AST 1430F
Observational Cosmology

V

Thermal History: Part II

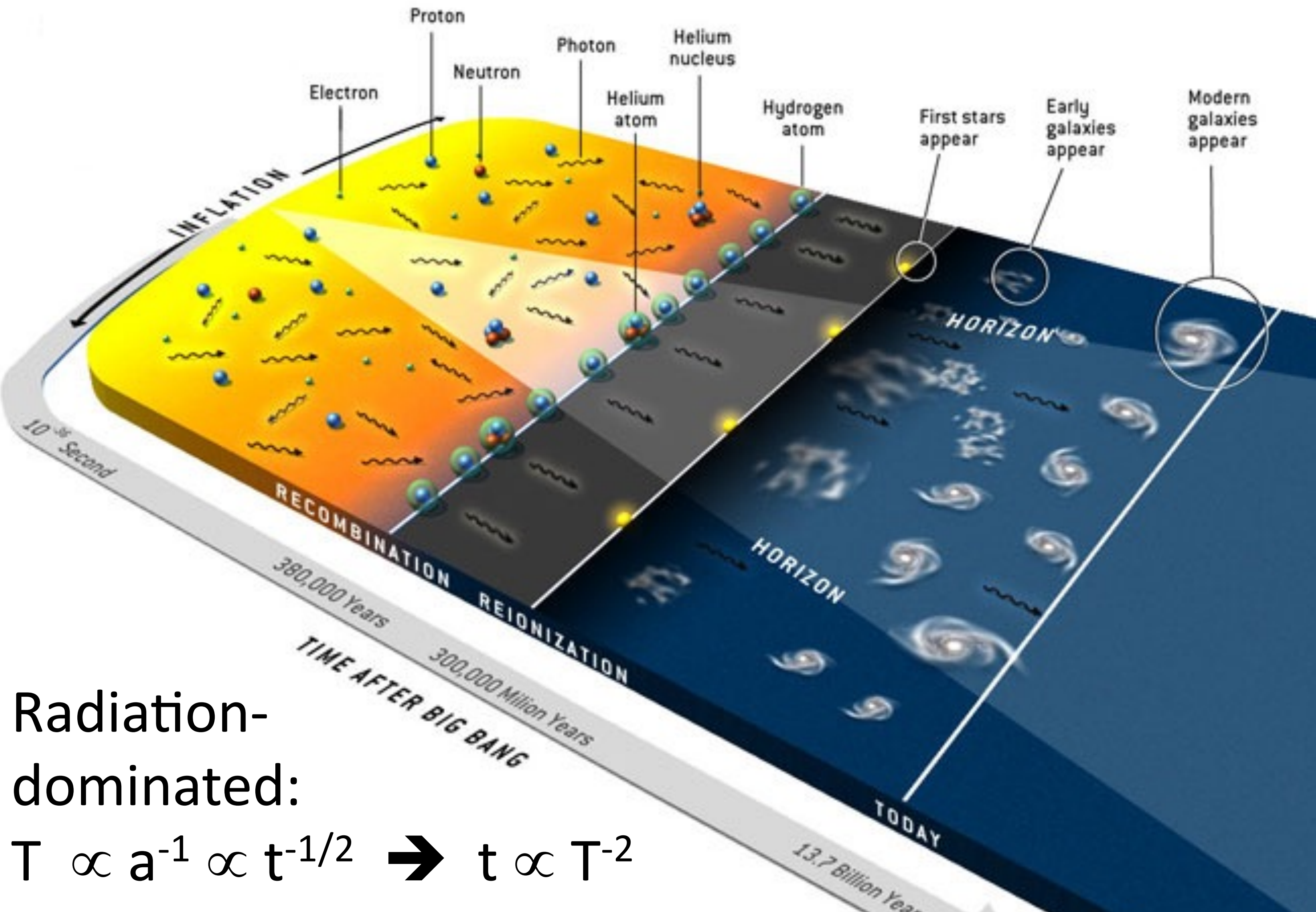
Logistics

- **Asst 1** due Tuesday!
- Change-of-venue Feb 1, Mar 1, Apr 5.
  - ~~Probably~~ AB113
- Please book presentations
  - Topics are first-come, first-served!

Some suggestions:

- Inflation models
- Inflation observables
- BBN observations
- ~~CMB B-mode polarization: observation status / prospects~~
- Dark Energy alternatives (quintessence?)
- Cosmic birefringence (CMB? Other tests?)
- Symmetry breaking / topological defects (cosmic strings, monopoles, …)
- Baryogenesis (more detail then I'll go into)
- Massive Neutrinos & the CMB

Context Reminder

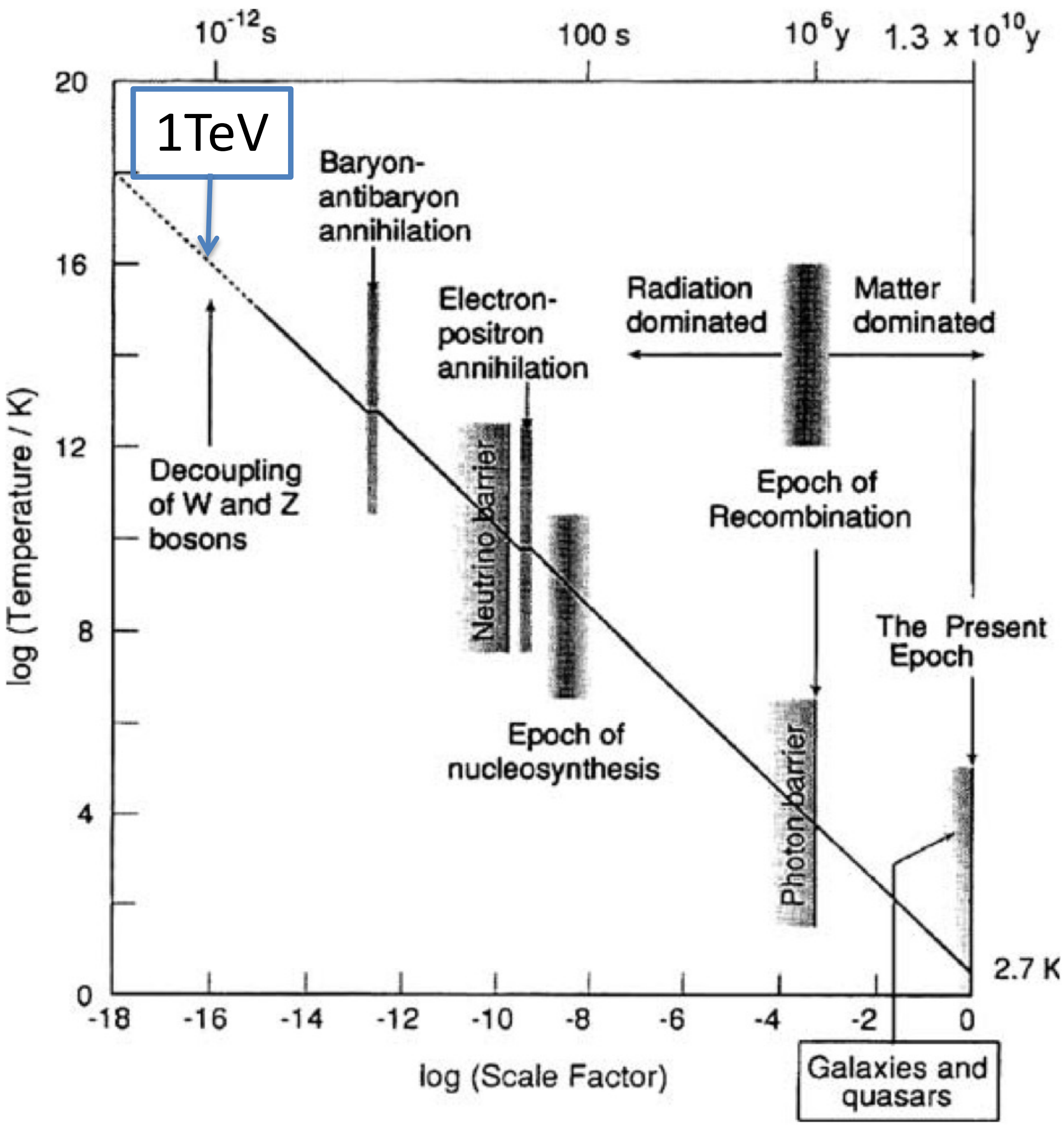


Radiation-dominated:

