

AST 1430

Cosmology

VI

Inflation

“Once, about 13.8 billion years ago, the universe began... It started cranking along and expanding in a fairly ordinary way, except that everywhere you look there [were] insanely high densities and energies. Then, around 10^{-36} seconds later something unexpected happened. In a tiny fraction of a second, the universe inflated to something like 10^{100} times its original size and then just as suddenly continued with its ordinary expansion as though nothing had happened.”

– Dave Goldberg, *Ask a Physicist*

Announcements

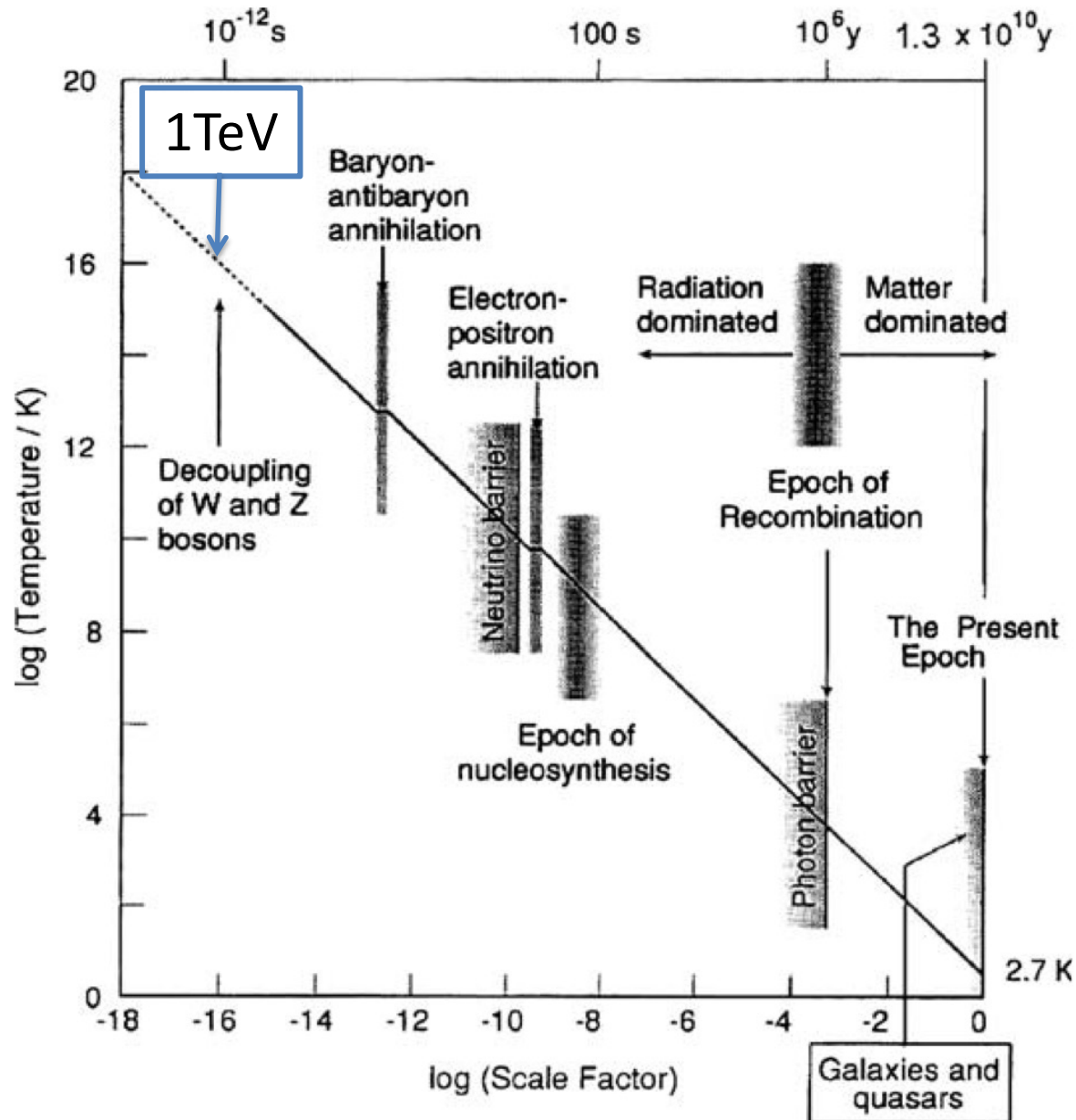
- Venue: back to AB88 for the rest of Feb
- Presentations
 - Feb 15: Anika – Baryogenesis
 - Feb 15: Caleb – CMB B-modes
 - Feb 17: Braden – ??
 - Feb 17: Anna – ??

Keith

Week Index	Dates	Topics	
Week 1	Jan 11, 13	logistics, introduction, basic observations	
Week 2	Jan 18, 20	Basic GR, RW metric, Distances, coordinates, Friedmann equations, cosmological models	
Week 3	Jan 25, 27	Consistency with observations, Early Hot Universe, BBN	asst1
Week 4	Feb 1, 3	Inflation, Perturbations & Structure pre-recombination	
Week 5	Feb 8, 10	CMB: basics, polarization, secondaries	
Week 6	Feb 15, 17	Early-Universe Presentations + Review	
Reading Week – No Class			asst2
Week 7	Mar 1, 3	Post-recombination growth of structure, formation of dark matter halos, halo mass function	
Week 8	Mar 8, 10	The relation between dark matter halos and galaxies	
Week 9	Mar 15, 17	Probing the cosmic density field / clustering	
Week 10	Mar 22, 24	Late-time cosmological observations: BAO, supernovae, weak lensing, etc.	asst3
Week 11	Mar 29, 31	H0 controversy: how fast exactly is the Universe expanding today?	
Week 12	Apr 5	Late Universe Presentations + Review	

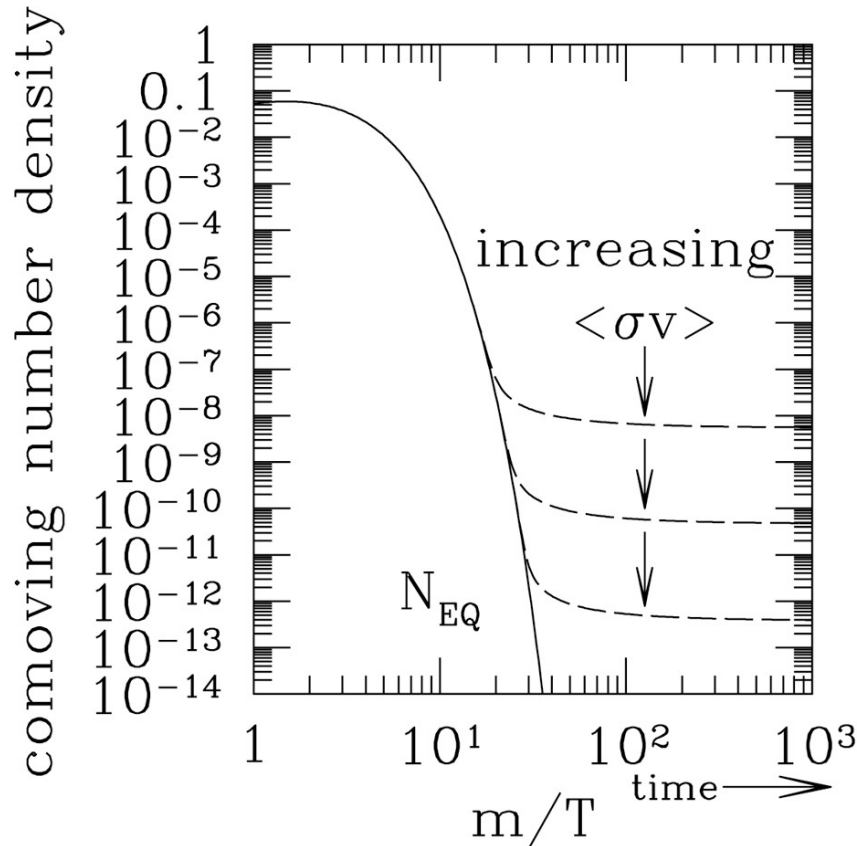
Jo

Life in the Early Universe



Pre-BBN

Neutron $n(T)$



n_N / n_P frozen when
neutrinos decouple @1s

Deuterium Bottleneck



$B \approx 2.2\text{MeV}$, so at early times ($T > 10^9\text{K}$, $t < 100\text{s}$), ${}^2\text{D}$ rapidly photo-dissociate.

(Think: Saha, recombination)

From freeze-out (1s) to $\approx 100\text{s}$, neutrons decay, $\tau \approx 890\text{s}$.

BBN Summary

- Neutrons in thermal equilibrium until neutrinos freeze out, $t \approx 1\text{s}$.
- Nucleosynthesis stalled by Deuterium bottleneck until $t \approx 100\text{s}$.
 - Timescale set by $\eta = n_b / n_\gamma$
- ~All remaining neutrons get sucked up into ${}^4\text{He}$.
- Small quantities of ${}^2\text{H}$, ${}^3\text{He}$, ${}^7\text{Li}$ remain.
- Nothing heavier has time to fuse.

BBN is a race against time. The conditions are only “right” briefly. Before 100s, too hot, dense, irradiated. After 15min, too cold, diffuse.

Y vs Parameters (I)

Fundamental constants

Higher neutron Γ
→ later freeze-out
→ fewer neutrons
→ smaller Y

Larger $\Delta m = m_n - m_p$
→ fewer neutrons
→ smaller Y

Composition of the Universe

→ Only weak dependence on the various Ω_i .
→ Comes in through $\eta = n_b / n_\gamma$
→ Larger η = shorter bottleneck
→ Less neutron decay = larger Y

^2H and ^3He

Produced on the path to ^4He .



Final abundances depend on η (i.e., Ω_b)

Larger Ω_b

- shorter bottleneck
- higher ^2H and ^3He reaction rate
- more ^4He
- as a fraction, *less* ^2D and ^3He

Lithium



Small quantity of ^7Li directly synthesized, mostly destroyed by free p.

Small quantity of unstable ^7Be is produced, decays later ($\tau \approx 50$ days).

Larger Ω_b

- shorter bottleneck
- higher ^2H and ^3He reaction rate
- more ^4He
- more ^7Li

Smaller Ω_b

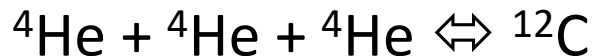
- longer bottleneck
- less ^2H and ^3He reaction
- higher ^2H and ^3He density
- more ^7Li

Final Abundances

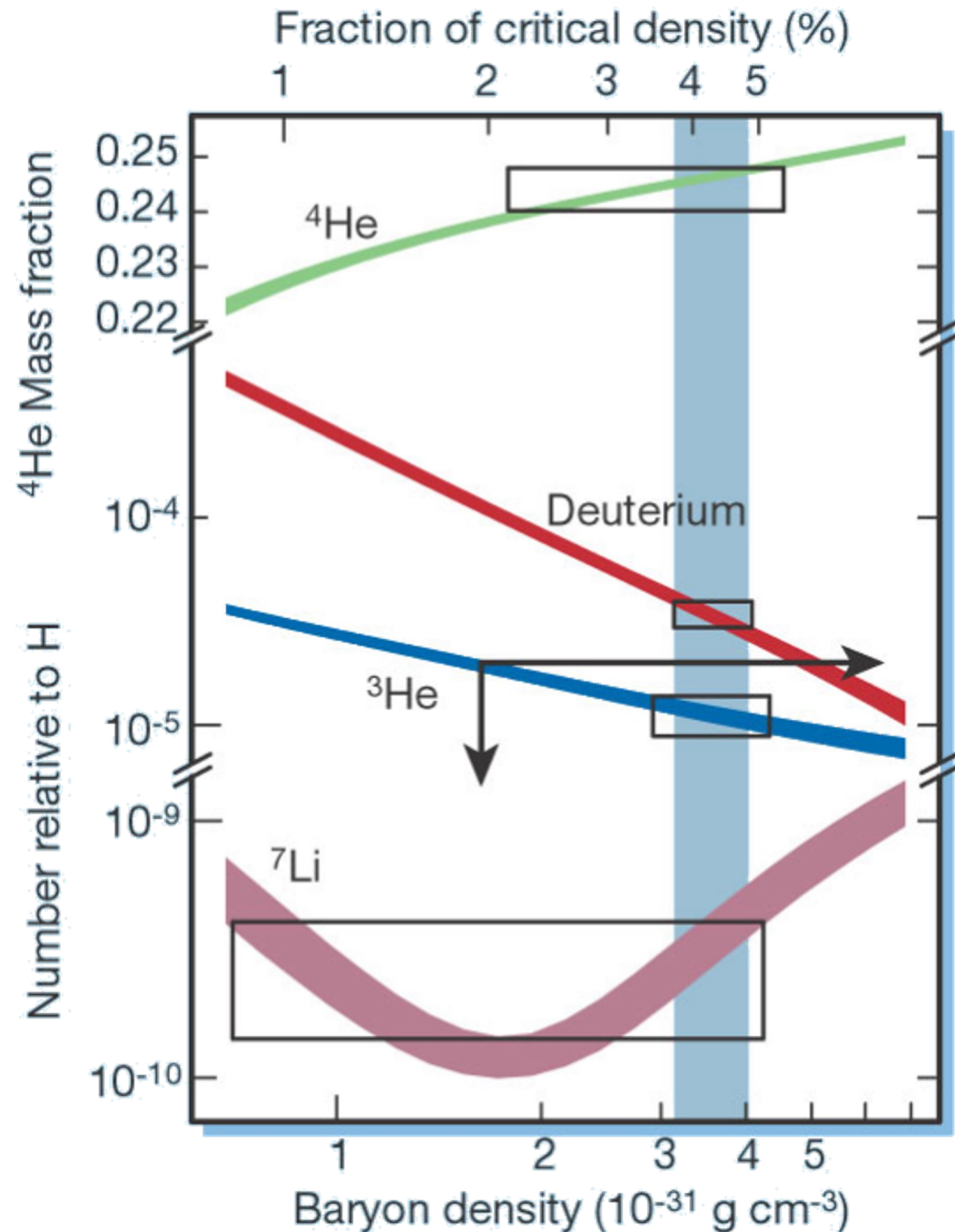
Helium is extremely stable compared to other light isotopes.

