

# Search

## Lecture 3, CMSC 170

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# Previously on CMSC 170

- **Rational agent:** maximize expected utility
- Reflex vs Planning Agent
- Types of Environments

# Today's Topics

- Search Problems
- Tree Search
- Uninformed Search
  - Depth-First Search
  - Breadth-First Search
  - Uniform Cost Search

# Brain: Decision-Making

- Planning vs Learning
- Simulation vs Memory

# Planning Agents

- Agents that **plan ahead** to solve problems
- Thinks about **consequences** of its actions by performing **simulations**

# Planning Agents

## Goal-Based

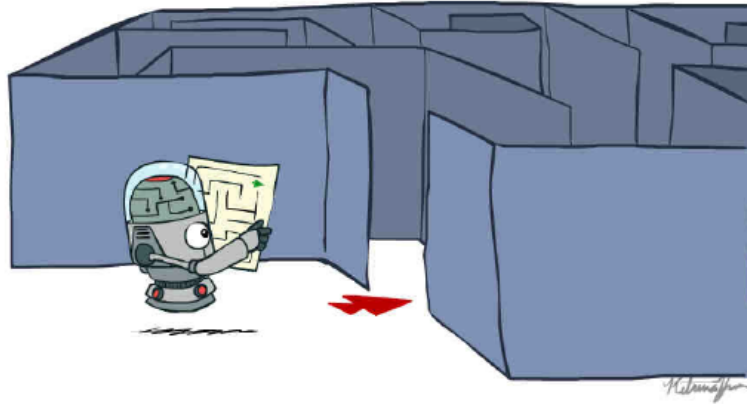
- finds a solution that satisfies goals

## Utility-Based

- find best possible solution

# Planning and Search

Search



# Search Problem

- How to reach **goal** state from **start** state?
- **Solution**: *sequence of actions* (**plan**);  
start state → goal state
- **Complexity** comes from having many  
possible states (**large search space**)



# Search Problem

## State

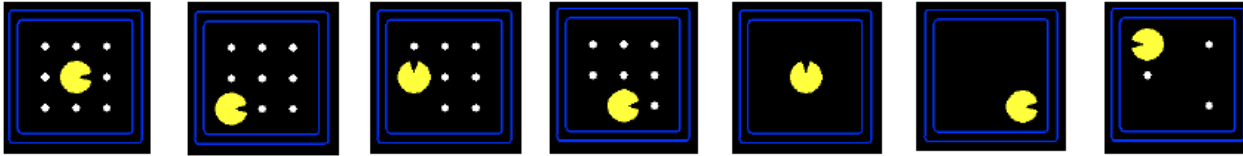
- encodes how the world is at a certain point

## Start State

- initial configuration of the world

# Search Problem

## States



# Search Problem

## Actions

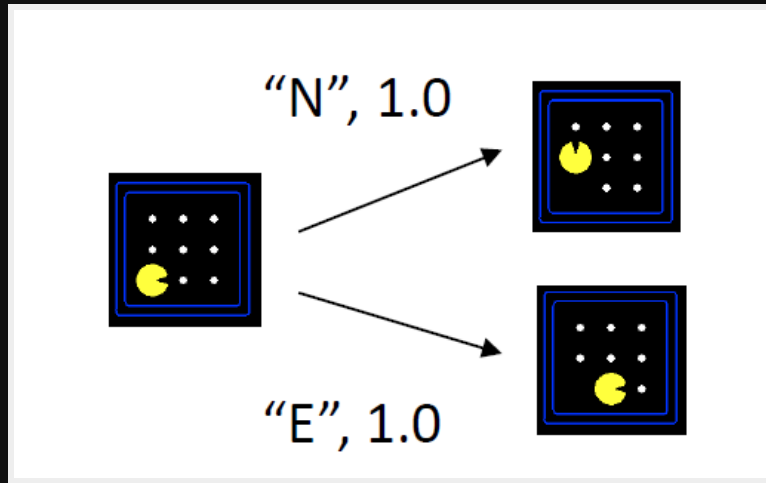
- valid steps the agent can perform

## Successor Function

- models how the world works
- $\text{state} \rightarrow (\text{action}, \text{cost}) \rightarrow \text{state}$

# Search Problem

## Successor Function

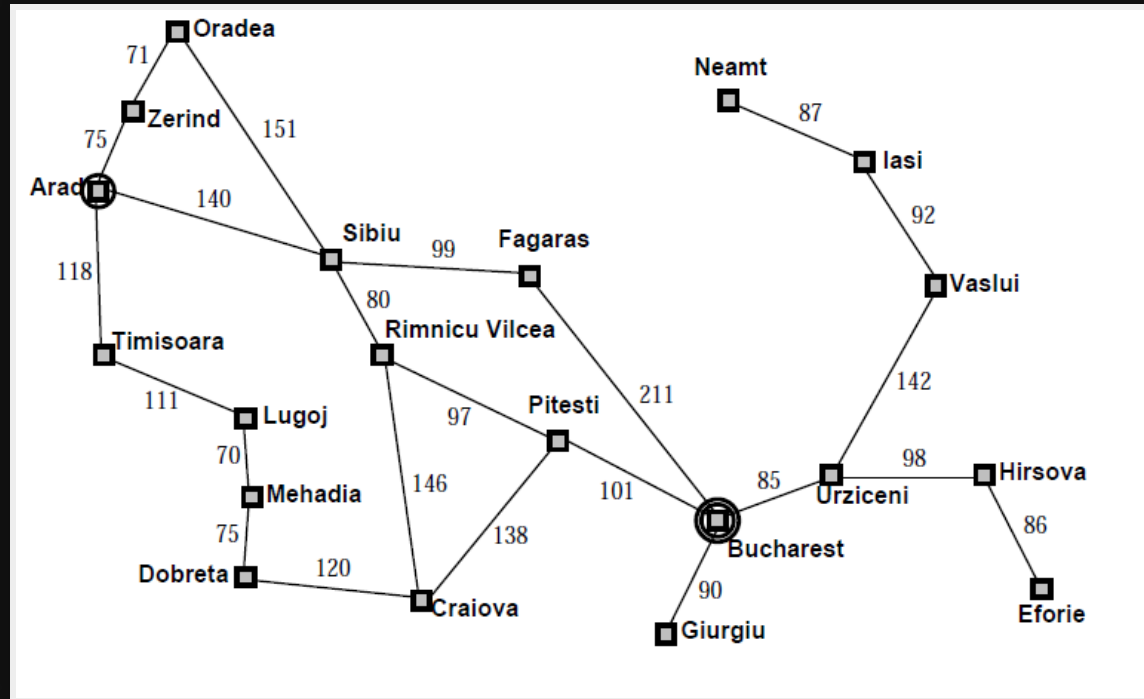


# Search Problem

## Goal Test

- checks if your **goal** has been *reached*
- not necessarily a goal state, could be a **description**

