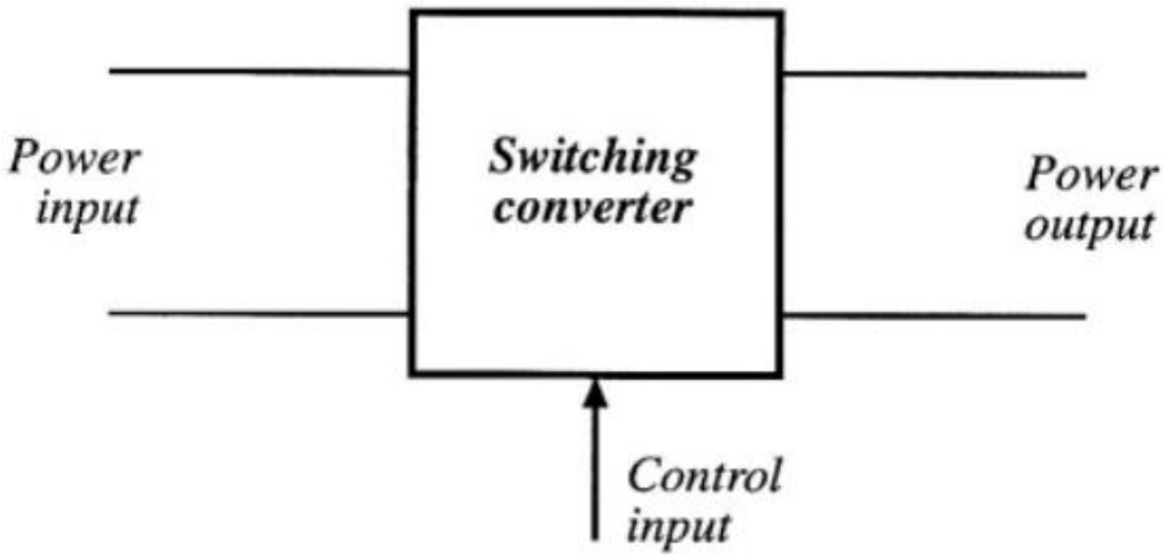
Chapter 1

DC-DC Converters

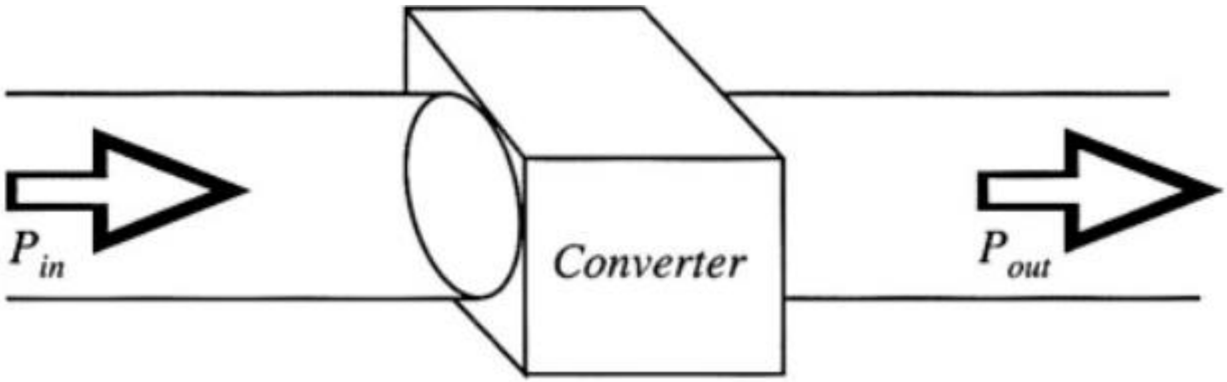
Buck Converter

Introduction



- Power electronics is concerned with the processing of electrical power using electronic devices.
- The key element is switching converter.

Converter Efficiency



$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{loss} = P_{in} - P_{out} = P_{out}\left(\frac{1}{\eta} - 1\right)$$

- High efficiency is essential in any power processing application. The key element is switching converter.
- The power loss is converted into heat which must be removed from the converter…. this leads to a large and expensive cooling system.

Efficiency

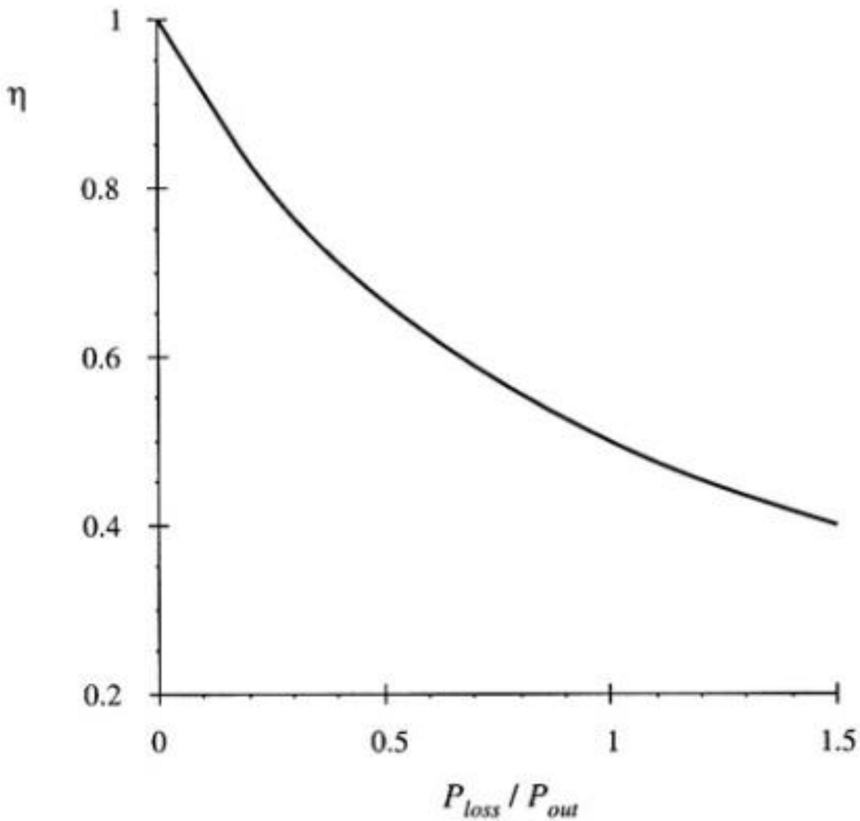


Fig. 1.3 Converter power loss vs. efficiency.

- Increasing the efficiency is the key to obtaining higher output powers.
- If converter can processes a large amount of power with very high efficiency, very little power is lost, the converter elements can be packaged with high density, leading to a converter of small size and weight, and of low temperature rise.

Converter Efficiency

- How can we build a circuit that changes the voltage with high efficiency ?

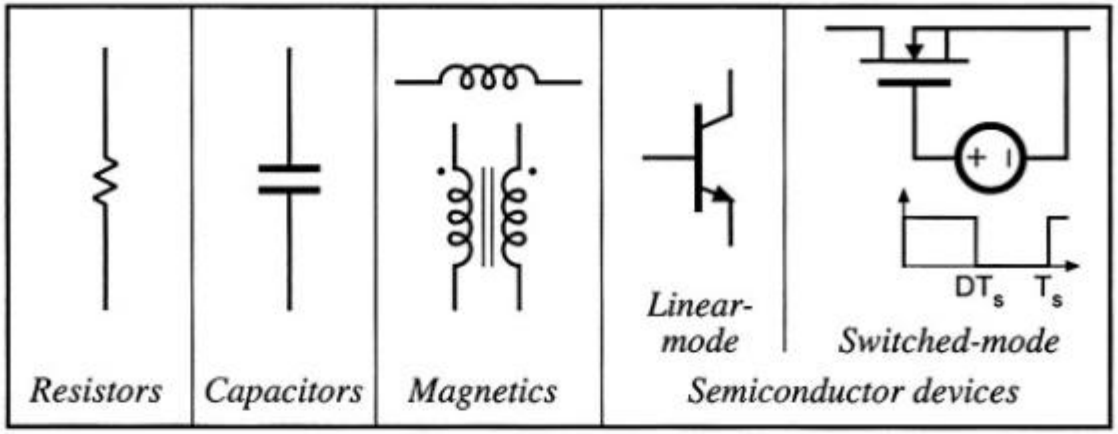


Fig. 1.5 Devices available to the circuit designer [2].

Efficiency

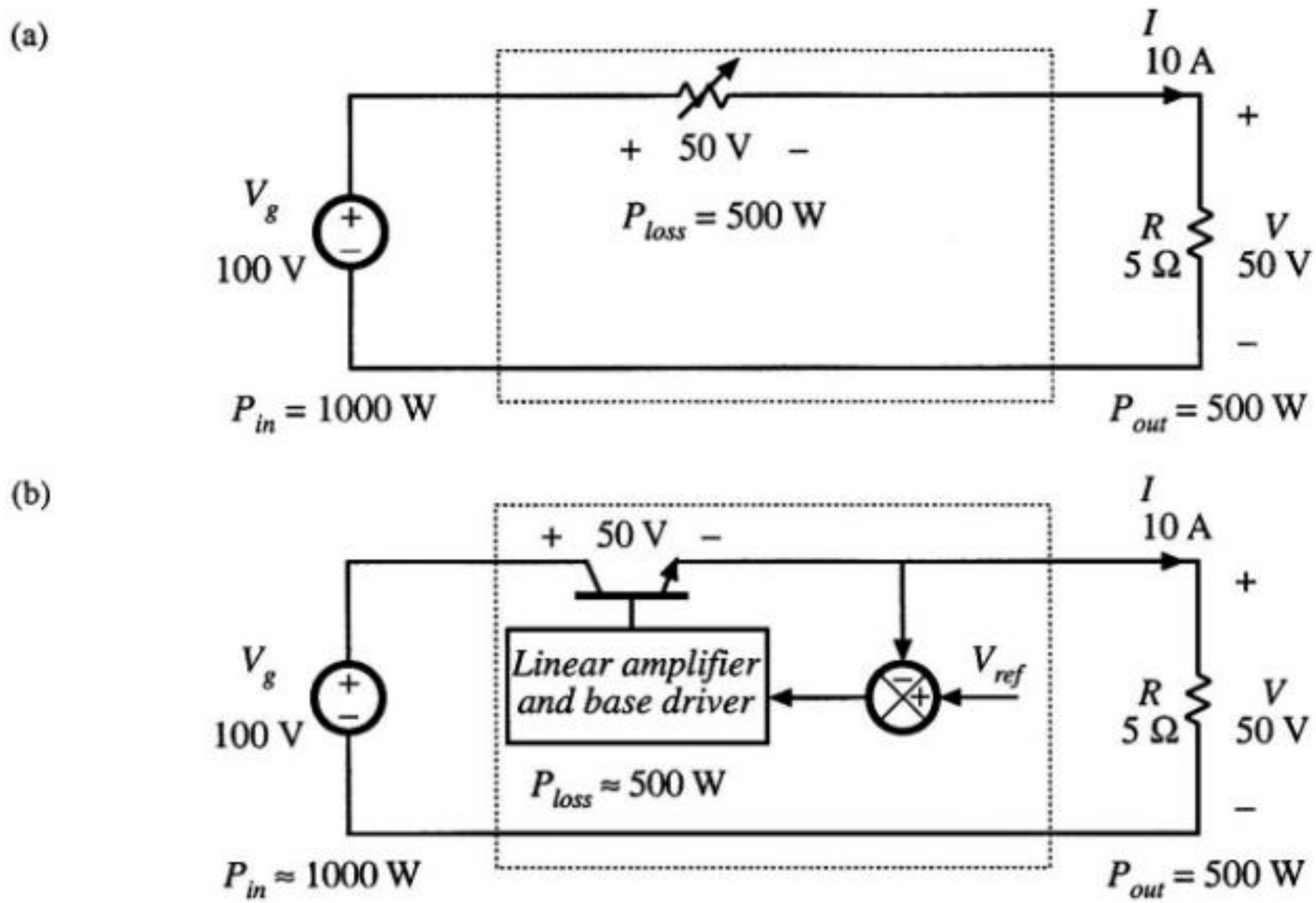


Fig. 1.7 Changing the dc voltage via dissipative means: (a) voltage divider, (b) series pass regulator.

