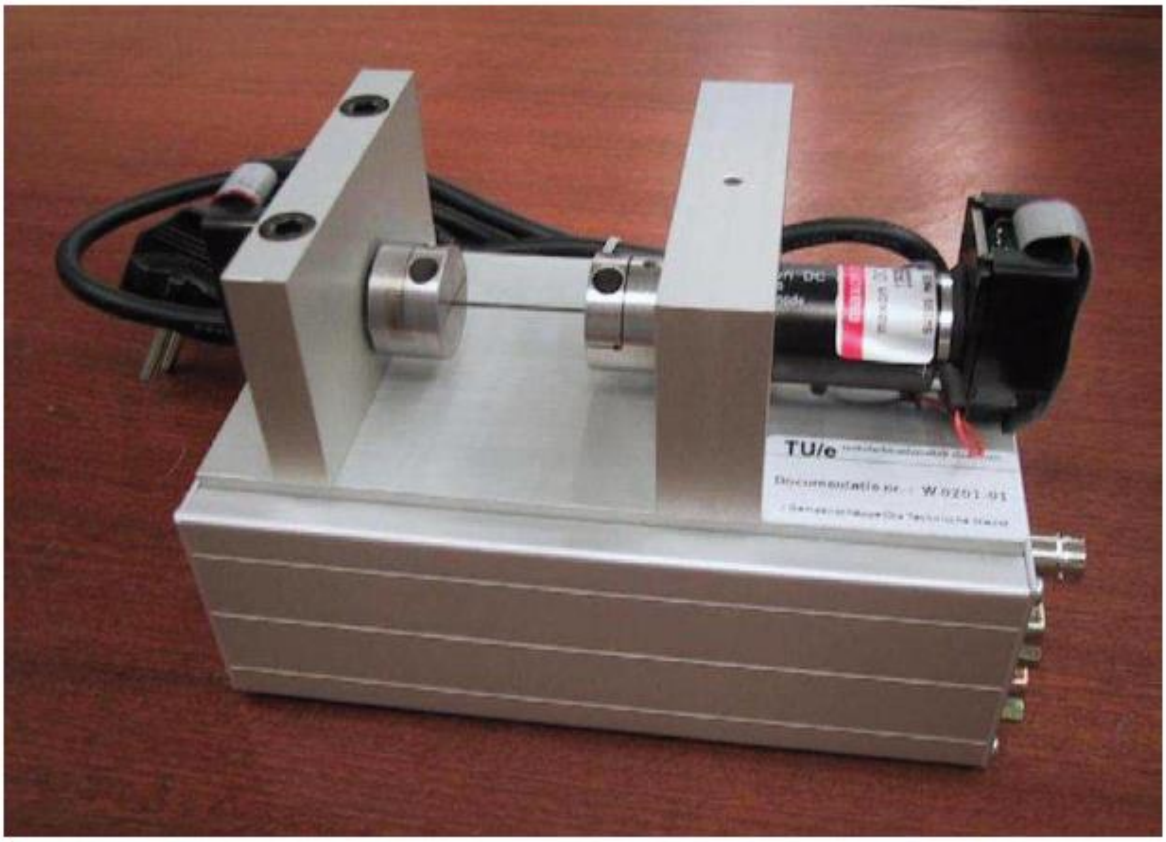
TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Report 2

Measurements preparation and report
4CM00: Control Engineering



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Content

1. Measurement plan

The goal of the experiment is to design a controller which controls the given motion setup (Figure 1) with an as high as possible accuracy.

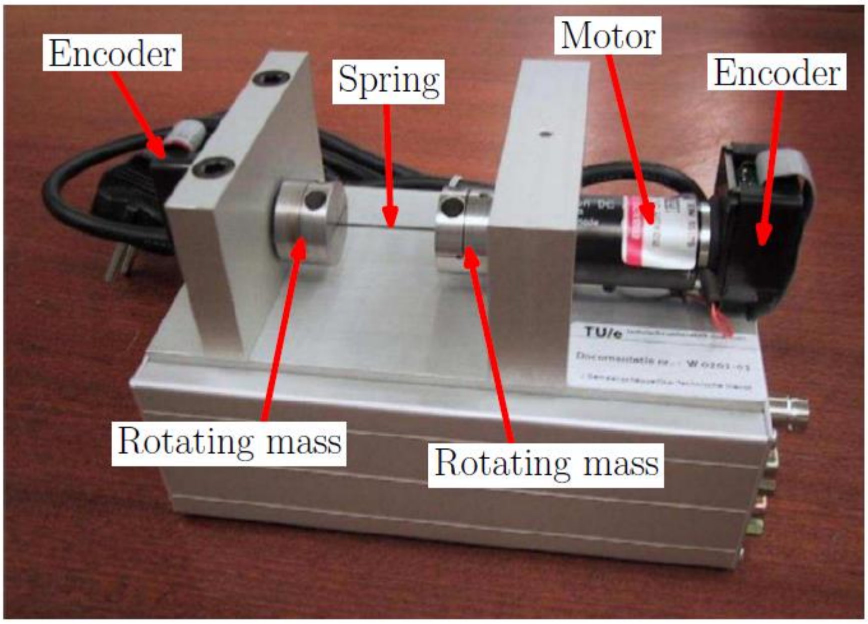

Figure 1 - Rotational motion setup used during the experiment. Source: 4CM00 - Exercise_X.pdf

1.1 – Introduction

The second non-co-located mass (left in Figure 1) must track the setpoint profile described in Figure 2 with an as low as possible tracking error. The second goal is to maximize the amount of constant velocity strokes (scans). The two goals for the setup can be described by the following requirements:

1. The "constant velocity" part has a velocity of 30 rad/s
2. The travelled distance during one constant velocity scan is equal to 25 revolutions
3. The "turn around" must take as less time as possible. This means that the acceleration and higher order differentials (jerk, …) must be as high as possible at the turn around.
4. The travelled distance during a turnaround has no maximum, however, from point 2) follows that when the acceleration is higher, the total distance travelled is decreasing.

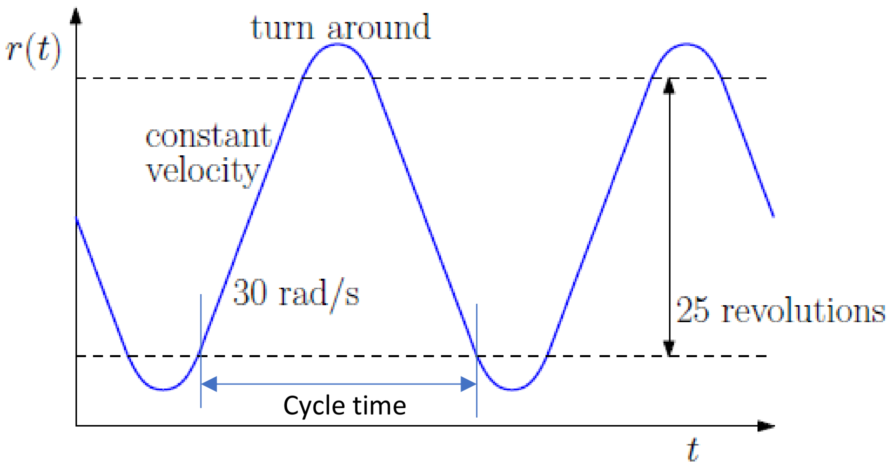

Figure 2 - Setpoint profile for the non-co-located mass. source: 4CM00 - Exercise_X.pdf

1.2 - Dynamic behavior of the system

The expected dynamic of the system is comparable to the system as given in exercise 1.5. In this exercise the system as described in Figure 3 is evaluated. This resulted in the bode plots as found in Figure 4. The difference between the system of exercise 1.5 and the motion system in this measurement plan is that the system of 1.5 is a translating system and the current system is a rotational system. The behavior is expected to be equal.

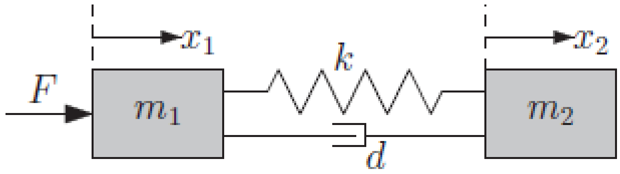

Figure 3 - Schematic overview of the motion system

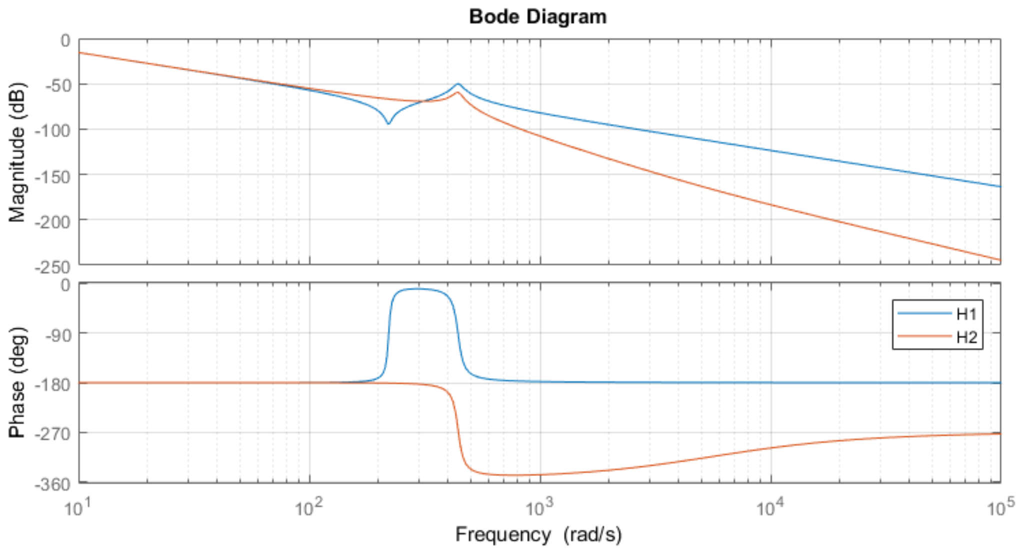

Figure 4 - Bode of H1 and H2 of the given motion system

1.3 - Given limitations on the setup

1. The amplifier has a limited voltage range between -2.5 Volt and +2.5 Volt
2. The sampling rate is limited to 4 kHz, thus the maximum measurable frequency is 2 kHz (Shannon-Nyquist sampling criterium)
3. The modulus margin is restricted to a maximum of 6 dB.

1.4 - Goal of the experiment

The goal is to minimize the tracking error. This tracking error is defined as the value of the error (reference signal – actual position (x2)) during the constant velocity part of the setpoint generator.

Maximize number of scans per time period. This will be measured by checking the time, indicated as "cycle time" in Figure 2) one full scan takes. Minimizing this time is better. This time consists of two parts. The first part is fixed, determined by the time two constant velocity parts take (approx. 5 seconds each). The second part is determined by the time two turn arounds take. The time this second part takes can be minimized by tuning a better controller and feedforward.

To reach the goal the following parts need to be created:

- Create a weak feedback controller with low bandwidth (for feedforward tuning)
- Create and tune feedforward controller
- Create feedback controller with a high bandwidth and good performance for this problem
- Combine and test the good feedback controller and feedforward controller

1.5 - Timeline for the experiment

The actual measurements on the setup can be performed during two sessions of 3,5 hour each. Before the first session and between the two sessions the team can process the data and prepare for the upcoming session.

Preparation before first session

- Create this measurement plan
- Create template for final report
- Create a theoretical model of the setup in order to understand the behavior and the limitations that can be expected.
- Create the following scripts/files:
    - Setpoint for the desired scanning behavior (according Figure 2)
    - Setpoint for tuning FF (1/3 constant acceleration, 1/3 constant velocity, 1/3 constant deacceleration, vice versa)
    - Create controller template with all possible controller blocks and tunable parameters so when a controller must be created the team can pick multiple controller blocks and multiply them, resulting in the desired controller.
    - Create script where the measured FRF (of the plant) can be imported and used to check the sensitivity, open loop Bode, Nyquist

During first measurement session

- Getting familiar with the Linux Realtime environment
- Validate proper functioning of the setup (i.e. check direction of encoders, check direction of motor, …)
- Perform two FRF measurements on the setup and save the data:
    - From output of the PC to mass 1 (H1, the co-located mass on the motor)
    - From output of the PC to mass 2 (H2, the non-co-located mass)
- Tune offline a weak controller for H2, which can be used to tune feed forward.
- Tune online the feed forward for system H2

Between the two measurement sessions

- Fit created model to measured FRF data and estimate TF of the setup
- Use measured FRF data to create multiple good feedback controllers (as defined in the goal)
- If time allows, try to create a more advanced control strategy:
    - Use a nested loop to control H1 and with that increasing the performance of H2
- Simulate the different controllers and strategies
- Update second report

During the second measurement session

- Testing of the designed feedback controllers
- Testing the combination of the feedback + feedforward controller
- Tuning and improving the controllers

2. Experiment results

This chapter describes the results of the measurement plan of chapter 1. First the plant is identified using two FRF measurements, followed by tuning the feedforward controller. The third step is tuning the feedback controller and finally the results of the total control loop are discussed.

2.1 – Models of the systems

To get reliable models of the system, several FRF measurements are performed and models are fitted to the measured data. The models are later used to create better controllers and get the desired performance.

2.1.1 – FRF measurements of the system

A measurement is done to obtain the Frequency Response Functions (FRF) of the plant. From this data, a model is created of the system. The created system describes the behavior of the system from the amplifier to the position of mass 2 (y2), called H2.

Because plant H2 is unstable on its own, a stabilizing controller is used during the measurements of this plant. For the measurement, the three-point method is used. This three-point measurement is done by injecting white noise as a disturbance at d in Figure 5 and measuring the error (e in Figure 5) and plant input (u in Figure 5). From the resulting measurement data the sensitivity function (Equation 1) and the process sensitivity function (Equation 2) are be determined. The response of the plant can then be found according Equation 3.

$$\text{Sensitivity } S\,(f) = \frac{u}{d}$$

Equation 1 - Sensitivity function

$$\text{Process sensitivity } PS\,(f) = \frac{e}{d}$$

Equation 2 - Process sensitivity

$$\text{Plant H } (f) = \frac{-PS}{S}$$

Equation 3 - Plant determined from PS and S

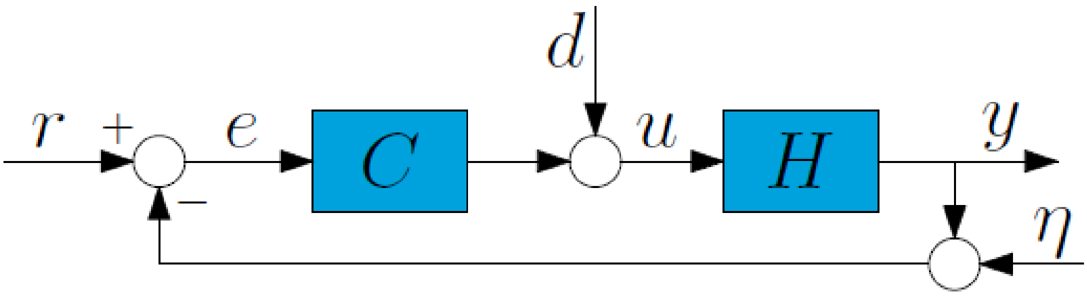

Figure 5 - Control loop with the measurement points for a three-point method

The reliability of the FRF measurement is determined by the value of the coherence. Between 5 Hz and 90 Hz the FRF is assumed to be reliable. In this frequency region the most interesting behavior of the plant can be found. The FRF data can therefore be used to estimate the models containing the most important behavior of the two plants. In Figure 6 the FRF measurements of the plant H2 is shown.

2.1.2 – Model fitting to the FRF data

The behavior of the plant is as expected (described in chapter 1.2) and is equal to Equation 4. The next step is to tune the constants in these equations in such a way that the model fits the measured data.

$$H_2(s) = \frac{a_1 \cdot s + a_2}{b_1 \cdot s^4 + b_2 \cdot s^3 + b_3 \cdot s^2 + b_4 \cdot s + b_5}$$

Equation 4 - Outline of the transfer function of plant H2

In the actual system a time delay is present due to the sampling delays, computation delays and other causes. This results in a linear phase delay which is visible in the FRF data. In order to fit the models to the FRF data, a 3th order pade approximation of the delay is added to the model. The transfer function of the delay is found in Equation 5.

$$H_{delay}(s) = e^{-Ts \cdot s} \approx \frac{-s^3 + 6667s^2 - 1.85 \cdot 10^7 \cdot s + 2.06 \cdot 10^{10}}{s^3 + 6667 \cdot s^2 + 1.85 \cdot 10^7 \cdot s + 2.06 \cdot 10^{10}}$$

Equation 5 – 3th order pade approximation for the delay

The time delay (Ts) is estimated to be approximately 1.8 milliseconds. With the fixed sample rate of 2048 samples/second this results in a delay of approximately 3.7 samples.