# Romania Vacation



# Romania Vacation

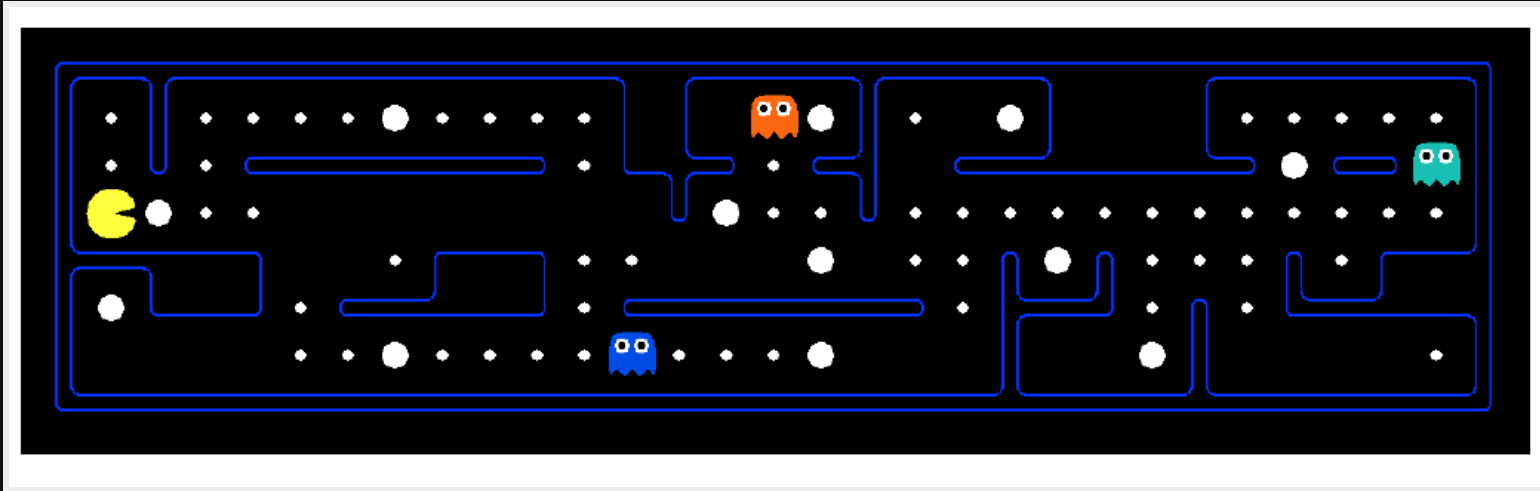
- **States:** cities
- **Successor Function:** roads  
(go to adjacent city, cost = distance)

# Romania Vacation

- **Start state:** Arad
- **Goal test:** is state == Bucharest?
- **Solution:** path from Arad to Bucharest



# Pacman



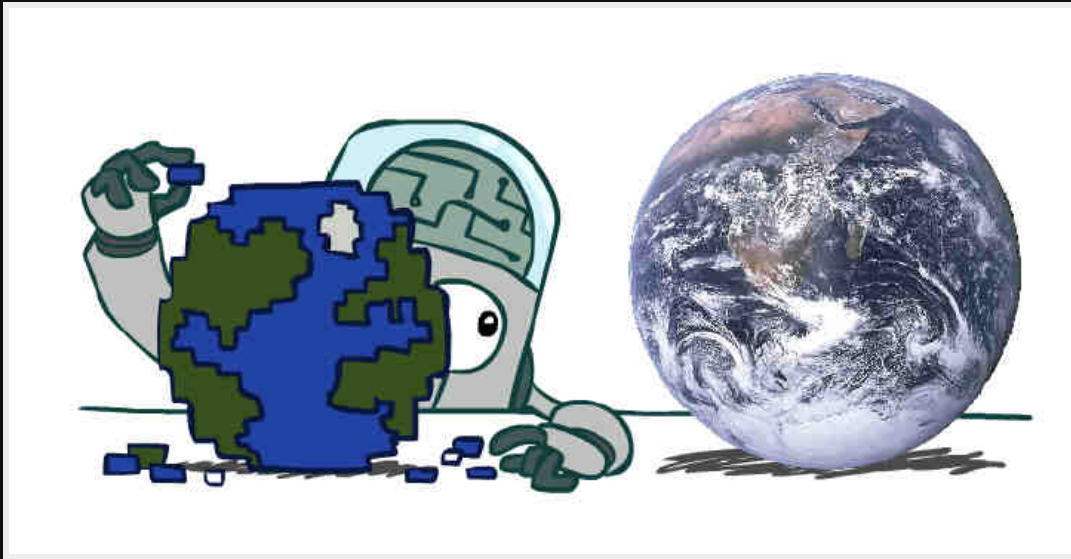
# Pacman: Pathing

- **States:** (x,y) location
- **Actions:** NEWS
- **Successor:** update location
- **Goal test:** is (x,y) == END?

# Pacman: Eat-All-Dots

- **States:**  $\{(x,y), \text{dot booleans}\}$
- **Actions:** NEWS
- **Successor:** Update location, dot boolean
- **Goal test:** dots all false?

