AST 1430W
Cosmology

IV

Cepheids, Distances;

Thermal History: Part I

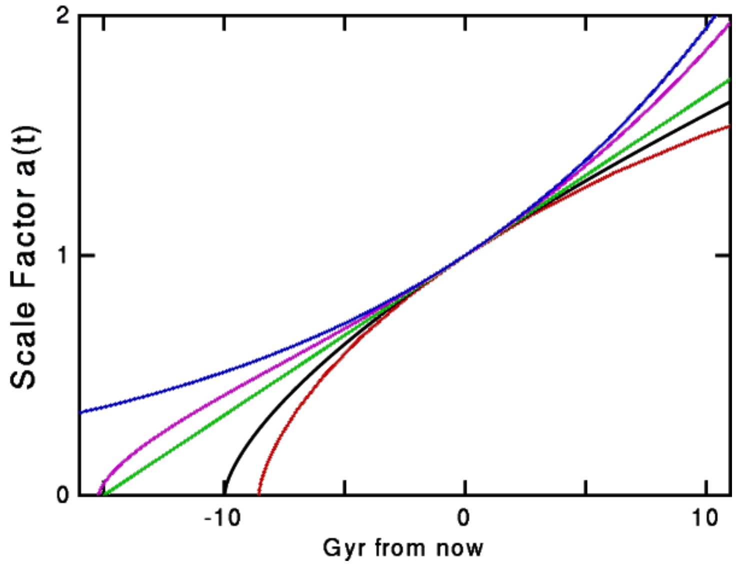
Announcements

1. Assignment #1 out due next Tue, Jan 31^{st}.
2. Presentations:
    - Feb 15 / 17^{th}: Early Universe
        - Anika Slizewski
        - Braden Gail
        - Caleb Lammers
        - Anna Tsai
    - Please send suggested topics! first come, first served.
    - 20min presentation + 5-10min Q&A + 1-2pg handout including refs

	Week Index	Dates	Topics	
Keith	~~Week 1~~	~~Jan 11, 13~~	~~logistics, introduction, basic observations~~	
	~~Week 2~~	~~Jan 18, 20~~	~~Basic GR, RW metric, Distances, coordinates, Friedmann equations, cosmological models~~	asst1
	Week 3	Jan 25, 27	Consistency with observations, Early Hot Universe, BBN	
	Week 4	Feb 1, 3	Inflation, Perturbations & Structure pre-recombination	
	Week 5	Feb 8, 10	CMB: basics, polarization, secondaries	
	Week 6	Feb 15, 17	Early-Universe Presentations + Review	asst2
	Reading Week – No Class			
Jo	**Week 7**	Mar 1, 3	Post-recombination growth of structure, formation of dark matter halos, halo mass function	
	Week 8	Mar 8, 10	The relation between dark matter halos and galaxies	
	Week 9	Mar 15, 17	Probing the cosmic density field / clustering	asst3
	Week 10	Mar 22, 24	Late-time cosmological observations: BAO, supernovae, weak lensing, etc.	
	Week 11	Mar 29, 31	H0 controversy: how fast exactly is the Universe expanding today?	
	Week 12	Apr 5	Late Universe Presentations + Review	

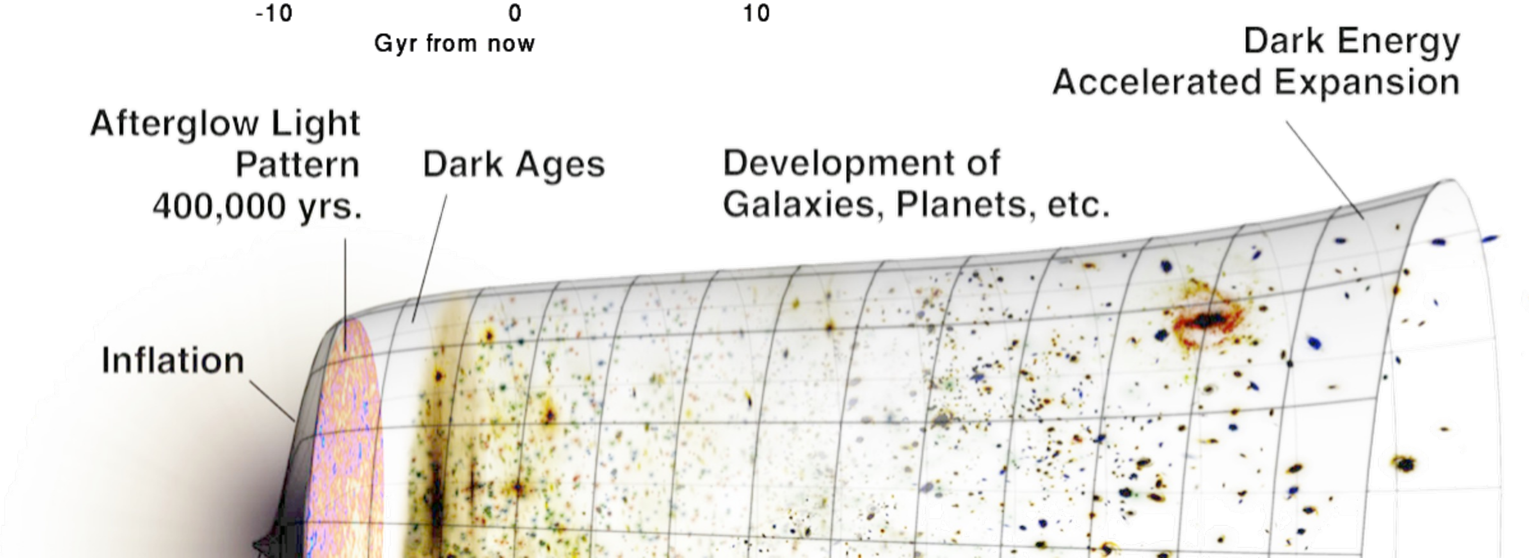
Context Reminder

$$\frac{\dot{a}^2}{a^2} = H^2 = H_0^2(\Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda)$$

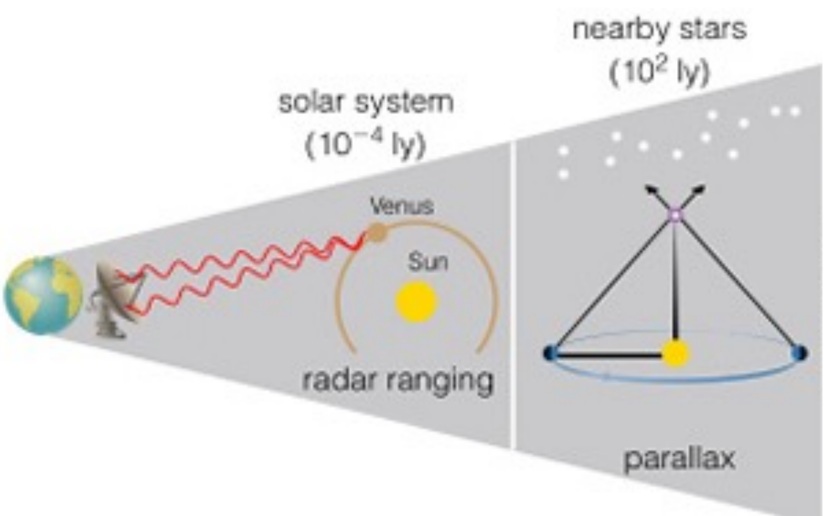


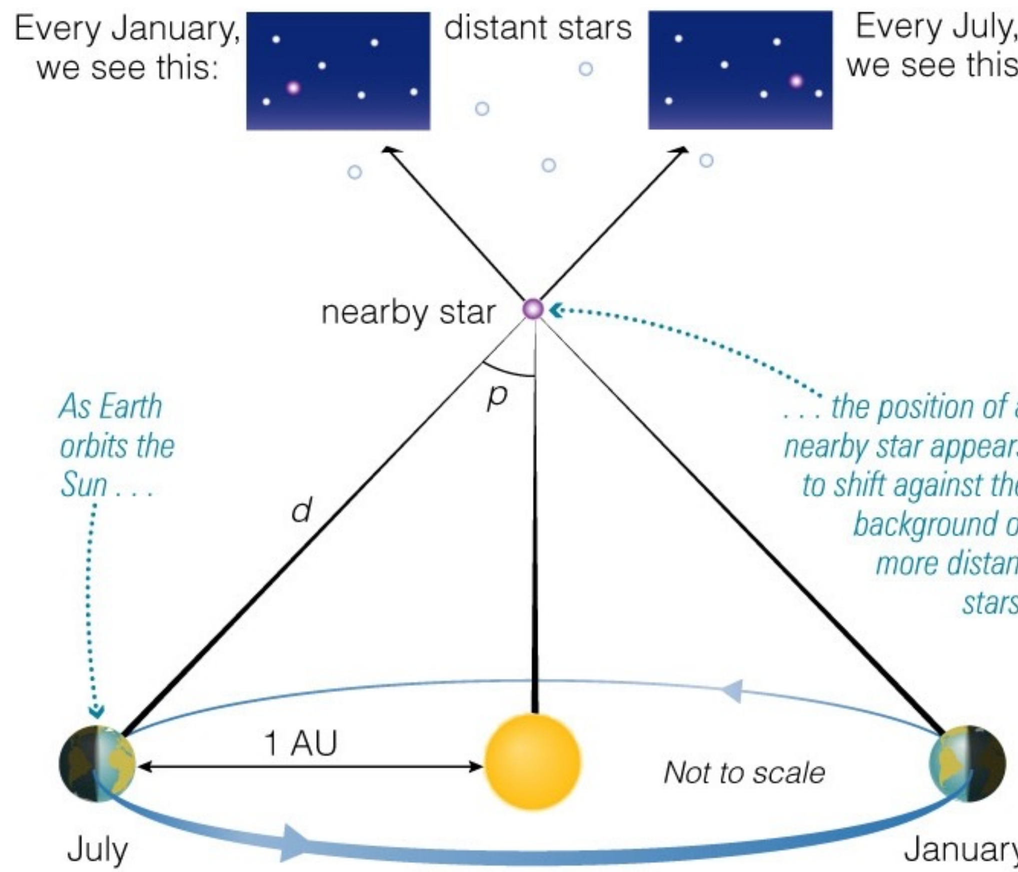
$D_A = D / (1+z)$

$D_L = D (1+z)$



The Distance Ladder



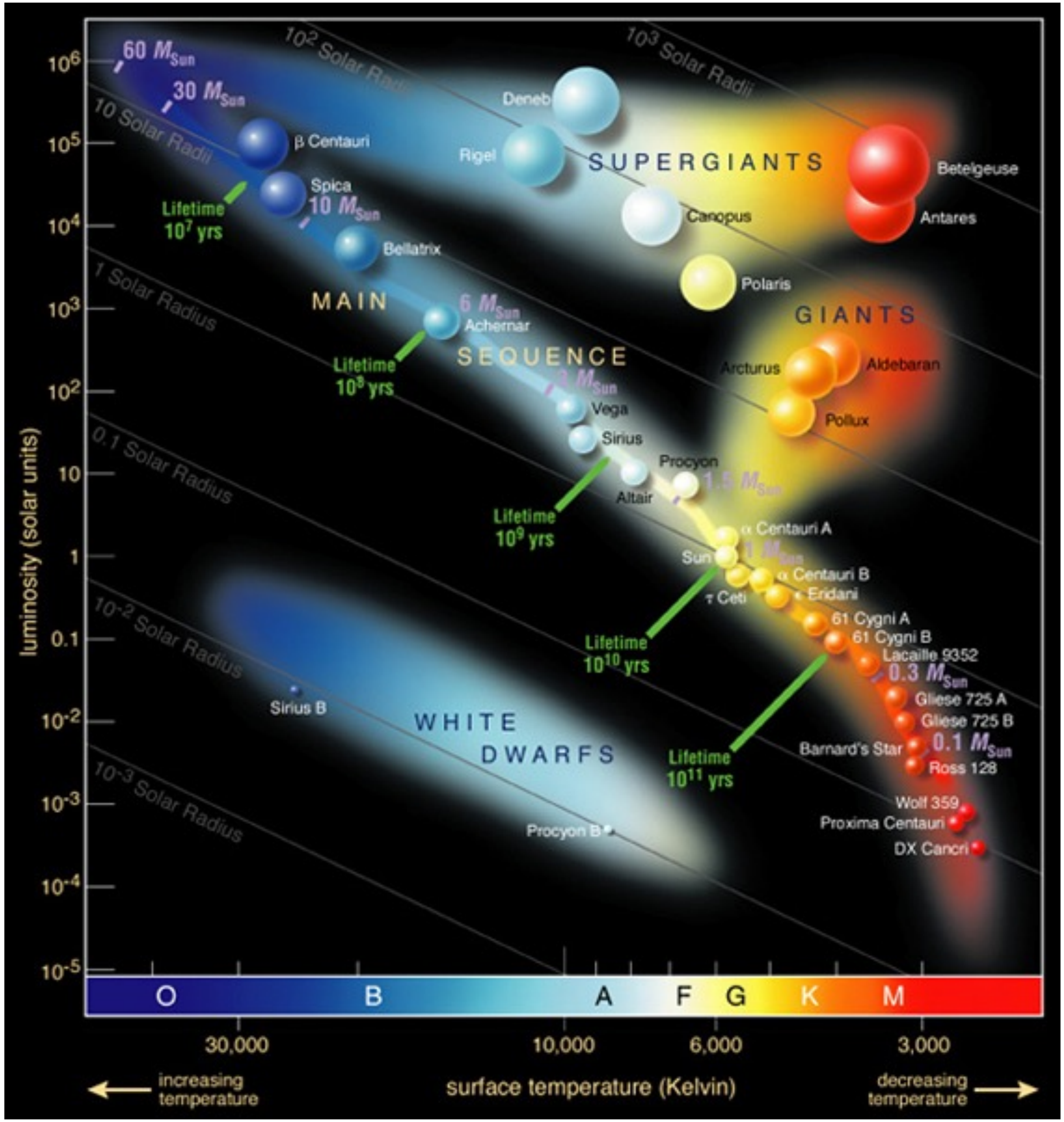


Standard Candles

If we know how much total light is given off by a source, that source is called a **standard candle**.

