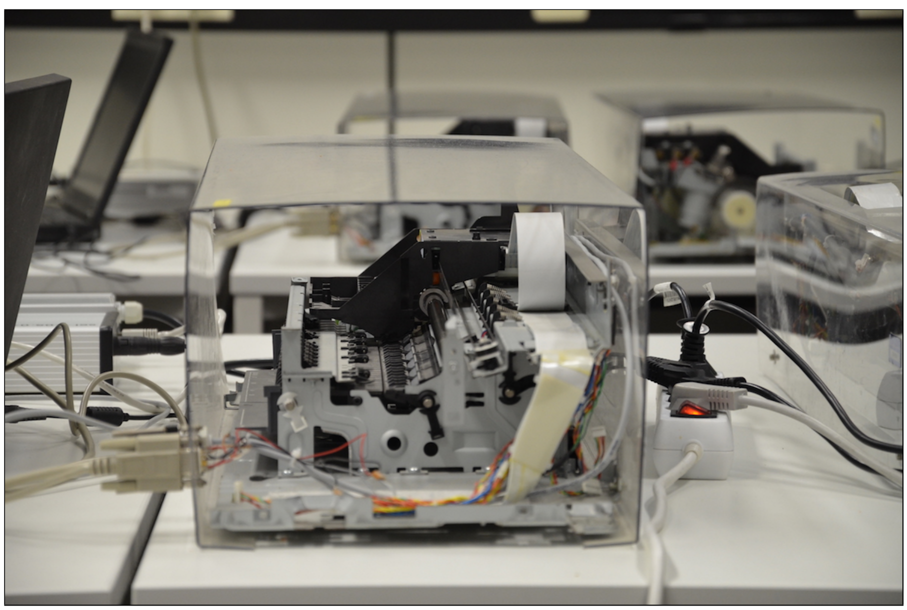
Hand-in date: Friday June 14, 2019, 17.00 hours.

- All exercises refer to the book [2] with ISBN978-0-470-01168-3. Make sure you use the correct version, since old versions of the book have different exercises.
- It is advised to use standard `Matlab` toolboxes. The use of other toolboxes, including `diet` and `shapeit` is not supported and not advised to be used.
- Besides providing the correct values, the following criteria are used for evaluation:
  1. please write down your answers in a concise way, and
  2. explain the techniques you use and interpret the answers (especially from a system theoretic point of view).

15. **Frequency domain ILC applied to a printer system**

In this exercise, a frequency domain ILC controller is designed and implemented on a simulation model of a printer system, see the figure, below. Indeed, repeating tasks are standard in printing systems, see also [1].

Before starting the exercise, load the frequency response function, the parametric model G and controller C. Inspect the results: continuous/discrete time model? sampling frequency/time? Nyquist frequency? Bode diagram: collocated/non-collocated2 mass-spring-damper system? model order? model stable? what kind of controller? controller stable? controller order? Closed-loop stable? bandwidth, gain margin, phase margin, modulus margin?



Printer system.

15.a Perform a simulation with the feedback controller implemented. Set `N_trial = 1` and use `SimILC.m`. Inspect the servo error. What is the achieved accuracy?

15.b Implement a mass feedforward on the simulation model, i.e., $f = C^{\text{ff}}\ddot{r}$ Derive a suitable value of m based on G. Implement the result by implementing the feedforward controller in `FeedforwardUpdate.m`, set `N_trial = 2`, and using `SimILC.m`. What accuracy can you achieve?

15.c Next, we proceed with designing a frequency domain ILC controller. Construct L using `zpetc.m`. Construct both the causal L_c and the non-causal L. Investigate the quality of the inverse using a Bode diagram. How large are magnitude and phase errors for L and L_c? How large is `p+d` and why? What is the order of L_c?

15.d Investigate whether the designed L leads to a convergent ILC scheme when evaluated for the model GS. Next, investigate whether the designed learning filter L leads to a convergent ILC scheme when evaluated for the frequency response function of GS. (hint: use `bodemag.m`.)

15.e Design a robustness filter Q to ensure that the convergence condition is met for both the model and the frequency response function (hint: use `butter.m`). Notice that the frequency response function should be ignored below 20 Hz due to a poor signal to noise ratio. Generate relevant (Bode) plots and explicitly indicate which function you plot.