The final fitted models are found in Figure 6, plotted over the measured FRF data.

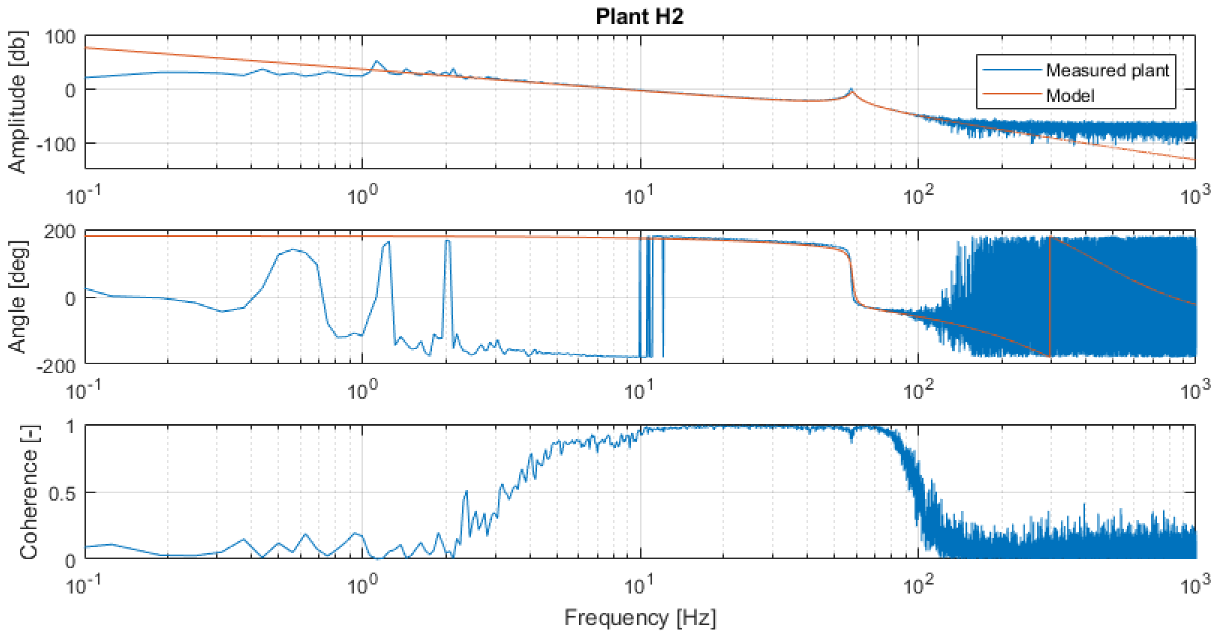

Figure 6 – FRF from the amplifier to mass 2

2.2 - Feedforward controller tuning

In order to reach the desired performance on H2, a feedforward control strategy is selected in combination with feedback. In this feedforward controller, three parameters are tunable and are found by experiments.

2.2.1 – Introduction to feed forward

The used feed forward method is based on three parameters: K_{fa}, K_{fv} and K_{fc}. The parameter K_{fa} is compensating for the lack of acceleration, caused by the inertia in the plant. This parameter is using the acceleration of the setpoint. The parameter K_{fv} compensates for the viscous friction in the plant and is using the velocity of the setpoint. To overcome dry Coulomb friction, K_{fc} is used. This parameter uses the sign of the velocity of the setpoint. In Figure 7 is illustrated how this feedforward controller is implemented.

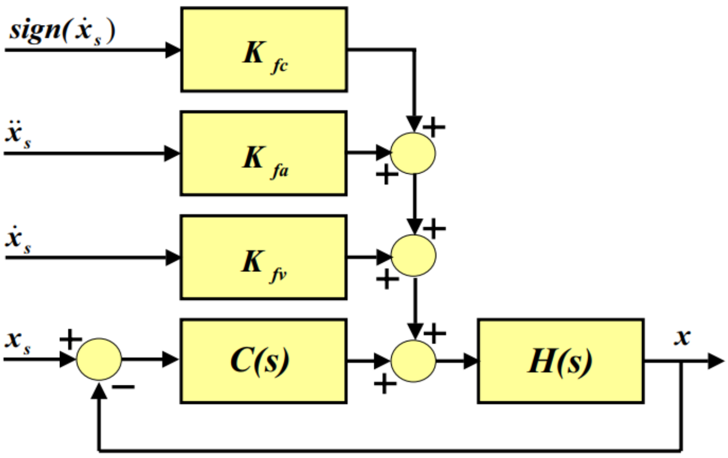

Figure 7 – Feedforward controller schematic overview – Source: 4CM00: Control Engineering Feedforward control

2.2.2 – Finding the feedforward parameters

The next step is determining the correct values for K_{fa}, K_{fv} and K_{fc}. This is done by applying a repeating setpoint to the input of the system with plant H2 and looking at the error. Because the plant is not stable on its own, a stabilizing controller is added to the control loop. Because the error profile must be distinguishable from noise, it must be large enough. To create a large error, the controller must have a relative low bandwidth. The controller also must not contain an integrator, because this eliminates the error over time.

The chosen controller is shown in Figure 8 and contains a PD controller for stability, a notch filter for the resonance and a lowpass filter. The resulting open loop response is also found in the same figure. The resulting bandwidth of the system is 3.4 Hz.

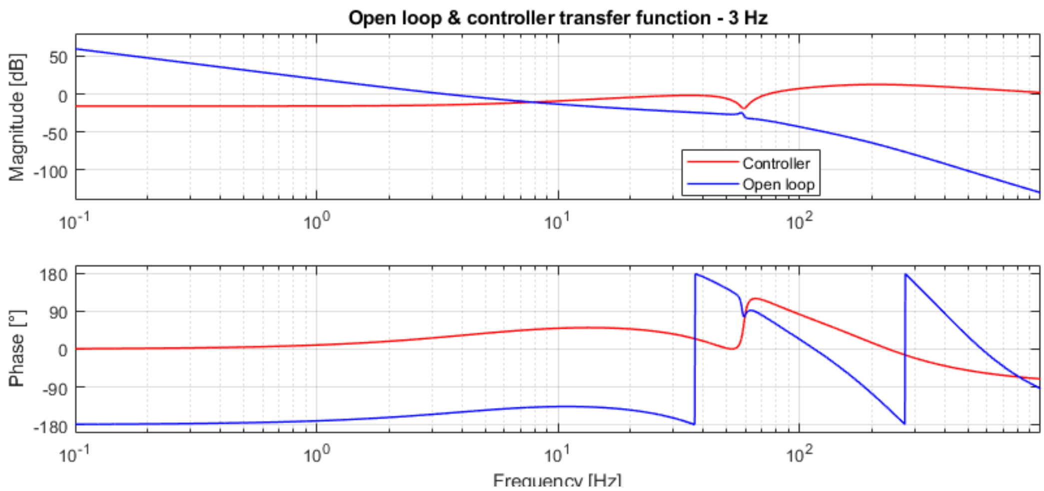

Figure 8 – PD controller used during feed forward tuning

Next, the optimal parameters for the feedforward controller are determined based on the error during a move. To get the right parameters, it is important to have a clear error profile. By using a setpoint profile where the acceleration and velocity part can be clearly identified, the corresponding feedforward parameters can be easily tuned.

The setpoint that is applied to the system during the feedforward tuning must have the following properties:

- At least 2nd order
- Forward and backward movement
- Challenging enough to create a clear error profile
- Approximately 1/3 of the time
    - Acceleration

- Constant velocity
- Deceleration

The used setpoint for feed forward tuning is shown in Figure 9. the parameters for acceleration, velocity and distance are shown in Table 1 - Parameters of the setpoint used for feed forward tuning. The profile moves forward and backward as desired, with a standstill time of one second between the two moves. In this way every part of the setpoint takes one second, resulting in a clear error response.

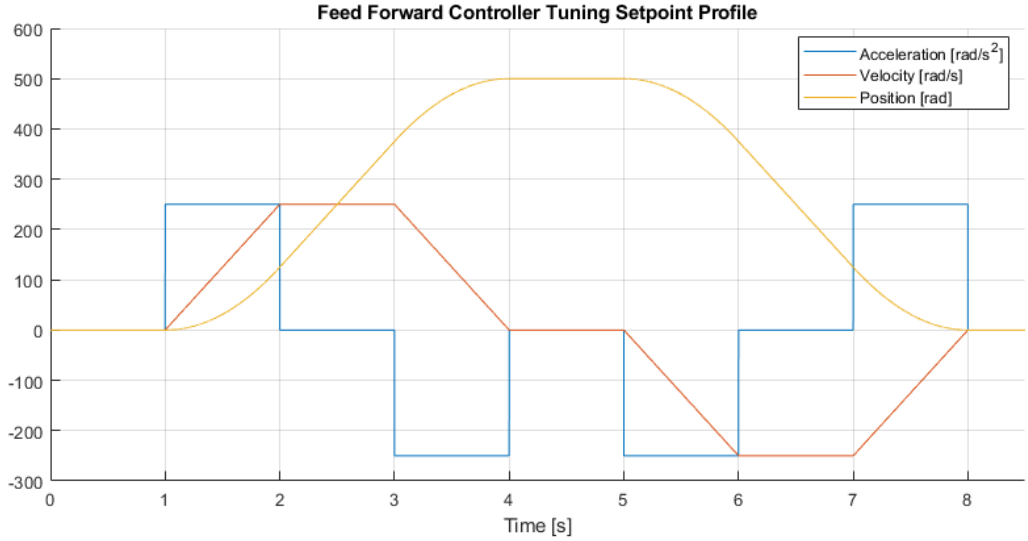

Figure 9 – Feed forward controller setpoint profile

Table 1 - Parameters of the setpoint used for feed forward tuning

Parameter	Value
Distance	500 [rad]
Max velocity	250 [rad/s]
Max acceleration	250 [rad/s^2]
Max jerk	1000000 [rad/s^3]

An example for the error profile is shown in Figure 10, where the different behavior of the error profile is shown as the feedforward controller parameters are adjusted. The parameters are adjusted in the order Kfc, Kfv and finally Kfa.

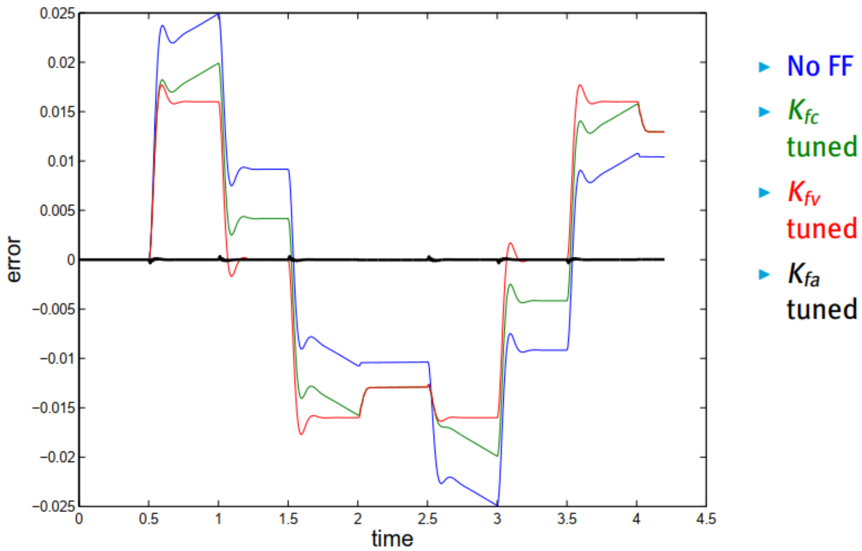

Figure 10 – Example of error profile during tuning of FF controller – Source: 4CM00: Control Engineering Feedforward control

2.2.3 – Feedforward parameters

The feedforward parameters for system H2 are tuned in the same order as described in paragraph 2.2.2. In Figure 11 the resulting error after tuning each feed forward parameter is shown. In Table 2 the final values for K_{fc}, K_{fv} and K_{fa} are found.

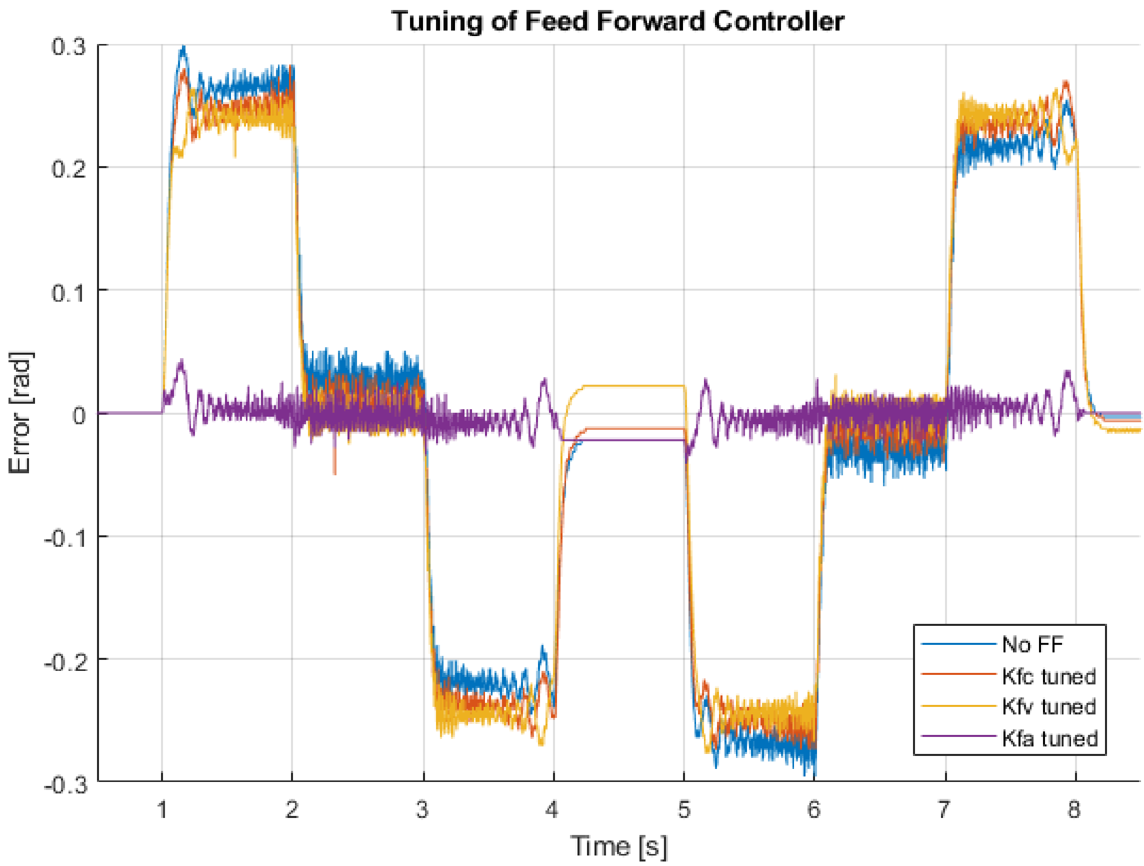

Figure 11 - Error after tuning each parameter of feed forward

Table 2 - Final feed forward parameters

Parameter	**Value**
K_{fc}	0.007
K_{fv}	0.00002
K_{fa}	0.00038

2.3 - Feedback controller design

Between the two measurement sessions, two controllers are designed. The first controller has a higher bandwidth compared to the second controller. The downside of the first controller is that it is more sensitive to noise. Both controllers are simulated and in chapter 2.4 both controllers are tested on the actual setup. The controllers are designed with the help of 'Shapeit', a graphical tool for loop shaping within Matlab. The only requirement regarding the controller is a modulus margin of at least 6 dB.

2.3.1 - Controller 1 with 23Hz bandwidth

The first designed controller has a high bandwidth of 23 Hz. The controller consists of six standardized blocks. In Table 3 the parameters of the blocks and the reason why the block is added to the controller is shown. The transfer functions of the open loop and controller are shown in Figure 12 and the Nyquist plot is shown in Figure 13.