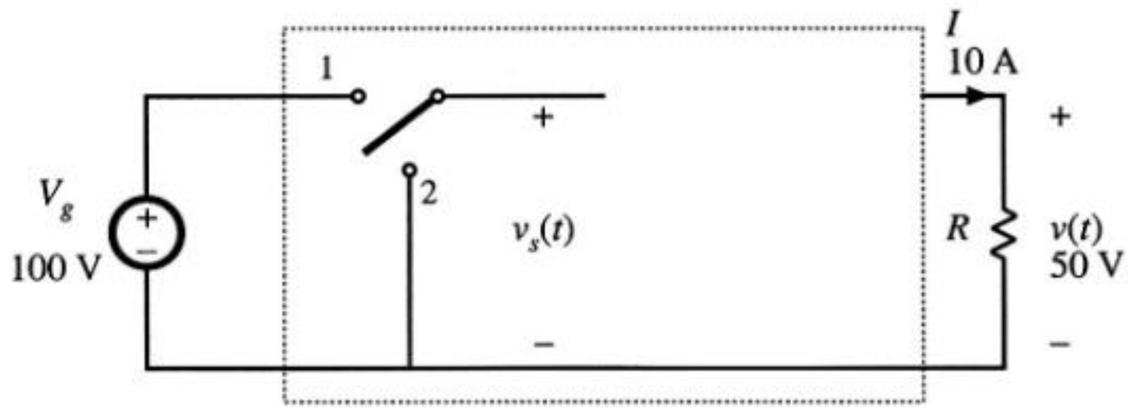


Fig. 1.8 Insertion of SPDT switch which changes the dc component of the voltage.

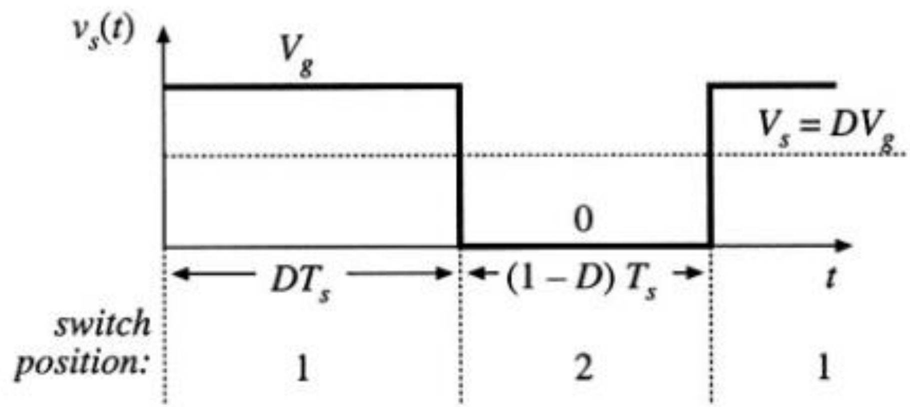


Fig. 1.9 Switch output voltage waveform $v_s(t)$.

Efficiency with Switched-Mode Semiconductor Devices

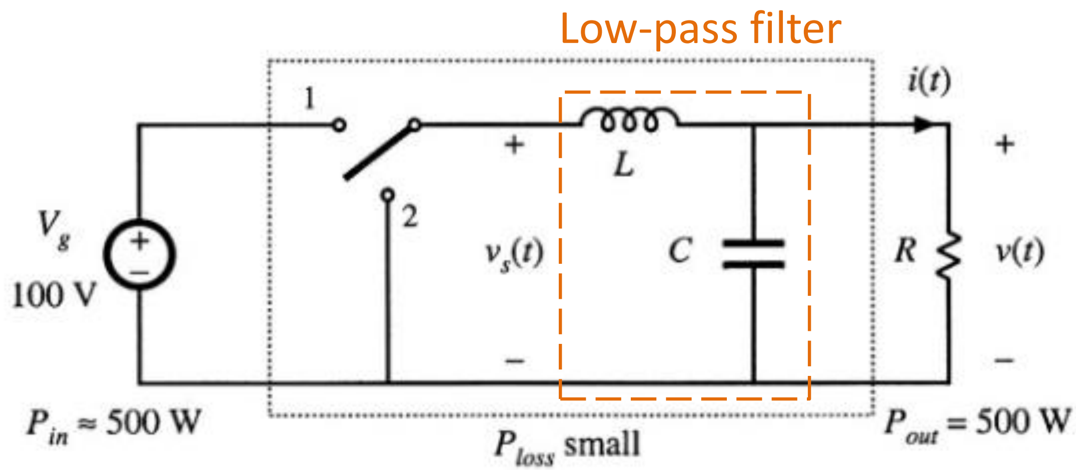


Fig. 1.10 Addition of L–C low-pass filter, for removal of switching harmonics.

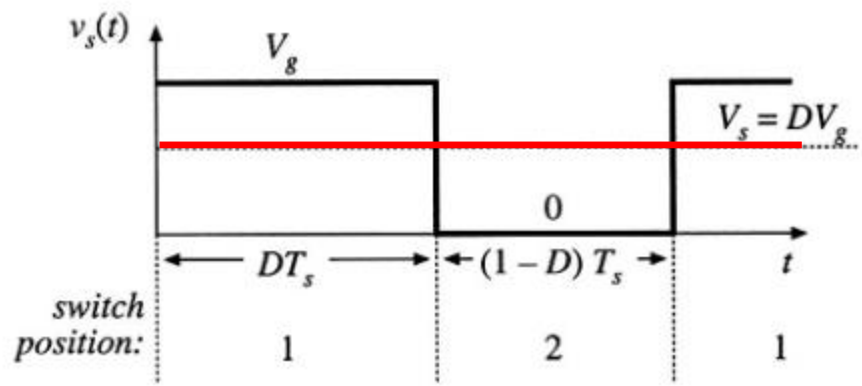


Fig. 1.9 Switch output voltage waveform $v_s(t)$.

- Power dissipated by the switched-mode semiconductor device is ideally zero.
- Capacitors and magnetic devices do not consume power.

- To the extent that the switch, inductor and capacitor elements are ideal, the efficiency of this dc-dc converter can approach 100%

Efficiency with Switched-Mode Semiconductor Devices

Low-pass filter

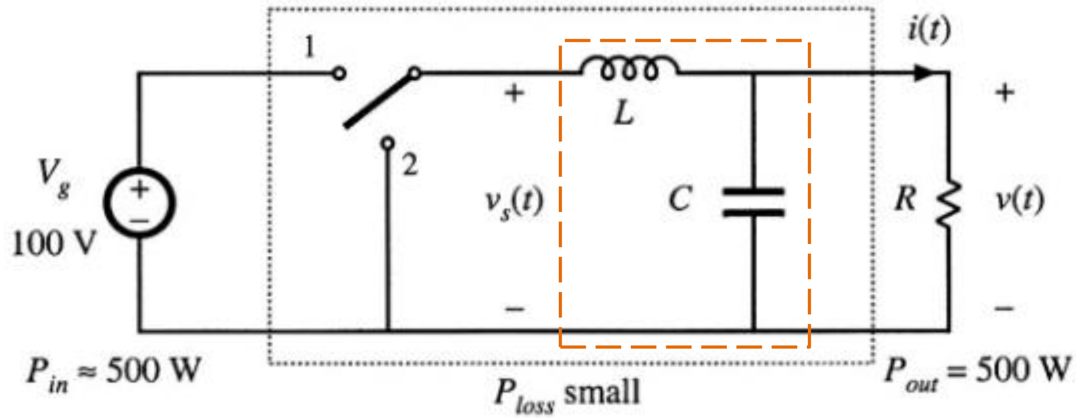


Fig. 1.10 Addition of *L–C* low-pass filter, for removal of switching harmonics.

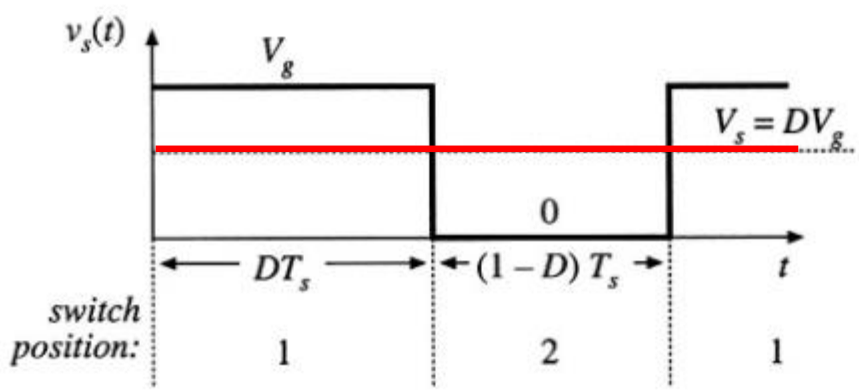


Fig. 1.9 Switch output voltage waveform $v_s(t)$.

- Power dissipated by the switched-mode semiconductor device is ideally zero.
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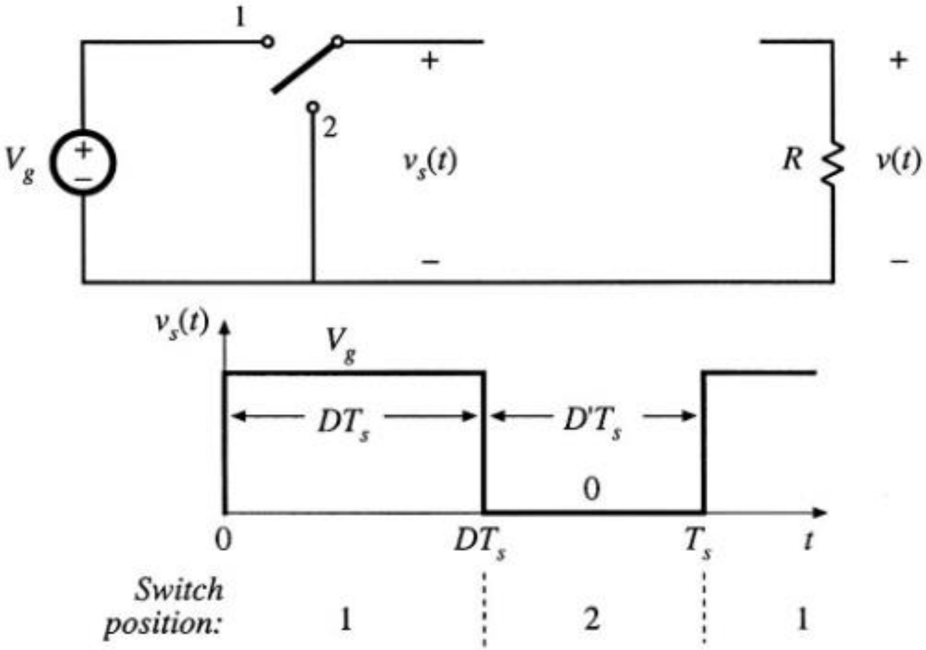
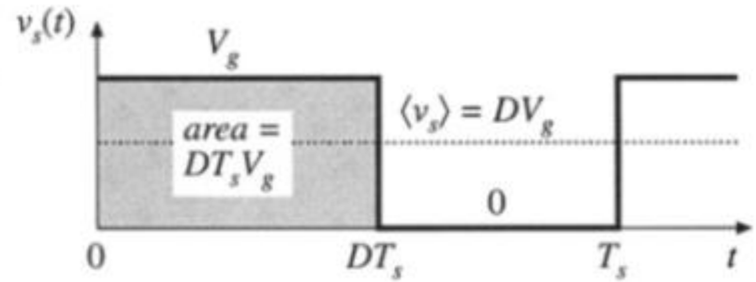
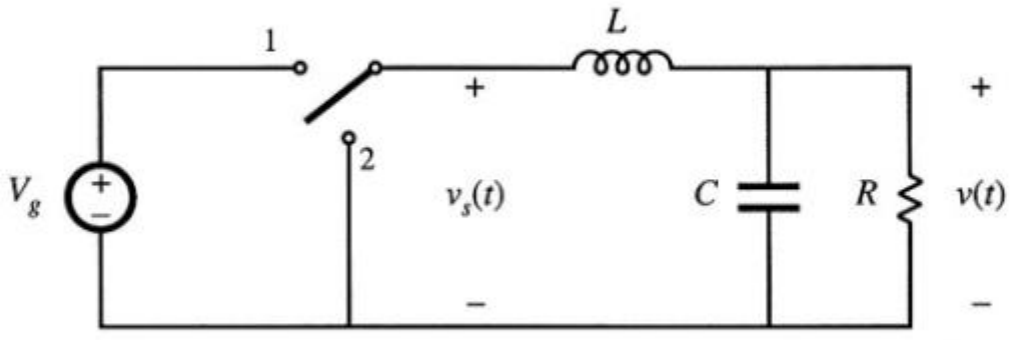
Buck Converter: Steady-State Converter Analysis





- *fs* is the switching frequency, generally lies in the range of 1 kHz to 1 MHz.
- Ts (=1/fs) is the switching period.
- D is the fraction of time that the switch spends in position 1 (0 <= D <= 1). D’ = 1-D

Buck Converter : Steady-State Converter Analysis



$$\langle v_s \rangle = \frac{1}{T_s}\int_0^{T_s} v_s(t)\,dt$$

$$\langle v_s \rangle = \frac{1}{T_s}\left(DT_sV_g\right) = DV_g$$

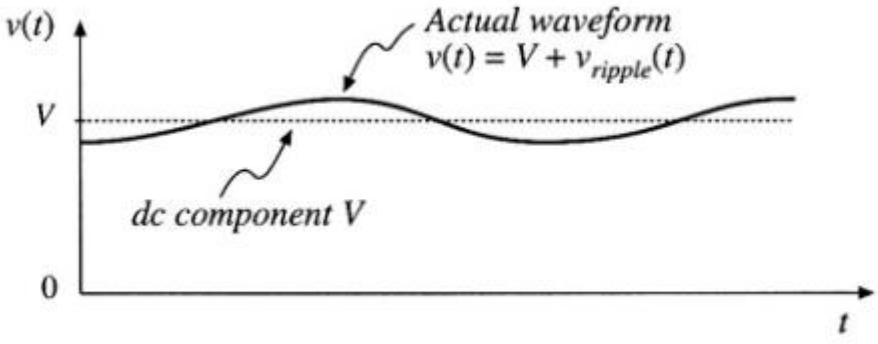
$$\boxed{v \approx \langle v_s \rangle = DV_g}$$

