AST 1430
Observational Cosmology

II

FLRW &

The Measure of a Metric

Week Index	Dates	Topics
~~Week 1~~	~~Jan 11, 13~~	~~logistics, introduction, basic observations~~
Week 2	Jan 18, 20	Basic GR, RW metric, Distances, coordinates, Friedmann equations
Week 3	Jan 25, 27	Cosmological models, consistency with observations, Early Hot Universe, BBN
Week 4	Feb 1, 3	Inflation, Perturbations & Structure pre-recombination
Week 5	Feb 8, 10	CMB: basics, polarization, secondaries
Week 6	Feb 15, 17	Early-Universe Presentations + Review
Reading Week – No Class		
Week 7	Mar 1, 3	Post-recombination growth of structure, formation of dark matter halos, halo mass function
Week 8	Mar 8, 10	The relation between dark matter halos and galaxies
Week 9	Mar 15, 17	Probing the cosmic density field / clustering
Week 10	Mar 22, 24	Late-time cosmological observations: BAO, supernovae, weak lensing, etc.
Week 11	Mar 29, 31	H0 controversy: how fast exactly is the Universe expanding today?
Week 12	Apr 5	Late Universe Presentations + Review

Keith

Jo

[Friedman-Lemaitre-] Robertson-Walker Metric

- What is the most general metric for homogenous and isotropic space?
- F/L each derived, R/W each proved the sol'n.

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa(r)^2 d\Omega^2 \right]$$

Scale Factor

$d\Omega^2$ (= $d\gamma^2$) = $d\theta^2 + \sin^2\theta \, d\phi^2$

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

(= f(r))

Radius of Curvature

Sign of Curvature

Times and Distances

Cosmology is a mess of conflicting and misleading terms.
Worse, "distance" and "time" are slippery concepts in relativity.

Comoving Coords => raw coordinates, defined to match proper positions at $t=t_0$

Proper Time => clock of a fundamental observer

Proper Distance => spatial interval between fundamental observers at a common proper time

Comoving Dist => proper distance at $t=t_0$

Effective Distance => "radius" of illumination, $D == a(t)S_\kappa(r)$ (for a flat universe, equal to proper dist)

Luminosity Dist => distance a photon has travelled $D_L = D\ (1+z)$

Angular diameter Dist => geometric (Euclidean) distance $D_A = D\ /\ (1+z)$

Ok… so…?

- We have two unknowns before we can make physical sense of the FLRW metric:
  - the “scale factor” $a(t)$.
  - $S_\kappa(r)$, which depends on the “curvature” κ/R^2

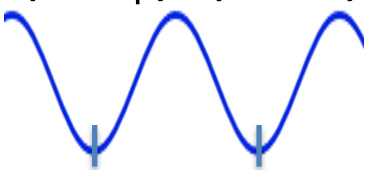
- Deriving these from physics requires more work, and results in the Friedmann Equations

- But: we can measure them directly, and see how/if the Universe behaves. (Do we even live in an RW Homogeneous & Isotropic Universe?)

The Evolution of Light

Emission

$(r,\theta,\phi)=(r,0,0)$



t_1 $\quad$ $t_1 + \Delta t_1$

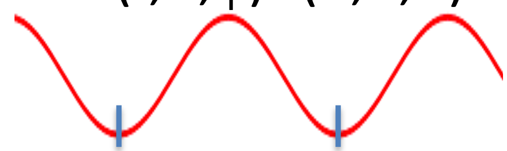
$a(t) = a(t_1)$

$$\Delta t_1 = \nu_1^{-1}$$

a(t)

Observation

$(r,\theta,\phi)=(0,0,0)$



t_0 $\quad$ $t_0 + \Delta t_0$

$a(t) = a(t_0) = 1$

$$\Delta t_0 = \nu_0^{-1}$$

$ds^2 = 0$ for light, so $\mathrm{d}t = -\frac{a(t)}{c}\,\mathrm{d}r$

$$\left(ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + S_\kappa(r)^2 d\Omega^2\right]\right)$$

Emission

$(r,\theta,\phi)=(r,0,0)$

t_1 $\quad$ $t_1 + \Delta t_1$

$a(t) = a(t_1)$

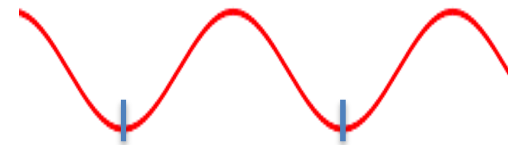
$$\Delta t_1 = \nu_1^{-1}$$

$$\xrightarrow{a(t)}$$

$$dt = -\frac{a(t)}{c}\, dr$$

Observation

$(r,\theta,\phi)=(0,0,0)$



t_0 $\quad$ $t_0 + \Delta t_0$

$a(t) = a(t_0) = 1$

$$\Delta t_0 = \nu_0^{-1}$$

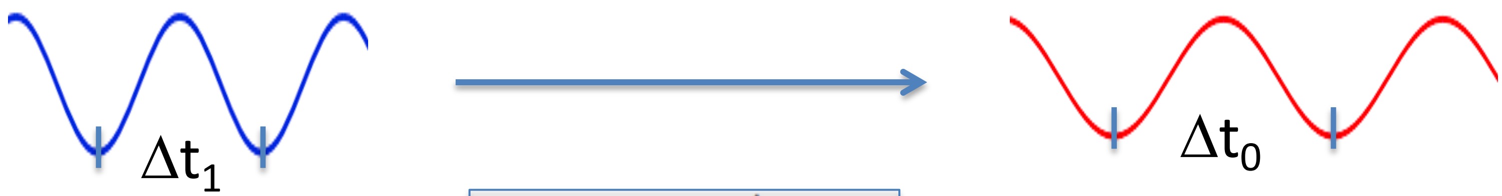
$$\left.\begin{aligned} \int_{t_1}^{t_0} \frac{c\, dt}{a(t)} &= -\int_r^0 dr \\ \int_{t_1+\Delta t_1}^{t_0+\Delta t_0} \frac{c\, dt}{a(t)} &= -\int_r^0 dr \end{aligned}\right\} \quad \int_{t_1}^{t_0} \frac{c\, dt}{a(t)} + \frac{c\,\Delta t_0}{a(t_0)} - \frac{c\,\Delta t_1}{a(t_1)} = \int_{t_1}^{t_0} \frac{c\, dt}{a(t)}$$

