AST 1430

Cosmology

Week Index	Dates	Topics
Week 1	Jan 11, 13	logistics, introduction, basic observations
Week 2	Jan 18, 20	Basic GR, RW metric, Distances, coordinates, Friedmann equations, cosmological models
Week 3	Jan 25, 27	Consistency with observations, Early Hot Universe, BBN
Week 4	Feb 1, 3	Inflation, Perturbations & Structure pre-recombination
Week 5	Feb 8, 10	CMB: basics, polarization, secondaries
Week 6	Feb 15 (2h)	Early-Universe Presentations
Reading Week – No Class		
Week 7	Mar 1, 3	Post-recombination growth of structure, formation of dark matter halos, halo mass function
Week 8	Mar 8, 10	The relation between dark matter halos and galaxies
Week 9	Mar 15, 17	Probing the cosmic density field / clustering
Week 10	Mar 22, 24	Late Universe Presentations; lcosmological observations: BAO, supernovae, weak lensing, etc.
Week 11	Mar 29, 31	H0 controversy: how fast exactly is the Universe expanding today?
Week 12	Apr 5	Review

Keith

Jo

asst1

asst2

asst3

Last week

Last week: growth of structure

- Late-time growth (z <~ 1000):
    - Dark matter: scale independent, $\delta \propto D_+(a)$, $D_+(a) = a$ for $\Omega_m=1$
    - Baryons: scale independent like DM about Jeans scale, pressure-stabilized on smaller scales
- Early-time growth (z >~ 1000):
    - Potential evolution described by transfer function T(k)
    - $T(k) = 1$ for $k < k_{eq}$, $k_{eq} \sim 0.01$/Mpc scale that enters the horizon at a_{eq}
    - $T(k) \sim 1/k^2$ for $k > k_{eq}$
- Statistics of late-time density fluctuations described by the matter power spectrum: Gaussian with scale-dependent variance

$$\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle = (2\pi)^3 \, P_m(k) \, \delta^{(3)}(\mathbf{k} - \mathbf{k}') \, .$$

$$P_m(k,a) = \frac{8\pi^2}{25} \, \frac{A_s \, k_p \, c^4}{\Omega_{0,m}^2 \, H_0^4} \, T^2(k) \, D_+^2(a) \left(\frac{k}{k_p}\right)^{n_s} .$$

Last week: formation of DM halos

- Dark matter halos form when regions of the Universe with $\delta >> 1$ decouple from expansion and gravitationally collapse

- Spherical collapse of top-hat overdensity: DM halo has formed when linear-theory $\delta = \delta_c \sim 1.686$ *for any reasonable Universe*

- Virialization: radial collapse is unstable, slight asymmetries cause shell crossing, violent relaxation, phase mixing, … —> virialized halo forms due to non-linear evolution

    - Non-linear overdensity $\Delta v \sim 18\,\pi^2 \sim 178 \sim 200$ in EdS, ~330 in LCDM today

- Halo mass function: combine statistics of smoothed linear over densities δ_S with $\delta_c \sim 1.686$ linear collapse threshold to derive halo mass function

$$\frac{\mathrm{d}n}{\mathrm{d}\ln M} = \sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}_{0,m}}{M}\,\frac{\delta_c}{\sigma}\,\exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)\frac{\mathrm{d}\ln\sigma^{-1}}{\mathrm{d}\ln M}$$

The extended Press-Schechter formalism

The extended Press-Schechter (EPS) formalism

- Previously, we derived the halo mass function in a somewhat ad hoc way using the Press-Schechter formalism
- The extended Press-Schechter formalism is a re-formulation of this formalism that is more rigorous (but still a bit ad hoc) and that allows more properties of halos to be determined (such as their merger history)
- Introduced by Bond et al. (1991) as the *excursion set formalism*

The basic insight of the EPS formalism

- Consider again the smoothed density field on a scale set by M or R

$$\delta_S(\mathbf{x}, t; M) = \int \mathrm{d}\mathbf{x}'\, \delta(\mathbf{x}', t)\, W(\mathbf{x} - \mathbf{x}'; R)\,.$$

- The variance of the difference between the density field smoothed on scale R and that smoothed on slightly smaller scale R-ΔR is in general

$$\langle [\delta_S(\mathbf{x}, t; R - \Delta R) - \delta_S(\mathbf{x}, t; R)]^2 \rangle = \sigma^2(R - \Delta R, t) + \sigma^2(R, t) - 2\, \frac{1}{2\pi^2} \int_0^\infty \mathrm{d}k\, k^2\, P_m(k, a)\, W_k(R)\, W_k(R - \Delta R)$$

- However, when using a sharp *k*-space filter (top-hat in Fourier space), this becomes

$$= \frac{1}{2\pi^2} \int_{1/R}^{1/(R-\Delta R)} \mathrm{d}k\, k^2\, P_m(k, a)\,,$$

- Thus, the variance of the difference only depends on power spectrum on intermediate scales

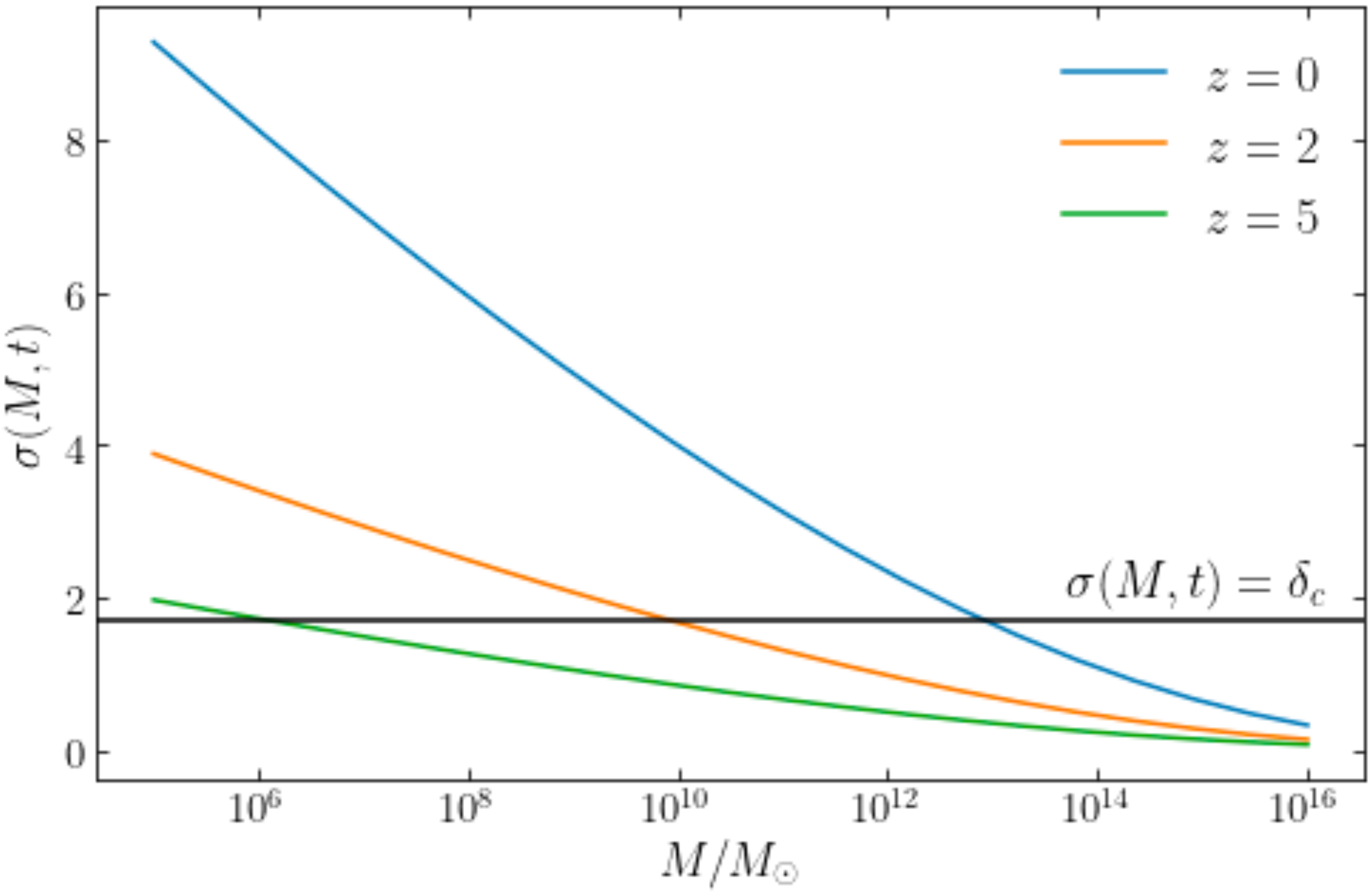
Halos in the EPS formalism

- As in Press-Schechter, mass element at **x** is in a halo with size larger than R if

$$\delta_S(\mathbf{x}, t; R) \geq \delta_c(t).$$

- In EPS, we can simulate what δ_S does as a function of R at a given **x**, starting from large scales and using the basic insight
  - On very large scales, $S \equiv \sigma^2(R, t),$ = 0 (when R = infinity)
  - Thus, $\delta_S(\mathbf{x},t;R=\mathrm{inf}) = 0$
  - Going from R=inf to R=inf-ΔR requires one to draw from a Gaussian with zero mean and variance S(inf-ΔR)-S(inf)

Statistics of smoothed overdensities



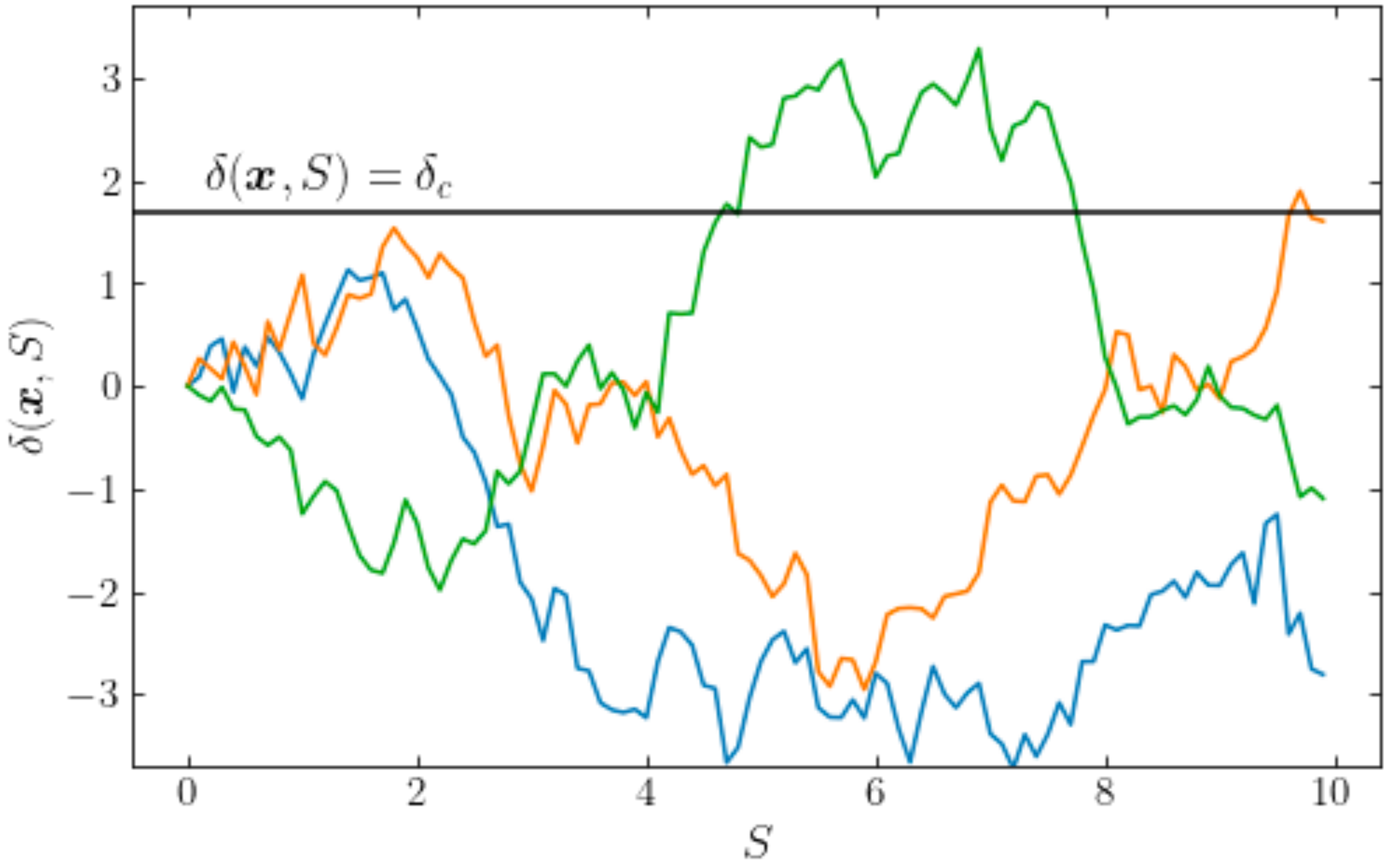
Halos in the EPS formalism

- As in Press-Schechter, mass element at **x** is in a halo with size larger than R if