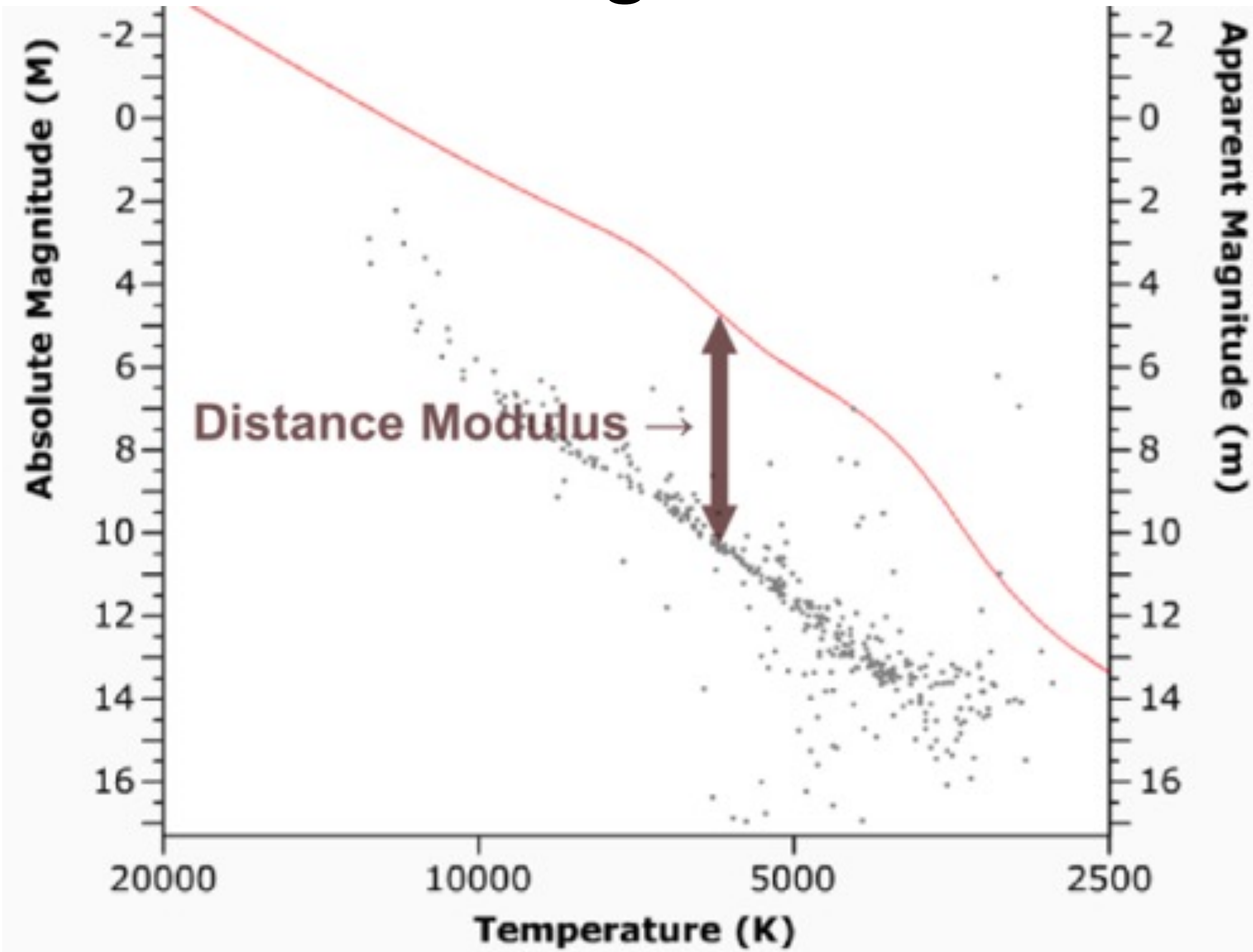
By measuring how much light makes it to us, we can calculate the candle's distance.

Luminosity
(Total Energy Radiated)



Color ∝ Temperature

Compare Absolute to Apparent Magnitude



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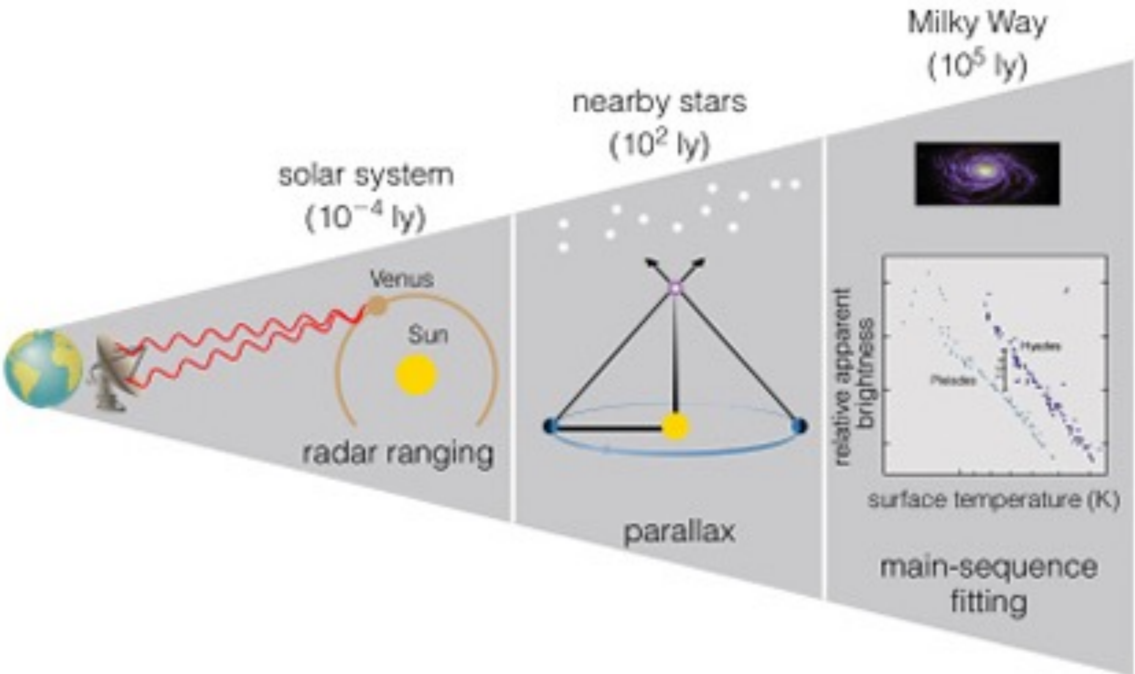
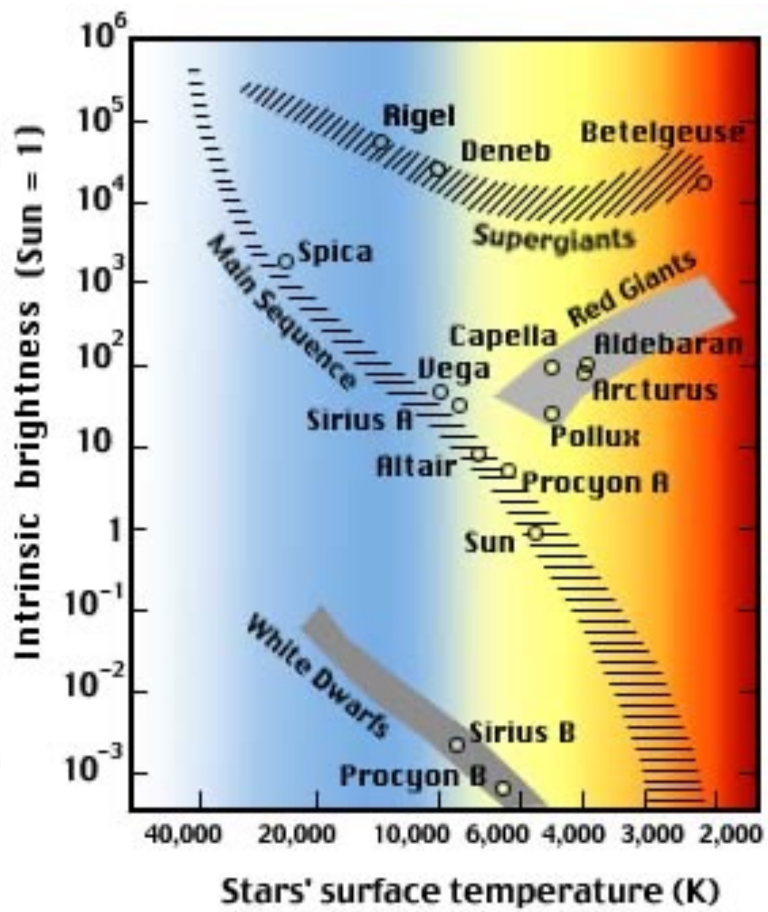




Seeing beyond the Galaxy

Henrietta Swan Leavitt

Took her first astro course in her final year of college.

Worked as a “computer”, cataloguing variable stars in Magellenic Clouds.

In the SMC, discovered “Leavitt’s Law”, aka the Period-Luminosity relation.

Cepheid Variables

Period of Oscillation (days)

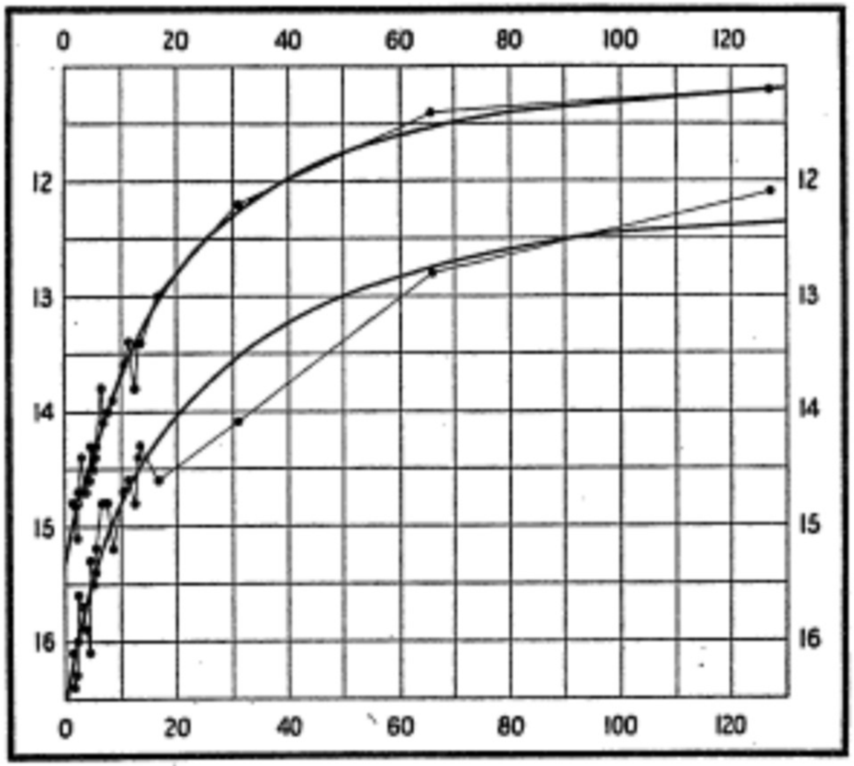


FIG. 1.

Luminosity (mag)

Log(Period)

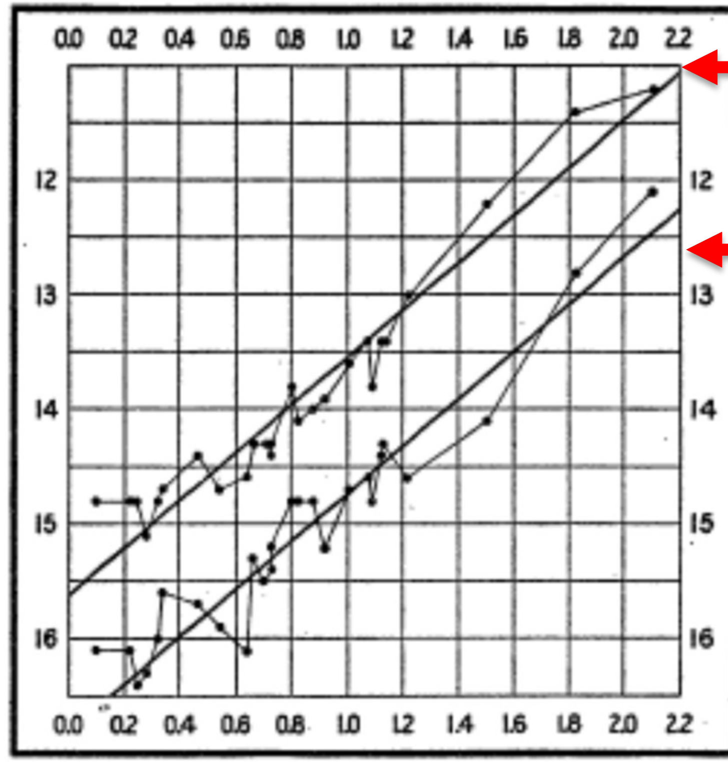


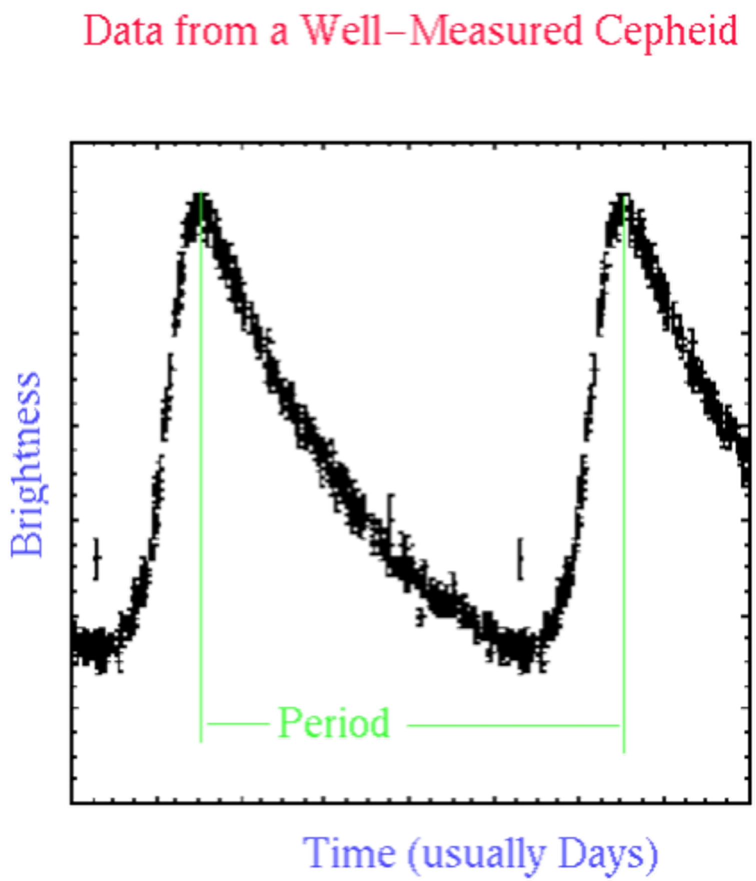
FIG. 2.