# Search Problems are Models



# Search Problems are Models

- Planning agent uses a **model** of the world
- **Search quality** (correctness, time, memory) is dependent on **model quality**

# Model Quality

- **Too abstract:** not enough details, can't solve the problem
- **Too detailed:** deal with all complexities of the world, search will take too long

# World State vs. Search State

## World State

- includes all details of the environment
- don't model over this, usually very large

# World State vs. Search State

## Search State

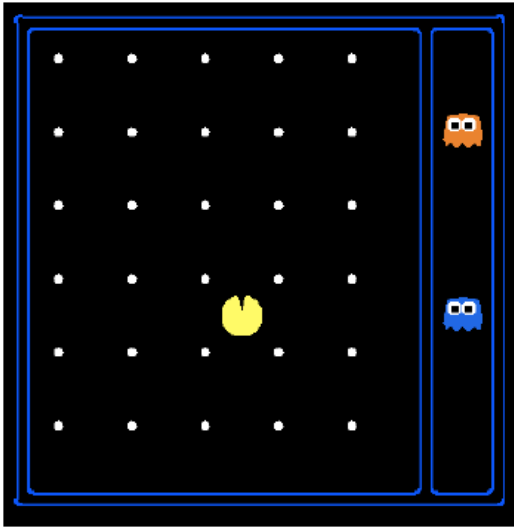
- only keeps details needed for planning  
(*abstraction*)
- depends on your search problem



# Search Problem

- **State space**: start state + actions + successor function
- **Complexity** of search problem depends on **size** of state space

# State Space Sizes



# State Space Sizes

## World state:

- Agent positions:  $10 \times 12 = 120$
- Food count: 30
- Ghost positions: 12
- Agent Facing: 4 (NEWS)

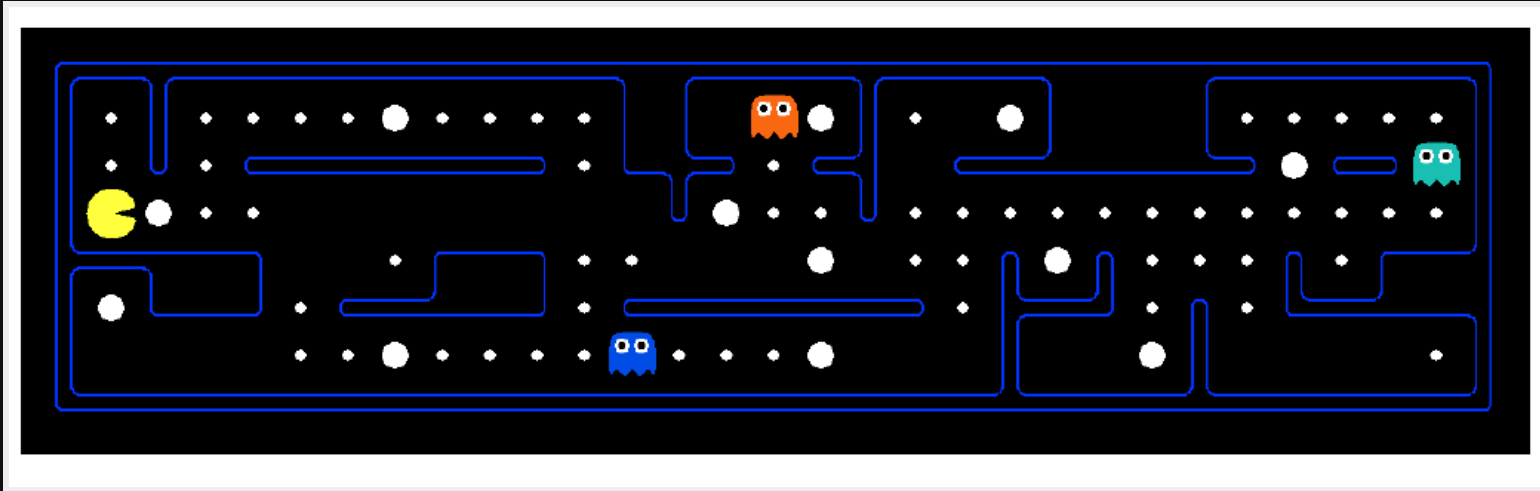
# State Space Sizes

- World States =  $120 * (2^{30}) * (12^2) * 4$
- States for pathing = 120
- States for eat-all-dots =  $120 * (2^{30})$

# State Space Sizes

*"In general, search space is so large that you will never be able to enumerate it."*

# Exercise



# Exercise

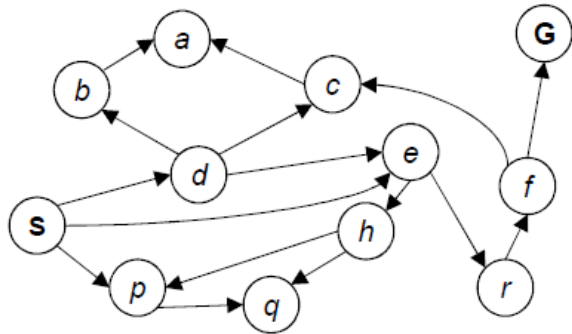
- *Problem:* Eat all dots while keeping ghosts scared
- *Question:* What does the state space have to specify?

# Answer

- Agent Position (x,y)
- Dot booleans
- Power pellet booleans
- Remaining scared time



# State Space Graphs



*Tiny search graph for a tiny  
search problem*

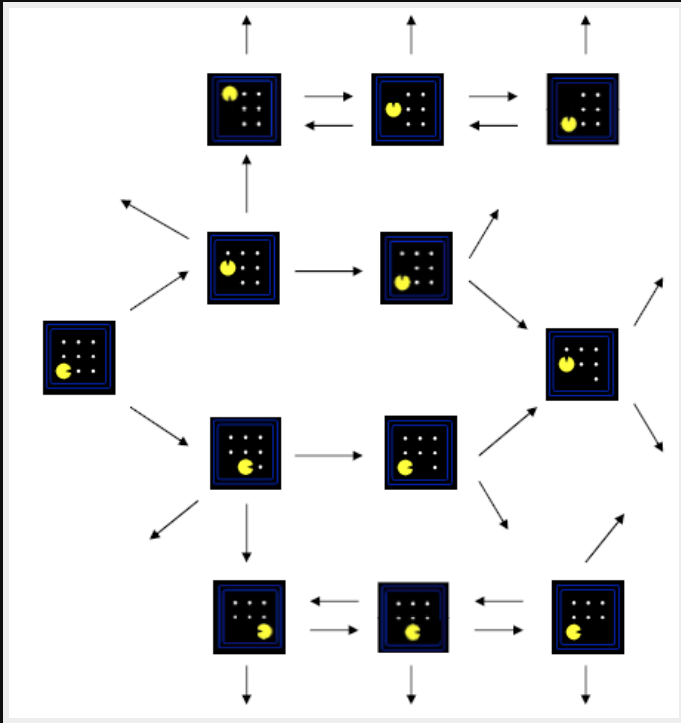
# State Space Graphs

- **Nodes**: *abstracted* world configurations (*states*)
- The more you can **abstract**, the more **efficient** your search will be

