AST1420 “Galactic Structure and Dynamics” Problem Set 1

Due on Oct. 9 at the start of class

Most of the exercises in this problem set must be solved on a computer and the best way to hand in the problem set is as an `ipython notebook`. Rather than sending me the notebook, you can upload it to `GitHub`, which will automatically render the notebook. Rather than starting a repository for a single notebook, you can upload your notebook as a `gist`, which are version-controlled snippets of code that can optionally be made private.

If you want to upload your notebook as a gist from the command-line, you can use the package at this http URL and use it as follows. Log into your `GitHub` account:

```
gist --login
```

and then upload your notebook `AST1420_2018_PS1_YOURNAME.ipynb` as

```
gist -p AST1420_2018_PS1_YOURNAME.ipynb
```

(the `-p` option will make the gist private). If you want to make further changes, you can clone your gist in a separate directory and use it as you would any other git repository. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before uploading it*; this will make sure that the input and output are fully consistent.

If you are unfamiliar with notebooks, you can also hand in a traditional write-up, but you also need to send in well-commented code for how you solved the problems. Thus, notebooks are strongly preferred :-)

Problem 1: The Local Group Timing Argument. This argument about the total mass of the Local Group by Kahn & Woltjer was very influential in early discussions of dark matter. The basic argument is given in the lecture notes, where we also performed a simple estimate of the mass of the Local Group implied by the current separation and relative velocity of Andromeda with respect to the Milky Way and the age of the Universe. We’ll do a better job here.

(a) As discussed in the notes, we can model the system as that of the displacement vector with respect to the center of mass in a point-mass potential with mass equal to the sum of the Milky Way and Andromeda masses. Solve the full Keplerian equation of motion for a radial orbit (eccentricity equal to 1) for a present-day separation of 740 kpc, relative velocity $-125\,\mathrm{km\,s^{-1}}$, and age of the Universe 13.7 Gyr. What is the exact mass of the Local Group implied by these values? What is the maximum separation between the Milky Way and Andromeda in the past? (Hint: use the parametric solution in terms of the eccentric anomaly; see BT08.)

(b) The modeling in the notes and in part (a) assumes a matter-only Universe. However, our Universe contains both matter and dark energy (and other components, but they can be neglected for the purpose of this problem). The contribution of dark energy to the gravitational acceleration is $\Lambda r/3$, where $\Lambda = 8\pi G\rho_\Lambda/c^2$, with ρ_Λ the cosmological dark energy density. By using the expression for the radial period of an orbit in a spherical potential, a similar expression for the time between apocenter and the present radius, and energy conservation, numerically solve for the mass of the Local Group. In this case, what is the maximum separation? Assume that $\Omega_\Lambda = 0.7$ and $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Problem 2: (Inspired by BT08 problem 3.6) Let's consider what happens when we add energy to an orbit in a spherical potential.

(a) Consider an orbit in a spherical isochrone potential with $b = 1$ (pick an orbit that explores $r \approx 1$ and is not too close to circular). Using orbit integration in `galpy`, add an instantaneous velocity offset when the orbit is at its pericenter radius. Investigate what happens to the pericenter radius of the resulting orbit. Is it larger or smaller than the original pericenter radius? It is useful to consider the special cases where (i) you only change the radial velocity and (ii) you only change the tangential velocity (or equivalently the angular momentum). Does the answer change if you consider different orbits?

(b) Argue why the behavior you saw in part (a) is true for *any* orbit in *any* spherical potential. (Hint: consider the special cases and what happens to the effective potential). You can illustrate your argument with the orbit(s) that you investigated in (a), but make it clear why the behavior is general.

(c) What happens to the apocenter radius and the eccentricity in (a)? Investigate numerically and explain what is happening.

(d) Does the behavior you see change if you apply the velocity offset at different parts of the orbit (e.g., at apocenter, or at a random point along the orbit)?

Problem 3: The velocity structure of dark matter halos. We have discussed the spherical Jeans equation and its dependence on the anisotropy parameter β. Let's investigate its effect on velocities in dark matter halos.

(a) The simplest model for a dark matter halo is the *logarithmic* model: $\Phi(r) = v_c^2 \ln r$. What is the radial velocity dispersion for a constant β?

(b) A more realistic model for the anisotropy accounts for the fact that orbits are more isotropic in the inner regions, where dark matter has been orbiting for many dynamical times, and more radial in the outer regions, where dark matter has only just been accreted onto the halo. In the Osipkov–Merritt model, the anisotropy depends on radius as

$$\beta(r) = \frac{r^2}{r^2 + r_a^2}\,, \tag{1}$$

where r_a is parameter that sets the transition between isotropic and radial orbits. Setting $r_a = 1$, compute the radial velocity dispersion for the logarithmic model with $v_c = 1$

(perhaps numerically). By comparing to the results in (a), does your solution make sense?

(c) As discussed in class, an often preferred model for the dark matter density profile is the NFW profile. Setting the parameters of the NFW profile such that the circular velocity is equal to one at $a/2$, where a is the scale radius, compute the radial velocity dispersion profile for $\beta = 0$, $\beta = 0.5$, $\beta = 1$, and the form of Equation (1) with $r_a = 40a/3$. Plot your solution on a logarithmic grid from $r = a/10$ to $r = 100a$, where a is the scale radius.