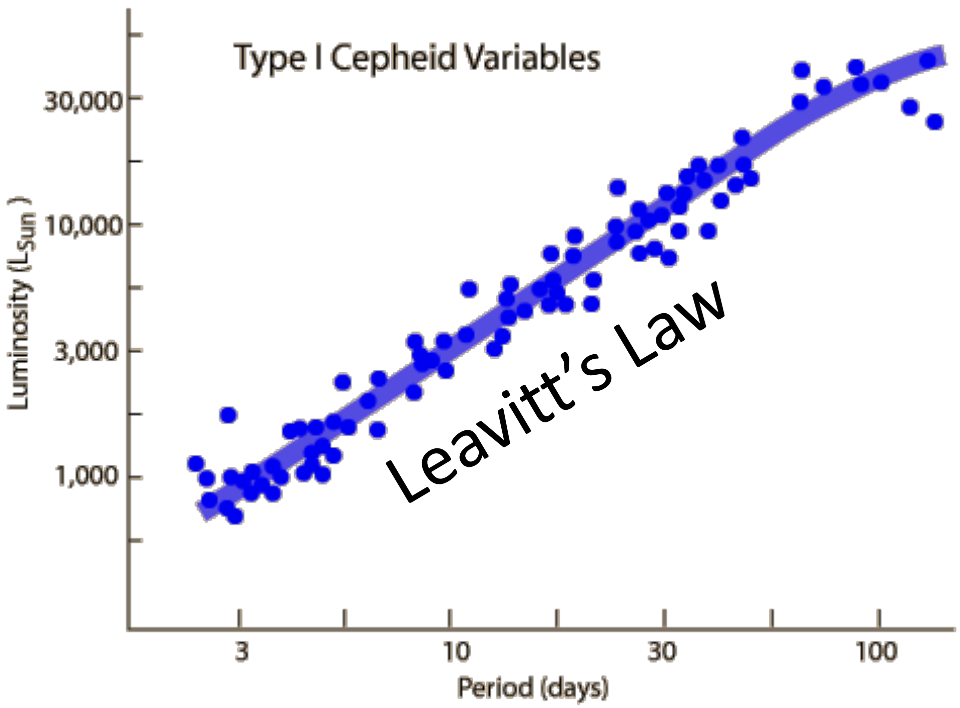
Maximum Luminosity

Minimum Luminosity

Leavitt et al., 1912

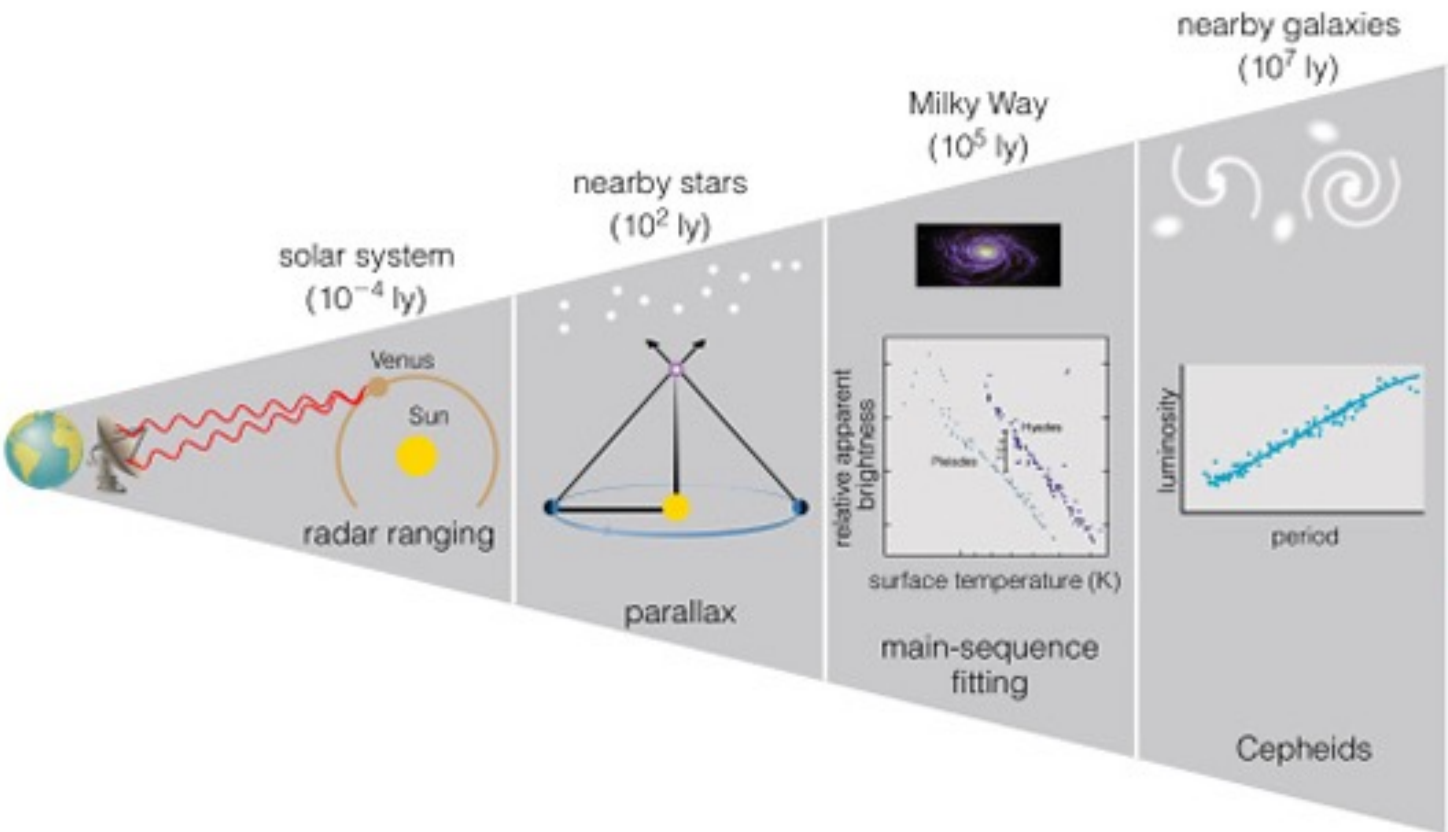
Cepheid Variables

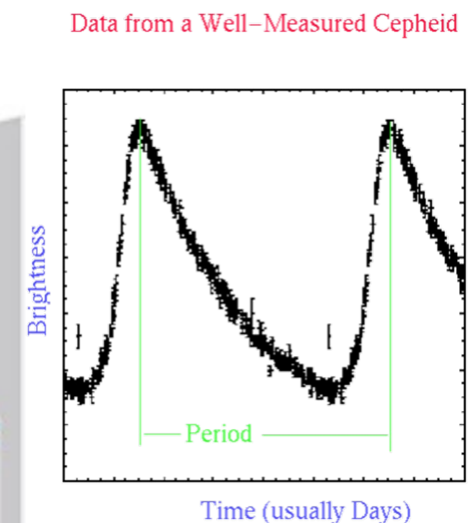


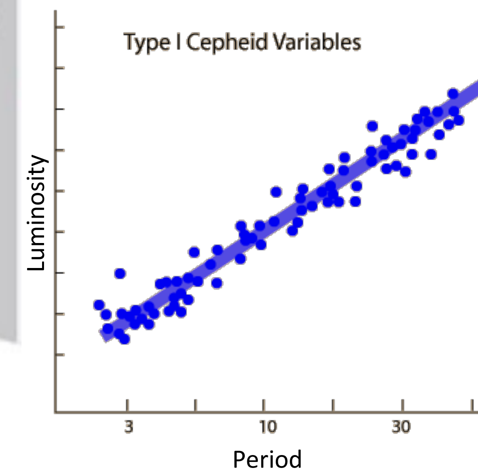


VERY bright stars, easy to infer absolute magnitudes!

The Distance Ladder







Measuring t_0

The Universe must be at least as old as anything in it:

- The earth is 4.5Gy old, so that’s a solid lower limit.
- Degenerate cooling: white dwarfs slowly cool from initial hot states. The coldest are ≈4000K, so ≈10Gy old.
- Fitting HR diagrams: the oldest clusters are ≈13Gy.
- Decay of long-lived radioactive elements: small abundances in the oldest, most metal-poor stars give ages ≈15Gy.

Note: These “direct” measures are all crude & only lower limits!

Time-Redshift Relation

$$E(z) \equiv \frac{H(z)}{H_0} = \sqrt{\cancel{\Omega}_R(1+z)^4 + \Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$$

(Radiation is negligible except at very large *z*)

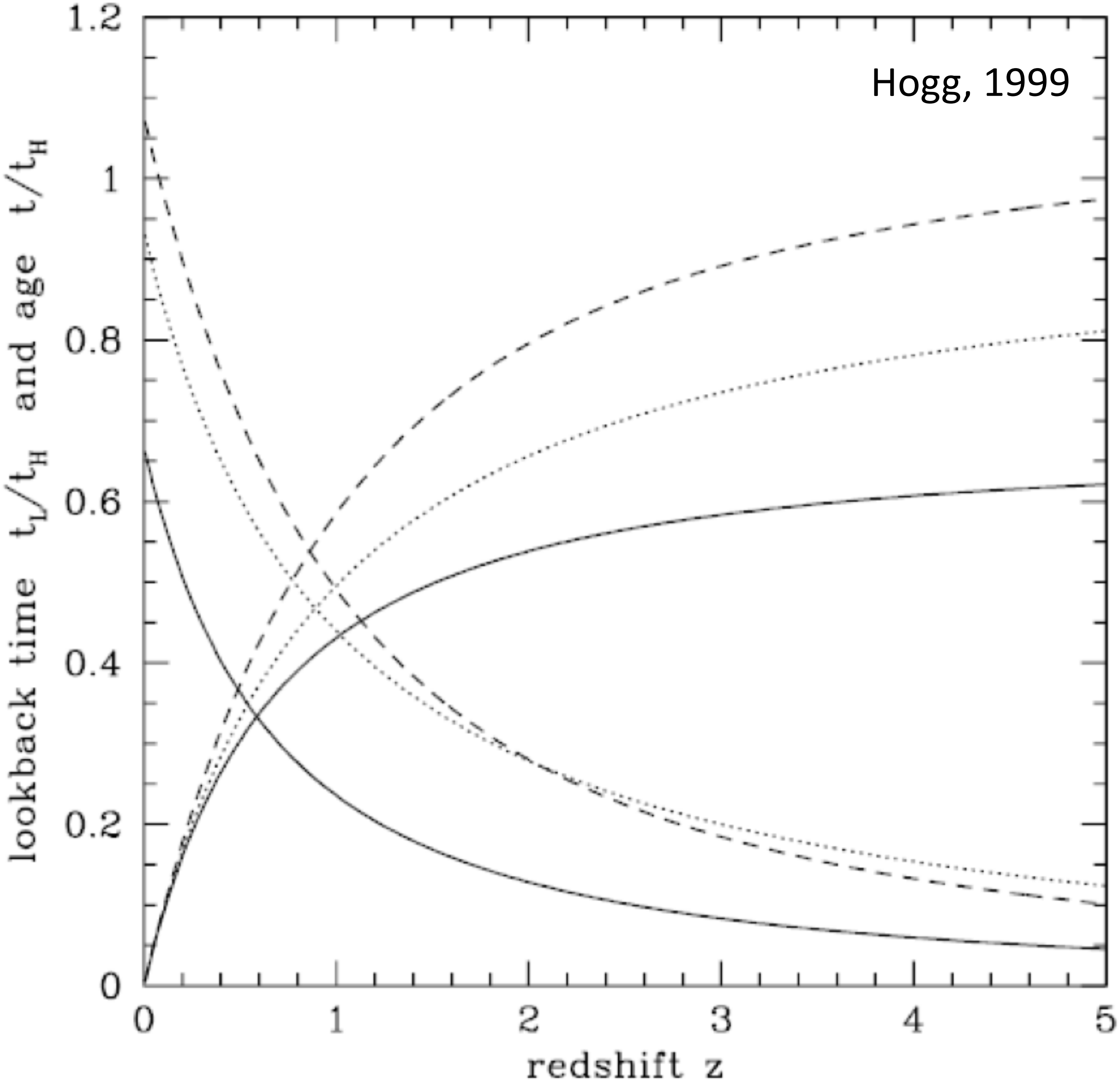
$$H(z) = \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{dt} = H_0 E(z) \Rightarrow \frac{dz}{dt} = -(1+z)H_0 E(z)$$

We can use this to compute z(t) – or, equivalently, t(z)

$$t(z) = \frac{1}{H_0}\int_\infty^z \frac{dz}{(1+z)E(z)}$$

Can solve this analytically in some cases, but it's not terribly informative.
Let's look at plots instead.

