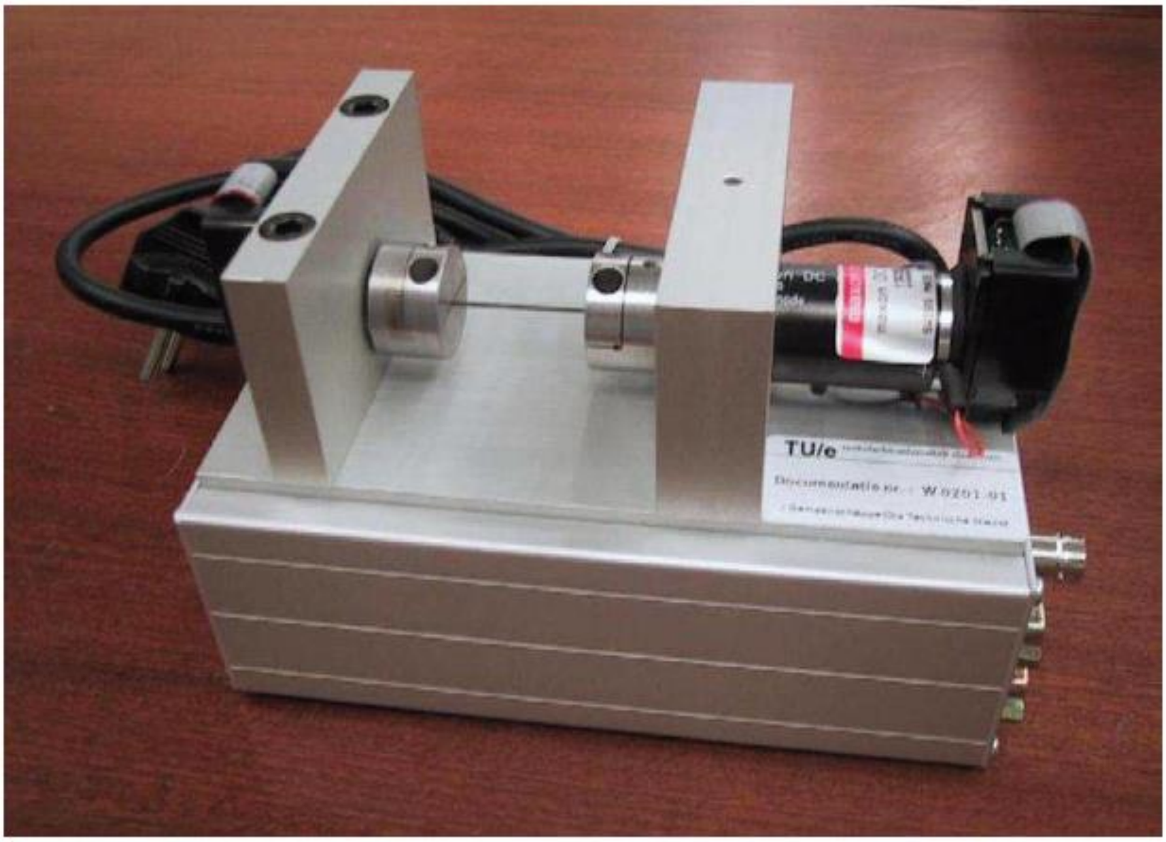
Technische Universiteit
Eindhoven
University of Technology

Report 1

Preparation for the measurements
4CM00: Control Engineering



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Content

Solutions of exercises marked with !!*!!

Exercise 1.2 - Interpreting a Bode diagram

The goal is to interpreted the two systems shown in Figure 1.

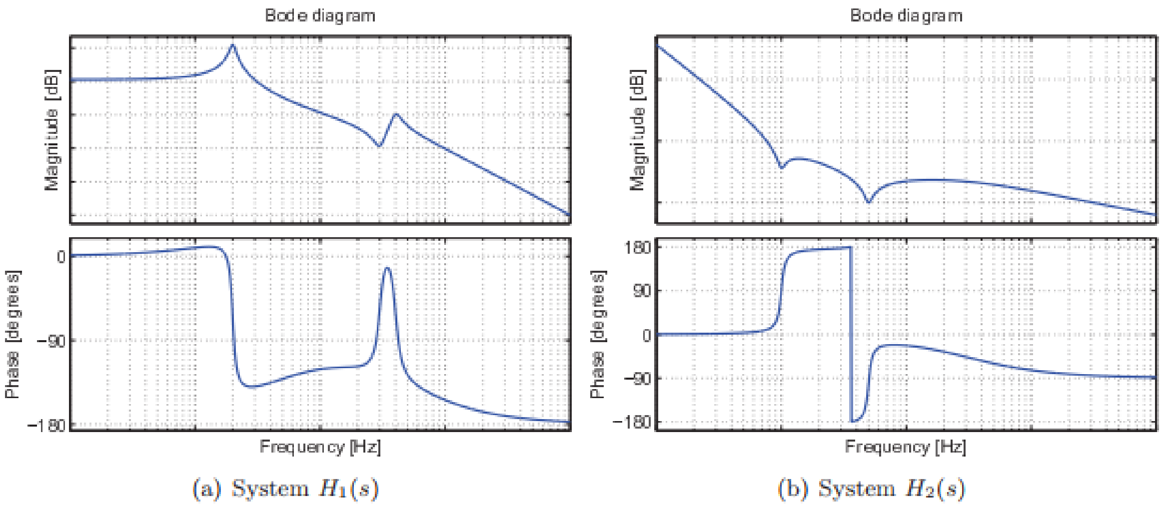

Figure 1 – Bode diagrams of systems H1(s) and H2(s)

System H1:

- Relative degree (#poles - # zeros): 2
  - Slope of -2 at high frequencies
  - Phase to -180 degrees at high frequencies
- Number of integrators: 0
  - Slope is 0 at low frequencies
  - Phase of 0 degree at low frequencies
- Total number of poles: 5
  - 2 LHP poles at frequency of first peak
  - 2 LHP poles at second peak
  - 1 LHP pole around the second peak (part of a lead filter)
- Total number of zeros: 3
  - 2 LHP zeros at dip between peaks
  - 1 LHP zero around the first peak (part of a lead filter)

System H2:

- Relative degree (#poles - # zeros): 1
  - Slope of -1 at high frequencies
  - Phase of -90 degrees
- Number of integrators: 4
  - Slope is -4 at low frequencies
  - 4x LHP pole, resulting in 0 (-360) degrees phase shift
- Number of poles: 5
  - 4 LHP poles at low frequencies (integrators)
  - 1 LHP pole slightly after second dip
- Number of zeros: 4
  - 2 LHP zeros at first dip
  - 2 LHP zeros at second dip

Exercise 1.3 - Estimating transfer functions

The systems H1 and H2 from the file frfdata.mat are plotted in Figure 2. System H2 has better controllability, because system H1 contains RHP poles and system H2 does not.

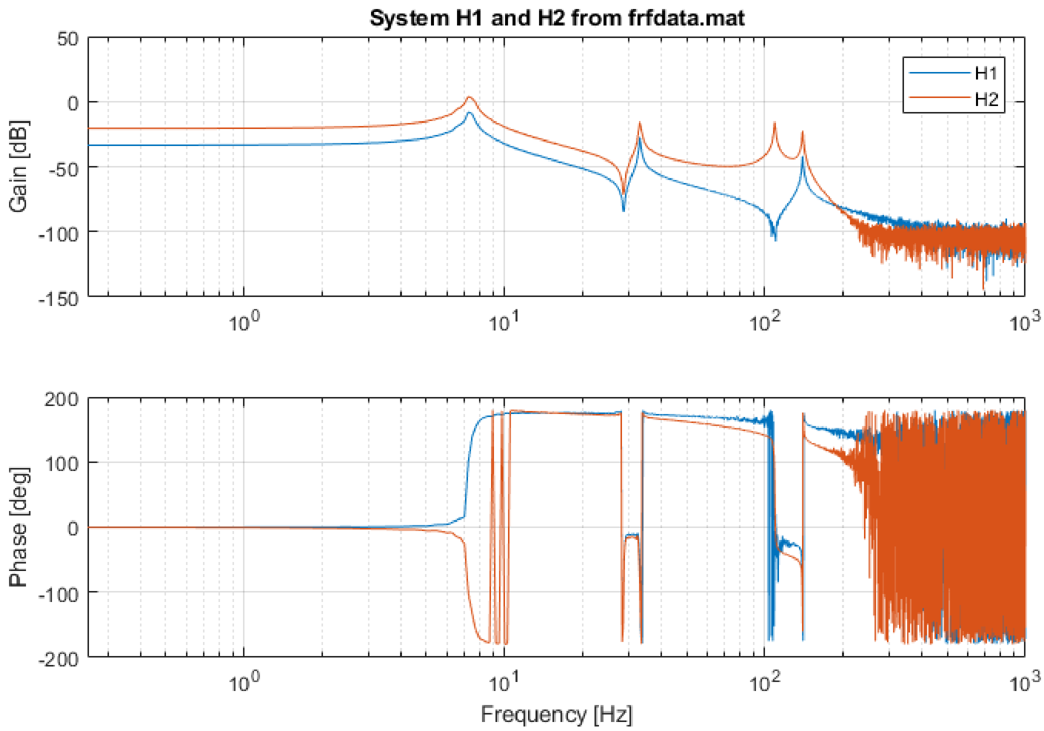

Figure 2 – System H1 and H2 from frfdata.mat

Exercise 2.3 - Closed loop FRF measurement

The closed loop sensitivity is determined by performing an 3-point FRF measurement on the simulation model shown in Figure 3.