No stable isotopes at $A=5, 8$.

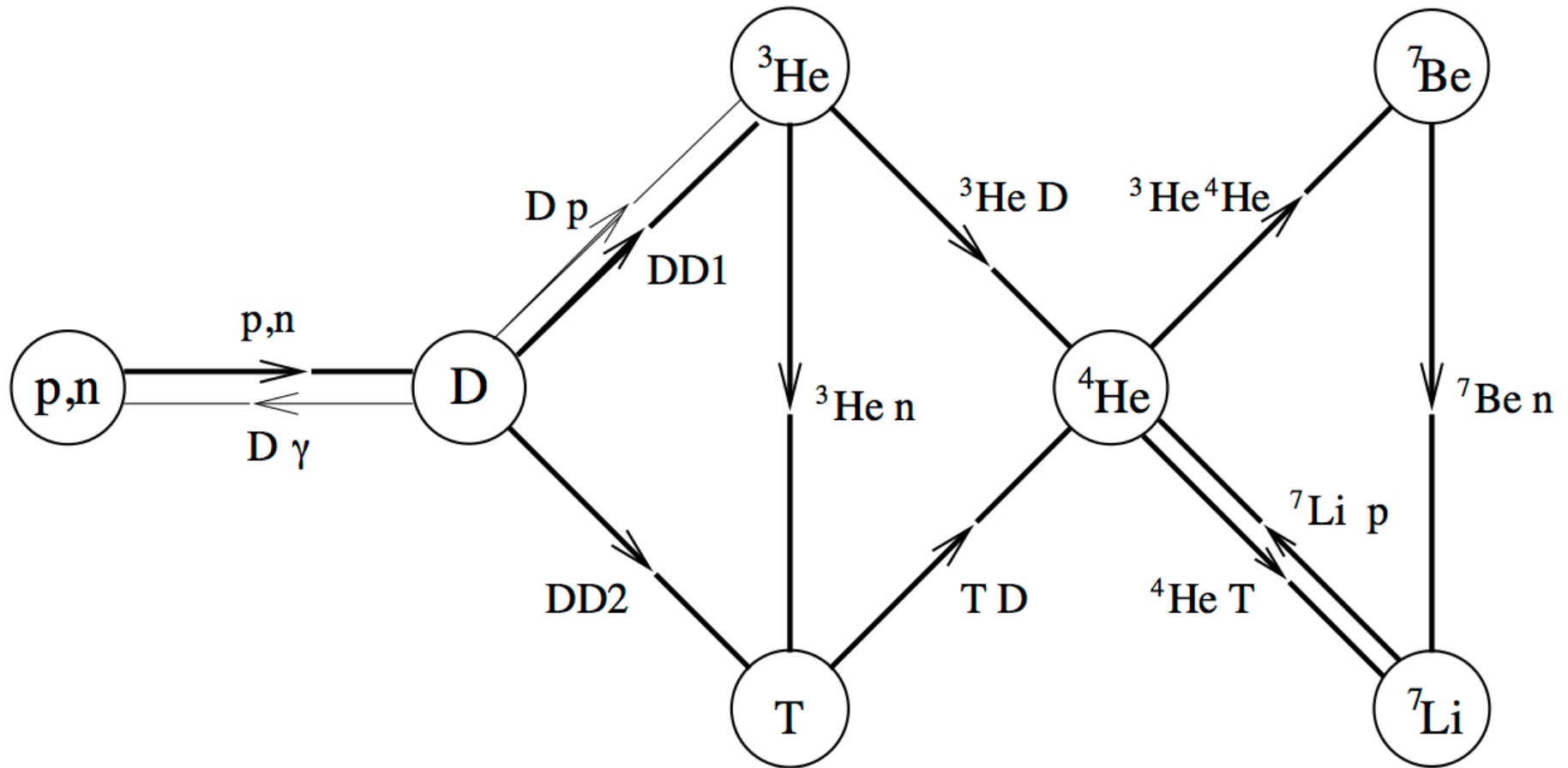
Triple-alpha rate is too low



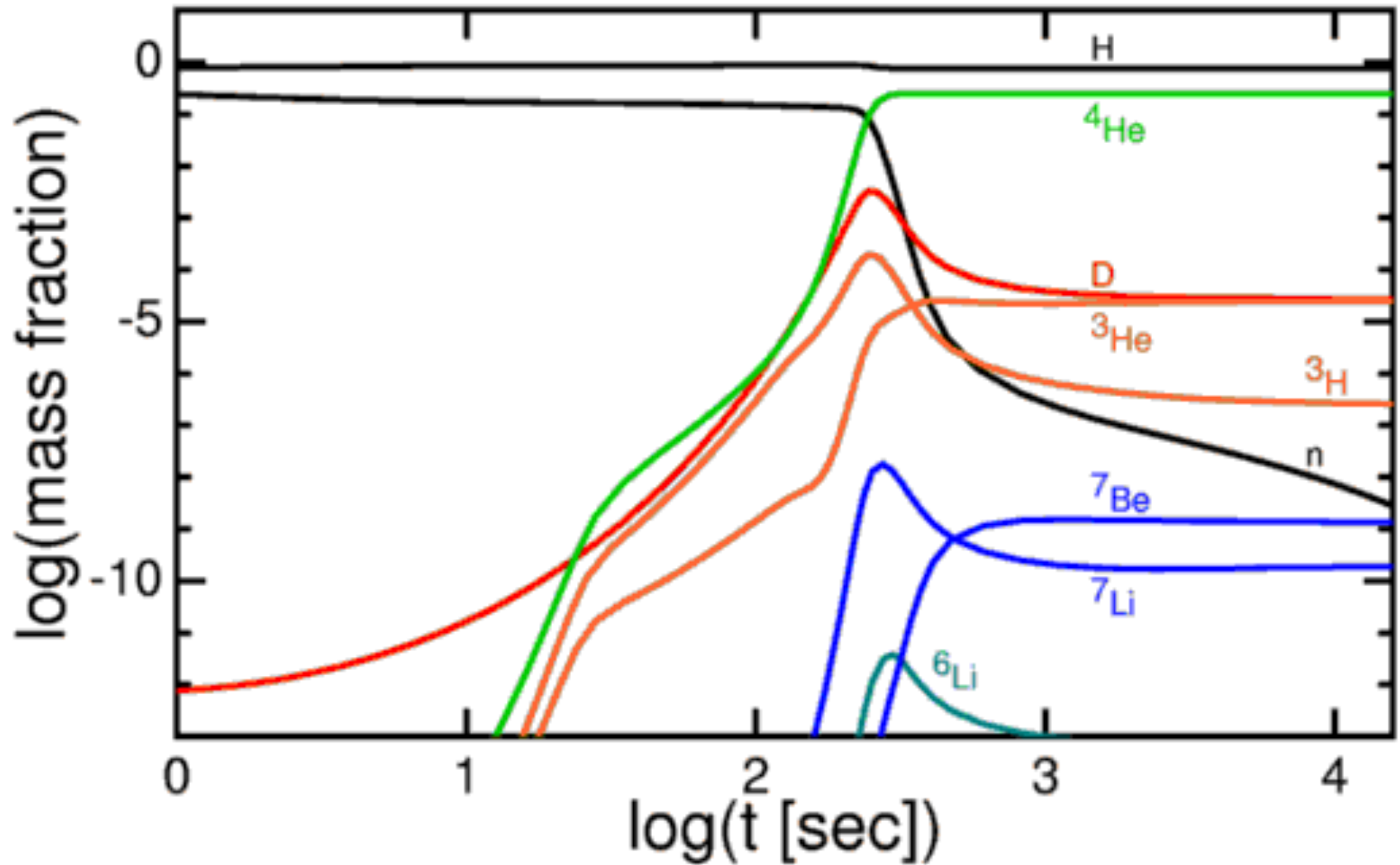
→ Nothing heavy produced before the Universe cools