$T \propto a^{-1} \propto t^{-1/2} \rightarrow t \propto T^{-2}$

Hu & White, Scientific American, 2004

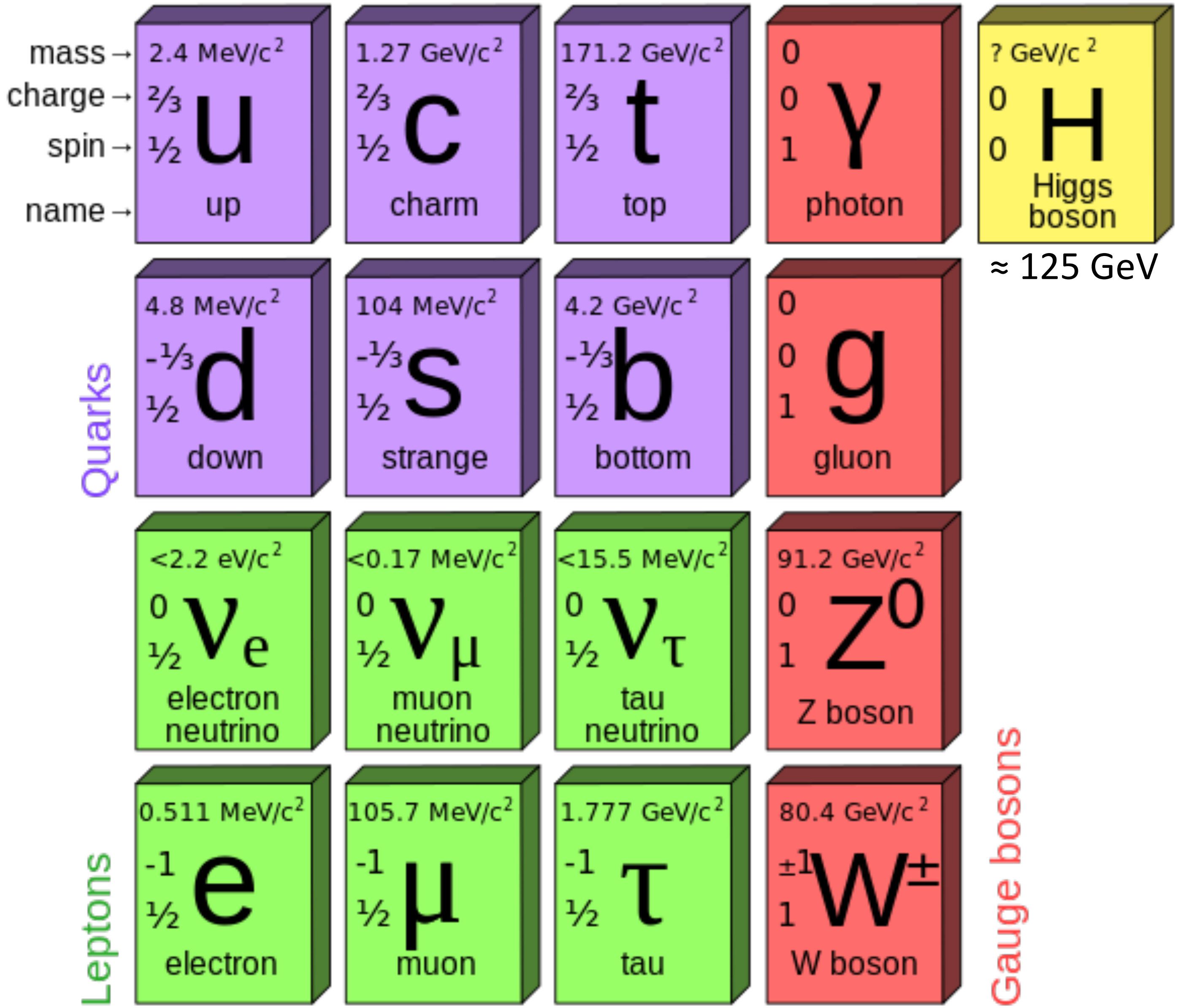
Life in the Early Universe

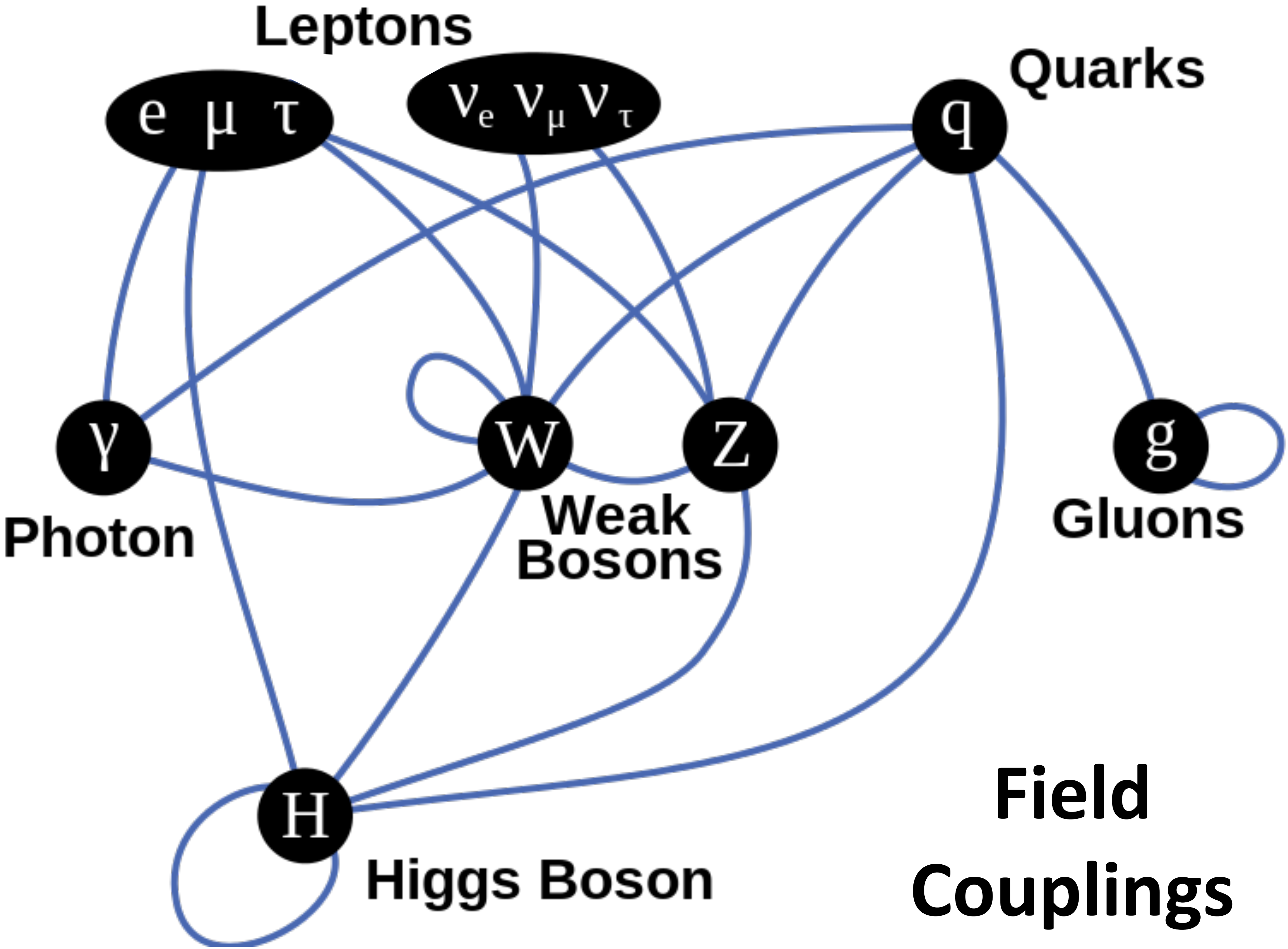


Sometimes handy to define the dimensionless $T_{10} = T / 10^{10}$K.

$\mathbf{T_{10}(t\!=\!1s) \approx 1}$

Quick Particle Physics




Leptons
e μ τ
νe νμ ντ
Quarks
q
γ
W
Z
g
Photon
Weak
Bosons
Gluons
H
Higgs Boson
Field
Couplings

Thermal Equilibrium

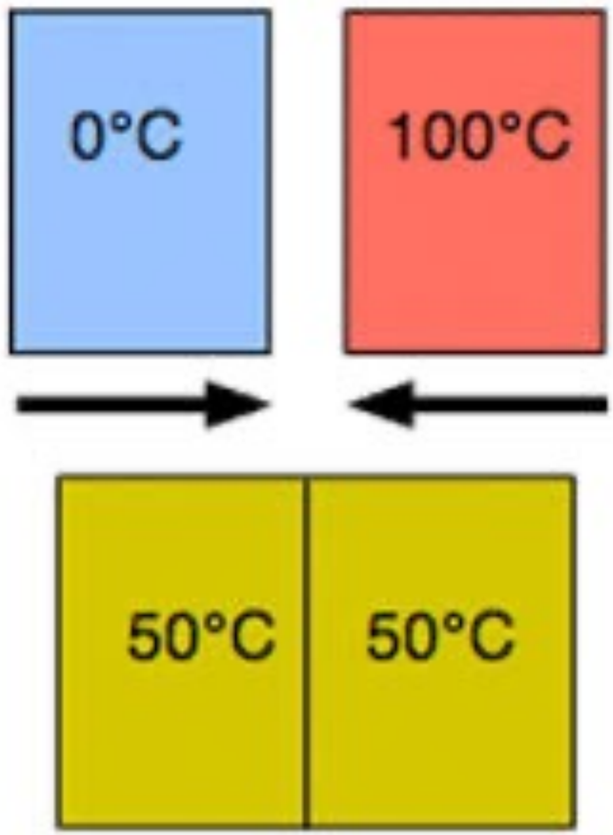
2nd Law of Thermodynamics:
Connected systems will tend toward equal temperatures.
(Entropy ≈ disorder, will be maximized for the available energy.)