$$\boxed{\Delta t_0 = \frac{\Delta t_1}{a(t_1)}}$$

$$\boxed{\nu_0 = \nu_1 a(t_1)}$$

Redshift, z

$$z == \nu_1 / \nu_0 - 1$$
$$= \lambda_0 / \lambda_1 - 1$$



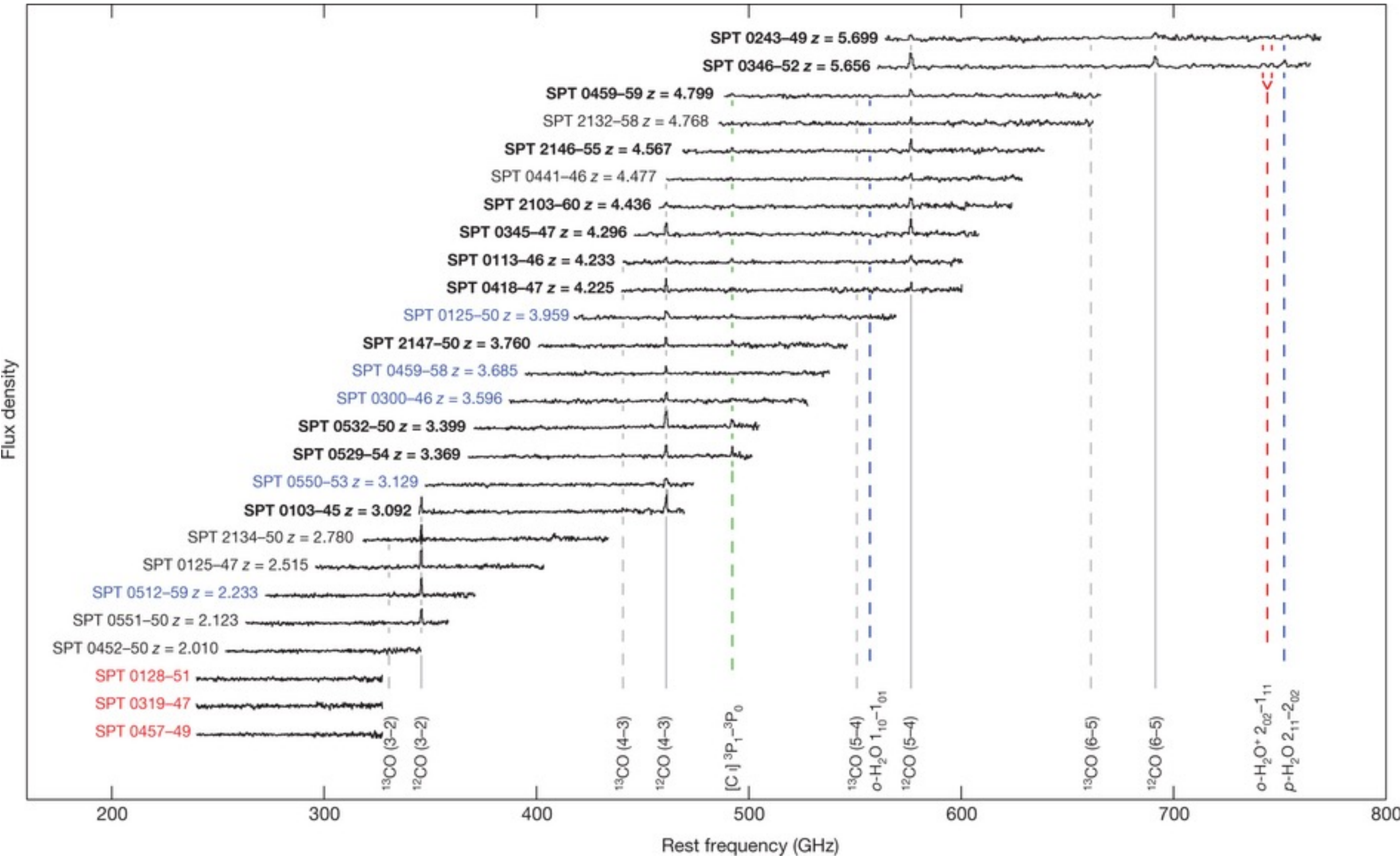
$$\boxed{\Delta t_0 = \frac{\Delta t_1}{a(t_1)}}$$

Redshift can be seen as *time dilation*.

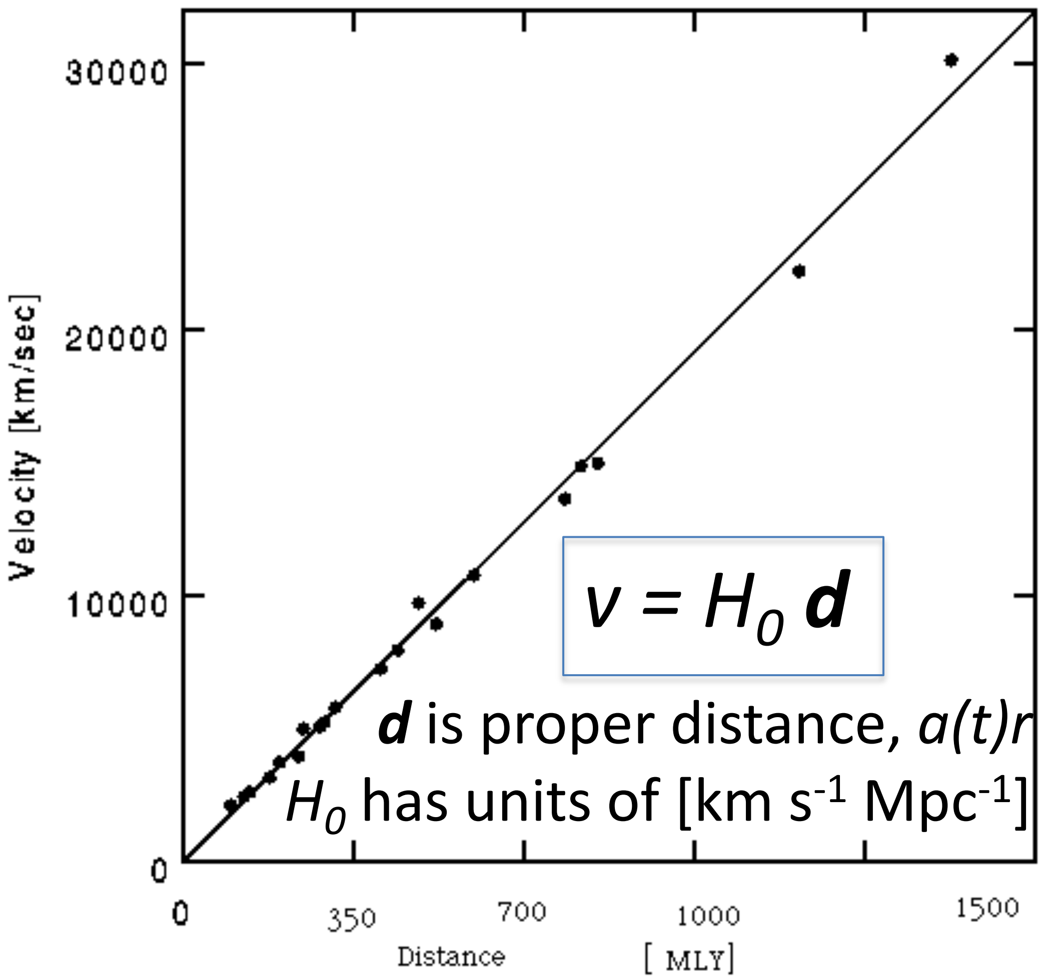
ALL processes show time dilation.

