AST1430 - Assignment 1

Due: Jan 31st, 11:59pm

1 Olber's Paradox (15%)

Suppose you are in a classical (static) universe with an average stellar density $n_\star = 10^9$ Mpc^{-3} with the average stellar radius equal to the Sun's, $R_\star = R_\odot = 7 \times 10^8$m. How far, on average, could you see in any direction before your sightline struck a star? What if the stars clump into galaxies with density $n_g = 1$ Mpc^{-3} and average radius $R_g = 2000$pc? Use the darkness of the sky to estimate a maximum allowable age for each case.

2 Tired Light (15%)

A hypothesis once used to explain the Hubble relation is "tired light." This model states that the universe is not expanding, but that through some unknown mechanism, photons simply lose energy as the move through space, as $\frac{dE}{dr} = -KE$, for some constant K. Show that this hypothesis gives a linear distance-redshift relation for $z \ll 1$. How would you distinguish between an expanding universe and tired light?

3 Ages of the Universe (45%)

Current measurements indicate the Universe today is spatially flat ($\kappa = 0$, or at least extremely close), expanding at a rate of $H_0 \approx 70$ km s^{-1} Mpc^{-1}, and dominated by three components $\Omega_\Lambda \approx 0.73$, $\Omega_M \approx 0.27$, and $\Omega_r \approx 5 \times 10^{-5}$.
a) How do the three densities (ρ_Λ, ρ_M, ρ_r) evolve with the Universe? Plot their evolution against the scale factor (use log axes). At what redshift did matter-radiation equality occur? Matter-Λ equality?
b) Assuming a single component dominates at any given time, what form does the evolution of $a(t)$ take during the epochs of radiation domination, matter domination, and Λ-domination?
c) Starting from the scale factor today ($a(t_0) \equiv 1$), plot a piecewise approximation of a(t), using the appropriate evolution in each epoch. Use logarithmic axes!

4 Direct Measurement of Deceleration (25%)

Suppose we live in an Einstein-de Sitter Universe. We observe an object at redshift z. Calculate the observed rate of change of redshift for the object. What fractional precision in observed frequency would be needed to detect cosmological deceleration in a decade?