Thermal equilibrium is a generalization of this concept, where energy is distributed to across *all properties* (temperatures, populations, etc) to reach a state of maximum entropy.

Quantum Statistical Mechanics
Equilibrium occupation number:

$$\langle f \rangle = \frac{1}{e^{(\epsilon-\mu)/kT} \pm 1}$$

Classical Example

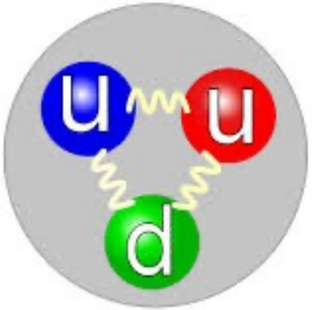


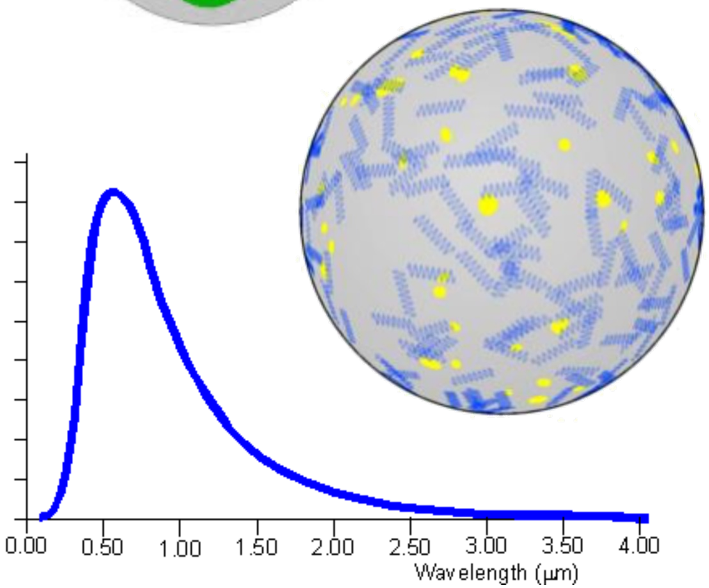
Thermal Equilibrium is a *statistical* property.

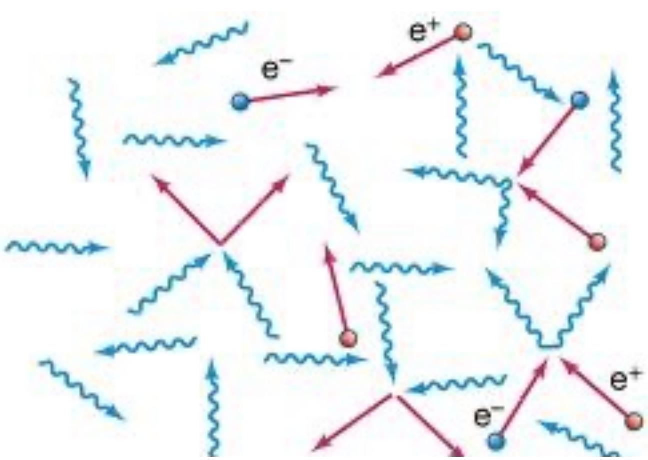
Once connected, energy will spread randomly throughout the systems, evening out the temperature.

Once they're the same temperature, it would be weird for lots of energy to coincidentally go to one side.

QFT Example







What is the most disordered way to distribute 1GeV of energy?

→ 1 proton? No, too simple…

→ Monochromatic light? Nope…

→ Blackbody? Much better.

→ What about particles? Some electrons, neutrinos, blackbody photons... Yes!

$$\langle f \rangle = \frac{1}{e^{(\epsilon - \mu)/kT} \pm 1}$$

Equilibrium

$E = (m^2c^4 + p^2c^2)^{1/2}$
– for Bosons
+ for Fermions
g is the degen. factor

$$N = \bar{N} = \frac{4\pi g}{h^3}\int_0^\infty \frac{p^2\,\mathrm{d}p}{e^{E/kT} \pm 1}$$

$kT >> mc^2$

$$n = \left(\frac{kT}{c}\right)^3 \frac{4\pi g}{(2\pi\hbar)^3}\int_0^\infty \frac{y^2 dy}{e^y \pm 1}$$

(where y = pc / kT)

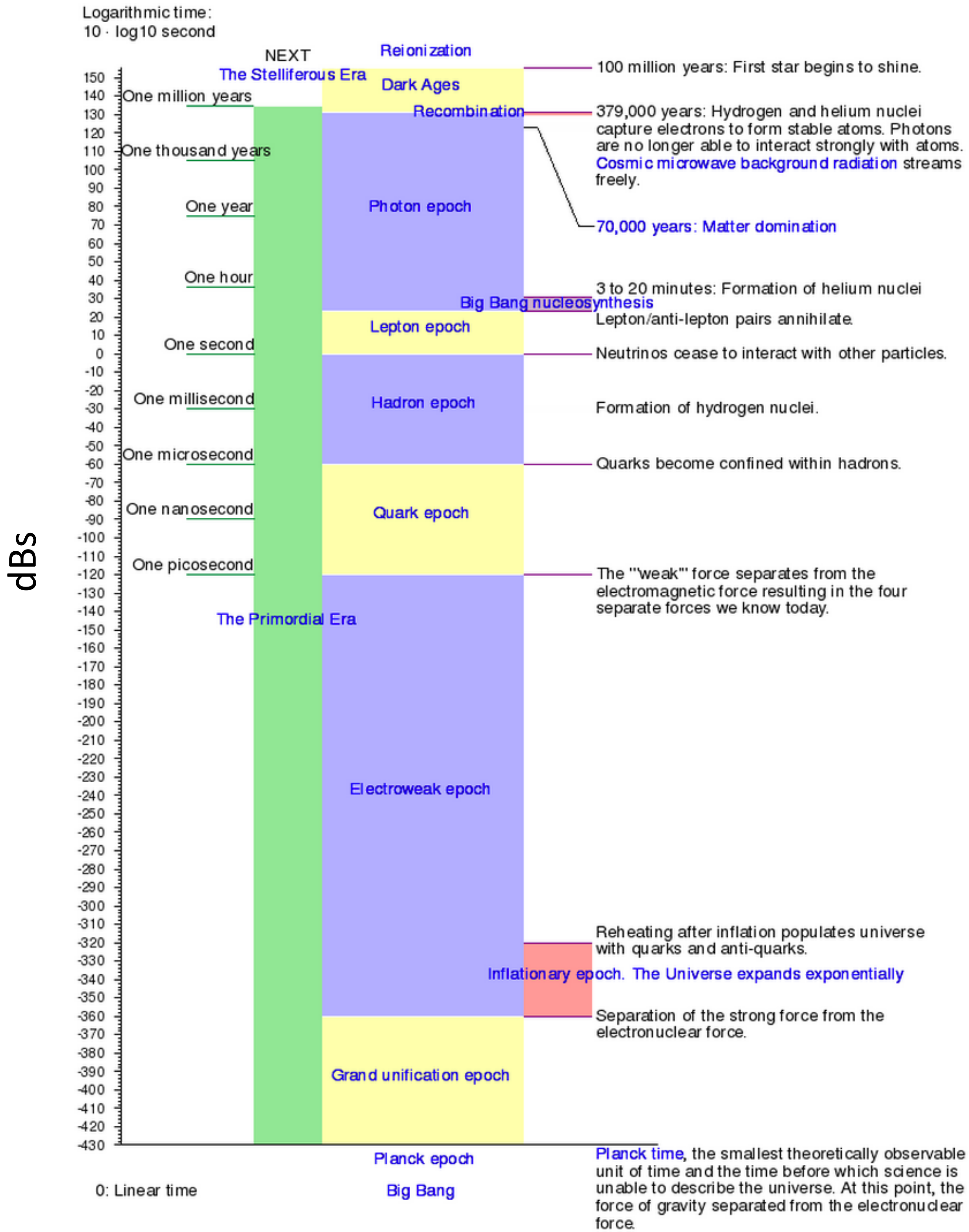
$kT << mc^2$

$$n = e^{-mc^2/kT}(2mkT)^{3/2}\frac{4\pi g}{(2\pi\hbar)^3}\underbrace{\int_0^\infty e^{-y^2} y^2 dy}_{=\frac{\sqrt{\pi}}{4} \approx 0.443}$$