The key specifications of this controller are:

- Bandwidth: 23.16 Hz
- Modulus margin: 6.0 dB
- Phase margin: 33 degrees
- Gain margin: 9.7 dB

Table 3 – 23 Hz controller blocks and parameters

Block	Parameter & value	Reasoning
Gain	0.4	Create gain to ensure bandwidth
Notch	Zeros: 58.1 Hz, damping 0.02 Poles: 80 Hz, damping 5	Zeros at resonance frequency of plant, poles at a higher frequency to create bigger stability margins
Integrator	Zero 5 Hz	Reducing error at low frequencies
PD	P: 1 D: 0.025	Adding phase at cross over frequency
Lead/lag	Zero: 6.67 Hz Pole: 90 Hz	Adding phase around the cross over frequency
Lowpass filter 1st order	Cutoff frequency 550 Hz	Making the system proper by adding an extra pole

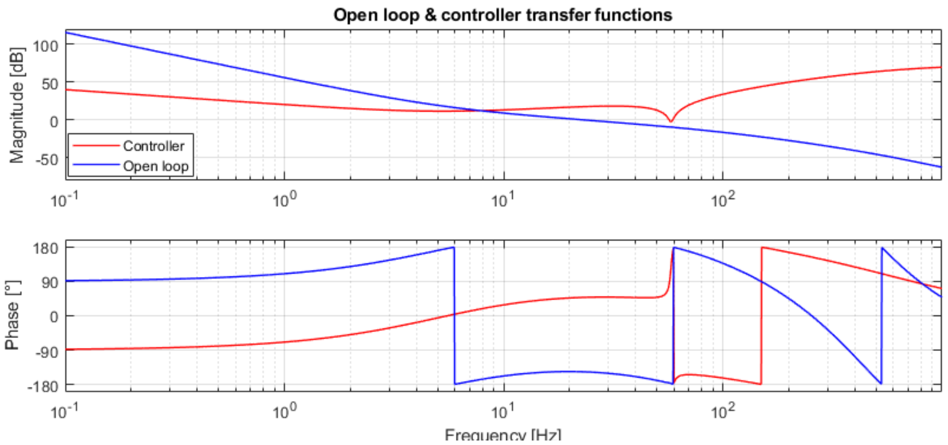

Figure 12 – Open loop and controller transfer functions of 23 Hz controller

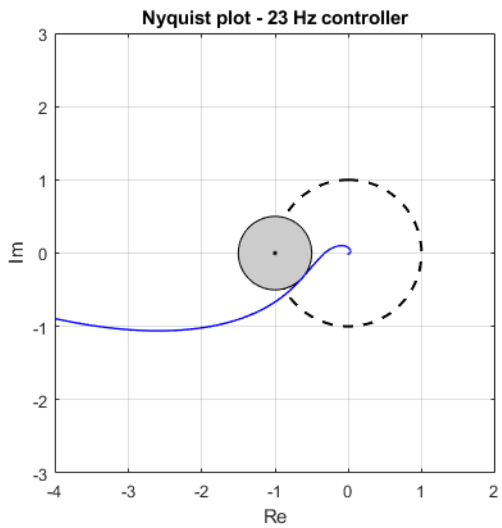

Figure 13 – Nyquist plot of 23 Hz controller

2.3.2 - Controller 2 with 10Hz bandwidth

The 10 Hz controller is designed as a simple and stable controller with a relatively low crossover frequency and large stability margins. The key specifications of this controller are:

- Bandwidth: 10.68 Hz
- Modulus margin: 3.8 dB
- Phase margin: 43.3 degrees
- Gain margin: 10.4 dB

The standardized controller blocks that are used to generate this controller are listed in Table 4. The open loop and controller transfer functions of the 10 Hz controller are illustrated in Figure 14 and the Nyquist plot is shown in Figure 15.

Table 4 – 10 Hz controller blocks and parameter values

Block	**Parameter & value**	**Reasoning**
Gain	5	Create gain to ensure bandwidth
Notch	Zeros: 59 Hz, damping 0.03 Poles: 59 Hz, damping 0.5	Notch resonance peak in plant at a frequency of 59 Hz. Damping is determined manually.
PD	P: 0.16 D: 0.005	Adding phase lead at the cross over frequency to get the desired phase- and modulus margins
Lowpass filter 2nd order	Cutoff frequency 200 Hz Damping 0.7	Reduce gain at higher frequencies when the gain is below 0 dB, creating proper system

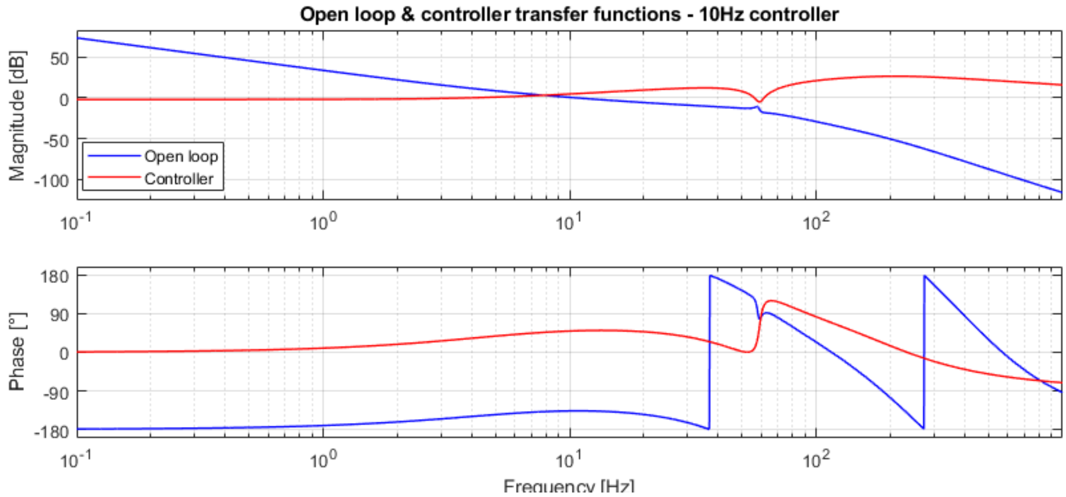

Figure 14 – Open loop and controller transfer functions of 10 Hz controller

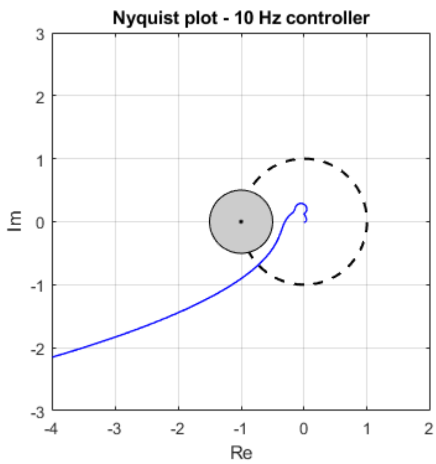

Figure 15 – Nyquist plot 10 Hz controller

2.4 – Testing and improving of the controller

Now the two possible controllers are designed and the feed forward controller is tuned. In this chapter the two controllers are tested on the actual setup, in combination with feedforward. The best performing controller is chosen and further improved.

2.4.1 – Setpoint profile

The controllers are tested on a given scan profile in which the speed of the second mass must be 30 rad/s for a displacement of at least 25 rad. To compensate for the velocity not being 30 rad/s at turnaround, a displacement of 26 rad is selected. The scan profile is illustrated in Figure 16 and has the following properties:

- Position to 26 rad and back to 0
- Velocity of 30 rad/s
- Acceleration of 1000 rad/s^2
- Jerk of 1,000,000 rad/s^3
- Total duration of 1.8 seconds

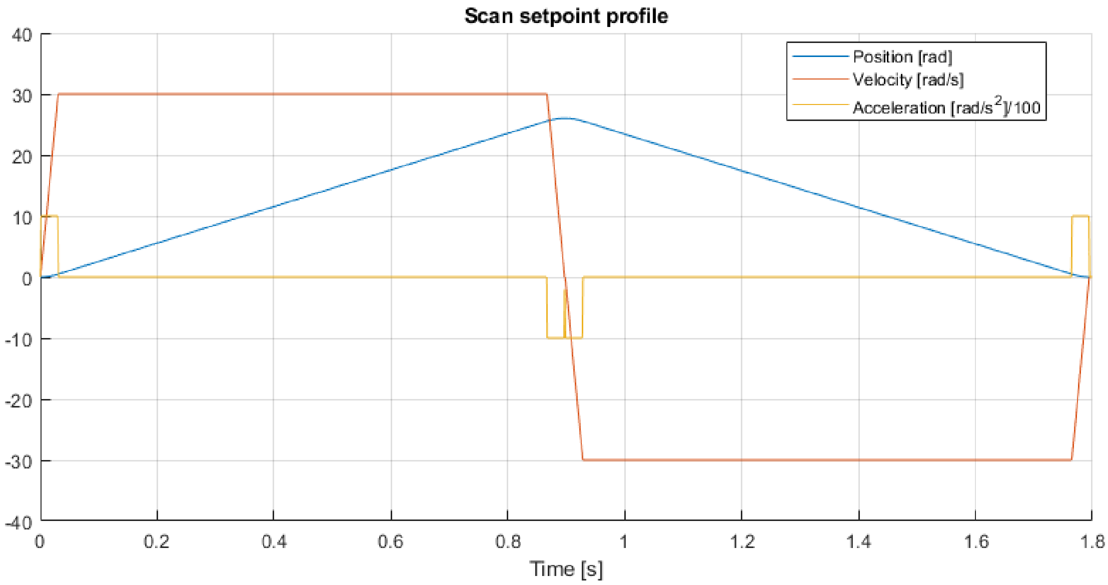

Figure 16 – Single scan setpoint profile

2.4.2 – Performance of controller 1 with and without feedforward

First the controller with a bandwidth of 23Hz is tested on the system. The system is tested with- and without feed forward and the setpoint profile as described in paragraph 2.4.1 is applied. The performance of the controller is judged by analyzing the error. In Figure 17 the error with feed forward is shown in blue, the error without feedforward is shown in red. Also, a scaled version of the position setpoint is plotted in orange for reference.

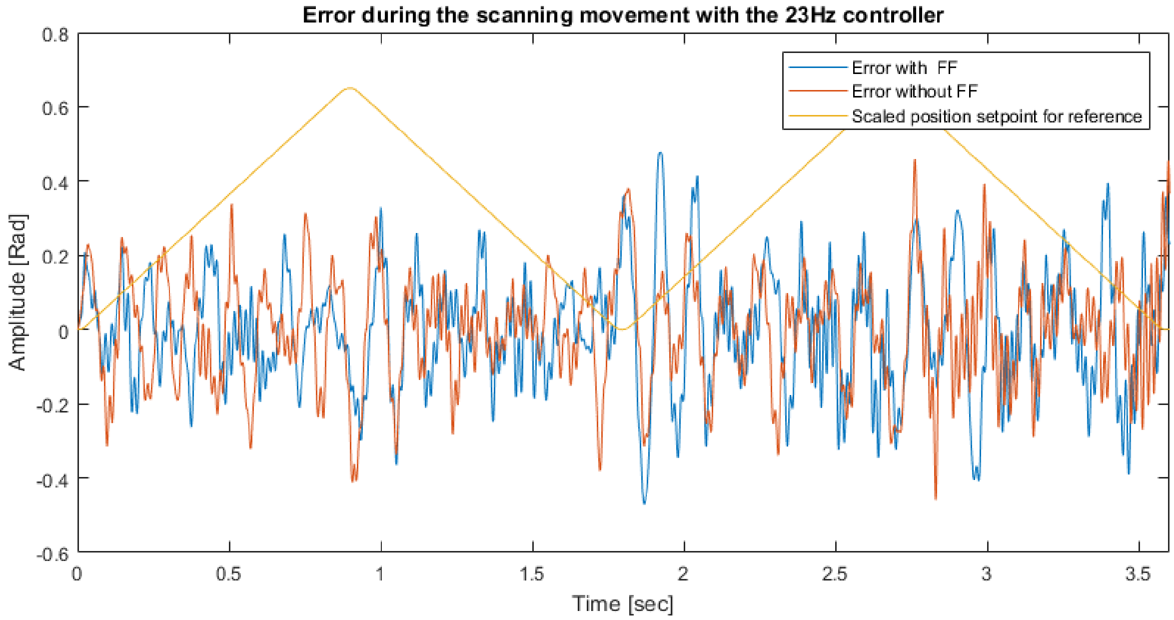

Figure 17 - Time domain error plot of controller 1 with 23Hz bandwidth

Because it is hard to identify the source of the error with only a time domain plot, a power spectral density (PSD) is made. Also, the cumulative sum of the PSD in forward and reverse direction is included in the plot, which can be found in Figure 18.

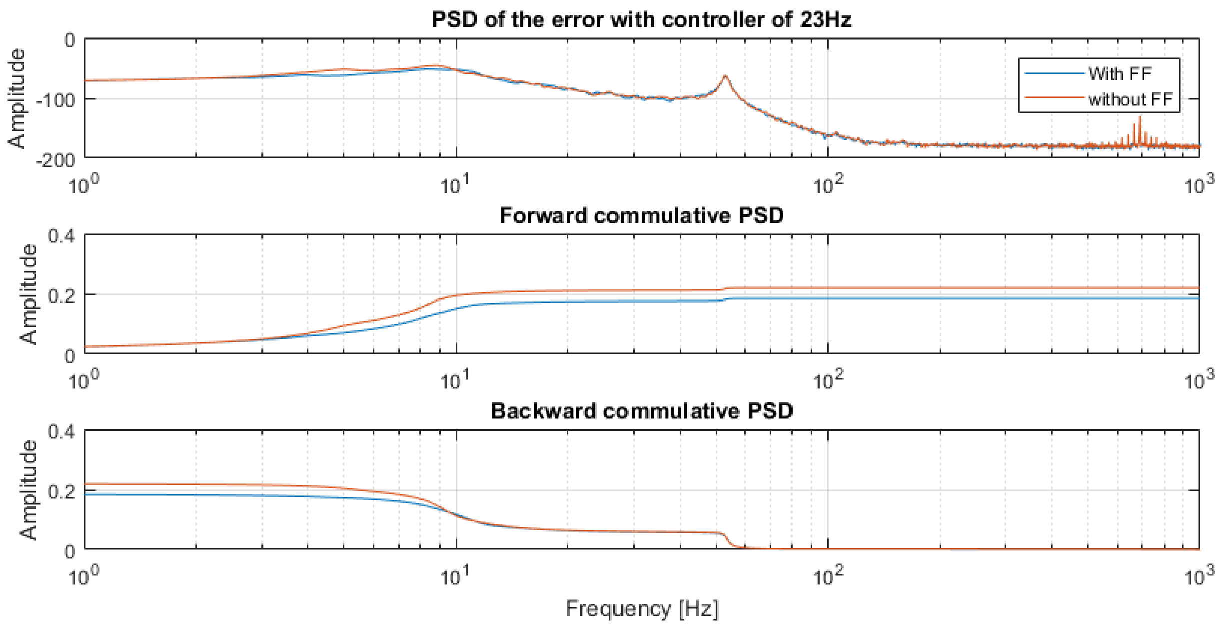

Figure 18 - PSD of the error when using controller 1 with a 23Hz bandwidth

From the plots it can be seen that the feed forward improves the tracking behavior, although the improvement is minimal. From the PSD it can be seen that the frequency ranges between 8 and 11 Hz and around the resonance of 57 Hz are the main cause for the error.

2.4.3 – Performance of controller 2 with and without feedforward

The second controller has a bandwidth of 10 Hz. This controller is also tested with and without feed forward and the error is analyzed. During testing, the same scanning setpoint is used as for controller 1, the setpoint that is described in paragraph 2.4.1.