“Lookback” time, t_0-$t(z)$



Ω_M=0.2, Ω_Λ = 0.8

Ω_M=0.05, Ω_Λ = 0.0

Ω_M=1.0, Ω_Λ = 0.0

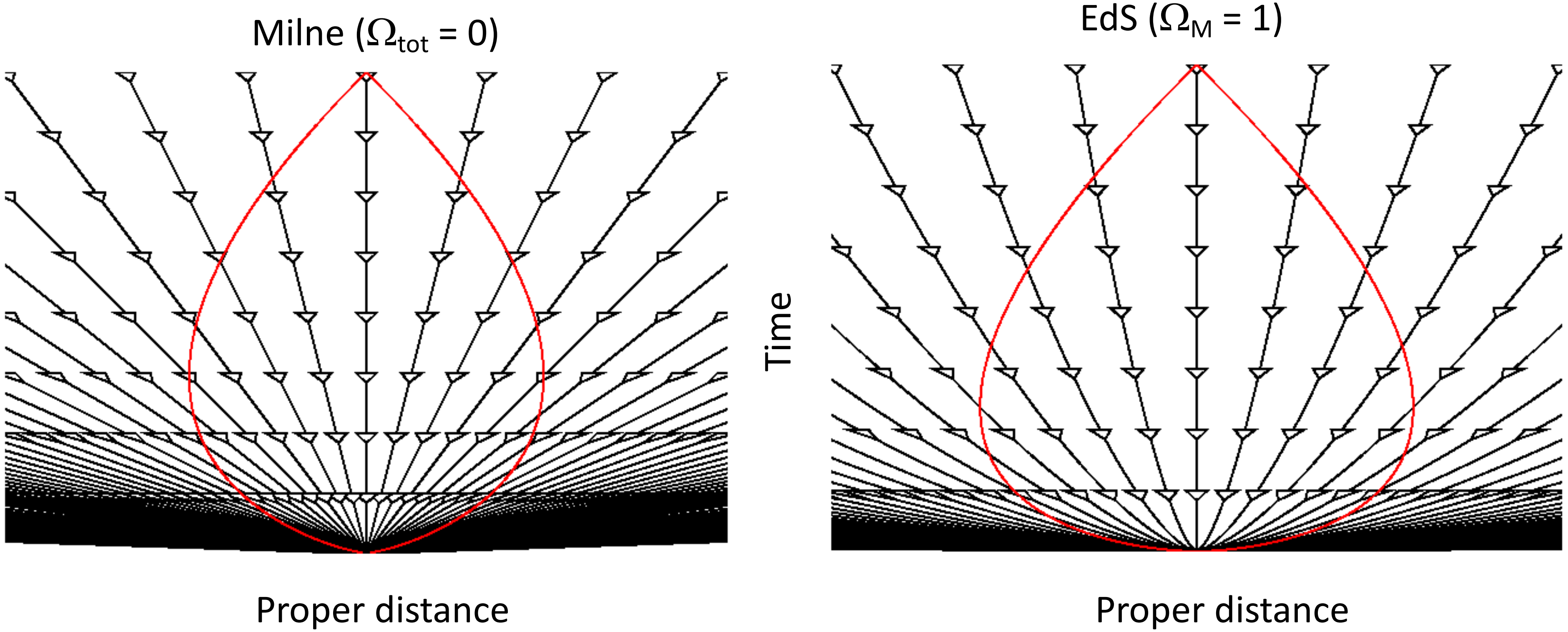
Ω_M	Ω_Λ
0.05	0.0
0.2	0.8
1.0	0.0

$t_H \approx$ 14Gy

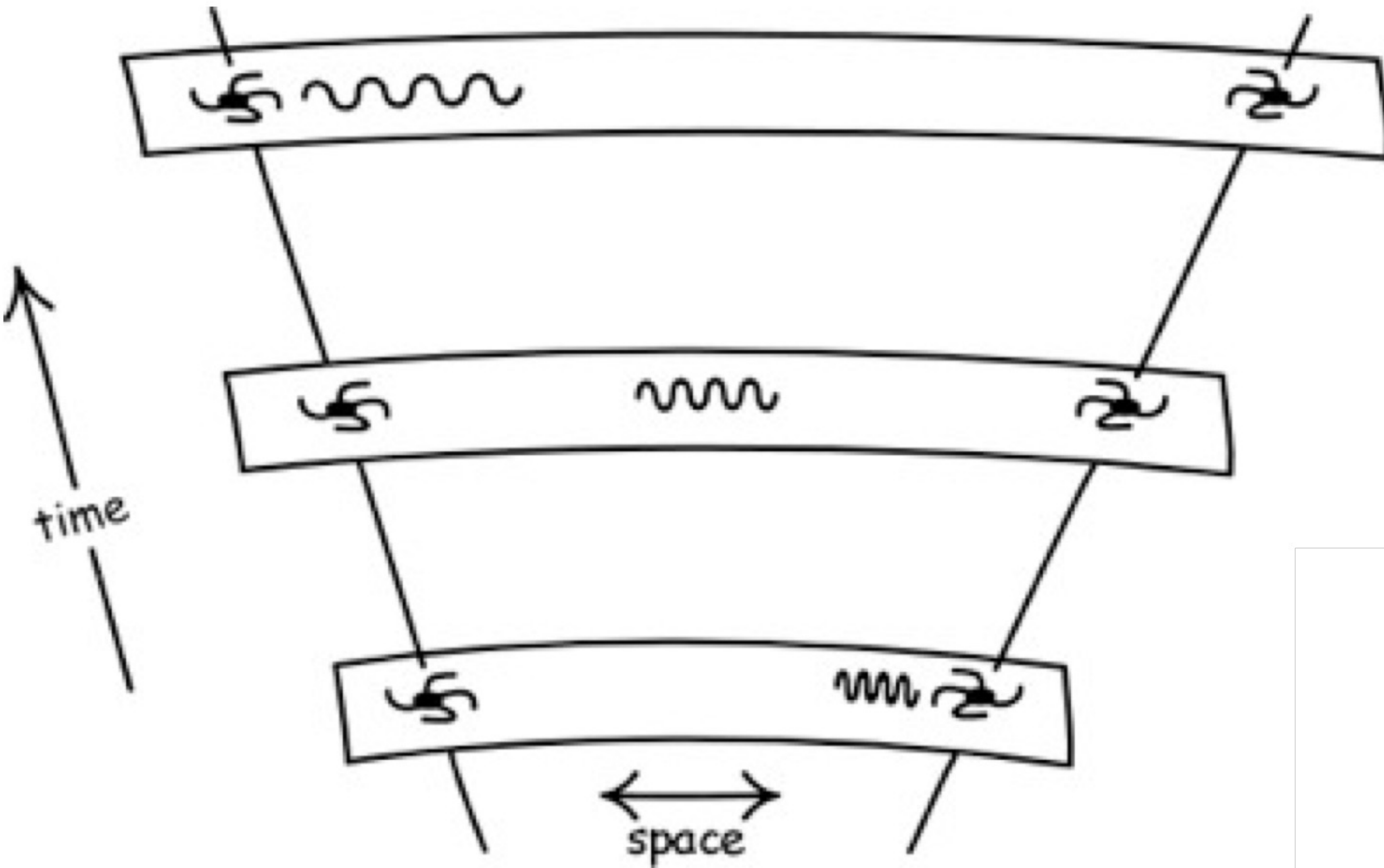
a(t) *vs.* **t** vs. ***d*** vs. ***z***

How do we measure *t* for something at redshift *z*?

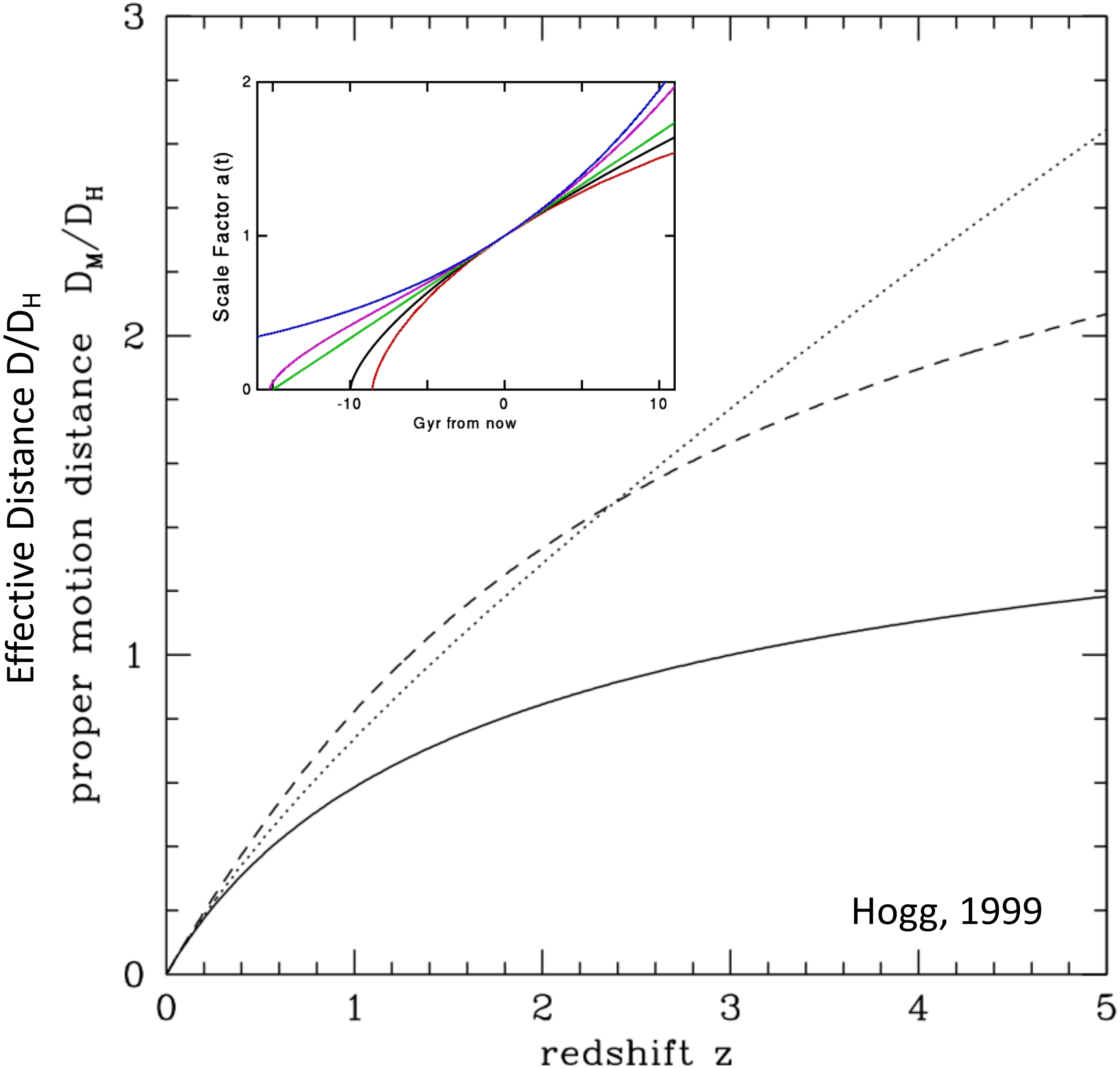
Light follows null intervals, ds^2=0. Distances at which things are observed depend on cosmology.



a(t) *vs.* ***t*** vs. ***d*** vs. ***z***



Effective Distance, D=S_κ(r)



