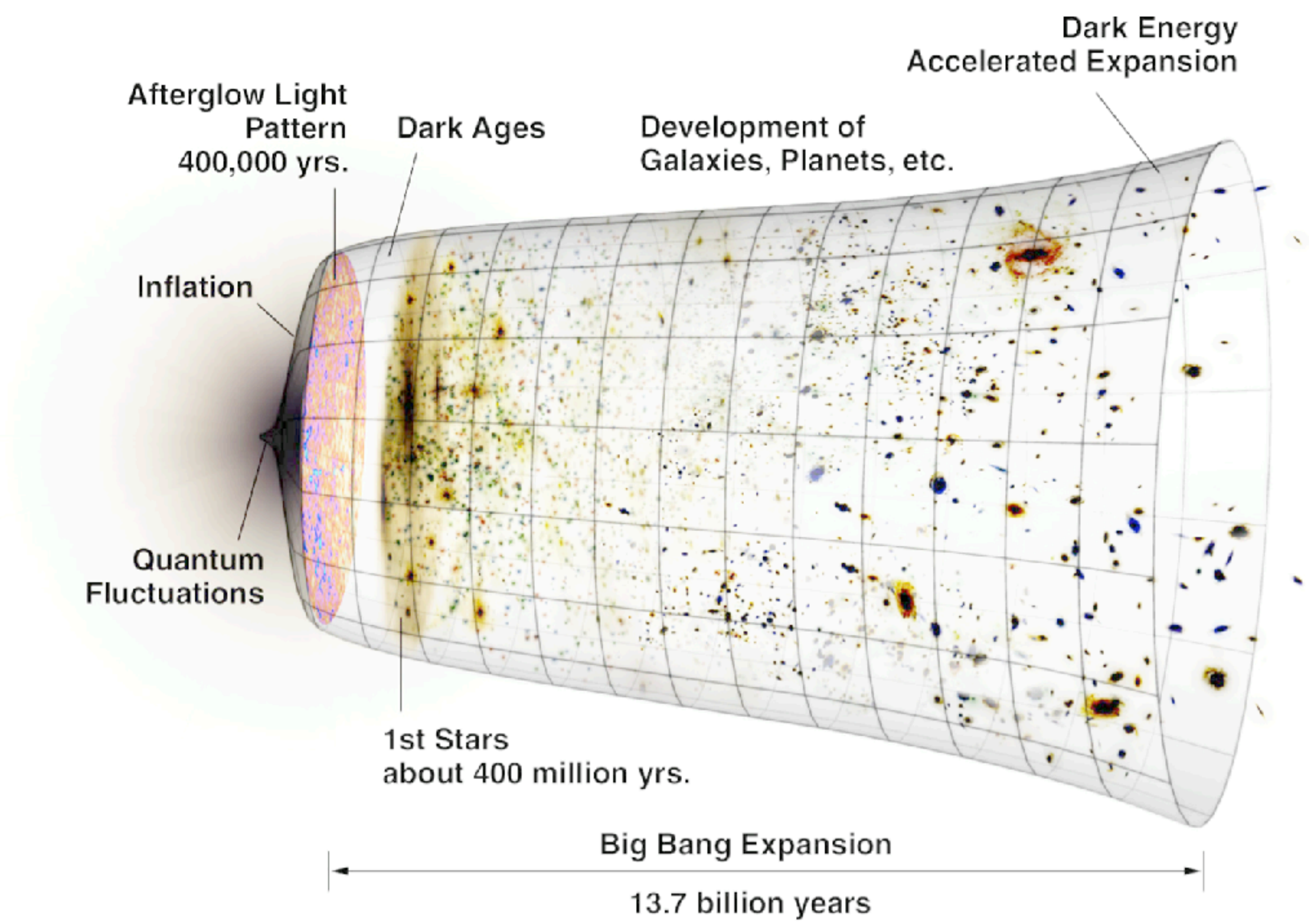

AST 1430
Cosmology

Review

Context

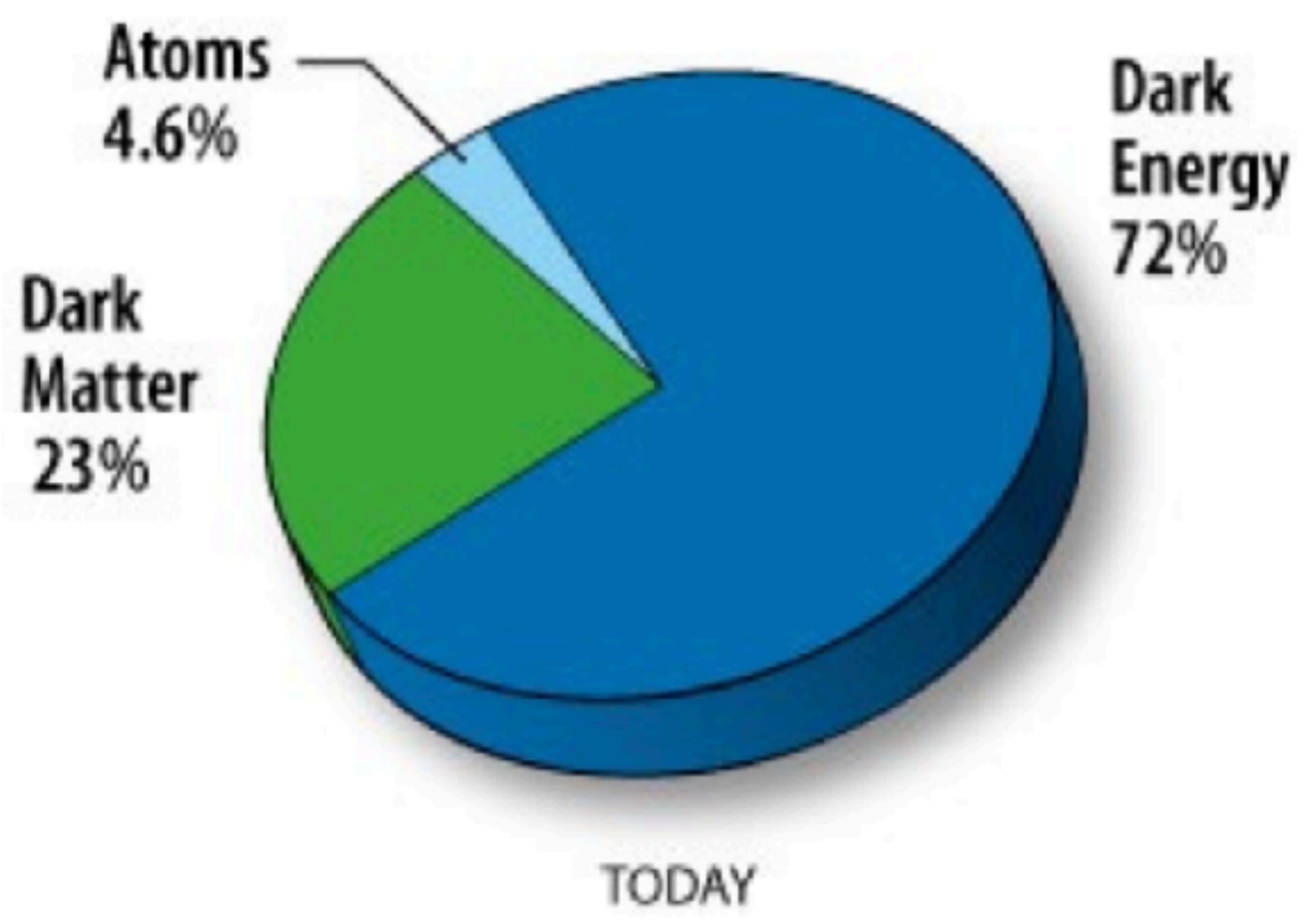


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Physical Cosmology

Concordance (ΛCDM) Model:

- The Universe began with a Big Bang, about 13,800,000,000 years ago
- The Universe is expanding
- The Expansion is accelerating
- The Universe is full of weird stuff



We Don't Know:

- What dark matter is, but have several ideas
- What dark energy is, and don't have much data about it

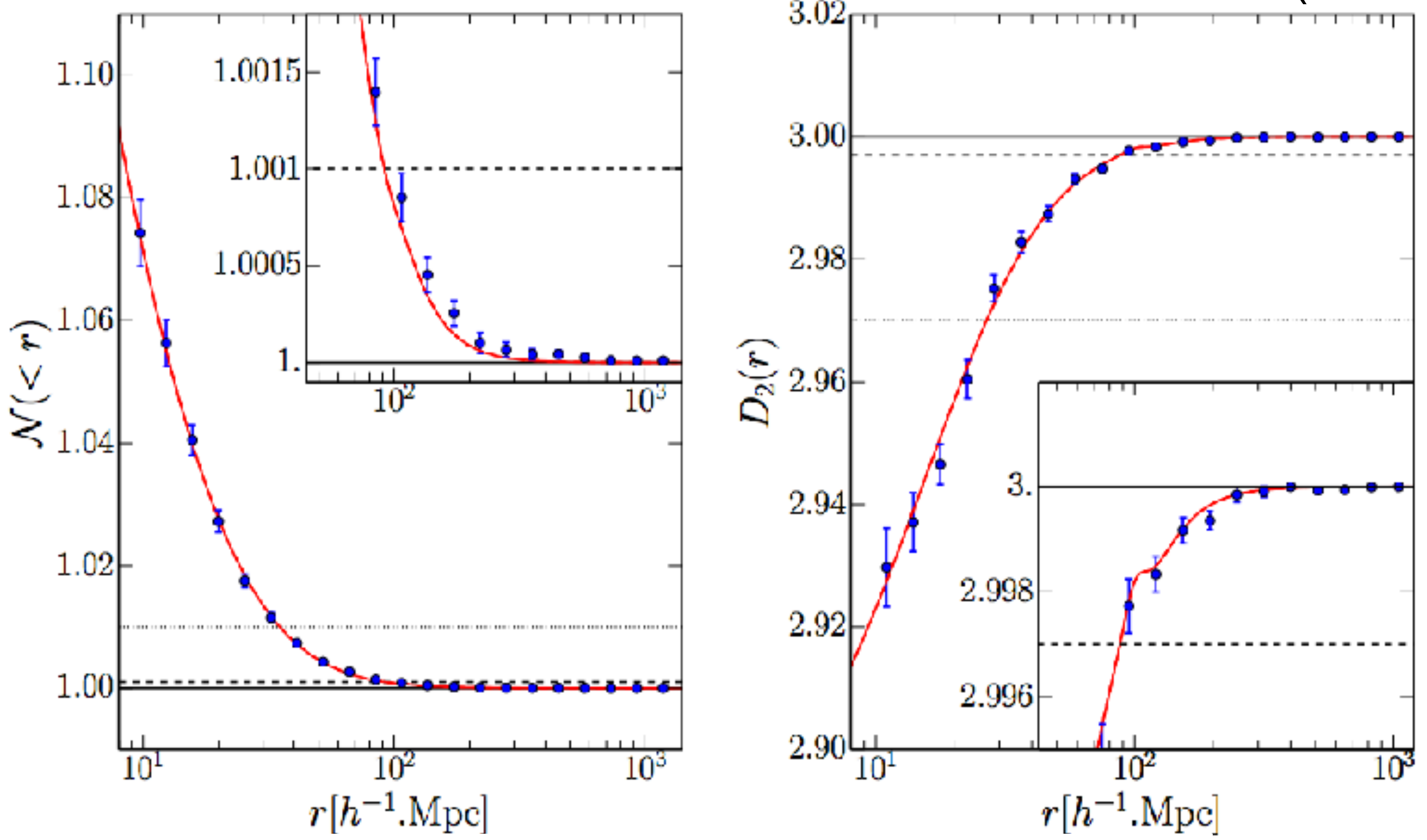
(Taking our best guess, a vacuum energy required to create empty space, when we try to calculate how big it should be from QM, the answer, known as "the worst prediction in physics" is off by a factor of roughly **1,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000** times.)

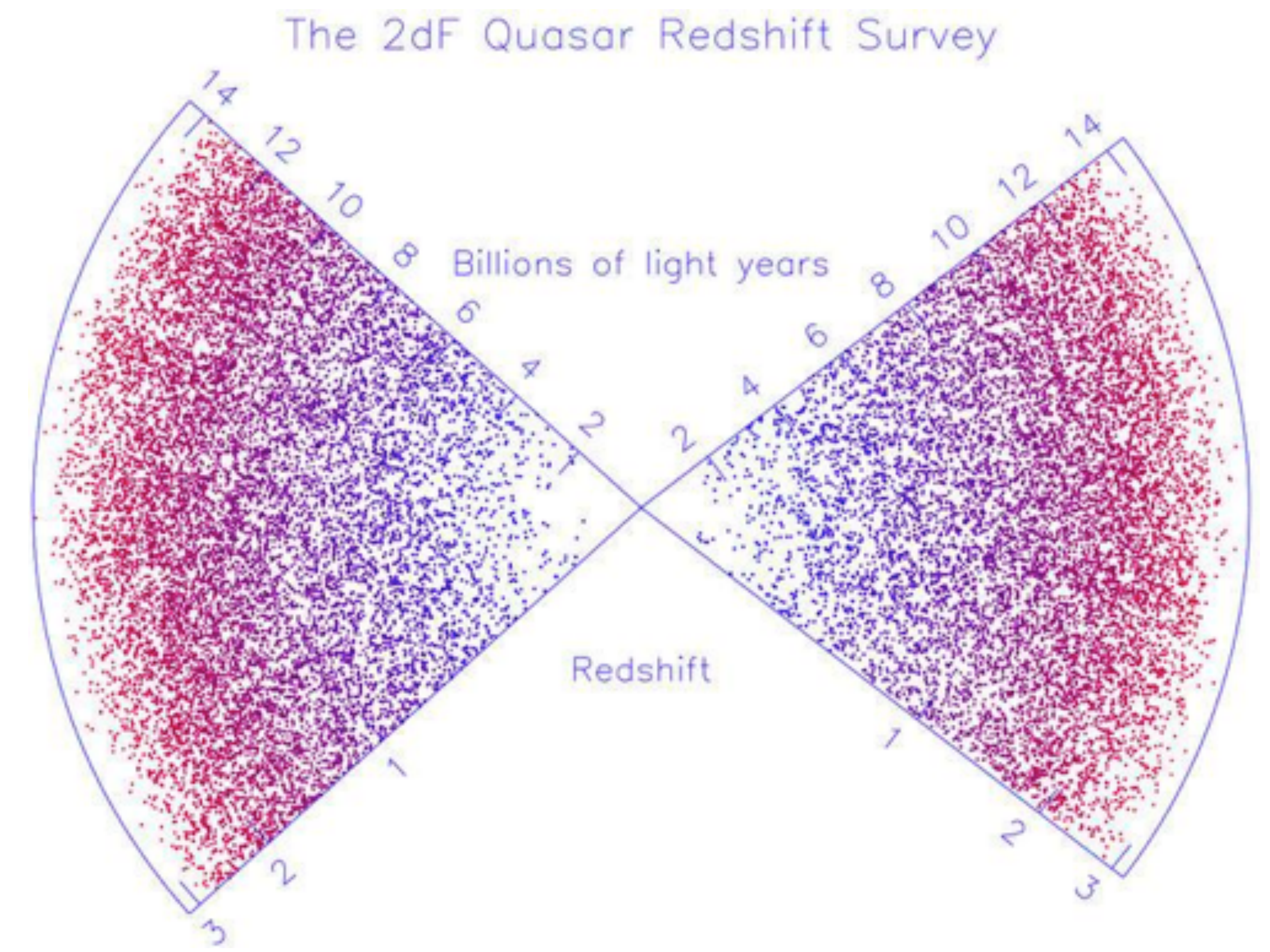
Basic cosmology observations

The Universe is homogeneous and isotropic

- Bedrock principle of modern cosmology
- Homogeneous: the Universe is (statistically) the same at any point in space
- Isotropic: the Universe is (statistically) the same in all directions
- On large scales!
- Scale of homogeneity: ~100 Mpc, well measured these days

Laurent et al. (2016)





Olbers' paradox

- Why is the sky dark?
- In infinite Universe, every line-of-sight ends on star similar to the Sun
- Surface brightness is conserved
- Thus, sky should be as bright as the surface of the Sun!
- Adding dust extinction does not help; would heat up to same temperature as a star —> same brightness!
- Conclusion: Universe has a finite age (Edgar Allen Poe)

Other fundamental cosmological observations

- Ages of oldest globular clusters in the Milky Way ~ 12 to 13 Gyr
- Cosmic Microwave Background has the same temperature everywhere to 1/10,000; spectrum is black-body
- Hubble's law
- Mass fraction of helium in stars and nebulae ~ 25% —> hot Big Bang

Cosmological models

GR describes the Universe on large scales

- GR is a metric theory of gravity: fundamental quantity is the metric, which gives the distance between points in space-time

