The arithmetic progression $5137n+1$, $n \geqslant 0$, contains infinitely many primes. A Euclidean proof

About this document

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We will prove that the arithmetic progression $\equiv 1 \pmod{5137}$ contains infinitely many primes. Equivalently, we will see that there are infinitely many primes of the form $5137n+1$, $n \geqslant 0$. To follow the proof, one must recall the expression of the discriminant of a polynomial.

Definition 1. The discriminant of a monic polynomial $A(x) = x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0$ is given, in terms of its roots $\{r_1, r_2, \ldots, r_m\} \subset \mathbb{C}$ (not necessarily distinct), by

$$\Delta(A) = \prod_{i<j}(r_i - r_j)^2, \quad 1 \leqslant i, j \leqslant m. \tag{1}$$

It will be useful to remember that the 5137th cyclotomic polynomial is $\Phi_{5137}(x) =$
$x^{4660}-x^{4659}+x^{4649}-x^{4648}+x^{4638}-x^{4637}+x^{4627}-x^{4626}+x^{4616}-x^{4615}+x^{4605}-x^{4604}+x^{4594}-$
$x^{4593}+x^{4583}-x^{4582}+x^{4572}-x^{4571}+x^{4561}-x^{4560}+x^{4550}-x^{4549}+x^{4539}-x^{4538}+x^{4528}-x^{4527}+$
$x^{4517}-x^{4516}+x^{4506}-x^{4505}+x^{4495}-x^{4494}+x^{4484}-x^{4483}+x^{4473}-x^{4472}+x^{4462}-x^{4461}+x^{4451}-$
$x^{4450}+x^{4440}-x^{4439}+x^{4429}-x^{4428}+x^{4418}-x^{4417}+x^{4407}-x^{4406}+x^{4396}-x^{4395}+x^{4385}-x^{4384}+$
$x^{4374}-x^{4373}+x^{4363}-x^{4362}+x^{4352}-x^{4351}+x^{4341}-x^{4340}+x^{4330}-x^{4329}+x^{4319}-x^{4318}+x^{4308}-$
$x^{4307}+x^{4297}-x^{4296}+x^{4286}-x^{4285}+x^{4275}-x^{4274}+x^{4264}-x^{4263}+x^{4253}-x^{4252}+x^{4242}-x^{4241}+$
$x^{4231}-x^{4230}+x^{4220}-x^{4219}+x^{4209}-x^{4208}+x^{4198}-x^{4197}+x^{4193}-x^{4192}+x^{4187}-x^{4186}+x^{4182}-$
$x^{4181}+x^{4176}-x^{4175}+x^{4171}-x^{4170}+x^{4165}-x^{4164}+x^{4160}-x^{4159}+x^{4154}-x^{4153}+x^{4149}-x^{4148}+$
$x^{4143}-x^{4142}+x^{4138}-x^{4137}+x^{4132}-x^{4131}+x^{4127}-x^{4126}+x^{4121}-x^{4120}+x^{4116}-x^{4115}+x^{4110}-$
$x^{4109}+x^{4105}-x^{4104}+x^{4099}-x^{4098}+x^{4094}-x^{4093}+x^{4088}-x^{4087}+x^{4083}-x^{4082}+x^{4077}-x^{4076}+$
$x^{4072}-x^{4071}+x^{4066}-x^{4065}+x^{4061}-x^{4060}+x^{4055}-x^{4054}+x^{4050}-x^{4049}+x^{4044}-x^{4043}+x^{4039}-$

$$x^{4038}+x^{4033}-x^{4032}+x^{4028}-x^{4027}+x^{4022}-x^{4021}+x^{4017}-x^{4016}+x^{4011}-x^{4010}+x^{4006}-x^{4005}+$$
$$x^{4000}-x^{3999}+x^{3995}-x^{3994}+x^{3989}-x^{3988}+x^{3984}-x^{3983}+x^{3978}-x^{3977}+x^{3973}-x^{3972}+x^{3967}-$$
$$x^{3966}+x^{3962}-x^{3961}+x^{3956}-x^{3955}+x^{3951}-x^{3950}+x^{3945}-x^{3944}+x^{3940}-x^{3939}+x^{3934}-x^{3933}+$$
$$x^{3929}-x^{3928}+x^{3923}-x^{3922}+x^{3918}-x^{3917}+x^{3912}-x^{3911}+x^{3907}-x^{3906}+x^{3901}-x^{3900}+x^{3896}-$$
$$x^{3895}+x^{3890}-x^{3889}+x^{3885}-x^{3884}+x^{3879}-x^{3878}+x^{3874}-x^{3873}+x^{3868}-x^{3867}+x^{3863}-x^{3862}+$$
$$x^{3857}-x^{3856}+x^{3852}-x^{3851}+x^{3846}-x^{3845}+x^{3841}-x^{3840}+x^{3835}-x^{3834}+x^{3830}-x^{3829}+x^{3824}-$$
$$x^{3823}+x^{3819}-x^{3818}+x^{3813}-x^{3812}+x^{3808}-x^{3807}+x^{3802}-x^{3801}+x^{3797}-x^{3796}+x^{3791}-x^{3790}+$$
$$x^{3786}-x^{3785}+x^{3780}-x^{3779}+x^{3775}-x^{3774}+x^{3769}-x^{3768}+x^{3764}-x^{3763}+x^{3758}-x^{3757}+x^{3753}-$$
$$x^{3752}+x^{3747}-x^{3746}+x^{3742}-x^{3741}+x^{3736}-x^{3735}+x^{3731}-x^{3730}+x^{3726}-x^{3724}+x^{3720}-x^{3719}+$$
$$x^{3715}-x^{3713}+x^{3709}-x^{3708}+x^{3704}-x^{3702}+x^{3698}-x^{3697}+x^{3693}-x^{3691}+x^{3687}-x^{3686}+x^{3682}-$$
$$x^{3680}+x^{3676}-x^{3675}+x^{3671}-x^{3669}+x^{3665}-x^{3664}+x^{3660}-x^{3658}+x^{3654}-x^{3653}+x^{3649}-x^{3647}+$$
$$x^{3643}-x^{3642}+x^{3638}-x^{3636}+x^{3632}-x^{3631}+x^{3627}-x^{3625}+x^{3621}-x^{3620}+x^{3616}-x^{3614}+x^{3610}-$$
$$x^{3609}+x^{3605}-x^{3603}+x^{3599}-x^{3598}+x^{3594}-x^{3592}+x^{3588}-x^{3587}+x^{3583}-x^{3581}+x^{3577}-x^{3576}+$$
$$x^{3572}-x^{3570}+x^{3566}-x^{3565}+x^{3561}-x^{3559}+x^{3555}-x^{3554}+x^{3550}-x^{3548}+x^{3544}-x^{3543}+x^{3539}-$$
$$x^{3537}+x^{3533}-x^{3532}+x^{3528}-x^{3526}+x^{3522}-x^{3521}+x^{3517}-x^{3515}+x^{3511}-x^{3510}+x^{3506}-x^{3504}+$$
$$x^{3500}-x^{3499}+x^{3495}-x^{3493}+x^{3489}-x^{3488}+x^{3484}-x^{3482}+x^{3478}-x^{3477}+x^{3473}-x^{3471}+x^{3467}-$$
$$x^{3466}+x^{3462}-x^{3460}+x^{3456}-x^{3455}+x^{3451}-x^{3449}+x^{3445}-x^{3444}+x^{3440}-x^{3438}+x^{3434}-x^{3433}+$$
$$x^{3429}-x^{3427}+x^{3423}-x^{3422}+x^{3418}-x^{3416}+x^{3412}-x^{3411}+x^{3407}-x^{3405}+x^{3401}-x^{3400}+x^{3396}-$$
$$x^{3394}+x^{3390}-x^{3389}+x^{3385}-x^{3383}+x^{3379}-x^{3378}+x^{3374}-x^{3372}+x^{3368}-x^{3367}+x^{3363}-x^{3361}+$$
$$x^{3357}-x^{3356}+x^{3352}-x^{3350}+x^{3346}-x^{3345}+x^{3341}-x^{3339}+x^{3335}-x^{3334}+x^{3330}-x^{3328}+x^{3324}-$$
$$x^{3323}+x^{3319}-x^{3317}+x^{3313}-x^{3312}+x^{3308}-x^{3306}+x^{3302}-x^{3301}+x^{3297}-x^{3295}+x^{3291}-x^{3290}+$$
$$x^{3286}-x^{3284}+x^{3280}-x^{3279}+x^{3275}-x^{3273}+x^{3269}-x^{3268}+x^{3264}-x^{3262}+x^{3259}-x^{3257}+x^{3253}-$$
$$x^{3251}+x^{3248}-x^{3246}+x^{3242}-x^{3240}+x^{3237}-x^{3235}+x^{3231}-x^{3229}+x^{3226}-x^{3224}+x^{3220}-x^{3218}+$$
$$x^{3215}-x^{3213}+x^{3209}-x^{3207}+x^{3204}-x^{3202}+x^{3198}-x^{3196}+x^{3193}-x^{3191}+x^{3187}-x^{3185}+x^{3182}-$$
$$x^{3180}+x^{3176}-x^{3174}+x^{3171}-x^{3169}+x^{3165}-x^{3163}+x^{3160}-x^{3158}+x^{3154}-x^{3152}+x^{3149}-x^{3147}+$$
$$x^{3143}-x^{3141}+x^{3138}-x^{3136}+x^{3132}-x^{3130}+x^{3127}-x^{3125}+x^{3121}-x^{3119}+x^{3116}-x^{3114}+x^{3110}-$$
$$x^{3108}+x^{3105}-x^{3103}+x^{3099}-x^{3097}+x^{3094}-x^{3092}+x^{3088}-x^{3086}+x^{3083}-x^{3081}+x^{3077}-x^{3075}+$$
$$x^{3072}-x^{3070}+x^{3066}-x^{3064}+x^{3061}-x^{3059}+x^{3055}-x^{3053}+x^{3050}-x^{3048}+x^{3044}-x^{3042}+x^{3039}-$$
$$x^{3037}+x^{3033}-x^{3031}+x^{3028}-x^{3026}+x^{3022}-x^{3020}+x^{3017}-x^{3015}+x^{3011}-x^{3009}+x^{3006}-x^{3004}+$$
$$x^{3000}-x^{2998}+x^{2995}-x^{2993}+x^{2989}-x^{2987}+x^{2984}-x^{2982}+x^{2978}-x^{2976}+x^{2973}-x^{2971}+x^{2967}-$$
$$x^{2965}+x^{2962}-x^{2960}+x^{2956}-x^{2954}+x^{2951}-x^{2949}+x^{2945}-x^{2943}+x^{2940}-x^{2938}+x^{2934}-x^{2932}+$$
$$x^{2929}-x^{2927}+x^{2923}-x^{2921}+x^{2918}-x^{2916}+x^{2912}-x^{2910}+x^{2907}-x^{2905}+x^{2901}-x^{2899}+x^{2896}-$$
$$x^{2894}+x^{2890}-x^{2888}+x^{2885}-x^{2883}+x^{2879}-x^{2877}+x^{2874}-x^{2872}+x^{2868}-x^{2866}+x^{2863}-x^{2861}+$$
$$x^{2857}-x^{2855}+x^{2852}-x^{2850}+x^{2846}-x^{2844}+x^{2841}-x^{2839}+x^{2835}-x^{2833}+x^{2830}-x^{2828}+x^{2824}-$$
$$x^{2822}+x^{2819}-x^{2817}+x^{2813}-x^{2811}+x^{2808}-x^{2806}+x^{2802}-x^{2800}+x^{2797}-x^{2795}+x^{2792}-x^{2789}+$$
$$x^{2786}-x^{2784}+x^{2781}-x^{2778}+x^{2775}-x^{2773}+x^{2770}-x^{2767}+x^{2764}-x^{2762}+x^{2759}-x^{2756}+x^{2753}-$$
$$x^{2751}+x^{2748}-x^{2745}+x^{2742}-x^{2740}+x^{2737}-x^{2734}+x^{2731}-x^{2729}+x^{2726}-x^{2723}+x^{2720}-x^{2718}+$$
$$x^{2715}-x^{2712}+x^{2709}-x^{2707}+x^{2704}-x^{2701}+x^{2698}-x^{2696}+x^{2693}-x^{2690}+x^{2687}-x^{2685}+x^{2682}-$$

