AST1420 "Galactic Structure and Dynamics" Final

Due at 5pm on Dec. 7

The full mark for the final includes the oral defense of the exercises listed here as well as your oral answers to the selected questions from the Astronomy & Astrophysics General Qualifying Exam, listed at the end of this document. The breakdown is: 50% written solutions, 25 % oral defense of solutions, 25 % Qualifying Exam questions. The points given for the different problems below only relate to the 50 % written-solutions part of the full mark.

Some of the exercises in this final must be solved on a computer and the best way to hand in the final is as an `jupyter notebook`. Rather than sending me the notebook, you can upload it to `GitHub`, which will automatically render the notebook. Rather than starting a repository for a single notebook, you can upload your notebook as a `gist`, which are version-controlled snippets of code that can optionally be made private.

If you want to upload your notebook as a gist from the command-line, you can use the package at this http URL and use it as follows. Log into your `GitHub` account:

```
gist --login
```

and then upload your notebook `AST1420_2018_FINAL_YOURNAME.ipynb` as

```
gist -p AST1420_2018_FINAL_YOURNAME.ipynb
```

(the `-p` option will make the gist private). If you want to make further changes, you can clone your gist in a separate directory and use it as you would any other git repository. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before uploading it*; this will make sure that the input and output are fully consistent.

If you are unfamiliar with notebooks, you can also hand in a traditional write-up, but you also need to send in well-commented code for how you solved the problems. Thus, notebooks are strongly preferred :-)

Problem 1: (25 points, 5 each) Short questions.

(a) Consider the following potential

$$\Phi(r) = -4\pi G\,\rho_0\,a^2\,\ln\left[1+\frac{a}{r}\right]\,, \tag{1}$$

where a and ρ_0 are constants. Without directly computing the full density profile, determine whether or not the total mass is finite and if it is finite, what the mass is.

(b) Without using `galpy` (or any other package aside from `numpy` or `scipy`), compute the rotation curve between 1 and 300 kpc for an NFW halo with a virial mass of $10^{12}\,M_\odot$ and a concentration of $c = 10$ in a ΛCDM Universe with $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, and

$H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ that virialized at $z = 2$.

(c) (BT08 3.14) Show that in a spherical potential the vertical and circular frequencies ν and Ω are equal.

(d; 2 parts of 3 points each) The observed relation between the oxygen abundance and the stellar mass of galaxies is shown in Figure 1. It is clear that there is a strong correlation between the stellar mass and the oxygen abundance for galaxies.

(d.1) Why is oxygen a good element to measure to investigate the relation between a galaxy's mass and its metal abundance?

(d.2) If you interpret this relation in the context of the leaky-box chemical-evolution model, what are two options for explaining the origin of this relation? Discuss these options in terms of what we have learned this semester about dynamics and the structure of galaxies.

Problem 2: (18 points; 3 each) A popular method for modifying gravity is replacing the Newtonian gravitational force with a *Yukawa force*. In such a model, particles feel a force mediated through a massive, scalar particle instead of the standard Newtonian $1/r^2$ force. Such an interaction gives rise to a Yukawa potential $\Phi_y(r)$; specifically, an object with mass M gives rise to a potential

$$\Phi_y(r) = -\frac{\alpha\,GM}{r}\,e^{-m_\phi\,r}\,, \tag{2}$$

where α parameterizes the strength compared to that of standard gravity and m_ϕ is the mass of the scalar mediator (in the exponential we are using units in which $\hbar = c = 1$). Let's investigate these types of models and how well they are constrained!

(a) Re-write Equation (2) as

$$\Phi_y(r) = -\frac{\alpha\,GM}{r}\,e^{-r/\lambda}\,, \tag{3}$$

by converting m_ϕ to a length scale using $\hbar$ and c. In particular, what m_ϕ does $10^{12}\,\mathrm{km}$ correspond to (express m_ϕ in eV)?

(b) Does Newton's first shell theorem hold if the gravitational force is of the Yukawa type rather than Newtonian? Explain why or why not.

(c) Explore what orbits look like in a Yukawa potential. You can do this by considering the effective potential and looking at orbits that are either near to or far from circular. Some cases to consider: $r \ll \lambda$, $r = \lambda$, $r = 2\lambda$, and $r \gg \lambda$.