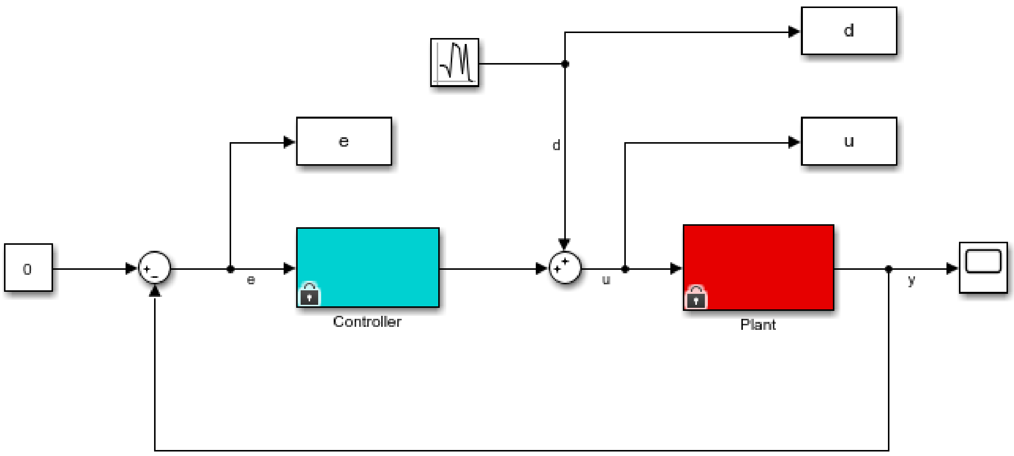

Figure 3 – frf_ex3.mdl simulation model

First the sensitivity of the simulation model is measured by injecting noise at d and measuring at u. This measurement resulted in the transfer function shown in Figure 4.

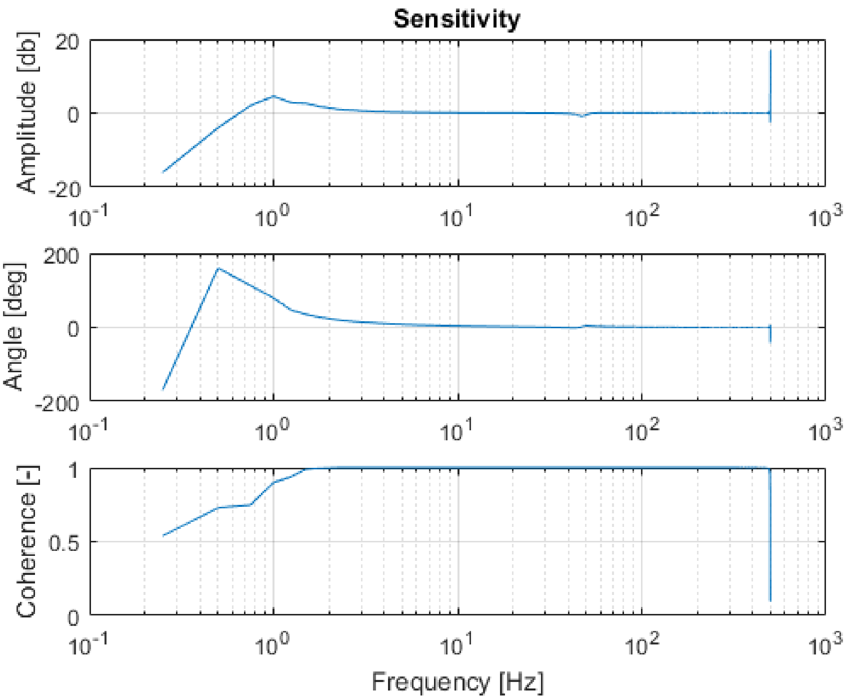

Figure 4 – Sensitivity FRF

The coherence is not 1 at lower frequencies because the sensitivity gain at lower frequencies is relatively low, caused by the controller which has high suppression on these lower frequencies. The coherence is 1 (or almost 1) between 2 and 500 Hz, indicating that the measurement is reliable in this frequency range. However, the coherence can be 1 even when the measurement is not reliable. In this exercise the coherence function corresponds to the expected result.

To determine the process sensitivity, another simulation is done using the model shown in Figure 3. Now the transfer function from d to e is measured, resulting in the FRF shown in Figure 5.

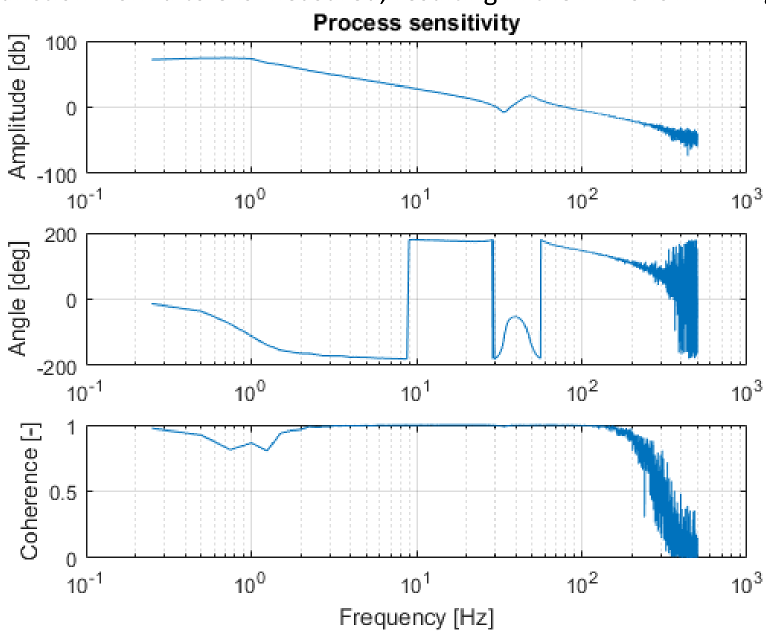

Figure 5 – Process sensitivity FRF

The coherence of the process sensitivity is flat between 2 and 100 Hz. At frequencies above 100 Hz the coherence is dropping fast, caused by the sensor noise. Because the gain of the process sensitivity is low at high frequencies (around -40 dB) the sensor noise gets relatively large, causing the gain and phase to be determined incorrectly.

The plant dynamics can be determined without measuring the controller by using the sensitivity and process sensitivity, using the following formula:

$$Plant(f) = \frac{Process\ sensitivity(f)}{Sensitivity(f)}$$

The plant FRF that is computed using this formula is shown in Figure 6.

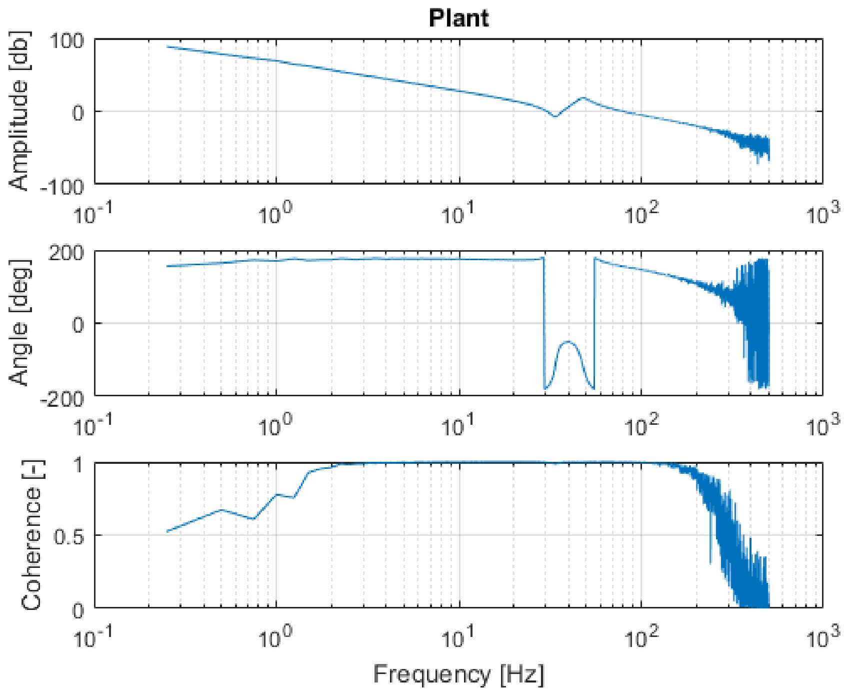

Figure 6 – Plant FRF

At higher frequencies there are two phenomena: bad amplitude estimation and phase drop. The phase drop can be explained by delay in the system, because the phase drop seems linear (when plotted on a linear scale). To determine the delay a ‘pade approximation’ is fitted to the phase angle, shown in Figure 7. This resulted in a delay of about 1 millisecond. The bad amplitude estimation can be explained by noise in the system which becomes more present as the plant gain decreases.

Now the FRF measurements of this exercise are repeated with different values for nfft: 1, 4 and 15. The variation of nfft resulted in slightly different FRF functions, shown in Figure 7. The result of different simulation times is that the coherence becomes 1 at all frequencies when the simulation time is the same as the window length. The cause of this phenomena is that there is only one window, meaning that there are no other windows to compare with. The number of possible windows is also decreasing when the simulation time is shortened. This will result in a more varying estimation.

