
AST 1430
Cosmology

Week Index	Dates	Topics
Week 1	Jan 11, 13	logistics, introduction, basic observations
Week 2	Jan 18, 20	Basic GR, RW metric, Distances, coordinates, Friedmann equations, cosmological models
Week 3	Jan 25, 27	Consistency with observations, Early Hot Universe, BBN
Week 4	Feb 1, 3	Inflation, Perturbations & Structure pre-recombination
Week 5	Feb 8, 10	CMB: basics, polarization, secondaries
Week 6	Feb 15 (2h)	Early-Universe Presentations
Reading Week – No Class		
Week 7	Mar 1, 3	Post-recombination growth of structure, formation of dark matter halos, halo mass function
Week 8	Mar 8, 10	The relation between dark matter halos and galaxies
Week 9	Mar 15, 17	Probing the cosmic density field / clustering
Week 10	Mar 22, 24	Late Universe Presentations; lcosmological observations: BAO, supernovae, weak lensing, etc.
Week 11	Mar 29, 31	H0 controversy: how fast exactly is the Universe expanding today?
Week 12	Apr 5	Review

Keith

Jo

asst1

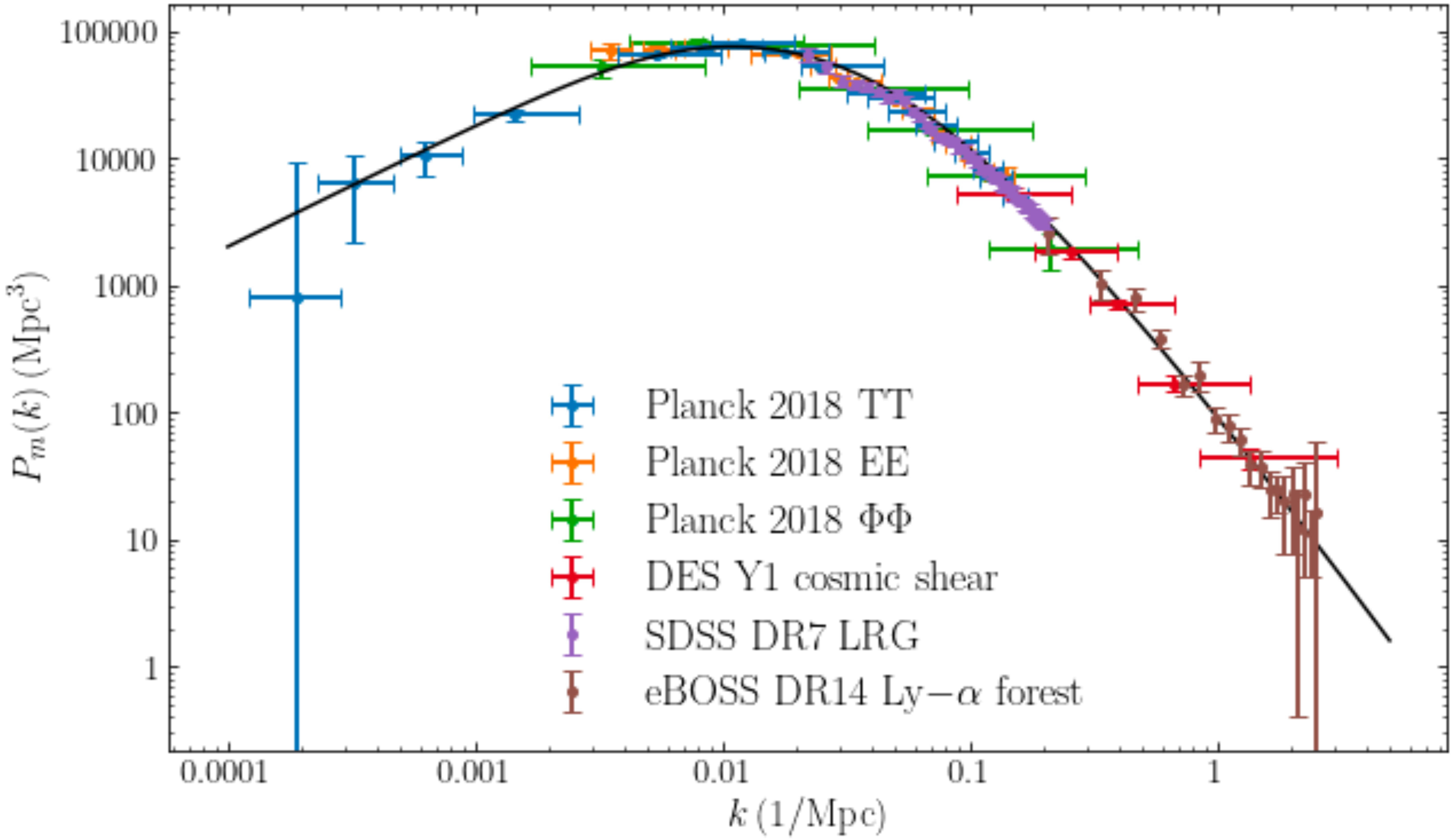
asst2

asst3

Overview

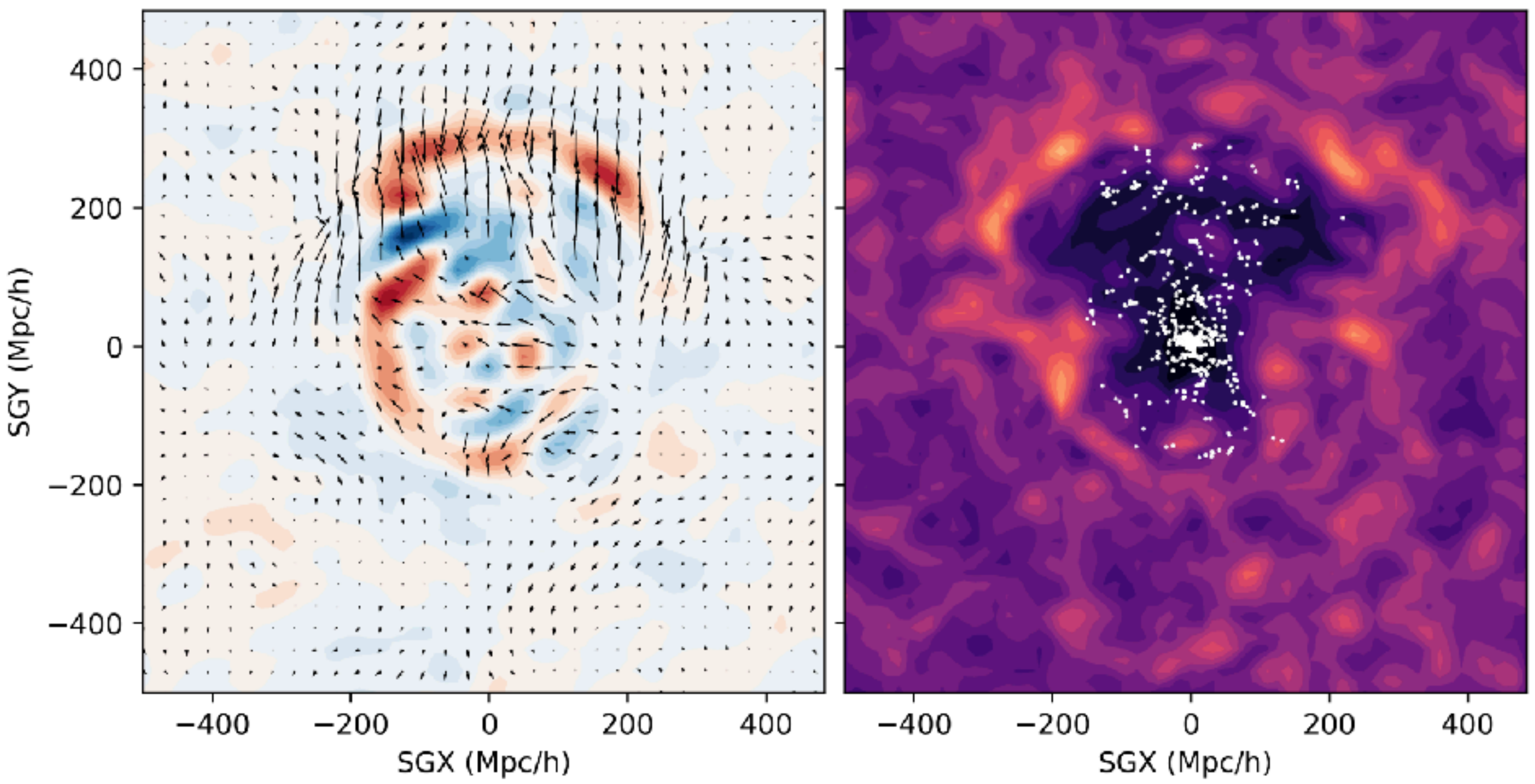
- Why probing the density field is important
- Non-linear evolution of the dark-matter field
- Bias of halos
- Galaxy clustering and bias
- Redshift space distortions
- BAO

Probing the cosmic density field



Probing the cosmic density field

- The distribution of matter on large scales in the low-redshift Universe contains much information about the Universe we live in
- We can probe this structure in different ways:
    - Galaxy clustering
    - Galaxy peculiar velocities
    - Gas correlation functions
    - Cross-correlations
    - Weak lensing
    - …



Courtois et al. (2023)

The two-point correlation function

- As we saw in previous weeks, power spectrum P(**k**,z) is the natural theoretical quantity describing the statistics of the density distribution
- However, we live in real space! In real-space, the natural description of an isotropic, homogeneous density distribution is through its *two-point correlation function*

$$\xi\ (|\mathbf{x}_1 - \mathbf{x}_2|) = \langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle,$$

- For example, for galaxies, this function gives the probability of finding a galaxy at a distance of $r = |\mathbf{x}_1 - \mathbf{x}_2|$ from another galaxy
- Correlation function is directly related to the power spectrum

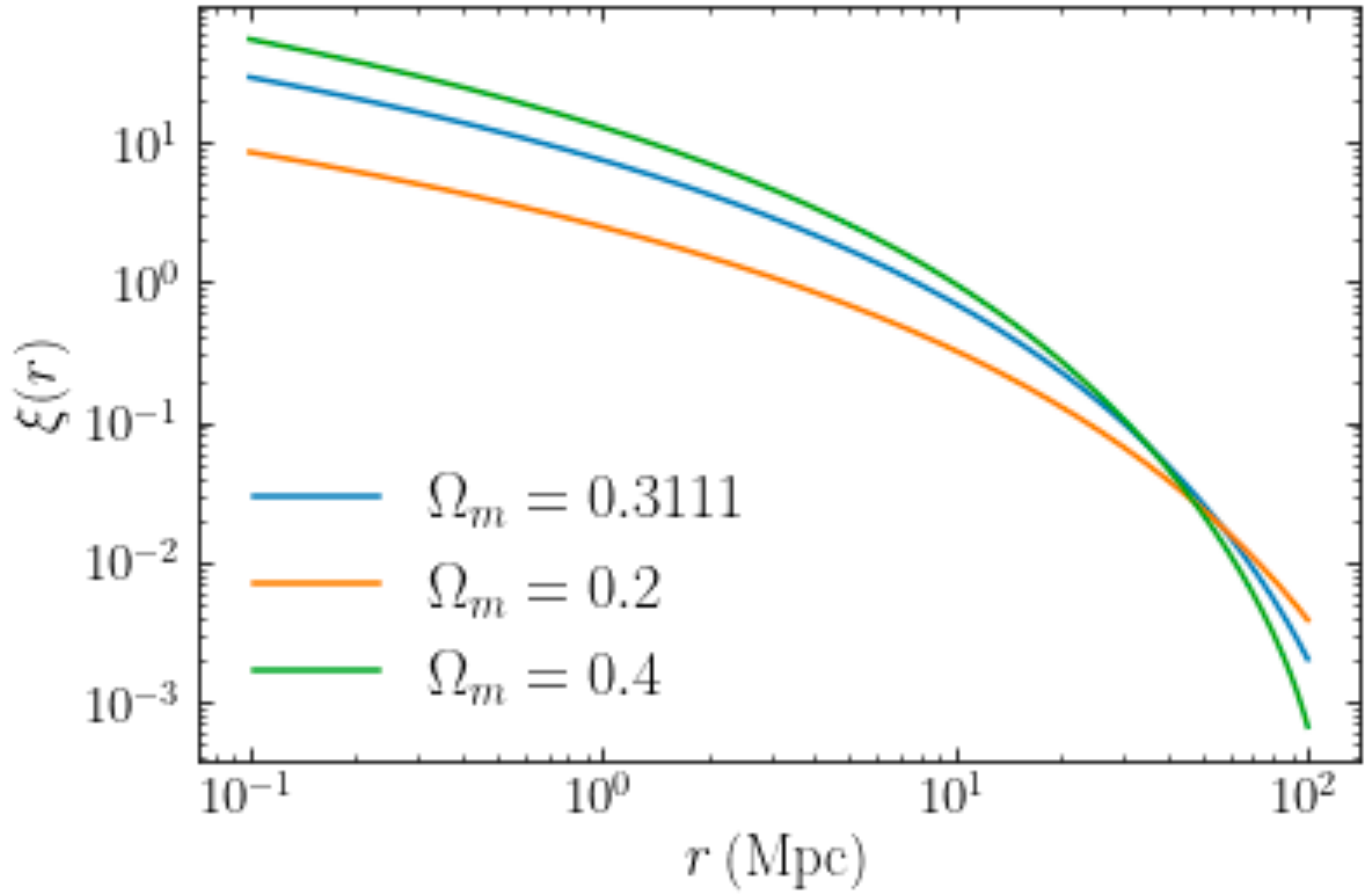
$$\xi(r) = \frac{1}{2\pi^2} \int dk\, k^2 P(k)\, \frac{\sin(kr)}{kr}$$

The two-point correlation function

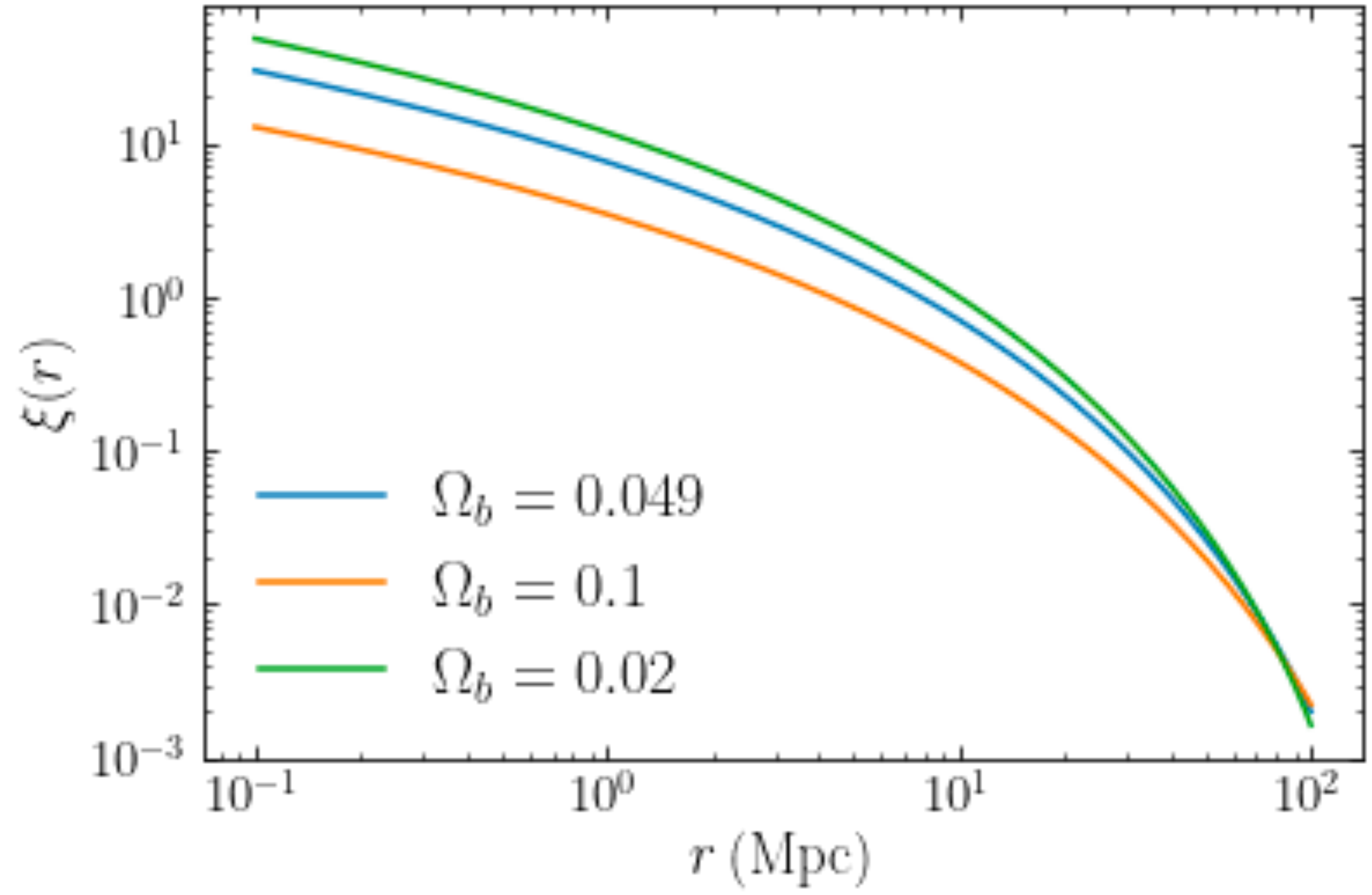
$$\xi\ (|\mathbf{x}_1 - \mathbf{x}_2|) = \langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle, \qquad \xi(r) = \frac{1}{2\pi^2}\int dk\, k^2 P(k)\, \frac{\sin(kr)}{kr}$$

- Like the power spectrum, for an isotropic/homogeneous density distribution, the two-point correlation function only depends on the distance $r = |\mathbf{x}_1 - \mathbf{x}_2|$
- Like for the power spectrum, for a Gaussian field, higher-order correlation functions are given directly in terms of the two-point correlation function
- Thus, the two-point correlation function is a full statistical description of the linear density field in positional space

The two-point correlation function



The two-point correlation function



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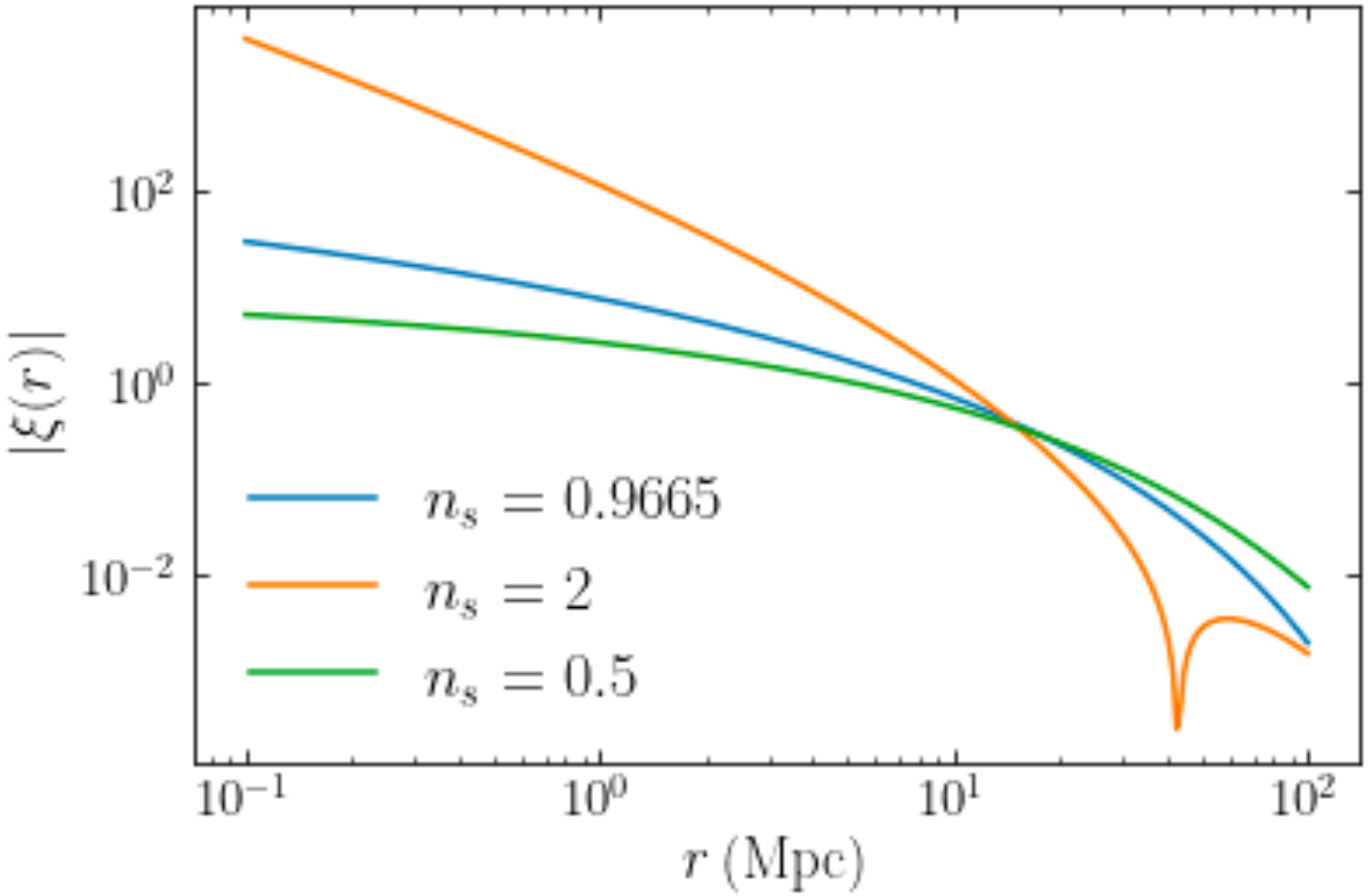

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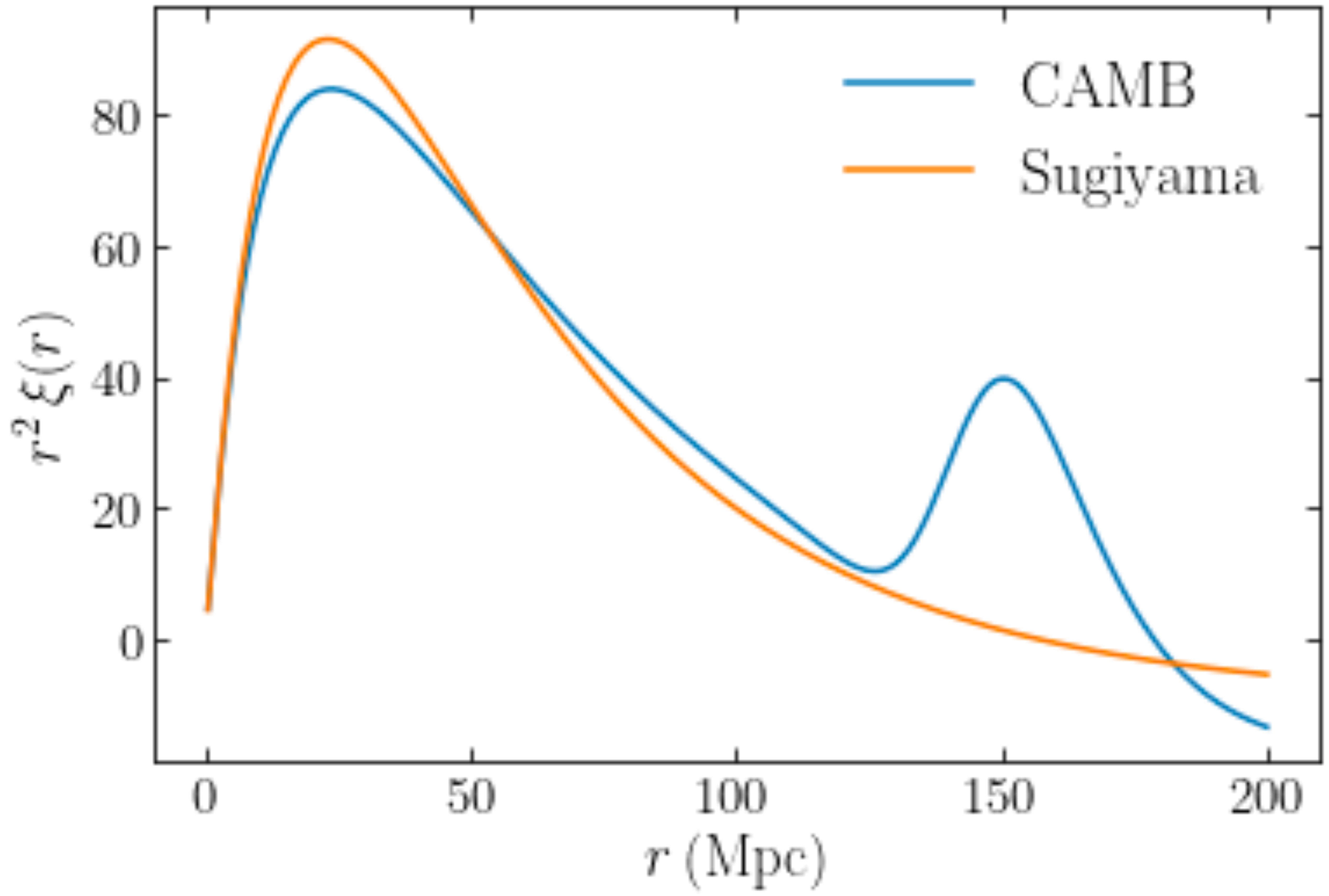

