AST1420 "Galactic Structure and Dynamics" Problem Set 1

Due on Oct. 6 at the start of class

Most of the exercises in this problem set must be solved on a computer and the best way to hand in the problem set is as an `ipython notebook`. Rather than sending me the notebook, you can upload it to `GitHub`, which will automatically render the notebook. Rather than starting a repository for a single notebook, you can upload your notebook as a `gist`, which are version-controlled snippets of code that can optionally be made private.

If you want to upload your notebook as a gist from the command-line, you can use the package at this http URL and use it as follows. Log into your `GitHub` account:

```
gist --login
```

and then upload your notebook `AST1420_2017_PS1_YOURNAME.ipynb` as

```
gist -p AST1420_2017_PS1_YOURNAME.ipynb
```

(the `-p` option will make the gist private). If you want to make further changes, you can clone your gist in a separate directory and use it as you would any other git repository. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before uploading it*; this will make sure that the input and output are fully consistent.

If you are unfamiliar with notebooks, you can also hand in a traditional write-up, but you also need to send in well-commented code for how you solved the problems. Thus, notebooks are strongly preferred :-)

Problem 1: The Hernquist vs. the NFW models. Both of these are popular models for the density profile of dark matter halos (the NFW profile somewhat more so than the Hernquist profile). Let's compare the two in some more detail.

(a) We have parameterized both models in terms of (ρ_0, a), but a different parameterization is the radius r_{max} and velocity v_{max} where the circular velocity peaks. For the Hernquist and NFW profiles, what is r_{max}/a? And what is v_{max} for a given (ρ_0, a)? Try to get as far as possible analytically, but you will need to solve some of the necessary equations numerically. Verify your derived relations by overplotting them over the example rotation curves from the notes where both models are normalized to have the same mass at $r = 12a$.

(b) Plot the effective potential for both the Hernquist and NFW models normalized to have the same mass at $r = 12a$ for an orbit with angular momentum that is 1/3 of the angular momentum of a circular orbit at $r = 12a$. What is the smallest pericenter radius that *any* bound orbit in the Hernquist model can have? What is the relation between the pericenter radii of orbits with this angular momentum in the Hernquist and the NFW models?

Problem 2: (Inspired by BT08 problem 3.6) Let's consider what happens when we add energy to an orbit in a spherical potential.

(a) Consider an orbit in a spherical isochrone potential with $b = 1$ (pick an orbit that explores $r \approx 1$ and is not too close to circular). Using orbit integration in `galpy`, add an instantaneous velocity offset when the orbit is at its pericenter radius. Investigate what happens to the pericenter radius of the resulting orbit. Is it larger or smaller than the original pericenter radius? It is useful to consider the special cases where (i) you only change the radial velocity and (ii) you only change the tangential velocity (or equivalently the angular momentum). Does the answer change if you consider different orbits?

(b) Argue why the behavior you saw in part (a) is true for *any* orbit in *any* spherical potential. (Hint: consider the special cases and what happens to the effective potential). You can illustrate your argument with the orbit(s) that you investigated in (a), but make it clear why the behavior is general.

(c) What happens to the apocenter radius and the eccentricity in (a)? Investigate numerically and explain what is happening.

(d) Does the behavior you see change if you apply the velocity offset at different parts of the orbit (e.g., at apocenter, or at a random point along the orbit)?

Problem 3: More on the virial theorem.

(a) In Sec. 4.1 of the lecture notes, we derived the virial theorem for a set of tracer particles in a point-mass potential and used it to estimate the Milky Way's mass out to about 40 kpc using globular clusters. Suppose instead that the potential is that corresponding to a power-law potential with $\rho(r) \propto r^{-1}$. Derive the virial estimator for the mass out to a certain radius r_c.

(b) Using this estimator, what is the mass of the Milky Way out to about 40 kpc implied by the globular cluster kinematics?

(c) A more realistic model for the Milky Way's mass distribution is that of a logarithmic potential $\Phi(r) = v_c^2 \ln r$, because this potential has a flat rotation curve. What is the virial theorem in this case?

(d) Using the logarithmic-potential virial theorem, what is the mass of the Milky Way out to about 40 kpc implied by the globular cluster kinematics?