- *The filter is designed to pass the dc component of vs(t), but to reject the components of vs(t) at the switching frequency.*
- The output voltage v(t) is then essentially equal to the dc component of vs(t)
- D is the fraction of time that the switch spends in position 1 (0 <= D <= 1). D' = 1-D

Small-Ripple Approximation and Inductor Volt-Sec Balance

Small-Ripple Approximation

- In reality, the low-pass filter is not perfect to pass dc component and completely remove the component at the switching frequency.
- So the low-pass filter must allow at least some small amount of the high-frequency components.
- v_{ripple} arising from the incomplete attenuation of the switching components by the low-pass filter.



$$v(t) = V + v_{ripple}(t)$$

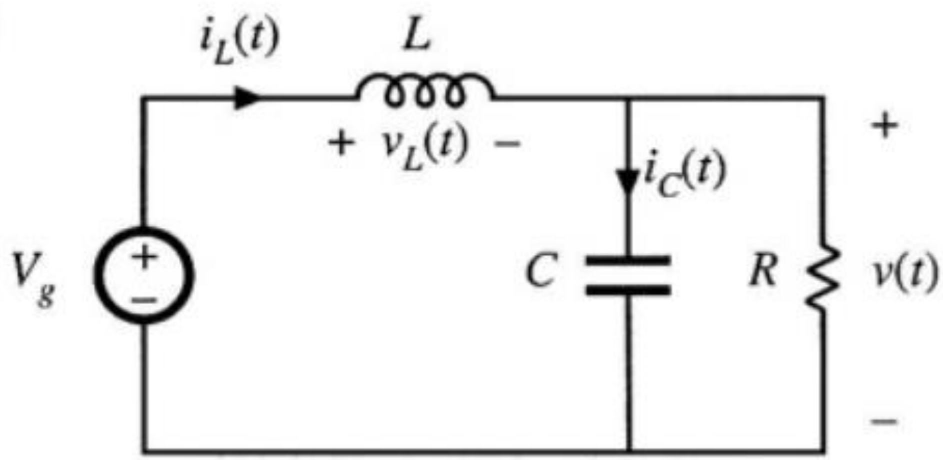
The output voltage ripple should be small in well-designed converter. It can be approximated that the ripple voltage is much smaller than the dc component…

$$\|v_{ripple}\| \ll V$$

$$\boxed{v(t) \approx V}$$

Steady-State Inductor Voltage & Current Waveforms

The switch is in position 1



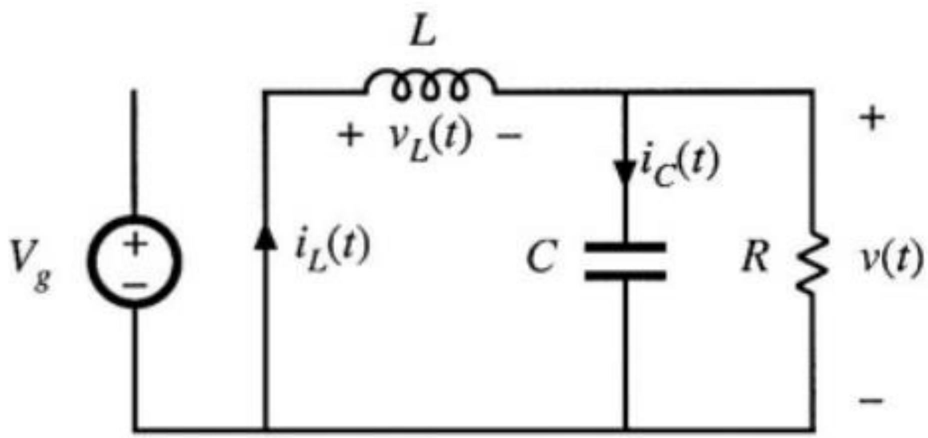
$$v_L = V_g - v(t)$$

$$v_L \approx V_g - V$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$\boxed{\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}}$$

The switch is in position 2

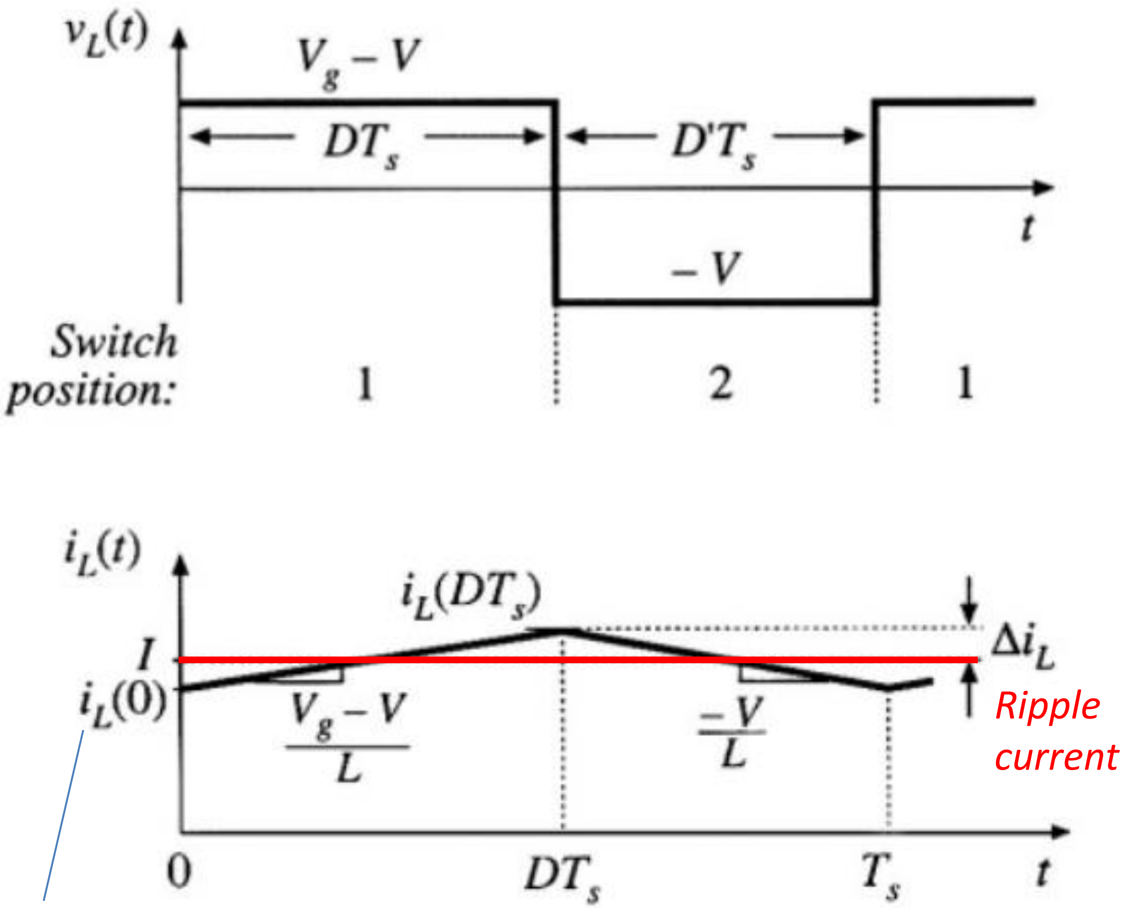


$$v_L(t) = -v(t)$$

$$v_L(t) \approx -V$$

$$\boxed{\frac{di_L(t)}{dt} = -\frac{V}{L}}$$

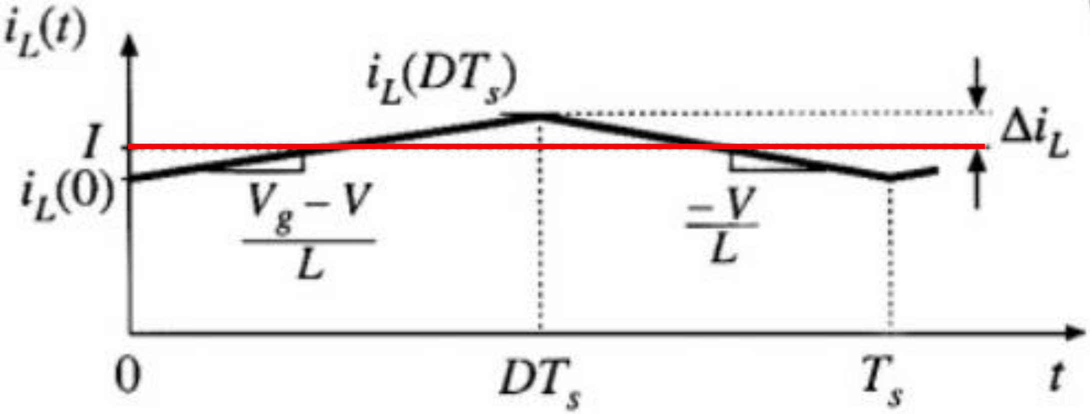
Steady-State Inductor Voltage & Current Waveforms



The inductor current begins at some initial value

- The peak inductor current is equal to the dc component I plus the peak-to-average ripple Δi_L .
- This peak current flows through not only the inductor, but also through the semiconductor devices that comprise the switch.
- Knowledge of the peak current is necessary when specifying the ratings of these devices.

Inductor Current Ripple & Inductor at Steady State



(change in i_L) = (slope)(length of subinterval)

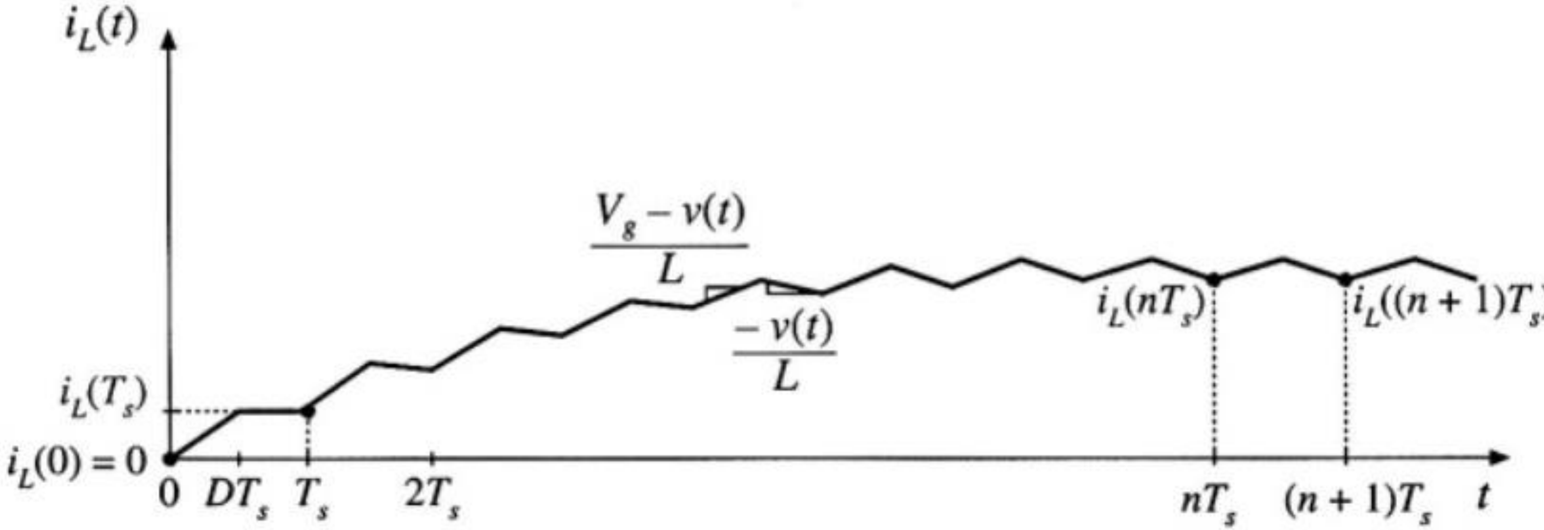
$$(2\Delta i_L) = \left(\frac{V_g - V}{L}\right)(DT_s)$$

$$\Delta i_L = \frac{V_g - V}{2L} DT_s$$

- *Typical values of ripple current lie in the range of 10% to 20% of the full-load value of the dc component I.*
- *By design the inductor current ripple is also usually small compared to the dc component I. The small ripple approximation* $i_L(t) \approx I$ *is usually justified for the inductor current.*