# State Space Graphs

- **Edges**: represent successors (action results)
- **Goal test**: set of *goal nodes* (maybe one)

# State Space Graphs



# State Space Graphs

- Each state occurs only *once*
- We can rarely build *full graph* in memory (*too big*)
- Most parts are **unreachable** during search

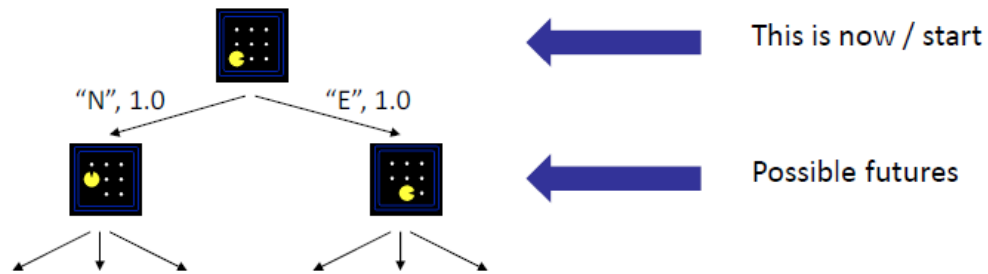
# Search Trees

- Contains **start state** and things that can happen from it
- A "what if" tree of **plans** and their **outcomes**

# Search Trees

- **Start state** is the *root node*
- *Children* correspond to **successors**

# Search Trees

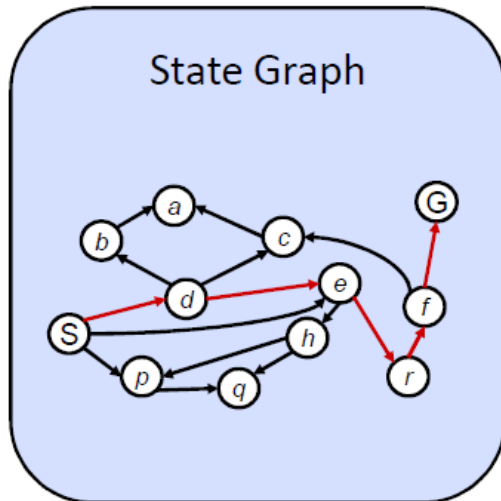




# Search Trees

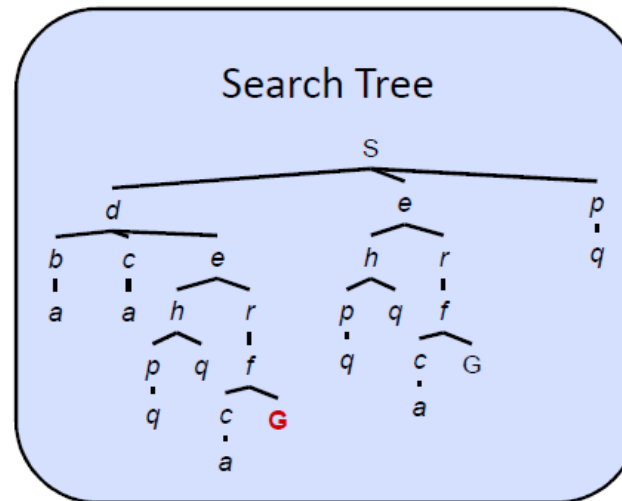
- Nodes show states, but correspond to **plans** that achieve those states
- For most problems, we can never actually build the *whole tree*

# State Graphs vs. Search Trees



Each **NODE** in in the search tree is an entire **PATH** in the problem graph.

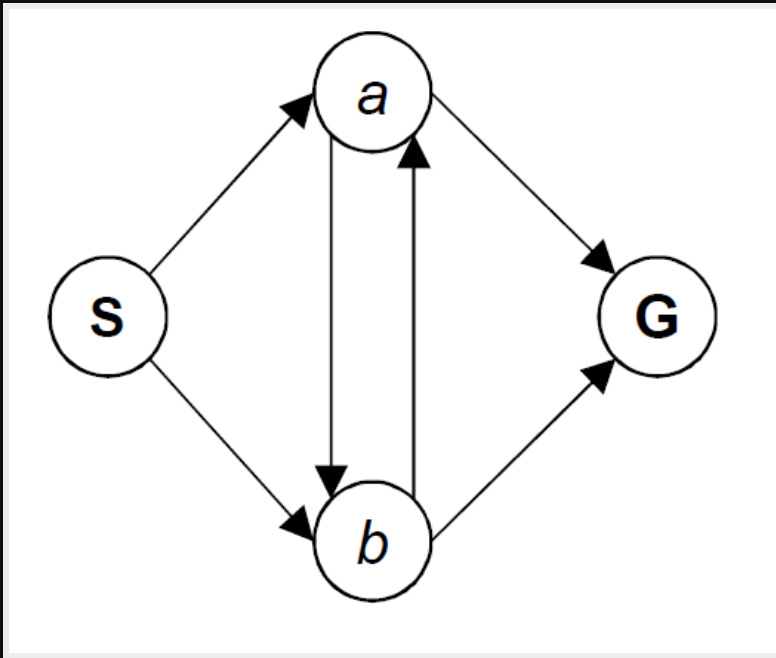
*We construct both  
on demand – and  
we construct as  
little as possible.*