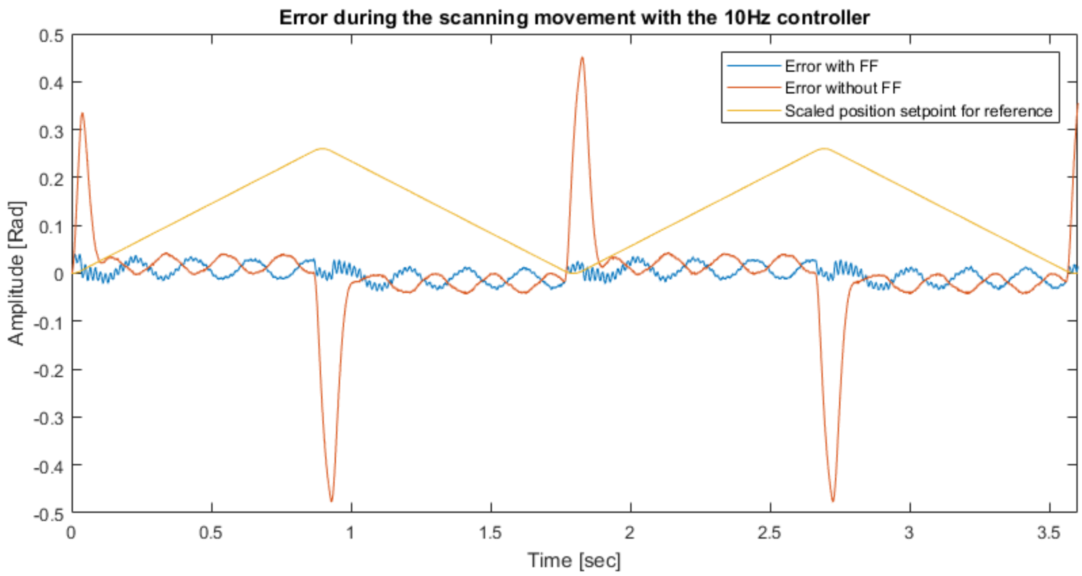

Figure 19 - Error when using controller 2 with a bandwidth of 10 Hz, with and without feed forward

The error in the time domain (Figure 19) is much smaller compared to the error obtained with controller 1. Also, it can be noticed that the feed forward has a major effect on the tracking error during the turnaround of the setpoint. In those points the inertia of the masses in the systems causes a tracking error and this is mainly compensated with the K_{fa} of the feed forward.
When the PSD of the error is analyzed in Figure 20 it can also be seen that the feed forward has a major effect on the low frequency part of the error. Also, it can be see that the resonance of the system at 57 Hz has a higher amplitude compared to not using feed forward. This has a minor effect on the power of the error. The reason is that the feedforward signal is inserted between the controller and the plant. The corresponding transfer function between this point and the error is equal to the process sensitivity, which has a peak at this resonance frequency. In Figure 21 the process sensitivity is shown.

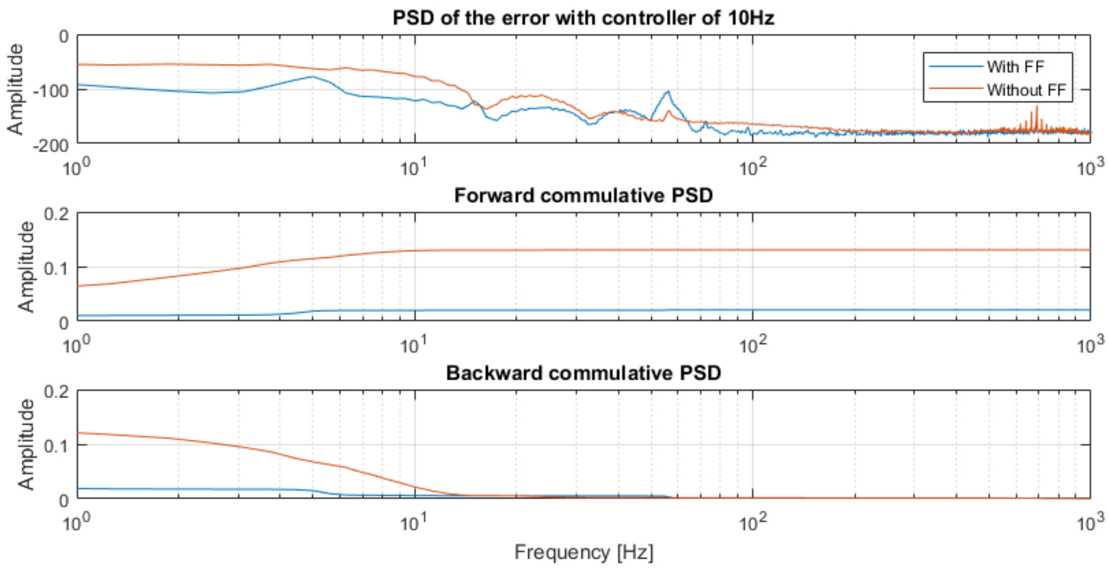

Figure 20 - PSD of the error when using controller 2 with a bandwidth of 10 Hz, with and without feed forward

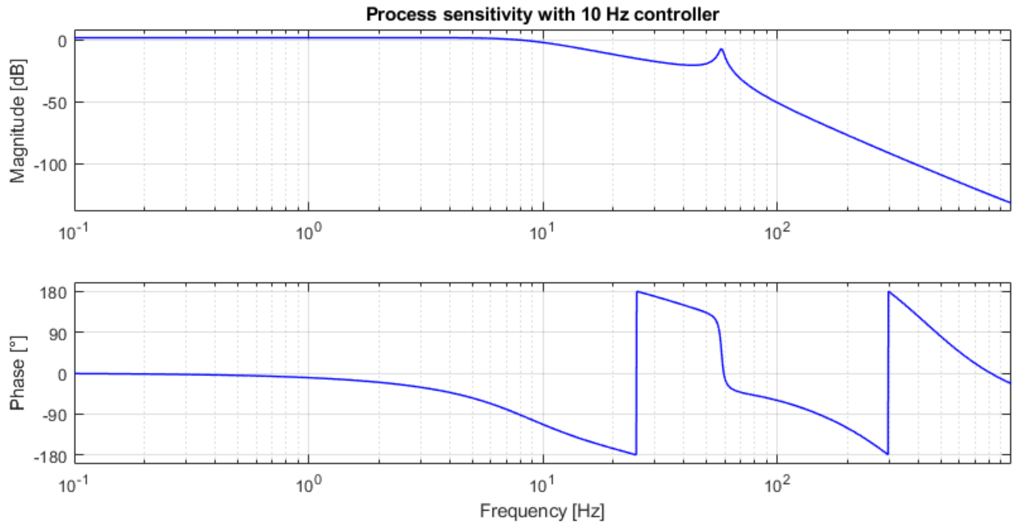

Figure 21 - Process sensitivity of the system with controller 2

When comparing the error of controller 1 and controller 2 it can be concluded that controller 2 has better performance. The bandwidth of the second controller is lower however it has also better suppression of low frequencies, and thus a lower amount of power in the low frequency part of the error. This region has the biggest contribution in the error and thus the total error is less.
When analyzing the PSD of the error from the second controller with feed forward it can be concluded that the error could be decreased even further.

2.4.4 – Performance of improved controller 2 with feedforward

Because Controller 2 is the best performing controller, this controller is used as basis to obtain an even better controller. The controller is fitted to an actual measurement of the plant at that moment. In Figure 22 the open loop response and the controller are shown. In Table 5 the used controller blocks are described and the used parameters are shown.

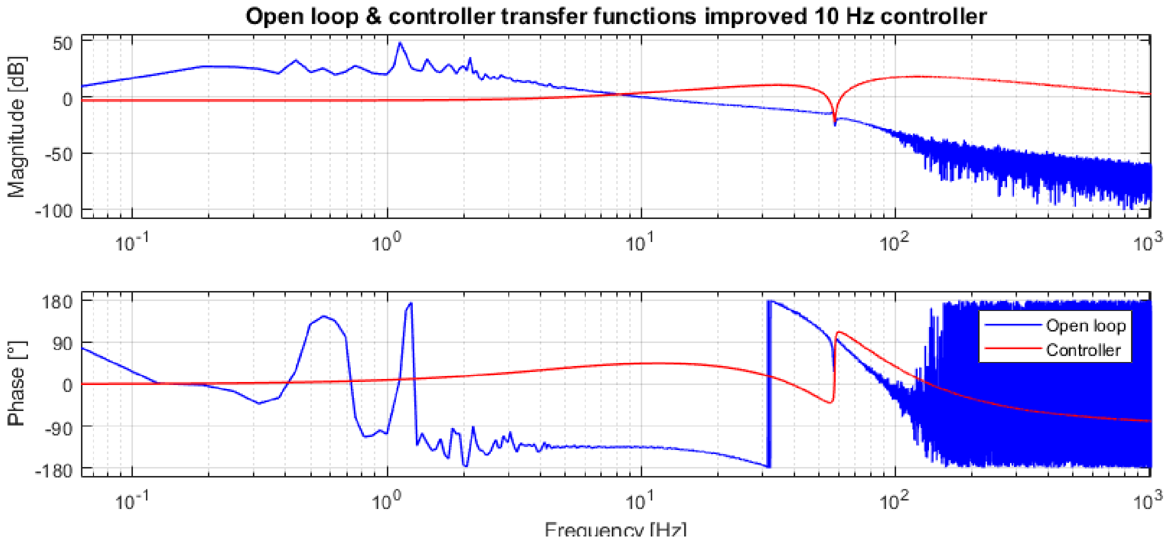

Figure 22 - Bode plot of controller and open loop of plant + controller

Table 5 - Improved controller 2 blocks and parameter values

Block	Parameter & value	Reasoning
Gain	0.7	Create gain to ensure bandwidth
Notch	Zeros: 58 Hz, damping 0.005 Poles: 59 Hz, damping 0.5	Notch resonance peak in plant at a frequency of 58 Hz. Damping is determined manually.
PD	P: 1 D: 0.03	Adding phase lead at the cross over frequency to get the desired phase- and modulus margins
Lowpass filter 2nd order	Cutoff frequency 100 Hz Damping 0.7	Reduce gain at higher frequencies when the gain is below 0 dB, creating proper system

The improved controller is tested on the system and the error is measured. This measurement is compared to the measurement obtained with controller 2. In both situations feed forward is used and the same setpoint profile. In Figure 23 the error of both measurements is shown. In blue the earlier used controller 2, in red the improved controller.

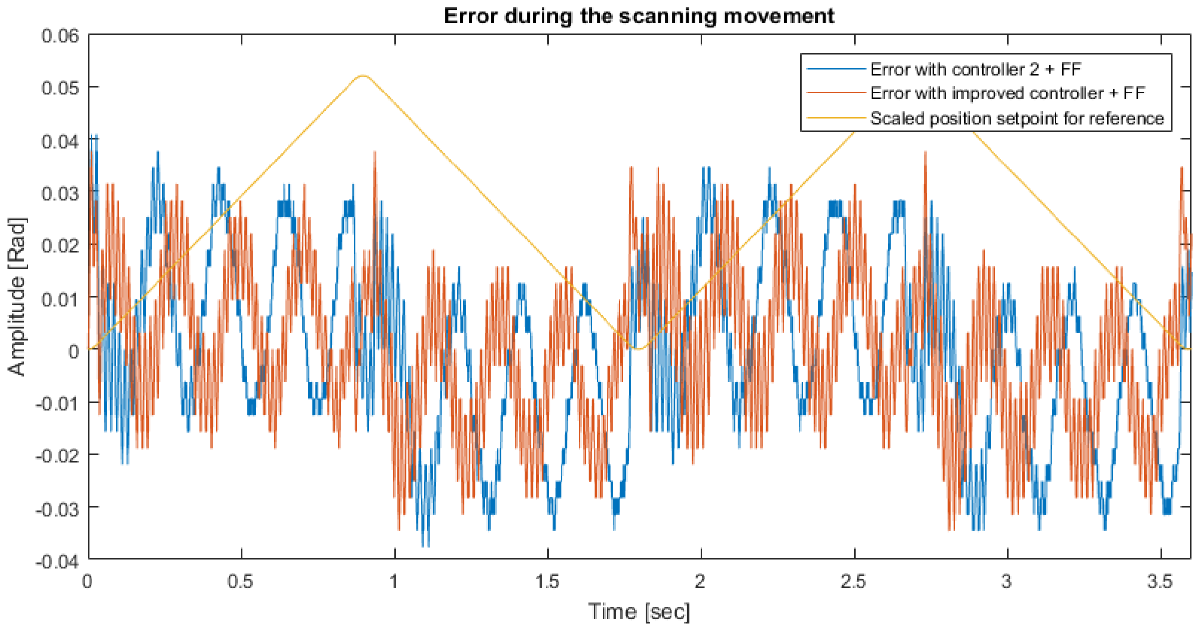

Figure 23 - Error in time domain of controller 2 and the improved controller

From the time domain plot the improvement is not visible. When looking at the cumulative sum of the PSD the improvement is visible. The PSD is shown in Figure 24.

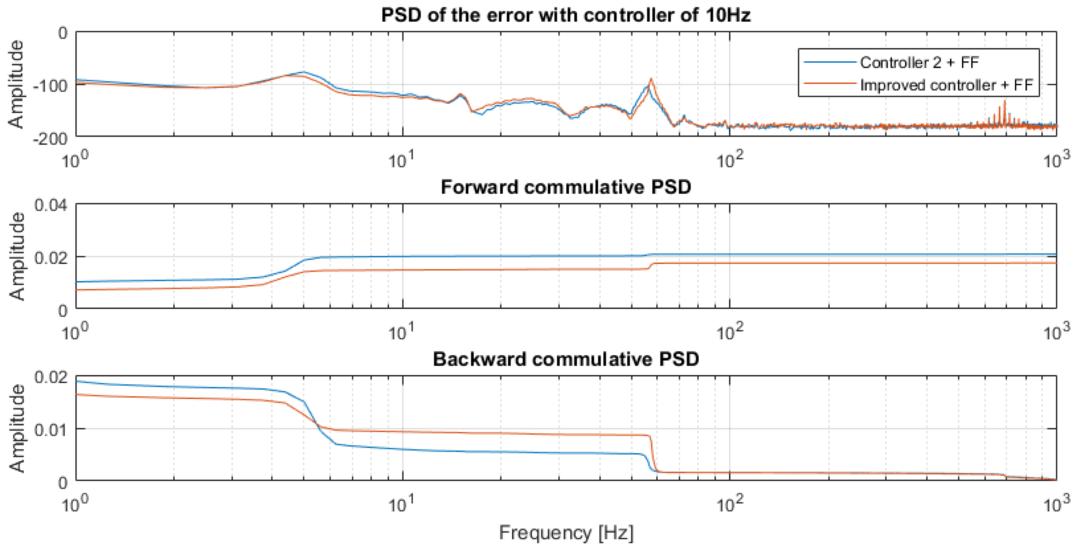

Figure 24 - PSD of the error when using controller 2 and the improved controller

The total sum of the power is less when using the improved controller. When comparing the forward and backward sum it can be concluded that at the resonance frequency of 57 Hz the performance decreased, while the performance around 5 Hz increased. The improvement at 5 Hz is larger compared to the decreased performance at 57 Hz and thus the total sum is less when using the improved controller.

2.5 – Results and conclusions

In total three controllers are made and tested on the system. Controller 1 and 2 are tested with and without feed forward, the improved controller 2 only with feed forward. The results are analyzed based on the time it takes to track the setpoint and the error during the setpoint.

2.5.1 – Performance of the system

The performance of the controllers is graded based on two criteria. The first criterium is the time it takes to complete one full scan cycle. The second criterium is the amount of error during a scan cycle. In all measurements the scanning profile setpoint is used.
To have the shortest scan cycle time, the turnaround time must be as short as possible. Therefore, the performance is based on the turnaround time, where lower is better. The turnaround time is measured by measuring the time between the moment the setpoint has reversed and has a constant speed and the moment the error is within 1% of the position of the setpoint. In the situation of the scanning profile the error must be less than 0.3 rad.
The amount of error during a scan cycle is measured by determining the total power in the error signal during a scan cycle. The power is measured by calculating the cumulative power in the PSD of the error signal. The results of the different controllers are listed in Table 6.

Control strategy		**Performance**	
Controller	**Feed forward**	**Turnaround time [ms]** (time required to get error within 0.3 rad after turnaround)	**Cumulative error [rad]**
1, 23Hz	No	> 0.7 seconds (never within margin during move)	0.219
1, 23Hz	Yes	> 0.7 seconds (never within margin during move)	0.185
2, 10Hz	No	19 ms	0.130
2, 10Hz	Yes	0 ms (always within margin)	0.021
Improved 2	Yes	0 ms (always within margin)	0.017

Table 6 – Controller performance comparison

From Table 6 it can be seen that the improved controller 2 has the best performance. This controller ensures the error is always within the desired 1% and the total power in the error signal is the least compared to the other controllers.

2.5.2 – Possible future improvements

Although the improved controller 2 has the best performance, the performance can even further be improved. The following options are available:

Suppressing error at 4.77 Hz

In the spectrum of the error signal as shown in Figure 20 and Figure 24 is a peak visible at 5Hz. In reality this peak is at 4.77 Hz, equal to the rotational speed of the mass (30 rad/s = 4.77 Hz). The exact mechanical cause is not clear. This could be caused by one of the bearings, an eccentricity in the inertias or some misalignment in the system.
This error could be minimized by adding an extra peak in the controller at this frequency. This will result in a local dip in the sensitivity function at this frequency and the error will decrease. Because the frequency is depending on the speed of the setpoint, the location of the peak must be adjusted when the setpoint is changed.
Another option is to add an extra term in the feed forward. This term is depending on the speed of the setpoint and compensated for the error caused by this speed.