(d) Circular orbits in a spherical potential are stable if orbits close to a circular orbit oscillate around the circular orbit. In the epicycle formalism, what constraint does this set on the epicycle frequency? (Hint: consider the relation between the epicycle frequency and the curvature of the effective potential.) Using this constraint, find the maximum r/λ where

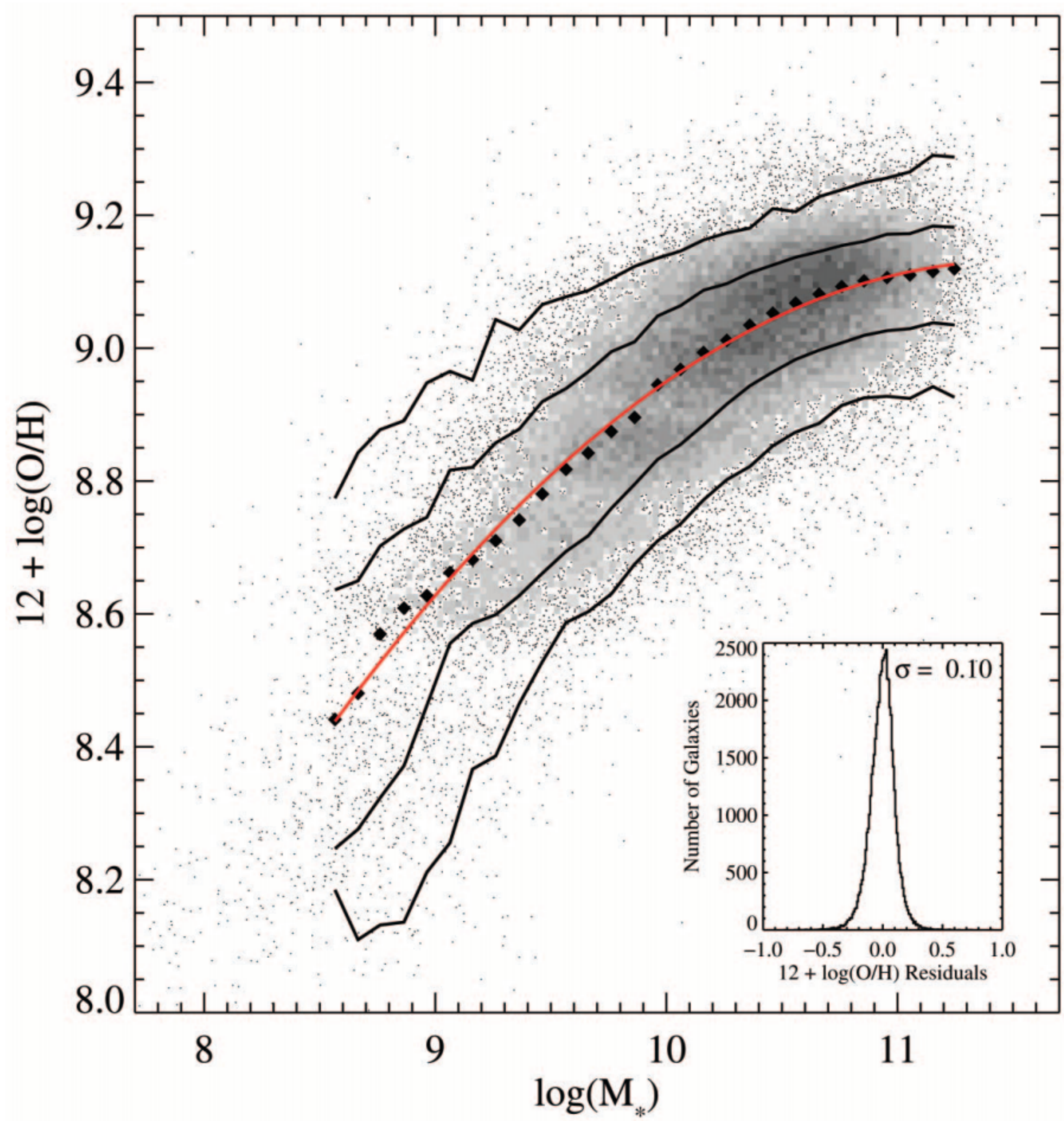

Figure 1: The observed mass–metallicity relation for star-forming galaxies in the Sloan Digital Sky Survey (Tremonti et al. 2004).

stable circular orbits exist in a Yukawa potential. Given that galaxy clusters appear to be gravitationally bound, what kind of constraint on λ can you derive from this (express in km and also in terms of m_ϕ in eV)?

