HW 6 P4.1, P4.2, P4.3, P4.4, P.4.21 (a,b)

(a) $e^{-2(t-1)}u(t-1)$ Equation $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

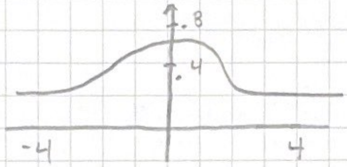
$$X(j\omega) = \int_{-\infty}^{\infty} e^{-2(t-1)}u(t-1)e^{-j\omega t}dt$$

$$X(j\omega) = \int_{1}^{\infty} e^{-2(t-1)}e^{-j\omega t}dt$$

$$X(j\omega) = e^{2}\int_{1}^{\infty} e^{(-j\omega-2)t}dt = -e^{2}\cdot\frac{1}{2+j\omega}\cdot\left(0-e^{-j\omega-2}\right)$$

$\boxed{X(j\omega) = \frac{e^{-j\omega}}{2+j\omega}}$ Magnitude $|X(j\omega)| = \left|\frac{1}{\sqrt{4+\omega^2}}\right|$

Sketch



(b) $e^{-2|t-1|}$ for negative $t-1<0$

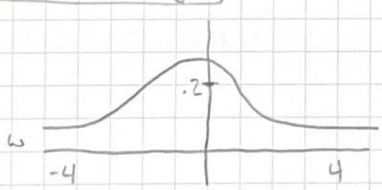
$$X_1(j\omega) = \int_{1}^{\infty} e^{-2(t-1)}e^{-j\omega t}dt \qquad X_2(j\omega) = \int_{-\infty}^{1} e^{-2(t-1)}e^{-j\omega t}dt$$

Part 1

$$X_1(j\omega) = \frac{e^{-j\omega}}{2+j\omega} \rightarrow X_2(j\omega) = e^{-2}\int_{\infty}^{1} e^{(2-j\omega)t}dt$$

$$X_2(j\omega) = \frac{e^{-2}}{2-j\omega}\cdot e^{(2-j\omega)}\Big|_{\infty}^{1} \rightarrow \frac{e^{-2}}{2-j\omega}\cdot\left(e^{2-j\omega-2}-0\right)$$

$\boxed{X_2(j\omega) = \frac{e^{-j\omega}}{2-j\omega}}$ So, $\frac{e^{-j\omega}}{2+j\omega} + \frac{e^{-j\omega}}{2-j\omega} = \boxed{\frac{4e^{-j\omega}}{4+\omega^2}}$ Sum



Magnitude

$$|X(j\omega)| = \left|\frac{4}{4+\omega^2}\right|$$

P4.2

(a) $\delta(t+1)+\delta(t-1)$

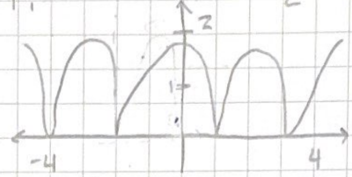
Plug into Equation

$$X(j\omega) = \int_{-\infty}^{\infty} \left(\delta(t+1)+\delta(t-1)\right) e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t+1) e^{-j\omega t} dt + \int_{-\infty}^{\infty} \delta(t-1) e^{-j\omega t} dt$$

$$X(j\omega) = e^{-j\omega t}\Big|_{-1}^{\infty} + e^{-j\omega t}\Big|_{1}^{\infty} = 2 \cdot \frac{e^{j\omega}+e^{-j\omega}}{2} = \boxed{2\cos(\omega)}$$

Sketch $|X(j\omega)| = 2|\cos(\omega)|$



(b) $x(t) = \frac{d}{dt}\left(u(-2-t)+u(t-2)\right) = -\delta(-2-t)+\delta(t-2)$

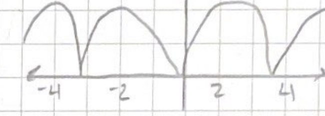
Same as (a)

$X(j\omega) = -e^{2j\omega} + e^{-2j\omega}$

$$X(j\omega) = -j2 \cdot \frac{e^{2j\omega}-e^{-2j\omega}}{j2} = \boxed{-2j\sin(\omega)}$$

Sketch

$|X(j\omega)| = 2|\sin(\omega)|$



P4.3 (a) $\sin(2\pi t + \frac{\pi}{4})$

$T = 1$ and $\omega = 2\pi$

$$x_1(t) = \sin\left(2\pi t + \frac{\pi}{4}\right) = \frac{e^{j(2\pi t+\frac{\pi}{4})} - e^{-j(2\pi t+\frac{\pi}{4})}}{2j}$$

Non-Zero FS

$$a_1 = \frac{e^{j\frac{\pi}{4}}}{2j},\ a_{-1} = -\frac{e^{-j\frac{\pi}{4}}}{2j} \qquad = \frac{e^{j\frac{\pi}{4}}}{2j} e^{j2\pi t} - \frac{e^{-j\frac{\pi}{4}}}{2j} e^{-j2\pi t} = a_1 e^{j2\pi t} + a_{-1} e^{-j2\pi t}$$

Plug into

$$X(j\omega) = \sum_{-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0) \qquad X(j\omega) = \sum_{-\infty}^{\infty} 2\pi a_k \delta(\omega - 2\pi k)$$

$$X_1(j\omega) = 2\pi \frac{e^{j\frac{\pi}{4}}}{2j} \delta(\omega-2\pi) - 2\pi \frac{e^{-j\frac{\pi}{4}}}{2j} \delta(\omega+2\pi)$$

$$\boxed{X_1(j\omega) = \frac{\pi}{j} e^{j\frac{\pi}{4}} \delta(\omega-2\pi) - \frac{\pi}{j} e^{-j\frac{\pi}{4}} \delta(\omega+2\pi)}$$