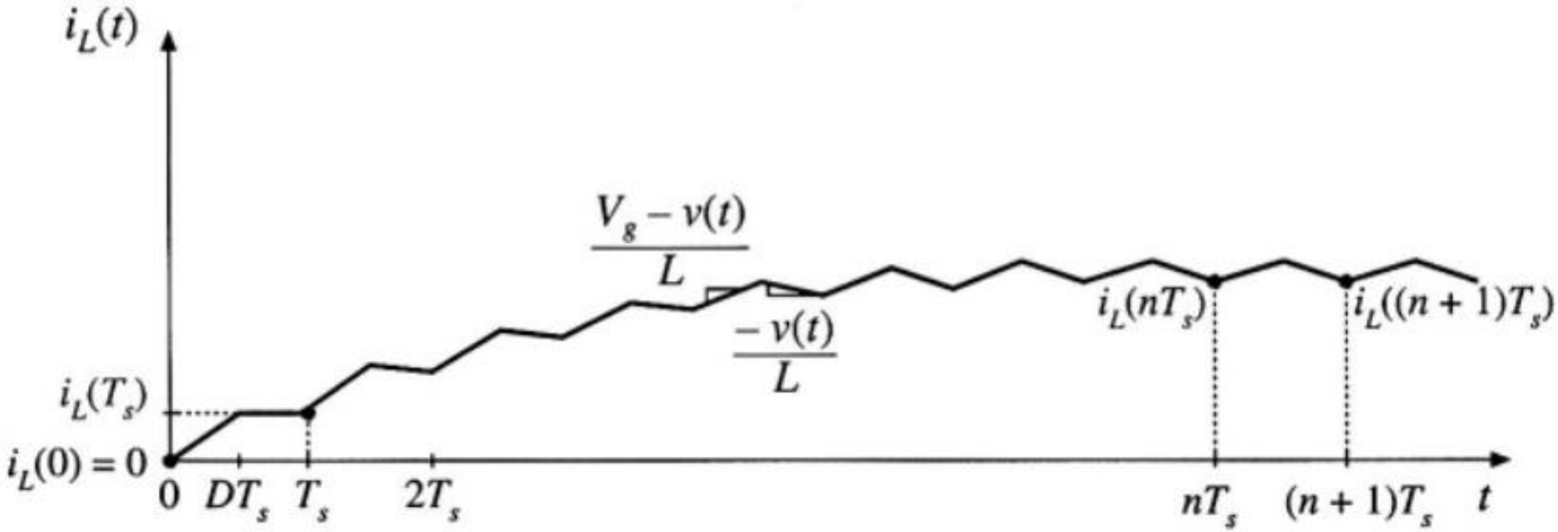
$$\boxed{L = \frac{V_g - V}{2\Delta i_L} DT_s}$$

Inductor Current Ripple & Inductor when First Turned On



- Suppose iL(0) =0 and v(0) =0 and input voltage is then applied.
- During the first subinterval (0 - DTs):
  switch in position 1 => inductor current will increase with a slope of Vg/L
  switch in position 2 => inductor-current slop is 0.
- ***There is a net increase in inductor current over the first switching period, because iL(Ts) > iL(0).***
- Since the inductor current flows to the output, the output capacitor will charge slightly, and v will increase slightly.

Inductor Current Ripple & Inductor when First Turned On



- The process repeats during the second and succeeding switching periods, with the inductor current increasing during each subinterval 1 and decreasing during each subinterval 2.
- As the output capacitor continues to charge and v increase :
  - the slope during subinterval 1 decrease (Vg-v)/L.
  - While the slope during subinterval 2 becomes more negative –v/L.
- ***Eventually, the increase in inductor current during subinterval 1 is equal to the decrease in inductor current during subinterval 2.***

Inductor Volt-Second Balance in Steady State

- At steady state => There is no net change in inductor current over a complete switching period. The converter waveforms are periodic : iL(nTs) = iL((n+1)Ts).
- In equilibrium, the net change in inductor current over one switching period be zero leads us to a way to find steady-state conditions in any switching converter : the principle of *inductor volt-second balance.*

$$v_L(t) = L\frac{di_L(t)}{dt}$$

Integration over one complete switching period, 0 to Ts :

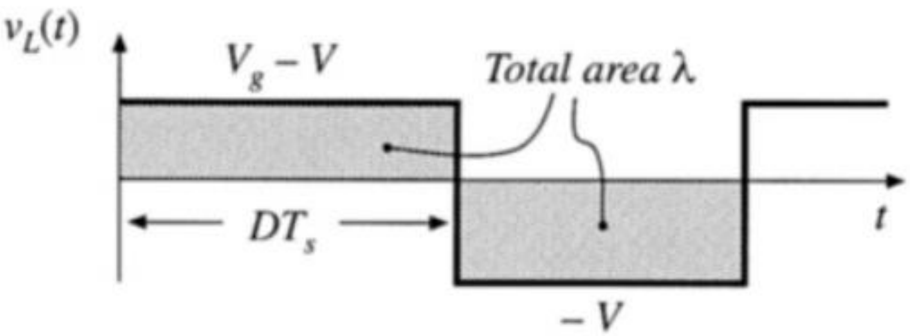
$$\underbrace{i_L(T_s) - i_L(0)}_{\text{Net change in inductor current over one switching period}} = \underbrace{\frac{1}{L}\int_0^{T_s} v_L(t)\,dt}_{\text{Integral of the applied inductor voltage over the interval}}$$

In steady state, the initial and final values of the inductor current are equal, hence $i_L(T_s) - i_L(0) = 0$

Therefore, in steady state, the integral of the applied inductor voltage must be zero:

$$\boxed{0 = \underbrace{\frac{1}{L}\int_0^{T_s} v_L(t)\,dt}_{\text{Has the unit of volt-second or flux linkage}}}$$

Inductor Volt-Second Balance in Steady State



$$\lambda = \int_0^{T_s} v_L(t)dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

The average value is therefore….

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Since $\langle v_g \rangle = 0 \quad and \quad D + D' = 1 \ldots$

$$\boxed{0 = DV_g - (D + D')V = DV_g - V}$$

$$V = DV_g$$

$$\boxed{0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)dt = \langle v_L \rangle}$$

Average value or DC component

- This equation states that, in steady state, the applied inductor voltage must have zero dc component (called Volt-Sec Balance) !!

- *What happens if a dc voltage is applied to inductor ????*

- This result coincides with the previous result. So the principle of inductor volt-second balance allow us to derive an expression for the dc component of the converter output voltage.

- An advance of this approach is its generality – it can be applied to any converter. One simply sketches the applied inductor voltage waveform and equates the average value to zero. This method is an analyzing tool for power electronics, and can be used to solve more complicated converters.

Capacitor Amp-Second Balance in Steady State

- Similar arguments can be applied to capacitors…

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Integration over one complete switching period, 0 to Ts :

$$\underbrace{v_C(T_s) - v_C(0)} = \underbrace{\frac{1}{C}\int_0^{T_s} i_C(t)dt}$$

- In steady state, the net change over one switching period of the capacitor voltage must be zero… There is no net change in capacitor change in stead state

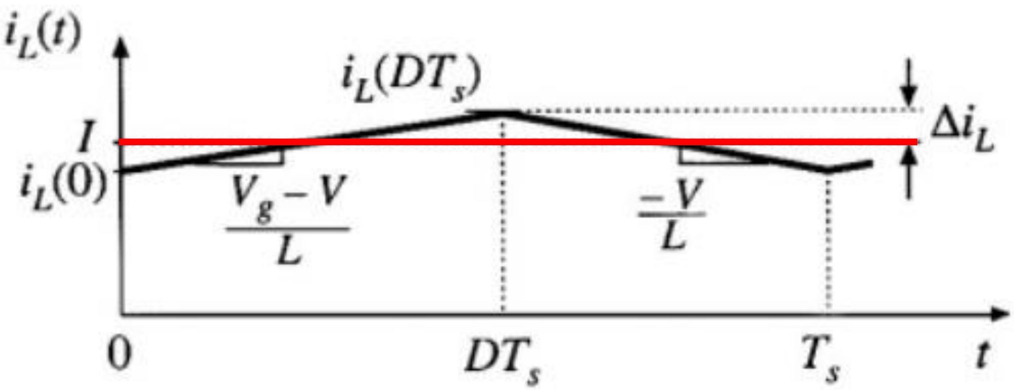
$$\boxed{0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)dt = \langle i_C \rangle}$$

Average value or DC component

- *What happens if a dc current is applied to capacitor ????*

Estimating the Output Voltage Ripple in Converters

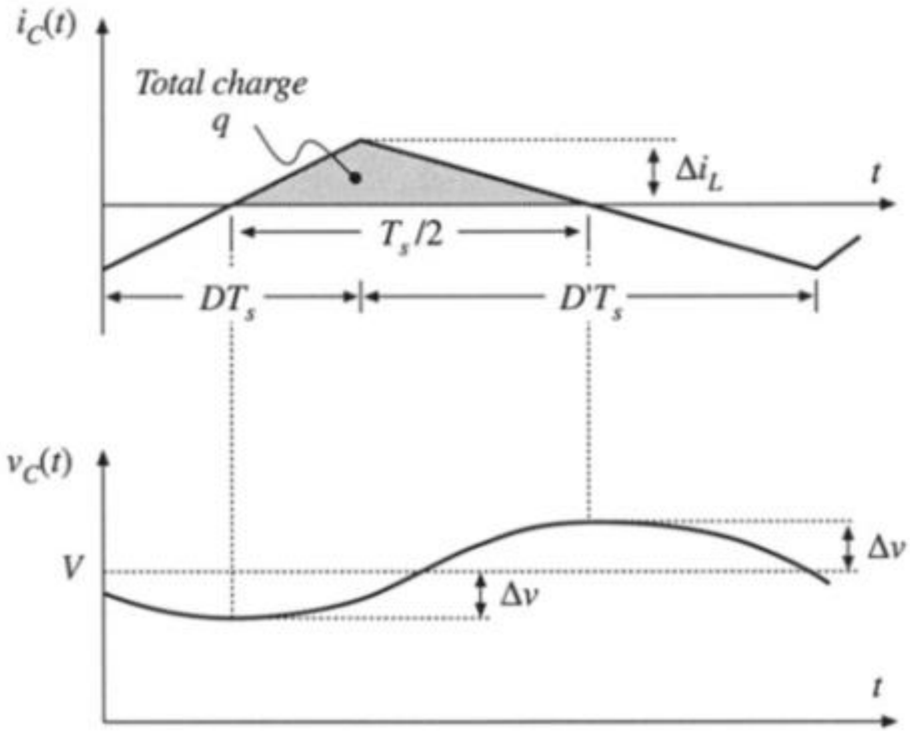
- For buck converter only component of output capacitor current is that arising from the inductor current ripple. cannot
- Hence, inductor current ripple be neglected when calculating the output capacitor voltage ripple…
- The capacitor voltage ripple can then be related to the total charge contained in the positive portion of the ic(t) waveform.



- Consider the buck converter, the inductor current waveform iL(t) contains a dc component I and linear ripple of peak magnitude ΔiL.
- *DC component I must flow entirely through the load resistance R (why?), while the ac switching ripple divides between the load resistance R and the filter capacitor C.*

- In a well-designed converter; *capacitance C is chosen large enough* that its impedance at switching frequency is much smaller than the load impedance R. *Hence nearly all of the inductor current ripple flows through the capacitor.*

Estimating the Output Voltage Ripple in Converters



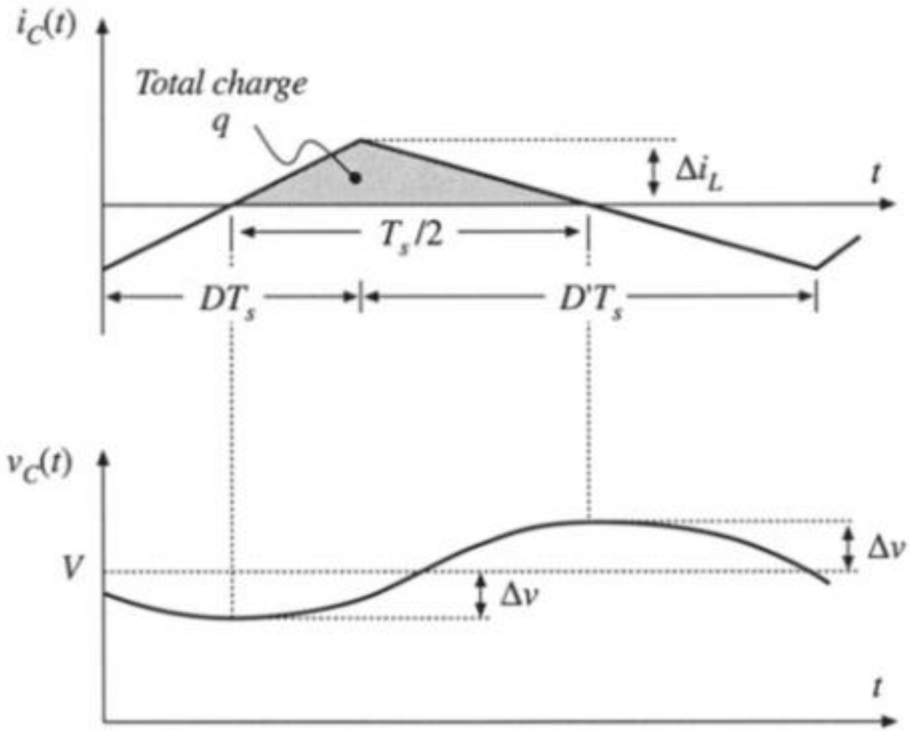
- The capacitor current waveform ic(t) is then equal to the inductor current waveform with the dc component removed. The current ripple is linear, with peak value ΔiL.

