AST 1430F
Observational Cosmology

X

Structure I – Perturbations

Announcements

- Feb 15th will be a 2hr session: 10am-12pm
- Presentations
  - Anika – Baryogenesis
  - Caleb – CMB B-modes
  - Braden – Alternative DE
  - Anna – Higgs Mechanism

Week Index	Dates	Topics
~~Week 1~~	~~Jan 11, 13~~	~~logistics, introduction, basic observations~~
~~Week 2~~	~~Jan 18, 20~~	~~Basic GR, RW metric, Distances, coordinates, Friedmann equations, cosmological models~~
~~Week 3~~	~~Jan 25, 27~~	~~Consistency with observations, Early Hot Universe, BBN~~
Week 4	Feb 1, 3	~~Inflation~~, Perturbations & Structure pre-recombination
Week 5	Feb 8, 10	CMB: basics, polarization, secondaries
Week 6	Feb 15, 17	Early-Universe Presentations + Review
Reading Week – No Class		
Week 7	Mar 1, 3	Post-recombination growth of structure, formation of dark matter halos, halo mass function
Week 8	Mar 8, 10	The relation between dark matter halos and galaxies
Week 9	Mar 15, 17	Probing the cosmic density field / clustering
Week 10	Mar 22, 24	Late-time cosmological observations: BAO, supernovae, weak lensing, etc.
Week 11	Mar 29, 31	H0 controversy: how fast exactly is the Universe expanding today?
Week 12	Apr 5	Late Universe Presentations + Review

Keith

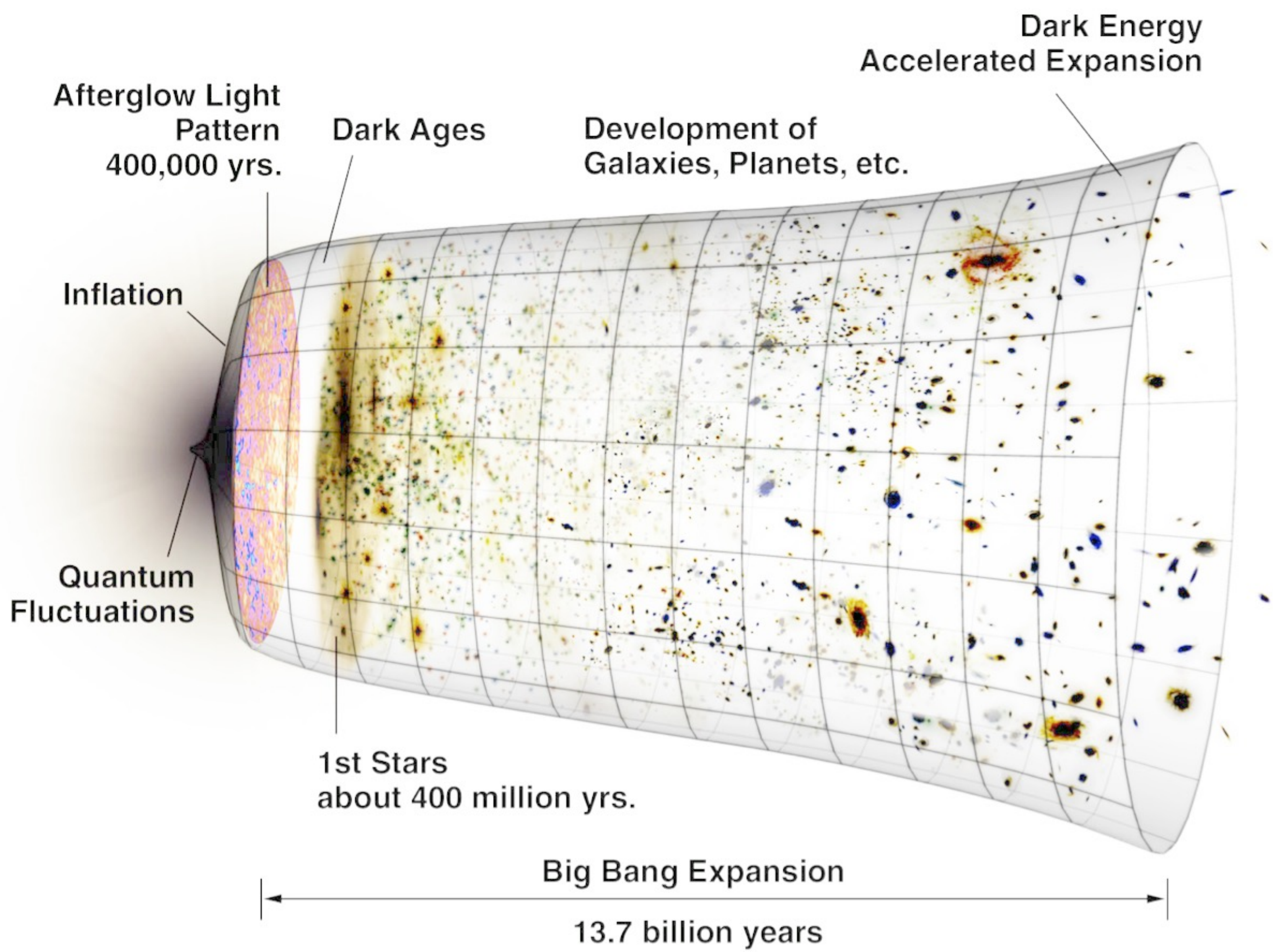
Jo

asst1

asst2

asst3

Context



NASA / WMAP Science Team

Perturbations

Consider a small perturbation to the density field, with Density Contrast $\Delta = \rho(x) / \langle\rho\rangle - 1$.

For example:

Galaxies	→	$\Delta \approx 10^6$
Galaxy Clusters	→	$\Delta \approx 10^3$
Superclusters	→	$\Delta \approx 10^0$

If Virialized / Bound:

$\Delta \approx 1$ at $z \approx (10^6)^{1/3} \approx 100$

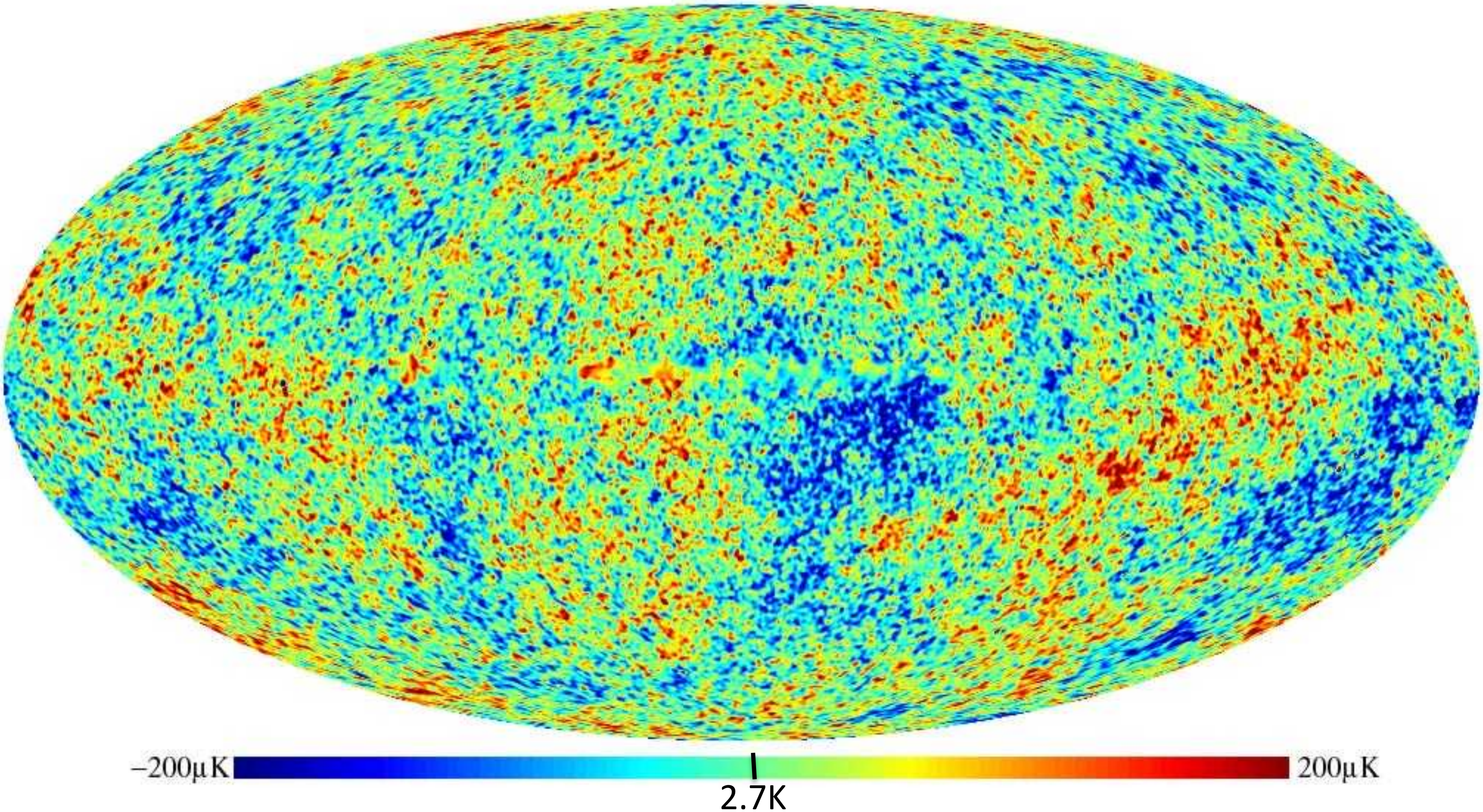
$\Delta \approx 1$ at $z \approx (10^3)^{1/3} \approx 10$

$\Delta \approx 1$ at $z \approx (10^0)^{1/3} \approx 1$

Bound structures cannot have separated before these redshifts. (Or they would be much more dense today.)

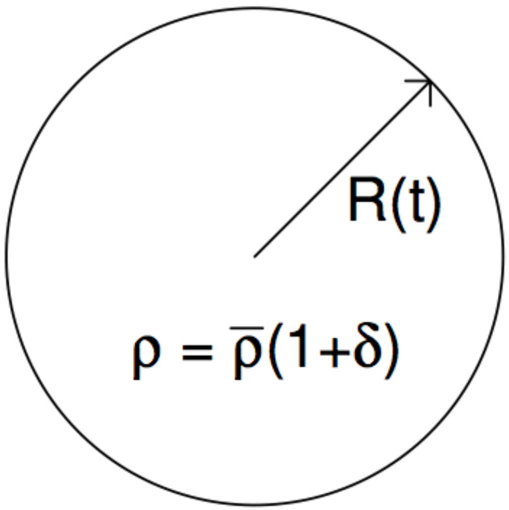
In the early Universe, $\Delta << 1$, linear approximations are adequate.