$$= g\frac{(2kTm\pi)^{3/2}}{(2\pi\hbar)^3}e^{-mc^2/kT}$$

Number density exponentially suppressed below $kT = mc^2$

$$\gamma + \gamma \Leftrightarrow P + \bar{P}$$

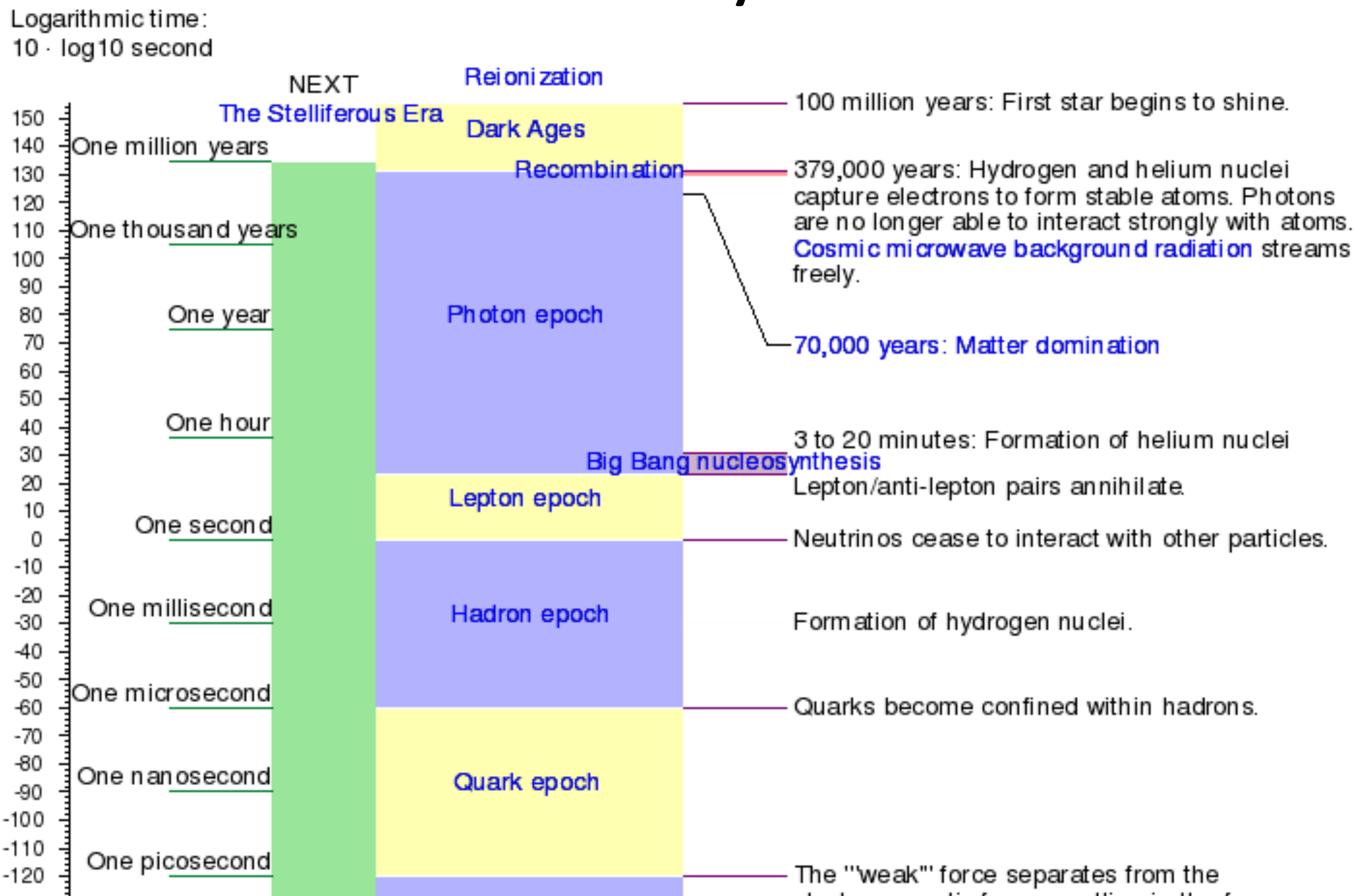


Present Day

Well Tested Physics!

Progressively More Speculative

The Early Eras



Quark-Gluon Plasma

$t < 1ps$ (10^{-12}s)
$T > 10^{16}$ K
$kT > 1$TeV

The Universe is a hot mess.

All particles of mass $< kT/c^2$ exist en masse.
Everything rapidly comes to an equilibrium.

This just includes the highest energy regime tested in accelerators.

The Hadron Era

$1\text{ps} < t < 1\mu\text{s}$

$10^{13}\text{ K} < T < 10^{16}\text{ K}$

$kT > 1\text{GeV}$

Hadrons are clumps of quarks.

- Isolated quarks no longer exist.
- Mesons are made of 1 quark, 1 anti-quark.
- (Anti-)Baryons are composed of 3 (anti-)quarks.
- Exotic species such as tetraquarks, pentaquarks, etc may exist.

At extreme temperatures, the Universe is filled with exotic hadrons, with typical masses of order $mc^2 \sim kT > 1\text{GeV}$.

As the Universe cools, there is a sea of particles in equilibrium:

γ, e^-, e^+, p, $\bar{p}$, n, $\bar{n}$, ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$, ν_τ, $\bar{\nu}_\tau$, μ^-, μ^+, τ^-, τ^+, etc etc

The Lepton Era

$1\mu s < t < 1s$
10^{10} K < T < 10^{13} K
kT > 1MeV

Exotic Mesons all decay or annihilate early on.
Things settle back into equilibrium, numbers even out.
…
Below 1GeV, baryons no longer pair-produce, but still annihilate.
The Universe is flooded with photons, 2 per b-$\bar{b}$ pair.
…
Taus and muons all decay to electrons and neutrinos (also, anti-).

As it cools, the sea of particles starts shrinking:

γ, e^-, e^+, ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$, ν_τ, $\bar{\nu}_\tau$

Freezing Out

$$\Gamma = n\, v\, \sigma$$

Scattering rate

Number density particles

Velocity of particles

Scattering cross section

If the reaction rate drops below the expansion rate, H(z), the species stop interacting: they decouple, or freeze out.

Heuristically:

For many reactions, $\Gamma \propto T^{n}$

Some integer, not number density!

Assuming a radiation-dominated Universe ($t \propto T^{-2}$),

$$N_{scatter} = \int_{t_1}^{\infty} \Gamma(t')dt' = \frac{1}{n-2}\frac{\Gamma}{H}\Big|_{t_1}$$

For $n>2$ (as in most reactions) a rate $\Gamma < H$ will result in fewer than one interaction between $t=t_1$ and $t=\infty$.

Electrons & Neutrinos

Above 1MeV, electrons remain coupled to the radiation via pair production & annihilation.

Neutrinos don't couple to the electromagnetic field. Instead, they are coupled to the electrons via the weak force.

Neutral-current weak interactions:

$e^+ + e^- \Leftrightarrow \nu_i + \bar{\nu}_i$

$\Gamma = n\, c\, \sigma$

$n \approx 2 \times 10^{-32}\, T_{10}^3\ \mathrm{cm}^{-3}$

$\sigma \approx 10^{-44}\, T_{10}^2\ \mathrm{cm}^{-3}$

Expansion timescale: $t/1\mathrm{s} \approx T_{10}^{-2}$ ➔ Matches interaction rate at $T_{10} \approx 5 \Leftrightarrow t \approx 40\mathrm{ms}$.

Charged-current weak interactions:

$p + e^- \Leftrightarrow n + \nu_e$

Rate matches expansion at $T_{10} \approx 1 \Leftrightarrow t \approx 1\mathrm{s}$.

The Cosmic Neutrino Background

Neutrinos froze out at $t \approx 1$ second.
(More on this next time – BBN.)

Low interaction cross-section – minimal annihilation.
At freeze-out, $n_\nu \approx n_\gamma$ – roughly equal numbers.

Relativistic → "hot dark matter."
Part of the radiation density of the Universe.
Neutrinos redshift just like photons $\rho_\nu = \rho_\nu(t_0)\, a(t)^4$

The Universe is filled with a CνB, just like the CMB.
Its present temperature is ≈1.9K. (*Colder* than the CMB.)
→ we have no real hope of observing it. ☹

Note that before freeze-out, the Universe is opaque to neutrinos!

Aside: Massive Neutrinos

A 1.9K background of neutrinos corresponds to ≈ 113 ν / cm^3

If each species had some mass, the density added would be:

$$\Omega_\nu h^2 \approx \frac{\sum m_{\nu,i} c^2}{93.5eV}$$

→ For Σm_ν > 16eV, the Universe would be closed!

Current cosmological observations limit to Σm_ν < 0.23 eV.

Beyond Equilibrium

To be rigorous, the number density of particles can be written:

$$n(t) = \frac{g}{(2\pi\hbar)^3} \int d^3p f(E, x, t)$$

Non-equilibrium populations can then be studied with the Boltzmann Equation:

$$\frac{\partial f}{\partial t} + (\dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}}) f + (\dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}}) f = \dot{f}_c$$

In English: Particles maintain their phase-space density, unless acted on by some collision.

This tends to get messy. For simple annihilation interactions:

$$a^{-3}\frac{d\left(n_1 a^3\right)}{dt} = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4}$$

$$\times\ (2\pi)^4 \delta^3(p_1 + p_2 - p_3 - p_4)\delta(E_1 + E_2 - E_3 - E_4)\,|\mathcal{M}|^2$$

$$\times\ \{f_3 f_4[1 \pm f_1][1 \pm f_2] - f_1 f_2[1 \pm f_3][1 \pm f_4]\}\,.$$

The Plasma Era

1s < t < 380,000y

10^{10} K < T < 3000 K

kT > ¼eV

With the neutrinos frozen out,
not much left in sea of particles:

γ, e^-, e^+

Below kT = 1MeV, electrons and positrons annihilate,
injecting some extra energy into the radiation.
A tiny fraction (10^{-9}) remain, but can't form HI.

Note: midway through the plasma era, the Universe
transitions from Radiation to Matter dominated!
(This changes the a(t) evolution.)