- The capacitor current ic(t) is positive, charge is deposited on the capacitor plates and the voltage vc(t) increase. Therefore, between the two zero-crossings of the capacitor current waveform, the capacitor voltage changes between its minimum and maximum extrema.

- *This change in capacitor voltage can be related to the total charge q contained in the positive portion of the capacitor current waveform. By the capacitor relation Q=CV,*

$$q = C\left(2\Delta v\right)$$

Estimating the Output Voltage Ripple in Converters



- The capacitor current waveform ic(t) is then equal to the inductor current waveform with the dc component removed. The current ripple is linear, with peak value ΔiL.

- The charge q is the integral of the current waveform between its zero crossings, the integral can be expressed as the area of the shaded triangle…

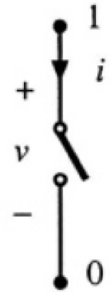
$$q = \frac{1}{2}\Delta i_L \frac{T_s}{2}$$

$$\boxed{\Delta v = \frac{\Delta i_L T_s}{8C}}$$

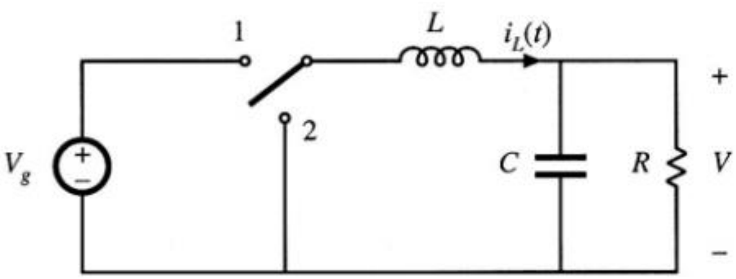
- *This expression can be used to select a value for the capacitance C such that a given voltage ripple Δv is obtained.*

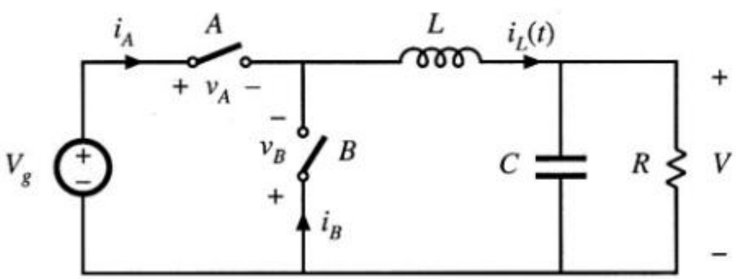
Switch Realization

Single Pole Single Throw (SPST) Switch



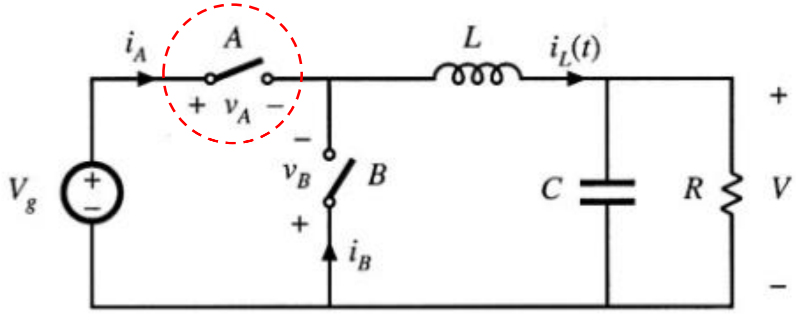
- Semiconductor power devices behave as single-pole single-throw (SPST) switches. The converter schematic containing SPST switches is more realistic.





- Be careful !!! The realization of a Single-Pole-Double-Throw (SPDT) switch using two SPST switches is not exactly equivalent…
  - *It is possible for both SPST switches to be simultaneously in the on state or in the off state, leading to behavior not predicted by the circuit with SPDT switch.*
  - *It is possible for the switch state to depend on the applied voltage and current waveforms …. for example discontinuous mode.*

A Single-Quadrant Switch



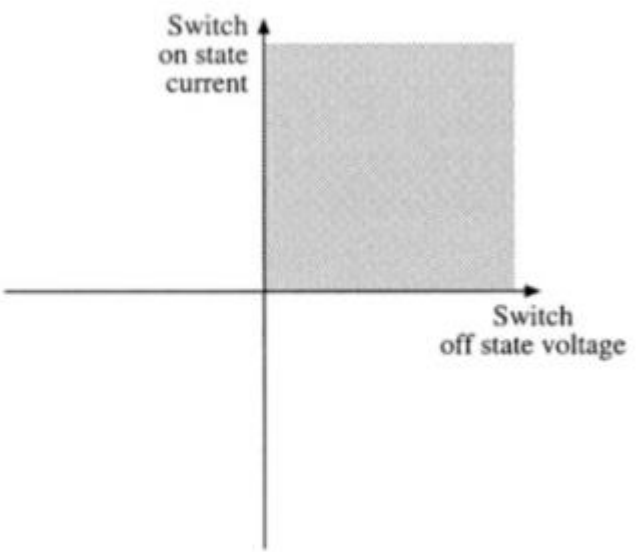


Fig. 4.3 A single-quadrant switch is capable of conducting currents of a single polarity, and of blocking voltages of a single polarity.

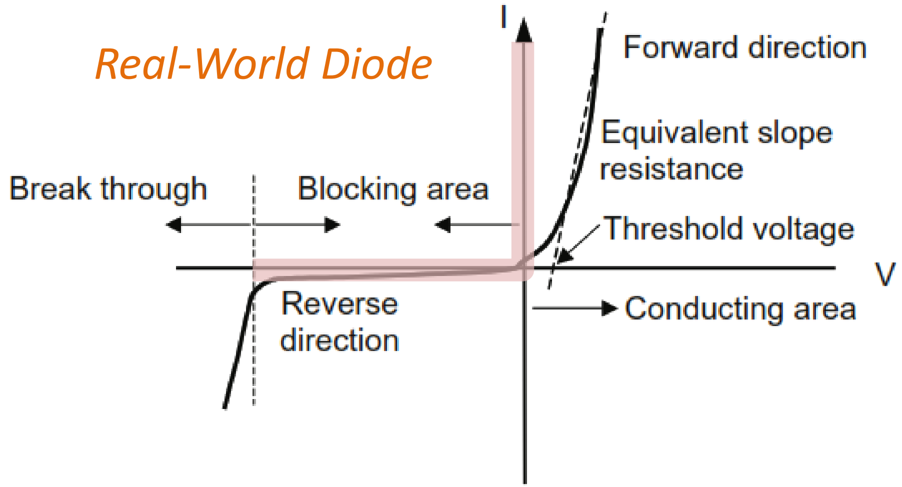
- An ideal switch can be realized using semiconductor devices depends on
  1) the polarity of the voltage that the devices must block in the off state, and
  2) on the polarity of the current that the devices must conduct in the on state.

  behave as single-throw (SPST) switches.

For example, switch A must block positive voltage Vg when in the off state, and must conduct positive current iL when in the on state....
the current and blocking voltage lie in a single quadrant of the plane.

What kinds of semiconductor devices fit in with this operation ?

Diode: Single-Quadrant Switch



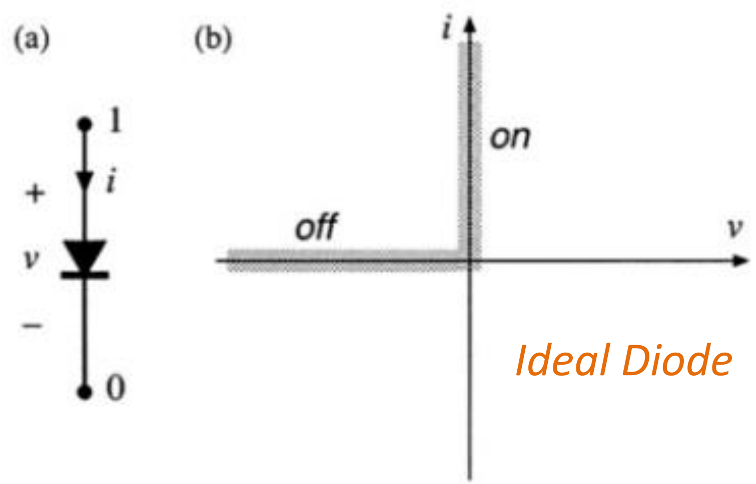


Fig. 4.4 Diode symbol (a), and its ideal characteristic (b).

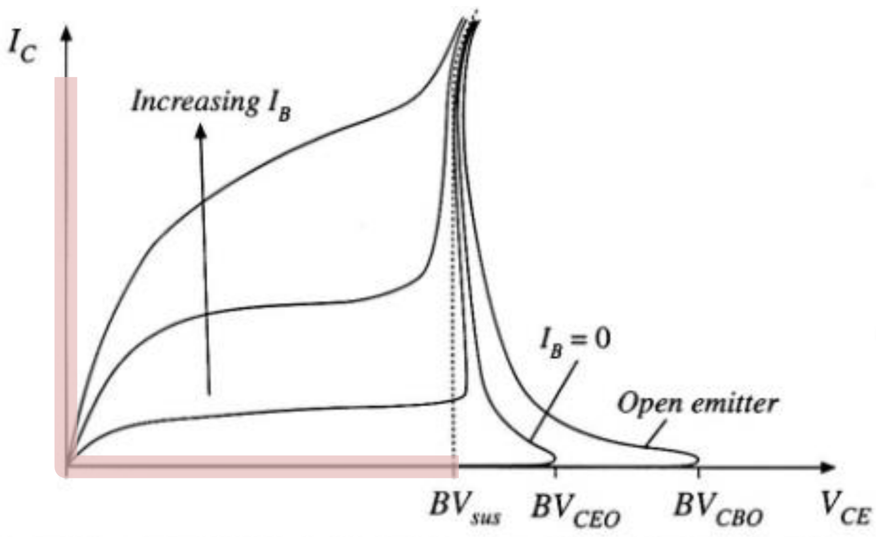
- Diode is a *passive* switch, which does not contain a control terminal, and *the state of switch is determined by the waveform i(t) and v(t) applied to terminals 0 and 1.*

- Diode is "off" (i=0) when v < 0
- Diode is "on" (v=0) when i > 0

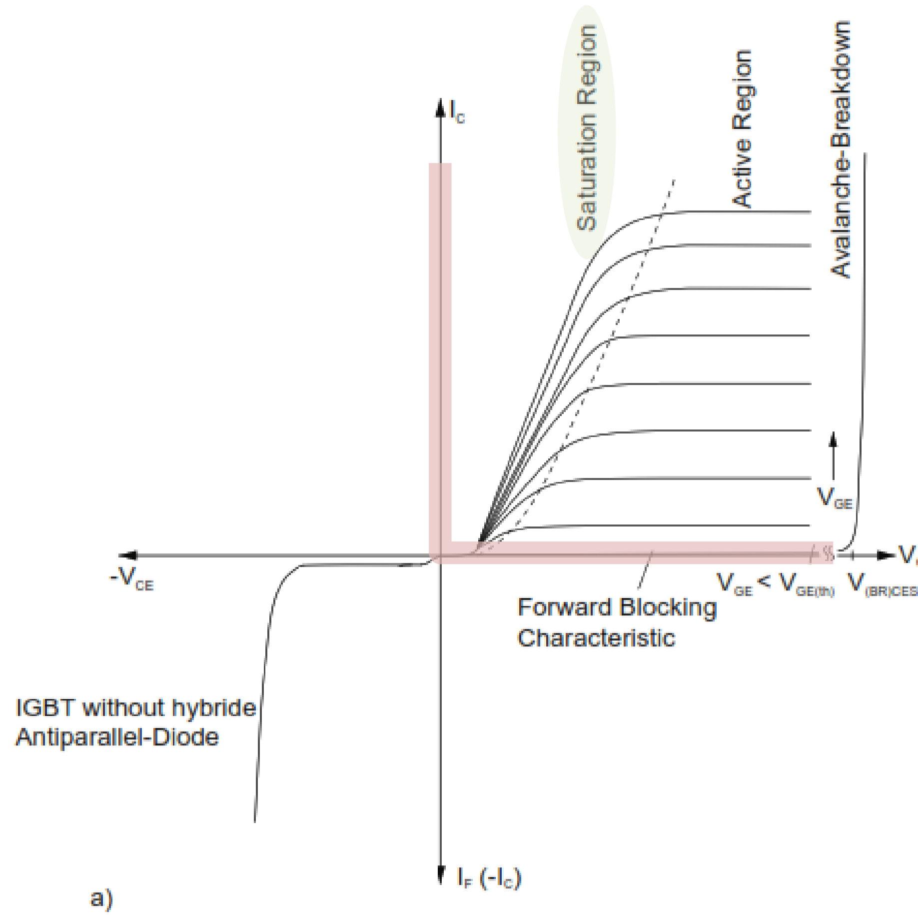
it can block negative voltage but not positive voltage.

