Exercise 2

Frequency response measurements

For all exercises you will need a Windows or Linux pc with Matlab/Simulink installed.

2.1 Frequency Response Function of a notch filter

A notch filter is a controller element which is meant to suppress a very specific frequency. It is defined, in continuous-time, as

$$F(s) = \frac{\dfrac{1}{(2\pi f_1)^2}s^2 + \dfrac{2\beta_1}{2\pi f_1}s + 1}{\dfrac{1}{(2\pi f_2)^2}s^2 + \dfrac{2\beta_2}{2\pi f_2}s + 1},$$

where f_1 and f_2 are the locations of the zeros and poles (in Hz) respectively, and β_1 and β_2 are the dampings of these zeros and poles. For a pure notch filter $f_1 = f_2$.

a. Create a Simulink model with a notch filter, e.g. with a standard 'Transfer Fcn'-block from the Simulink library. Apply a broadband noise ('Random number') as input to the filter. Place the zeros (numerator) and the poles (denominator) of the notch at 40 Hz, with a damping of 0.01 for the zeros and a damping of 1 for the poles.

b. Simulate the notch over a time interval of 30 s with a fixed step solver (e.g. `ode1` or `ode3`) with a step size of 0.001 s. Save both the input and output of the filter to the Matlab workspace. Compare the filter response with its input in time domain. Do you see any difference?

c. Compare the input and output in frequency domain by calculating their auto power spectral densities. What do you see now? *Hint: see Matlab functions* `fft.m`, `pwelch.m` *or* `periodogram.m`.

d. Determine the FRF of the notch filter using the ratio between the appropriate cross and auto power spectral densities. Also, determine the corresponding coherence function. Does the outcome match your expectation? *Hint: see Matlab functions* `cpsd.m` *and* `mscohere.m`.

e. Increase the damping of the zeros by a factor 10. How do the FRF and coherence change? Explain what you see.

2.2 Frequency Response Function of a mass system

In this exercise you will perform a closed-loop FRF measurement of a simple mass, controlled by a PD. Mathematically, this plant and controller are defined as

$$H(s) = \frac{1}{ms^2} \qquad \text{and} \qquad C(s) = P + Ds,$$

where s is the Laplace variable.

a. Assume the mass is 1 kg. Using the above definitions, calculate appropriate P and D so that the bandwidth (cross-over frequency of the open loop $C(s)H(s)$) is 10 Hz. To this end, place the zero of the controller at the desired bandwidth, then adjust the controller gain so that the open-loop gain is 1 (i.e. 0 dB) at the same frequency. Demonstrate your design by making a Bode plot of the open loop $C(s)H(s)$.

b. Create a Simulink model with a 1 kg mass and your calculated PD-controller. Inject a noise signal in the closed loop just behind the controller block.

c. Simulate the model over 120 s with the fixed step Euler solver with step size 0.001 s. Save the relevant signals for a closed-loop sensitivity FRF measurement to the Matlab workspace. Study these signals in both time domain and frequency domain.

d. Determine the coherence function and the FRF from the data obtained in part b. Does the outcome match your expectation?

e. Compute the open-loop FRF from the sensitivity FRF. How does it compare to your design of $C(s)H(s)$ in part a.? Explain what happens with the phase of the FRF for high frequencies.

f. Perform a separate FRF-measurement of the PD-controller only and determine the quotient of the open-loop FRF from part d. and the newly measured controller FRF. To what extend does the obtained result resemble a double integrator behavior?

2.3 Closed loop FRF measurement !!⋆!!

Download the Simulink file `frf_ex3.mdl` from OASE (this file is meant for Matlab r2006b or higher; Matlab 7.0.4 users can download `frf_ex3_m7.mdl`, Matlab 6.1 users can download `frf_ex3_m6.mdl`).

1. Open the Simulink file `frf_ex3.mdl`. This model simulates a real experiment, where the plant FRF has to be determined. It contains an unknown low-bandwidth controller, and an unknown plant. Assume that the controller cannot be measured separately. Check whether the simulation parameters are set to the right values: fixed-step solver, ode1 (Euler), simulation time of 30 seconds with a step size of 0.001.

2. Determine the sensitivity of the closed loop system by performing an FRF measurement, i.e. by inserting noise at the right place and measuring the right signals. Use the Matlab function `tfe.m` or `tfestimate.m`, and choose `nfft` such that the resolution of the frequency vector is 0.25 Hz. Choose `noverlap` to be half this size. Make sure to plot

the amplitude, the phase and the coherence of the sensitivity. Why is the coherence not equal to 1 at low frequencies? So between which frequencies is this measurement reliable?

3. Which signals have to be measured to determine the process sensitivity of the closed loop? Perform this measurement. Make sure to plot the amplitude, the phase and the coherence of the process sensitivity. What happens to the coherence at high frequencies? Can you explain this? Between which frequencies is this measurement reliable?

4. How can you determine the plant dynamics using only sensitivity and process sensitivity (without measuring the controller)? Determine the plant FRF and plot amplitude and phase. Keeping in mind that this is a simulation of a real experiment, what could cause the two high frequent phenomena: bad amplitude estimate and phase drop? How much is this phase drop and can you explain this?

