University of south Asia

Assignment on : 01

Assignment Name : Midterm Assignment (**Discrete Mathematics).**

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Answer The question on: 1.a

Discrete Mathematics: Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly" the objects studied in discrete mathematics - such as integers, graphs, and statements in logic [1] - donot vary smoothly in this way, but have distinct, separted values. Discrete objects can often be enumerated by Integers.

Discrete mathematics is important for CSE students:
Discrete objects can often be Enumerated by intgers. More formally, Discrete mathematics has been characterized as the branch of mathematics dealing with countable sets[4].
Research in discrete mathematics increased in the latter half of the Twentieth century partly due to the development of digital computers which operate in discrete steps and store data in discrete bits concept and notations from discrete mathematics are useful in studying and describing objects and problems in branches of Computer science, such as computer algorithms, Programming Languages, software Developments.

Discrete math Developments our Personal Life:-

An analog clock has gears inside and the size/teeth needed for correct timekeeping are determined using discrete math. wiring a computer network using the least amount of cable is a minimum-weight spanning tree Problem. Encryption and decryption are part of cryptography, which is Part of discrete mathemati

Answer The question on: 1.b

Define Proposition: Yes, Khulna is one of the major divisional cities of Bangladesh. It is the 3rd Largest city with a population of 1.5 million After Dhaka and cortogram situated in the south western region of the country. The city is around 45 square kilometers in Area and situated along river Bahia. the railway line and National Highway in parallel to river act as the spine of urban development in the city. These give the city a linar form. major urban development took place along this road network Later several major road were constructed and together from the arterial road network of the city.

Study of Area: Khulna is the 2nd Port and 3rd Largest city of Bangladesh. It is in the south west part of country. (figure no; 1.1). Khulna is a Low laying linear city Located on the bank of the Rush and Bahia river. Geographically Kholna lies between 22°49' north Latitude and 84°34' east longitude and its elevation is 7 feet above the mean sea Level. there are 31 wards in Khulna city. So the efficients steps need to undertake to make the bus services more reliable to the other city.

Answer The question on. 1-c

implication: Let p and q be Propositions. The Proposition "p implies q" denoted by $p \to q$ is called implication. It is false when p is true and q is false and is true otherwise. In $p \to q$, p ~~and~~ is called The hypothesis and q is called the conclusion.

Inverse of Implication:

- Implication: $p \to q$
- Inverse : $\neg p \to \neg q$
  - Implication: If I am going to town it is raining
  - Inverse: If I am not going to town, Then it is not raining.
- The inverse of an implication has The same Truth table as The converse of That implication

example bi conditional Proposition:

- "if and only if" iff "
  - $p \leftrightarrow q$
  - I am going to town if and only if it is raining.
  - Both p and q must have The same Truth value for the assertion to be ~~true~~ True.

Answer The question no: 2.a

Truth Table Conjunction: The conjunction AND is a logical operator p: I am going to town, q: It is raining.
p∧q: I am going to town and it is raining.
Both p and q must be True for the conjuction to be true

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table Disjunction: Inclusive or - only one Proposition needs to be true for the disjunction to be True, p: I am going to town, q: It is raining.
p∨q: I am going to town or it is raining.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Answer The question on : 2·b

Logical Equivalence : In logical & mathematics statements P and q are said to be logical equivalence if they are Provable from each other under a set of axioms.

Show That: $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$:

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	F	T	F	F
F	T	F	T	F	T	T	T
F	F	T	T	F	T	F	F
F	F	F	F	F	F	F	F

Since colums corresponding to ~~pa~~ $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$ match. The Propositions are logically equivalent This Particular equivalence is know as The Distributive Law.

Answer The question on; 2.e

Tautology and contradiction:

A formula is said to be a Tautology if every Truth assignments to its component Statements results in The formula being true.

A formula is said to be contradiction if every truth assignments to its components statements results in The formula being false.

example:-

- A tautology is a Proposition which is always true, $p \lor \neg p$
- A contradication is a Proposition That is alwayes false. $p \land \neg p$.

3a

Answer The question no: 3.a

Predication: A predicate is a statements That contains varibles and That May be true or false depending on That valus of These variable.

