The Study of The N-Links Swimmer for Monoflagellate Micro-swimmers

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- Microrobotics is a field with diverse applications, such as environmental monitoring, medical procedures, and micro-assembly.
- Designing microrobots that can efficiently swim in fluids is challenging due to the dominance of viscosity over inertia at the microscale. Researchers have looked to biological microorganisms like bacteria and sperm cells,
- which have evolved effective swimming mechanisms for low-Reynolds number.
- One of the pioneering works in this field taylor [7] who established the mathematical settings for the problem of biological self-propulsion powered by thin undilating filaments.

Reynolds Number

In this work, we focus on low Reynolds numbers, where viscous forces dominate over inertial forces, making all movement reversible.

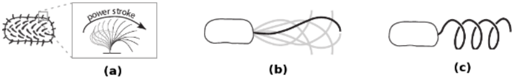


Figure: (a) : Locomotion methods at Micro-scale [1] : Propulsion by cilliary beating, (b):Propulsion by flagellar beating, (c) : Corkscrew-type rotating propulsion

- I studied the 3D model of N-segment micro-swimmers based on the thesis [2].
- I worked on validating the model using an already implemented code.
- The validation was carried out in two steps:
    - First, by studying the planar movement [3].
    - Then, by examining the non-planar movement [6].
- I performed numerical simulations to analyze the factors influencing the micro-swimmer's movement.
- Subsequently, I focused on the optimal control of flagellar movement.
- For the tools used during this internship, I used Python, GitHub, and Slack.

- The main challenge was balancing model accuracy for reliable swimmer movement prediction with numerical simplicity for optimal control.
- We used resistive force theory (RFT), introduced by Gray and Hancock [5], to model hydrodynamics and discretized the tail into N thin rods.

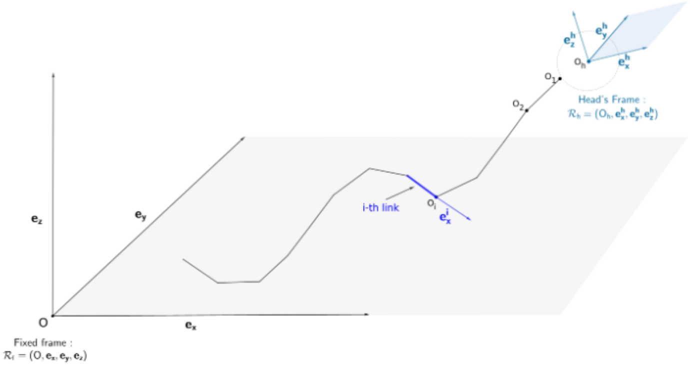


Figure: The 3D swimmer model features an N-link discretization of the tail [2]. The swimmer's head is oriented relative to a fixed Galilean reference frame. Each link i, is oriented relative to the head frame R_h.

- The model uses a moving frame of reference attached to the head of the swimmer $R_{\text{head}} = (X, e_x^h, e_y^h, e_z^h)$.
- The position of each segment O_{i+1} is given by $O_{i+1} = O_i - l e_x^i$.
- The rotational matrix $R_{\text{head}} \in SO(3)$ is defined by $R_{\text{head}} e_x = e_x^h$, $R_{\text{head}} e_y = e_y^h$, $R_{\text{head}} e_z = e_z^h$.
- Using rotation angles θ_x, θ_y, θ_z, $R_{\text{head}} = R_x(\theta_x) R_y(\theta_y) R_z(\theta_z)$.

- The position of the i-th link: $O_i = X - rR_{\text{head}}e_x - l\sum_{k=1}^{i-1} R_{\text{head}}R_k e_x$.
- Angular velocity vectors Ω_h for the head and Ω_i for the i-th link are determined by antisymmetric matrices.
- Hydrodynamic force on the head: $F_{\text{head}} = -R_{\text{head}} \begin{pmatrix} k_{\parallel} & 0 & 0 \\ 0 & k_{\perp} & 0 \\ 0 & 0 & k_{\perp} \end{pmatrix} R_{\text{head}}^T \dot{X}$.
- Torques on the head: $T_{\text{head}} = -k_r \Omega_{\text{h}}$.