5. Repeat this exercise with different values for `nfft` en different simulation times in Simulink. What differences do you notice and how can you explain these differences?

Exercise 3

Stability

3.1 Modelling and control of a motion system !!⋆!!

Again consider the classic mass-spring-damper-mass system from Exercise 1.5, depicted in figure 1.1. For this exercise we consider the motor feedback case, i.e. the input is F and the output is x_1, using the parameters mentioned in Exercise 1.5.

- Make a simple stabilizing controller for this system such that a bandwidth (0db of open-loop) of 20 Hz is achieved, and $|S| < 6$dB (and reasonable phase and gain margins).
- Implement the system and the controller in a Simulink (closed loop) block scheme. *Hint: you can use blocks like 'Transfer Fcn' (in 'Simulink - Continuous') or 'LTI system' (in 'Control System Toolbox')*
- Use a step as a reference signal, and perform a simulation. Plot the resulting error.
- Try to find out what the influence of the frequency domain specifications *bandwidth* and *phase margin* is on time domain performance like overshoot, rise time and settling time. Do this by designing and applying different controllers (with varying bandwidth and phase margins) and comparing the resulting error plots. Try to explain these influences.
- Apply the controller from the first step to the non co-located plant $H_2(s)$. Is the resulting closed loop stable? Explain why or why not.

3.2 Inverted pendulum !!⋆!!

Consider the inverted pendulum problem, see figure 3.1. The objective of this problem is to keep the pendulum in upward vertical position $\phi = 0$, by only applying a horizontal force F to the cart. It is easy to see that this system is non-linear. However, around the equilibrium $\phi = 0$ the system can be linearized, resulting in the following transfer function (you might want to show this yourself):

$$H(s) = \frac{\Phi(s)}{F(s)} = \frac{s}{-\frac{1}{6}(4M+m)Ls^3 - \frac{2}{3}Lbs^2 + g(M+m)s + gb}, \tag{3.1}$$

where L is the length of the pendulum and b is the friction between the cart and the floor. Furthermore, assume that

$$M = 0.05, \quad m = 0.04, \quad b = 0.02, \quad g = 9.81, \quad L = 0.15.$$

Now answer the following questions:

a) Draw a Bode diagram of this system. By looking at this plot, argue whether this system is stable or unstable. Verify your result by calculating the poles of (3.1).

b) Consider the simple controllers $C_1 = 1$ and $C_2 = -0.55$. Draw the Nyquist plots of the open loops C_1H and C_2H. Would you label the closed loops stable or unstable? Why? Check your answer by making time plots of the step response of the closed loop $T_i = \frac{C_iH}{1+C_iH}$. *Hint: read Section 6.3 (Nyquist Stability Criterion) of the book of Franklin and Powell.*

c) Design a stabilizing controller with a bandwidth of 10 Hz. Prove the closed loop stability by making a step response plot and analyzing the Nyquist plot properly. *Hint: use Franklin's formula $Z = N + P$, and design your controller such that you have the right amount of counterclockwise encirclements of the point -1.*

d) Suppose the cart is on a slope with a small (unknown but constant) angle, causing the cart to drift away. Show how this changes the closed loop by drawing a block scheme. Does this change the closed loop stability? Is your controller still able to keep the pendulum in upright position?

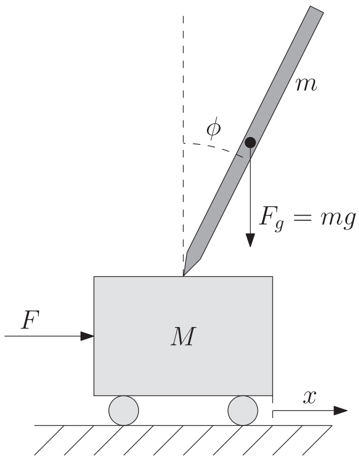

Figure 3.1: Schematic of the inverted pendulum example

3.3 Non-minimum phase systems

Consider the following third order systems

$$H_1(s) = 1000\frac{s+6}{s^3+20s^2+5000s} \quad \text{and} \quad H_2(s) = 1000\frac{s-6}{s^3+20s^2+5000s}.$$

- Draw the frequency response of both systems in one Bode diagram.
- Compare the magnitude and phase diagrams. What difference do you notice? Explain what causes this difference.
- System H_2 is called *non-minimum phase*. Explain why.
- Design a simple stabilizing controller C_1 for H_1. Put the bandwidth around 30Hz and attain a robustness margin $|S| < 3.5$dB. What's your phase margin?
- Apply the same controller C_1 to H_2 and draw the open loop Bode diagram. What's the phase margin?
- Draw the Nyquist curve of $C_1(j\omega)H_2(j\omega)$. Would you label this system stable or unstable? Why? Verify your result by drawing a time response plot.
- Design a new stabilizing controller C_2 for H_2. Try to achieve a high as possible bandwidth while attaining $|S| < 6$dB. What is the bandwidth-limiting factor in this case?