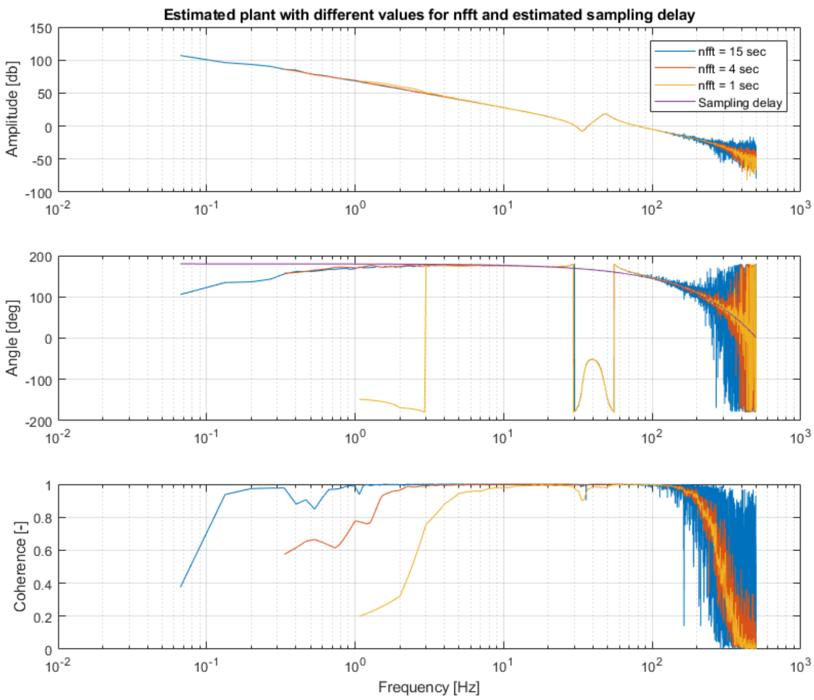

Figure 7 – Estimated plant with different values for nfft and estimated sampling delay

Exercise 3.1 - Modelling and control of a motion system

In this exercise a classic mass-spring-damper-mass system is used to create a feedback loop. In Figure 8 this system is shown. The parameters of the system are:

- $m_1 = 0.015$ kg
- $m_2 = 0.045$ kg
- $d = 0.4$ Ns/m
- $k = 2200$ N/m

The input of the system is motor force F, the output of the system is position x_1.

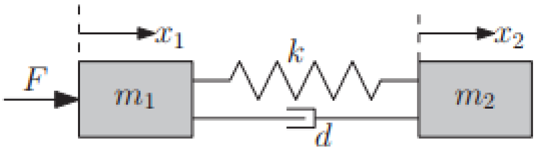

Figure 8 – Mass-spring-damper-mass system

The transfer function from F to x_1 is:

$$H_1(s) = \frac{x_1}{F} = \frac{m_2 s^2 + ds + k}{m_1 m_2 s^4 + d(m_1 + m_2)s^3 + k(m_1 + m_2)s^2} = \frac{1000s + 6000}{s^3 + 20s^2 + 5000s}$$

From F to x_2 this becomes:

$$H_2(s) = \frac{x_2}{F} = \frac{ds + k}{m_1 m_2 s^4 + d(m_1 + m_2)s^3 + k(m_1 + m_2)s^2} = \frac{1000s - 6000}{s^3 + 20s^2 + 5000s}$$

First, a simple stabilizing controller is created with a bandwidth of 20 Hz and a modulus margin of 2.2 dB. The resulting open loop bode is found in Figure 9 and the sensitivity function in Figure 10. The step response is plotted in Figure 11.

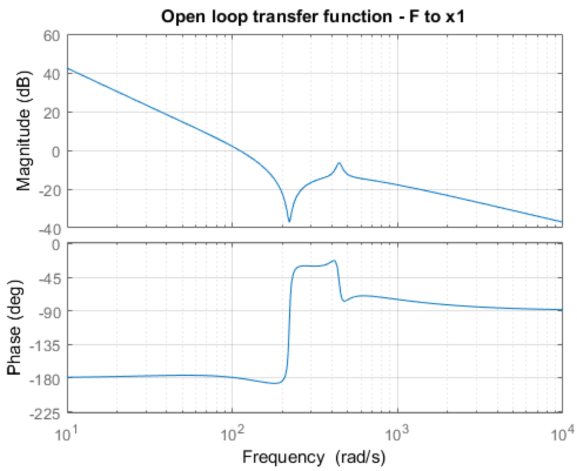

*Figure 9 – Bode of the open loop transfer function H1*C*

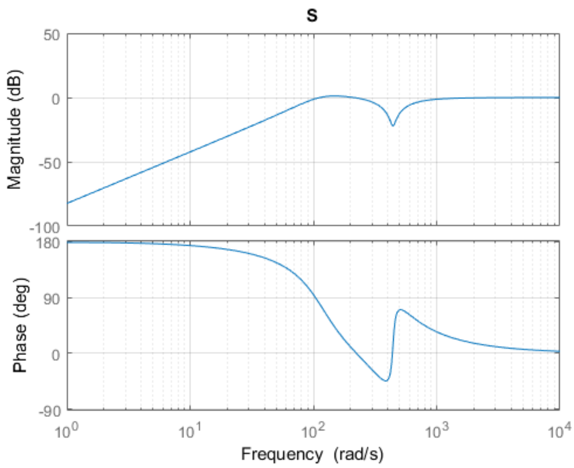

*Figure 10 – Bode of the sensitivity function of H1*C*

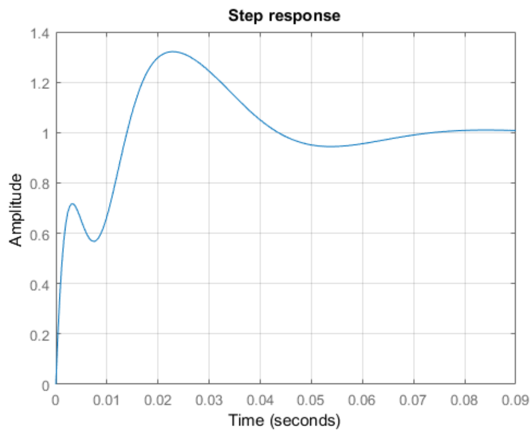

Figure 11 – Step response of H1· C

Now the system and controller are implemented in Simulink as closed loop, which is shown in Figure 12.

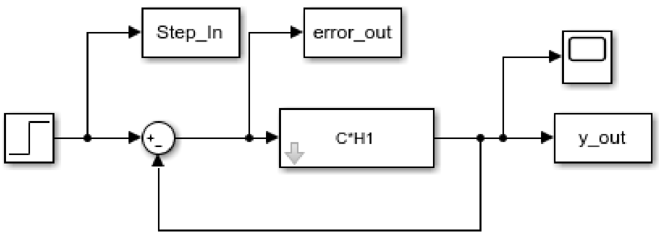

Figure 12 – Simulation schematic of mass-spring-damper-mass control loop

With this model the influence of the bandwidth and phase margin is investigated. First the bandwidth of the controller is varied, while maintaining the phase margin (Figure 13). Secondly the phase margin is varied with a fixed bandwidth (Figure 14). The error is plotted in both cases.

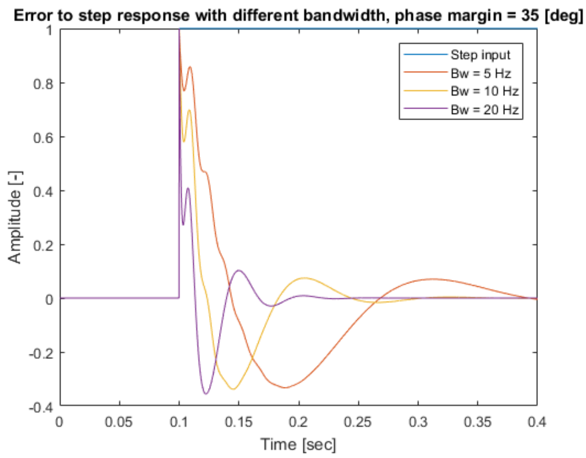

Figure 13 – Error when using different bandwidth

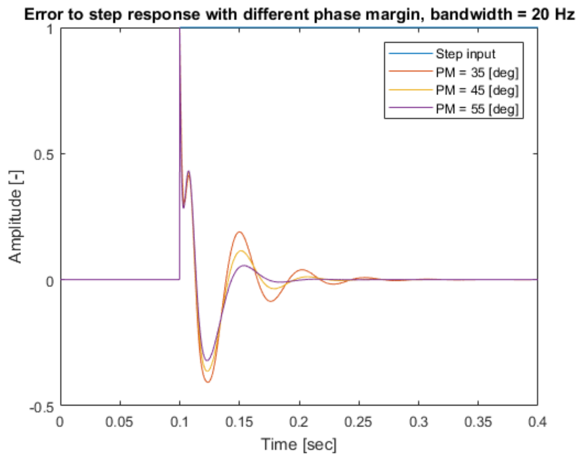

Figure 14 – Error when using different phase margins

The influence of the bandwidth and phase margin onto the overshoot, rise time and settling time is described in Table 1.

Table 1 - Influence of the bandwidth and phase margin onto the overshoot, rise time and settling time

System parameter	**Overshoot**	**Rise time**	**Settling time**
Increasing Bandwidth	No difference	Decreasing	Decreasing
Increasing Phase margin	Decreasing	Equal	Decreasing

Increasing the bandwidth has a positive effect on the rise time and settling time because this allows the system to use more and higher frequencies to follow the step input. The Fourier analysis of a step (block wave) has an infinite number of frequencies. Being able to use more frequencies (higher bandwidth) to track the step reference, the step response of the system converges to the actual step input, and thus has a better tracking behavior.
By increasing the phase margin, the effective damping of the system is increased. This damping determines the amount of overshoot. When the system has a more overshoot, the response needs more time to get within a certain percentage of the final value, which determines the settling time. Less overshoot will thus result in a shorter settling time.