# Exercise

**State graph** = 4 nodes

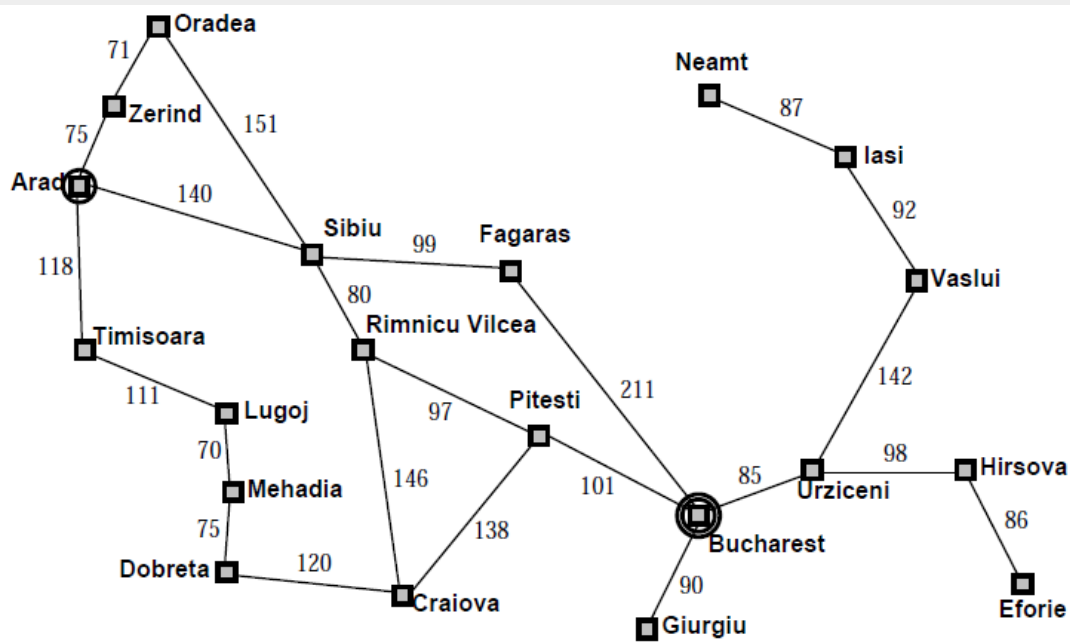
How big is the **search tree** (from S)?



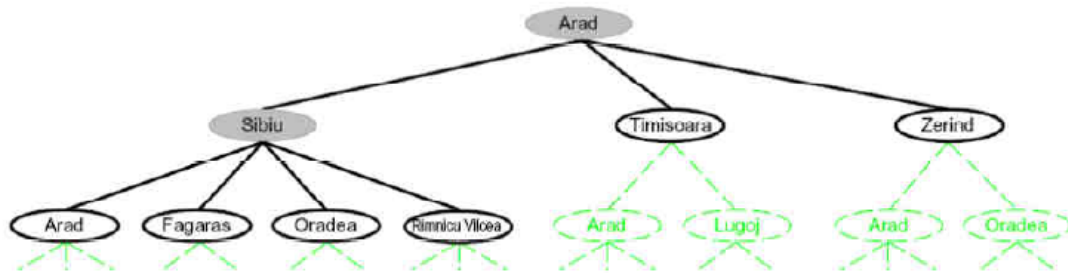
# Answer

- $\infty$
- *Important:* Lots of **repeated** structure in the search tree

# Search Example: Romania



# Searching with a Search Tree



# Search

- **Expand** out potential plans (tree nodes)
- Maintain a **fringe** of partial plans under consideration
- Try to expand as *few* tree nodes as possible

# General Tree Search

Basic Idea:

- **Offline, simulated** exploration of state space
- Generating **successors** of already-explored states (*expanding*)



# General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

# General Tree Search

Important ideas:

- **Fringe**: all of the plans that may yet to work
- **Expansion**: picking something out of fringe and expanding
- **Exploration strategy**: which fringe nodes to explore next?

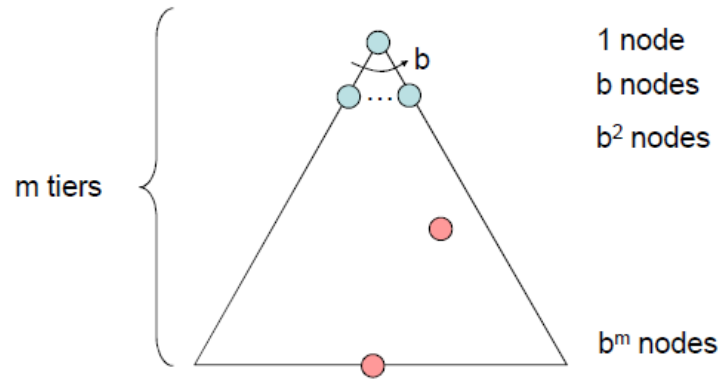
# Uninformed Search

- Depth-First Search
- Breadth-First Search
- Uniform-Cost Search

# Questions

- **Complete**? Can it find solution?
- **Optimal**? Best solution?
- **Time** complexity?
- **Space** complexity?

# Search Tree



- search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths
- Number of nodes in entire tree?
  - $1 + b + b^2 + \dots + b^m = O(b^m)$

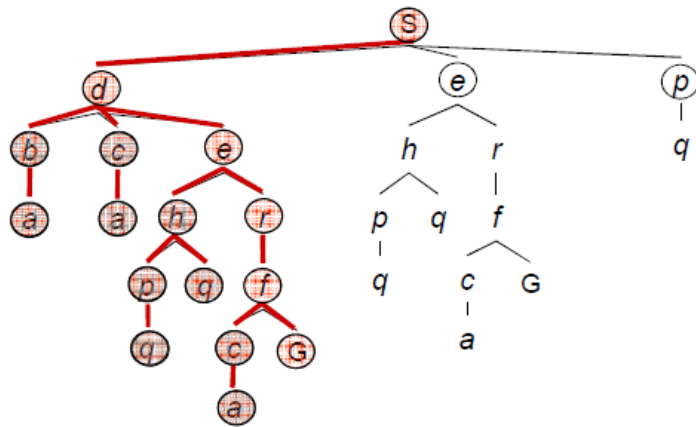
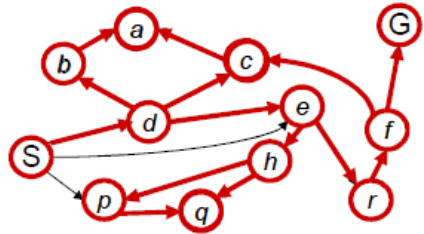
# Depth-First Search



