Maxwell equations applied to Mie scattering theory

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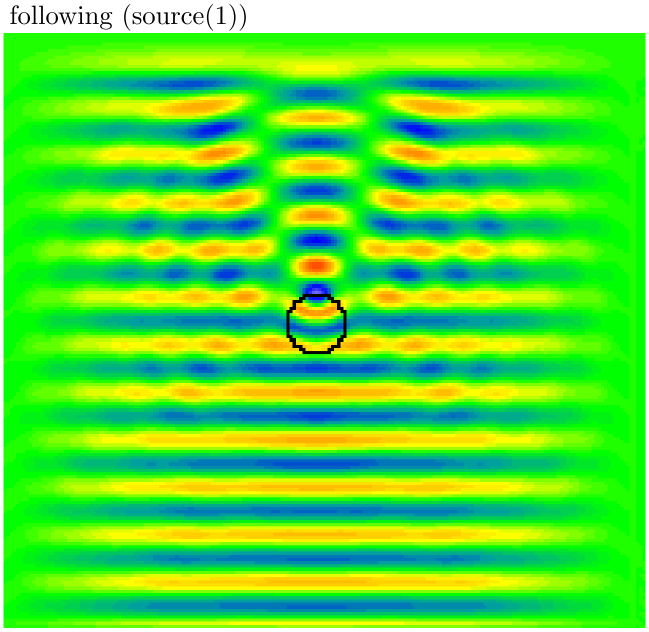
1 Objective

In wave optics physics, the Mie theory, also known as Lorenz-Mie theory, is a theory of light scattering by spherical particles larger than the wavelength. This theory is a particular solution of Maxwell's equations. We may use this theory, for example, to measure the concentration of particles suspended in a liquid or a gas. The objective of this project is to create a simple model to simulate Maxwell's equations in the context of Mie scattering theory similar to the following (source(1))



2 Model Description

2.1 Mathematical Equations

To simulate Maxwell's equations in 2D, we take what is proposed in the article (source (2)).

$$(1) : \mu_r \frac{\partial H_x}{\partial t} = -\frac{\partial E_z}{\partial y}$$

$$(2) : \mu_r \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

$$(3) : \epsilon_r \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$

With ϵ_r and μ_r being the relative permittivity and permeability witch allows us to modify the reflective index of the particle.

To model this problem I am using FEEL++ and more specifically the CFPDE toolbox which allows me to solve PDE's of the following form:

$$d\frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

In my case the interpretation of (1), (2) and (3) using CFPDE toolbox is :

1. $d = \mu_r; \gamma = (0, E_z)$
2. $d = \mu_r; \gamma = (-E_z, 0)$
3. $d = \epsilon_r; \gamma = (-H_y, H_x)$

2.2 Boundary Condition

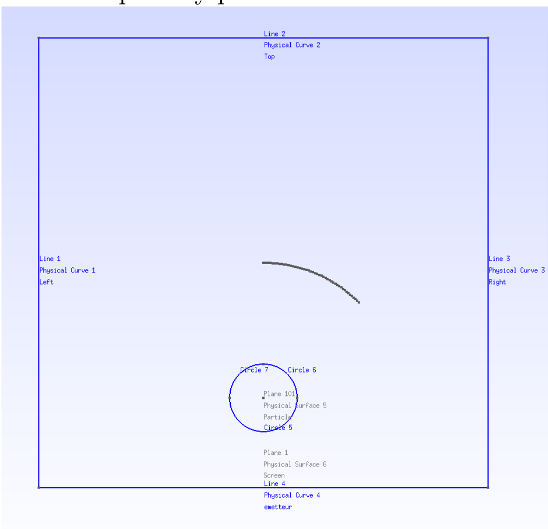
I have used two different boundary condition in this model.