Looking at $z \approx 1100$



At recombination, $\Delta = \delta\rho / \rho = dT / T \approx 300\mu K / 3K \approx 10^{-4}$

Gravitational Instability



$$\ddot{R} = -\frac{G(\Delta M)}{R^2} = -\frac{G}{R^2}\left(\frac{4\pi}{3}R^3\bar{\rho}\delta\right)$$

$$\rightarrow \quad \frac{\ddot{R}}{R} = -\frac{4\pi G\bar{\rho}}{3}\delta(t) \underbrace{\approx -\frac{1}{3}\ddot{\delta}}_{\text{mass conservation}}$$

$$\delta(t) = A_1 e^{t/t_{\rm dyn}} + A_2 e^{-t/t_{\rm dyn}} \qquad t_{\rm dyn} = \frac{1}{(4\pi G\bar{\rho})^{1/2}}$$

More commonly, $\Delta(t)$

Jeans' Length & Mass

As the sphere collapses, a pressure gradient will build, supporting it.

Pressure propagates at the speed of sound, $c_s^2 = dp/d\rho$.

To achieve equilibrium, pressure must build faster than the collapse.

$$t_{\rm pre} \sim \frac{R}{c_s} \ < t_{\rm dyn}$$

Overdensities can be stable if smaller than the Jeans length λ_J.

$$\lambda_J = c_s \left(\frac{\pi c^2}{G\bar{\varepsilon}}\right)^{1/2} = 2\pi c_s t_{\rm dyn}$$

$M_J \approx 4/3\ \pi\ (\lambda_J/2)^3\ \rho$

Cosmological Complication: a(t)

In an expanding Universe, things are more complicated.

Equations of Fluid Dynamics:

Continuity → $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

Motion (Euler) → $$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\nabla p/\rho - \nabla \phi$$

Gravity (Poisson) → $$\nabla^2 \phi = 4\pi G \rho$$

Generally, hard to solve. Let's look at 1st order of perturbations,

$v \rightarrow v + dv$ $\quad \rho \rightarrow \rho + d\rho$

$p \rightarrow p + dp$ $\quad \phi \rightarrow \phi + d\phi$

Perturbations

v → v + ∂v   ρ → ρ + ∂ρ

p → p + ∂p   ϕ → ϕ + ∂ϕ

Continuity → $$\frac{d\Delta}{dt} \equiv \dot{\Delta} = -\nabla \cdot \partial v$$ Δ = ∂ρ / ρ

Motion (Euler) → $$\frac{d(\partial v)}{dt} + (\partial v \cdot \nabla) v = -\nabla \partial p / \rho - \nabla \partial \Phi$$

Gravity (Poisson) → $$\nabla^2 \partial \Phi = 4\pi G \rho \Delta$$

x = a(t) r ; c_s^2 = ∂p/∂ρ (adiabatic)

… (see text e.g. Longair p316, Peacock p460-464)

$$\frac{d^2\Delta}{dt^2} + 2\left(\frac{\dot{a}}{a}\right)\frac{d\Delta}{dt} = 4\pi G \partial\rho + \frac{c_s^2}{\rho a^2}\nabla^2 \partial\rho$$

Perturbations

$$\frac{d^2\Delta}{dt^2}+2\left(\frac{\dot{a}}{a}\right)\frac{d\Delta}{dt}=4\pi G\partial\rho+\frac{c_s^2}{\rho a^2}\nabla^2\partial\rho$$

$$\Delta(r,t) = A \sum e^{ikr}\, \Delta_k(t)$$

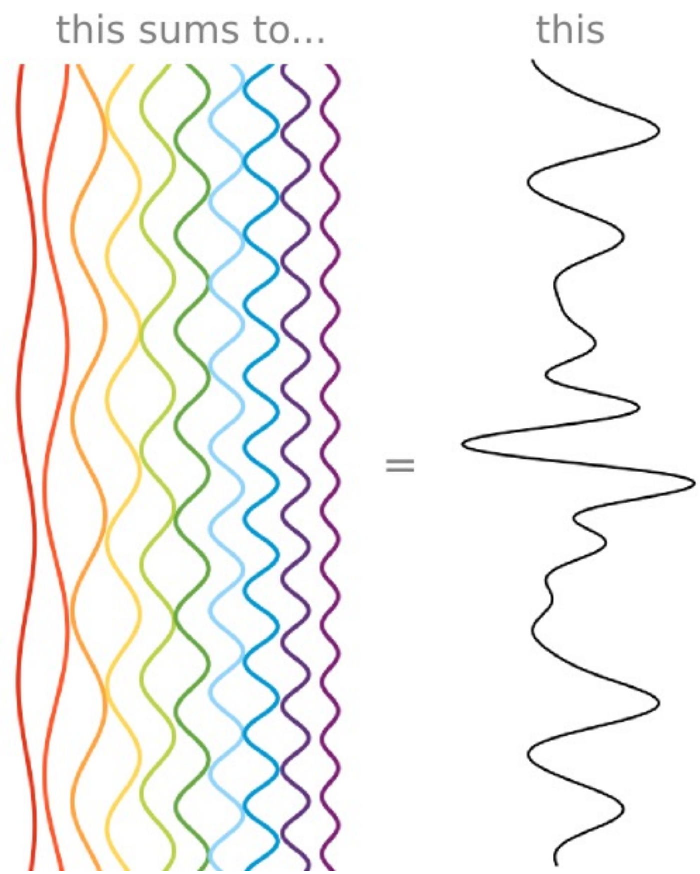
$$\boxed{\ddot{\Delta}+2\frac{\dot{a}}{a}\dot{\Delta}=\Delta\left(4\pi G\bar{\rho}-\frac{c_s^2k^2}{a^2}\right)}$$

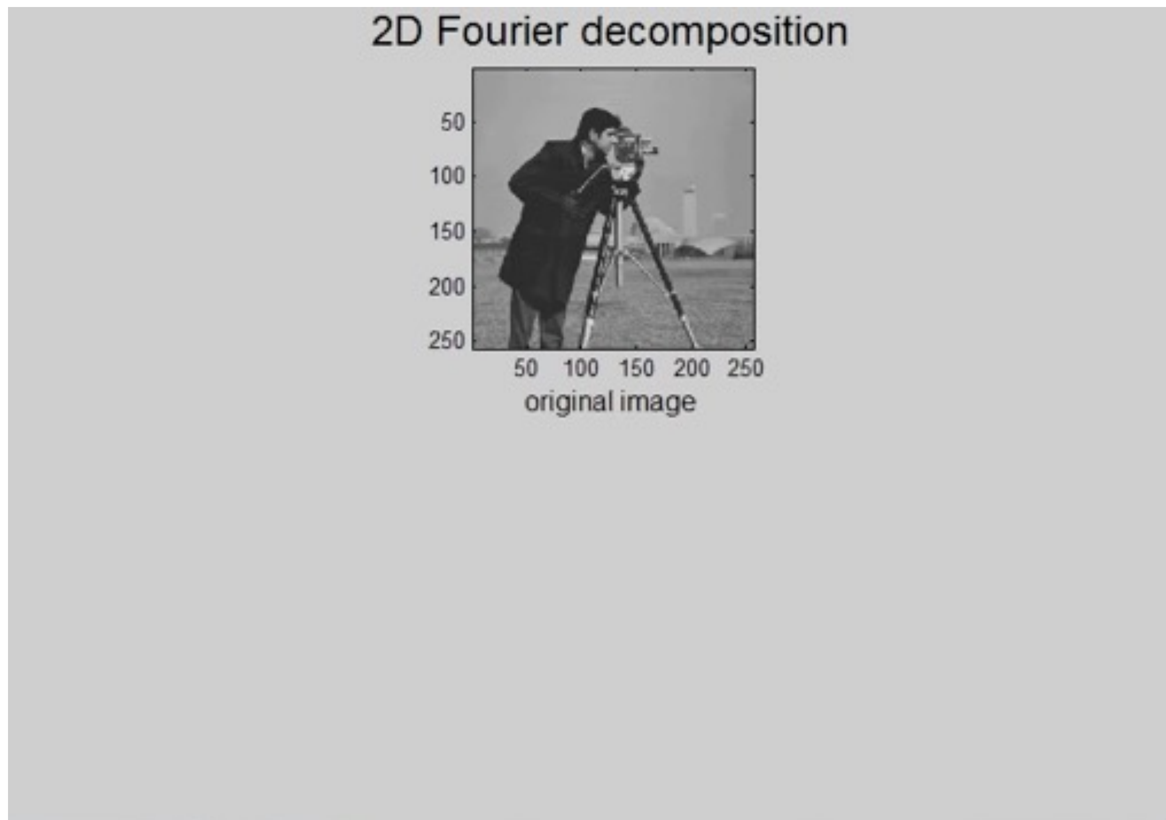
Wavenumber,
Not curvature!

Perturbations

$$\boxed{\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right)}$$

Wavenumber,
Not curvature!





Classical Space

$$\boxed{\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right)}$$

$\dot{a} = 0$ → $$\ddot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right)$$

$$\Delta(t) = e^{\pm t/\tau}, \tau = 1/\sqrt{4\pi G\rho - c_s^2 k^2}$$

Behaviour depends on scale of perturbation, relative to Jeans Length $\lambda_J = 2\pi c_s t_{\rm dyn}$

$\lambda = 2\pi/k > \lambda_J$ → exponential collapse

$\lambda = 2\pi/k < \lambda_J$ → stable oscillation

Empty (Milne) Universe

$$\boxed{\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right)}$$

Assume pressure gradients are negligible, $\lambda >> \lambda_J$ (i.e., $k<<1/ c_s$)

$a \propto t$

$\rho \approx 0$

$$\ddot{\Delta} + \frac{2}{t}\dot{\Delta} = 0$$

Solutions of form $\Delta \propto t^n$

where $n = 0, -1$

→ No growth possible

Matter Dominated, flat

$$\boxed{\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right)}$$

$a \propto t^{2/3}$

$\rho \propto a^3$

$$\ddot{\Delta} + \frac{4}{3t}\dot{\Delta} = \Delta\frac{2}{3t^2}$$

Solutions of form $\Delta \propto t^n$ → $n = 2/3, -1$

Perturbations grow slowly in time, $\Delta \propto a$.

Radiation Dominated, flat

A few changes in the derivation lead to a similar equation:

$$\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right) \quad \rightarrow \quad \boxed{\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\frac{32\pi}{3}G\rho}$$

$a \propto t^{1/2}$

$\rho \propto a^4$

$c_s \approx c / \sqrt{3}$

Solutions of form $\Delta \propto t^n$ → $n = \pm 1$

Perturbations grow, only slightly faster, still not exp, $\Delta \propto a^2$.

Baryonic Perturbations

Consider perturbations in **baryonic** matter.

$$M_J \approx 4/3\ \pi\ (\lambda_J/2)^3\ \rho_b = \pi\lambda_J^3\rho_b\ /\ 6$$

Radiation dominated Universe: $\lambda_J = c_s \sqrt{\dfrac{3\pi}{8G\rho}} \propto a^2$

$\rho_b \propto a^{-3}$ → $\boxed{M_J \propto a^3}$

For $M > M_J$ → $\Delta \propto a^2$

For reference:

$M_J \approx M_{sun}$ @ $z \approx 3\text{x}10^9$

$M_J \approx M_{gal} \approx 10^{11}\ M_{sun}$ @ $z \approx 10^6$

Note: in rad-dom flat universe,

$\lambda_J = c/\sqrt{3}(\pi/H) \approx 1.8c/H$

$r_H = 2c/H$

Recombination

At recombination, matter-radiation coupling breaks down.