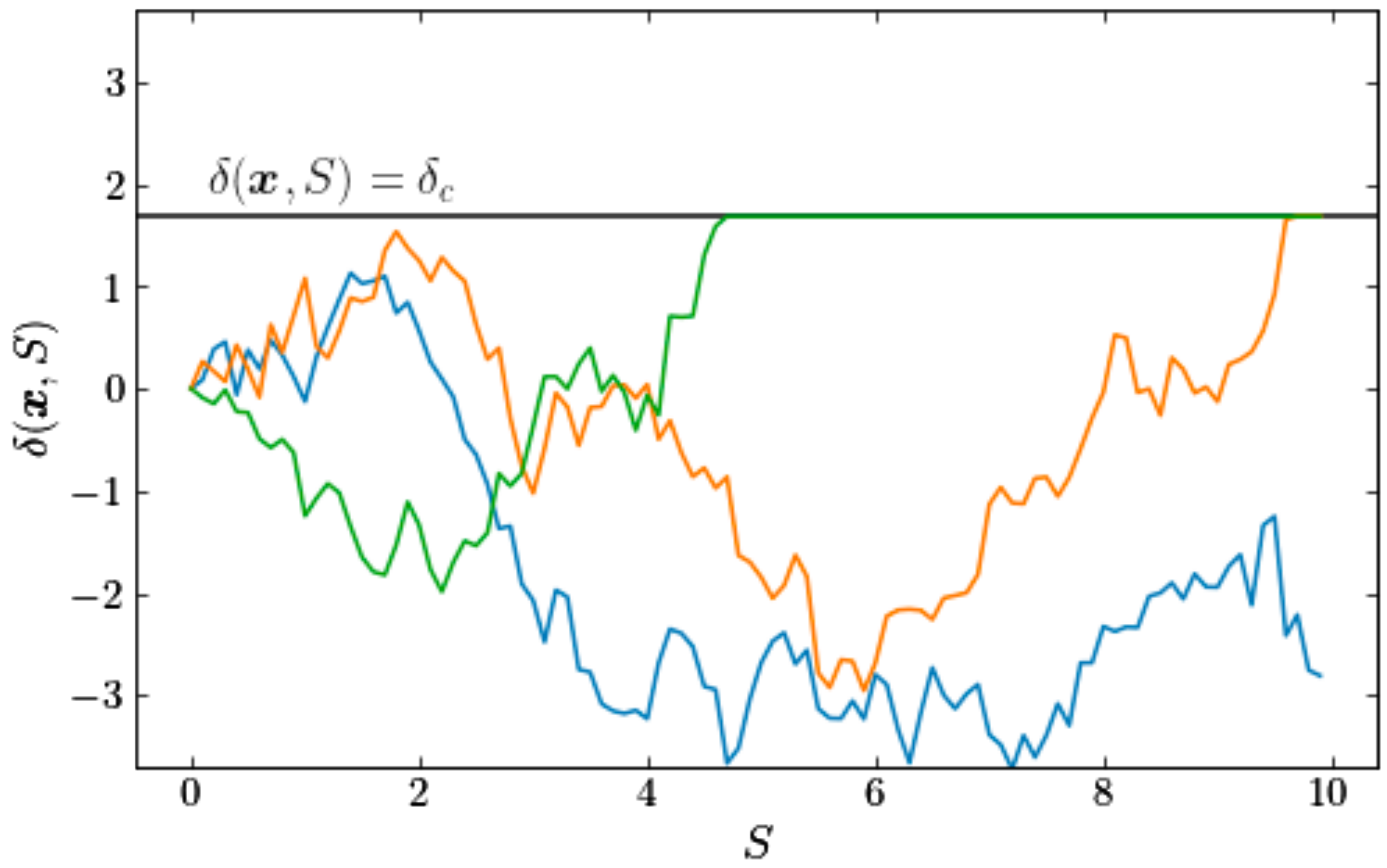
$$\delta_S(\mathbf{x}, t; R) \geq \delta_c(t).$$

- **Hold on!** In CDM-like models, we can just work with S instead of R or M, thus we do
  - On very large scales, $S \equiv \sigma^2(R, t),$ = 0 (when R = infinity)
  - Thus, $\delta_S(\mathbf{x},t;S=0) = 0$
  - Going from S=0 to S=ΔS requires one to draw from a Gaussian with zero mean and variance ΔS
  - Generally, going from S=S' to S=S'+ΔS requires one to draw from a Gaussian with zero mean and variance ΔS

Halo formation is a random walk in EPS



Halo formation is a random walk in EPS *with an absorbing barrier*



The halo mass function in EPS

- From the statistics of random walks, it is straightforward to compute the probability that a mass element is part of a halo with scale S

- In the absence of the absorbing barrier, we simply have

$$p(\delta[\mathbf{x}, S]) = \frac{1}{\sqrt{2\pi\, S}}\, \exp\left(-\frac{\delta^2[\mathbf{x}, S]}{2S}\right)$$

- But with the absorbing barrier this becomes (Chandrasekhar 1943)

$$p(\delta[\mathbf{x}, S]) = \frac{1}{\sqrt{2\pi\, S}} \left[\exp\left(-\frac{\delta^2[\mathbf{x}, S]}{2S}\right) - \exp\left(-\frac{\{2\delta_c - \delta[\mathbf{x}, S]\}^2}{2S}\right)\right]$$

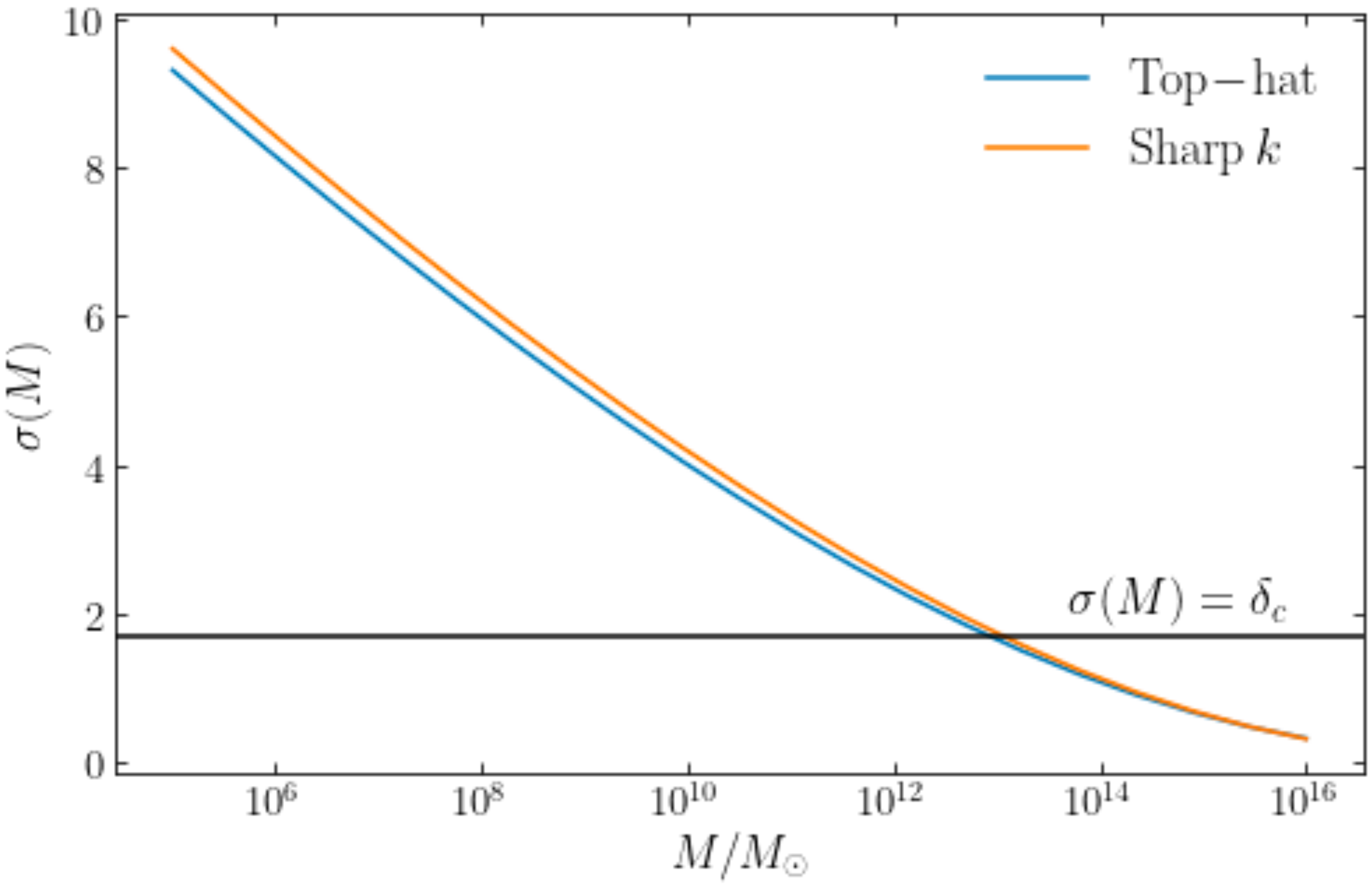
- Fraction of mass elements in halos with size smaller than R (larger than S)

$$P[\delta(\mathbf{x}, S[R]) < \delta_c] = \operatorname{erf}\left(\frac{\delta_c}{\sqrt{2S}}\right)$$

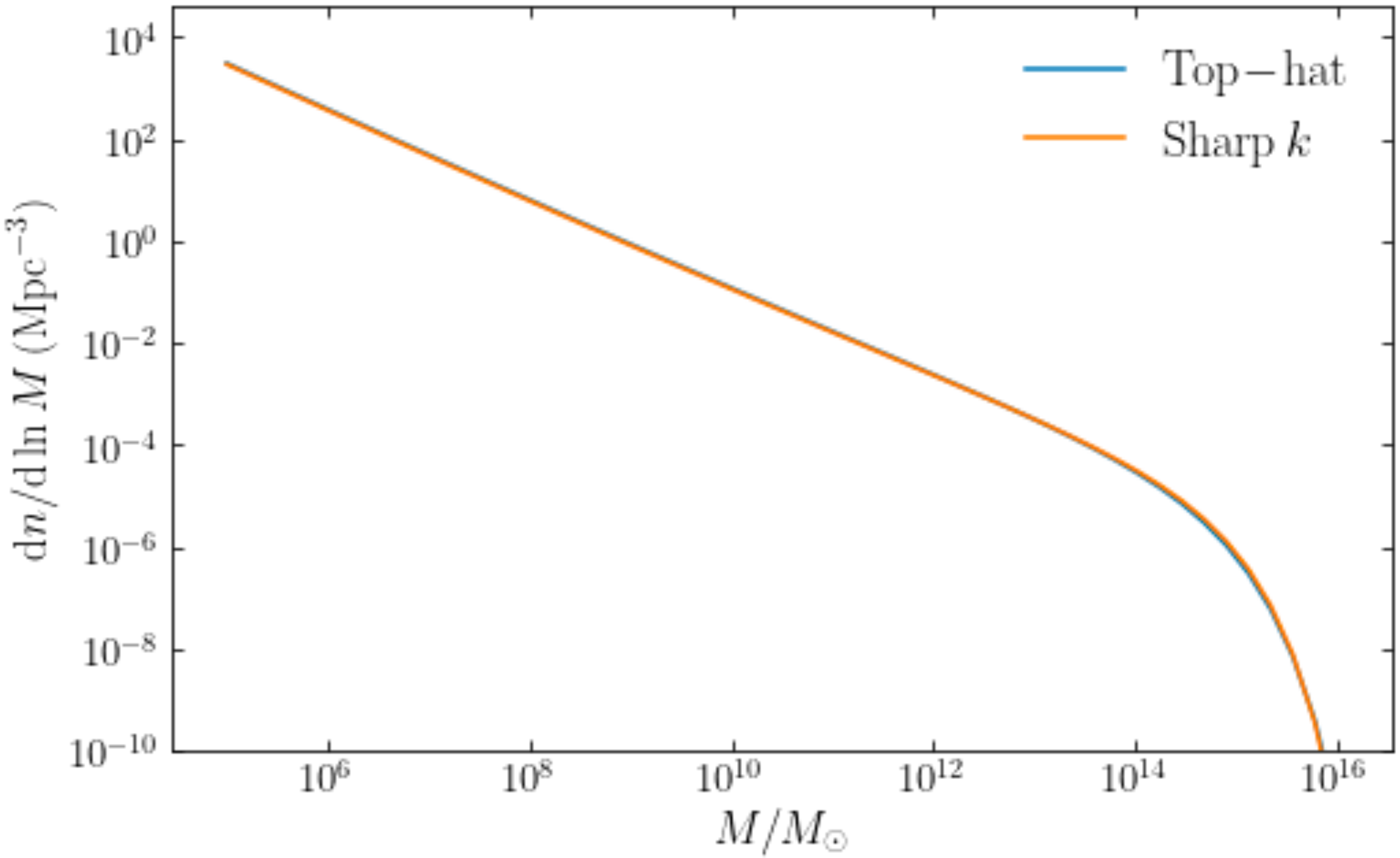
- PS mass function *including the factor of two* immediately follows