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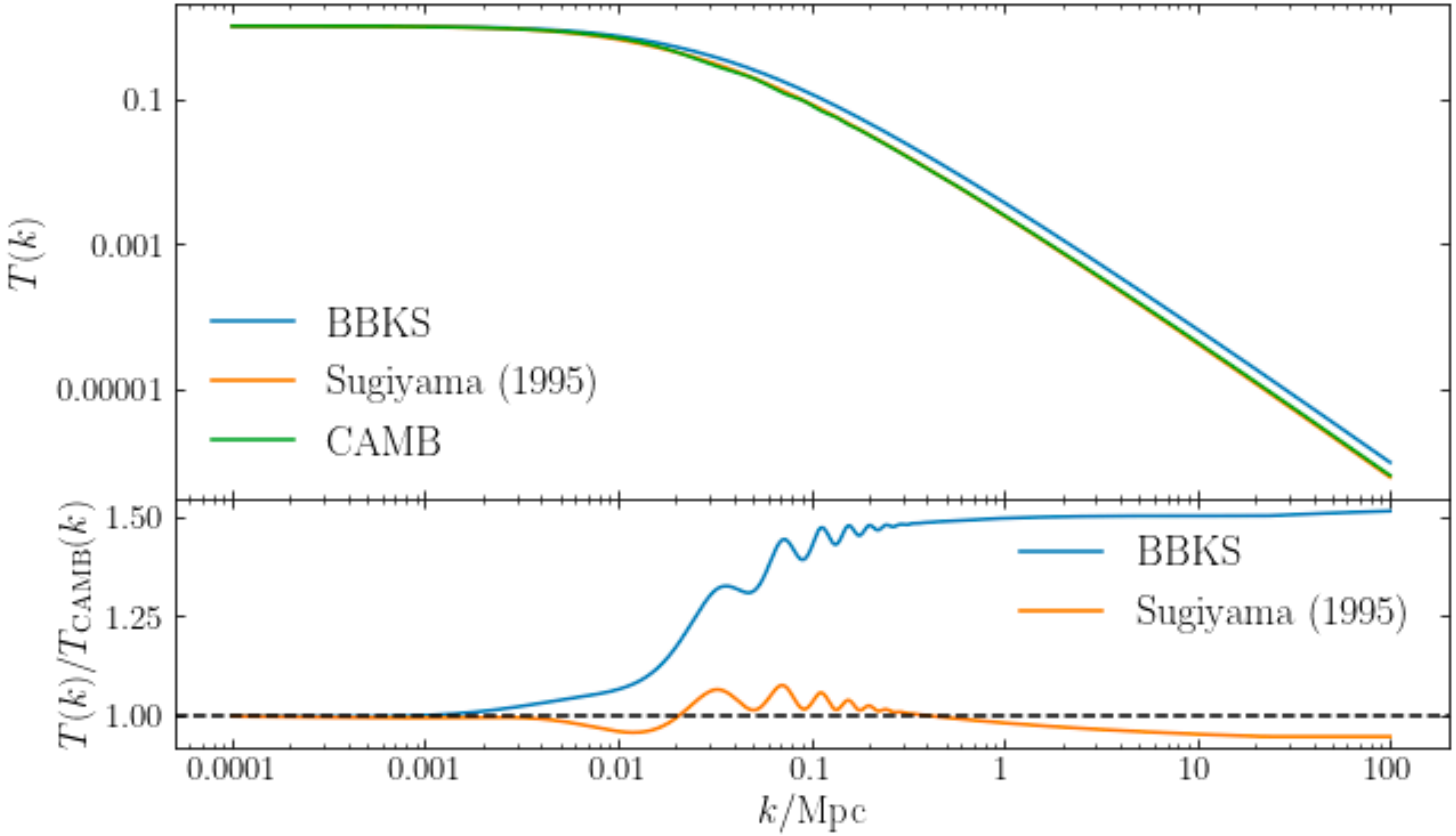
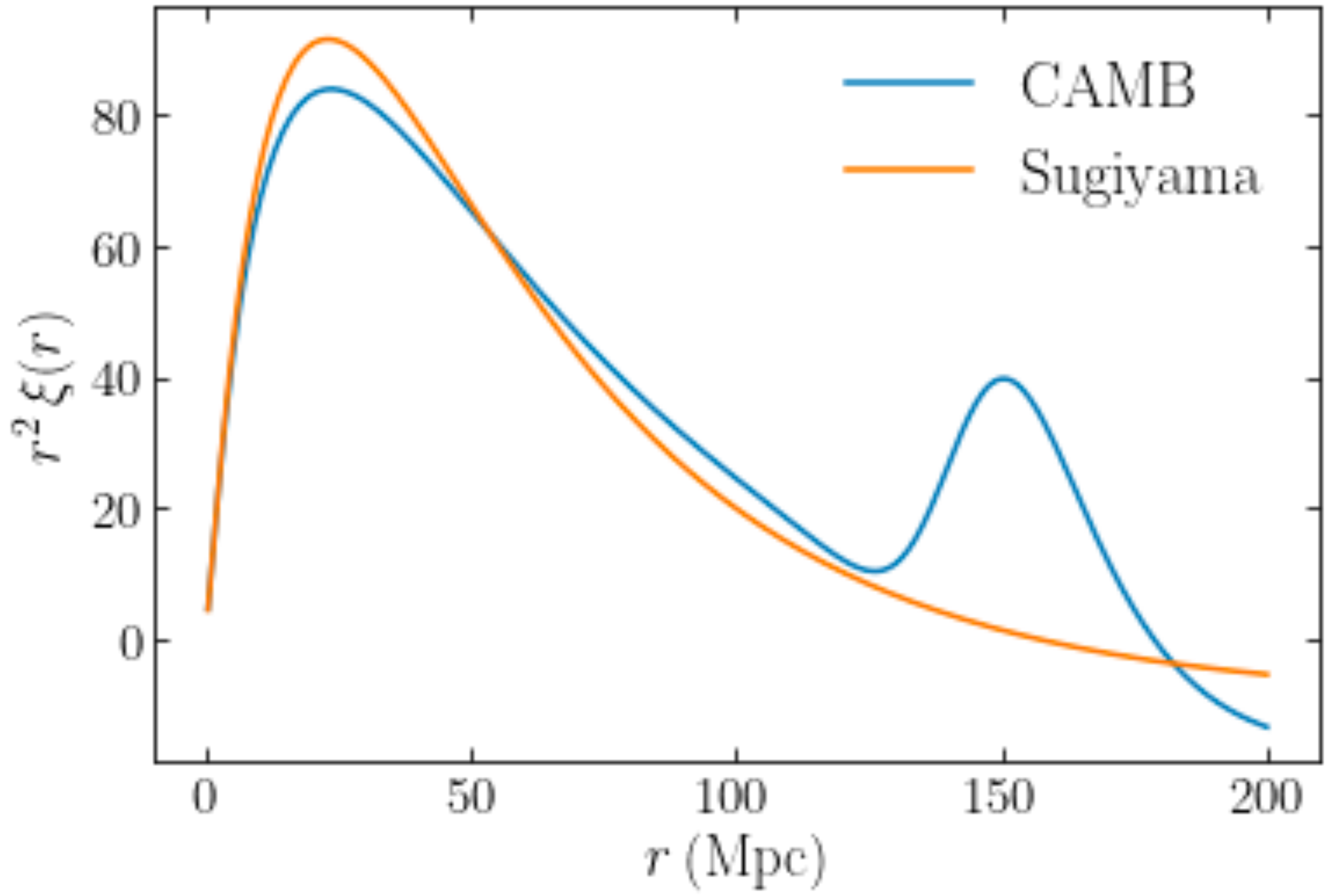


The two-point correlation function



Doing cosmology with the two-point correlation function

- Three big issues:
    - Non-linear evolution of the DM, especially on smaller scales (halos form)
    - Galaxy formation when using the galaxy correlation function as a proxy
    - Redshift distortions: we don't observe distance, but redshift instead
- Bias: relate observed density field back to the linear density field:

  $\delta(k) = b(k)\ \delta_{lin}(k)$

    - Non-linear evolution produces bias
    - Distribution of DM halos is biased representation of density field
    - Galaxies live in DM halos —> different types of galaxies ~ different DM halos —> different bias

Non-linear evolution of the matter power spectrum

Non-linear evolution of the matter power spectrum

Perturbation theory

- Non-linear perturbation theory: compute second, third, … order density solution

- Works as long as δ <~ 1

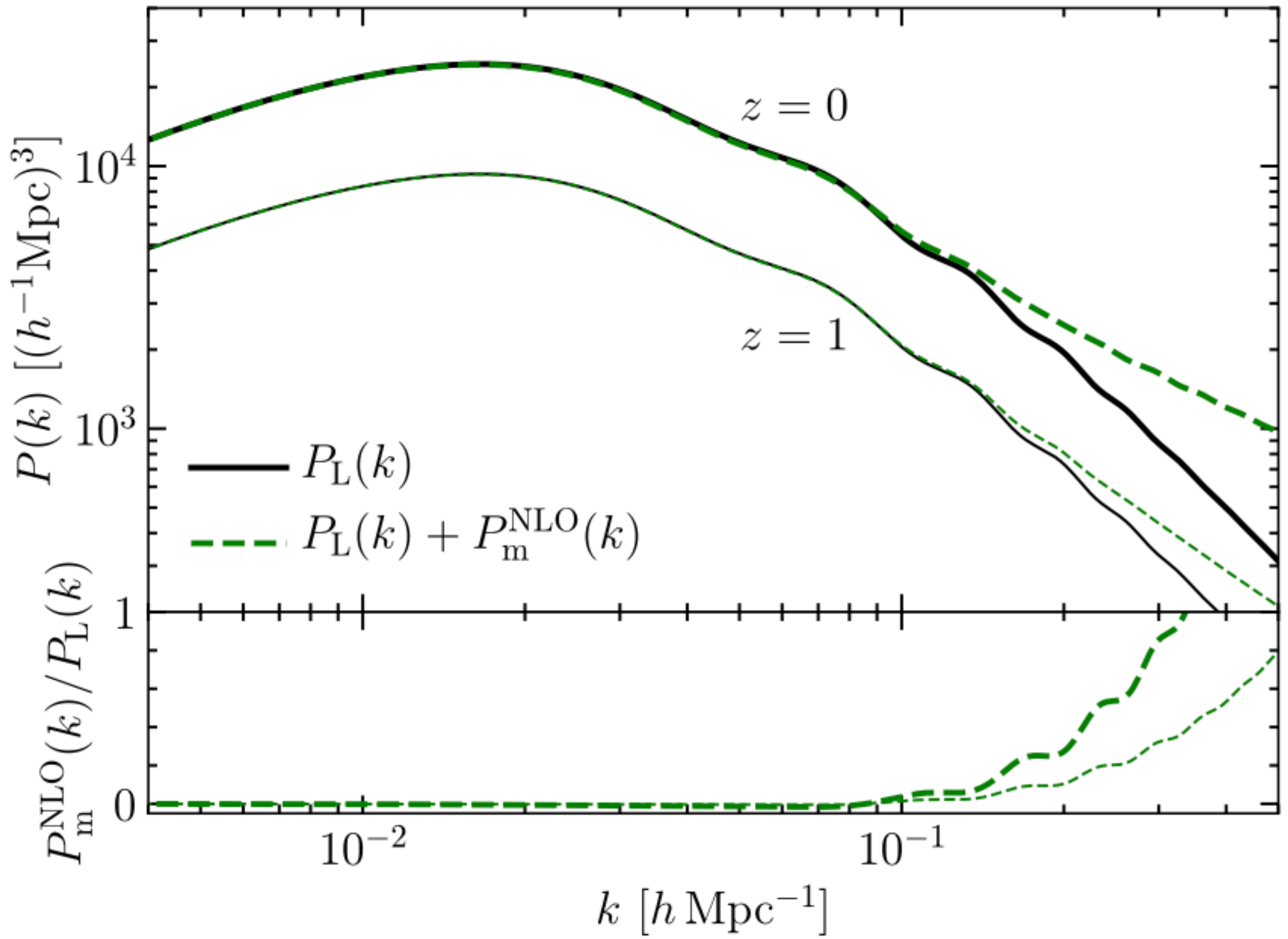
- Relatively easy to calculate, because potential remains small even when $\delta \sim 1$ (potentials don't linearly grow in matter domination, decay in DM domination)

- Eulerian: solve Euler equation at higher orders (Dodelson & Schmidt 2020 sec. 12.2)

- Lagrangian: Zeldovich approximation

Non-linear evolution of the matter power spectrum

Perturbation theory



Dodelson & Schmidt 2020; Fig. 12.4

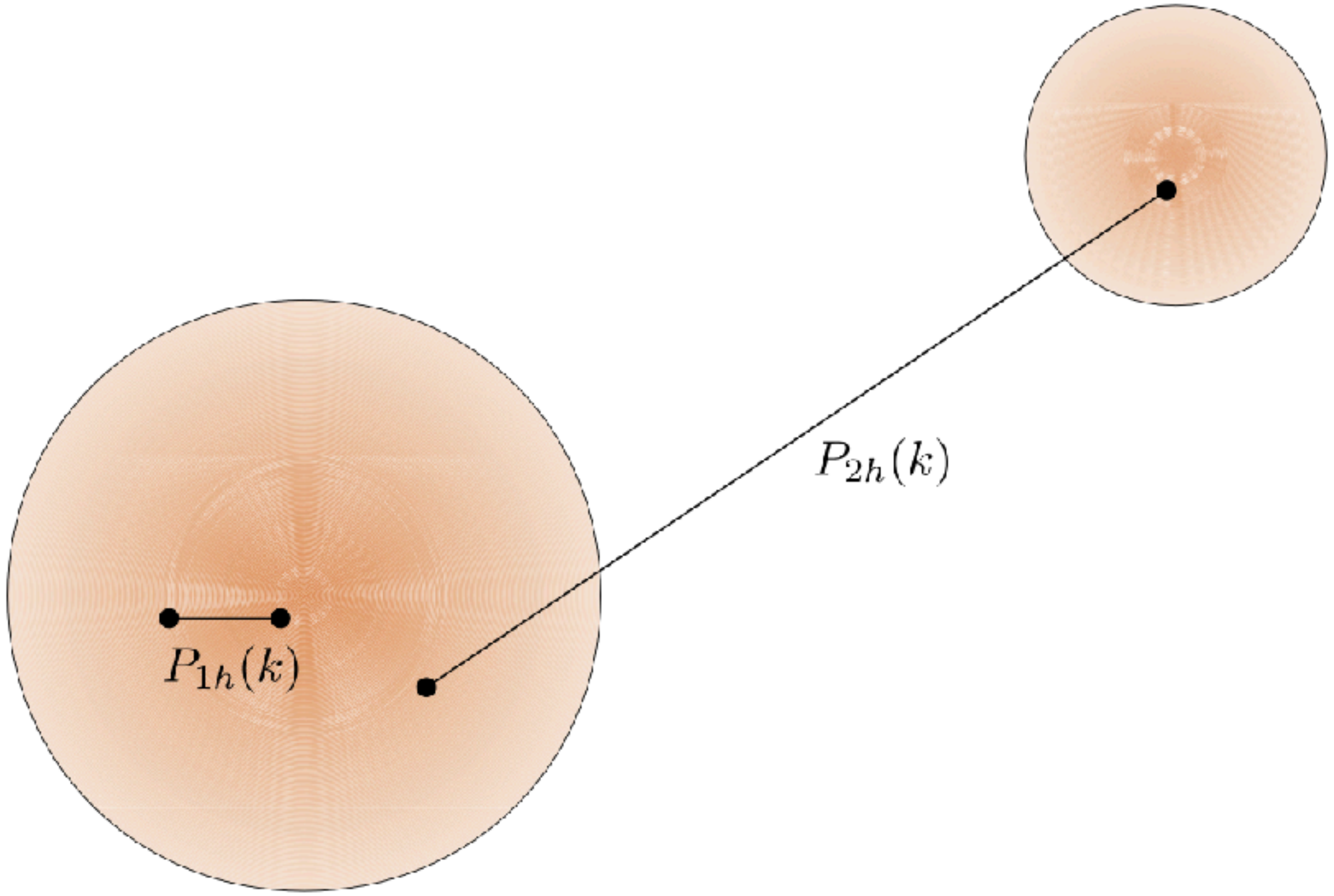
Non-linear evolution of the matter power spectrum

Halo model

- A simple yet accurate way to understand the non-linear clustering of DM is through the *halo model* (review: Cooray & Sheth 2002)
- Assumes that all DM is in a halo of some mass (~Press-Schechter), then uses
    - Internal DM halo profile (NFW)
    - Halo mass function (from Press-Schechter, e.g., the Sheth-Tormen form)
    - Halo clustering at fixed mass ($\propto P_{lin}[k]$)
- Non-linear DM power spectrum is then the sum of
    - *Two-halo term*: a convolution of halo clustering at fixed mass over the halo mass function and the internal DM profile
    - *One-halo term:* correlation of DM within one halo