Speed of sound c_s drops from $c/\sqrt{3}$ to speed in gas $\approx (5kT/3m_H)^{1/2}$

Jeans mass drops by a factor of 10^{10}!

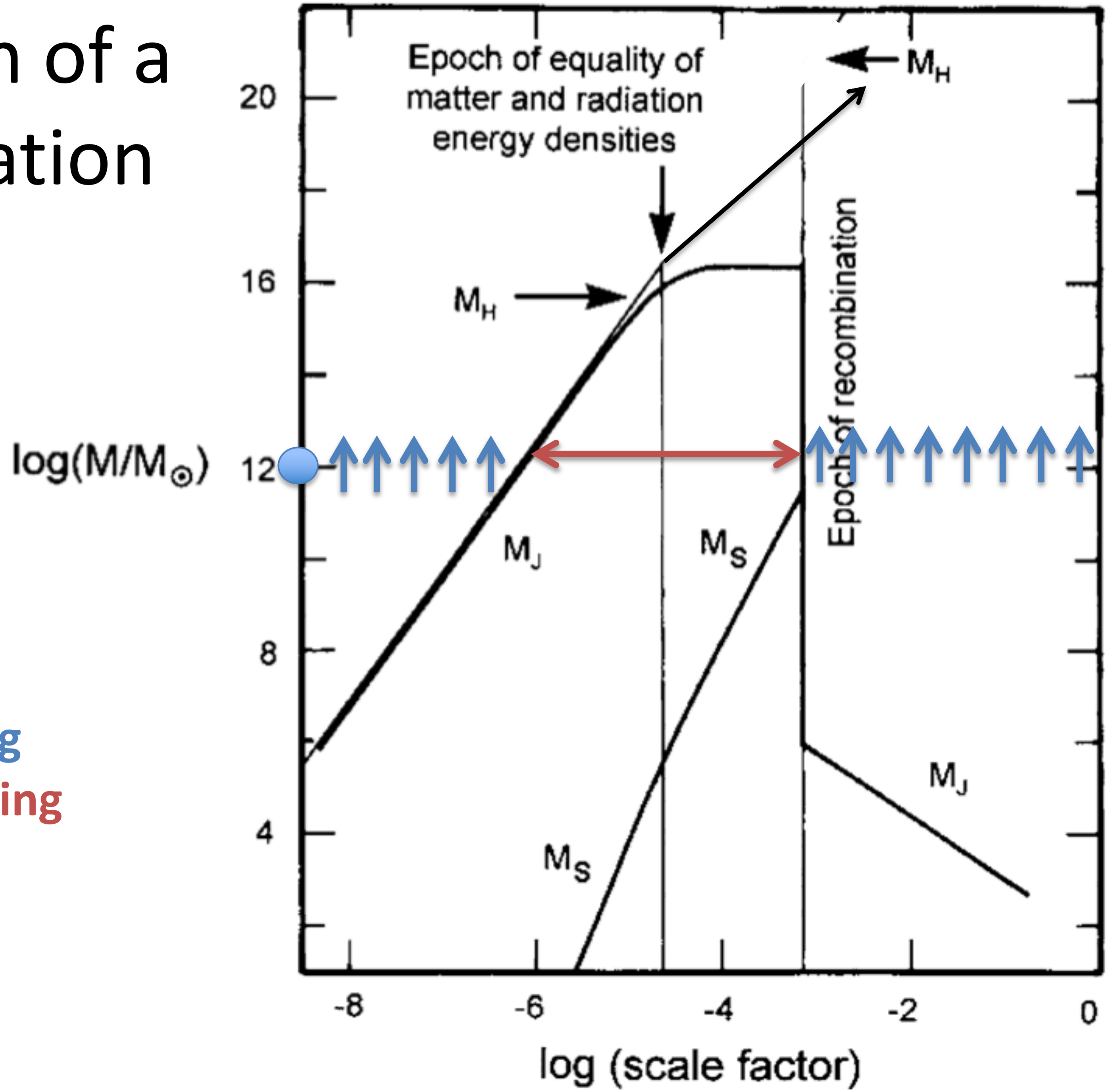
All perturbations with mass > $10^5\ M_{sun}$ begin growing, $\Delta \propto a(t)$.

$M_{\text{globular cluster}} \approx 10^5\ M_{sun}$

Evolution: $T \propto a^{-2}$; $c_s \propto 1/a$ → $\lambda_J \propto a^{-1/2}$

→ $M_J \propto a^{-3/2}$

Evolution of a Perturbation



Growing
Oscillating

Silk Damping

Photon-baryon coupling not perfect,
maintained by Thompson scattering of e^- and γ.

Mean free path of photons: $\lambda_T \approx 1 / n_e\sigma_T$

Diffusion theory: $D=\lambda_T c/3$

$\lambda_D \approx (Dt)^{1/2} = (\lambda_T ct/3)^{1/2}$

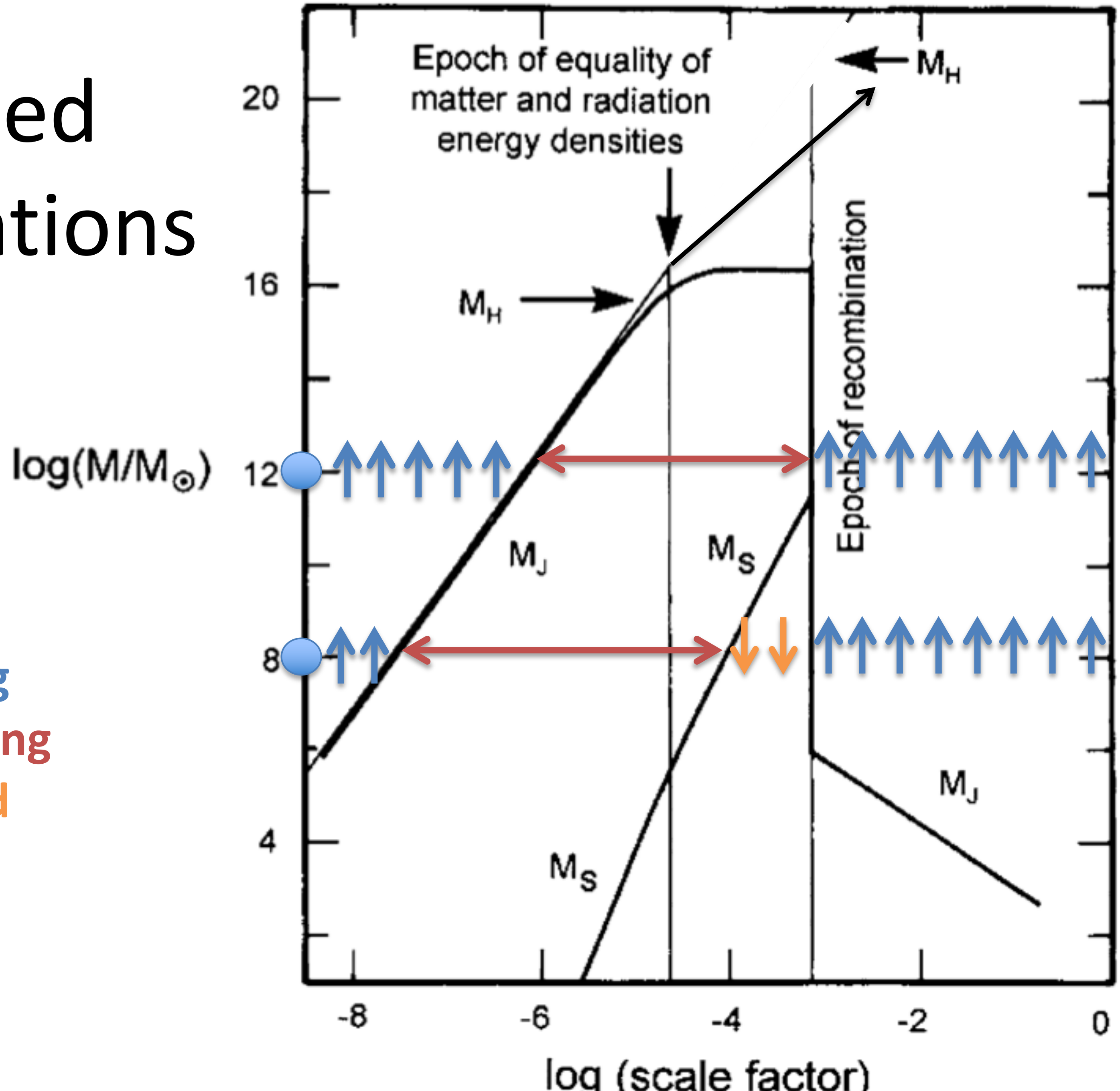
During rad domination, $t \approx 3\pi G\rho_{rel} / 32 \approx 2.4 \text{x} 10^{19} \text{s}\ a^2$

$n_e \approx \Omega_b\rho_c(1+z)^3 / m_p \approx 2 \text{x} 10^{-7} \text{cm}^{-3} a^{-3}$

Mass enclosed in diffusion radius sets threshold for "Silk" damping

$\rightarrow M_S \approx 3.5 \text{ x } 10^{26} M_{sun}\ a^{9/2}$

Damped Perturbations



Growing
Oscillating
Damped

Other Perturbations

So far, we're only considering adiabatic (curvature) perturbations.

Isothermal perturbations decouple baryonic and radiation densities.

Isocurvature perturbations maintain total density.

Adiabatic	→	constant entropy
	→	all densities rise & fall together
	→	"curvature" perturbations
Isothermal	→	constant temperature
Isocurvature	→	density variations offset each other

INTERMISSION

Perturbations

$$\Delta(r,t) = A \sum e^{ikr} \Delta_k(t)$$

$$\boxed{\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right)}$$

Wavenumber,
Not curvature!