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\nu \mathrm{d}x^\nu \, .$$

  - Objects move on geodesics (‘gravity’)
  - Metric is determined by the energy-momentum tensor through the field equations

$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu} \, .$$

The FLRW metric

- Requirement of homogeneity and isotropy leads to the FLRW metric (unique set of possible models)

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left\{\frac{1}{1-kr^2/R_0^2}\,\mathrm{d}r^2 + r^2\,\left[\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right]\right\}$$

- k=-1,0,1 for open, flat, and closed Universe
- a(t) is the scale factor

Distances in cosmology

- Various distances derive from the FLRW metric
- Comoving distance: distance χ at same time with a=1

  Flat: $$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left\{\mathrm{d}\chi^2 + \chi^2\left[\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right]\right\}$$

  k=+1: $$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left\{\mathrm{d}\chi^2 + R_0^2\sin^2(\chi/R_0)\left[\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right]\right\}$$

- Comoving angular diameter distance $D_M(z)$:

$$\begin{cases} \frac{c}{H_0\sqrt{\Omega_{0,k}}}\sin\left(\chi\,\frac{H_0\sqrt{\Omega_{0,k}}}{c}\right), & k=+1 \\ \chi\,, & k=0 \\ \frac{c}{H_0\sqrt{-\Omega_{0,k}}}\sinh\left(\chi\,\frac{H_0\sqrt{-\Omega_{0,k}}}{c}\right), & k=-1 \end{cases}.$$

  - Angular diameter distance: $D_A(z) = a\,D_M(z)$ as in $\theta = r/D_A(z)$
  - Luminosity distance: $D_L(z) = D_M(z)/a$ as in $f = L/(4\pi[D_L]^2)$

Friedmann equations

- Apply field equations to the FLRW metric:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{c^2 k}{a^2 R_0^2},$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda c^2}{3},$$

- Augmented by *equation of state*: p(ρ), generally as $p = w\,\rho\,c^2$

General Fluids, w_i

$$\rho_i \propto a^{-3(1+w_i)}$$

Cold matter:

$w_i = 0 \quad \Rightarrow \quad \rho \propto a^{-3}$

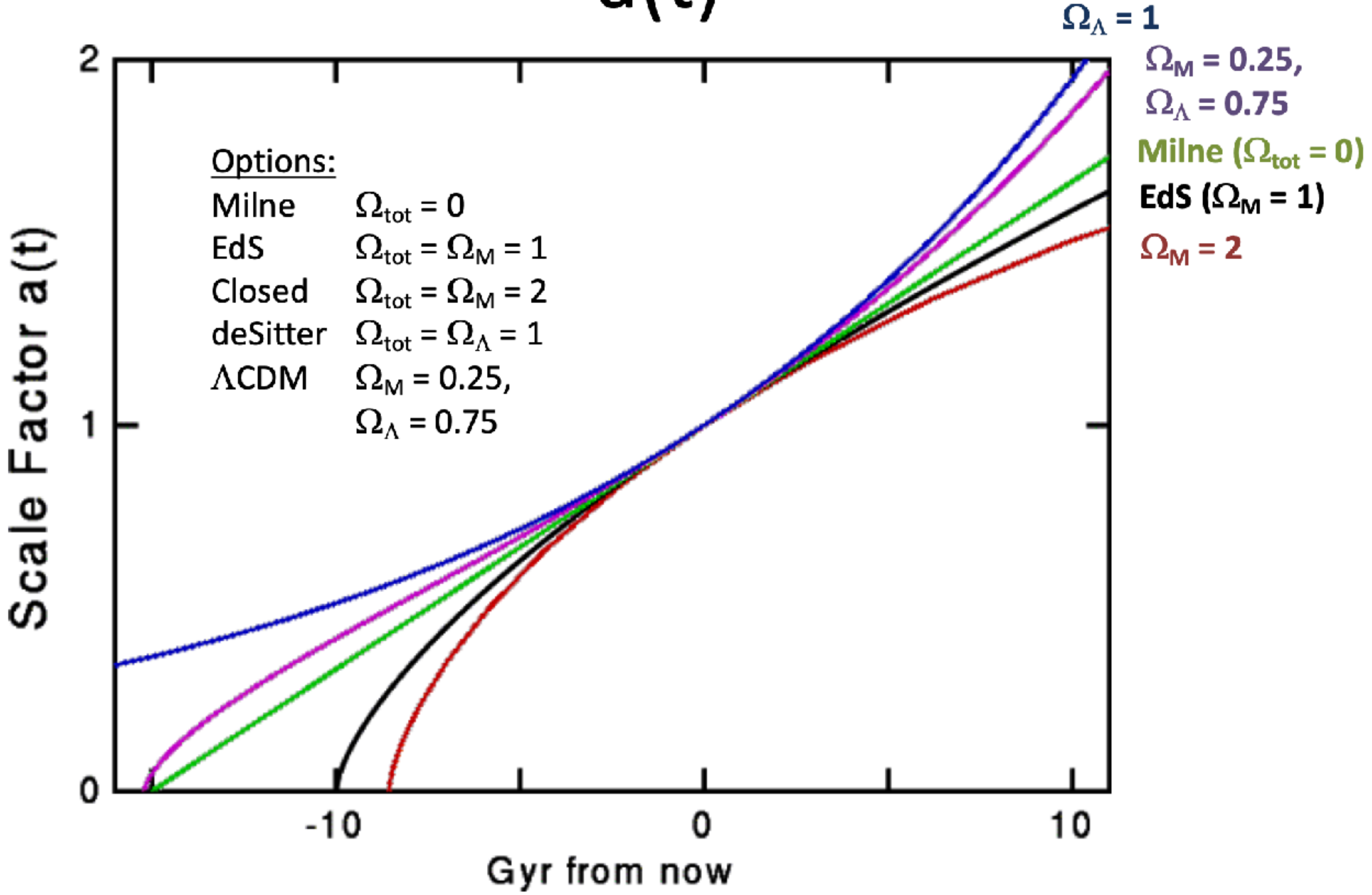
Hot matter / radiation:

$w_i = 1/3 \Rightarrow \quad \rho \propto a^{-4}$

Cosmological Constant Λ:

$w_i = -1 \quad \Rightarrow \quad \rho = \text{const}$

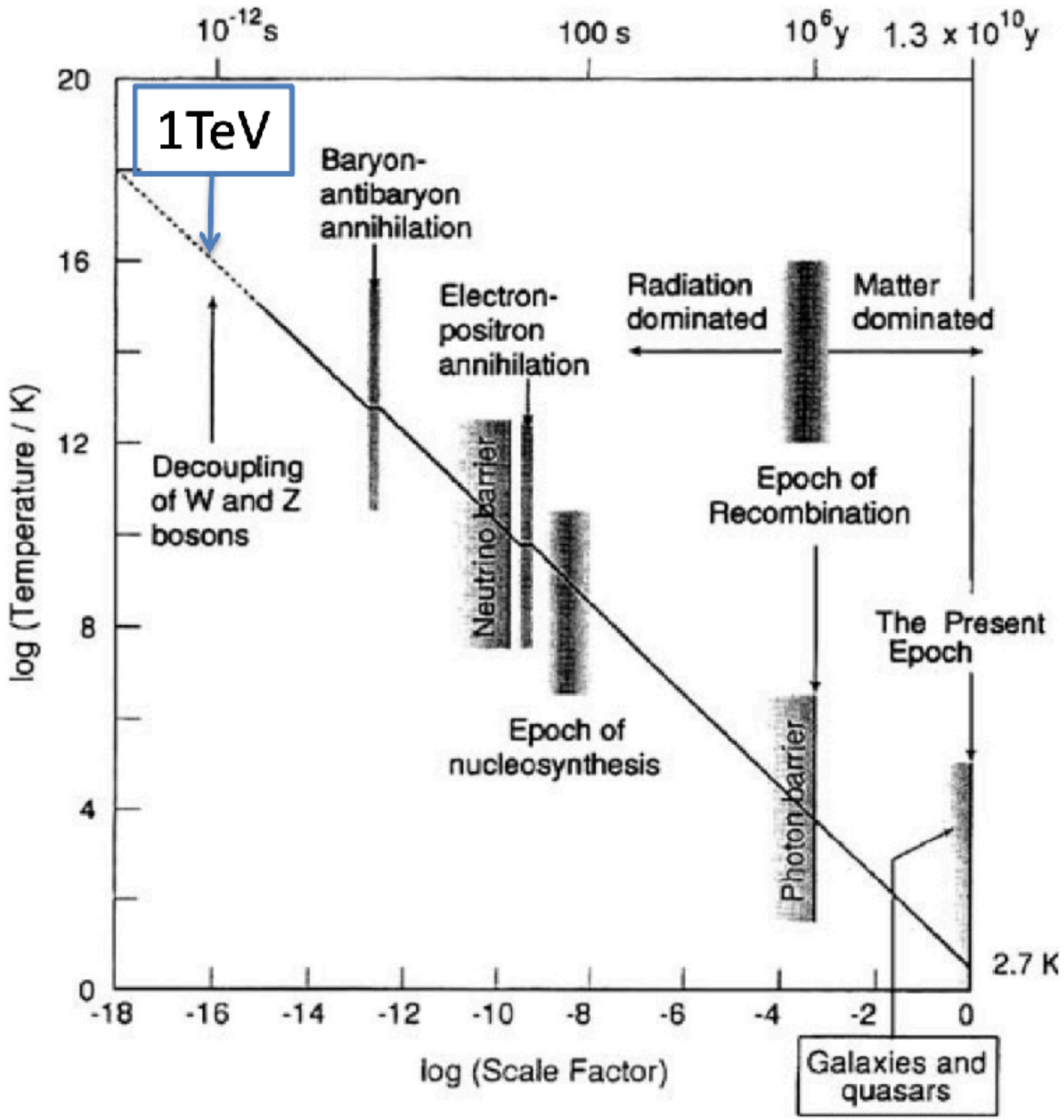
a(t)



Scale Factor a(t)
2
1
0
-10
0
10
Gyr from now
Options:
Milne $\Omega_{tot} = 0$
EdS $\Omega_{tot} = \Omega_M = 1$
Closed $\Omega_{tot} = \Omega_M = 2$
deSitter $\Omega_{tot} = \Omega_\Lambda = 1$
ΛCDM $\Omega_M = 0.25$, $\Omega_\Lambda = 0.75$
$\Omega_\Lambda = 1$
$\Omega_M = 0.25$, $\Omega_\Lambda = 0.75$
Milne ($\Omega_{tot} = 0$)
EdS ($\Omega_M = 1$)
$\Omega_M = 2$

Thermal history

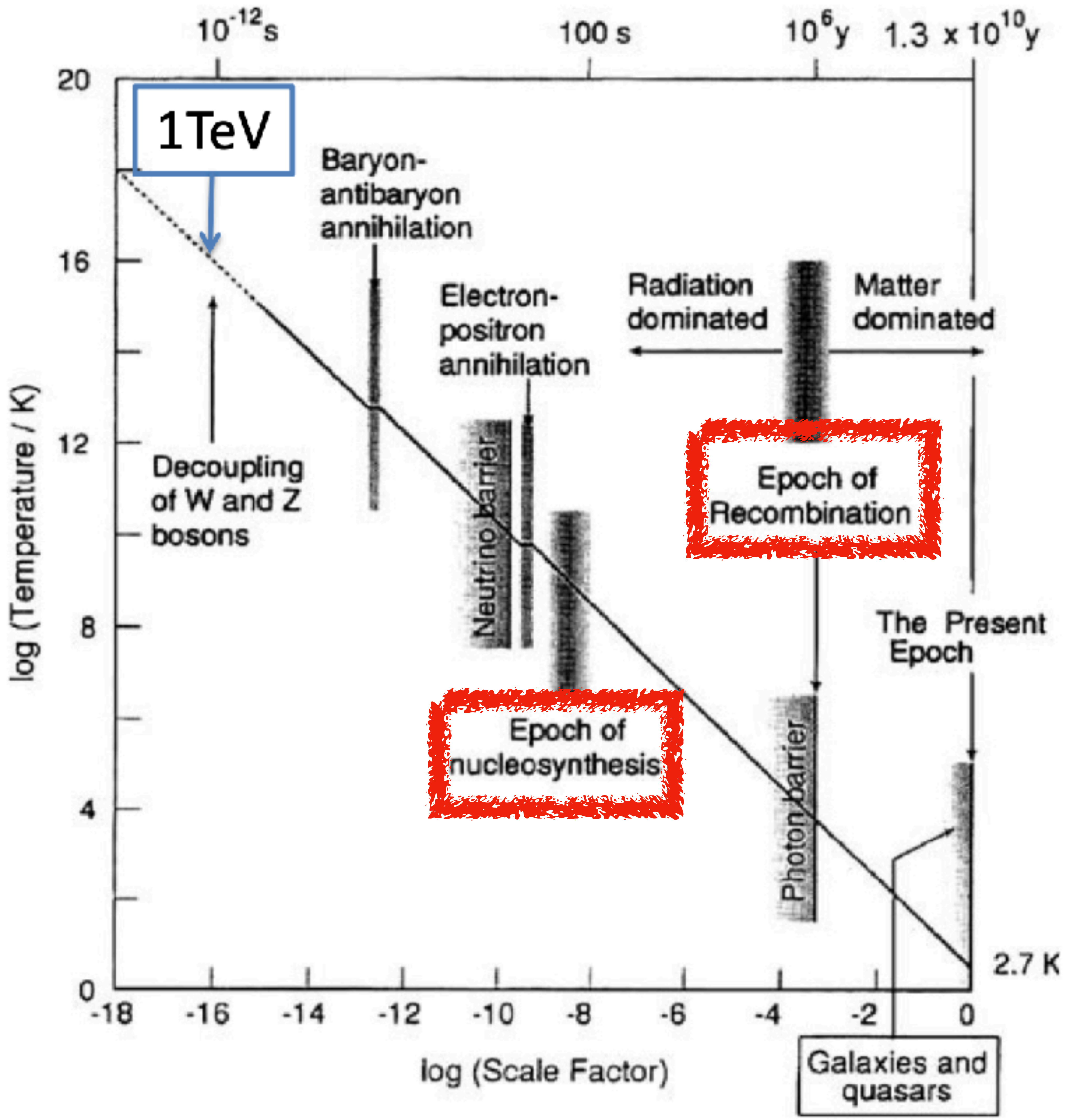
Life in the Early Universe



Sometimes handy to define the dimensionless $T_{10} = T / 10^{10}$K.

$\mathbf{T_{10}}$**(**t=1s**) ≈ 1**

Life in the Early Universe



Sometimes handy to define the dimensionless $T_{10} = T / 10^{10}$K.