When the controller of H1 is used to control the system H2, the output becomes unstable (Figure 15). This can be explained by looking at the bode plot of the system (Figure 16). The phase margin at the crossover frequency is negative (phase is less than -180 degree). This will result in an unstable system. Decreasing the gain will shift the crossover frequency to a lower value, where the system has more phase, resulting in more phase margin.

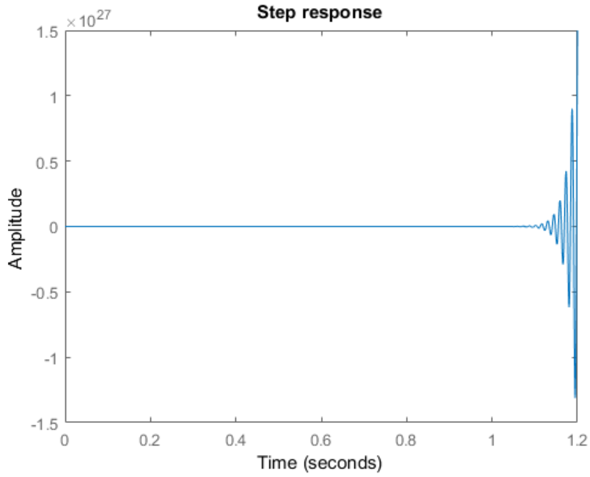

Figure 15 - Step response when using C1 to H2

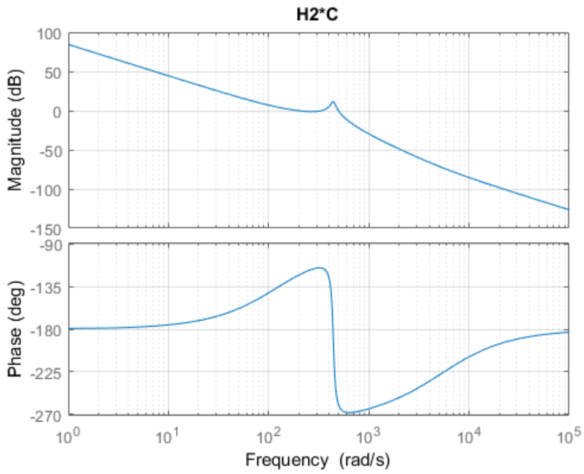

Figure 16 - Bode when C1 is applied to H2

Exercise 3.2 - Inverted pendulum

The considered system is a linearized inverse pendulum (Figure 17).

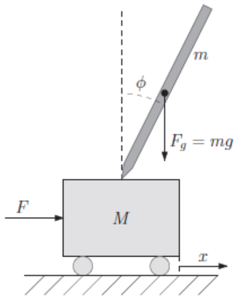

Figure 17 - Schematic overview of the inverse pendulum setup

The transfer function of this system is equal to:

$$H_{(s)} = \frac{\Phi_{(s)}}{F_{(s)}} = \frac{s}{-\frac{1}{6}(4M+m)Ls^3 - \frac{2}{3}Lbs^2 + g(M+m)s + gb}$$

where L is the length of the pendulum and b is the friction between the cart and the floor. Furthermore, assume that

M = 0.05, m = 0.04, b = 0.02, g = 9.81, L = 0.15

This results in a transfer function equal to

$$H_{(s)} = \frac{\Phi_{(s)}}{F_{(s)}} = \frac{-18 \cdot s}{0.108 \cdot s^3 + 0.036 \cdot s^2 - 15.89 \cdot s - 3.532}$$

And the Bode plot of this system is equal to Figure 18, with the corresponding pole-zero map, showing the position of the poles and zeros in the complex plane.

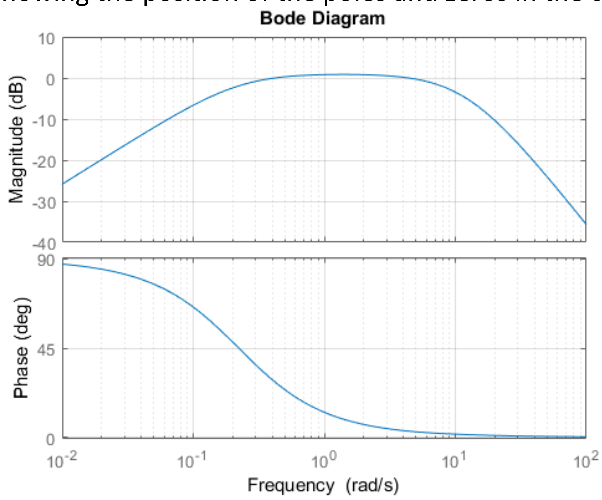

Figure 18 - Bode of the inverse pendulum system

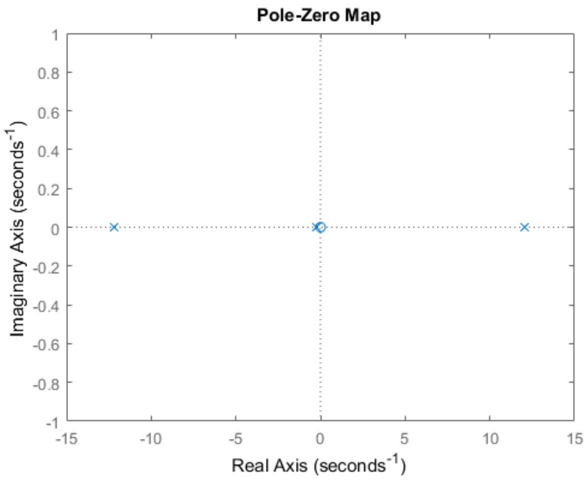

Figure 19 - Pole-zero map of the inverse pendulum system

By looking at the bode of the system it is clear that this system is unstable. At the frequency of 10 rad/s the amplitude starts having a slope of -2, corresponding to two poles. At the same frequency the phase stays at 0 degree, when a drop of -180 degree is expected. This can be explained by one pole laying in the right halve plane, generating a phase lead of 90 degree.

When two controllers with C1 = 1 (Figure 20 and Figure 22) and C2 = -0.55 (Figure 21 and Figure 23) are tested on the system, both systems are still unstable. This is because at least one counterclockwise encirclement is needed around (-1,0) to compensate for the RHP pole (Z = N+P). The difference between the two systems is that when the gain of the controller is negated, the Nyquist plot is flipped over the y axis. The step response is also neglected. In the step response it is also clearly visible that both systems are unstable.

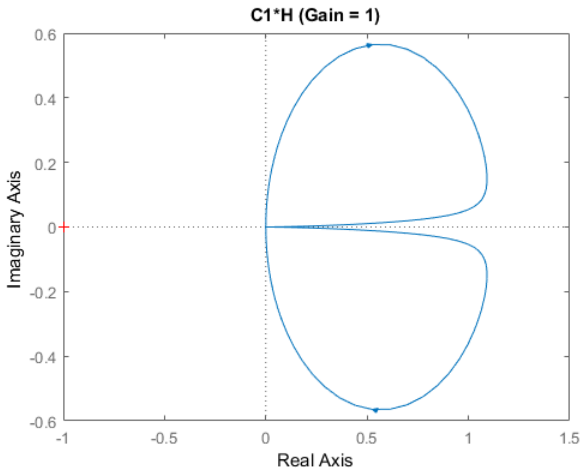

Figure 20 - Nyquist of the inverse pendulum sytem with C1

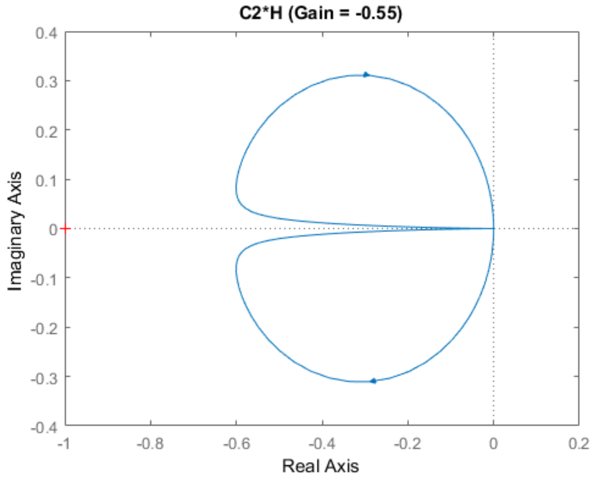

Figure 21 - Nyquist of the inverse pendulum sytem with C2

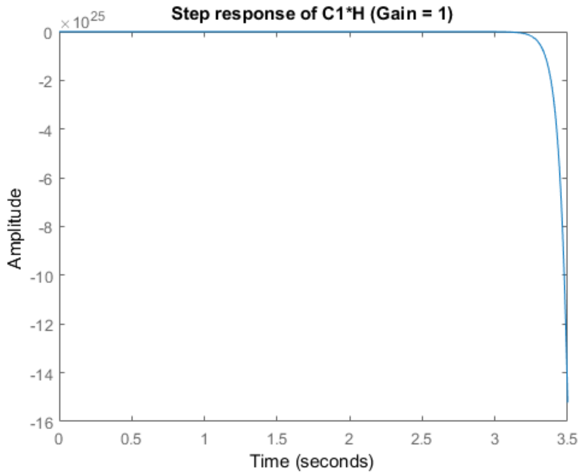

Figure 22 – Closed loop step response of the system with C1

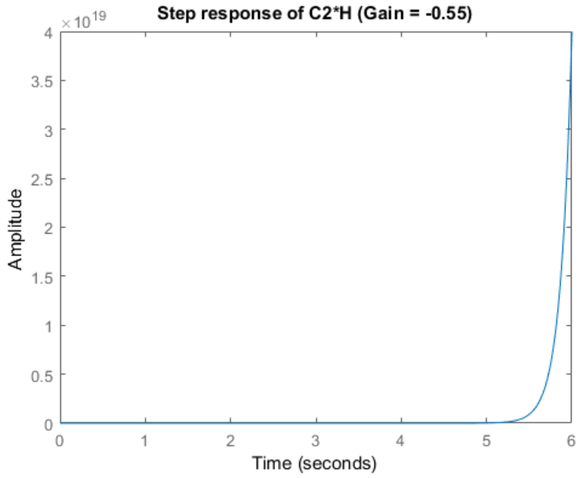

Figure 23 - Closed loop step response of the system with C1

In order to stabilize the system, a counterclockwise encirclement of the point (-1,0) is needed. This is achieved by entering the unit circle in the 3th quadrant (between -90 degrees and -180 degrees, approximately an -1 slope in amplitude). The used controller is consisting of the following parts:

Table 2 - Parts of the stabilizing controller of the inverse pendulum system

Block	**Parameter 1**	**Parameter 2**	**Reasoning**
Gain	Gain = -4		Create gain to ensure bandwidth, flip phase
Lead filter	Pole = bandwidth*3	Pole = bandwidth/3	Create phase around bandwidth
Differentiator	Zero at bandwidth		Create extra phase around bandwidth
2x Integrator	Pole at bandwidth/5		Create high gain at low frequencies
Lowpass filter	Pole at bandwidth*5		Create proper system

The resulting Open loop bode is found in Figure 24. In this bode plot it can be seen that around the crossover frequency, the slope is -1 and the phase is between -90 degrees and -180 degrees. In the Nyquist plot (Figure 25) a counterclockwise encirclement of (-1,0) is visible, which stabilizes the system.

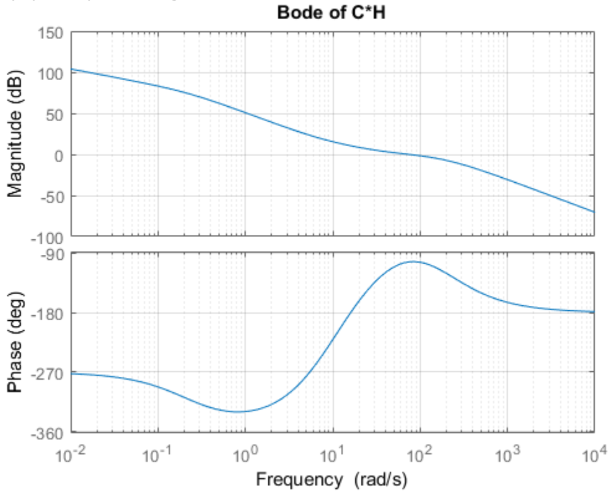

Figure 24 - Open loop bode of system and controller

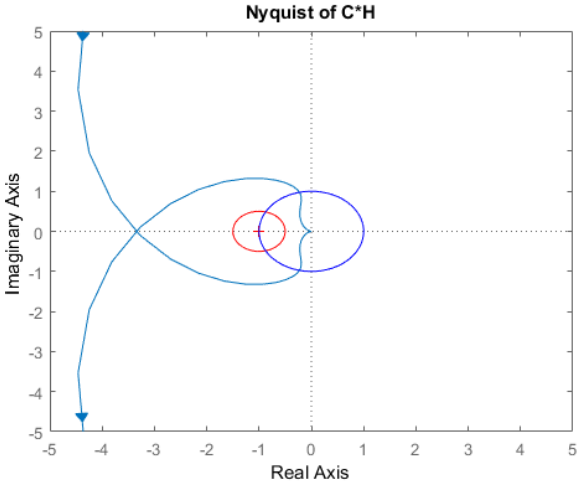

Figure 25 - Nyquist of system with controller

The step response is a stable output. In Figure 26 this step response is shown. It takes approximately 0.5 seconds to reach a stable output.

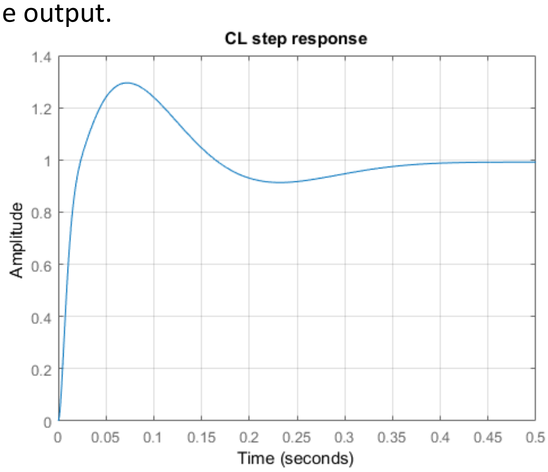

Figure 26 - Closed loop step response of the inverse pendulum system

In the case the chart is on a small slope, there is a constant force reacting on the chart, next to the controller output. This can be modelled in a block diagram as shown in Figure 27. The disturbance influences the system between the controller and the plant.

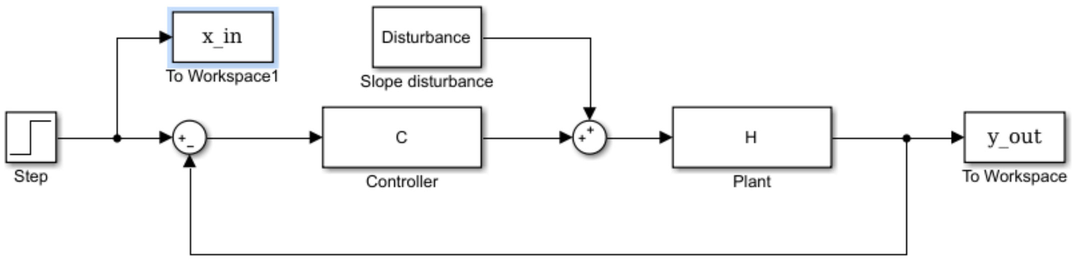

Figure 27 - Block diagram of the inverse pendulum system with a disturbance

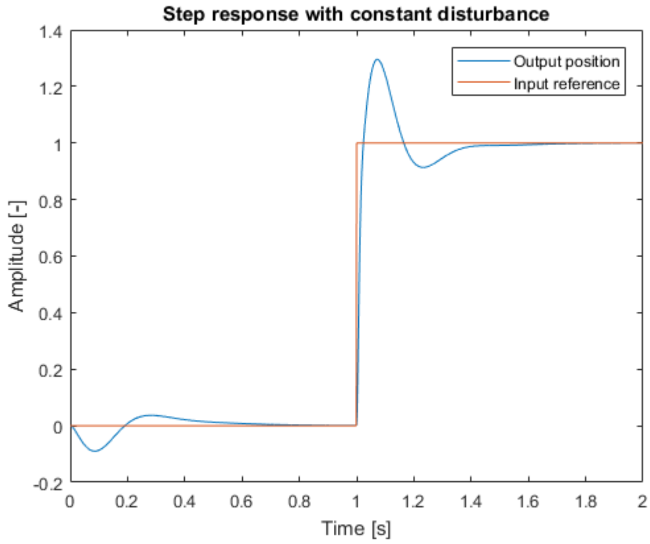

Figure 28 - Output response of the Closed loop system to an input reverence step with constant disturbance

The output of the system is found in Figure 28. In this plot a step reference is applied after 1 second. In the first second of the plot it is visible that the controller compensated for the constant disturbance of 1 [Newton]. After approximately 0.5 seconds this error is too little to see. After 1 second the input of the system (the reference) is changed with the step. The error due to the reference is also minimized after 0.5 second and the system is still in the upright position. The stability of the system itself is not changed, this is because the disturbance is an external input and is not part of the closed loop stability.

Exercise 4.2 - Non-collocated plant

A simple stabilizing controller is created by combining the following parts:

- Gain of 1
- Notch on 52.5 Hz with a damping of 0.01 for the nominator and 0.5 for the denominator. This notch minimizes the resonance peak
- Differentiator with the pole at 10 Hz to create phase margin around the bandwidth

The resulting open loop bode and step response of (Controller + Plant) can be found in Figure 29 and Figure 30:

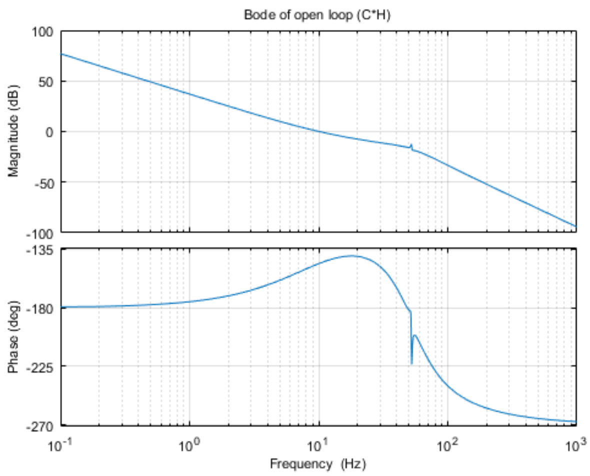

Figure 29 - Open loop bode of a simple stabilizing controller and plant (H) of exercise 4.2