# Depth-First Search

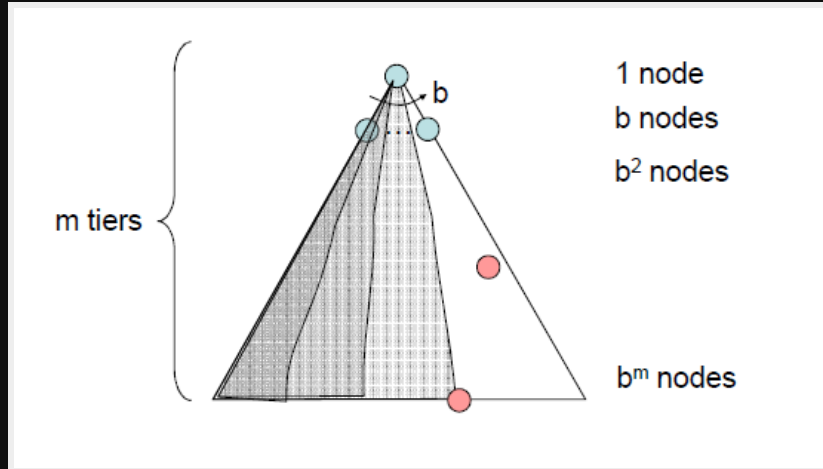
- **Strategy**: Expand a *deepest* node first
- **Implementation**: Fringe is a LIFO **stack**

# Depth-First Search





# DFS Properties



# DFS Properties

What nodes does DFS **expand**?

- Some **left prefix** of the tree
- Could process the *whole* tree!
- If  $m$  is finite, takes  **$O(b^m)$**  *time (exponential)*

# DFS Properties

How much **space** does *fringe* take?

- Only keeps *siblings* on path to root
- Takes  **$O(bm)$**  *space (polynomial)*

# DFS Properties

## Completeness

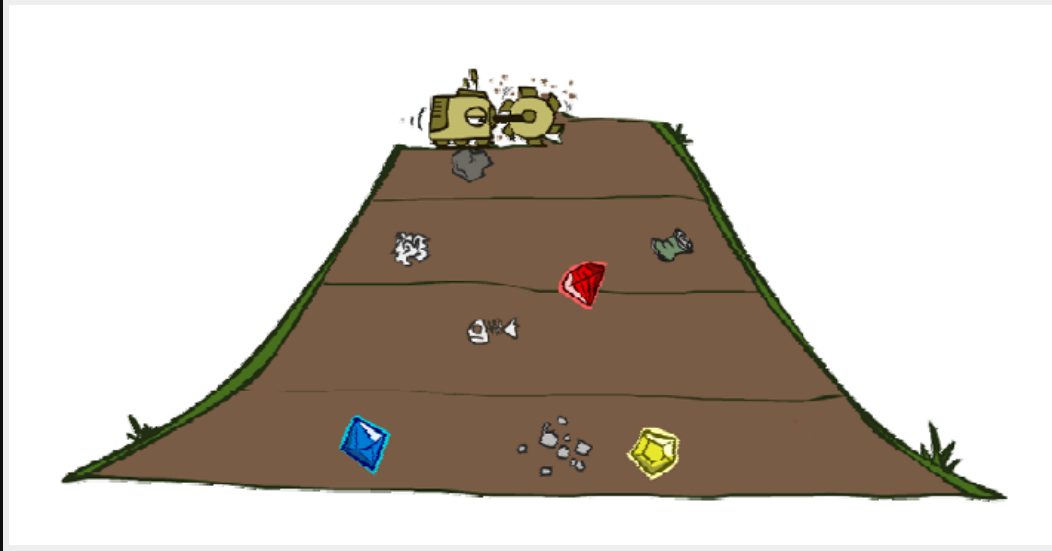
- m could be infinite (*cycles*)
- DFS is *complete* only if we **prevent cycles**

# DFS Properties

## Optimal

- not optimal
- finds the **leftmost** solution regardless of *depth* or *cost*

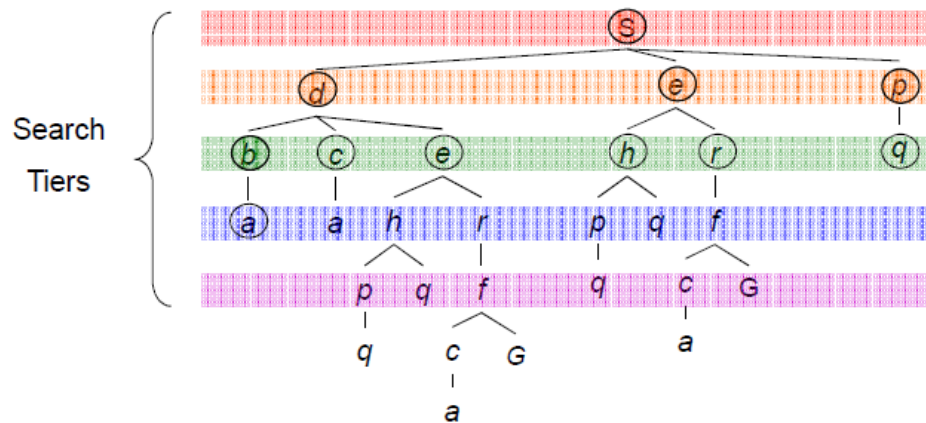
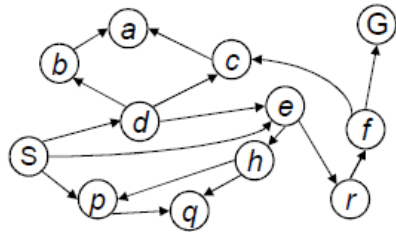
# Breadth-First Search



