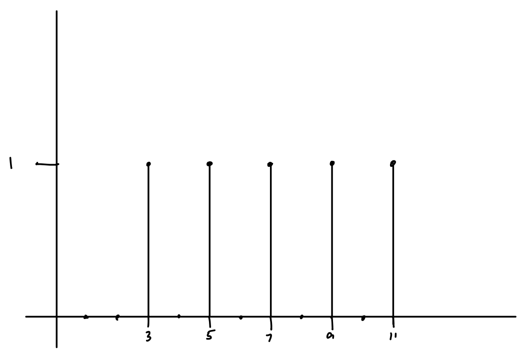
$$Y(e^{jw}) = e^{jw}\left(e^{jw2} + e^{jw4} + e^{jw6} + e^{jw8} + e^{jw10}\right)$$

$$= e^{jw3} + e^{jw5} + e^{jw7} + e^{jw9} + e^{jw11}$$

$$\downarrow$$

$$y[n] = \delta(n-3] + \delta[n-5] + \delta[n-7] + \delta[n-9] + \delta[n-11]$$

sketch:



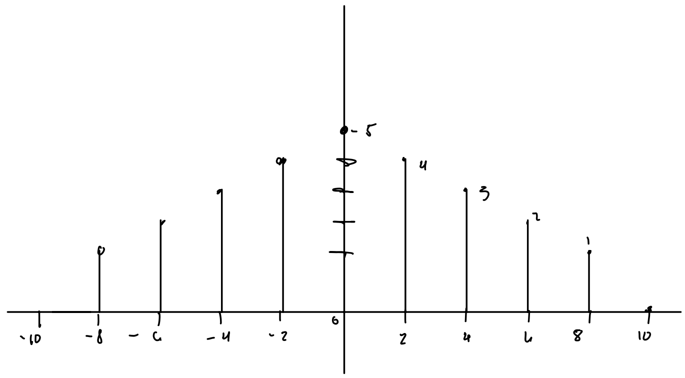
c) Define $Z(e^{jw}) = |X(e^{jw})|^2$. Compute & sketch the inverse FT of $Z(e^{jw})$

$$\uparrow$$

$$|X(e^{jw})|^2 = X(e^{jw})\ X^*(e^{jw})$$

$$= \left(e^{-jw2} + e^{-jw4} + e^{-jw6} + e^{-jw8} + e^{-jw10}\right)\left(e^{jw2} + e^{jw4} + e^{jw6} + e^{jw8} + e^{jw10}\right)$$

Sketch:



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(2) $s[n] = \left(\frac{1}{2}\right)^n u[n-2]$

a) Determine the System Function $H(z)$ for the system.

$$s[n] = \left(\tfrac{1}{2}\right)^n u[n-2]$$

$$\hookrightarrow S(z) = Z\{(\tfrac{1}{2})^n u[n-2]\}$$

$$X(z) \rightsquigarrow \frac{1}{1-z^{-1}}, \quad |z| > 1$$

$$H(z) = \frac{S(z)}{X(z)}$$

$$\downarrow$$

$$S(z) \Rightarrow 0.25 \cdot \sum_{n=0}^{\infty} 0.5^n z^{-n} u(n+2)$$

$$H(z) = \left(0.25 - \frac{0.25}{z}\right) \cdot \sum_{n=0}^{\infty} 0.5^n z^{-n}$$

b) what are the poles and zeroes

$$H(z) = \left(0.25 - \frac{0.25}{z}\right) \cdot \sum_{n=0}^{\infty} 0.5^{n} z^{-n}$$