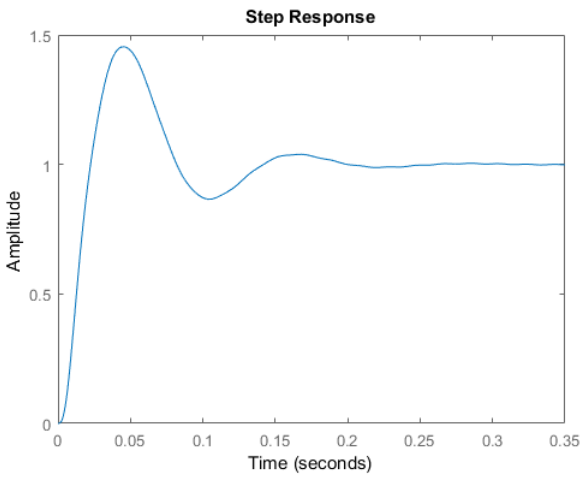

Figure 30 - Step response of a simple stabilizing controller and plant (H) of exercise 4.2

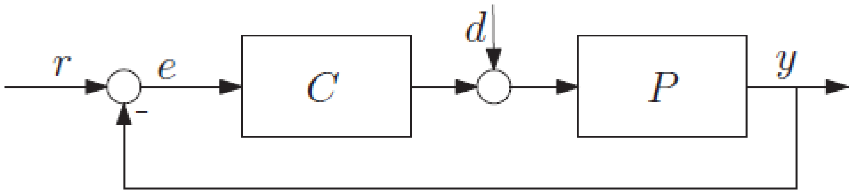

Figure 31 - Block scheme of exercise 4.2

When a 2 Hz sinus with an amplitude of 1 is applied as a disturbance at d (Figure 31) and the tracking error due to this disturbance (around e) must be evaluated, the interesting transfer function is equal to the process sensitivity. This process sensitivity is equal to:

$$PS = \frac{H}{1 + HC}$$

The goal is to minimize the error after 0.5 second (1 period) to be less than 0.01. This means that the Process Sensitivity at this frequency must be less than -40 dB. This is ensured by adding a notch filter at this particular frequency, creating a peak in the controller behavior at this frequency. In order to let the error converge to zero, when time approaches infinity, the process sensitivity must go to zero for low frequencies. Because the process sensitivity at low frequencies is depending on the inverse of the controller behavior, the controller must have an infinite gain at frequency zero. Adding an integrator to the controller creates this behavior. The controller, used blocks and responses are found in Figure 32 till Figure 36 and in Table 3.

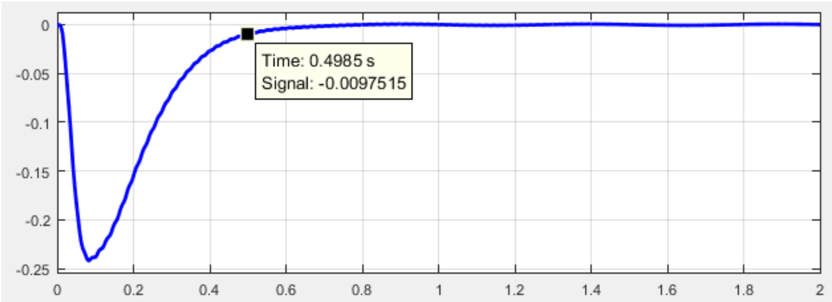

Figure 32 - Error response due to the 2 Hz disturbance

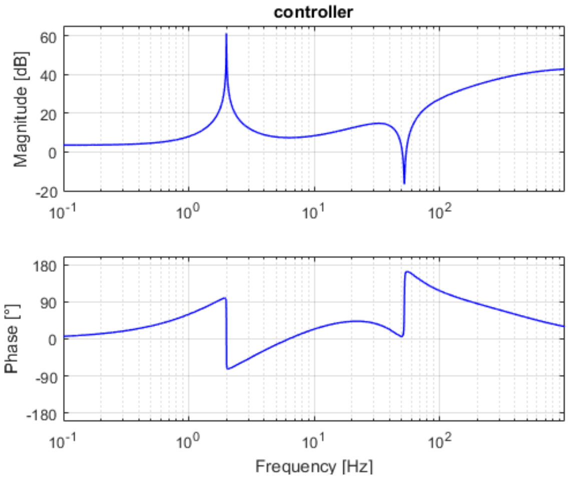

Figure 33 - Bode of the controller

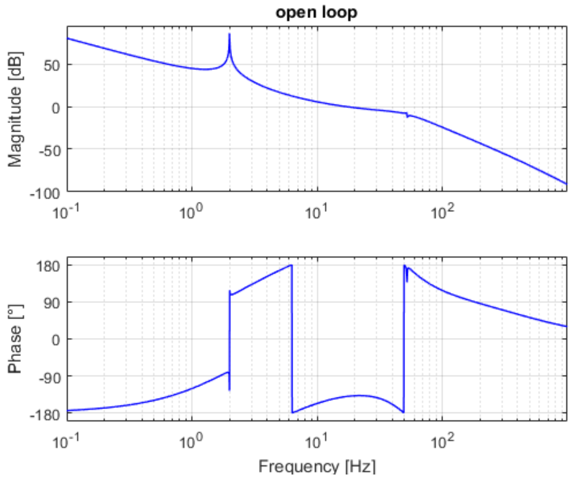

Figure 34 - Open loop bode

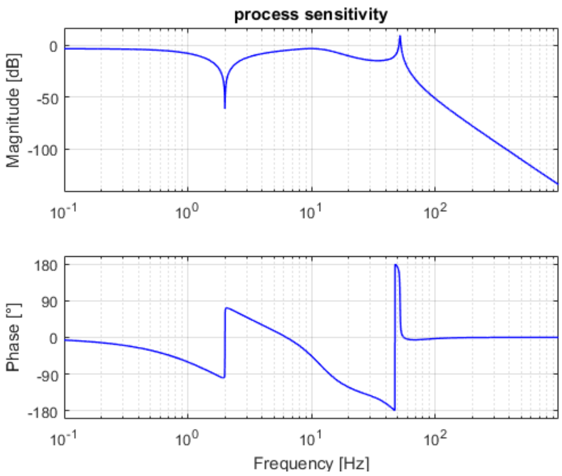

Figure 35 - Bode of the process sensitivity

Figure 36 - Stability margins of the system

Table 3 – Controller overview

Block	Parameter 1	Parameter 2	Parameter 3	Reasoning
Gain	Gain = 15			Create gain to ensure bandwidth
Notch	Zero/Pole = 52.5 Hz	B1 = 0.005	B2 = 0.5	Minimize resonance peak
PD	D = 1/(2*pi*80) = 0.002			Create phase margin
Lead	Zero = 30 Hz	Pole = 500 Hz		Create extra phase around Bandwidth
Notch	Zero/Pole = 2 Hz	B2 = 0.001	B1 = 1	Create steep dip in PS around 2 Hz
Lowpass filter	Pole = 50 Hz			Create proper system

Exercise 5.4 - Feedforward design

In this exercise a feedforward controller is tuned, which is applied in the simulation schematic shown in Figure 37. In this schematic a feedback- and feedforward controller are used to control the behavior of a plant.

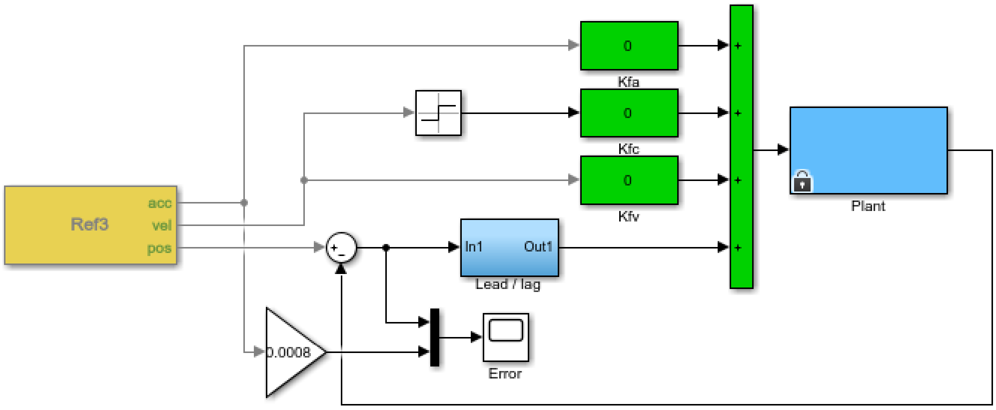

Figure 37 – Simulation schematic feedforward design exercise

The feedforward controller exists of three different parameters, which control the following system properties:

- K_{fa}: lack of force to accelerate
- K_{fc}: lack of force to overcome the Coulomb friction/dry friction
- K_{fv}: lack of force to overcome the viscous friction

The feedback controller is a lead/lag controller, consisting of a pole, zero and a gain. The simulation schematic of the feedback controller is shown in Figure 38.

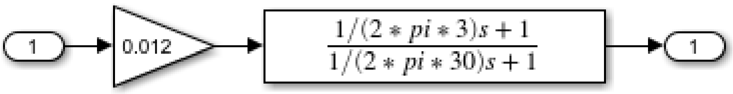

Figure 38 – feedback controller

Furthermore, the plant of the system is unknown.

When using a feedforward controller in combination with a feedback controller, the feedback controller must initially have a relatively low bandwidth, must not have an I-action and the closed loop system must be stable.

If the bandwidth of the feedback controller is low, the error is relatively large which makes it easier to determine the feedforward parameters K_{fc}, K_{fv} and K_{fa}.

If there is an I-action in the feedback controller, the error gets reduced towards zero over time. The I-action makes it impossible to distinguish the mass acceleration, viscous damping and dry friction from the error profile, thus making it impossible to determine the correct feedforward controller parameters.

To find the correct feedforward controller parameters, profiles shown in Figure 39 are used as setpoints for acceleration, velocity and position. This profile contains 1/3 of the time constant acceleration, 1/3 of the time constant velocity and 1/3 of the time constant deceleration.

