Exercise 1

Systems and controller design

1.1 Interpreting a transfer function

Consider the system described by the following transfer function

$$P(s) = \frac{(s+2)^2}{s^2(s+3)(s^2+s+25)} \tag{1.1}$$

a) How many poles and zeros does this system have?
b) What is the order of this system? What is its relative degree?
c) An integrator is formally defined as a pole at $s = 0$. How many integrators does the system have?

In general, the frequency response of a system can simply be obtained by substituting $s = j\omega$ into the transfer function. For system (1.1) this results in

$$P(j\omega) = \frac{(j\omega+2)^2}{-\omega^2(j\omega+3)(-\omega^2+j\omega+25)}, \tag{1.2}$$

which is a complex function. The expressions for the magnitude and phase of its real and imaginary parts are the exact definitions of the Bode diagrams. These expressions can be very complicated (you might want to try this yourself as an exercise). Fortunately, the asymptotes (slopes) for limit cases like $\omega \to 0$ and $\omega \to \infty$ can also be determined directly from $P(s)$ with $s \to 0$ and $s \to \infty$.

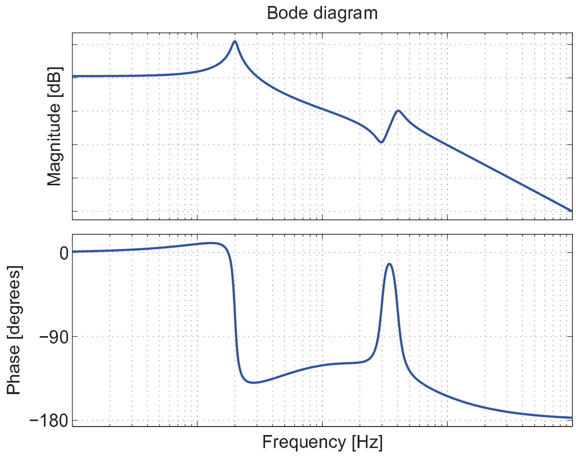
d) For the limit case $\omega \to 0$, determine the asymptote of $P(s)$ in terms of s. To what slope does this correspond?
e) For the limit case $\omega \to \infty$, determine the asymptote of $P(s)$ in terms of s. To what slope does this correspond?
f) Compare the answers to b, c with d, e and draw your conclusions.
g) Verify your results by drawing a Bode diagram.

1.2 Interpreting a Bode diagram !!⋆!!

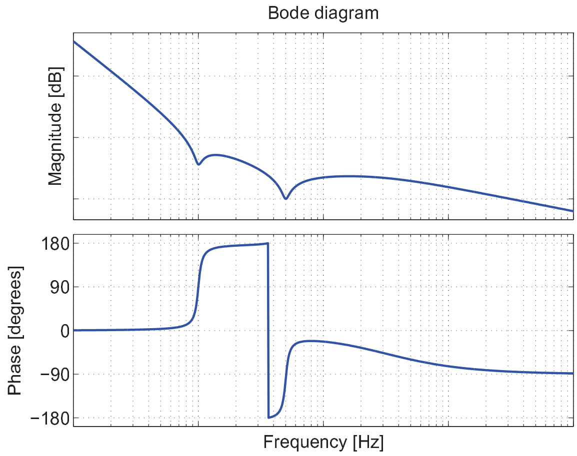
Exercise 1.1 indicated that you can predict parts of the Bode diagram by using system properties like the number of poles or the relative degree. Now let's turn this around.
For the Bode diagrams below, determine

- the relative degree
- the number of integrators
- the total number of poles and zeros

Motivate your answers (e.g. point out at which frequencies the poles and zeros are located).



(a) System $H_1(s)$



(b) System $H_2(s)$

Download the frequency response fitting toolbox `frffit.zip` from OASE, extract the files to a Matlab directory (e.g. `$matlabroot\toolbox\frffit`) and add this directory to the Matlab path (e.g. by using `pathtool.m`). In the end type `>>rehash toolbox`.

