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the Ensemble Kalman Filter (EnKF)

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Overview

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Contexte and Objectives

Context

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Study of the application of the Ensemble Kalman Filter (EnKF) to nonlinear dynamic systems, specifically the Lorenz system.

The Ensemble Kalman Filter[1] is a powerful tool for state estimation in nonlinear and high-dimensional systems, enhancing forecast accuracy by integrating observational data.

Objectives

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1. To determine how effective the Ensemble Kalman Filter (EnKF) is in tracking and predicting the behavior of the Lorenz system, which is intrinsically chaotic.
2. To examine how different parameters, such as observation noise levels and ensemble size, influence the performance of the EnKF in forecasting the future states of the system.

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Background

Lorenz system and ODEs solvers

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$$\begin{cases} \frac{dx}{dt} = \sigma(y - x) \\ \frac{dy}{dt} = x(\rho - z) - y \\ \frac{dz}{dt} = xy - \beta z \end{cases}$$

where:[2]

- x is the rate of convective overturning, [2]
- y is the horizontal temperature variation,
- z is the vertical temperature variation,
- σ is the Prandtl number,
- ρ is the Rayleigh number,
- β is a geometrical factor.

Explicit Euler Method

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Method Description:

- Simple and effective for step-by-step approximation.
- Uses the derivative to estimate state changes.

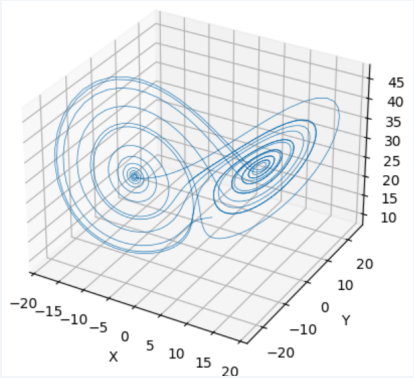

Figure: Lorenz system solved with Explicit Euler.

Ensemble Kalman Filter (EnKF)

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The Ensemble Kalman Filter [3] works by :

- **Propagation through the Model** we use our dynamic model to predict future states by running multiple state samples through it.
- **Update with Observations** we adjust these predictions based on actual observations to get a more accurate estimate of the system's state.

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Implementation and Results

EnKF Algorithm Steps

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1. **Initialization:** Generate initial state samples $\{x_i^0\}_{i=1}^N$.
2. **Prediction:** Forecast the next state for each ensemble member [4]:

$$x_i^{\text{pred}} = f(x_i^{\text{upd}}) + w_i$$

3. **Update:** Adjust predictions based on observations:
   - Compute predicted observation:

$$z_i^{\text{pred}} = h(x_i^{\text{pred}}) + v_i$$

   - Calculate Kalman gain:

$$K_k = P_k^{\text{pred}} H^T (H P_k^{\text{pred}} H^T + R)^{-1}$$

   - Update state estimates:

$$x_i^{\text{upd}} = x_i^{\text{pred}} + K_k(z_k - h(x_i^{\text{pred}}))$$

Results

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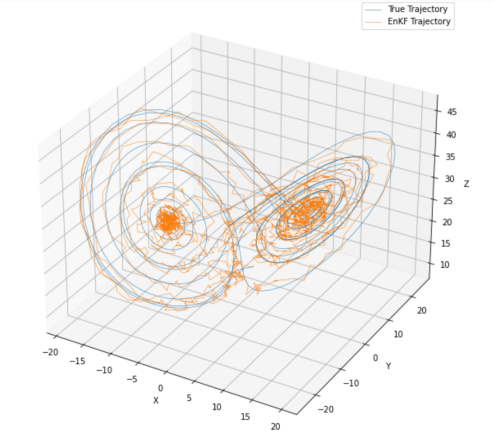
EnKF Trajectories and Deviations



Results

Uncertainty of Estimates and Absolute Errors

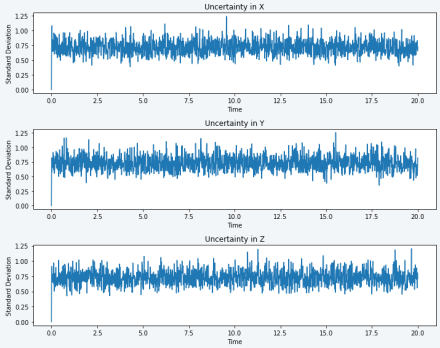


Figure: Uncertainty

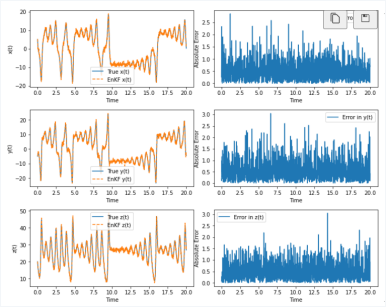


Figure: Individual Trajectories and Absolute Errors

Results

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Impact of Observation Covariance Matrix (R)

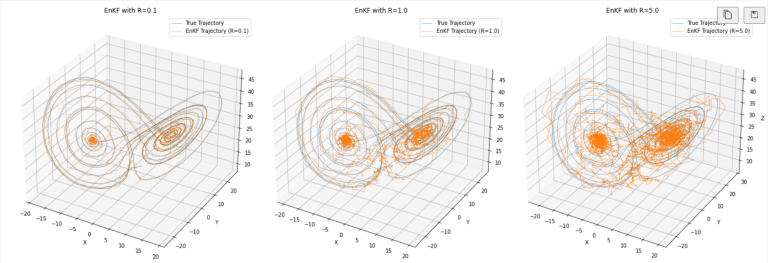

Figure: Lorenz System Analysis Using Ensemble Kalman Filter for Different R Values

Results

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Impact of Ensemble Size (N)

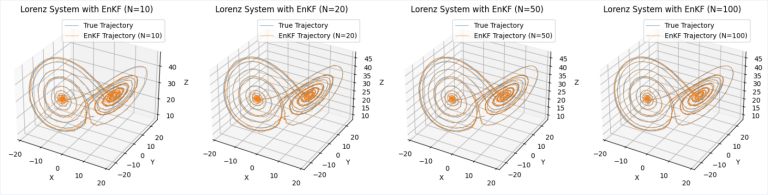

Figure: Influence of the Number of Ensembles N

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Conclusion

Conclusion

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Effectiveness of EnKF:

- The Ensemble Kalman Filter (EnKF) has proven effective in estimating the states of the Lorenz system, even in the presence of uncertainties and noise.

Sensitivity to Parameters:

- The performance of the EnKF is influenced by observation noise levels and ensemble size.

References I

[1] W. contributors, *Ensemble kalman filter*, [Online]. Available: https://en.wikipedia.org/wiki/Ensemble_Kalman_filter.

[2] Wikipédia, *Système dynamique de lorenz*, [Online]. Available: https://fr.wikipedia.org/wiki/Syst%C3%A8me_dynamique_de_Lorenz.

[3] R. R. L. Jr, *Kalman and bayesian filters in python*, May 2020. [Online]. Available: https://github.com/rlabbe/Kalman-and-Bayesian-Filters-in-Python/.

[4] M. A. Iglesias, K. J. Law, and A. M. Stuart, "Ensemble kalman methods for inverse problems," *Inverse Problems*, 2013.

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Thank you for your attention

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