$$x^{2679}+x^{2676}-x^{2674}+x^{2671}-x^{2668}+x^{2665}-x^{2663}+x^{2660}-x^{2657}+x^{2654}-x^{2652}+x^{2649}-x^{2646}+$$
$$x^{2643}-x^{2641}+x^{2638}-x^{2635}+x^{2632}-x^{2630}+x^{2627}-x^{2624}+x^{2621}-x^{2619}+x^{2616}-x^{2613}+x^{2610}-$$
$$x^{2608}+x^{2605}-x^{2602}+x^{2599}-x^{2597}+x^{2594}-x^{2591}+x^{2588}-x^{2586}+x^{2583}-x^{2580}+x^{2577}-x^{2575}+$$
$$x^{2572}-x^{2569}+x^{2566}-x^{2564}+x^{2561}-x^{2558}+x^{2555}-x^{2553}+x^{2550}-x^{2547}+x^{2544}-x^{2542}+x^{2539}-$$
$$x^{2536}+x^{2533}-x^{2531}+x^{2528}-x^{2525}+x^{2522}-x^{2520}+x^{2517}-x^{2514}+x^{2511}-x^{2509}+x^{2506}-x^{2503}+$$
$$x^{2500}-x^{2498}+x^{2495}-x^{2492}+x^{2489}-x^{2487}+x^{2484}-x^{2481}+x^{2478}-x^{2476}+x^{2473}-x^{2470}+x^{2467}-$$
$$x^{2465}+x^{2462}-x^{2459}+x^{2456}-x^{2454}+x^{2451}-x^{2448}+x^{2445}-x^{2443}+x^{2440}-x^{2437}+x^{2434}-x^{2432}+$$
$$x^{2429}-x^{2426}+x^{2423}-x^{2421}+x^{2418}-x^{2415}+x^{2412}-x^{2410}+x^{2407}-x^{2404}+x^{2401}-x^{2399}+x^{2396}-$$
$$x^{2393}+x^{2390}-x^{2388}+x^{2385}-x^{2382}+x^{2379}-x^{2377}+x^{2374}-x^{2371}+x^{2368}-x^{2366}+x^{2363}-x^{2360}+$$
$$x^{2357}-x^{2355}+x^{2352}-x^{2349}+x^{2346}-x^{2344}+x^{2341}-x^{2338}+x^{2335}-x^{2333}+x^{2330}-x^{2327}+x^{2325}-$$
$$x^{2322}+x^{2319}-x^{2316}+x^{2314}-x^{2311}+x^{2308}-x^{2305}+x^{2303}-x^{2300}+x^{2297}-x^{2294}+x^{2292}-x^{2289}+$$
$$x^{2286}-x^{2283}+x^{2281}-x^{2278}+x^{2275}-x^{2272}+x^{2270}-x^{2267}+x^{2264}-x^{2261}+x^{2259}-x^{2256}+x^{2253}-$$
$$x^{2250}+x^{2248}-x^{2245}+x^{2242}-x^{2239}+x^{2237}-x^{2234}+x^{2231}-x^{2228}+x^{2226}-x^{2223}+x^{2220}-x^{2217}+$$
$$x^{2215}-x^{2212}+x^{2209}-x^{2206}+x^{2204}-x^{2201}+x^{2198}-x^{2195}+x^{2193}-x^{2190}+x^{2187}-x^{2184}+x^{2182}-$$
$$x^{2179}+x^{2176}-x^{2173}+x^{2171}-x^{2168}+x^{2165}-x^{2162}+x^{2160}-x^{2157}+x^{2154}-x^{2151}+x^{2149}-x^{2146}+$$
$$x^{2143}-x^{2140}+x^{2138}-x^{2135}+x^{2132}-x^{2129}+x^{2127}-x^{2124}+x^{2121}-x^{2118}+x^{2116}-x^{2113}+x^{2110}-$$
$$x^{2107}+x^{2105}-x^{2102}+x^{2099}-x^{2096}+x^{2094}-x^{2091}+x^{2088}-x^{2085}+x^{2083}-x^{2080}+x^{2077}-x^{2074}+$$
$$x^{2072}-x^{2069}+x^{2066}-x^{2063}+x^{2061}-x^{2058}+x^{2055}-x^{2052}+x^{2050}-x^{2047}+x^{2044}-x^{2041}+x^{2039}-$$
$$x^{2036}+x^{2033}-x^{2030}+x^{2028}-x^{2025}+x^{2022}-x^{2019}+x^{2017}-x^{2014}+x^{2011}-x^{2008}+x^{2006}-x^{2003}+$$
$$x^{2000}-x^{1997}+x^{1995}-x^{1992}+x^{1989}-x^{1986}+x^{1984}-x^{1981}+x^{1978}-x^{1975}+x^{1973}-x^{1970}+x^{1967}-$$
$$x^{1964}+x^{1962}-x^{1959}+x^{1956}-x^{1953}+x^{1951}-x^{1948}+x^{1945}-x^{1942}+x^{1940}-x^{1937}+x^{1934}-x^{1931}+$$
$$x^{1929}-x^{1926}+x^{1923}-x^{1920}+x^{1918}-x^{1915}+x^{1912}-x^{1909}+x^{1907}-x^{1904}+x^{1901}-x^{1898}+x^{1896}-$$
$$x^{1893}+x^{1890}-x^{1887}+x^{1885}-x^{1882}+x^{1879}-x^{1876}+x^{1874}-x^{1871}+x^{1868}-x^{1865}+x^{1863}-x^{1860}+$$
$$x^{1858}-x^{1854}+x^{1852}-x^{1849}+x^{1847}-x^{1843}+x^{1841}-x^{1838}+x^{1836}-x^{1832}+x^{1830}-x^{1827}+x^{1825}-$$
$$x^{1821}+x^{1819}-x^{1816}+x^{1814}-x^{1810}+x^{1808}-x^{1805}+x^{1803}-x^{1799}+x^{1797}-x^{1794}+x^{1792}-x^{1788}+$$
$$x^{1786}-x^{1783}+x^{1781}-x^{1777}+x^{1775}-x^{1772}+x^{1770}-x^{1766}+x^{1764}-x^{1761}+x^{1759}-x^{1755}+x^{1753}-$$
$$x^{1750}+x^{1748}-x^{1744}+x^{1742}-x^{1739}+x^{1737}-x^{1733}+x^{1731}-x^{1728}+x^{1726}-x^{1722}+x^{1720}-x^{1717}+$$
$$x^{1715}-x^{1711}+x^{1709}-x^{1706}+x^{1704}-x^{1700}+x^{1698}-x^{1695}+x^{1693}-x^{1689}+x^{1687}-x^{1684}+x^{1682}-$$
$$x^{1678}+x^{1676}-x^{1673}+x^{1671}-x^{1667}+x^{1665}-x^{1662}+x^{1660}-x^{1656}+x^{1654}-x^{1651}+x^{1649}-x^{1645}+$$
$$x^{1643}-x^{1640}+x^{1638}-x^{1634}+x^{1632}-x^{1629}+x^{1627}-x^{1623}+x^{1621}-x^{1618}+x^{1616}-x^{1612}+x^{1610}-$$
$$x^{1607}+x^{1605}-x^{1601}+x^{1599}-x^{1596}+x^{1594}-x^{1590}+x^{1588}-x^{1585}+x^{1583}-x^{1579}+x^{1577}-x^{1574}+$$
$$x^{1572}-x^{1568}+x^{1566}-x^{1563}+x^{1561}-x^{1557}+x^{1555}-x^{1552}+x^{1550}-x^{1546}+x^{1544}-x^{1541}+x^{1539}-$$
$$x^{1535}+x^{1533}-x^{1530}+x^{1528}-x^{1524}+x^{1522}-x^{1519}+x^{1517}-x^{1513}+x^{1511}-x^{1508}+x^{1506}-x^{1502}+$$
$$x^{1500}-x^{1497}+x^{1495}-x^{1491}+x^{1489}-x^{1486}+x^{1484}-x^{1480}+x^{1478}-x^{1475}+x^{1473}-x^{1469}+x^{1467}-$$
$$x^{1464}+x^{1462}-x^{1458}+x^{1456}-x^{1453}+x^{1451}-x^{1447}+x^{1445}-x^{1442}+x^{1440}-x^{1436}+x^{1434}-x^{1431}+$$
$$x^{1429}-x^{1425}+x^{1423}-x^{1420}+x^{1418}-x^{1414}+x^{1412}-x^{1409}+x^{1407}-x^{1403}+x^{1401}-x^{1398}+x^{1396}-$$
$$x^{1392}+x^{1391}-x^{1387}+x^{1385}-x^{1381}+x^{1380}-x^{1376}+x^{1374}-x^{1370}+x^{1369}-x^{1365}+x^{1363}-x^{1359}+$$
$$x^{1358}-x^{1354}+x^{1352}-x^{1348}+x^{1347}-x^{1343}+x^{1341}-x^{1337}+x^{1336}-x^{1332}+x^{1330}-x^{1326}+x^{1325}-$$