Bipolar Junction Transistor (BJT) and Insulated Gate Bipolar Transistor (IGBT) : Single-Quadrant Switches

Real-World BJT



Real-World IGBT



BJT & IGBT: Ideal Single-Quadrant Switches

- BJT and IGBT are *active* switches.
- *The conducting state of an active switch is determined by the signal applied to the control terminal C. The state does not directly depend on the waveforms v(t) and i(t) applied to terminal 0 and 1.*

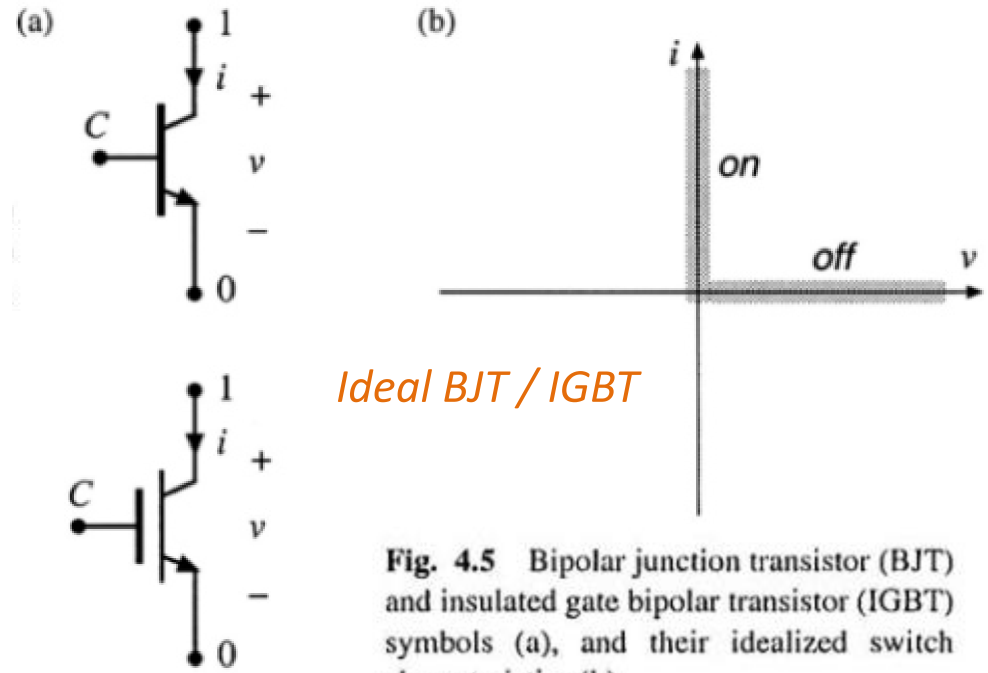
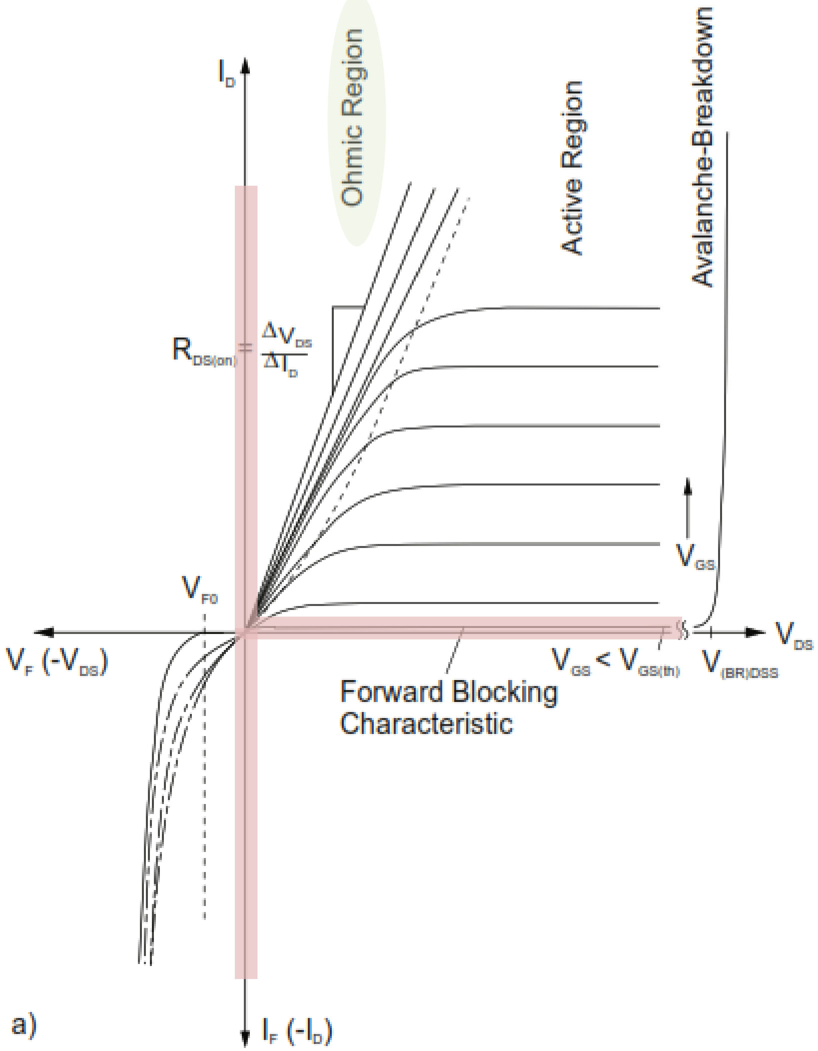


Fig. 4.5 Bipolar junction transistor (BJT) and insulated gate bipolar transistor (IGBT) symbols (a), and their idealized switch characteristics (b).

- When the control terminal causes the transistor to be in the off state, i=0 and the device is capable of blocking positive voltage: v >= 0.
- When the control terminal causes the transistor to be in the on state, v ≈ 0 and the device is capable of conducting positive current: i>=0.
- *The reverse-conducting and reverse-blocking characteristics of the BJT and IGBT are poor and nonexistent, and have no application in the power converter area.*

Metal Oxide Semiconductor Field-Effect Transistor: Two-Quadrant Switch

Real-World MOSFET



Ideal MOSFET

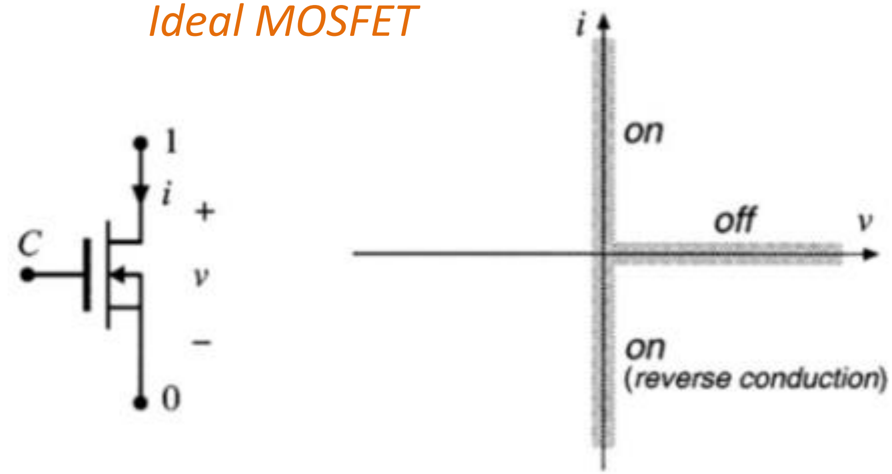


Fig. 4.6 Power MOSFET symbol (a), and its idealized switch characteristics (b).

- *MOSFET has similar characteristics to that of BJT and IGBT, except that it is able to conduct current in the reverse direction.*
- MOSFET is normally operated with i >=0 , in the same manner as the BJT and IGBT. So an active SPST switch can be realized using a BJT, IGBT or MOSFET.

Switches Realization for Buck Converter

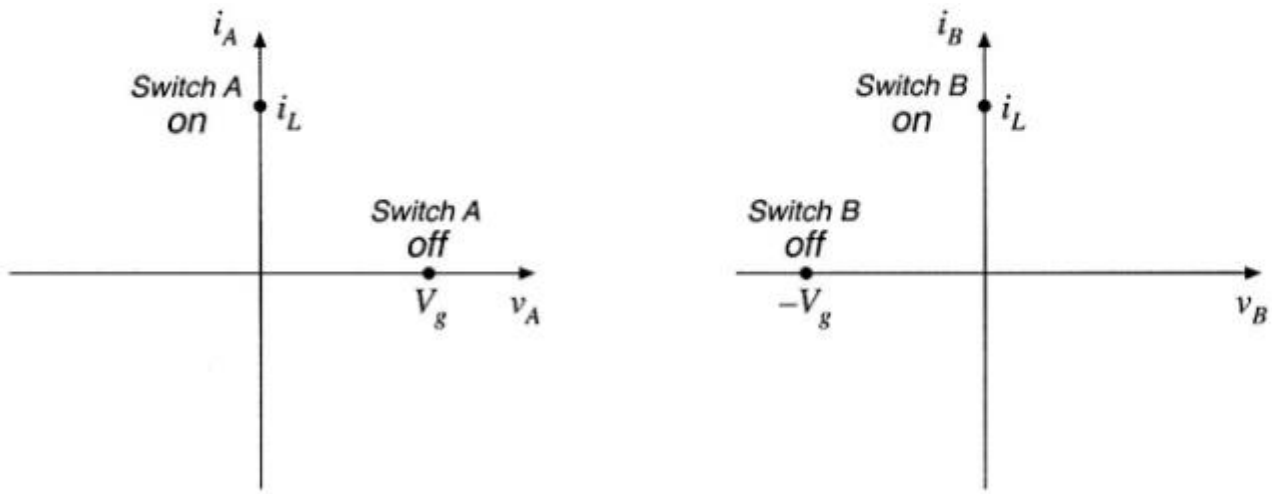


Fig. 4.7 Operating points of switch A, (a), and switch B, (b), in the buck converter of Fig. 4.2(b).

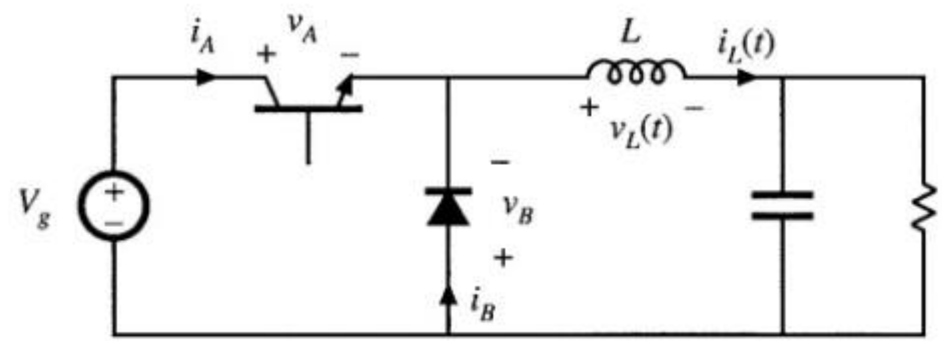
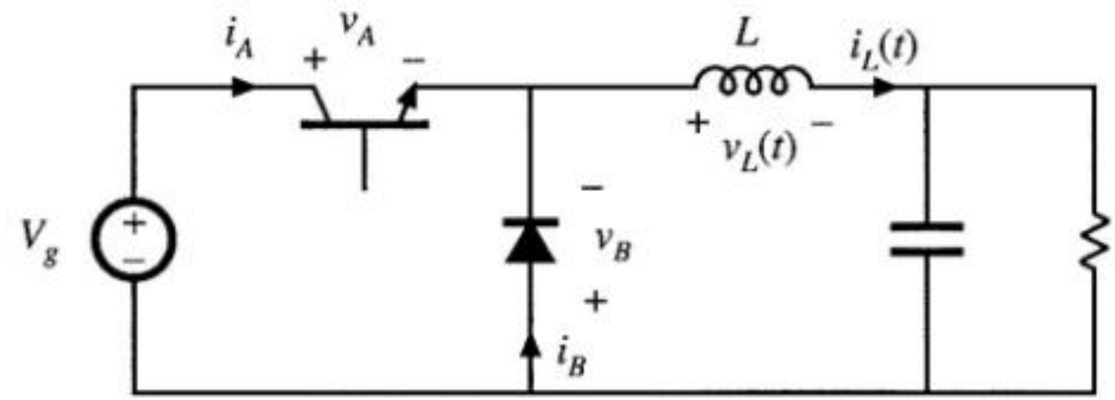


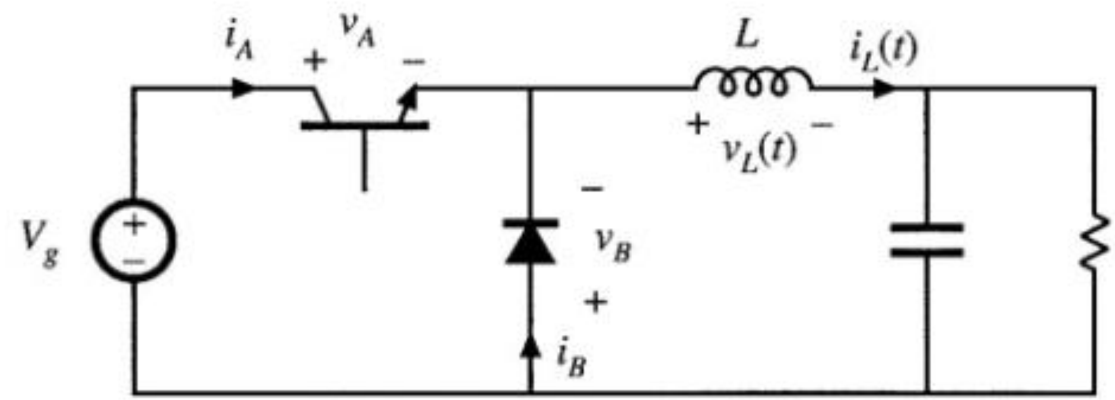
Fig. 4.8 Implementation of the SPST switches of Fig. 4.2(b) using a transistor and diode.