$\mathbf{T_{10}(t=1s) \approx 1}$

Freeze out

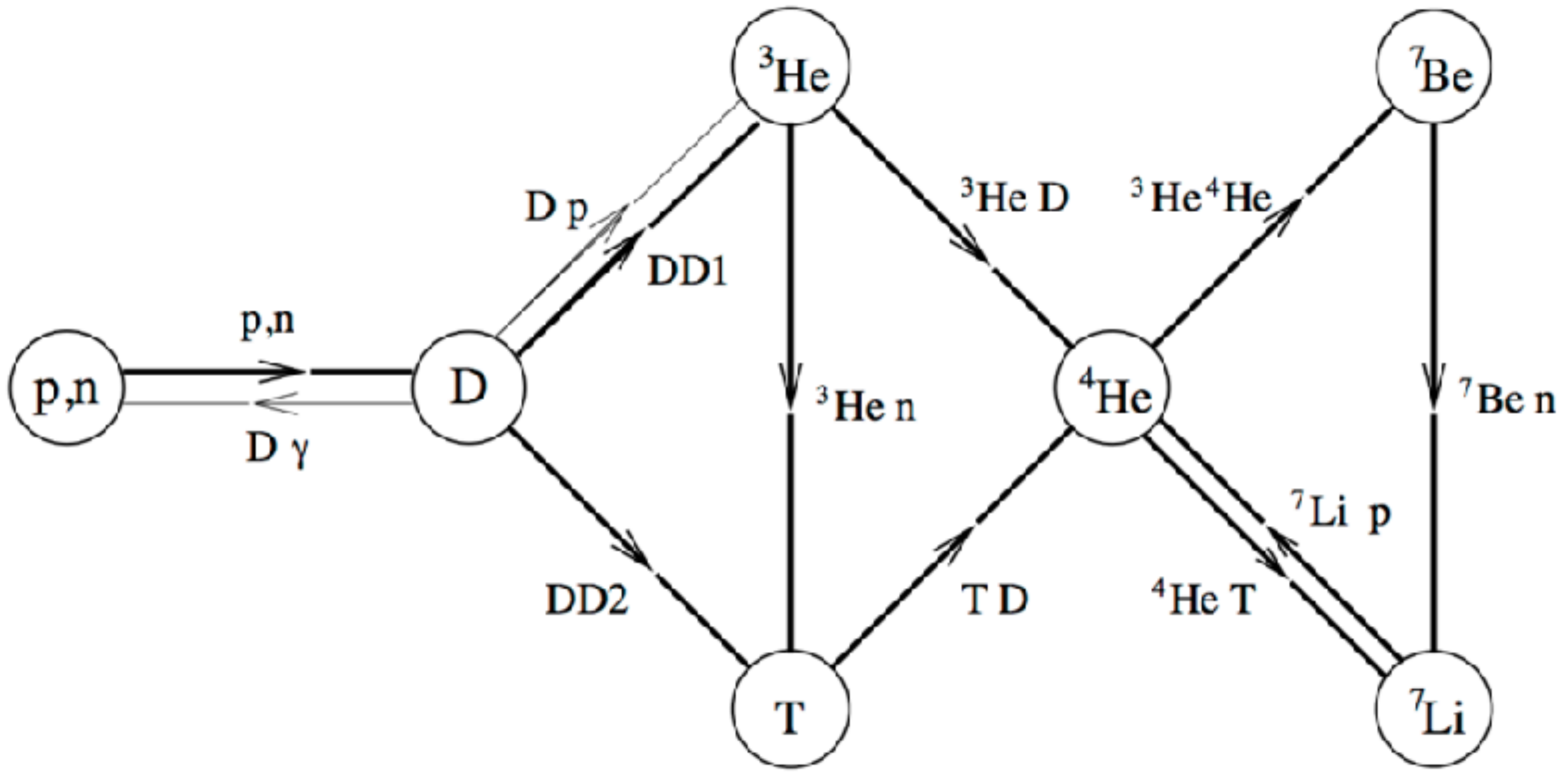
- Early Universe:
  - Reactions convert particles into one another (e.g., protons <-> neutrons)
  - At some point expansion cools Universe enough that the reaction ceases —> particles 'freeze out' in their final proportion
- Example: proton-neutron fraction
  - Reaction $p + e^- \Leftrightarrow n + \nu_e$
  - Freezes out at t~1s, T~10^{10} K

Big Bang Nucleosynthesis

Summary

- Neutrons in thermal equilibrium until neutrinos freeze out, t≈1s.
- Nucleosynthesis stalled by Deuterium bottleneck until t≈100s.

  → Timescale set by $\eta = n_b / n_\gamma$
- Nearly all remaining neutrons get sucked up into ^{4}He.
- Small quantities of ^{2}H, ^{3}He, ^{7}Li remain.
- Nothing heavier has time to fuse.



BBN is a race against time. The conditions are only “right” briefly.
Before 100s, too hot, dense, irradiated. After 15min, too cold, diffuse.

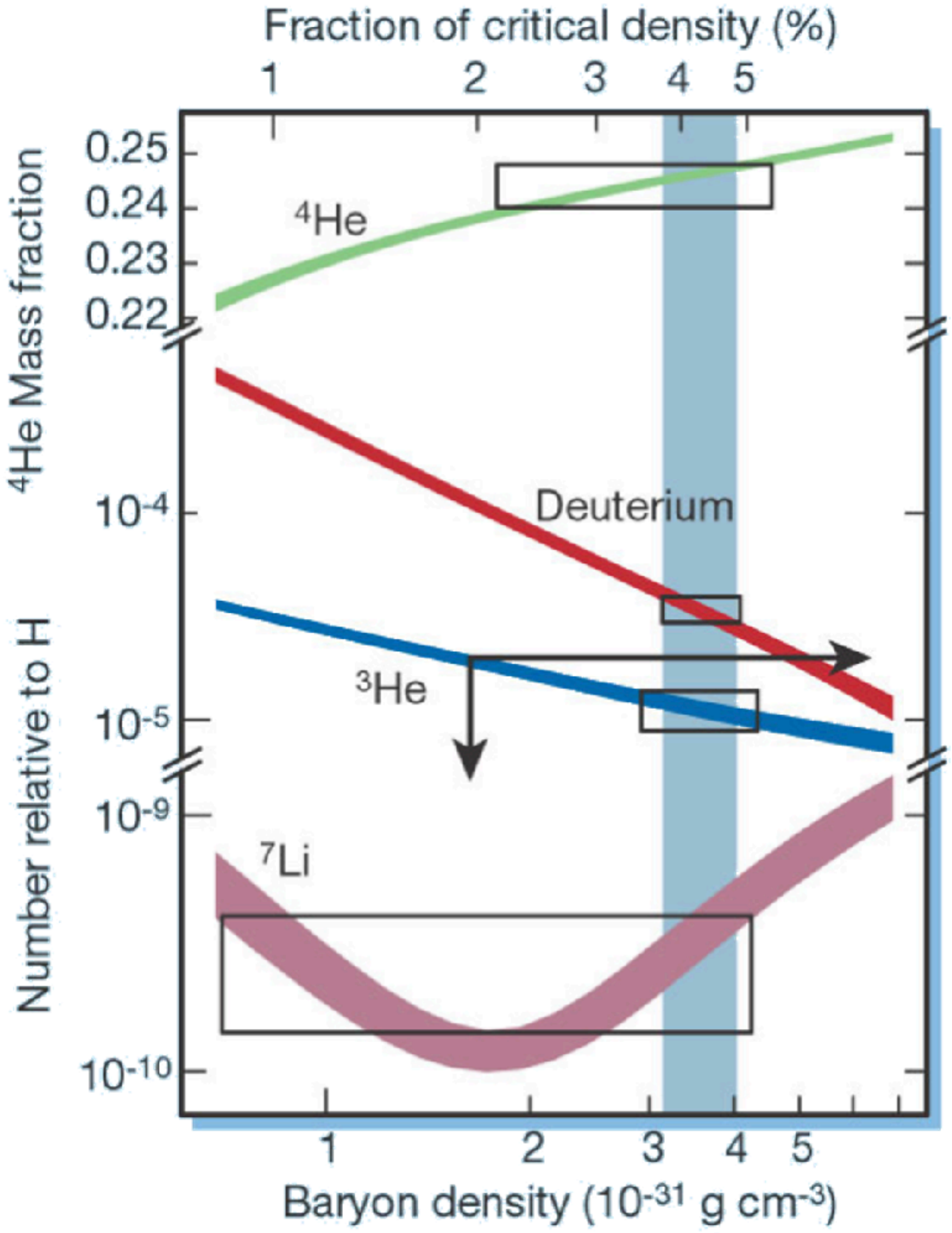
BBN Final Abundances

Helium is extremely stable compared to other light isotopes.

No stable isotopes at A=5, 8.

Triple-alpha rate is too low
$^4He + {}^4He + {}^4He \Leftrightarrow {}^{12}C$

→ Nothing heavy produced before the Universe cools



Growth of structure pre-recombination

Structure at z ≈ 1100

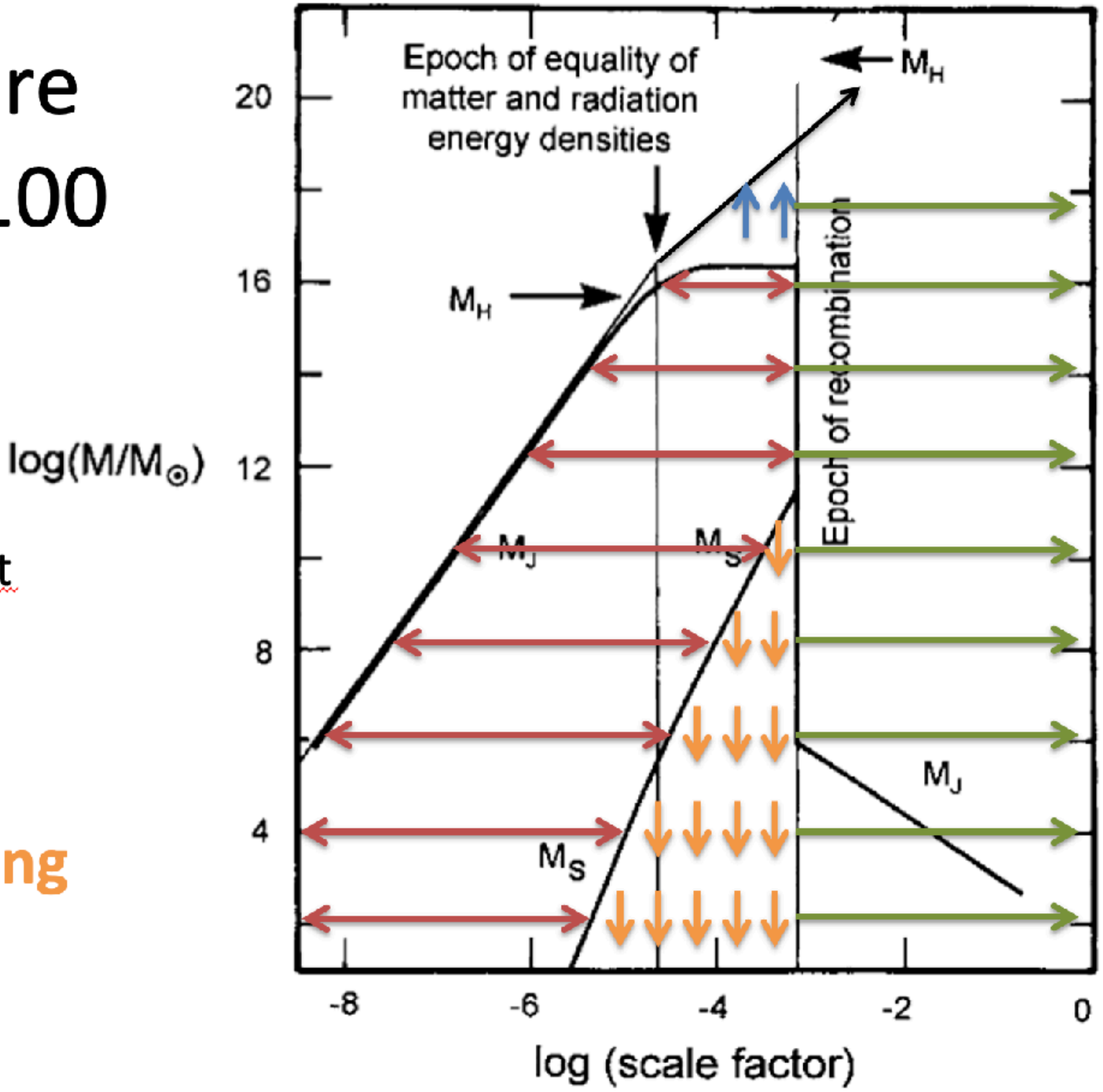
Assume Harrison-Zel'dovich, $\Delta_k(\mathrm{hor}) = \mathrm{const}$

Growing

Oscillating

Silk Damping

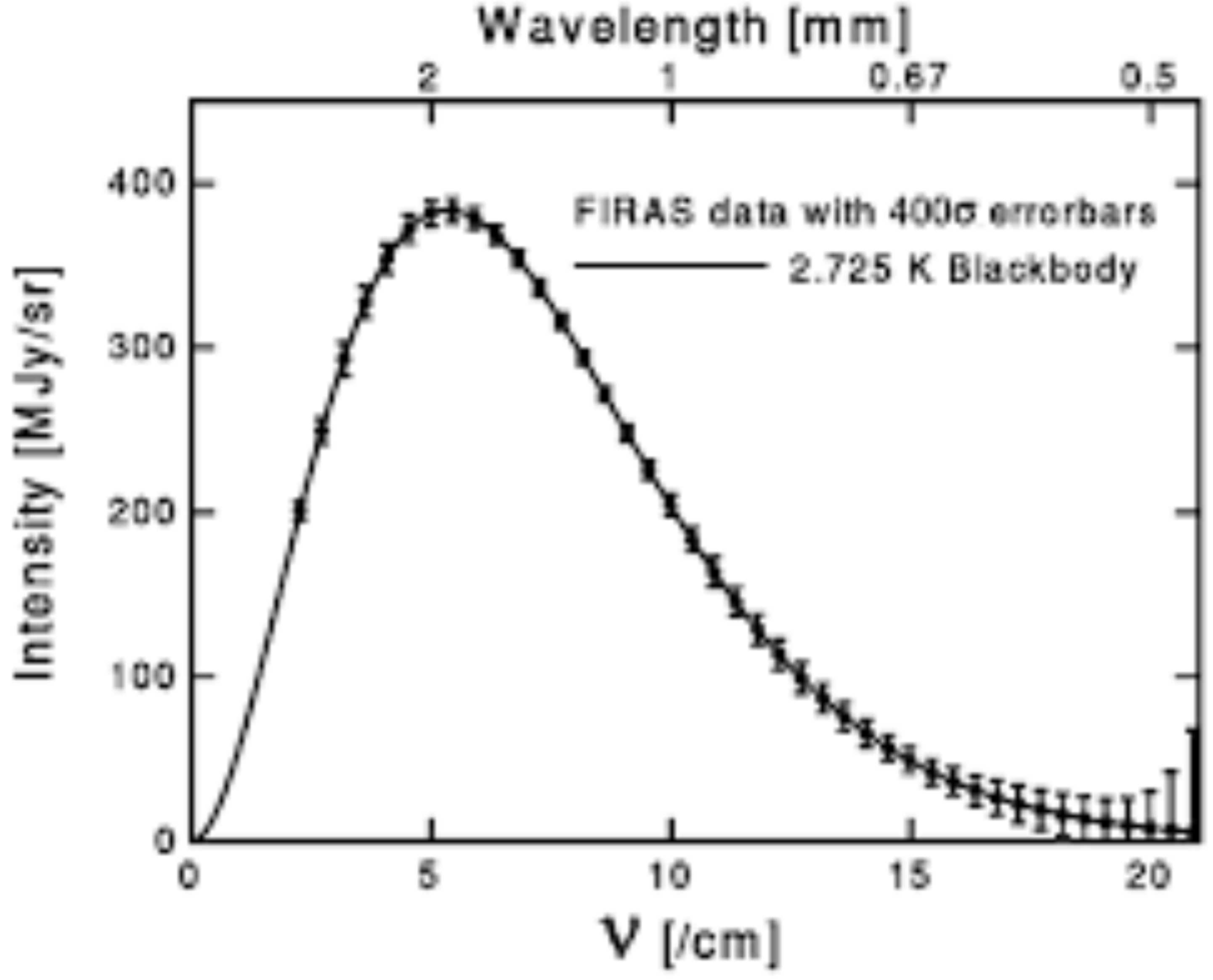
Frozen In



Cosmic microwave background

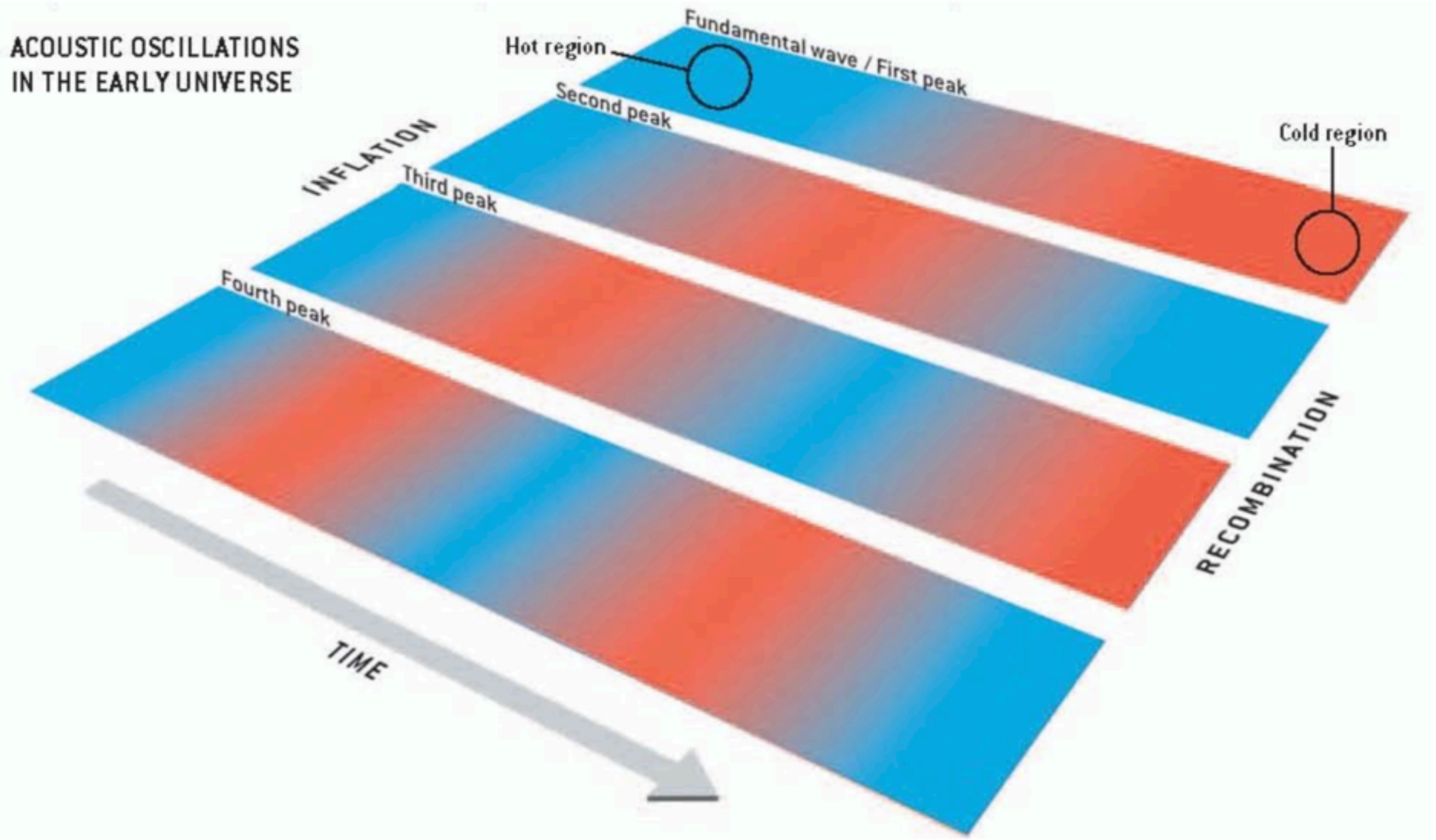
Cosmic microwave background

- Photons redshift as Universe expand, photon temperature T ~ 1/a
- Scattering of photons and baryons at z > 1100 causes the photons to be in thermal equilibrium and the Universe is opaque
- This is like the 'classical cavity with a hole' (the hole being recombination) —> spectrum of CMB photons is that of a black body
- Temperature: 2.72548 +/- 0.00057 K (Fixsen 2009)