15.f Implement the controller in `FeedforwardUpdate.m`, set `N_trial` to your desired value, and use `SimILC.m` to run simulations. Interpret the results. Do the results coincide with your expectations?

15.g Incorporate a learning gain α, i.e. replace L by αL. Explain the purpose of α. What is a suitable range for α given its purpose? What is the admissible range of α to ensure convergence (assuming $L = (GS)^{-1}$ and $Q = 1$)? Investigate the influence of the learning gain on the convergence speed using simulations.

16. **Frequency domain ILC applied to a printer system: Experiments**

Suggested exercise. This exercise is NOT compulsary. An alternative exercise 16 is provided at the end of this document that does not require performing experiments. It is advised to do the experiments, since they are interesting and typically do not take too much time.

16.a (**Experiment.**) Implement the obtained mass feedforward of Exercise b on the experimental setup. Implement the feedforward controller in `feedForwardUpdate.m`, set `N_trial = 2`, and using `ExpILC.m`. Compare the result without (trial 1) and with (trial 2) feedforward. What is the achieved accuracy? What is the performance increase/decrease? Compare this with the result of Exercise b and explain the observed differences. Try to tune the value of m further, what accuracy can you achieve?

16.b (**Experiment**) Implement the designed controller in Exercise f and g experimentally. Start with a small learning gain α. Interpret the results, is convergence achieved? What is the peak servo error and 2-norm of the servo error? Investigate the dependence of the results on Q and α. What is the best servo performance that can be achieved in terms of peak and 2-norm?

17. **Lifted ILC applied to the printer system**

In this exercise, lifted ILC is applied to the printer system.

17.a For lifted ILC, a model of the system is required in the lifted domain. In particular, the matrix J will contain the impulse response of GS. Set the trial length N equal to 100 and construct J (hint: use `dimpulse` and `toeplitz`. In addition, set small elements of J equal to zero to facilitate the later computations). Inspect its structure (hint: use `mesh` and `imagesc`). Is the matrix triangular? Toeplitz? Is the diagonal equal to zero? How does this relate to system properties, e.g., causality, time (in)variance, properness?

17.b Next, a lifted ILC controller will be designed. Keep $N = 100$, to facilitate fast computations. Set $W_e = w_e I$, $W_f = w_f I$, and $W_{\Delta f} = w_{\Delta f} I$, and choose as initial values, e.g., $w_e = 10^3$, $w_f = 10^{-3}$, $w_{\Delta f} = 10^{-1}$. Compute the optimal lifted ILC controller L and Q. Inspect the resulting matrices L and Q (hint: use `plot(diag(X,i))`) to plot the i^{th} diagonal of the matrix X (use negative i to obtain lower diagonals))). What is the structure? How does this relate to causality and time (in)variance? Vary the values of w_e, w_f, $w_{\Delta f}$. What happens to L and Q if w_f is set to zero? How should the values be chosen to guarantee $e_\infty = 0$?

17.c Investigate convergence and monotonic convergence properties of the lifted ILC controller.

17.d Set $N = 4501$, i.e., the length of the servo task. Implement and simulate the lifted ILC controller. Investigate the influence of w_e, w_f, $w_{\Delta f}$ on convergence speed and ILC performance.

17.e Compare the lifted ILC controller to the frequency domain ILC controller. Comment on

(a) how to design for zero converged error,
(b) how to attenuate trial varying disturbances,
(c) how to address model errors,
(d) computational speed for the used implementation (note that there are more efficient implementations of lifted ILC, see lecture slides), and
(e) causality and time-(in)variance of the ILC controller.

18. Let $J = 1$ with trial length $N = 1$. Assume the ILC controller $f_{j+1} = Qf_j + Le_j$ with $Q = -0.8$, $L = -0.4$. Set the reference $r = 1$ and perform 20 iterations. Plot e_k and f_k. What can you conclude from the plots regarding convergence? Monotonic convergence?

19. In lifted ILC, we used the notion of monotonic convergence with respect to the common vector 2-norm. Of course, other choices are valid, e.g., the vector 1-norm and ∞-norm. Does monotonic convergence in one of these norms imply monotonic convergence in other norms? Analyze (preferably using a solid theory, using, e.g., results of [2], otherwise consider numerical simulation), the following cases:

1.
$$J = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} \\ 0 & -\frac{3}{4} \end{bmatrix} \tag{1}$$

2.
$$J = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} -\frac{1}{4} & 0 \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix} \tag{2}$$

20. Consider the system
$$J(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|c} 0.5 & 1 \\ \hline 1 & 1 \end{array}\right] \tag{3}$$

and the ILC controller $f_{j+1} = Qf_j + Le_j$, with

$$Q = 1 \tag{4}$$

$$L = \alpha, \quad \alpha \in \mathbb{R}. \tag{5}$$

In terms of the underlying feedback loop, the closed-loop system corresponds to $J = SP$ and $C = 1$.