$$x^{1321}+x^{1319}-x^{1315}+x^{1314}-x^{1310}+x^{1308}-x^{1304}+x^{1303}-x^{1299}+x^{1297}-x^{1293}+x^{1292}-x^{1288}+$$
$$x^{1286}-x^{1282}+x^{1281}-x^{1277}+x^{1275}-x^{1271}+x^{1270}-x^{1266}+x^{1264}-x^{1260}+x^{1259}-x^{1255}+x^{1253}-$$
$$x^{1249}+x^{1248}-x^{1244}+x^{1242}-x^{1238}+x^{1237}-x^{1233}+x^{1231}-x^{1227}+x^{1226}-x^{1222}+x^{1220}-x^{1216}+$$
$$x^{1215}-x^{1211}+x^{1209}-x^{1205}+x^{1204}-x^{1200}+x^{1198}-x^{1194}+x^{1193}-x^{1189}+x^{1187}-x^{1183}+x^{1182}-$$
$$x^{1178}+x^{1176}-x^{1172}+x^{1171}-x^{1167}+x^{1165}-x^{1161}+x^{1160}-x^{1156}+x^{1154}-x^{1150}+x^{1149}-x^{1145}+$$
$$x^{1143}-x^{1139}+x^{1138}-x^{1134}+x^{1132}-x^{1128}+x^{1127}-x^{1123}+x^{1121}-x^{1117}+x^{1116}-x^{1112}+x^{1110}-$$
$$x^{1106}+x^{1105}-x^{1101}+x^{1099}-x^{1095}+x^{1094}-x^{1090}+x^{1088}-x^{1084}+x^{1083}-x^{1079}+x^{1077}-x^{1073}+$$
$$x^{1072}-x^{1068}+x^{1066}-x^{1062}+x^{1061}-x^{1057}+x^{1055}-x^{1051}+x^{1050}-x^{1046}+x^{1044}-x^{1040}+x^{1039}-$$
$$x^{1035}+x^{1033}-x^{1029}+x^{1028}-x^{1024}+x^{1022}-x^{1018}+x^{1017}-x^{1013}+x^{1011}-x^{1007}+x^{1006}-x^{1002}+$$
$$x^{1000}-x^{996}+x^{995}-x^{991}+x^{989}-x^{985}+x^{984}-x^{980}+x^{978}-x^{974}+x^{973}-x^{969}+x^{967}-x^{963}+$$
$$x^{962}-x^{958}+x^{956}-x^{952}+x^{951}-x^{947}+x^{945}-x^{941}+x^{940}-x^{936}+x^{934}-x^{930}+x^{929}-x^{925}+$$
$$x^{924}-x^{919}+x^{918}-x^{914}+x^{913}-x^{908}+x^{907}-x^{903}+x^{902}-x^{897}+x^{896}-x^{892}+x^{891}-x^{886}+$$
$$x^{885}-x^{881}+x^{880}-x^{875}+x^{874}-x^{870}+x^{869}-x^{864}+x^{863}-x^{859}+x^{858}-x^{853}+x^{852}-x^{848}+$$
$$x^{847}-x^{842}+x^{841}-x^{837}+x^{836}-x^{831}+x^{830}-x^{826}+x^{825}-x^{820}+x^{819}-x^{815}+x^{814}-x^{809}+$$
$$x^{808}-x^{804}+x^{803}-x^{798}+x^{797}-x^{793}+x^{792}-x^{787}+x^{786}-x^{782}+x^{781}-x^{776}+x^{775}-x^{771}+$$
$$x^{770}-x^{765}+x^{764}-x^{760}+x^{759}-x^{754}+x^{753}-x^{749}+x^{748}-x^{743}+x^{742}-x^{738}+x^{737}-x^{732}+$$
$$x^{731}-x^{727}+x^{726}-x^{721}+x^{720}-x^{716}+x^{715}-x^{710}+x^{709}-x^{705}+x^{704}-x^{699}+x^{698}-x^{694}+x^{693}-$$
$$x^{688}+x^{687}-x^{683}+x^{682}-x^{677}+x^{676}-x^{672}+x^{671}-x^{666}+x^{665}-x^{661}+x^{660}-x^{655}+x^{654}-x^{650}+$$
$$x^{649}-x^{644}+x^{643}-x^{639}+x^{638}-x^{633}+x^{632}-x^{628}+x^{627}-x^{622}+x^{621}-x^{617}+x^{616}-x^{611}+x^{610}-$$
$$x^{606}+x^{605}-x^{600}+x^{599}-x^{595}+x^{594}-x^{589}+x^{588}-x^{584}+x^{583}-x^{578}+x^{577}-x^{573}+x^{572}-x^{567}+$$
$$x^{566}-x^{562}+x^{561}-x^{556}+x^{555}-x^{551}+x^{550}-x^{545}+x^{544}-x^{540}+x^{539}-x^{534}+x^{533}-x^{529}+x^{528}-$$
$$x^{523}+x^{522}-x^{518}+x^{517}-x^{512}+x^{511}-x^{507}+x^{506}-x^{501}+x^{500}-x^{496}+x^{495}-x^{490}+x^{489}-x^{485}+$$
$$x^{484}-x^{479}+x^{478}-x^{474}+x^{473}-x^{468}+x^{467}-x^{463}+x^{462}-x^{452}+x^{451}-x^{441}+x^{440}-x^{430}+x^{429}-$$
$$x^{419}+x^{418}-x^{408}+x^{407}-x^{397}+x^{396}-x^{386}+x^{385}-x^{375}+x^{374}-x^{364}+x^{363}-x^{353}+x^{352}-x^{342}+$$
$$x^{341}-x^{331}+x^{330}-x^{320}+x^{319}-x^{309}+x^{308}-x^{298}+x^{297}-x^{287}+x^{286}-x^{276}+x^{275}-x^{265}+x^{264}-$$
$$x^{254}+x^{253}-x^{243}+x^{242}-x^{232}+x^{231}-x^{221}+x^{220}-x^{210}+x^{209}-x^{199}+x^{198}-x^{188}+x^{187}-x^{177}+$$
$$x^{176}-x^{166}+x^{165}-x^{155}+x^{154}-x^{144}+x^{143}-x^{133}+x^{132}-x^{122}+x^{121}-x^{111}+x^{110}-x^{100}+x^{99}-$$
$$x^{89}+x^{88}-x^{78}+x^{77}-x^{67}+x^{66}-x^{56}+x^{55}-x^{45}+x^{44}-x^{34}+x^{33}-x^{23}+x^{22}-x^{12}+x^{11}-x+1.$$

We shall also define what a *prime divisor* of a given polynomial is.

Definition 2. Let $A(x) \in \mathbb{Z}[x]$ be a polynomial. We say that a prime number p is a *prime divisor* of A (or simply that p *divides* A) if there exists $m \in \mathbb{Z}$ such that p divides $A(m)$.

1 The main Theorem

We are now able to show that there exist infinitely many primes $\equiv 1 \pmod{5137}$. For this purpose, consider the polynomial

