University of south Asia

Assignment on : 01

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Course Title : Numerical Methods
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1. $\int_0^1 \frac{xdx}{\sqrt{1+x^2}}$ = $\frac{1}{2}\int_0^1 \frac{2xdx}{\sqrt{1+x^2}}$

= $\frac{1}{2}\int_0^1 \frac{d(1+x^2)}{\sqrt{1+x^2}}$

= $\frac{1}{2}\int 2\sqrt{1+x^2}\int_0^1$

=$\sqrt{1+1^2}-\sqrt{1+0^2}$

=$\sqrt{2}$ - $\sqrt{1}$

=$\sqrt{2}$ - $\sqrt{1}$ **Ans:**

2. **Let, I=** $\int_0^{log2} \frac{e^x}{1+e^x}\,dx$

Put, $1+e^x$

= e^x **dx = dy**

Limit:

When, x=0 **Then, Y=2**

When, x=log2 **Then, y=3**

I = $\int_2^3 \frac{dy}{y}$

=$\left| Logy \quad \right|$

= Log3 –Log2

= log $\frac{3}{2}$

Hence $\int_0^{log2} \frac{e^x}{1+e^x}\,dx$ **= log** $\frac{3}{2}$ **Ans:**

3. $\int \frac{dx}{x(logx)^2}$

Sol: **Let, I =** $\int \frac{dx}{x(logx)^2}$

Put, log x=y

$\therefore \frac{1}{2} dx = dy$

$\therefore I = \int \frac{dy}{y^2}$

$= \int y^{-2}\, dy$

$= \frac{y^{-2+1}}{-2+1} + c$

$= -\frac{1}{y} + c$

$= -\frac{1}{logx} + c$ **Ans:**

FINITE DIFFERENCES :

Assume that we have a table of values ($x_{i,}y_i$), i=0,1,2,3,……….,nof any function y=f(x) , the of x being equally spaced, ie, x_i=x_0+ih, i=0,1,2,3,……….,n. Supposed that we are required to recover the values of f(x) for some intermediate values of x, or to obtain the derivative of f(x) for some x in the range $x_0 \le x \le x_n$. The methods of the solution of these problems are based on the concept of the differences of a function which we now proceed to define.

Forward Differences :

If $y_0, y_1, y_2, \dots \dots \dots .., y_n$ denote a set of values of y, then $y_1 - y_{0,}\ y_2 - y_{1,}$ ………………….., $y_{n-}y_{n-1}$ are Called the differences of y. Denoted these differences by $\Delta y_0, \Delta y_1, \dots \dots \dots \dots .., \Delta y_{n-1}$ respectively. We have, Δy_0=$y_1 - y_{0,}\Delta y_1 = y_2 - y_{1,} \dots \dots \dots .., \Delta y_{n-1} = y_{n-}y_{n-1}$ **where Δ is called the forward difference operator and** $\Delta y_0, \Delta y_1, \dots \dots \dots \dots .., \Delta y_{n-1}$ are called first **forward differences. The differences of the first forward differences are called second forward differences and are denoted by** $\Delta^2{}_{y_0}, \Delta^2{}_{y_1}, \dots \dots \dots \dots$.... Similarly, one can define third forward differences, forth forward differences,
Etc. hus

$$\begin{aligned}
\Delta^2{}_{y_0} &= \Delta y_1 - \Delta y_0 \\
&= y_2 - y_{1,} - (y_1 - y_{0,}) \\
&= y_2 - y_{1,} - y_1 + y_{0,} \\
&= y_2 - 2y_1 + y_0
\end{aligned}$$

$$\begin{aligned}
\Delta^3{}_{y_0} &= \Delta^2{}_{y_1} - \Delta^2{}_{y_0} \\
&= y_3 - 2y_2 + y_0 - (y_2 - 2y_1 + y_0) \\
&= y_3 - 2y_2 + y_0 - y_2 + 2y_1 - y_0 \\
&= y_3 - 3y_2 + 3y_1 - y_0
\end{aligned}$$

and $\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$

$$= y_4 - 3y_3 + 3y_2 - y_1 - (y_3 - 3y_2 + 3y_1 - y_0)$$
$$= y_4 - 3y_3 + 3y_2 - y_1 - y_3 + 3y_2 - 3y_1 + y_0$$
$$= y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$$

It is therefore clear that any higher order difference can easily be expressed in terms of the ordinates, since the coefficients occurring on the right side are the binomial coefficients.

The following table shows how the forward differences of all orders can be formed :

X	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
x_0	y_0						
		Δy_0					
x_1	y_1		$\Delta^2 y_0$				
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$		
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$	
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$		$\Delta^6 y_0$
		Δy_3		$\Delta^3 y_2$		$\Delta^5 y_1$	
x_4	y_4		$\Delta^2 y_3$		$\Delta^4 y_2$		
		Δy_4		$\Delta^3 y_3$			
x_5	y_5		$\Delta^2 y_4$				
		Δy_5					
x_6	y_6						