Existence of parabolic minimizers to the total variation flow on metric measure spaces

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Abstract

In this talk I will discuss some fine properties and existence of the variational solutions to the Total Variation Flow. Instead of the classical Euclidean setting, we intend to work mostly in the general setting of metric measure spaces. During the past two decades, a theory of Sobolev functions and BV functions has been developed in this abstract setting. A central motivation for developing such a theory has been the desire to unify the assumptions and methods employed in various specific spaces, such as weighted Euclidean spaces, Riemannian manifolds, Heisenberg groups, graphs, etc.

The total variation flow can be understood as a process diminishing the total variation using the gradient descent method. This idea can be reformulated using parabolic minimizers, and it gives rise to a definition of variational solutions. The advantages of the approach using a minimization formulation include much better convergence and stability properties. This is a very essential advantage as the solutions naturally lie only in the space of BV functions.

We give an existence proof for variational solutions u associated to the total variation flow. Here, the functions being considered are defined on a metric measure space (X, d, μ). For such parabolic minimizers that coincide with a time-independent Cauchy-Dirichlet datum u_0 on the parabolic boundary of a space-time-cylinder $\Omega \times (0, T)$ with $\Omega \subset X$ an open set and $T > 0$, we prove existence in the weak parabolic function space $L^1_w(0, T; BV(\Omega))$. In this paper, we generalize results from a previous work by Bögelein, Duzaar and Marcellini and argue completely on a variational level. This is a join project with Vito Buffa and Michael Collins, from Friedrich-Alexander-Universität Erlangen-Nürnberg.