Suppressing resonance at 57 Hz

At 57Hz the plant has a resonance. By compensating for this resonance in the controller, i.e. adding a notch, the peak disappears in the open loop transfer, but remains at the process sensitivity. Because the feed forward terms are inserted between the controller and the plant, this signal excited the resonance in the process sensitivity.
A possible solution is to increase the overall gain of the controller. This reduces the gain of the process sensitivity and the feed forward term creates less excitation of this frequency. However, the controller must be tuned again in order to maintain good stability margins with the higher gain.

Improving the feed forward terms

From the error plot in Figure 23 it can be seen that the error when moving in the positive direction has a positive offset. When the move is in the negative direction the error has an offset in the negative direction. This offset can be eliminated by better tuning the feed forward terms for the viscose friction of coulombs friction, respectively K_{fc} or K_{fv}.

Increasing of the acceleration and jerk of the setpoint profile

The ultimate goal of the system is to achieve a turnaround time which is as short as possible. With controller 2 + feed forward and with the improved controller + feed forward the turnaround time is solely determined by the setpoint only. The error during a turnaround is always within the specified margin of 1%. By increasing the acceleration and jerk of the setpoint the time required to reach the desired speed is decreased and thus the turnaround time is decreased. Increasing the acceleration and jerk will increase the error during this turnaround, but this can again be decreased by improving the controller.

3. Solutions of exercises marked with !!*!!

Exercise 1.2 - Interpreting a Bode diagram

The goal is to interpreted the two systems shown in Figure 25.

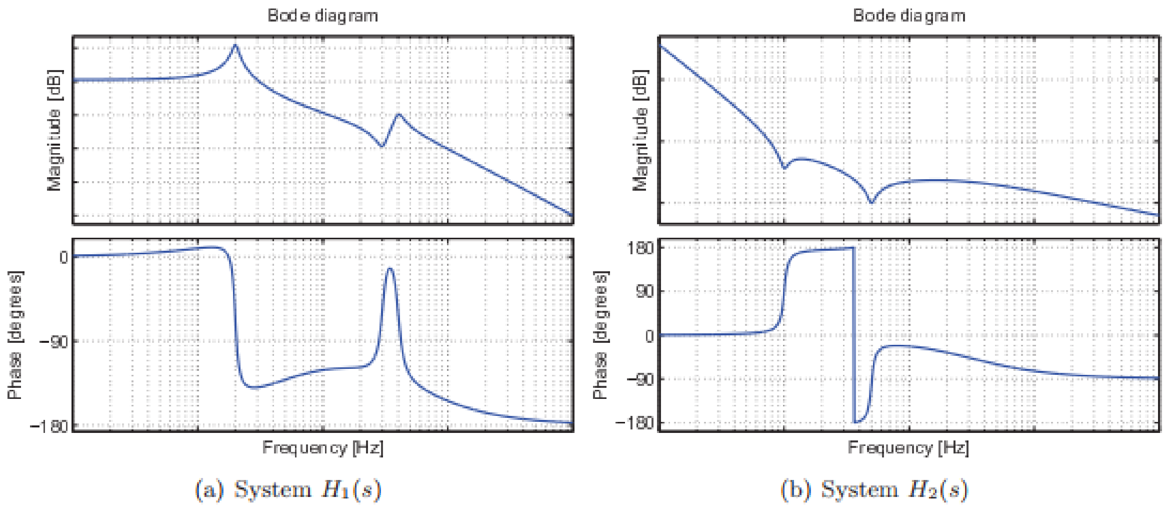

Figure 25 – Bode diagrams of systems H1(s) and H2(s)

System H1:

- Relative degree (#poles - # zeros): 2
    - Slope of -2 at high frequencies
    - Phase to -180 degrees at high frequencies
- Number of integrators: 0
    - Slope is 0 at low frequencies
    - Phase of 0 degree at low frequencies
- Total number of poles: 5
    - 2 LHP poles at frequency of first peak
    - 2 LHP poles at second peak
    - 1 LHP pole around the second peak (part of a lead filter)
- Total number of zeros: 3
    - 2 LHP zeros at dip between peaks
    - 1 LHP zero around the first peak (part of a lead filter)

System H2:

- Relative degree (#poles - # zeros): 1
    - Slope of -1 at high frequencies
    - Phase of -90 degrees
- Number of integrators: 4
    - Slope is -4 at low frequencies
    - 4x LHP pole, resulting in 0 (-360) degrees phase shift
- Number of poles: 5
    - 4 LHP poles at low frequencies (integrators)
    - 1 LHP pole slightly after second dip
- Number of zeros: 4
    - 2 LHP zeros at first dip
    - 2 LHP zeros at second dip

Exercise 1.3 - Estimating transfer functions

The systems H1 and H2 from the file frfdata.mat are plotted in Figure 26. System H2 has better controllability, because system H1 contains RHP poles and system H2 does not.

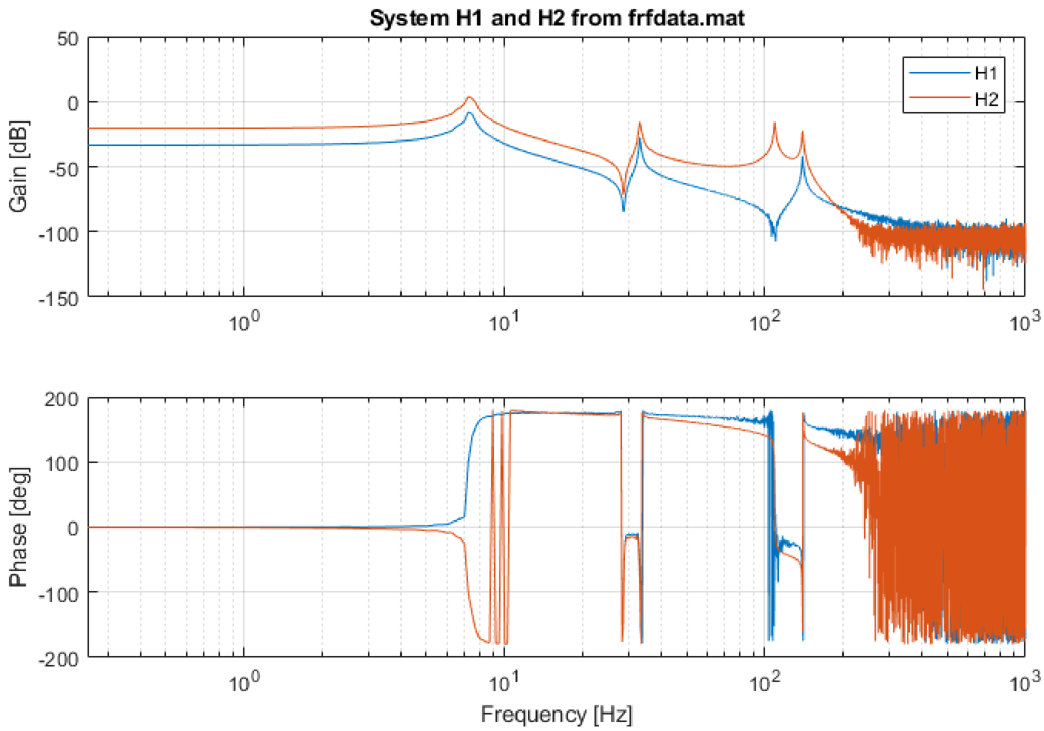

Figure 26 – System H1 and H2 from frfdata.mat

Exercise 2.3 - Closed loop FRF measurement

The closed loop sensitivity is determined by performing an 3-point FRF measurement on the simulation model shown in Figure 27.

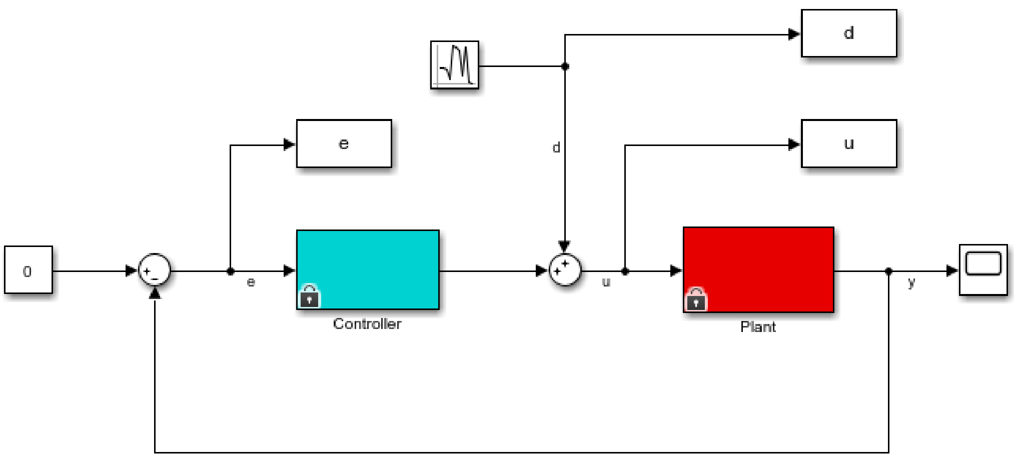

Figure 27 – frf_ex3.mdl simulation model

First the sensitivity of the simulation model is measured by injecting noise at d and measuring at u. This measurement resulted in the transfer function shown in Figure 28.

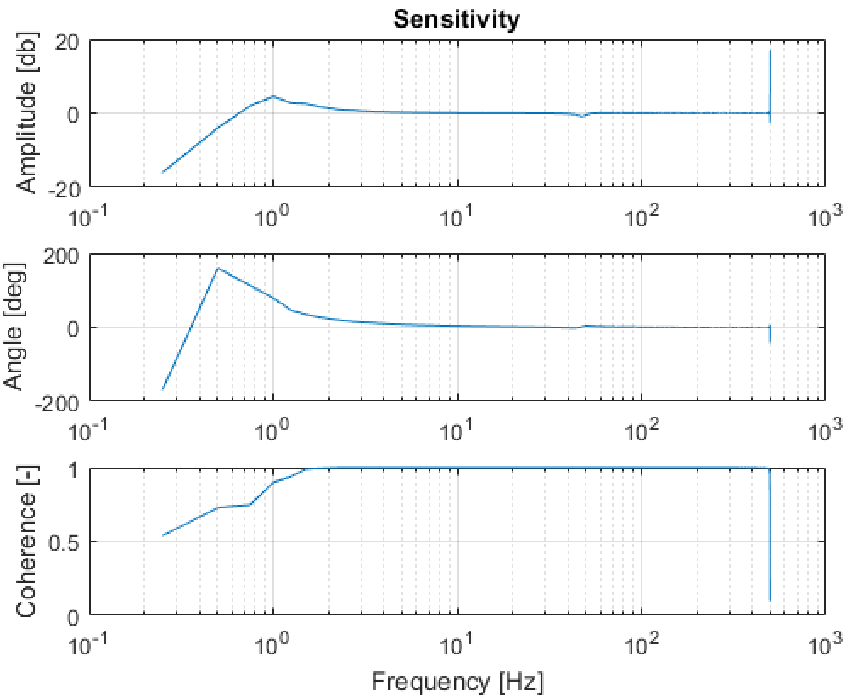

Figure 28 – Sensitivity FRF

The coherence is not 1 at lower frequencies because the sensitivity gain at lower frequencies is relatively low, caused by the controller which has high suppression on these lower frequencies. The coherence is 1 (or almost 1) between 2 and 500 Hz, indicating that the measurement is reliable in this frequency range. However, the coherence can be 1 even when the measurement is not reliable. In this exercise the coherence function corresponds to the expected result.

To determine the process sensitivity, another simulation is done using the model shown in Figure 27. Now the transfer function from d to e is measured, resulting in the FRF shown in Figure 29.

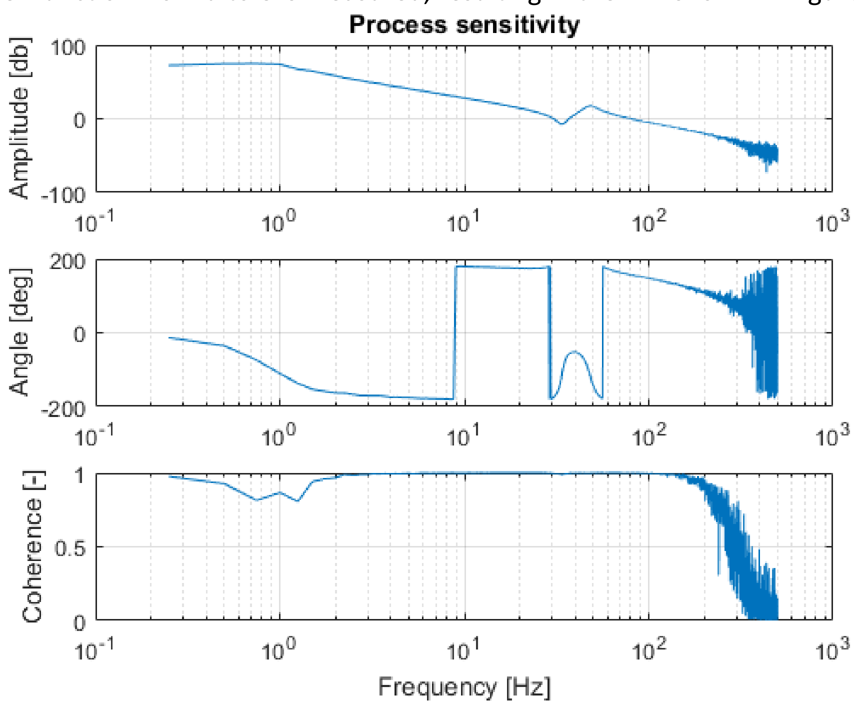

Figure 29 – Process sensitivity FRF

The coherence of the process sensitivity is flat between 2 and 100 Hz. At frequencies above 100 Hz the coherence is dropping fast, caused by the sensor noise. Because the gain of the process sensitivity is low at high frequencies (around -40 dB) the sensor noise gets relatively large, causing the gain and phase to be determined incorrectly.

The plant dynamics can be determined without measuring the controller by using the sensitivity and process sensitivity, using the following formula:

$$Plant(f) = \frac{Process\ sensitivity(f)}{Sensitivity(f)}$$

The plant FRF that is computed using this formula is shown in Figure 30.

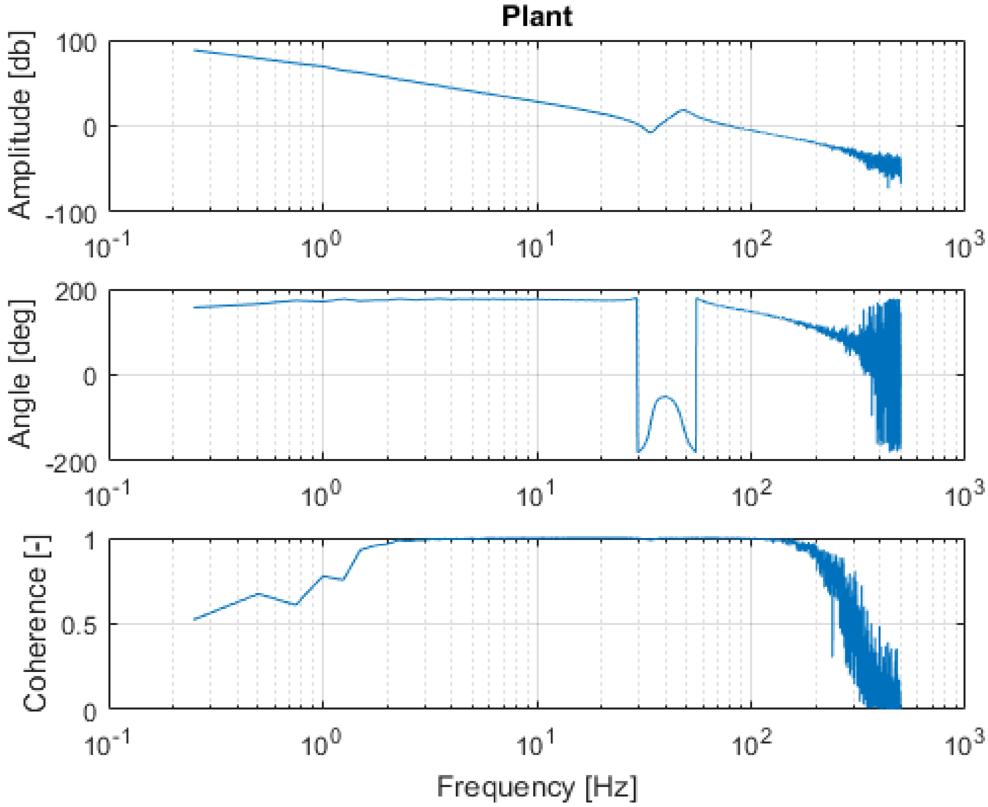

Figure 30 – Plant FRF

At higher frequencies there are two phenomena: bad amplitude estimation and phase drop. The phase drop can be explained by delay in the system, because the phase drop seems linear (when plotted on a linear scale). To determine the delay a 'pade approximation' is fitted to the phase angle, shown in Figure 31. This resulted in a delay of about 1 millisecond. The bad amplitude estimation can be explained by noise in the system which becomes more present as the plant gain decreases.