Sharp *k*-space filter vs. Real-space top hat

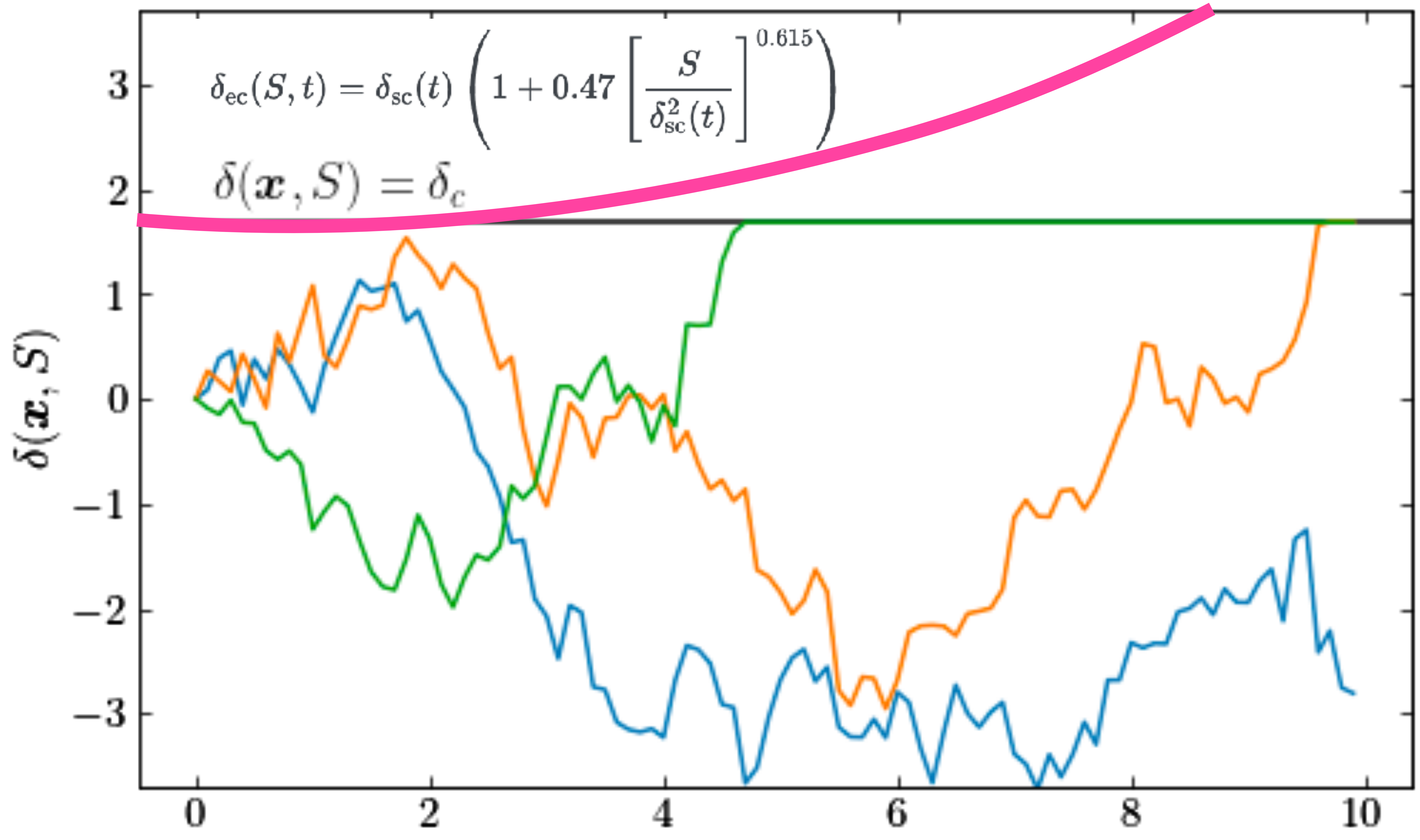
$$R = 0.9\,\mathrm{Mpc}\left(\frac{M}{10^{12}\,M_\odot}\right)^{1/3},$$



Sharp *k*-space filter vs. Real-space top hat



Halo formation assuming ellipsoidal collapse



Sheth, Mo, & Tormen (2001)

$$f_{\rm ST}(\nu) = 0.3222\sqrt{\frac{2}{\pi}}\,\nu'\,e^{-\nu'^2/2}\,\left(1+\nu'^{-0.6}\right)\,,$$ with ν’ = ν

History of mass elements in EPS

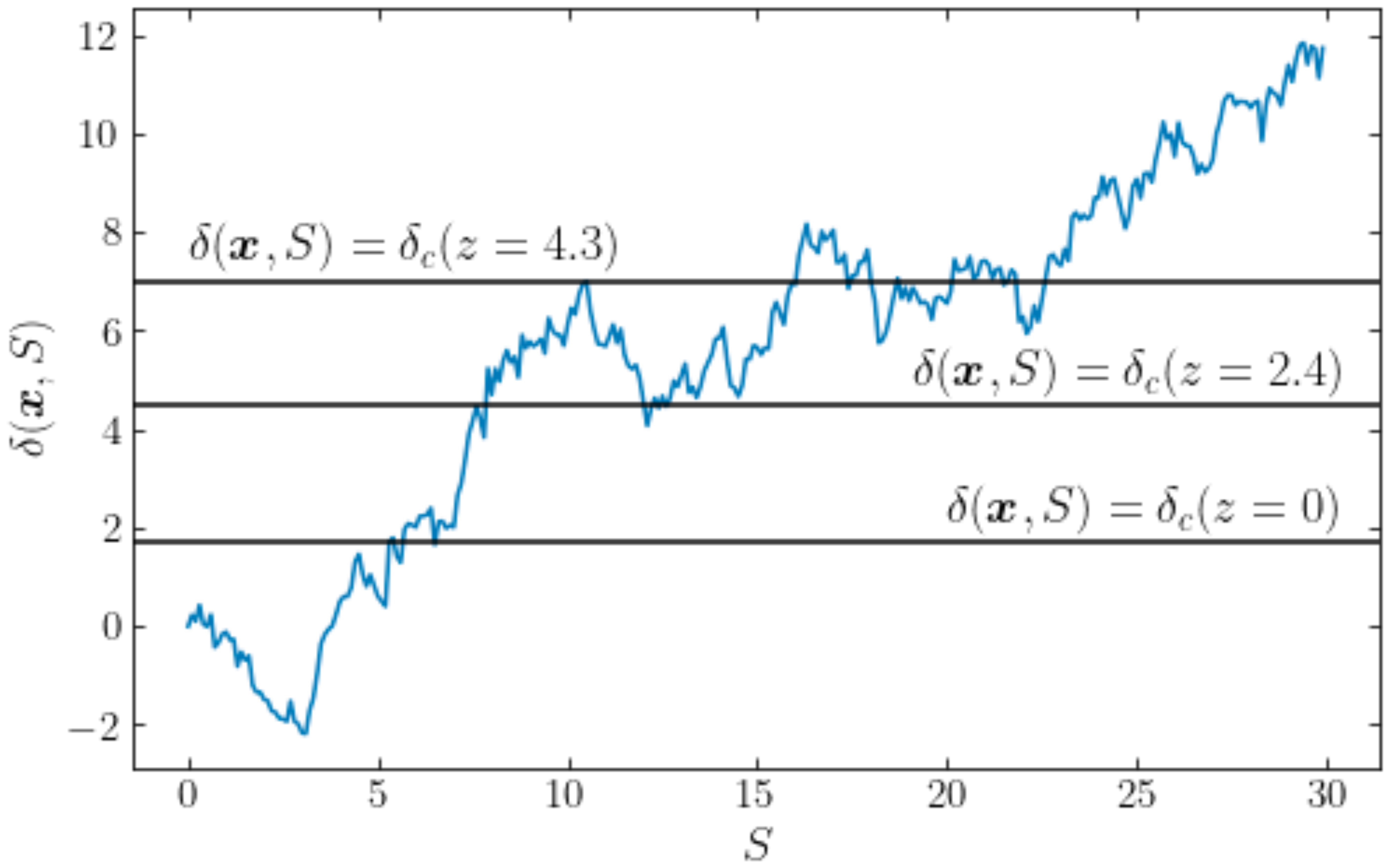
- Because σ(R,t) depends on time, halos at earlier times form differently
- However, the only time dependence in σ(R,t) comes from the growth factor and EPS only depends on $\sigma(R,t)/\delta_c(t)$, so we can absorb the growth factor into the denominator

$$\delta_c(a) = \delta_c/[D_+(a)/D_+(1)].$$

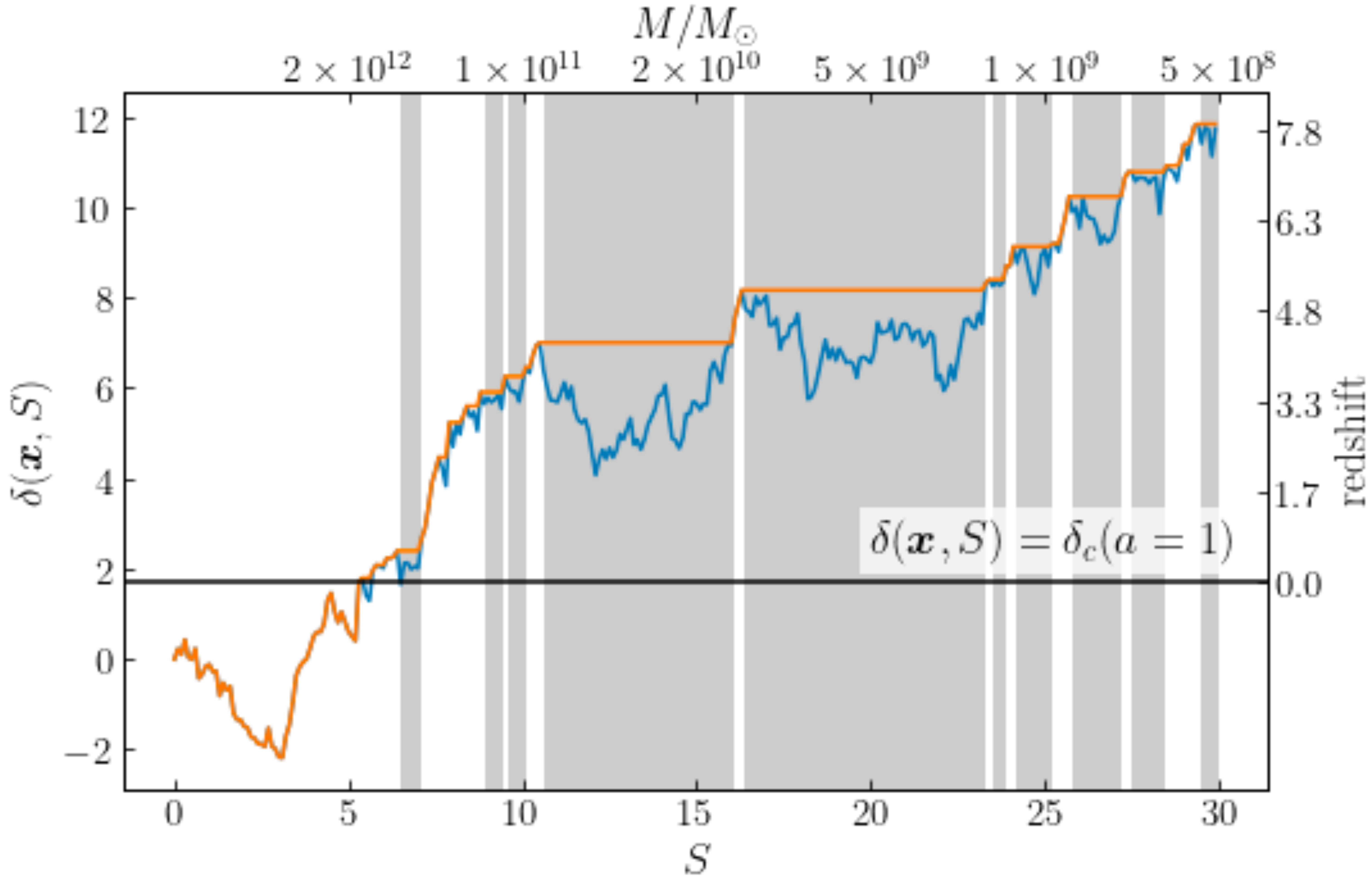
  and now always use S(M) at redshift 0

- The random walk from before can then be interpreted as the whole history of the mass element: was it part of a halo at redshift 1, 2, 10?

