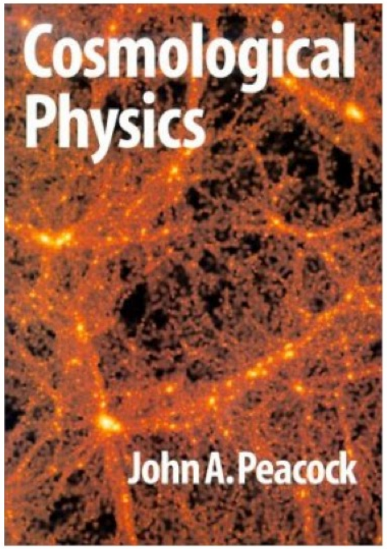
AST 1430
Graduate Cosmology

I

Context, Background, Overview

Recommended Text: *Cosmological Physics*



John Peacock

In my opinion, an excellent cosmology text and reference. Easy to read, Peacock does a great job building a physical understanding of what's going on. The internet, however, seems to disagree, complaining that most derivations are simply statements of the answer. I like that it doesn't get bogged down in the math, and it is my go-to reference, but your mileage may vary.

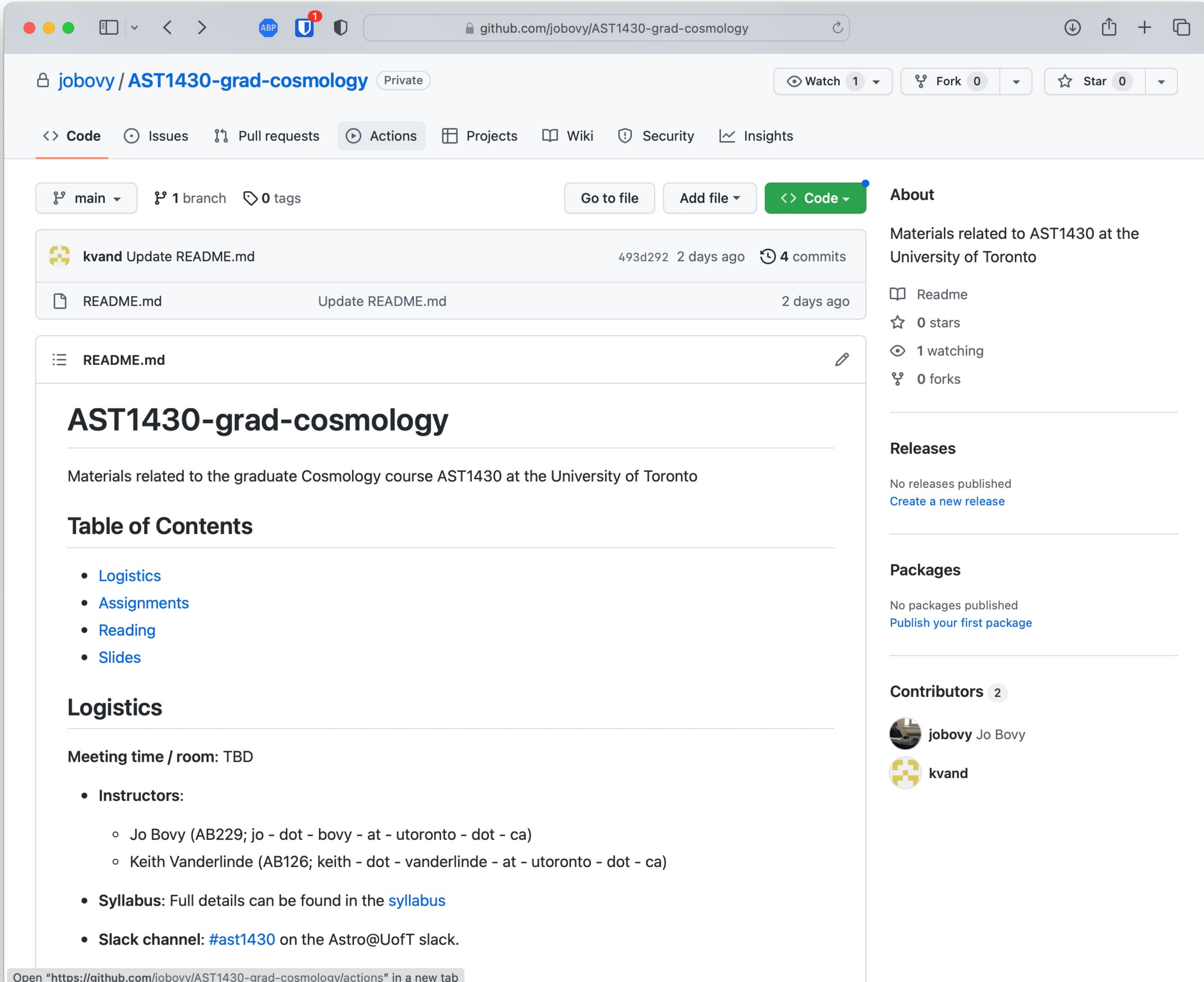
Other suggested texts:

- Dodelson, *Modern Cosmology* → Heavy on the math, but widely liked
- Kolb & Turner, *The Early Universe* → Classic, old but still good for pre-recombination
- Longair, *Galaxy Formation* → Not my cup of tea, but covers most of this stuff
- Ryden, *Introduction to Cosmology* → Undergrad-level, but supremely readable

Also note: co-teaching! Jo will have other suggestions for his half!

	Week Index	Dates	Topics
Keith	**Week 1**	Jan 11, 13	logistics, introduction, basic observations
Keith	**Week 2**	Jan 18, 20	Basic GR, RW metric, Distances, coordinates, Friedmann equations
Keith	**Week 3**	Jan 25, 27	Cosmological models, consistency with observations, Early Hot Universe, BBN
Keith	**Week 4**	Feb 1, 3	Inflation, Perturbations & Structure pre-recombination
Keith	**Week 5**	Feb 8, 10	CMB: basics, polarization, secondaries
	Week 6	Feb 15, 17	Early-Universe Presentations + Review
	Reading Week – No Class		
Jo	**Week 7**	Mar 1, 3	Post-recombination growth of structure, formation of dark matter halos, halo mass function
Jo	**Week 8**	Mar 8, 10	The relation between dark matter halos and galaxies
Jo	**Week 9**	Mar 15, 17	Probing the cosmic density field / clustering
Jo	**Week 10**	Mar 22, 24	Late-time cosmological observations: BAO, supernovae, weak lensing, etc.
Jo	**Week 11**	Mar 29, 31	H0 controversy: how fast exactly is the Universe expanding today?
	Week 12	Apr 5	Late Universe Presentations + Review

Course Website – Coming soon!



Cosmology

(According to Wikipedia, “Not to be confused with Cosmetology.”)

From the Greek “Kosmos Logia” (κόσμος λογία), “Order Study”

The study of the universe as a whole (eg all of existence):

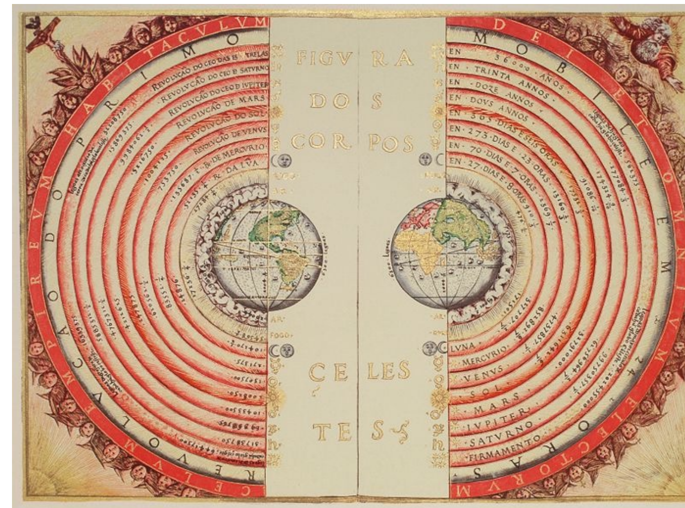
Provides explanations of different aspects of the universe
- the origin
- the history
- the composition
- the evolution
- the future and fate

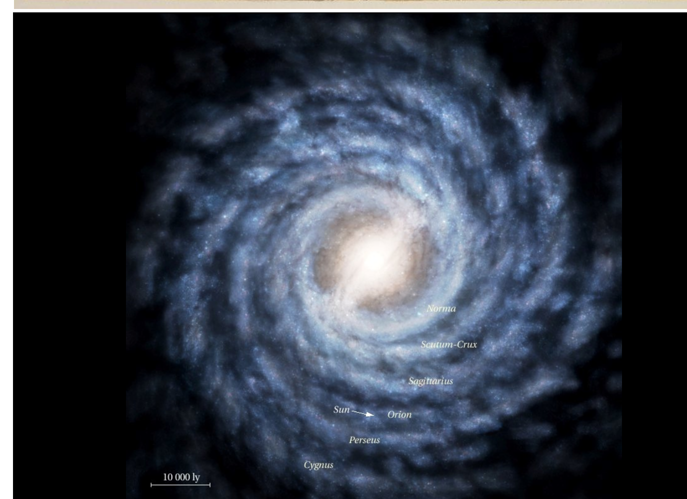
Often (though not necessarily) concentrates on the composition of the heavens, closely tied to astronomy.

Historical [Western] Cosmology

For most of history, Cosmology was strictly a philosophical/religious pursuit.

- <1600CE Aristotelian, Ptelomaic (**geocentric**)
- Eternally static, unchanging surroundings
- >1600CE Copernican (**heliocentric**)
- Expansion, contraction, static existence…?
- 1838: Stars – a whole galaxy, the Milky Way
- 1924: Other galaxies!
- 1930: redshift & the expanding universe

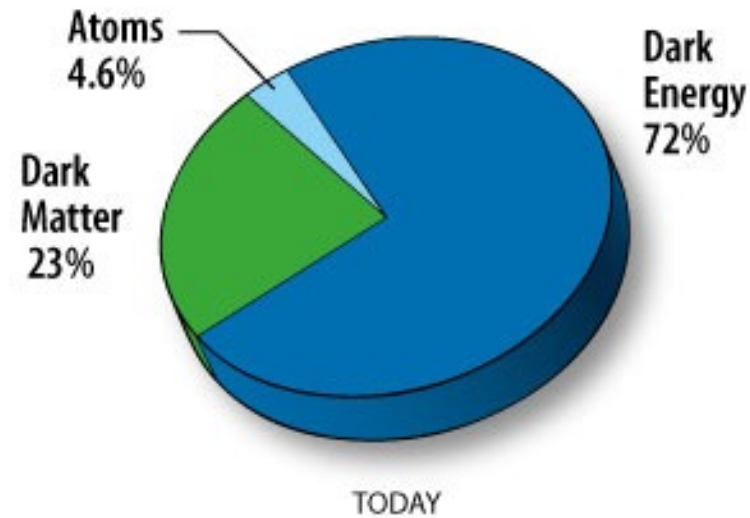




Physical Cosmology

Concordance (ΛCDM) Model:

- The Universe began with a Big Bang, about 13,800,000,000 years ago
- The Universe is expanding
- The Expansion is accelerating
- The Universe is full of weird stuff



We Don't Know:

- What dark matter is, but have several ideas
- What dark energy is, and don't have much data about it

(Taking our best guess, a vacuum energy required to create empty space, when we try to calculate how big it should be from QM, the answer, known as "the worst prediction in physics" is off by a factor of roughly **1,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000** times.)

Units in Cosmology

Time: 1 Gy = 10^9 years

Distance: 1 Mpc = 10^6 pc

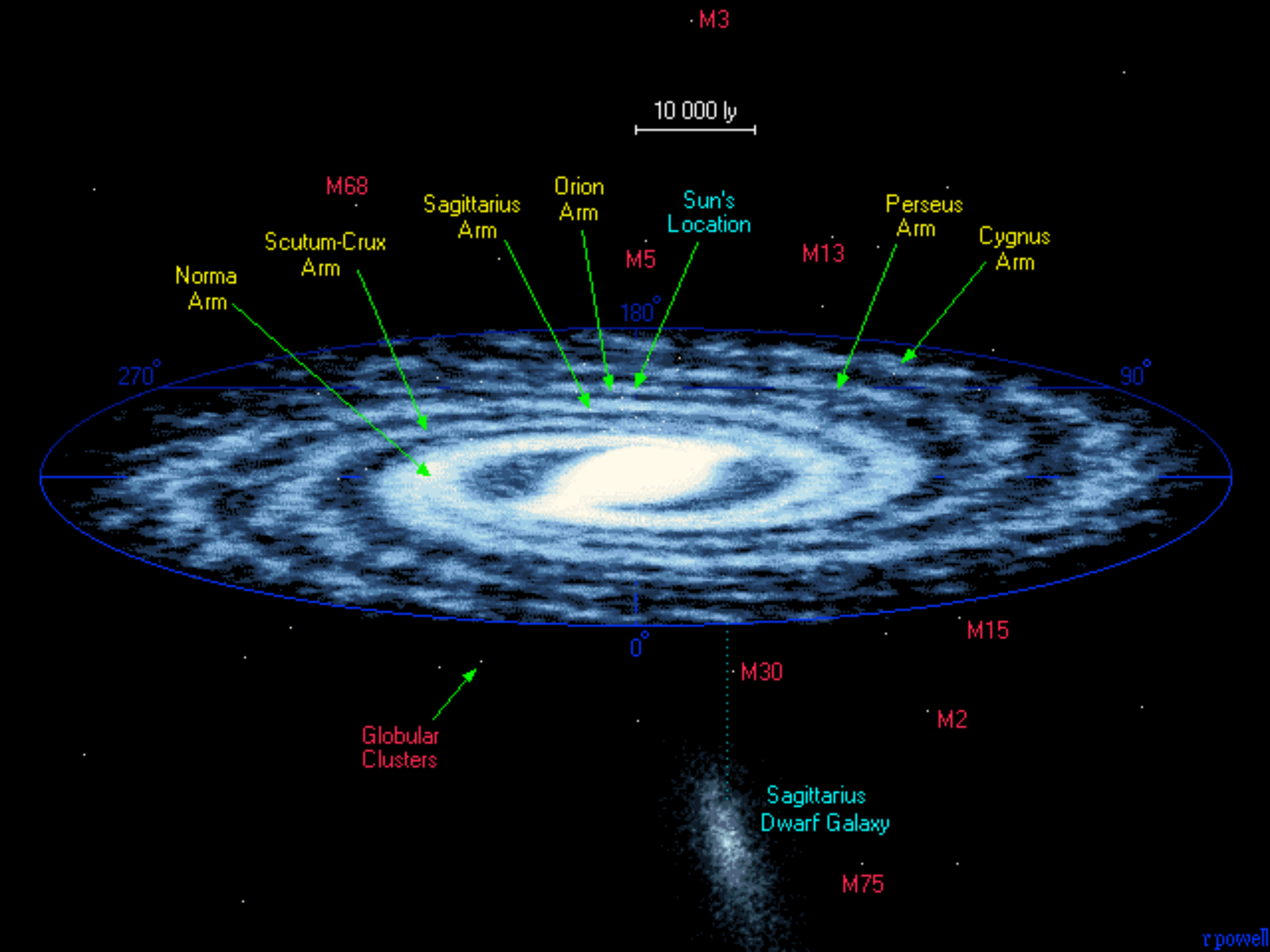
Mass: 1 $M_{sun} \approx 2 \times 10^{30}$ kg

"Natural" Units occasionally used.

$$c = \hbar = \mu_0 = \varepsilon_0 = 1$$

Comoving vs. proper vs. conformal coordinates.

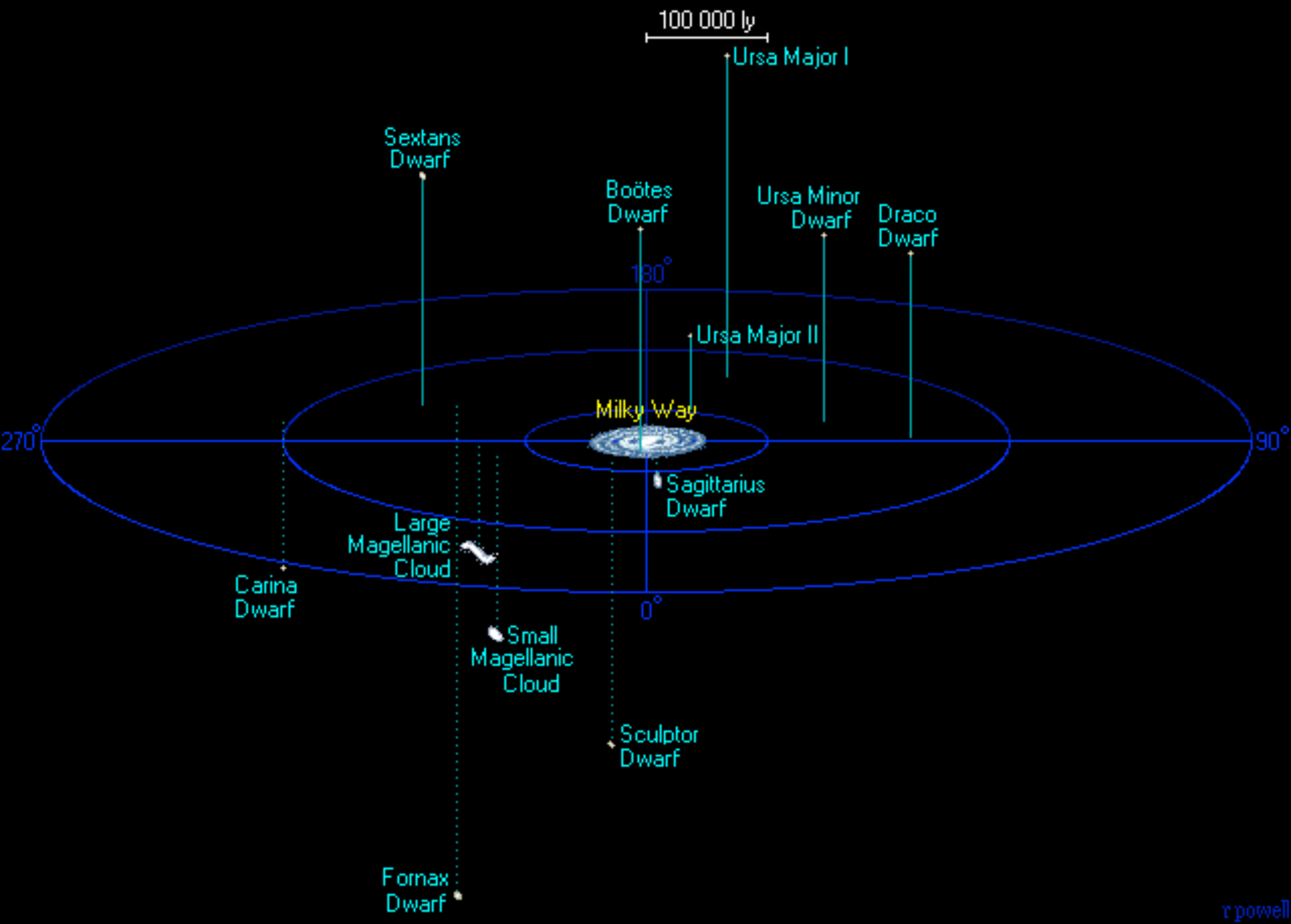
(More on this later.)


M3
10 000 ly
M68
Sagittarius
Arm
Orion
Arm
Sun's
Location
Perseus
Arm
Cygnus
Arm
Scutum-Crux
Arm
Norma
Arm
M5
M13
180°
270°
90°
0°
M15
M30
M2
Globular
Clusters
Sagittarius
Dwarf Galaxy
M75
r powell

“Island Universes”

The Great Andromeda Nebula

A galaxy, the size of our own, twenty-billion-billion kilometers distant.