$$\Phi_{5137}(x) = x^{4660} - x^{4659} + x^{4649} - x^{4648} + x^{4638} - x^{4637} + x^{4627} - x^{4626} + x^{4616} - x^{4615} +$$
$$x^{4605} - x^{4604} + x^{4594} - x^{4593} + x^{4583} - x^{4582} + x^{4572} - x^{4571} + x^{4561} - x^{4560} + x^{4550} - x^{4549} +$$
$$x^{4539} - x^{4538} + x^{4528} - x^{4527} + x^{4517} - x^{4516} + x^{4506} - x^{4505} + x^{4495} - x^{4494} + x^{4484} - x^{4483} +$$
$$x^{4473} - x^{4472} + x^{4462} - x^{4461} + x^{4451} - x^{4450} + x^{4440} - x^{4439} + x^{4429} - x^{4428} + x^{4418} - x^{4417} +$$
$$x^{4407} - x^{4406} + x^{4396} - x^{4395} + x^{4385} - x^{4384} + x^{4374} - x^{4373} + x^{4363} - x^{4362} + x^{4352} - x^{4351} +$$
$$x^{4341} - x^{4340} + x^{4330} - x^{4329} + x^{4319} - x^{4318} + x^{4308} - x^{4307} + x^{4297} - x^{4296} + x^{4286} - x^{4285} +$$
$$x^{4275} - x^{4274} + x^{4264} - x^{4263} + x^{4253} - x^{4252} + x^{4242} - x^{4241} + x^{4231} - x^{4230} + x^{4220} - x^{4219} +$$
$$x^{4209} - x^{4208} + x^{4198} - x^{4197} + x^{4193} - x^{4192} + x^{4187} - x^{4186} + x^{4182} - x^{4181} + x^{4176} - x^{4175} +$$
$$x^{4171} - x^{4170} + x^{4165} - x^{4164} + x^{4160} - x^{4159} + x^{4154} - x^{4153} + x^{4149} - x^{4148} + x^{4143} - x^{4142} +$$
$$x^{4138} - x^{4137} + x^{4132} - x^{4131} + x^{4127} - x^{4126} + x^{4121} - x^{4120} + x^{4116} - x^{4115} + x^{4110} - x^{4109} +$$
$$x^{4105} - x^{4104} + x^{4099} - x^{4098} + x^{4094} - x^{4093} + x^{4088} - x^{4087} + x^{4083} - x^{4082} + x^{4077} - x^{4076} +$$
$$x^{4072} - x^{4071} + x^{4066} - x^{4065} + x^{4061} - x^{4060} + x^{4055} - x^{4054} + x^{4050} - x^{4049} + x^{4044} - x^{4043} +$$
$$x^{4039} - x^{4038} + x^{4033} - x^{4032} + x^{4028} - x^{4027} + x^{4022} - x^{4021} + x^{4017} - x^{4016} + x^{4011} - x^{4010} +$$
$$x^{4006} - x^{4005} + x^{4000} - x^{3999} + x^{3995} - x^{3994} + x^{3989} - x^{3988} + x^{3984} - x^{3983} + x^{3978} - x^{3977} +$$
$$x^{3973} - x^{3972} + x^{3967} - x^{3966} + x^{3962} - x^{3961} + x^{3956} - x^{3955} + x^{3951} - x^{3950} + x^{3945} - x^{3944} +$$
$$x^{3940} - x^{3939} + x^{3934} - x^{3933} + x^{3929} - x^{3928} + x^{3923} - x^{3922} + x^{3918} - x^{3917} + x^{3912} - x^{3911} +$$
$$x^{3907} - x^{3906} + x^{3901} - x^{3900} + x^{3896} - x^{3895} + x^{3890} - x^{3889} + x^{3885} - x^{3884} + x^{3879} - x^{3878} +$$
$$x^{3874} - x^{3873} + x^{3868} - x^{3867} + x^{3863} - x^{3862} + x^{3857} - x^{3856} + x^{3852} - x^{3851} + x^{3846} - x^{3845} +$$
$$x^{3841} - x^{3840} + x^{3835} - x^{3834} + x^{3830} - x^{3829} + x^{3824} - x^{3823} + x^{3819} - x^{3818} + x^{3813} - x^{3812} +$$
$$x^{3808} - x^{3807} + x^{3802} - x^{3801} + x^{3797} - x^{3796} + x^{3791} - x^{3790} + x^{3786} - x^{3785} + x^{3780} - x^{3779} +$$
$$x^{3775} - x^{3774} + x^{3769} - x^{3768} + x^{3764} - x^{3763} + x^{3758} - x^{3757} + x^{3753} - x^{3752} + x^{3747} - x^{3746} +$$
$$x^{3742} - x^{3741} + x^{3736} - x^{3735} + x^{3731} - x^{3730} + x^{3726} - x^{3724} + x^{3720} - x^{3719} + x^{3715} - x^{3713} +$$
$$x^{3709} - x^{3708} + x^{3704} - x^{3702} + x^{3698} - x^{3697} + x^{3693} - x^{3691} + x^{3687} - x^{3686} + x^{3682} - x^{3680} +$$
$$x^{3676} - x^{3675} + x^{3671} - x^{3669} + x^{3665} - x^{3664} + x^{3660} - x^{3658} + x^{3654} - x^{3653} + x^{3649} - x^{3647} +$$
$$x^{3643} - x^{3642} + x^{3638} - x^{3636} + x^{3632} - x^{3631} + x^{3627} - x^{3625} + x^{3621} - x^{3620} + x^{3616} - x^{3614} +$$
$$x^{3610} - x^{3609} + x^{3605} - x^{3603} + x^{3599} - x^{3598} + x^{3594} - x^{3592} + x^{3588} - x^{3587} + x^{3583} - x^{3581} +$$
$$x^{3577} - x^{3576} + x^{3572} - x^{3570} + x^{3566} - x^{3565} + x^{3561} - x^{3559} + x^{3555} - x^{3554} + x^{3550} - x^{3548} +$$
$$x^{3544} - x^{3543} + x^{3539} - x^{3537} + x^{3533} - x^{3532} + x^{3528} - x^{3526} + x^{3522} - x^{3521} + x^{3517} - x^{3515} +$$
$$x^{3511} - x^{3510} + x^{3506} - x^{3504} + x^{3500} - x^{3499} + x^{3495} - x^{3493} + x^{3489} - x^{3488} + x^{3484} - x^{3482} +$$
$$x^{3478} - x^{3477} + x^{3473} - x^{3471} + x^{3467} - x^{3466} + x^{3462} - x^{3460} + x^{3456} - x^{3455} + x^{3451} - x^{3449} +$$
$$x^{3445} - x^{3444} + x^{3440} - x^{3438} + x^{3434} - x^{3433} + x^{3429} - x^{3427} + x^{3423} - x^{3422} + x^{3418} - x^{3416} +$$
$$x^{3412} - x^{3411} + x^{3407} - x^{3405} + x^{3401} - x^{3400} + x^{3396} - x^{3394} + x^{3390} - x^{3389} + x^{3385} - x^{3383} +$$
$$x^{3379} - x^{3378} + x^{3374} - x^{3372} + x^{3368} - x^{3367} + x^{3363} - x^{3361} + x^{3357} - x^{3356} + x^{3352} - x^{3350} +$$
$$x^{3346} - x^{3345} + x^{3341} - x^{3339} + x^{3335} - x^{3334} + x^{3330} - x^{3328} + x^{3324} - x^{3323} + x^{3319} - x^{3317} +$$
$$x^{3313} - x^{3312} + x^{3308} - x^{3306} + x^{3302} - x^{3301} + x^{3297} - x^{3295} + x^{3291} - x^{3290} + x^{3286} - x^{3284} +$$
$$x^{3280} - x^{3279} + x^{3275} - x^{3273} + x^{3269} - x^{3268} + x^{3264} - x^{3262} + x^{3259} - x^{3257} + x^{3253} - x^{3251} +$$
$$x^{3248} - x^{3246} + x^{3242} - x^{3240} + x^{3237} - x^{3235} + x^{3231} - x^{3229} + x^{3226} - x^{3224} + x^{3220} - x^{3218} +$$
$$x^{3215} - x^{3213} + x^{3209} - x^{3207} + x^{3204} - x^{3202} + x^{3198} - x^{3196} + x^{3193} - x^{3191} + x^{3187} - x^{3185} +$$
$$x^{3182} - x^{3180} + x^{3176} - x^{3174} + x^{3171} - x^{3169} + x^{3165} - x^{3163} + x^{3160} - x^{3158} + x^{3154} - x^{3152} +$$
$$x^{3149} - x^{3147} + x^{3143} - x^{3141} + x^{3138} - x^{3136} + x^{3132} - x^{3130} + x^{3127} - x^{3125} + x^{3121} - x^{3119} +$$
$$x^{3116} - x^{3114} + x^{3110} - x^{3108} + x^{3105} - x^{3103} + x^{3099} - x^{3097} + x^{3094} - x^{3092} + x^{3088} - x^{3086} +$$
$$x^{3083} - x^{3081} + x^{3077} - x^{3075} + x^{3072} - x^{3070} + x^{3066} - x^{3064} + x^{3061} - x^{3059} + x^{3055} - x^{3053} +$$
$$x^{3050} - x^{3048} + x^{3044} - x^{3042} + x^{3039} - x^{3037} + x^{3033} - x^{3031} + x^{3028} - x^{3026} + x^{3022} - x^{3020} +$$

We will specifically show that every prime divisor p of $\Phi_{5137}(x)$ either belongs to the finite set

$$T = \{11, 467\}$$

or satisfies $p \equiv 1 \pmod{5137}$. To see this, consider the set $S := \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10,$ 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465, 466, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 738, 739, 740, 741, 742, 743,

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4569, 4570, 4571, 4572, 4573, 4574, 4575, 4577, 4578, 4579, 4580, 4581, 4582, 4583, 4584, 4585, 4586, 4588, 4589, 4590, 4591, 4592, 4593, 4594, 4595, 4596, 4597, 4599, 4600, 4601, 4602, 4603, 4604, 4605, 4606, 4607, 4608, 4610, 4611, 4612, 4613, 4614, 4615, 4616, 4617, 4618, 4619, 4621, 4622, 4623, 4624, 4625, 4626, 4627, 4628, 4629, 4630, 4632, 4633, 4634, 4635, 4636, 4637, 4638, 4639, 4640, 4641, 4643, 4644, 4645, 4646, 4647, 4648, 4649, 4650, 4651, 4652, 4654, 4655, 4656, 4657, 4658, 4659, 4660, 4661, 4662, 4663, 4665, 4666, 4667, 4668, 4669, 4671, 4672, 4673, 4674, 4676, 4677, 4678, 4679, 4680, 4681, 4682, 4683, 4684, 4685, 4687, 4688, 4689, 4690, 4691, 4692, 4693, 4694, 4695, 4696, 4698, 4699, 4700, 4701, 4702, 4703, 4704, 4705, 4706, 4707, 4709, 4710, 4711, 4712, 4713, 4714, 4715, 4716, 4717, 4718, 4720, 4721, 4722, 4723, 4724, 4725, 4726, 4727, 4728, 4729, 4731, 4732, 4733, 4734, 4735, 4736, 4737, 4738, 4739, 4740, 4742, 4743, 4744, 4745, 4746, 4747, 4748, 4749, 4750, 4751, 4753, 4754, 4755, 4756, 4757, 4758, 4759, 4760, 4761, 4762, 4764, 4765, 4766, 4767, 4768, 4769, 4770, 4771, 4772, 4773, 4775, 4776, 4777, 4778, 4779, 4780, 4781, 4782, 4783, 4784, 4786, 4787, 4788, 4789, 4790, 4791, 4792, 4793, 4794, 4795, 4797, 4798, 4799, 4800, 4801, 4802, 4803, 4804, 4805, 4806, 4808, 4809, 4810, 4811, 4812, 4813, 4814, 4815, 4816, 4817, 4819, 4820, 4821, 4822, 4823, 4824, 4825, 4826, 4827, 4828, 4830, 4831, 4832, 4833, 4834, 4835, 4836, 4837, 4838, 4839, 4841, 4842, 4843, 4844, 4845, 4846, 4847, 4848, 4849, 4850, 4852, 4853, 4854, 4855, 4856, 4857, 4858, 4859, 4860, 4861, 4863, 4864, 4865, 4866, 4867, 4868, 4869, 4870, 4871, 4872, 4874, 4875, 4876, 4877, 4878, 4879, 4880, 4881, 4882, 4883, 4885, 4886, 4887, 4888, 4889, 4890, 4891, 4892, 4893, 4894, 4896, 4897, 4898, 4899, 4900, 4901, 4902, 4903, 4904, 4905, 4907, 4908, 4909, 4910, 4911, 4912, 4913, 4914, 4915, 4916, 4918, 4919, 4920, 4921, 4922, 4923, 4924, 4925, 4926, 4927, 4929, 4930, 4931, 4932, 4933, 4934, 4935, 4936, 4937, 4938, 4940, 4941, 4942, 4943, 4944, 4945, 4946, 4947, 4948, 4949, 4951, 4952, 4953, 4954, 4955, 4956, 4957, 4958, 4959, 4960, 4962, 4963, 4964, 4965, 4966, 4967, 4968, 4969, 4970, 4971, 4973, 4974, 4975, 4976, 4977, 4978, 4979, 4980, 4981, 4982, 4984, 4985, 4986, 4987, 4988, 4989, 4990, 4991, 4992, 4993, 4995, 4996, 4997, 4998, 4999, 5000, 5001, 5002, 5003, 5004, 5006, 5007, 5008, 5009, 5010, 5011, 5012, 5013, 5014, 5015, 5017, 5018, 5019, 5020, 5021, 5022, 5023, 5024, 5025, 5026, 5028, 5029, 5030, 5031, 5032, 5033, 5034, 5035, 5036, 5037, 5039, 5040, 5041, 5042, 5043, 5044, 5045, 5046, 5047, 5048, 5050, 5051, 5052, 5053, 5054, 5055, 5056, 5057, 5058, 5059, 5061, 5062, 5063, 5064, 5065, 5066, 5067, 5068, 5069, 5070, 5072, 5073, 5074, 5075, 5076, 5077, 5078, 5079, 5080, 5081, 5083, 5084, 5085, 5086, 5087, 5088, 5089, 5090, 5091, 5092, 5094, 5095, 5096, 5097, 5098, 5099, 5100, 5101, 5102, 5103, 5105, 5106, 5107, 5108, 5109, 5110, 5111, 5112, 5113, 5114, 5116, 5117, 5118, 5119, 5120, 5121, 5122, 5123, 5124, 5125, 5127, 5128, 5129, 5130, 5131, 5132, 5133, 5134, 5135, 5136} and the values ζ^s, with $s \in S$ and $\zeta := e^{2\pi i/5137}$, a 5137th primitive root of unity (thus a root of $\Phi_{5137}(x)$). A simple calculation shows that