It's been seen in SNe light curves.

Redshift in Practice



Hubble Diagram & Constant



$v = H_0\, \boldsymbol{d}$

d is proper distance, *a(t)r*

H_0 has units of [km s^{-1} Mpc^{-1}]

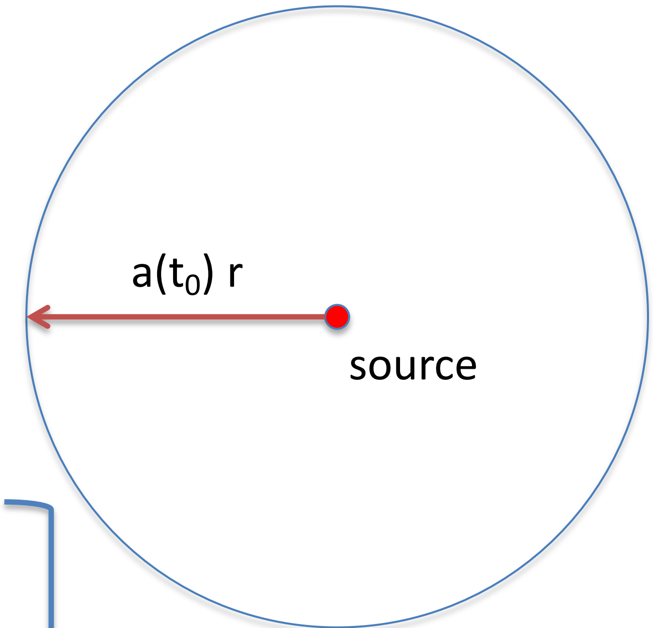
$$
\begin{aligned}
v &= d(\boldsymbol{d}(t))/dt \\
&= r\, d(a(t))/dt \\
&= r\, \dot{a}(t) \\
&= [\dot{a}(t)/a(t)]\ \ \boldsymbol{d}(t)
\end{aligned}
$$

$$H(t) = \dot{a}(t)/a(t)$$

Things expanding away from us this way are said to be caught in the "Hubble Flow"

Apparent Intensity

Source at proper distance $a(t_0)r$
(i.e., comoving distance r).



Bolometric Luminosity is L_{bol}
what is the flux on earth?

Rate of photon arrival dilated:
$n / \Delta t_0 = n\, a(t_1)/\Delta t_1 = (n/\Delta t_1) / (1+z)$

Energy per photon decreased:
$E_0 = h\nu_0 = h\nu_1/(1+z) = E_1 / (1+z)$

Area of the sphere being illuminated:

$$A = \int_{d\Omega} dl^2 = \int a(t)(S_\kappa(r)d\Omega^2) = 4\pi\, \underbrace{a(t_0)^2\, S_\kappa(r)^2}_{== D^2}$$

"Effective Distance"

$$\text{flux} = L_{bol} / (1+z)^2 / A$$
$$= L_{bol} / (4\pi D^2 (1+z)^2)$$
$$= L_{bol} / (4\pi D_L{}^2)$$

$$\boxed{D_L == D\,(1+z)}$$

"Luminosity Distance"

Observing Luminosity

L_{bol} is integrated over all ν (or λ), difficult to observe in practice.

Instead, measure f_ν (f_λ), flux per frequency (wavelength), based on L_ν (L_λ), luminosity per freqency (wavelength):

f_ν:

Bandwidth stretch

$$\Delta\nu_1 = \Delta\nu_0\,(1+z)$$

$$f_\nu = L_{\nu 0} / 4\pi D_L^2$$

$$L_{\nu 1} / (1+z)\, 4\pi D^2$$

f_λ:

Bandwidth compresison

$$\Delta\lambda_0 = \Delta\lambda_1\,(1+z)$$

$$f_\lambda = L_{\lambda 0} / 4\pi D_L^2$$

$$L_{\lambda 1} / (1+z)^3 / 4\pi D^2$$

K-correction

When observing,
1. correct to rest-frame ν (λ)
2. correct for bandwidth compression

In Magnitude units, these are additive. For monochromatic light,

$$\Delta m(\nu) = K(z,\nu) = -2.5 \log \left((1+z) \quad L_\nu(\nu(1+z)) \; / \; L_\nu(\nu) \right)$$
$$\Delta m(\lambda) = K(z,\lambda) = -2.5 \log \left(1/(1+z) \; L_\lambda(\lambda/(1+z)) \; / \; L_\lambda(\lambda) \right)$$

For a finite bandwidth, we have to integrate the SED.