100 000 ly
Ursa Major I
Sextans Dwarf
Boötes Dwarf
Ursa Minor Dwarf
Draco Dwarf
180°
Ursa Major II
Milky Way
270°
90°
Sagittarius Dwarf
Large Magellanic Cloud
Carina Dwarf
0°
Small Magellanic Cloud
Sculptor Dwarf
Fornax Dwarf
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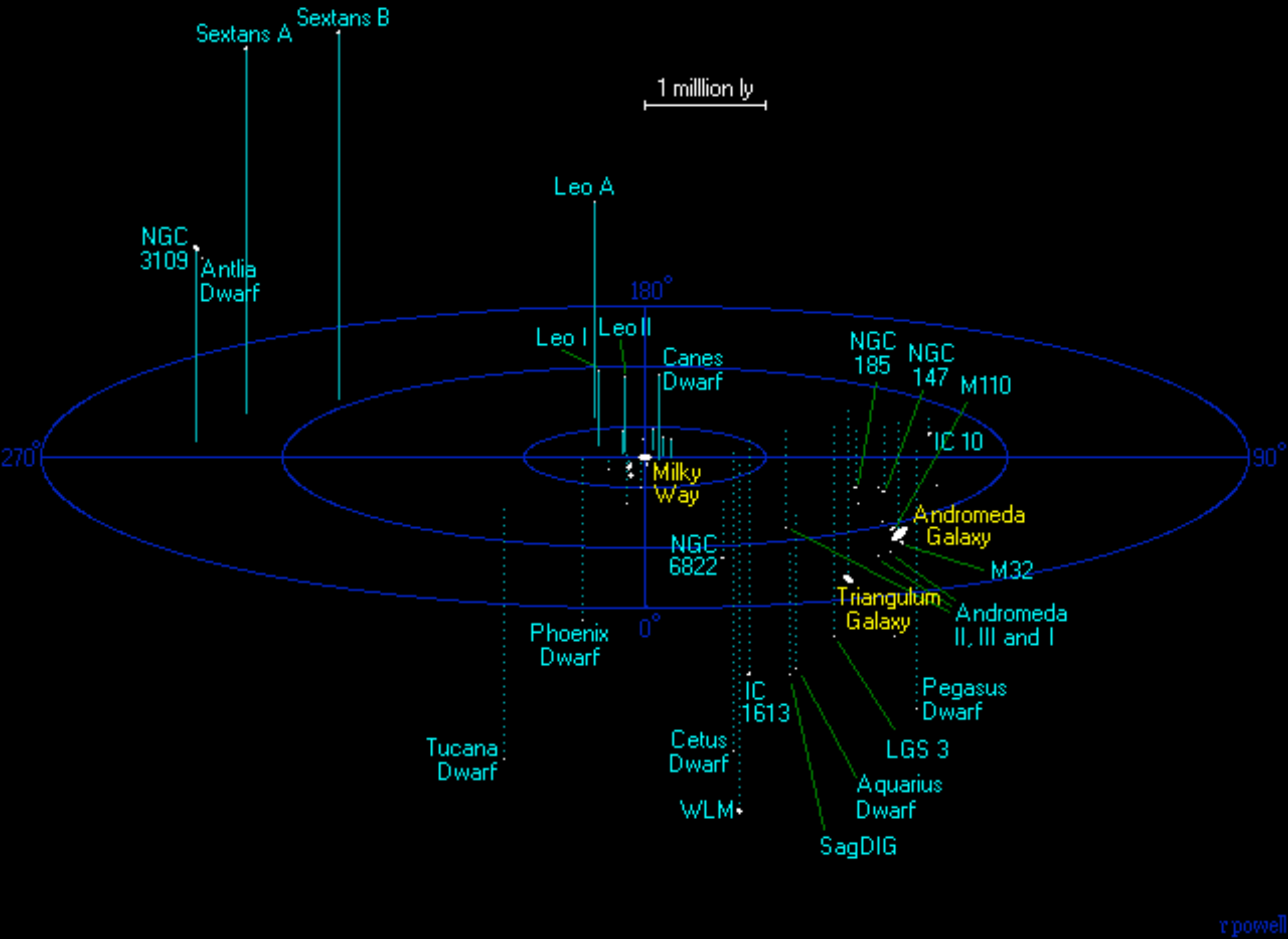
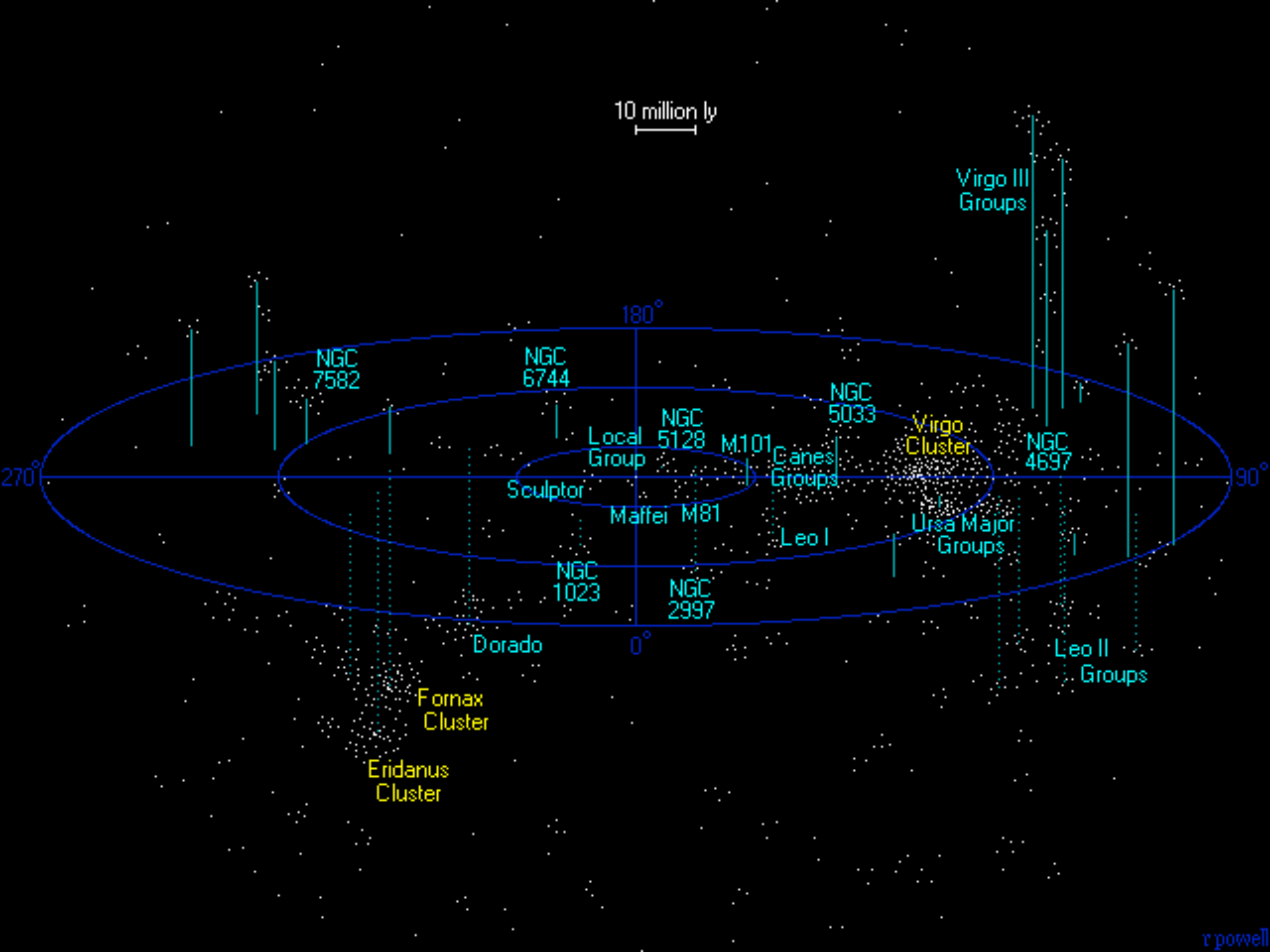
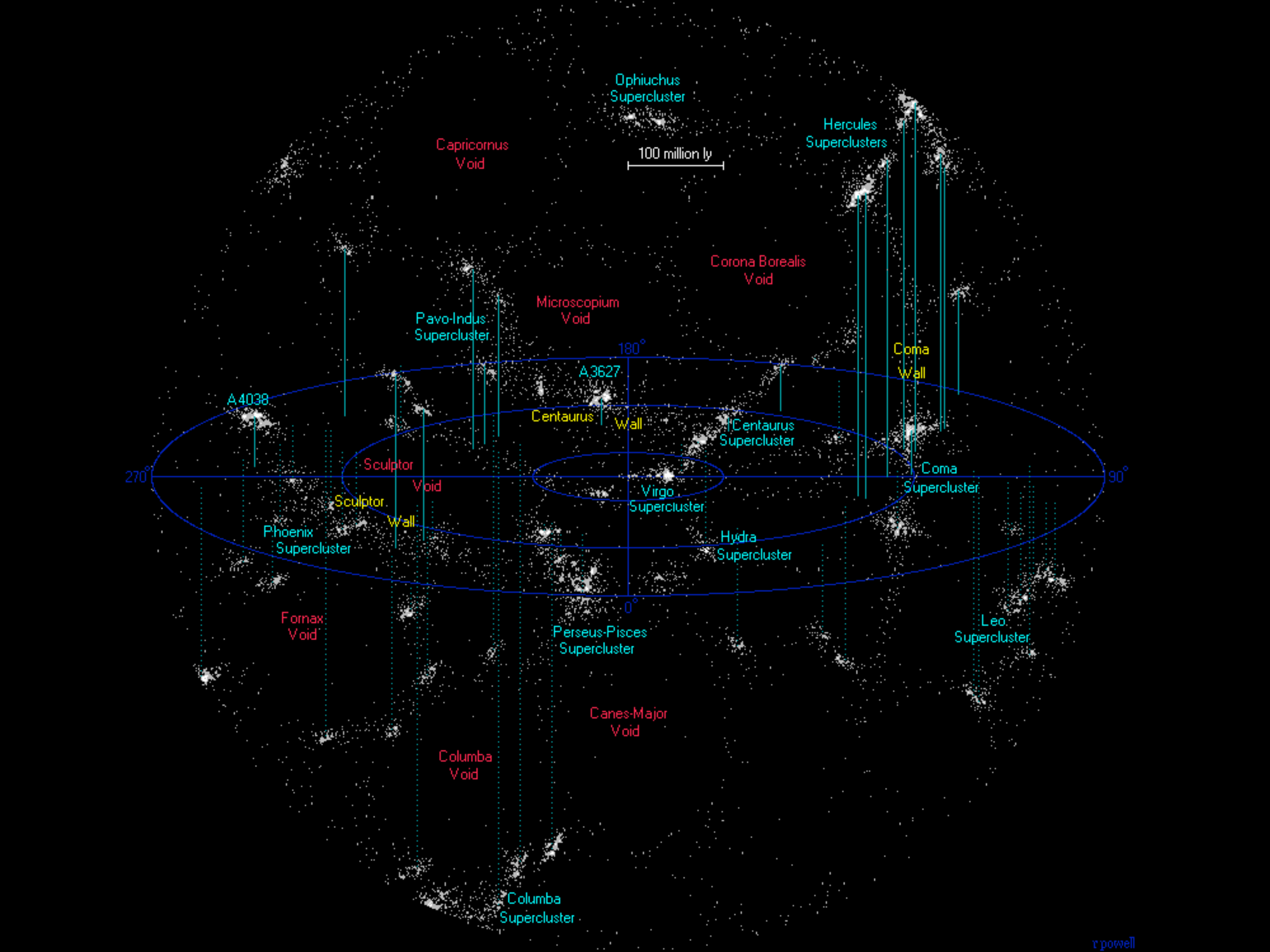


Sextans A
Sextans B
1 milllion ly
Leo A
NGC 3109
Antlia Dwarf
180°
Leo I
Leo II
Canes Dwarf
NGC 185
NGC 147
M110
IC 10
270°
90°
Milky Way
Andromeda Galaxy
NGC 6822
M32
Triangulum Galaxy
Andromeda II, III and I
Phoenix Dwarf
0°
IC 1613
Pegasus Dwarf
Tucana Dwarf
Cetus Dwarf
LGS 3
Aquarius Dwarf
WLM
SagDIG
r powell


10 million ly
Virgo III Groups
180°
NGC 7582
NGC 6744
NGC 5033
NGC 5128
Local Group
M101
Virgo Cluster
NGC 4697
Canes Groups
270°
90°
Sculptor
Maffei
M81
Leo I
Ursa Major Groups
NGC 1023
NGC 2997
Dorado
0°
Leo II Groups
Fornax Cluster
Eridanus Cluster
r powell


Ophiuchus Supercluster
Hercules Superclusters
Capricornus Void
100 million ly
Corona Borealis Void
Microscopium Void
Pavo-Indus Supercluster
180°
Coma Wall
A3627
A4038
Centaurus Wall
Centaurus Supercluster
Sculptor Void
270°
90°
Coma Supercluster
Sculptor Wall
Virgo Supercluster
Phoenix Supercluster
Hydra Supercluster
0°
Fornax Void
Perseus-Pisces Supercluster
Leo Supercluster
Canes-Major Void
Columba Void
Columba Supercluster
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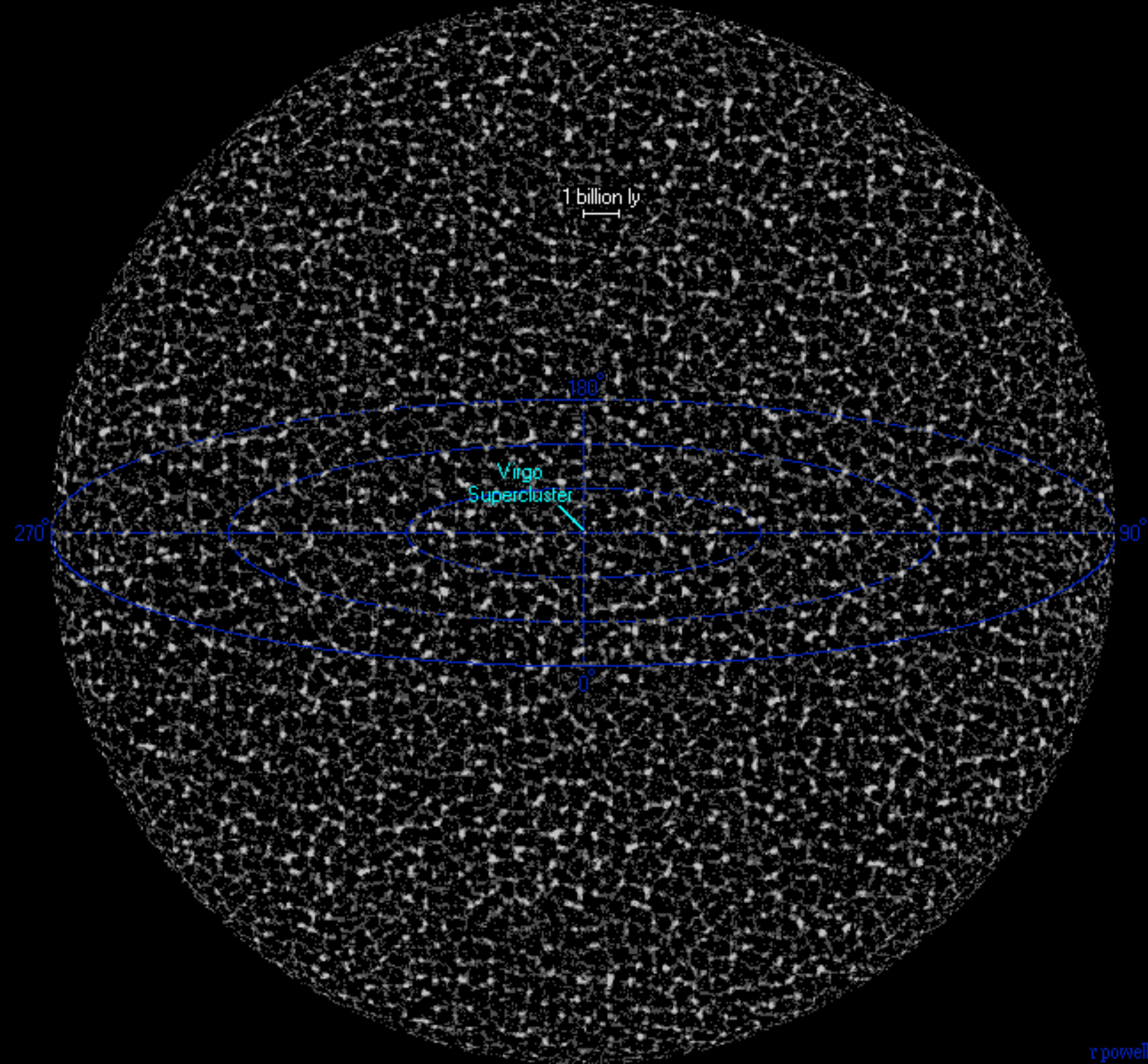


1 billion ly
180°
Virgo
Supercluster
270°
90°
0°
r powell

Cosmological Principle

You're not special.

(There are no [spatially] privileged observers.)

Implies:

1) Homogeneity
2) Isotropy

"Perfect Cosmological Principle" claimed temporal uniformity – a static universe. Observations forced us to discard this.

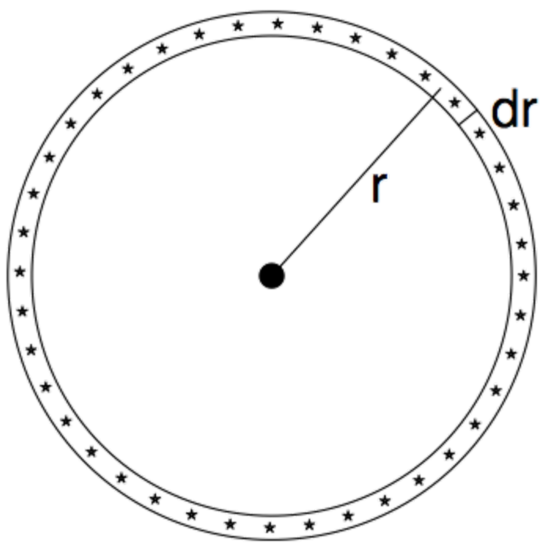
Early Observations

1. People exist.

2. The (night) sky is dark.

The Classical Universe

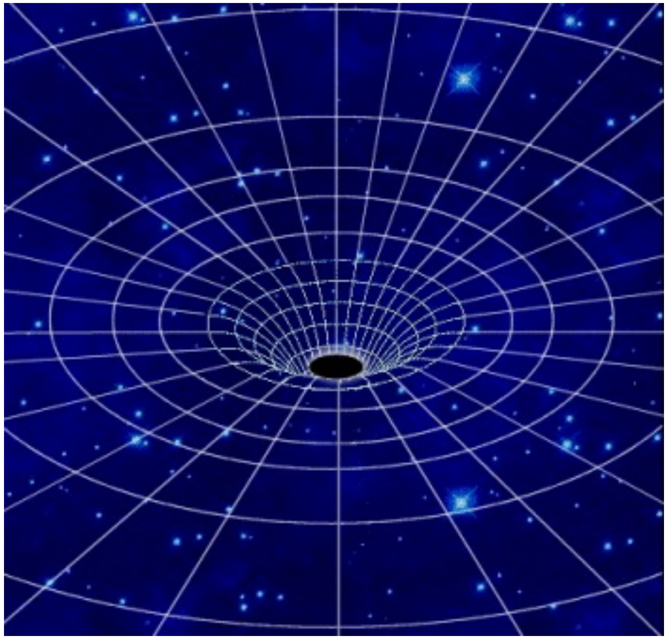
- Historically, static Universe was preferred.
- Newton/Bentley: a static Universe requires an infinite Universe
- Olber's paradox: The Sky is Dark.



$$dJ(r) = \frac{L}{4\pi r^2} \cdot n \cdot r^2 dr = \frac{nL}{4\pi} dr$$

$$J = \int_{r=0}^{\infty} dJ = \frac{nL}{4\pi} \int_0^{\infty} dr = \infty$$

General Relativity (1915)



- Newton: gravity balanced against other gravity.
- Olber: dark, so not infinite…
- Newton can’t explain large scale dynamics.
- Einstein: General Relativity!
- Spacetime bends, twists, stretches and distorts.
- Any geometry is possible, and space can grow and shrink.
- But: Gravitationally Unstable.

The Cosmological Constant (1917)

Einstein’s Greatest Blunder

(Only it turned out to be correct)

- Einstein thought the Universe should be static and eternal.

- Added Λ, the “Cosmological Constant”.

- Λ counteracted gravity, pushing things apart just enough to prevent collapse.

- Λ was otherwise unmotivated, and Einstein later called it his “greatest blunder”.