- Position of a point $x_i(s)$ on the i-th segment:
  $x_i(s) = X - rR_{\text{head}}e_x - l\sum_{k=1}^{i-1} R_{\text{head}}R_k e_x - sR_{\text{head}}R_i e_x$.
- Hydrodynamic force density $f_i(s)$: $f_i(s) = S_i\dot{x}_i(s)$, where
  $$S_i = (R_i R_{\text{head}})D(R_{\text{head}}R_i)^T \text{ and } D = -\begin{pmatrix} k_\parallel & 0 & 0 \\ 0 & k_\perp & 0 \\ 0 & 0 & k_\perp \end{pmatrix}.$$
- Linear hydrodynamic force: $f_i(s) = A_i(s)\begin{pmatrix} \dot{X} \\ \dot{\Theta} \\ \dot{\Phi} \end{pmatrix}$.
- Hydrodynamic force on the i-link: $F_i = B_i\begin{pmatrix} \dot{X} \\ \dot{\Theta} \\ \dot{\Phi} \end{pmatrix}$.
- Torques on the i-link:
  $$T_i = -\left(\left[rR_{\text{head}}e_x + \sum_{k=1}^{i-1} lR_{\text{head}}R_k e_x\right] B_i\right)\begin{pmatrix} \dot{X} \\ \dot{\Theta} \\ \dot{\Phi} \end{pmatrix} - \left(\left[R_{\text{head}}R_i\right] C_i\right)\begin{pmatrix} \dot{X} \\ \dot{\Theta} \\ \dot{\Phi} \end{pmatrix}.$$

The equation of the swimmer's motion, relying on self-propulsion, is given by:

$$\begin{cases} F_h + \sum_{i=1}^{N} F_i = 0, \\ T_h + \sum_{i=1}^{N} T_i = 0. \end{cases} \tag{1}$$

This system can be rewritten in matrix form as:

$$M(\Theta, \Phi) \begin{pmatrix} \dot{X} \\ \dot{\Theta} \\ \dot{\Phi} \end{pmatrix} = 0, \tag{2}$$

where M is a matrix of dimension $6 \times (2N + 6)$.

The matrix $M(\Theta,\Phi) \in \mathbb{R}^{6\times(2N+6)}$ can be subdivided into $M_{X,\Theta}(\Theta,\Phi) \in \mathbb{R}^{6\times 6}$ and $M_{\Phi}(\Theta,\Phi) \in \mathbb{R}^{6\times 2N}$:

$$M(\Theta,\Phi) = \left(M_{X,\Theta}(\Theta,\Phi) \mid N_{\Phi}(\Theta,\Phi)\right). \tag{3}$$

Assuming $\dot{\Phi}(t)$ values are given, the position and orientation of a self-propelled swimmer can be obtained by solving:

$$\begin{pmatrix} \dot{X} \\ \dot{\Theta} \end{pmatrix} = M_{X,\Theta}^{-1}(\Theta,\Phi) N_{\Phi}(\Theta,\Phi)\dot{\Phi}. \tag{4}$$

The dynamics of the swimmer can be described by a linear control system where $u(t)$ represents the derivatives of Φ:

$$\begin{pmatrix} \dot{\Phi} \\ \dot{X} \\ \dot{\theta} \end{pmatrix} = \tilde{M}_{X,\Theta}^{-1}(\Theta,\Phi)\tilde{M}_{\Phi}(\Theta,\Phi)u(t). \tag{5}$$

The deformation of the tail over time

The flagellum's position $r(s,t)$ is determined by:

$$r(s,t) = \frac{L_{\text{head}}}{2} e_1(t) + \int_0^s \left[\cos(\psi(u,t)) e_1(t) + \sin(\psi(u,t)) e_2(t)\right] du,$$

where $\psi(s,t)$ is:

$$\psi(s,t) = K_0 s + 2A_0 s \cos(\omega t - \frac{2\pi s}{\lambda}),$$

with λ as the wavelength, ω the angular frequency, K_0 the asymmetry measure, and A_0 the curvature amplitude.