* expressing it without since it converges

$$= \left(0.25 - \frac{0.25}{z}\right) \frac{1}{1-0.5z^{-1}}$$

zeroes : $z = 1$

poles : $z = 0.5$

c) write a difference equation for the system

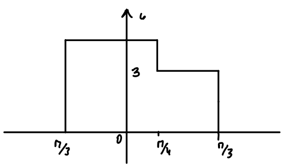
$$H(z) = \left(0.25 - \frac{0.25}{z}\right) \cdot \sum_{n=0}^{\infty} 0.5^{n} z^{-n}$$

$$\left(0.25 - \frac{0.25}{z}\right) \cdot \frac{1}{1-0.5z^{-1}}$$

$$\frac{0.25}{1-0.5z^{-1}} \cdot \frac{1}{z(1-0.5z^{-1})}$$

$$y[n] = 0.25\, x[n] + 0.125\, x[n-1] - 0.25\, y[n-1]$$

3



a) $\sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \frac{1}{2\pi}\int_{-\pi/3}^{\pi/4} |6|^2 \, d\omega + \int_{\pi/4}^{\pi/3} |3|^2 \, d\omega$$

$$= \frac{1}{2\pi}\left[36\left(\frac{\pi}{4}+\frac{\pi}{3}\right) + 9\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\right]$$

$$= \frac{1}{2\pi}\left(36\,\frac{3\pi}{4} + 9\left(\frac{\pi}{4}\right)\right]$$

$$= 39/4$$

b) compute $\sum_{n=-\infty}^{\infty} j^n x(n)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \longrightarrow e^{j\pi/2} = j$$

$$\downarrow$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} j^n x(n)$$

$$\sum_{n=-\infty}^{\infty} j^n x(n) = 0$$

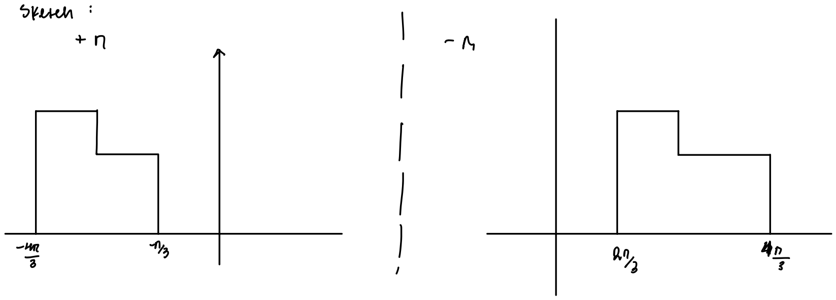
c) $y(n) = (-1)^n x(n)$

$$FT \Rightarrow Y(\omega) = \sum_{n=-\infty}^{\infty} y(n)\, e^{j\omega n}$$

$$y(n) = e^{j\pi n} x(n) \longrightarrow \cos\pi + j\sin\pi = -1$$

$$= (-1)^n x(n) \rightsquigarrow \text{FT properties} \rightsquigarrow = X(\omega \pm \pi)$$

Sketch :

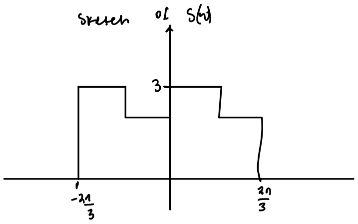


d) $S(n) = x(n) \left[\frac{e^{j\omega\pi/3}}{2} + \frac{e^{-j\omega\pi/3}}{2} \right.$

$$= \tfrac{1}{2}\left(x(n)\, e^{j\omega\frac{\pi}{3}} + x(n)\, e^{-j\omega\frac{\pi}{3}}\right)$$

$$= \tfrac{1}{2}\left[X(\omega - \pi/3) + X(\omega + \pi/3)\right]$$

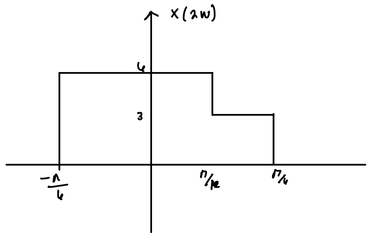
Sketch of S(n)



c) $t(n) \begin{cases} x(n/2), & \text{if } n = \text{multiple of } 2 \\ 0, & \text{o.w.} \end{cases}$