(Einstein didn’t live long enough to find out that it was actually right, or that he was completely wrong about quantum mechanics.)

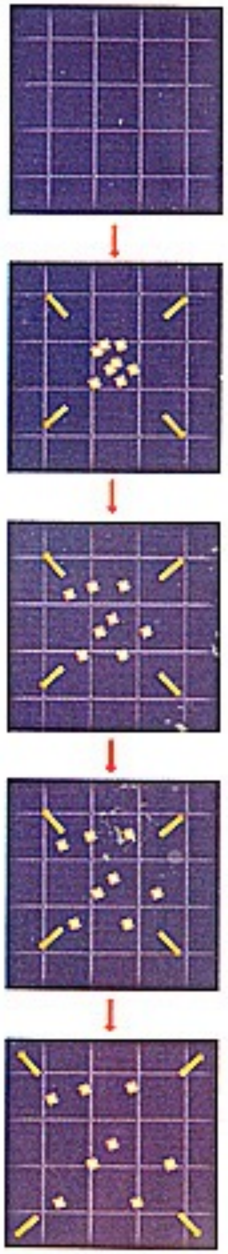
The [Friedmann-Lemaitre-] Robertson-Walker Metric

- Independently derived by all 4 people.
- Provides the general metric for all homogenous, isotropic spacetimes

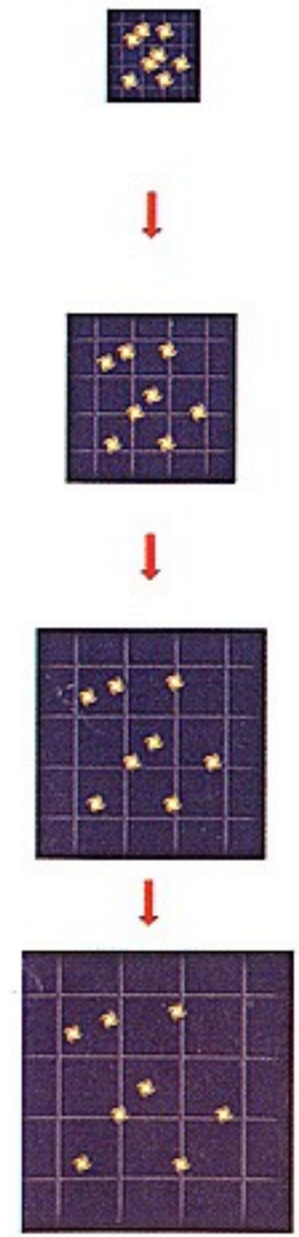
$$ds^2 = -c^2dt^2 + \underbrace{a(t)^2}_{\text{Scale Factor}}\left[dr^2 + \underbrace{S_\kappa(r)^2}_{\text{Curvature } (\approx 0)}d\Omega^2\right]$$

The Scale Factor

Classical Expansion



Scale Factor



In relativity, space and time are not fixed. They are free to stretch and twist, distort and compress.

For a uniform system, only the scale and overall geometry need be to considered.

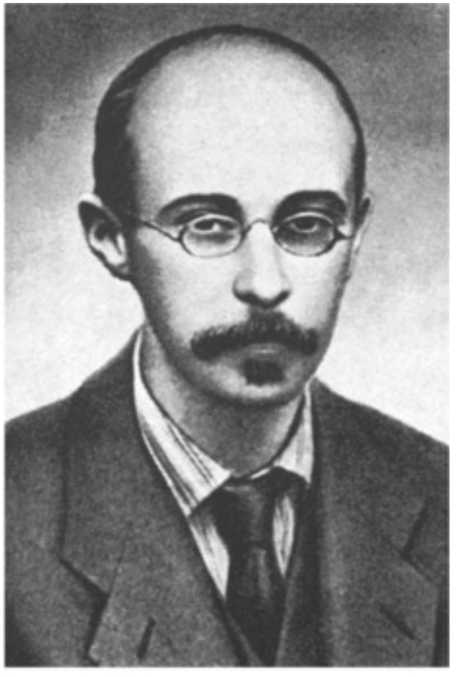
The Scale Factor is a measure of the relative size of the universe, a measure of the size of space itself.

Friedmann's Equations

- Alexander Friedmann took GR, assumed isotropy and homogeneity.

- This produced a simpler set of equations for describing its evolution.

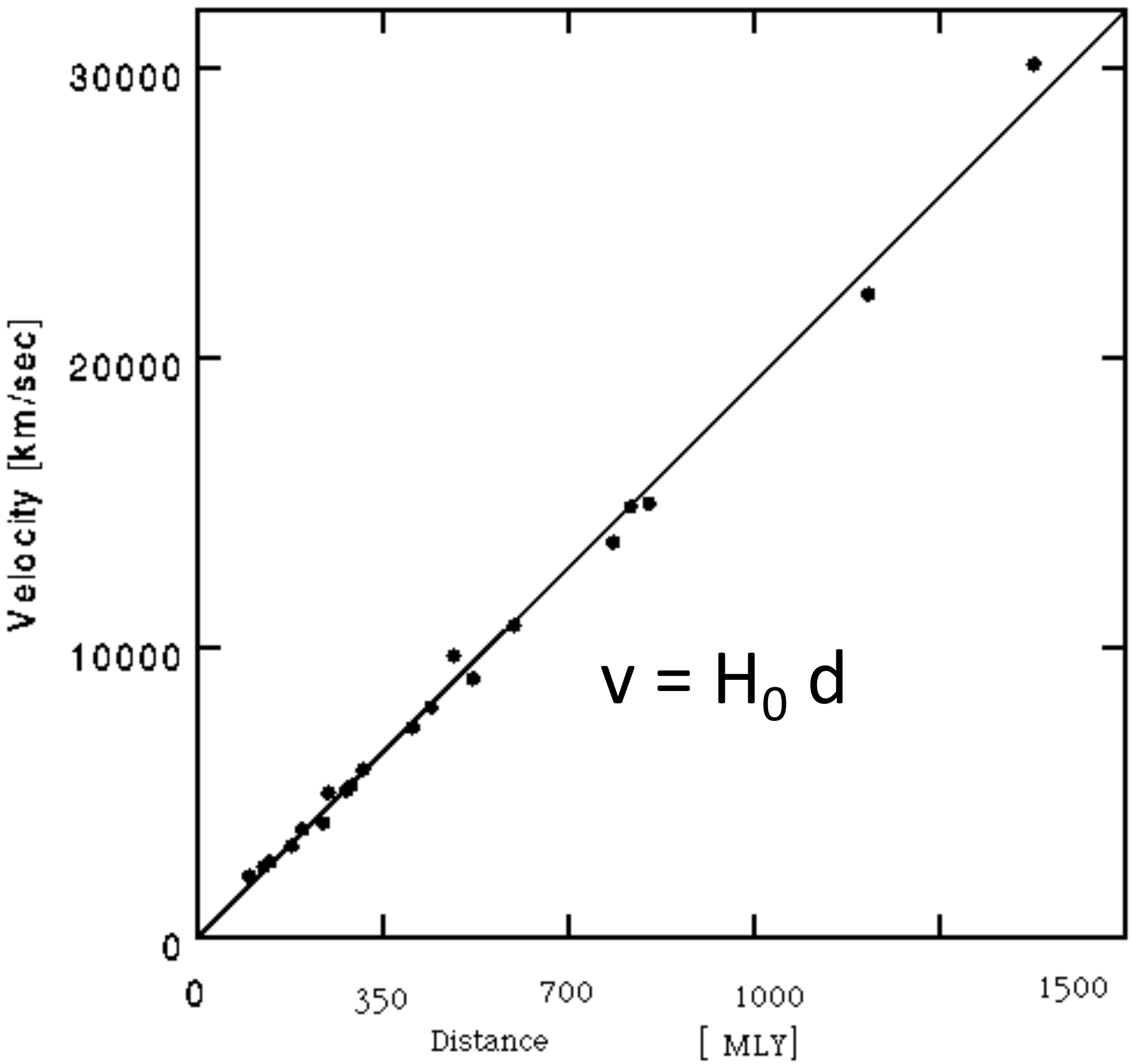
- Friedmann Equations are the basis of modern cosmology.



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

Hubble’s Diagram (1929)

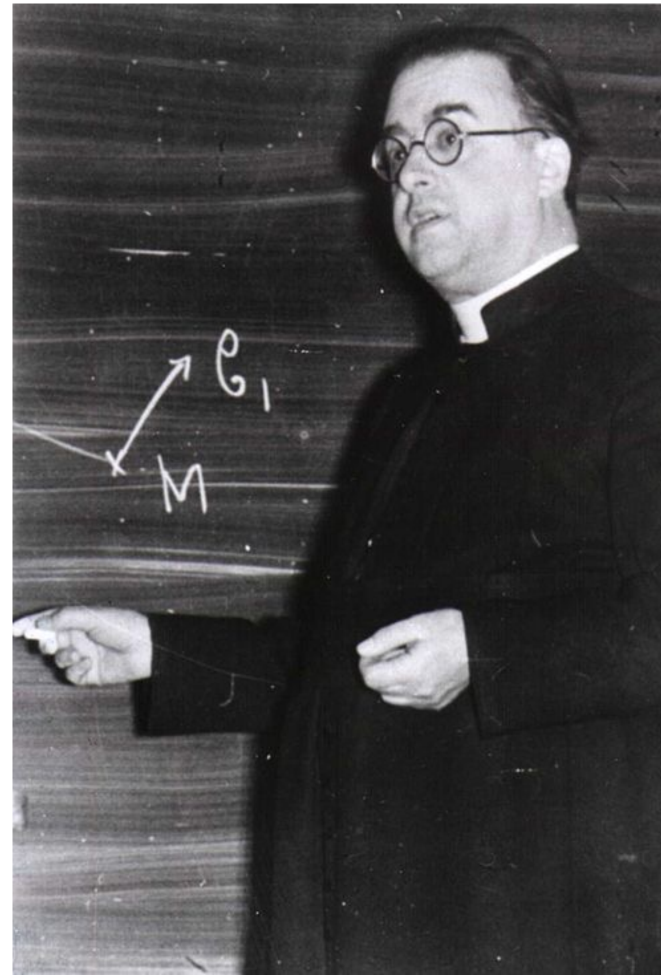


Georges Lemaître

Belgian priest who discovered & published the Hubble law 2 years before Hubble (1927), *and* explained it.

“Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extragalactiques.”

("A homogeneous Universe of constant mass and growing radius accounting for the radial velocity of extragalactic nebulae.”)



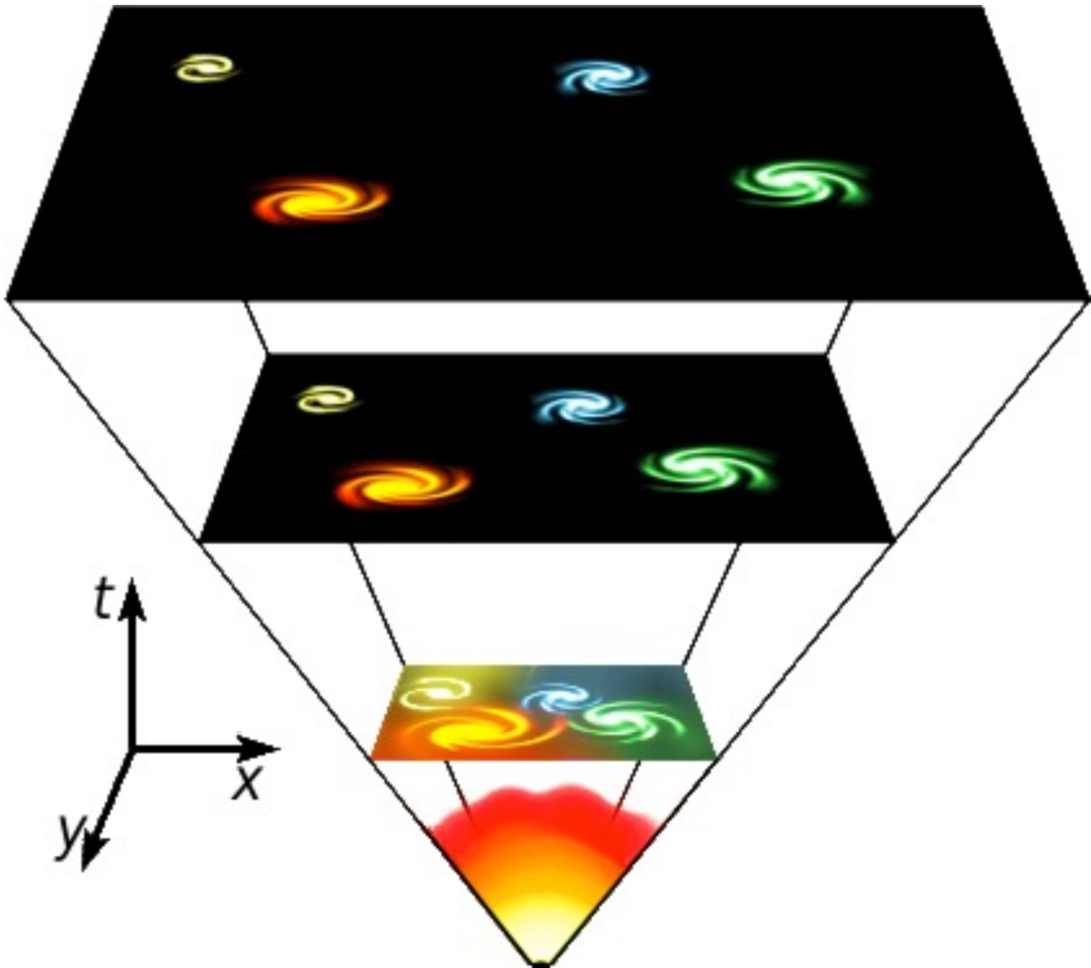
The Big Bang (1931)

Lemaître: if space is expanding, it used to be smaller.

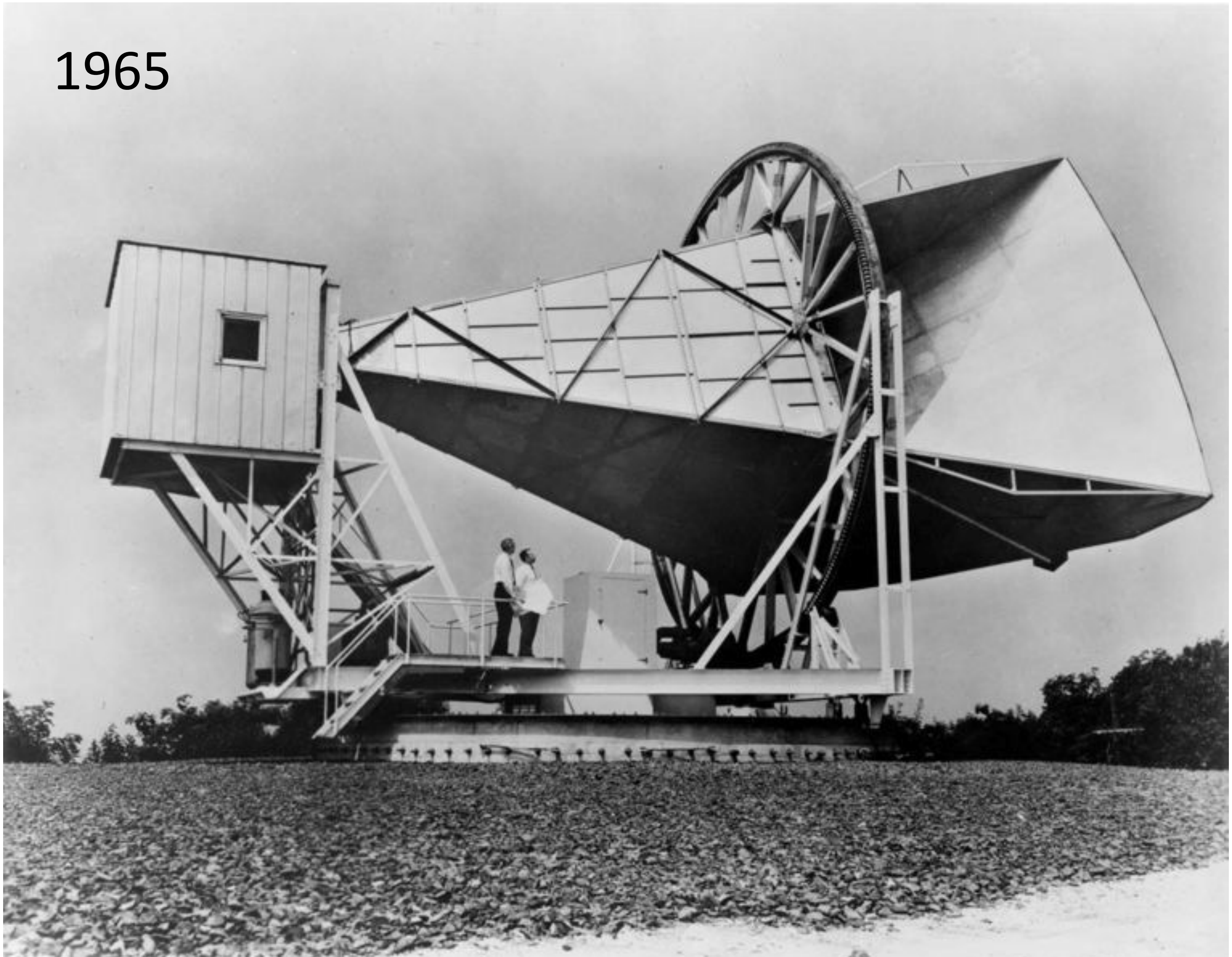
Far enough back, things would have been incredibly dense and hot.

This meant a singularity some finite time in the past, a **primeval atom**.

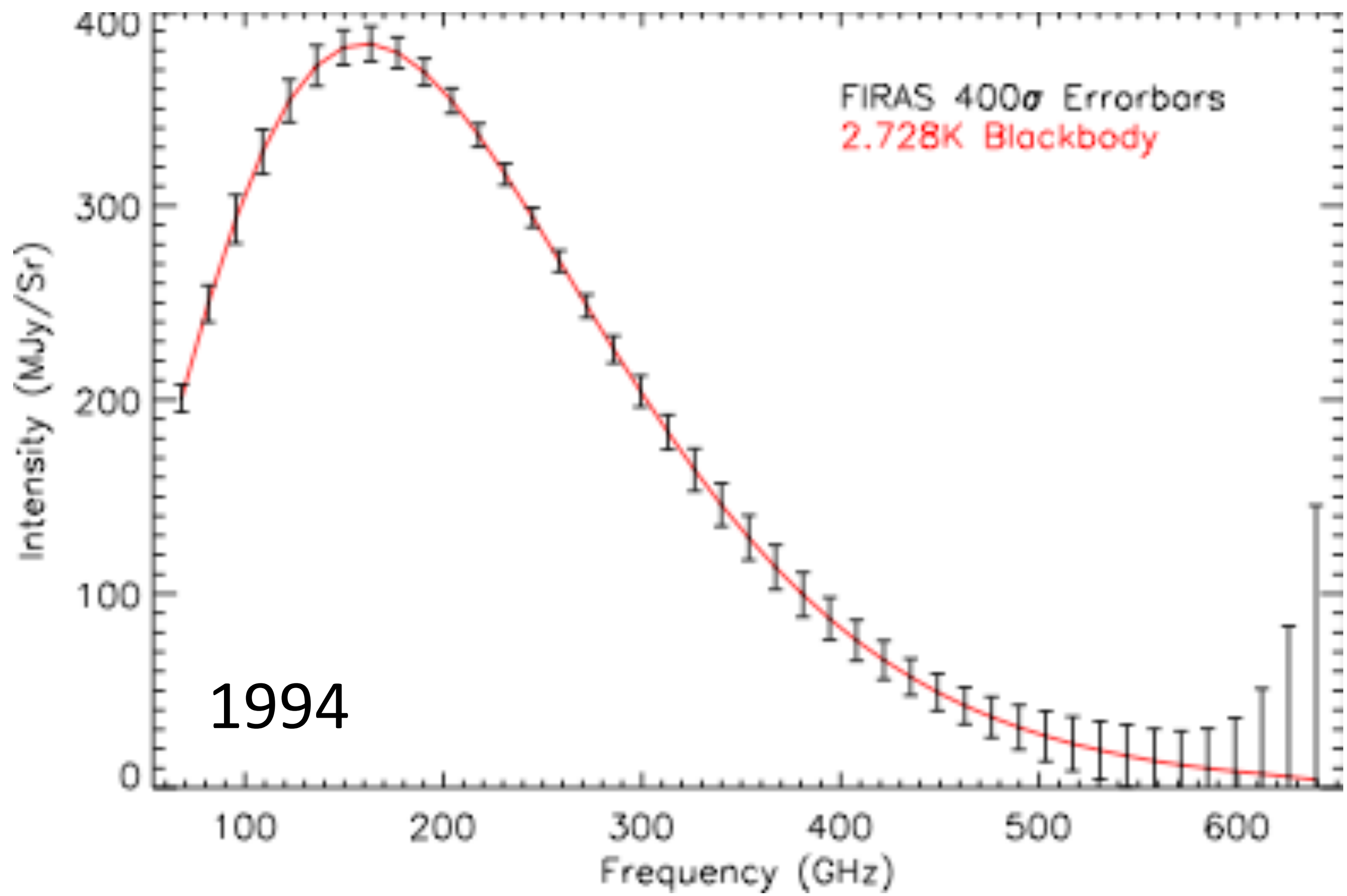
That event was mockingly dubbed the **Big Bang**.