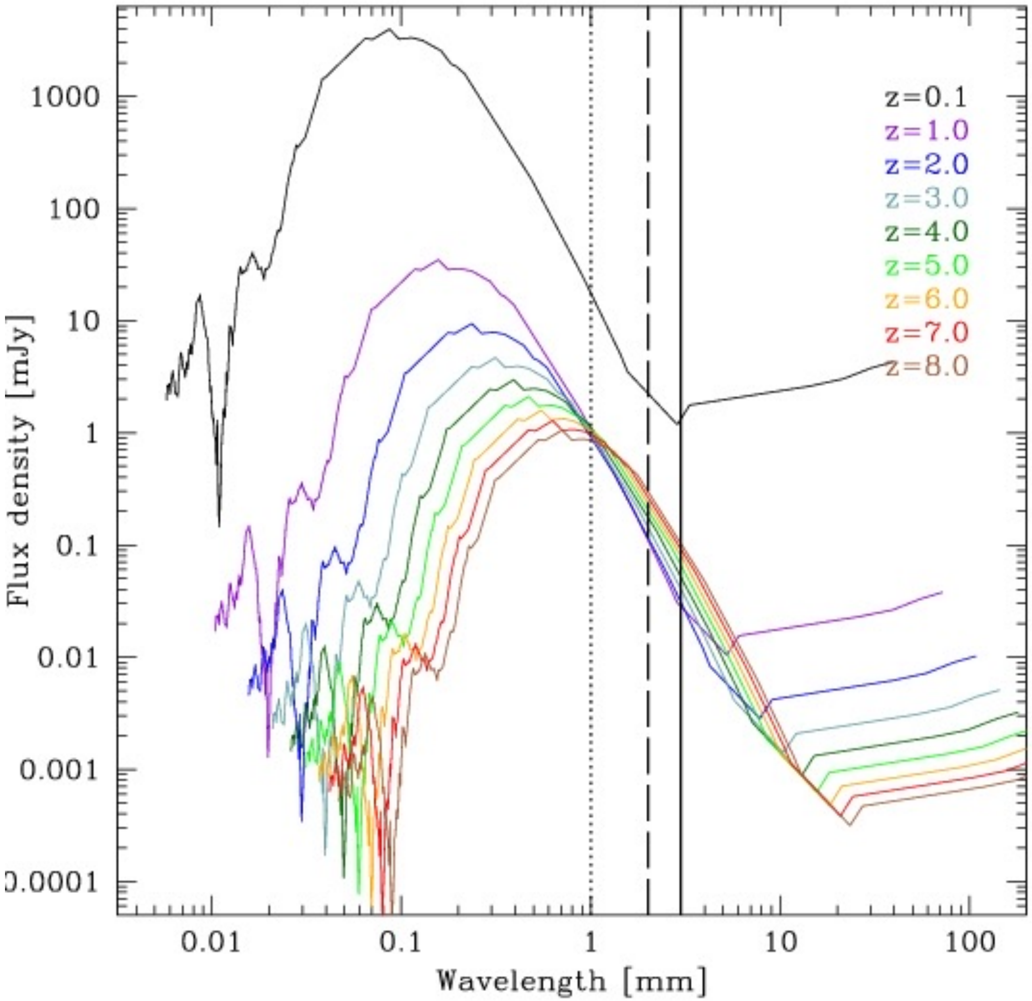
Distance Modulus

$$DM = 5 \log (D_L(z) \; / \; 10pc) = 5 \log (D \, (1+z) \; / \; 10pc)$$

Convenient Conversion from Absolute Magnitude $M(\nu)$ to apparent magnitude $m(\nu,z)$

$$m(\lambda,z) = M(\lambda) + DM + K(\lambda,z)$$

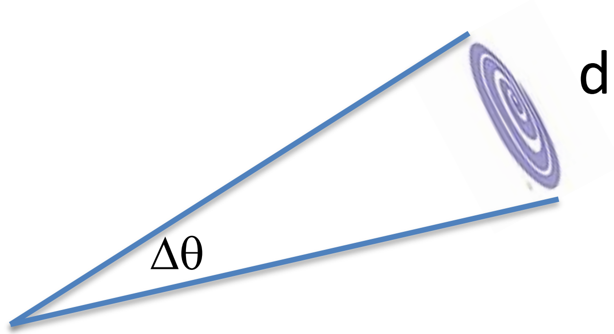
Negative K-correction



For steeply falling spectra, ($f_\lambda \propto \lambda^{-3}$), K-correction can cancel Distance Modulus, leading to flux independent of redshift.

Angular Sizes

Proper size of an object at time t is $dl^2 = a(t)^2\,(dr^2 + S_\kappa(r)^2\,d\Omega^2)$



$$d = \int dl$$

$$= S_\kappa(r)a(t)\int d\theta$$

$$= S_\kappa(r)a(t)\Delta\theta$$

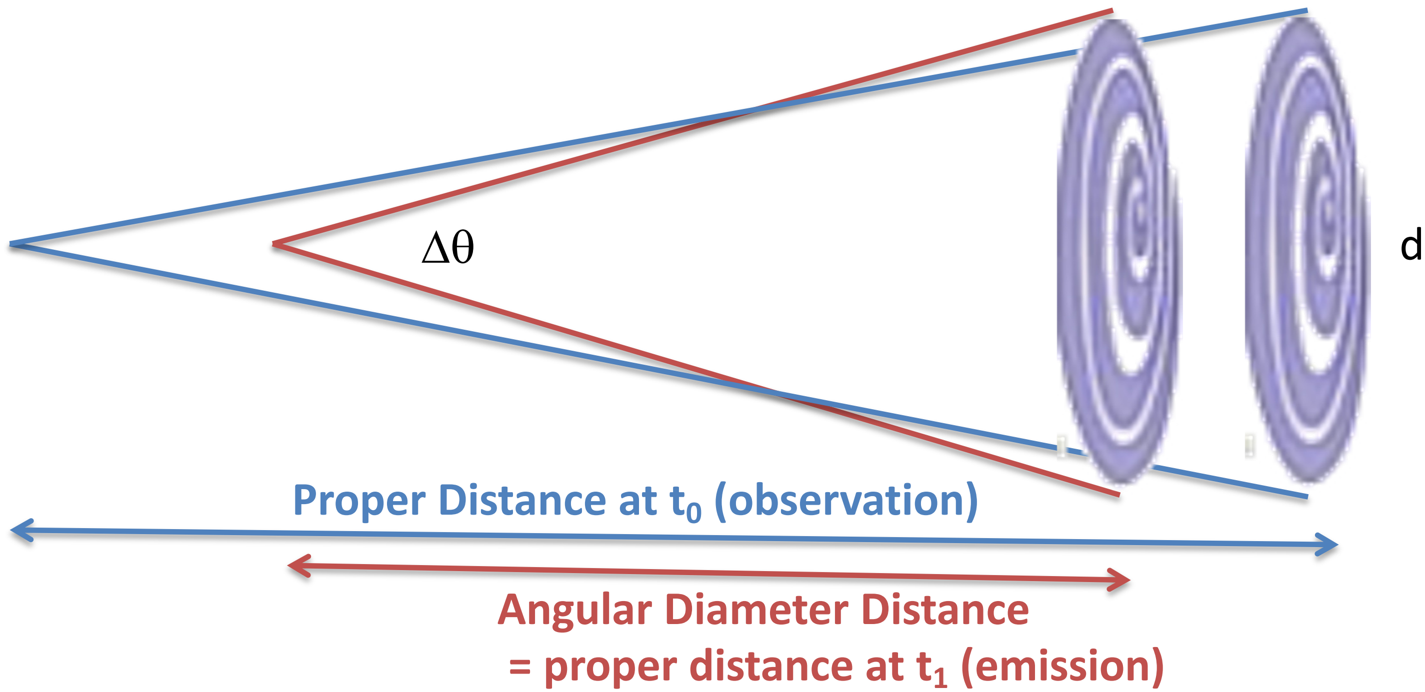
"Angular Diameter Distance" $D_A = d/\Delta\theta = S_\kappa(r)a(t)$

Recall: $D = a(t_0)S_\kappa(r)$,
$a(t)^{-1} = (1+z)$

=> $\boxed{D_A = D\,/\,(1+z)}$

(Distance to an object implied by Euclidean geometry.)