Quantifiers: A Predicate is an expression of one or more variables defined on some specific domain. A Predicate with variables can be made a Proposition by either assigning a values to value to the variables by quantifying the variables.

Two types of quantifiers: ⊞ universal quantifier: $\forall$
⊞ Existential quantifier: $\exists$

$\forall x P(x)$ $\forall x P(x)$ is read as for every values of x $P(x)$ is true.
Example: universal- "man is mortal" can be transform into the Propositional from $\forall x (P)(x)$ $\forall x P(x)$ where $P(x)$ is mortal and the universe of discourse is all man.
$\exists x P(x)$ $\exists x P(x)$ is read as for Some values of x $P(x)$ is true.
Example: "Some people are dishonest" can be Transformed into the Propositional from: $\exists x P(x)$ where $P(x)$ is the Predicate some People.

8.2

Answer the question no: 3.b

Truth value of $\forall x P(x)$:

ⓐ If $\exists! x P(x)$ is true, Then There exists a unique value x for which $P(x)$ is true. Thus there exists a value x for which $P(x)$ is true and thus $\exists x P(x)$ is true. And Thus we then have That $\exists! x P(x) \rightarrow \exists x P(x)$ is true.

ⓑ If $\forall x P(x)$ is true, then for every value x we have That $P(x)$ is true. Thus There does not exists a unique value x for which $P(x)$ is true if The domain consists of more then 1 elements. And Thus we Then have That $\forall x P(x) \rightarrow \exists! x P(x)$ is false if The domain consists of more Than 1 element

ⓐ True

ⓑ False if the domain consists of more Than 1 element

$x^2 < 10$:

$P(x)$ is The statements "$x^2 < 10$" and The domain consists of the Positive integers not exceeding 4:

The statements $\forall x P(x)$ is The Same as The conjunction $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$ $P(1) = 12 < 10 = 1 < 10$ $P(2) = 22 < 10 = 4 < 10$ $P(3) = 32 < 10 = 9 < 10$ $P(4) = 42 < 10$ Solution Because $P(4)$ is false. it follows That $\forall x P(x)$ is false

Answer The question no: 3.c

Quantifiers: "$x \leq 5 \wedge x > 3 x \leq 5 \wedge x > 3$" is true for $x=4 x=4$ and false for $x=6 x=6$. compare this with The statements "For every $x x$, $x \leq 5 \wedge x > 3 x \leq 5 \wedge x > 3$" which is definitely false and the statements "There exists an $x x$ such ~~as~~ That $x \leq 5 \wedge x > 3 x \leq 5 \wedge x > 3$".

Translation with example:

Translating nested Quantifiers into English

Example: Translate the statements $\forall x (C(x) \vee \exists y (C(y) \wedge F(x, y)))$ where $C(x)$ is "x has a computer," and $F(x, y)$ is "x and y are friends and the domain for both x and y consists of all students in your school, Page 3 of 9 © 2020. J Perepelitsa Example:

Translate the statements $\exists x \forall y \forall z ((F(x, y) \wedge F(x, z) \wedge (y \neq z)) \rightarrow F(y, z))$

Answer The question no : 4.a

Theorem and Lemma with example :

- A theorem is a valid logical assertion which can be proved using.
  - Axioms : statements which are given to be true.
  - Rules of inference ; logical rules allowing the deduction of conclusions from peremises.

- A lemma is a 'Pre-theorem' or a result which is needed to prove a theorem.

- A corollary is a 'post-theorem' or a result which follows directly from a theorem.

Answer The question no: 4.6

Type of Proof methods:

- Direct Proof
- Indirect Proof
- Vacuous Proof
- Trivial Proof
- Proof by Contradiction
- Proof by cases
- Existence Proof.

explains contradiction Proof:

- Sometimes called "Reductio ad absurdum"
- we want to prove P. we do That by assuming The opposite, $\neg p$ and show That That implies a contradiction q
- Mathematics definition of the Proof Find a contradiction q such That

$\neg P \to q \Leftrightarrow \neg P \to F \Leftrightarrow \neg (\neg P) \Leftrightarrow P$

direct Proof methods

- Assume The hypotheses are true
- Use rules of inference and any logical equivalences to establish The truth of the ~~example of~~ a conclusion.
- How To PROVE:

$p \to q$ – If P is true, Then q has to be true for $p \to q$ to be true.