$\Phi_{5137}(x)$ can be written as

$$\Phi_{5137}(x) = \prod_{s\in S}\left(x-\zeta^s\right) = x^{4660} - x^{4659} + x^{4649} - x^{4648} + x^{4638} - x^{4637} + x^{4627} - x^{4626} +$$
$$x^{4616} - x^{4615} + x^{4605} - x^{4604} + x^{4594} - x^{4593} + x^{4583} - x^{4582} + x^{4572} - x^{4571} + x^{4561} - x^{4560} +$$
$$x^{4550} - x^{4549} + x^{4539} - x^{4538} + x^{4528} - x^{4527} + x^{4517} - x^{4516} + x^{4506} - x^{4505} + x^{4495} - x^{4494} +$$
$$x^{4484} - x^{4483} + x^{4473} - x^{4472} + x^{4462} - x^{4461} + x^{4451} - x^{4450} + x^{4440} - x^{4439} + x^{4429} - x^{4428} +$$
$$x^{4418} - x^{4417} + x^{4407} - x^{4406} + x^{4396} - x^{4395} + x^{4385} - x^{4384} + x^{4374} - x^{4373} + x^{4363} - x^{4362} +$$
$$x^{4352} - x^{4351} + x^{4341} - x^{4340} + x^{4330} - x^{4329} + x^{4319} - x^{4318} + x^{4308} - x^{4307} + x^{4297} - x^{4296} +$$
$$x^{4286} - x^{4285} + x^{4275} - x^{4274} + x^{4264} - x^{4263} + x^{4253} - x^{4252} + x^{4242} - x^{4241} + x^{4231} - x^{4230} +$$
$$x^{4220} - x^{4219} + x^{4209} - x^{4208} + x^{4198} - x^{4197} + x^{4193} - x^{4192} + x^{4187} - x^{4186} + x^{4182} - x^{4181} +$$
$$x^{4176} - x^{4175} + x^{4171} - x^{4170} + x^{4165} - x^{4164} + x^{4160} - x^{4159} + x^{4154} - x^{4153} + x^{4149} - x^{4148} +$$
$$x^{4143} - x^{4142} + x^{4138} - x^{4137} + x^{4132} - x^{4131} + x^{4127} - x^{4126} + x^{4121} - x^{4120} + x^{4116} - x^{4115} +$$
$$x^{4110} - x^{4109} + x^{4105} - x^{4104} + x^{4099} - x^{4098} + x^{4094} - x^{4093} + x^{4088} - x^{4087} + x^{4083} - x^{4082} +$$
$$x^{4077} - x^{4076} + x^{4072} - x^{4071} + x^{4066} - x^{4065} + x^{4061} - x^{4060} + x^{4055} - x^{4054} + x^{4050} - x^{4049} +$$
$$x^{4044} - x^{4043} + x^{4039} - x^{4038} + x^{4033} - x^{4032} + x^{4028} - x^{4027} + x^{4022} - x^{4021} + x^{4017} - x^{4016} +$$
$$x^{4011} - x^{4010} + x^{4006} - x^{4005} + x^{4000} - x^{3999} + x^{3995} - x^{3994} + x^{3989} - x^{3988} + x^{3984} - x^{3983} +$$
$$x^{3978} - x^{3977} + x^{3973} - x^{3972} + x^{3967} - x^{3966} + x^{3962} - x^{3961} + x^{3956} - x^{3955} + x^{3951} - x^{3950} +$$
$$x^{3945} - x^{3944} + x^{3940} - x^{3939} + x^{3934} - x^{3933} + x^{3929} - x^{3928} + x^{3923} - x^{3922} + x^{3918} - x^{3917} +$$
$$x^{3912} - x^{3911} + x^{3907} - x^{3906} + x^{3901} - x^{3900} + x^{3896} - x^{3895} + x^{3890} - x^{3889} + x^{3885} - x^{3884} +$$
$$x^{3879} - x^{3878} + x^{3874} - x^{3873} + x^{3868} - x^{3867} + x^{3863} - x^{3862} + x^{3857} - x^{3856} + x^{3852} - x^{3851} +$$
$$x^{3846} - x^{3845} + x^{3841} - x^{3840} + x^{3835} - x^{3834} + x^{3830} - x^{3829} + x^{3824} - x^{3823} + x^{3819} - x^{3818} +$$
$$x^{3813} - x^{3812} + x^{3808} - x^{3807} + x^{3802} - x^{3801} + x^{3797} - x^{3796} + x^{3791} - x^{3790} + x^{3786} - x^{3785} +$$
$$x^{3780} - x^{3779} + x^{3775} - x^{3774} + x^{3769} - x^{3768} + x^{3764} - x^{3763} + x^{3758} - x^{3757} + x^{3753} - x^{3752} +$$
$$x^{3747} - x^{3746} + x^{3742} - x^{3741} + x^{3736} - x^{3735} + x^{3731} - x^{3730} + x^{3726} - x^{3724} + x^{3720} - x^{3719} +$$
$$x^{3715} - x^{3713} + x^{3709} - x^{3708} + x^{3704} - x^{3702} + x^{3698} - x^{3697} + x^{3693} - x^{3691} + x^{3687} - x^{3686} +$$
$$x^{3682} - x^{3680} + x^{3676} - x^{3675} + x^{3671} - x^{3669} + x^{3665} - x^{3664} + x^{3660} - x^{3658} + x^{3654} - x^{3653} +$$
$$x^{3649} - x^{3647} + x^{3643} - x^{3642} + x^{3638} - x^{3636} + x^{3632} - x^{3631} + x^{3627} - x^{3625} + x^{3621} - x^{3620} +$$
$$x^{3616} - x^{3614} + x^{3610} - x^{3609} + x^{3605} - x^{3603} + x^{3599} - x^{3598} + x^{3594} - x^{3592} + x^{3588} - x^{3587} +$$
$$x^{3583} - x^{3581} + x^{3577} - x^{3576} + x^{3572} - x^{3570} + x^{3566} - x^{3565} + x^{3561} - x^{3559} + x^{3555} - x^{3554} +$$
$$x^{3550} - x^{3548} + x^{3544} - x^{3543} + x^{3539} - x^{3537} + x^{3533} - x^{3532} + x^{3528} - x^{3526} + x^{3522} - x^{3521} +$$
$$x^{3517} - x^{3515} + x^{3511} - x^{3510} + x^{3506} - x^{3504} + x^{3500} - x^{3499} + x^{3495} - x^{3493} + x^{3489} - x^{3488} +$$
$$x^{3484} - x^{3482} + x^{3478} - x^{3477} + x^{3473} - x^{3471} + x^{3467} - x^{3466} + x^{3462} - x^{3460} + x^{3456} - x^{3455} +$$
$$x^{3451} - x^{3449} + x^{3445} - x^{3444} + x^{3440} - x^{3438} + x^{3434} - x^{3433} + x^{3429} - x^{3427} + x^{3423} - x^{3422} +$$
$$x^{3418} - x^{3416} + x^{3412} - x^{3411} + x^{3407} - x^{3405} + x^{3401} - x^{3400} + x^{3396} - x^{3394} + x^{3390} - x^{3389} +$$
$$x^{3385} - x^{3383} + x^{3379} - x^{3378} + x^{3374} - x^{3372} + x^{3368} - x^{3367} + x^{3363} - x^{3361} + x^{3357} - x^{3356} +$$
$$x^{3352} - x^{3350} + x^{3346} - x^{3345} + x^{3341} - x^{3339} + x^{3335} - x^{3334} + x^{3330} - x^{3328} + x^{3324} - x^{3323} +$$
$$x^{3319} - x^{3317} + x^{3313} - x^{3312} + x^{3308} - x^{3306} + x^{3302} - x^{3301} + x^{3297} - x^{3295} + x^{3291} - x^{3290} +$$
$$x^{3286} - x^{3284} + x^{3280} - x^{3279} + x^{3275} - x^{3273} + x^{3269} - x^{3268} + x^{3264} - x^{3262} + x^{3259} - x^{3257} +$$
$$x^{3253} - x^{3251} + x^{3248} - x^{3246} + x^{3242} - x^{3240} + x^{3237} - x^{3235} + x^{3231} - x^{3229} + x^{3226} - x^{3224} +$$
$$x^{3220} - x^{3218} + x^{3215} - x^{3213} + x^{3209} - x^{3207} + x^{3204} - x^{3202} + x^{3198} - x^{3196} + x^{3193} - x^{3191} +$$
$$x^{3187} - x^{3185} + x^{3182} - x^{3180} + x^{3176} - x^{3174} + x^{3171} - x^{3169} + x^{3165} - x^{3163} + x^{3160} - x^{3158} +$$
$$x^{3154} - x^{3152} + x^{3149} - x^{3147} + x^{3143} - x^{3141} + x^{3138} - x^{3136} + x^{3132} - x^{3130} + x^{3127} - x^{3125} +$$
$$x^{3121} - x^{3119} + x^{3116} - x^{3114} + x^{3110} - x^{3108} + x^{3105} - x^{3103} + x^{3099} - x^{3097} + x^{3094} - x^{3092} +$$
$$x^{3088} - x^{3086} + x^{3083} - x^{3081} + x^{3077} - x^{3075} + x^{3072} - x^{3070} + x^{3066} - x^{3064} + x^{3061} - x^{3059} +$$
$$x^{3055} - x^{3053} + x^{3050} - x^{3048} + x^{3044} - x^{3042} + x^{3039} - x^{3037} + x^{3033} - x^{3031} + x^{3028} - x^{3026} +$$
$$x^{3022} - x^{3020} + x^{3017} - x^{3015} + x^{3011} - x^{3009} + x^{3006} - x^{3004} + x^{3000} - x^{2998} + x^{2995} - x^{2993} +$$
$$x^{2989} - x^{2987} + x^{2984} - x^{2982} + x^{2978} - x^{2976} + x^{2973} - x^{2971} + x^{2967} - x^{2965} + x^{2962} - x^{2960} +$$

The discriminant of $\Phi_{5137}(x)$ can be calculated[1] to be $\Delta(\Phi_{5137}) = 11^{4194} \cdot 467^{4650}$.

Now, suppose that p is a prime divisor of Φ_{5137} such that $p \notin T$. Next, consider a field $\mathbb{F}$ containing both the finite field $\mathbb{F}_p$ and ζ[2]. Since p divides Φ_{5137}, working in $\mathbb{F}$, there exists $a \in \mathbb{Z}$ such that

$$\Phi_{5137}(a) = \prod_{s' \in S} \left(a - \zeta^{s'}\right) = 0.$$

Since $\mathbb{F}$ is a field, there exists some $s \in S$ such that $a = \zeta^s$.