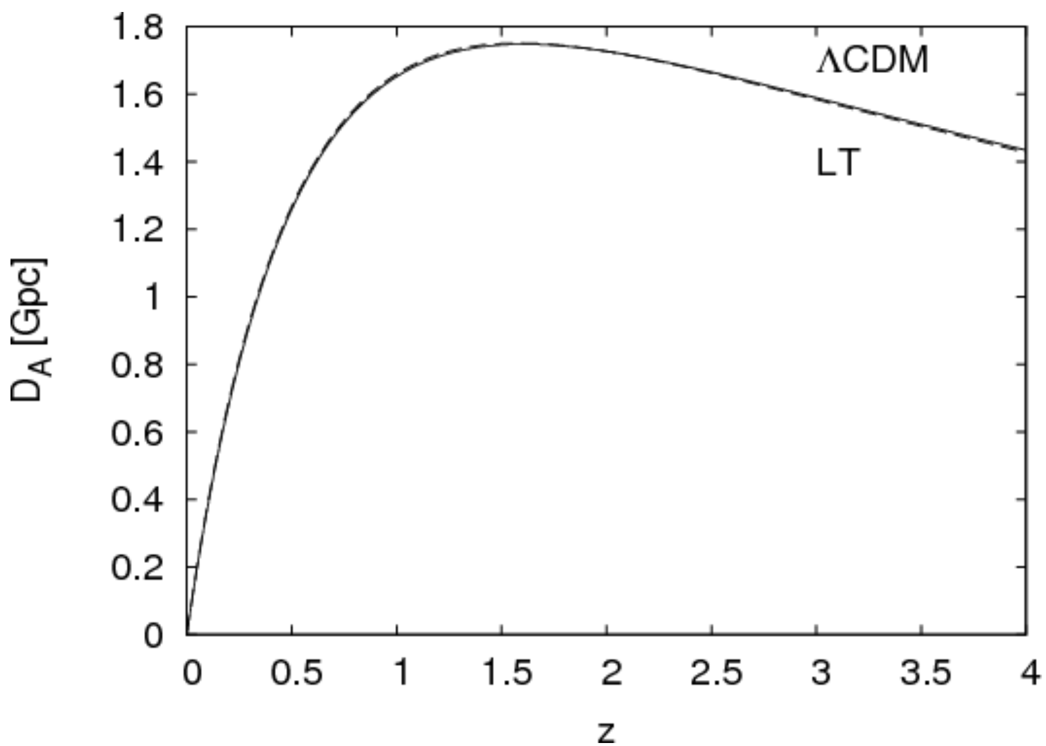
Angular Diameter Distance, D_A

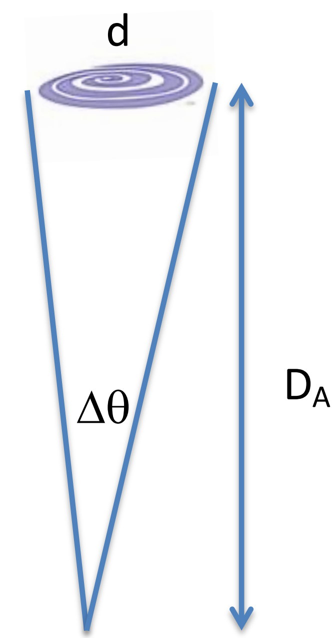


Angular sizes are frozen at emission!
For bound structures (don't expand with Hubble flow), can calculate physical sizes from angular sizes with D_A

Angular Size

Note: D_A is (1+z) *smaller* than D, the "effective distance."





$$\Delta\theta = d / D_A = d\,(1+z) / D$$

Bound objects get LARGER past z≈1.5!

Measurement Review

(r,θ,ϕ), (x,y,z), etc => raw coordinates, used to define the metric
(these don't change with expansion)

Comoving Coords => raw coordinates, but defined to match proper positions at $t=t_0$

Proper Time => clock of a fundamental observer

Proper Distance => spatial interval between fundamental observers at a common proper time

Comoving Dist => proper distance at $t=t_0$

Effective Distance => "radius" of illumination, $D == a(t)S_\kappa(r)$
(for a flat universe, equal to proper dist)

Luminosity Dist => Euclidean dist implied by diminution of flux
$D_L = D\,(1+z)$

Angular diameter Dist => geometric (Euclidean) distance
$D_A = D\,/\,(1+z)$

Surface Brightness

$$\mu \propto fA^{-1}D^{-2}$$

Static Flat Universe: $\mu \propto f\, D^2\, D^{-2}$

Expanding Universe: $\mu \propto f\, D_A{}^2\, D_L{}^{-2}$

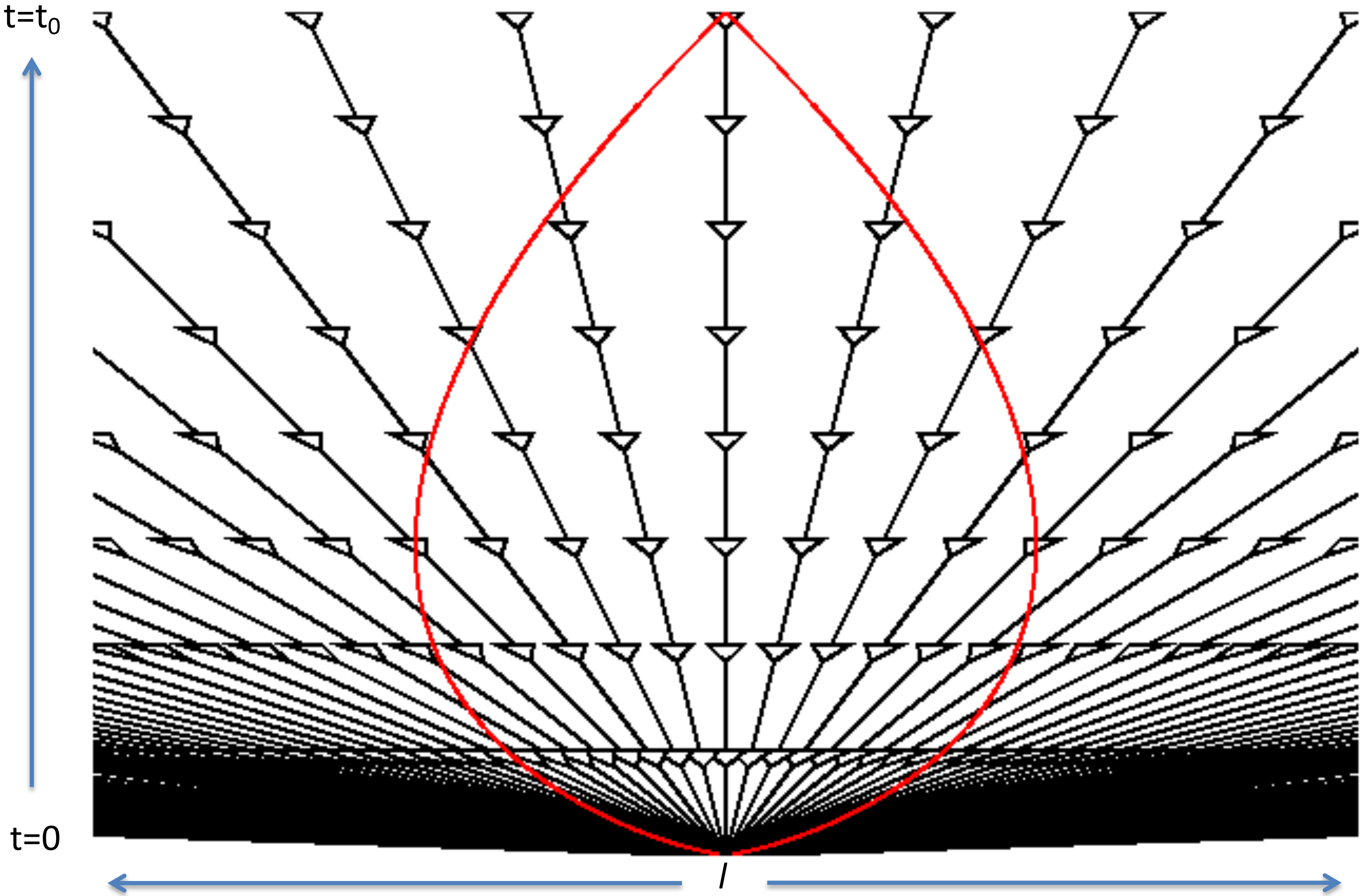
$$\mu_{bol0} \sim \mu_{bol1} / (1+z)^4$$ ("Tolman Effect")

