Geometric View Factors For Radiative Thermal Simulation

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Introduction

Managing a building's heat performance is important for sustainable design and saving energy. Heat equations are essential for understanding how to manage heat. Heat flow has been categorised into three different modes

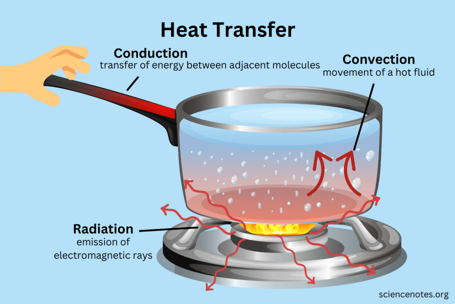


Figure: Heat transfer modes

Heat Transfer and View Factors

Heat transfer can occur by the three previous modes. Thermal conduction is induced by a temperature gradient between two entities in physical contact.

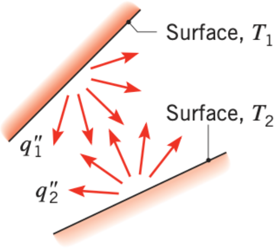


Figure: Net radiation heat exchange between two surfaces

Heat Transfer and View Factors

Radiative heat transfer between two surfaces (noted 1 and 2) is the radiation leaving the first surface for the other minus the one arriving from the second surface. The Stefan-Boltzmann law yields:

$$\phi_{1-2} = \sigma A_1 F_{1-2} \left(T_1^4 - T_2^4\right) \tag{1}$$

where σ denotes the Stefan-Boltzmann constant, A_1 denotes the area of the first surface and F_{1-2} denotes the view factor from 1 to 2 (unitless).

View Factor Between Two Infinitesimal Areas

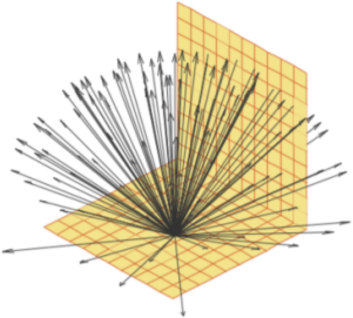
The view factor, F_{ij}, between two small areas, A_i and A_j, is the fraction of radiation emitted by area A_i that is intercepted by area A_j. given by the formula:

$$dF_{ij} = \frac{\cos(\theta_i)\cos(\theta_j)}{\pi R_{ij}^2} dA_i dA_j \tag{2}$$

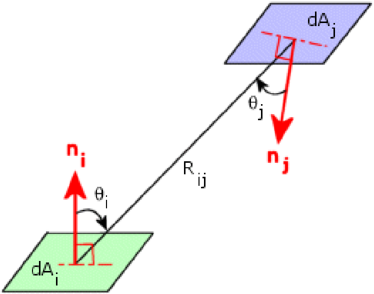
If the two areas are finite, then the view factor given by:

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos(\theta_i)\cos(\theta_j)}{\pi R_{ij}^2} dA_i dA_j \tag{3}$$

View Factor Between Two Infinitesimal Areas



(a) Uniform hemispherical model

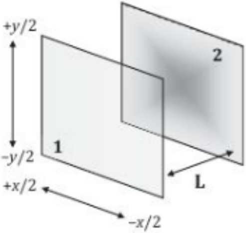


(b) Configuration of two surface elements

Computing View Factors

While view factors can be computed in closed form for certain classes of canonical surfaces without obstacles. For example:

① [a]Finite Parallel Plates [1,2]



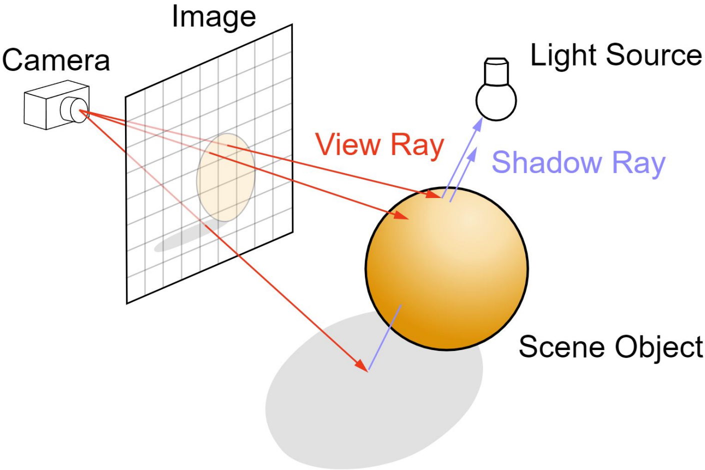
$$A = \frac{x}{L} \quad B = \frac{y}{L}$$

$$F_{1-2} = \frac{2}{\pi AB}\left[\ln\left(\frac{(1+A^2)(1+B^2)}{1+A^2+B^2}\right)^{0.5} + \left(A\sqrt{1+B^2}\right)\cdot \tan^{-1}\left(\frac{A}{\sqrt{1+B^2}}\right) + \left(B\sqrt{1+A^2}\right)\cdot \tan^{-1}\left(\frac{B}{\sqrt{1+A^2}}\right) - A\tan^{1}A - B\tan^{-1}B\right]$$

Figure: View factor between two parallel rectangles

Ray Tracing

Ray tracing is a technique used to compute the view factor between two surfaces.