Ω_M=0.05, Ω_Λ = 0.0

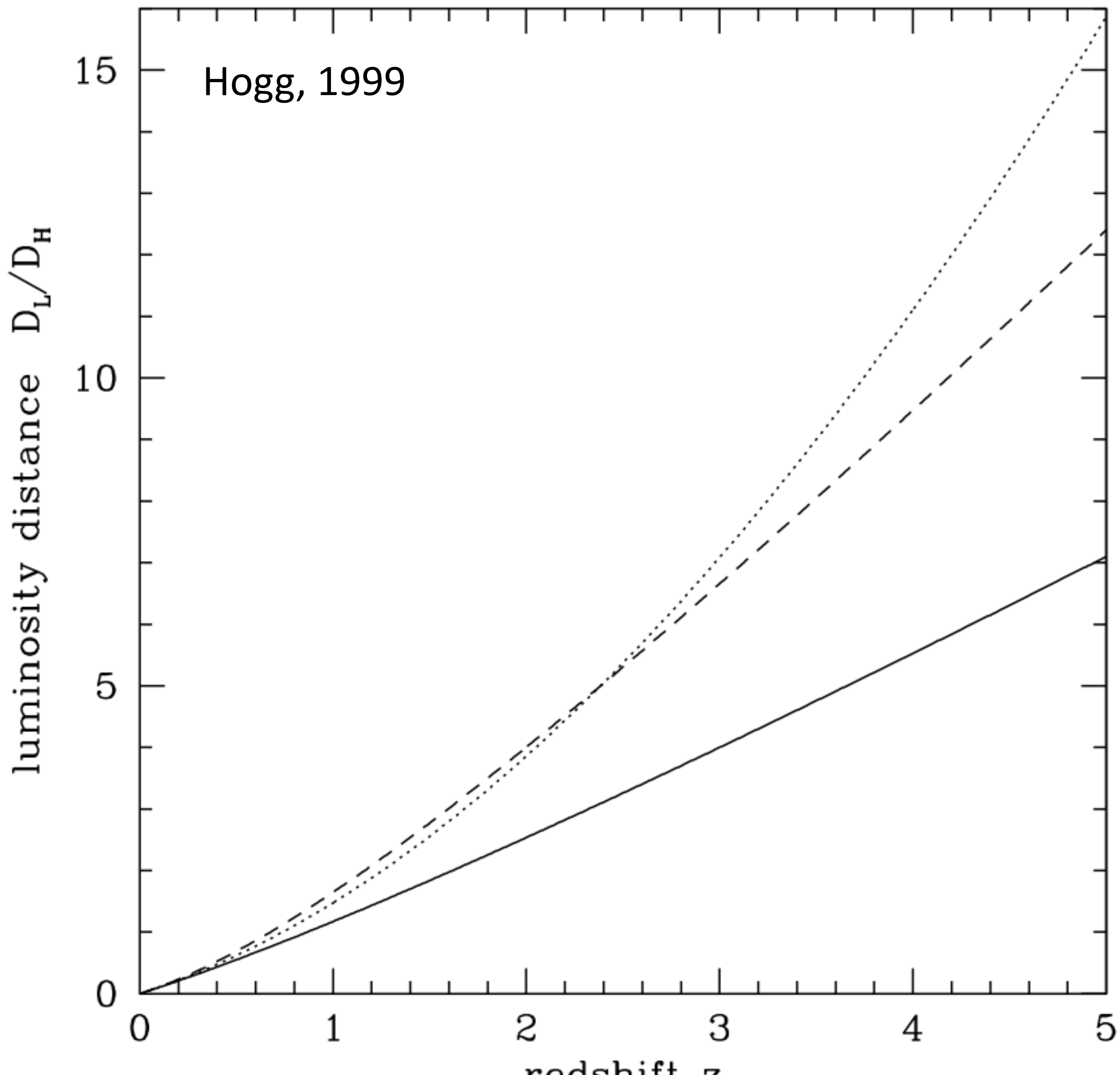
Ω_M=0.2, Ω_Λ = 0.8

Ω_M=1.0, Ω_Λ = 0.0

Ω_M	Ω_Λ
0.05	0.0
0.2	0.8
1.0	0.0

$D_H \approx$ 4Gpc

Luminosity Distance, $D_L = (1+z)D$



Ω_M=0.05, Ω_Λ = 0.0

Ω_M=0.2, Ω_Λ = 0.8

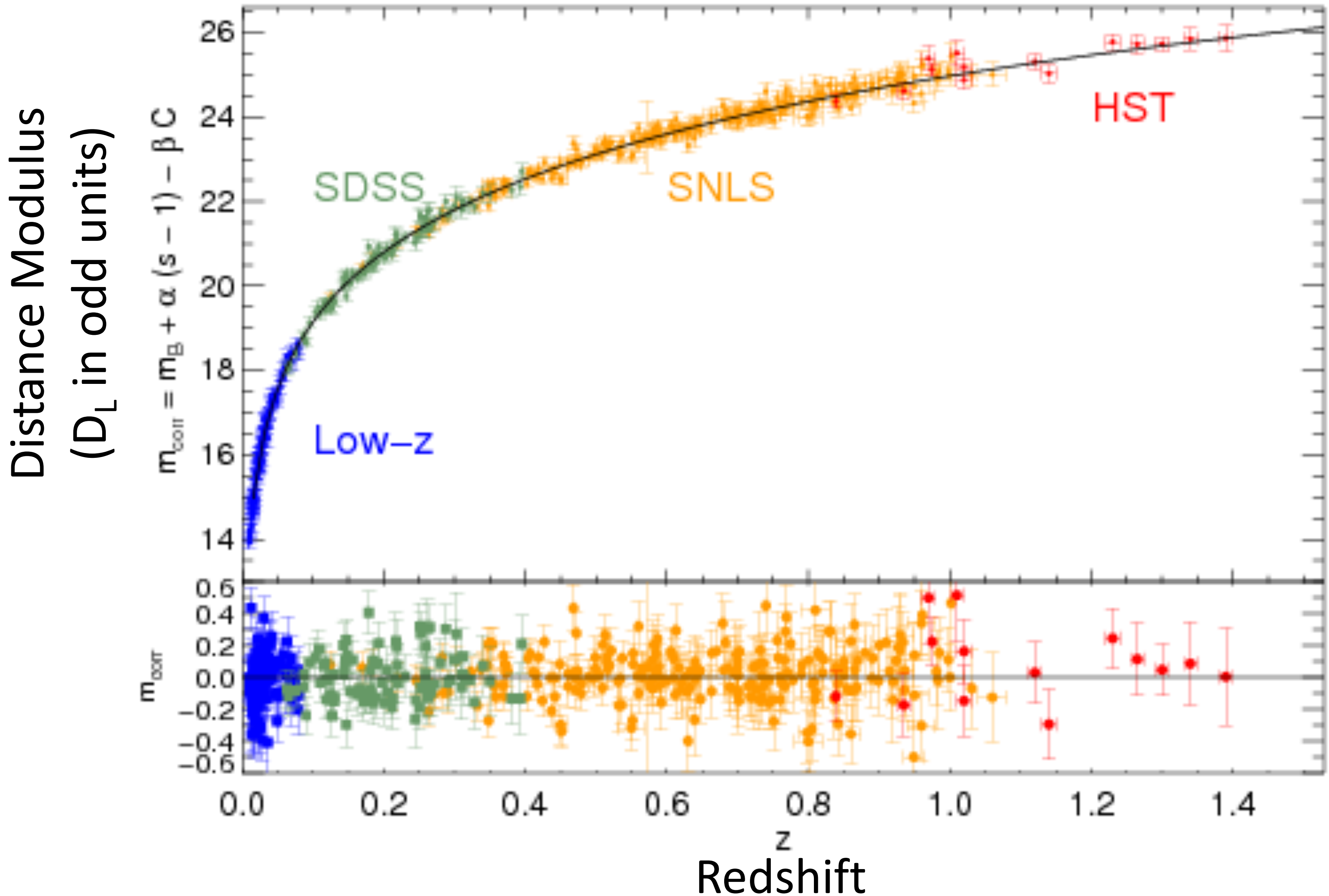
Ω_M=1.0, Ω_Λ = 0.0

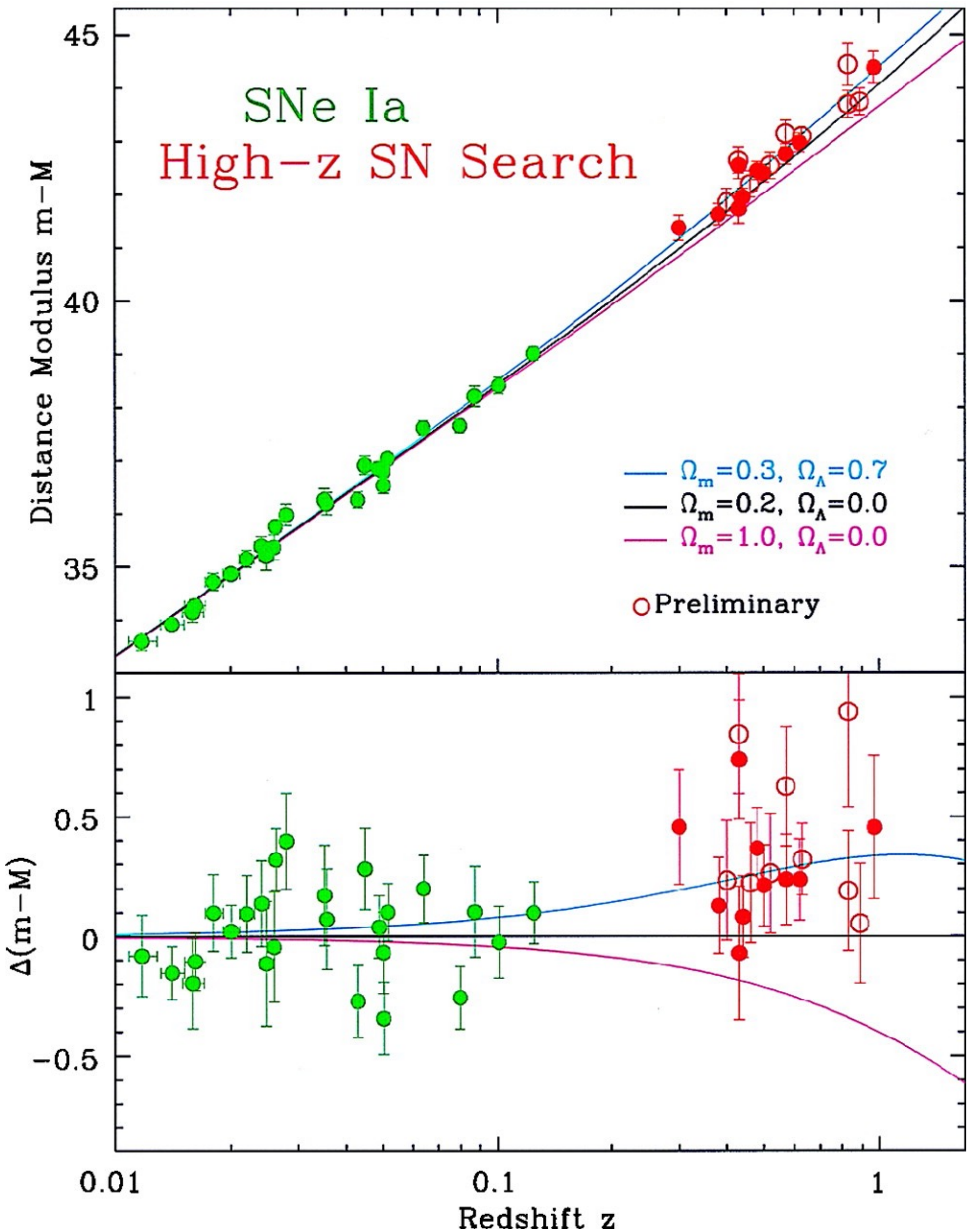
Ω_M	Ω_Λ
0.05	0.0
0.2	0.8
1.0	0.0

$D_H \approx$ 4Gpc

Distance-Redshift

Current SNe Constraints





Ω_Λ

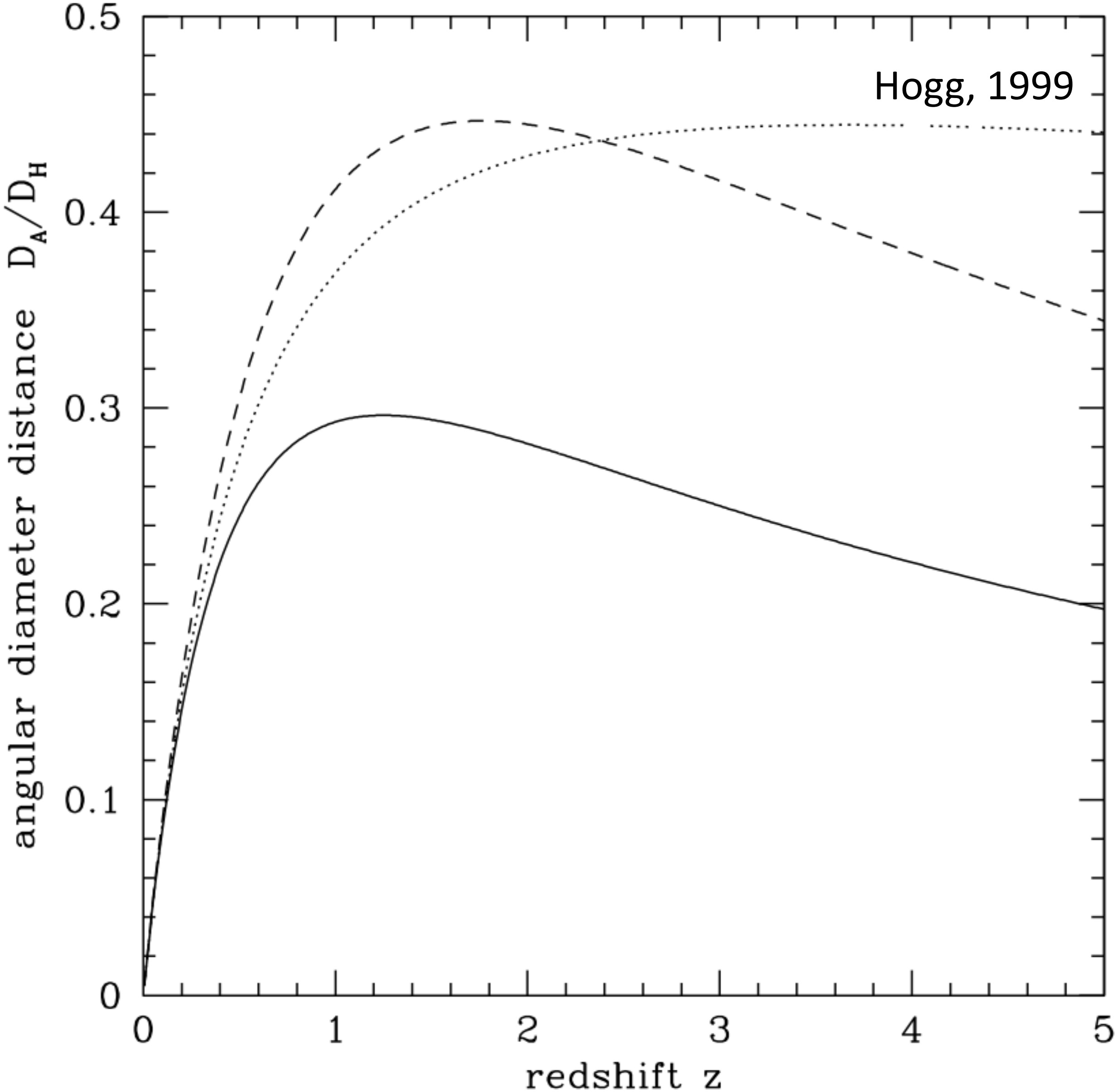
Discovered in 1999 by measuring distance-vs-redshift for SNe

Distance Modulus =

$$5 \log (D_L(z) / 10\text{pc})$$

$D_L = D(1+z) = S_\kappa(r)(1+z)$

Angular Diameter Distance, D_A=D/(1+z)