Non-linear evolution of the matter power spectrum

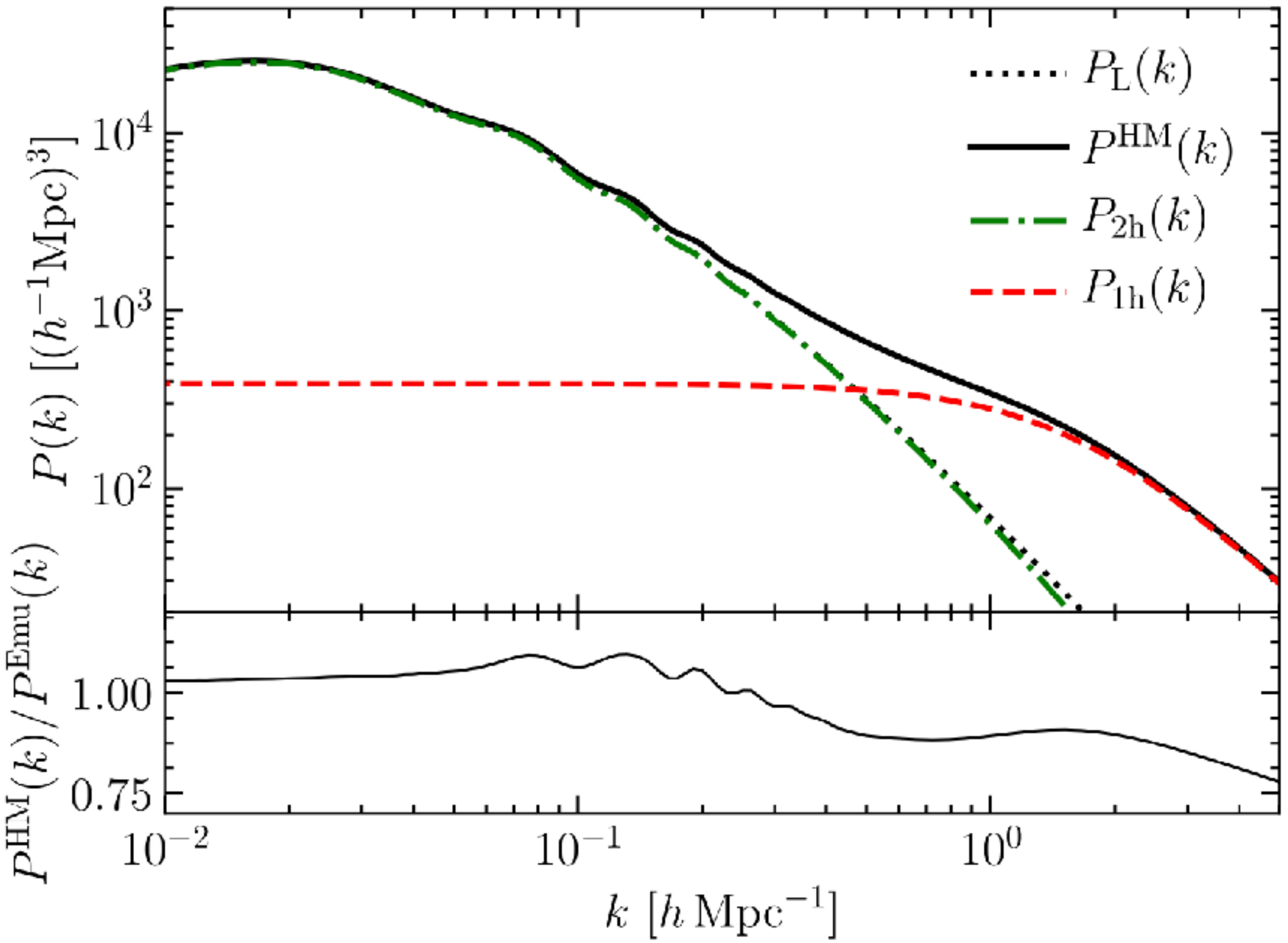
Halo model



Dodelson & Schmidt 2020; Fig. 12.11

Non-linear evolution of the matter power spectrum

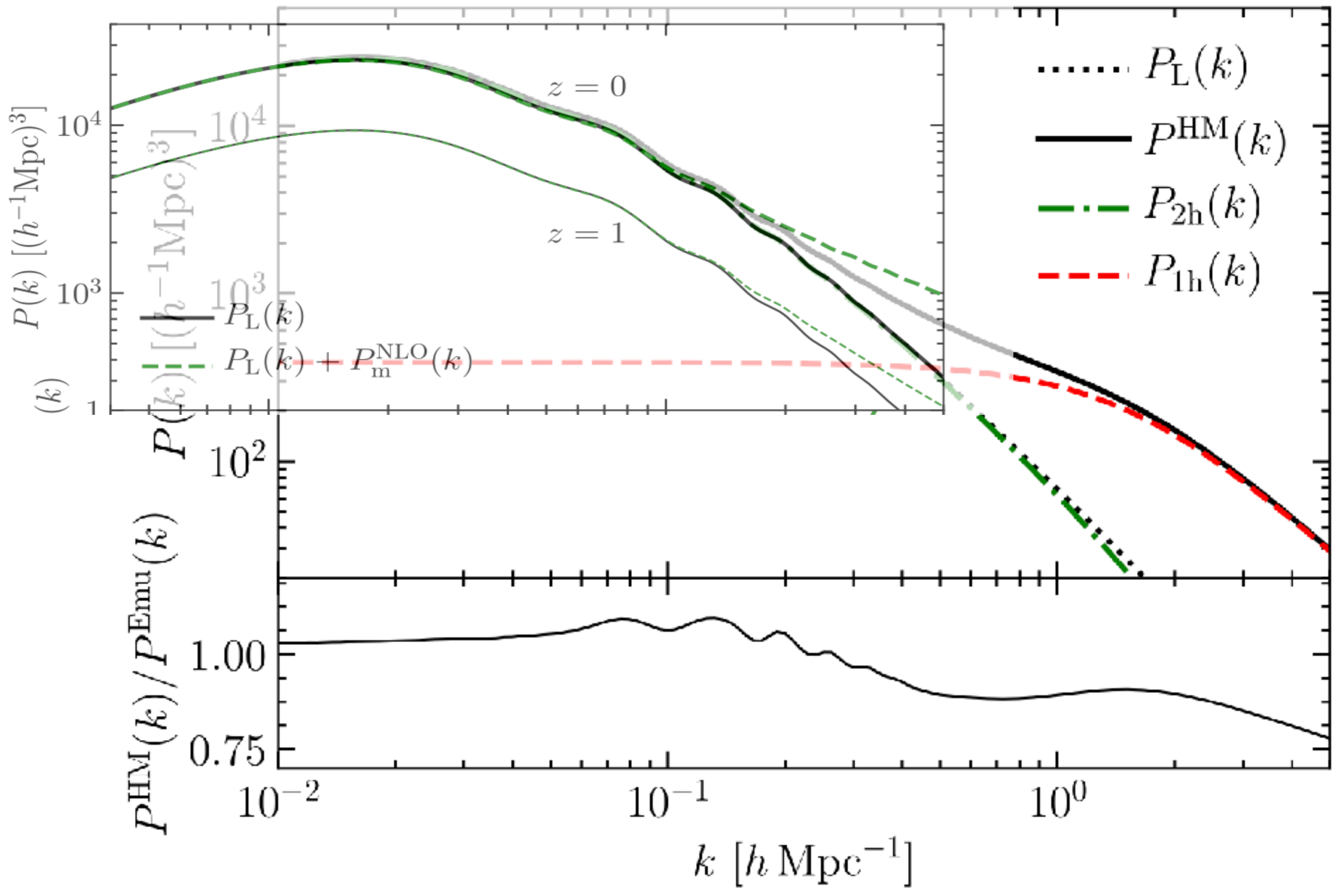
Halo model



Dodelson & Schmidt 2020; Fig. 12.12

Non-linear evolution of the matter power spectrum

Halo model

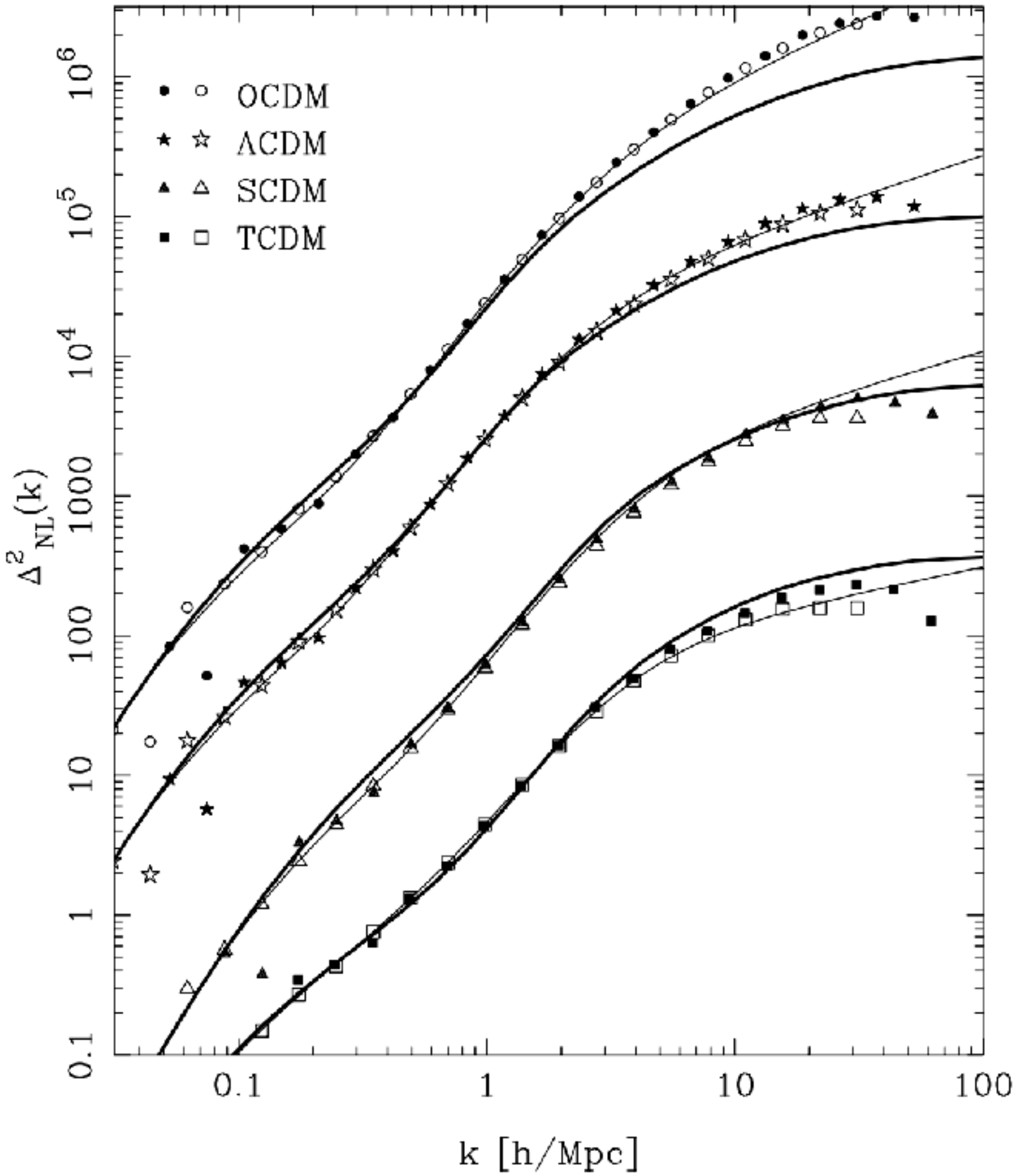


Dodelson & Schmidt 2020; Fig. 12.12

Non-linear evolution of the matter power spectrum

N-body simulations

- Full understanding of non-linear evolution requires *N*-body simulations
- Can use the halo model to come up with fitting functions applicable for a wide variety of cosmologies (e.g., HALOFIT; Smith et al. 2003, Takahashi et al. 2012)
- Halo model on its own does very well on scales k <~ 5 / Mpc



Smith et al. (2003)

Non-linear evolution of the matter power spectrum

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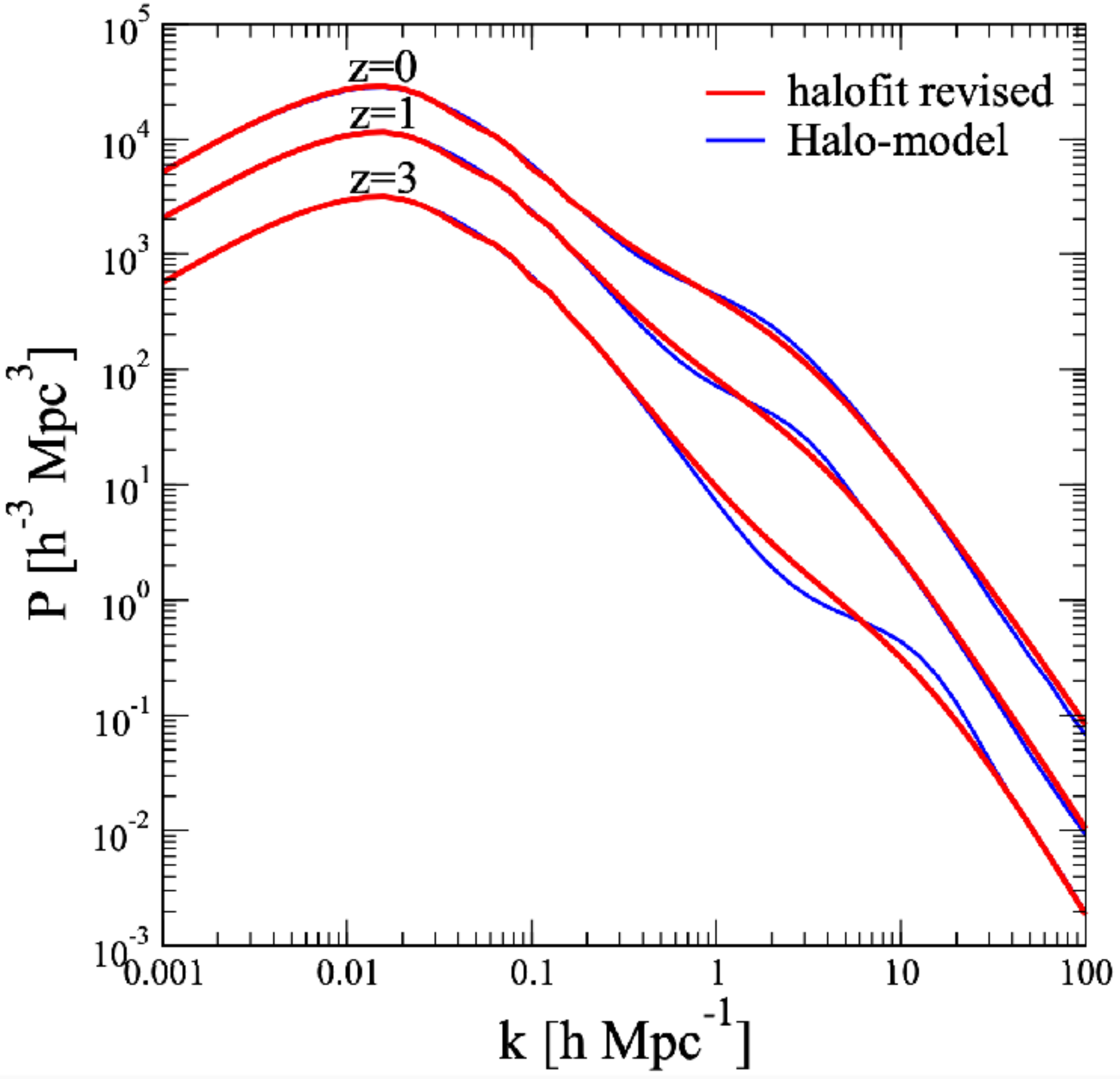
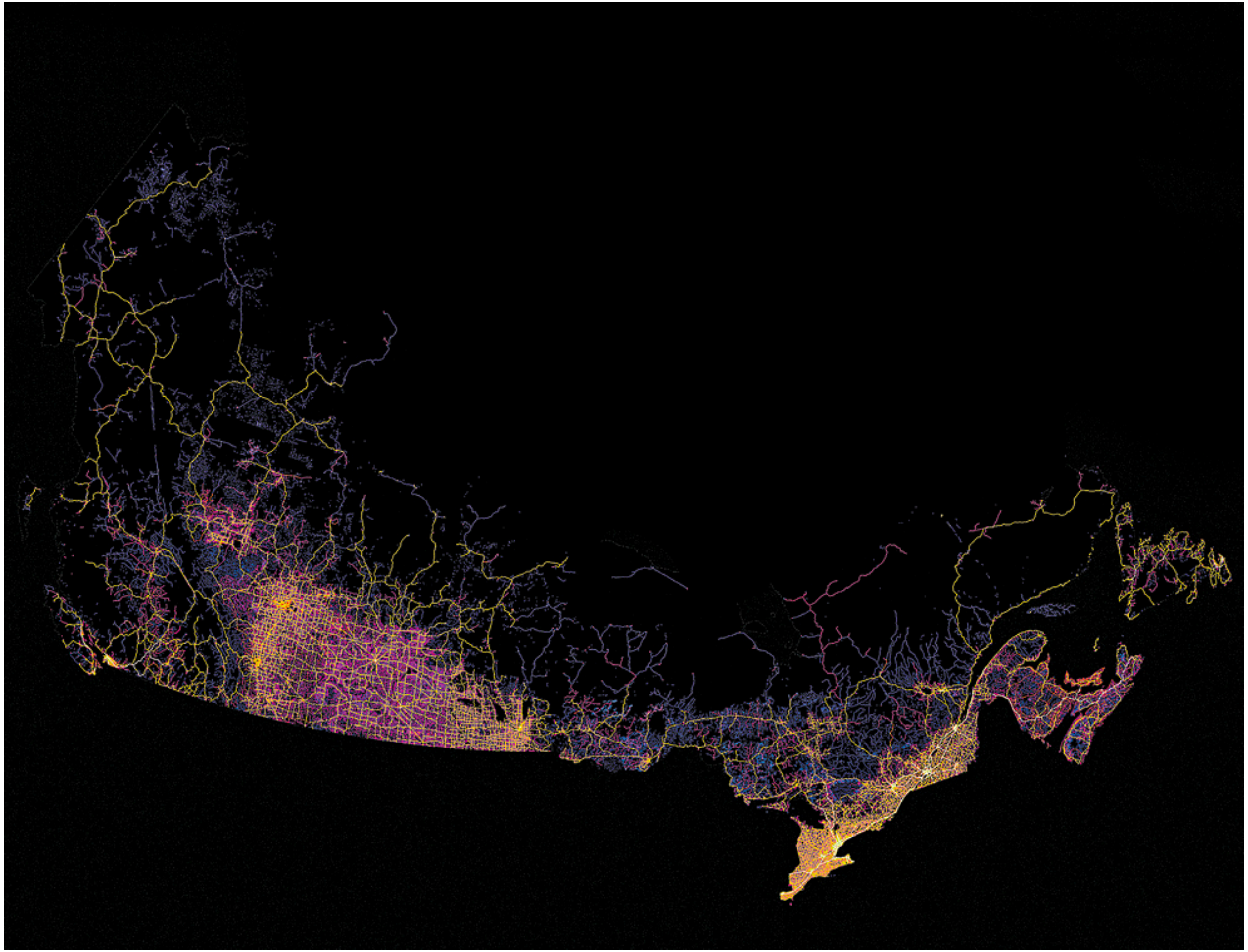


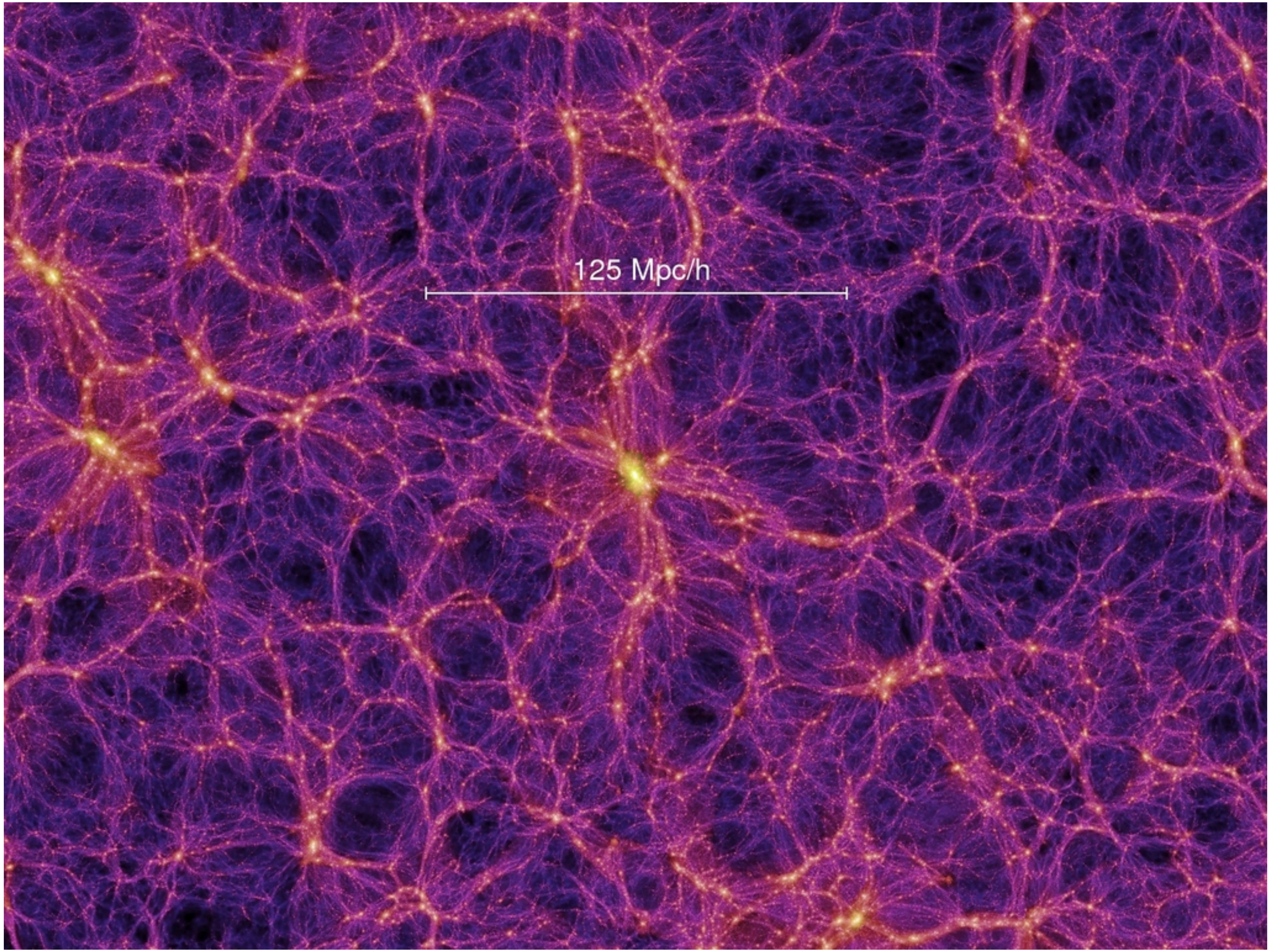
Fornengo & Regis (2013)

Bias



Maclean’s

Google Earth



V. Springel / Virgo Consortium

Bias of dark-matter halos vs. the linear density field

- To first approximation, the density of dark-matter halos can be written as (where b(M) is the *linear bias*)

$$\delta_h(M) = b(M) \times \delta_{\rm lin}$$

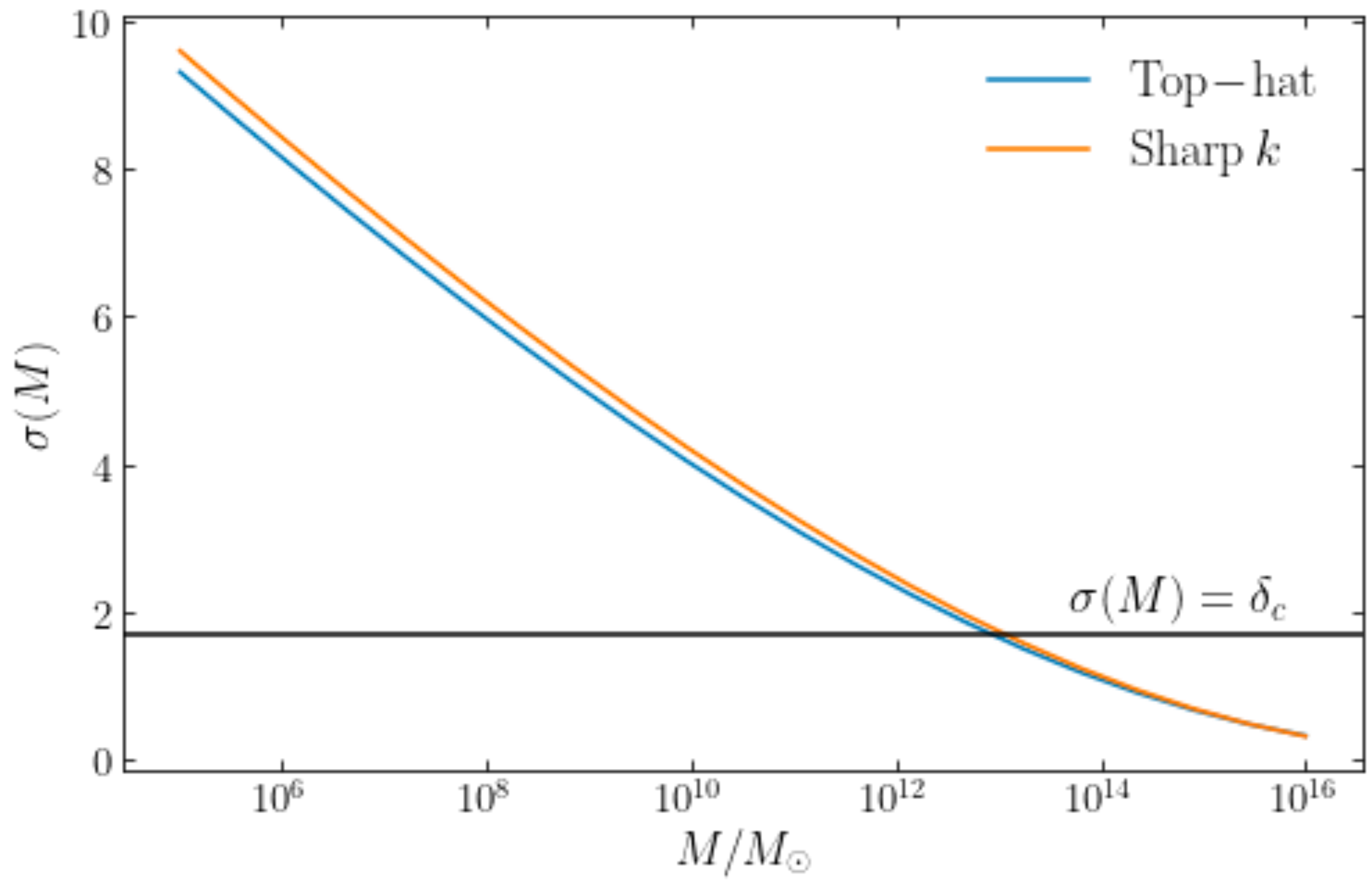
- Simple approximate form from the extended Press-Schechter formalism:
  - Conditional mass function $$\frac{\mathrm{d}n(M_1 \text{ at } z_1 | M_2 \text{ at } z_2)}{\mathrm{d}M_1} = \frac{M_2}{M_1} \frac{(\delta_1 - \delta_2)}{\sqrt{2\pi\,(S_1 - S_2)^3}} \exp\left(-\frac{[\delta_1 - \delta_2]^2}{2[S_1 - S_2])}\right) \left|\frac{\mathrm{d}S_1}{\mathrm{d}M_1}\right|$$
  - Re-interpret to mean 'M_2 halo' is large (S_2 << S_1) uncollapsed region
  - Then gives the number density of halos in region of the Universe with linear overdensity δ_2
  - Compare to the overall mean density of halos from PS, assuming S_2=0<<S_1 and δ_2 << δ_1

$$b(M) = 1 + \left(\frac{\nu^2(M) - 1}{\delta_c(t)}\right)$$

Bias of dark-matter halos vs. the linear density field

$$b(M) = 1 + \left(\frac{\nu^2(M) - 1}{\delta_c(t)}\right)$$

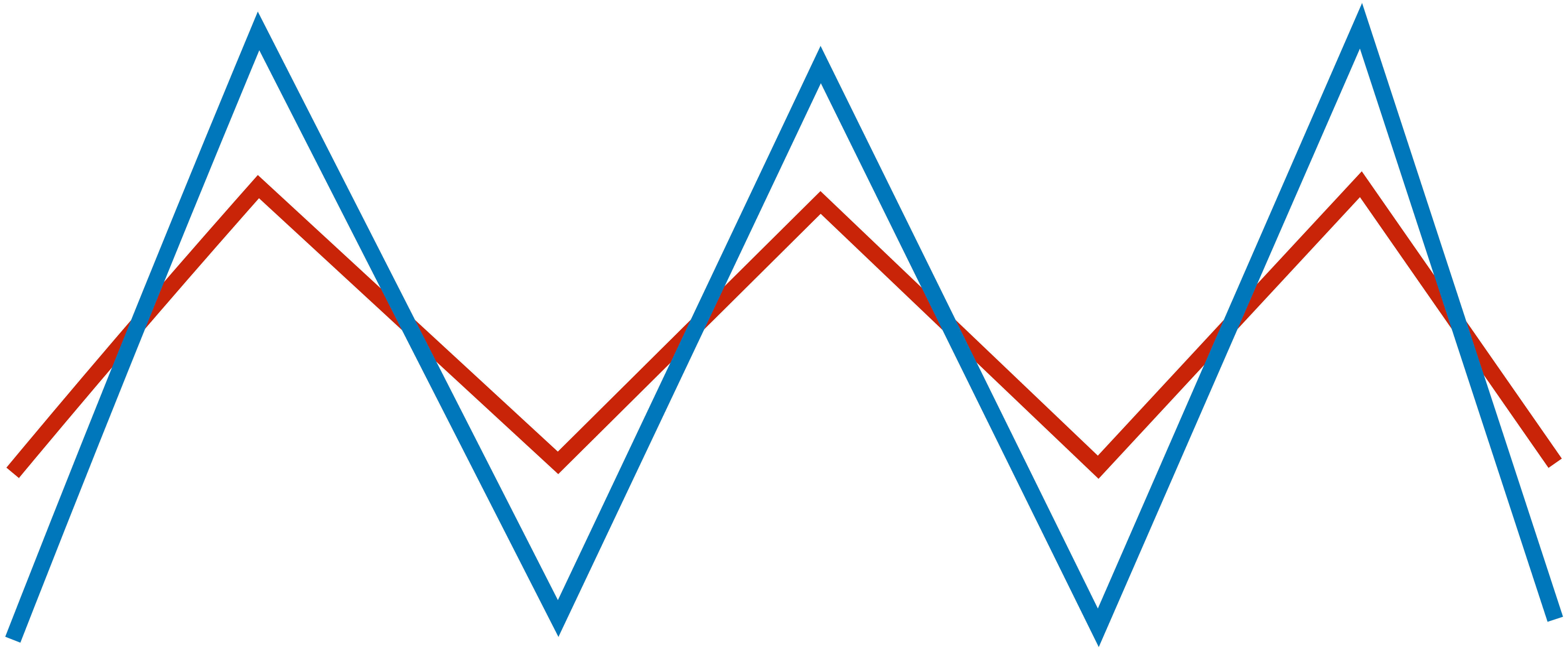
- $\nu(M_*) = 1$ when $\sigma(M_*) = \delta_c(a)$
- So halos with
    - $M > M_*$ have $b > 1$
    - $M < M_*$ have $b < 1$
    - Note that b never becomes negative