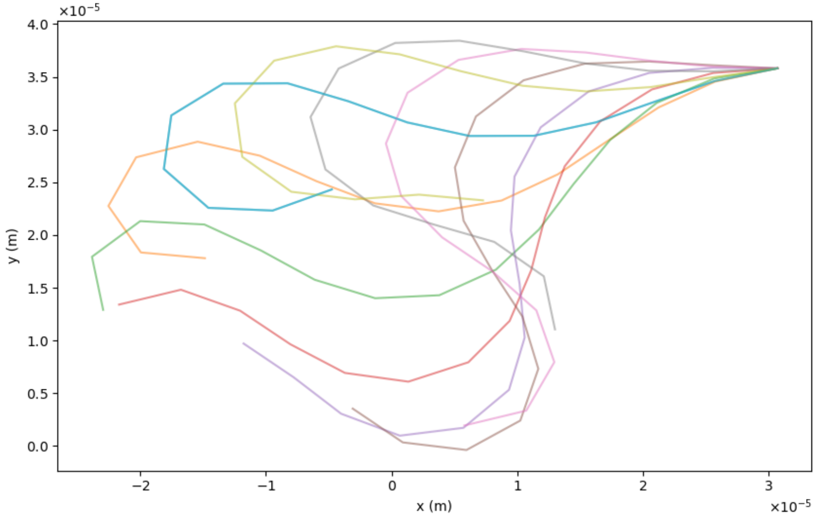


Figure: the deformation of the tail over the time

The Motion of the Head over the time

By solving the equation associated with the swimmer, we obtain:

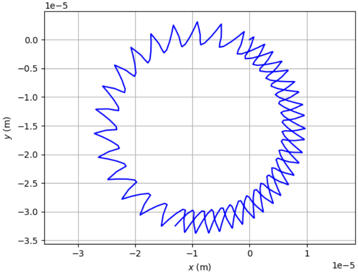
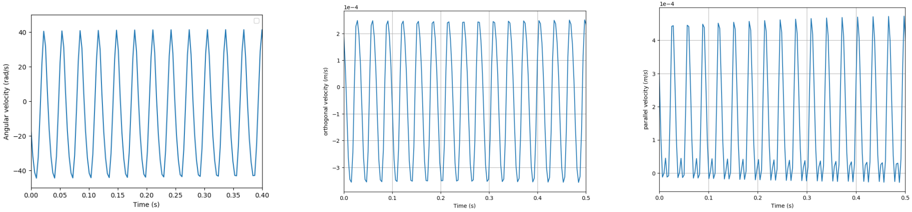
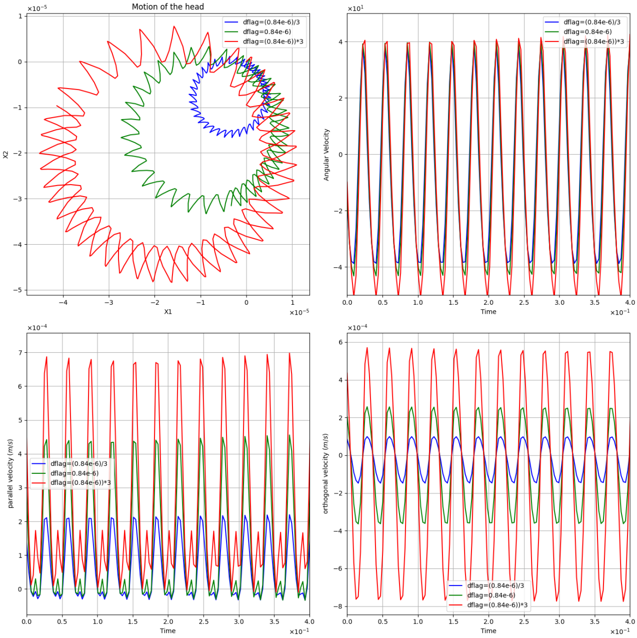


Figure: the motion of the head in (e_x, e_y) frame



(a) (b) (c)

Figure: (a) Angular velocity of the swimmer head, (b) orthogonal velocity of the swimmer head and (c) parallel velocity of the swimmer head.


