AST1420 "Galactic Structure and Dynamics" Problem Set 3

Due on Nov. 13 at the start of class

Most of the exercises in this problem set must be solved on a computer and the best way to hand in the problem set is as an `jupyter notebook`. Rather than sending me the notebook, you can upload it to `GitHub`, which will automatically render the notebook. Rather than starting a repository for a single notebook, you can upload your notebook as a `gist`, which are version-controlled snippets of code that can optionally be made private.

If you want to upload your notebook as a gist from the command-line, you can use the package at this http URL and use it as follows. Log into your `GitHub` account:

```
gist --login
```

and then upload your notebook `AST1420_2018_PS3_YOURNAME.ipynb` as

```
gist -p AST1420_2018_PS3_YOURNAME.ipynb
```

(the `-p` option will make the gist private). If you want to make further changes, you can clone your gist in a separate directory and use it as you would any other git repository. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before uploading it*; this will make sure that the input and output are fully consistent.

If you are unfamiliar with notebooks, you can also hand in a traditional write-up, but you also need to send in well-commented code for how you solved the problems. Thus, notebooks are strongly preferred :-)

Problem 1: (10 points) Growth of perturbations in the Universe. We discussed the growth of dark matter and baryonic perturbations in class and in the notes; here we dig a little deeper into how these perturbations evolve.

(a) We argued that as the Universe transitions from being matter dominated at $z \gtrsim 1$ to being dark-energy dominated at $z \lesssim 0.5$ perturbations in the linear regime stop growing. Show this explicitly by computing and plotting the growing mode for our Universe (which you can approximate as consisting of dark energy and dark matter with density parameters today of $\Omega_{\Lambda,0} = 0.7$ and $\Omega_{m,0} = 0.3$) from $a = 10^{-3}$ to $a = 10$ and compare it to growth in a matter-dominated Universe. Normalize the growing mode such that $\delta_m(a = 1) = 1$. Also overplot the scale factor at which dark energy and dark matter have equal densities. Discuss.

(b) We discussed how on large scales—scales much larger than the Jeans scale—baryonic perturbations follow the same differential equation as dark matter perturbations. However, this does not directly imply that baryonic perturbations and dark matter perturbations are equal, because their initial conditions differ. In particular, before recombination, baryons are coupled to photons, undergo baryon acoustic oscillations, and they therefore do not grow, while dark matter can already start growing. To investigate what happens, consider the evolution equation for baryons in the limit of zero pressure and using the approximation that the gravitational potential is contributed by the *dark matter* only. Directly integrate the differential equation (you can use `scipy.odeint` for this) starting with zero baryon perturbation and velocity at $t = 10^{-4.5}$ in an Einstein–de-Sitter Universe (where $t = 1$ corresponds to $a = 1$) and integrate until $t = 100$ (you want to use a logarithmic time grid). Compare the solution to that of the dark matter. What happens?

(c) On scales smaller than the Jeans scale, baryonic perturbations cannot grow. Again working in the limit that the gravitational potential is sourced by the dark matter alone, show that in an Einstein–de-Sitter Universe where the baryon equation of state is $P \propto \rho^{4/3}$ a solution of the equation is given by

$$\delta_b(\vec{k}, t) = \frac{\delta_c(\vec{k}, t)}{1 + k^2/k_J^2}\,, \tag{1}$$

where k_J is the comoving Jeans wavenumber and $k = |\vec{k}|$. Plot δ_b/δ_c as a function of k (you will have to compute the sound speed for this, which you can do using the knowledge that at $z \approx 1,000$ the temperature was $\approx 3,000\,\mathrm{K}$). Compare this to what this function looks like before recombination, when baryons are coupled to photons. Discuss.

(d) (extra credit) A more realistic equation of state is that of a mono-atomic gas. Still working in the Einstein–de-Sitter Universe with the approximation that dark matter alone sources the gravitational potential, explicitly integrate the differential equation for $\delta_b(\vec{k}, t)$ from $a = 10^{-3}$ to $a = 10$ starting from a vanishing density perturbation and plot the resulting function as a function of time for a few well-chosen k and as a function of k for a few well-chosen times. Compare to the behavior you found in (c) and discuss in terms of what you saw in (b) and (c). Hint: To check your code, it's a good idea to test that you recover the result in (c). For obtaining numerical values, use that the Hubble constant is $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Problem 2: (10 points) Chaos in the Milky Way? In class, we discussed the Henon & Heiles (1964) potential and how it exhibits chaotic regions. Let's see whether more realistic galactic potentials have chaotic regions!

(a) An older, but useful for this exercise, model for the Milky Way potential was presented by Helmi (2004; MNRAS **351**, 643). This model consists of a Hernquist bulge, a Miyamoto-Nagai disk, and a logarithmic halo with a core radius. These are all basic `galpy` potentials. Setup this model in `galpy`, plot the rotation curve, and compare to the top panel of Fig. 1 of Helmi (2004).

(b) Let's investigate orbits in this potential. Before continuing, it's easiest to turn off the physical outputs of the potential that you setup in (a):

```
galpy.potential.turn_physical_off(YOUR_POTENTIAL_LIST).
```

Write functions that (i) compute the energy $E_c(R_c)$ of a circular orbit at a given radius R_c

and (ii) the z-component of the angular momentum $L_c(E_c)$ for this orbit. What is the energy and angular momentum of a circular orbit at $R_c = 30\,\mathrm{kpc}$? Use the function from the notes to setup an orbit with a given radius, radial velocity, energy, and angular momentum. Check that you are correctly computing the energy and angular momentum for the circular orbit with $R_c = 30\,\mathrm{kpc}$ by setting up an orbit using this function, integrating it, and displaying it in (x, y) and (R, z).

(c) Investigate the surface of section (R, v_R) (using $z = 0$ as the surface at which to record points) for orbits with $E = E_c(R_c = 30\,\mathrm{kpc})$ and $L_z = 0.15\,L_c(E_c)$ in a similar manner to how we did this in the notes in Chapter 11. Try to find interesting parts of the surface of section and discuss these.

(d) Repeat the analysis in (c) for orbits closer to the solar neighborhood, at $R_c = 9\,\mathrm{kpc}$ (again using $L_z = 0.15\,L_c(E_c)$). Again try to find interesting parts of the surface of section and discuss what you find.

(e) Repeat the analysis in (c), but for a prolate halo: that is, set the logarithmic halo's flattening parameter to $q = 1.25$. Again try to find interesting parts of the surface of section and discuss what you find.

Note for all of the above: like in the notes, to trace an orbit in the surface of section, you have to integrate for a long time and obtain high time resolution. A time sequence like `ts= numpy.linspace(0.,10000.,1000001)` appears to work okay, but you might have to adjust this for some orbits.