History of mass elements in EPS

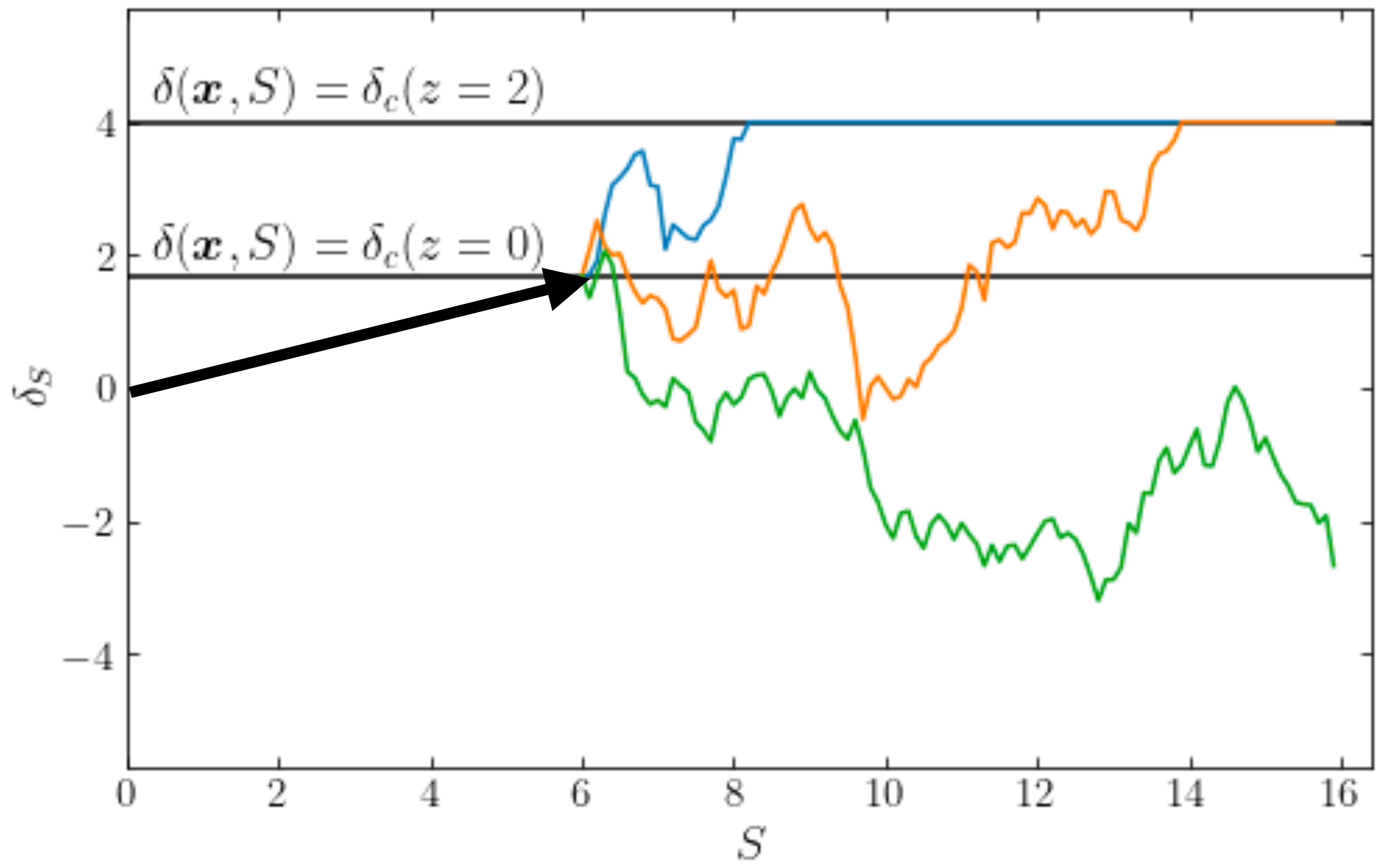


Merging history of mass elements in EPS



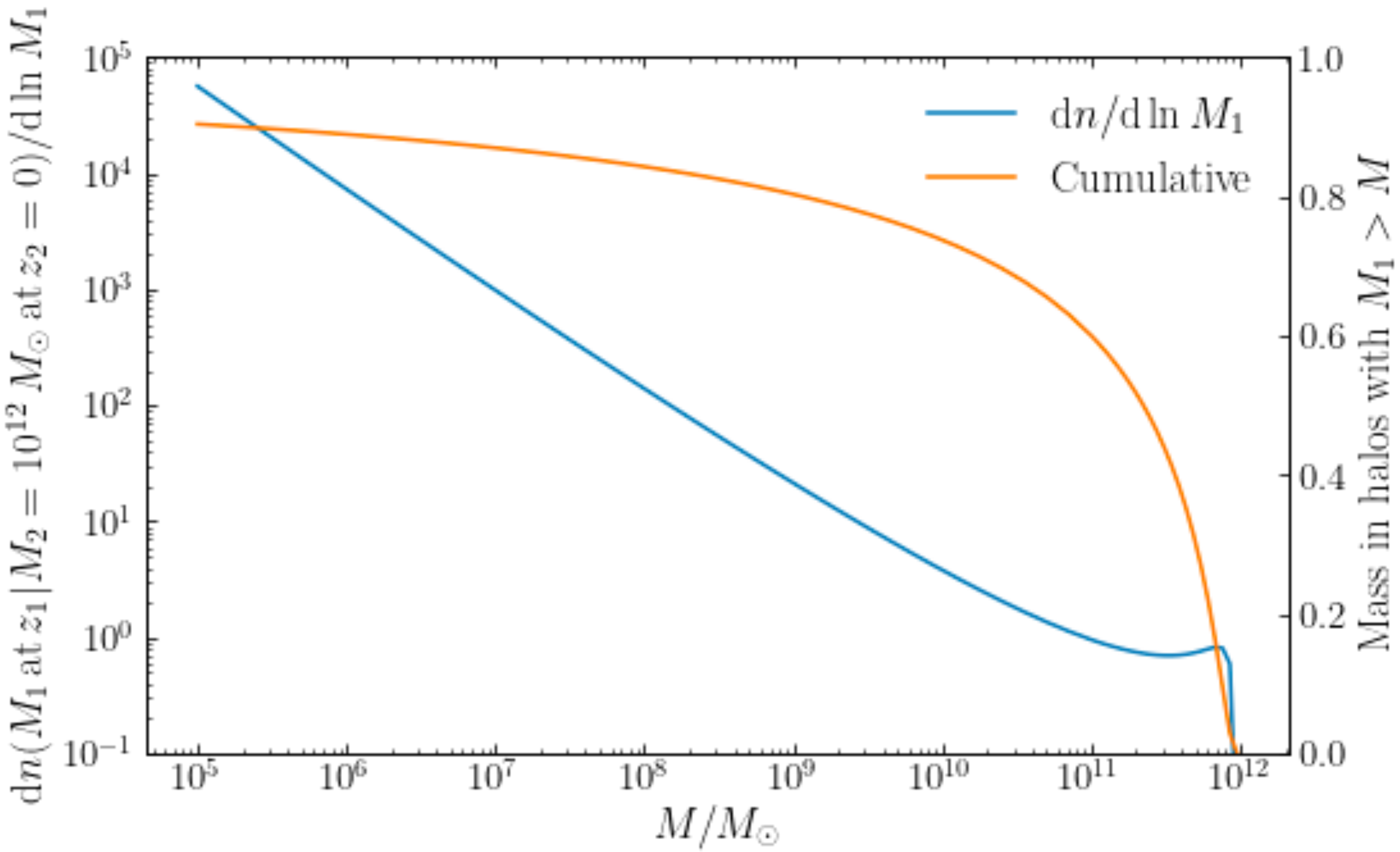
Conditional histories

Use EPS to determine the history of a z=0 halo



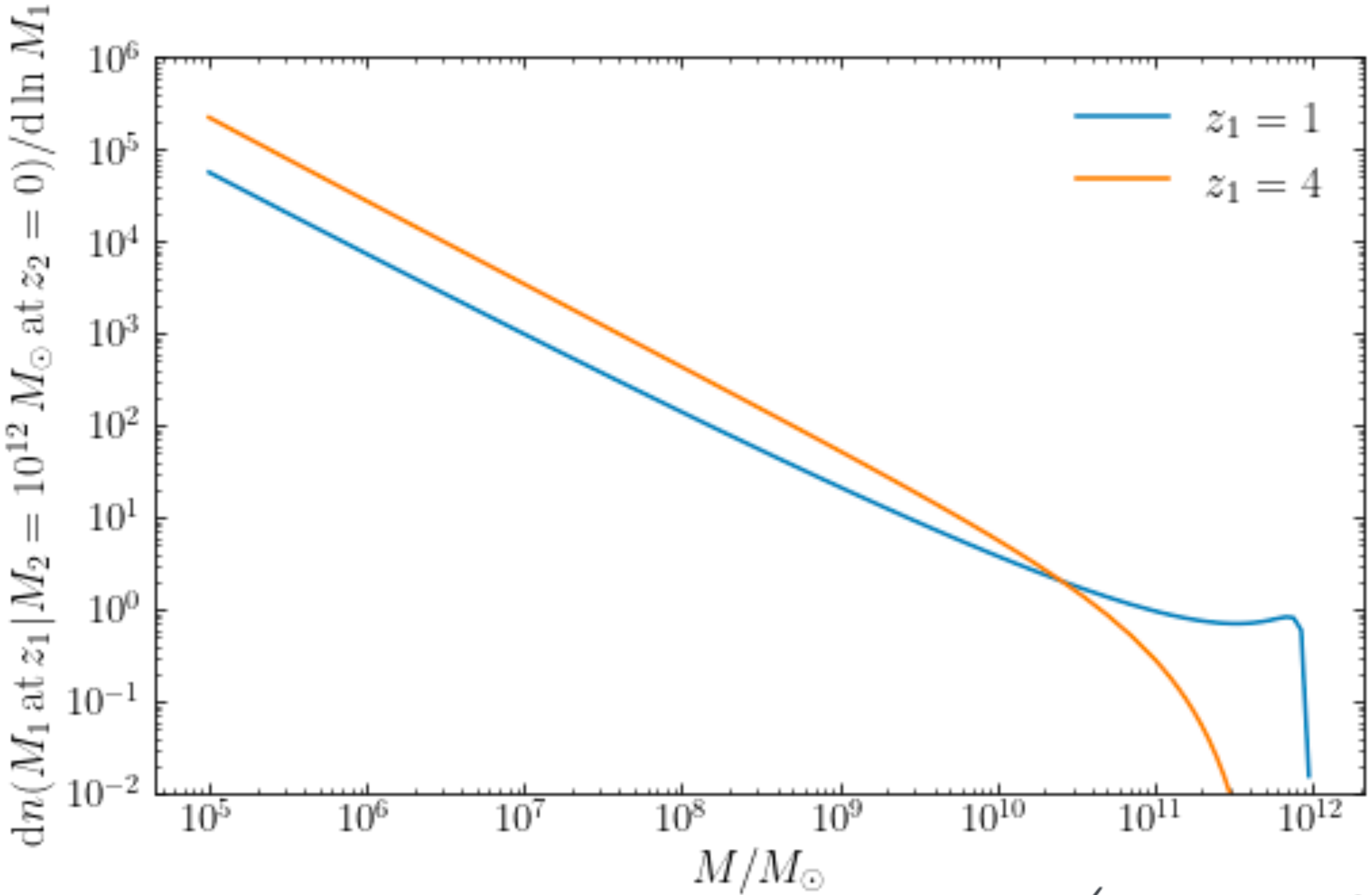
$$p[\delta(\mathbf{x}, S_1) = \delta_1 | \delta(\mathbf{x}, S_2) = \delta_2] = \frac{(\delta_1 - \delta_2)}{\sqrt{2\pi\,(S_1 - S_2)^3}}\ \exp\left(-\frac{[\delta_1 - \delta_2]^2}{2[S_1 - S_2])}\right)$$

Conditional mass function (z=0, 10^{12} M_{sun})



$$\frac{\mathrm{d}n(M_1 \,\mathrm{at}\, z_1 | M_2 \,\mathrm{at}\, z_2)}{\mathrm{d}M_1} = \frac{M_2}{M_1} \frac{(\delta_1 - \delta_2)}{\sqrt{2\pi\,(S_1 - S_2)^3}} \exp\left(-\frac{[\delta_1 - \delta_2]^2}{2[S_1 - S_2])}\right) \left|\frac{\mathrm{d}S_1}{\mathrm{d}M_1}\right|$$

Conditional mass function (z=0, 10^{12} M_{sun})