Motion of the head
dflag=(0.84e-6)/3
dflag=0.84e-6)
dflag=(0.84e-6))*3
X1
X2
×10⁻⁵
Angular Velocity
Time
×10¹
×10⁻¹
parallel velocity (m/s)
×10⁻⁴
orthogonal velocity (m/s)

The equation of the motion in non planar movement is expressed by :

$$\begin{cases} \frac{d\mathbf{r}(\mathbf{s},\mathbf{t})}{ds} & = \psi_1(s,t), \\ \frac{d\psi_1(s,t)}{ds} & = k_f \psi_3(s,t), \\ \frac{d\psi_2(s,t)}{ds} & = \tau_f \psi_3(s,t), \\ \frac{d\psi_3(s,t)}{ds} & = -k_f \psi_1(s,t) + \tau_f \psi_2(s,t). \end{cases} \tag{6}$$

with $\psi_i \in \mathbb{R}^3$, $i \in \{1,2,3\}$, k_f and τ_f are the flagellar curvature and twist, respectively. The expression is given by :

$$k_f(l,t) = K_0 + B\cos(\omega_0 t - \lambda l), \tag{7}$$

The motion of the head over the time

The resolution of the equation above for each t provides us with the position of the swimmer's tail over time, as well as ψ, and then the derivatives Φ

$$\phi_y^i = \arctan\left(\frac{(\psi_1(0^i, t))_2}{(\psi_1(O^i, t))_1}\right), \tag{8}$$

$$\phi_z^i = \arctan\left(\frac{(\psi_3(O^i, t))_2}{(\psi_3(O^i, t))_1}\right), \tag{9}$$

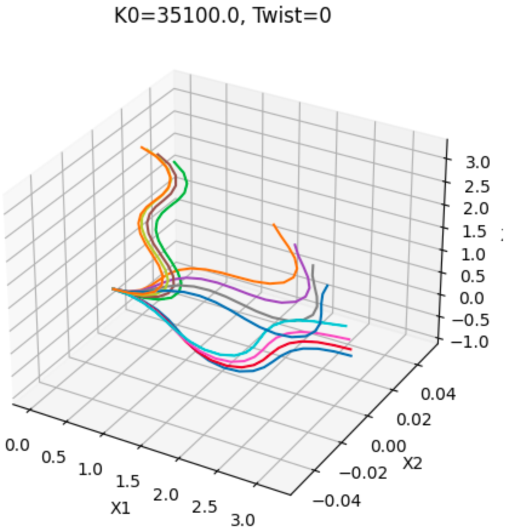
The angular derivative is approximated by a central finite difference. We consider Δt to be the time step size defined as $\frac{T_{\text{final}}}{NT-1}$, where T_{final} is the total simulation time.

$$\dot{\phi}_j(t) = \frac{\phi_j(t+\Delta t) - \phi_j(t-\Delta t)}{2\Delta t} + O(\Delta t^2),$$

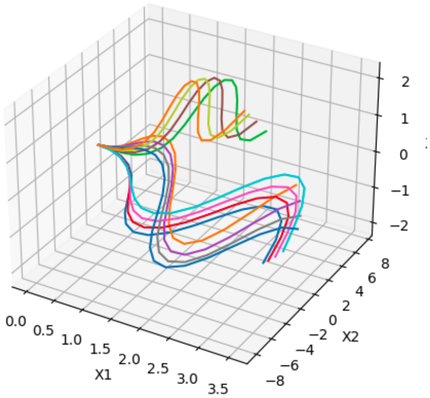
and for $t = 0$, we have:

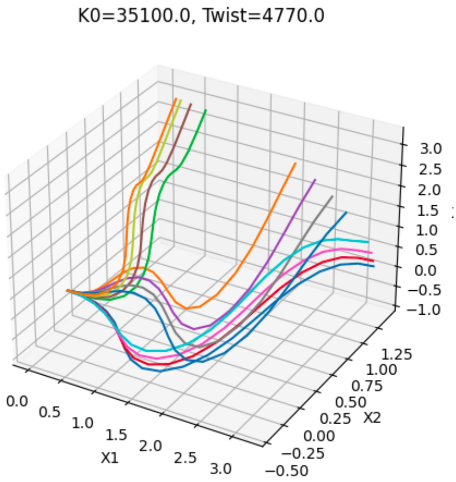
$$\dot{\phi}_j(0) = \frac{\phi_j(\Delta t) - \phi_j(-\Delta t)}{2\Delta t}.$$

