AST1420 "Galactic Structure and Dynamics" Final

Due on Apr. 23 by 5pm

The full mark for the final includes the oral defense of the problems and questions listed here. The breakdown is: 80% written solutions and 20 % oral defense of solutions. The points given for the different problems below only relate to the 80 % written-solutions part of the full mark.

Some of the exercises in this problem set must be solved on a computer and a good way to hand in the problem set is as a `jupyter notebook`. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before sending it in*; this will make sure that the input and output are fully consistent. You can also send in a traditional write-up in LaTeX as a PDF, but then you also need to send in well-commented code for how you solved the numerical problems. Thus, notebooks are preferred :-)

Problem 1: (25 points) Evidence for dark matter. In this class, we discussed the evidence for dark matter in galaxies and galaxy clusters that historically led to the acceptance of the dark matter hypothesis. Among these are Zwicky's observations of Coma cluster galaxies, the Local Group Timing argument, measurements of the rotation curves of spiral galaxies in the nearby Universe, and theoretical arguments regarding the stability of galactic disks. In about a page to a page-and-a-half, discuss each of these pieces of evidence in detail, their relative strengths and weaknesses in supporting the dark matter hypothesis, and how these measurements have stood up to further observational and theoretical developments.

Problem 2: (30 points, 6 each) Short questions.

(a) IC 2574 is a low-surface-brightness galaxy whose mass budget near the center is dominated by dark matter. Its rotation curve has been measured and found to be linearly rising, $v_c \propto r$, within about 6 kpc from its center. What type of density profile does that imply for the dark matter at the center of IC 2574?

(b) A set of galaxies is orbiting in the potential of a galaxy cluster with mass M. If we were to increase the mass M by a factor of two, while somehow keeping the galaxies' distance to the center the same, what would happen to their velocities?

(c) Show that the central value of the gravitational potential of an arbitrary razor-thin, axisymmetric disk with surface density $\Sigma(R)$ is given by

$$\Phi(0,0) = -2\pi\, G \int_0^\infty \mathrm{d}R\, \Sigma(R)\,.$$

(d) The observed relation between the oxygen abundance and the stellar mass of galaxies is shown in Figure 1. It is clear that there is a strong correlation between the stellar mass and the oxygen abundance for galaxies. Why is oxygen a good element to measure to investigate