Bias of dark-matter halos vs. the linear density field

$$b(M) = 1 + \left(\frac{\nu^2(M) - 1}{\delta_c(t)}\right)$$

M > M*



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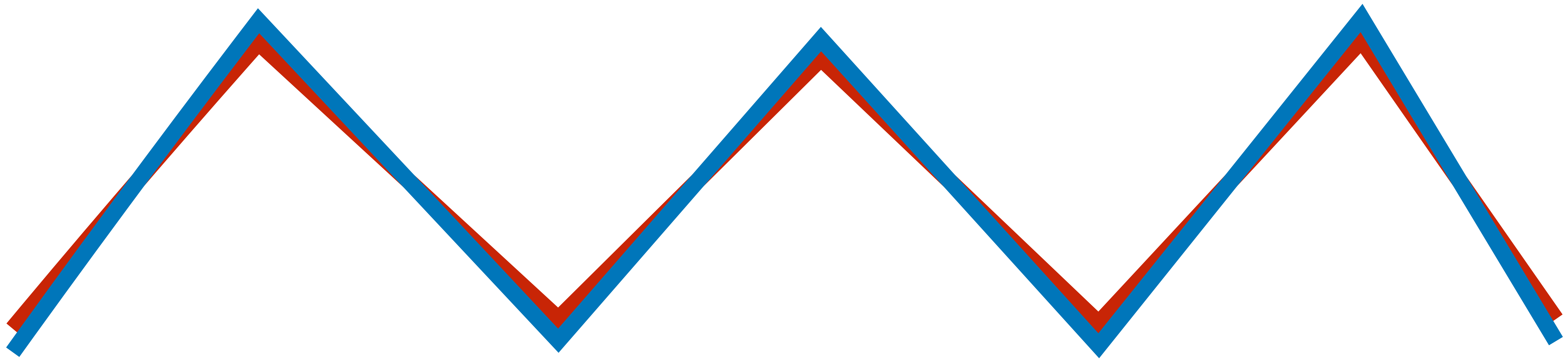

Bias of dark-matter halos vs. the linear density field

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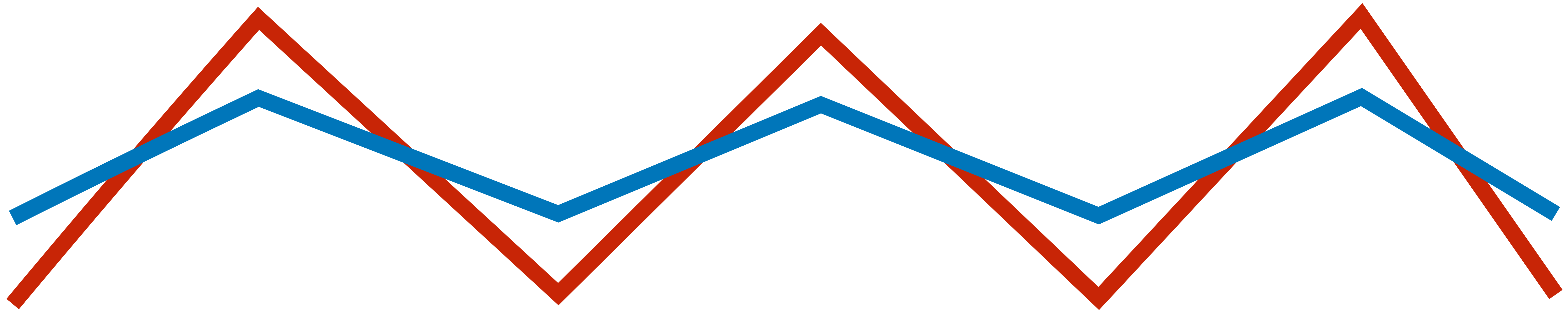


Bias of dark-matter halos vs. the linear density field

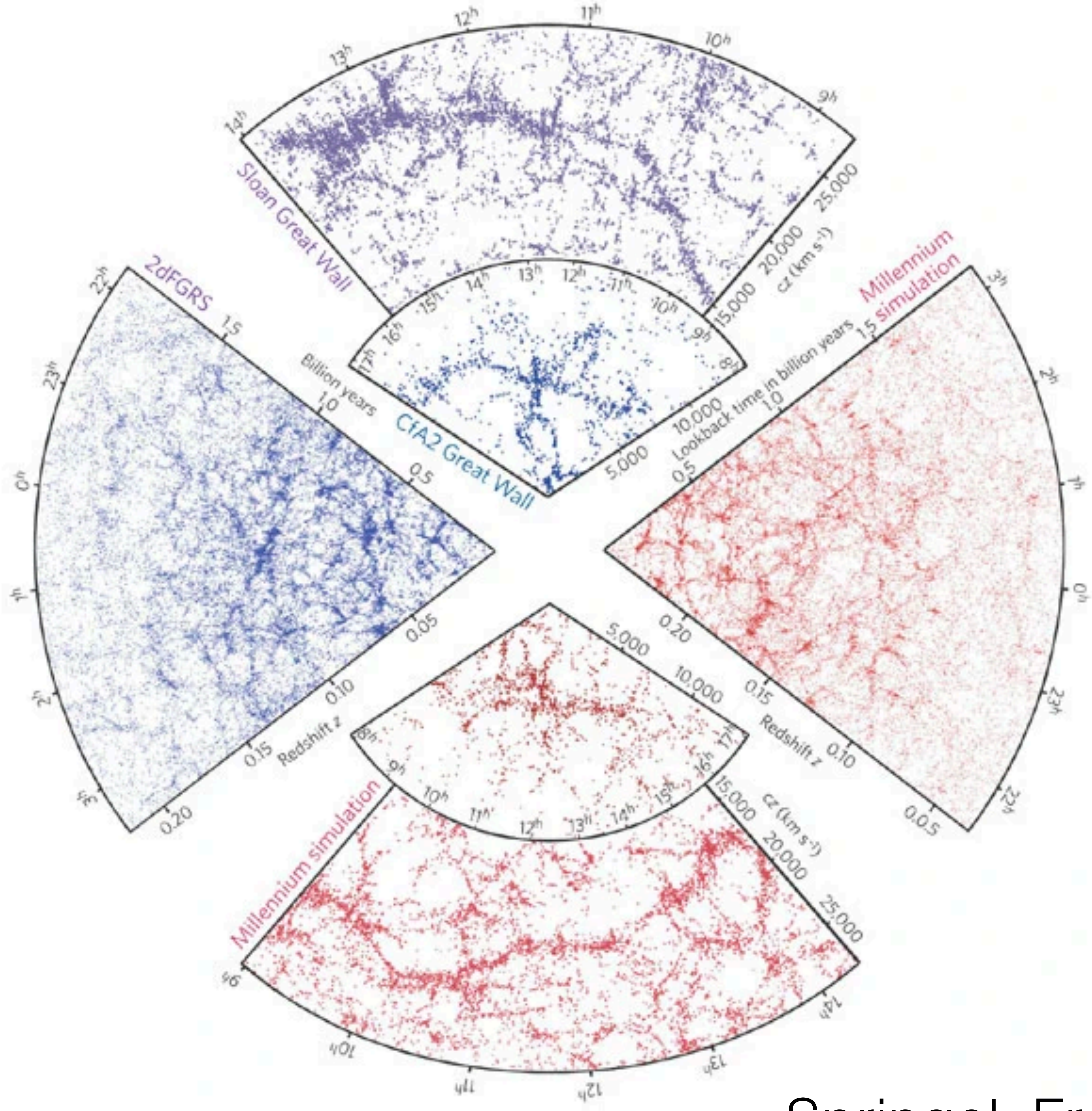
Ellipsoidal collapse

$$b_{\rm h}(M_1,\delta_1,t) = 1 + \frac{1}{D(t)\delta_1}\left[\nu_1'^{\,2} + b\nu_1'^{\,2(1-c)} - \frac{\nu_1'^{\,2c}/\sqrt{a}}{\nu'^{2c} + b(1-c)(1-c/2)}\right], \qquad (7.106)$$

where $\nu_1' = \sqrt{a}\nu_1$, $a = 0.707$, $b = 0.5$ and $c = 0.6$ (Sheth et al., 2001). Numerical simulations

Galaxy clustering - bias

Galaxy clustering



Springel, Frenk, & White (2006)

Galaxy clustering

- Clustering of dark-matter halos can be traced through clustering of galaxies that form within dark-matter halos
- Like dark-matter halos, the galaxy over density field is biased with respect to the linear matter density field
- Galaxy clustering is generally probed through the two-point correlation function

$$\xi\ (|\mathbf{x}_1 - \mathbf{x}_2|) = \langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle,$$

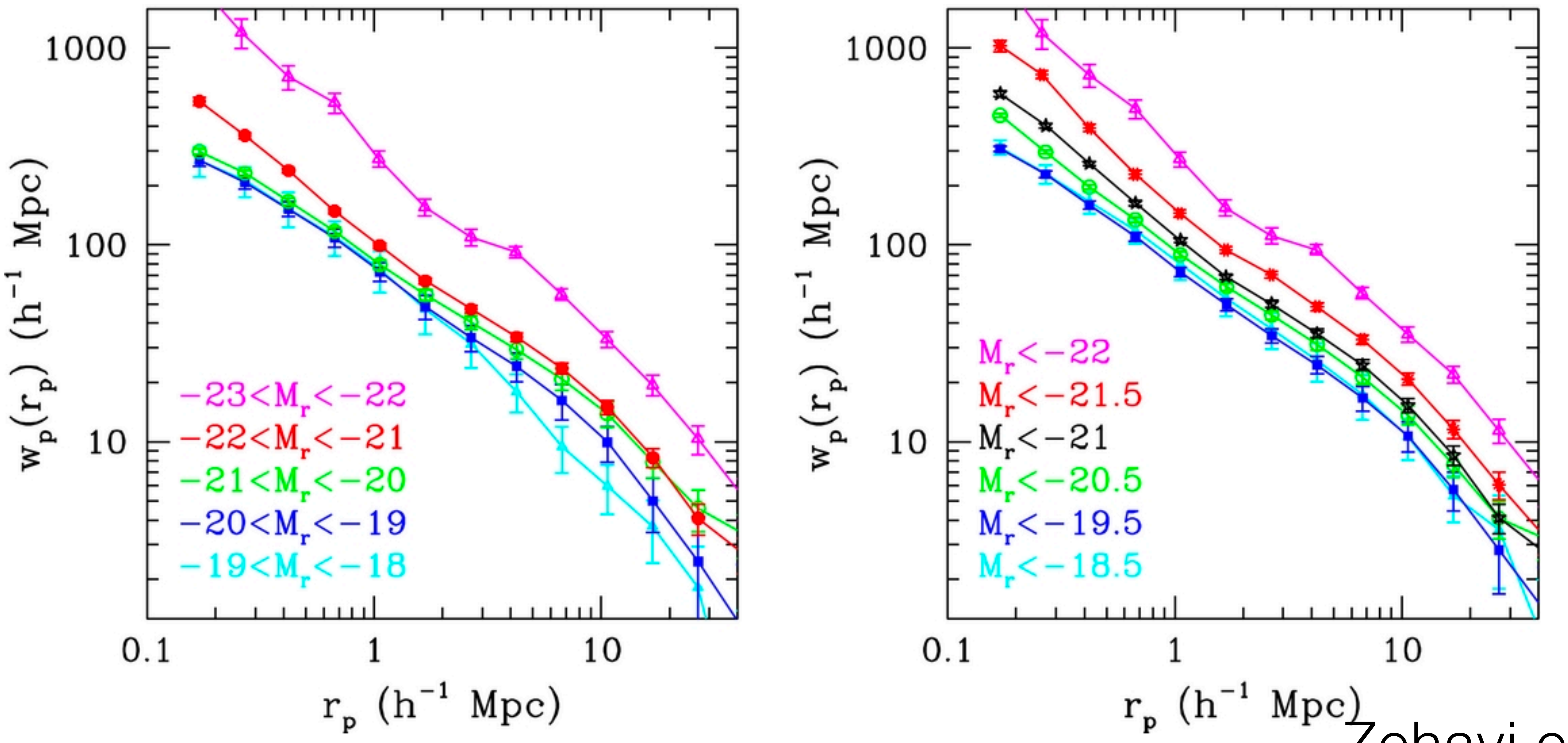
- Assuming a bias b(r;galaxy props g)

$$\xi_g(r) = b^2(r;g) \times \xi_{\rm lin}(r)$$

- Bias may depend on scale

Galaxy clustering
Relative bias

- Like for DM halos, galaxy bias depends on galaxy mass (luminosity)

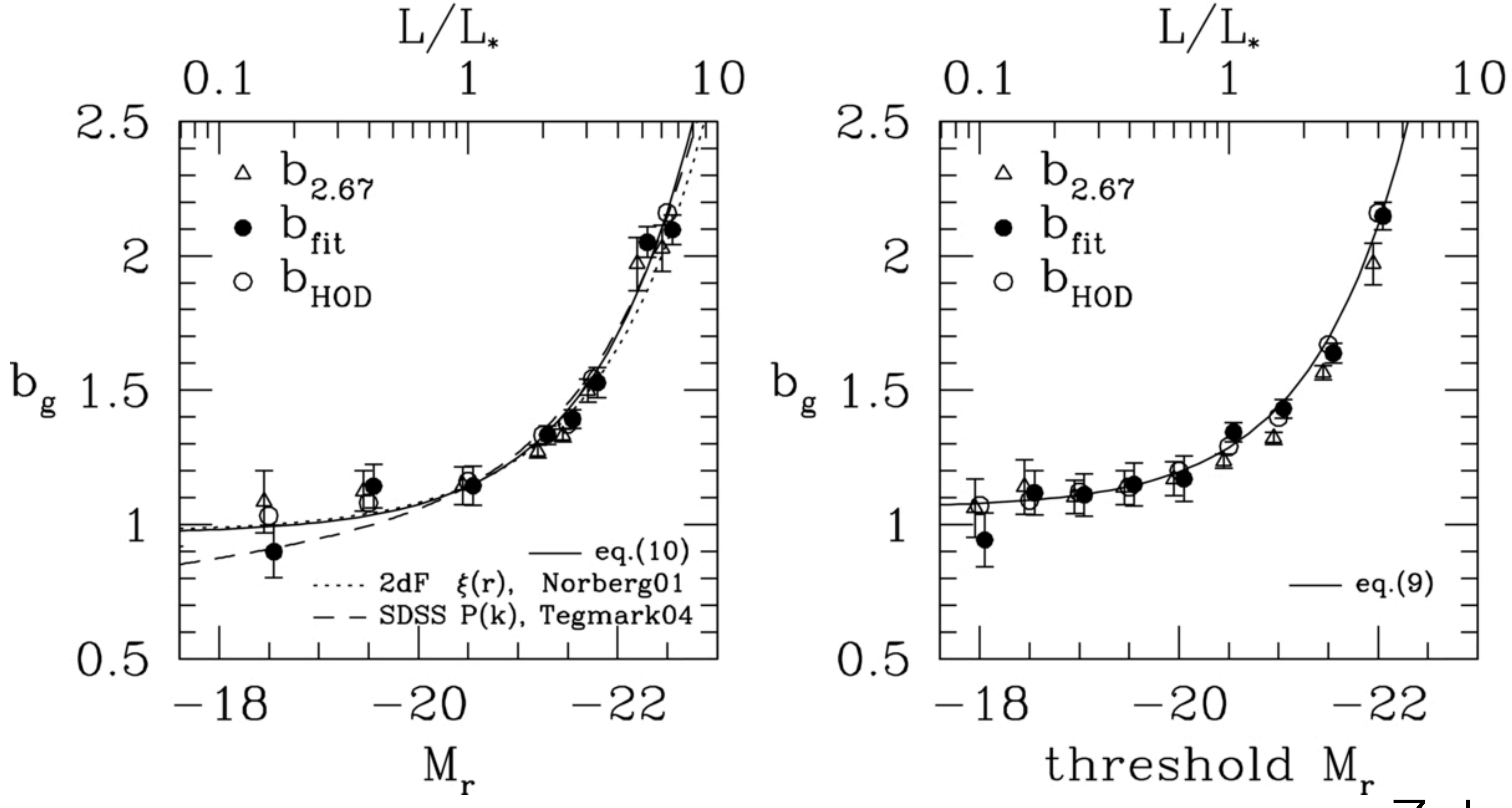


Zehavi et al. (2011)

Galaxy clustering

Relative bias

- Like for DM halos, galaxy bias depends on galaxy mass (luminosity)

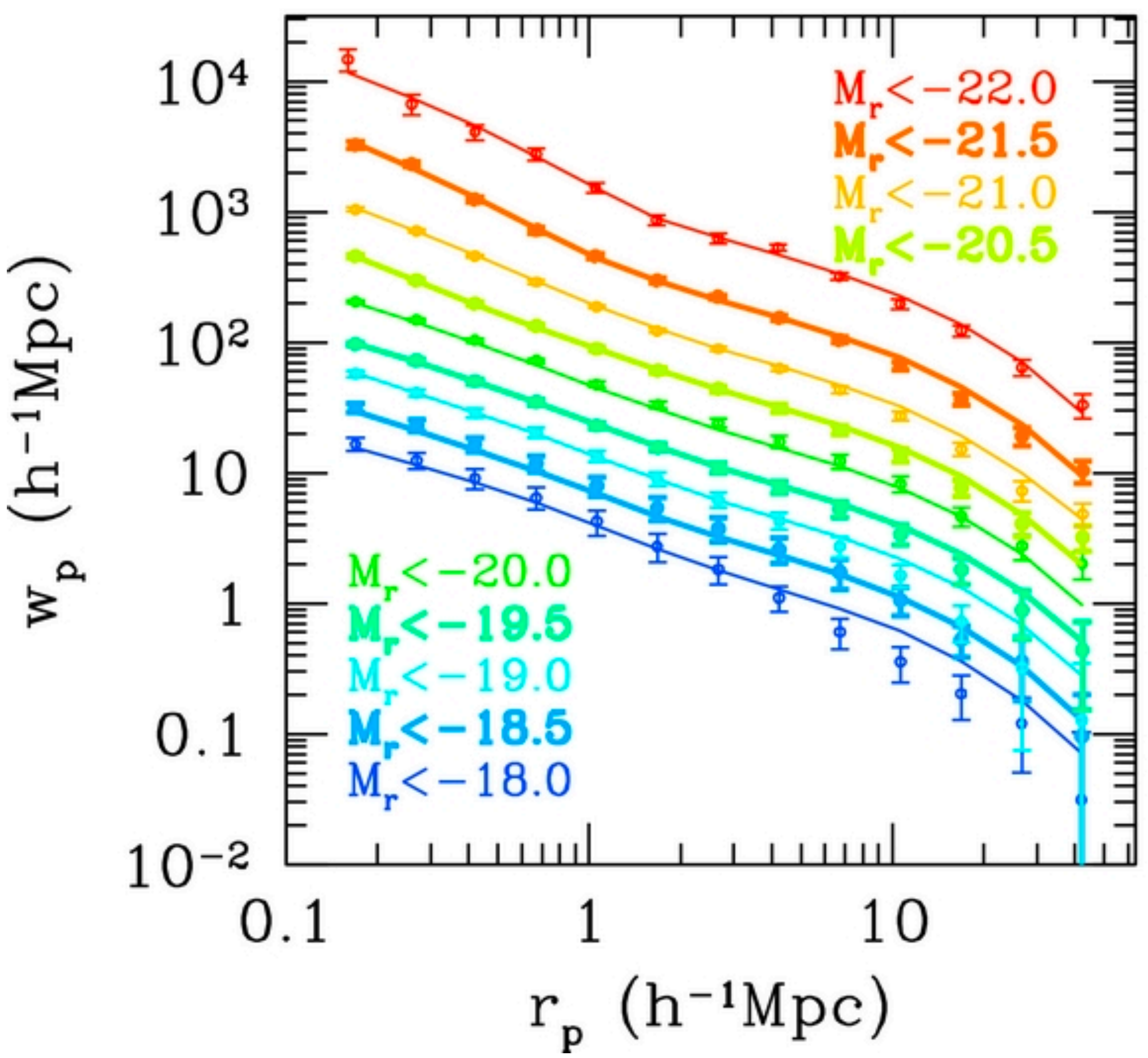


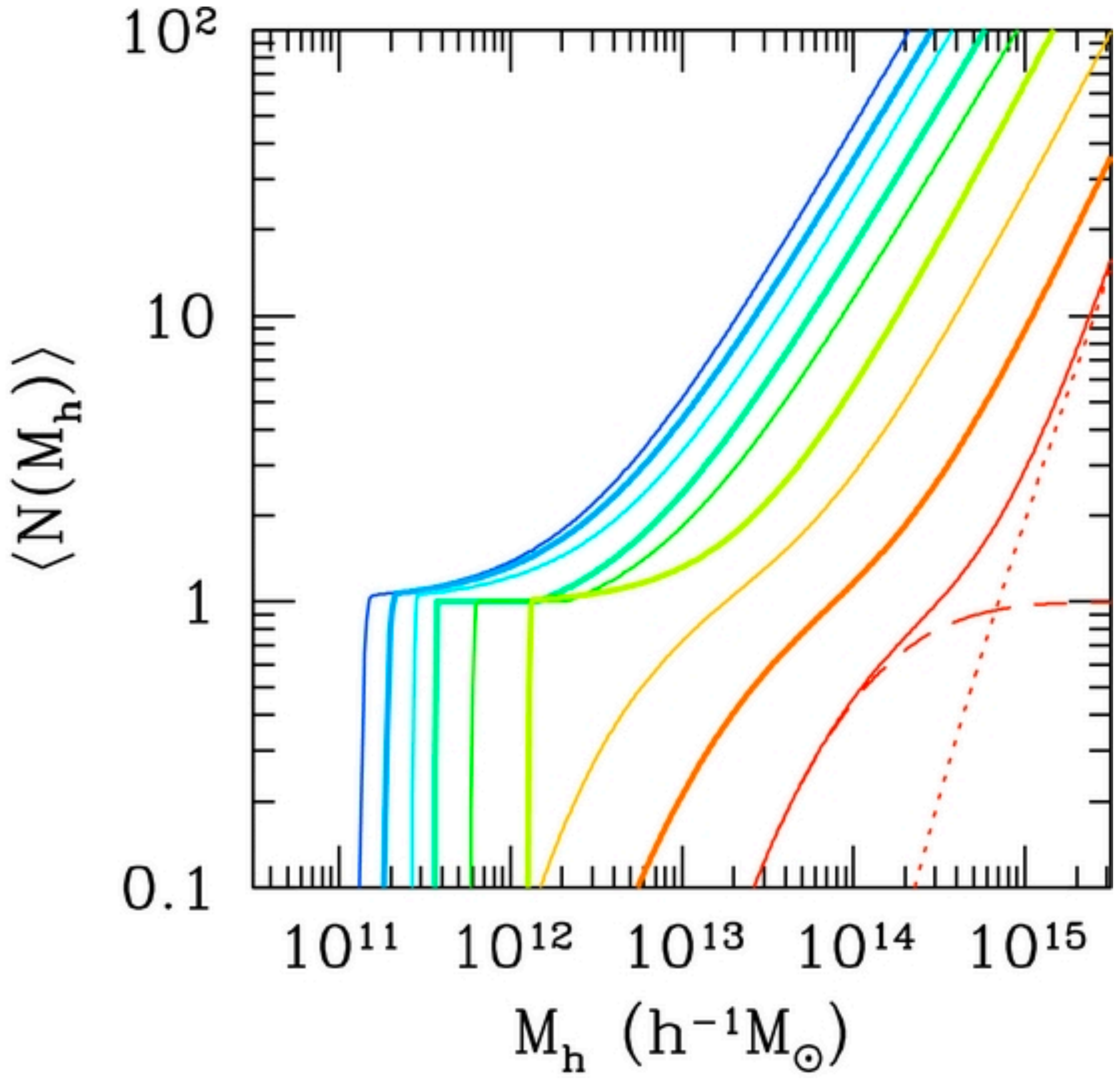
Zehavi et al. (2011)