The Dark Ages

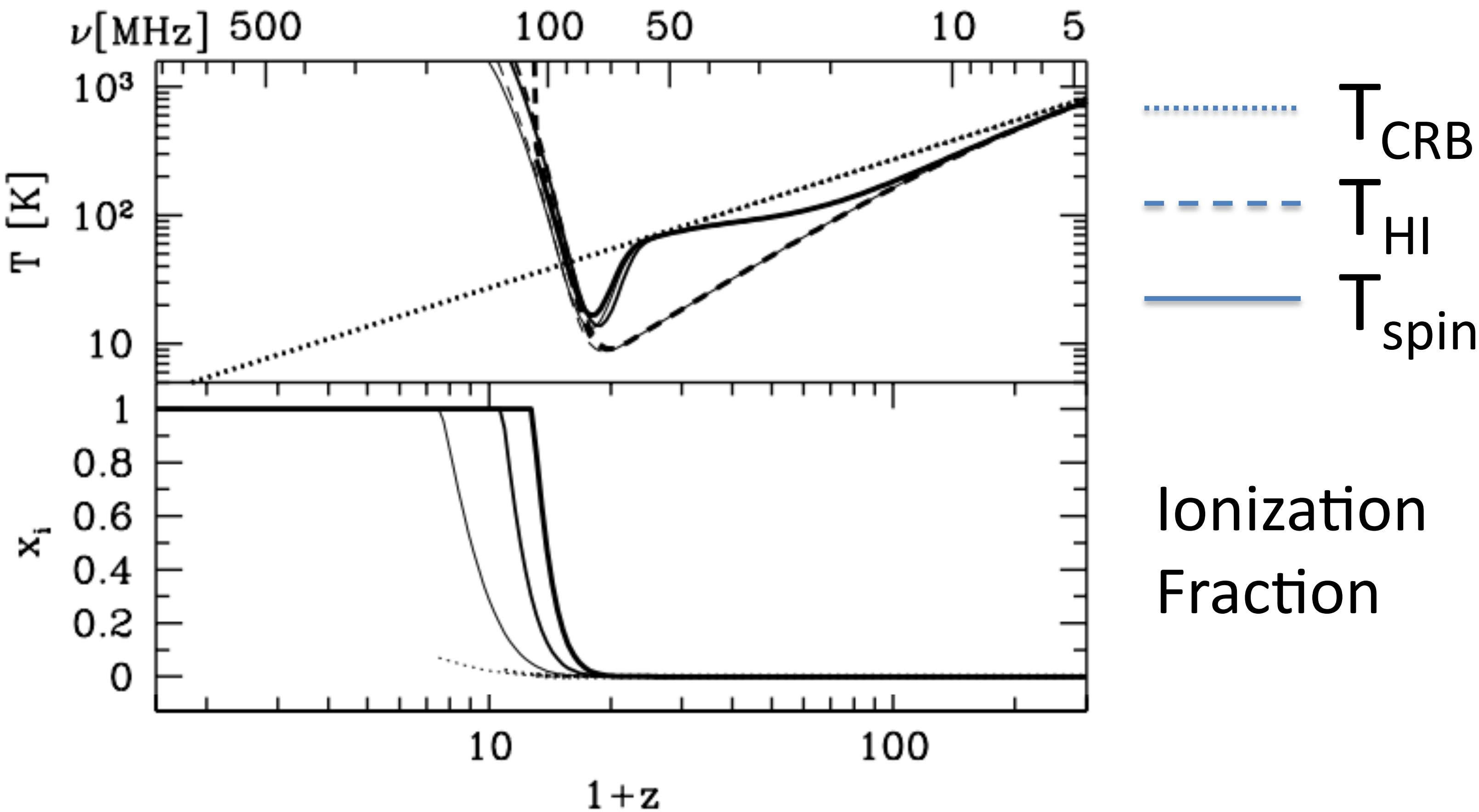
$t > 300{,}000y$
$T < 3000$ K
$kT < \frac{1}{4}$eV

After recombination, things are kind of dull.

- No more unstable species.
- Relic photons and neutrinos are streaming freely.
- Baryons & electrons have bound.
- Universe doesn't have many photons left with the energy to unbind them.

Luckily, cosmological principle is wrong – not perfectly Homogeneous.
Small perturbations in the density field collapse under gravity.
Universe continued to expand and cool, structures collapsed, heated.
At $t \approx 500$My, stars ignited and began spraying out UV radiation.
By $z \approx 5$, the Universe was re-ionized! (But now mostly transparent.)

Thermal History Post-Recombination



The Modern Universe

$t > 1Gy$

$T < 20$ K

Cosmologically, not much has happened since reionization:

1) Structures have continued to grow.
2) Dark Energy (Λ) became the dominant actor.

(And really, the 1st doesn't matter except that it made us.)

INTERMISSION

Nucleosynthesis

Loose protons are more massive than agglomerations.

But: Nuclei are positively charged, repelled by EM.

Nucleons are bound by the [short-range] strong force.

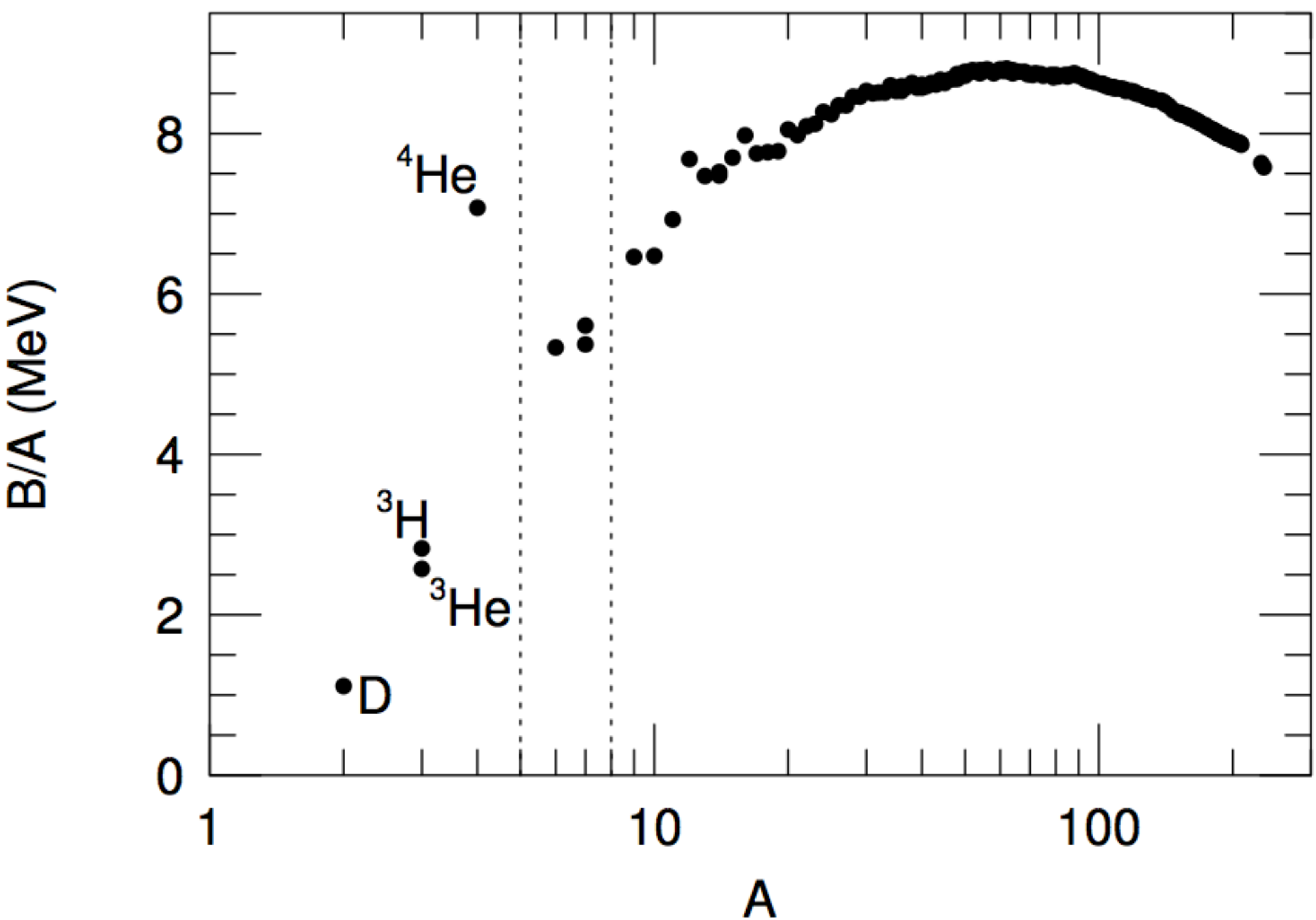
Fusion requires overcoming [tunneling] the Coulomb barrier.

→ High density and high temperature.

We're used to this happening in stars.