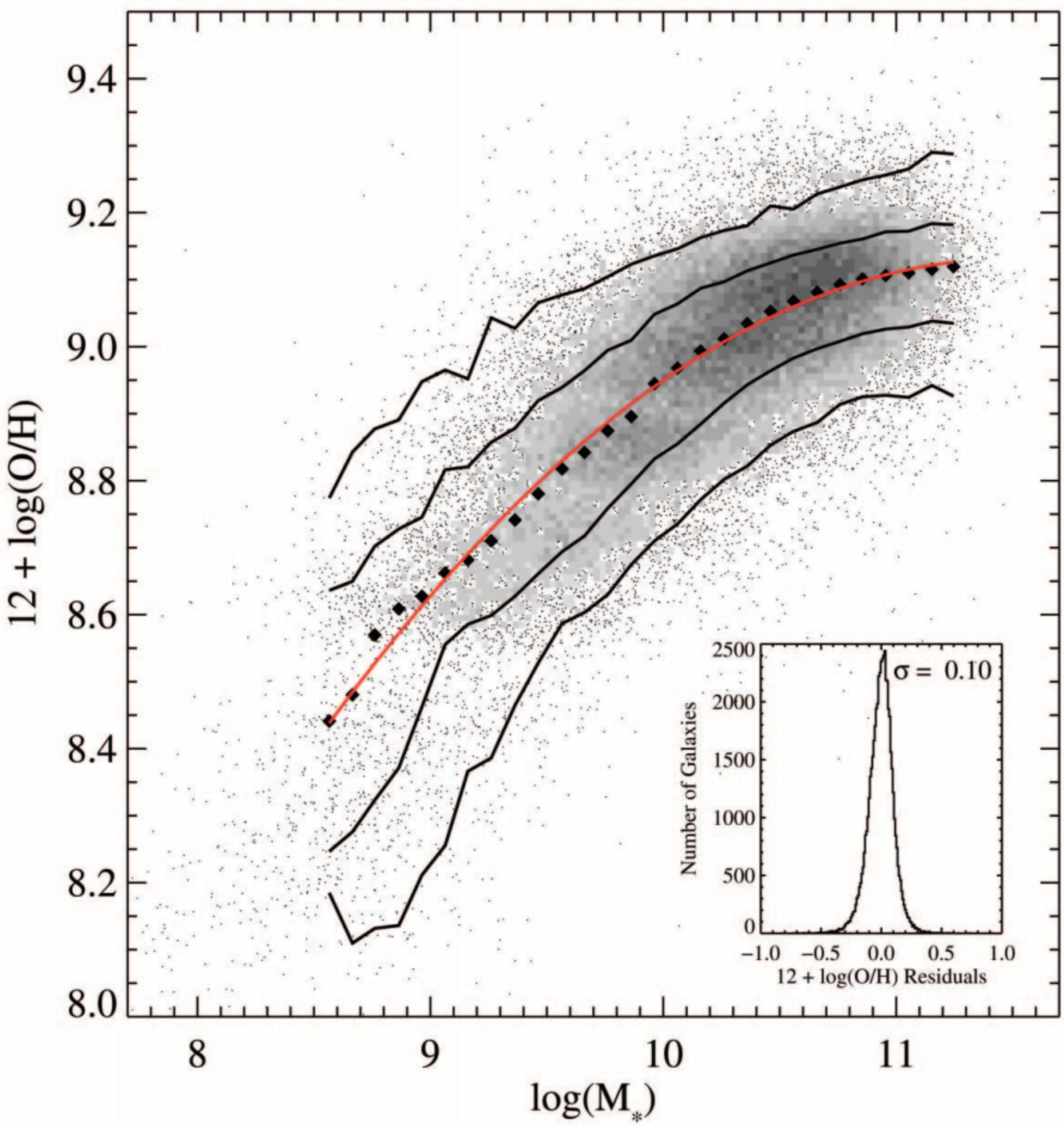

Figure 1: The observed mass–metallicity relation for star-forming galaxies in the Sloan Digital Sky Survey (Tremonti et al. 2004).

the relation between a galaxy's mass and its metal abundance? If you interpret this relation in the context of the leaky-box chemical-evolution model, what are two options for explaining the origin of this relation? Discuss these options in terms of what we have learned this semester about dynamics and the structure of galaxies.

(e) We derived the dynamical-friction time scale t_{DF} in Equation (20.74) for an orbit on a circular orbit. For suitably-chosen parameters, numerically investigate the effect of eccentricity on t_{DF}. That is, compare t_{DF} for objects that spiral in on circular orbits and objects on eccentric orbits with the same initial angular momentum. Interpret your results using the properties of eccentric orbits and of the dynamical-friction deceleration formula.

Problem 3: (25 points; 5 each) A popular method for modifying gravity is replacing the Newtonian gravitational force with a *Yukawa force*. In such a model, particles feel a force mediated through a massive, scalar particle instead of the standard Newtonian $1/r^2$ force. Such an interaction gives rise to a Yukawa potential $\Phi_y(r)$; specifically, an object with mass M gives rise to a potential

$$\Phi_y(r) = -\frac{\alpha\, GM}{r}\, e^{-m_\phi\, r}\,, \tag{1}$$

where α parameterizes the strength compared to that of standard gravity and m_ϕ is the mass of the scalar mediator (in the exponential we are using units in which $\hbar = c = 1$). Let's investigate these types of models and how well they are constrained!

(a) Re-write Equation (1) as

$$\Phi_y(r) = -\frac{\alpha\, GM}{r}\, e^{-r/\lambda}\,, \tag{2}$$

by converting m_ϕ to a length scale using $\hbar$ and c. In particular, what m_ϕ does $10^{12}\,\mathrm{km}$ correspond to (express m_ϕ in eV)?

(b) Does Newton's first shell theorem hold if the gravitational force is of the Yukawa type rather than Newtonian? Explain why or why not.

(c) Explore what orbits look like in a Yukawa potential. You can do this by considering the effective potential and looking at orbits that are either near to or far from circular. Some cases to consider: $r \ll \lambda$, $r = \lambda$, $r = 2\lambda$, and $r \gg \lambda$.

(d) Yukawa-type forces can be constrained by using the fact that the orbit of the Moon around the Earth or the orbits of planets around the Sun close to within measurement uncertainties—after accounting for small deviations stemming from the general theory of relativity and the Earth/Sun's quadrupole moment. Compute, analytically, the precession $2\pi - \Delta\psi$ of the apocenter in a Yukawa potential for a near-circular orbit in terms of α, λ, and the radius r of the circular orbit. From observations of Mercury and Mars' orbits, we can determine that $|2\pi - \Delta\psi| \lesssim 100$ nanoradians. For $\alpha \approx 1$—i.e., gravity is fully Yukawa—what constraint does this put on λ (express in km) and, thus, m_ϕ (express in eV)? For reference, the LIGO constraints on m_ϕ are $m_\phi \lesssim 10^{-22}\,\mathrm{eV}$ (these come from the fact that a massive graviton leads to frequency-dependent travel-times for gravitational waves, constrained by the observed binary-black-hole mergers, and to a slower propagation speed of gravitational waves compared to the speed of light, constrained by the near-coincidence of the gravitational-wave and electromagnetic emission in the binary neutron-star merger GW170817).

(e) Rather than being a full-on replacement of Newtonian gravity, Yukawa-type forces may also be present *in addition* to gravity. For example, in some dark matter models, dark matter feels an additional Yukawa force (with strength α relative to Newtonian gravity) in addition to standard gravity, while ordinary matter does not feel the additional force. At scales $r \gg m_\phi^{-1}$, such a force acts as an additional gravitational force between dark-matter particles and can therefore be modeled as a simple change in the gravitational constant for such particles: $\tilde{G} = G\,(1+\alpha)$. Discuss how this additional interaction affects the following types of observations if we interpret them assuming that gravity is the only force (that is, how does the inferred mass relate to the true mass):

- The matter distribution in disk galaxies inferred from their rotation curves.
- The mass of the Milky Way inferred from the velocities of halo globular clusters.
- The mass of the Milky Way inferred by assuming that a distant satellite (e.g., Leo I) moves at the escape velocity.
- The mass of the Local Group determined using the timing argument.