Galaxy clustering

Relative bias and halo occupation models

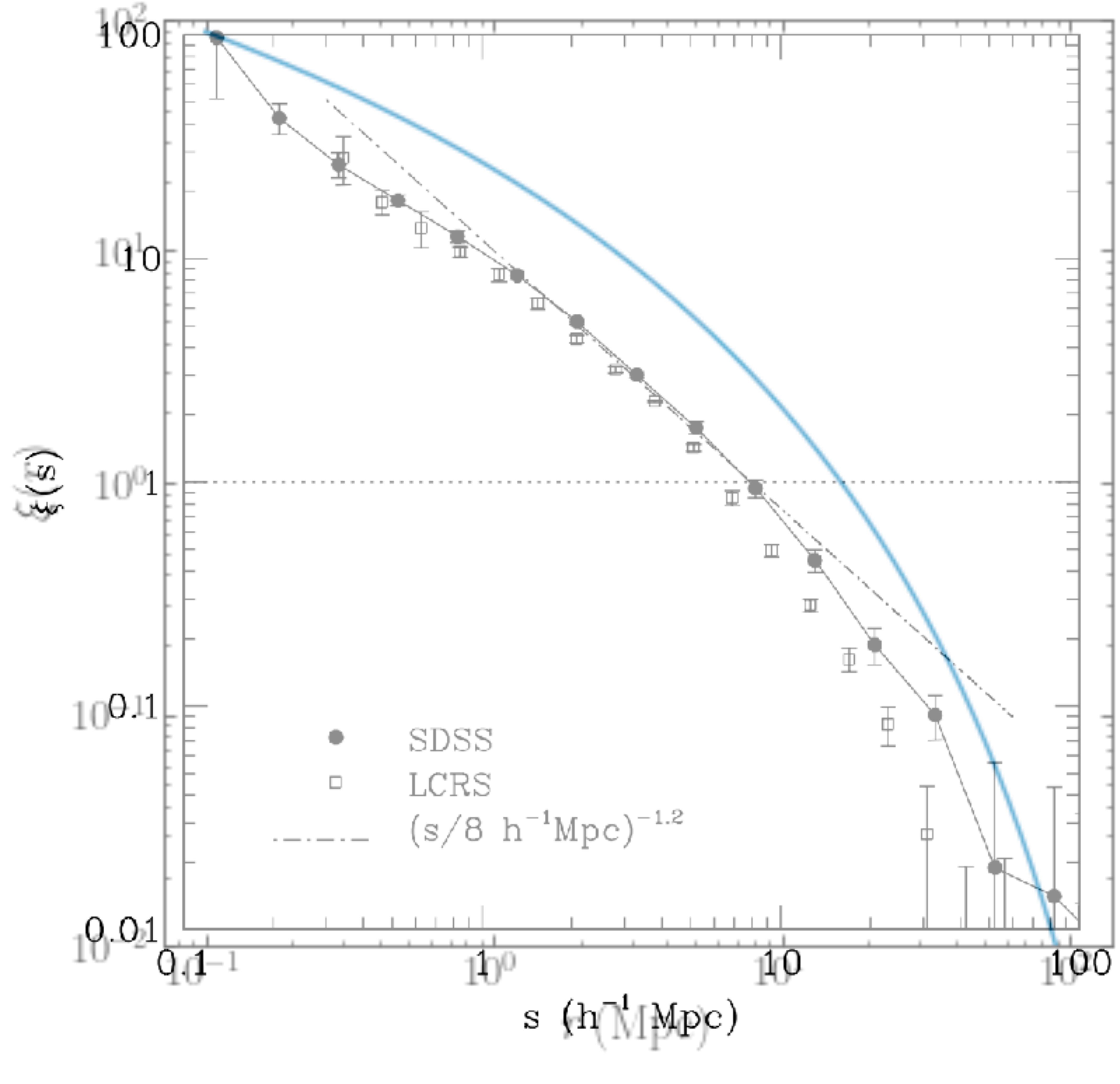
- Like for DM halos, galaxy bias depends on galaxy mass (luminosity)





Zehavi et al. (2011)

Galaxy bias must depend on scale … on small scales



Zehavi et al. (2001)

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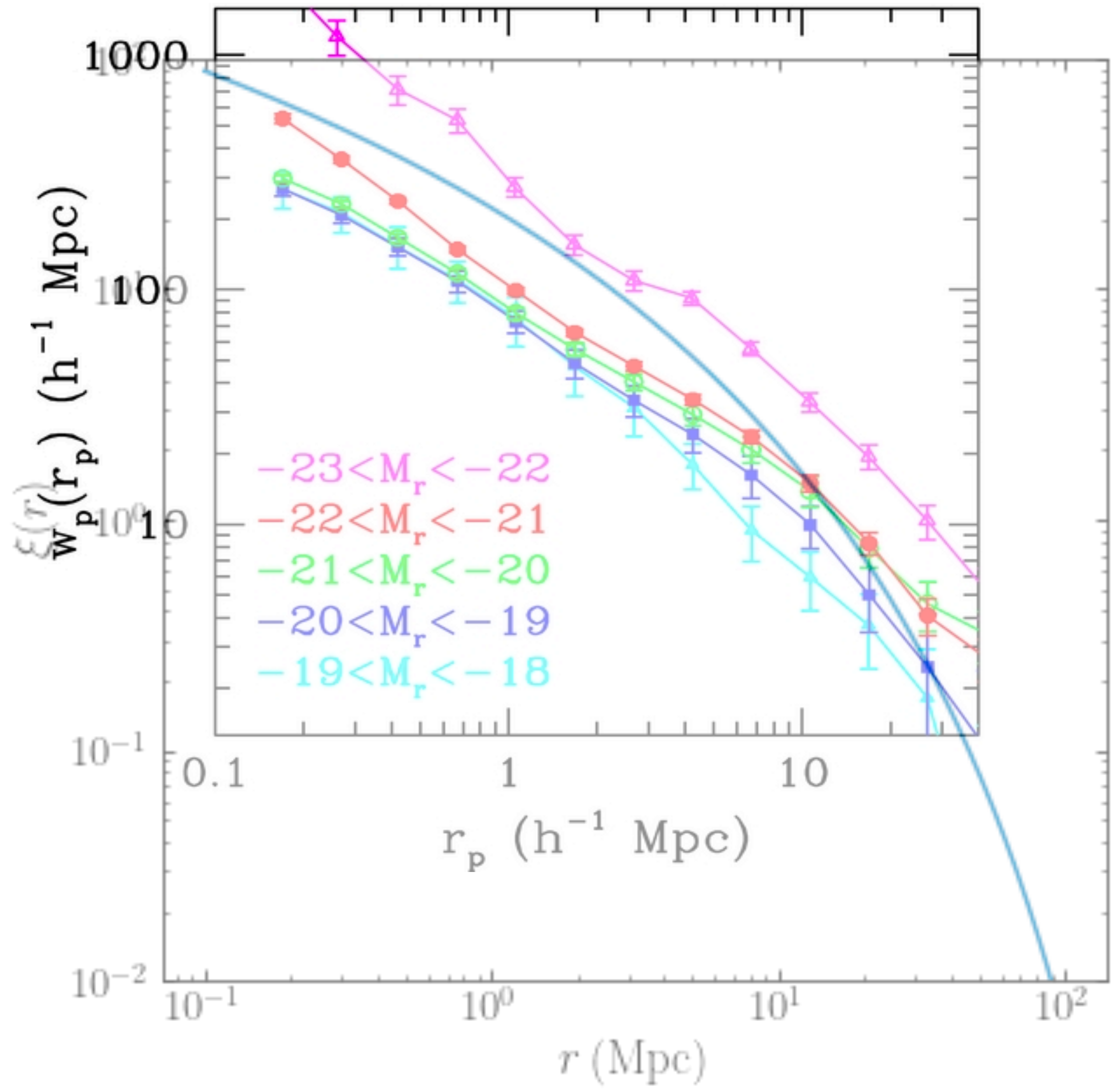

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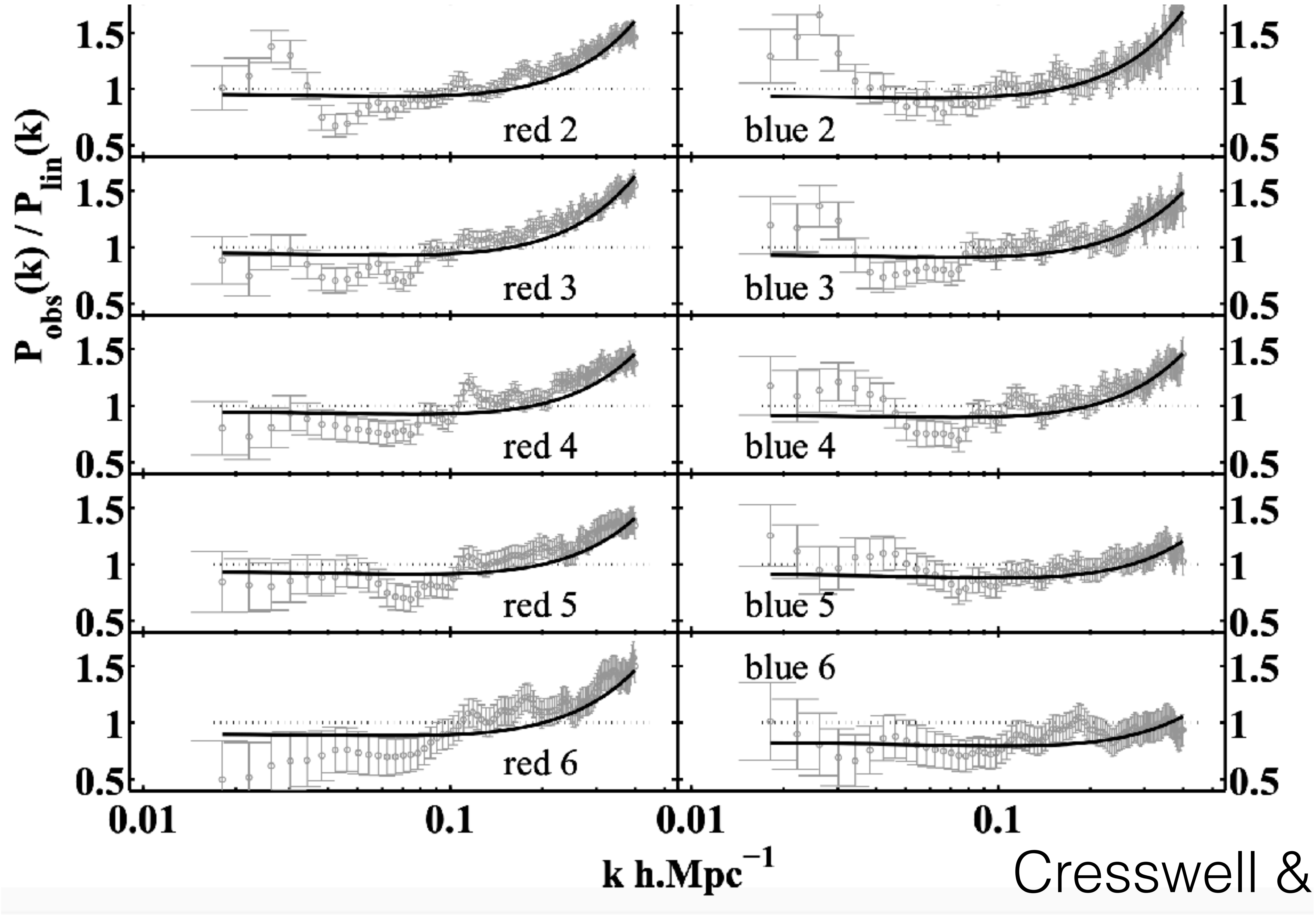
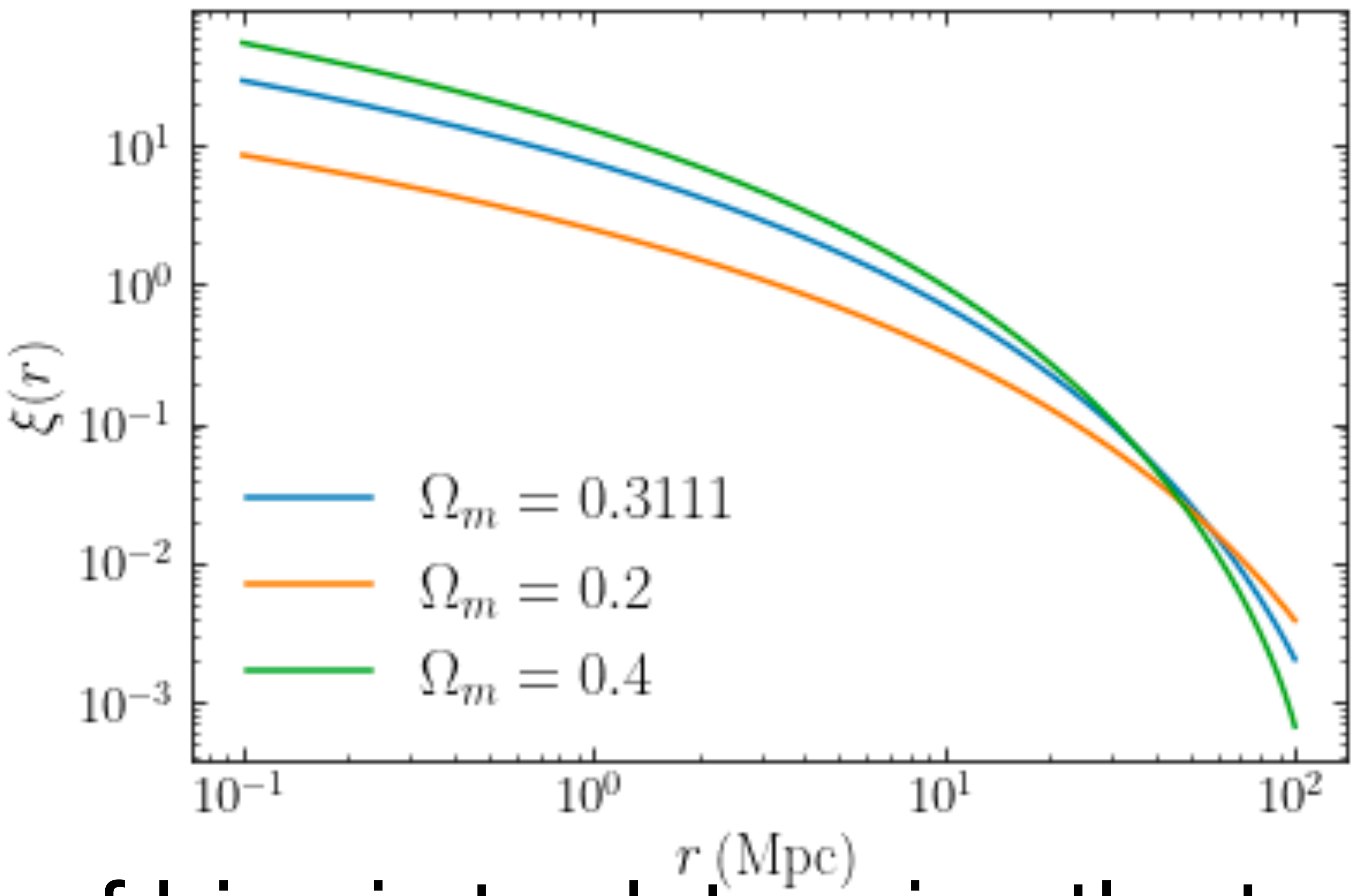


Cresswell & Percival (2008)

Galaxy clustering and bias

- Scale dependence on small scales due to galaxy formation makes it difficult to use the galaxy correlation function to do cosmology
- On large scales (k <~ 0.1 / Mpc, r >~ 10 Mpc), galaxy bias is ~constant, so shape of correlation function / power spectrum can constrain cosmology



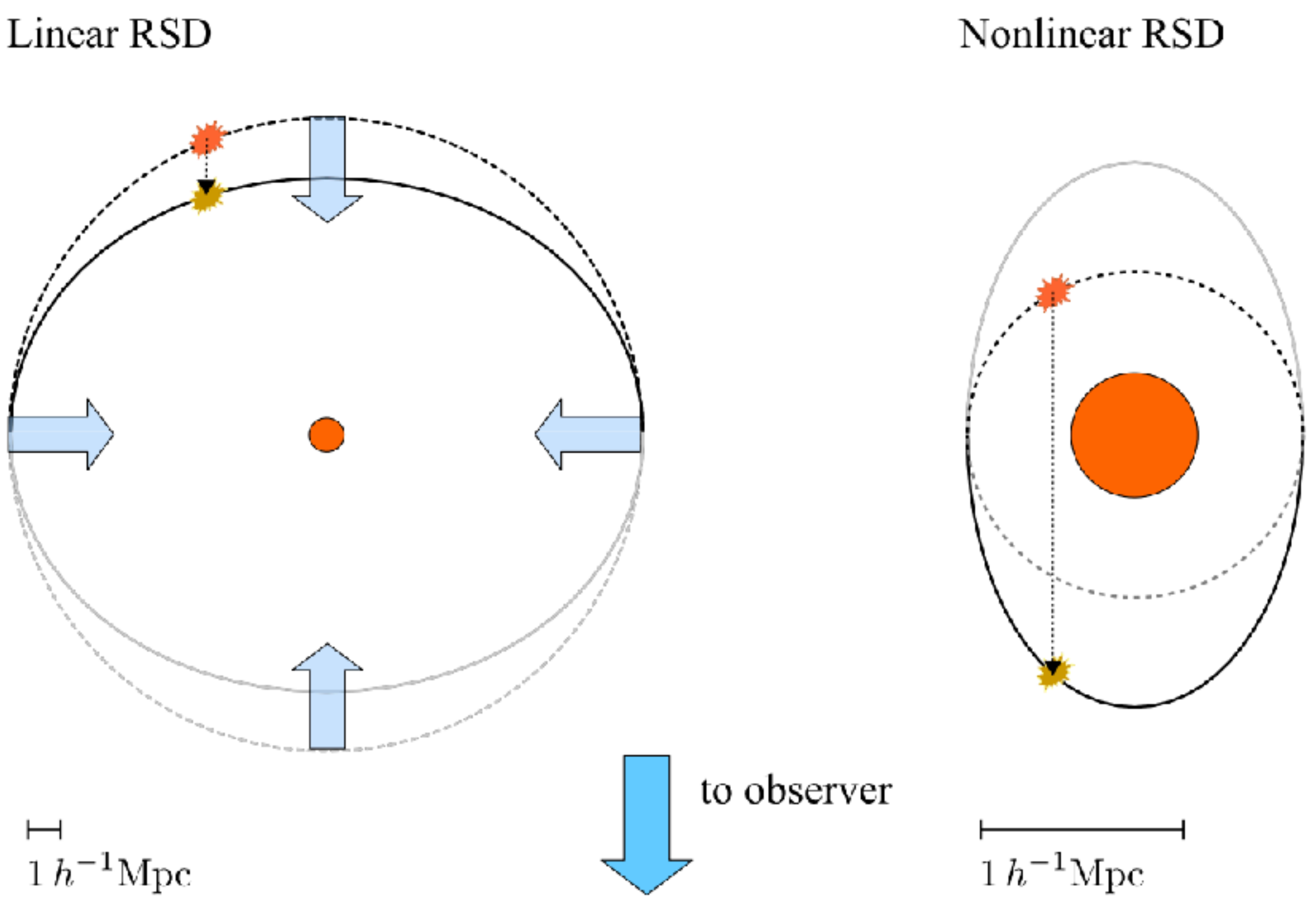
- Another useful application of bias is to determine the typical halo mass of objects, since bias depends on halo mass

Galaxy clustering - redshift space distortions

Redshift space distortions (RSD)

- Theory correlation function depends on 3D positions
- But surveys determine distance using redshift and Hubble law
- Redshift = cosmological expansion + peculiar velocity
- Peculiar velocity causes distortions in the clustering signal
    - Clustering becomes non-isotropic (angular coordinates unaffected)
- Not just a nuisance! Scale of peculiar velocities depends on matter content (primarily Ω_m), so can constrain cosmology with RSD!