Acoustic oscillations in the baryon-photon plasma

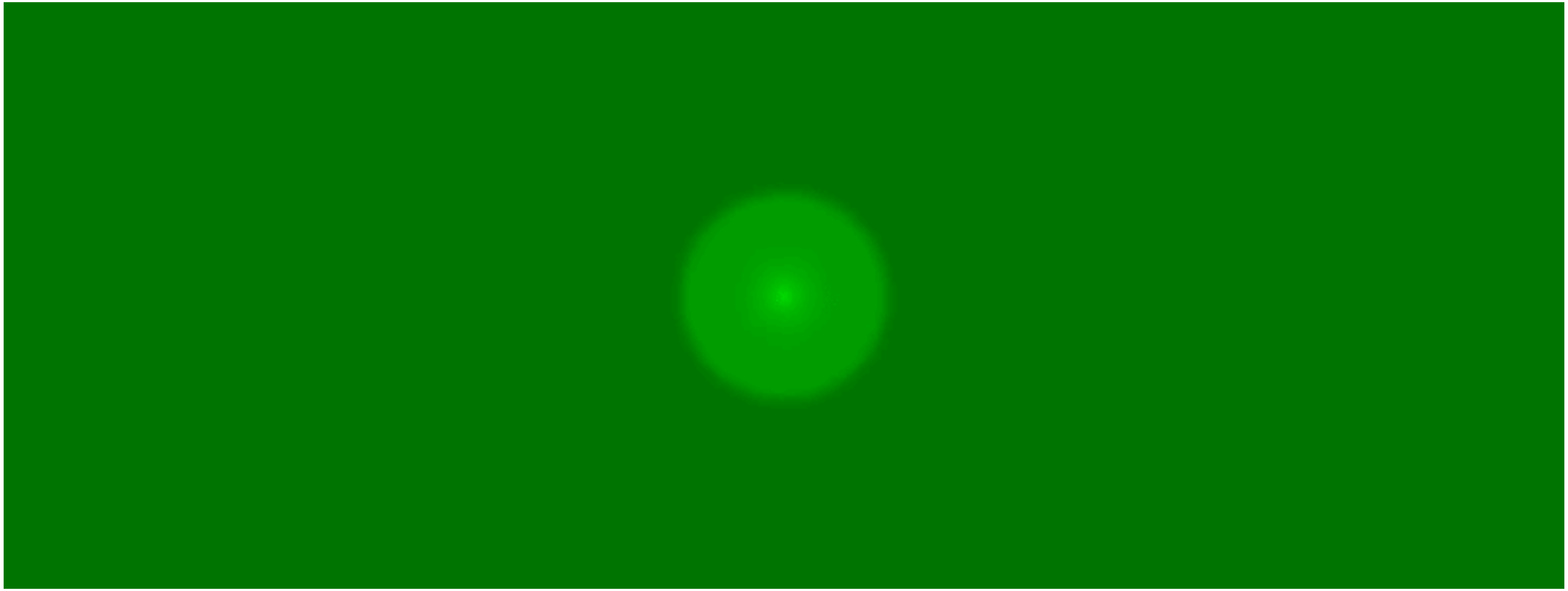
- Early Universe: baryons and photons are coupled through Thomson scattering and form a plasma
- Sound waves propagate in this plasma one perturbations enter the horizon
- Ends when Universe cools enough to become neutral and Thomson scattering ends
- Strong imprint of potential variations on scale where sound wave performs 0.5, 1, 1.5, etc. oscillations from t=0 to $t=t_{recomb}$



Scientific American, 2004

The cosmic microwave background

- Imprint of potential fluctuations as the Universe recombines
- Sachs-Wolfe effect:
    - Photons climbing out of potential wells have a gravitational redshift, appear cooler
    - Photons enter potential wells have a gravitational blueshift, appear hotter
    - Photons ‘free stream’ until they are detected today
    - So temperature anisotropies of the cosmic microwave background show potential fluctuations at recombination



Planck

CMB analysis

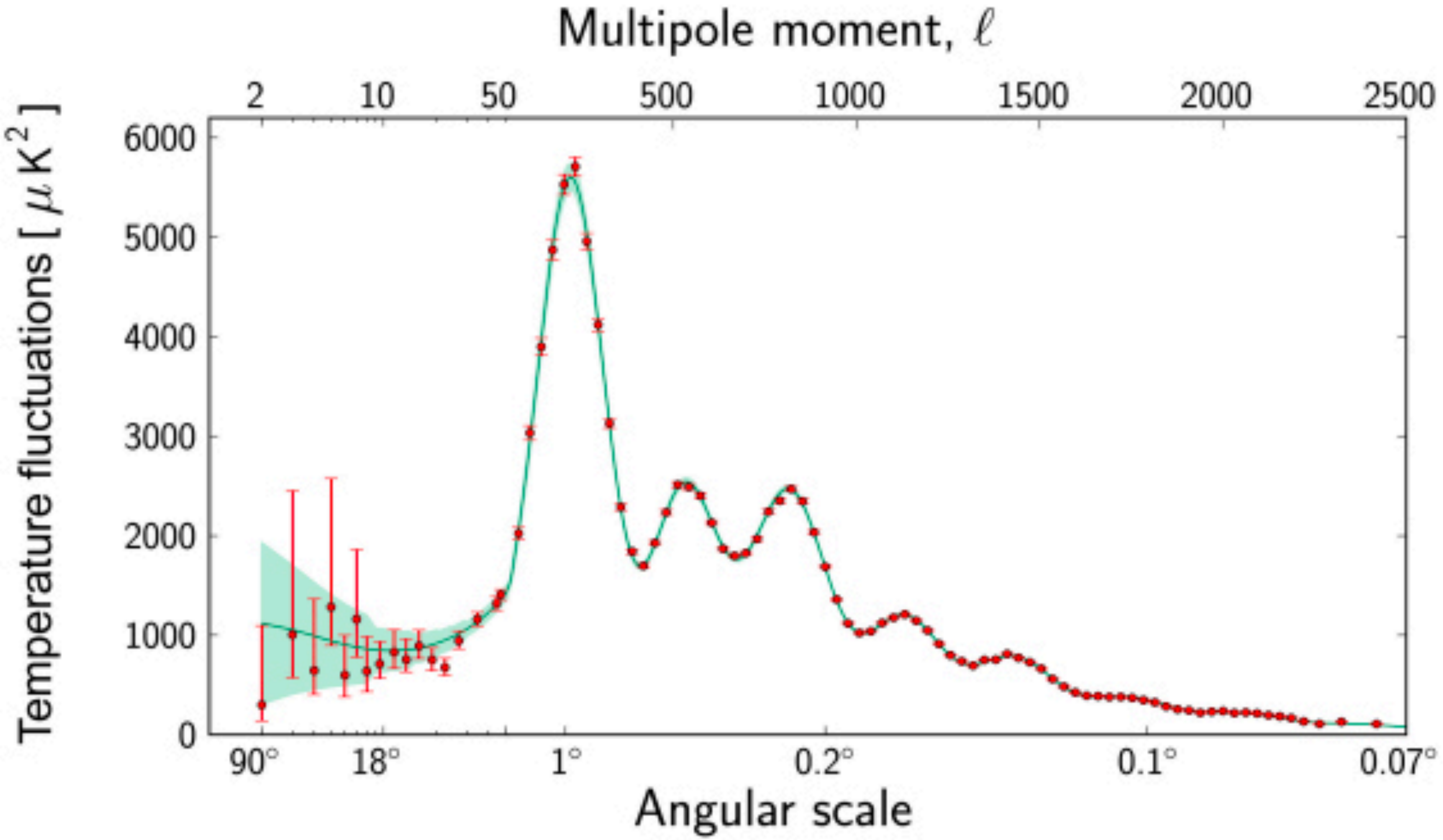
- Decompose temperature fluctuation pattern into spherical harmonics

$$T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta, \phi)$$

- m describes orientation of the mode, which statistically doesn't matter in an isotropic Universe, so real cosmological information in

$$C_l \equiv \frac{1}{2l+1} \sum_m a_{lm} a_{lm}^* = \left\langle |a_{lm}|^2 \right\rangle$$

- Basic CMB observable



Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

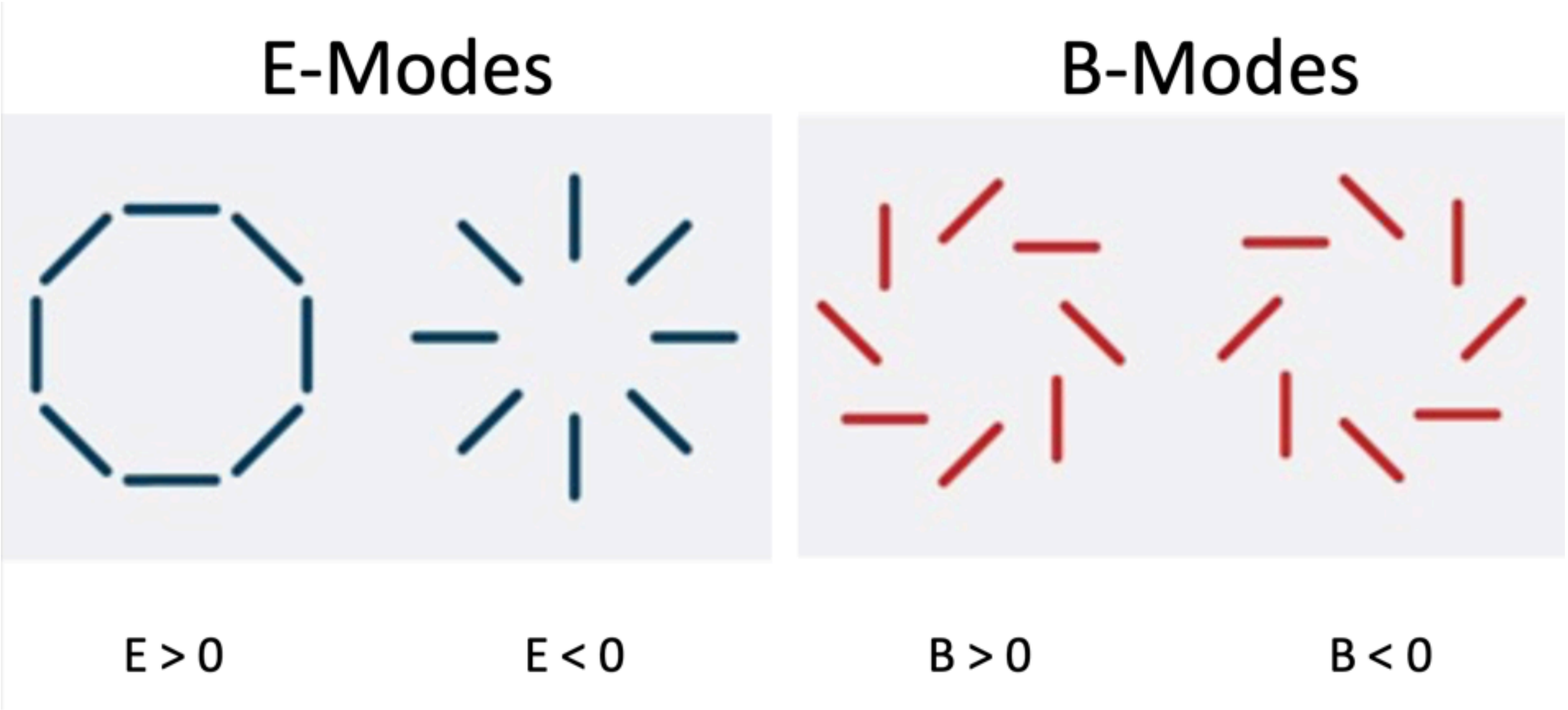
Planck

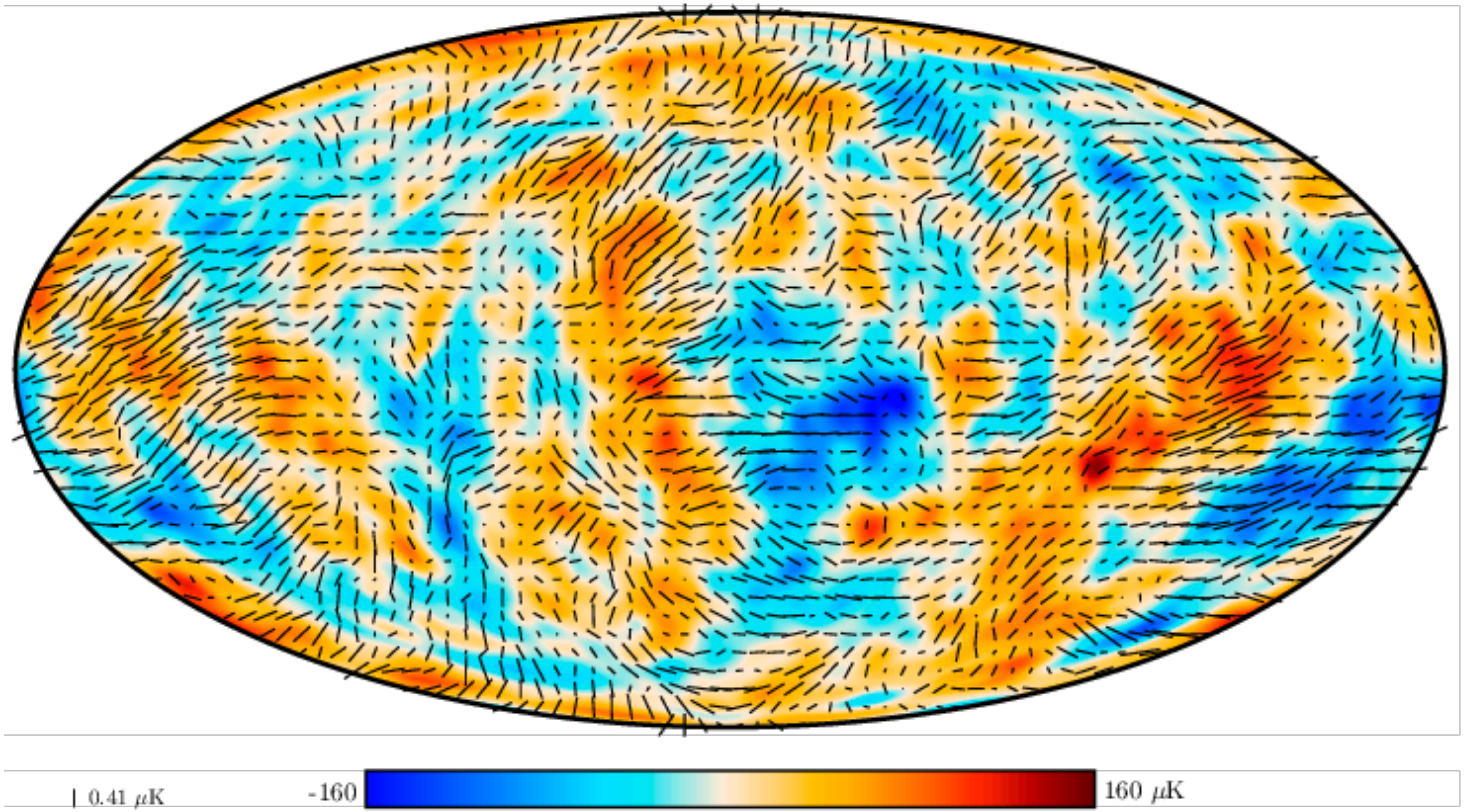
What do the peaks measure?