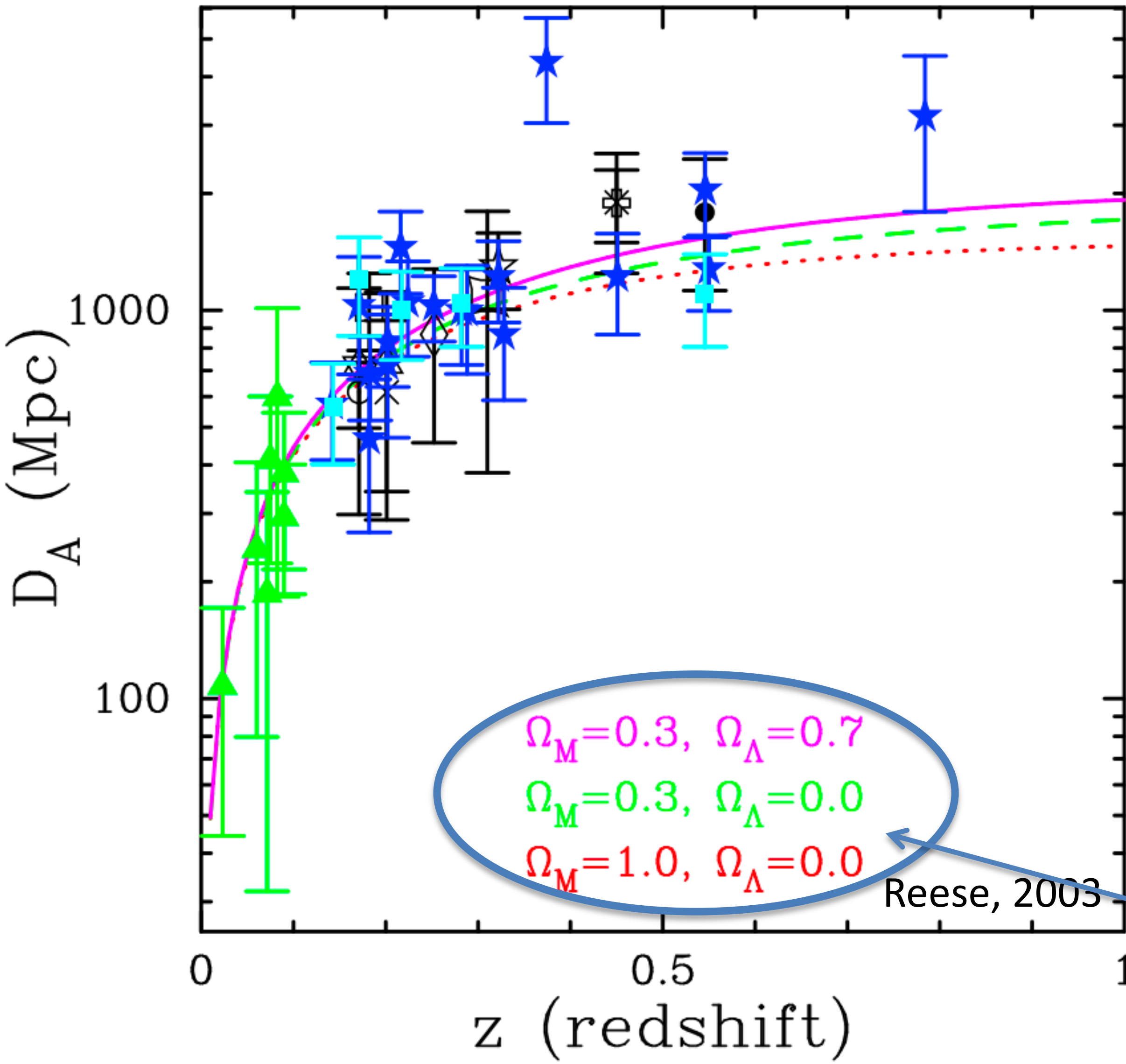
Ω_M=0.05, Ω_Λ = 0.0

Ω_M=0.2, Ω_Λ = 0.8

Ω_M=1.0, Ω_Λ = 0.0

$D_H \approx$ 4Gpc

Galaxy Clusters: X-ray + SZ



$$S_x \sim \int d\ell n_e^2 \Lambda_{eH}$$

$$\Delta T_{SZE} \sim \int d\ell n_e T_e$$

$$D_A \propto \frac{(\Delta T_0)^2 \Lambda_{eH0}}{S_{x0} T_{e0}^2} \frac{1}{\theta_c}$$

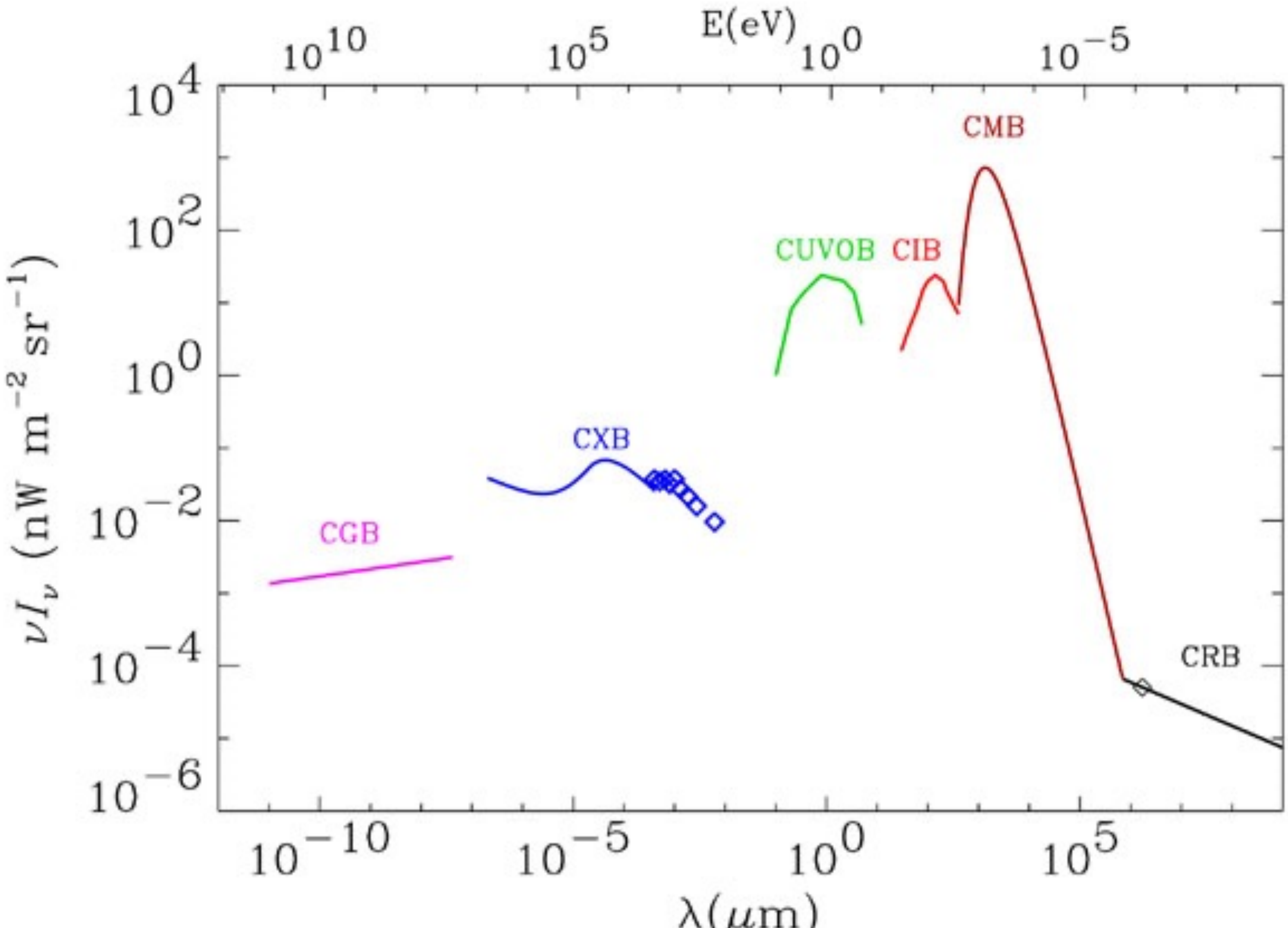
"Geometric" methods depend on expansion history, so cosmology!

INTERMISSION

Ω_R?

Waveband	Energy density of radiation (eV m^{-3})	Number density of photons (m^{-3})
Radio (300 MHz)	10^{-2}	$\sim 10^4$
Cosmic Microwave Background	2.6×10^5	4×10^8
Infrared (140–1000 μm)	4×10^3	3×10^5
UV-optical-near IR (0.16–3.5 μm)	$\sim 10^4$	$\sim 10^4$
X-ray ($\sim$ 10 keV)	20	3×10^{-3}
γ-ray ($\sim$ 1 MeV)	10	$\sim 10^{-5}$
γ-ray ($\geq$ 10 MeV)	0.5	$\sim 3 \times 10^{-8}$

Ω_R?



E(eV)
10^{10}
10^{5}
10^{0}
10^{-5}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
νI_ν (nW m^{-2} sr^{-1})
CMB
CUVOB
CIB
CXB
CGB
CRB
10^{-10}
10^{-5}
10^{0}
10^{5}
$\lambda(\mu m)$

T(a)

Recall: $T \propto a^{-2}$ for non-relativistic fluids (cold matter)
$T \propto a^{-1}$ for relativistic (light, neutrinos)

A blackbody emitted at $t=t_1$ and temperature T_1

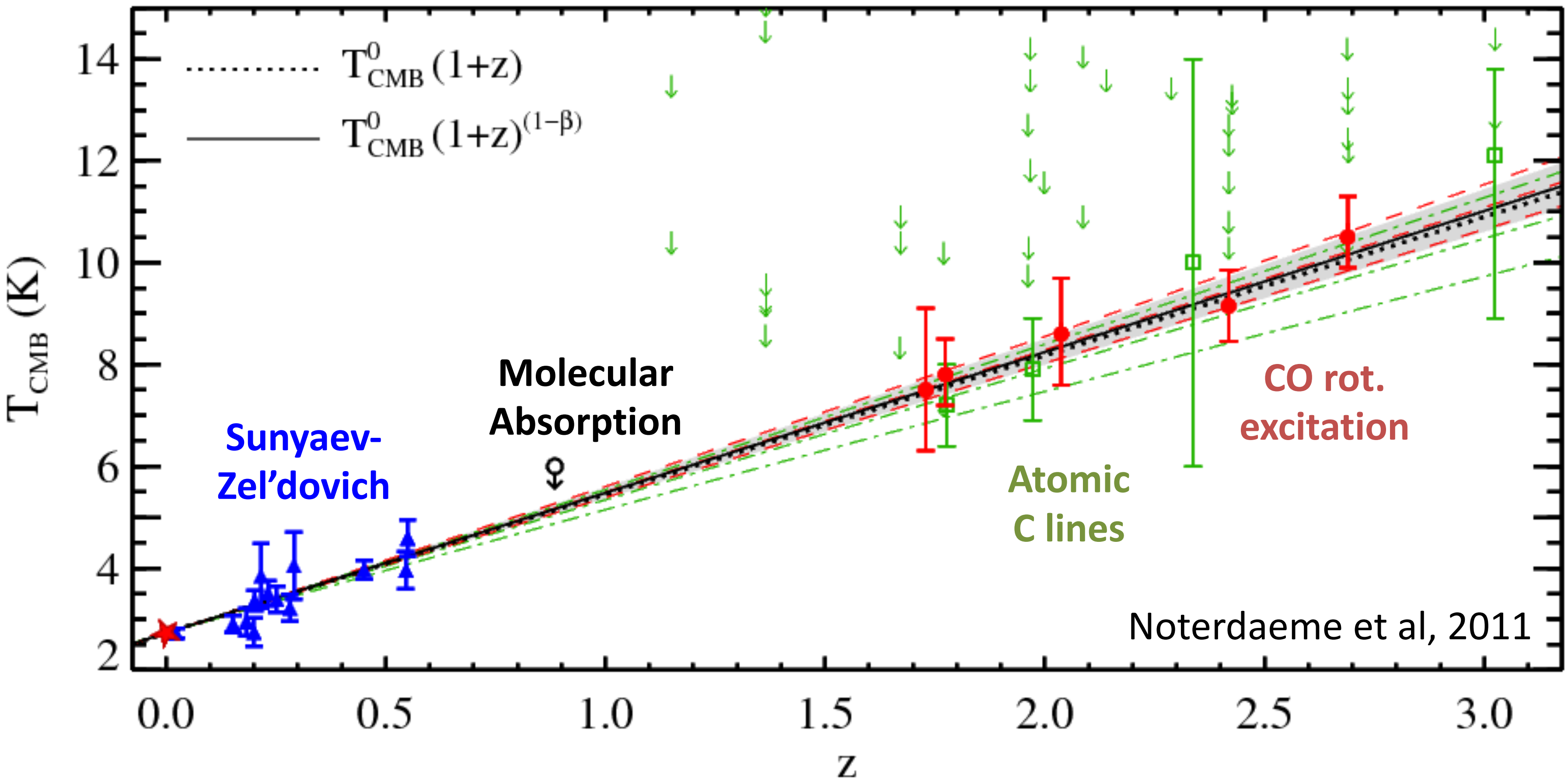
$$\varepsilon(\nu_1)\,\mathrm{d}\nu_1 = \frac{8\pi h\nu_1^3}{c^3}\frac{1}{\mathrm{e}^{h\nu_1/kT_1}-1}\,\mathrm{d}\nu_1$$

$$= \frac{8\pi h\nu_0^3}{c^3}\frac{1}{\mathrm{e}^{h\nu_0/kT_0}-1}(1+z)^4\,\mathrm{d}\nu_0$$

$$= (1+z)^4\,\varepsilon(\nu_0)\,\mathrm{d}\nu_0$$

Spectral shape is still Blackbody, $T_0 = (1+z)T_1$

Measuring T(z)



Matter-vs-Radiation

Matter contributes much more to the energy density of the Universe today than does radiation.

$$(\rho_R / \rho_M)|_{z=0} = \Omega_R / \Omega_M \approx 1.6 \times 10^{-4}$$

Radiation Evolution: $\rho_R(z) = (4/c)\ \sigma T^4(z) = (4/c)\ \sigma T_0^{\,4}\,(1+z)^4$