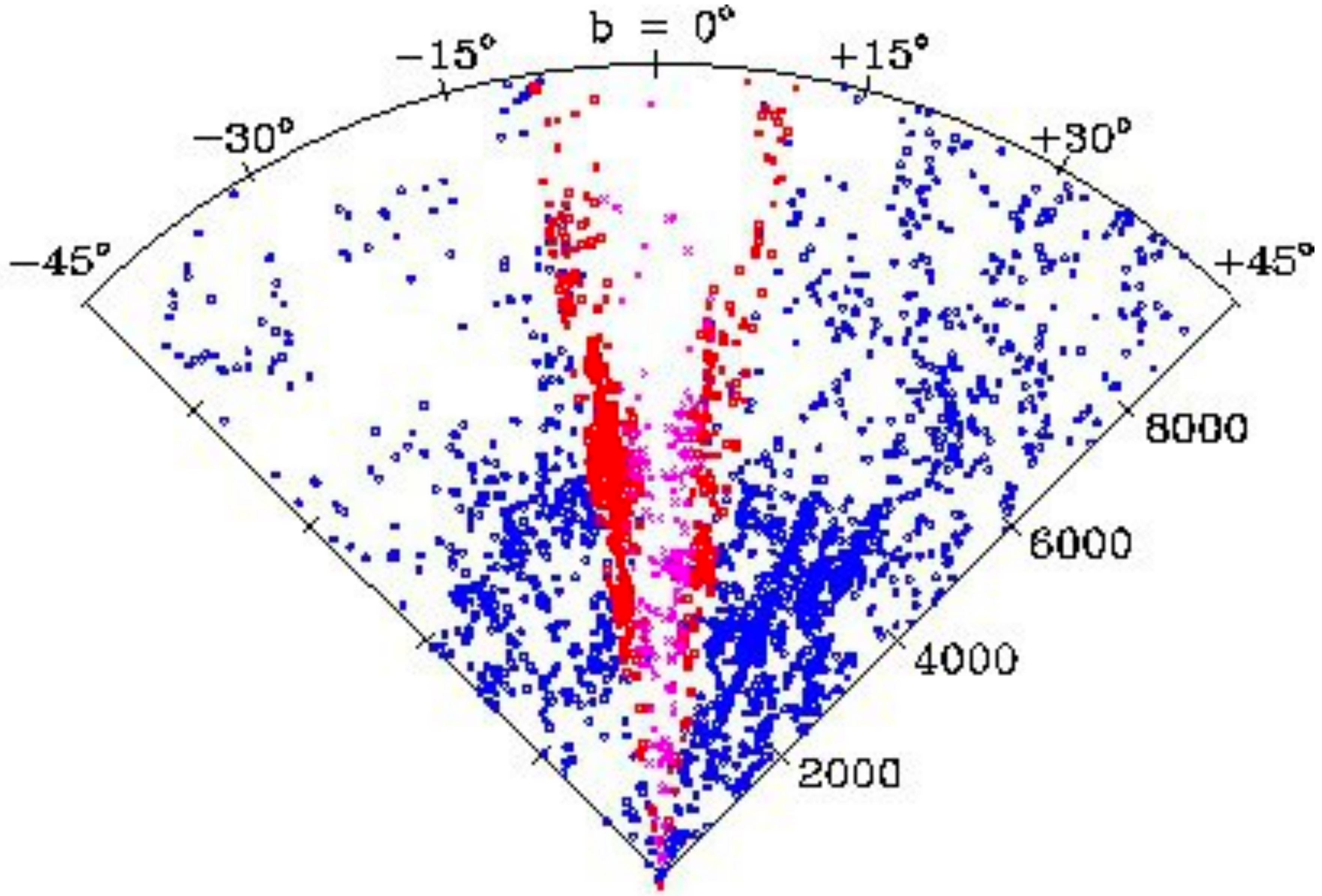
Redshift space distortions (RSD)

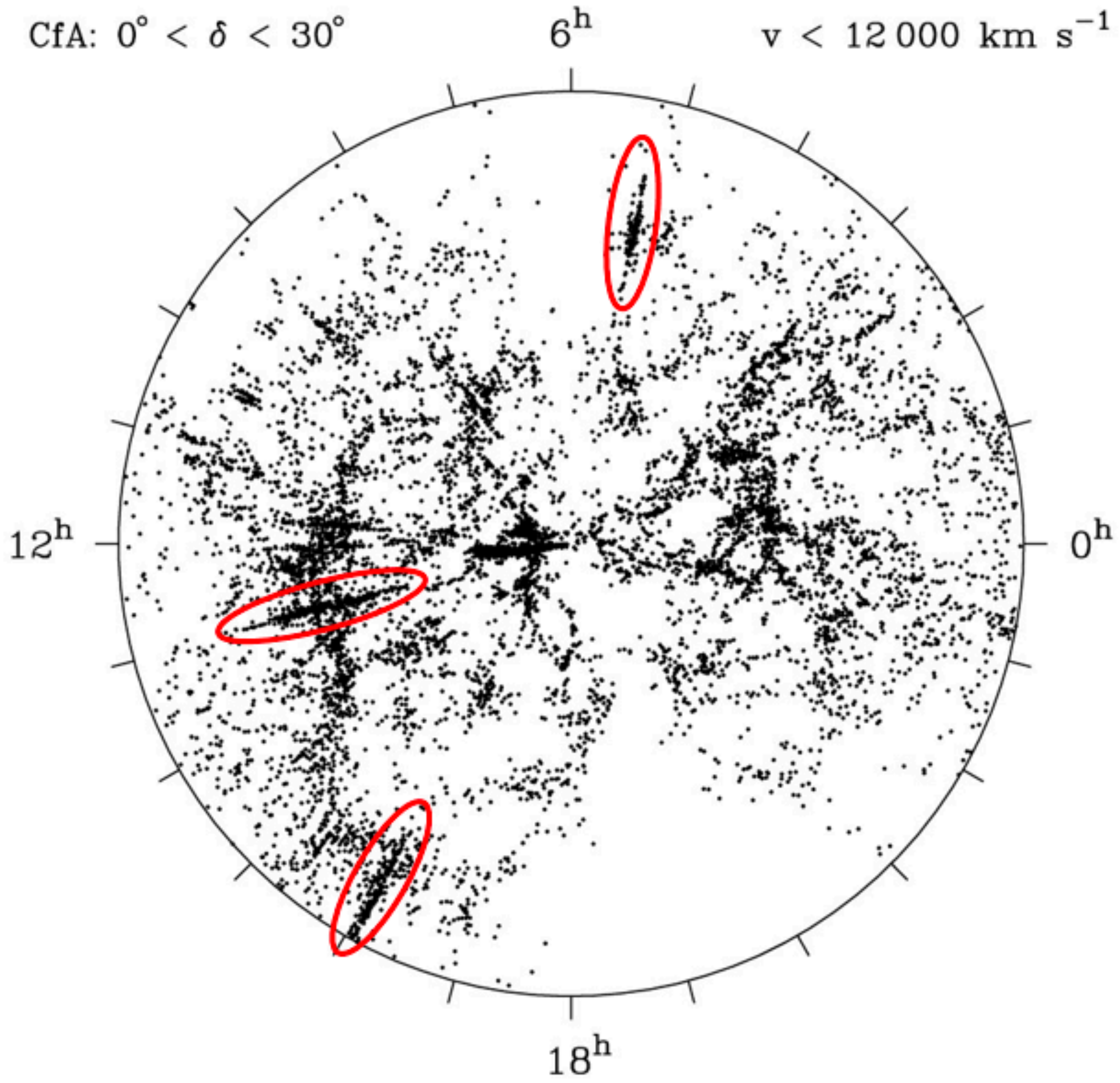


Dodelson & Schmidt 2020; Fig. 11.3

Fingers of God

Large RSD in galaxy clusters due to large internal motions





RSD on large scales

- Peculiar velocities on large scales are smaller, so RSD is more subtle
- Peculiar velocities follow from the continuity equation

$$\frac{\partial \delta}{\partial t} + a^{-1} \nabla \cdot \langle \mathbf{v} \rangle = 0\,,$$

- In Fourier space (and using dot for time derivative, v = <v>)

$$\dot{\delta} + i\, a^{-1}\, k\, v = 0$$

- Using δ ∝ $D_+(a)$ ==>

$$v = i\, \frac{\vec{k}}{k^2}\, a^2\, H\, f\, \delta$$

- Where the linear growth rate f: $f \equiv \dfrac{\mathrm{d}\ln D_+(a)}{\mathrm{d}\ln a}$; f ~ 1 for LCDM

RSD on large scales

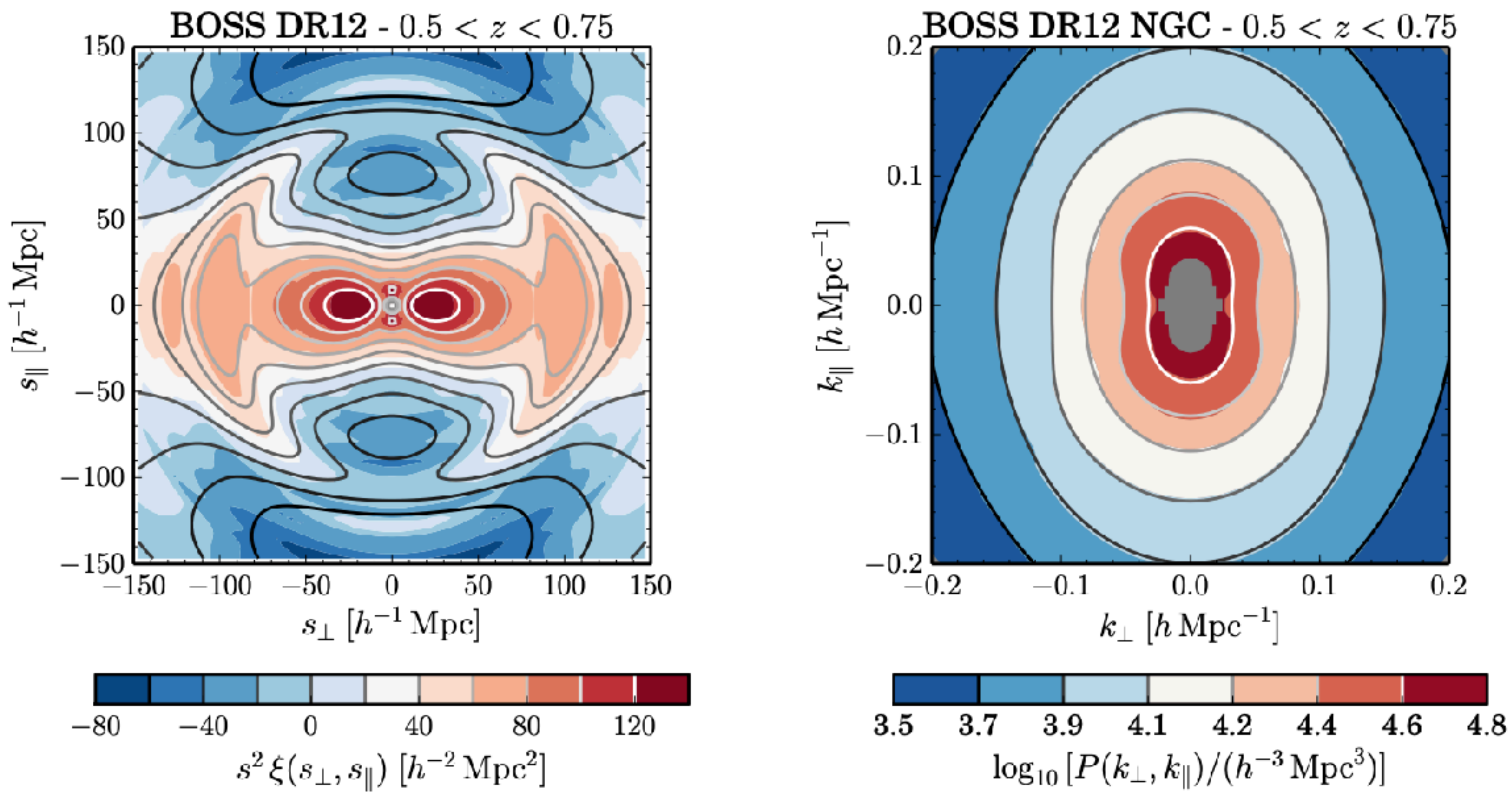
- Power spectrum and clustering becomes dependent on angle

$$P_{\mathrm{g,RSD}}(k, \mu_k, \bar{z}) = P_{\mathrm{L}}(k, \bar{z}) \left[b_1 + f \mu_k^2 \right]^2$$

- Measure either through:
    - Full 2D mapping of redshift-angle clustering (2 angles still isotropic)
    - Measuring monopole and quadrupole of the correlation function or power spectrum

RSD on large scales: full anisotropic clustering

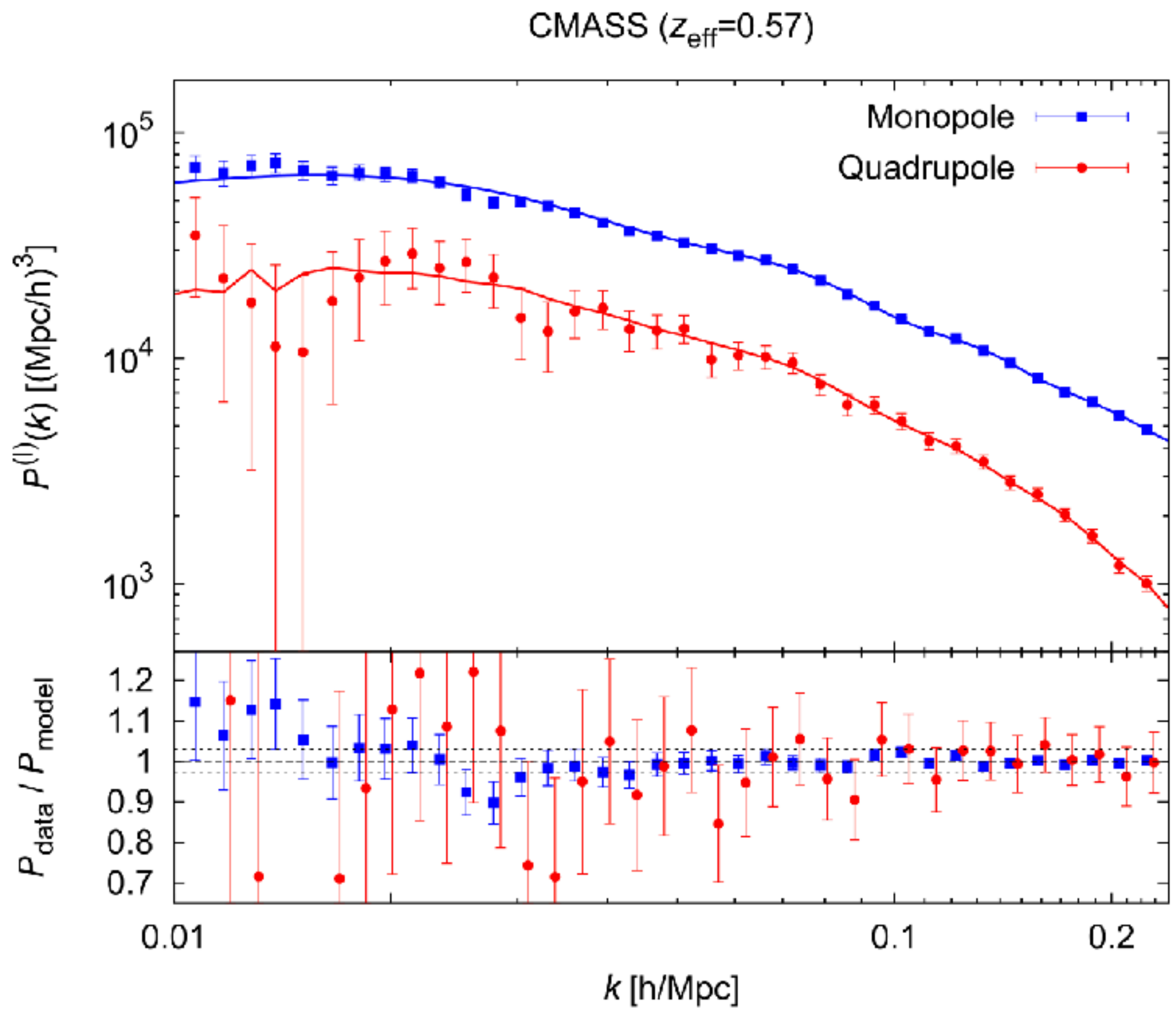
$$P_{\mathrm{g,RSD}}(k, \mu_k, \bar{z}) = P_{\mathrm{L}}(k, \bar{z}) \left[b_1 + f \mu_k^2 \right]^2$$



Alam et al. (2016; final SDSS-III)

RSD on large scales: monopole, quadruple

$$P_{\mathrm{g,RSD}}(k, \mu_k, \bar{z}) = P_{\mathrm{L}}(k, \bar{z}) \left[b_1 + f \mu_k^2 \right]^2$$



Dodelson & Schmidt (2020; Fig. 11.4); originally from Gil-Marin et al. (2016; BOSS)

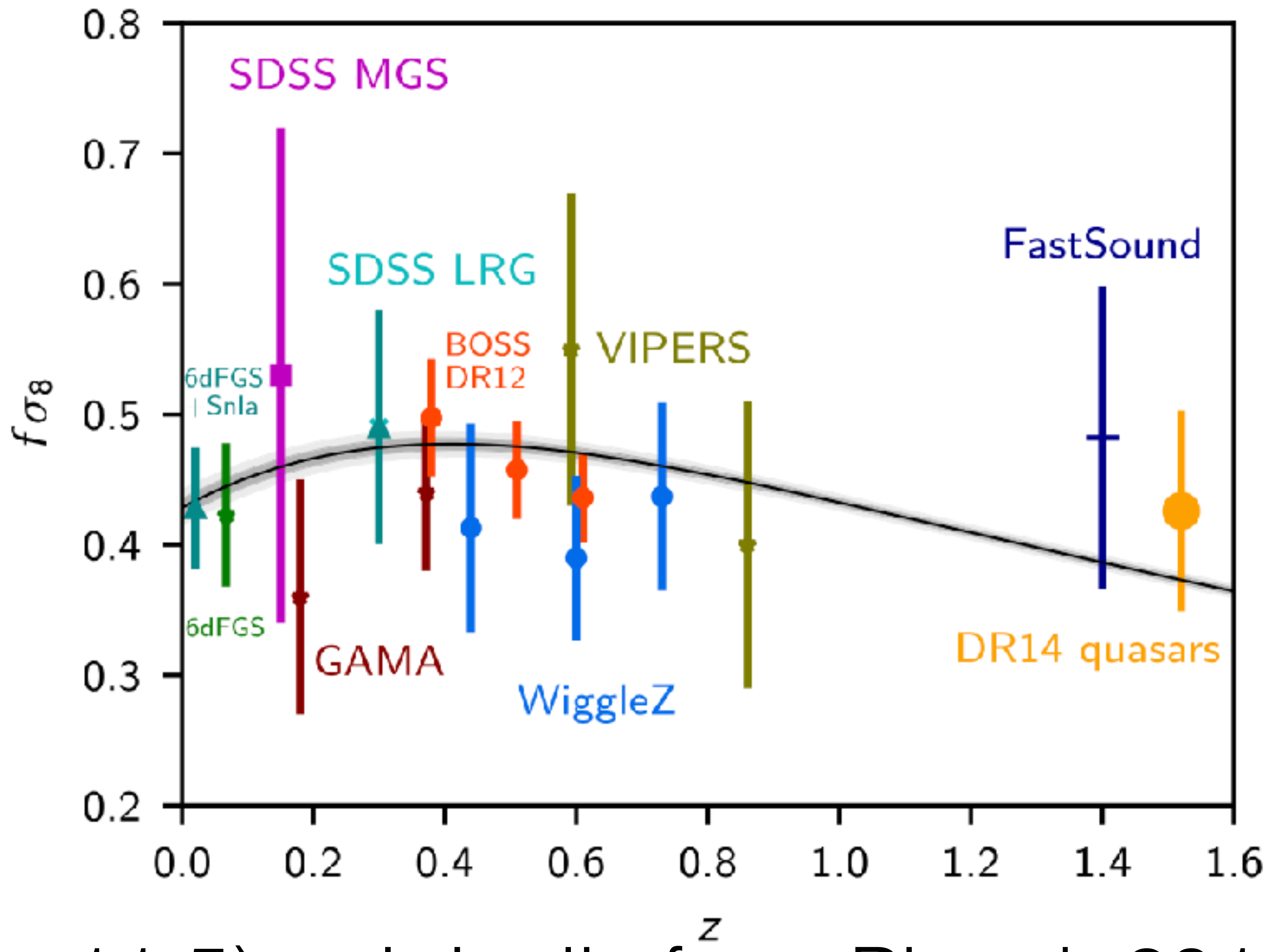
Cosmological constraints from RSD

$$P_{\mathrm{g,RSD}}(k, \mu_k, \bar{z}) = P_{\mathrm{L}}(k, \bar{z}) \left[b_1 + f \mu_k^2 \right]^2$$

- Amplitude of anisotropic part is proportional to f^2 x amplitude of the primordial power spectrum

- Amplitude of the primordial power spectrum generally parameterized through σ_8

- So we can measure $f\sigma_8$ from RSD



Dodelson & Schmidt (2020; Fig. 11.5); originally from Planck 2018

Galaxy clustering - Alcock-Paczynski effect and BAO

The Alcock-Paczynski effect

- In RSD discussion, we assumed we were able to map redshift to the correct comoving distance
- But we don't know the correct mapping, because it depends on the cosmological model
- Using the wrong mapping affects line-of-sight and transverse directions differently, so induces anisotropy in the galaxy clustering signal
- Anisotropy depends on the shape of the power-spectrum, so can be disentangled from RSD (which only affected the amplitude)
- First pointed out by Alcock & Paczynski (1979)

The Alcock-Paczynski effect in more detail

(See Dodelson & Schmidt 2020 Sec. 11.1.3)

- Define coordinate system such that origin is at the angular and redshift centre of the survey

- Transverse components (χ is the comoving distance, but more generally should really be the angular diameter distance / a):

$$(x^1, x^2) = \chi(z) \times (\theta^1, \theta^2) = \left[1 - \frac{\delta\chi(z)}{\chi_{\rm fid}(z)}\right](x^1_{\rm obs}, x^2_{\rm obs}) .$$

- Line-of-sight component:

$$x^3(z) \simeq \frac{1}{H(\bar{z})}(z - \bar{z}) = \frac{H_{\rm fid}(\bar{z})}{H(\bar{z})} x^3_{\rm obs}.$$

- So need to evaluate intrinsic correlation function / power spectrum at

$$\boldsymbol{x}(\boldsymbol{x}_{\rm obs}) = \left([1 - \alpha_\perp]x^1_{\rm obs},\ [1 - \alpha_\perp]x^2_{\rm obs},\ [1 - \alpha_\parallel]x^3_{\rm obs}\right),$$

$$\alpha_\perp = \frac{\delta\chi}{\chi_{\rm fid}}\Big|_{\bar{z}}; \quad \alpha_\parallel = \frac{\delta H}{H_{\rm fid}}\Big|_{\bar{z}}.$$