The Cosmic Microwave Background

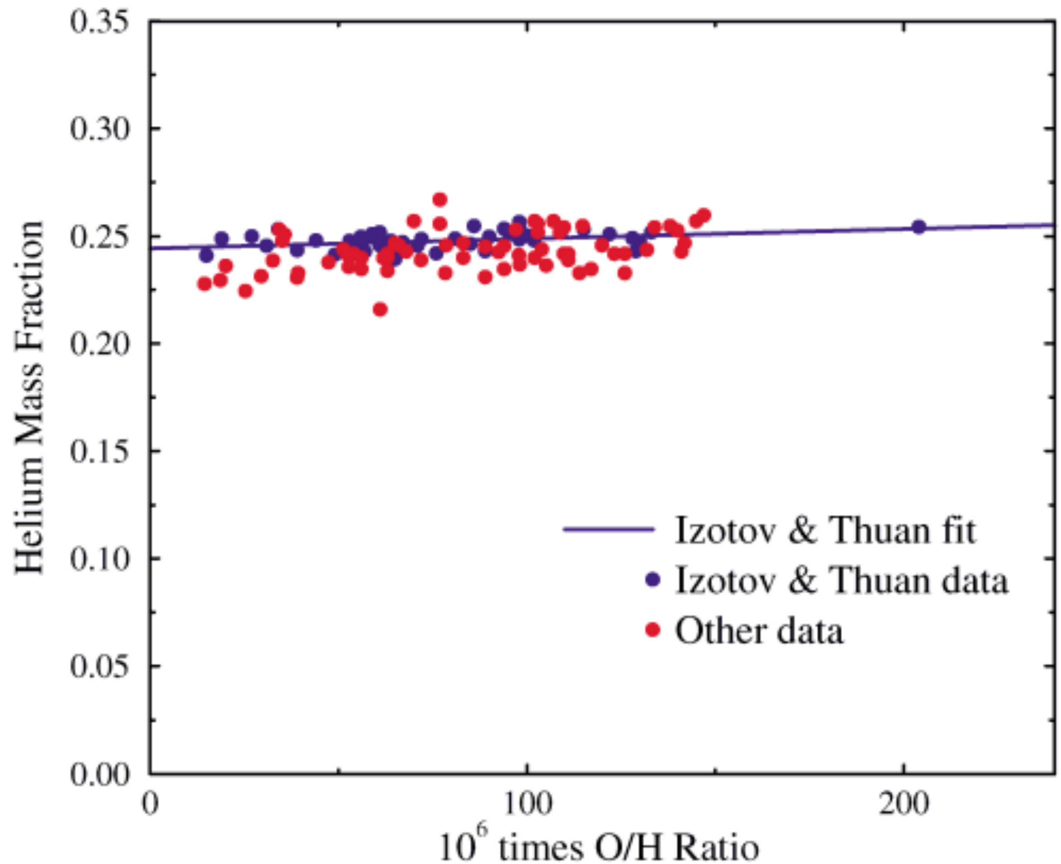


The CMB is a Perfect Blackbody

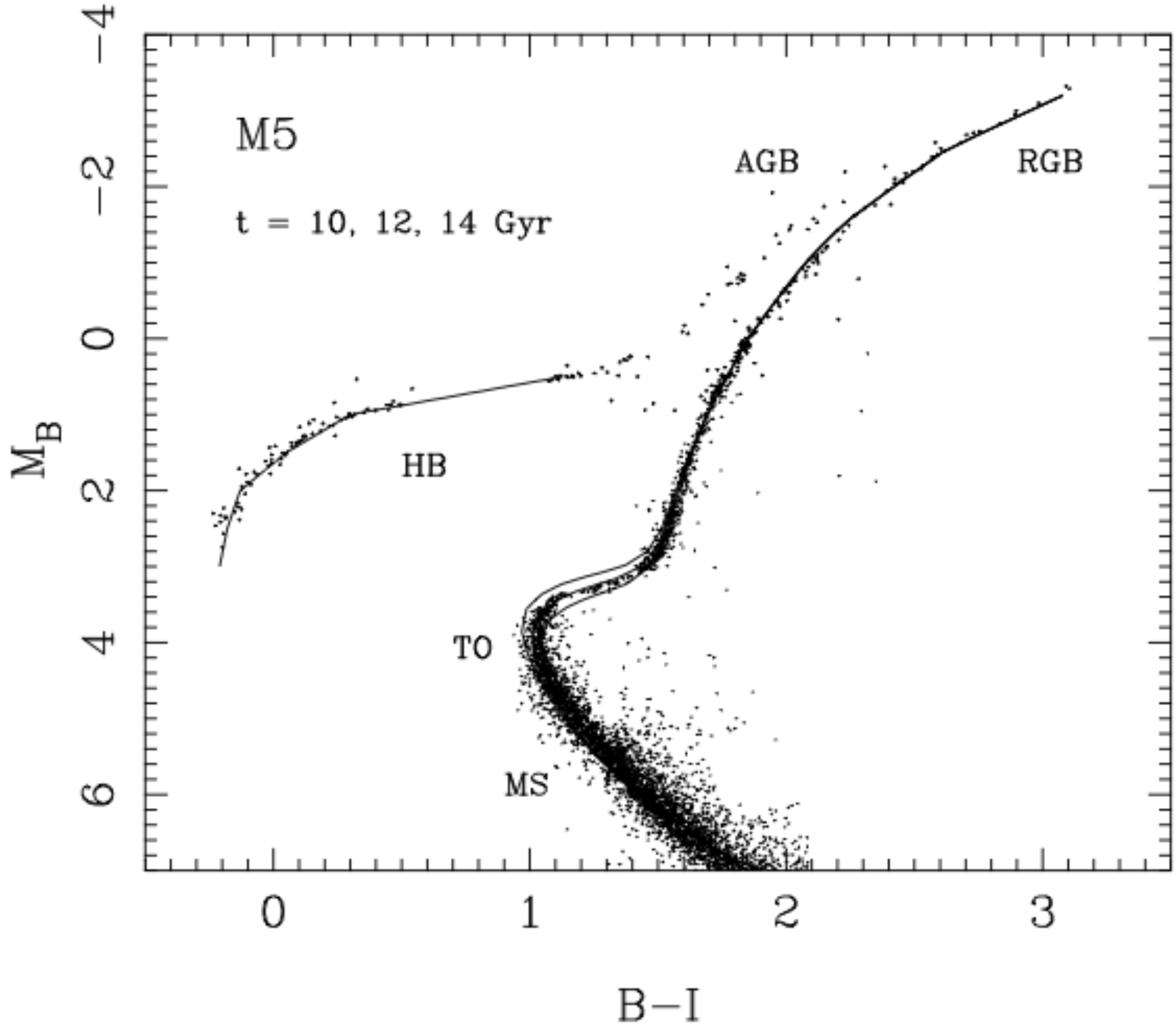


Hydrogen/Helium Mass Fraction is 3:1

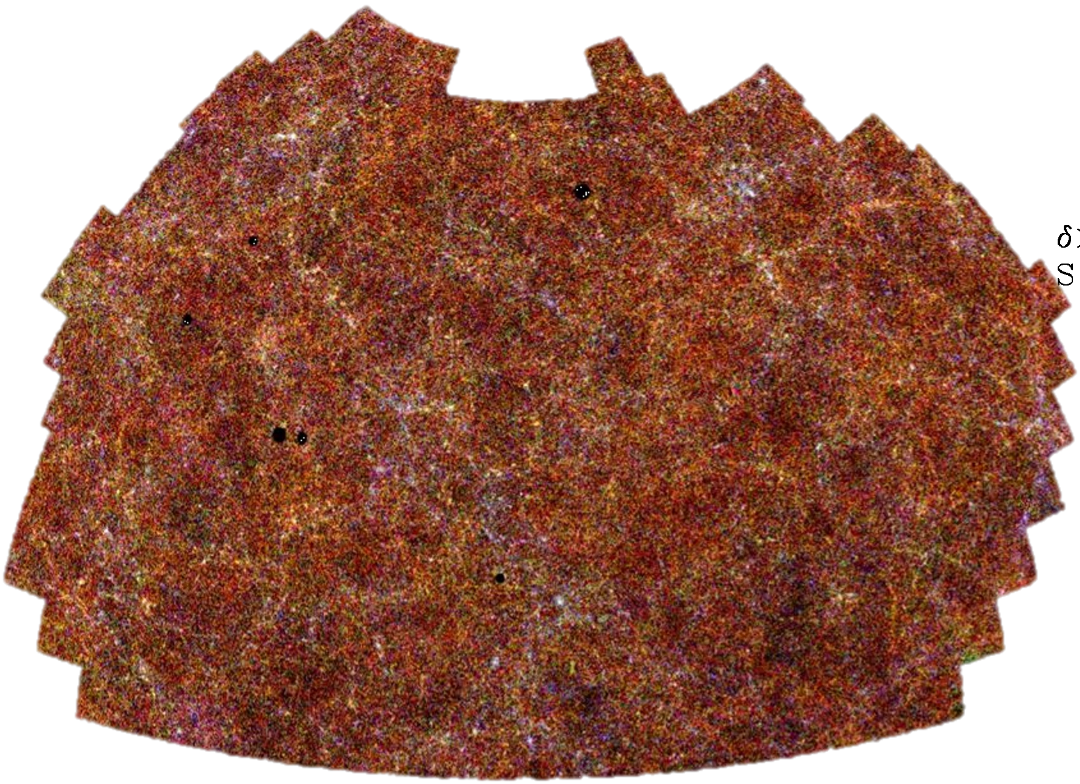
Very constant, independent of stellar age & processing.
Other elements vary by many orders of magnitude depending on environment.