$$\mu_{\nu 0} \sim \mu_{\nu 1} / (1+z)^3$$

$$\mu_{\lambda 0} \sim \mu_{\lambda 1} / (1+z)^5$$

These are strictly a consequence of expanding spacetime, completely independent of what makes it expand (GR).

Thinking In Expanding Spacetime



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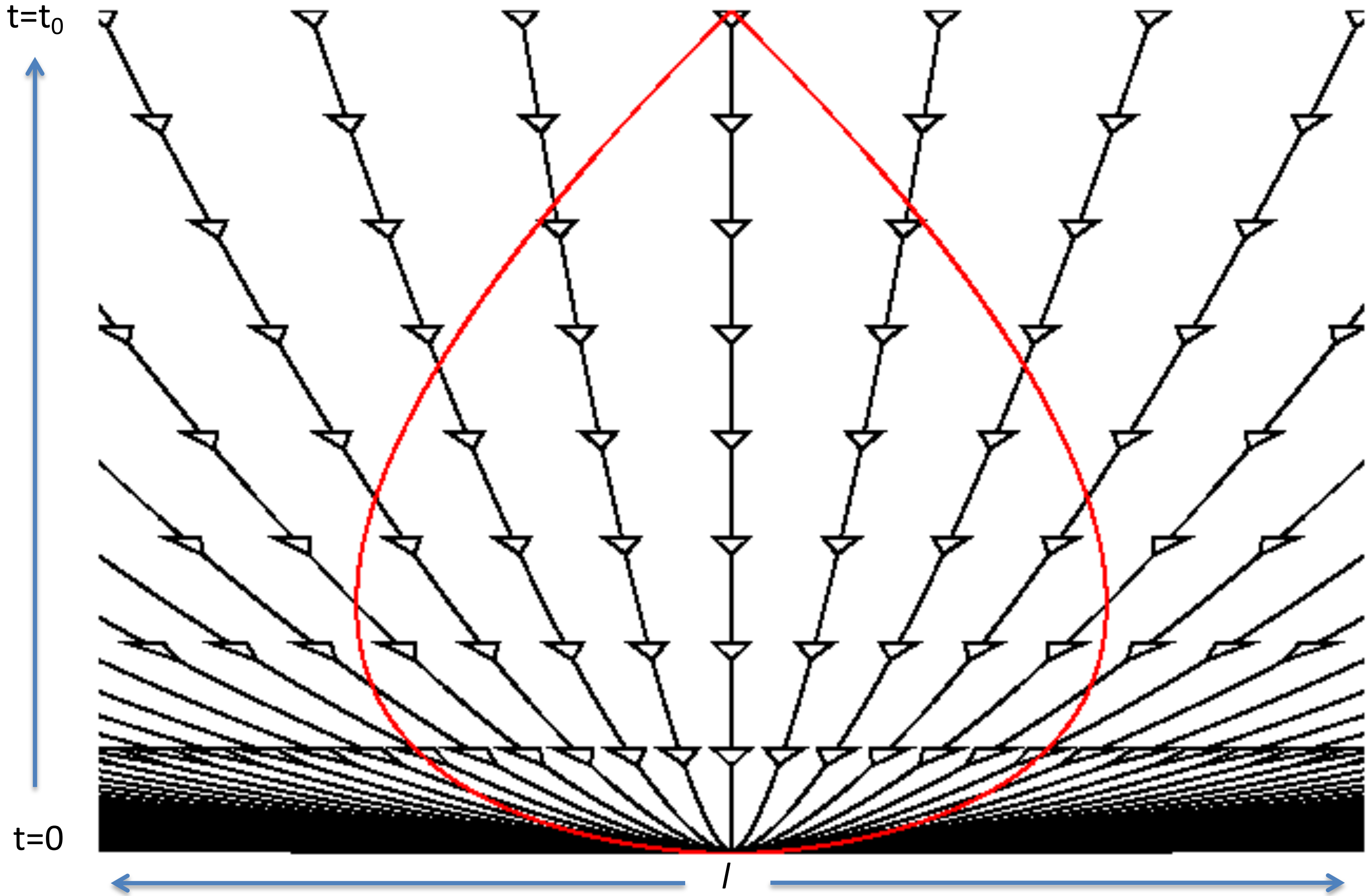
Decelerating Expansion



