AST1420 "Galactic Structure and Dynamics" Final

Due at 5pm on Dec. 13

The full mark for the final includes the oral defense of the exercises listed here as well as your oral answers to the selected questions from the Astronomy & Astrophysics General Qualifying Exam, listed at the end of this document. The breakdown is: 50% written solutions, 25 % oral defense of solutions, 25 % Qualifying Exam questions. The points given for the different problems below only relate to the 50 % written-solutions part of the full mark.

Some of the exercises in this final must be solved on a computer and the best way to hand in the final is as an `jupyter notebook`. Rather than sending me the notebook, you can upload it to `GitHub`, which will automatically render the notebook. Rather than starting a repository for a single notebook, you can upload your notebook as a `gist`, which are version-controlled snippets of code that can optionally be made private.

If you want to upload your notebook as a gist from the command-line, you can use the package at this http URL and use it as follows. Log into your `GitHub` account:

```
gist --login
```

and then upload your notebook `AST1420_2017_FINAL_YOURNAME.ipynb` as

```
gist -p AST1420_2017_FINAL_YOURNAME.ipynb
```

(the `-p` option will make the gist private). If you want to make further changes, you can clone your gist in a separate directory and use it as you would any other git repository. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before uploading it*; this will make sure that the input and output are fully consistent.

If you are unfamiliar with notebooks, you can also hand in a traditional write-up, but you also need to send in well-commented code for how you solved the problems. Thus, notebooks are strongly preferred :-)

Problem 1: (25 points, 5 each) Short questions.

(a) Consider the following potential

$$\Phi(r) = -4\pi G\, \rho_0\, a^2\, \ln\left[1 + \frac{a}{r}\right]\,, \tag{1}$$

where a and ρ_0 are constants. Without directly computing the full density profile, determine (i) whether or not the total mass is finite and if it is finite, what the mass is; (ii) the large r power-law behavior limit of the density.

(b) Give an initial phase-space position in cylindrical coordinates for an orbit with eccentricity 0.50 that reaches a maximum height of 20 kpc above the $z = 0$ plane in an NFW potential with total mass $M = 10^{12}\,M_\odot$ and scale radius $a = 16\,\mathrm{kpc}$. Verify your answer by setting up this problem in `galpy`, integrating the orbit, and computing its eccentricity and maximum height using the orbit methods `e` and `zmax`.

(c) (BT08 3.15) Prove that at any point in an axisymmetric system at which the local density is negligible, the epicycle, vertical, and circular frequencies κ, ν, and Ω are related by $\kappa^2 + \nu^2 = 2\Omega^2$.

(d) In certain dark-matter models, dark matter particles feel an additional long-range force mediated through a scalar particle. Such an interaction gives rise to a Yukawa potential $V_y(r)$

$$V_y(r) = -\frac{g^2}{4\pi\, r}\, e^{-m_\phi\, r}\,, \tag{2}$$

where m_ϕ is the mass of the scalar mediator and g is a coupling constant (we are using units in which $\hbar = c = 1$ here). At scales $r \gg m_\phi^{-1}$, such a force acts as an additional gravitational force between dark-matter particles and can therefore be modeled as a simple change in the gravitational constant for such particles: $\tilde{G} = G\,(1+\beta^2)$, where $\beta = g\, m_P/\sqrt{4\pi} m_{\mathrm{DM}}$ (with m_P the Planck mass). Discuss how this additional interaction affects the following (that is, how does the inferred mass relate to the true mass):

- The matter distribution in disk galaxies inferred from their rotation curves.
- The mass of the Milky Way inferred from the velocities of halo globular clusters.
- The mass of the Milky Way inferred by assuming that a distant satellite (e.g., Leo I) moves at the escape velocity.
- The mass of the Local Group determined using the timing argument.

(e) In the Milky Way, the ratio of the epicycle-to-angular frequency $\kappa/\Omega \approx 4/3$. Assuming that the rotation curve depends on radius as $v_c \propto r^\alpha$ and that κ/Ω is constant at the above value, what is the outer Lindblad radius R_{OLR} of an $m = 2$ bar with $\Omega_b = 1.85\,\Omega_0$ relative to the Solar radius R_0 (where the angular velocity is Ω_0)?