Now the FRF measurements of this exercise are repeated with different values for nfft: 1, 4 and 15. The variation of nfft resulted in slightly different FRF functions, shown in Figure 31. The result of different simulation times is that the coherence becomes 1 at all frequencies when the simulation time is the same as the window length. The cause of this phenomena is that there is only one window, meaning that there are no other windows to compare with. The number of possible windows is also decreasing when the simulation time is shortened. This will result in a more varying estimation.

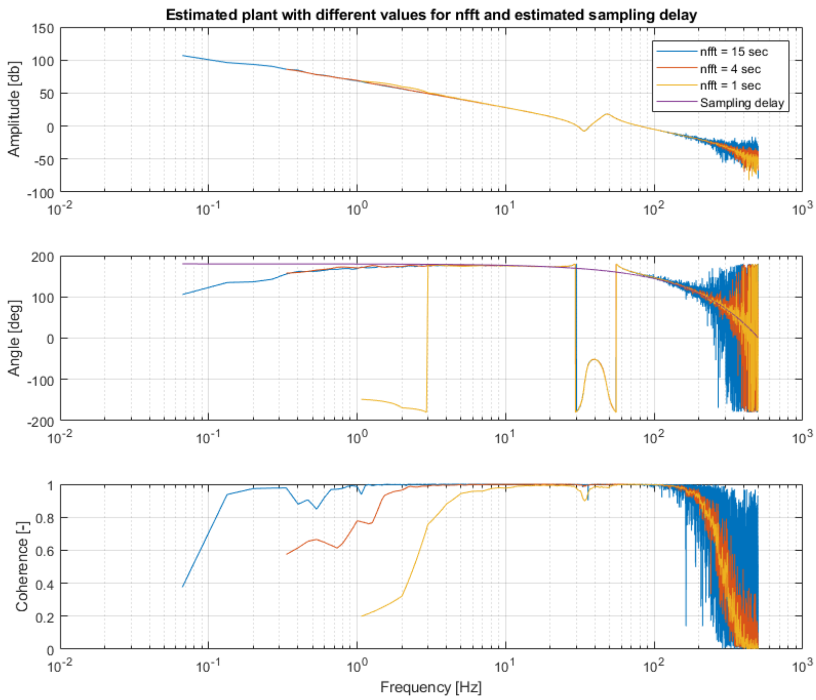

Figure 31 – Estimated plant with different values for nfft and estimated sampling delay

Exercise 3.1 - Modelling and control of a motion system

In this exercise a classic mass-spring-damper-mass system is used to create a feedback loop. In Figure 32 this system is shown. The parameters of the system are:

- $m_1 = 0.015$ kg
- $m_2 = 0.045$ kg
- $d = 0.4$ Ns/m
- $k = 2200$ N/m

The input of the system is motor force F, the output of the system is position x_1.

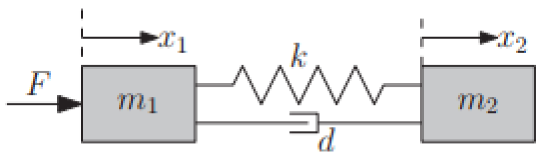

Figure 32 – Mass-spring-damper-mass system

The transfer function from F to x_1 is:

$$H_1(s) = \frac{x_1}{F} = \frac{m_2 s^2 + ds + k}{m_1 m_2 s^4 + d(m_1 + m_2)s^3 + k(m_1 + m_2)s^2} = \frac{1000s + 6000}{s^3 + 20s^2 + 5000s}$$

From F to x_2 this becomes:

$$H_2(s) = \frac{x_2}{F} = \frac{ds + k}{m_1 m_2 s^4 + d(m_1 + m_2)s^3 + k(m_1 + m_2)s^2} = \frac{1000s - 6000}{s^3 + 20s^2 + 5000s}$$

First, a simple stabilizing controller is created with a bandwidth of 20 Hz and a modulus margin of 2.2 dB. The resulting open loop bode is found in Figure 33 and the sensitivity function in Figure 34. The step response is plotted in Figure 35.

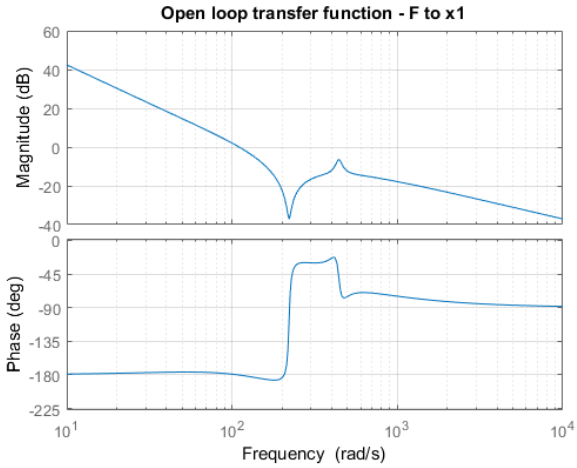

Figure 33 – Bode of the open loop transfer function H1*C

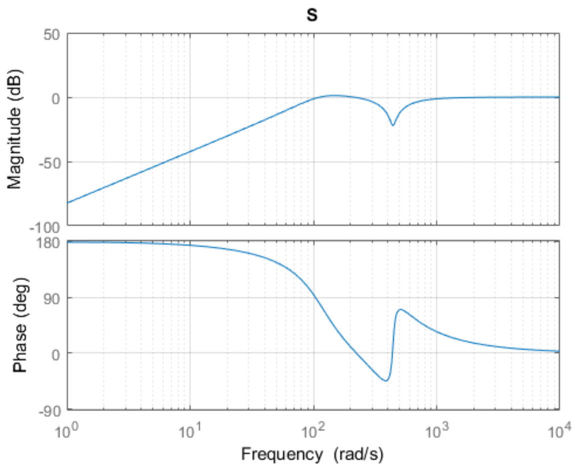

Figure 34 – Bode of the sensitivity function of H1*C

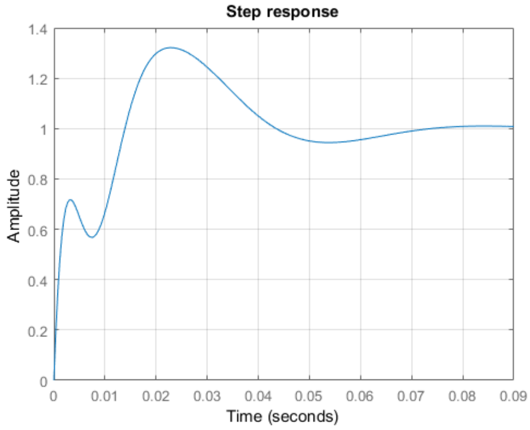

Figure 35 – Step response of H1· C

Now the system and controller are implemented in Simulink as closed loop, which is shown in Figure 36.

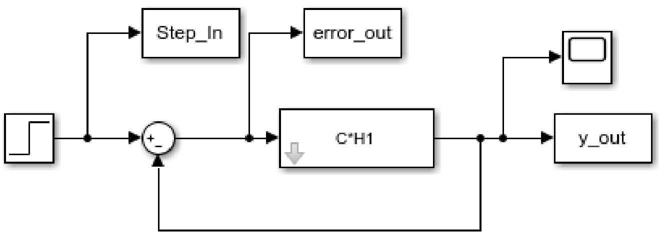

Figure 36 – Simulation schematic of mass-spring-damper-mass control loop

With this model the influence of the bandwidth and phase margin is investigated. First the bandwidth of the controller is varied, while maintaining the phase margin (Figure 37). Secondly the phase margin is varied with a fixed bandwidth (Figure 38). The error is plotted in both cases.

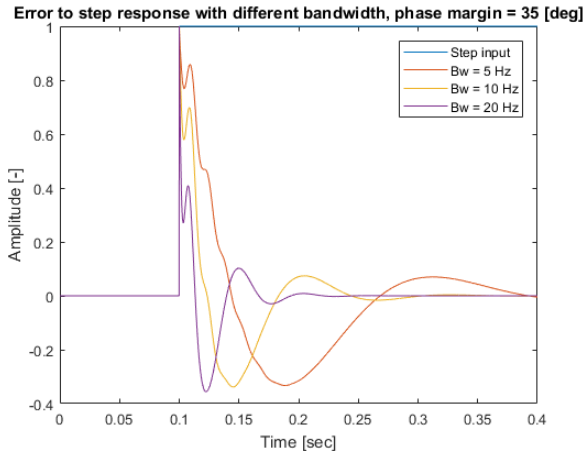

Figure 37 – Error when using different bandwidth

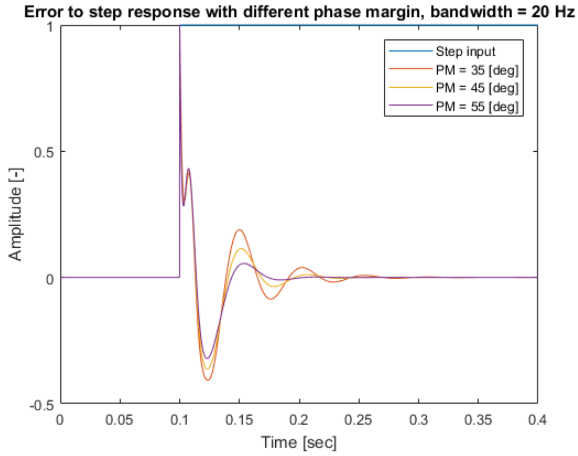

Figure 38 – Error when using different phase margins

The influence of the bandwidth and phase margin onto the overshoot, rise time and settling time is described in Table 7.

Table 7 - Influence of the bandwidth and phase margin onto the overshoot, rise time and settling time

System parameter	Overshoot	Rise time	Settling time
Increasing Bandwidth	No difference	Decreasing	Decreasing
Increasing Phase margin	Decreasing	Equal	Decreasing

Increasing the bandwidth has a positive effect on the rise time and settling time because this allows the system to use more and higher frequencies to follow the step input. The Fourier analysis of a step (block wave) has an infinite number of frequencies. Being able to use more frequencies (higher bandwidth) to track the step reference, the step response of the system converges to the actual step input, and thus has a better tracking behavior.
By increasing the phase margin, the effective damping of the system is increased. This damping determines the amount of overshoot. When the system has a more overshoot, the response needs more time to get within a certain percentage of the final value, which determines the settling time. Less overshoot will thus result in a shorter settling time.

When the controller of H1 is used to control the system H2, the output becomes unstable (Figure 39). This can be explained by looking at the bode plot of the system (Figure 40). The phase margin at the crossover frequency is negative (phase is less than -180 degree). This will result in an unstable system. Decreasing the gain will shift the crossover frequency to a lower value, where the system has more phase, resulting in more phase margin.

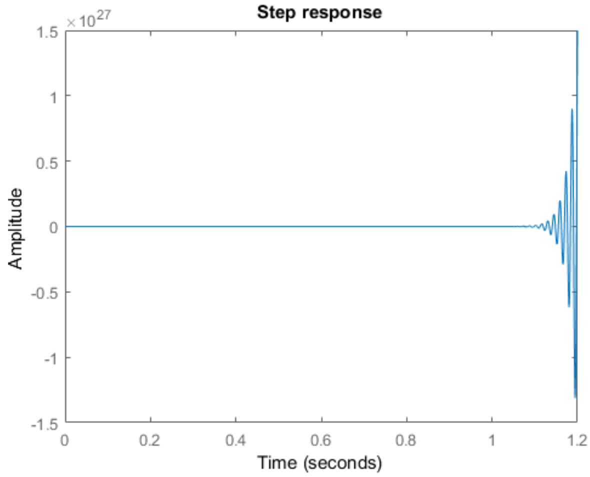

Figure 39 - Step response when using C1 to H2

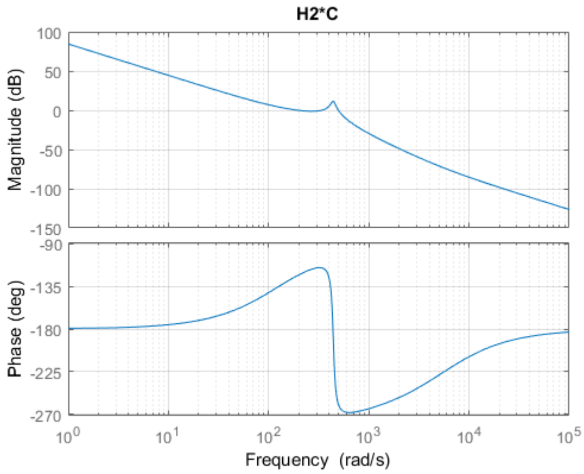

Figure 40 - Bode when C1 is applied to H2

Exercise 3.2 - Inverted pendulum

The considered system is a linearized inverse pendulum (Figure 41).

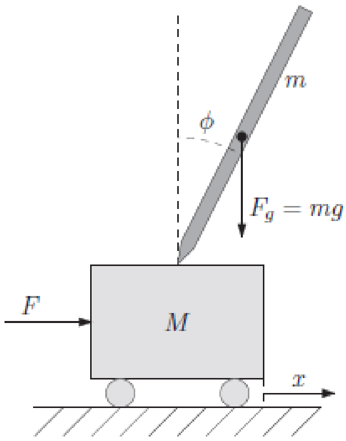

Figure 41 - Schematic overview of the inverse pendulum setup

The transfer function of this system is equal to:

$$H_{(s)} = \frac{\Phi_{(s)}}{F_{(s)}} = \frac{s}{-\frac{1}{6}(4M+m)Ls^3 - \frac{2}{3}Lbs^2 + g(M+m)s + gb}$$

where L is the length of the pendulum and b is the friction between the cart and the floor. Furthermore, assume that

M = 0.05, m = 0.04, b = 0.02, g = 9.81, L = 0.15

This results in a transfer function equal to

$$H_{(s)} = \frac{\Phi_{(s)}}{F_{(s)}} = \frac{-18 \cdot s}{0.108 \cdot s^3 + 0.036 \cdot s^2 - 15.89 \cdot s - 3.532}$$

And the Bode plot of this system is equal to Figure 42, with the corresponding pole-zero map, showing the position of the poles and zeros in the complex plane.

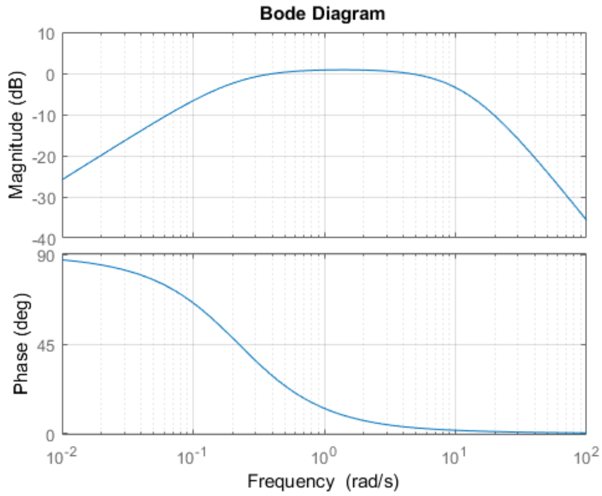

Figure 42 - Bode of the inverse pendulum system

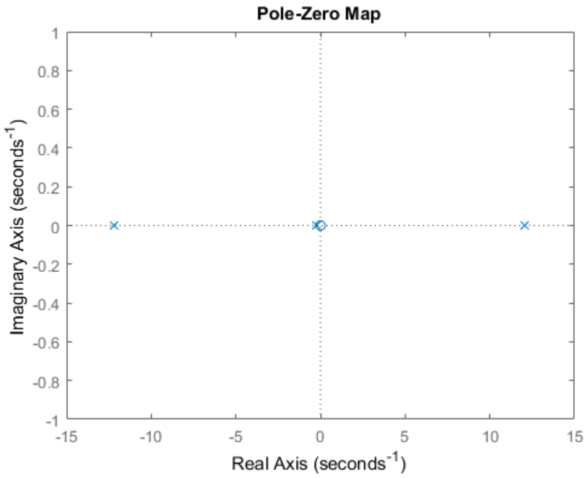

Figure 43 - Pole-zero map of the inverse pendulum system

By looking at the bode of the system it is clear that this system is unstable. At the frequency of 10 rad/s the amplitude starts having a slope of -2, corresponding to two poles. At the same frequency the phase stays at 0 degree, when a drop of -180 degree is expected. This can be explained by one pole laying in the right halve plane, generating a phase lead of 90 degree.