Combined effect of RSD and the Alcock-Paczynski effect

- Observed power spectrum becomes

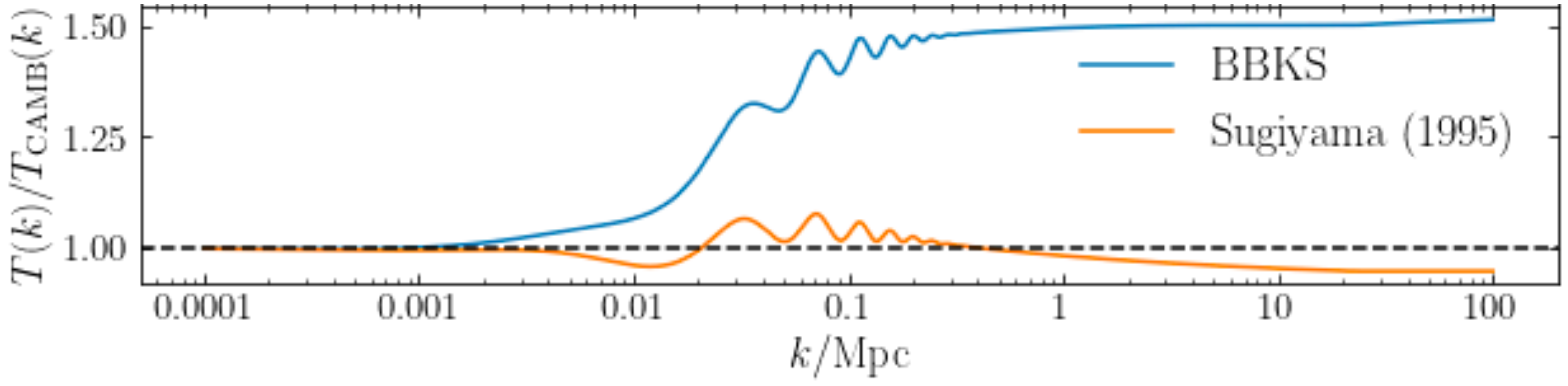
$$P_{\mathrm{g,obs}}(\boldsymbol{k}_{\mathrm{obs}}, \bar{z}) = \left(P_{\mathrm{L}}(k, \bar{z}) \left[b_1 + f \mu_k^2 \right]^2 \right) \Big|_{\boldsymbol{k}=\left([1+\alpha_\perp]k^1_{\mathrm{obs}},\ [1+\alpha_\perp]k^2_{\mathrm{obs}},\ [1+\alpha_\parallel]k^3_{\mathrm{obs}}\right)}$$

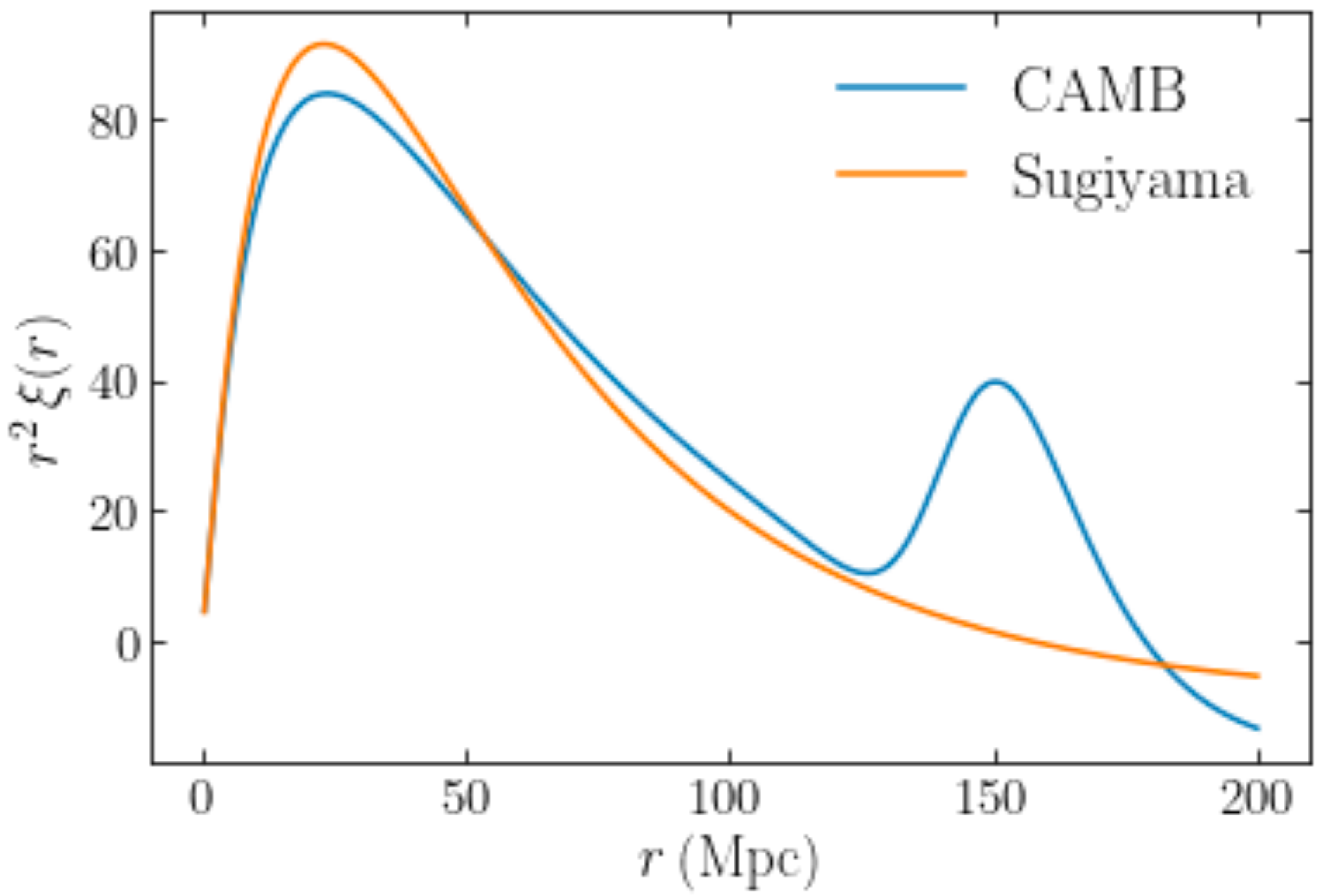
- So can measure $f\sigma_8$, $D_A(z)$, and $H(z)$ —> all contain valuable information about the cosmological parameters

- Because RSD only affects the amplitude of the anisotropy, while AP changes its shape, can disentangle the two

Baryon acoustic oscillations

- Determining the AP effect not straightforward for smooth power spectrum / correlation function
- Luckily, the acoustic oscillations from the early Universe survive at a small level in the power spectrum / correlation function today
- These oscillations set a true scale that is shifted by the AP effect





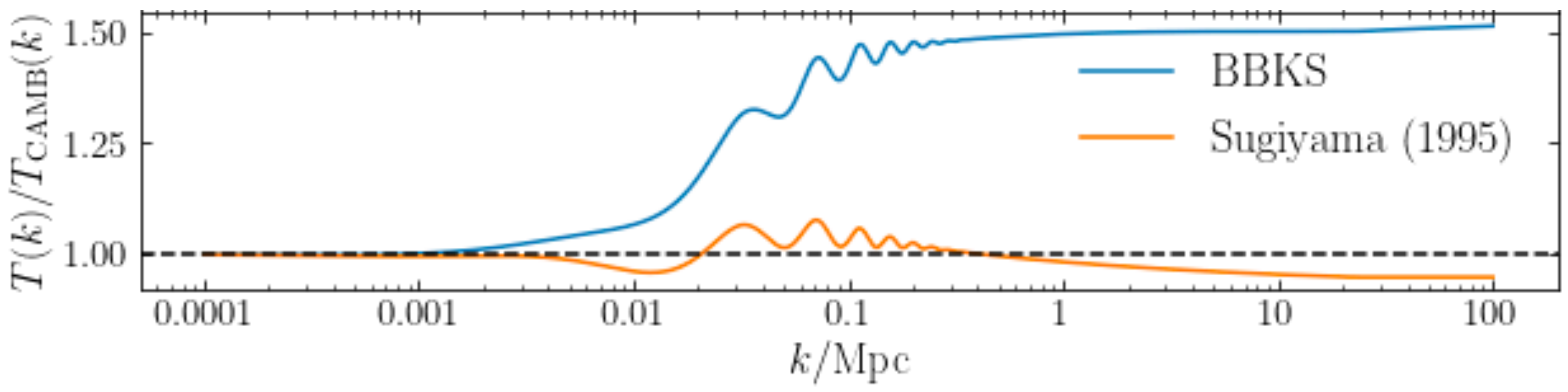
Baryon acoustic oscillations

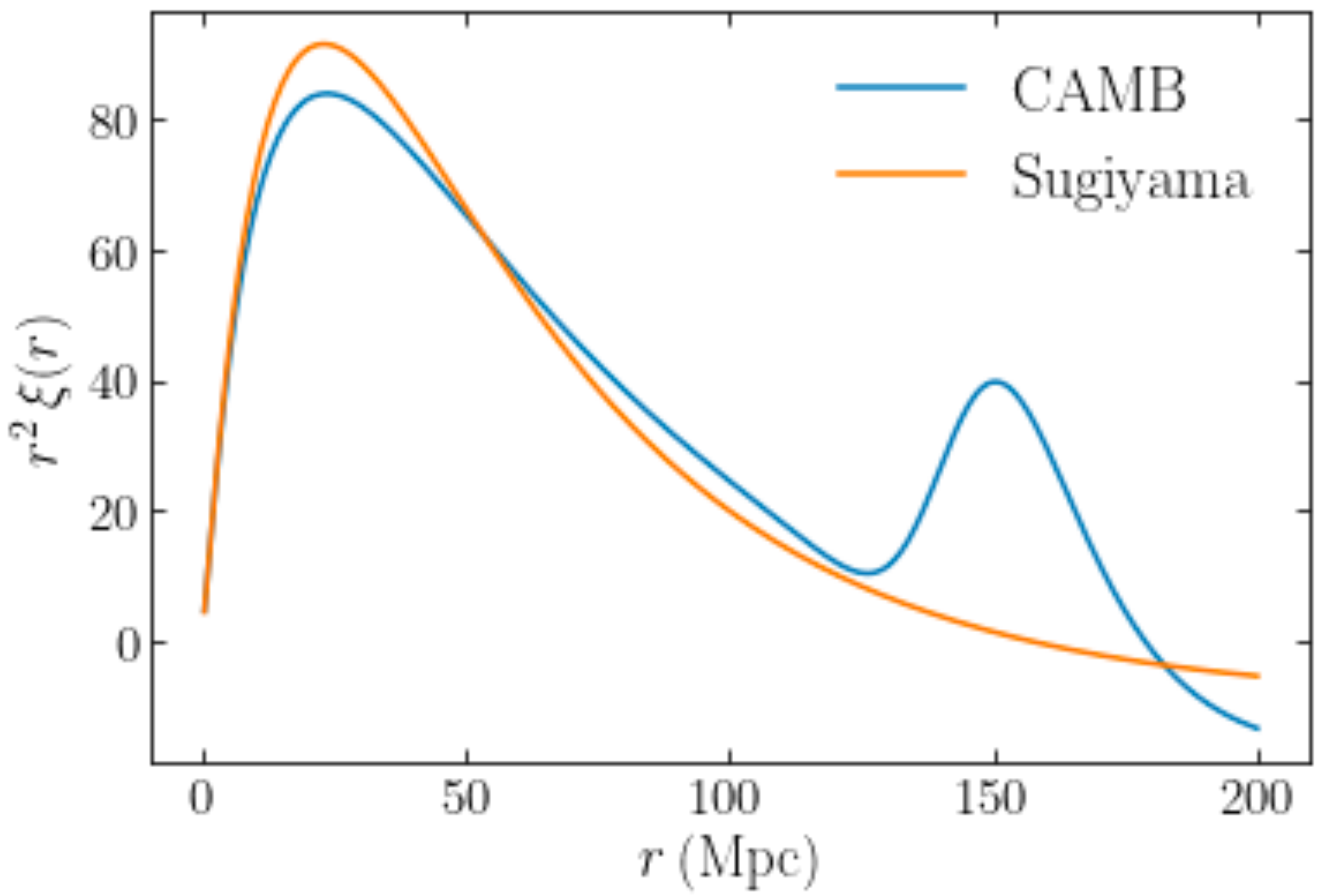
- Acoustic oscillations ~ $\cos(kr_s)$, where r_s is the sound horizon at recombination (~150 comoving Mpc) —> frozen in at later times

- Measuring the oscillation scale at low redshift z gives:

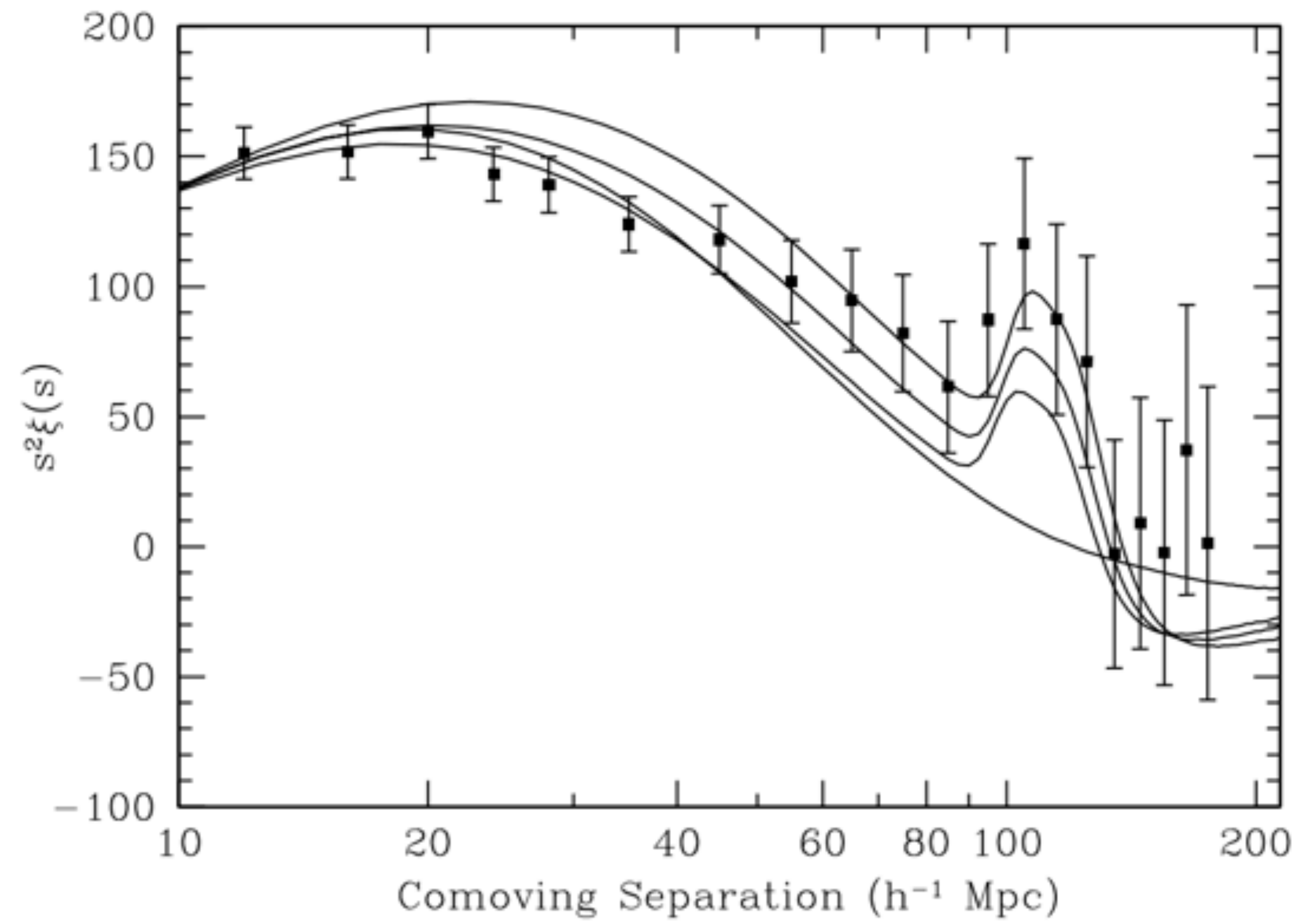
  - Isotropic part: $$D_V(z) = \left[D_M(z)^2 \frac{cz}{H(z)} \right]^{1/3},$$ $D_M(z) = D_A(z)[1+z]$

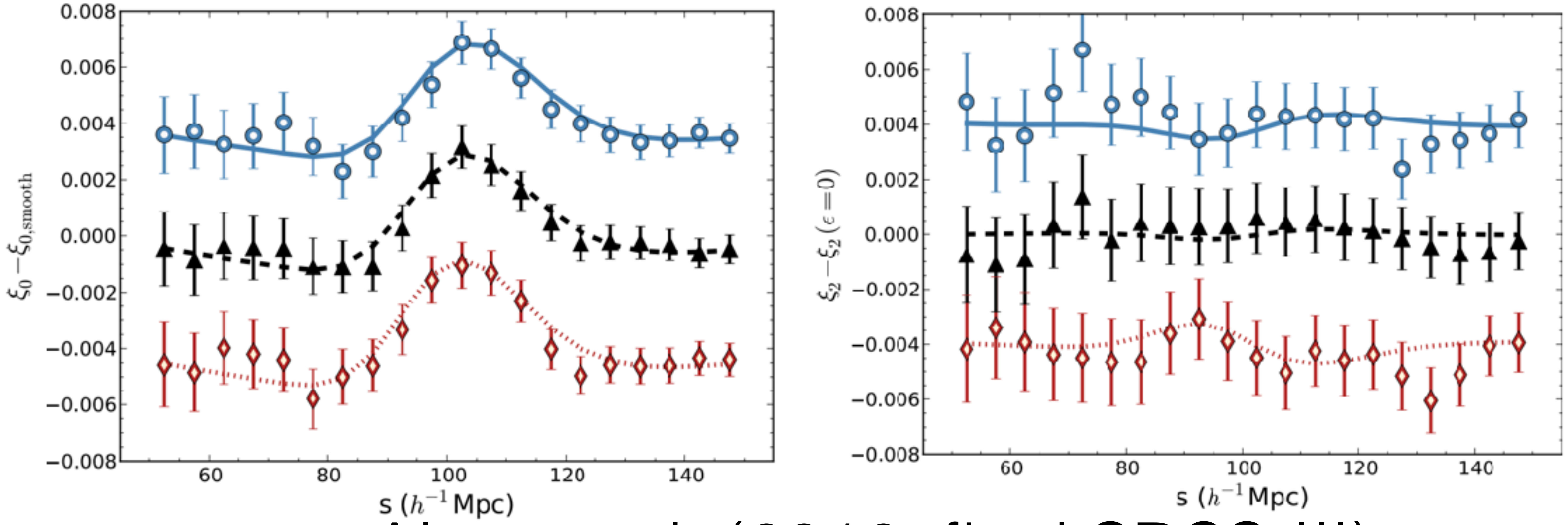
  - Anisotropic: $D_A(z)$ and $H(z)$ separately





Baryon acoustic oscillations

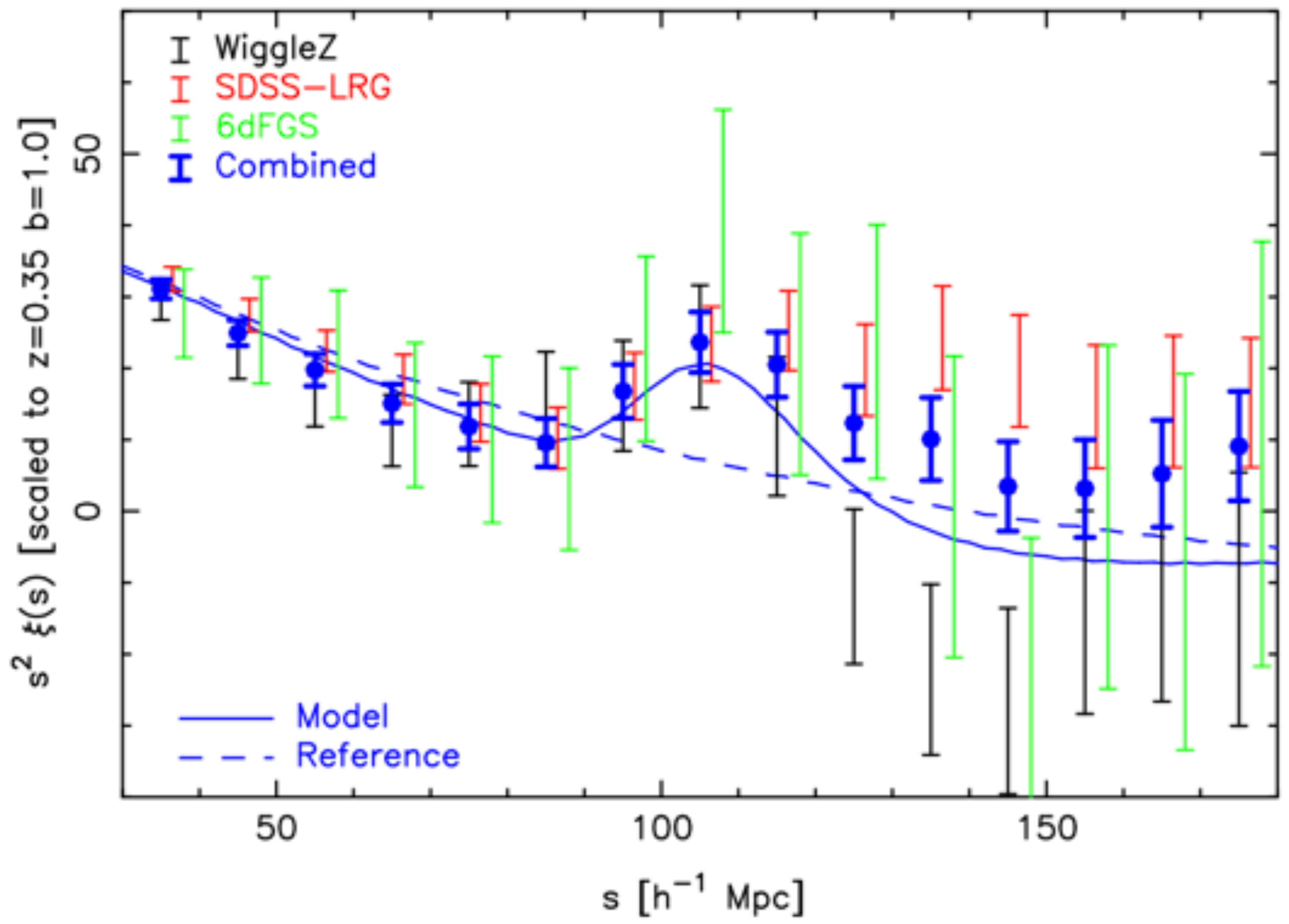




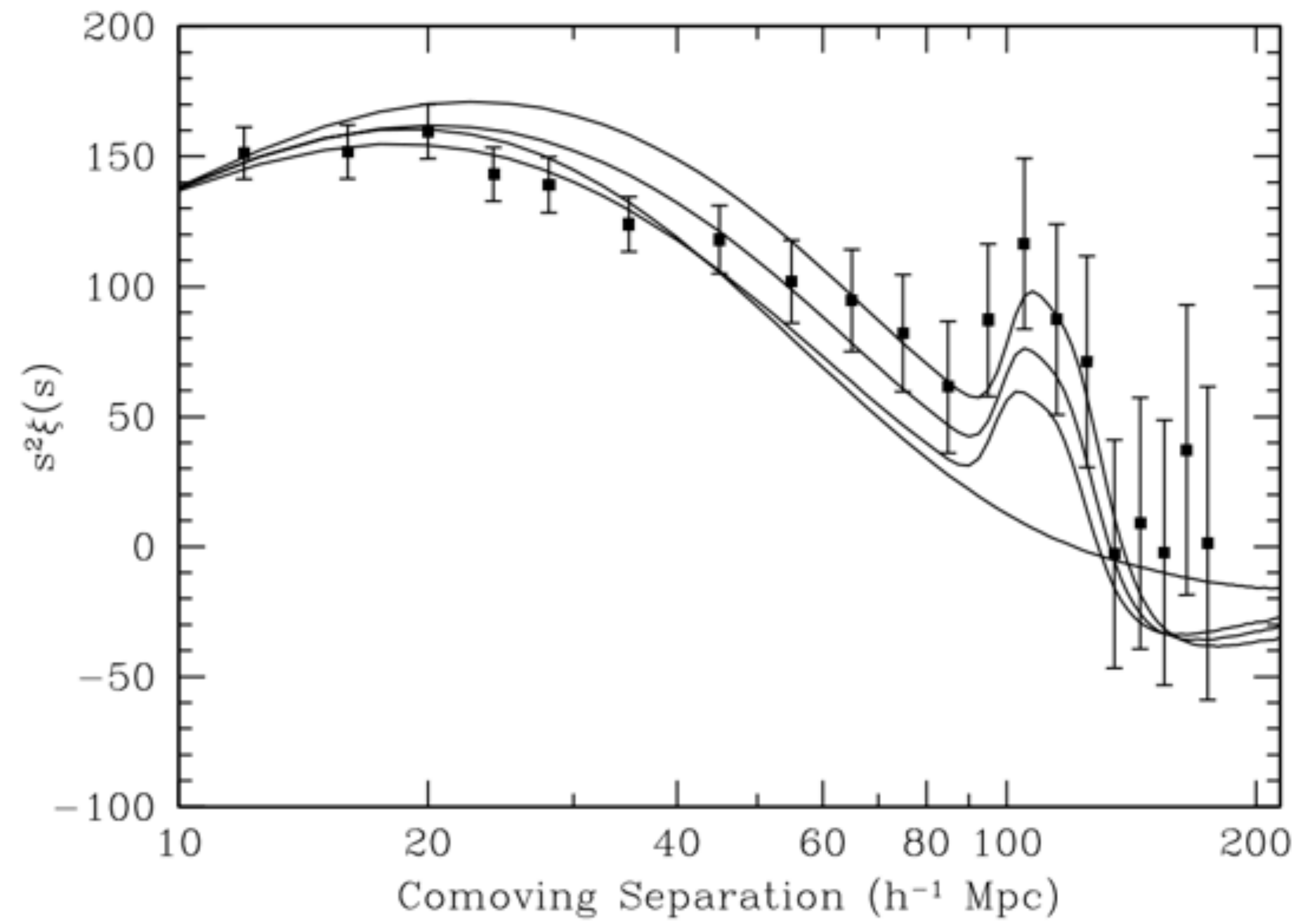
Alam et al. (2016; final SDSS-III)