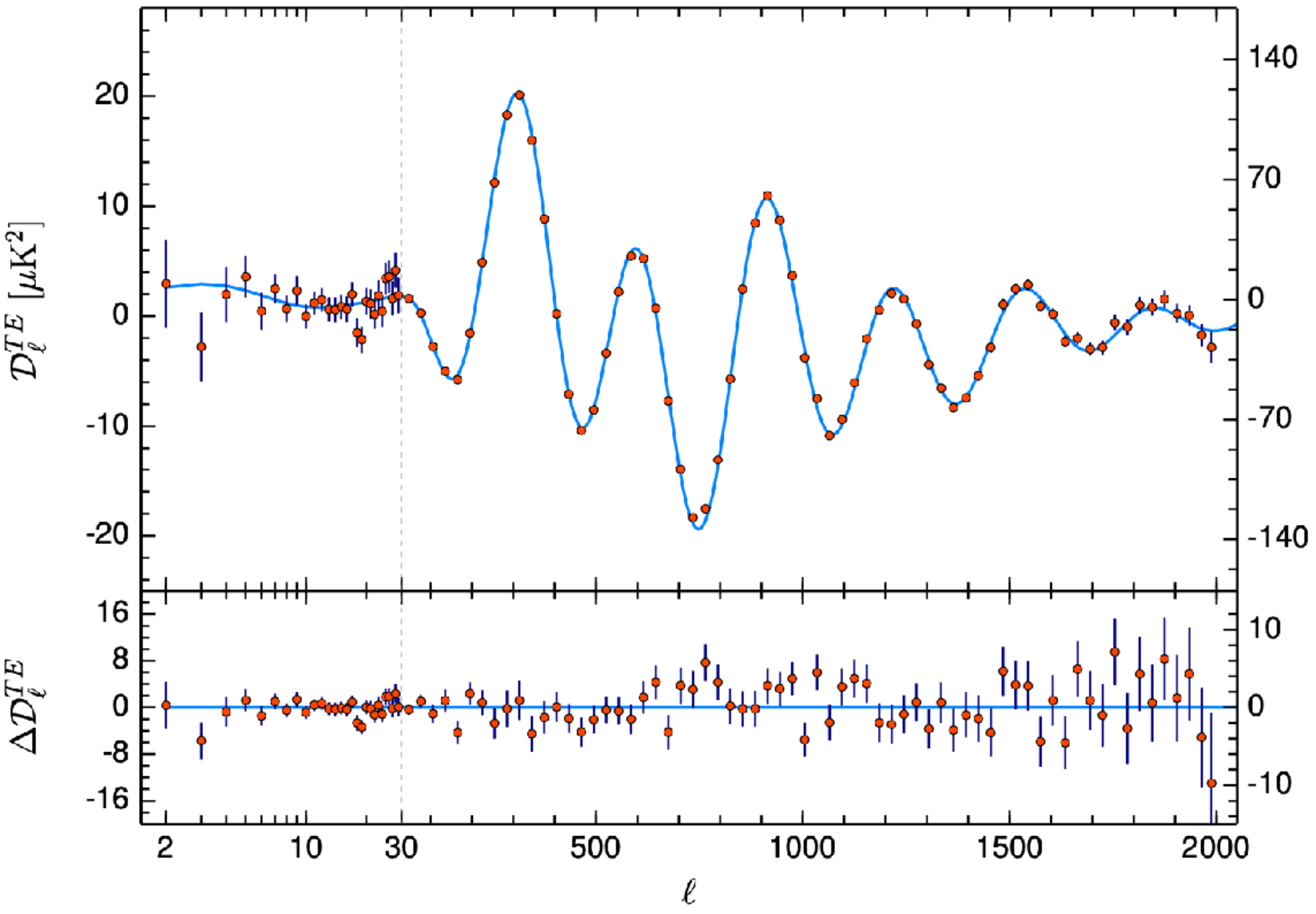
- First peak: ‘acoustic scale’: sound horizon at recombination (decoupling)
    - Observed value of ~1 degree is close to saying that the Universe is flat, but not perfect correspondence
- Second peak: baryon density parameter, sets damping of damped oscillator
- Third peak: dark-matter density parameter, limits damping
- Overall amplitude and shape: amplitude and shape of initial power spectrum

Polarization

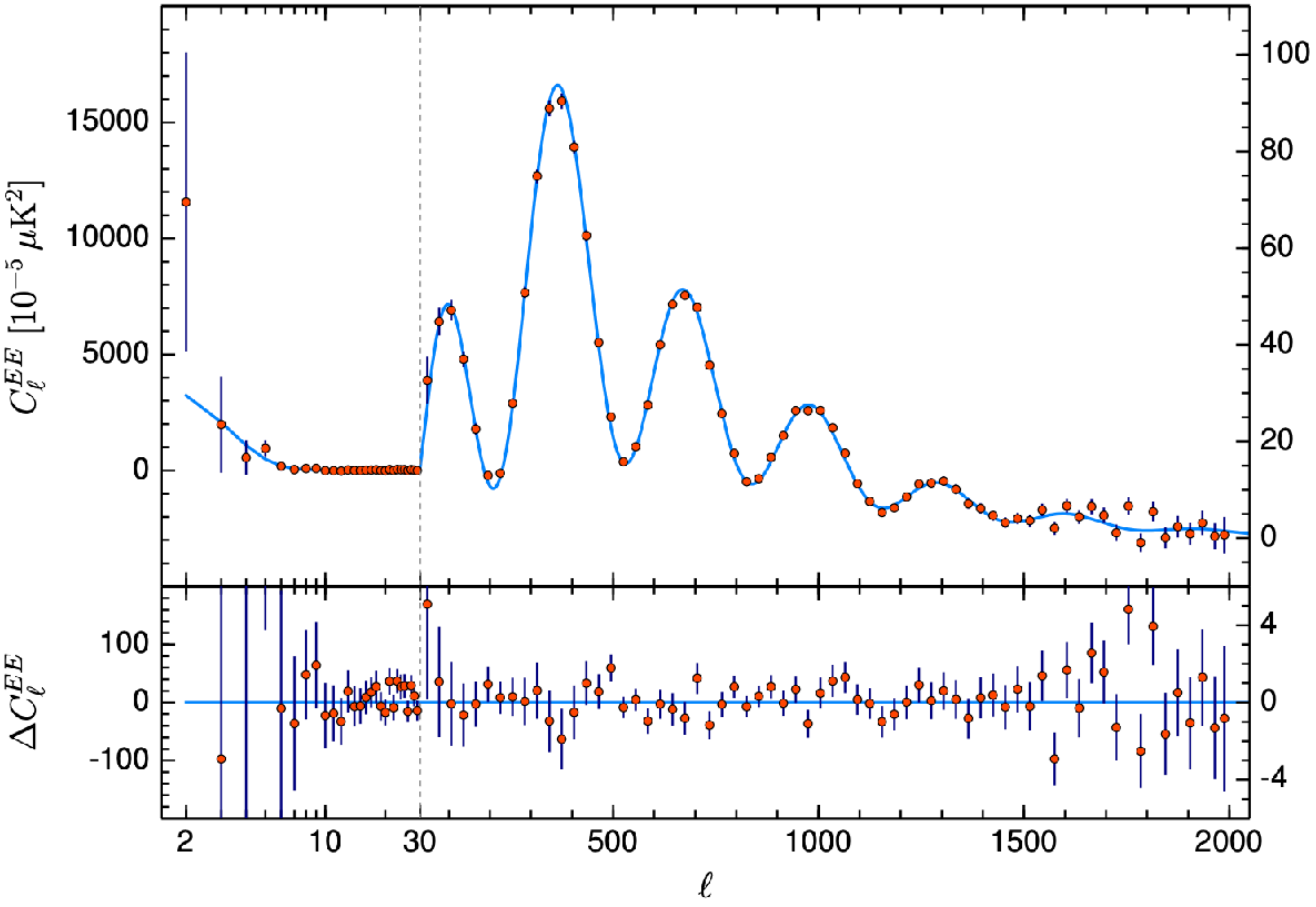
- CMB is linearly polarized because Thomson scattering polarizes light if the intensity distribution has a quadrupole
- Polarization at 10% level
- Polarization expressed in terms of “E modes” and “B modes”




0.41 μK
-160
160 μK



Planck 2018 VI (2020)



Planck 2018 VI (2020)

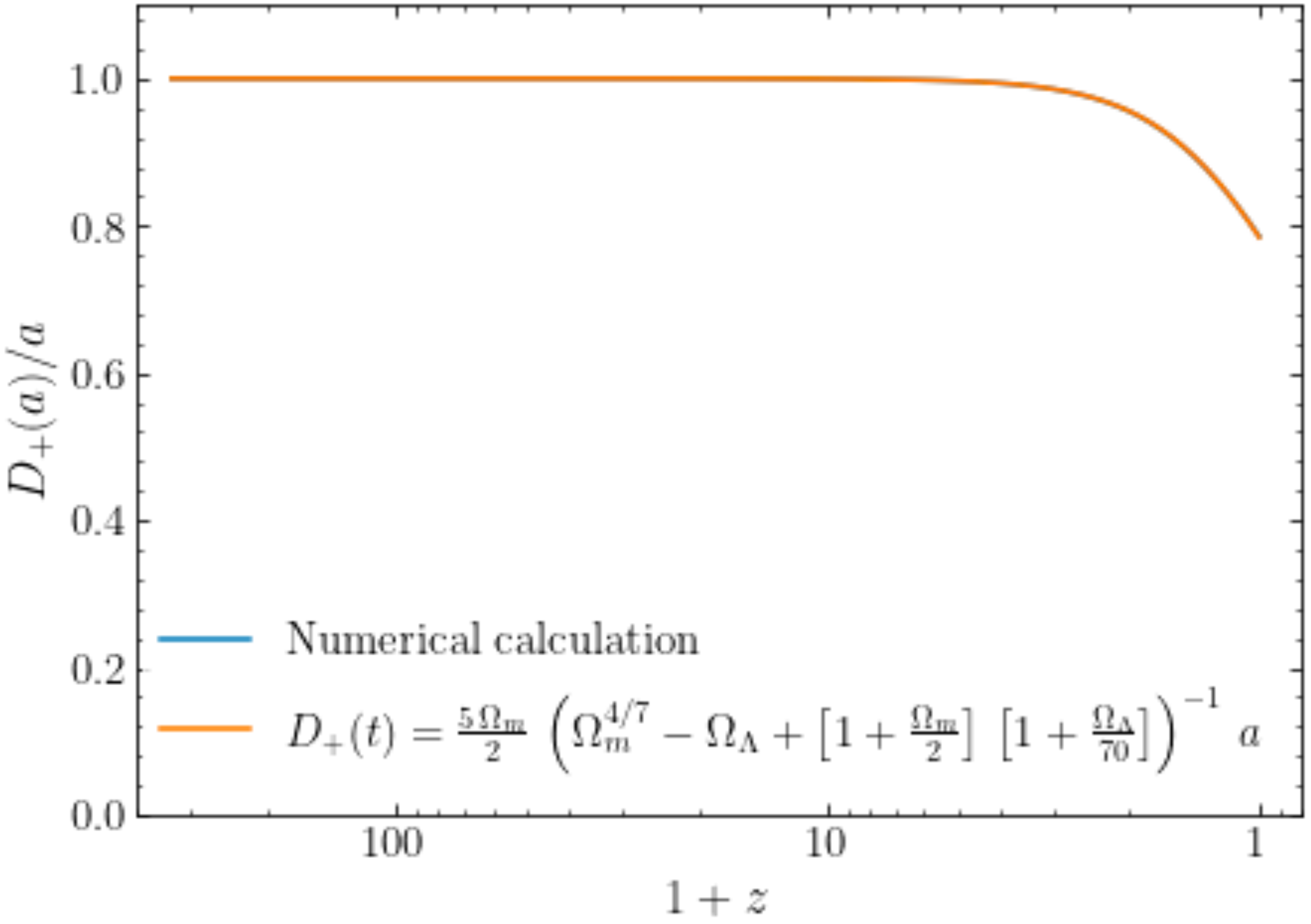
Secondary anisotropies

- CMB anisotropies also receive (small) contributions from processes between recombination and observation:
    - Reionization causes polarization
    - Lensing by the matter distribution
    - Sunyaev-Zeldovich effect: inverse Compton scattering of CMB photons by high-energy electrons in galaxy clusters
        - Thermal: from random motions of electrons in clusters
        - Kinetic: from bulk motion of cluster wrt the CMB

Growth of structure

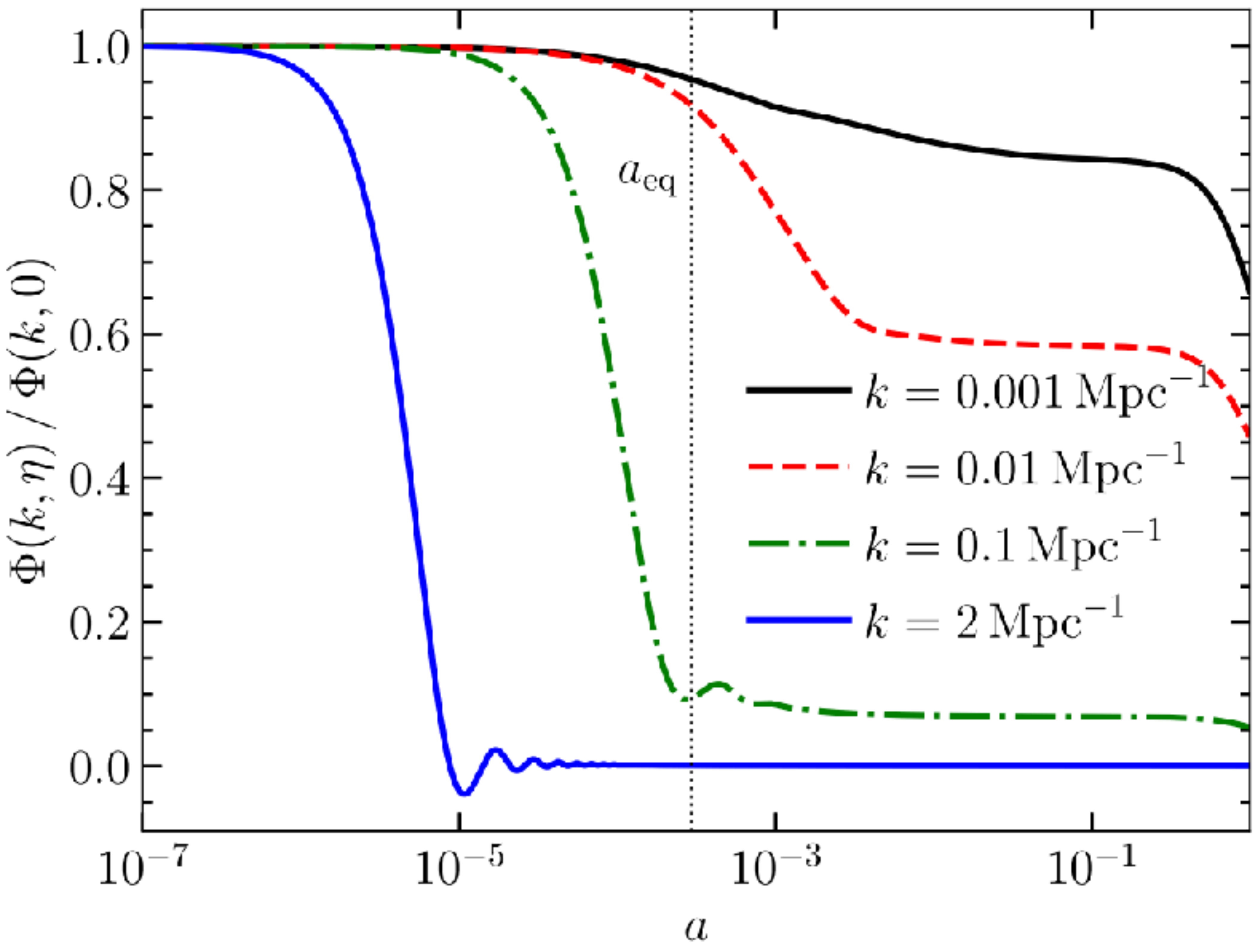
Growth of structure

- Late-time growth (z <~ 1000):
    - Dark matter: scale independent, $\delta \propto D_+(a)$, $D_+(a) = a$ for Ω_m=1
    - Baryons: scale independent like DM about Jeans scale, pressure-stabilized on smaller scales