# Breadth-First Search

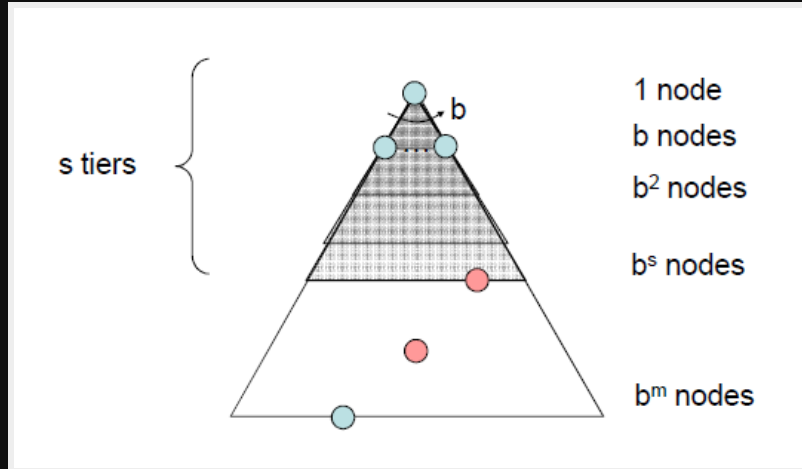
- **Strategy**: Expand a *shallowest* node first
- **Implementation**: Fringe is a FIFO **queue**
- Search the tree in tiers / **layers**

# Breadth-First Search





# BFS Properties



# BFS Properties

What nodes does BFS **expand**?

- Processes **all nodes** above *shallowest* solution
- Let depth of shallowest solution =  $s$
- Search takes  **$O(b^s)$**  time (*exponential*)

# BFS Properties

How much **space** does *fringe* take?

- Keeps *all nodes* from the last layer
- Takes  **$O(b^s)$**  *space* (exponential)

# BFS Properties

## Completeness

- s must be finite if a solution exists, so yes

## Optimal

- only if costs are all 1
- solution deeper into the tree might cost less than shallowest solution

# BFS

- **Space** is the big problem
- Can easily generate nodes at 100MB/sec
- 24 hrs = 8640GB

# Question

- When will BFS outperform DFS?
- When will DFS outperform BFS?

# Answer

- If solutions are relatively *shallow*, **BFS**
- If solutions are down at the *bottom*, **DFS**

# Iterative Deepening

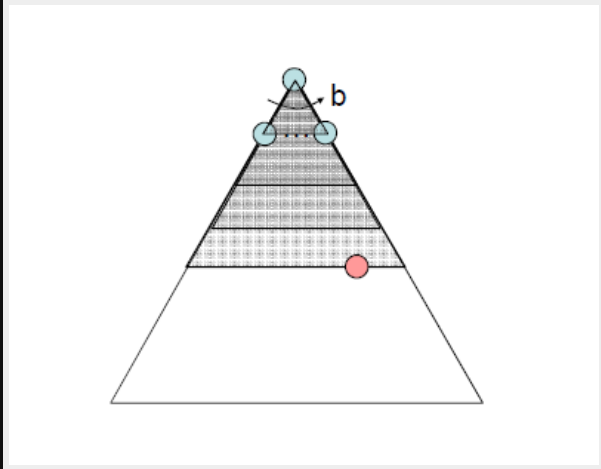
- Best of both worlds!
- *Idea*: Combine DFS' **space** advantage with BFS' **time** / shallow-solution advantages



# Iterative Deepening

- Run a DFS with depth limit 1
- If no solution, run a DFS with depth limit 2
- If no solution, run a DFS with depth limit 3
- And so on..

# Iterative Deepening



# Iterative Deepening

- **RT**:  $O(b^s)$
- **Space**:  $O(bs)$
- **Complete**? Yes
- **Optimal**? Yes, if step cost = 1 (like BFS)

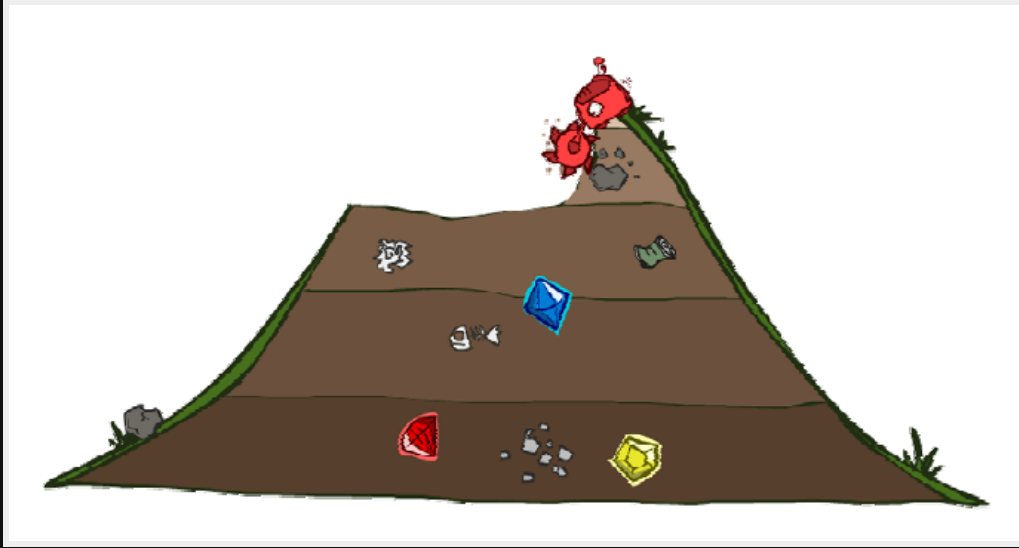
# Iterative Deepening

- Doing DFS lots of times
- Isn't this wastefully redundant?
- Generally, most work happens in the lowest level searched, so **not so bad!**

# Cost-Sensitive Search

- BFS finds *shortest path* in terms of **number of actions**
- It doesn't find the **least-cost path**