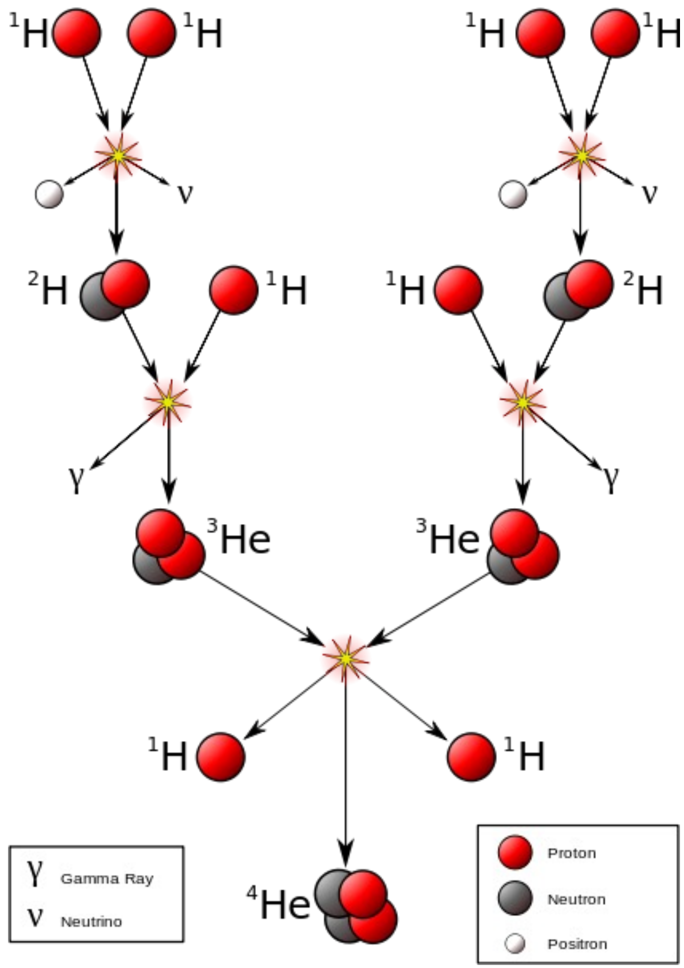
The early Universe was also capable of it.

Binding Energy

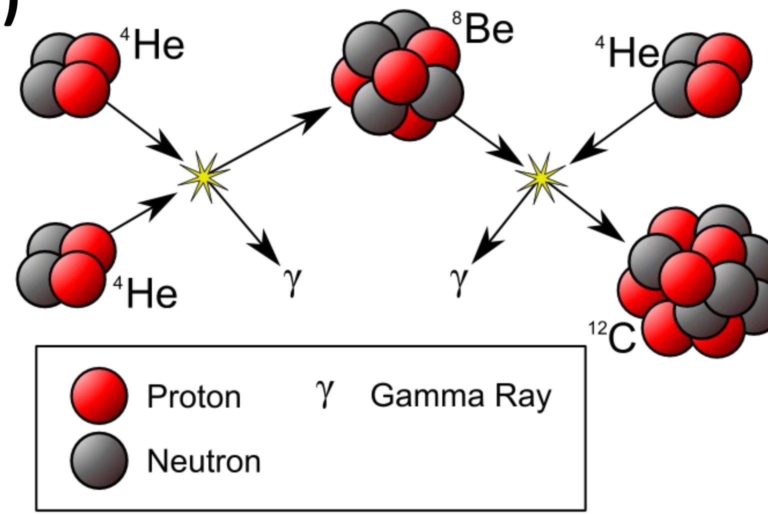


Stellar Nucleosynthesis

(1)



(2)



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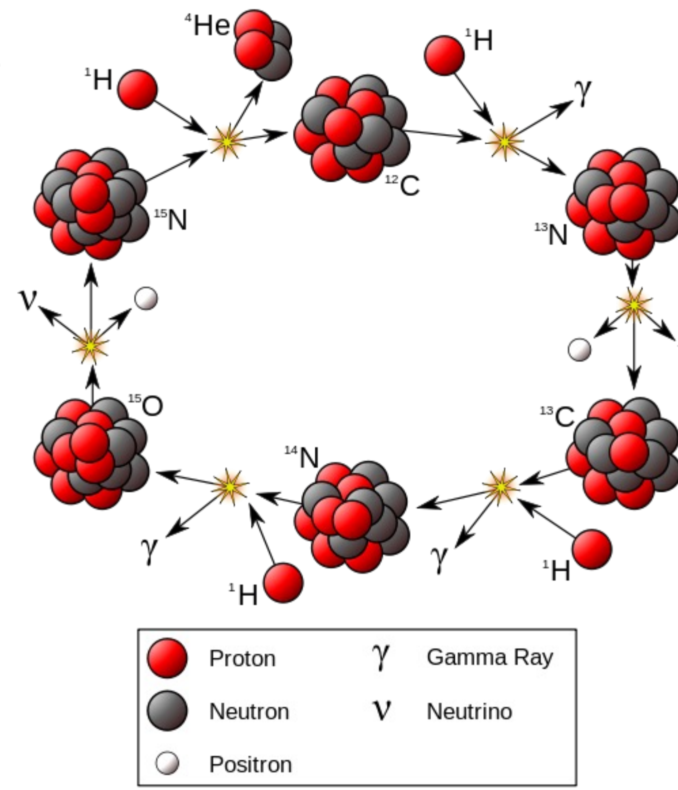
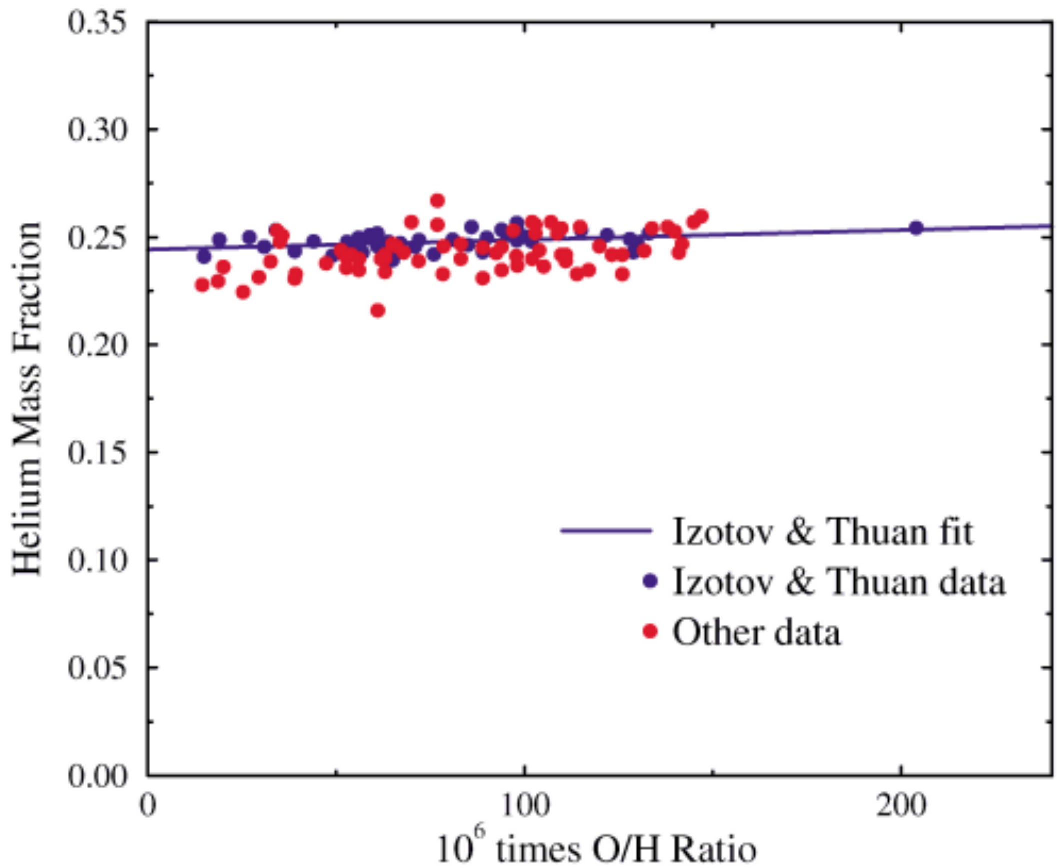


The Light Elements

≈25% of Baryonic mass appears to be in Helium nuclei.

13Gy of L_{MW} would fuse <1%.

^{2}H, ^{3}He, ^{7}Li don't survive in stars, but are seen in ISM.



Primordial (BB) Nucleosynthesis

Stellar Nucleosynthesis:

- small dT/dt, lots of time
- all species reach thermal equilibrium
- stats depend on mass of star

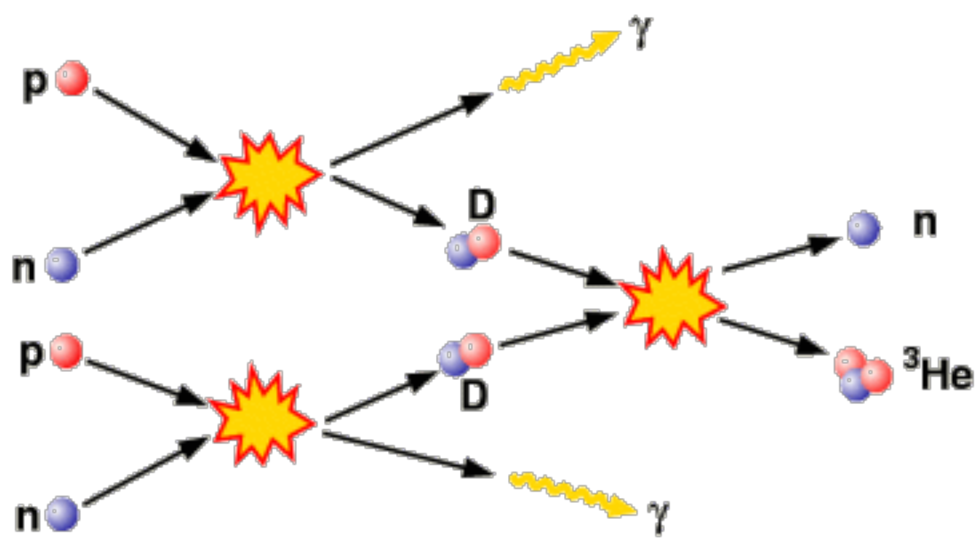
BB Nucleosynthesis:

- large dT/dt, limited time
- only the first few elements form before freezing out
- stats depend on expansion history

The Neutrons

Proton-Proton merger ($p + p \rightarrow {}^2H + e^+ + \nu_e$) is extremely rare.