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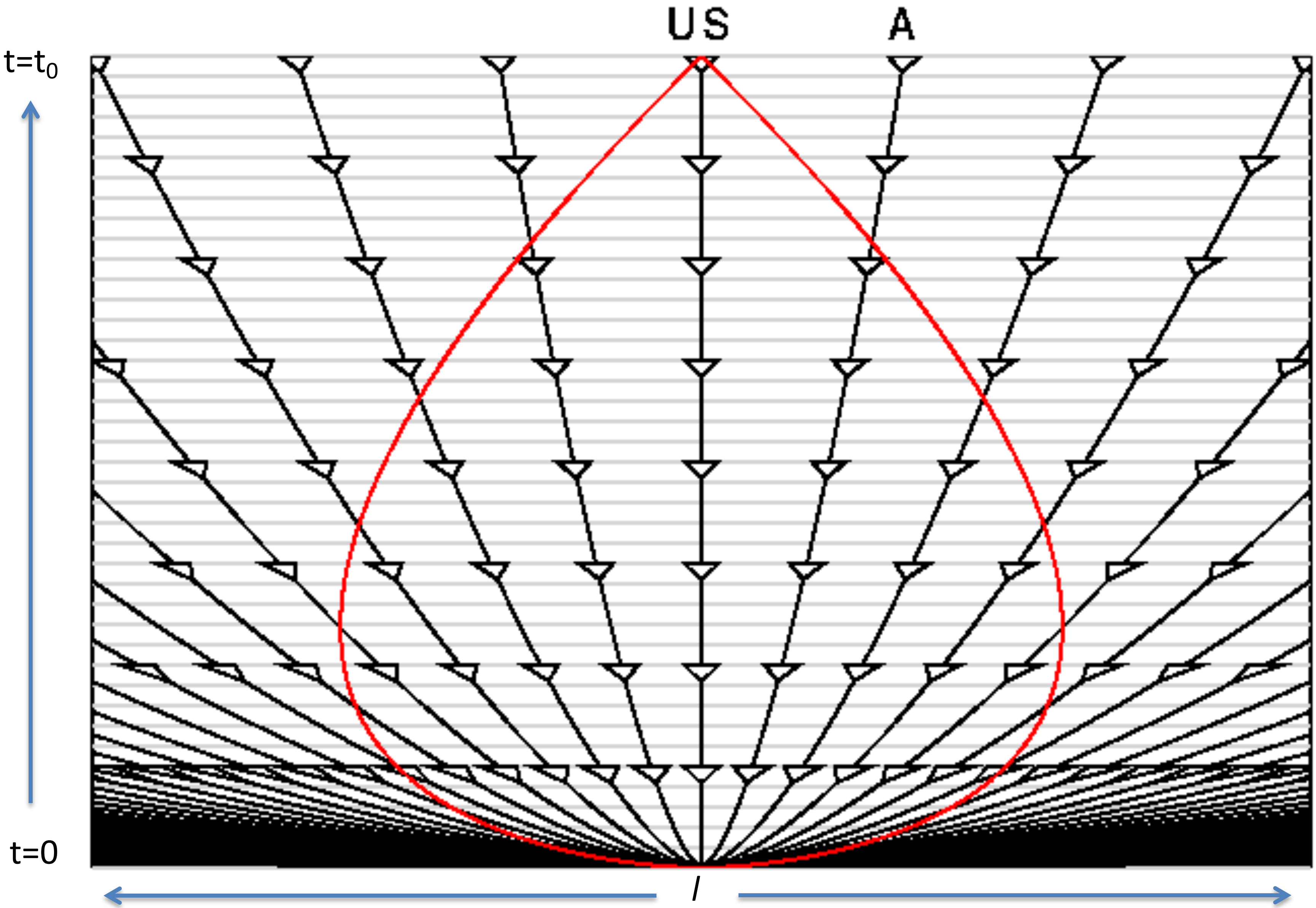
Changing Perspective



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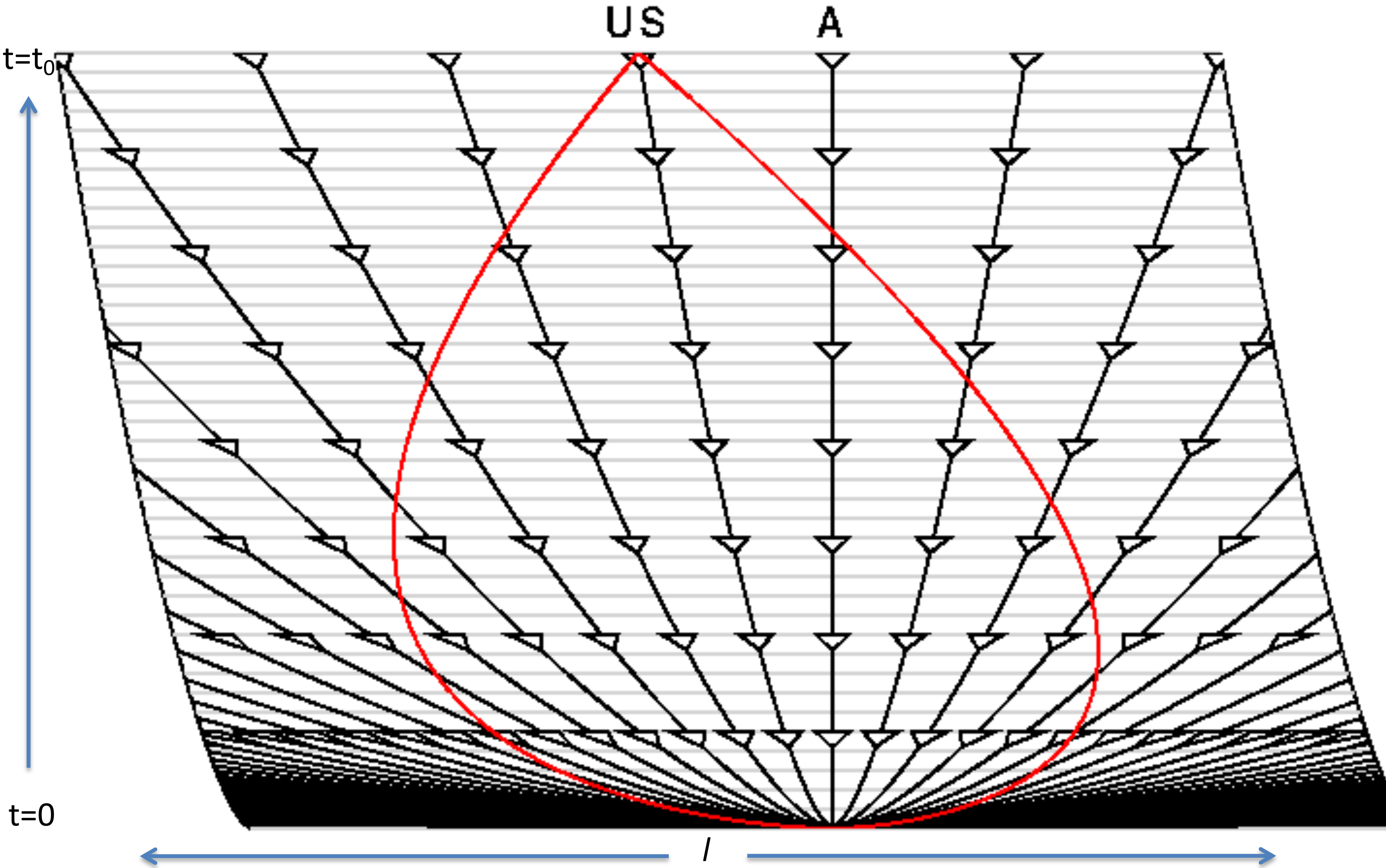
Changing Perspective