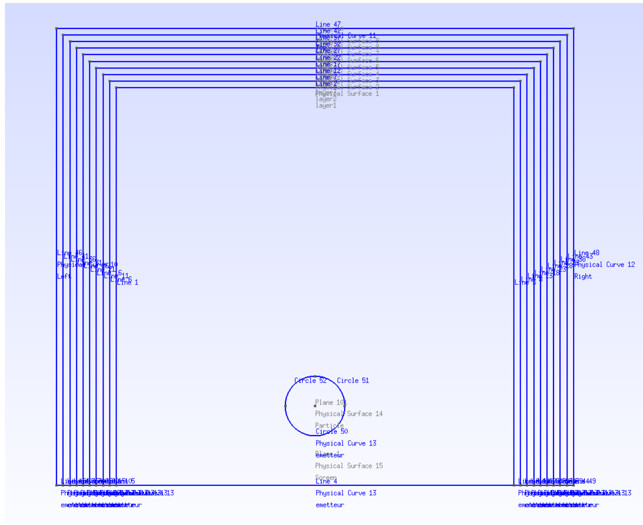
The first one are the boundary condition given in source (2):
$H \wedge n = 0$ and $\langle H, n \rangle = 0$
These boundary condition works fine but in my case led to some reflection.

The other boundary condition that I have designed are absorbing layer that consist in incrementally increasing the reflective index in each layers, which is done by increasing μ_r and ϵ_r.
Absorbing boundary condition allows to have little to none reflection but slows down significantly calculation mainly do to mesh enlargement.

2.3 Mesh

Mesh are Done on GMSH.
There are two background meshes, one for classic BC and one for absorbing BC.
Then we can put any particle we want inside the screen.





2.4 Initial Condition

Initial conditions are simply set to 0 everywhere on the Mesh

3 Use the Model

The Model relies on 3 files: .json file, .cfg file and .geo file, In this section I will explain the purpose of these file and how to modify them to use the model.

3.1 .geo

You have to choose first between Absorbing BC and Classic BC then you can add one particle and attach it to a physical surface named "Particle"

3.2 .json

In the .json file you have to make sure that you import your .geo file and you can adjust multiple variable in the Parameters section:
n and n1 are respectively the environment's reflective index and the particle's reflective index, lambdar is the wave length of your signal, dr is the real diameter of the particle and d is the diameter of the particle in the mesh.

3.3 .cfg

Make sure that the .json file is the corect one and adjust if needed the time-related settings

4 Results

Following is en example of what we can simulate when a signal hit a particle with a higher reflective index using absorbing BC this exmple is inspired by https://www.researchgate.net/publication/243492286

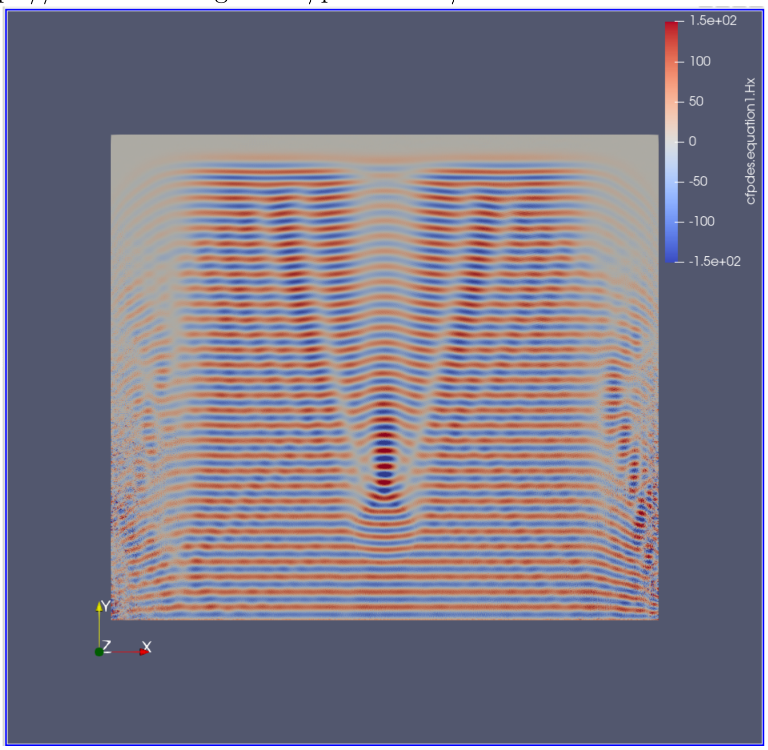


Figure 1: total field (incident plus scattered)