Switches Realization for Buck Converter



- When the controller turns the transistor on…
  - the diode becomes reverse-biased since vB = - Vg. It is required that Vg be positive; otherwise, the diode will be forward-biased.
  - The transistor conducts current iL. This current should also be positive, so that the transistor conducts in the forward direction.

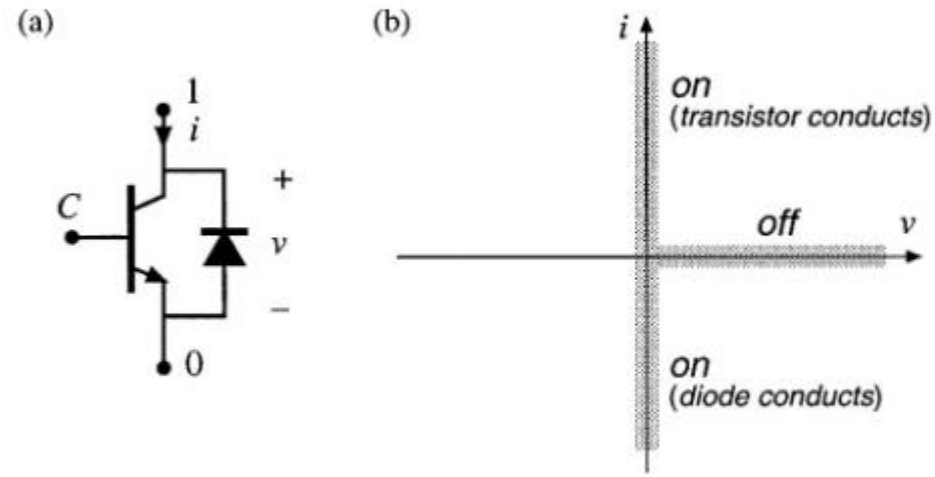
Switches Realization for Buck Converter



- When the controller turns the transistor off…
  - The diode must turn on so that the inductor current can continue to flow. Turning the transistor off causing the induction current iL(t) to decrease. Since vL = L diL/dt, the inductor voltage becomes sufficiently negative to forward-bias the diode, and the diode turns on.
  - Diodes that operate in this manner are sometimes called *freewheeling diodes.*
  - It is recalled that iL be positive; otherwise, the diode cannot be forward-biased since iB = iL. The transistor blocks voltage Vg; this voltage should be positive to avoid operating the transistor in the reverse blocking mode.

Current-Bidirectional Two-Quadrant Switches

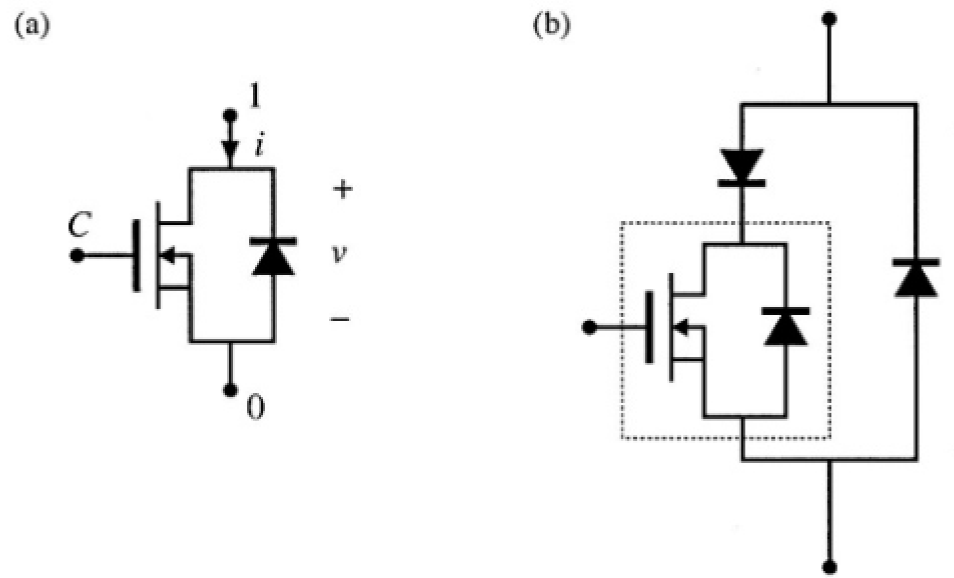
Fig. 4.9 A current-bidirectional two-quadrant SPST switch: (a) implementation using a transistor and antiparallel diode, (b) idealized switch characteristics.



- The switch elements conduct currents of both polarity, but block only positive voltages.
- *A Current-Bidirectional two-quadrant SPST switch of this type can be realized using a transistor and diode, connected in an antiparallel manner.*

MOSFET: Current-Bidirectional Two-Quadrant Switches

Fig. 4.10 The power MOSFET inherently contains a built-in body diode: (a) equivalent circuit, (b) addition of external diodes to prevent conduction of body diode.



- MOSFET is also a two-quadrant switch, practical power MOSFETS inherently contain a built-in diode, often called the *body diode*.
- The switching speed of the body diode is much slower than that of the MOSFET, if this body diode is allowed to conduct, then high peak currents can occur during the diode turn-off transition. Most MOSFETs are not rated to handle these currents, and device failure can occur.
- To avoid this situation, external series and antiparallel diodes can be added.

Example of Converter with Current-Bidirectional Two-Quadrant Switches

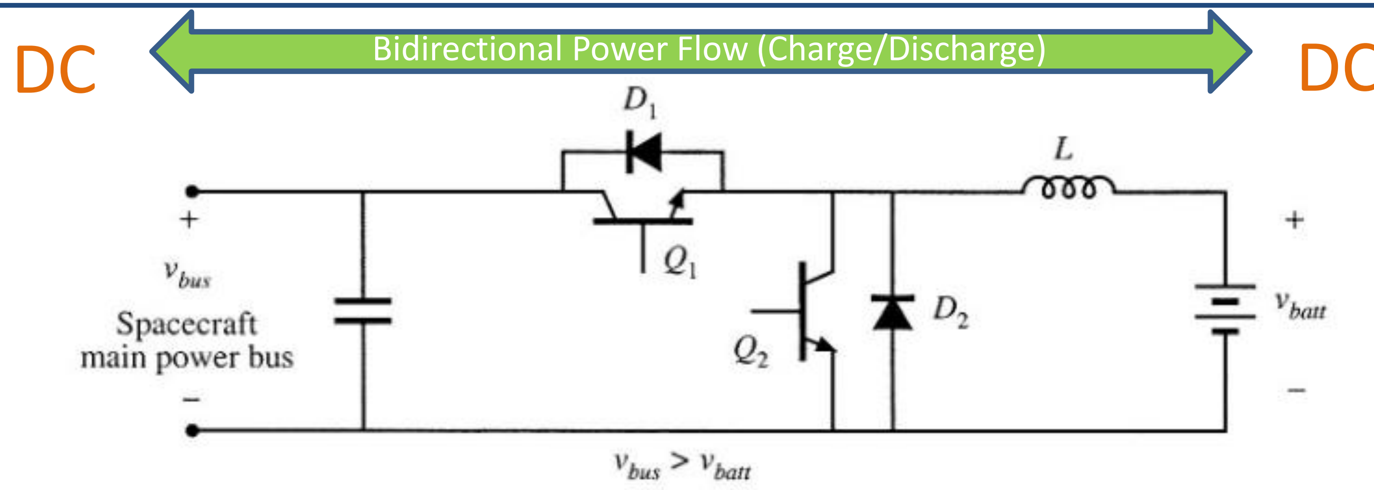


Fig. 4.15 Bidirectional battery charger/discharger, based on the dc-dc buck converter.

- This converter interfaces a battery to the main DC power supply. Both the dc bus vbus and the battery voltage vbat are always positive. *The semiconductor switch elements block positive voltage vbus.*
  - When the battery is being charged, iL is positive, and Q1 and D2 alternately conduct.
  - When the battery is being discharge, iL is negative, and Q2 and D1 alternately conduct.
- Although, this is a DC-DC converter, it requires two-quadrant switches because the power can flow in either direction.

Voltage-Bidirectional Two-Quadrant Switches

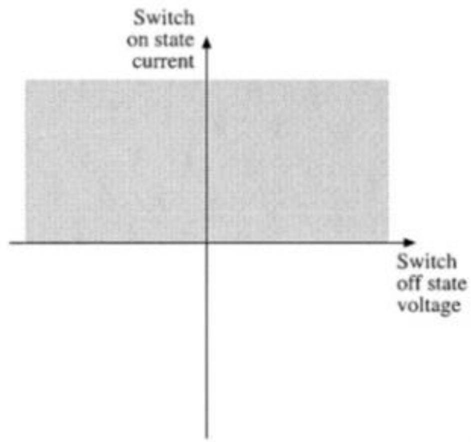


Fig. 4.16 Voltage-bidirectional two-quadrant switch properties.

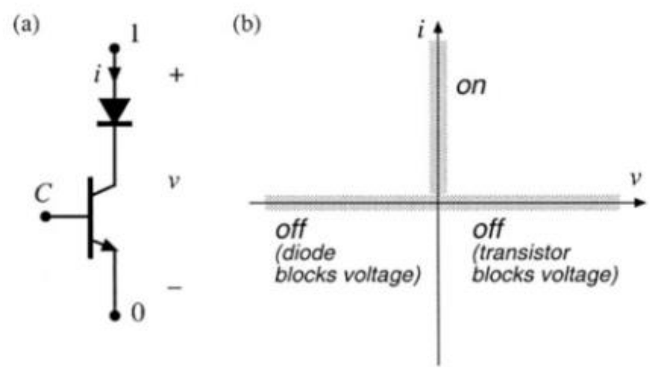
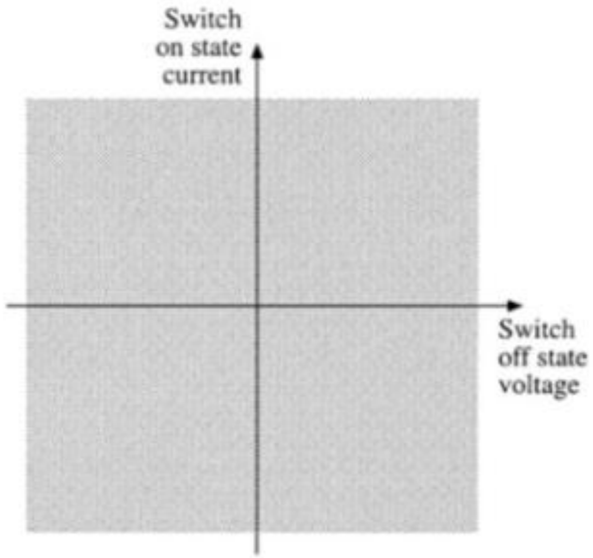


Fig. 4.17 A voltage-bidirectional two-quadrant SPST switch: (a) implementation using a transistor and series diode, (b) idealized switch characteristics.