Example: the Proof we did earlier about cows not eating artichokes was an example of a direct Proof.

Answer the question no: 4.e

Exhaustive proof : An Exhaustive Proof is a special type of proof by cases where each case involves checking a single example.

Prove That $(n+1)^3 \geq 3^n$: Given That n is a positive integer and $n \leq 4$: Prove That $(n+1)^3 \geq 3^n$
Here we have only 4 cases and each case involves a specific value of n: 1, 2, 3 and 4

- For $n=1$, $(n+1)^3 = 8$ and $3^n = 3$
- For $n=2$ $(n+1)^3 = 27$ and $3^n = 9$
- For $n=3$, $(n+1)^3 = 64$ and $3^n = 27$
- For $n=4$, $(n+1)^3 = 125$ and $3^n = 81$

These 4 case exhaust all of the possibilities: QED.

Answer The question no 9.d

Mistakes in Proof :-

* Sometimes we cause mistake in our proofs by making a faulty assumption.
* For Example, there is a famous - proof " That 2=1 That is based on a faulty assumption.
* Given That a & b are positive integers and $a=b$, what is wrong with the following prof.

Step	Reason
$a=b$	hypothesis.
$a^2=ab$	Multiply both sides by a
$a^2-b^2=ab-b^2$	subtract b^2 from both sides
$(a-b)(a+b)=b(a-b)$	Factor both side
$a+b=b$	Divide by $(a-b)$
$2b=b$	step 1, replace & simplify.
$2=1$	Divide by b

Answer The question no; 5.a

Set: A set is an unordered collection of different elements. A set can be written explicitly by listing its elements using set bracket. If the order of The elements is changed, it does not make any changes in the set.

Example: A set of all positive integers.

Empty set or Null set: An empty set contains no elements. It is denoted by $\emptyset$. As The number of elements in an empty set is finite. empty set is a finite set. the cardinality of empty set or Null sets is Zero.

Example: $S = \{x \mid x \in N\} = \{x \mid x \in N \text{ and } 7 < x < 8\} = \emptyset$

Elements of set: The order in which element occur in sets is irrelevant. For example the following two sets are equal.

$\{a, b, c, d, e\}$
$\{e, c, a, d, b\}$

union of sets: * union of Two set A and B is denoted by $A \cup B$ * $A \cup B$ contains elements that are either in A or in B or in both.

* $A \cup B = \{x \mid x \in A \vee x \in B\}$
* $A = \{1, 3, 5\}, B = \{2, 3, 4\}$
* $A \cup B = \{1, 2, 3, 4, 5\}$

intersection of set: * Intersection of Two sets A and B is denoted by $A \cap B$

* $A \cap B$ contains element that are in both A and B * $A \cap B = \{x \mid x \in A \wedge x \in B\}$ * $A = \{1, 3, 5\}, B = \{1, 2, 3\}$

$A \cap B = \{1, 3\}$.

Disjoint sets: A Two sets are called disjoint if Their intersection is the empty set.

$A = \{1, 3, 5\}, B = \{1, 2, 3\}, C = \{6, 7, 8\}$

* Are A and B disjoint? No
* Are A and C are disjoint? yes,

universal set: A universal set is a set which contains all the elements or objects of other sets including its own elements. It is usually denoted by the symbol 'U'

Example: $A = \{1, 3, 6, 8\}, B = \{2, 3, 4, 5\}, C = \{5, 8, 9\}$

Ans: $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Complements of set:

The complement of a set, denoted A' is the set of all elements in the given universe set U that are not in A, Example: $U' = \emptyset$
The complements of the universe is the empty set.

Power set:

The power set of S is the of all subset of S.
The power set of S is represents by $P(S)$
example: if $S = \{a, b\}$ then.

$P(S) = \{\{a\}, \{b\}, \{a, b\}, \emptyset\}$