** $x(n/k) = x(e^{j\omega k})$ **

$t(n) = x(n/2) \longleftrightarrow T(\omega) = X(2\omega)$

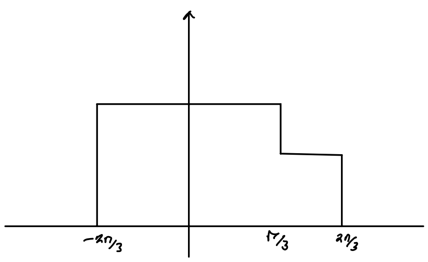


f) $z(n) = x(2n)$

* $x(n/k) \longleftrightarrow x(k\omega)$

$z(n) = x(2n)$

$Z(\omega) = X(\omega/2)$



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(4)

a) M = 201 ⟶ the impulse response will be symmetric

100 samples on each side

$$hd[n] = \frac{1}{2\pi} \int_{-0.7\pi}^{-0.9\pi} e^{jn\omega} d\omega + \frac{1}{2\pi} \int_{0.4\pi}^{0.7\pi} e^{jn\omega} d\omega$$

b) Hamming window method : * $w[n] = 0.54 + 0.46 \cos(2\pi n / M-1)$, $-\frac{M-1}{2} \le n \le \frac{M-1}{2}$

* reduces side lobes in frequency response

to do this :

① since its non-causal, we have to shift it :

$h'd[n] = hd[n - \frac{M-1}{2}]$ $0 \le n \le M-1$

2. multiply by Hamming window

$h[n] = h'd[n] \cdot w[n] \qquad 0 \le n \le M-1$

3. normalize by making sure the sum of the coefficients = 1

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5. $x(n) = \{1, 3, 5, 3\} \qquad y(n) = \{1, 2, 3, 4\}$

a) compute the 4-point circular convolution $z(n) = x(n) \circledast y(n)$ for $0 \le n \le 3$

$$z(n) = x(n) \circledast y(n)$$

$z(0) = 4 + 4 + 5 + 4$
$z(1) = 2 + 12 + 10 + 3$
$z(2) = 6 + 20 + 6 + 1$
$z(3) = 10 + 12 + 2 + 3$

$z(n) = \{21, 27, 33, 27\}$

b) $\sum_{n=0}^{N-1} x(n) e^{-\frac{2\pi}{N} kn}$

$X(k) = \{12, -4, 0, -4\}$

$Y(k) = \{7, 3, 1, 3\}$

c) $Z(k) = X(k) \cdot Y(k)$

$= \{107, -12, 0, -12\}$

d) $= \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{-j 2\pi nk / N}$

$\downarrow$

$z(n) = \{33, 27, 21, 27\}$

6

a) explain how to construct the linear convolution result $y[n]$ & determine all the values of $y[n]$ for $-\infty < n < \infty$

> add the linear convolutions $y_1[n]$ & $y_2[n]$

↳ $h[n]$ length $= 5$,

↓

overlapping will be 4 points

> add last 4 values of $y_1[n]$ to first 4 of $y_2[n]$

> first 4 of $y[n]$ will = the first 4 of $y_1[n]$

the next 4 will be sum of the $y_1[n]$ & $y_2[n]$

the last 5 of $y[n]$ will = the last 5 of $y_2[n]$

↓

$y[n] = [1, 0, 2, 2, -1, 1, -2, -1, 1, 0, 2, 3, 2, 3, 4]$

b) given $y[n]$ and $h[n] = \{1, 1, 1, 1, 1\}$

↓

$y[0] = x[0]$

$y[1] = x[0] + x[1]$

$y[2] = x[0] + x[1] + x[2]$

...

$y[5] = x[0] + x[1] + x[2] + x[3] + x[4] + x[5]$

... * continuing to end of $y[n]$

$x[n] = \{1, -1, 2, 0, -3, 2, -3, 1, 2, -1, 2\}$