Lemma 3. *The equality $\zeta^s = \zeta^{ps}$ holds in $\mathbb{F}$.*

Proof. Observe that the following calculation holds in $\mathbb{F}$:

$$\zeta^s = a = a^p = \zeta^{ps}, \tag{2}$$

where we have used Fermat's little theorem in the second equality. ■

Therefore, equality (2) means that $\zeta^{ps} = \zeta^s$ is a root of $\overline{\Phi_{5137}(x)} \in \mathbb{F}[x]$.

Lemma 4. *ζ^{ps} is also a root of $\Phi_{5137}(x)$ in $\mathbb{Q}(\zeta)$ (the smallest subfield of $\mathbb{C}$ containing ζ).*

Proof. Begin by noting that the value ζ^{ps} only depends on the value of $ps \pmod{5137}$ since it only appears as an exponent of ζ. Since p does not divide 5137 and s is coprime to 5137, ps is coprime to 5137 (so $ps \pmod{5137}$ is coprime to 5137) and hence ζ^{ps} is a primitive 5137th root of unity. Thus, ζ^{ps} is a root of $\Phi_{5137}(x)$ in $\mathbb{Q}(\zeta)$. ■

Lemma 5. *ζ^{ps} and ζ^s are the same root of $\Phi_{5137}(x)$ in $\mathbb{Q}(\zeta)$.*

Proof. If ζ^{ps} and ζ^s were two distinct roots of $\Phi_{5137}(x)$ in $\mathbb{Q}(\zeta)$, we know because of (2) that they would be the same in $\mathbb{F}$. Therefore, observing expression (1), it follows that $\Delta(\Phi_{5137} \pmod p) = \Delta(\Phi_{5137}) \pmod p = 0$, so p divides $\Delta(\Phi_{5137}) = 11^{4194} \cdot 467^{4650}$. This is a contradiction with our choice of p. Thus, ζ^{ps} and ζ^s are in fact the same root of $\Phi_{5137}(x)$ in $\mathbb{Q}(\zeta)$. ■

Therefore, the equality

$$\zeta^{ps} = \zeta^s \tag{3}$$

holds in $\mathbb{Q}(\zeta)$.

Lemma 6. *The fact that (3) holds implies that $p \pmod{5137} = 1$.*

[1] One way of calculating $\Delta(\Phi_{5137})$ is via the resultant of Φ_{5137} and Φ'_{5137}.

[2] For instance, consider $\mathbb{F} = \mathbb{F}_{p^n}$ with a suitable integer $n \geqslant 1$ such that Φ_{5137} has a root ζ.

Proof. Write the above equation in terms of $\theta := \zeta^s$. This change yields

$$\theta^p = \theta.$$

The right-hand side of the equation above does not depend on p. The left-hand side only depends on the value of $p \pmod{5137}$, since p only appears as an exponent of θ. In conclusion, expression (3) only holds if $p \pmod{5137} = 1$, that is, if $p \equiv 1 \pmod{5137}$. ■

In conclusion, every prime divisor p of Φ_{5137} either belongs to the finite set

$$T = \{11, 467\}$$

or satisfies $p \equiv 1 \pmod{5137}$. In Section 2 we will establish that the polynomial Φ_{5137} has infinitely many prime divisors. But, from the remark above, all these prime divisors must be $\equiv 1 \pmod{5137}$ (except for those $p \in T$). This concludes the proof that there are infinitely many primes $\equiv 1 \pmod{5137}$.

2 Property of the polynomial $\Phi_{5137}(x)$

We just need the following lemma to complete the proof of the main Theorem in Section 1.

Lemma 7. *The cyclotomic polynomial $\Phi_{5137}(x) \in \mathbb{Z}[x]$ has infinitely many prime divisors.*

Proof. There is obviously at least one prime divisor of Φ_{5137}, since the case $\Phi_{5137}(x) =$
$x^{4660}-x^{4659}+x^{4649}-x^{4648}+x^{4638}-x^{4637}+x^{4627}-x^{4626}+x^{4616}-x^{4615}+x^{4605}-x^{4604}+x^{4594}-$
$x^{4593}+x^{4583}-x^{4582}+x^{4572}-x^{4571}+x^{4561}-x^{4560}+x^{4550}-x^{4549}+x^{4539}-x^{4538}+x^{4528}-x^{4527}+$
$x^{4517}-x^{4516}+x^{4506}-x^{4505}+x^{4495}-x^{4494}+x^{4484}-x^{4483}+x^{4473}-x^{4472}+x^{4462}-x^{4461}+x^{4451}-$
$x^{4450}+x^{4440}-x^{4439}+x^{4429}-x^{4428}+x^{4418}-x^{4417}+x^{4407}-x^{4406}+x^{4396}-x^{4395}+x^{4385}-x^{4384}+$
$x^{4374}-x^{4373}+x^{4363}-x^{4362}+x^{4352}-x^{4351}+x^{4341}-x^{4340}+x^{4330}-x^{4329}+x^{4319}-x^{4318}+x^{4308}-$
$x^{4307}+x^{4297}-x^{4296}+x^{4286}-x^{4285}+x^{4275}-x^{4274}+x^{4264}-x^{4263}+x^{4253}-x^{4252}+x^{4242}-x^{4241}+$
$x^{4231}-x^{4230}+x^{4220}-x^{4219}+x^{4209}-x^{4208}+x^{4198}-x^{4197}+x^{4193}-x^{4192}+x^{4187}-x^{4186}+x^{4182}-$
$x^{4181}+x^{4176}-x^{4175}+x^{4171}-x^{4170}+x^{4165}-x^{4164}+x^{4160}-x^{4159}+x^{4154}-x^{4153}+x^{4149}-x^{4148}+$
$x^{4143}-x^{4142}+x^{4138}-x^{4137}+x^{4132}-x^{4131}+x^{4127}-x^{4126}+x^{4121}-x^{4120}+x^{4116}-x^{4115}+x^{4110}-$
$x^{4109}+x^{4105}-x^{4104}+x^{4099}-x^{4098}+x^{4094}-x^{4093}+x^{4088}-x^{4087}+x^{4083}-x^{4082}+x^{4077}-x^{4076}+$
$x^{4072}-x^{4071}+x^{4066}-x^{4065}+x^{4061}-x^{4060}+x^{4055}-x^{4054}+x^{4050}-x^{4049}+x^{4044}-x^{4043}+x^{4039}-$
$x^{4038}+x^{4033}-x^{4032}+x^{4028}-x^{4027}+x^{4022}-x^{4021}+x^{4017}-x^{4016}+x^{4011}-x^{4010}+x^{4006}-x^{4005}+$
$x^{4000}-x^{3999}+x^{3995}-x^{3994}+x^{3989}-x^{3988}+x^{3984}-x^{3983}+x^{3978}-x^{3977}+x^{3973}-x^{3972}+x^{3967}-$
$x^{3966}+x^{3962}-x^{3961}+x^{3956}-x^{3955}+x^{3951}-x^{3950}+x^{3945}-x^{3944}+x^{3940}-x^{3939}+x^{3934}-x^{3933}+$
$x^{3929}-x^{3928}+x^{3923}-x^{3922}+x^{3918}-x^{3917}+x^{3912}-x^{3911}+x^{3907}-x^{3906}+x^{3901}-x^{3900}+x^{3896}-$