BVH

Bounding Volume Hierarchy (BHV) is a technique used to accelerate the ray tracing computation.

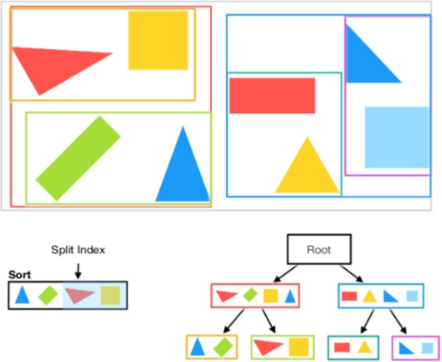
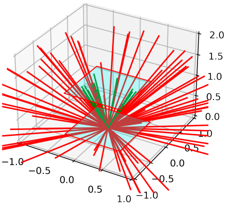


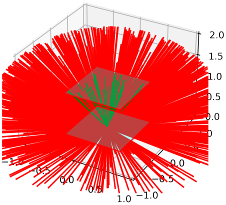
Figure: BHV construction

Monte Carlo Method

Monte Carlo methods are a set of statistical simulation techniques used to estimate numerical results based on random sampling



(a) 100 rays



(b) 1000 rays

Tests : Triangular cavity

The geometry consists of three rectangles made of highly conductive material, mutually isolated by small regions of thermal insulator at the corners of the cavity

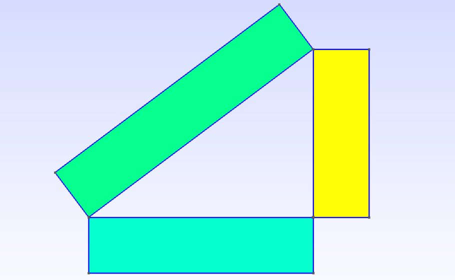


Figure: Triangular cavity

Results: Triangular cavity

The results of the computation are shown in the following matrices:

$$\text{Approximation value} = \begin{pmatrix} 0 & 0.250018 & 0.748253 \\ 0.333427 & 0 & 0.665082 \\ 0.599727 & 0.399716 & 0 \end{pmatrix}$$

$$\text{Exact value} = \begin{pmatrix} 0 & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{3}{5} & \frac{2}{5} & 0 \end{pmatrix}$$

$$\text{Error value} = \begin{pmatrix} 0 & 1.8 \times 10^{-5} & 1.747 \times 10^{-3} \\ 9.4 \times 10^{-5} & 0 & 1.585 \times 10^{-3} \\ 2.73 \times 10^{-4} & 2.84 \times 10^{-4} & 0 \end{pmatrix}$$

Applications: Triangular cavity

The required temperatures (fluxes) are imposed as Dirichlet (Neumann) boundary conditions on the external surfaces of the rectangles which are parallel to the cavity boundaries. Homogeneous Neumann conditions are imposed on the remaining surfaces of the rectangles.

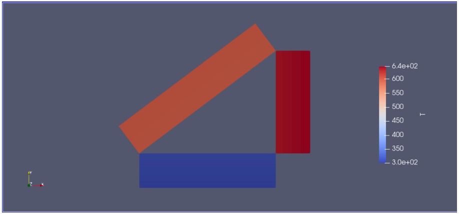
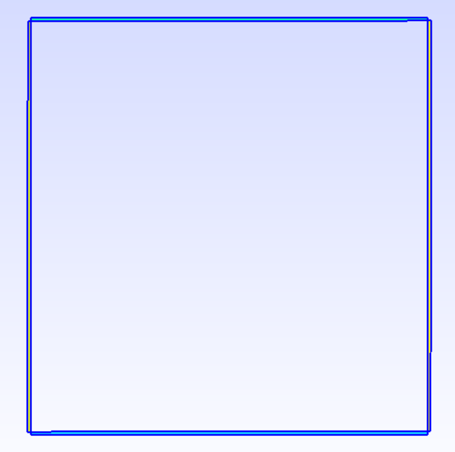


Figure: Temperature distribution in the triangular cavity

Tests : Rectangular cavity

The geometry consists of four rectangles made from highly conductive material. They are separated by small regions of thermal insulator at the corners of the cavity.