Overdensities stable if smaller than Jeans length λ_J / mass M_J

Non-relativistic

$$\lambda_J = c_s \left(\frac{\pi c^2}{G\bar{\varepsilon}}\right)^{1/2} \propto a^2$$

Relativistic

$$\lambda_J = c_s \sqrt{\frac{3\pi}{8G\rho}} \propto a^2$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad M_J \approx 4/3\,\pi\,(\lambda_J/2)^3\,\rho_b = \pi\lambda_J^3\rho_b / 6 \propto a^3$$

$M < M_J \rightarrow$ stable (oscillating)

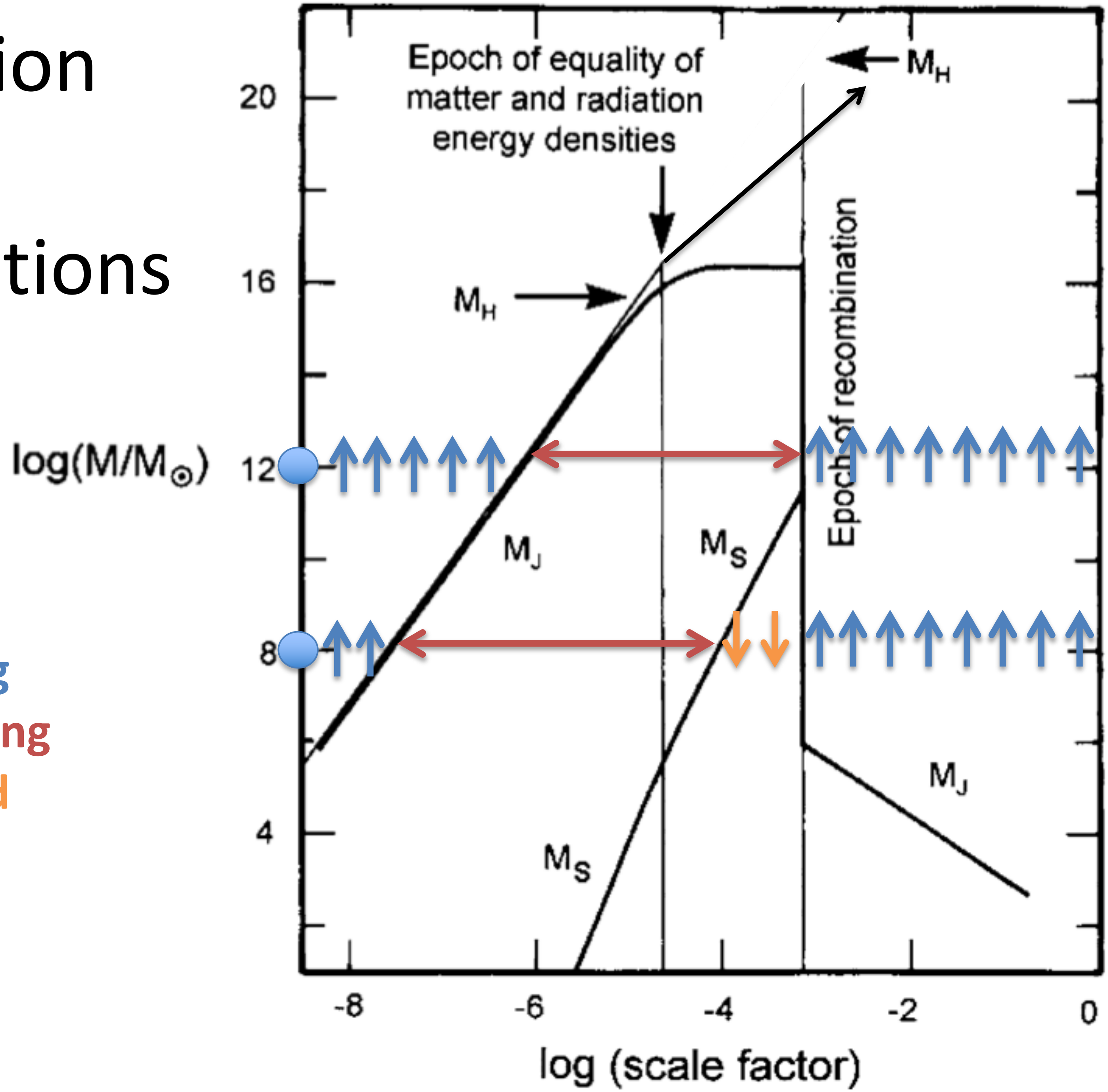
$M > M_J \rightarrow \Delta \propto a^2$

Evolution of Perturbations

Note: linear regime! $\Delta << 1$, so we decompose $\Delta(t) = \sum \Delta_k(t)$

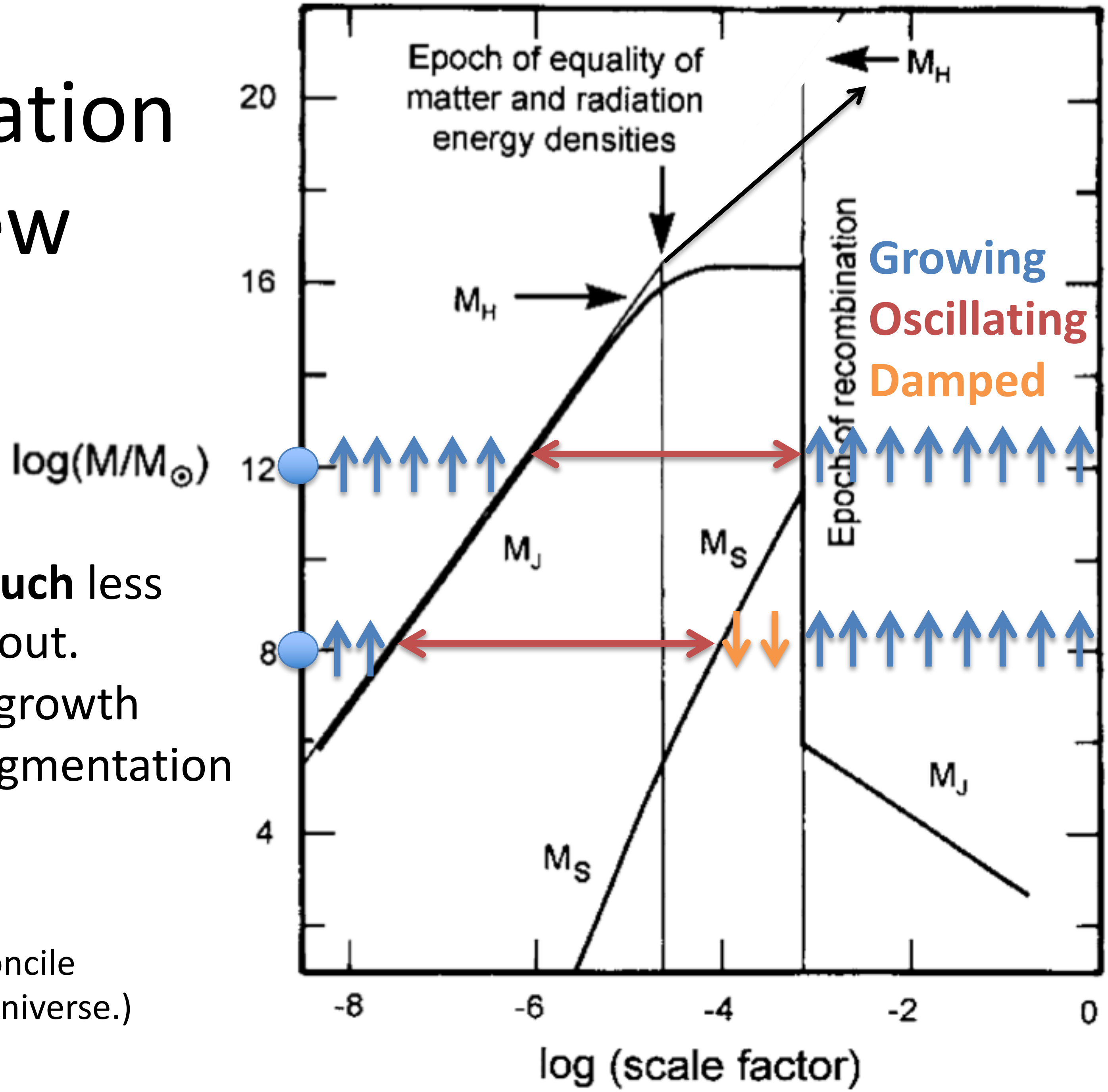
	Ideal ($c_s = 0$)	Classical ($\dot{a} = 0$)	Empty ($a \propto t$)	Radiation Dominated	Matter Dominated	Post Re-combination
λ_J	N/A	$c_s\, t_{dyn}$	∞	a	const	$a^{-1/2}$
M_J	N/A	λ_J^3	∞	a^3	const	$a^{-3/2}$
M_s	N/A		N/A	$a^{9/2}$	$a^{15/4}$	N/A
$\Delta(t)$ $k < 1/c_s$ $M > M_J$	$e^{\pm t/\tau}$	$e^{\pm t/\tau}$	a^0 a^{-1}	a^2 a^{-2}	a $a^{-3/2}$	a
$\Delta(t)$ $k > 1/c_s$ $M < M_J$	N/A	$e^{\pm iwt}$	N/A	osc	osc	osc

Evolution of Perturbations



Growing
Oscillating
Damped

Perturbation Review



Small Δ evolve **much** less and are damped out.

→ "Top-down" growth

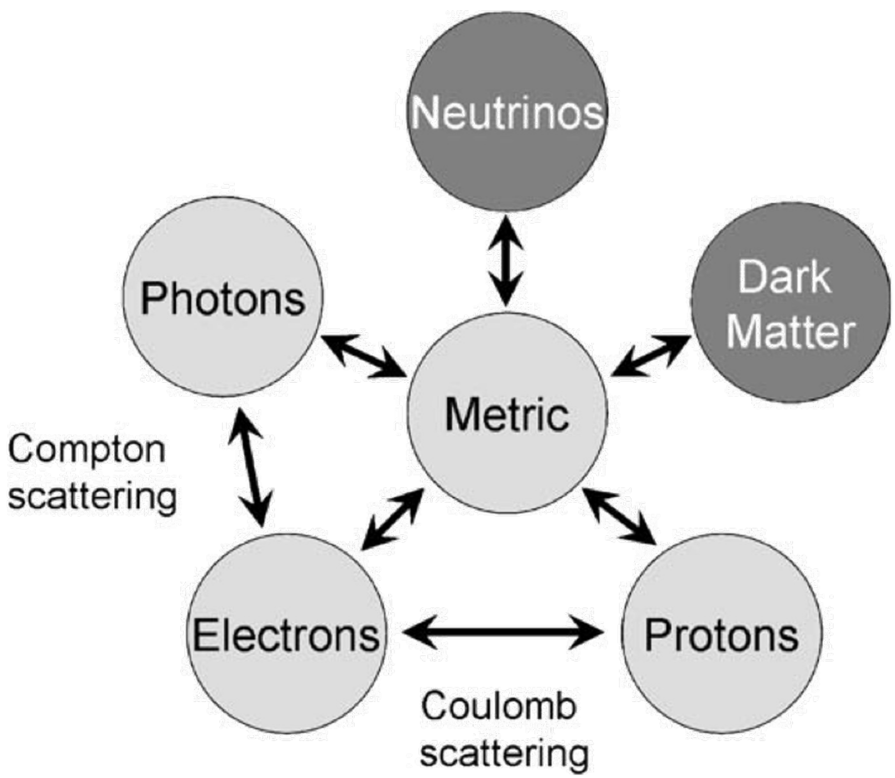
→ Driven by fragmentation

(This is tricky to reconcile with the observed Universe.)

The Missing Bit

We considered structure in a world we understand, baryons + radiation.

But it doesn't look right.