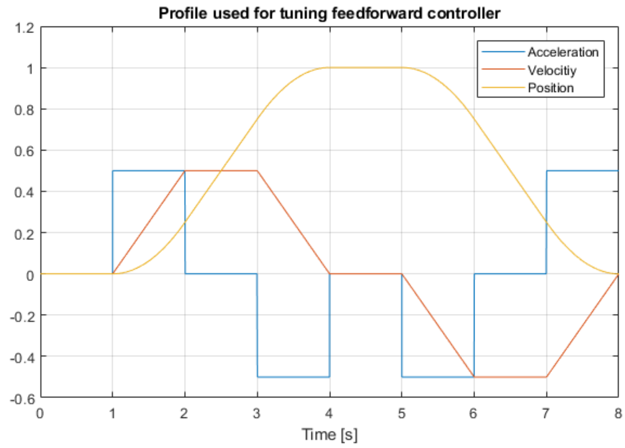

Figure 39 – Input profile used for tuning feedforward controller

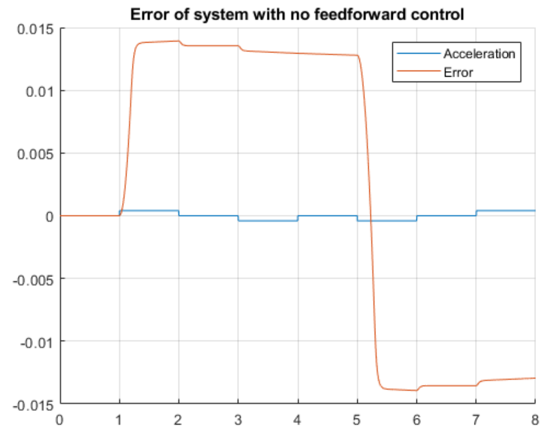

Figure 40 – Error of the system with no feedforward control

Using the position profile from Figure 39 on the input and no feedforward controller, only a feedback controller, resulted in the error profile shown in Figure 40.

First, K_{fc} is tuned. K_{fc} can be found by slowly increasing the value until the error remains constant when the velocity is becoming constant up until it starts changing again. In this exercise this takes place at second 4 and at second 5. The resulting profile is shown in Figure 41, where K_{fc} was set to 1.6e-4.

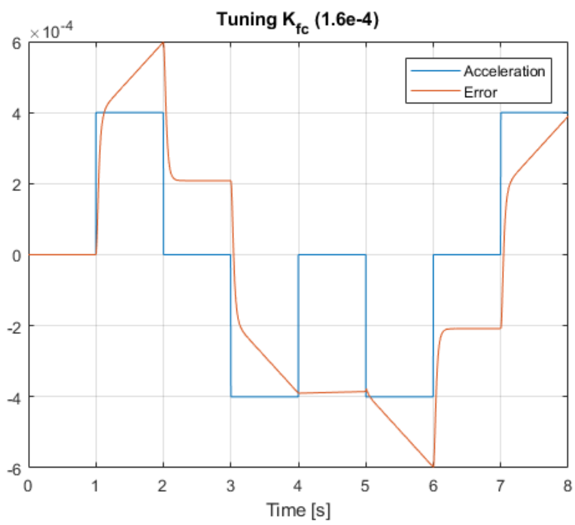

Figure 41 – Tuning of feedforward parameter Kfc with determined value 1.6e-4

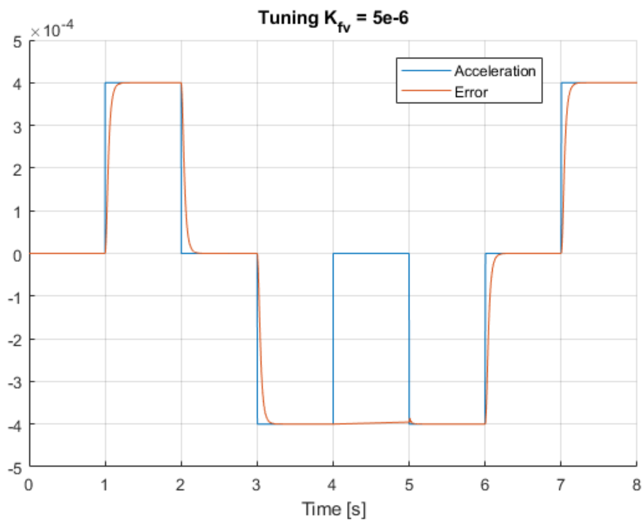

Figure 42 – Tuning of feedforward parameter Kfv with determined value 5e-6

The next step is to tune K_{fv}, which is done by increasing K_{fv} until the error is constant during acceleration. This resulted in K_{fv} = 5e-6 and the error profile shown in Figure 42.

Finally, K_{fa} needs to be tuned. This is done by increasing K_{fa} until the error is almost constant, only small peaks are allowed during change of acceleration. In Figure 43 is the tuning result shown with the determined value of K_{fa} = 9.6e-6.

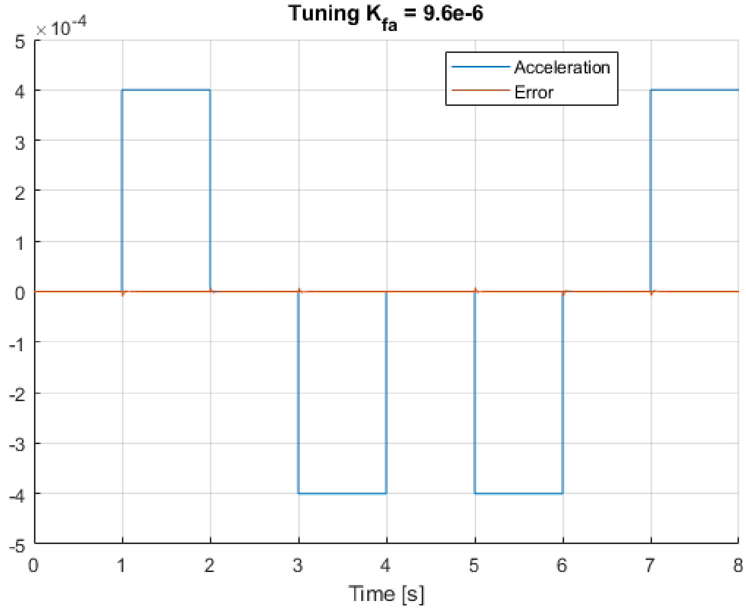

Figure 43 – Tuning of feedforward parameter K_{fa} with determined value 9.6e-6

To show the improvement of the system, the error without feedforward control and the error with feedforward control are plotted in Figure 44. From this comparison can be concluded that feedforward control decreases the error by a large amount.

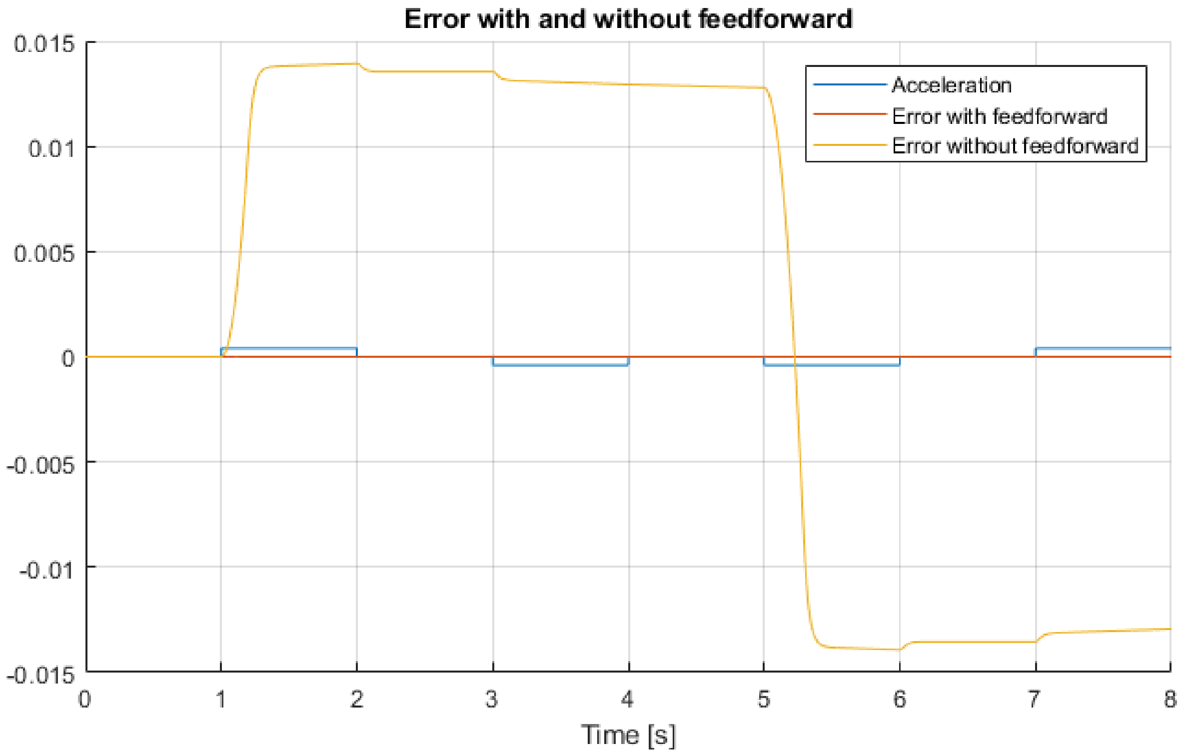

Figure 44 – Error without feedforward versus error with feedforward

Measurement plan

The goal of the experiment is to design a controller which controls the given motion setup (Figure 45) with an as high as possible accuracy.