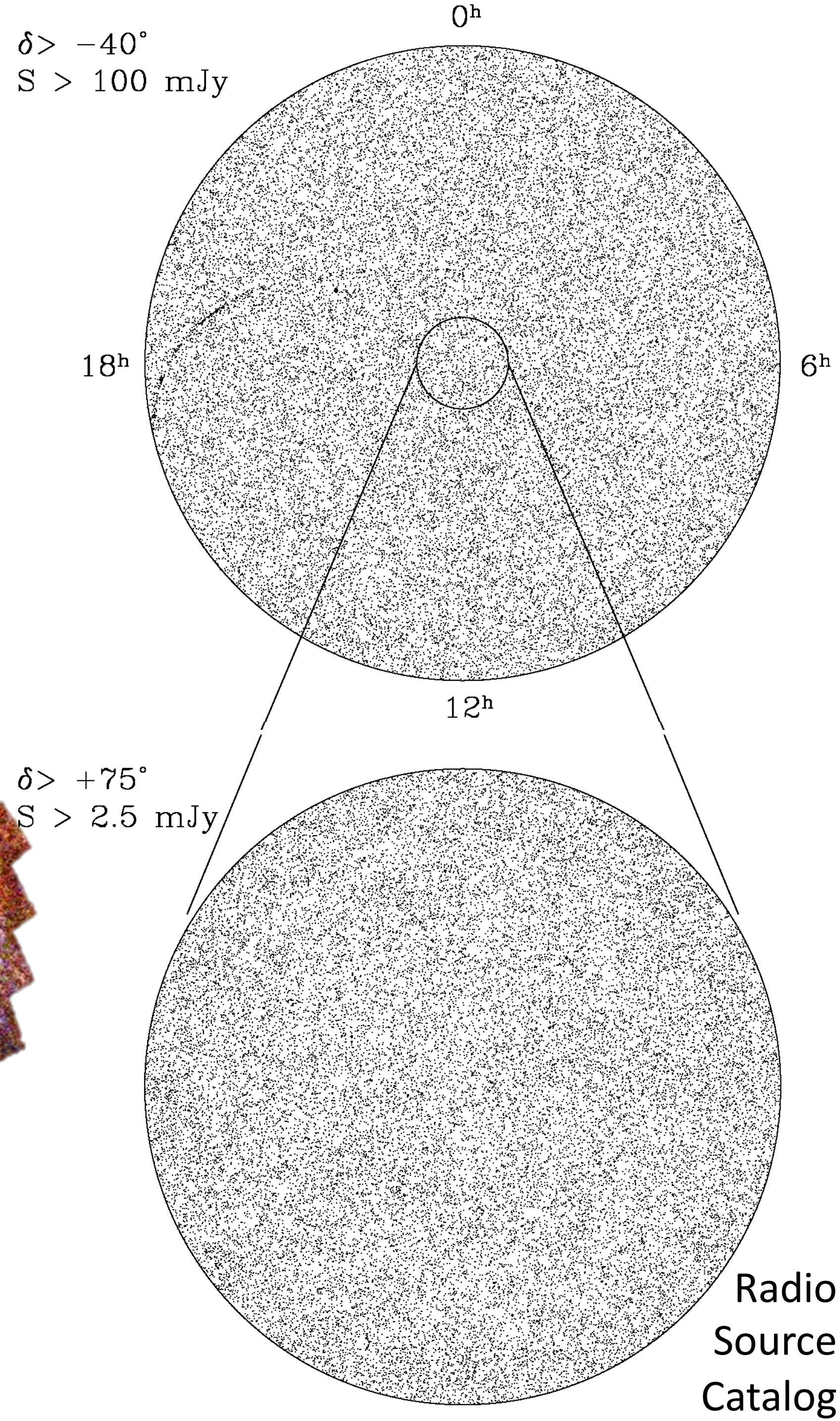
The oldest stars are ~12Gy



The Universe Really i Isotropic

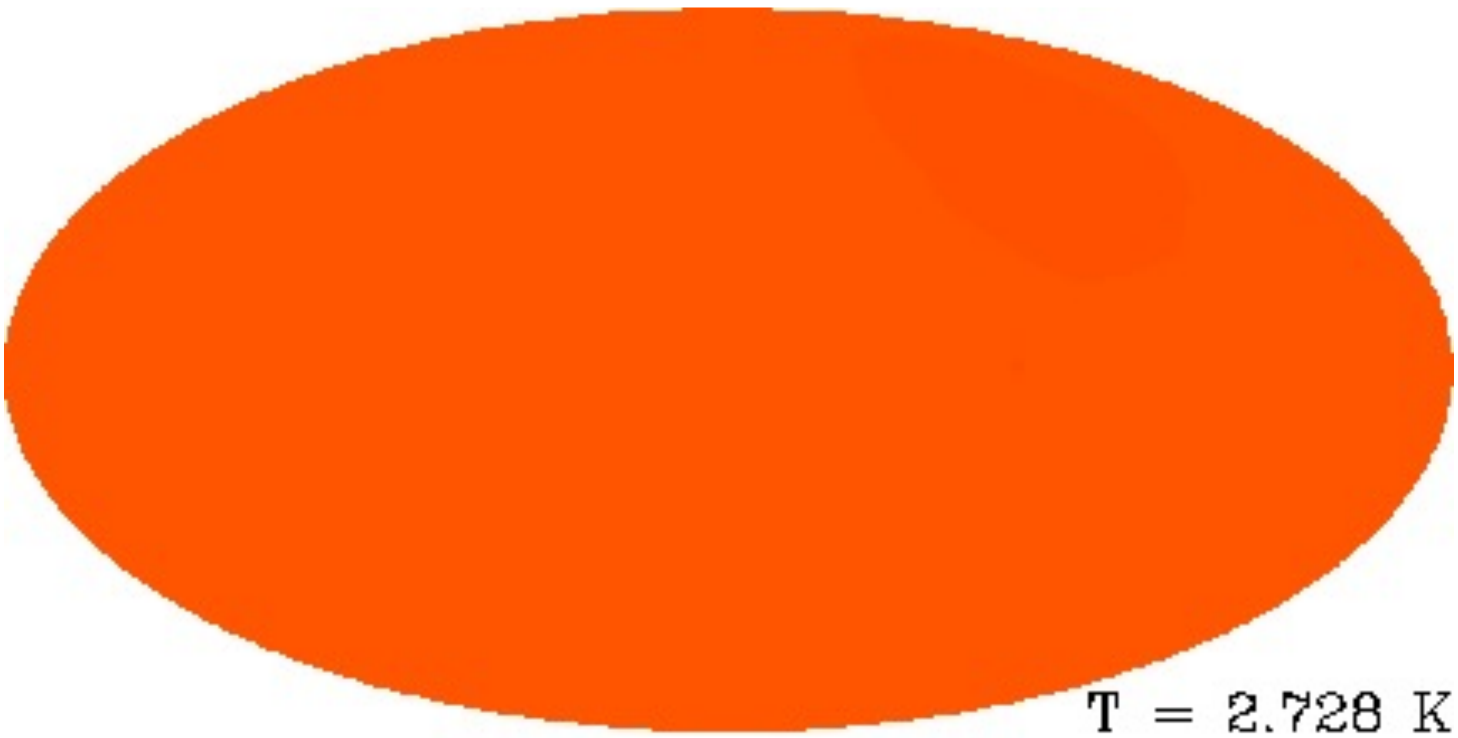


APM Galaxy Survey

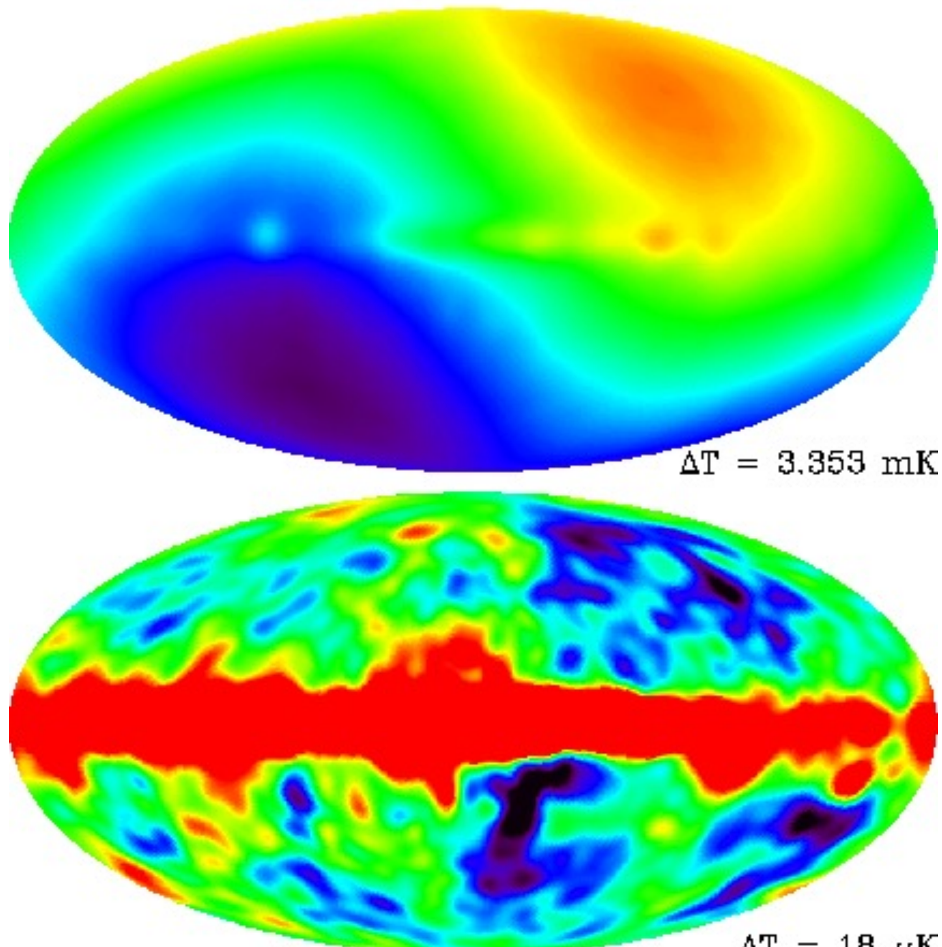


Radio Source Catalog

The CMB is Isotropic

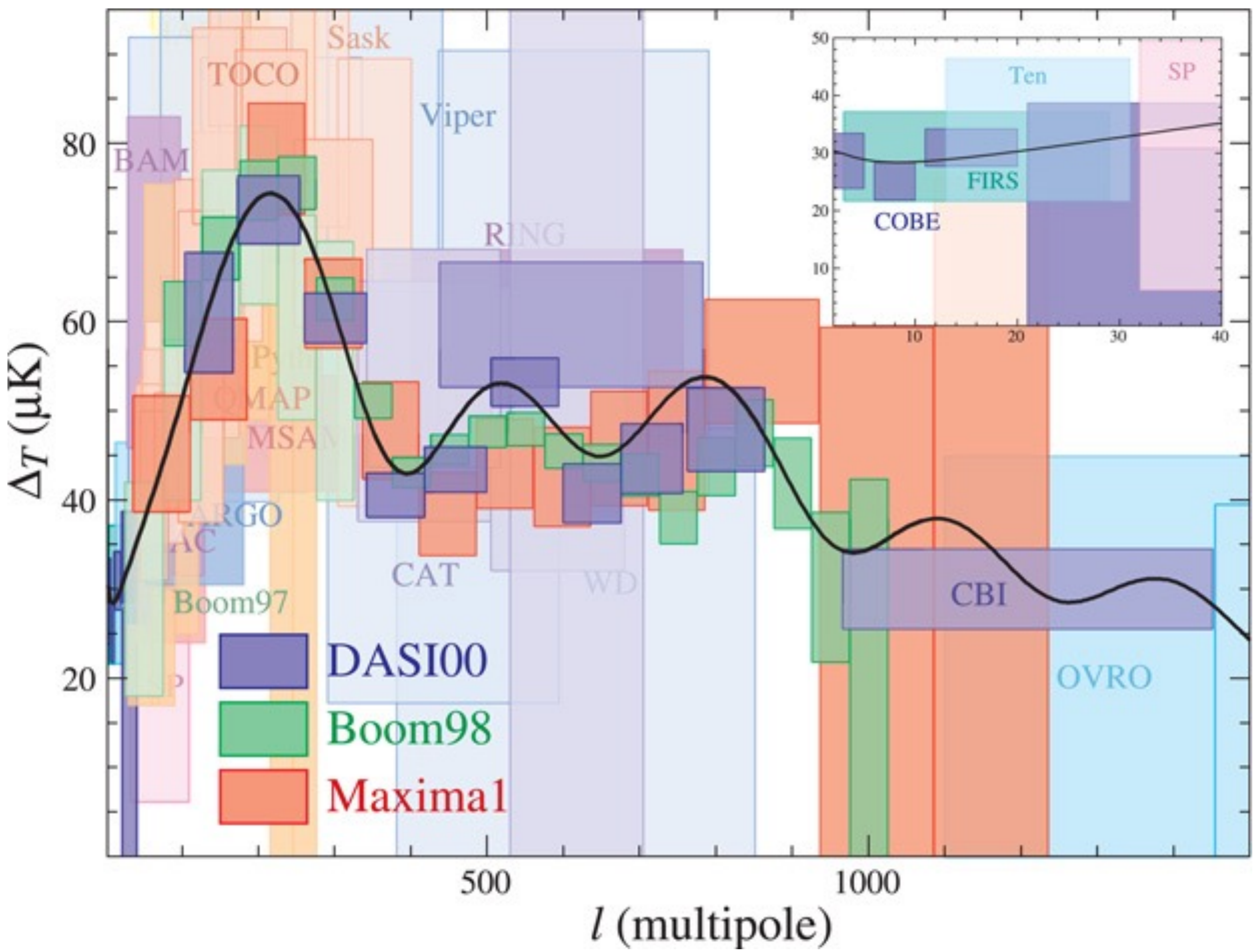


The CMB is Isotropic …almost

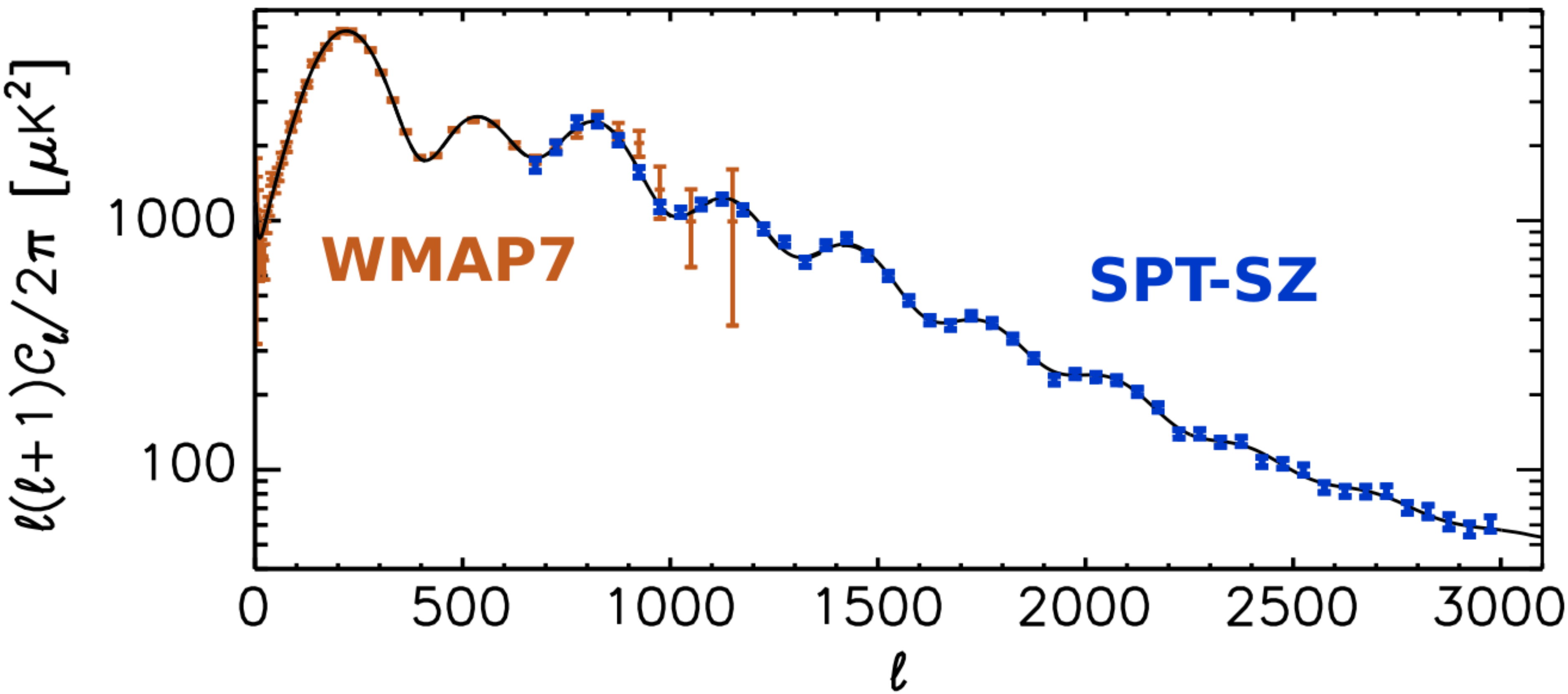


The CMB is Anisotropic

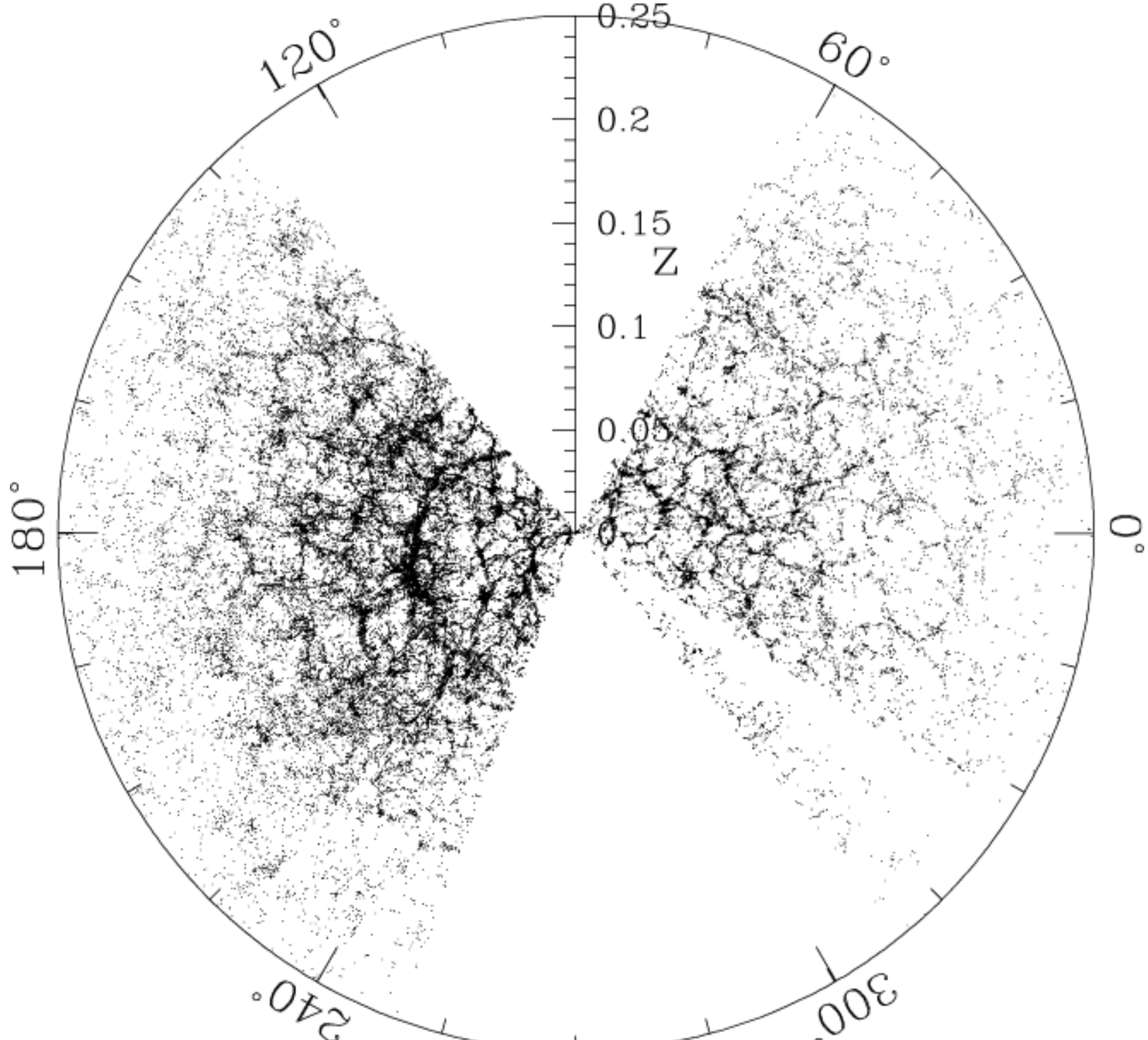
Circa 2002



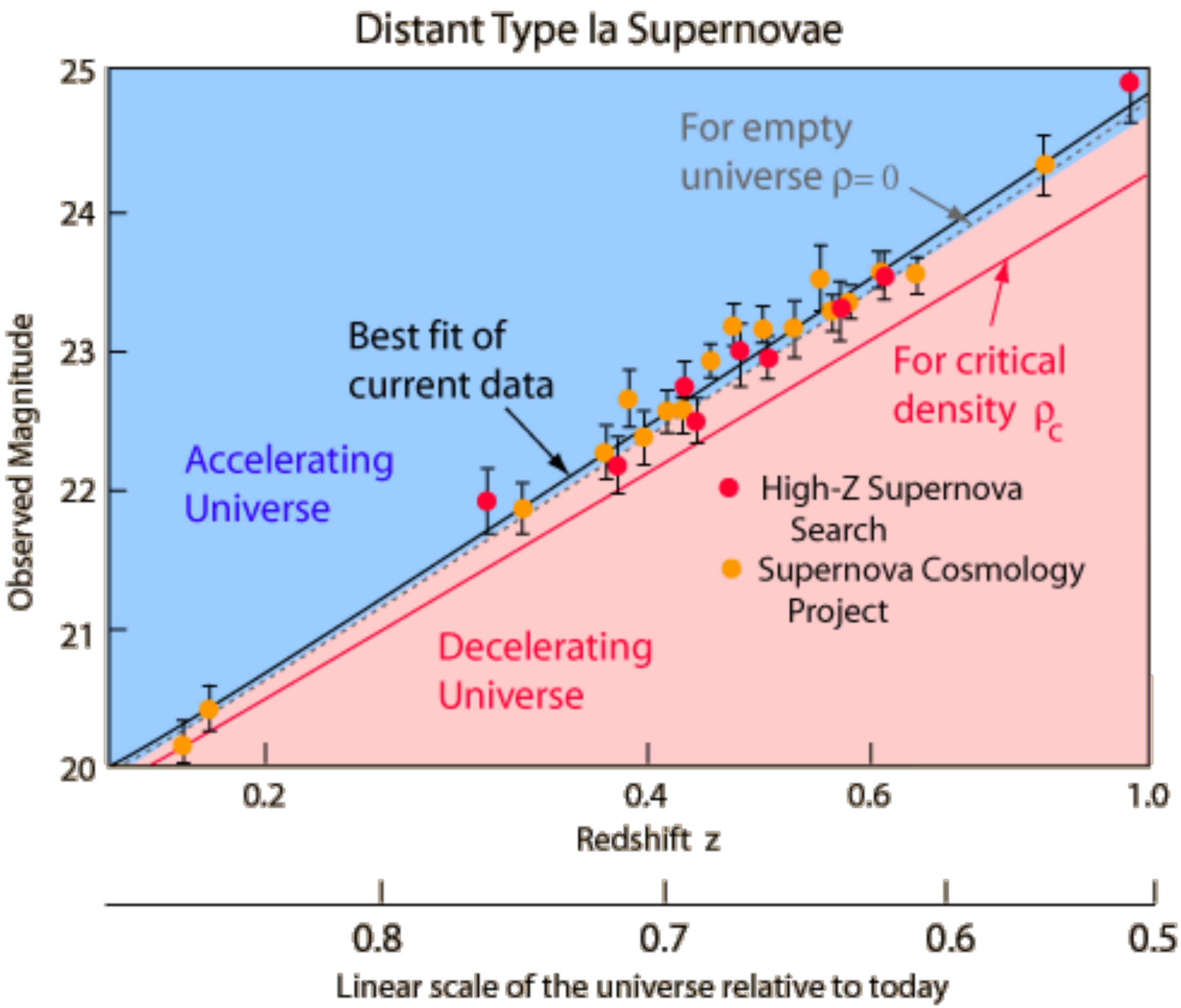
CMB Anisotropy circa 2012



The Universe Contains Large Scale Structure

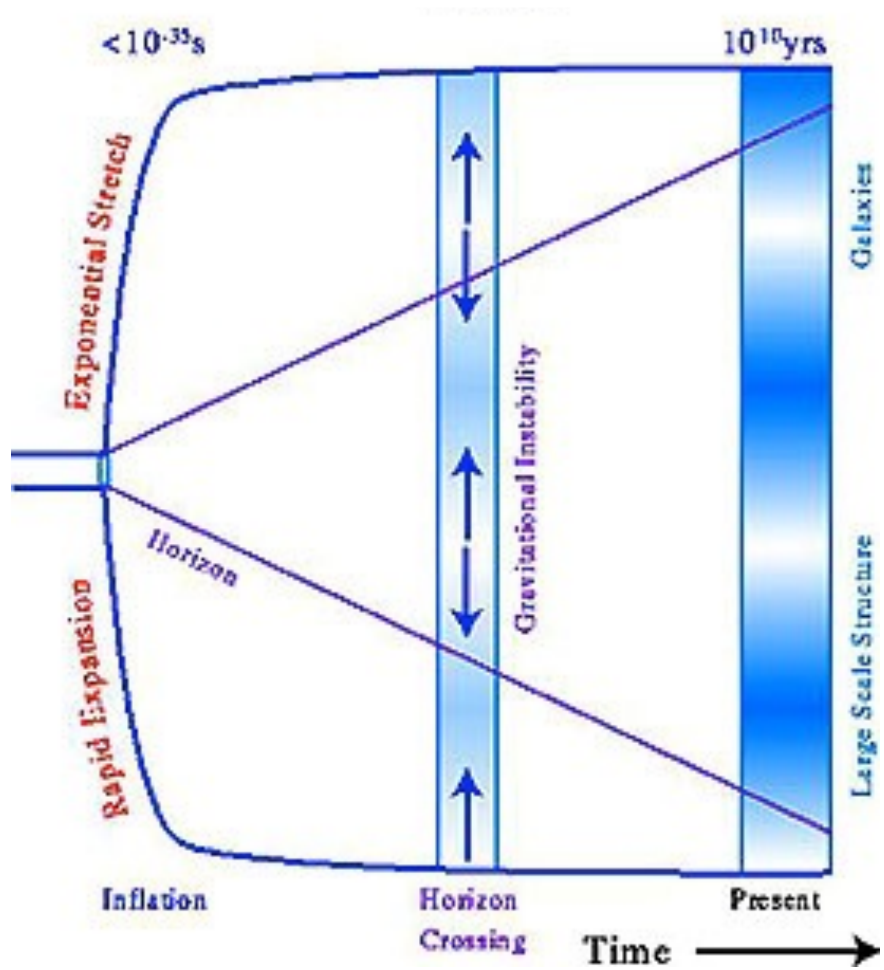


The Expansion is Accelerating

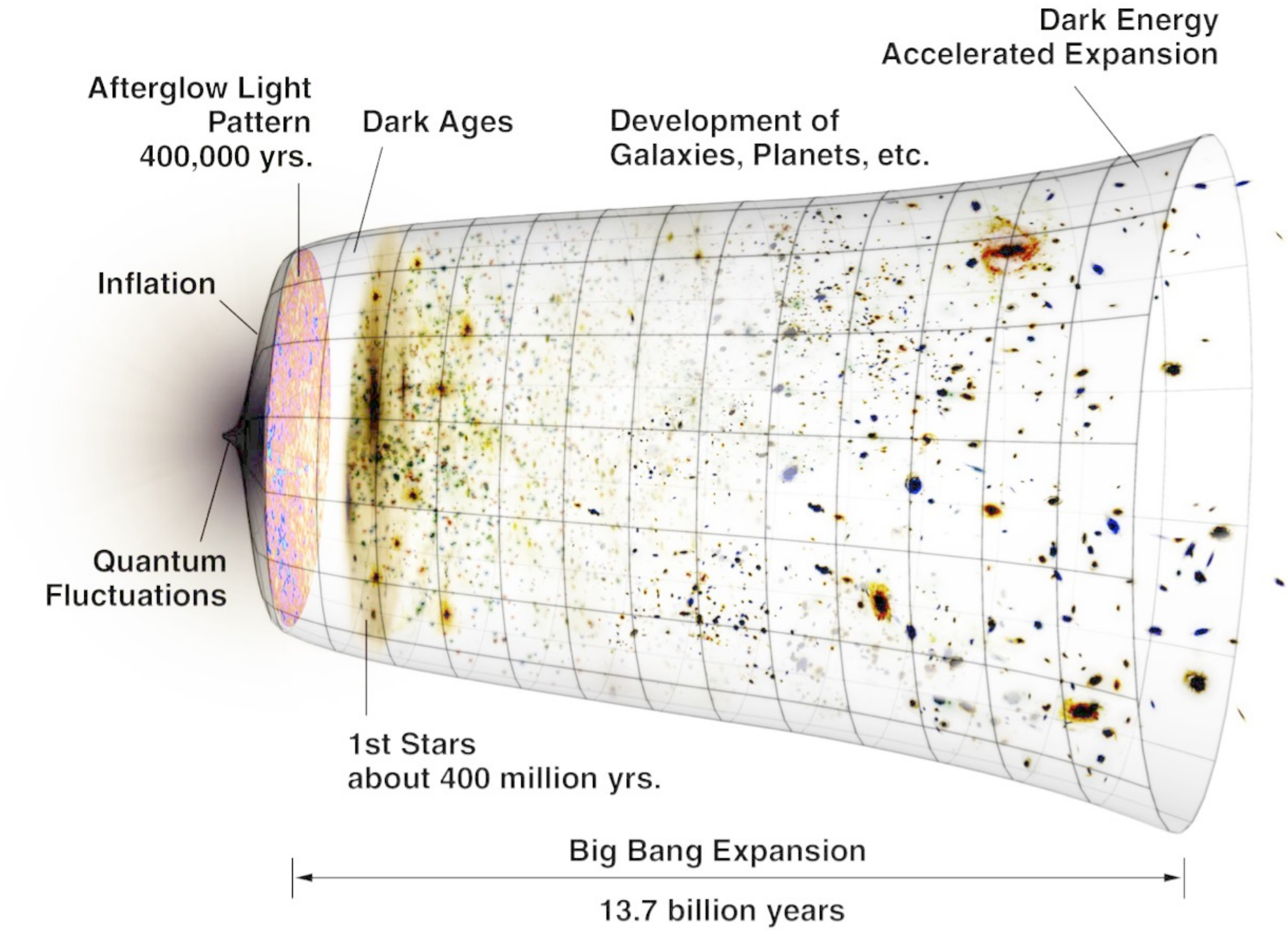


There was Probably Inflation?