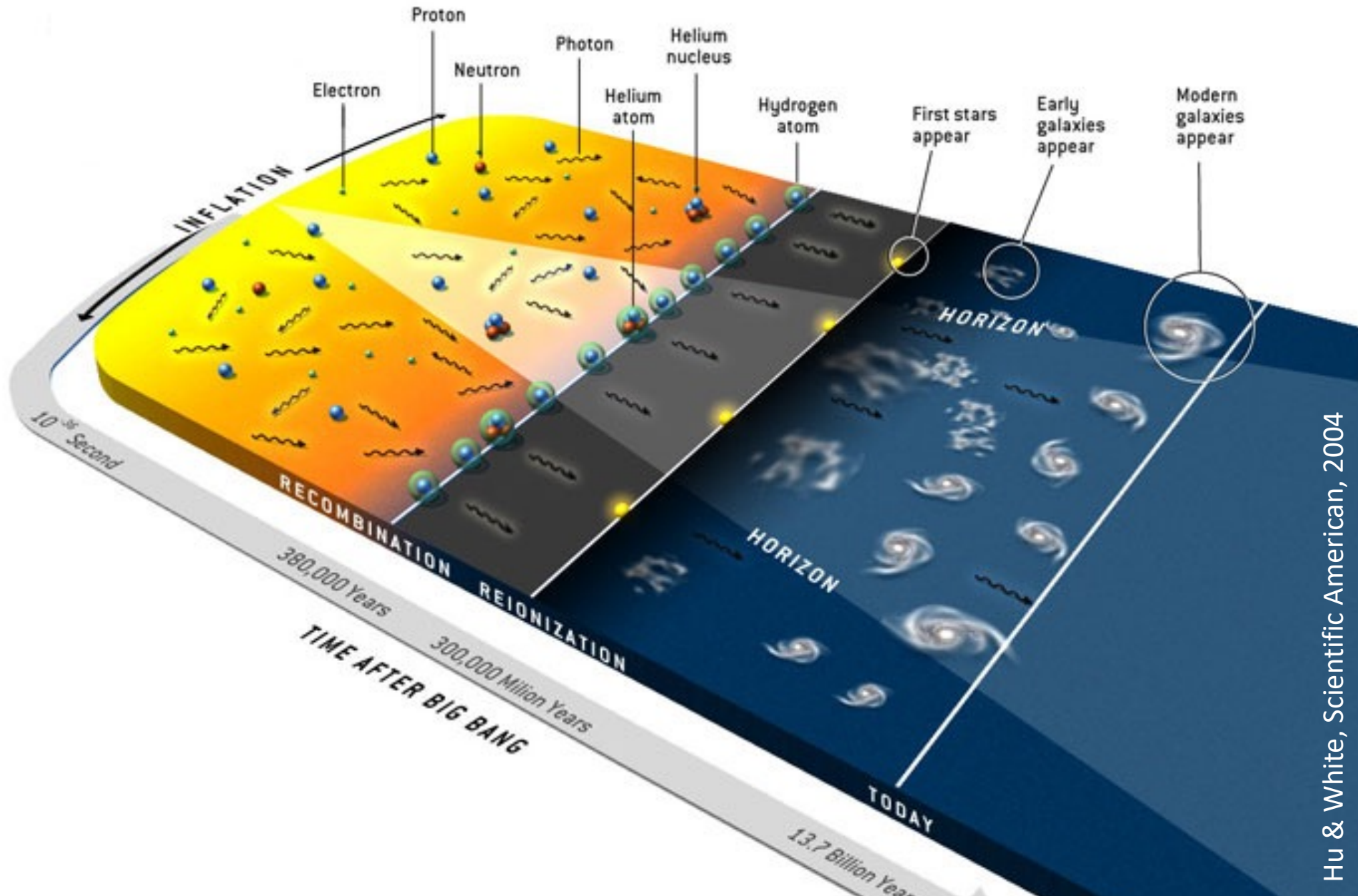
Pathways



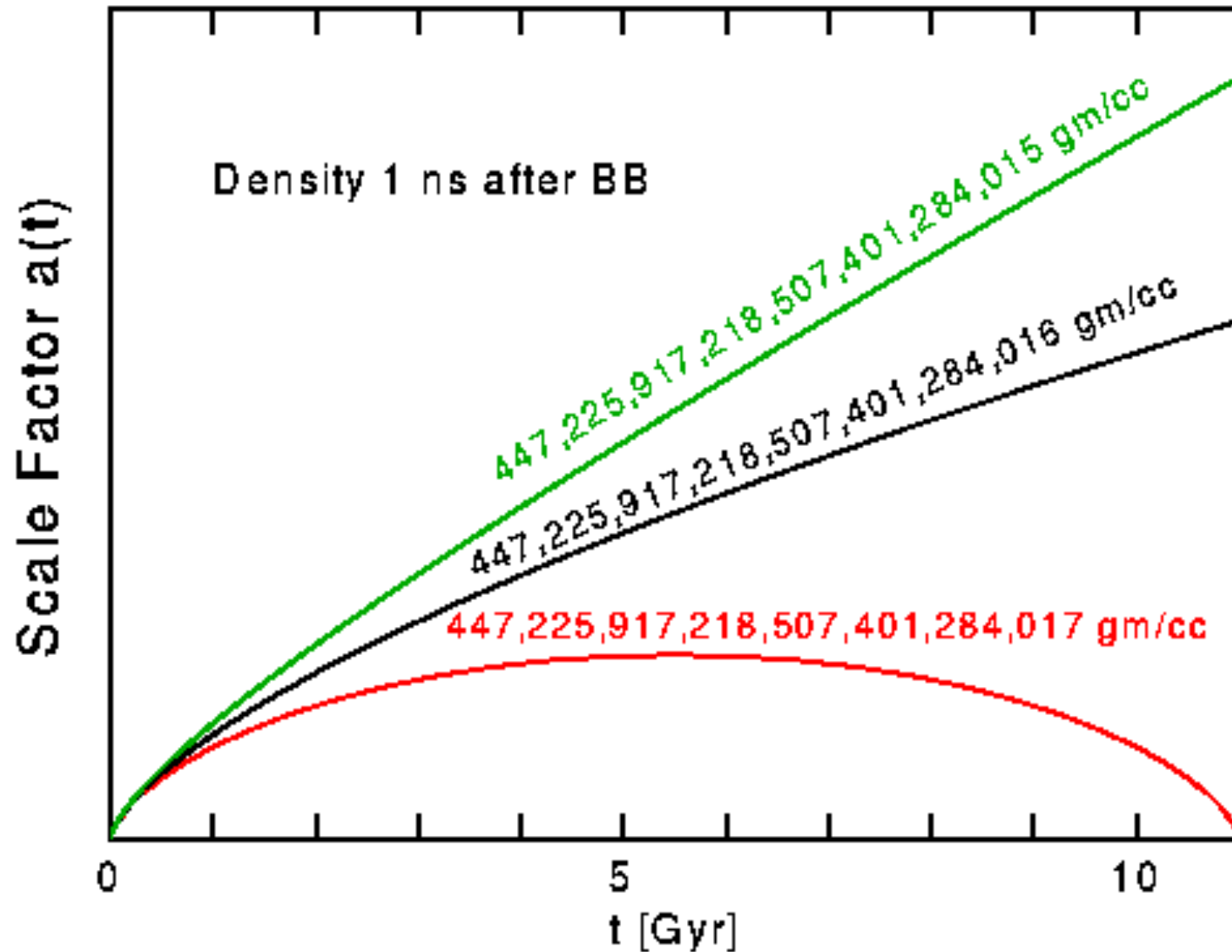
BBN Timeline



Context Reminder



Problem #1 Flatness



Flatness

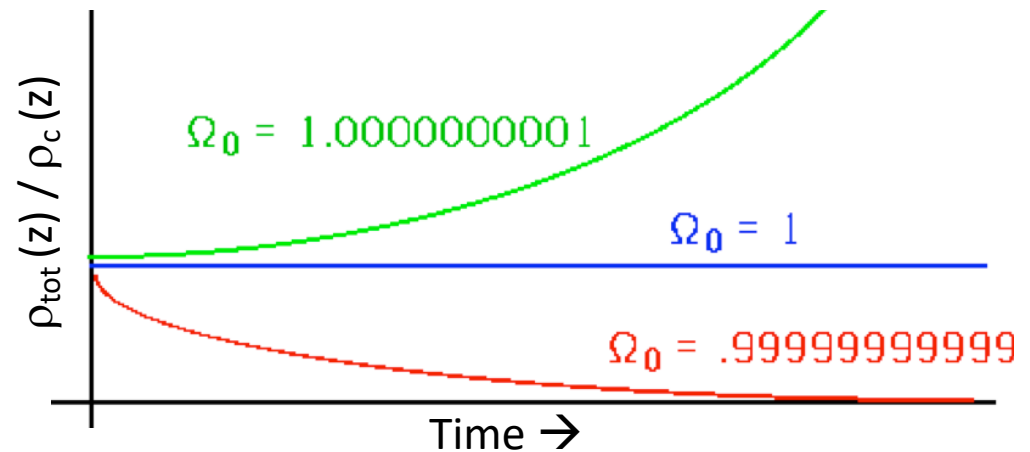
$$H^2 = H_0^2(\Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda)$$

Radiation-dominated:

$$\rho_k(t) / \rho_c(t) \propto a^2 \propto t$$

Matter-dominated:

$$\rho_k(t) / \rho_c(t) \propto a \propto t^{2/3}$$



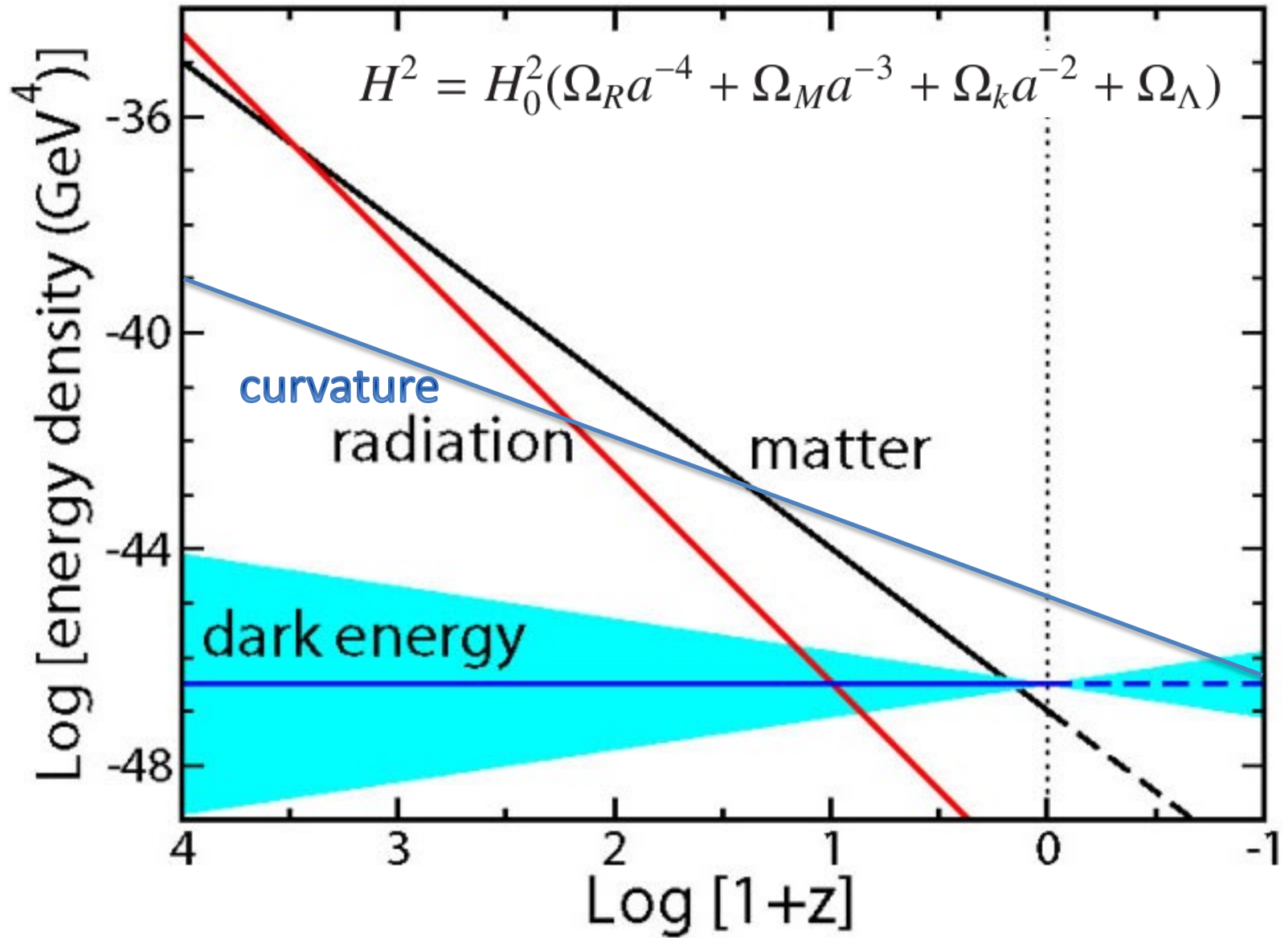
$\Omega_{\text{tot}} = \rho / \rho_c \approx 1 \pm 0.2 \quad \rightarrow \text{During BBN (} t \approx 5\text{min)}$

$$\rho / \rho_c \approx 1 \pm 10^{-14}$$

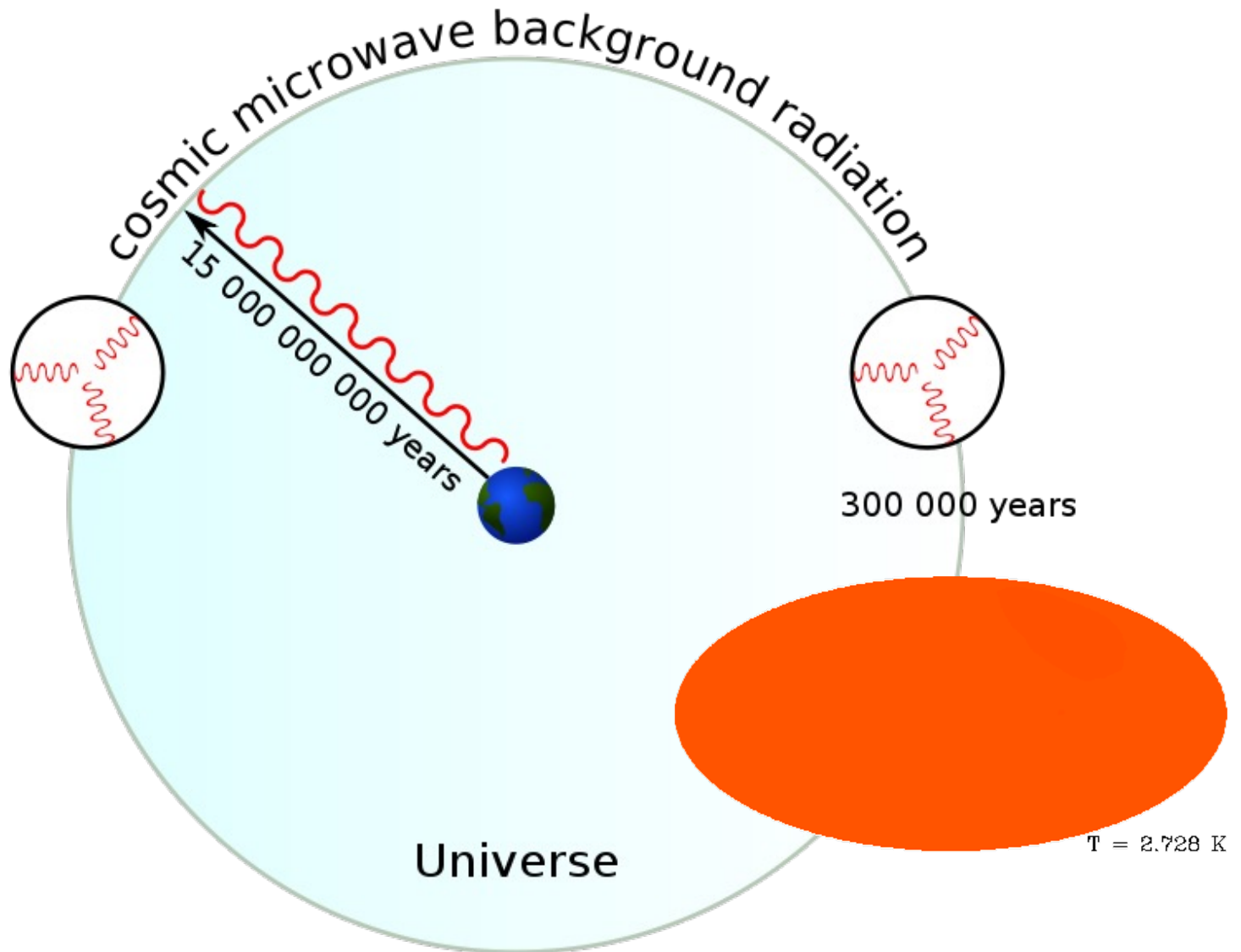
$\rightarrow \text{At a the Planck time, } 5 \times 10^{-44}\text{s}$