Growth of structure

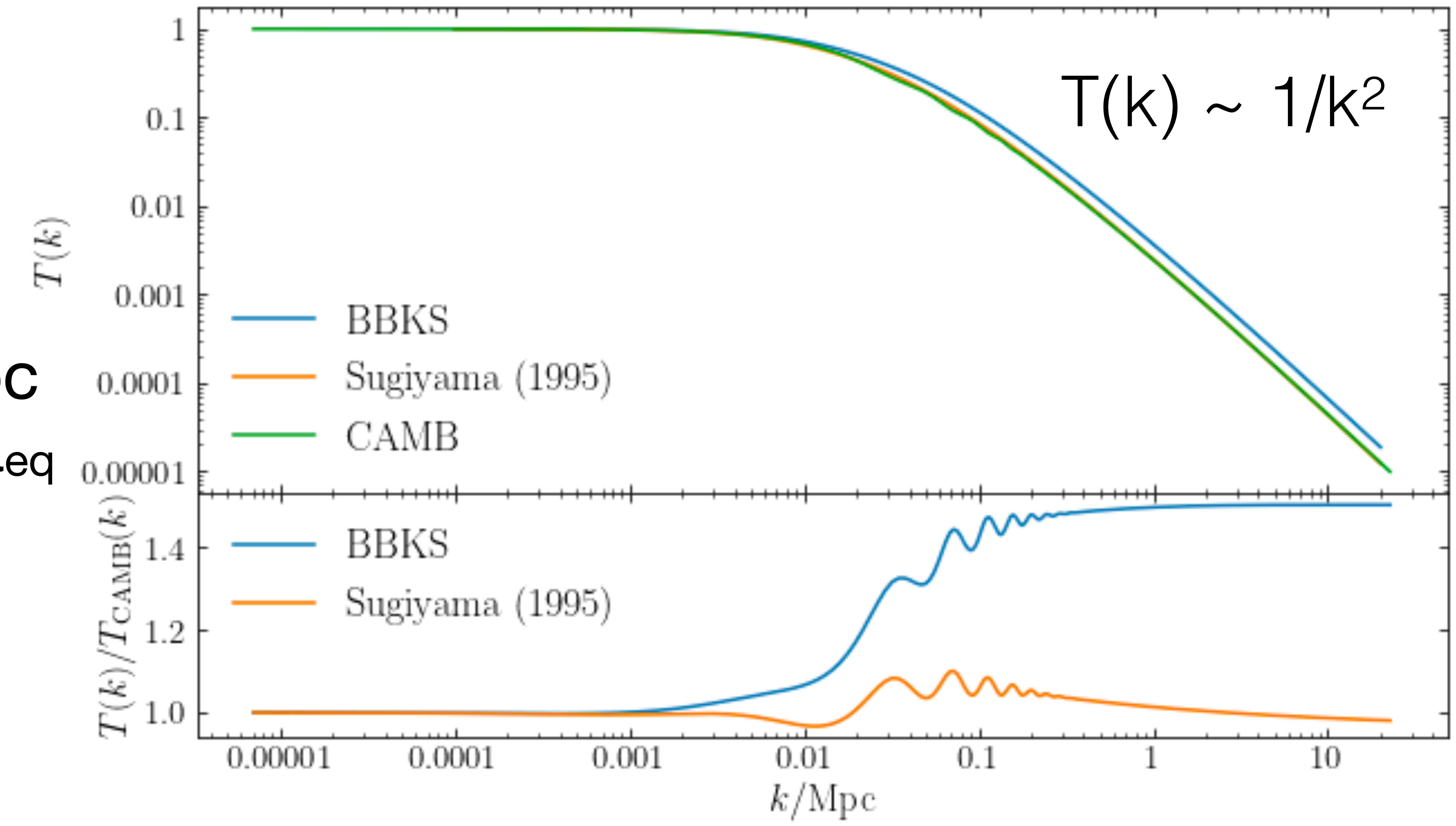
- Early-time growth (z >~ 1000):
  - Potential evolution described by transfer function T(k)
  - T(k) = 1 for k < k_{eq}, k_{eq} ~ 0.01/Mpc scale that enters the horizon at a_{eq}
  - T(k) ~ $1/k^2$ for k > k_{eq}



Dodelson & Schmidt 2020; Fig. 8.1

Growth of structure

- Early-time growth (z >~ 1000):
  - Potential evolution described by transfer function T(k)
  - $T(k) = 1$ for $k < k_{eq}$, $k_{eq} \sim 0.01$/Mpc scale that enters the horizon at a_{eq}
  - $T(k) \sim 1/k^2$ for $k > k_{eq}$



Growth of structure

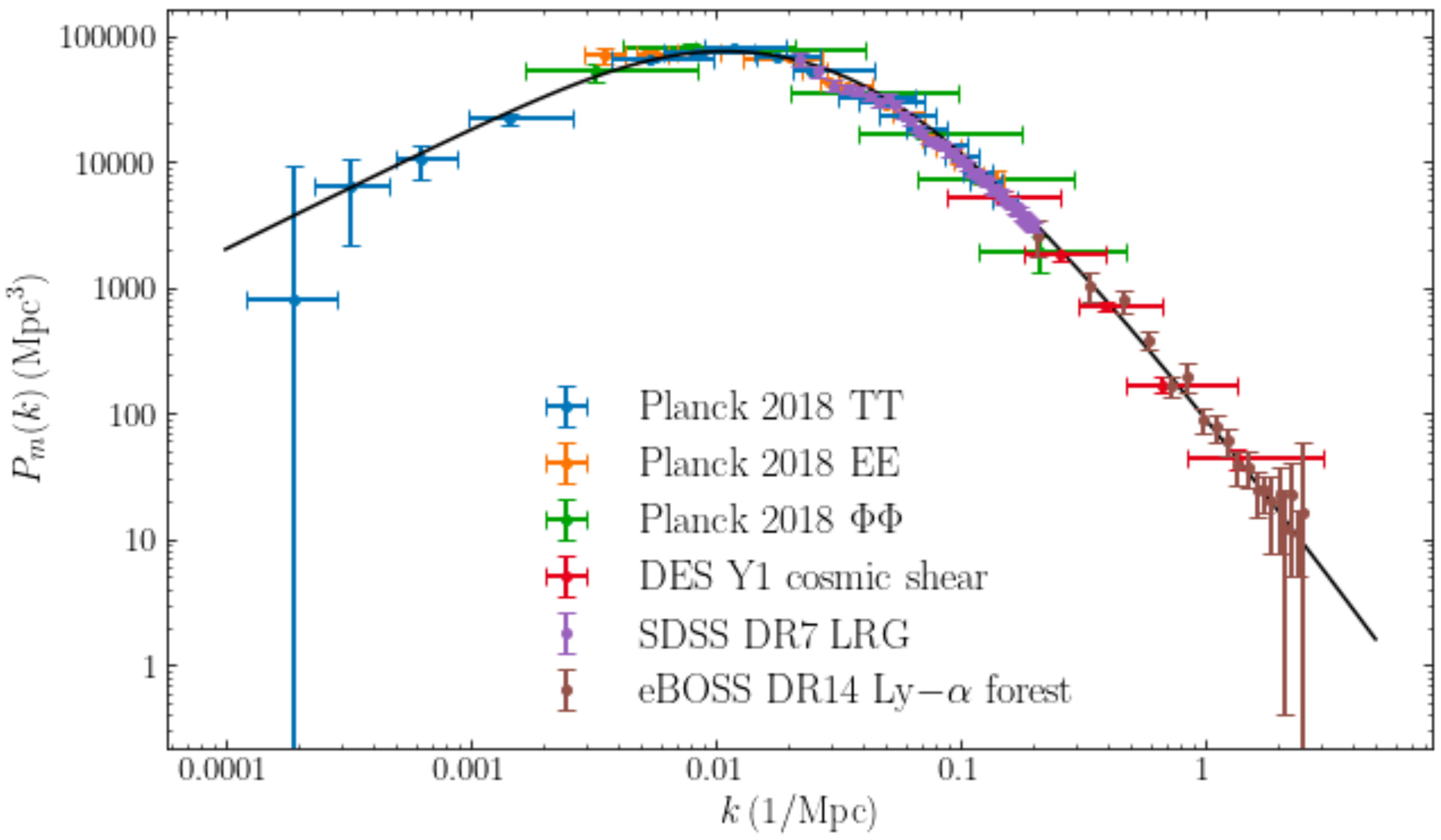
Peak at k~k_{eq}

- Statistics of late-time density fluctuations described by the matter power spectrum: Gaussian with scale-dependent variance

$$\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle = (2\pi)^3 \, P_m(k) \, \delta^{(3)}(\mathbf{k} - \mathbf{k}') \, .$$

- And power spectrum

$$P_m(k,a) = \frac{8\pi^2}{25} \frac{A_s \, k_p \, c^4}{\Omega_{0,m}^2 \, H_0^4} T^2(k) \, D_+^2(a) \left(\frac{k}{k_p} \right)^{n_s} .$$



Dark matter halos

Formation of DM halos

- Dark matter halos form when regions of the Universe with $\delta >> 1$ decouple from expansion and gravitationally collapse
- Spherical collapse of top-hat overdensity: DM halo has formed when linear-theory $\delta = \delta_c \sim 1.686$ *for any reasonable Universe*
- Virialization: radial collapse is unstable, slight asymmetries cause shell crossing, violent relaxation, phase mixing, … —> virialized halo forms due to non-linear evolution
    - Non-linear overdensity $\Delta v \sim 18\,\pi^2 \sim 178 \sim 200$ in EdS, ~330 in LCDM today

Halo mass function

- Halo mass function: Comoving number density of DM halos as a function of mass

- Derivation using (extended) Press-Schechter formalism:

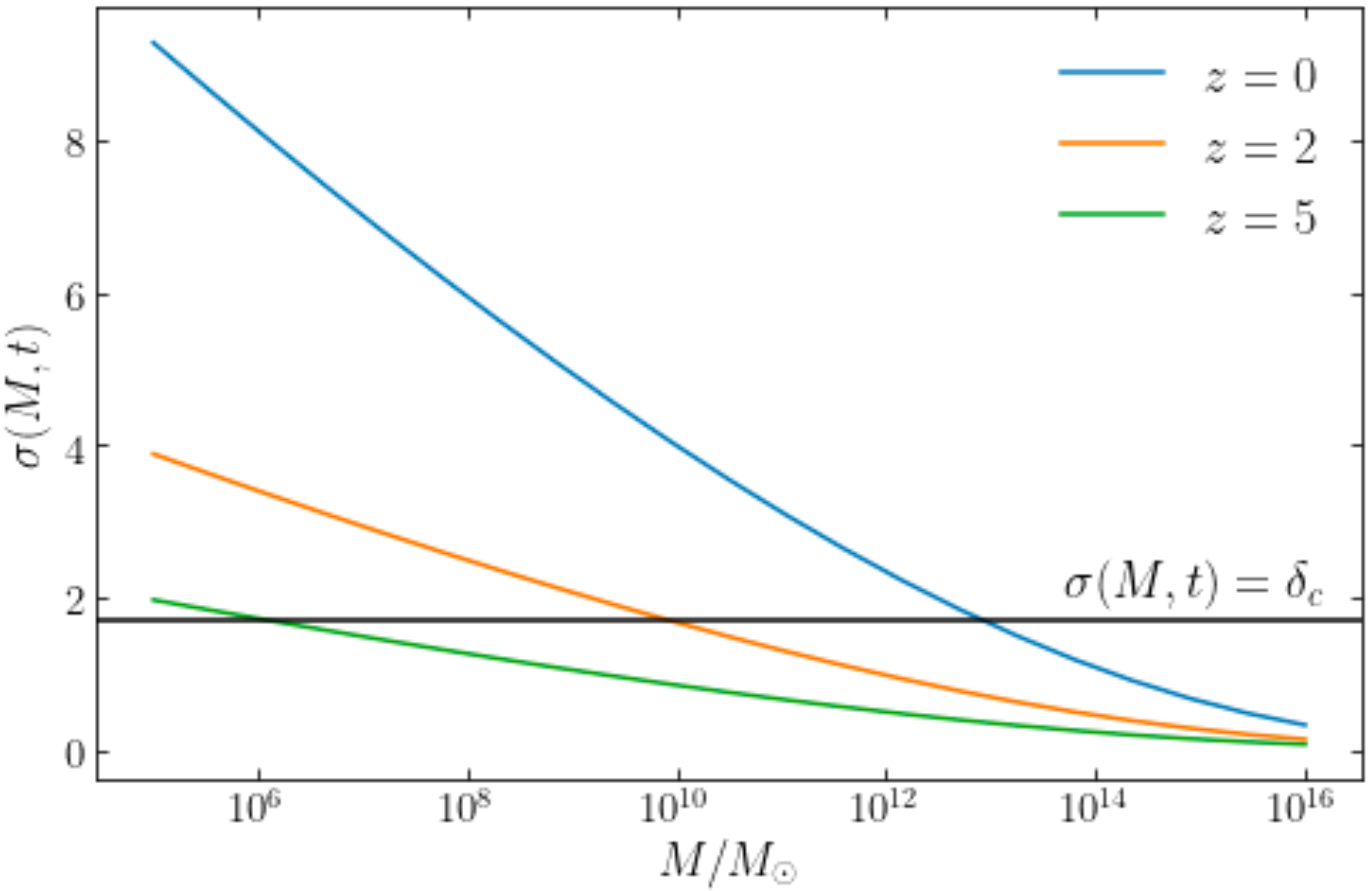
  Combine statistics of smoothed linear over densities δ_S with $\delta_c \sim 1.686$ linear collapse threshold to derive halo mass function

$$\frac{\mathrm{d}n}{\mathrm{d}\ln M} = \sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}_{0,m}}{M}\,\frac{\delta_c}{\sigma}\,\exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)\frac{\mathrm{d}\ln\sigma^{-1}}{\mathrm{d}\ln M}$$

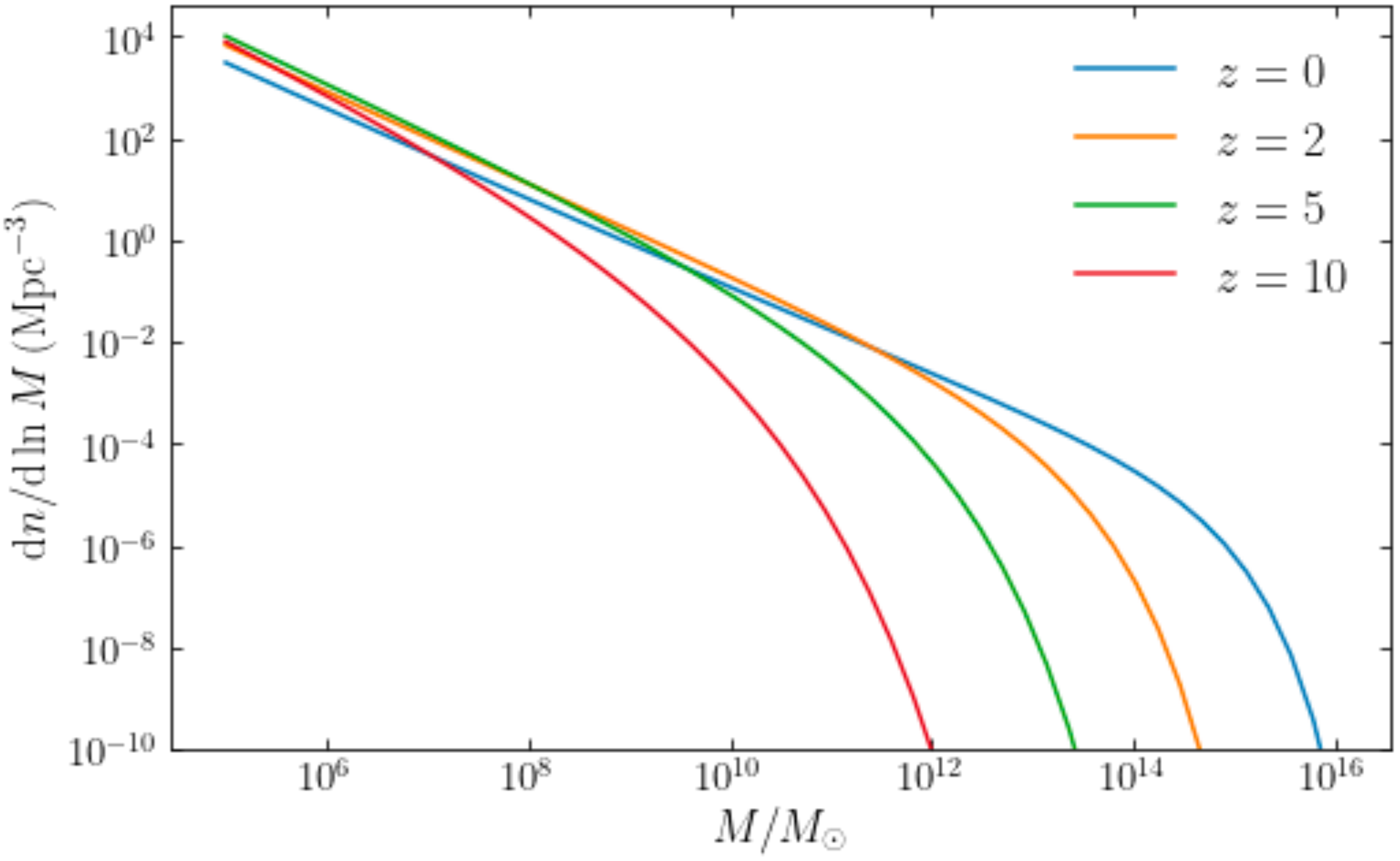
- Where

$$\sigma^2(M,t) = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\,k^2\,P_m(k,a)\,W_k^2(R)\,,$$