Matter Evolution: $\rho_M(z) = \Omega_M(z)\rho_C(z)\,(1+z)^3$

$$(\rho_R / \rho_M)(z) \approx 2.48 \times 10^{-5}\ (1+z) / \Omega_M h^2$$

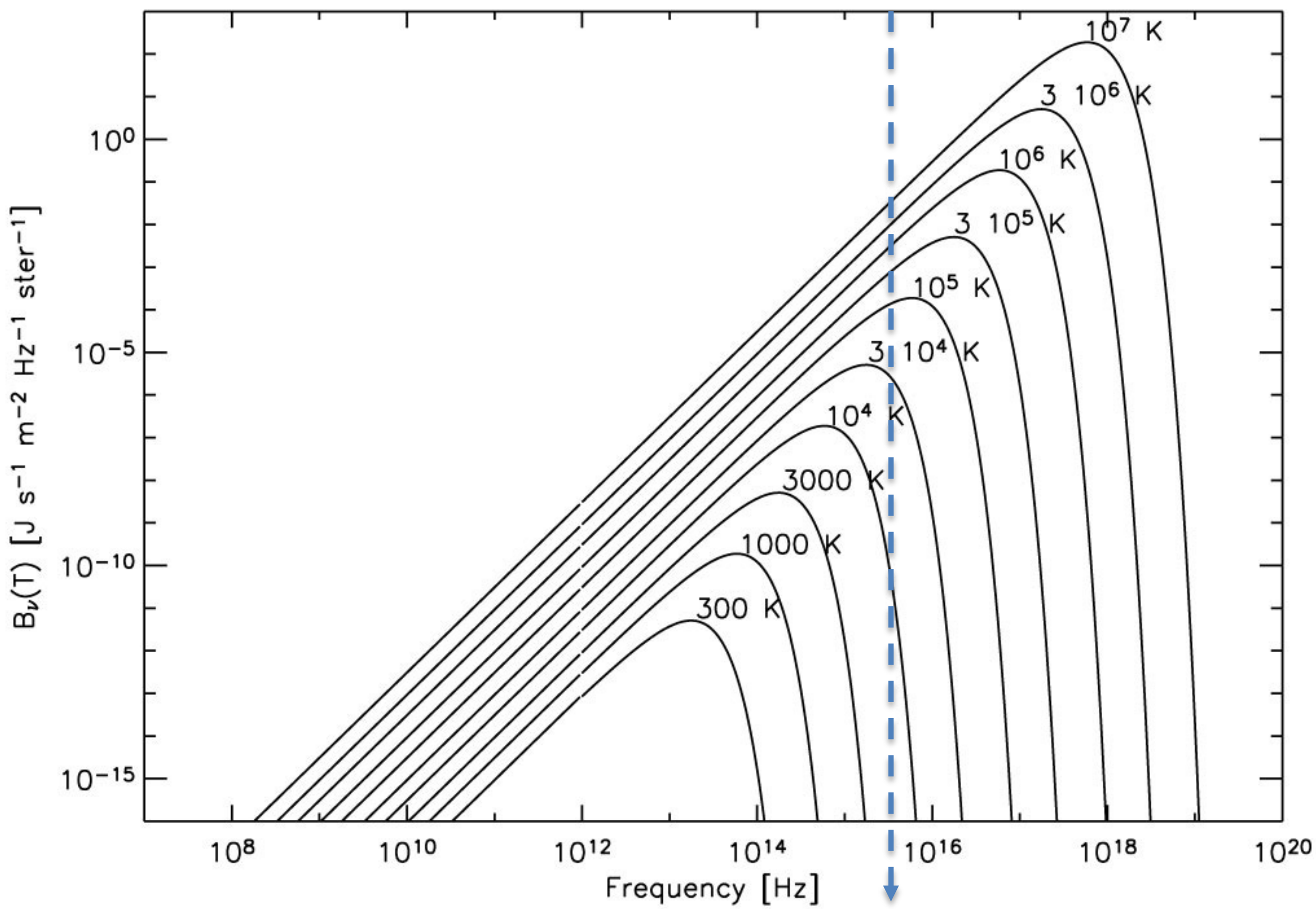
Grows with redshift!

For $z \gg 4 \times 10^4\ \Omega_M h^2 \approx 6000$, radiation dominates over matter.

Temperatures

$$T(z=6000) = 2.78\,(1+6000) = 16{,}680\text{K}$$

That seems quite hot. Is it enough to ionize the matter?



$h\nu = 13.6\text{eV}$

Quantum Stats Review

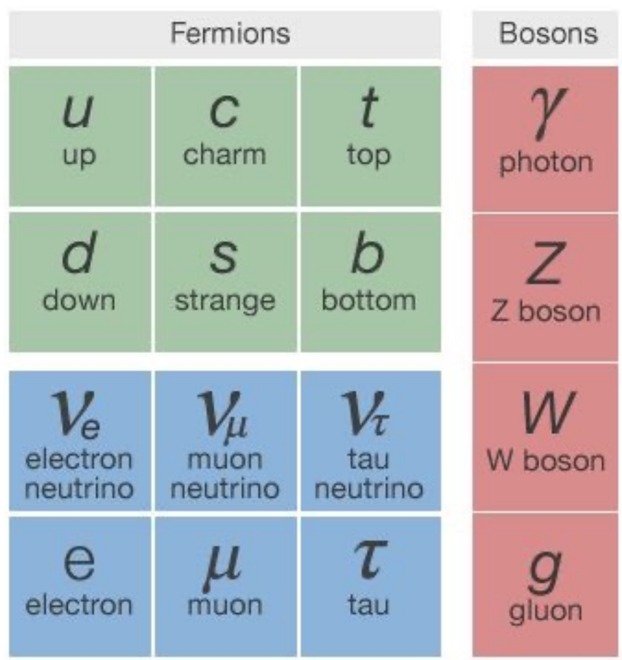
Equilibrium occupation number: $\langle f \rangle = \dfrac{1}{e^{(\epsilon-\mu)/kT} \pm 1}$

$$dn = g\frac{4\pi p^2 dp}{(2\pi\hbar)^3}\frac{1}{e^{\epsilon(p)/kT} \pm 1}$$

– for Bosons
+ for Fermions
g is the degeneracy factor
$\epsilon(p) = \sqrt{m^2c^4 + p^2c^2}$

Fermions

- Anti-symmetric wavefunction
- Pauli Exclusion
- ½-integer spin
- Includes all standard matter



Bosons

- Symmetric wavefunction
- integer spin
- Force carriers
- Photon is the most familiar

Baryons-vs-Radiation

How many particles are actually floating around?

Baryons: $N_B = \Omega_B \rho_c / c^2 / m_p$

Photons: $$N = \frac{4\pi g}{h^3} \int_0^\infty \frac{p^2\,\mathrm{d}p}{e^{E/kT} \pm 1}$$

$$N_\gamma \approx 0.244(2\pi kT/hc)^3$$

@T≈3K: $N_\gamma / N_B = 3.7 \times 10^7 / \Omega_B h^2 \approx 1.5 \times 10^9$

CMB photons outnumber baryons a-billion-to-one!

Ionized?

~~$h\nu = kT = 13.6\text{eV}$ $\Rightarrow$ $T \approx 13.6\text{eV} / k \approx 150{,}000\text{K}$~~

~~$\Rightarrow$ $z \approx 150{,}000\text{K} / 2.7\text{K} \approx 55{,}000$~~

Number of ionizing photons:
$$n(\geq E) = \int_{E/h}^{\infty} \frac{8\pi\nu^2}{c^3} \frac{d\nu}{e^{h\nu/kT}}$$
$$= \frac{1}{\pi^2}\left(\frac{2\pi kT}{hc}\right)^3 e^{-x}(x^2 + 2x + 2)$$

x=13.6eV/kT

Ionizing fraction of photons: $N(>E) / N_\gamma \approx e^{-x}(x^2+2x+2) / 0.244\pi^2$

For equal numbers of baryons and ionizing photons, $T \approx 5600$K,
$z \approx 2000$.

Recombination

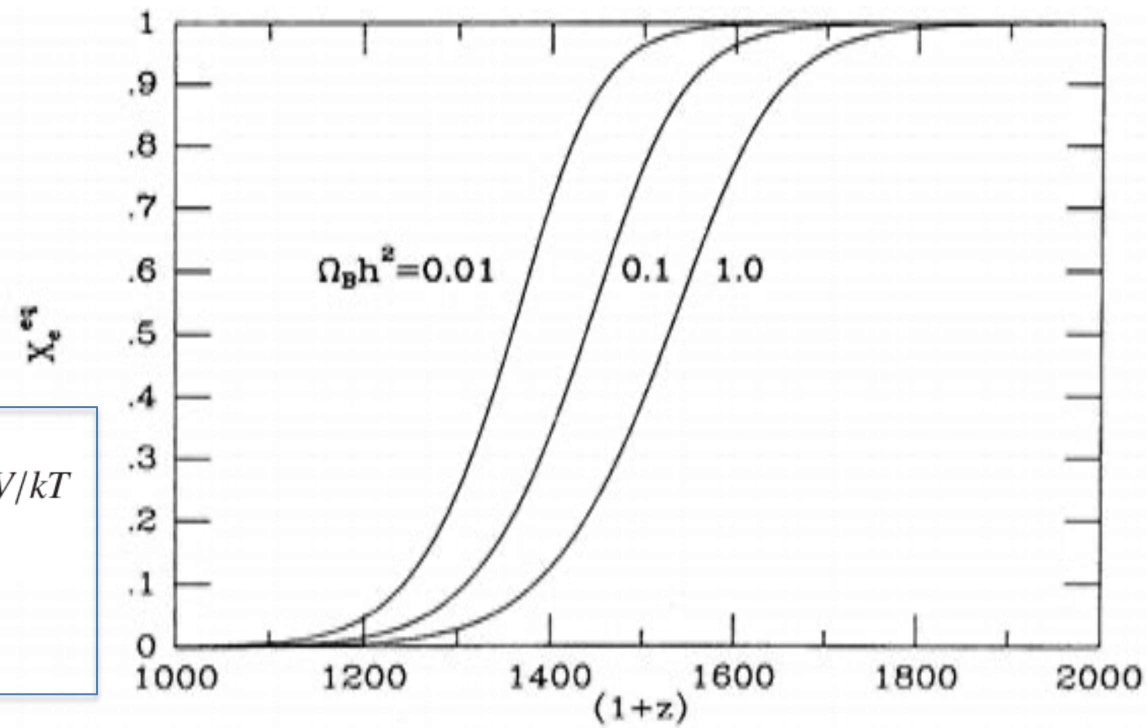
X(z) = n_p / (n_p + n_H) = ionization fraction

$$dn = g\frac{4\pi p^2 dp}{(2\pi\hbar)^3}\frac{1}{e^{\epsilon(p)/kT} \pm 1} \quad \rightarrow \quad n = \frac{4\pi g}{(2\pi\hbar)^3}\int_0^\infty \frac{p^2 dp}{e^{\epsilon(p)/kT} \pm 1}$$

In equilibrium,
$e^- + p \Leftrightarrow H + \gamma$

$$\frac{X^2}{1-X} = \frac{(2\pi m_e kT)^{3/2}}{(n_e + n_H)(2\pi\hbar)^3} e^{-13.6eV/kT}$$

Saha Equation



(Note: nothing "re"combined – this was the first combining of e^- and p^+.)

Baryon-Photon Coupling

Compton scattering transfers energy between photons and baryons.

(Actually, photons & electrons, the latter couple to baryons via Coulomb scattering.)

$$\Gamma_\gamma = n_e v \sigma_T$$

Scattering rate

Number density of free electrons

Velocity of free electrons

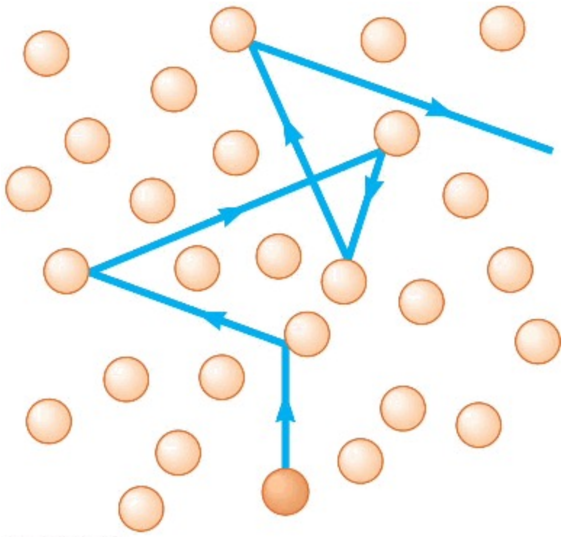
Thompson cross section $\approx 6.65 \times 10^{-25}$ cm^2

We'll come back to this later.

Optical Depth

Thompson scattering limits how far a photon can free stream.

We can characterize this as the optical depth τ, per differential redshift.



$$\frac{d\tau_T}{dz} = \sigma_T N_e(z)\frac{dx}{dz} = \sigma_T N_e(z) c \frac{dt}{dz}$$

Recall dz/dt from last lecture… taking z >> 1

$$\frac{dz}{dt} \approx -H_0 \Omega_0^{1/2} z^{5/2}$$

Optical Depth in the Early Universe

Plugging in, $$\frac{d\tau_T}{dz} = -\frac{\sigma_T N_e(z) c}{H_0 \Omega_0^{1/2} z^{5/2}}$$

For N_e, assume neutrality of the Universe, ionization fraction X(z).

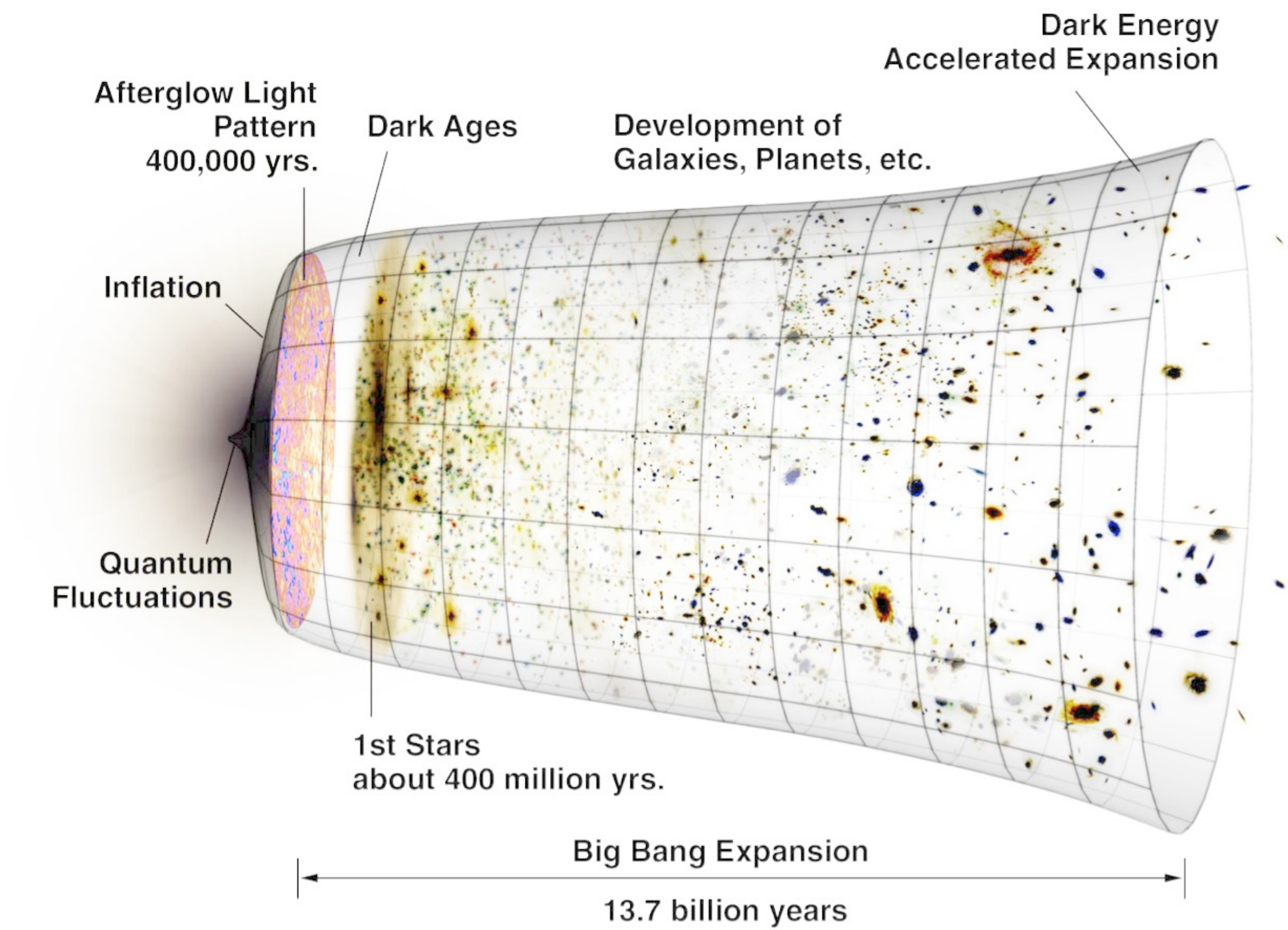
$N_e = X(z) N_H = X(z) \rho_B / m_p$

We define $\Omega_B = \dfrac{8\pi G \rho_B}{3 H_0^2}$

$$\tau_T = \frac{9 \sigma_T H_0 c}{32 \pi G m_p} \frac{\Omega_B}{\Omega_M^{1/2}} \int \frac{z^3 X(z)}{z^{5/2}} dz$$

Careful calcs show that $\tau \approx 1$ at $z \approx 1100$.
This barrier forms the "surface of last scattering."
When $X(z) \rightarrow 0$, $\tau_t \rightarrow 0$.
The Universe is transparent.

