AST1420 "Galactic Structure and Dynamics" Problem Set 3

Due on Nov. 17 at the start of class

Most of the exercises in this problem set must be solved on a computer and the best way to hand in the problem set is as an `jupyter notebook`. Rather than sending me the notebook, you can upload it to `GitHub`, which will automatically render the notebook. Rather than starting a repository for a single notebook, you can upload your notebook as a `gist`, which are version-controlled snippets of code that can optionally be made private.

If you want to upload your notebook as a gist from the command-line, you can use the package at this http URL and use it as follows. Log into your `GitHub` account:

```
gist --login
```

and then upload your notebook `AST1420_2017_PS3_YOURNAME.ipynb` as

```
gist -p AST1420_2017_PS3_YOURNAME.ipynb
```

(the `-p` option will make the gist private). If you want to make further changes, you can clone your gist in a separate directory and use it as you would any other git repository. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before uploading it*; this will make sure that the input and output are fully consistent.

If you are unfamiliar with notebooks, you can also hand in a traditional write-up, but you also need to send in well-commented code for how you solved the problems. Thus, notebooks are strongly preferred :-)

Problem 1: (10 points) Deviations from circular rotation in external galaxies. In chapter 7, we derived the two-dimensional velocity fields of external galaxies for various rotation curves. In this derivation, we assumed that the gas in these galaxies is on circular orbits. In this problem, we will explore what happens when this is not the case and what observational limits we can place on deviations from circular motion.

(a) Following the derivation in Chapter 7.1.2, derive the expression for $V(x, y)$ when gas also has a radial component $v_R(R)$. For simplicity, assume the same geometry as in Chapter 7.1.2 and that the radial component does not depend on θ.

(b) Using this expression, discuss what happens to the two-dimensional velocity field when the rotation curve is that of solid-body rotation, but $v_R(R)$ is some constant fraction $f = 0.1$ of $v_c(R)$ [that is, $v_R(R) = f\, v_c(R)$].

(c) How does the behavior you see in (b) generalize to other rotation curves? That is, what is the relation between the two-dimensional velocity field for $f = 0$ and for general f? When you have photometry (or HI emission) *and* kinematics for an external galaxy (like for NGC

3198 in the notes), how can you combine these observations to test for $f \neq 0$?

(d) Plot the two-dimensional velocity field for the three basic cases considered in the notes (solid-body rotation, flat rotation curve, peaked rotation curve) for $f = 0.1$. Discuss what happens in terms of (c) above.

(e) Use the HI emission and two-dimensional velocity field of NGC 3198 from Walter et al. (2008; see notes) and the above behavior to estimate an upper limit on f for this galaxy (without having the actual data in hand, simply provide a rough estimate based on the figure in the notes).

Problem 2: (10 points) Chaos in the Milky Way? In class, we discussed the Henon & Heiles (1964) potential and how it exhibits chaotic regions. Let's see whether more realistic galactic potentials have chaotic regions!

(a) An older, but useful for this exercise, model for the Milky Way potential was presented by Helmi (2004; MNRAS **351**, 643). This model consists of a Hernquist bulge, a Miyamoto-Nagai disk, and a logarithmic halo with a core radius. These are all basic `galpy` potentials. Setup this model in `galpy`, plot the rotation curve, and compare to the top panel of Fig. 1 of Helmi (2004).

(b) Let's investigate orbits in this potential. Before continuing, it's easiest to turn off the physical outputs of the potential that you setup in (a):

```
galpy.potential.turn_physical_off(YOUR_POTENTIAL_LIST).
```

Write functions that (i) compute the energy $E_c(R_c)$ of a circular orbit at a given radius R_c and (ii) the z-component of the angular momentum $L_c(E_c)$ for this orbit. What is the energy and angular momentum of a circular orbit at $R_c = 30\,\mathrm{kpc}$? Use the function from the notes to setup an orbit with a given radius, radial velocity, energy, and angular momentum. Check that you are correctly computing the energy and angular momentum for the circular orbit with $R_c = 30\,\mathrm{kpc}$ by setting up an orbit using this function, integrating it, and displaying it in (x, y) and (R, z).

(c) Investigate the surface of section (R, v_R) (using $z = 0$ as the surface at which to record points) for orbits with $E = E_c(R_c = 30\,\mathrm{kpc})$ and $L_z = 0.15\,L_c(E_c)$ in a similar manner to how we did this in the notes in Sections 9.2 and 9.3. Try to find interesting parts of the surface of section and discuss these.

(d) Repeat the analysis in (c) for orbits closer to the solar neighborhood, at $R_c = 9\,\mathrm{kpc}$ (again using $L_z = 0.15\,L_c(E_c)$). Again try to find interesting parts of the surface of section and discuss what you find.

(e) Repeat the analysis in (c), but for a prolate halo: that is, set the logarithmic halo's flattening parameter to $q = 1.25$. Again try to find interesting parts of the surface of section and discuss what you find.

Note for all of the above: like in the notes, to trace an orbit in the surface of section, you have to integrate for a long time and obtain high time resolution. A time sequence like `ts= numpy.linspace(0.,10000.,1000001)` appears to work okay, but you might have to adjust this for some orbits.