- Simple idea: early on, a very very fast period of expansion.
- Solves a few major problems and explains the structure, but mostly post hoc!
- Most cosmologists take it for granted, but inflation is as yet largely untested.

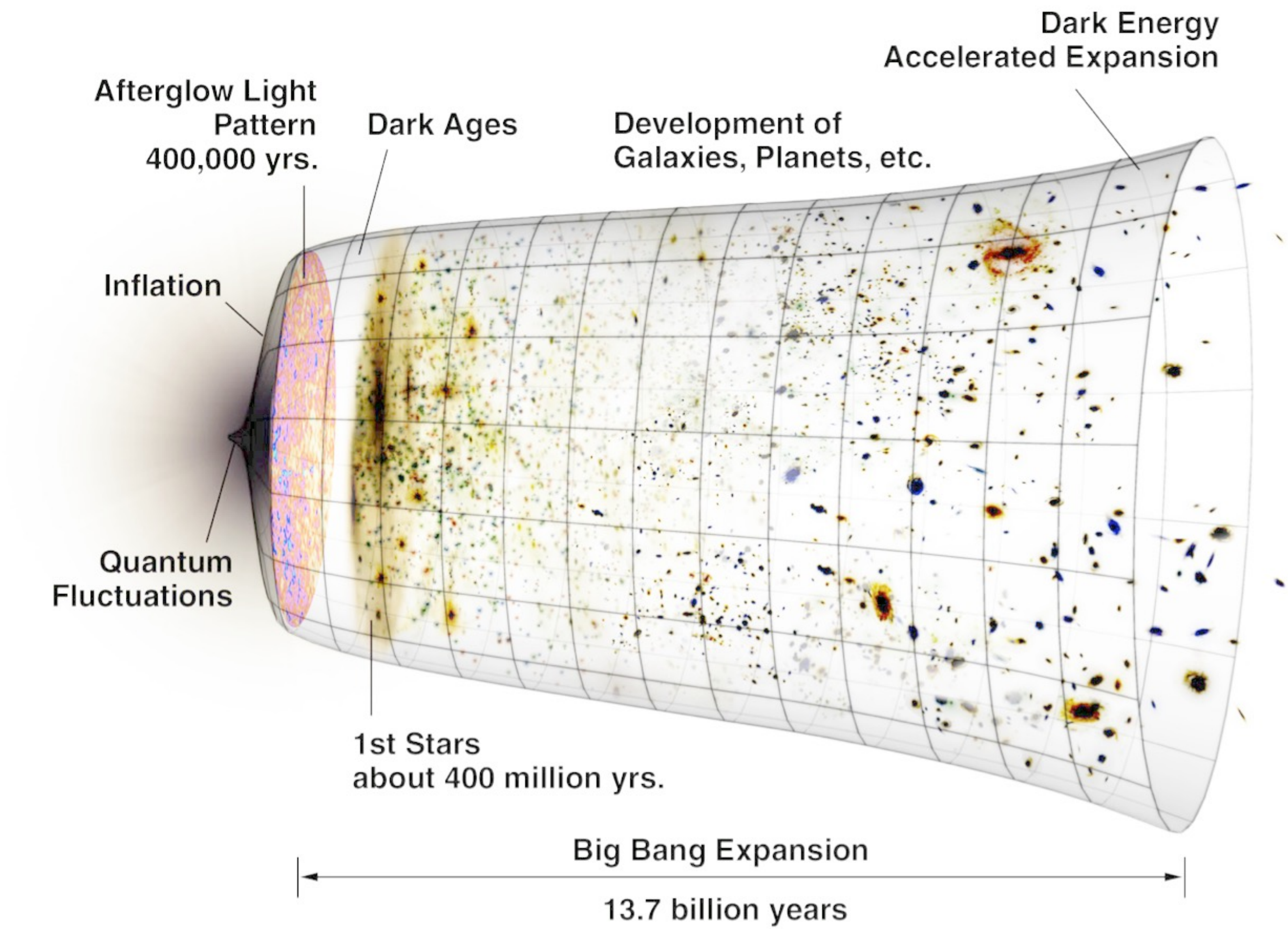


ΛCDM


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

INTERMISSION

Context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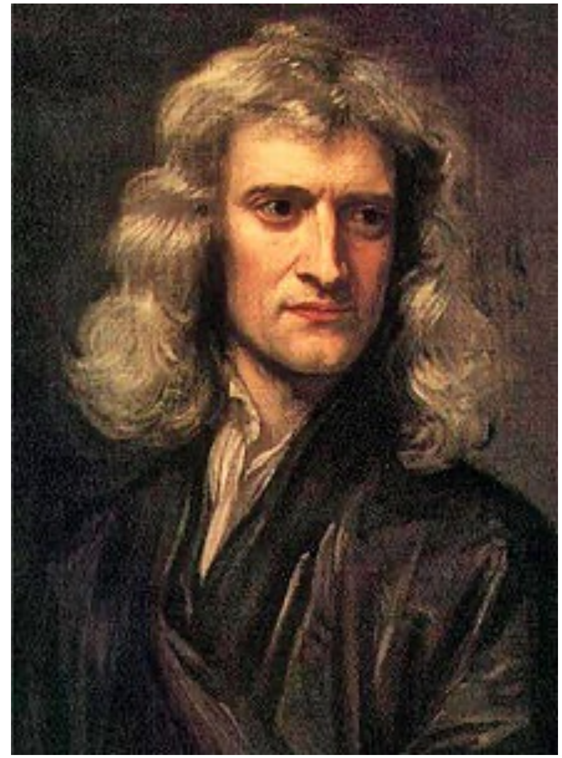
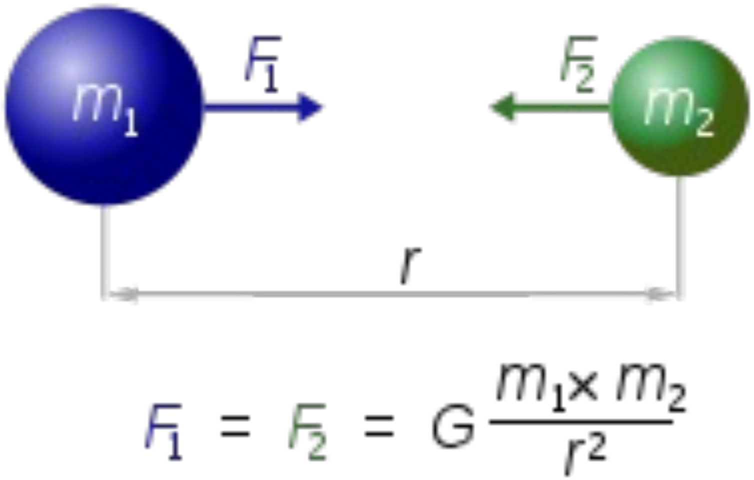
Newton’s First Law of Motion

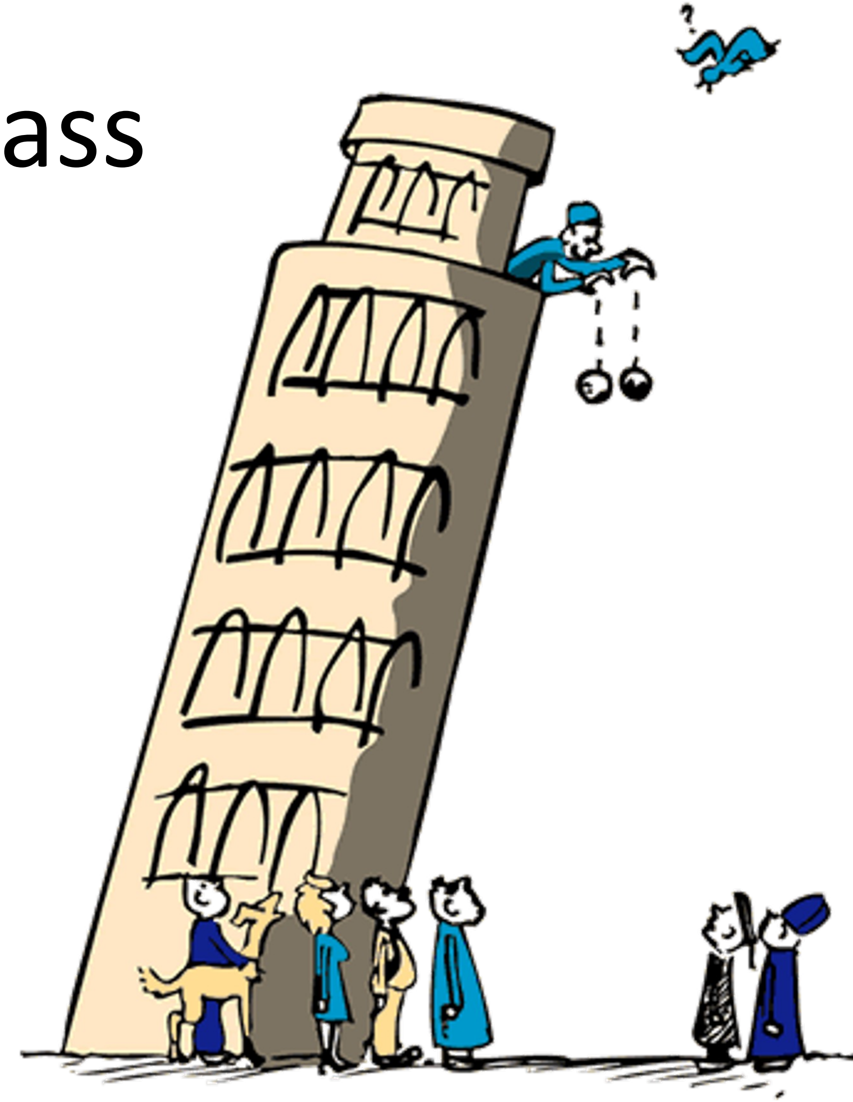


Unless acted on by an outside force:
An object at rest will remain at rest; and an object in motion will continue at constant velocity in a straight line.

Mass

Newton's 2nd Law: $F=ma$

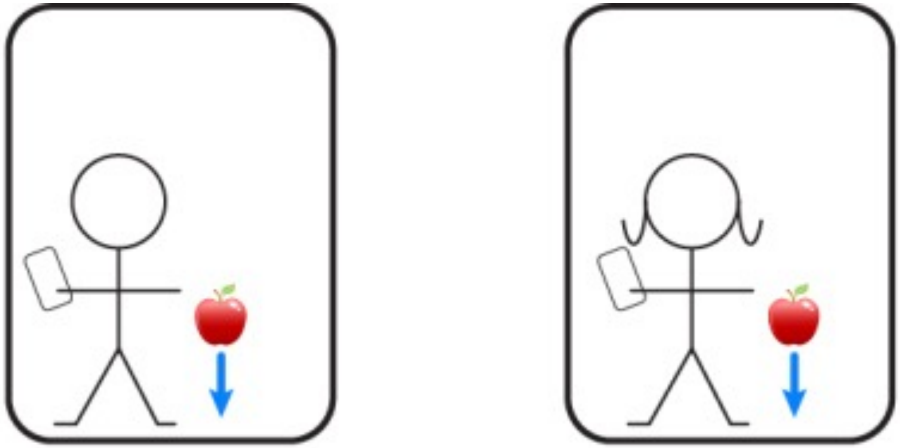




Acceleration is independent of mass.
Inertial mass = Gravitational mass.
Why?

The Equivalence Principle

If I can't tell two things apart, maybe they're the same thing.



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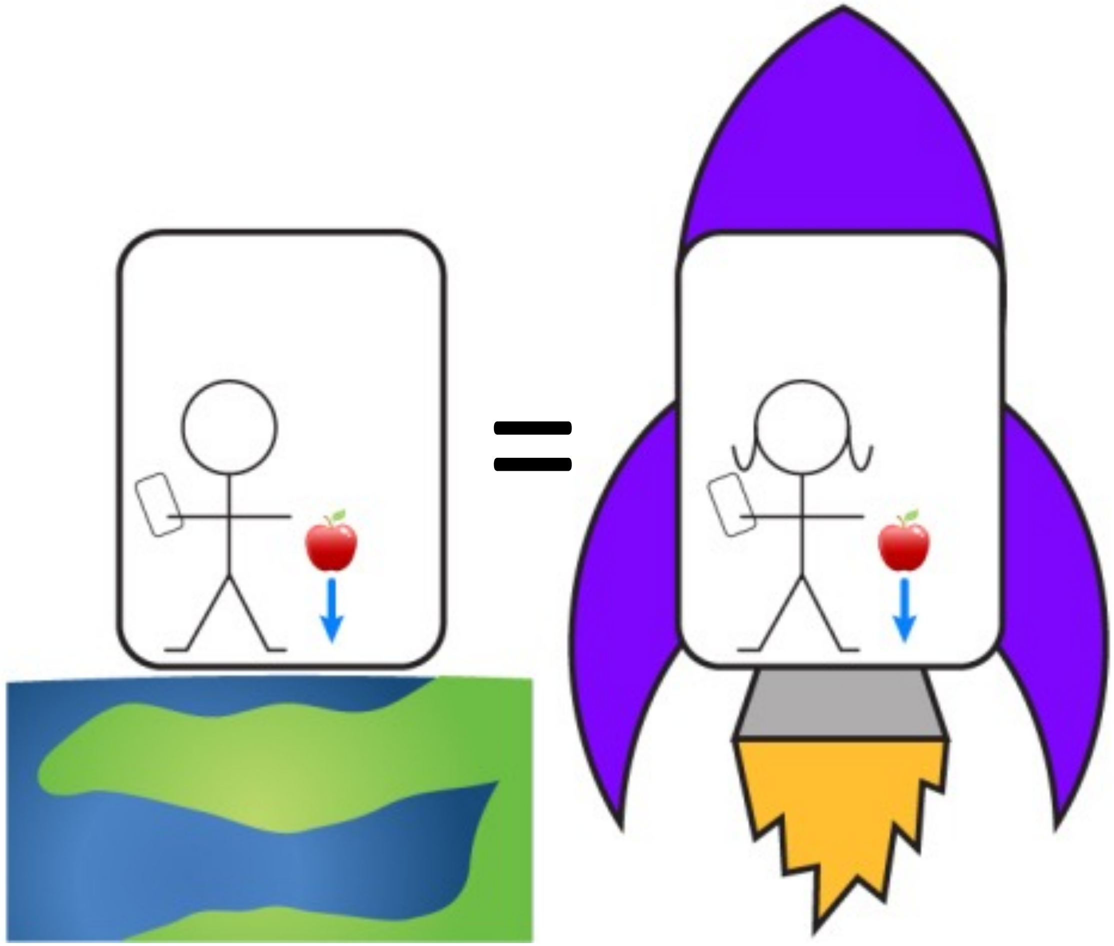
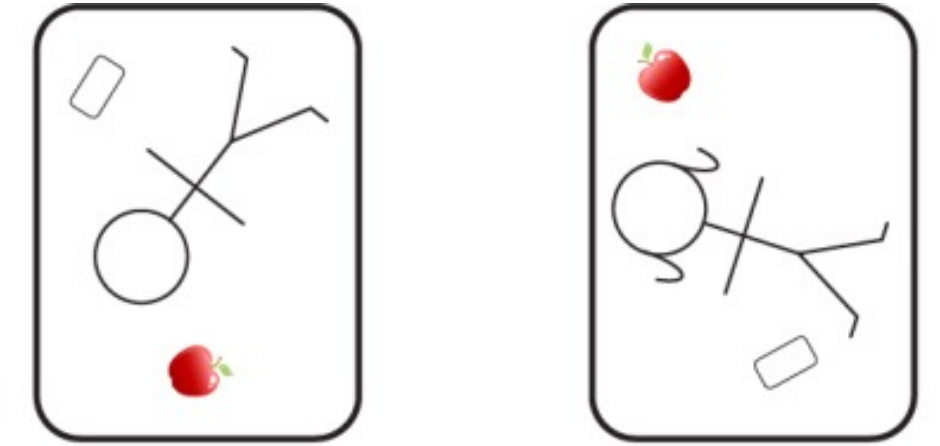
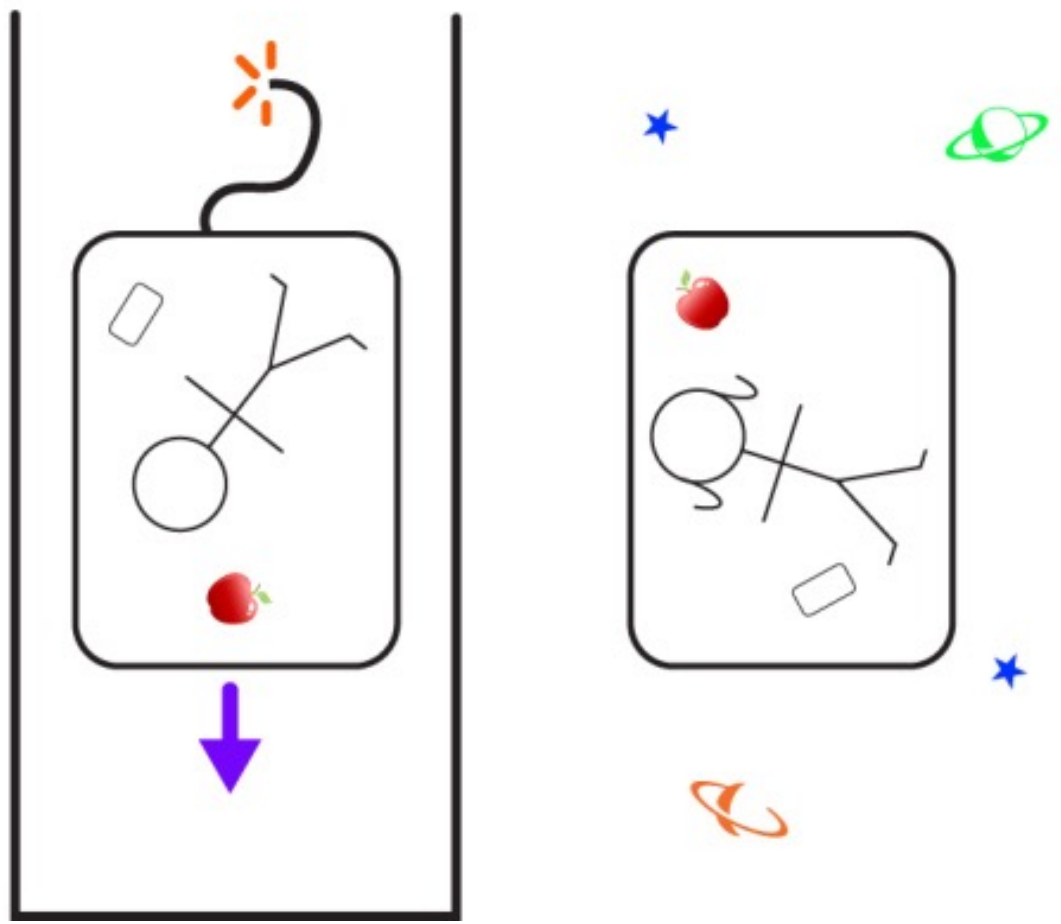


What we call "rest" on earth behaves exactly like an accelerating frame. There is no way to distinguish between an accelerating reference frame, and one in a gravitational field.