We was able to have the results of the deformation of the tail which are similary to those presented in this article [6]

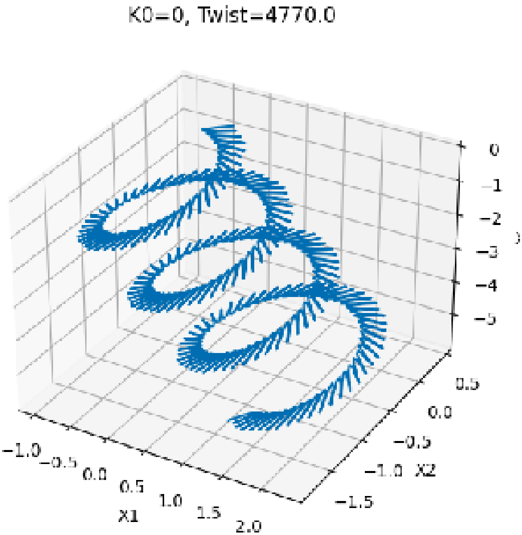

K0=35100.0, Twist=0
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.04
0.02
0.00
X2
−0.02
−0.04
0.0
0.5
1.0
1.5
2.0
X1
2.5
3.0

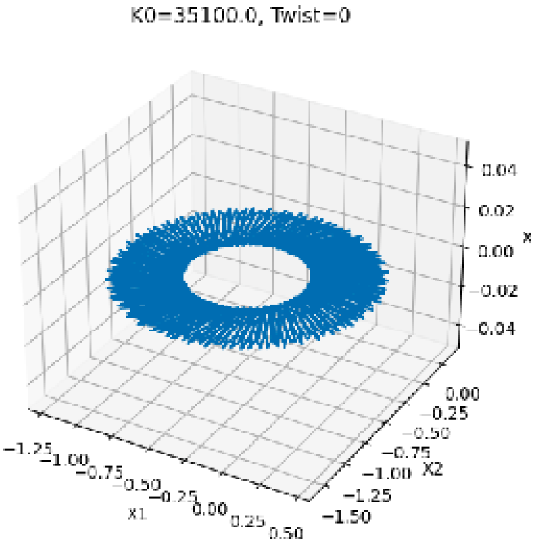
K0=0, Twist=4770.0

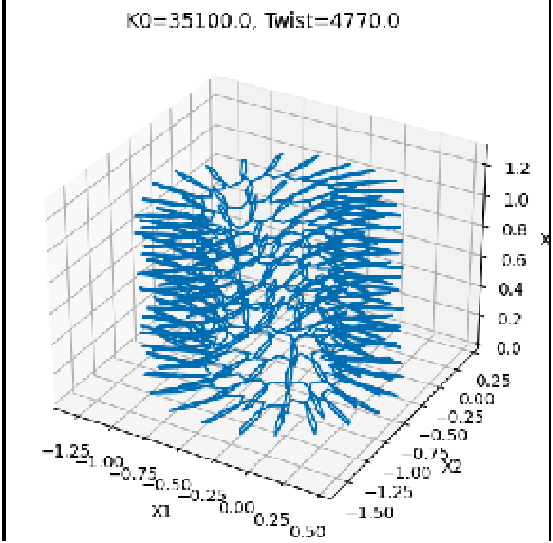



K0=35100.0, Twist=4770.0
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
X1
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
X2

The motion of the head over the time




K0=35100.0, Twist=0
0.04
0.02
0.00
−0.02
−0.04
x
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
X2
−1.25
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
X1


K0=35100.0, Twist=4770.0
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
X2
−1.25
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
X1