Last Scattering Surface



The Early Universe

$$\gamma + \gamma \Leftrightarrow P + \bar{P}$$

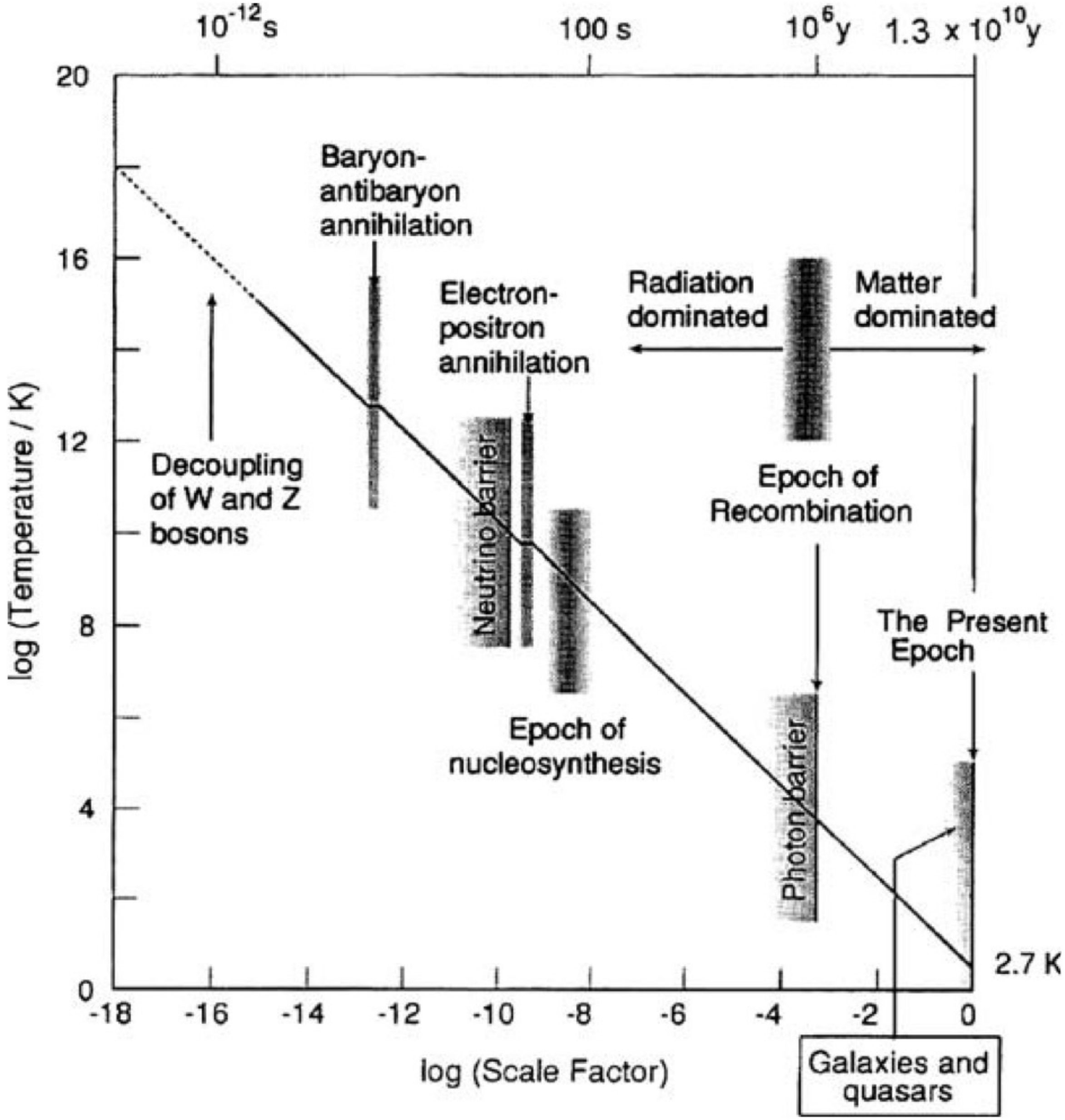
$$N = \bar{N} = \frac{4\pi g}{h^3} \int_0^\infty \frac{p^2\,\mathrm{d}p}{e^{E/kT} \pm 1}$$

Photons $\quad g = 2\,, \quad N = 0.244 \left(\frac{2\pi kT}{hc}\right)^3 \mathrm{m}^{-3}$

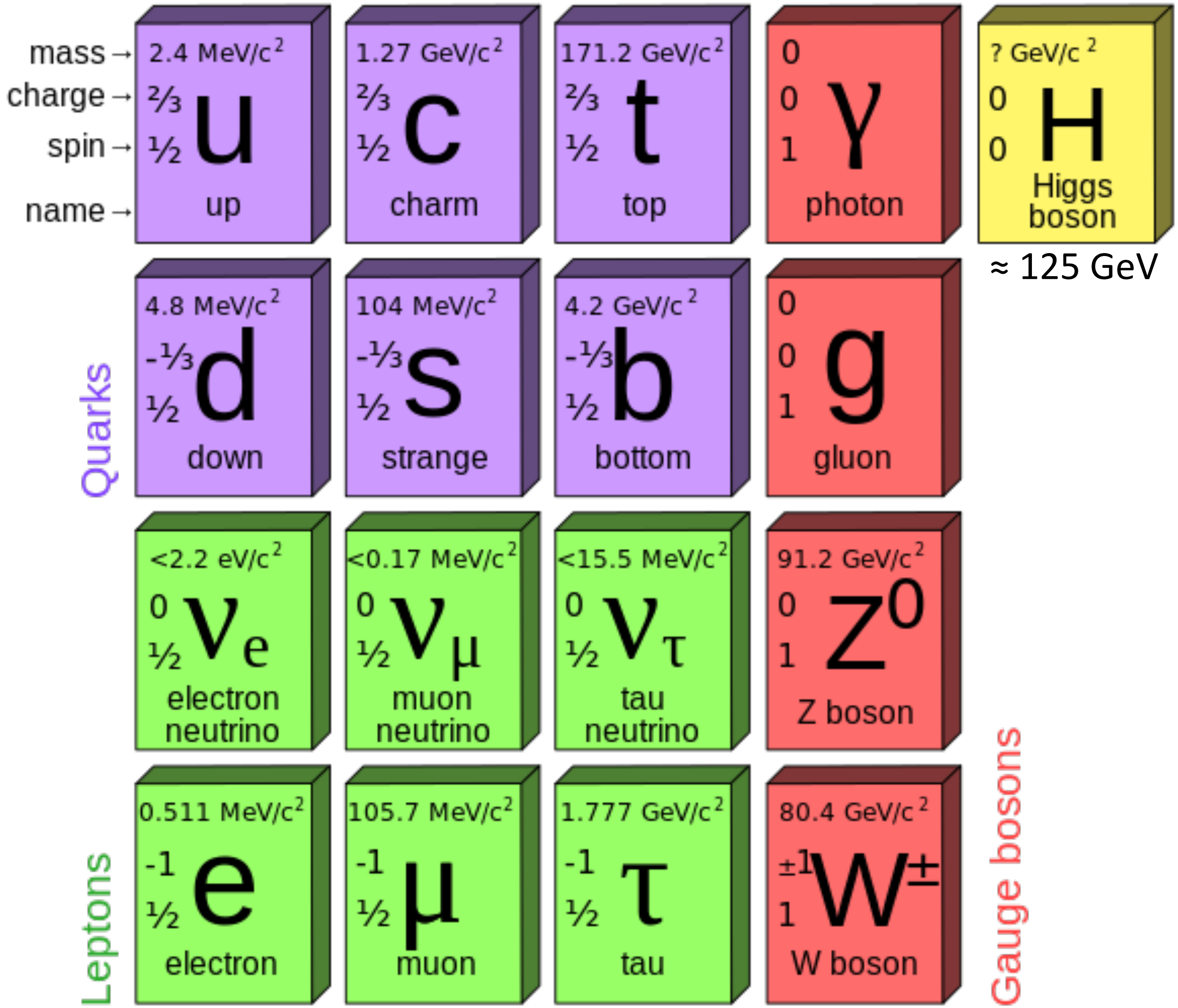
Electrons, Protons, Neutrons $\quad g = 2\,, \quad N = \bar{N} = 0.183 \left(\frac{2\pi kT}{hc}\right)^3 \mathrm{m}^{-3}$

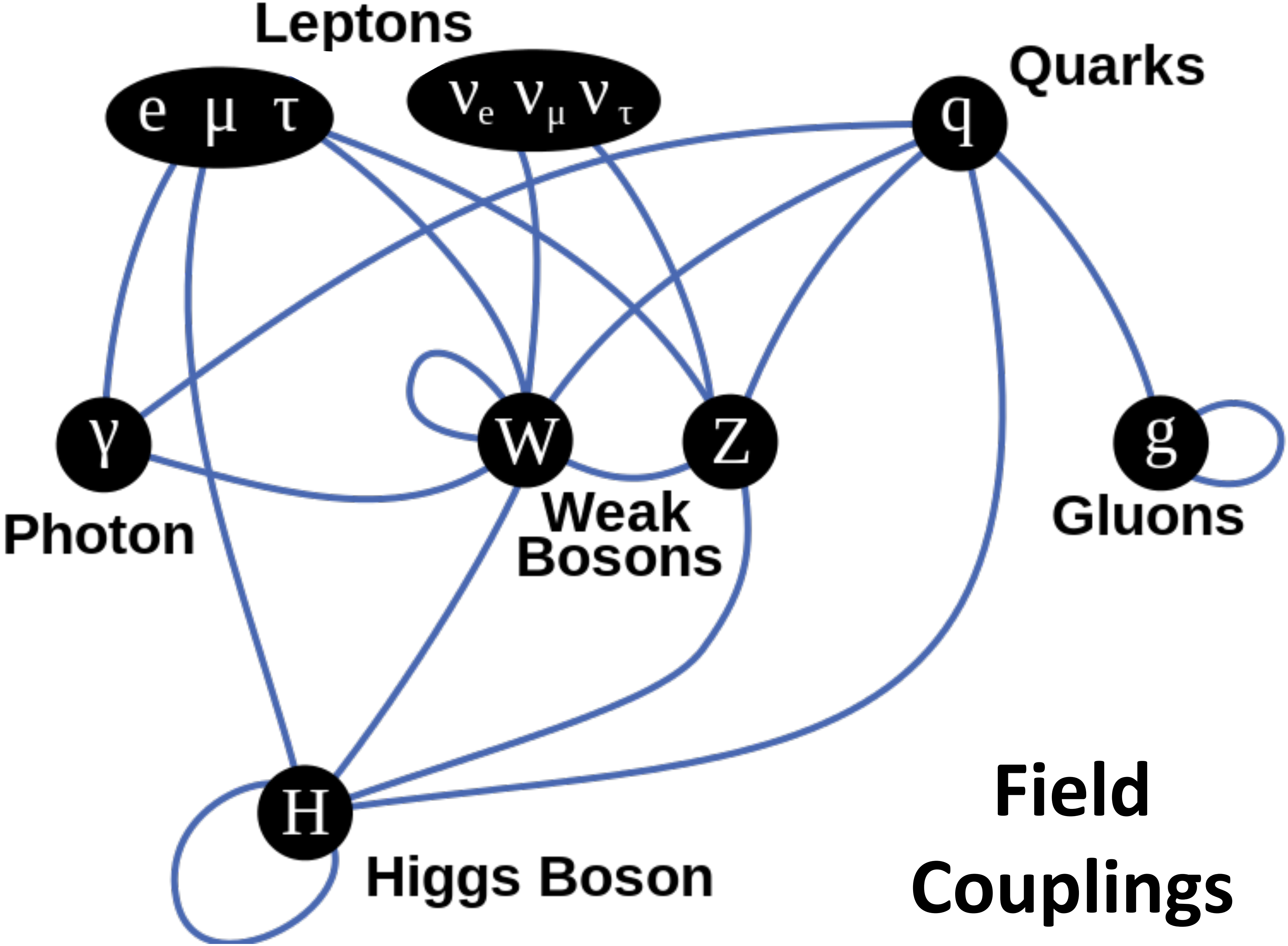
Neutrinos $\quad g = 1\,, \quad N = \bar{N} = 0.091 \left(\frac{2\pi kT}{hc}\right)^3 \mathrm{m}^{-3}$

Eras of the Early Universe



Quick Particle Physics





Field Couplings

Next Time:

Working through the Eras