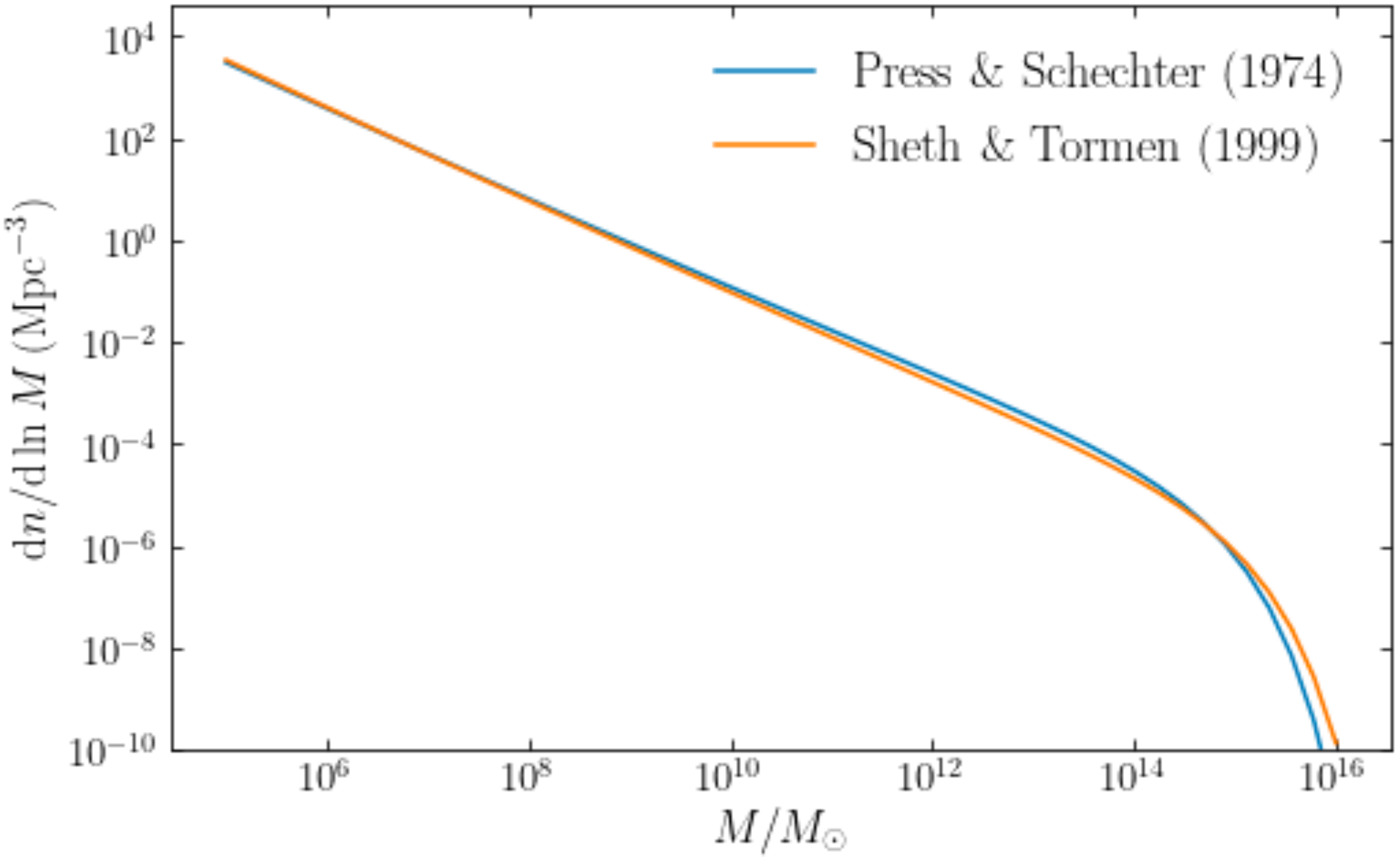
- W_k is the Fourier transform of the window function used to define halos of mass M



$z = 0$
$z = 2$
$z = 5$
$\sigma(M, t) = \delta_c$
$\sigma(M, t)$
0
2
4
6
8
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
$M/M_\odot$



10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
10^{16}
$M/M_{\odot}$
$\mathrm{d}n/\mathrm{d}\ln M\ (\mathrm{Mpc}^{-3})$
$z = 0$
$z = 2$
$z = 5$
$z = 10$



$$f_{\rm PS}(\nu) = \sqrt{\frac{2}{\pi}}\,\nu\, e^{-\nu^2/2}\,, \quad \longrightarrow \quad f_{\rm ST}(\nu) = 0.3222\sqrt{\frac{2}{\pi}}\,\nu'\, e^{-\nu'^2/2}\,\left(1+\nu'^{-0.6}\right)\,, \qquad \text{where} \quad \nu' = 0.707\nu.$$

Galaxy clustering

Clustering

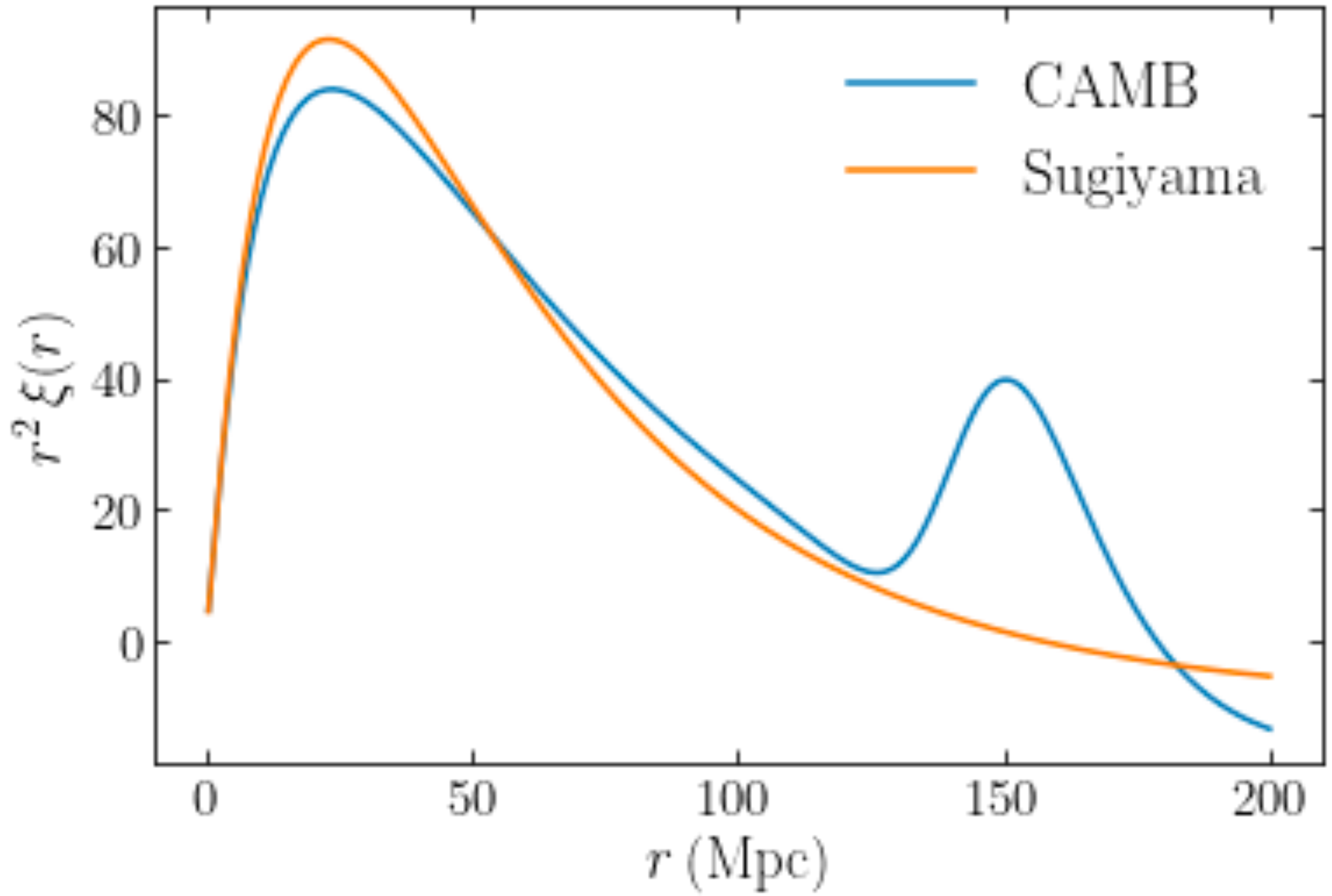
- 'Diagonal' power spectrum $P_m(\underline{\mathbf{k}})$ translates to clustering in real space:

  Halos/galaxies more likely to form near each other, as halos more likely to exceed the critical collapse threshold in an volume where the density is higher than average

- Clustering in real space is described by the *two-point correlation function:* the function giving the probability of finding a halo/galaxy/overdensity at a distance of r = $|\mathbf{x}_1-\mathbf{x}_2|$ from another halo/galaxy/overdensity

$$\xi\ (|\mathbf{x}_1 - \mathbf{x}_2|) = \langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle,$$

$$\xi(r) = \frac{1}{2\pi^2}\int dk\, k^2\, P(k)\, \frac{\sin(kr)}{kr}$$



CAMB
Sugiyama
$r^2\,\xi(r)$
r (Mpc)
0
20
40
60
80
0
50
100
150
200

Bias

- Bias: relation between density distribution of DM halos / galaxy (of a certain type) and the linear density field

- E.g., halos

$$\delta_h(M) = b(M) \times \delta_{\mathrm{lin}}$$

where

$$b(M) = 1 + \left(\frac{\nu^2(M) - 1}{\delta_c(t)} \right)$$

Galaxy bias

- In terms of the correlation function

$$\xi_g(r) = b^2(r; g) \times \xi_{\rm lin}(r)$$

- Galaxy formation makes bias *scale-dependent* on small scales (small separations) —> hard to use cosmologically ('full shape of the correlation function')

- Large-scale bias (r > 10 Mpc) scale-independent, but still function of galaxy type (~trace DM halo bias)

—> Can use large-scale correlation function to do robust cosmology

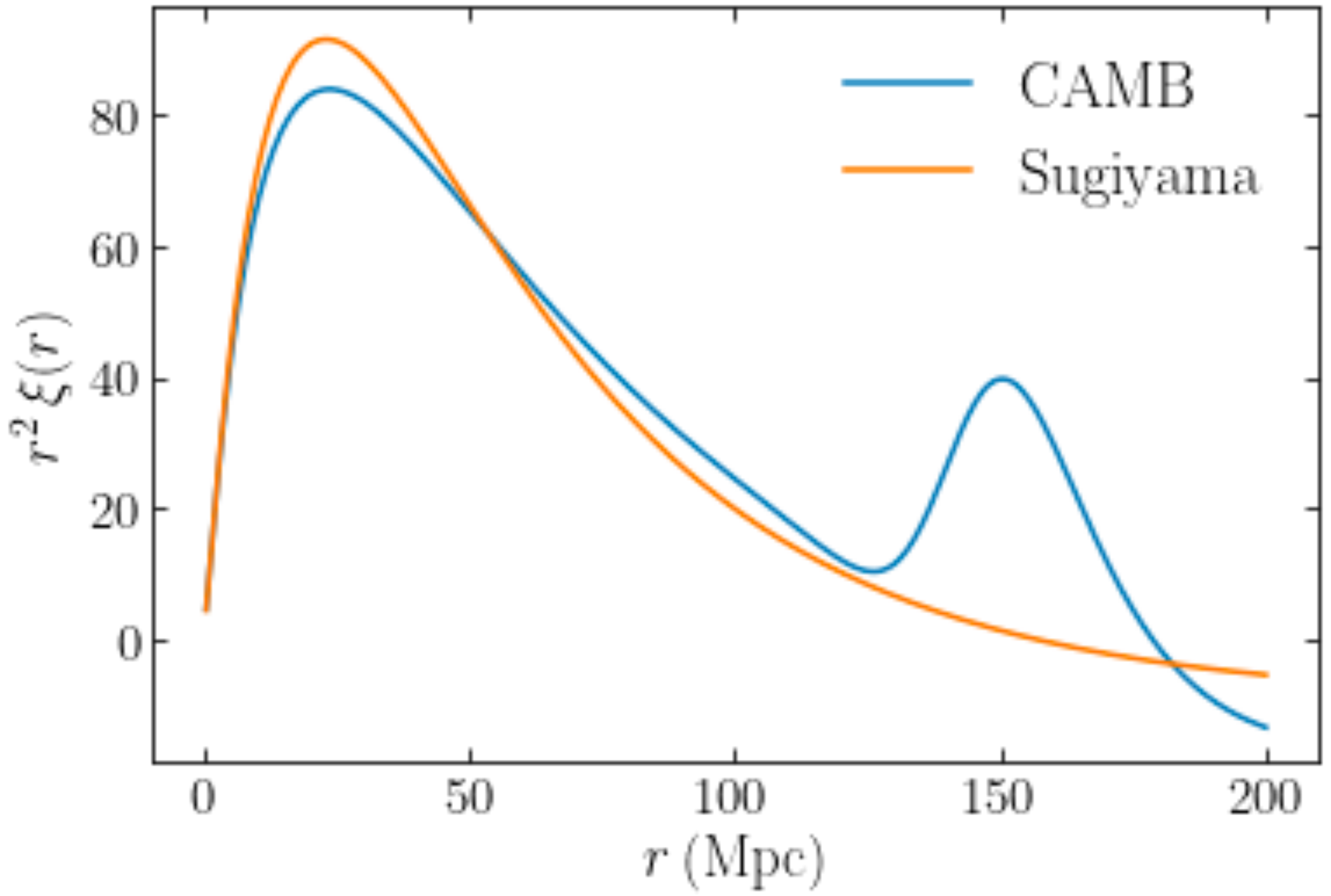
Cosmology using galaxy clustering

- Two very robust measurements:
  - Redshift-space distortions: $P_{g,\mathrm{RSD}}(k, \mu_k, \bar{z}) = P_{\mathrm{L}}(k, \bar{z}) \left[b_1 + f \mu_k^2 \right]^2$

    —> measure $f\sigma_8$, where f is the growth rate: $f \equiv \dfrac{\mathrm{d} \ln D_+(a)}{\mathrm{d} \ln a}$

    BAO / Alcock-Paczynski effect: sound horizon r_d at decoupling is known scale, shows up in the correlation function

