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the Ensemble Kalman Filter (EnKF)

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Overview

Contexte and Objectives

Context

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Study of the application of the Ensemble Kalman Filter (EnKF) to nonlinear dynamic systems, specifically the Lorenz system.

The Ensemble Kalman Filter[1] is a powerful tool for state estimation in nonlinear and high-dimensional systems.

Objectives

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1. To determine how effective the Ensemble Kalman Filter (EnKF) is in tracking and predicting the behavior of the Lorenz system.
2. To examine how different parameters, such as observation noise levels and ensemble size, influence the performance of the EnKF.

Background

Lorenz system

$$\begin{cases} \dfrac{dx}{dt} = \sigma(y - x) \\ \dfrac{dy}{dt} = x(\rho - z) - y \\ \dfrac{dz}{dt} = xy - \beta z \end{cases}$$

where:[2]

- x is the rate of convective overturning,
- y is the horizontal temperature variation,
- z is the vertical temperature variation,
- σ,ρ, β three physical parameters.

Explicit Euler Method

Method Description:

- Simple and effective for step-by-step approximation.
- Uses the derivative to estimate state changes.

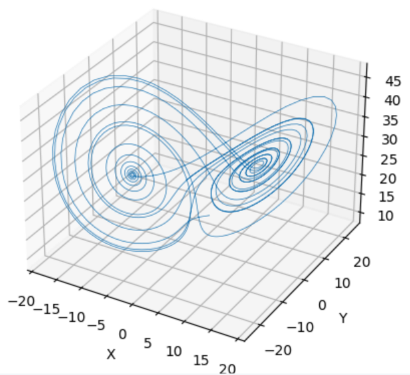

Figure: Lorenz system solved with Explicit Euler

Ensemble Kalman Filter (EnKF)

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The Ensemble Kalman Filter [3] works by :

- **Propagation through the Model** we use our dynamic model to predict future states by running multiple state samples through it.
- **Update with Observations** we adjust these predictions based on actual observations to get a more accurate estimate of the system's state.

Implementation and Results

EnKF Algorithm Steps

1. Initialization: Generate initial state samples $\{x_i^0\}_{i=1}^N$.
2. Prediction: each ensemble member is projected to the next state using the dynamic model. [4]:

$$x_i^{\text{pred}} = f(x_i^{\text{upd}}) + w_i$$

3. Update: Adjust predictions based on observations:

- Compute predicted observation:

$$z_i^{\text{pred}} = h(x_i^{\text{pred}}) + v_i$$

- Calculate Kalman gain:

$$K_k = P_k^{\text{pred}} H^T (H P_k^{\text{pred}} H^T + R)^{-1}$$

- Update state estimates:

$$x_i^{\text{upd}} = x_i^{\text{pred}} + K_k(z_k - h(x_i^{\text{pred}}))$$

Results

EnKF Trajectories and Deviations

The filter follows the true path but shows some deviations due to noisy observations.

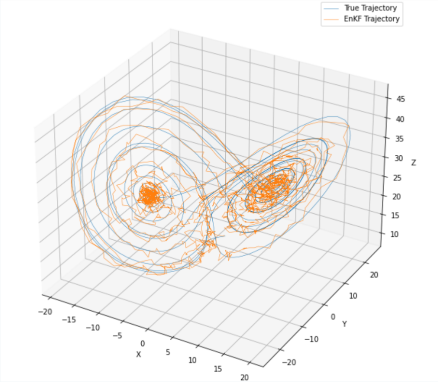

Figure: Lorenz system with EnKF

Results

Uncertainty of Estimates and Absolute Errors

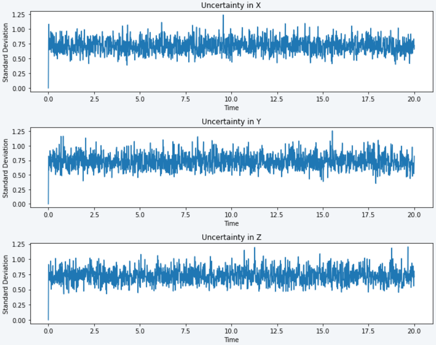

Figure: Uncertainty

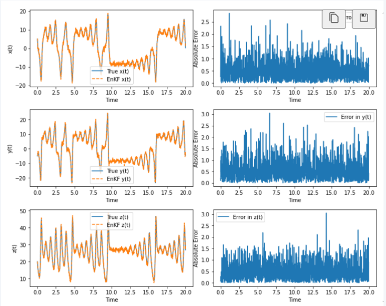

Figure: Individual Trajectories and Absolute Errors

Results

Impact of Observation Covariance Matrix (R)

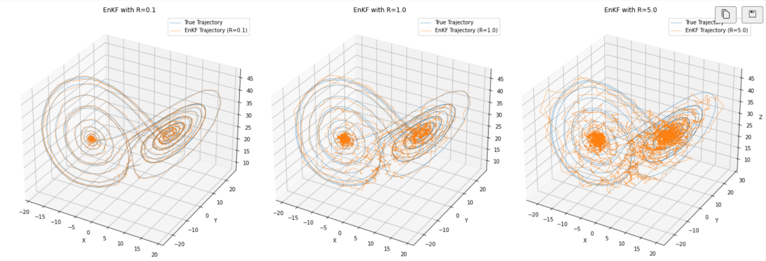

Figure: Lorenz System Analysis Using Ensemble Kalman Filter for Different R Values

- When R is low, the filter heavily relies on observations.
- When R is high, the filter gives less weight to the observations.

Results

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Impact of Ensemble Size (N)

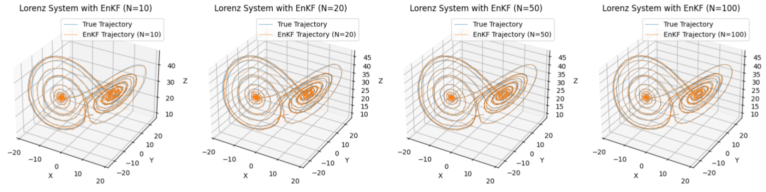

Figure: Influence of the Number of Ensembles N

- When N is small, the EnKF has trouble capturing the system's complexity.
- With a larger N, we cover more states.

Conclusion

Conclusion

Effectiveness of EnKF:

- The Ensemble Kalman Filter (EnKF) has proven effective in estimating the states of the Lorenz system, even in the presence of uncertainties and noise.

Sensitivity to Parameters:

- The performance of the EnKF is influenced by observation noise levels and ensemble size.

References I

[1] W. contributors, *Ensemble kalman filter*, [Online]. Available: https://en.wikipedia.org/wiki/Ensemble_Kalman_filter.

[2] Wikipédia, *Système dynamique de lorenz*, [Online]. Available: https://fr.wikipedia.org/wiki/Syst%C3%A8me_dynamique_de_Lorenz.

[3] R. R. L. Jr, *Kalman and bayesian filters in python*, May 2020. [Online]. Available: https://github.com/rlabbe/Kalman-and-Bayesian-Filters-in-Python/.

[4] M. A. Iglesias, K. J. Law, and A. M. Stuart, "Ensemble kalman methods for inverse problems," *Inverse Problems*, 2013.

Thank you for your attention

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