When two controllers with C1 = 1 (Figure 44 and Figure 46) and C2 = -0.55 (Figure 45 and Figure 47) are tested on the system, both systems are still unstable. This is because at least one counterclockwise encirclement is needed around (-1,0) to compensate for the RHP pole (Z = N+P). The difference between the two systems is that when the gain of the controller is negated, the Nyquist plot is flipped over the y axis. The step response is also neglected. In the step response it is also clearly visible that both systems are unstable.

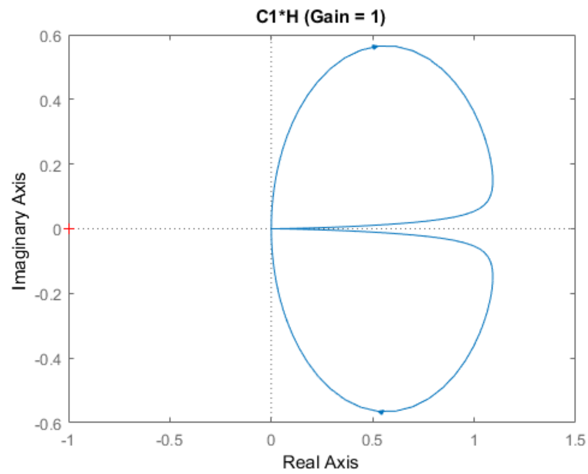

Figure 44 - Nyquist of the inverse pendulum sytem with C1

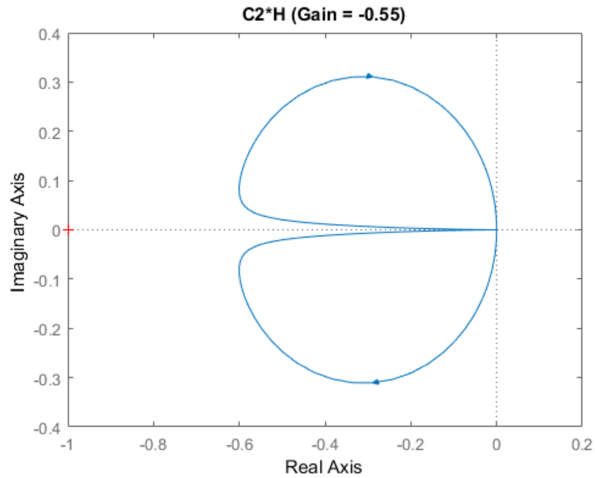

Figure 45 - Nyquist of the inverse pendulum sytem with C2

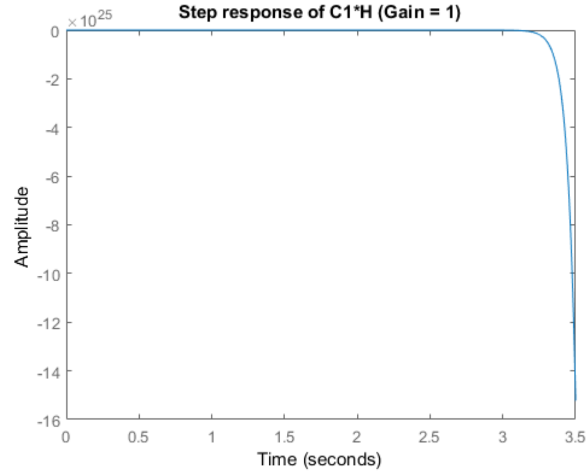

Figure 46 – Closed loop step response of the system with C1

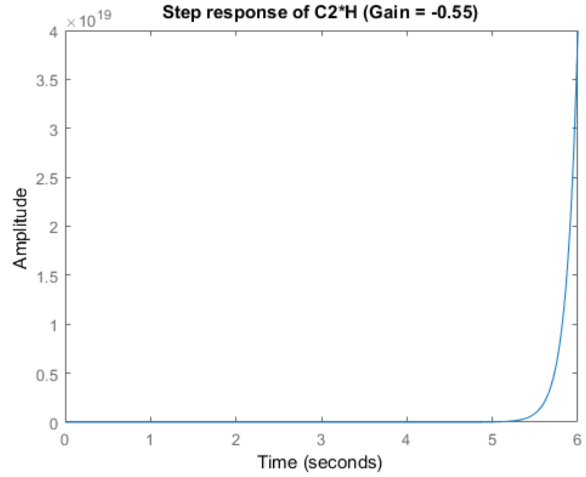

Figure 47 - Closed loop step response of the system with C1

In order to stabilize the system, a counterclockwise encirclement of the point (-1,0) is needed. This is achieved by entering the unit circle in the 3th quadrant (between -90 degrees and -180 degrees, approximately an -1 slope in amplitude). The used controller is consisting of the following parts:

Table 8 - Parts of the stabilizing controller of the inverse pendulum system

Block	**Parameter 1**	**Parameter 2**	**Reasoning**
Gain	Gain = -4		Create gain to ensure bandwidth, flip phase
Lead filter	Pole = bandwidth*3	Pole = bandwidth/3	Create phase around bandwidth
Differentiator	Zero at bandwidth		Create extra phase around bandwidth
2x Integrator	Pole at bandwidth/5		Create high gain at low frequencies
Lowpass filter	Pole at bandwidth*5		Create proper system

The resulting Open loop bode is found in Figure 48. In this bode plot it can be seen that around the crossover frequency, the slope is -1 and the phase is between -90 degrees and -180 degrees. In the Nyquist plot (Figure 49) a counterclockwise encirclement of (-1,0) is visible, which stabilizes the system.

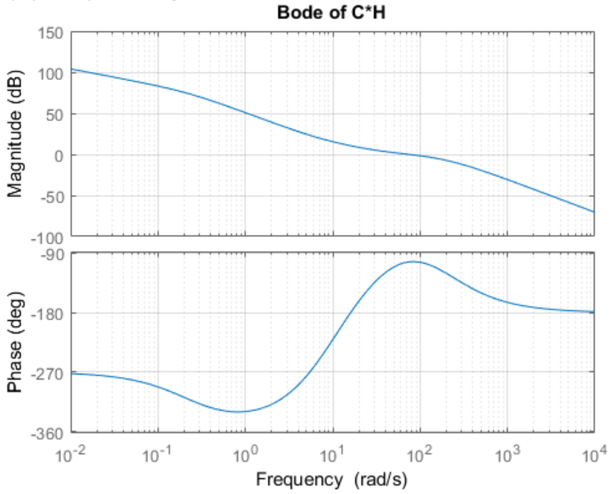

Figure 48 - Open loop bode of system and controller

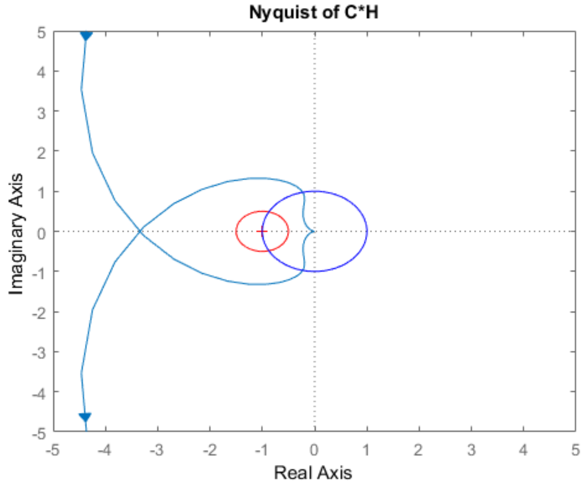

Figure 49 - Nyquist of system with controller

The step response is a stable output. In Figure 50 this step response is shown. It takes approximately 0.5 seconds to reach a stable output.

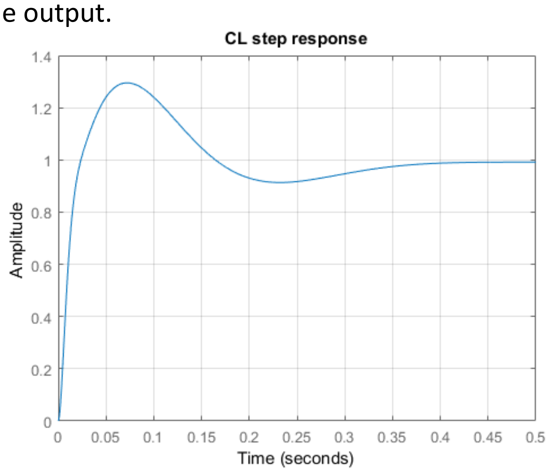

Figure 50 - Closed loop step response of the inverse pendulum system

In the case the chart is on a small slope, there is a constant force reacting on the chart, next to the controller output. This can be modelled in a block diagram as shown in Figure 51. The disturbance influences the system between the controller and the plant.

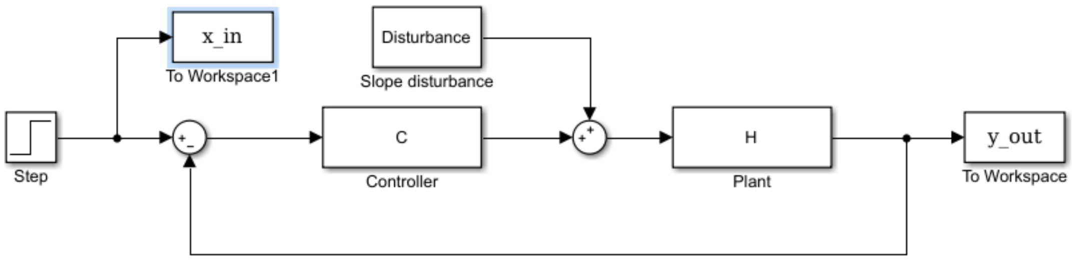

Figure 51 - Block diagram of the inverse pendulum system with a disturbance

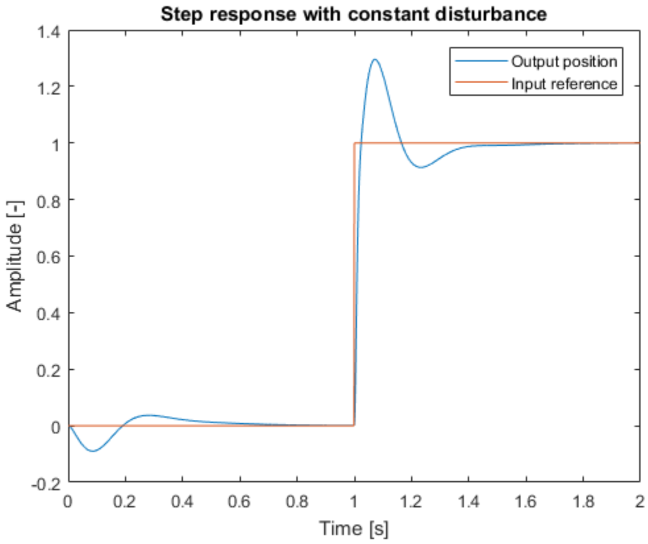

Figure 52 - Output response of the Closed loop system to an input reverence step with constant disturbance

The output of the system is found in Figure 52. In this plot a step reference is applied after 1 second. In the first second of the plot it is visible that the controller compensated for the constant disturbance of 1 [Newton]. After approximately 0.5 seconds this error is too little to see. After 1 second the input of the system (the reference) is changed with the step. The error due to the reference is also minimized after 0.5 second and the system is still in the upright position. The stability of the system itself is not changed, this is because the disturbance is an external input and is not part of the closed loop stability.

Exercise 4.2 - Non-collocated plant

A simple stabilizing controller is created by combining the following parts:

- Gain of 1
- Notch on 52.5 Hz with a damping of 0.01 for the nominator and 0.5 for the denominator. This notch minimizes the resonance peak
- Differentiator with the pole at 10 Hz to create phase margin around the bandwidth

The resulting open loop bode and step response of (Controller + Plant) can be found in Figure 53 and Figure 54:

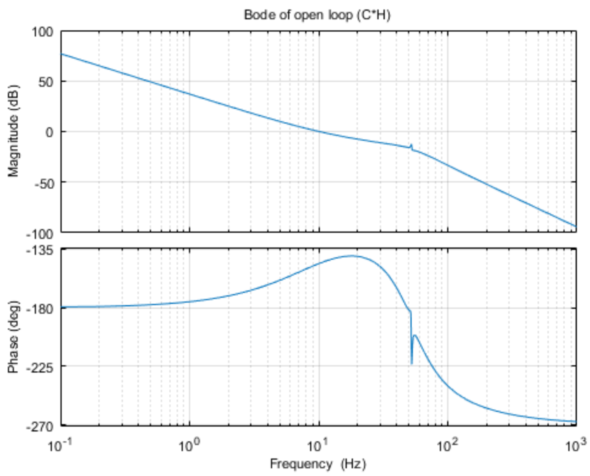

Figure 53 - Open loop bode of a simple stabilizing controller and plant (H) of exercise 4.2

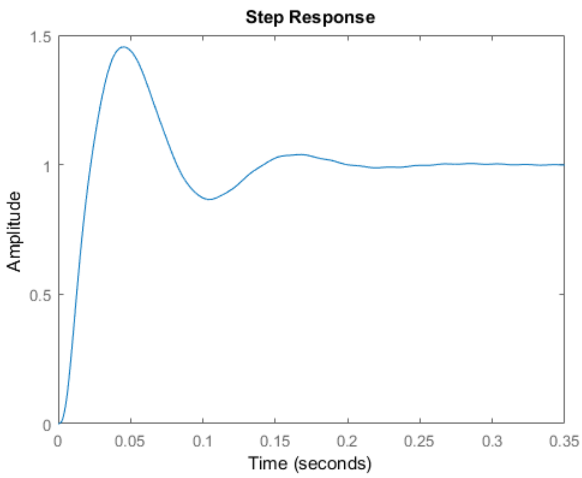

Figure 54 - Step response of a simple stabilizing controller and plant (H) of exercise 4.2

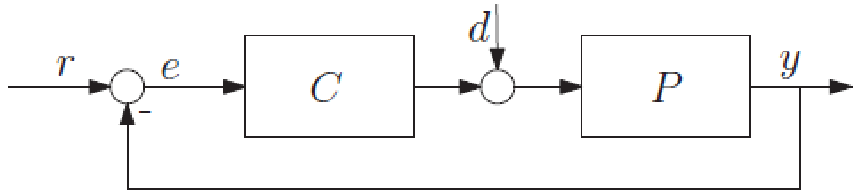

Figure 55 - Block scheme of exercise 4.2

When a 2 Hz sinus with an amplitude of 1 is applied as a disturbance at d (Figure 55) and the tracking error due to this disturbance (around e) must be evaluated, the interesting transfer function is equal to the process sensitivity. This process sensitivity is equal to:

$$PS = \frac{H}{1 + HC}$$

The goal is to minimize the error after 0.5 second (1 period) to be less than 0.01. This means that the Process Sensitivity at this frequency must be less than -40 dB. This is ensured by adding a notch filter at this particular frequency, creating a peak in the controller behavior at this frequency. In order to let the error converge to zero, when time approaches infinity, the process sensitivity must go to zero for low frequencies. Because the process sensitivity at low frequencies is depending on the inverse of the controller behavior, the controller must have an infinite gain at frequency zero. Adding an integrator to the controller creates this behavior. The controller, used blocks and responses are found in Figure 56 till Figure 60 and in Table 9.