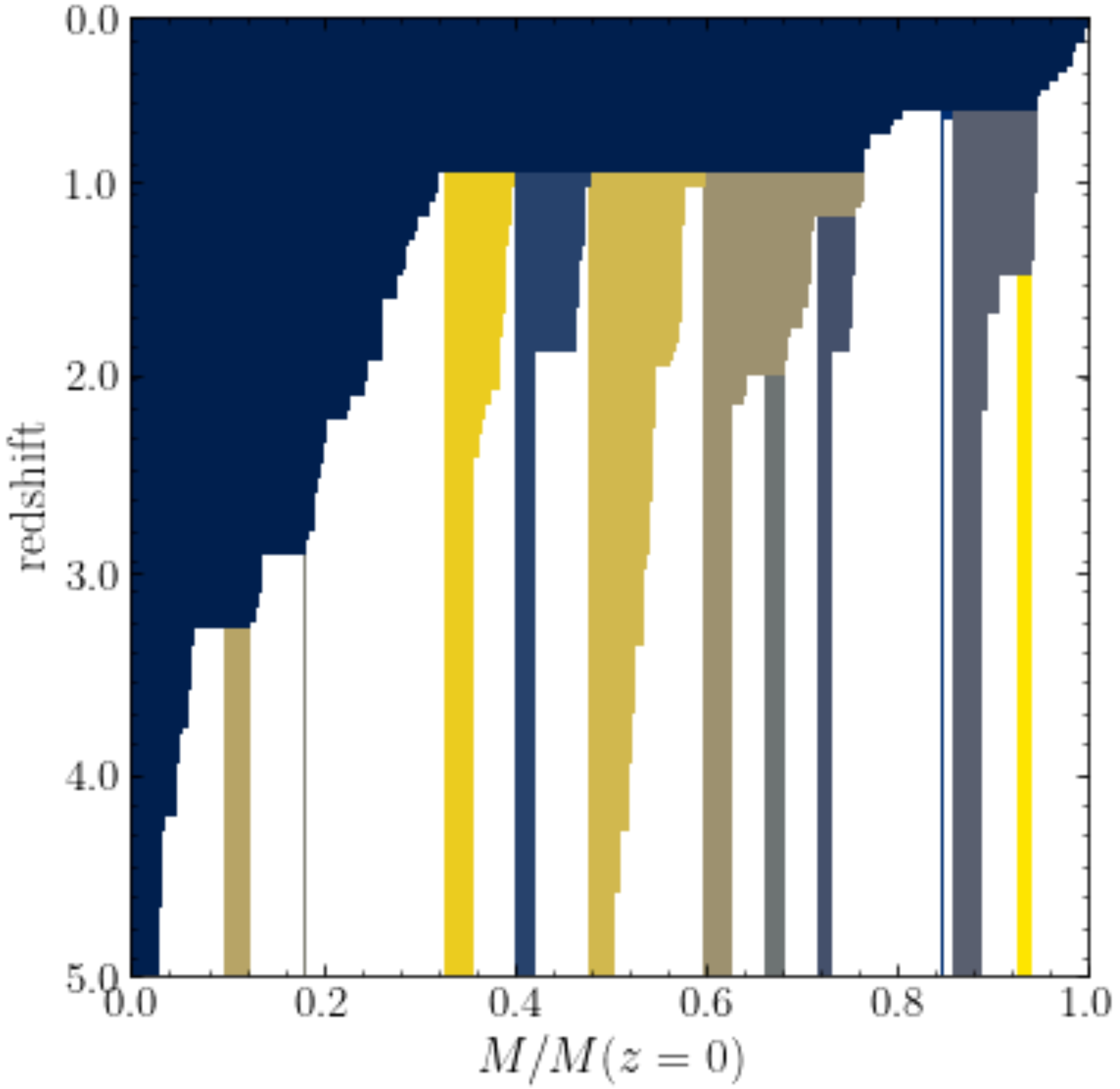


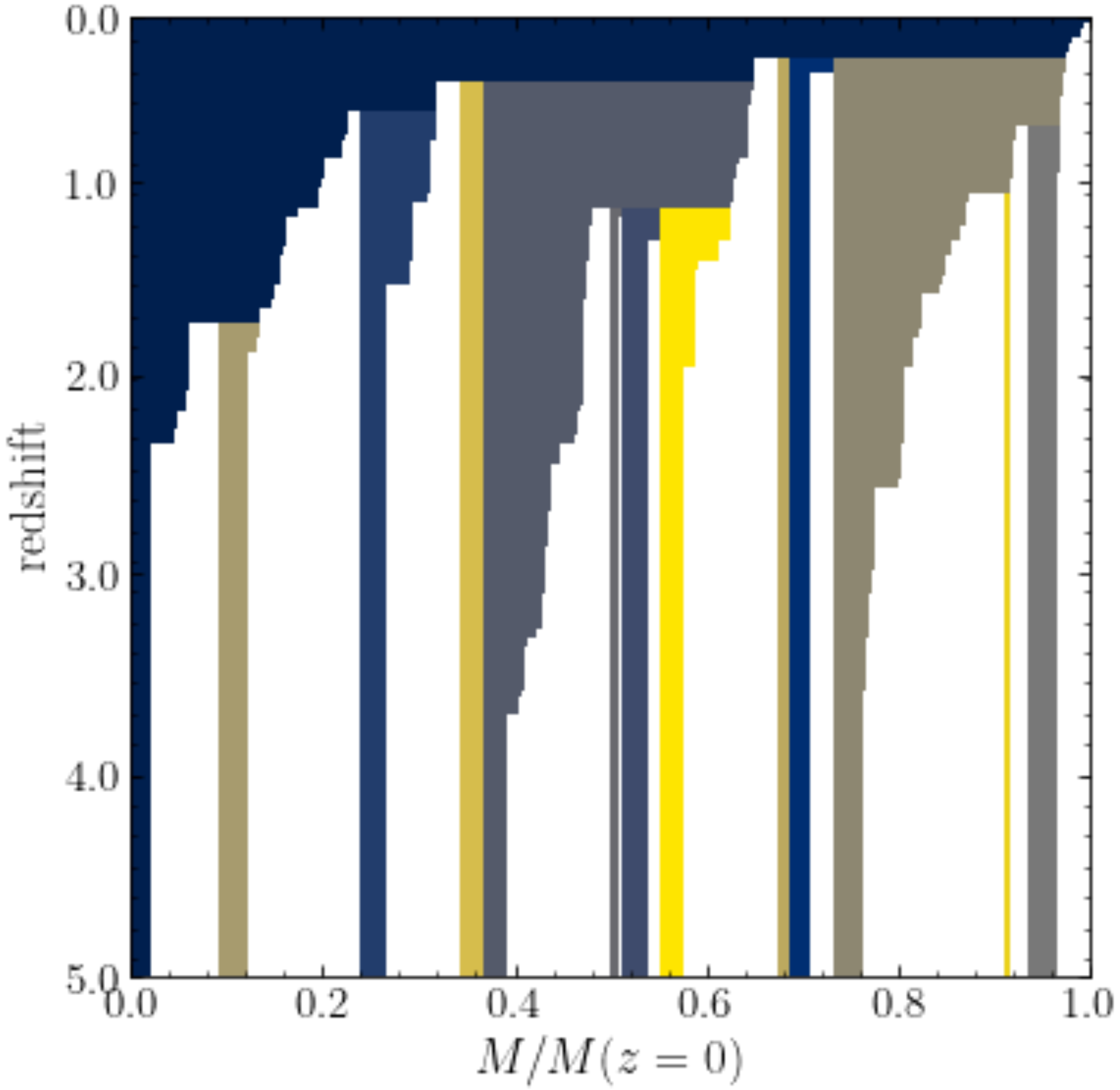
$$\frac{\mathrm{d}n(M_1 \,\mathrm{at}\, z_1 | M_2 \,\mathrm{at}\, z_2)}{\mathrm{d}M_1} = \frac{M_2}{M_1} \frac{(\delta_1 - \delta_2)}{\sqrt{2\pi\,(S_1 - S_2)^3}} \exp\left(-\frac{[\delta_1 - \delta_2]^2}{2[S_1 - S_2])}\right) \left|\frac{\mathrm{d}S_1}{\mathrm{d}M_1}\right|$$

Merger trees

- The EPS formalism allows one to compute the backwards history of a given halo
    - Very recent: merger rate
    - In the past: conditional mass function
- Can use EPS analytical result to create algorithm to generate full merger history of progenitors, and progenitors' progenitors, and progenitors' progenitors' progenitors, etc.
- Can't be done exactly, because it's impossible to exactly satisfy mass conservation and the conditional mass function. But algorithms get very close.

Merger trees



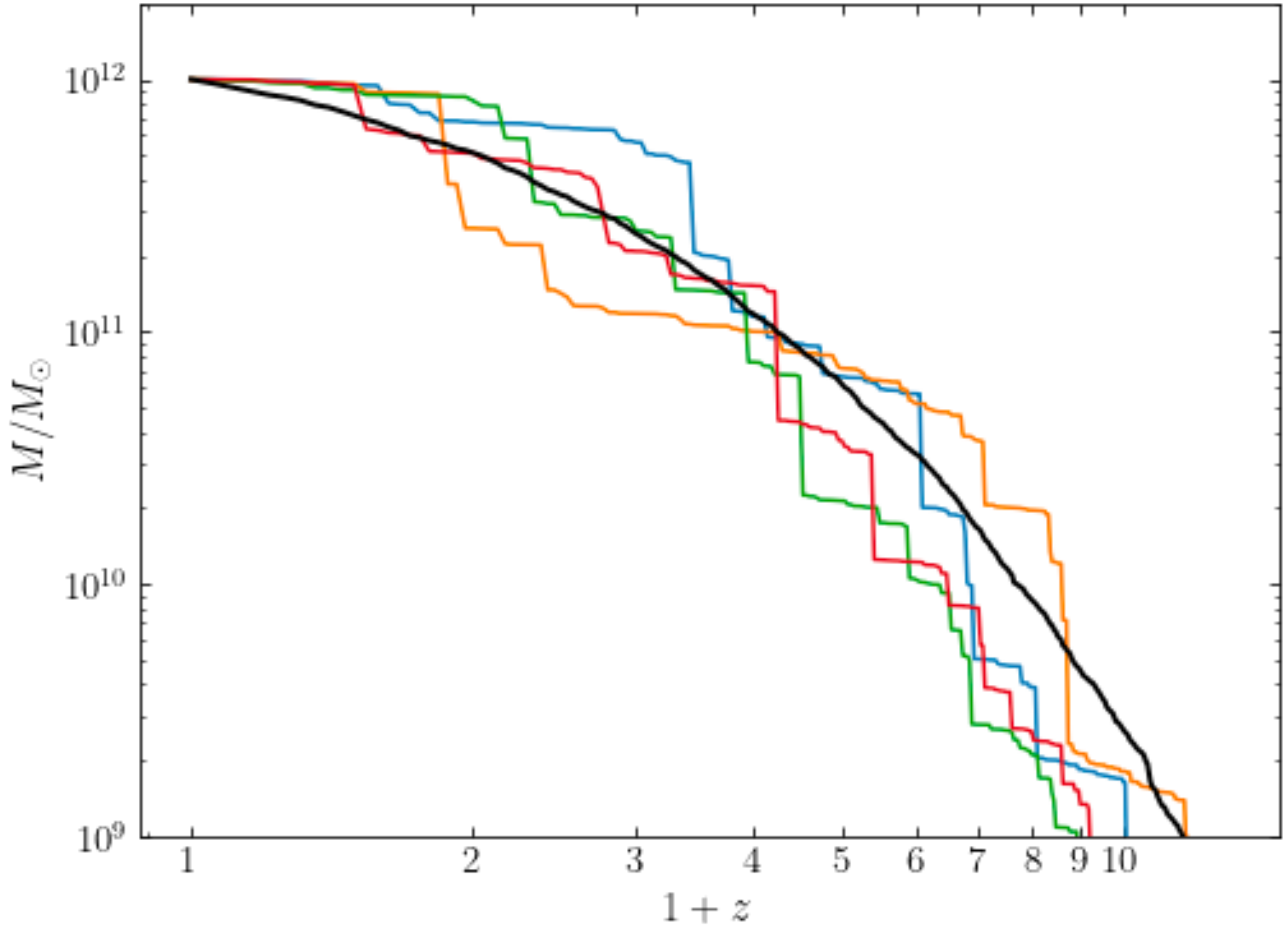


Using the Somerville & Kolatt (1999) merger-tree algorithm

Merger trees statistics

- Can use ensembles of merger trees to study likely histories for a given halo

Mass accretion for
Milky-Way-like halo at z=0



Semi-analytic galaxy formation

- EPS merger trees can be combined with prescriptions for galaxy formation to predict full galaxy populations:
    - Gas cooling
    - Star formation
    - Feedback
    - …
- Mainly useful for learning about galaxy formation, not cosmology!
- Mostly replaced by large-volume hydrodynamical N-body simulations these days

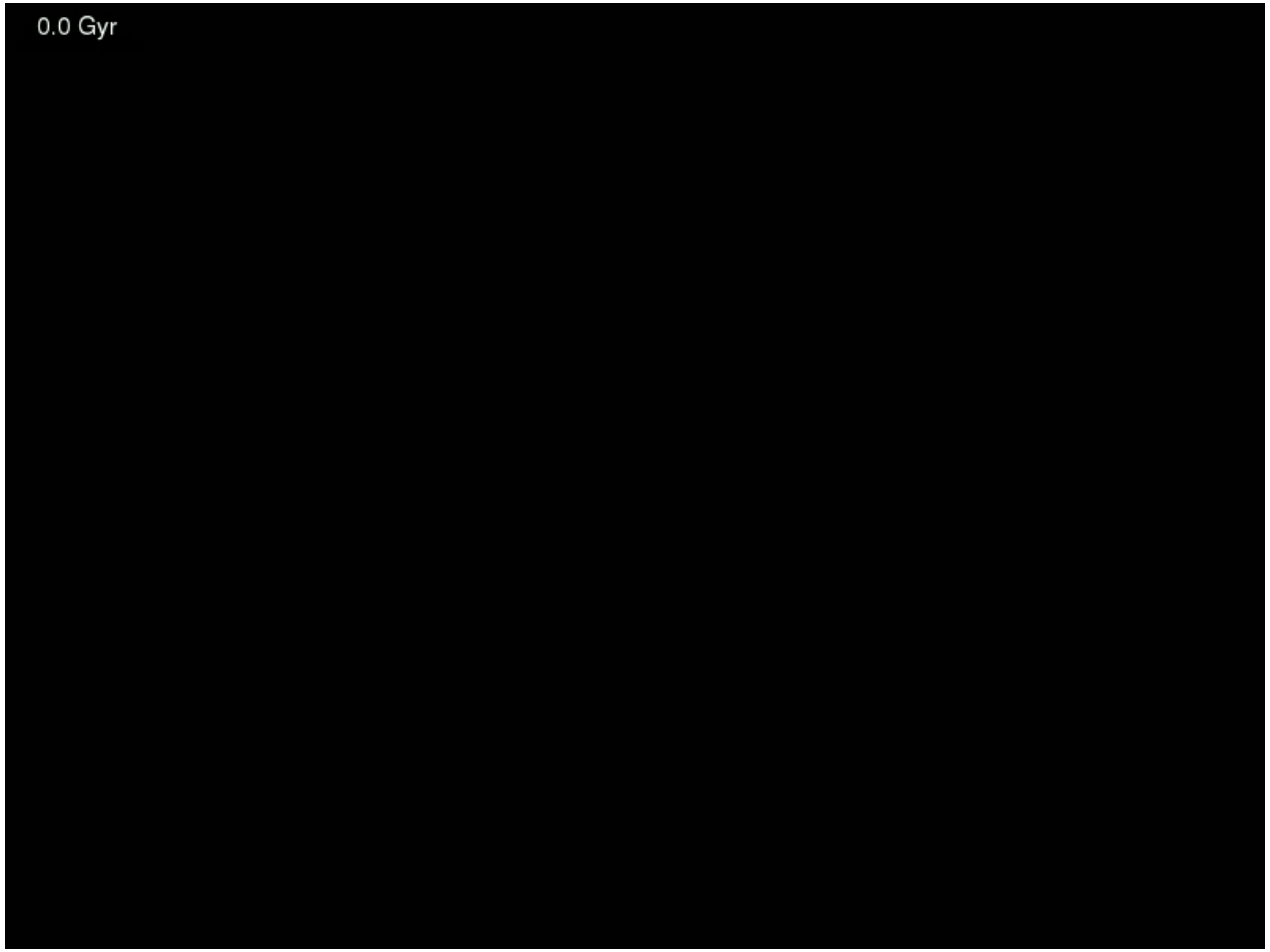
The relation between dark matter halos and galaxies

Why the relation between dark matter halos and galaxies?