$$\rho / \rho_c \approx 1 \pm 10^{-60}$$

Flatness

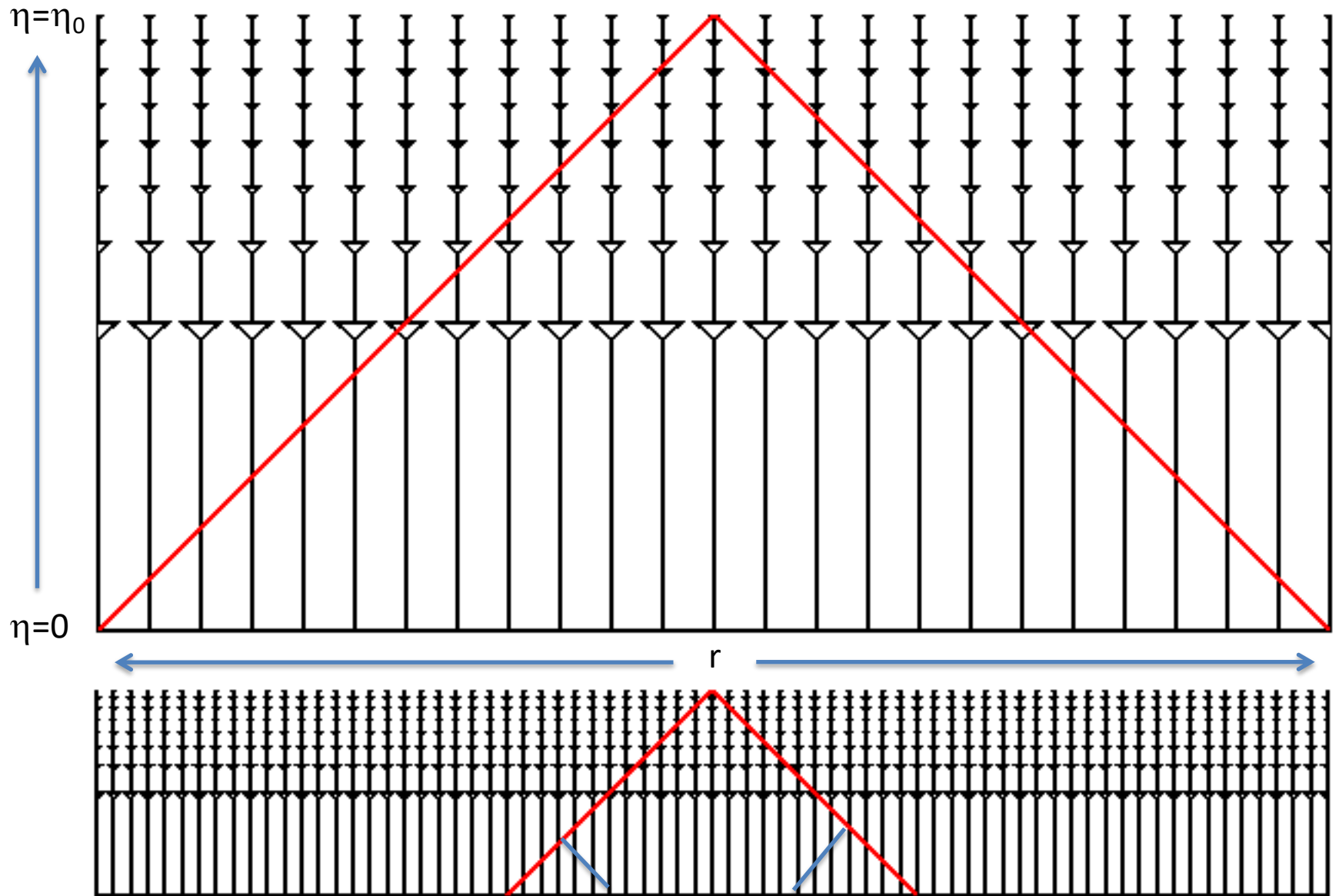


Problem #2: Horizon



Conformal Time

$$d\eta = dt / a(t)$$



Horizons

What is the ***maximum*** comoving distance light can travel?

$$d_{hor}(t_0) = \int_0^{t_0} \frac{c}{a(t)} dt$$

Can integrate for any given $a(t)$.

This is called the **particle horizon**.

It's the farthest signals can travel since $t=0$.

Defines the **observable Universe**.

Distances in Rad Dom

$$a(t) = (t/t_0)^{1/2}$$

$$\begin{aligned} d_p(t_0) &= \int_{t_e}^{t_0} \frac{cdt}{a(t)} \\ &= c \int_{t_e}^{t_0} \frac{dt}{(t/t_0)^{1/2}} \\ &= \frac{c}{H_0} \frac{(1+z)^2 - 1}{(1+z)^2} \end{aligned}$$

Horizon at $d_{\text{hor}}(t_0) = c/H_0$

Distances in EdS

$$a(t) = (t/t_0)^{2/3}$$

$$\begin{aligned} d_p(t_0) &= \int_{t_e}^{t_0} \frac{cdt}{a(t)} \\ &= c \int_{t_e}^{t_0} \frac{dt}{(t/t_0)^{2/3}} \\ &= \frac{2c}{H_0} \left[1 - \frac{1}{\sqrt{1+z}} \right] \end{aligned}$$

Horizon at $d_{\text{hor}}(t_0) = 2c/H_0$

Problem #3: Monopoles

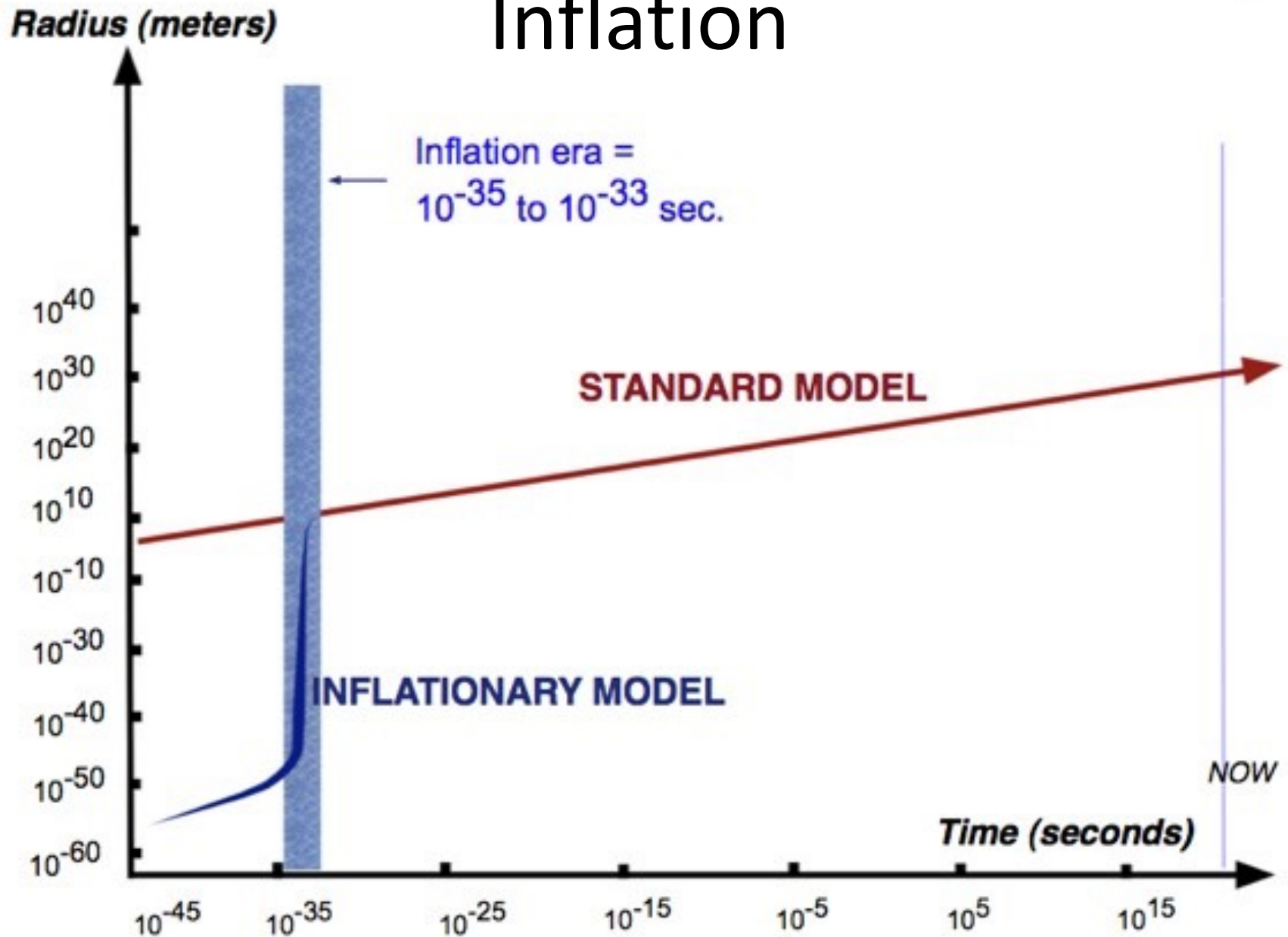
Separation of strong and electro-weak forces requires spontaneous symmetry breaking.

We expect topological defects at borders of causally disconnected regions, $r \approx ct_{\text{GUT}}$

Magnetic monopoles and cosmic strings are examples of defects.
 $n_{\text{MM}} \approx 10^{82} \text{ m}^{-3}$ at the moment of symmetry breaking.

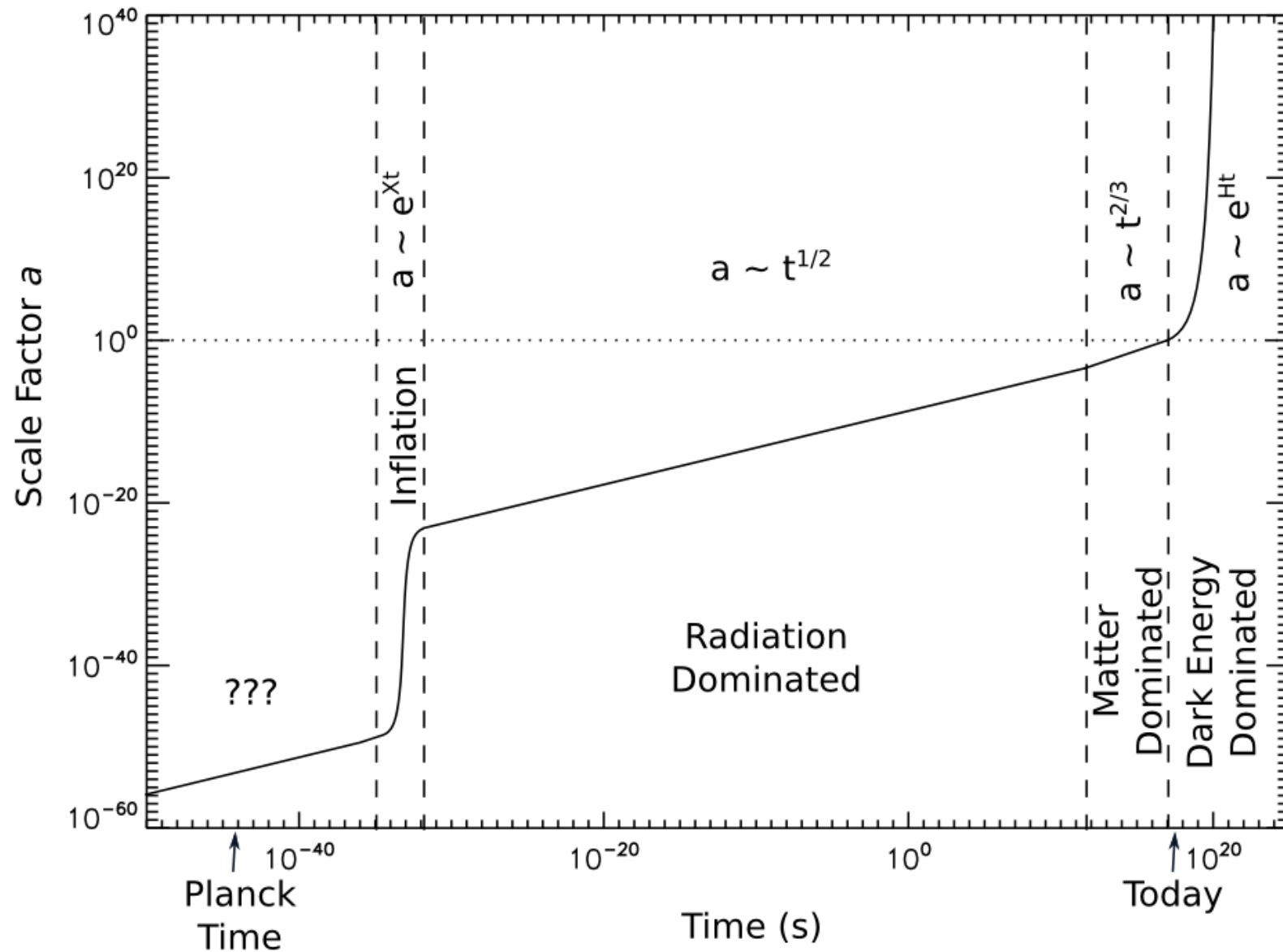
They would quickly dominate the density of the Universe.
(Observation: there aren't any.)