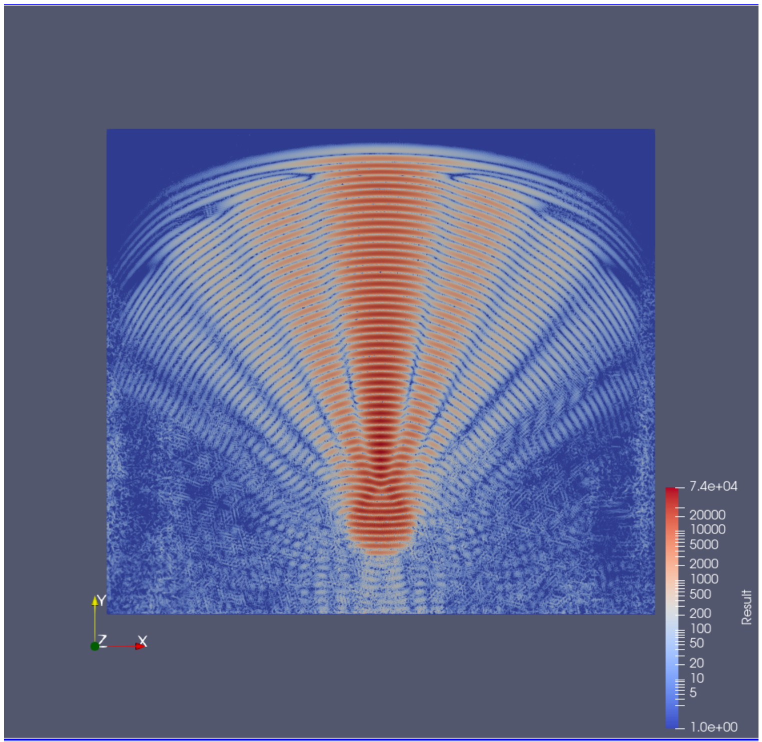


Figure 2: Scattered field (total minus incident) on a log scale

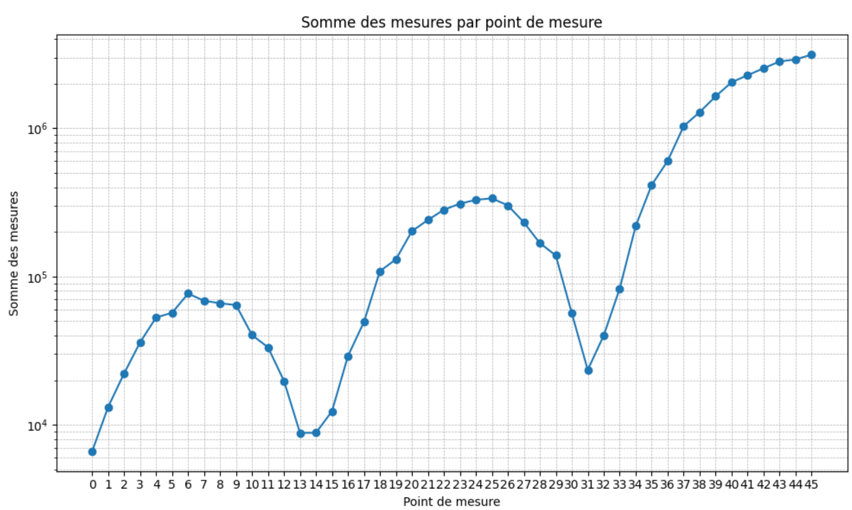


Figure 3: The angular scattering distribution for incident light

The results seems to be going in the right direction but fail to be exactly similar to the studie.

5 Sources

1. https://www.met.reading.ac.uk/clouds/maxwell/
2. https://arxiv.org/pdf/2302.02860 (chapter 2.1)
3. https://docs.feelpp.org/toolboxes/latest/cfpdes/manual.html
4. https://www.techno-science.net/glossaire-definition/Theorie-de-Mie.html
5. https://www.researchgate.net/publication/243492286