Optimal Control

The problem considered in this Chapter is as follows: Let $(n, m) \in \mathbb{N}^*$, I be an interval of $\mathbb{R}$, and f a function from $\mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m$ to $\mathbb{R}^n$ chosen to be sufficiently regular. Let U be a subset of $\mathbb{R}^m$ and let $x_0 \in \mathbb{R}^n$. The linear control system of interest is:

$$\begin{cases} \dot{x}(t) = f(t, x(t), u(t)) \quad \forall t \in I \\ x(0) = x_0. \end{cases} \tag{10}$$

where the set of controls u considered is the set of measurable and bounded functions on I, taking values in the subset U. Assuming that the system is controllable, it is defined by:

$$\begin{cases} \inf\limits_{u \in U_{\text{ad}}} C(u,t) = g(x(T),T) + \int_0^T f(x(t),u(t))\,dt, \\ \dot{x}(t) = f(t,x,u) \\ x(0) = x_0. \end{cases} \tag{11}$$

wth U_{ad} represents the set of all admissible controls that can be applied to a system.

Optimal Control Problems for Train Motion

we use a rectilinear motion of a train, we first maximize the distance traveled from the origin while considering energy expended:

$$\inf C(u, T) = -x(T) + \int_0^T \|u(t)\|_2^2 \, dt$$

with

$$\begin{cases} u \in U_{\text{ad}} = \{u \in L^2([0, T], \mathbb{R})\} \\ \dot{x}(t) = y(t) \\ y(t) = u(t) \quad \forall t \in [0, T] \\ x(0) = 0 \\ y(0) = 0. \end{cases}$$

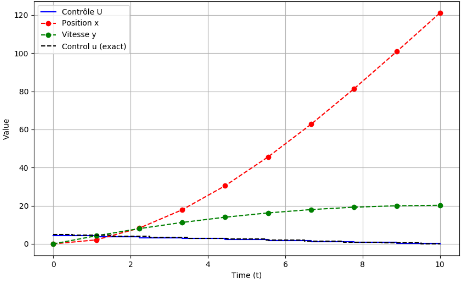
The exact solution is $u(t) = \frac{T-t}{2}$.

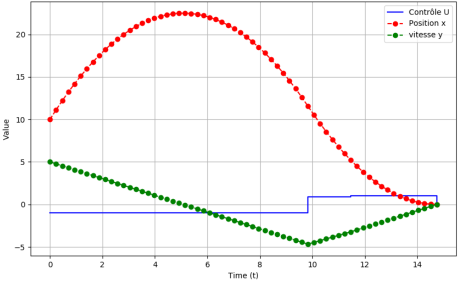
Next, we minimize the time to return the train to a station while avoiding a crash, imposing a control constraint:

$$\inf C(u, T) = T$$

with

$$\begin{cases} u \in U_{\text{ad}} = \{u \in L^2([0, T], \mathbb{R}) \text{ such that } |u(t)| \leq 1 \quad \forall t \in [0, T]\} \\ \dot{x}(t) = y(t) \\ y(t) = u(t) \quad \forall t \in [0, T] \\ x(0) \neq 0 \\ y(0) \neq 0 \\ x(T) = 0 \\ y(T) = 0. \end{cases}$$


Contrôle U
Position x
Vitesse y
Control u (exact)
Value
Time (t)
0
20
40
60
80
100
120
0
2
4
6
8
10


Contrôle U
Position x
vitesse y
Value
Time (t)
20
15
10
5
0
−5
0
2
4
6
8
10
12
14

Optimal Control of N-link Micro-swimmers

Optimal control of N-link micro-swimmers in low Reynolds number fluids is crucial. In [4], controllability and optimal movement strategies are explored to achieve objectives like displacement or orientation while minimizing energy or time.
The equation is:

$$c(u, T) = -x(T) + \int_0^T \|u(t)\|^2 \, dt$$

where $x(T)$ is the final state to maximize, and $\|u(t)\|^2$ is the control effort to minimize.
Full implementation was not achieved due to time constraints.

Conclusion and Future Work

Summary of Contributions:

- **Model Understanding:** Comprehensive study and comparison of the model with existing literature.
- **Simulations:** Conducted simulations to observe the influence of drag coefficients on the movement of the flagellum.

Future Work:

- **Optimal Control:** Implement optimal control strategies for micro-swimmers.
- **Extended Simulations:** Further simulations to validate control strategies and explore new scenarios.

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