Inflation



Observed flatness requires expansion by ≈ 60 e-foldings.

Scale vs Time



So how do you inflate a Universe?

We've seen one candidate: **Dark Energy**

$$\frac{H^2}{H_0^2} = \cancel{\Omega_R a^{-4}} + \cancel{\Omega_M a^{-3}} + \cancel{\Omega_k a^{-2}} + \Omega_\Lambda \Rightarrow a(t) = C e^{H_0 t}$$

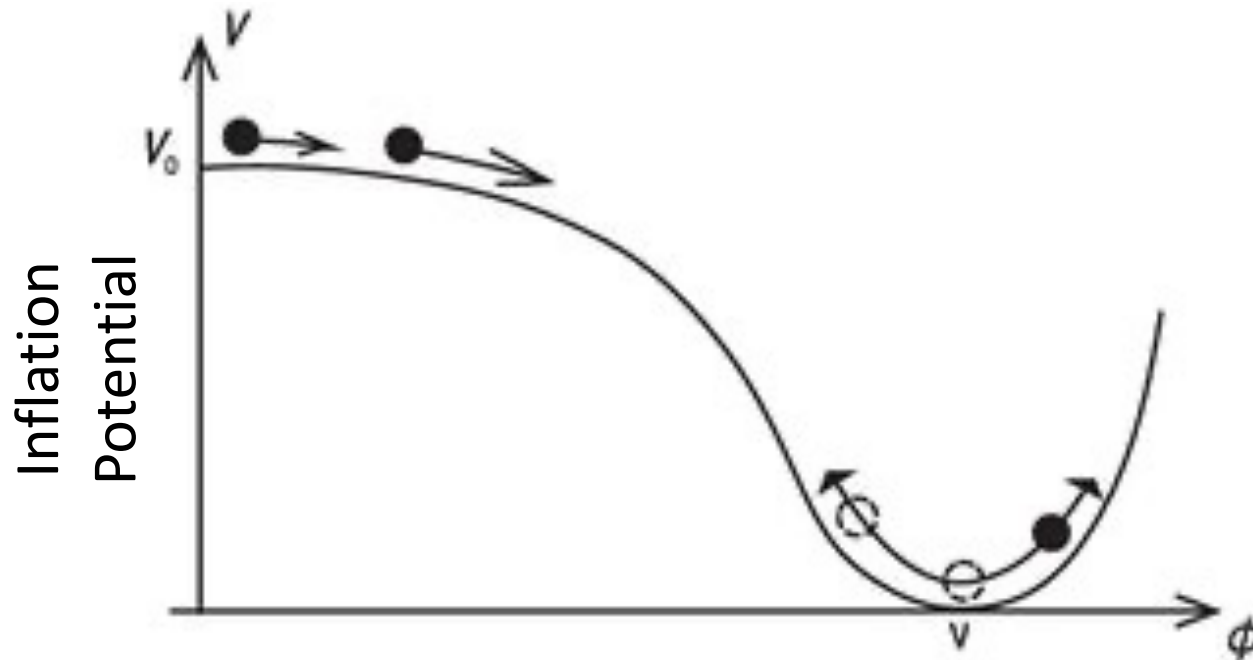
Remember: DE behaves like a vacuum energy density.

To inflate a Universe, we need a vacuum with some energy density built in!

How do you stop it?

Λ -driven accelerated expansion will continue indefinitely.
How do we stop inflation from doing the same thing?

Make the energy density of the vacuum move to zero at some point.

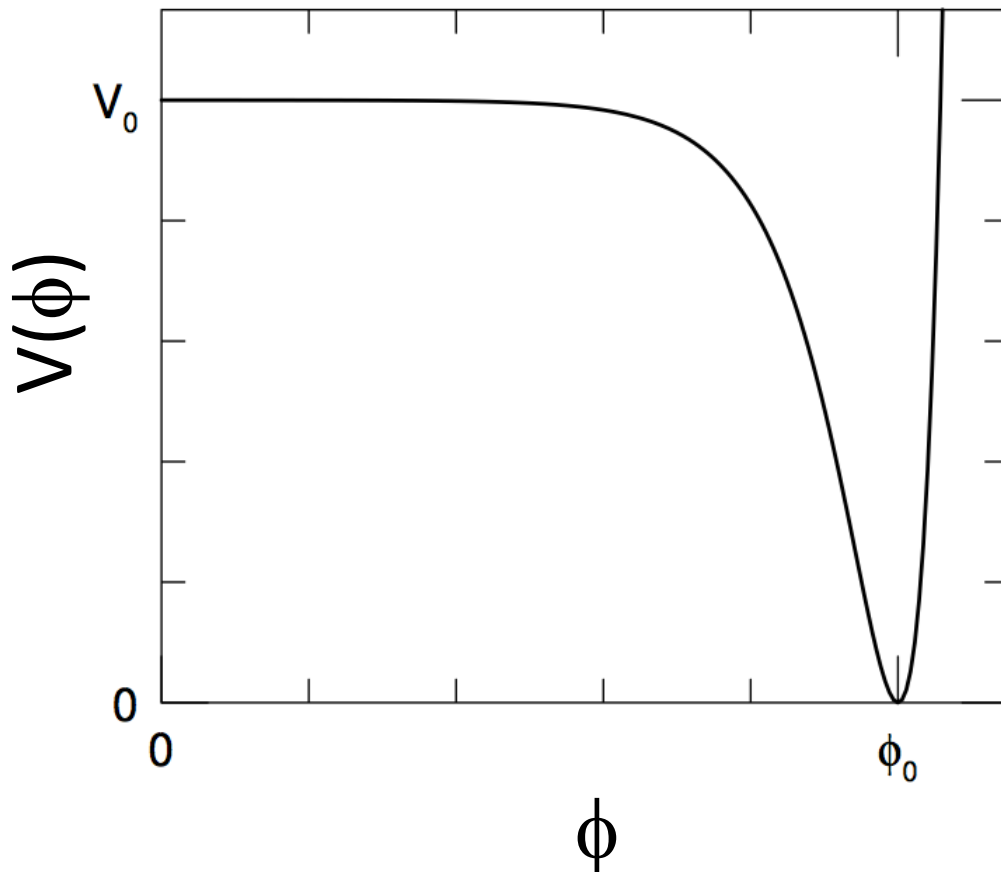


Single-field slow-roll Inflation

How do you Inflate a Universe?

A Time-evolving Scalar field!

$$a(t) \propto \begin{cases} \sinh Ht & (k = -1) \\ \cosh Ht & (k = +1) \\ \exp Ht & (k = 0), \end{cases}$$



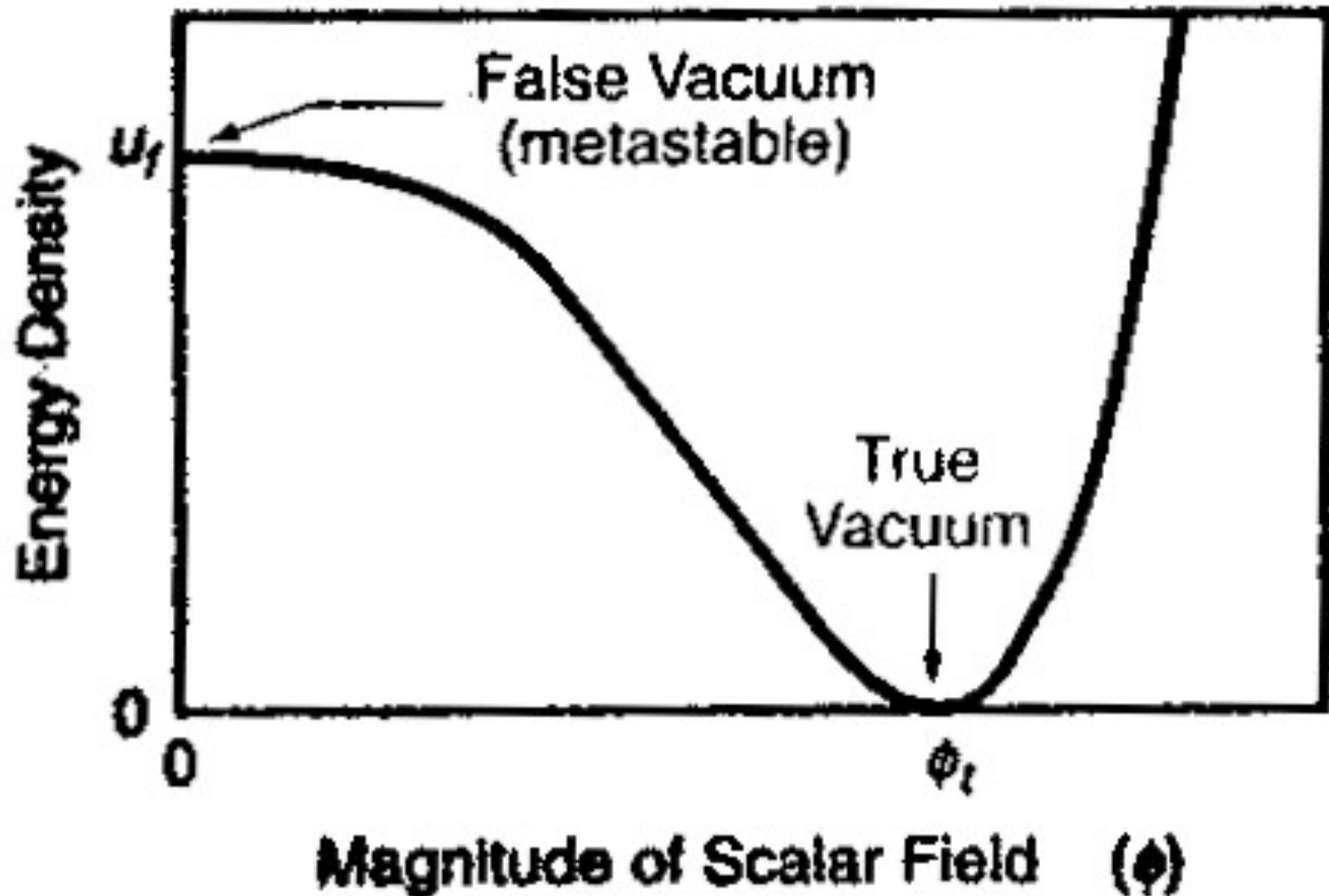
$$\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

2nd Friedmann eq:

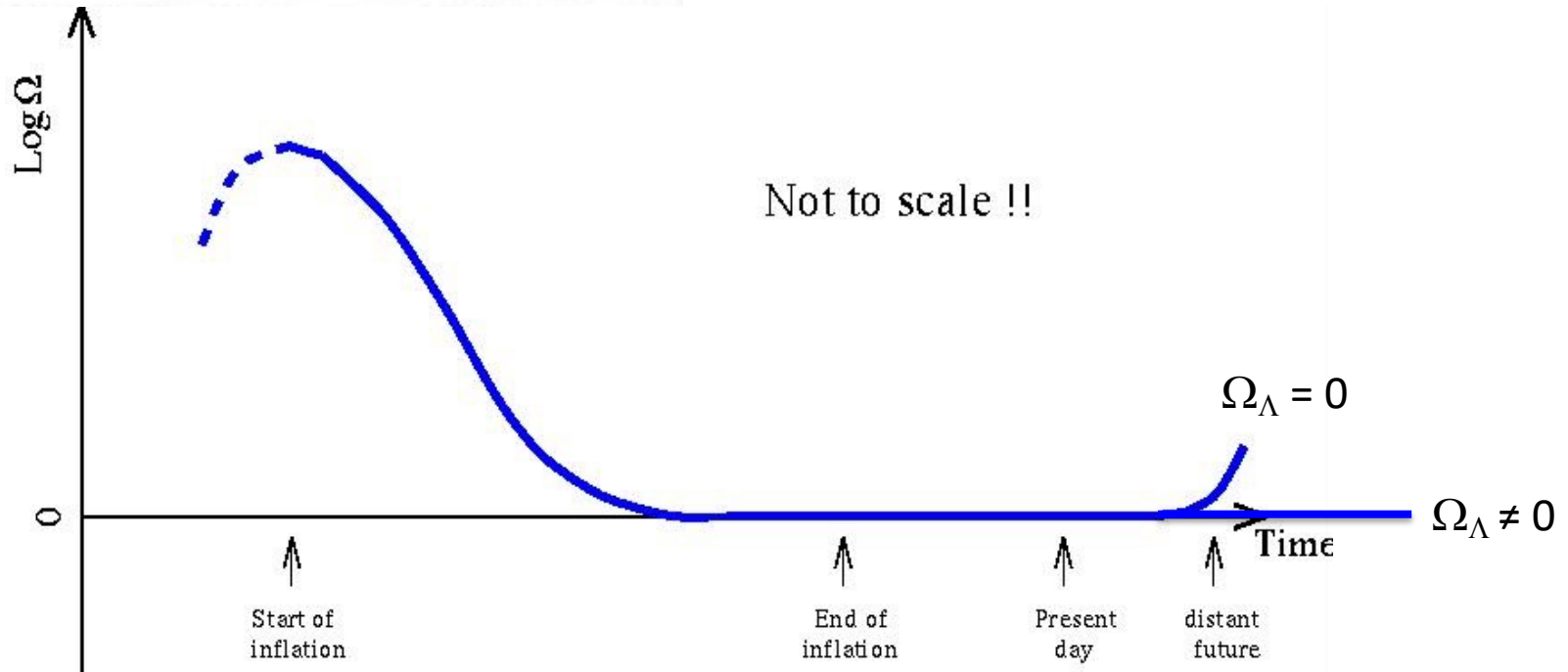
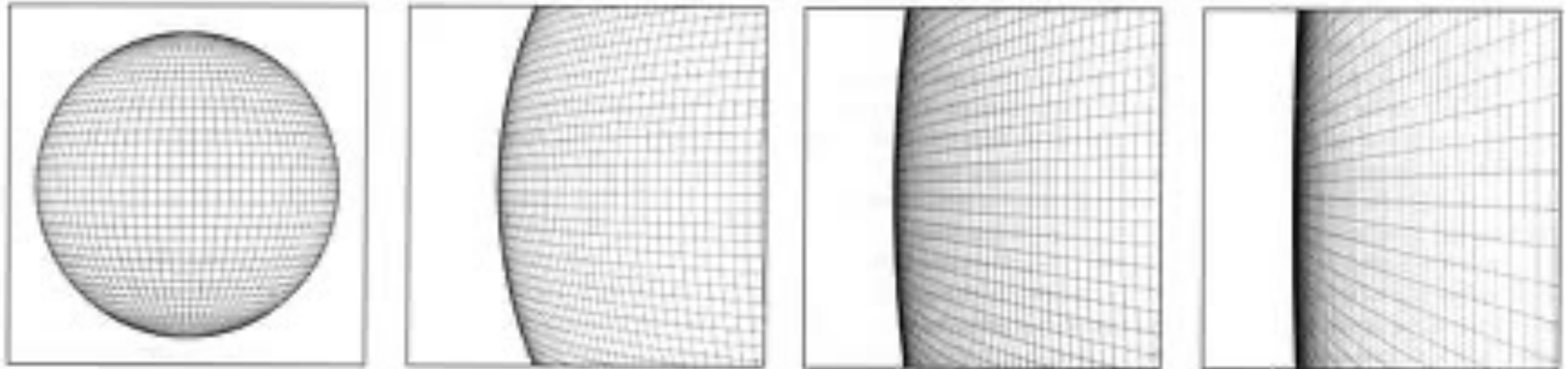
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0$$

True and False Vacua



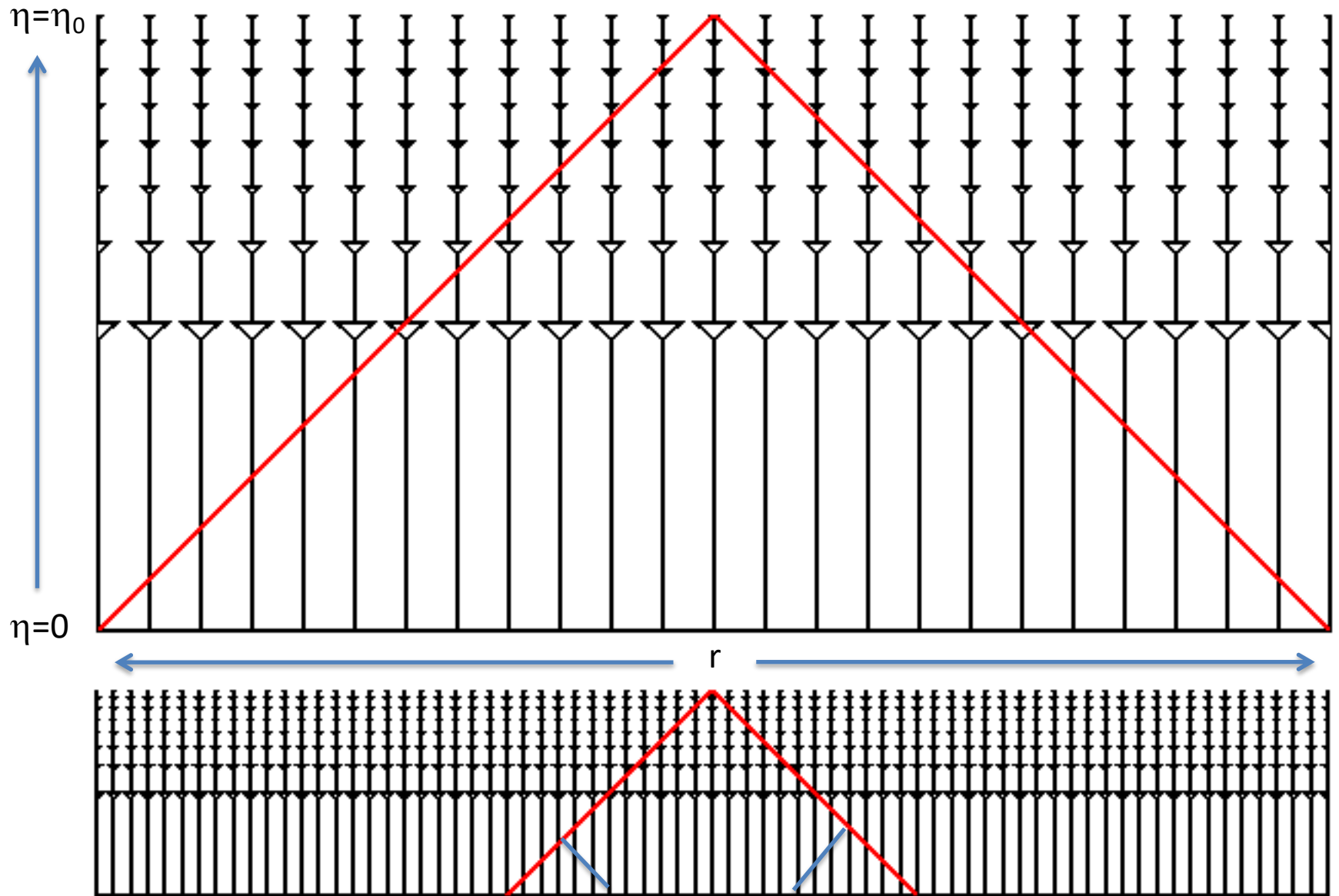
Flatness: Fixed

In a cosmological-constant-dominated Universe, $\rho_{\kappa} / \rho_{\Lambda} \rightarrow 0$



Conformal Time

$$d\eta = dt / a(t)$$



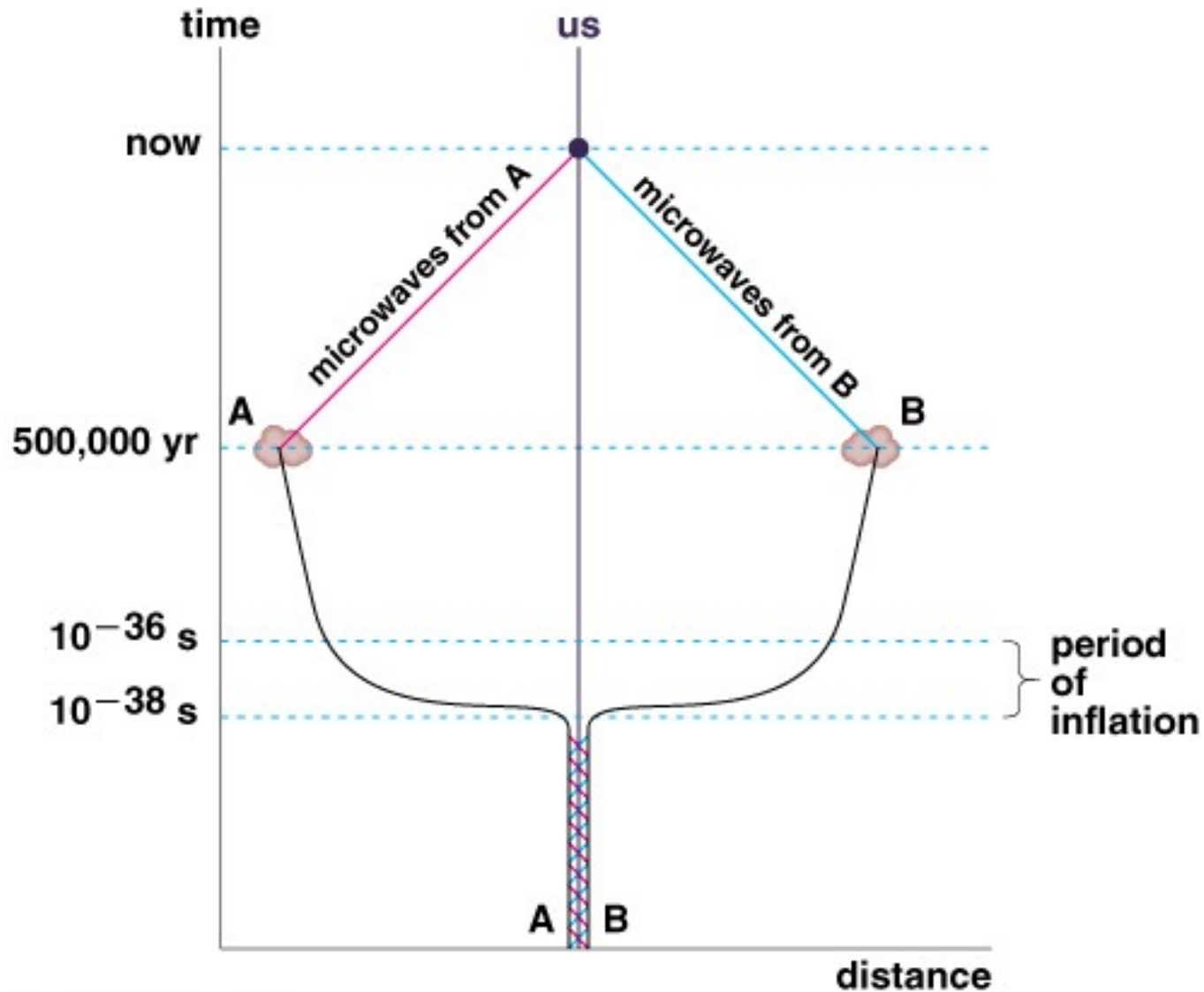
Distances in de Sitter

$$a(t) = e^{H_0(t-t_0)}$$

$$\begin{aligned} d_p(t_0) &= \int_{t_e}^{t_0} \frac{cdt}{a(t)} \\ &= \frac{c}{H_0} \left[e^{H_0(t_0-t_e)} - 1 \right] \\ &= cz/H_0 \end{aligned}$$

No particle horizon!