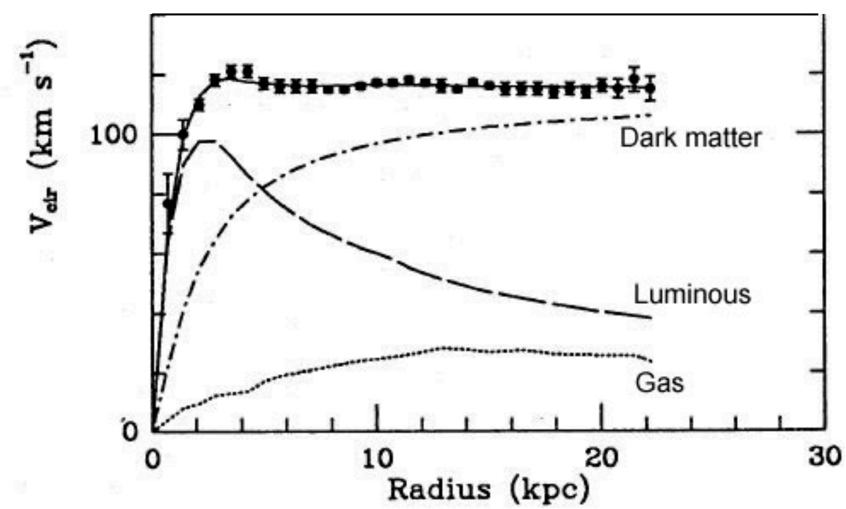
We need to include the dark matter!

Evidence for Dark Matter

If all mass is in stars and gas, expect M/L ≈ 2-3
(where M_{sun} / L_{sun} == 1)

Way back in 1930s
→ Oort: galactic plane stars have >> escape velocity
→ Zwicky: Coma cluster M/L ≈ 50x too high for n_{gal}
→ Babcock: rotation curve in Andromeda…

Fast-forward to 1960s
→ Volders, Rubin: Galaxies definitely rotate wrong…



More M/L Evidence for DM

Virial Theorem:

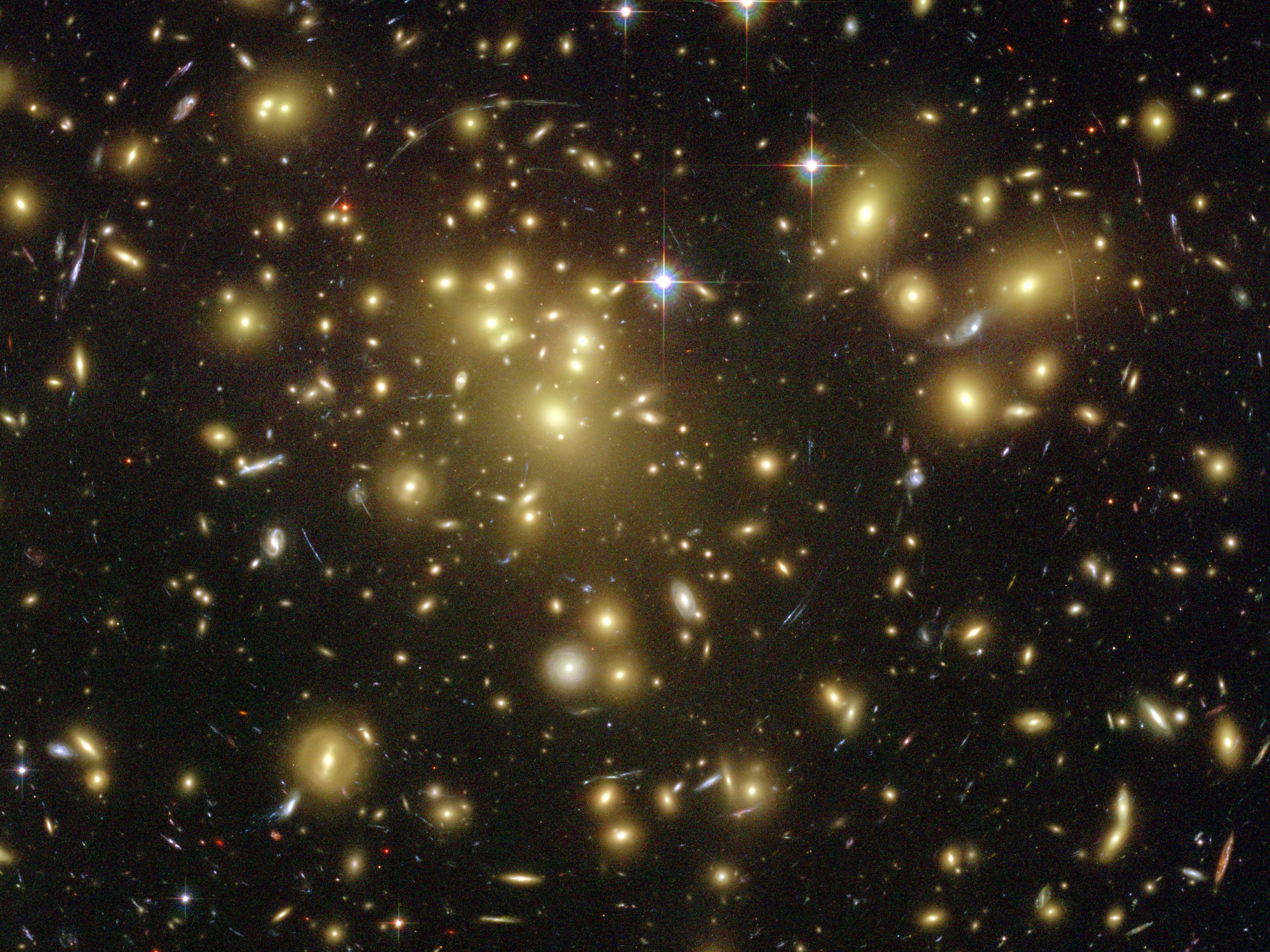
Look at galaxy clusters, measure velocity dispersion

→ $GM/R \approx 3\sigma^2$

Gravitational Lensing:

Can no only give the mass, but a detailed profile.

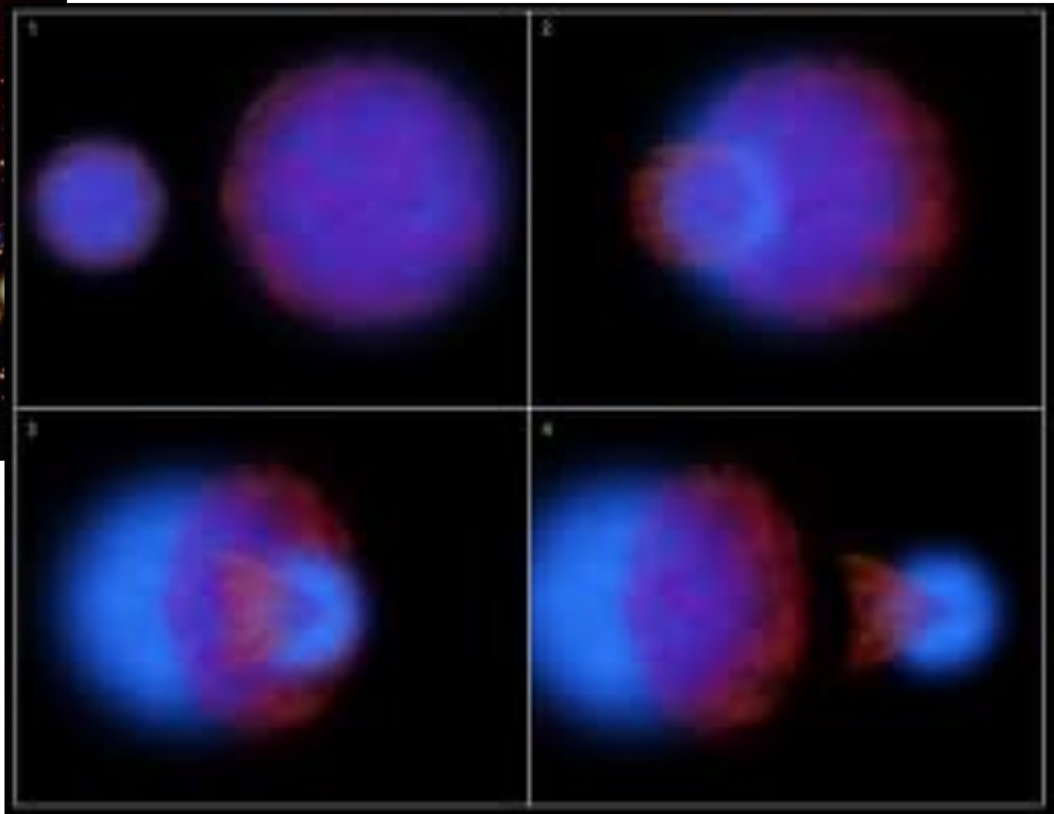
Typical $M/L \approx 200$ (remember: baryons predict ≈ 2-3)



The Bullet Cluster

Blue: WL potential (DM)
Red: Hot gas (Baryons)

Model of Bullet Interaction



Neutrinos?
(Hot Dark Matter)

If DM relativistic when it freezes out, called “hot” DM.

Evolves as radiation that is not coupled to baryons.

“Leaks” out of halos, damping analogous to Silk damping.

$$r_{\mathrm{FS}} = \int_0^t \frac{v(t')}{a(t')}\,\mathrm{d}t'$$

→ $M_{FS} \approx (m_\nu c^2/\mathrm{eV})^{-2}\ 4 \times 10^{17} M_{sun}$

→ All but the very very largest structures are quickly damped out once oscillations begin. (Like Silk!)

DM Perturbations

Recall the evolution of matter, in the case of $\lambda >> \lambda_J$

$$\ddot{\Delta} + 2\frac{\dot{a}}{a}\dot{\Delta} = \Delta\left(4\pi G\bar{\rho} - \cancel{\frac{c_s^2 k^2}{a^2}}\right)$$

Include two components to the density, baryonic and dark:

$$\ddot{\Delta}_\mathrm{B} + 2\left(\frac{\dot{a}}{a}\right)\dot{\Delta}_\mathrm{B} = A\varrho_\mathrm{B}\Delta_\mathrm{B} + A\varrho_\mathrm{D}\Delta_\mathrm{D}$$