During the early universe, there are plenty of neutrons around, so dominant mode of fusion is instead p + n.



Neutrons

In equilibrium, neutron and proton densities are related by

$$\frac{N_n}{N_p} = e^{-\Delta mc^2/kT} \approx e^{-1.5/T_{10}}$$

Equilibrium maintained through charged-current weak interactions: $p + e^- \Leftrightarrow n + \nu_e$

$\Gamma = \langle \sigma v \rangle N$

Rate matches expansion at $T_{10} \approx 1 \Leftrightarrow t \approx 1s$.

Without freeze out, N_n rapidly becomes $\ll N_p$.

Boltzmann

$$\frac{\partial f}{\partial t} + (\dot{\mathbf{x}} \cdot \cancel{\nabla_{\mathbf{x}}}) f + (\dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}}) f = \dot{f}_c$$

$$\frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p} = \dot{f}_c = -\int \langle \sigma v \rangle f \bar{f} \, d^3\bar{p}$$

H = ṗ / p

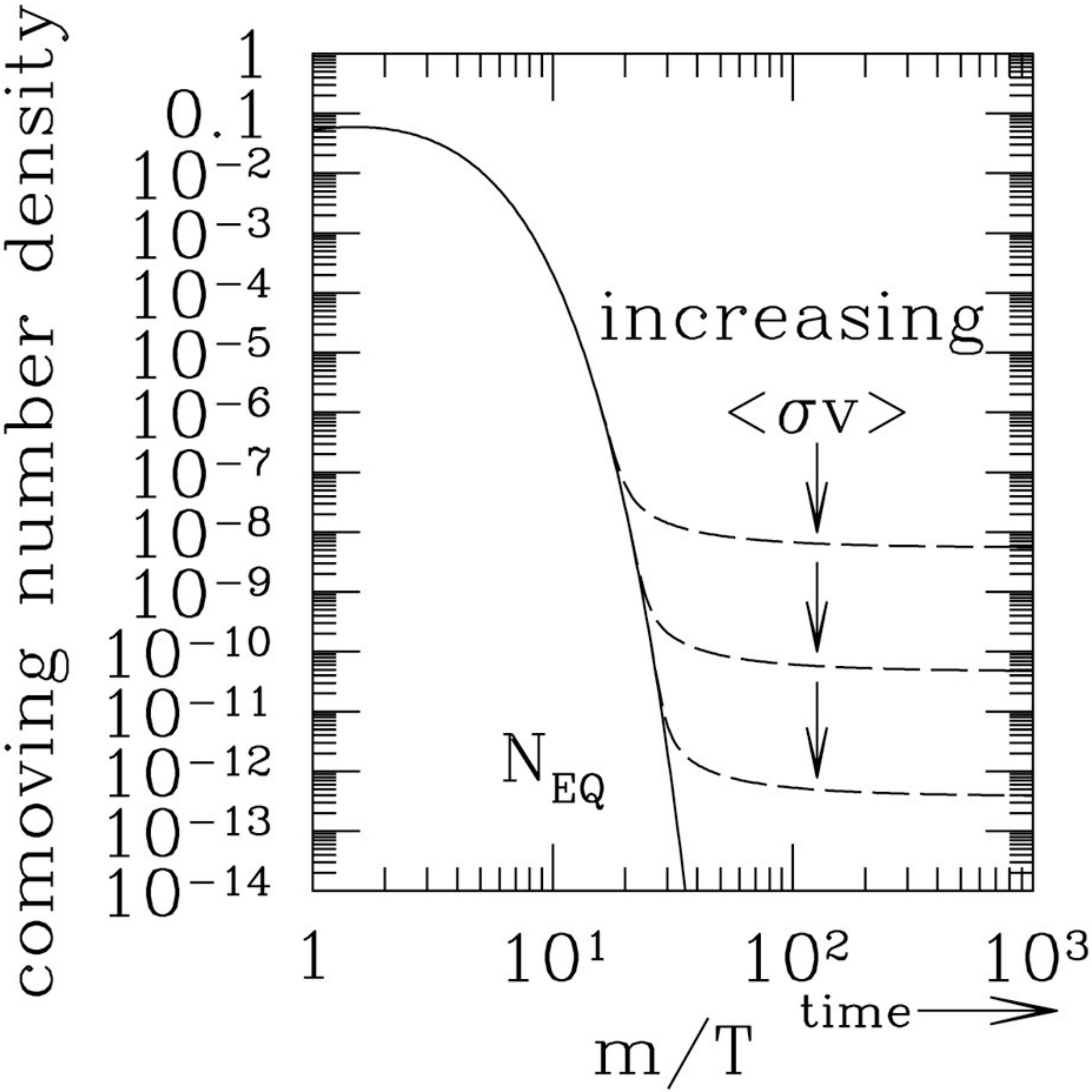
“Hubble Drag”

Annihilation + Creation

$$\dot{n} + 3Hn = -\langle \sigma v \rangle n^2 + \langle \sigma v \rangle n_T^2$$

$$\boxed{\frac{d\ln N}{d\ln a} = -\frac{\Gamma}{H}\left[1 - \left(\frac{N_T}{N}\right)^2\right]}$$

Freeze Out



Charged-current weak interactions have $\Gamma \approx T_{10}^{-5}\ s^{-1}$

$N_n/N_p \approx 0.34$

Deuterium Synthesis

$p + n \Leftrightarrow {}^2D + \gamma$

B ≈ 2.2MeV, so at early times
($T > 10^9$K, $t < 100$s),
^2D rapidly photo-dissociate.

Note: just like recombination, kT << B!

Until t ≈ 100s, nucleosynthesis is stuck at this stage.
"Deuterium Bottleneck"

From freeze-out (1s), neutrons decay, $\tau \approx 890$s.

For a larger $\Omega_b/\Omega\gamma$, higher T required, shorter bottleneck.

Helium-4

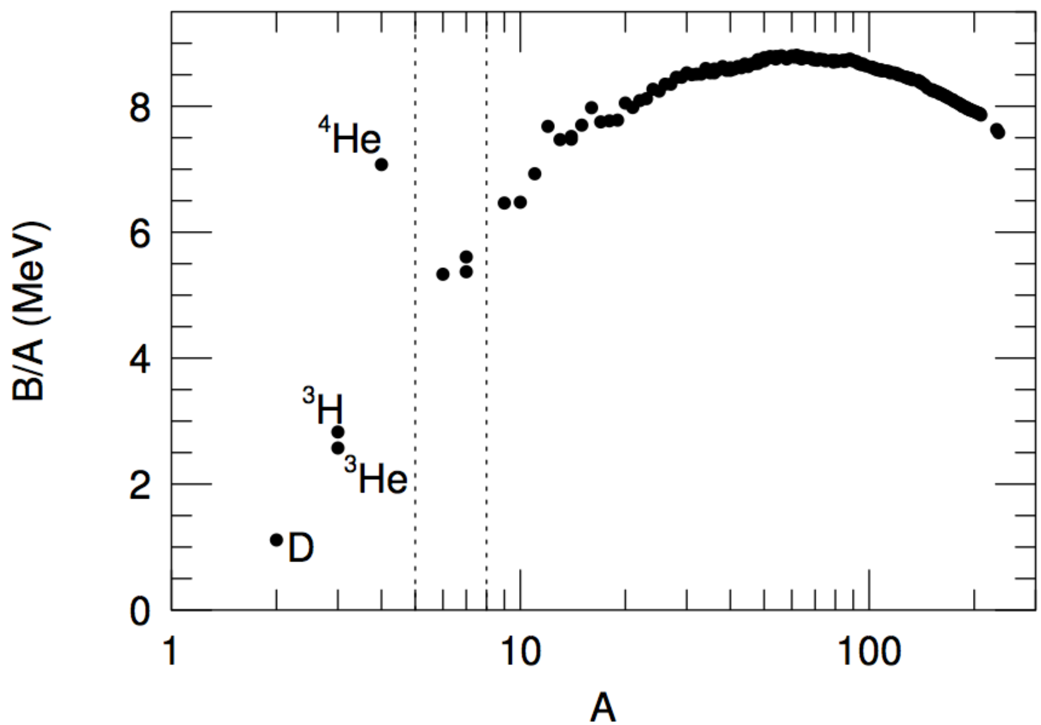
The rest of the chain proceeds rapidly, depleting ^2D as soon as it is produced:

$^2D + {}^2D \Leftrightarrow {}^3He + n$

$^3He + n \Leftrightarrow {}^3T + p$

$^2D + {}^3T \Leftrightarrow {}^4He + n$

Helium is extremely stable compared to other light isotopes.