$$x^{3895}+x^{3890}-x^{3889}+x^{3885}-x^{3884}+x^{3879}-x^{3878}+x^{3874}-x^{3873}+x^{3868}-x^{3867}+x^{3863}-x^{3862}+$$
$$x^{3857}-x^{3856}+x^{3852}-x^{3851}+x^{3846}-x^{3845}+x^{3841}-x^{3840}+x^{3835}-x^{3834}+x^{3830}-x^{3829}+x^{3824}-$$
$$x^{3823}+x^{3819}-x^{3818}+x^{3813}-x^{3812}+x^{3808}-x^{3807}+x^{3802}-x^{3801}+x^{3797}-x^{3796}+x^{3791}-x^{3790}+$$
$$x^{3786}-x^{3785}+x^{3780}-x^{3779}+x^{3775}-x^{3774}+x^{3769}-x^{3768}+x^{3764}-x^{3763}+x^{3758}-x^{3757}+x^{3753}-$$
$$x^{3752}+x^{3747}-x^{3746}+x^{3742}-x^{3741}+x^{3736}-x^{3735}+x^{3731}-x^{3730}+x^{3726}-x^{3724}+x^{3720}-x^{3719}+$$
$$x^{3715}-x^{3713}+x^{3709}-x^{3708}+x^{3704}-x^{3702}+x^{3698}-x^{3697}+x^{3693}-x^{3691}+x^{3687}-x^{3686}+x^{3682}-$$
$$x^{3680}+x^{3676}-x^{3675}+x^{3671}-x^{3669}+x^{3665}-x^{3664}+x^{3660}-x^{3658}+x^{3654}-x^{3653}+x^{3649}-x^{3647}+$$
$$x^{3643}-x^{3642}+x^{3638}-x^{3636}+x^{3632}-x^{3631}+x^{3627}-x^{3625}+x^{3621}-x^{3620}+x^{3616}-x^{3614}+x^{3610}-$$
$$x^{3609}+x^{3605}-x^{3603}+x^{3599}-x^{3598}+x^{3594}-x^{3592}+x^{3588}-x^{3587}+x^{3583}-x^{3581}+x^{3577}-x^{3576}+$$
$$x^{3572}-x^{3570}+x^{3566}-x^{3565}+x^{3561}-x^{3559}+x^{3555}-x^{3554}+x^{3550}-x^{3548}+x^{3544}-x^{3543}+x^{3539}-$$
$$x^{3537}+x^{3533}-x^{3532}+x^{3528}-x^{3526}+x^{3522}-x^{3521}+x^{3517}-x^{3515}+x^{3511}-x^{3510}+x^{3506}-x^{3504}+$$
$$x^{3500}-x^{3499}+x^{3495}-x^{3493}+x^{3489}-x^{3488}+x^{3484}-x^{3482}+x^{3478}-x^{3477}+x^{3473}-x^{3471}+x^{3467}-$$
$$x^{3466}+x^{3462}-x^{3460}+x^{3456}-x^{3455}+x^{3451}-x^{3449}+x^{3445}-x^{3444}+x^{3440}-x^{3438}+x^{3434}-x^{3433}+$$
$$x^{3429}-x^{3427}+x^{3423}-x^{3422}+x^{3418}-x^{3416}+x^{3412}-x^{3411}+x^{3407}-x^{3405}+x^{3401}-x^{3400}+x^{3396}-$$
$$x^{3394}+x^{3390}-x^{3389}+x^{3385}-x^{3383}+x^{3379}-x^{3378}+x^{3374}-x^{3372}+x^{3368}-x^{3367}+x^{3363}-x^{3361}+$$
$$x^{3357}-x^{3356}+x^{3352}-x^{3350}+x^{3346}-x^{3345}+x^{3341}-x^{3339}+x^{3335}-x^{3334}+x^{3330}-x^{3328}+x^{3324}-$$
$$x^{3323}+x^{3319}-x^{3317}+x^{3313}-x^{3312}+x^{3308}-x^{3306}+x^{3302}-x^{3301}+x^{3297}-x^{3295}+x^{3291}-x^{3290}+$$
$$x^{3286}-x^{3284}+x^{3280}-x^{3279}+x^{3275}-x^{3273}+x^{3269}-x^{3268}+x^{3264}-x^{3262}+x^{3259}-x^{3257}+x^{3253}-$$
$$x^{3251}+x^{3248}-x^{3246}+x^{3242}-x^{3240}+x^{3237}-x^{3235}+x^{3231}-x^{3229}+x^{3226}-x^{3224}+x^{3220}-x^{3218}+$$
$$x^{3215}-x^{3213}+x^{3209}-x^{3207}+x^{3204}-x^{3202}+x^{3198}-x^{3196}+x^{3193}-x^{3191}+x^{3187}-x^{3185}+x^{3182}-$$
$$x^{3180}+x^{3176}-x^{3174}+x^{3171}-x^{3169}+x^{3165}-x^{3163}+x^{3160}-x^{3158}+x^{3154}-x^{3152}+x^{3149}-x^{3147}+$$
$$x^{3143}-x^{3141}+x^{3138}-x^{3136}+x^{3132}-x^{3130}+x^{3127}-x^{3125}+x^{3121}-x^{3119}+x^{3116}-x^{3114}+x^{3110}-$$
$$x^{3108}+x^{3105}-x^{3103}+x^{3099}-x^{3097}+x^{3094}-x^{3092}+x^{3088}-x^{3086}+x^{3083}-x^{3081}+x^{3077}-x^{3075}+$$
$$x^{3072}-x^{3070}+x^{3066}-x^{3064}+x^{3061}-x^{3059}+x^{3055}-x^{3053}+x^{3050}-x^{3048}+x^{3044}-x^{3042}+x^{3039}-$$
$$x^{3037}+x^{3033}-x^{3031}+x^{3028}-x^{3026}+x^{3022}-x^{3020}+x^{3017}-x^{3015}+x^{3011}-x^{3009}+x^{3006}-x^{3004}+$$
$$x^{3000}-x^{2998}+x^{2995}-x^{2993}+x^{2989}-x^{2987}+x^{2984}-x^{2982}+x^{2978}-x^{2976}+x^{2973}-x^{2971}+x^{2967}-$$
$$x^{2965}+x^{2962}-x^{2960}+x^{2956}-x^{2954}+x^{2951}-x^{2949}+x^{2945}-x^{2943}+x^{2940}-x^{2938}+x^{2934}-x^{2932}+$$
$$x^{2929}-x^{2927}+x^{2923}-x^{2921}+x^{2918}-x^{2916}+x^{2912}-x^{2910}+x^{2907}-x^{2905}+x^{2901}-x^{2899}+x^{2896}-$$
$$x^{2894}+x^{2890}-x^{2888}+x^{2885}-x^{2883}+x^{2879}-x^{2877}+x^{2874}-x^{2872}+x^{2868}-x^{2866}+x^{2863}-x^{2861}+$$
$$x^{2857}-x^{2855}+x^{2852}-x^{2850}+x^{2846}-x^{2844}+x^{2841}-x^{2839}+x^{2835}-x^{2833}+x^{2830}-x^{2828}+x^{2824}-$$
$$x^{2822}+x^{2819}-x^{2817}+x^{2813}-x^{2811}+x^{2808}-x^{2806}+x^{2802}-x^{2800}+x^{2797}-x^{2795}+x^{2792}-x^{2789}+$$
$$x^{2786}-x^{2784}+x^{2781}-x^{2778}+x^{2775}-x^{2773}+x^{2770}-x^{2767}+x^{2764}-x^{2762}+x^{2759}-x^{2756}+x^{2753}-$$
$$x^{2751}+x^{2748}-x^{2745}+x^{2742}-x^{2740}+x^{2737}-x^{2734}+x^{2731}-x^{2729}+x^{2726}-x^{2723}+x^{2720}-x^{2718}+$$
$$x^{2715}-x^{2712}+x^{2709}-x^{2707}+x^{2704}-x^{2701}+x^{2698}-x^{2696}+x^{2693}-x^{2690}+x^{2687}-x^{2685}+x^{2682}-$$
$$x^{2679}+x^{2676}-x^{2674}+x^{2671}-x^{2668}+x^{2665}-x^{2663}+x^{2660}-x^{2657}+x^{2654}-x^{2652}+x^{2649}-x^{2646}+$$
$$x^{2643}-x^{2641}+x^{2638}-x^{2635}+x^{2632}-x^{2630}+x^{2627}-x^{2624}+x^{2621}-x^{2619}+x^{2616}-x^{2613}+x^{2610}-$$
$$x^{2608}+x^{2605}-x^{2602}+x^{2599}-x^{2597}+x^{2594}-x^{2591}+x^{2588}-x^{2586}+x^{2583}-x^{2580}+x^{2577}-x^{2575}+$$
$$x^{2572}-x^{2569}+x^{2566}-x^{2564}+x^{2561}-x^{2558}+x^{2555}-x^{2553}+x^{2550}-x^{2547}+x^{2544}-x^{2542}+x^{2539}-$$

$$x^{2536}+x^{2533}-x^{2531}+x^{2528}-x^{2525}+x^{2522}-x^{2520}+x^{2517}-x^{2514}+x^{2511}-x^{2509}+x^{2506}-x^{2503}+$$
$$x^{2500}-x^{2498}+x^{2495}-x^{2492}+x^{2489}-x^{2487}+x^{2484}-x^{2481}+x^{2478}-x^{2476}+x^{2473}-x^{2470}+x^{2467}-$$
$$x^{2465}+x^{2462}-x^{2459}+x^{2456}-x^{2454}+x^{2451}-x^{2448}+x^{2445}-x^{2443}+x^{2440}-x^{2437}+x^{2434}-x^{2432}+$$
$$x^{2429}-x^{2426}+x^{2423}-x^{2421}+x^{2418}-x^{2415}+x^{2412}-x^{2410}+x^{2407}-x^{2404}+x^{2401}-x^{2399}+x^{2396}-$$
$$x^{2393}+x^{2390}-x^{2388}+x^{2385}-x^{2382}+x^{2379}-x^{2377}+x^{2374}-x^{2371}+x^{2368}-x^{2366}+x^{2363}-x^{2360}+$$
$$x^{2357}-x^{2355}+x^{2352}-x^{2349}+x^{2346}-x^{2344}+x^{2341}-x^{2338}+x^{2335}-x^{2333}+x^{2330}-x^{2327}+x^{2325}-$$
$$x^{2322}+x^{2319}-x^{2316}+x^{2314}-x^{2311}+x^{2308}-x^{2305}+x^{2303}-x^{2300}+x^{2297}-x^{2294}+x^{2292}-x^{2289}+$$
$$x^{2286}-x^{2283}+x^{2281}-x^{2278}+x^{2275}-x^{2272}+x^{2270}-x^{2267}+x^{2264}-x^{2261}+x^{2259}-x^{2256}+x^{2253}-$$
$$x^{2250}+x^{2248}-x^{2245}+x^{2242}-x^{2239}+x^{2237}-x^{2234}+x^{2231}-x^{2228}+x^{2226}-x^{2223}+x^{2220}-x^{2217}+$$
$$x^{2215}-x^{2212}+x^{2209}-x^{2206}+x^{2204}-x^{2201}+x^{2198}-x^{2195}+x^{2193}-x^{2190}+x^{2187}-x^{2184}+x^{2182}-$$
$$x^{2179}+x^{2176}-x^{2173}+x^{2171}-x^{2168}+x^{2165}-x^{2162}+x^{2160}-x^{2157}+x^{2154}-x^{2151}+x^{2149}-x^{2146}+$$
$$x^{2143}-x^{2140}+x^{2138}-x^{2135}+x^{2132}-x^{2129}+x^{2127}-x^{2124}+x^{2121}-x^{2118}+x^{2116}-x^{2113}+x^{2110}-$$
$$x^{2107}+x^{2105}-x^{2102}+x^{2099}-x^{2096}+x^{2094}-x^{2091}+x^{2088}-x^{2085}+x^{2083}-x^{2080}+x^{2077}-x^{2074}+$$
$$x^{2072}-x^{2069}+x^{2066}-x^{2063}+x^{2061}-x^{2058}+x^{2055}-x^{2052}+x^{2050}-x^{2047}+x^{2044}-x^{2041}+x^{2039}-$$
$$x^{2036}+x^{2033}-x^{2030}+x^{2028}-x^{2025}+x^{2022}-x^{2019}+x^{2017}-x^{2014}+x^{2011}-x^{2008}+x^{2006}-x^{2003}+$$
$$x^{2000}-x^{1997}+x^{1995}-x^{1992}+x^{1989}-x^{1986}+x^{1984}-x^{1981}+x^{1978}-x^{1975}+x^{1973}-x^{1970}+x^{1967}-$$
$$x^{1964}+x^{1962}-x^{1959}+x^{1956}-x^{1953}+x^{1951}-x^{1948}+x^{1945}-x^{1942}+x^{1940}-x^{1937}+x^{1934}-x^{1931}+$$
$$x^{1929}-x^{1926}+x^{1923}-x^{1920}+x^{1918}-x^{1915}+x^{1912}-x^{1909}+x^{1907}-x^{1904}+x^{1901}-x^{1898}+x^{1896}-$$
$$x^{1893}+x^{1890}-x^{1887}+x^{1885}-x^{1882}+x^{1879}-x^{1876}+x^{1874}-x^{1871}+x^{1868}-x^{1865}+x^{1863}-x^{1860}+$$
$$x^{1858}-x^{1854}+x^{1852}-x^{1849}+x^{1847}-x^{1843}+x^{1841}-x^{1838}+x^{1836}-x^{1832}+x^{1830}-x^{1827}+x^{1825}-$$
$$x^{1821}+x^{1819}-x^{1816}+x^{1814}-x^{1810}+x^{1808}-x^{1805}+x^{1803}-x^{1799}+x^{1797}-x^{1794}+x^{1792}-x^{1788}+$$
$$x^{1786}-x^{1783}+x^{1781}-x^{1777}+x^{1775}-x^{1772}+x^{1770}-x^{1766}+x^{1764}-x^{1761}+x^{1759}-x^{1755}+x^{1753}-$$
$$x^{1750}+x^{1748}-x^{1744}+x^{1742}-x^{1739}+x^{1737}-x^{1733}+x^{1731}-x^{1728}+x^{1726}-x^{1722}+x^{1720}-x^{1717}+$$
$$x^{1715}-x^{1711}+x^{1709}-x^{1706}+x^{1704}-x^{1700}+x^{1698}-x^{1695}+x^{1693}-x^{1689}+x^{1687}-x^{1684}+x^{1682}-$$
$$x^{1678}+x^{1676}-x^{1673}+x^{1671}-x^{1667}+x^{1665}-x^{1662}+x^{1660}-x^{1656}+x^{1654}-x^{1651}+x^{1649}-x^{1645}+$$
$$x^{1643}-x^{1640}+x^{1638}-x^{1634}+x^{1632}-x^{1629}+x^{1627}-x^{1623}+x^{1621}-x^{1618}+x^{1616}-x^{1612}+x^{1610}-$$
$$x^{1607}+x^{1605}-x^{1601}+x^{1599}-x^{1596}+x^{1594}-x^{1590}+x^{1588}-x^{1585}+x^{1583}-x^{1579}+x^{1577}-x^{1574}+$$
$$x^{1572}-x^{1568}+x^{1566}-x^{1563}+x^{1561}-x^{1557}+x^{1555}-x^{1552}+x^{1550}-x^{1546}+x^{1544}-x^{1541}+x^{1539}-$$
$$x^{1535}+x^{1533}-x^{1530}+x^{1528}-x^{1524}+x^{1522}-x^{1519}+x^{1517}-x^{1513}+x^{1511}-x^{1508}+x^{1506}-x^{1502}+$$
$$x^{1500}-x^{1497}+x^{1495}-x^{1491}+x^{1489}-x^{1486}+x^{1484}-x^{1480}+x^{1478}-x^{1475}+x^{1473}-x^{1469}+x^{1467}-$$
$$x^{1464}+x^{1462}-x^{1458}+x^{1456}-x^{1453}+x^{1451}-x^{1447}+x^{1445}-x^{1442}+x^{1440}-x^{1436}+x^{1434}-x^{1431}+$$
$$x^{1429}-x^{1425}+x^{1423}-x^{1420}+x^{1418}-x^{1414}+x^{1412}-x^{1409}+x^{1407}-x^{1403}+x^{1401}-x^{1398}+x^{1396}-$$
$$x^{1392}+x^{1391}-x^{1387}+x^{1385}-x^{1381}+x^{1380}-x^{1376}+x^{1374}-x^{1370}+x^{1369}-x^{1365}+x^{1363}-x^{1359}+$$
$$x^{1358}-x^{1354}+x^{1352}-x^{1348}+x^{1347}-x^{1343}+x^{1341}-x^{1337}+x^{1336}-x^{1332}+x^{1330}-x^{1326}+x^{1325}-$$
$$x^{1321}+x^{1319}-x^{1315}+x^{1314}-x^{1310}+x^{1308}-x^{1304}+x^{1303}-x^{1299}+x^{1297}-x^{1293}+x^{1292}-x^{1288}+$$
$$x^{1286}-x^{1282}+x^{1281}-x^{1277}+x^{1275}-x^{1271}+x^{1270}-x^{1266}+x^{1264}-x^{1260}+x^{1259}-x^{1255}+x^{1253}-$$
$$x^{1249}+x^{1248}-x^{1244}+x^{1242}-x^{1238}+x^{1237}-x^{1233}+x^{1231}-x^{1227}+x^{1226}-x^{1222}+x^{1220}-x^{1216}+$$
$$x^{1215}-x^{1211}+x^{1209}-x^{1205}+x^{1204}-x^{1200}+x^{1198}-x^{1194}+x^{1193}-x^{1189}+x^{1187}-x^{1183}+x^{1182}-$$

