

Figure 2.6.1. (a) A nonsingular matrix \mathbf{A} maps a vector space into one of the same dimension. The vector \mathbf{x} is mapped into \mathbf{b} , so that \mathbf{x} satisfies the equation $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$. (b) A singular matrix \mathbf{A} maps a vector space into one of lower dimensionality, here a plane into a line, called the "range" of \mathbf{A} . The "nullspace" of \mathbf{A} is mapped to zero. The solutions of $\mathbf{A} \cdot \mathbf{x} = \mathbf{d}$ consist of any one particular solution plus any vector in the nullspace, here forming a line parallel to the nullspace. Singular value decomposition (SVD) selects the particular solution closest to zero, as shown. The point \mathbf{c} lies outside of the range of \mathbf{A} , so $\mathbf{A} \cdot \mathbf{x} = \mathbf{c}$ has no solution. SVD finds the least-squares best compromise solution, namely a solution of $\mathbf{A} \cdot \mathbf{x} = \mathbf{c}'$, as shown.

In the discussion since equation (2.6.6), we have been pretending that a matrix either is singular or else isn't. That is of course true analytically. Numerically, however, the far more common situation is that some of the w_j 's are very small but nonzero, so that the matrix is ill-conditioned. In that case, the direct solution methods of LU decomposition or Gaussian elimination may actually give a formal solution to the set of equations (that is, a zero pivot may not be encountered); but the solution vector may have wildly large components whose algebraic cancellation, when multiplying by the matrix $\bf A$, may give a very poor approximation to the right-hand vector $\bf b$. In such cases, the solution vector $\bf x$ obtained by zeroing the

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