Inertial Motion



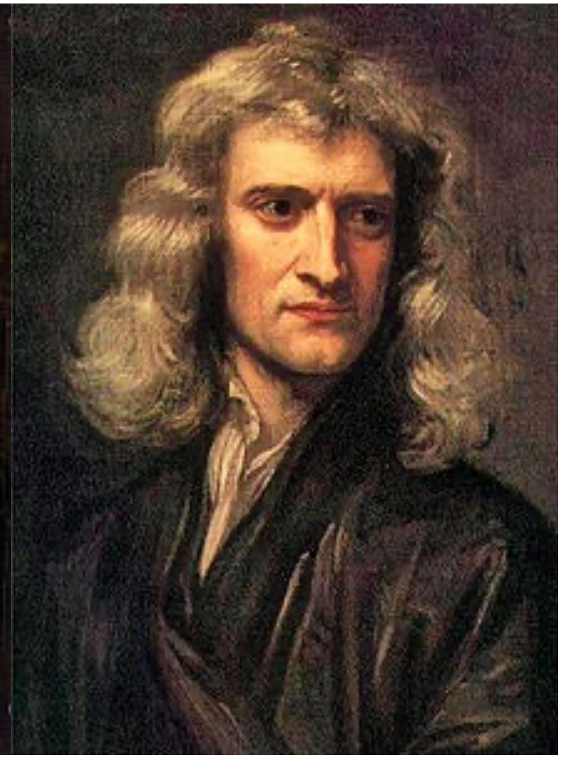
… or Free-fall?

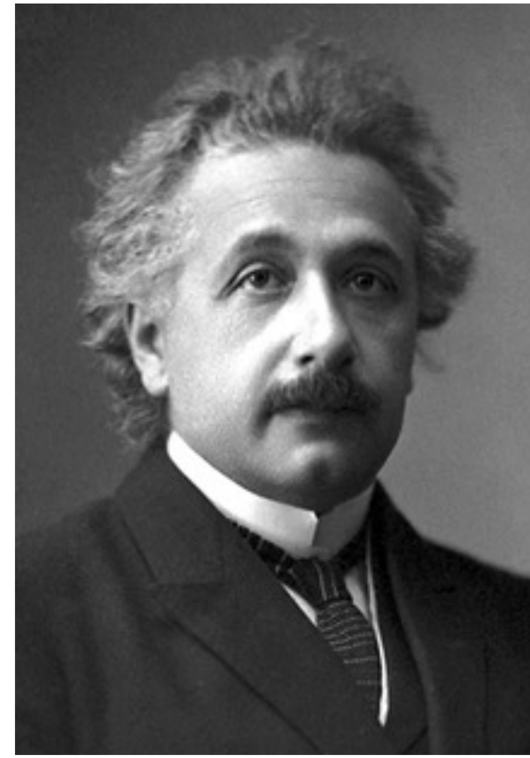


Orbits: Perpetual Free-fall

Equivalence Principle: this is Inertial motion.

Momentum & Motion



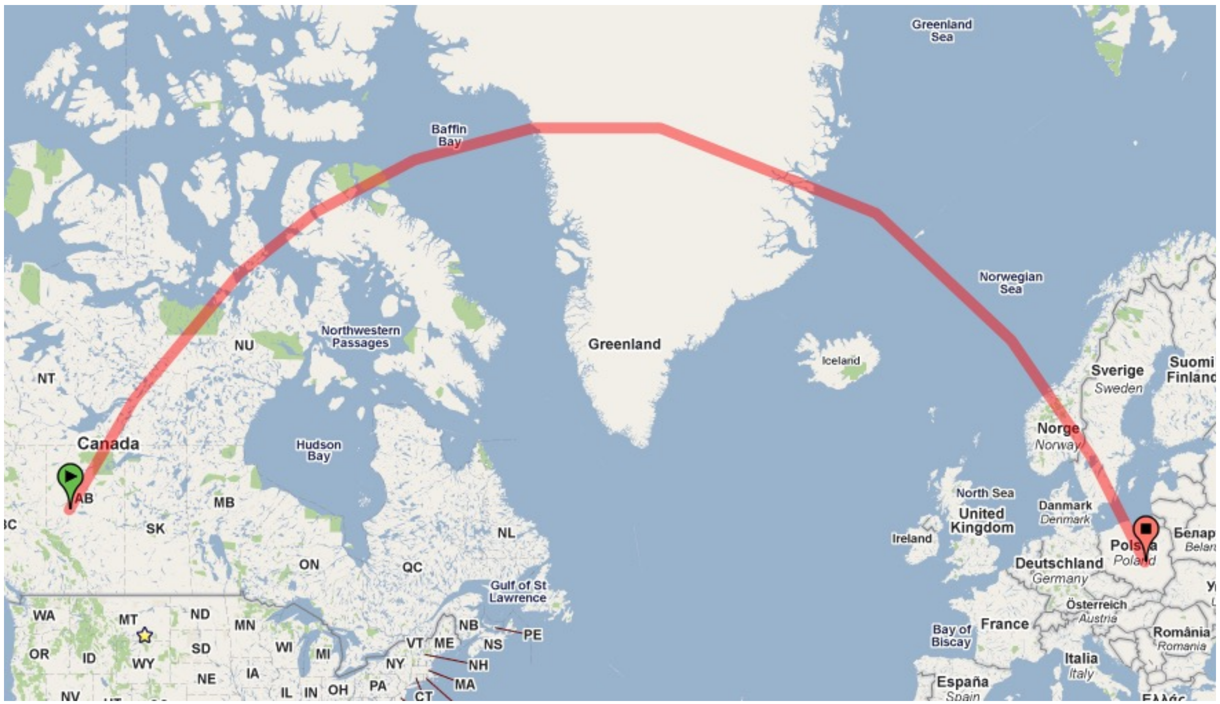


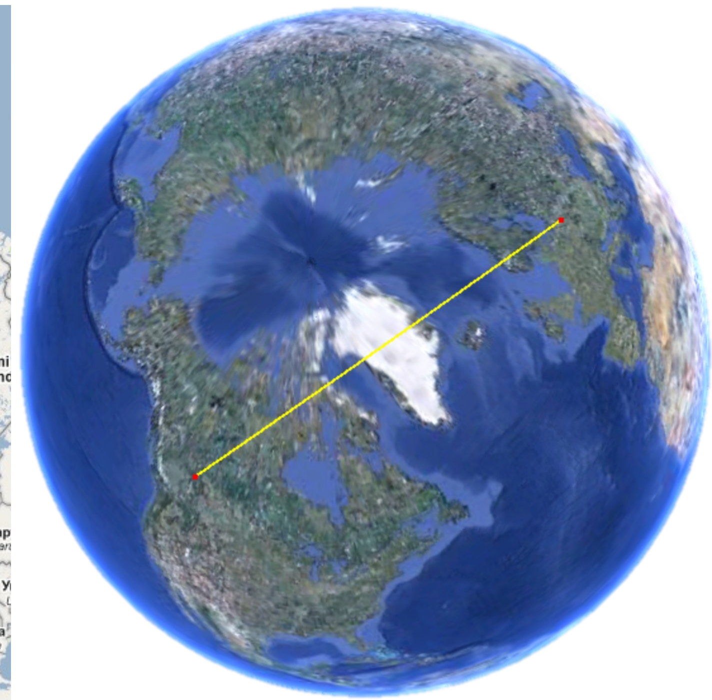
Unless acted on by an outside force:
An object at rest will remain at rest; and an object in motion will continue at constant velocity in a straight line.

Some straight lines are straighter than other straight lines.

Geodesics

On curved surfaces, “straight” lines don’t always look straight.
A geodesic is the shortest path between two points.





This is why longhaul flights often follow odd paths over the arctic.

Just like the 2d surface of the earth is curved,
the 4d hypersurface of spacetime is curved.

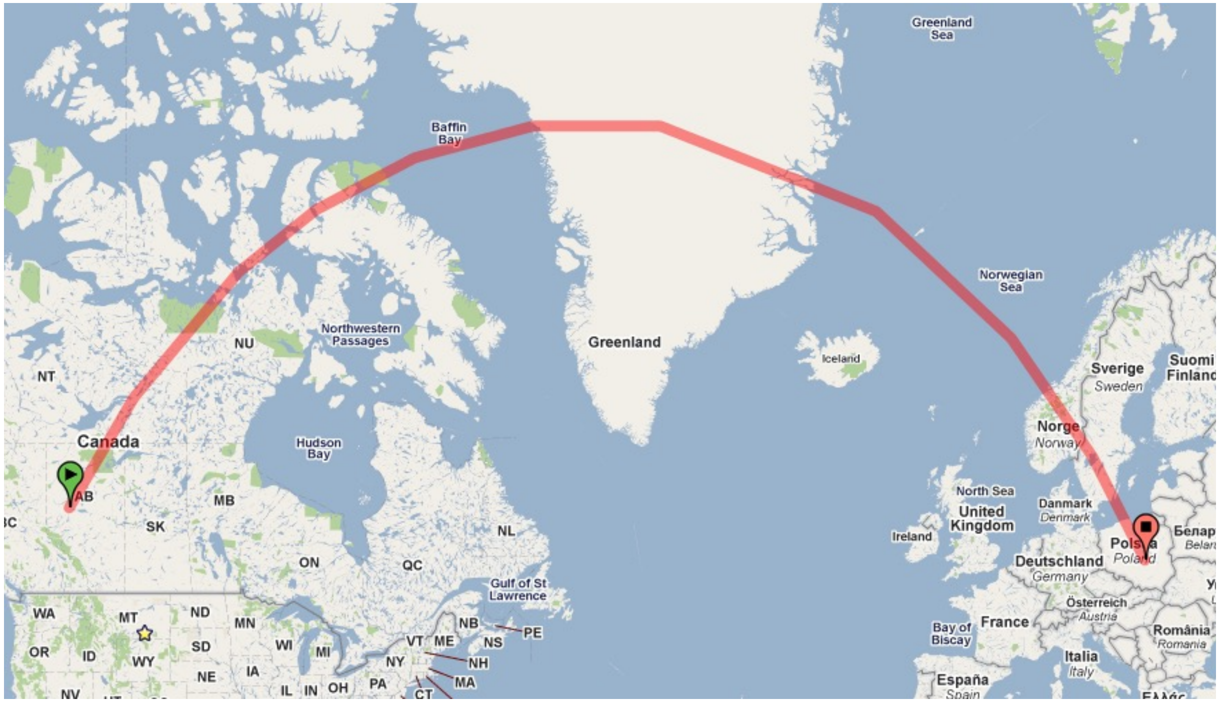
Curved Spacetime

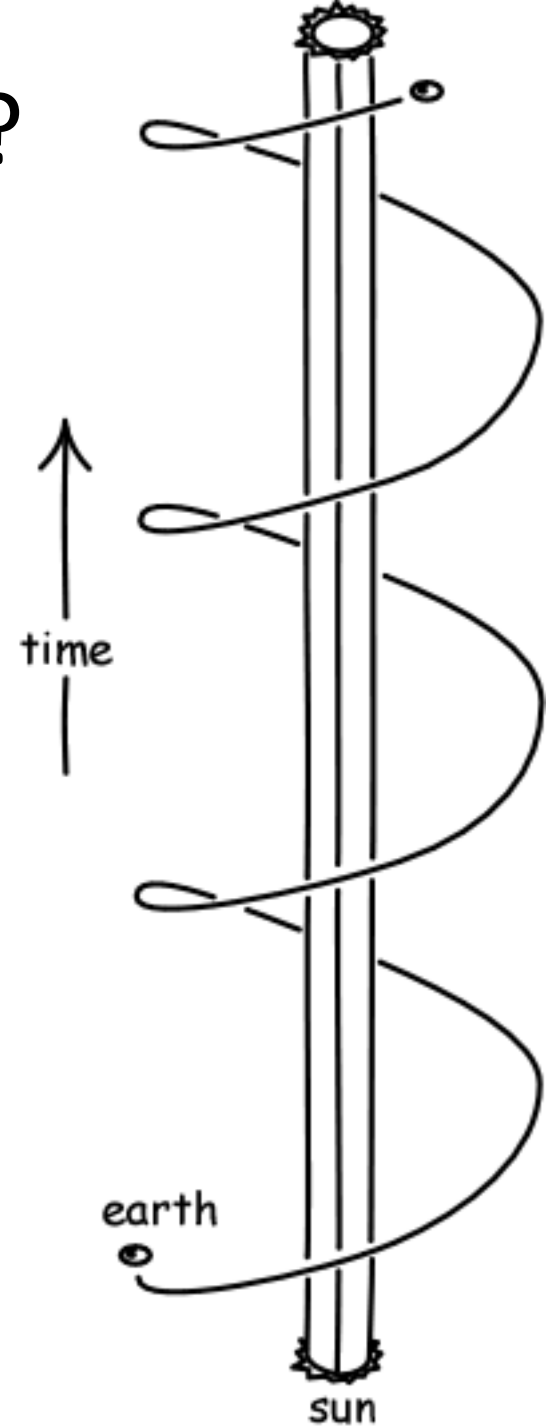
Newton & Galileo were very close to right.
In full generality, unforced objects follow geodesics.
In standard rectilinear Euclidean space, lines are geodesics.

Einstein pointed out that spacetime might not be Euclidean.
A curved spacetime would naturally have curvy geodesics.

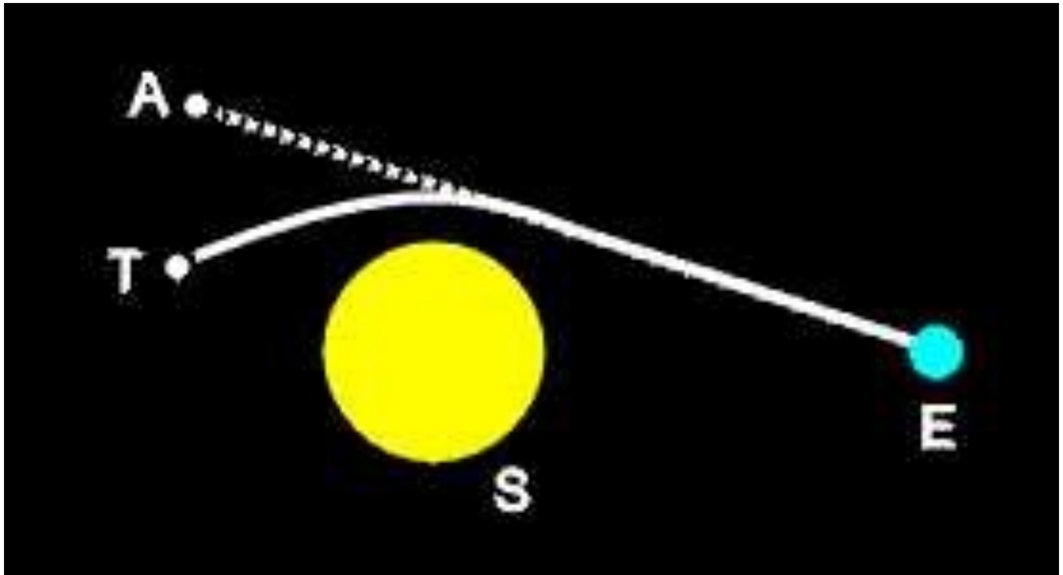
This curvy-ness is what we call gravity.

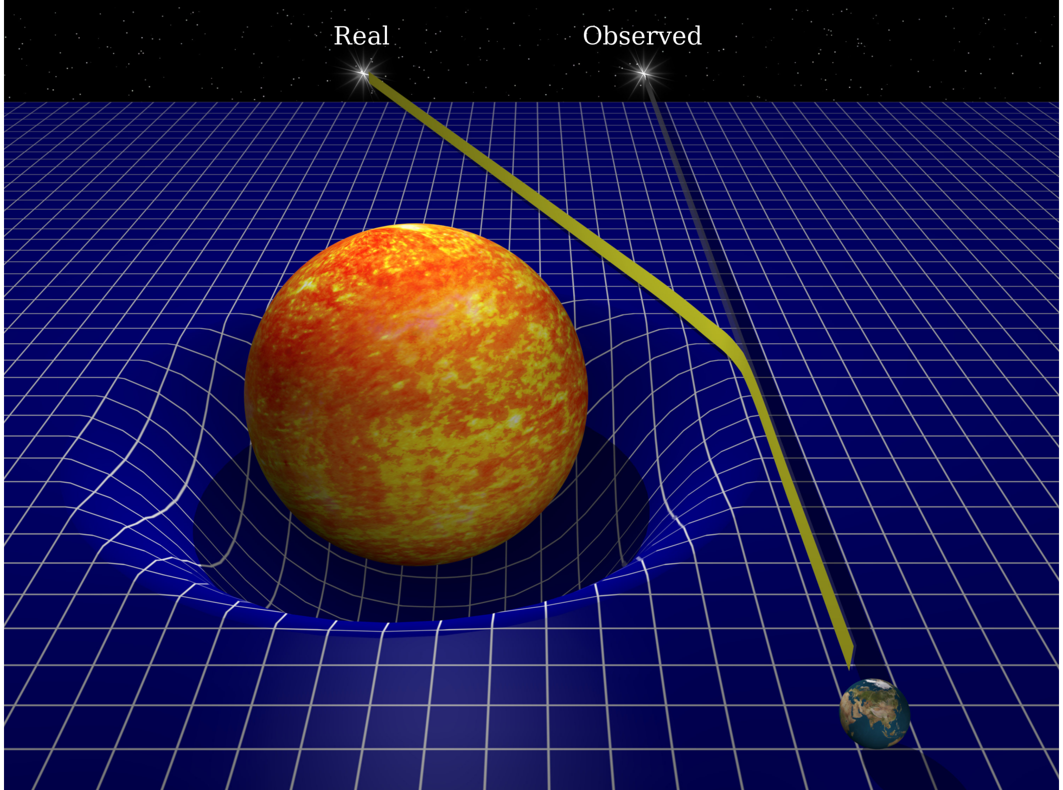
What? Gravity is geodesics?





Just like the red line is a geodesic in 2D, the earth's worldline, orbiting around the sun, is a geodesic in 4D spacetime.
This is what inertial motion looks like.





LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

General Relativity, Formally

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Ten equations, all rolled up into one line, using Tensor notation.
You'll never need this in this class, but it's worth seeing.

$G_{\alpha\beta}$ is the Einstein Tensor. $T_{\alpha\beta}$ is the Stress-Energy Tensor.

The left side describes the geometry of spacetime.
The right side describes the contents of spacetime.

General Relativity, in Prose

The Way of Newton:

Mass tells gravity how to exert a force ($F = -GMm/r^2$),
Force tells mass how to accelerate ($F = ma$).

The Way of Einstein:

Mass-energy tells space-time how to curve,
Curved space-time tells mass-energy how to move.

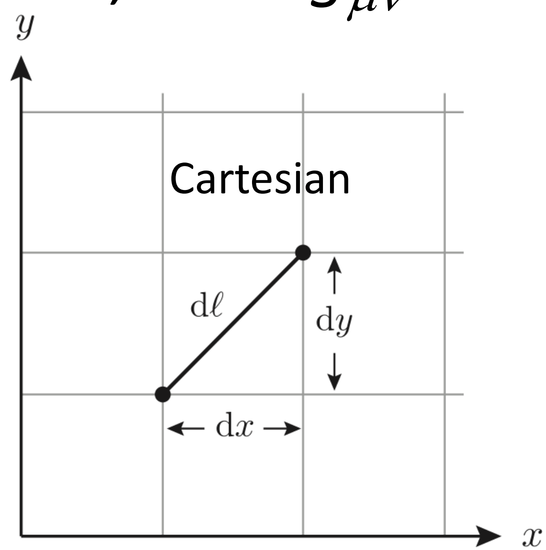
General Relativity, in Practice

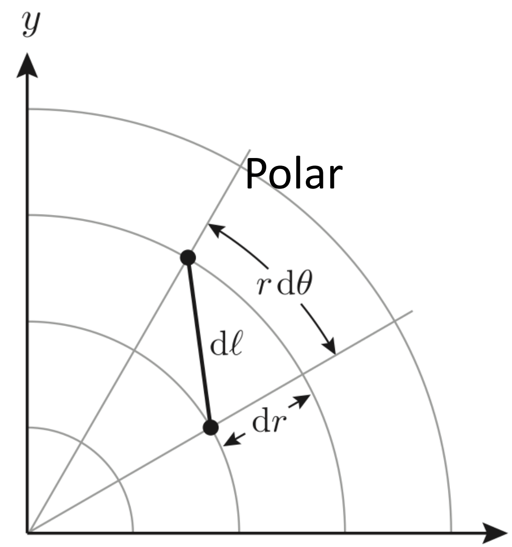
- Barring intervention, things move along **geodesics**. (Objects in motion/at rest tend to stay in motion/at rest unless acted upon.)
- Near a massive body, spacetime is curved so that geodesics point “down”.
- The ground gets in our way (EM and QM), acting on us and pushing back up.