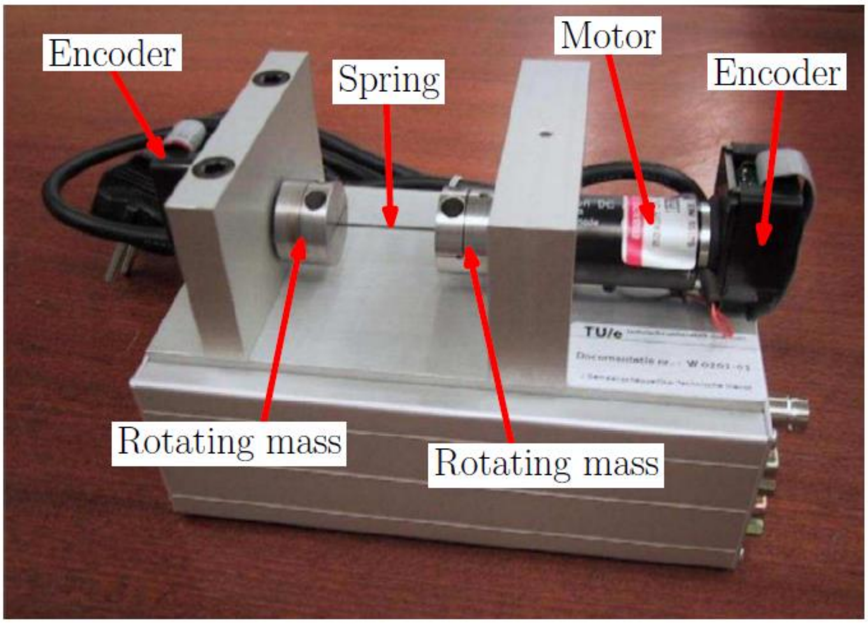

Figure 45 - Rotational motion setup used during the experiment. Source: 4CM00 - Exercise_X.pdf

Introduction

The second non-co-located mass (left in Figure 45) must track the setpoint profile described in Figure 46 with an as low as possible tracking error. The second goal is to maximize the amount of constant velocity strokes (scans). The two goals for the setup can be described by the following requirements:

1. The "constant velocity" part has a velocity of 30 rad/s
1. The travelled distance during one constant velocity scan is equal to 25 revolutions
2. The "turn around" must take as less time as possible. This means that the acceleration and higher order differentials (jerk, …) must be as high as possible at the turn around.
3. The travelled distance during a turnaround has no maximum, however, from point 2) follows that when the acceleration is higher, the total distance travelled is decreasing.

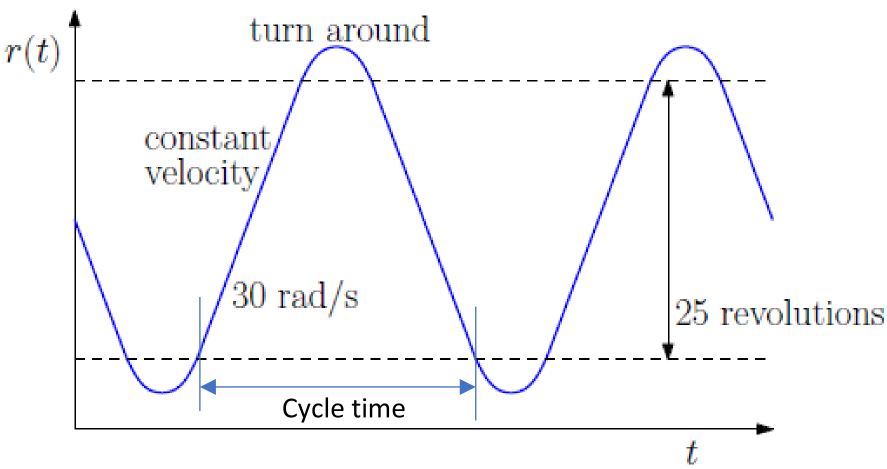

Figure 46 - Setpoint profile for the non-co-located mass. source: 4CM00 - Exercise_X.pdf

Dynamic behavior of the system

The expected dynamic of the system is comparable to the system as given in exercise 1.5. In this exercise the system as described in Figure 47 is evaluated. This resulted in the bode plots as found in Figure 48. The difference between the system of exercise 1.5 and the motion system in this measurement plan is that the system of 1.5 is a translating system and the current system is a rotational system. The behavior is expected to be equal.

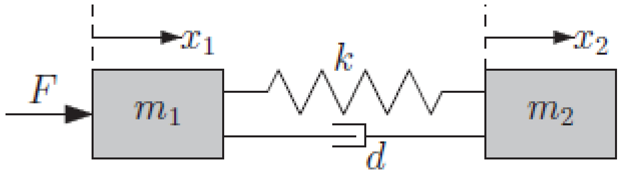

Figure 47 - Schematic overview of the motion system

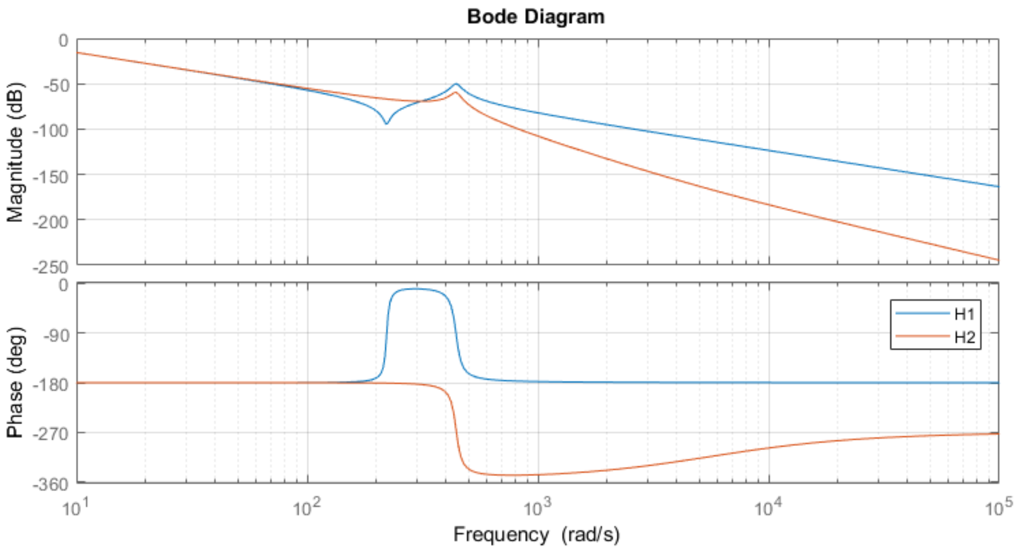

Figure 48 - Bode of H1 and H2 of the given motion system

Given limitations on the setup

1. The amplifier has a limited voltage range between -2.5 Volt and +2.5 Volt
2. The sampling rate is limited to 4 kHz, thus the maximum measurable frequency is 2 kHz (Shannon-Nyquist sampling criterium)
3. The modulus margin is restricted to a maximum of 6 dB.

Goal of the experiment

The goal is to minimize the tracking error. This tracking error is defined as the value of the error (reference signal – actual position (x2)) during the constant velocity part of the setpoint generator.

Maximize number of scans per time period. This will be measured by checking the time, indicated as “cycle time” in Figure 46) one full scan takes. Minimizing this time is better. This time consists of two parts. The first part is fixed, determined by the time two constant velocity parts take (approx. 5 seconds each). The second part is determined by the time two turn arounds take. The time this second part takes can be minimized by tuning a better controller and feedforward.

To reach the goal the following parts need to be created:

- Create a weak feedback controller with low bandwidth (for feedforward tuning)
- Create and tune feedforward controller
- Create feedback controller with a high bandwidth and good performance for this problem
- Combine and test the good feedback controller and feedforward controller

Timeline for the experiment

The actual measurements on the setup can be performed during two sessions of 3,5 hour each. Before the first session and between the two sessions the team can process the data and prepare for the upcoming session.

Preparation before first session

- Create this measurement plan
- Create template for final report
- Create a theoretical model of the setup in order to understand the behavior and the limitations that can be expected.
- Create the following scripts/files:
    - Setpoint for the desired scanning behavior (according Figure 46)
    - Setpoint for tuning FF (1/3 constant acceleration, 1/3 constant velocity, 1/3 constant deacceleration, vice versa)
    - Create controller template with all possible controller blocks and tunable parameters so when a controller must be created the team can pick multiple controller blocks and multiply them, resulting in the desired controller.
    - Create script where the measured FRF (of the plant) can be imported and used to check the sensitivity, open loop Bode, Nyquist

During first measurement session

- Getting familiar with the Linux Realtime environment
- Validate proper functioning of the setup (i.e. check direction of encoders, check direction of motor, …)
- Perform two FRF measurements on the setup and save the data:
    - From output of the PC to mass 1 (H1, the co-located mass on the motor)
    - From output of the PC to mass 2 (H2, the non-co-located mass)
- Tune offline a weak controller for H2, which can be used to tune feed forward.
- Tune online the feed forward for system H2

Between the two measurement sessions

- Fit created model to measured FRF data and estimate TF of the setup
- Use measured FRF data to create multiple good feedback controllers (as defined in the goal)
- If time allows, try to create a more advanced control strategy:
    - Use a nested loop to control H1 and with that increasing the performance of H2
- Simulate the different controllers and strategies
- Update second report

During the second measurement session

- Testing of the designed feedback controllers
- Testing the combination of the feedback + feedforward controller
- Tuning and improving the controllers