Horizon: Fixed



Monopoles: Fixed

Diluted enormously \rightarrow expect $\ll 1$ in observable Universe

This sets a timing requirement on inflation!
It must be active *after* $T < T_{\text{GUT}}$ to get rid of the monopoles.

$$t_{\text{inflation}} \approx 10^{-34} - 10^{-32} \text{ seconds}$$

Why didn't everything redshift away?

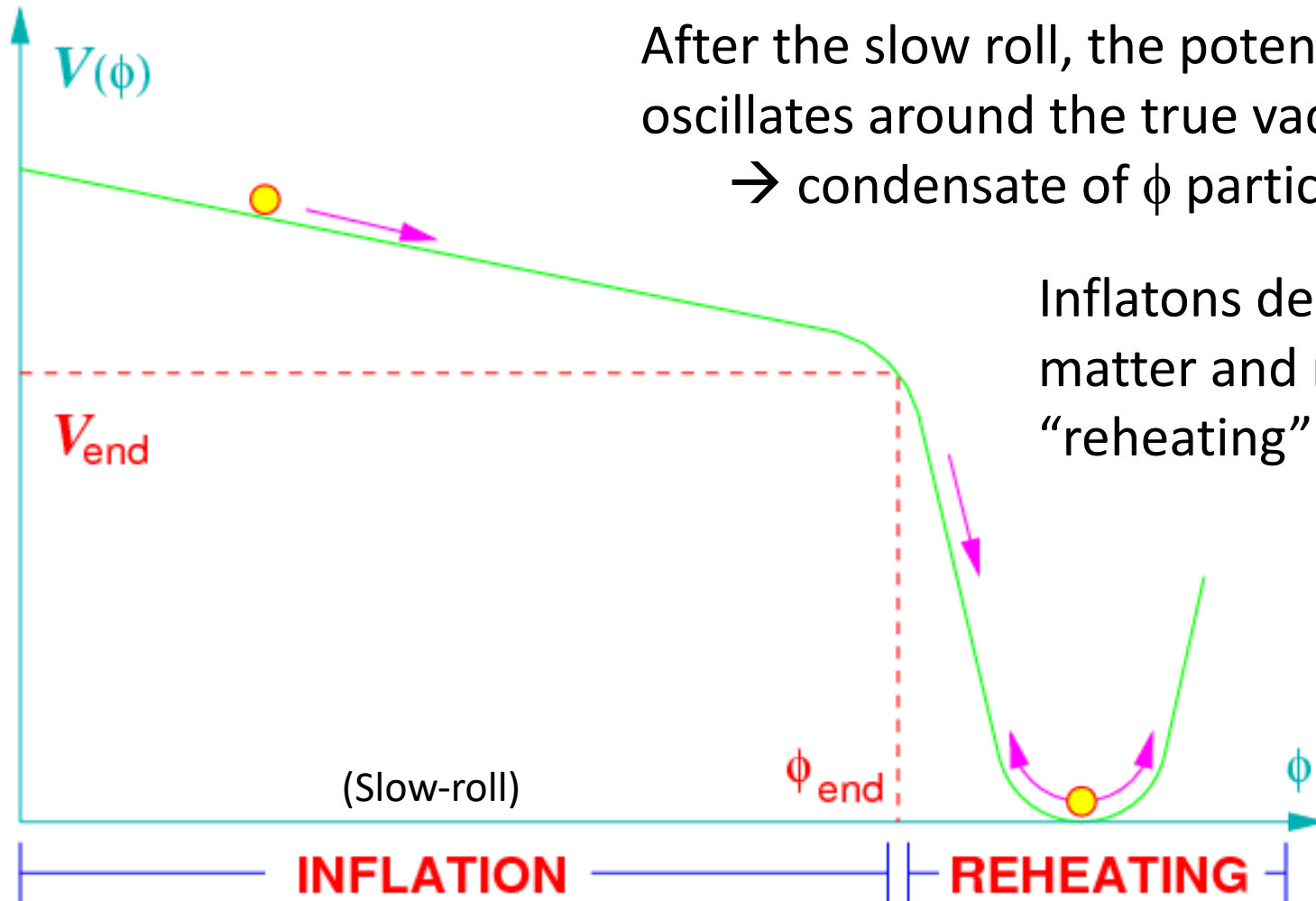
Trick question: It did.

Inflation creates a Universe with
nothing but the inflation field in it.

Reheating

$$T_{\text{inflation}} \approx 10^{14} \text{ GeV} \rightarrow 1\text{K}$$

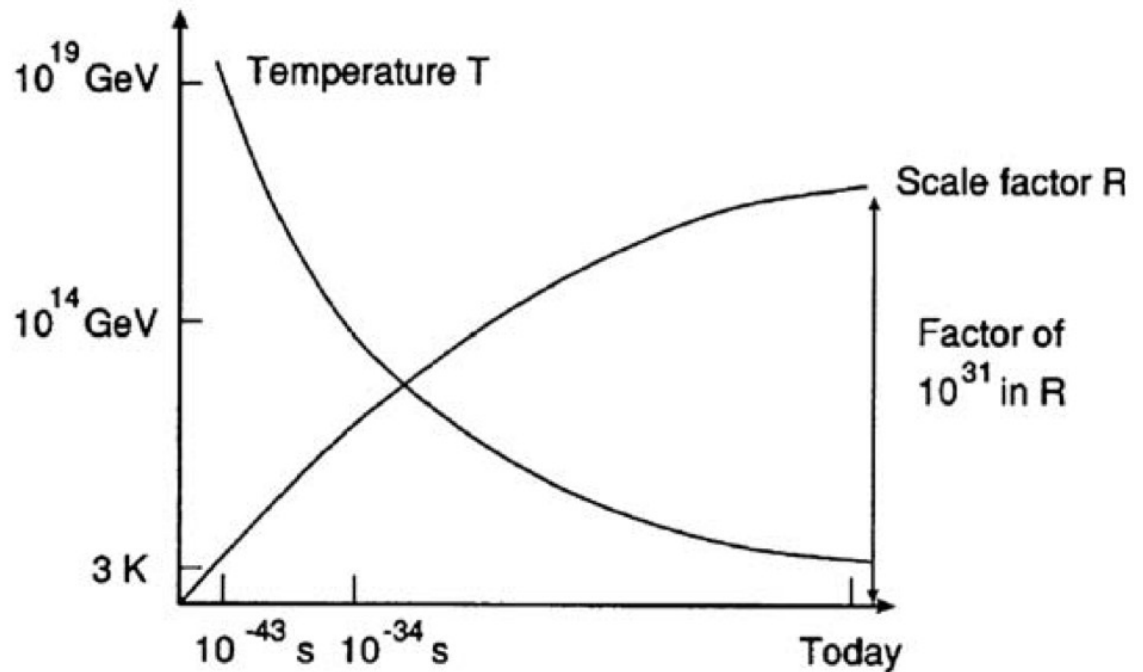
And if everything redshifted away, why is there anything?



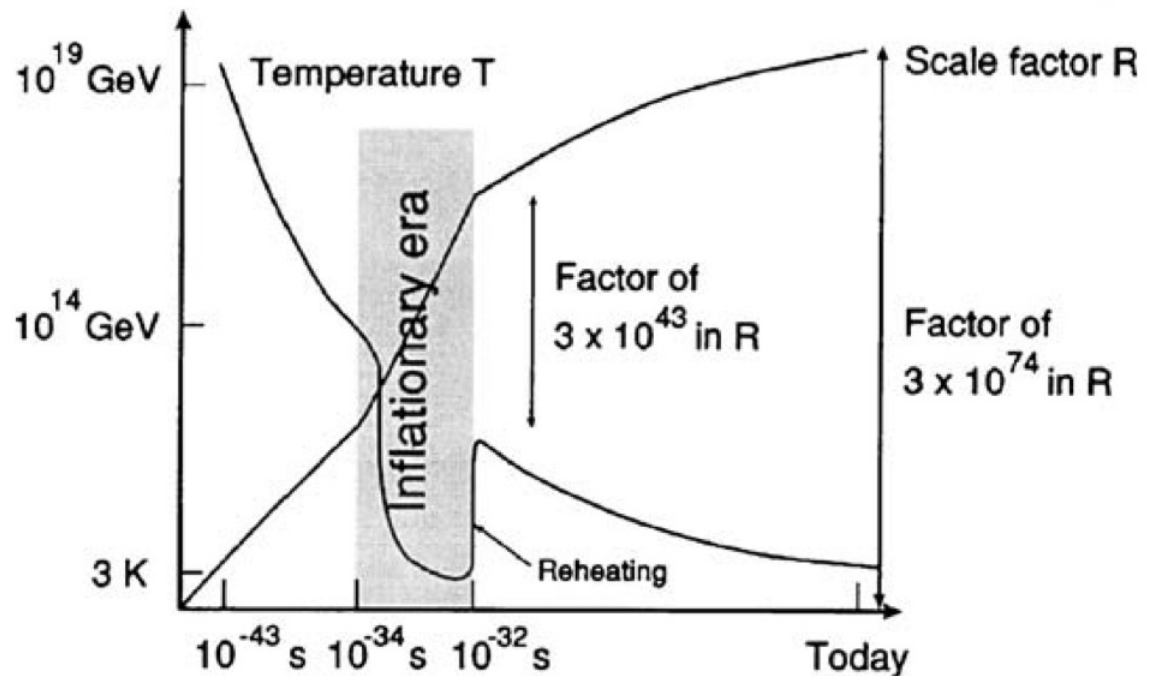
After the slow roll, the potential oscillates around the true vacuum state
→ condensate of ϕ particles, “inflaton”

Inflatons decay, producing matter and radiation, “reheating” the Universe.

Without Inflation



With Inflation



Baryogenesis

Reheating gives a convenient moment to invoke baryogenesis.

Sakharov's Rules describe the requirements:

- Baryon Number must be violated
- C [charge] and CP [charge/parity] must be violated
- Asymmetry must happen under non-equilibrium conditions

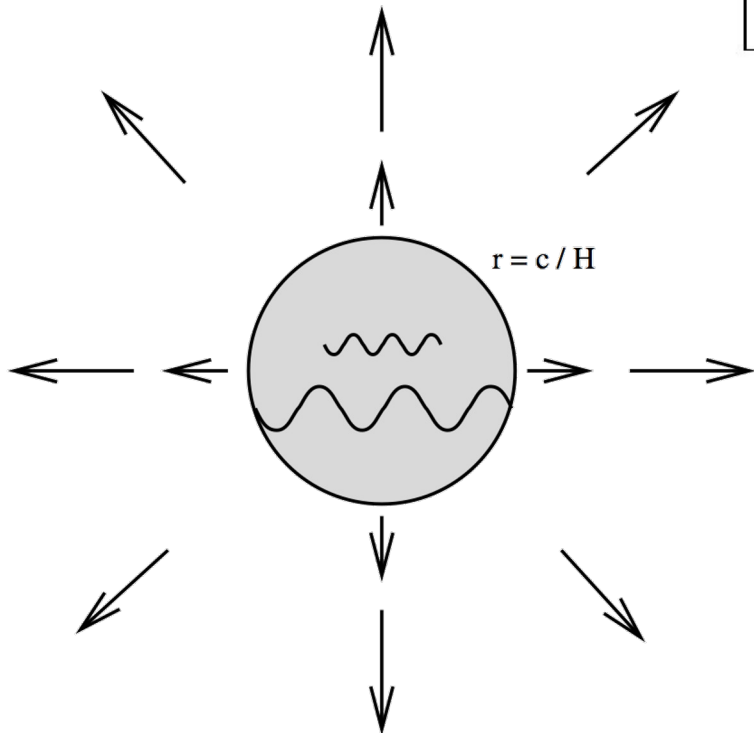
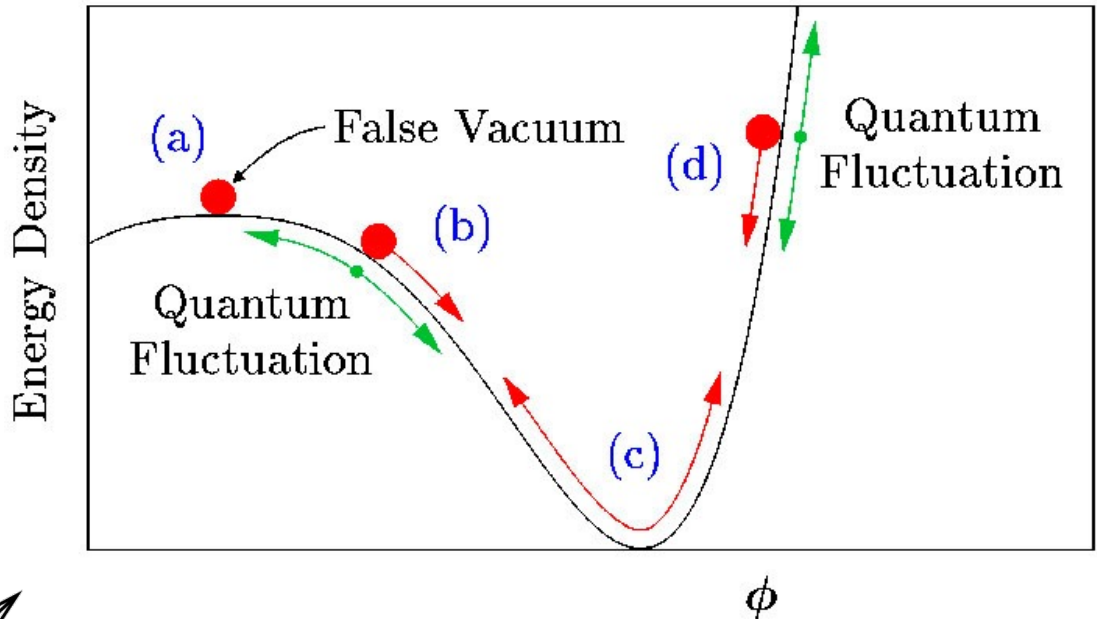
How this actually plays out is still unclear...