- Baryons are a small (~20%) fraction of the total matter content of the Universe
- and they behave in annoying ways (that are good for us!)
- Ideally, we would use the DM distribution to learn about cosmology
- But DM is difficult to observe (exception: lensing) so need to rely on baryonic tracers to 'see' the matter distribution
- Baryons exist as:
    - Galaxies: grow in DM halos by (gas cooling —> star formation) + (mergers), largely condensed at the centre of DM halos (~ DM size / 10)
      —> galaxy clustering
    - Gas between galaxies —> Lya forest, 21 cm

Galaxy formation 101

- Process of galaxy formation is highly complex, non-linear, multi-scale, multi-physics, …
- For cosmology, need simple yet accurate picture that allows galaxies to be related to matter distribution
- Luckily, despite their complexity, on the whole galaxies are quite regular and obey simple scaling relations, especially at the massive end that is most useful for cosmology
- This allows us to derive cosmologically-useful, simple methods for parameterizing the relation between DM halos and galaxies



Credit: Greg Stinson, MUGS (http://mugs.mcmaster.ca/)

Galaxy formation and cosmology

- Cosmological observations such as the large-scale clustering of galaxies tell us about *both* ***galaxy formation*** *and* ***cosmology***
- Major goal of observational cosmology in the past decades has been to do this in a way that makes few assumptions about galaxy formation when learning about cosmology
- Simple way to think about what's happening in the next many slides:
    - Derive simple relations between galaxies and dark matter halos *based on one-point statistics* (halo mass function, luminosity functions, …)
    - Use these to predict *two-point statistics* (and higher) of the galaxy distribution which tells us about cosmology (e.g,, the two-point correlation function)

The galaxy luminosity function

The galaxy luminosity function

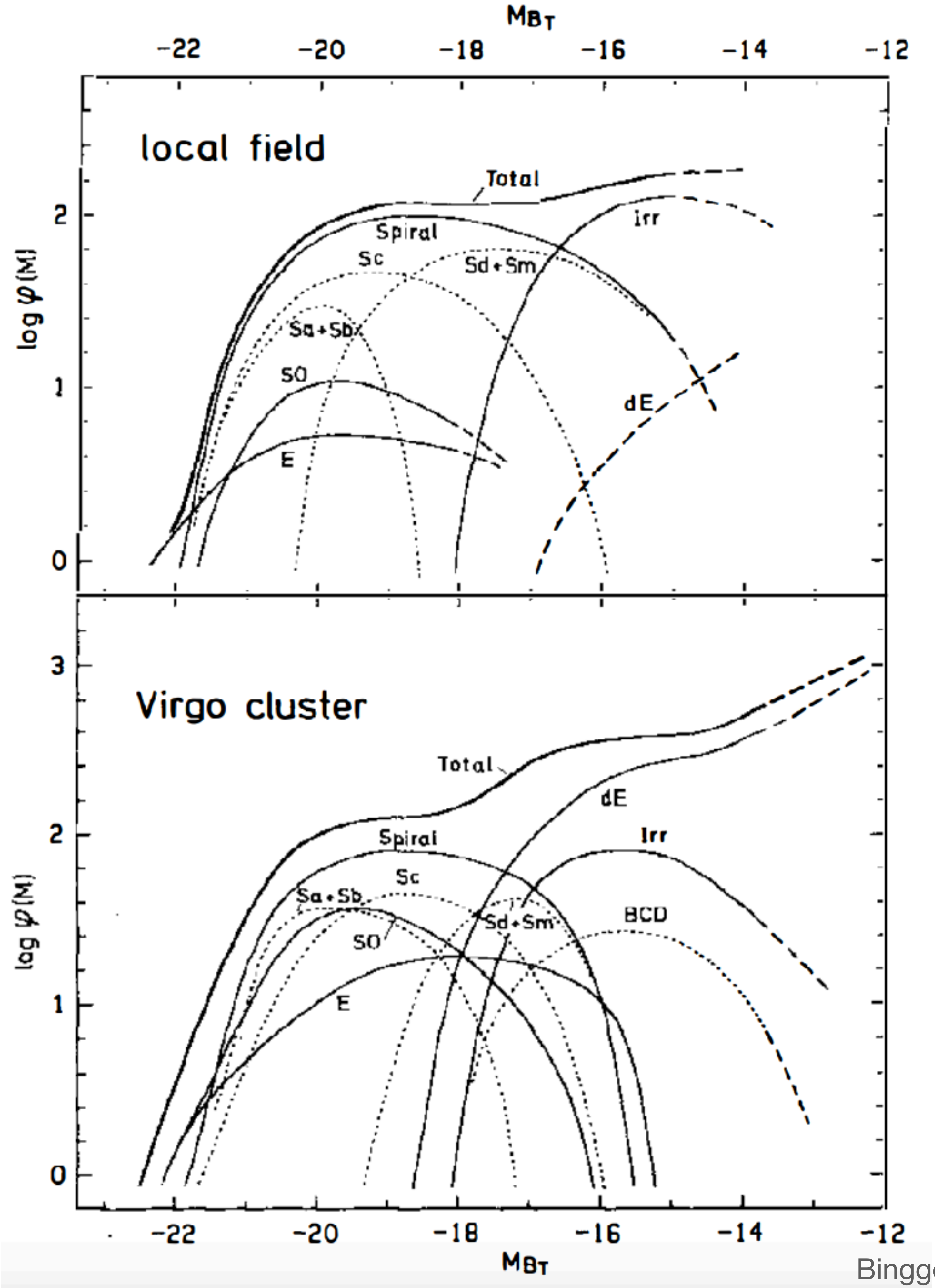
- The galaxy luminosity function is the simplest way to characterize the galaxy distribution: number of galaxies per unit luminosity and per unit volume

$$\Phi(L) = \frac{\mathrm{d}n(L)}{\mathrm{d}L} ,$$

- Using spectroscopic redshifts to go from m to M, essentially directly observable
- Luminosity generally measured in a broad photometric band (U, g, Ks, …)
    - Because different bands probe different physics, learn about stellar mass, star formation, depending on band
- Because we observe light, galaxy luminosity function is essential in going from cosmological model to observations of galaxies (e.g., clustering)

Galaxy luminosity function

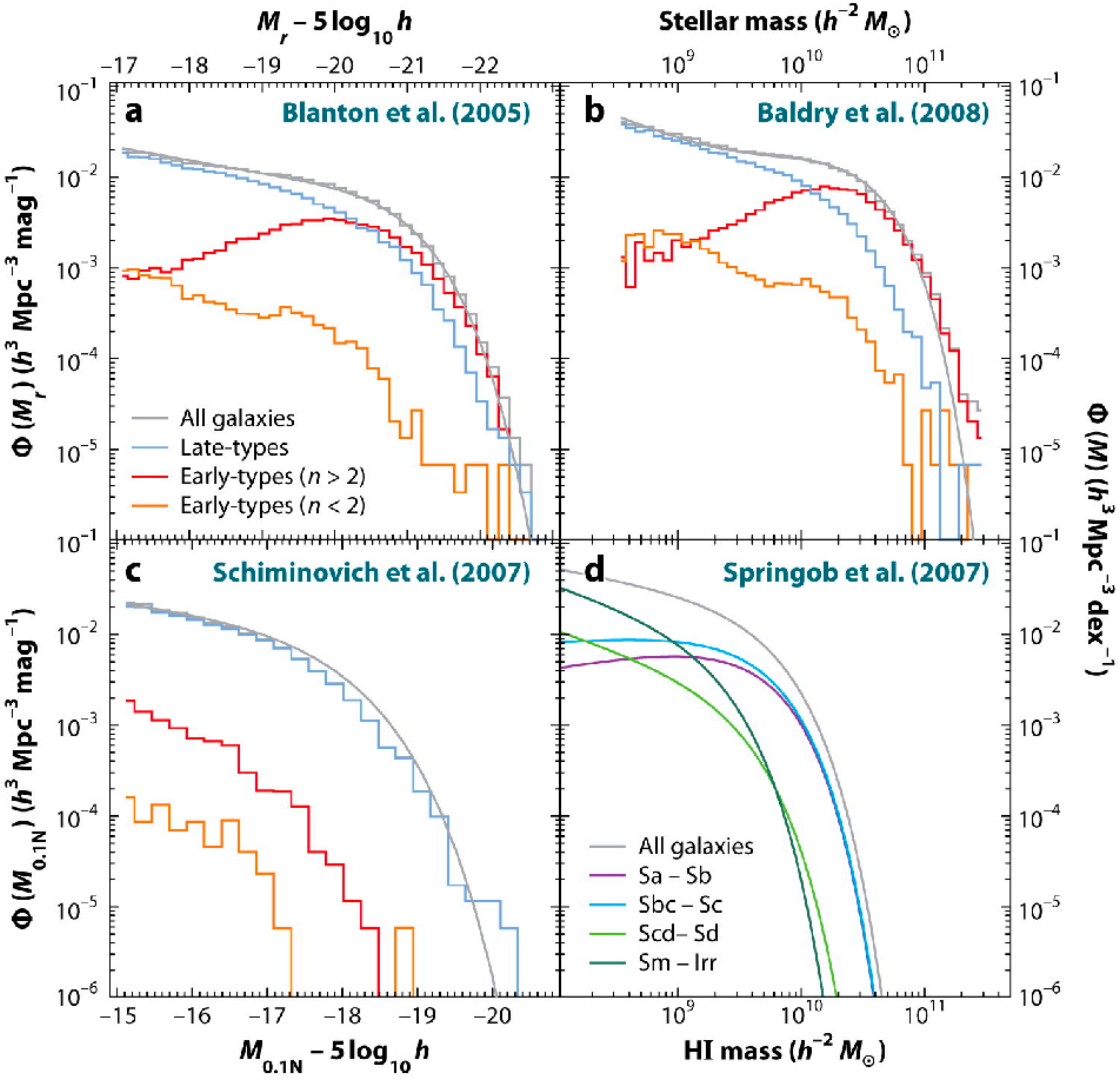
$$\Phi(M) = (0.4 \ln 10) \, L \, \Phi(L) \, ,$$



Binggeli et al. (1988)

Galaxy luminosity function

$$\Phi(M) = (0.4 \ln 10)\, L\, \Phi(L)\,,$$



Blanton MR, Moustakas J. 2009.
Annu. Rev. Astron. Astrophys. 47:159–210

Schechter function

- Galaxy luminosity function are generally expressed as a Schechter (1976) function or a variation of it

$$\Phi(L) = \frac{\Phi^*}{L^*} \exp\left(-\frac{L}{L^*}\right) \left(\frac{L}{L^*}\right)^{\alpha},$$

- Or as a function of absolute magnitude

$$\Phi(M) = (0.4 \ln 10)\, \Phi^* \exp\left(-10^{0.4\,[M^*-M]}\right) 10^{0.4\,[\alpha+1]\,[M^*-M]}$$

- Doesn't work so well at the low-luminosity end, so use 'double Schechter' function

$$\Phi(L) = \frac{1}{L^*} \exp\left(-\frac{L}{L^*}\right) \left[\Phi_1^*\left(\frac{L}{L^*}\right)^{\alpha_1} + \Phi_2^*\left(\frac{L}{L^*}\right)^{\alpha_2}\right]$$

Luminosity density

- Luminosity weighted integral of the luminosity function:

$$j = \int_0^\infty \mathrm{d}L \, L \, \Phi(L)$$

- Double Schechter:

$$j = L^* \left[\Phi_1^* \Gamma(\alpha_1 + 2) + \Phi_2^* \Gamma(\alpha_2 + 2)\right]$$

- Observed value:

$$j_r \approx 10^8 \, L_\odot \, \mathrm{Mpc}^{-3} \, .$$

- Convert to baryon mass density using $M/L_r \sim 3$ —> ~5% of the baryon density parameter
- So most baryons in the Universe are *not* stars

The galaxy stellar mass function

- Number of galaxies per unit stellar mass and unit volume

$$\Phi(M) = \frac{\mathrm{d}n(M)}{\mathrm{d}M}$$

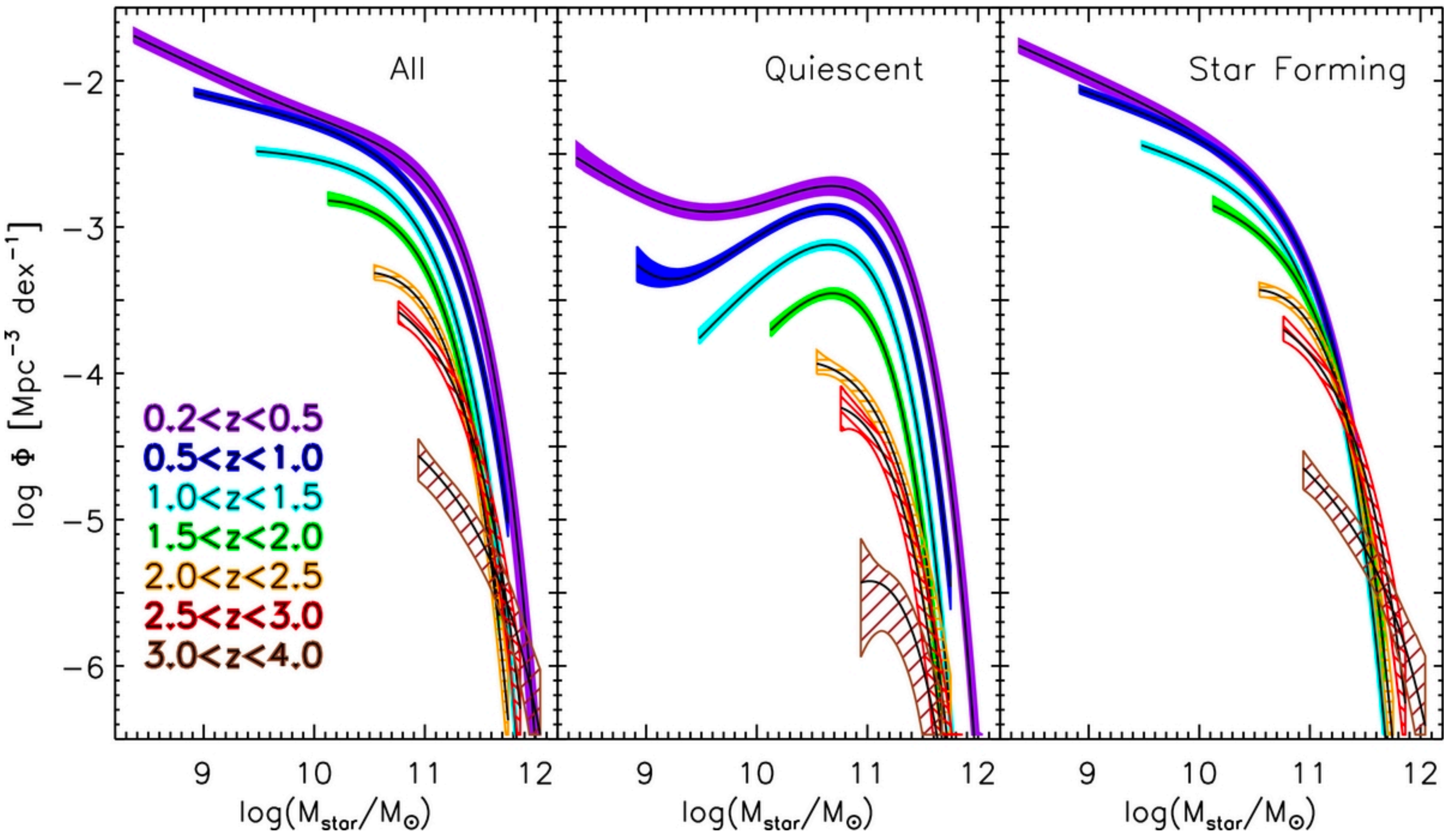
- Could convert luminosity function to stellar-mass function using M/L(L), but usually one estimates stellar masses for each galaxy and then directly derive the stellar mass function