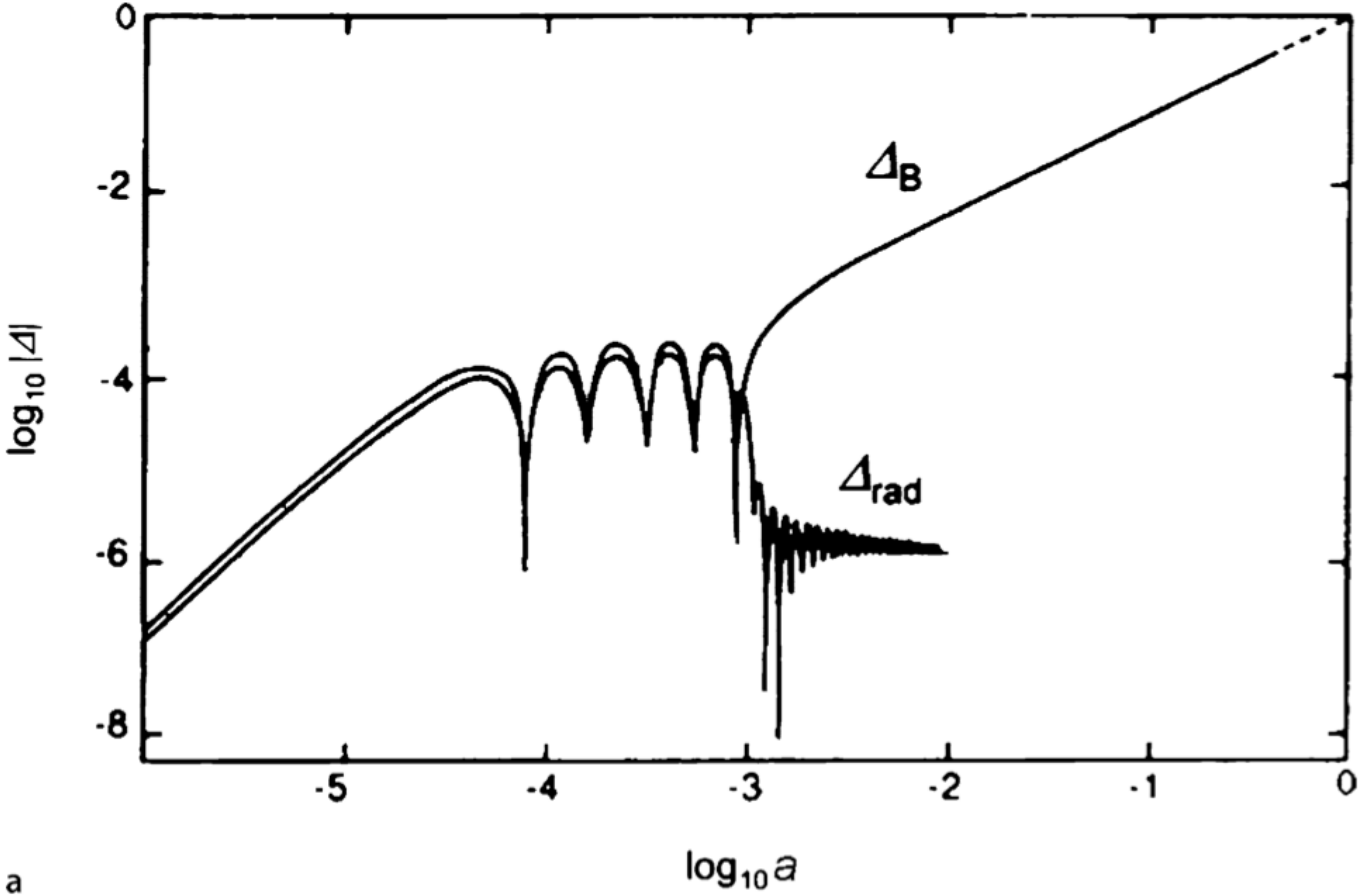
≈ 0

$$\ddot{\Delta}_\mathrm{D} + 2\left(\frac{\dot{a}}{a}\right)\dot{\Delta}_\mathrm{D} = A\varrho_\mathrm{B}\Delta_\mathrm{B} + A\varrho_\mathrm{D}\Delta_\mathrm{D} \rightarrow \Delta_D \propto a(t)$$

Plugging in and solving, $\Delta_B \propto \Delta_D\,(1-a_{ref}/a)$

→ Δ_B rapidly approaches Δ_d
→ baryons “fall in” to DM wells

Growth without Dark Matter



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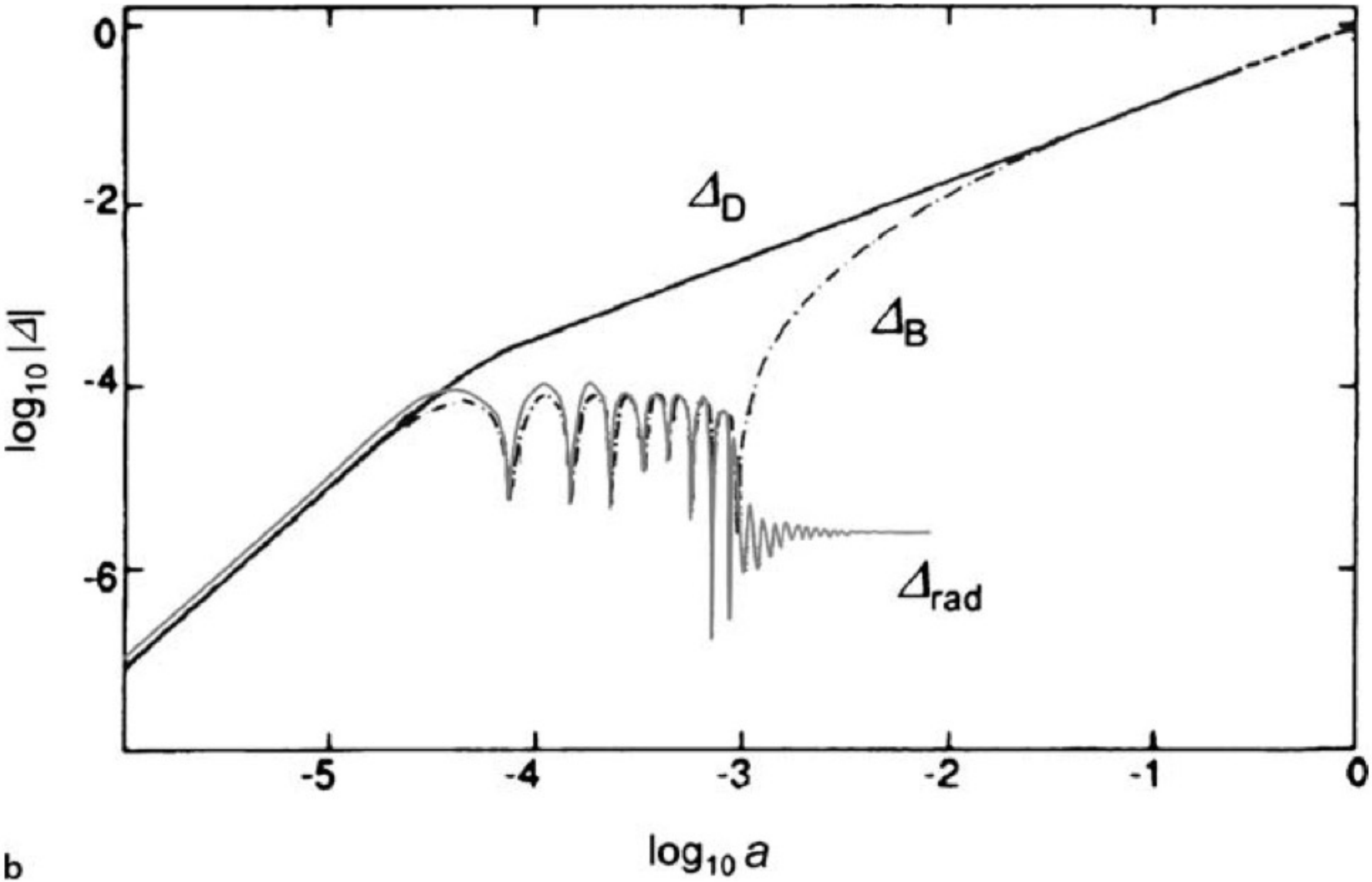
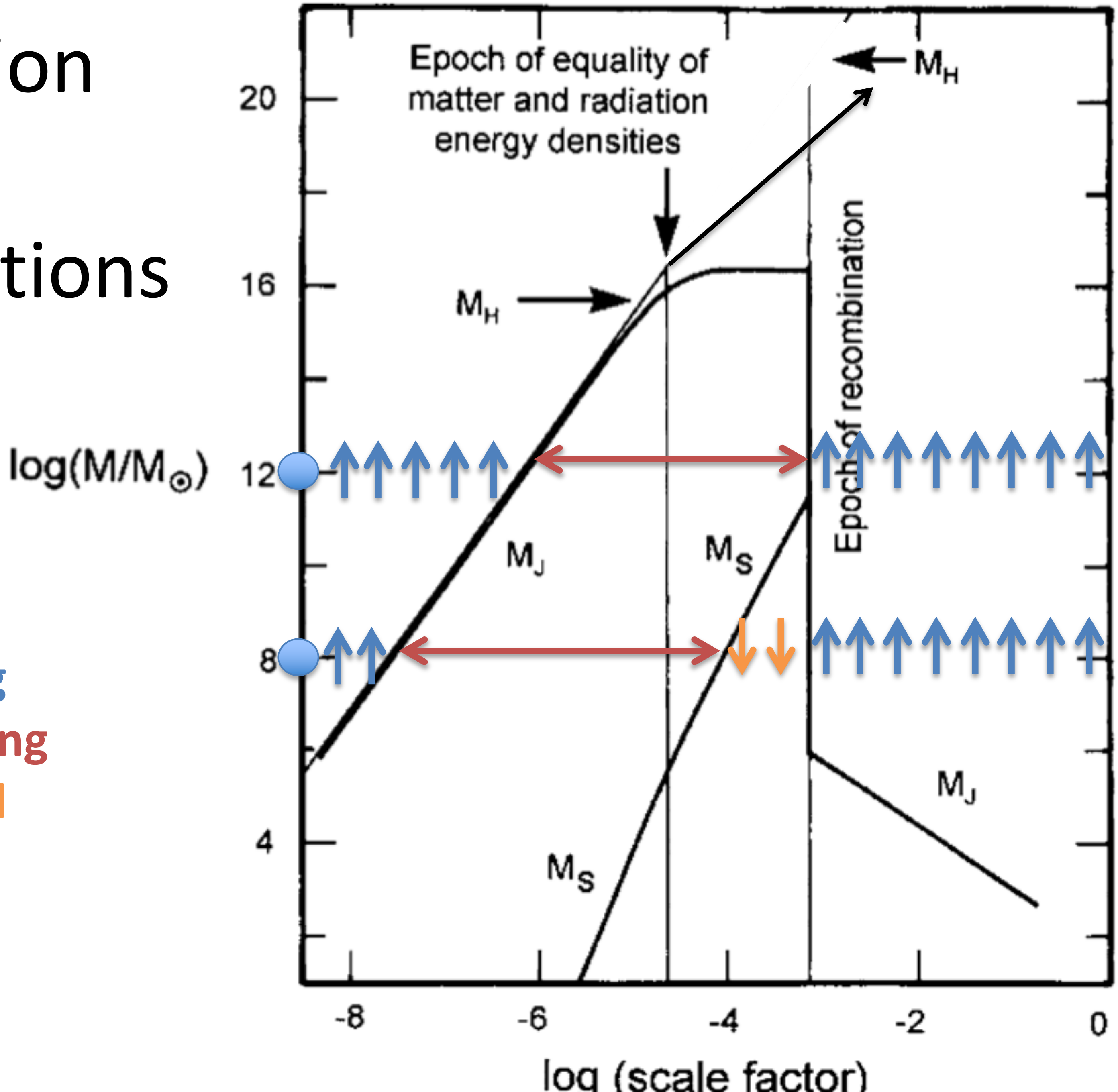
Growth with Dark Matter



Evolution of Perturbations



Growing
Oscillating
Damped

Power Spectra

So far, only considered a single mode Δ_k at at time.