    —> measure: $D_A(z)$ and $H(z)$ at median redshift of a survey



Cosmology using weak lensing

- Lensing by the gravitational potential Φ causes shapes of galaxies to be systematically distorted —> shear field
- Can correlating the shear field with itself or with galaxies
- Current standard weak lensing analyses use '3x2' analysis:
    - Shear-shear correlation
    - Shear-galaxy correlation ('galaxy-galaxy lensing')
    - Galaxy clustering
- Growth-of-structure probe, very sensitive to combination:

$$S_8 = \sigma_8 \sqrt{\Omega_m / 0.3}$$

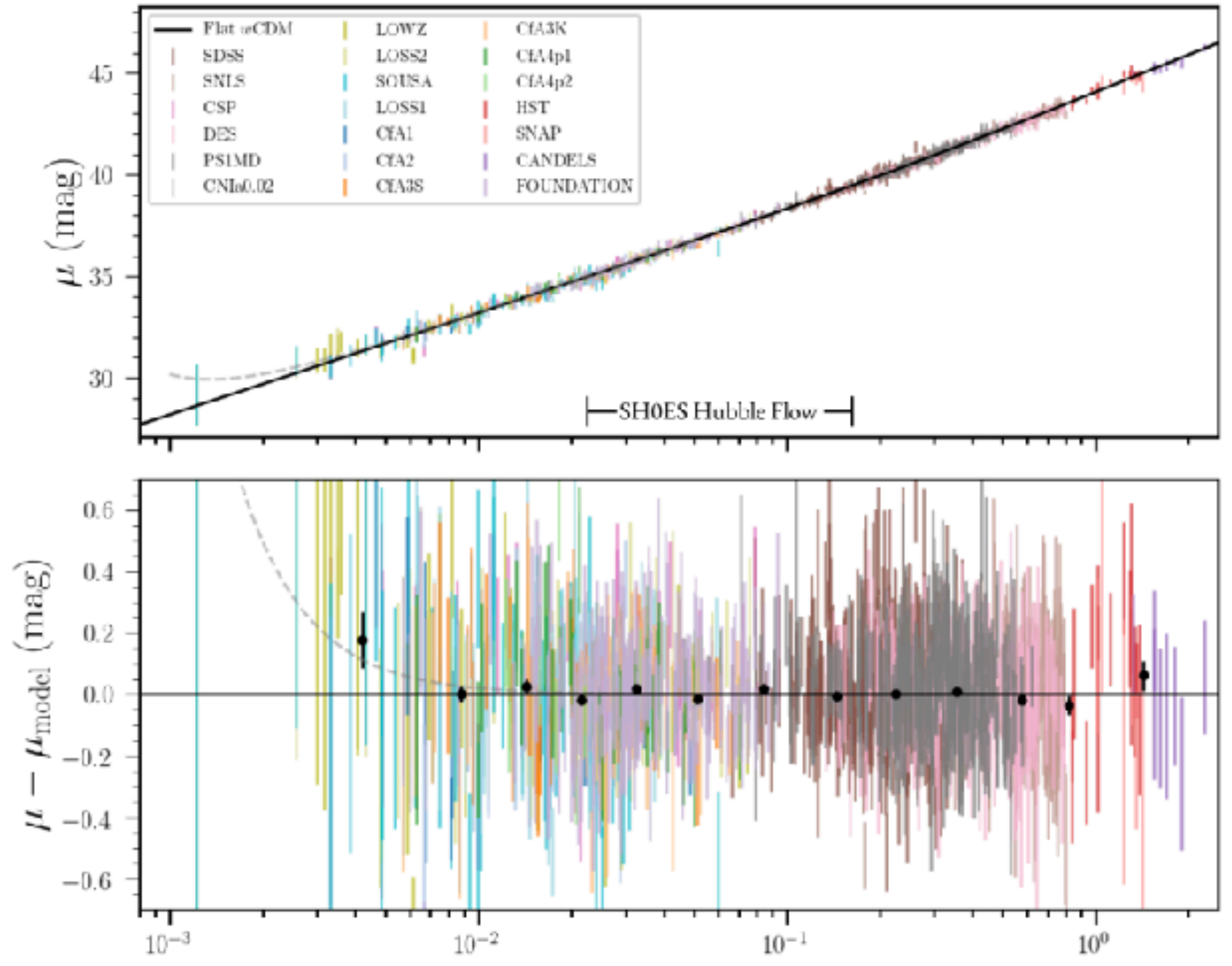
Cosmological constraints

CMB constraints

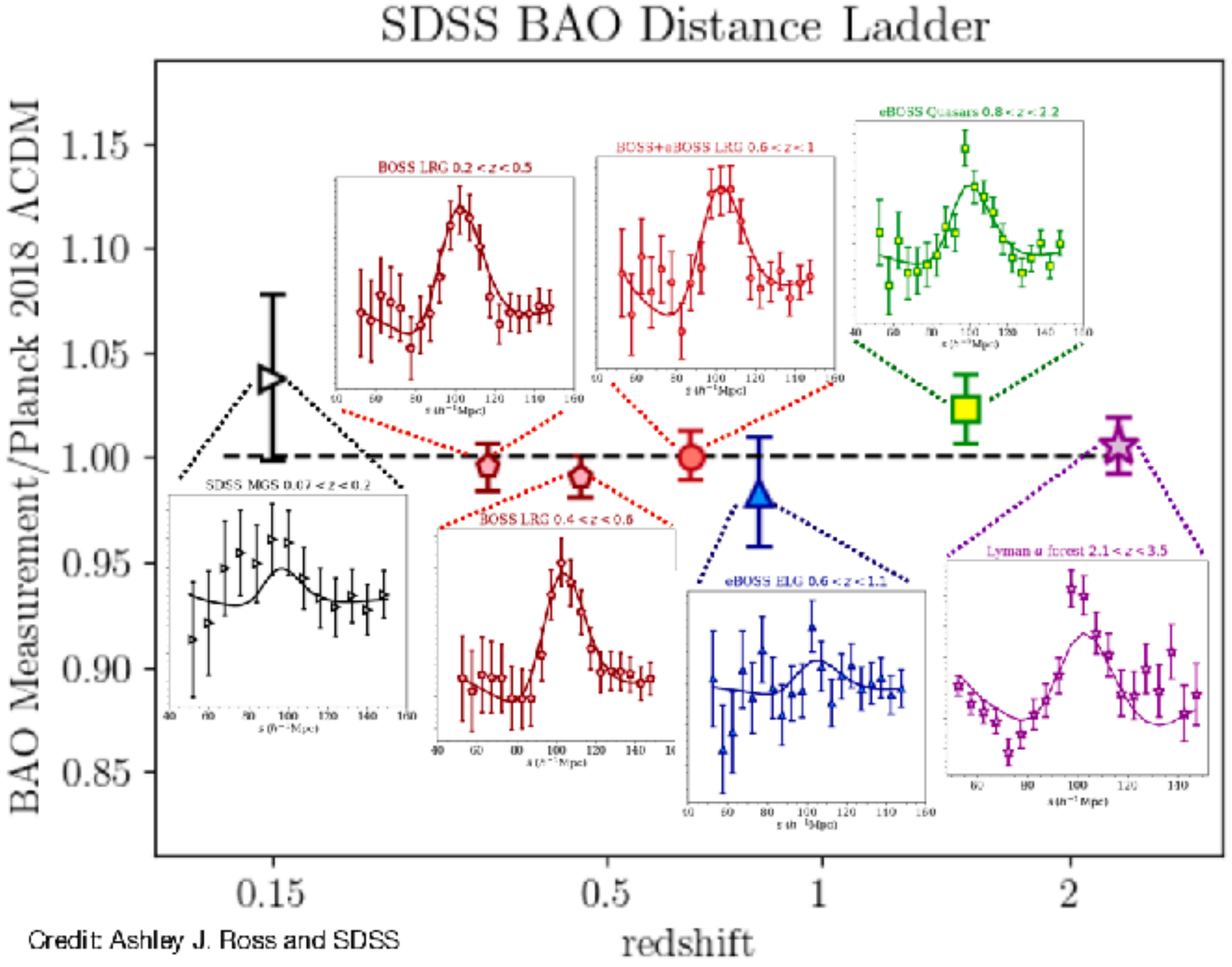
- Geometry vs. Growth-of-structure
- Geometry:
  - Physical baryon and dark-matter density: $\Omega_b h^2$ and $\Omega_c h^2$
  - Angular acoustic scale
  - Angular diameter distance to last scattering: $D_M(z=1090)/r_d$
- Growth-of-structure:
  - Amplitude and shape of the initial curvature power spectrum: n_s, A_s
  - Optical depth to reionization

Late-time constraints

- Type Ia supernovae: luminosity distance vs. redshift
    - Dense sampling of distance vs. redshift from z~0 to 2
    - Great for measuring cosmic acceleration
- BAO: H(z) and $D_A(z)$
    - High-precision distances to a few redshifts
- RSD: $f\sigma_8$
- Weak lensing: $S_8 \sim f\sigma_8$



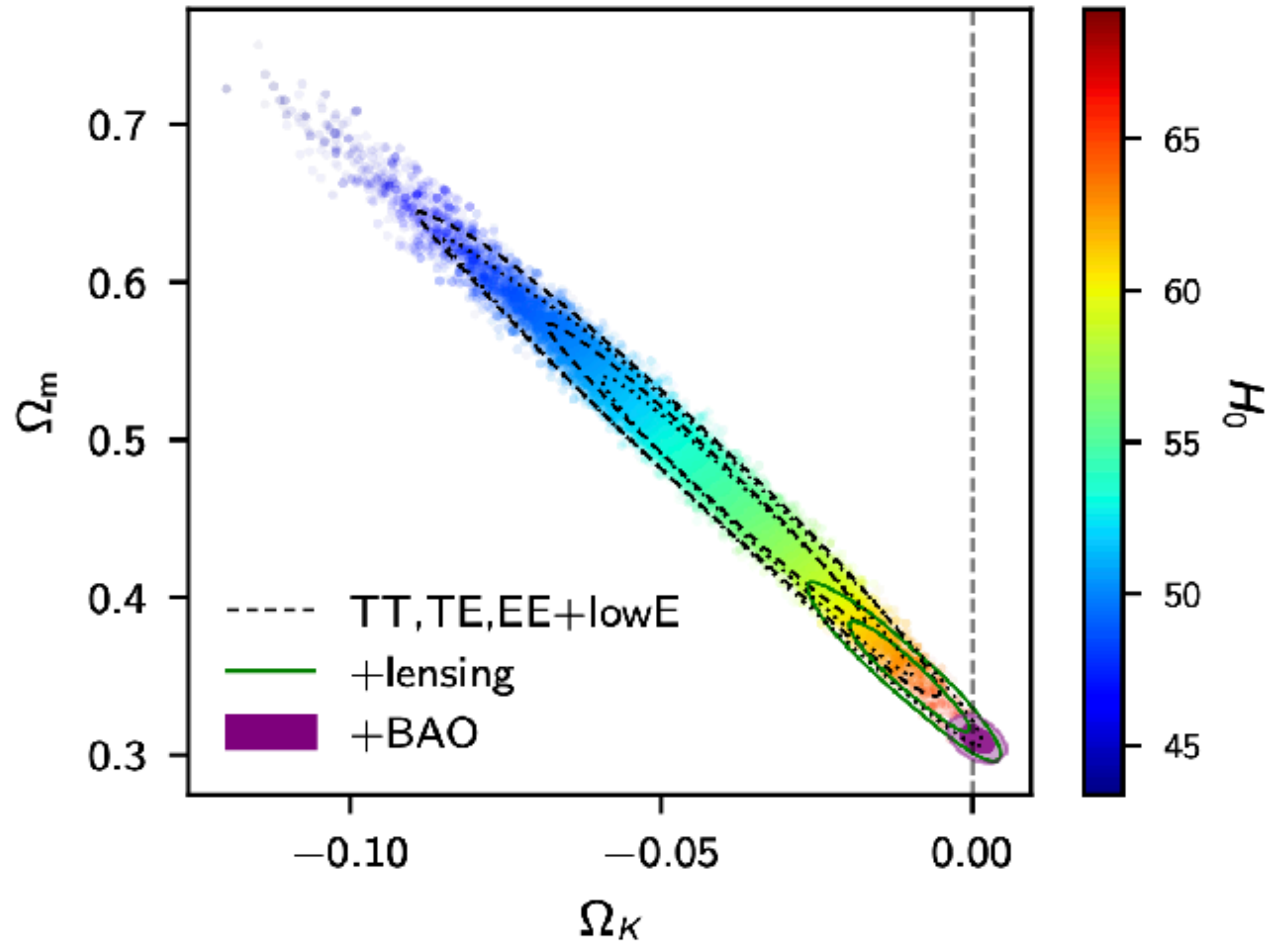
Brout et al. (2022: Pantheon+)



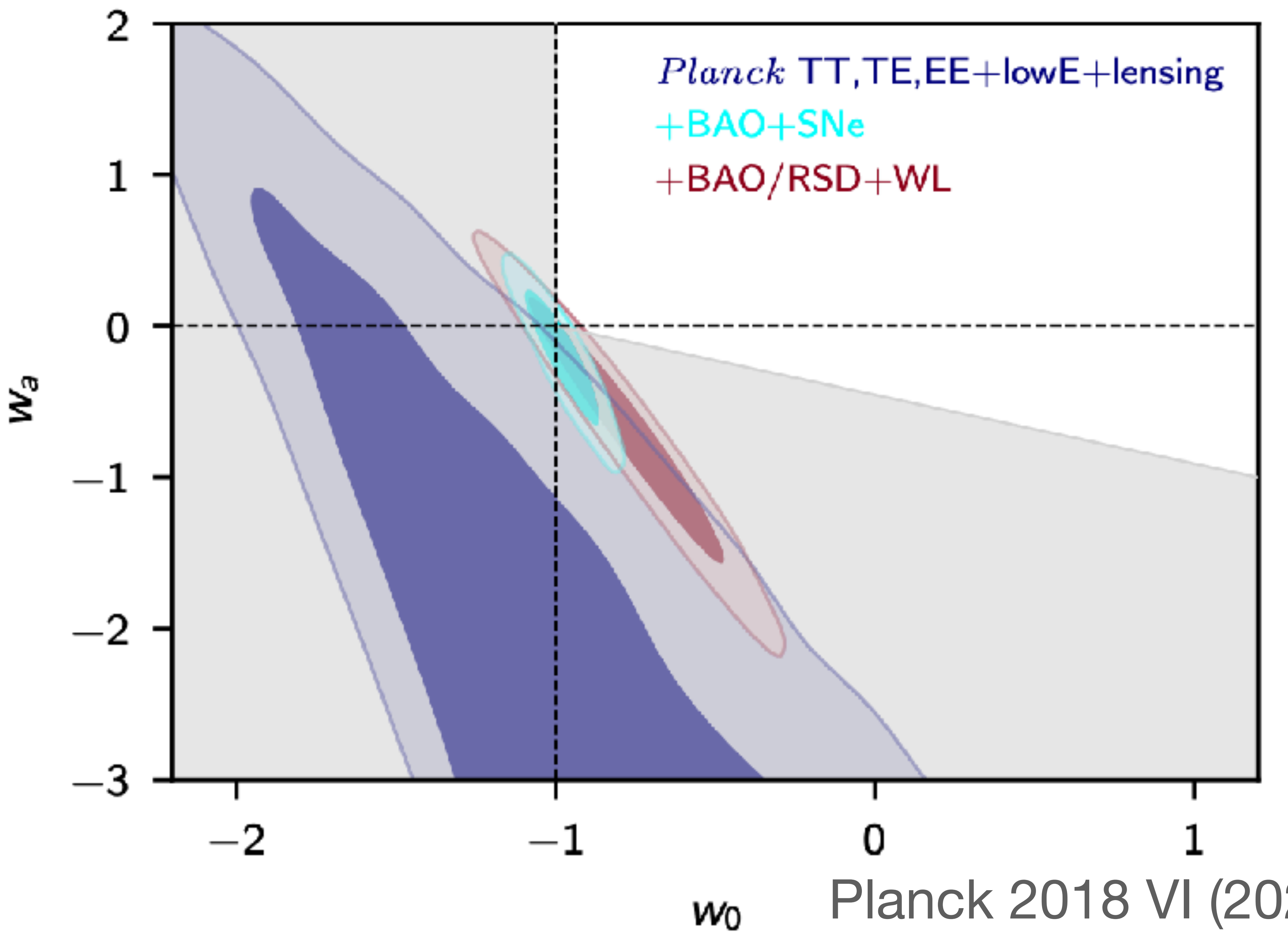
Credit: Ashley J. Ross and SDSS

Summary of combined constraints

- Flat LCDM fits all the data well
- No evidence for any deviation (aside from the H_0 tension)



Parameter	TT+lowE	TT, TE, EE+lowE	TT, TE, EE+lowE+lensing	TT, TE, EE+lowE+lensing+BAO
Ω_K	$-0.056^{+0.044}_{-0.050}$	$-0.044^{+0.033}_{-0.034}$	$-0.011^{+0.013}_{-0.012}$	$0.0007^{+0.0037}_{-0.0037}$
Σm_ν [eV]	< 0.537	< 0.257	< 0.241	< 0.120
$N_{\rm eff}$	$3.00^{+0.57}_{-0.53}$	$2.92^{+0.36}_{-0.37}$	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
$Y_{\rm P}$	$0.246^{+0.039}_{-0.041}$	$0.240^{+0.024}_{-0.025}$	$0.239^{+0.024}_{-0.025}$	$0.242^{+0.023}_{-0.024}$
$dn_s/d\ln k$	$-0.004^{+0.015}_{-0.015}$	$-0.006^{+0.013}_{-0.013}$	$-0.005^{+0.013}_{-0.013}$	$-0.004^{+0.013}_{-0.013}$
$r_{0.002}$	< 0.102	< 0.107	< 0.101	< 0.106
w_0	$-1.56^{+0.60}_{-0.48}$	$-1.58^{+0.52}_{-0.41}$	$-1.57^{+0.50}_{-0.40}$	$-1.04^{+0.10}_{-0.10}$



Planck 2018 VI (2020)