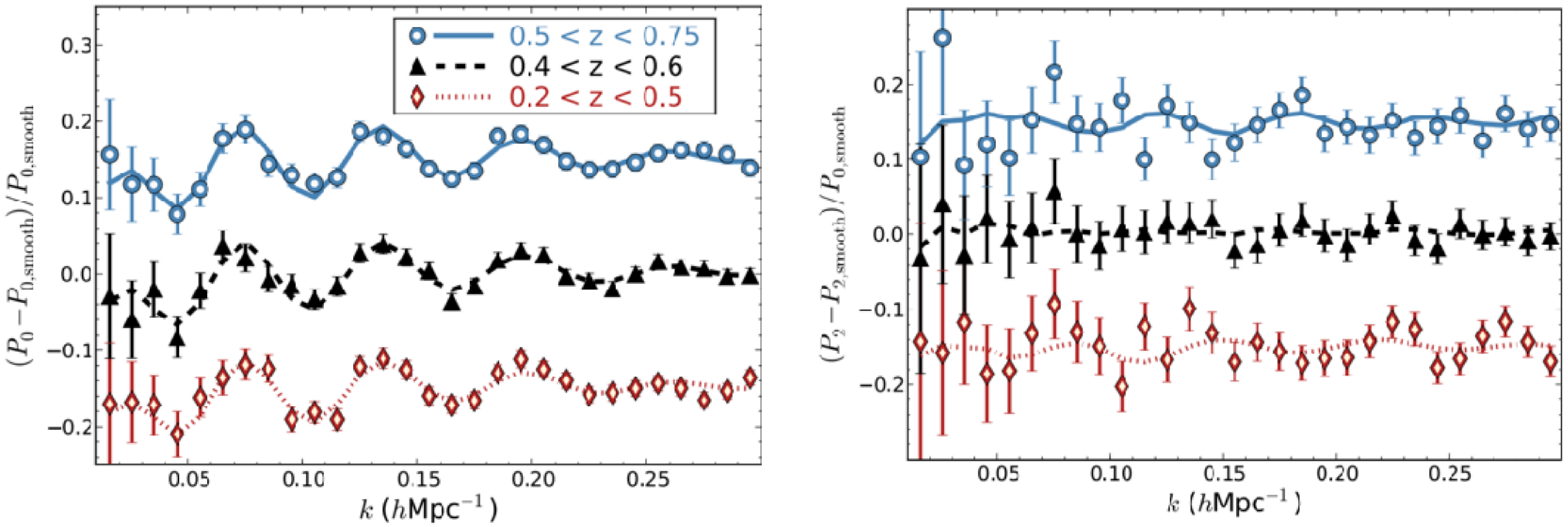
Eisenstein et al. (2005; SDSS LRGs)
See also Cole et al. (2005; 2dFGRS)

Blake et al. (2012; WiggleZ)



Baryon acoustic oscillations





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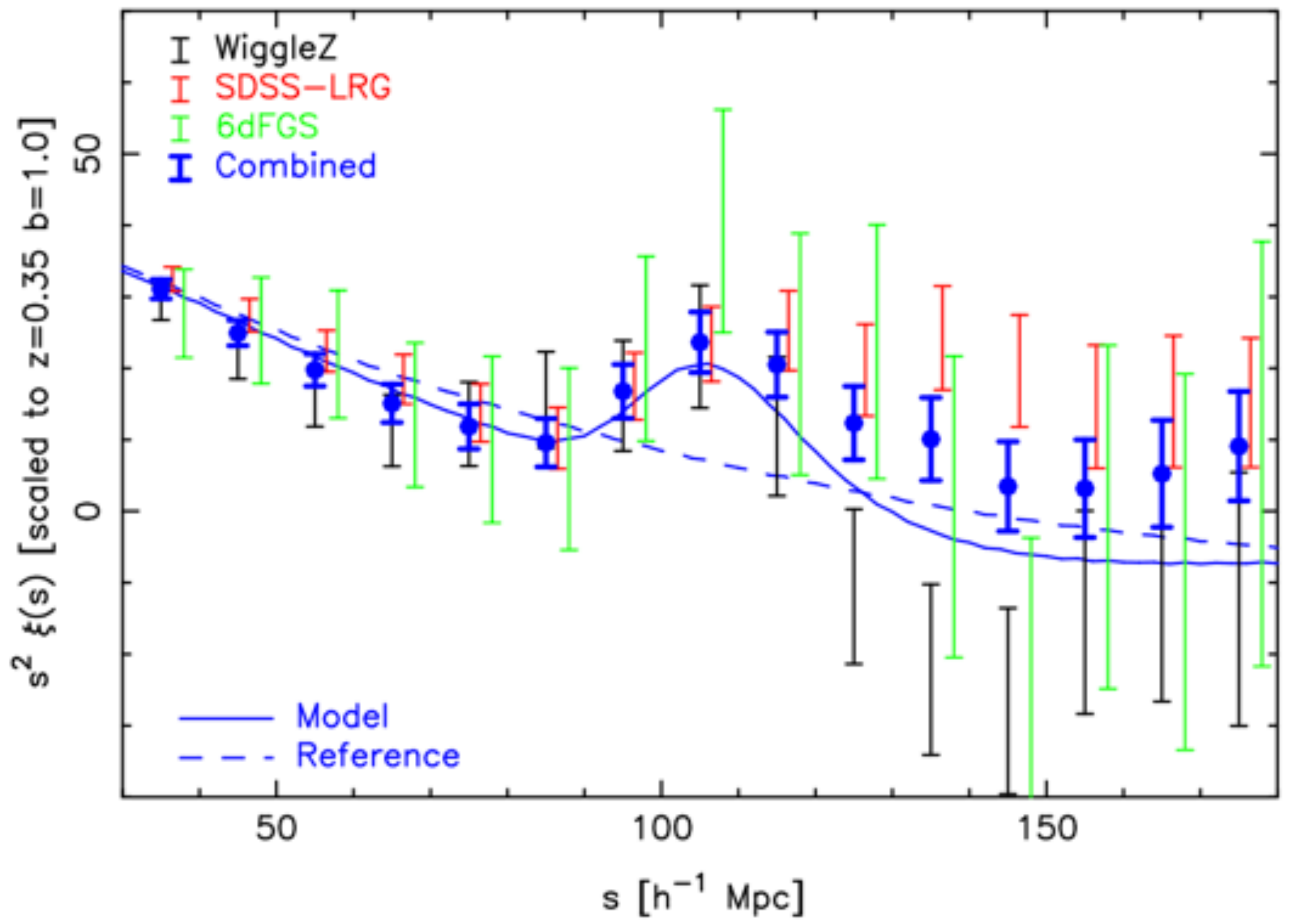

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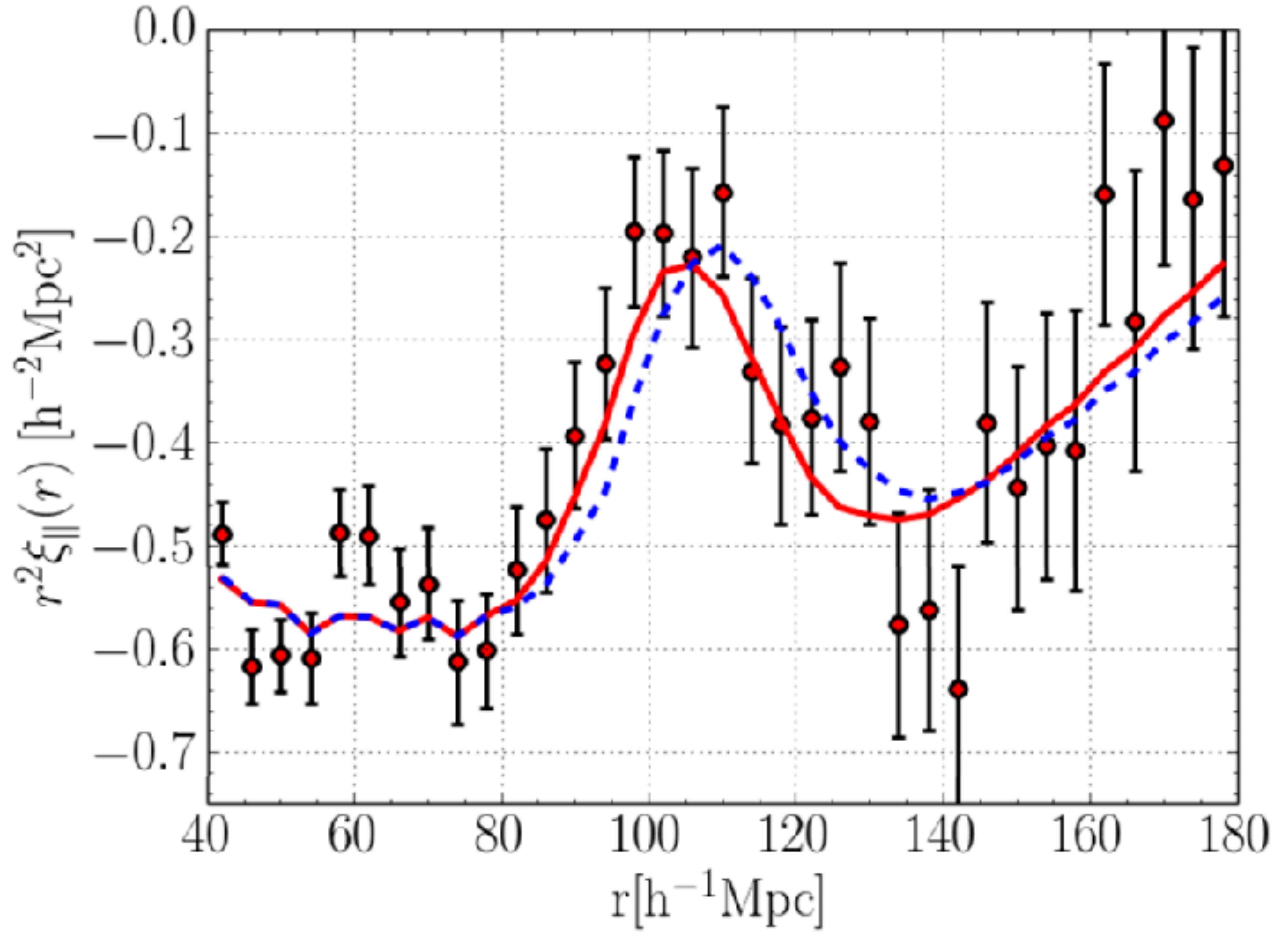
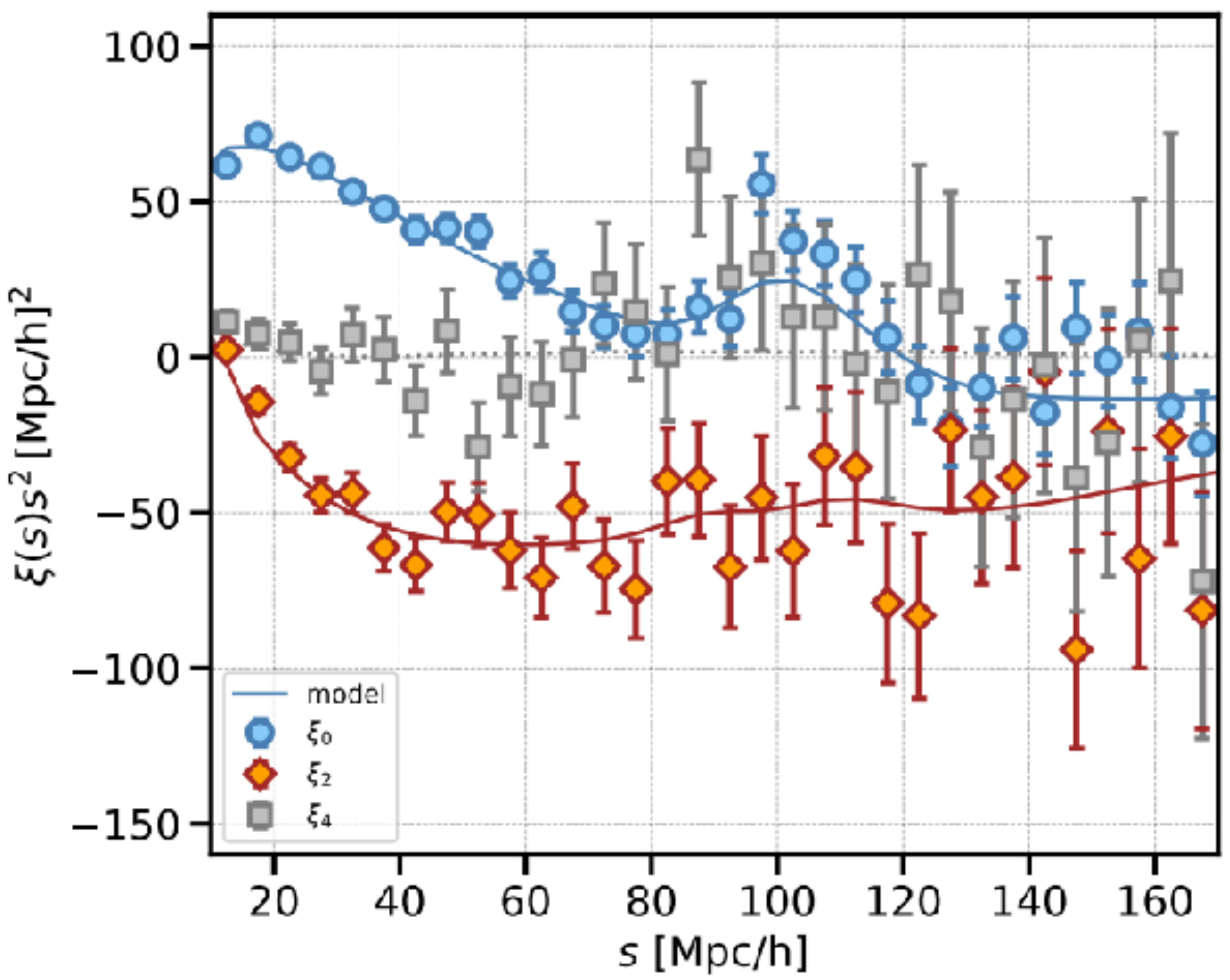
Baryon acoustic oscillations

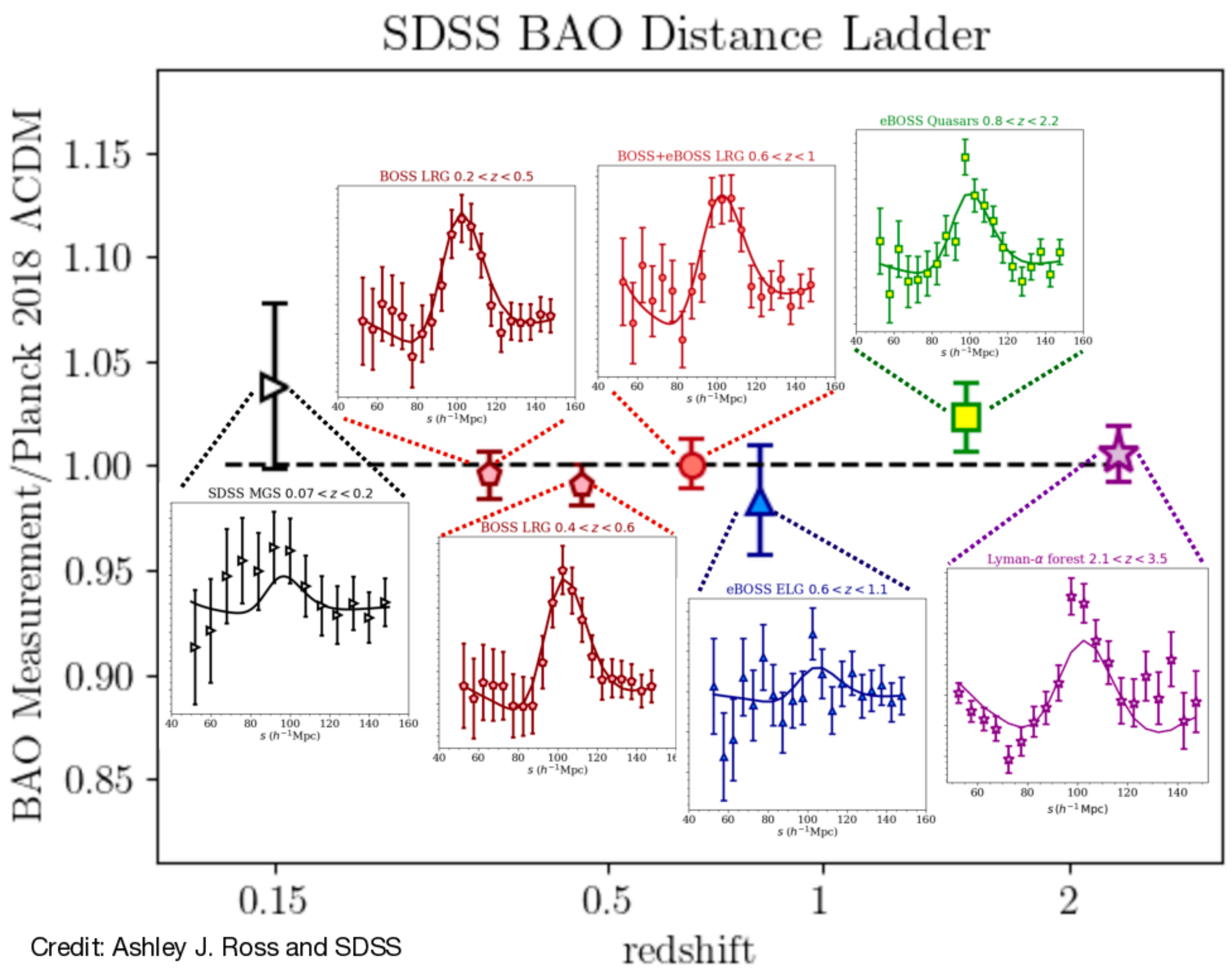




Hou et al. (2021; eBOSS z~1.5 quasars)

Delubac et al. (2015; BOSS z~2 Lya forest)

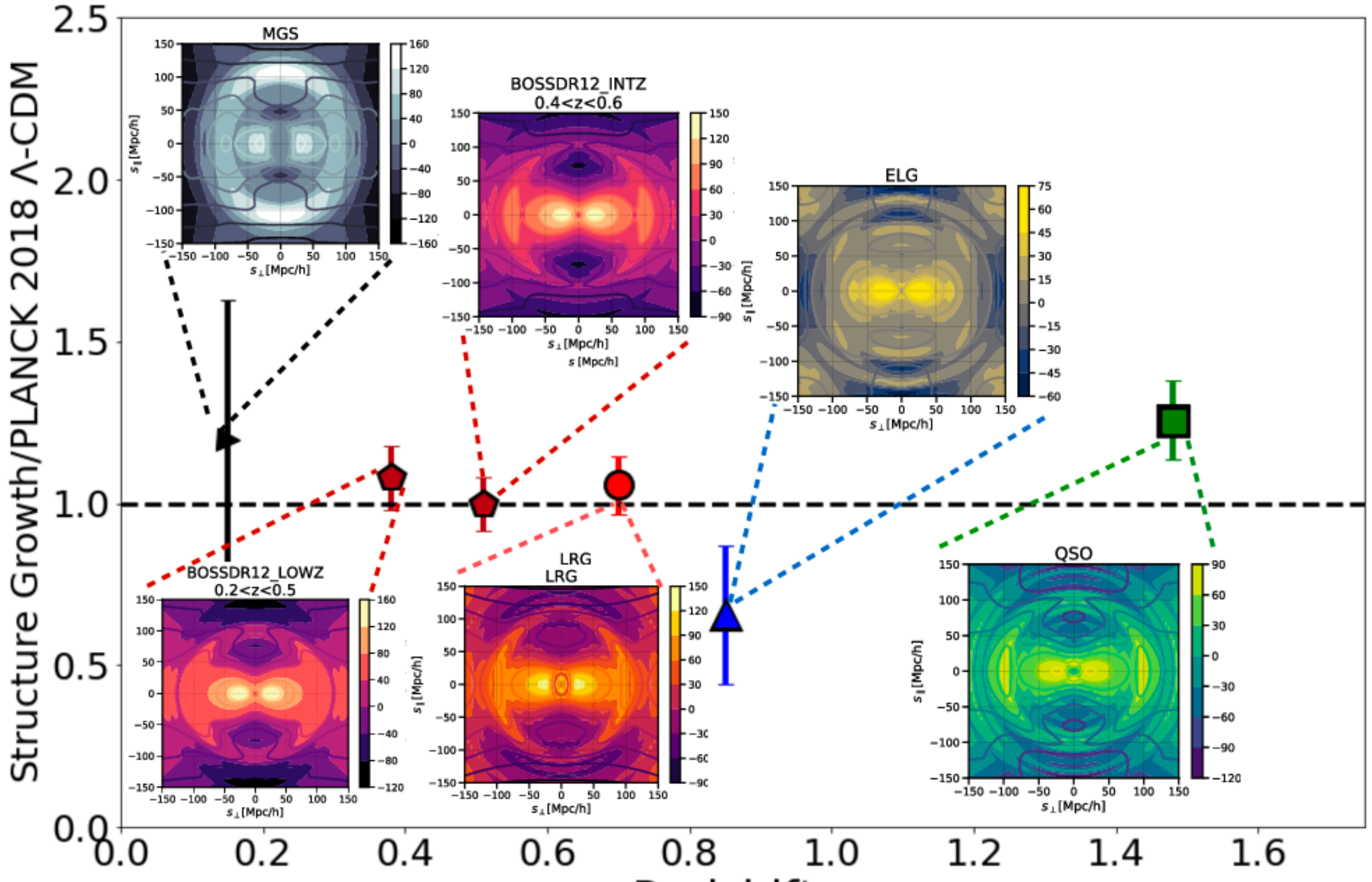
Baryon acoustic oscillations



Credit: Ashley J. Ross and SDSS

https://www.sdss4.org/science/cosmology-results-from-eboss/

RSD growth of structure



Credit: M.Vargas-Magaña and SDSS.

https://www.sdss4.org/science/cosmology-results-from-eboss/