The Metric, $g_{\mu\nu}$

- Contains all information about the geometry of spacetime in General Relativity, providing a notion of "distance"
  - *Label* points with coordinates, eg. r,θ,ϕ or t,x,y,z.
  - The metric converts coordinates to physical distances, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

g = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$





g = $\begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}$

The Interval

- Metric is often presented in terms of the spacetime interval, $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$

For example:

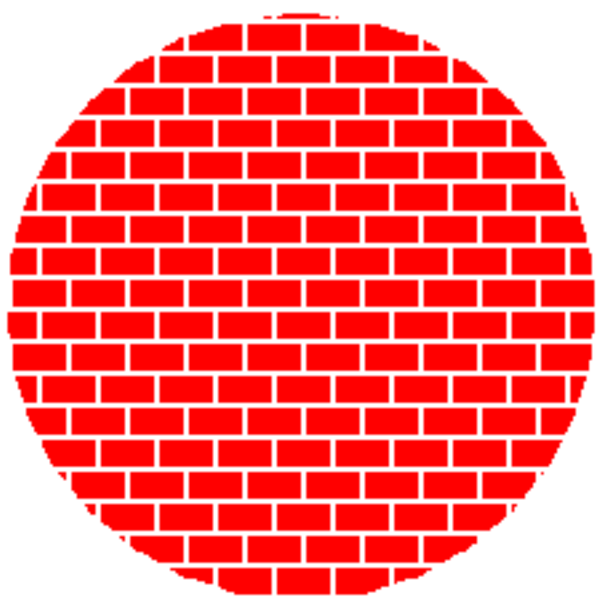
- Cartesian: $ds^2 = dx^2 + dy^2 + dz^2$
- Spherical: $ds^2 = r^2 + r^2(d\theta^2 + sin^2\theta\ d\phi^2)$
- Minkowski: $ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$

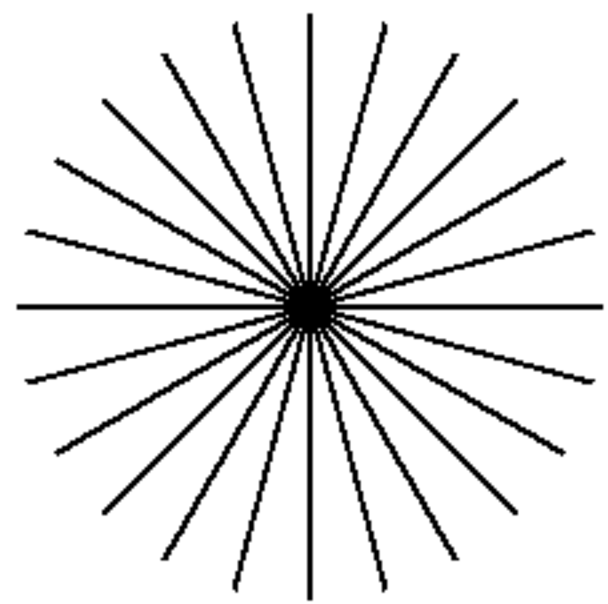
  $= c^2dt^2 - dl^2$

Reminder:
The Cosmological Principle

No special places or directions in the Universe.

It is Homogeneous and Isotropic.





[Friedman-Lemaitre-] Robertson-Walker Metric

- What is the most general metric for homogenous and isotropic space?
- F/L each derived, R/W each proved the

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa(r)^2 d\Omega^2 \right]$$

Scale Factor

$d\Omega^2$ (= $d\gamma^2$) = $d\theta^2 + \sin^2\theta \, d\phi^2$

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

(= f(r))

Radius of Curvature

Sign of Curvature

Curvature Exercise!

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa(r)^2 d\Omega^2\right]$$

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$

Imagine the Universe today were positively curved.

$a(t)=a(t_0)=1$ $\quad$ $R=\rho/\pi$ $\quad$ $\kappa=+1$

What is the shortest instantaneous ($\Delta t=0$) interval between $A(r,\theta,\phi) = (0,0,0)$ and $B(r,\theta,\phi) = (\rho,0,0)$?

Between A and $C(r,\theta,\phi) = (\rho,\pi,0)$?

Between B and C?

Curvature Exercise!

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa(r)^2 d\Omega^2 \right]$$

$$S_\kappa(r) = \begin{cases} R \sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R \sinh(r/R) & (\kappa = -1) \end{cases}$$

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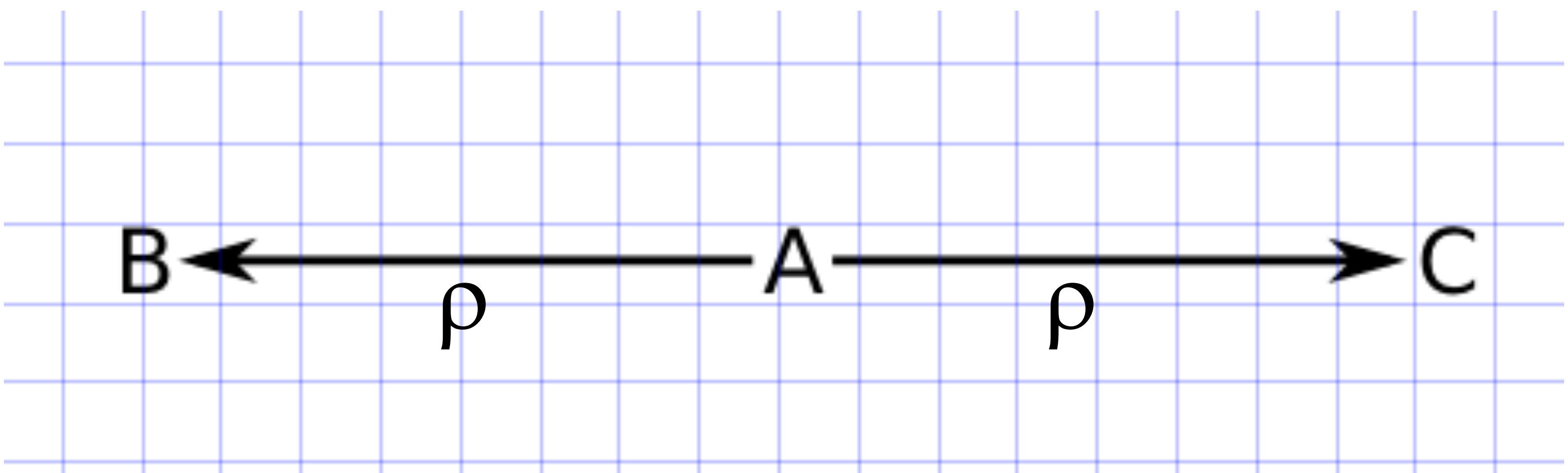
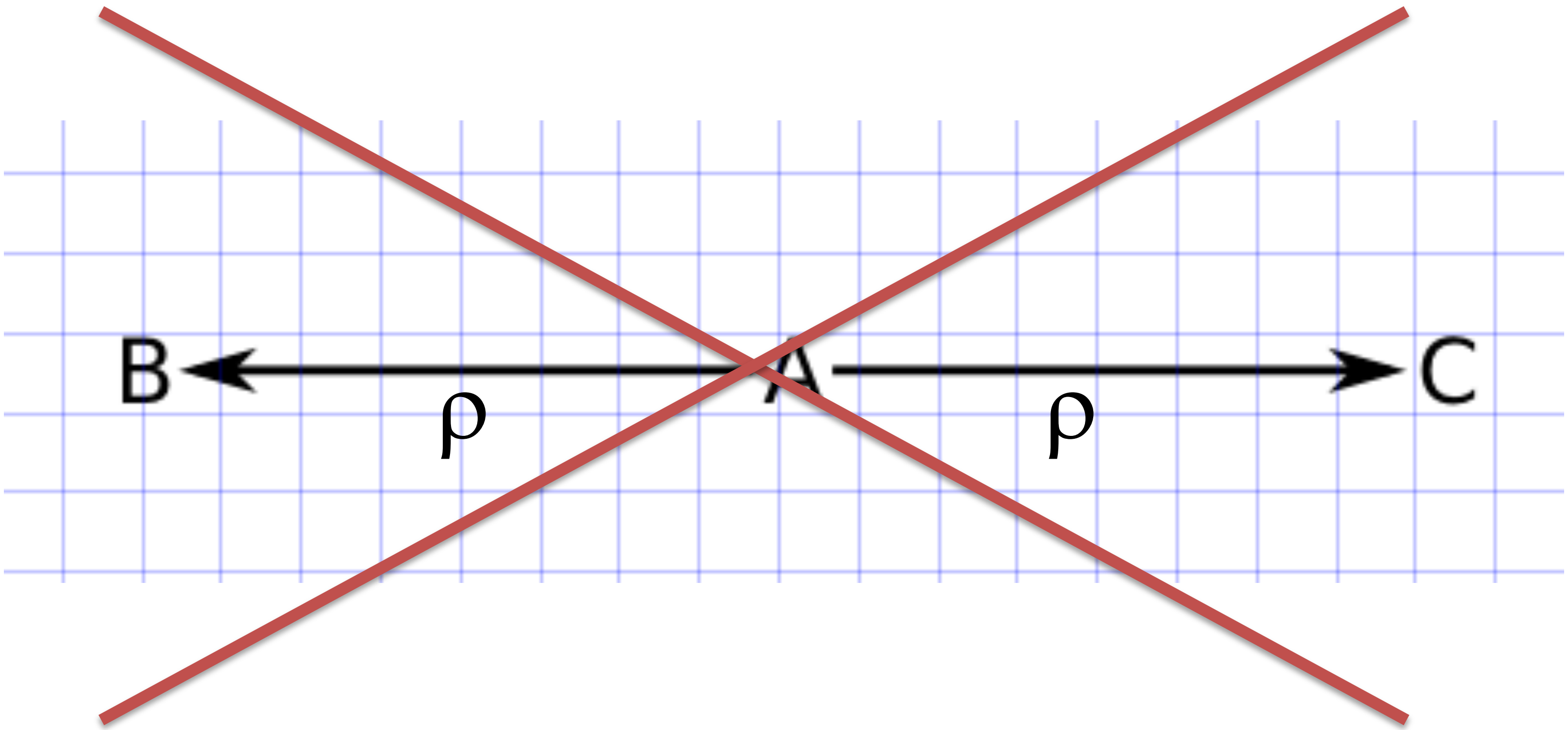
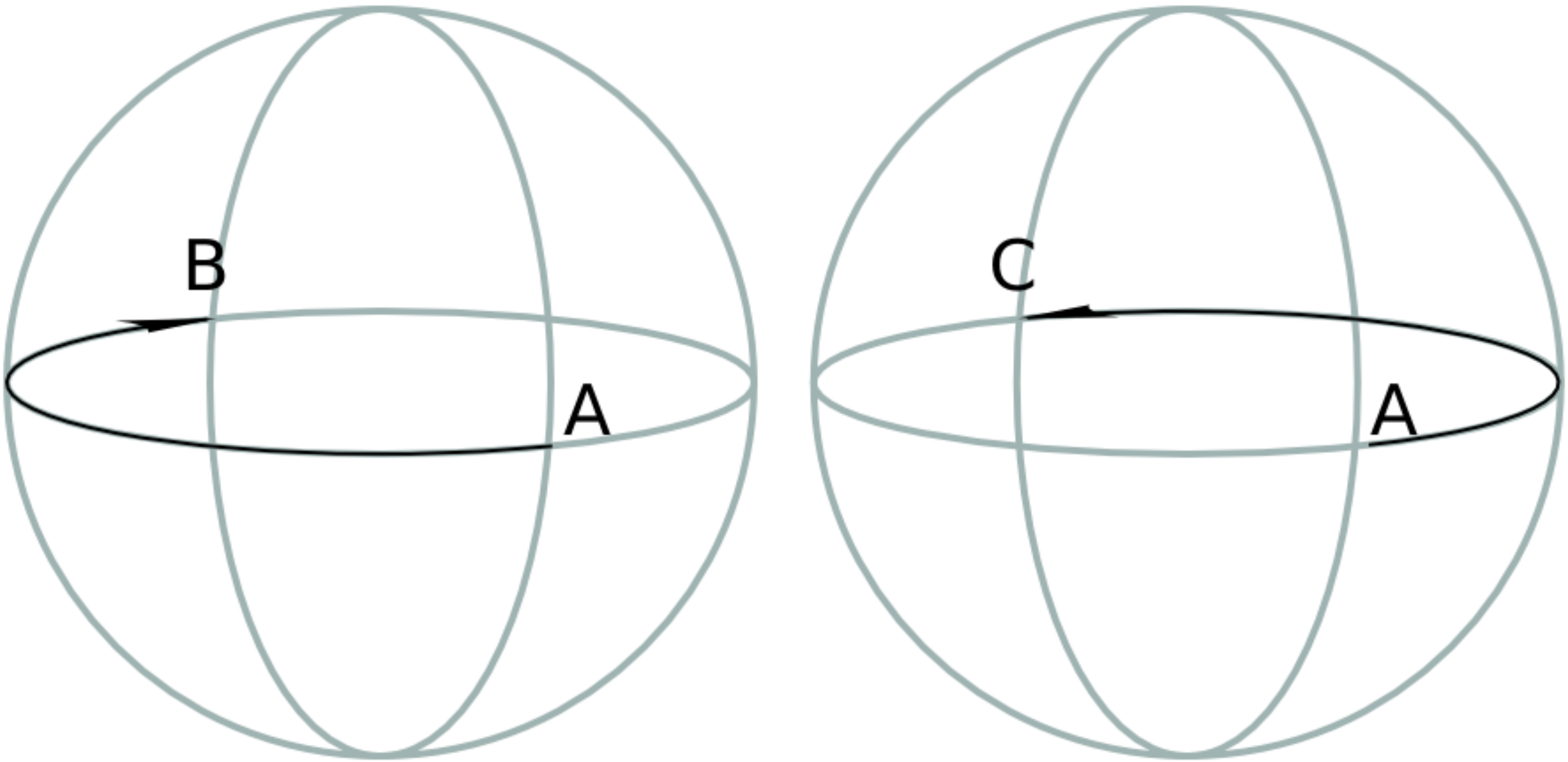


Curvature Exercise!



This is Cartesian! No good in curved space.

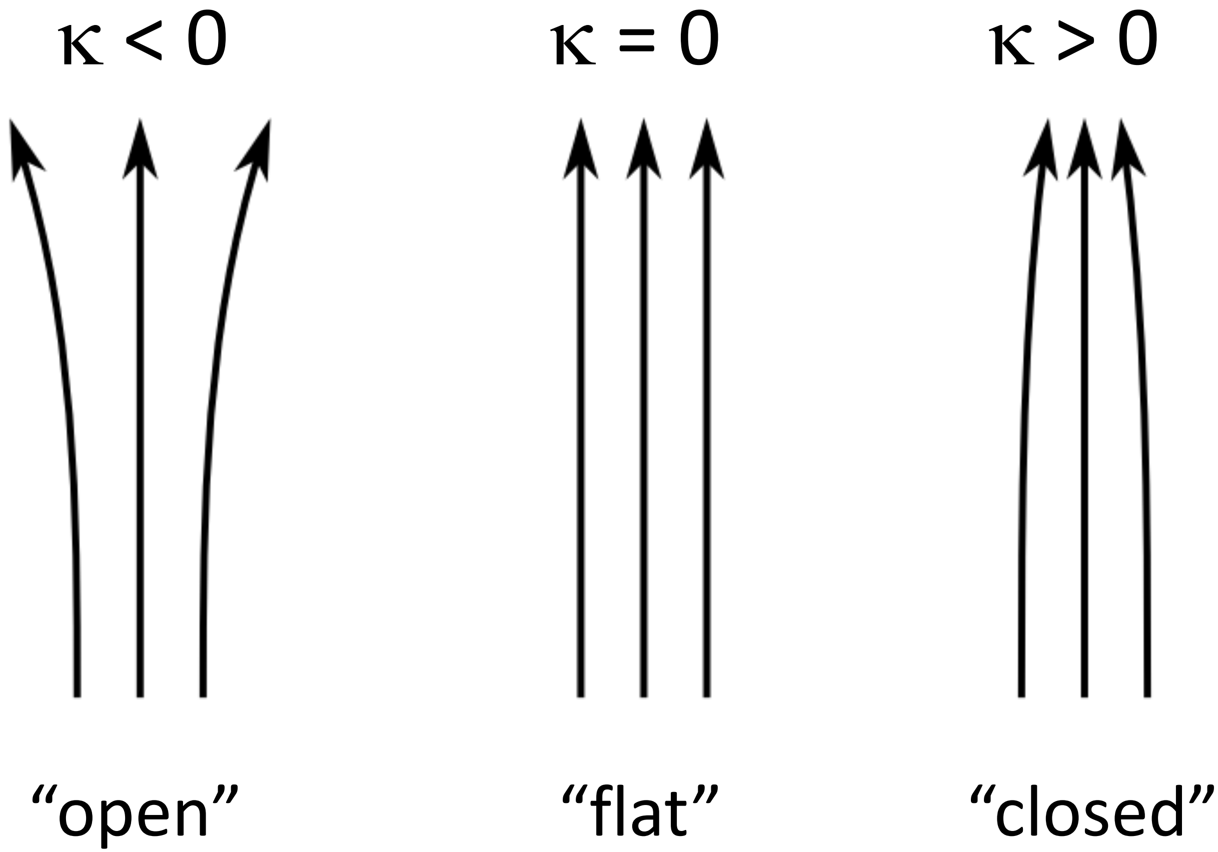
Curvature Exercise!



This is a “closed” universe.

(All points with a radial coordinate ρ are the SAME POINT.)

Curvature



The Many Faces of RW

Many different forms, we just chose a convenient one for astronomy.

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dx^2}{1 - \kappa x^2 / R_0^2} + x^2 d\Omega^2 \right] \quad , \quad x \equiv S_\kappa(r)$$

$d\Omega^2 = d\theta^2 + \sin^2\theta \; d\phi^2$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{a^2(t)}{c^2}[\underbrace{\mathrm{d}r^2}_{\text{Radial Distance}} + \underbrace{\Re^2 \sin^2(r/\Re)\underbrace{(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2)}_{d\Omega^2}}_{\text{Transverse Distance}}]$$

Scale Factor

Curvature
$R = R_c(t) / a(t)$

Relativistic Schmelativistic

- Special Relativity: no absolute frame of reference
- Isotropic, homogenous → preferred ref frame!
- Define “fundamental observers,” at rest WRT preferred frame (and matter, energy, etc)
- Clocks run at the same speed, possible to sync them
- Proper time of fundamental observers is shorthanded as “time”
- We invent “proper distance”, the distance between two fundamental observers at a common instant in proper time.

Times and Distances

Cosmology is a mess of conflicting and misleading terms.
Worse, “distance” and “time” are slippery concepts in relativity.

Comoving Coords => raw coordinates, defined to match proper positions at $t=t_0$

Proper Time => clock of a fundamental observer

Proper Distance => spatial interval between fundamental observers at a common proper time

Comoving Dist => proper distance at $t=t_0$

Effective Distance => “radius” of illumination, $D == a(t)S_\kappa(r)$ (for a flat universe, equal to proper dist)

Luminosity Dist => distance a photon has travelled $D_L = D\ (1+z)$

Angular diameter Dist => geometric (Euclidean) distance $D_A = D\ /\ (1+z)$

Wednesday:

More Metric.