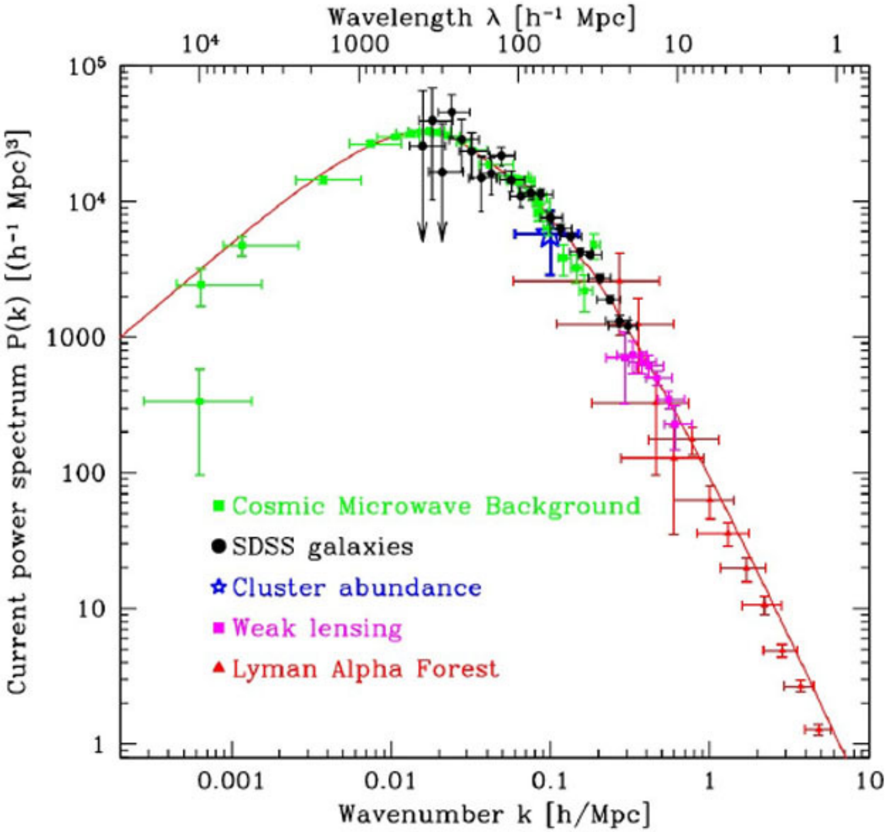
Instead of looking at a single mode over time,
look at fixed time over modes.

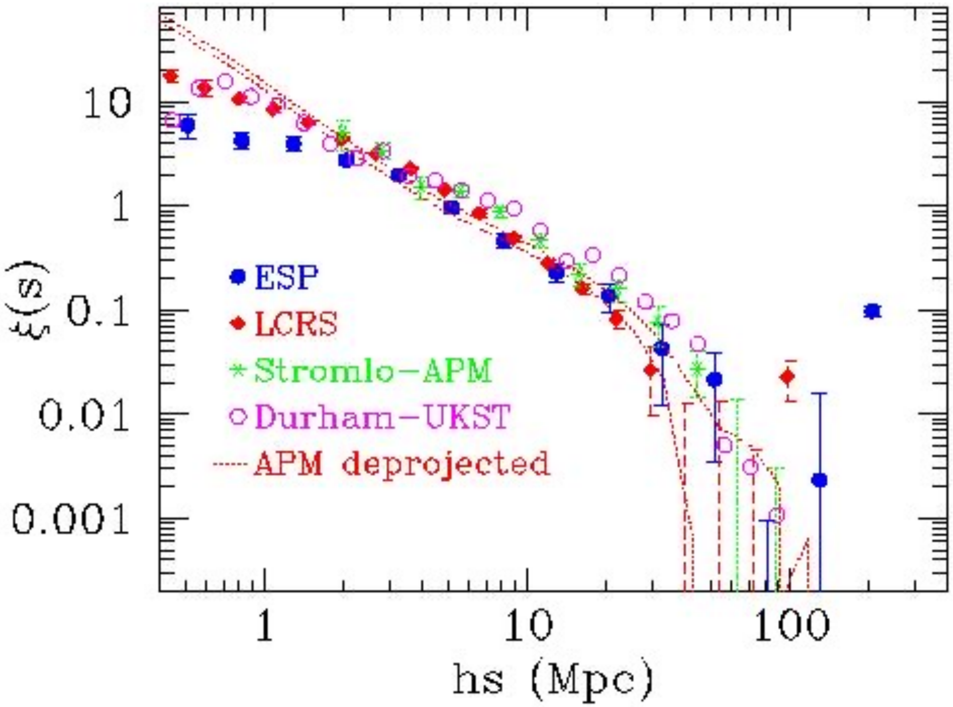
The **power spectrum** $P(k) = |\Delta_k|^2$ shows the power in each mode.

2-pt **correlation function** $\xi(r) = \dfrac{\frac{1}{V}\int_V n(x)\cdot n(x+r)d^3x}{\frac{1}{V}\int_V n^2(x)d^3x}$ $= \langle\Delta(x)\Delta(x+r)\rangle$

$= \sum_k |\Delta_k|^2 e^{ikr}$

Measured Spectra





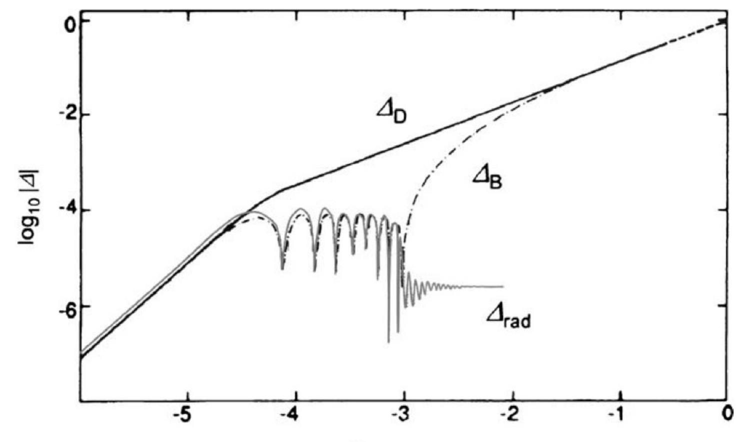
Growth and Transfer Function

Recall: we decomposed $\Delta(r,t) = A \sum e^{ikr} \Delta_k(t)$

We calculated the time evolution of $\Delta_k \propto a$ after recombination.

That also gives us the time evolution of $P(k) = |\Delta_k|^2$

- For the first while (rad dom), Δ_k grows as a^2
- After entering horizon (dropping below Jeans threshold), DM osc damped by freestreaming
- After recombination, Δ_k grows linearly with a



b

The **growth function** f(z) gives the growth rate, $\propto$ a.
The **transfer function** T(k) allows insertion of scale dependence.

$$\Delta_k(z=0) = T(k)f(z)\Delta_k(z)$$

Initial Spectrum

Set by the particulars of inflation & QM.

We assume a power law, $P(k) = |\Delta_k|^2 \propto k^n$

$n = 1$ → Harrison-Zeldovich, constant power
"Perturbations have same amplitude when they cross the particle horizon" (≈ Jeans mass)

Bias

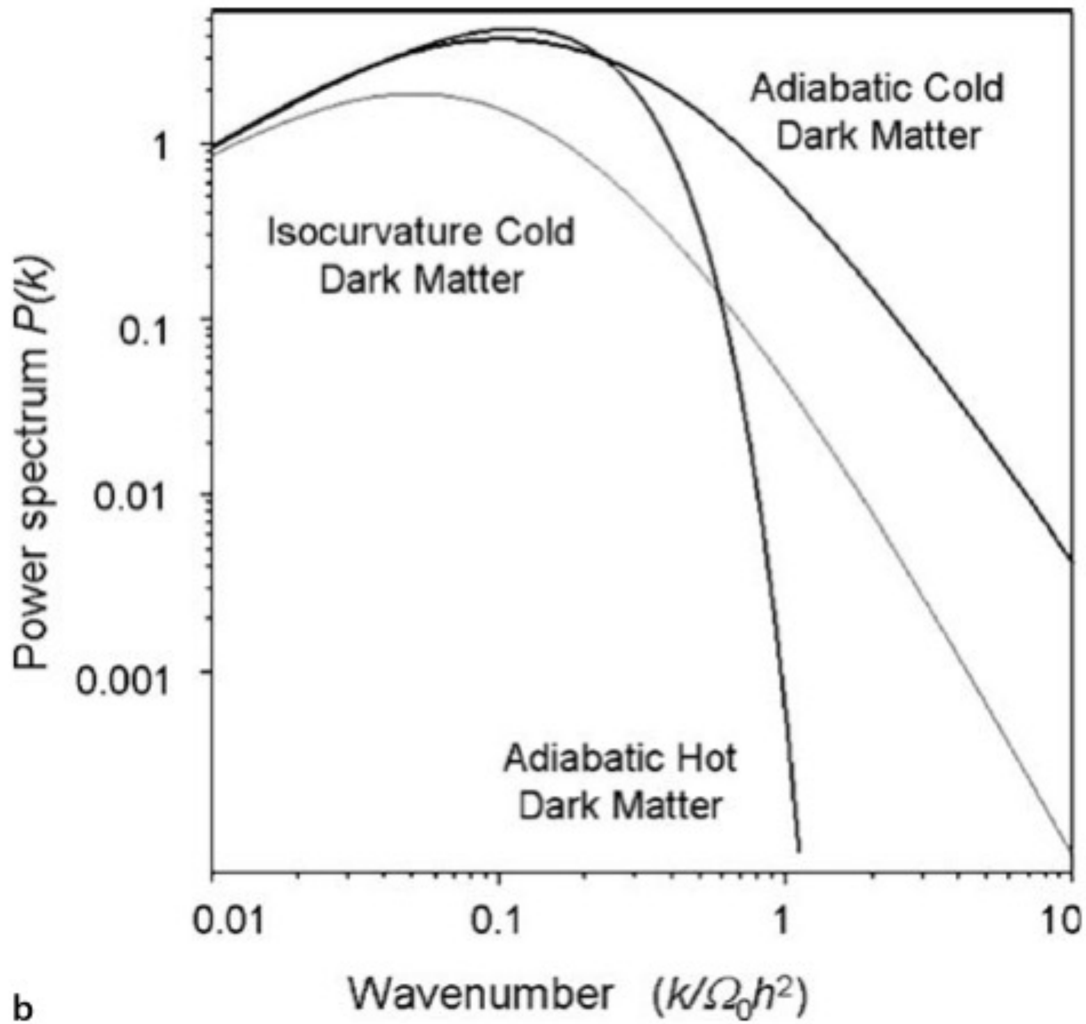
We can only observe baryons, may not perfectly trace DM halos.