$$x^{1178}+x^{1176}-x^{1172}+x^{1171}-x^{1167}+x^{1165}-x^{1161}+x^{1160}-x^{1156}+x^{1154}-x^{1150}+x^{1149}-x^{1145}+x^{1143}-x^{1139}+x^{1138}-x^{1134}+x^{1132}-x^{1128}+x^{1127}-x^{1123}+x^{1121}-x^{1117}+x^{1116}-x^{1112}+x^{1110}-x^{1106}+x^{1105}-x^{1101}+x^{1099}-x^{1095}+x^{1094}-x^{1090}+x^{1088}-x^{1084}+x^{1083}-x^{1079}+x^{1077}-x^{1073}+x^{1072}-x^{1068}+x^{1066}-x^{1062}+x^{1061}-x^{1057}+x^{1055}-x^{1051}+x^{1050}-x^{1046}+x^{1044}-x^{1040}+x^{1039}-x^{1035}+x^{1033}-x^{1029}+x^{1028}-x^{1024}+x^{1022}-x^{1018}+x^{1017}-x^{1013}+x^{1011}-x^{1007}+x^{1006}-x^{1002}+x^{1000}-x^{996}+x^{995}-x^{991}+x^{989}-x^{985}+x^{984}-x^{980}+x^{978}-x^{974}+x^{973}-x^{969}+x^{967}-x^{963}+x^{962}-x^{958}+x^{956}-x^{952}+x^{951}-x^{947}+x^{945}-x^{941}+x^{940}-x^{936}+x^{934}-x^{930}+x^{929}-x^{925}+x^{924}-x^{919}+x^{918}-x^{914}+x^{913}-x^{908}+x^{907}-x^{903}+x^{902}-x^{897}+x^{896}-x^{892}+x^{891}-x^{886}+x^{885}-x^{881}+x^{880}-x^{875}+x^{874}-x^{870}+x^{869}-x^{864}+x^{863}-x^{859}+x^{858}-x^{853}+x^{852}-x^{848}+x^{847}-x^{842}+x^{841}-x^{837}+x^{836}-x^{831}+x^{830}-x^{826}+x^{825}-x^{820}+x^{819}-x^{815}+x^{814}-x^{809}+x^{808}-x^{804}+x^{803}-x^{798}+x^{797}-x^{793}+x^{792}-x^{787}+x^{786}-x^{782}+x^{781}-x^{776}+x^{775}-x^{771}+x^{770}-x^{765}+x^{764}-x^{760}+x^{759}-x^{754}+x^{753}-x^{749}+x^{748}-x^{743}+x^{742}-x^{738}+x^{737}-x^{732}+x^{731}-x^{727}+x^{726}-x^{721}+x^{720}-x^{716}+x^{715}-x^{710}+x^{709}-x^{705}+x^{704}-x^{699}+x^{698}-x^{694}+x^{693}-x^{688}+x^{687}-x^{683}+x^{682}-x^{677}+x^{676}-x^{672}+x^{671}-x^{666}+x^{665}-x^{661}+x^{660}-x^{655}+x^{654}-x^{650}+x^{649}-x^{644}+x^{643}-x^{639}+x^{638}-x^{633}+x^{632}-x^{628}+x^{627}-x^{622}+x^{621}-x^{617}+x^{616}-x^{611}+x^{610}-x^{606}+x^{605}-x^{600}+x^{599}-x^{595}+x^{594}-x^{589}+x^{588}-x^{584}+x^{583}-x^{578}+x^{577}-x^{573}+x^{572}-x^{567}+x^{566}-x^{562}+x^{561}-x^{556}+x^{555}-x^{551}+x^{550}-x^{545}+x^{544}-x^{540}+x^{539}-x^{534}+x^{533}-x^{529}+x^{528}-x^{523}+x^{522}-x^{518}+x^{517}-x^{512}+x^{511}-x^{507}+x^{506}-x^{501}+x^{500}-x^{496}+x^{495}-x^{490}+x^{489}-x^{485}+x^{484}-x^{479}+x^{478}-x^{474}+x^{473}-x^{468}+x^{467}-x^{463}+x^{462}-x^{452}+x^{451}-x^{441}+x^{440}-x^{430}+x^{429}-x^{419}+x^{418}-x^{408}+x^{407}-x^{397}+x^{396}-x^{386}+x^{385}-x^{375}+x^{374}-x^{364}+x^{363}-x^{353}+x^{352}-x^{342}+x^{341}-x^{331}+x^{330}-x^{320}+x^{319}-x^{309}+x^{308}-x^{298}+x^{297}-x^{287}+x^{286}-x^{276}+x^{275}-x^{265}+x^{264}-x^{254}+x^{253}-x^{243}+x^{242}-x^{232}+x^{231}-x^{221}+x^{220}-x^{210}+x^{209}-x^{199}+x^{198}-x^{188}+x^{187}-x^{177}+x^{176}-x^{166}+x^{165}-x^{155}+x^{154}-x^{144}+x^{143}-x^{133}+x^{132}-x^{122}+x^{121}-x^{111}+x^{110}-x^{100}+x^{99}-x^{89}+x^{88}-x^{78}+x^{77}-x^{67}+x^{66}-x^{56}+x^{55}-x^{45}+x^{44}-x^{34}+x^{33}-x^{23}+x^{22}-x^{12}+x^{11}-x+1=\pm 1$$

only happens for a finite number of integer values of x. Now suppose Φ_{5137} has only a finite number of prime divisors, say $p_1, p_2, \ldots, p_k$ and let $Q := p_1 p_2 \cdots p_k$.

Observe that $\deg(\Phi_{5137}) = 4660$ and $\Phi_{5137}(0) = 1 \neq 0$. Then, $\Phi_{5137}(Qx) = g(x)$ for some $g(x) \in \mathbb{Z}[x]$ of the form $1 + c_1 x + \cdots + c_{4660} x^{4660}$, $c_i \in \mathbb{Z}$, satisfying $Q \mid c_i$ for every $1 \leqslant i \leqslant 4660$. This polynomial g must also have at least one prime divisor, say p, for the same reason as before. Therefore, p divides $g(m)$ for some $m \in \mathbb{Z}$, and this implies that p divides $\Phi_{5137}(Qm)$. Since $m' := Qm \in \mathbb{Z}$, it follows that p is a prime divisor of Φ_{5137}. But p does not divide Q, since p dividing Q would mean that p divides c_i, for every $1 \leqslant i \leqslant 4660$ (recall that Q divides every c_i). This, together with the fact that p divides $g(m)$, would imply that p divides $g(m) - \sum_{i=1}^{4660} c_i m^i = 1$, which means $p = 1$, a contradiction.

Now, p is a prime divisor of Φ_{5137}, but p is not any of the primes $p_1, p_2, \ldots, p_k$, since we just proved that p does not divide $p_1 p_2 \cdots p_k = Q$. Thus, we found a new prime divisor of Φ_{5137} not in our list. Since this argument can be repeated indefinitely, one concludes

that Φ_{5137} has infinitely many prime divisors. ■