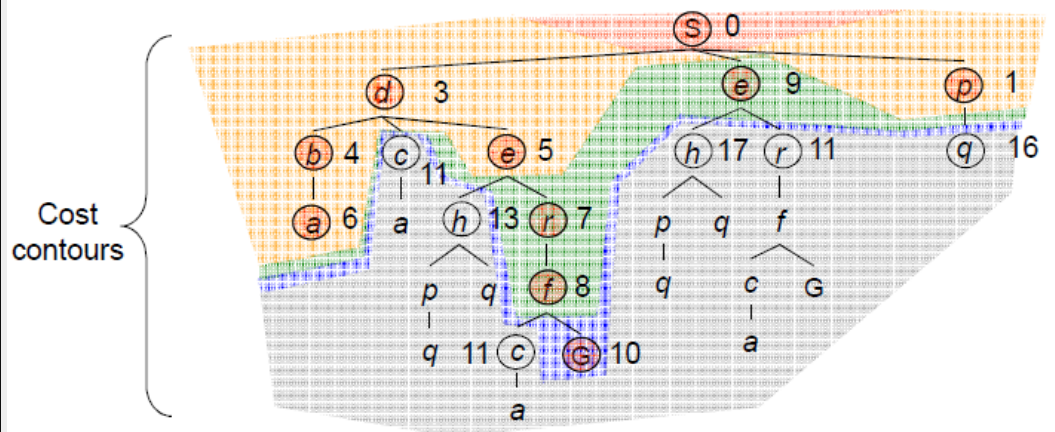
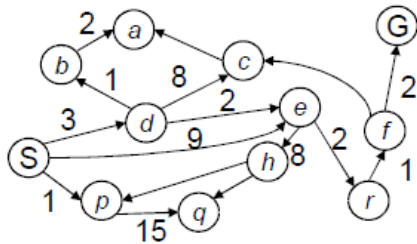
# Uniform Cost Search



# Uniform Cost Search

- **Strategy**: Expand a *cheapest* node first
- **Implementation**: Fringe = **priority queue**  
(priority: *cumulative cost*)

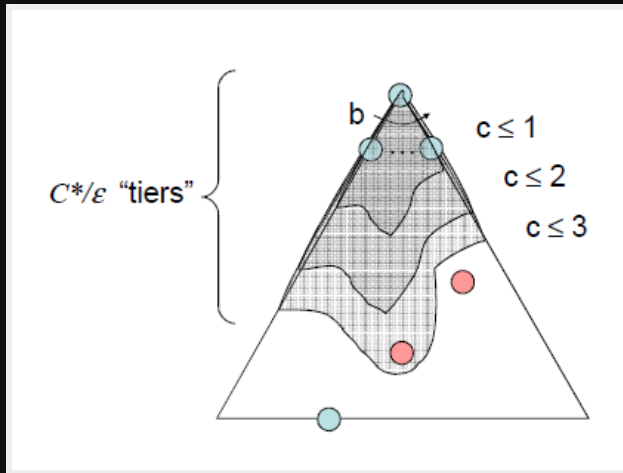
# Uniform Cost Search





# UCS Properties

$c^*$  = least-cost solution



# UCS Properties

What nodes does UCS **expand**?

- Processes *all nodes* with cost less than cheapest solution
- Let solution cost =  $C^*$ , arcs cost at least  $\epsilon$ ,
- Effective depth  $\sim$  roughly  **$C^*/\epsilon$**
- **$O(b^{C^*/\epsilon})$**  time (*exponential* in effective depth)

# UCS Properties

How much **space** does *fringe* take?

- Has roughly the last layer
- Takes  $O(b^{c^*/\epsilon})$  *space* (exponential)

# UCS Properties

## Completeness

- Assuming best solution has finite cost and minimum arc cost is positive, yes!

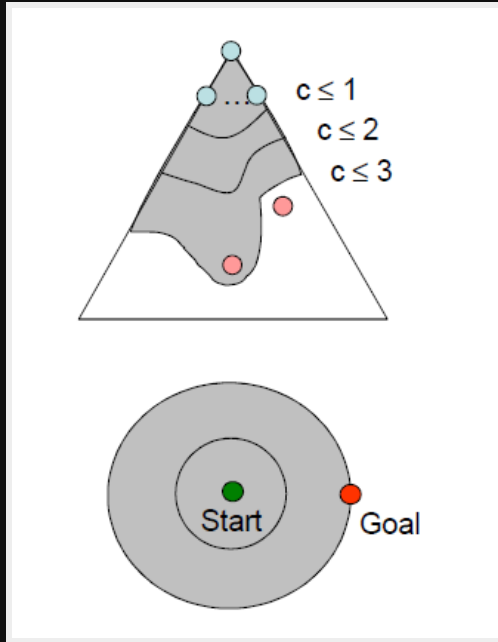
## Optimal

- Yes!

# UCS Issues

- UCS explores increasing cost contours
- **Good**: UCS is **complete** and **optimal**
- **Bad**: It explores options in *every direction*
- **No information** about *goal location*  
(**uninformed**)

# UCS Issues



# Search Algorithms

- Essentially the same except for **fringe** strategies
- **DFS**: Stack
- **BFS**: Queue
- **UCS**: Priority Queue

# Comparison

	<b>Finds</b>	<b>Time</b>	<b>Space</b>	<b>Fringe</b>
<b>DFS</b>	Leftmost solution	$O(b^m)$	$O(bm)$	Stack
<b>BFS</b>	Shallowest solution	$O(b^s)$	$O(b^s)$	Queue
<b>UCS</b>	Least-cost solution	$O(b^{c^*/\epsilon})$	$O(b^{c^*/\epsilon})$	Priority Queue



# Demo

- DFS
- BFS
- UCS

# Search and Models

- Search operates over **models** of the world
- Agent doesn't actually try plans in real world
- Planning is all in **simulation**
- Search is *only as good as* your model

# Search

Works when environment is:

- Fully observable
- Deterministic
- Discrete
- Benign
- Static

# Summary

- **Search Problems:** states, actions, successor function, start state, goal test
- Search quality depends on **model quality**
- **Tree Search Algorithms:** DFS, BFS, UCS

# Next Meeting

- Informed Search
- Heuristics
- Greedy Search
- A\* Search
- Graph Search

# Announcements

- *Assignment 2: Search*, next meeting
- *MP 1: Pacman Search*, next week

# References

- *Artificial Intelligence: A Modern Approach, 3rd Edition*, S. Russell and P. Norvig, 2010
- CS 188 Lec 2 slides, Dan Klein, UC Berkeley

# Questions?