Characterized by the "galaxy bias" parameter b ≈ 1:

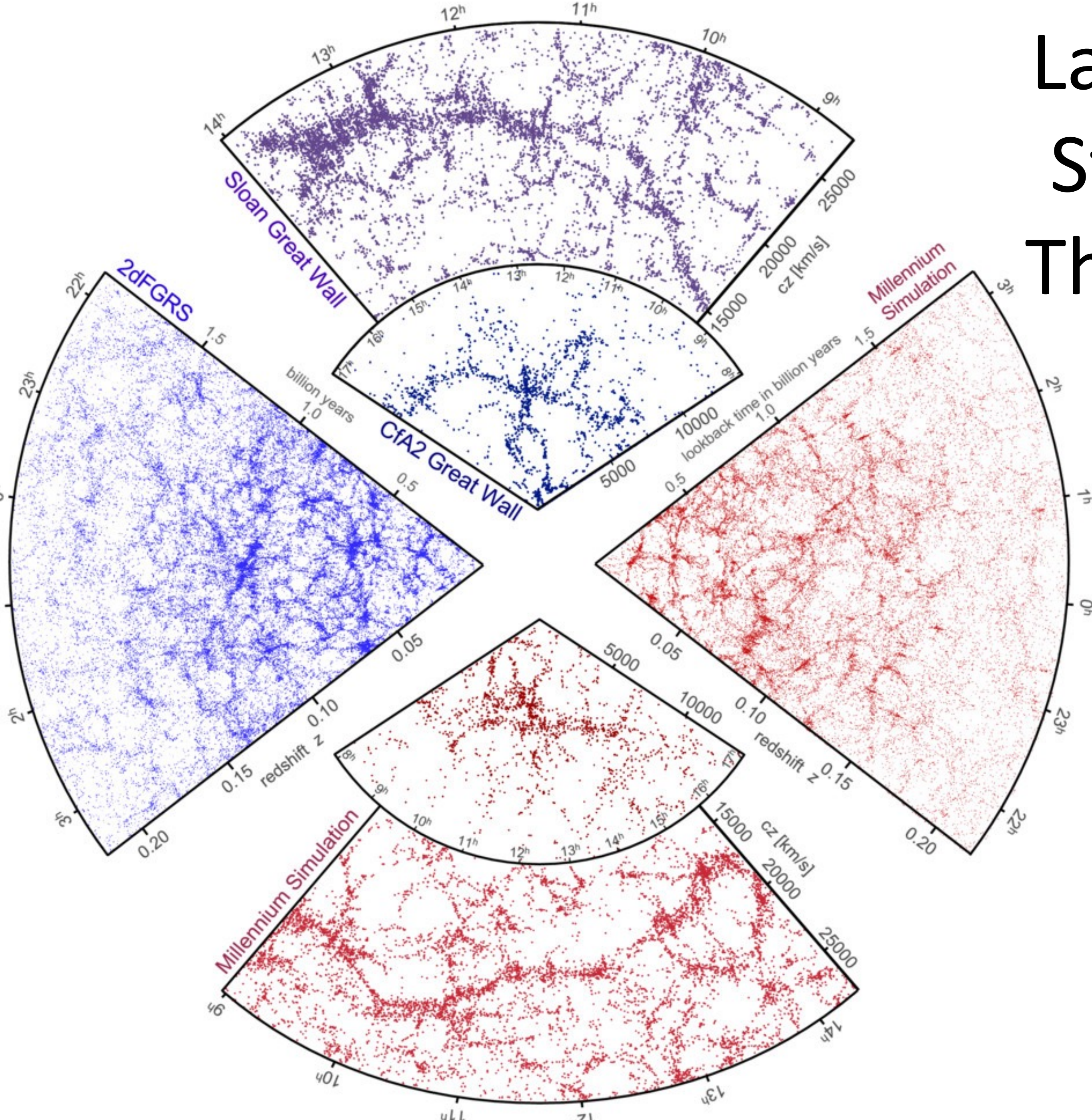
$$\Delta_{gal} = b\,\Delta_D$$
$$P_{gal}(k) = b^2\,P_D(k)$$
$$\xi_{gal}(r) = b^2\,\xi_D(r)$$

Must be accounted for when observing structure indirectly!

Distinguishing Cosmologies



Large Scale Structure: The Cosmic Web



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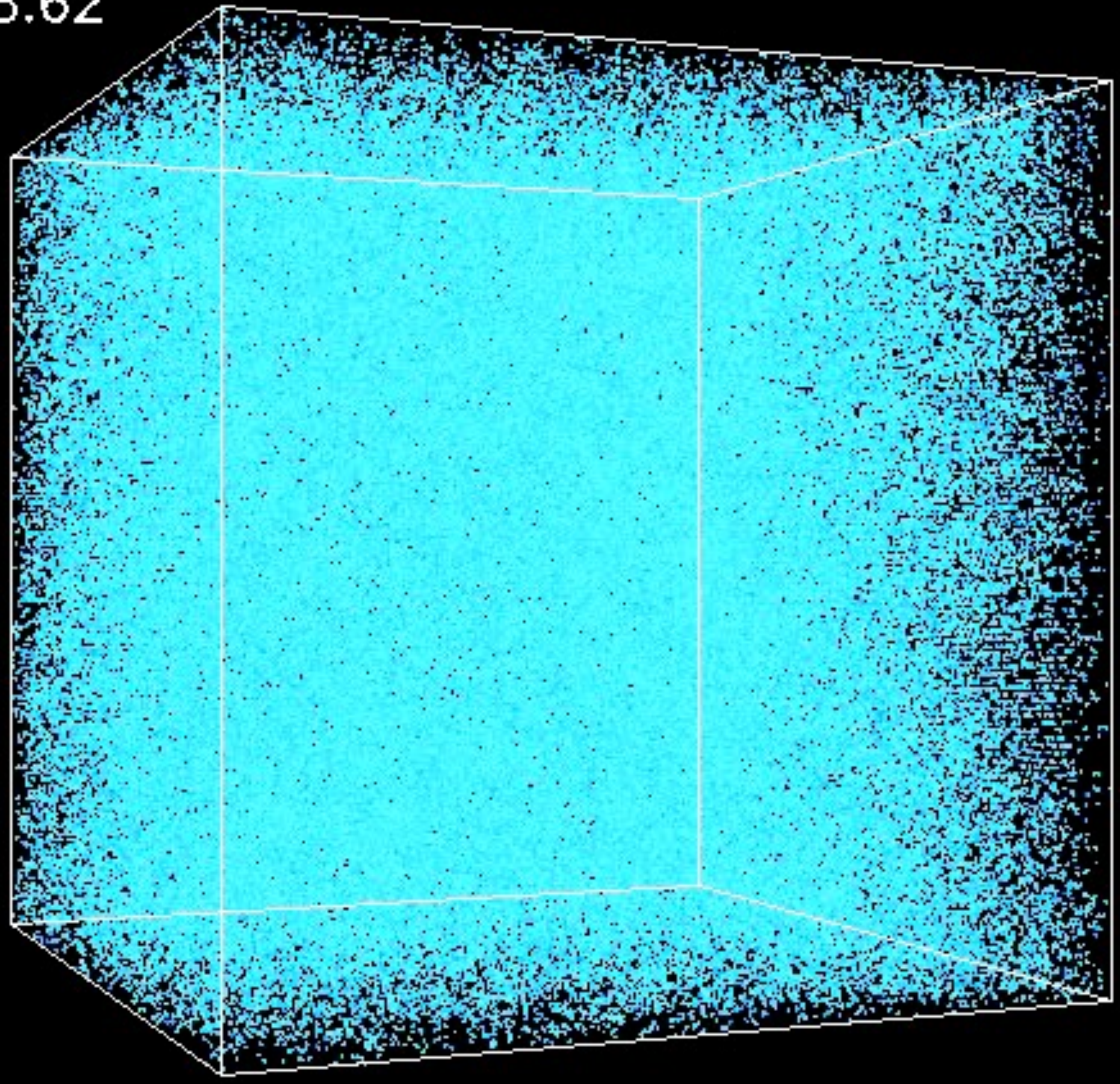

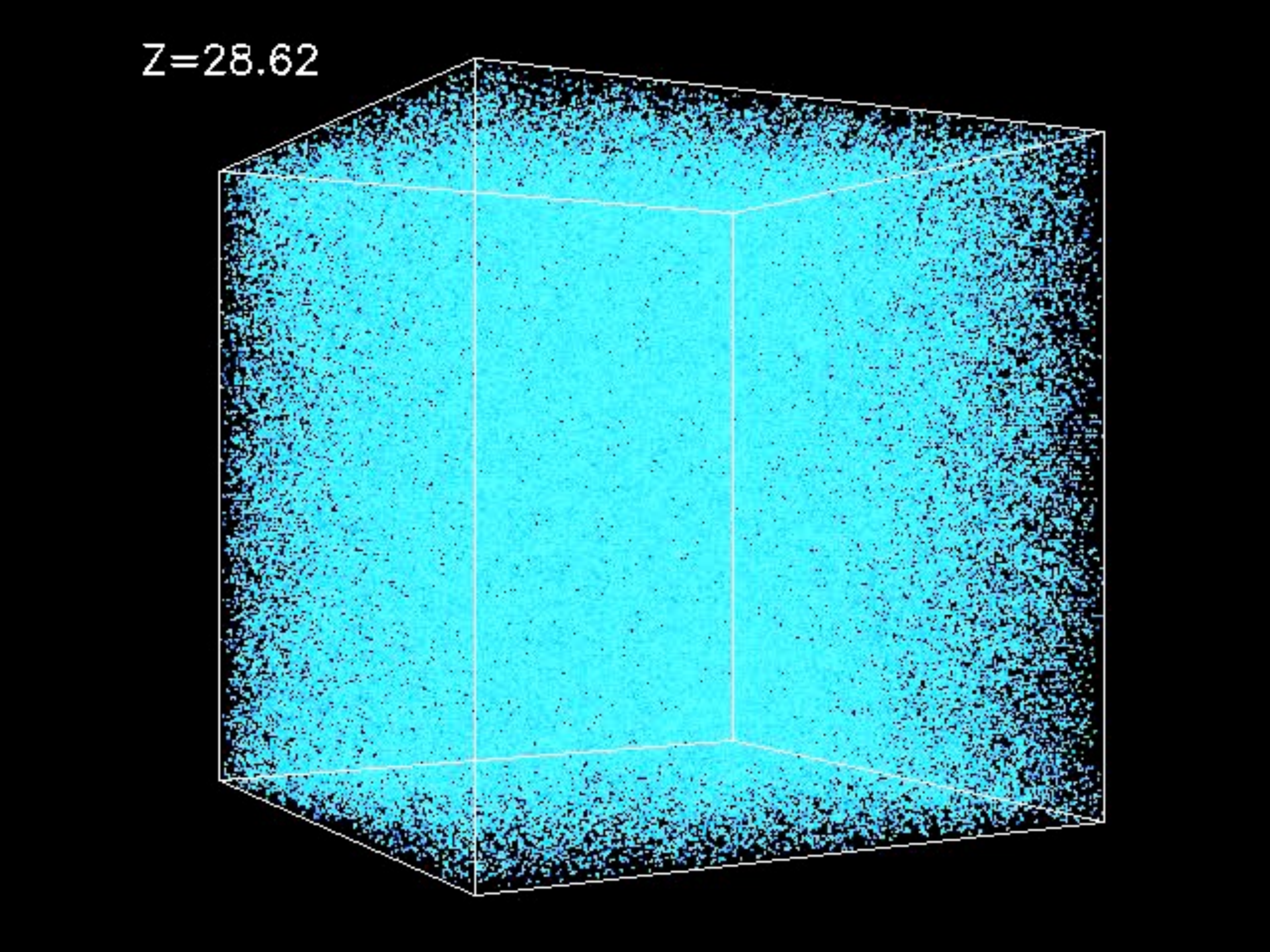
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Next Time:

CMB