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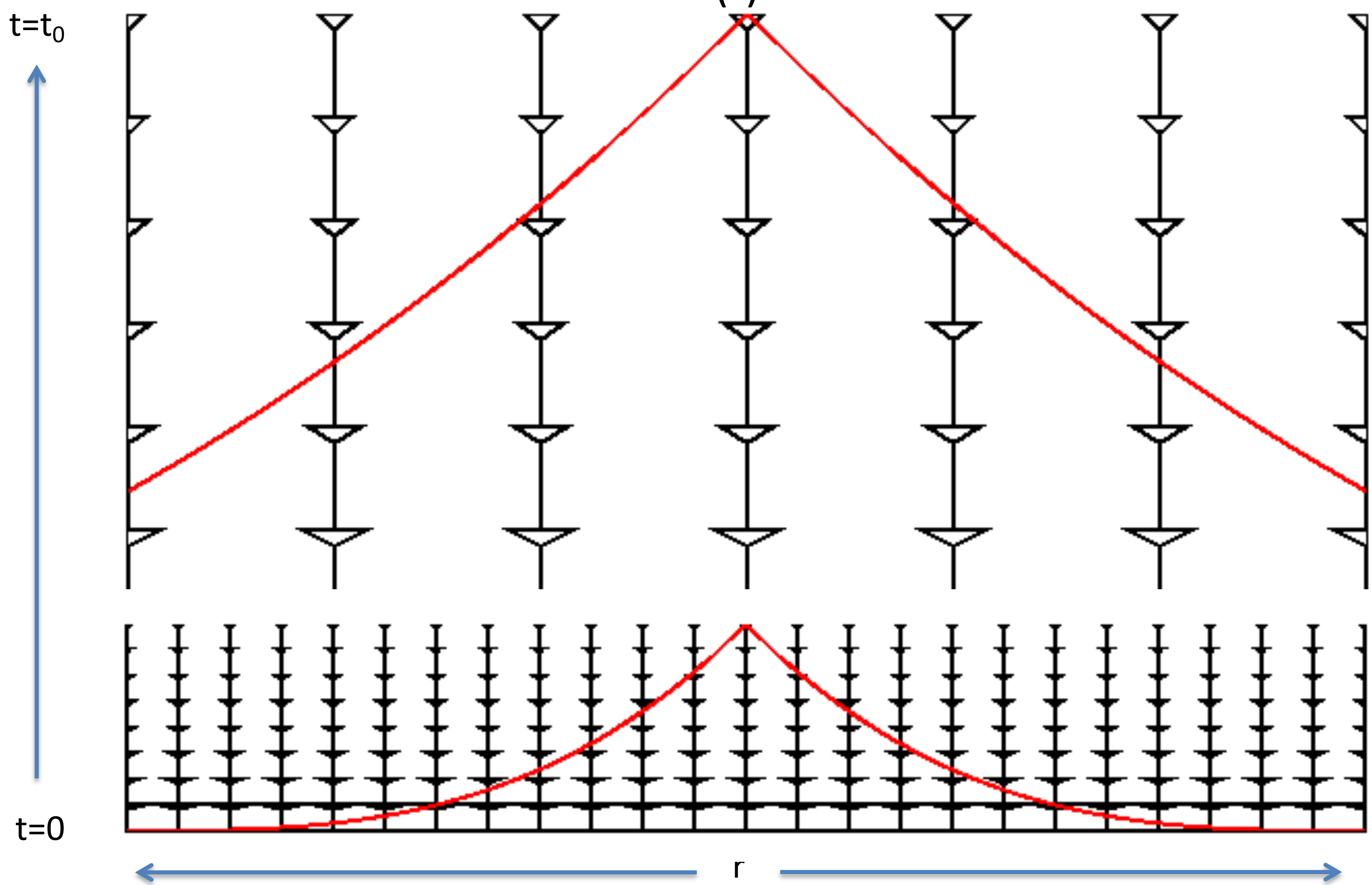
Comoving Coordinates

$r = a(t)\,d$



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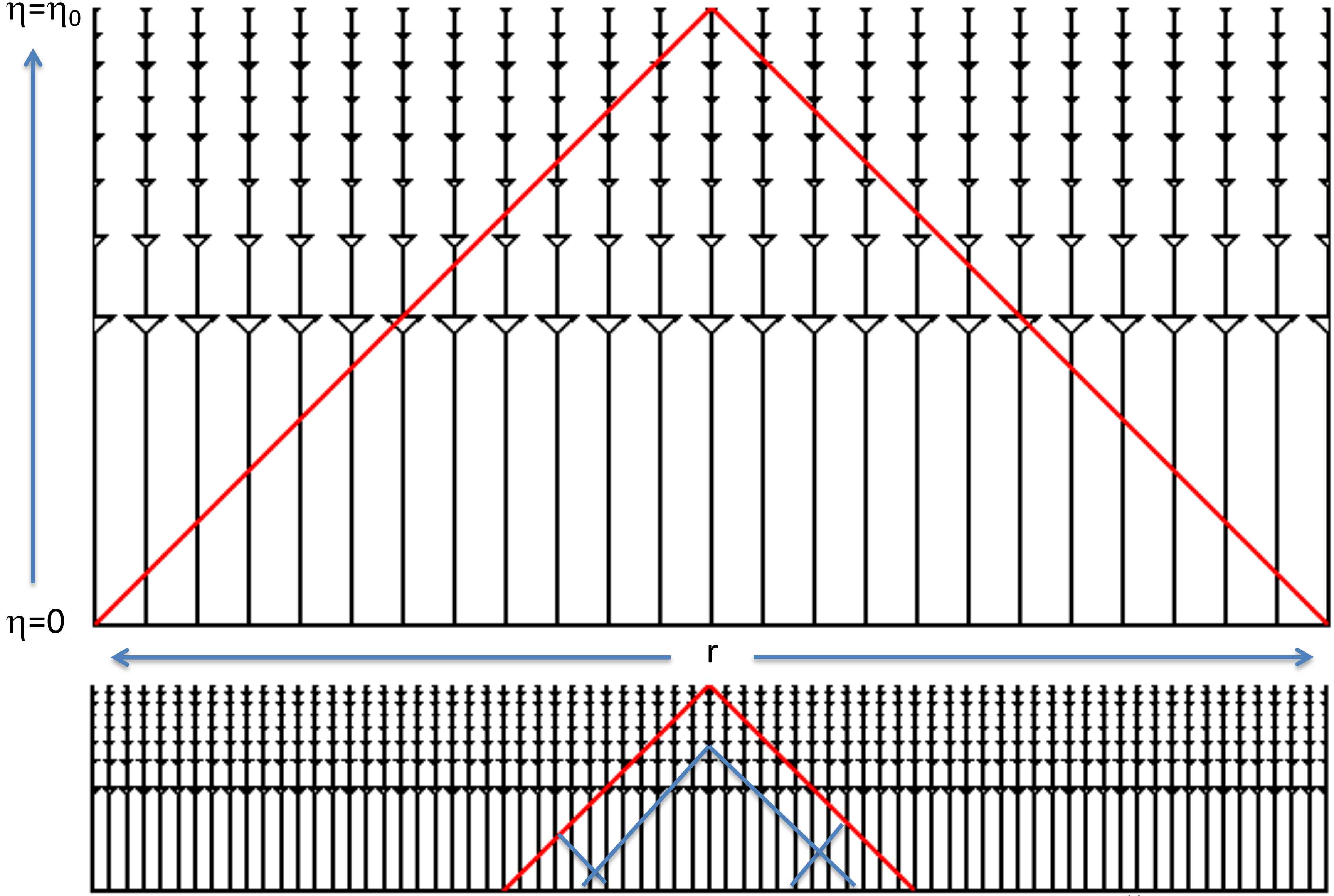
Conformal Time

$$\partial\eta = dt / a(t)$$



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Next Time:

Measuring $a(t)$ and $S_{\kappa}(r)$