Problem 2: (15 points; 3 each) Disk galaxies as collisionless sytems? One of the first topics that we discussed and something we relied on throughout the course is that galaxies are *collisionless systems*, that is, their two-body relaxation time t_{relax} is orders of magnitude larger than the current age of the Universe. We derived this result for a simple, uniform density spherical system and argued that there are enough orders of magnitude to spare in this argument, such that these simplifying assumptions do not affect the result. Let's look into that a little more closely now.

(a) We derived the relaxation time t_{relax} for a spherical system, but many galaxies are *disk galaxies* that are far from spherical. Derive the relaxation time in the two-dimensional limit, that is, for a uniform density, razor-thin galactic disk with size R and typical velocity v.

What is the ratio of $t_{\rm relax}$ to $t_{\rm cross}$?

(b) Explain the result in (a) physically: why is the situation in case (a) fundamentally different from the spherical case? (Hint: Newton's theorems and the potential of razor-thin disks).

(c) The result in (a) might surprise you. But the situation gets even more interesting. Galactic disks are not just close to being two dimensional, they are also cold in the sense that the random velocity σ of stars is a small fraction of the orbital velocity v; let's call $\alpha = \sqrt{2}\sigma/v$ the typical relative velocity between two stars. How does the relaxation time that you derived in (a) change for a cold disk, assuming a constant α?

(d) In reality, galactic disks are not razor-thin, but have some finite thickness z_d in addition to their two-dimensional size R_d. Derive the relaxation time for a cold ($\alpha < 1$) finite-thickness disk. What is this relaxation time relative to that for a spherical system? Remember: we are only interested in an order-of-magnitude estimate of the relaxation time, so do not attempt a complicated derivation, but rather make reasonable simplifying assumptions.

(e) Using typical parameters for the Milky Way disk, is the disk collisionless? How many particles N are necessary to simulate its dynamical evolution as a collisionless system?

Problem 3: (10 points) Conservation laws and numerical integration. Consider an N-body system under unsoftened, Newtonian gravity. Write a simple N-body integrator that uses i) direct summation to obtain the forces and ii) the modified Euler, leapfrog, or fourth-order Runge-Kutta method for orbit integration of the N bodies. Investigate the conservation of total angular momentum $\sum_i m_i \vec{x}_i \times \vec{v}_i$ when using a constant (small) time step. (Note: you are free to choose N, but do not make it *too* small). You should evolve your system for about 100 dynamical times. Explain what you see in terms of the theory behind the different integrators.

Selected questions from the General Qualifying Exam:

After having this course, you should be able to answer the following questions from the General Qualifying Exam:

- What is the total mass (in both dark matter and in stars) of the Milky Way galaxy? How does this compare to M31 and to the LMC? How is this mass determined?
- Define and describe globular clusters. Where are they located? What are their typical ages, and how is this determined?
- Describe a dynamical method used in the determination of the mass of a galaxy cluster. (Note: full question asks for three methods, not solely dynamical).
- What are galaxy clusters? What are their basic properties (e.g., mass, size). [We'll skip this part: List and explain three ways they can be detected.]
- Describe the orbits of stars in a galactic disk and in a galactic spheroid.
- Galactic stars are described as a collisionless system. Why?
- What is the difference between a globular cluster and a dwarf spheroidal galaxy?
- The stars in the solar neighborhood, roughly the 300 pc around us, have a range of ages, metallicities, and orbital properties. How are those properties related?
- What is dynamical friction? Explain how this operates in the merger of a small galaxy into a large one.
- Sketch the rotation curve for a typical spiral galaxy. SHow that a flat rotation curve implies the existence of a dark matter halo with a density profile that drops off as $1/r^2$.
- Characterize the stellar populations in the following regions: i) the Galactic bulge, ii) the Galactic disk, outside of star clusters, iii) open star clusters, iv) globular clusters, v) a typical elliptical galaxy.
- What is the G-dwarf problem in the solar neighborhood?