5LMA0 – Project heat diffusion

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# Introduction

In this project a model for hyperthermia is considered which describes how a temperature distribution on a locally heated surface evolves over time. The model is described by the following partial differential equation:

where

|  |  |  |  |
| --- | --- | --- | --- |
| **Symbol** | **Description** | **Unit** | **Value** |
| Lx | Length | [m] | 0.2 |
| Ly | Width | [m] | 0.15 |
|  | Material densities (yellow, orange area) | [kg/m3] | 1100, 1000 |
|  | Heat capacities (yellow, orange area) | [J/(kg K)] | 3890, 3350 |
|  | Thermal conductivities (yellow, orange area) | [W/(m K)] | 0.31, 0.31 |
| (X1,Y1) | Location heat flux 1 | ([m],[m]) | (Lx/4, Ly/2) |
| (X2,Y2) | Location heat flux 2 | ([m],[m]) | (3Lx/4, Ly/2) |
| W | Actuator width | [m] | 0.05 |
| Tamb | Ambient temperature | [K] | 309 |

Table 1 – Model properties

Furthermore, the input of the model is a heat flux u(t) at position (x,y) and the output of the model is the temperature of the surface in degrees Kelvin.

The purpose of this project is to derive a fast and simple model that accurately describes the temperature evolution of this system.

# Exercise 1

In this exercise is proven whether the system is linear or nonlinear and whether the system is time-variant or time-invariant. While doing so, the two cases where the model is homogeneous or non-homogeneous are distinguished.

If the model is **homogeneous**, , c and K do not depend on the position (x,y). The system is linear because , c and K remain constant over time and position, resulting in the same temperature profile when the position of the input heat flux changes. The system is time-invariant because the output is the same to the same input at different moments in time.

When the model is **non-homogeneous**, , c and K do depend on the position (x,y). The system is now non-linear because , c and K are changing over position, resulting in different temperatures when the position of the input heat flux changes. The system is still time-invariant.

# Exercise 2

For now it is assumed that the model is homogeneous, meaning that (no orange area) and the model is homogeneous meaning that and are positive constants.

Show that under these conditions, (equation 1 from exercise) admits a solution of the form whenever . Here and are scalar-valued functions on , and , respectively, that satisfy the separated differential equations

for suitable constants and .

When , this results in the following equation:

Now is substituted by , resulting in the following equation:

Which can be rewritten in the following three equations:

Now the following lambdas are defined:

Using these lambdas results in the following equations:

These can be substituted which results in:

These lambdas are the initial conditions of the simplified model.

# Exercise 3

In this exercise is shown that for any and the set is an orthonormal set of functions in . Because and have the values this indicates that and .

First the functions for and are defined:

With these functions can be defined that:

Now the inner product space , based on , is computed:

To proof that is an orthonormal set of functions in the following statement has to hold:

The following is defined for any and and :

Where is the remainder of a division of x and returns the nearest integer less than or equal to y.

The next step is to define , which is done for different conditions based on and :

* If :
* If and :
* If and :
* If and :

Also is defined for different conditions based on and :

* If :
* If and :
* If and :
* If and :

When computing the inner product space (3, fix automatische verwijzing) the outcome is 1 if and 0 when , leading to the conclusion that the set is an orthonormal set of functions in for any and .

# Exercise 4

Use the Garlekin projection to derive, for arbitrary and , an explicit ordinary differential equation for the coefficient function in the spectral expansion (formula 2 of assignment).

(see modred7 for inspiration)

# Exercise 5

Explain simulation TODO

# Exercise 6

Do some simulations TODO

# Exercise 7

# Exercise 8

# Exercise 9

# Exercise 10