(b) $1 + \cos\left(6\pi t + \frac{\pi}{8}\right) \quad T = 1/3 \Rightarrow \omega = 6\pi$

$$x_2(t) = 1 + \cos\left(6\pi t + \frac{\pi}{8}\right)$$

$$= 1 + \frac{e^{j(6\pi t + \frac{\pi}{8})} + e^{-j(6\pi t + \pi/8)}}{2} = 1 + \frac{e^{j\pi/8}}{2} e^{j6\pi t} + \frac{e^{-j\pi/8}}{2} e^{-j6\pi t}$$

$$= a_1 e^{j6\pi t} + a_1 e^{-j6\pi t}$$

$$a_0 = 1, \; a_1 = \frac{e^{j\pi/8}}{2}, \; a_1 = \frac{e^{-j\pi/8}}{2}$$

$$X_2(j\omega) = 2\pi\delta(\omega) + 2\pi \frac{e^{j\pi/8}}{2}\delta(\omega - 6\pi) + 2\pi \frac{e^{-j\pi/8}}{2}\delta(\omega + 6\pi)$$

$$\boxed{X_2(j\omega) = 2\pi\delta(\omega) + \pi e^{j\pi/8}\delta(\omega - 6\pi) + \pi e^{-j\pi/8}\delta(\omega + 6\pi)}$$

P4.4

(a) $X_1(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)\right) e^{j\omega t} d\omega$$

$$x(t) = \int_{-\infty}^{\infty} \delta(\omega) e^{j\omega t} d\omega + 0.5\int_{-\infty}^{\infty} \delta(\omega - 4\pi) e^{j\omega t} d\omega + 0.5\int_{-\infty}^{\infty} \delta(\omega + 4\pi) e^{j\omega t} d\omega$$

$$x(t) = e^{j0t} + 0.5 e^{j4\pi t} + 0.5 e^{-j4\pi t}$$

$$x(t) = 1 + \frac{e^{j4\pi t} + e^{-j4\pi t}}{2} = \boxed{1 + \cos(4\pi t)} \qquad |X(j\omega)| = 2|\cos(\omega)|$$

(b) $X(j\omega) = \begin{cases} 2; & 0 \geq \omega \geq 2 \\ -2; & -2 \geq \omega < 0 \\ 0; & |\omega| > 2 \end{cases} \qquad x(t) = \frac{1}{2\pi}\left(-2\int_{-2}^{0} e^{j\omega t} d\omega + 2\int_{0}^{2} e^{j\omega t} d\omega\right) = -2\frac{1}{jt\pi} e^{j\omega t},$

$$x(t) = \frac{1}{jt\pi}\left(e^{-2jt} - 1 + e^{2jt} - 1\right), \quad \boxed{x(t) = \frac{e^{-2jt} - 2 + e^{2jt}}{jt\pi}}$$

P4.21 (a) $[e^{-\alpha t}\cos\omega_0 t]u(t),\ a>0$

$$\cos(\omega_0 t) = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$$X(j\omega) = \int_0^\infty \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}\left(e^{-j\omega t}\right)dt$$

$$X(j\omega) = \frac{1}{2}\left(\int_0^\infty e^{-(\alpha+j(\omega-\omega_0))t}dt + \int_0^\infty e^{-(a+j(\omega+\omega_0))t}dt\right)$$

$$X(j\omega) = -\frac{1}{2}\left(\frac{e^{-(a+j(\omega-\omega_0))t}}{a+j(\omega-\omega_0}\right)_0^\infty - \left(\frac{e^{-(a+j(\omega+\omega_0))t}}{a+j(\omega+\omega_0)}\right)_0^\infty$$

$$\boxed{X(j\omega) = \frac{1}{2}\left(\frac{1}{a+j(\omega-\omega_0)} + \frac{1}{a+j(\omega+\omega_0)}\right)}$$

(b) written as

$t\geq 0$ and $t<0$, $e^{-3|t|}\sin 2t = e^{-3t}\sin 2t\,u(t) + e^{3t}\sin 2t\,u(-t)$

Linearity $X = X_1 + X_2$ $\quad X_1 = \frac{e^{-3t}}{2j}e^{j2t} - \frac{e^{-3t}}{2j}e^{-j2t}$

$$X_1(j\omega) = \int_0^\infty \left(\frac{e^{-3t}}{2j}e^{j2t} - \frac{e^{-3t}}{2j}e^{-j2t}\right)e^{-j\omega t}dt$$

$$X_1(j\omega) = \frac{1}{2j}\left(\int_0^\infty e^{-3+j(2-\omega))t}dt - \int_0^\infty e^{-3-j(2+\omega))t}dt\right)$$

$$X_1(j\omega) = \frac{1}{2j}\left(\frac{e^{-3+j(2-\omega))t}}{3-j2+j\omega}\right)_0^\infty - \left(\frac{e^{-3(-j(2+\omega))t}}{3+j2+j\omega}\right)_0^\infty$$

$$X_1(j\omega) = \frac{1}{2j(3-j2+j\omega)} - \frac{1}{2j(3+j2+j\omega)}$$

AND

$$X_2(j\omega) = -X_1(-j\omega) = \frac{1}{2j(3-j2-j\omega)} - \frac{1}{2j(3+j2-j\omega)}$$

$$X(j\omega) = X_1(j\omega) + X_2(j\omega) =$$

$$\boxed{\frac{3j}{9+(\omega+2)^2} - \frac{3j}{9+(\omega-2)^2}}$$