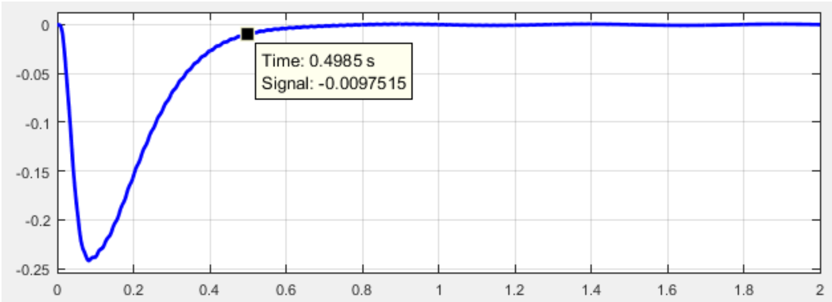

Figure 56 - Error response due to the 2 Hz disturbance

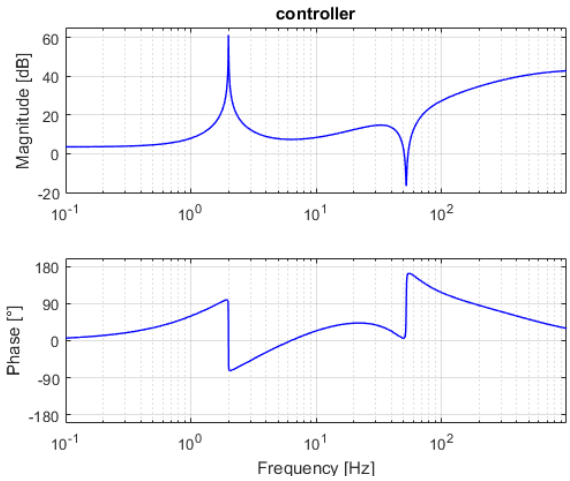

Figure 57 - Bode of the controller

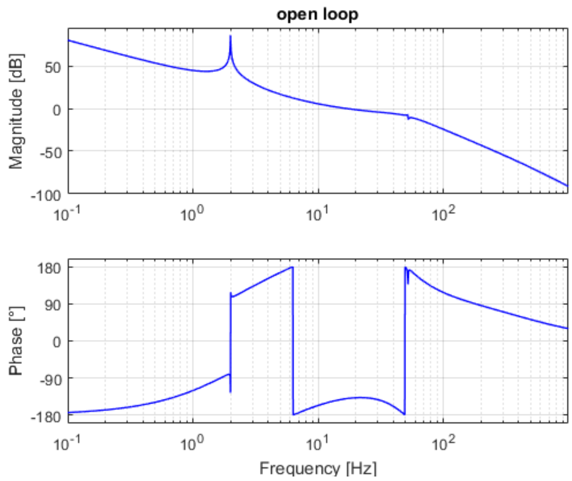

Figure 58 - Open loop bode

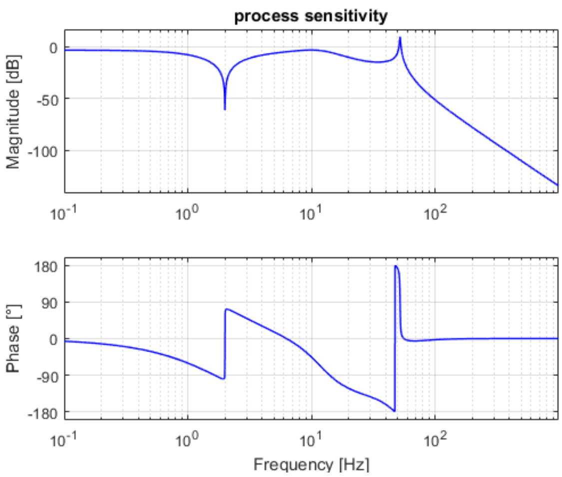

Figure 59 - Bode of the process sensitivity

Bandwidth	17.15	~	10
Modulus margin	5.1	<	6
Phase margin	40.4	>	30
Gain margin	7.7	>	6

Figure 60 - Stability margins of the system

Table 9 – Controller overview

Block	Parameter 1	Parameter 2	Parameter 3	Reasoning
Gain	Gain = 15			Create gain to ensure bandwidth
Notch	Zero/Pole = 52.5 Hz	B1 = 0.005	B2 = 0.5	Minimize resonance peak
PD	D = 1/(2*pi*80) = 0.002			Create phase margin
Lead	Zero = 30 Hz	Pole = 500 Hz		Create extra phase around Bandwidth
Notch	Zero/Pole = 2 Hz	B2 = 0.001	B1 = 1	Create steep dip in PS around 2 Hz
Lowpass filter	Pole = 50 Hz			Create proper system

Exercise 5.4 - Feedforward design

In this exercise a feedforward controller is tuned, which is applied in the simulation schematic shown in Figure 61. In this schematic a feedback- and feedforward controller are used to control the behavior of a plant.

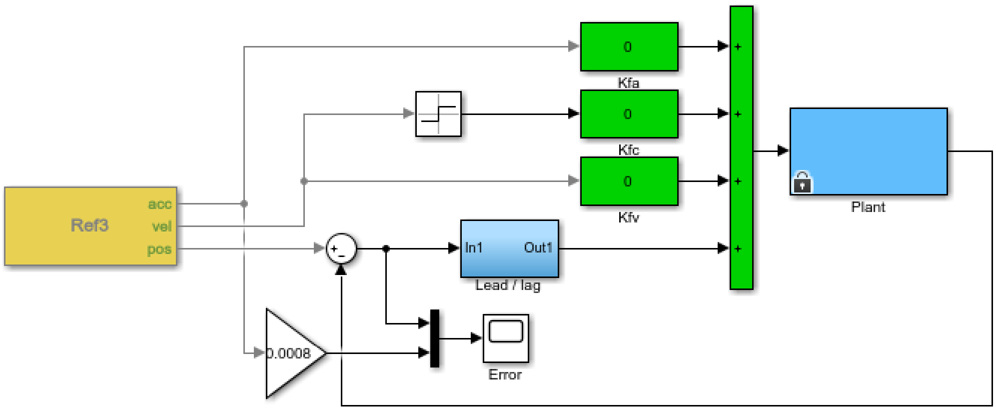

Figure 61 – Simulation schematic feedforward design exercise

The feedforward controller exists of three different parameters, which control the following system properties:

- K_{fa}: lack of force to accelerate
- K_{fc}: lack of force to overcome the Coulomb friction/dry friction
- K_{fv}: lack of force to overcome the viscous friction

The feedback controller is a lead/lag controller, consisting of a pole, zero and a gain. The simulation schematic of the feedback controller is shown in Figure 62.

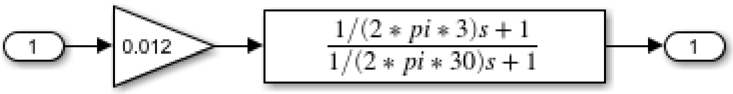

Figure 62 – feedback controller

Furthermore, the plant of the system is unknown.

When using a feedforward controller in combination with a feedback controller, the feedback controller must initially have a relatively low bandwidth, must not have an I-action and the closed loop system must be stable.

If the bandwidth of the feedback controller is low, the error is relatively large which makes it easier to determine the feedforward parameters K_{fc}, K_{fv} and K_{fa}.

If there is an I-action in the feedback controller, the error gets reduced towards zero over time. The I-action makes it impossible to distinguish the mass acceleration, viscous damping and dry friction from the error profile, thus making it impossible to determine the correct feedforward controller parameters.

To find the correct feedforward controller parameters, profiles shown in Figure 63 are used as setpoints for acceleration, velocity and position. This profile contains 1/3 of the time constant acceleration, 1/3 of the time constant velocity and 1/3 of the time constant deceleration.

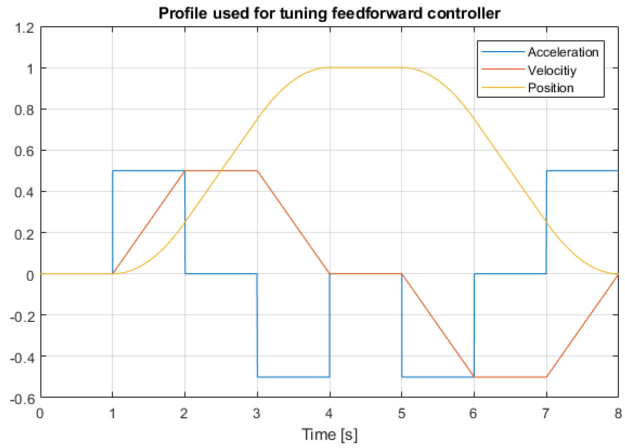

Figure 63 – Input profile used for tuning feedforward controller

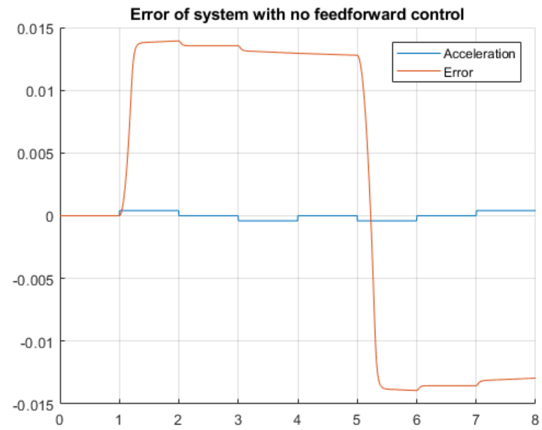

Figure 64 – Error of the system with no feedforward control

Using the position profile from Figure 63 on the input and no feedforward controller, only a feedback controller, resulted in the error profile shown in Figure 64.

First, K_{fc} is tuned. K_{fc} can be found by slowly increasing the value until the error remains constant when the velocity is becoming constant up until it starts changing again. In this exercise this takes place at second 4 and at second 5. The resulting profile is shown in Figure 65, where K_{fc} was set to 1.6e-4.

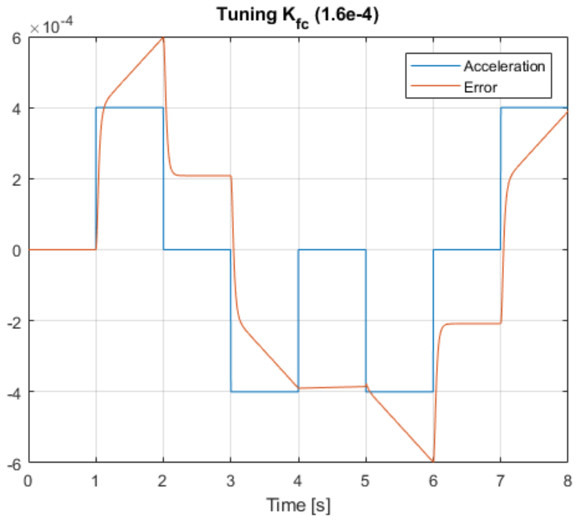

Figure 65 – Tuning of feedforward parameter Kfc with determined value 1.6e-4

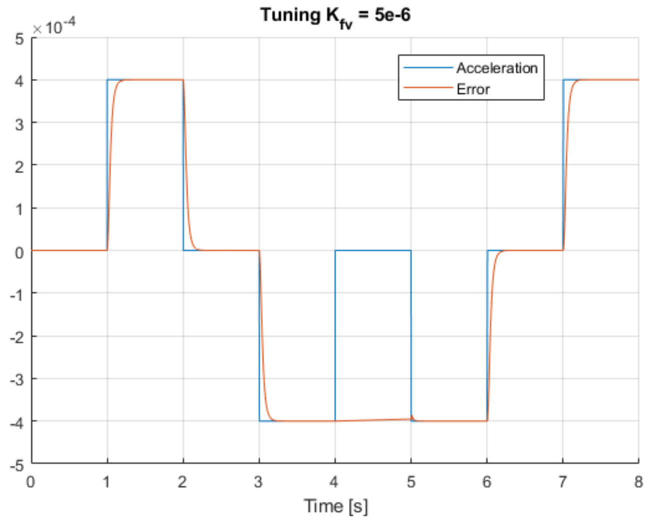

Figure 66 – Tuning of feedforward parameter Kfv with determined value 5e-6

The next step is to tune K_{fv}, which is done by increasing K_{fv} until the error is constant during acceleration. This resulted in K_{fv} = 5e-6 and the error profile shown in Figure 66.

Finally, K_{fa} needs to be tuned. This is done by increasing K_{fa} until the error is almost constant, only small peaks are allowed during change of acceleration. In Figure 67 is the tuning result shown with the determined value of K_{fa} = 9.6e-6.

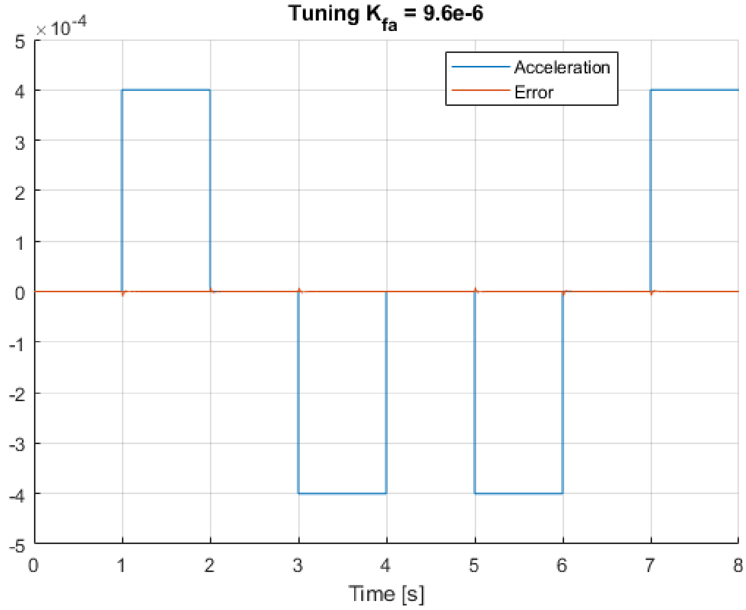

Figure 67 – Tuning of feedforward parameter K_{fa} with determined value 9.6e-6

To show the improvement of the system, the error without feedforward control and the error with feedforward control are plotted in Figure 68. From this comparison can be concluded that feedforward control decreases the error by a large amount.

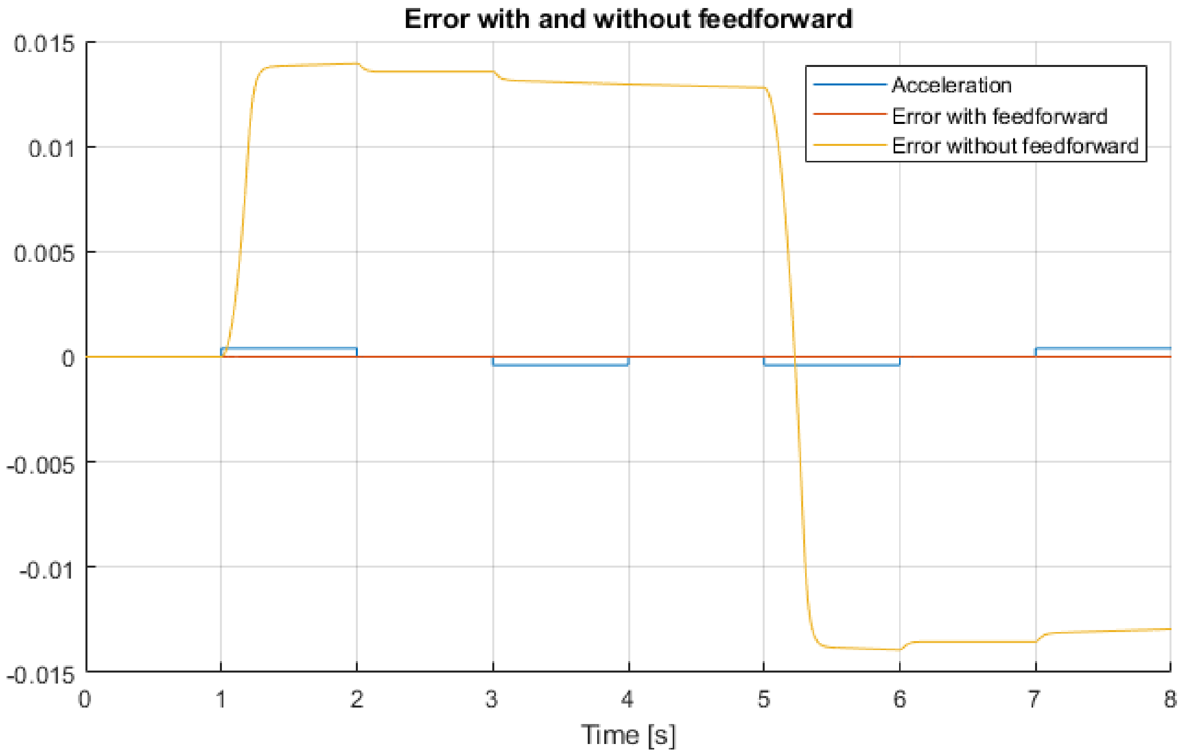

Figure 68 – Error without feedforward versus error with feedforward