1. Consider an infinite time interval, i.e., frequency-domain ILC. For which values of α is the system convergent?
2. Next, the ILC scheme (4), (5) is implemented in finite time with $N = 2$. What is the admissible range of α for convergence? And for monotonic convergence? Note: to analyze monotonic convergence, you are allowed to pursue a numerical approach by gridding α.
3. What is the admissible range of α to guarantee convergence (not necessarily monotonic) on an arbitrary finite time interval $N \in \mathbb{N}$?
4. Suppose your colleague designed a lifted ILC $L = 0.75 \cdot I \in \mathbb{R}^{2\times 2}$, i.e., for trial length $N = 2$. However, the ILC was implemented in an RC setting, i.e., the states of $J(z)$ are *not* reset after every trial, and the system is excited by a periodic $r(k)$, where $r(k+N) = r(k)$. Can you derive a condition for stability of the closed-loop system? Is the condition sufficient? Is the condition necessary? Apply the obtained condition, can you conclude whether the system is stable?
5. Same question as (d), but now for $N = 3$.

99. **ALTERNATIVE EXERCISE FOR EXERCISE 16**

This exercise is not recommended, but provided as an alternative to Exercise 16 and does not require doing experiments. It is advised to do the experiments, since they are interesting and typically do not take too much time. In any case, ONLY hand in this one or the experimental one.

Consider the system

$$G_c(s) = \frac{1}{s^2} \tag{18}$$

that is in open-loop, i.e., $C = 0$. The discretisation of G_c using a zero-order-hold is given by

$$G(z) = \frac{h^2(z+1)}{2(z-1)^2}, \tag{19}$$

where h denotes the sampling time (convince yourself of the result (19)).

1. Are $G_c(s)$ and $G(z)$ stable and minimum phase? Compute the corresponding Toeplitz matrix J for $N = 10$, $h = 1$ and provide the result.
2. Consider the ILC criterion

$$\mathcal{J}(f_{j+1}) = e_{j+1}^T W_e e_{j+1} + f_{j+1}^T W_f f_{j+1} + (f_{j+1} - f_j)^T W_{\Delta f}(f_{j+1} - f_j) \tag{20}$$

Set

$$W_e = I \tag{21}$$

$$W_f = 0 \cdot I \tag{22}$$

$$W_{\Delta f} = 10^{-2} \cdot I. \tag{23}$$

Compute L, Q. Explain the results. Investigate monotonic convergence and interpret the results.

3. Let

$$r = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T \tag{24}$$
$$f_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T. \tag{25}$$
$$\tag{26}$$

Implement the optimal ILC controller, and perform 100 experiments. Plot the resulting cost function $\mathcal{J}$ as function of the experiments j. Plot both f and e corresponding to iteration 100. Can you conclude from these figures whether the optimal ILC controller is causal? Why (not)?

4. Try to simulate the underlying continuous time model, while the ILC operates in a sampled-data way with sampling time $h = 1$, e.g., using simulink or a numerical integration scheme for solving ODEs. Compare the result with the results of Exercise 3. Notice that the model (18) represents the rigid-body motion of a mass of 1 kg. Can you give a physical explanation of the resulting behavior of the system?

5. The actual goal in Exercise 3 was a point-to-point motion, where the system should be approximately at rest at $t = N = 10$. Is this achieved? If not, can you suggest a way to achieve this, e.g., by changing $W_e, W_f, W_{\Delta f}, r, f_0$? Notice that the output at $t = 10$ should equal 1, and the velocity of the continuous time system should equal 0.

References

[1] J. Bolder, T. Oomen, S. Koekebakker, and M. Steinbuch. Using iterative learning control with basis functions to compensate medium deformation in a wide-format inkjet printer. *Mechatronics*, 24(8):944–953, 2014.

[2] S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control: Analysis and Design*. John Wiley & Sons, West Sussex, UK, second edition, 2005.