1.3 Estimating transfer functions

Download `frfdata.mat` from OASE and load it in your workspace. It contains two plant measurements, stored in a frequency vector `hz` and complex response vectors `H1` and `H2`.

- Plot the frequency reponses of both measurements in a Bode diagram. Which of the two plants is unstable? Predict the location of the unstable poles (in Hz) from the Bode diagram.

- Make models of the two measurements using either `frsfit.m` (stable fit) or `frfit.m` (possibly unstable fit). Choose an appropriate structure (number of poles, zeros and integrators) and choose an inverse weighting function (type `>>help frsfit` first).

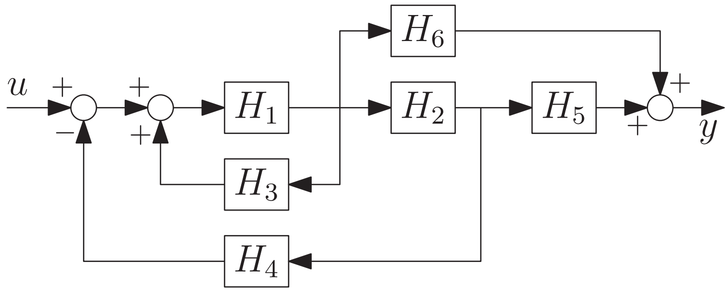
- What are the unstable poles of your model? Does this match your previous result?

- Create a system from the obtained model parameters (e.g. by using `ss.m` or `tf.m`). Plot your systems together with the data in a Bode diagram and comment on them.

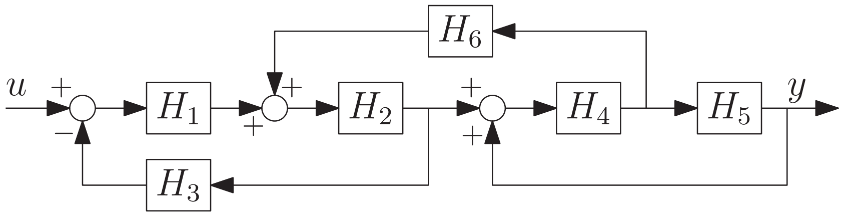
!!⋆!! Without designing a controller, which of the two plants do you think is easier to control (i.e. with a small controller order) to a 20Hz bandwidth? Explain why.

1.4 Working with block diagrams

Assuming all dynamic systems H_i in the block diagrams below are linear, derive the transfer function from input u to output y.



(a)



(b)

1.5 Modeling of a motion system

Consider the classic mass-spring-damper-mass system, depicted in figure 1.1. A motor applies a force F to a mass m_1, resulting in translations x_1 and x_2 of both masses m_1 and m_2.

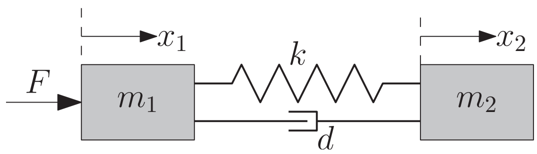


Figure 1.1: Mass-spring-damper-mass system

- Model this system and derive the transfer function $H_1(s)$ from F to x_1. This situation is called *co-location*, since actuation and measurement is 'on the same side'.
- Derive the expression for the undamped resonance and anti-resonance frequencies.
- Derive the transfer function $H_2(s)$ from F to x_2. Now there is dynamics between actuation and measurement, so we speak of *non co-location* (or load feedback).

Now take $m_1 = 0.015$kg, $m_2 = 0.045$kg, $d = 0.4$Ns/m and $k = 2200$N/m.

- Draw the Bode diagrams of $H_1(s)$ and $H_2(s)$ in one figure, and explain the similarities and differences between the two.
- Determine whether $H_1(s)$ and/or $H_2(s)$ are stable by only examining their Bode plots.