- Also well represented using (double) Schechter function

$$\Phi(M) = \frac{1}{M^*} \exp\left(-\frac{M}{M^*}\right) \left[\Phi_1^*\left(\frac{M}{M^*}\right)^{\alpha_1} + \Phi_2^*\left(\frac{M}{M^*}\right)^{\alpha_2}\right] , \qquad (19.24)$$

with $\quad M^* = 10^{10.66}\, M_\odot \approx 4.6 \times 10^{10}\, M_\odot, \quad \Phi_1^* = 3.96 \times 10^{-3}\,\mathrm{Mpc}^{-3}, \quad \alpha_1 = -0.35,$
$\Phi_2^* = 0.79 \times 10^{-3}\,\mathrm{Mpc}^{-3}$, and $\alpha_2 = -1.47$. As for the luminosity density, the stellar-

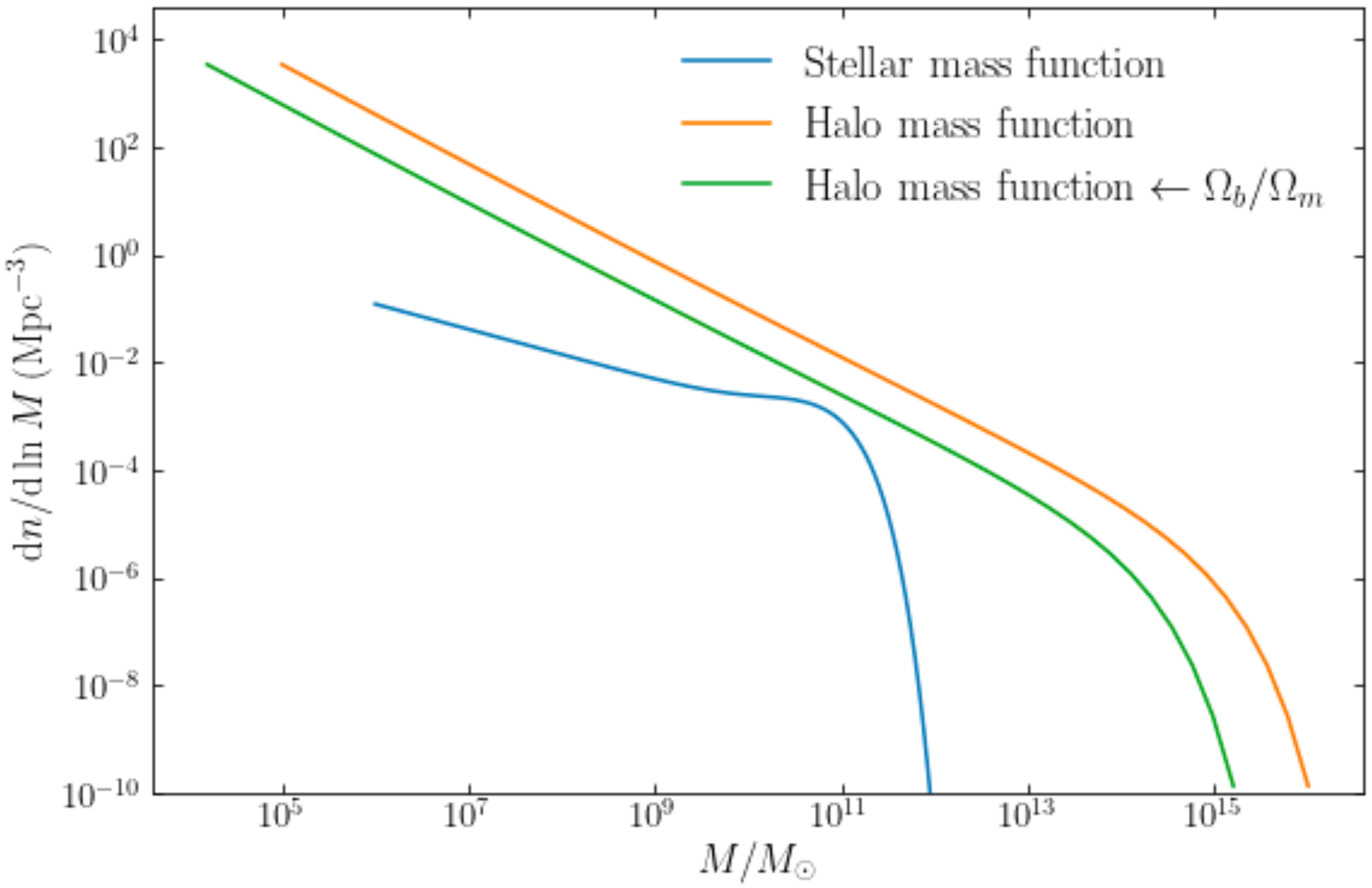
The galaxy stellar mass function



Muzzin et al. (2013)

The relation between the stellar mass and halo mass functions

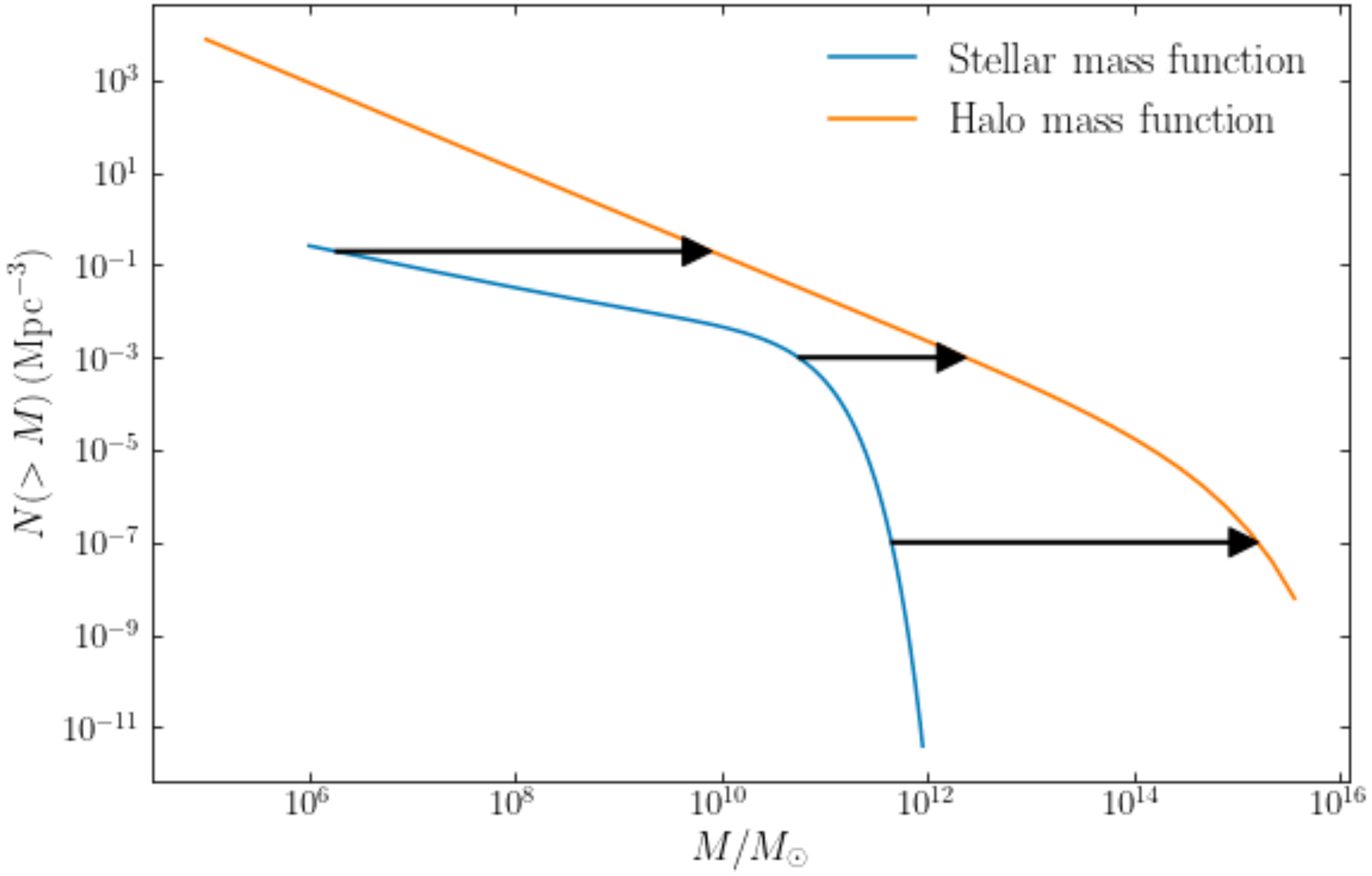
- Naive expectation: stellar mass = Ω_b/Ω_m —> stellar mass function is shifted version of halo mass function



The stellar-mass — halo-mass relation

Subhalo abundance matching (SHAM)

- By assuming that the most massive galaxy sits in the most massive halo, and the next-most-massive galaxy in the next-most-massive halo, etc., one can derive the relation between stellarmass and halo mass



The stellar-mass — halo-mass relation

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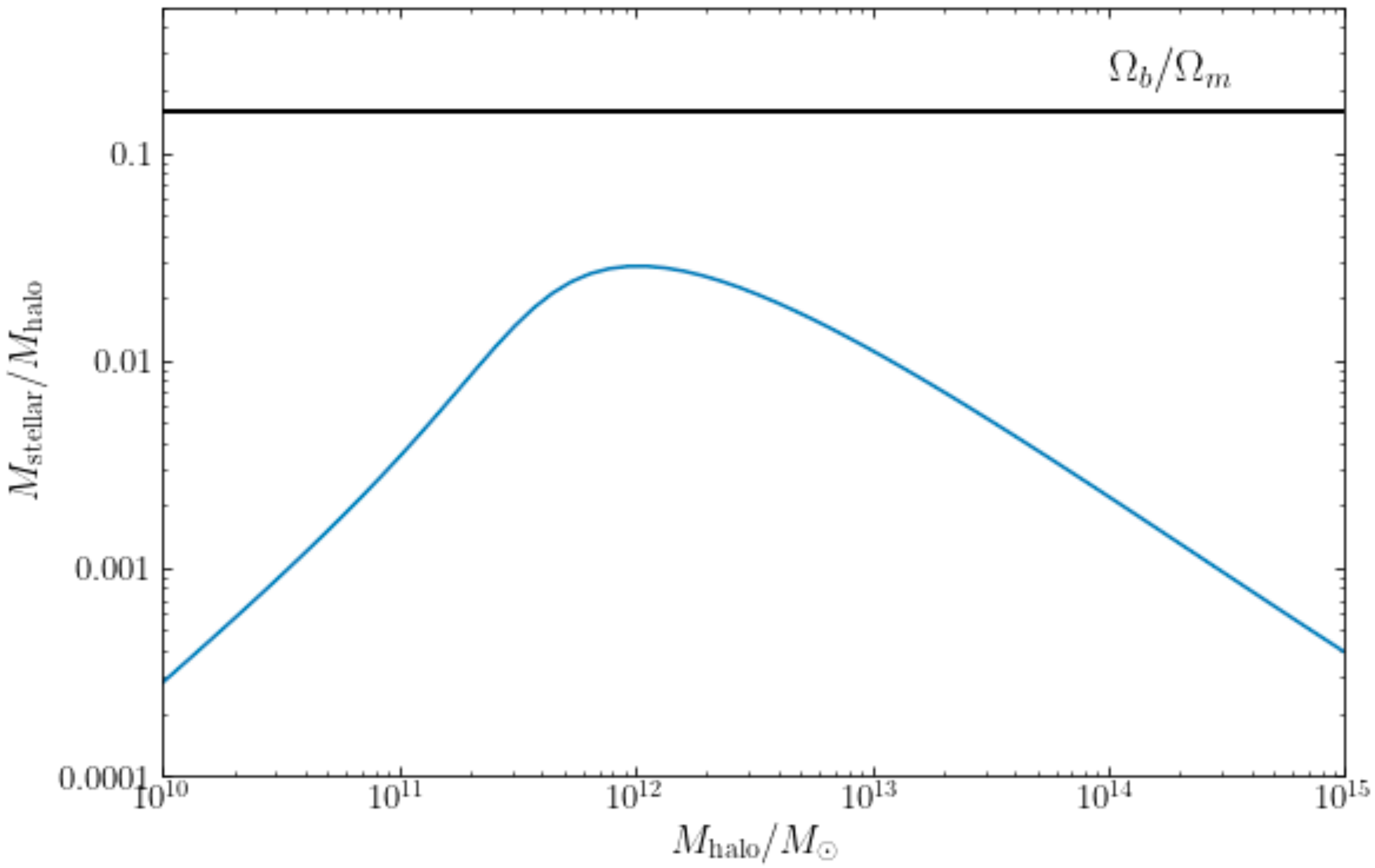