- CP symmetry is violated in electroweak interactions
- B non-conservation has never been observed
- Very short window for non-equilibrium conditions...

Structure

Heisenberg:

$$\Delta E \Delta t \geq \hbar/2$$



- During inflation, ϕ field varies randomly on short timescales.
- These scalar perturbations produce density variations.
- These variations get dragged along with inflating space.
- Once outside causal contact, they become frozen in.

Other Inflations

We've been discussing only one model,

→ Single-field, slow-roll inflation.

Many others work just as well.

- multiple field
- Chaotic inflation
- etc...

A Big Reset Button for the Cosmos

Summary:

Inflation takes an arbitrarily messy Universe, and

- 1) Puts the observable part in causal contact.
- 2) Empties it of all the junk.
- 3) Smooths it out nicely.
- 4) Fills it back up with stuff we know & love.

Good enough to make it widely accepted,
but it's all post-hoc. (For now...)

Are we Inflating again?

The energy density of the inflation field is the same as the energy density of the early Universe – very very high!

DE / Λ is much much weaker than the inflationary field.
 $10^{27} = 1,000,000,000,000,000,000,000,000,000,000$ x weaker.

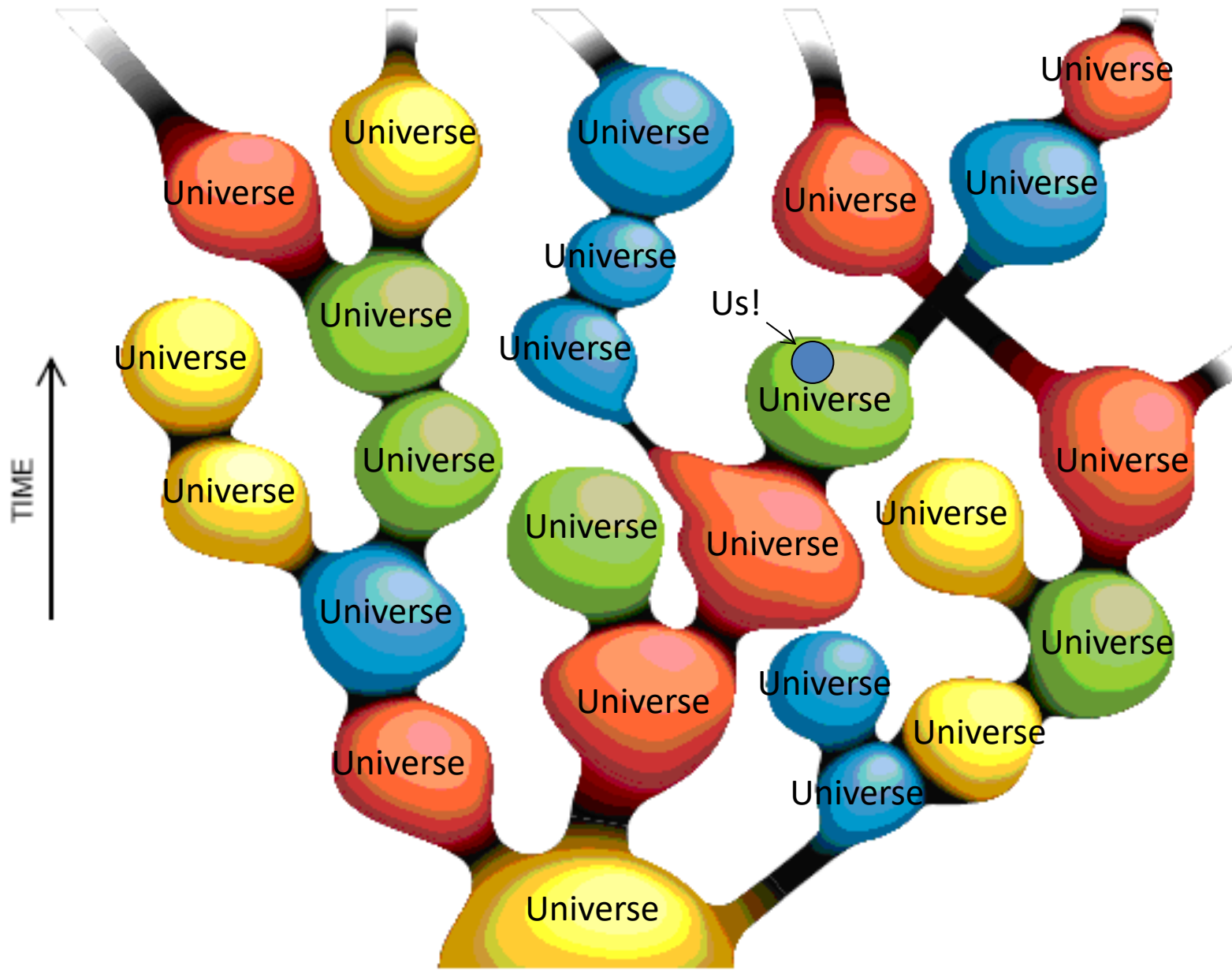
If DE inflates the Universe, redshifts everything away, flattens things out, rips causally connected stuff apart superluminally, then decays, it will only reheat the Universe very weakly.

The post-DE Universe will not contain any of the particles we're familiar with. Probably none of the forces either.

If we're in another inflation, it's to a **very** different place.

Warning: Wild Speculation!

Does this keep happening?





INTERMISSION



Perturbations

Consider a small perturbation to the density field,
with Density Contrast $\Delta = \rho(x) / \langle \rho \rangle - 1$.

For example:

Galaxies $\rightarrow \Delta \approx 10^6$

Galaxy Clusters $\rightarrow \Delta \approx 10^3$

Superclusters $\rightarrow \Delta \approx 10^0$

If Virialized / Bound:

$\Delta \approx 1$ at $z \approx (10^6)^{1/3} \approx 100$

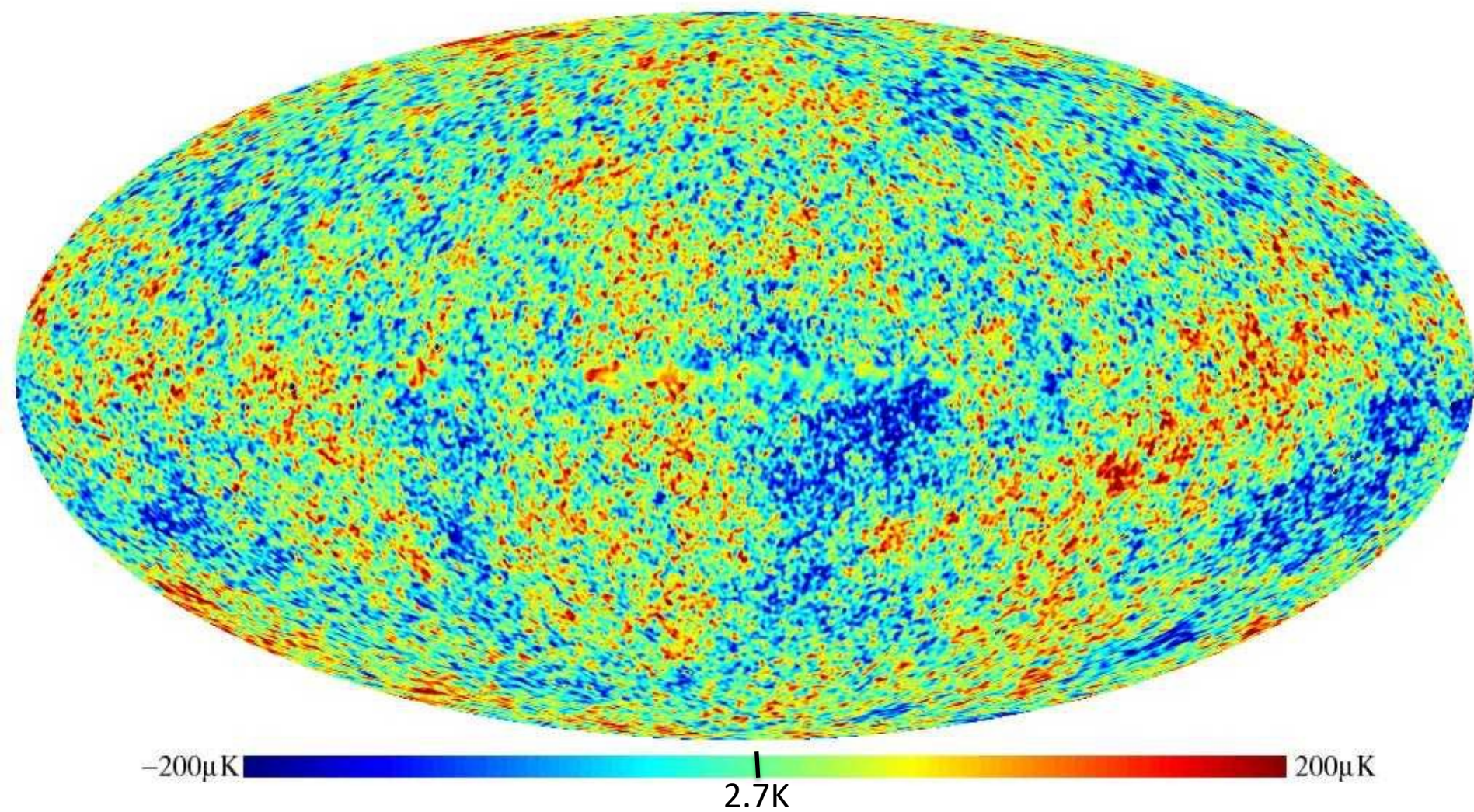
$\Delta \approx 1$ at $z \approx (10^3)^{1/3} \approx 10$

$\Delta \approx 1$ at $z \approx (10^0)^{1/3} \approx 1$

Bound structures cannot have separated before these redshifts.
(Or they would be much more dense today.)

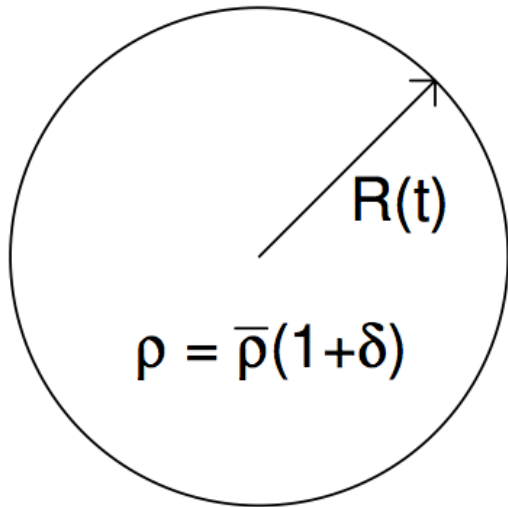
In the early Universe, $\Delta \ll 1$, linear approximations are adequate.

Looking at $z \approx 1100$



At recombination, $\Delta = \delta\rho / \rho = dT / T \approx 300\mu\text{K} / 3\text{K} \approx 10^{-4}$

Gravitational Instability



$$\ddot{R} = -\frac{G(\Delta M)}{R^2} = -\frac{G}{R^2} \left(\frac{4\pi}{3} R^3 \bar{\rho} \delta \right)$$

$$\rightarrow \frac{\ddot{R}}{R} = -\frac{4\pi G \bar{\rho}}{3} \delta(t) \approx -\underbrace{\frac{1}{3} \ddot{\delta}}_{\text{mass conservation}}$$

mass conservation

$$\delta(t) = A_1 e^{t/t_{\text{dyn}}} + A_2 e^{-t/t_{\text{dyn}}}$$

$$t_{\text{dyn}} = \frac{1}{(4\pi G \bar{\rho})^{1/2}}$$

More commonly, $\Delta(t)$

Next Time:

Structure