(e) Yukawa-type forces can be constrained by using the fact that the orbit of the Moon around the Earth or the orbits of planets around the Sun close to within measurement uncertainties—after accounting for small deviations stemming from the general theory of relativity and the Earth/Sun's quadrupole moment. Compute, analytically, the precession $2\pi - \Delta\psi$ of the apocenter in a Yukawa potential for a near-circular orbit in terms of α, λ, and the radius r of the circular orbit. From observations of Mercury and Mars' orbits, we can determine that $|2\pi - \Delta\psi| \lesssim 100$ nanoradians. For $\alpha \approx 1$—i.e., gravity is fully Yukawa—what constraint does this put on λ (express in km) and, thus, m_ϕ (express in eV)? For reference, the LIGO constraints on m_ϕ are $m_\phi \lesssim 10^{-22}\,\mathrm{eV}$ (these come from the fact that a massive graviton leads to frequency-dependent travel-times for gravitational waves, constrained by the observed binary-black-hole mergers, and to a slower propagation speed of gravitational waves compared to the speed of light, constrained by the near-coincidence of the gravitational-wave and electromagnetic emission in the binary neutron-star merger GW170817).

(f) Rather than being a full-on replacement of Newtonian gravity, Yukawa-type forces may also be present *in addition* to gravity. For example, in some dark matter models, dark matter feels an additional Yukawa force (with strength α relative to Newtonian gravity) in addition to standard gravity, while ordinary matter does not feel the additional force. At scales $r \gg m_\phi^{-1}$, such a force acts as an additional gravitational force between dark-matter particles and can therefore be modeled as a simple change in the gravitational constant for such particles: $\tilde{G} = G\,(1+\alpha)$. Discuss how this additional interaction affects the following types of observations if we interpret them assuming that gravity is the only force (that is, how does the inferred mass relate to the true mass):

- The matter distribution in disk galaxies inferred from their rotation curves.
- The mass of the Milky Way inferred from the velocities of halo globular clusters.
- The mass of the Milky Way inferred by assuming that a distant satellite (e.g., Leo I) moves at the escape velocity.
- The mass of the Local Group determined using the timing argument.

Problem 3: (7 points) Conservation laws and numerical integration. Consider an N-body system under unsoftened, Newtonian gravity. Write a simple N-body integrator that uses i) direct summation to obtain the forces and ii) both the leapfrog and fourth-order Runge-Kutta method for orbit integration of the N bodies. Compare the conservation of total angular momentum $\sum_i m_i\,\vec{x}_i \times \vec{v}_i$ when using a constant (small) time step between the leapfrog and Runge-Kutta integrators. (Note: you are free to choose N, but do not make it *too* small). You should evolve your system for about 100 dynamical times. Explain what you see in terms of the theory behind the different integrators.

Selected questions from the General Qualifying Exam:

After having this course, you should be able to answer the following questions from the General Qualifying Exam:

- What is the total mass (in both dark matter and in stars) of the Milky Way galaxy? How does this compare to M31 and to the LMC? How is this mass determined?
- Define and describe globular clusters. Where are they located? What are their typical ages, and how is this determined?
- Describe a dynamical method used in the determination of the mass of a galaxy cluster. (Note: full question asks for three methods, not solely dynamical).
- Draw the spectral energy distribution (SED) of a galaxy formed by a single burst of star formation at the ages of 10 Myrs, 2 Gyrs, and 10 Gyr. Please highlight the change over time in the 4000 Angstrom break.
- What are galaxy clusters? What are their basic properties (e.g., mass, size). [We'll skip this part: List and explain three ways they can be detected.]
- Describe the orbits of stars in a galactic disk and in a galactic spheroid.
- Galactic stars are described as a collisionless system. Why?
- The stars in the solar neighborhood, roughly the 300 pc around us, have a range of ages, metallicities, and orbital properties. How are those properties related?
- What is dynamical friction? Explain how this operates in the merger of a small galaxy into a large one.
- Sketch the rotation curve for a typical spiral galaxy. Show that a flat rotation curve implies the existence of a dark matter halo with a density profile that drops off as $1/r^2$.
- Characterize the stellar populations in the following regions: i) the Galactic bulge, ii) the Galactic disk, outside of star clusters, iii) open star clusters, iv) globular clusters, v) a typical elliptical galaxy.
- What is the G-dwarf problem in the solar neighborhood?
- Describe the general characteristics of spiral structure in galaxies.