No stable isotopes at A=5, 8.

Triple-alpha rate is too low

$^4He + {}^4He + {}^4He \Leftrightarrow {}^{12}C$

To 1st order, all neutrons end up bound in ^{4}He.

$$Y_p \equiv \frac{\rho(^4He)}{\rho_b}$$

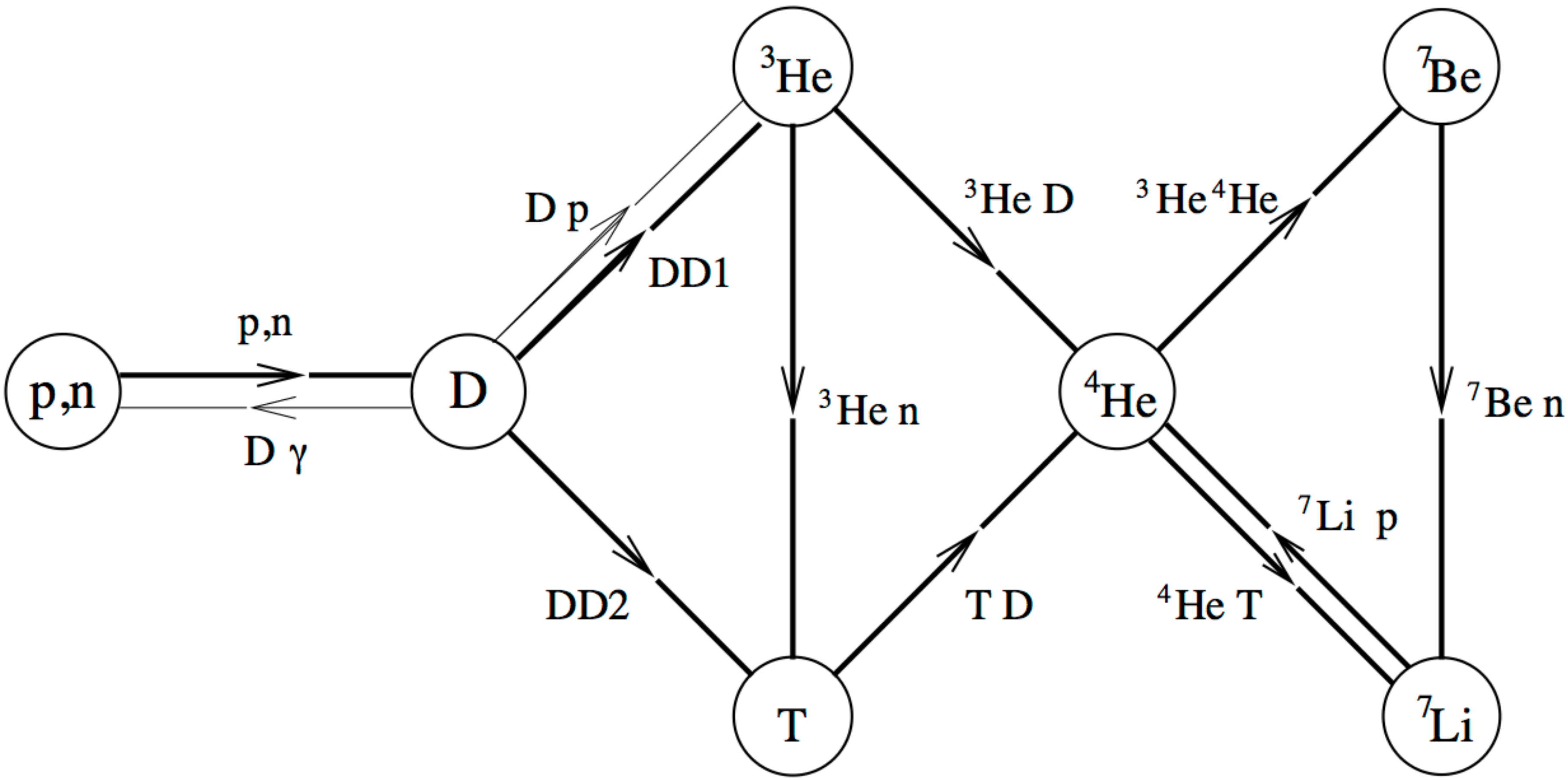
Summary

- Neutrons in thermal equilibrium until neutrinos freeze out, $t \approx 1s$.
- Nucleosynthesis stalled by Deuterium bottleneck until $t \approx 100s$.

  → Timescale set by $\eta = n_b / n_\gamma$
- Nearly all remaining neutrons get sucked up into ^4He.
- Small quantities of ^2H, ^3He, ^7Li remain.
- Nothing heavier has time to fuse.

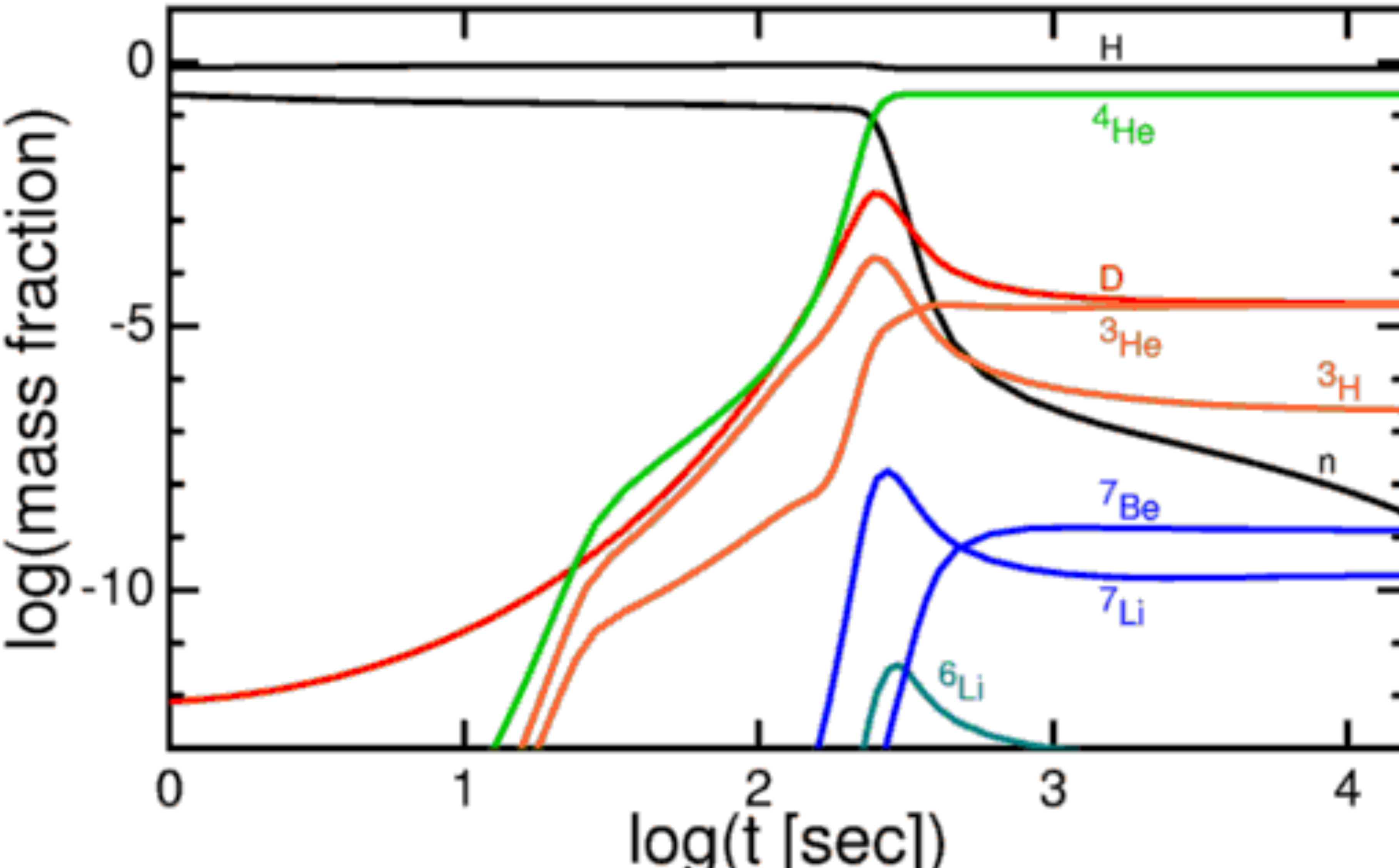
BBN is a race against time. The conditions are only "right" briefly.
Before 100s, too hot, dense, irradiated. After 15min, too cold, diffuse.

Pathways



p,n
D
^{3}He
T
^{4}He
^{7}Be
^{7}Li
p,n
D γ
D p
DD1
DD2
^{3}He n
^{3}He D
T D
^{3}He ^{4}He
^{7}Be n
^{7}Li p
^{4}He T

BBN Timeline



Next Time:

Inflation