The stellar-mass — halo-mass relation

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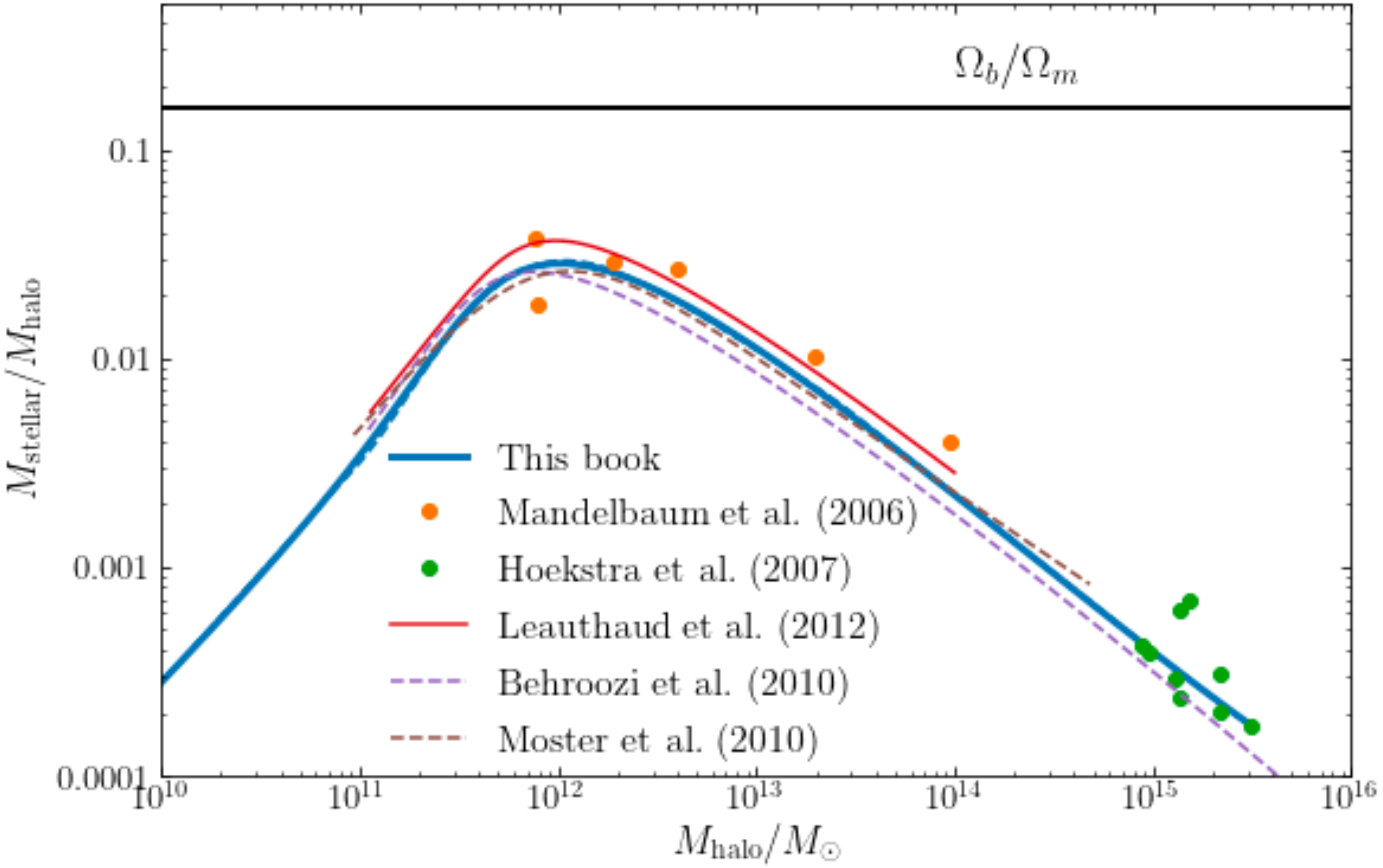


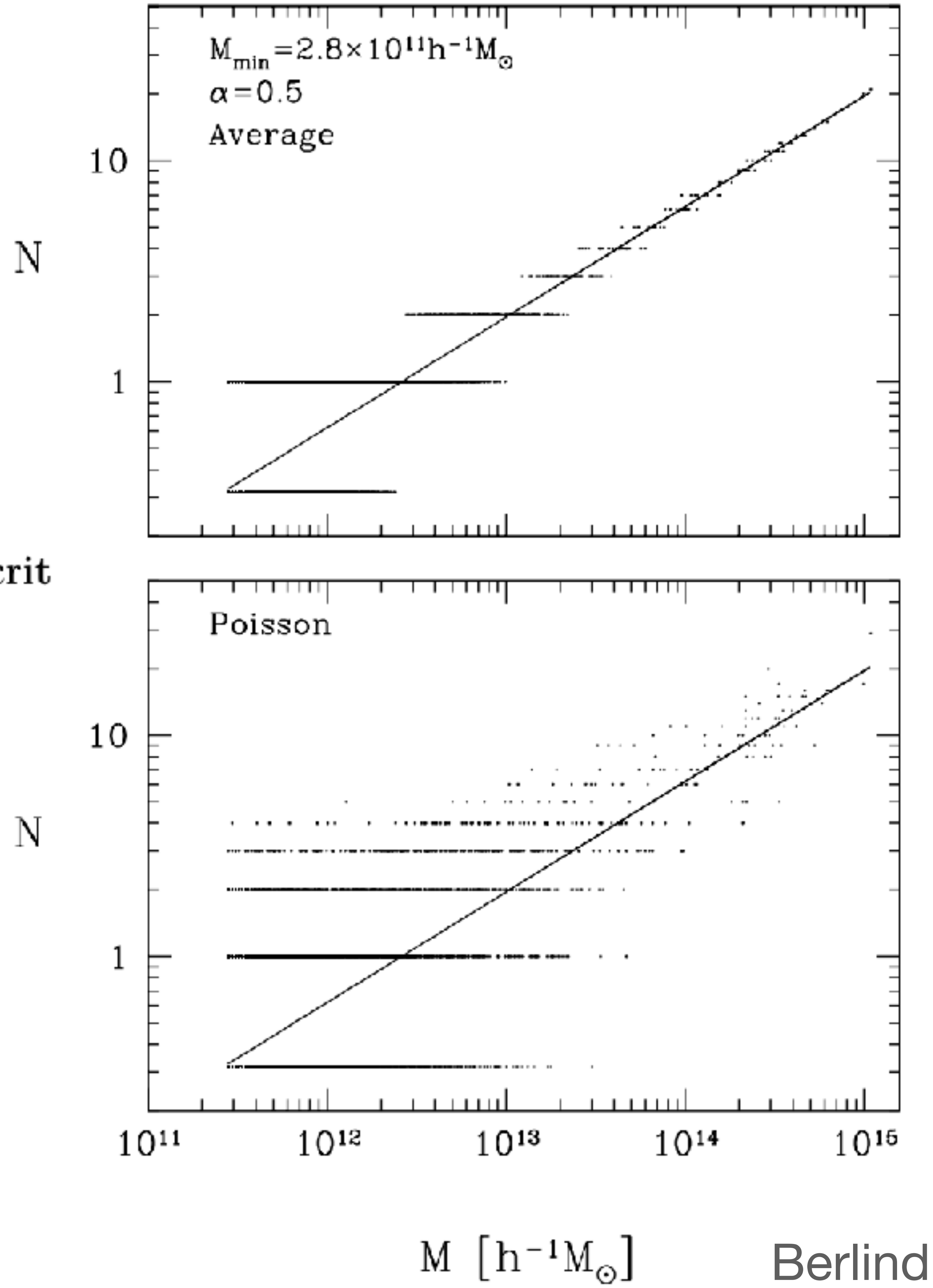
The halo occupation distribution (HOD)

Halo occupation distribution

- A simple, empirical connection between galaxies and dark-matter halos is given by the *halo occupation distribution*
- Simple model that gives
    - p(N|M): probability that a halo of mass M hosts N galaxies (of some type)
    - Distribution of galaxies in a halo
    - Distribution of galaxy velocities in a halo
- If N = 1: galaxy sits at rest at the centre of the halo
- If N > 1: central galaxy at centre, other galaxies distributed as satellites

Halo occupation distribution

$$N_{\rm avg} = \begin{cases} 0 & \text{if} \quad M < M_{\rm min} \\ (M/M_1)^{\alpha} & \text{if} \quad M_{\rm min} \le M \le M_{\rm crit} \\ (M/M_1')^{\beta} & \text{otherwise,} \end{cases}$$

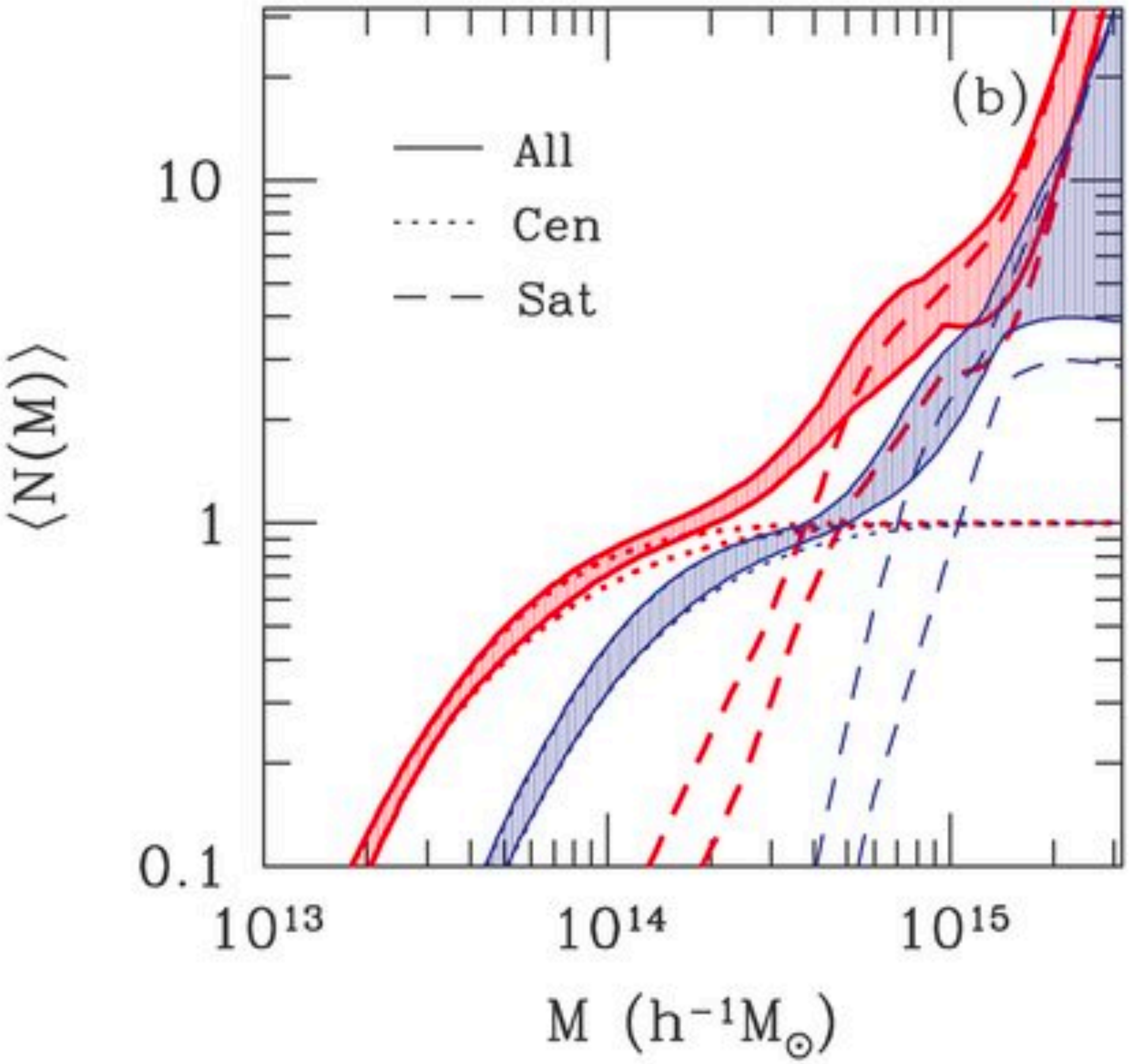


Berlind & Weinberg (2002)

Halo occupation distribution

- Easy to apply to sub-samples of galaxies, such as “Luminous red galaxies” —> (used by SDSS to do cosmology)

- Can also assign luminosities using the *conditional luminosity function*: luminosity function of galaxies residing in halos with mass M



Zheng et al. (2009)

Next week: Probing the cosmic density field / clustering

- Some of the most sensitive probes of cosmology at late time are *galaxy clustering* and *galaxy-galaxy lensing* (weak lensing)
- Cosmologically, these derive from the matter density field with following complications:
    - Non-linear evolution of the matter distribution, almost entirely due to halo formation
    - *Relation between galaxies and dark matter* telling us how the density field of galaxies relates to that of dark matter