Results: Rectangular cavity

The results of the computation are shown in the following matrices:

$$\text{Approximation} = \begin{pmatrix} 0 & 0.292696 & 0.413725 & 0.292696 \\ 0.292696 & 0 & 0.292696 & 0.413725 \\ 0.413725 & 0.292696 & 0 & 0.292696 \\ 0.292696 & 0.413725 & 0.292696 & 0 \end{pmatrix}$$

$$\text{Exact value} = \begin{pmatrix} 0 & 1-\frac{\sqrt{2}}{2} & \sqrt{2}-1 & 1-\frac{\sqrt{2}}{2} \\ 1-\frac{\sqrt{2}}{2} & 0 & 1-\frac{\sqrt{2}}{2} & \sqrt{2}-1 \\ \sqrt{2}-1 & 1-\frac{\sqrt{2}}{2} & 0 & 1-\frac{\sqrt{2}}{2} \\ 1-\frac{\sqrt{2}}{2} & \sqrt{2}-1 & 1-\frac{\sqrt{2}}{2} & 0 \end{pmatrix}$$

$$\text{Error value} = \begin{pmatrix} 0 & 2.93\times10^{-2} & 1.37\times10^{-1} & 2.93\times10^{-2} \\ 2.93\times10^{-2} & 0 & 2.93\times10^{-2} & 1.37\times10^{-1} \\ 1.37\times10^{-1} & 2.93\times10^{-2} & 0 & 2.93\times10^{-2} \\ 2.93\times10^{-2} & 1.37\times10^{-1} & 2.93\times10^{-2} & 0 \end{pmatrix}$$

Applications: Triangular cavity

The required temperatures (fluxes) are imposed as Dirichlet (Neumann) boundary conditions on the external surfaces of the rectangles which are parallel to the cavity boundaries. Homogeneous Neumann conditions are imposed on the remaining surfaces of the rectangles.

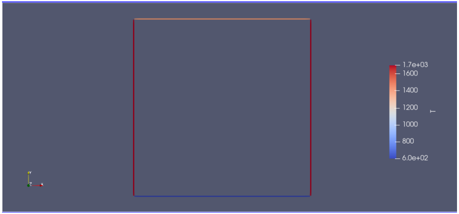


Figure: Temperature distribution in the rectangular cavity

Tests : Cylindrical cavity

The geometry consists of three cylinders/annuli made of highly conductive material. They are mutually isolated by small regions of thermal insulator at the corners of the cavity.

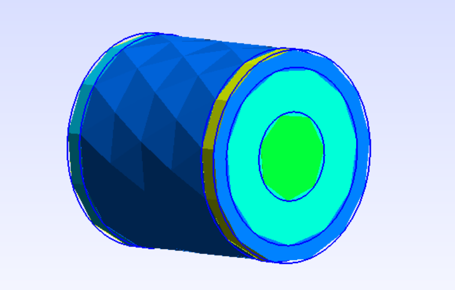


Figure: Cylindrical cavity

Results: Cylindrical cavity

The results of the computation are shown in the following matrices:

$$\text{Approximation} = \begin{pmatrix} 0 & 0.0548874 & 0.945113 \\ 0.0412309 & 0 & 0.944355 \\ 0.0886256 & 0.117885 & 0.764229 \end{pmatrix}$$

$$\text{Exact value} = \begin{pmatrix} 0 & 9 - \frac{\sqrt{320}}{2} & \sqrt{320} - 8 \\ 9 - \frac{\sqrt{320}}{2} & 0 & \frac{\sqrt{320}}{2} - 8 \\ \frac{1}{10}\left(\frac{\sqrt{320}}{2} - 8\right) & \frac{3}{25}\left(\frac{\sqrt{320}}{2} - 8\right) & 1 - \frac{11}{50}\left(\frac{\sqrt{320}}{2} - 8\right) \end{pmatrix}$$

$$\text{Error value} = \begin{pmatrix} 0 & 5.49 \times 10^{-2} & 5.49 \times 10^{-2} \\ 5.49 \times 10^{-2} & 0 & 5.49 \times 10^{-2} \\ 5.49 \times 10^{-2} & 5.49 \times 10^{-2} & 5.49 \times 10^{-2} \end{pmatrix}$$

Applications: Cylindrical cavity

The required temperatures are imposed as Dirichlet boundary conditions on the external surfaces which are parallel to the cavity boundaries. Homogeneous Neumann conditions are imposed on the remaining surfaces.

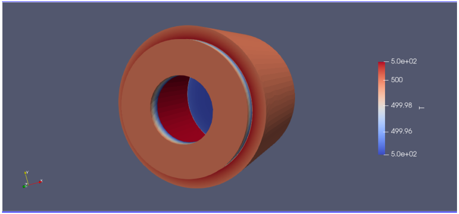


Figure: Temperature distribution in the rectangular cavity

Conclusion

The more accurately the view factor is calculated, the better our accurately of radiative heat transfer within the geometry .

Thank you for your attention !