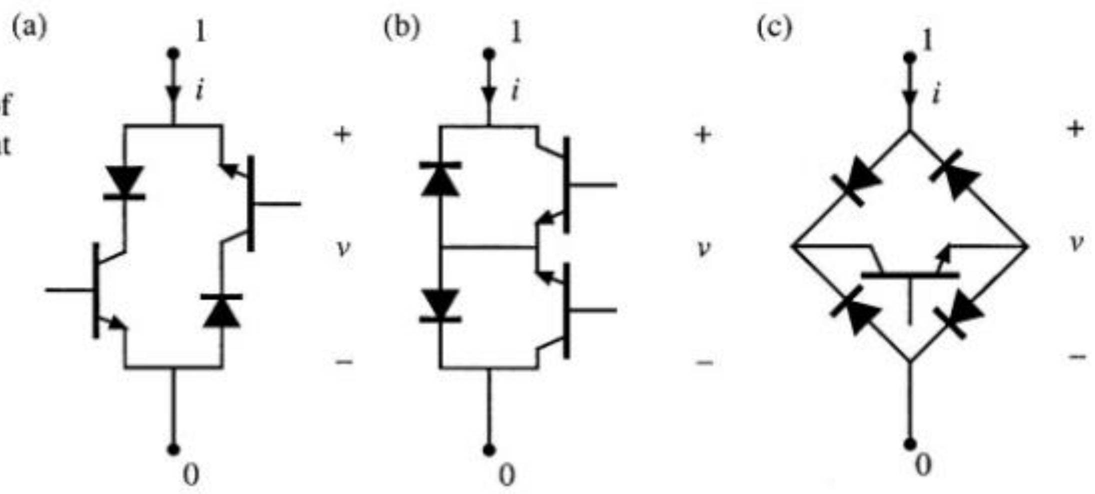
- The switch block both positive and negative voltages, but conduct only positive current, an SPST switch can be constructed using a series-connected transistor and diode.
  - When it is intend that the switch be in the off state, the controller turns the transistor off.
  - The diode then blocks negative voltage, and the transistor blocks positive voltage. The series connection can block negative voltages up to the diode voltage rating, and positive voltages up to the transistor voltage rating.

Four-Quadrant Switches



- More general type of switch is the four quadrant switch, capable of conducting currents of either polarity and blocking voltages of either polarity.
- There are several ways of construction:
  - (a) antiparallel connection of two voltage-bidirectional two-quadrant switches.
  - (b) two current-bidirectional two-quadrant switches connected in back-to-back fashion.
  - (c) using only one transistor but additional diodes.

Fig. 4.20 Three ways of implementing a four-quadrant SPST switch.



Example of Converter with Four-Quadrant Switches

AC — Bidirectional Power Flow (Charge/Discharge) — AC

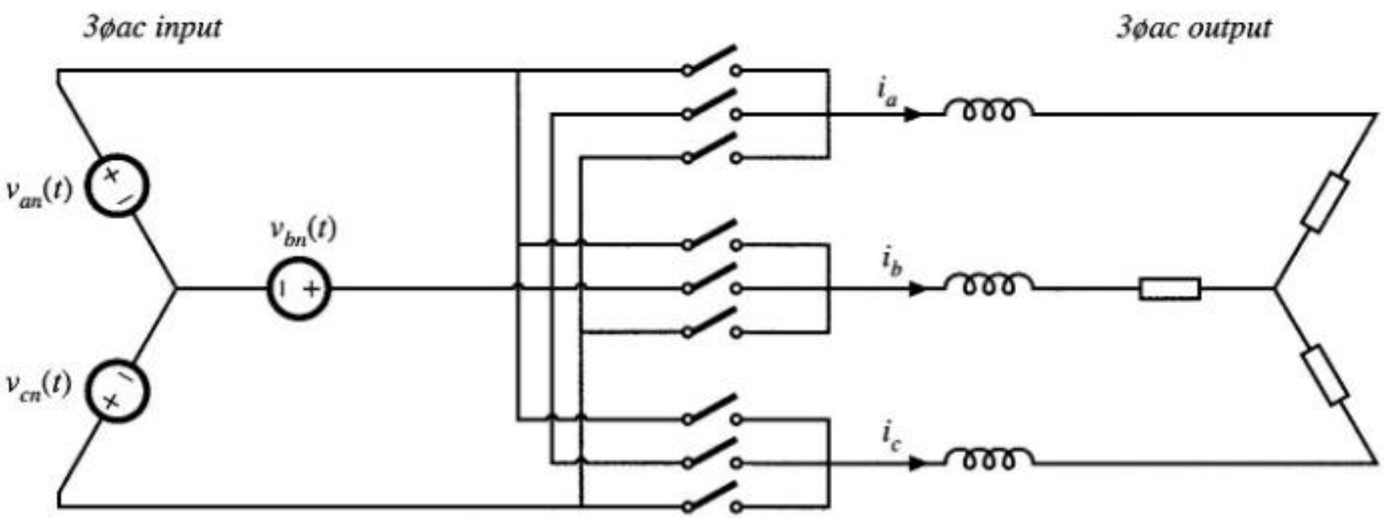


Fig. 4.21 A 3øac–3øac matrix converter, which requires nine SPST four-quadrant switches.

Discontinuous Conduction Mode

Origin of Discontinuous Conduction Mode and Mode Boundary

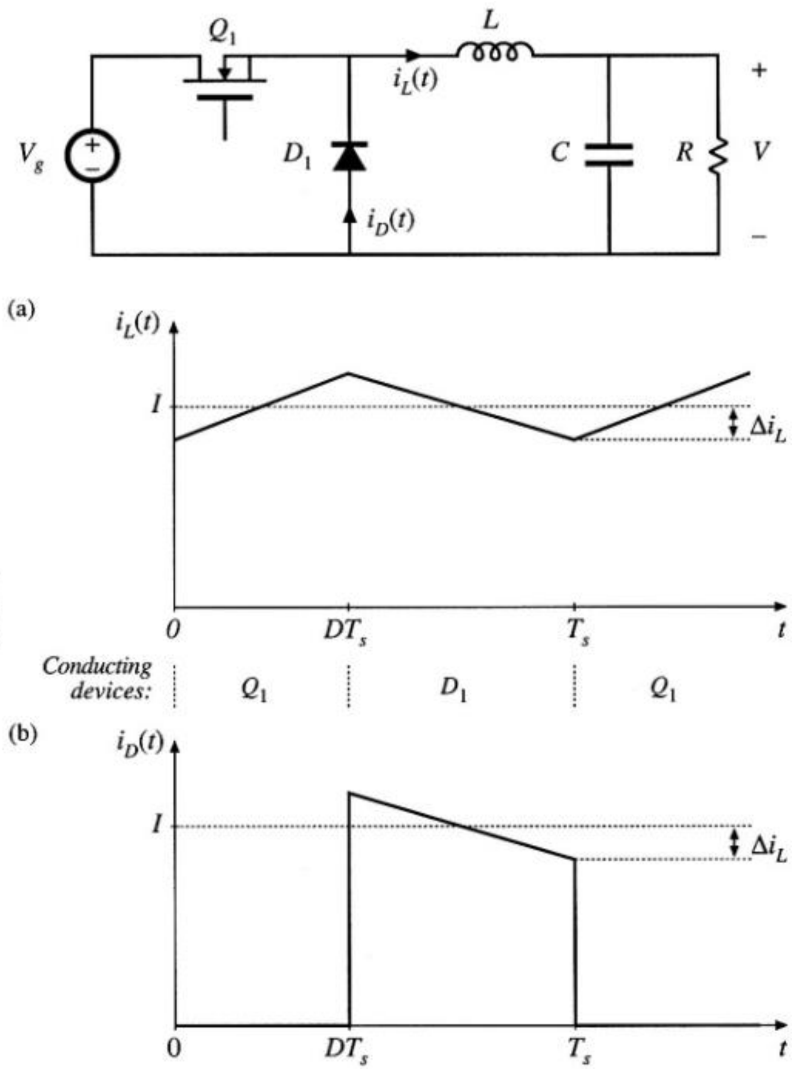


Fig. 5.1 Buck converter example.

Fig. 5.2 Buck converter waveforms in the continuous conduction mode: (a) inductor current $i_L(t)$, (b) diode current $i_D(t)$.

- The inductor current waveform contains a dc component I, plus switching ripple of peak amplitude diL.
  - During the second subinterval, the diode current is identical to the inductor current.
  - The minimum diode current during the second subinterval is equal to (I-diL); since the diode is a single-quadrant switch, operation in the continuous mode requires that this current remain positive.

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g D D' T_s}{2L}$$

  - *Ripple magnitude* depends on the applied voltage, on the inductance L, and on the DTs, but *does not depends on the load resistance R.*

Origin of Discontinuous Conduction Mode and Mode Boundary

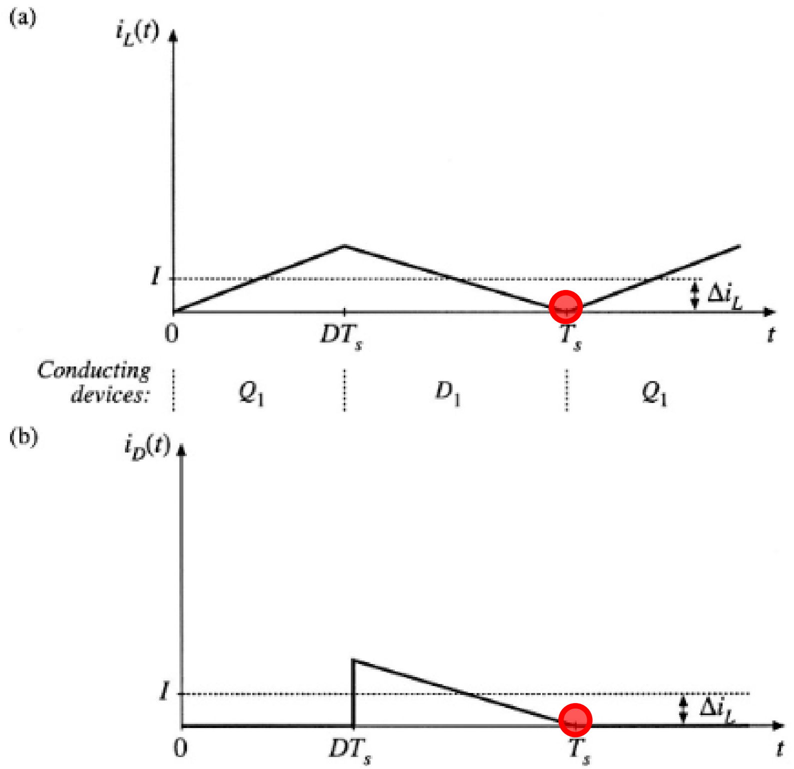


Fig. 5.3 Buck converter waveforms at the boundary between the continuous and discontinuous conduction modes: (a) inductor current $i_L(t)$, (b) diode current $i_D(t)$.

- If we continue to increase R, eventually the point is reached where I = diL.
- It can be seen that the inductor current iL(t) and the diode current id(t) are both zero at the end of the switching period.
- *What happens if we continue to increase the load resistance R ?*

Buck Converter in Discontinuous Conduction Mode

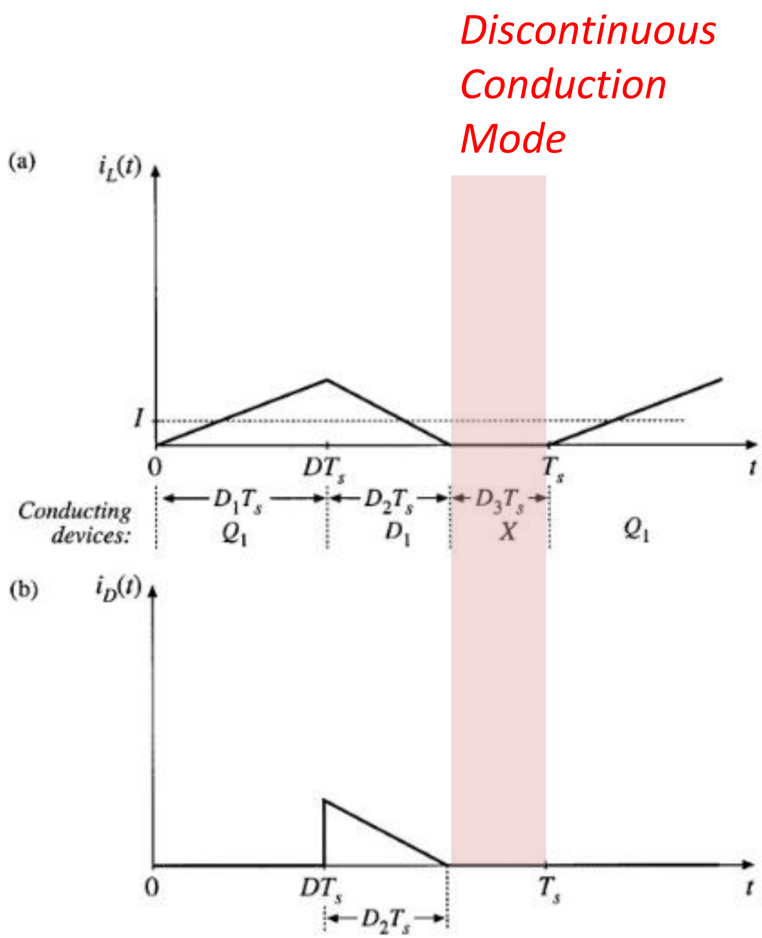


Fig. 5.4 Buck converter waveforms in the discontinuous conduction mode: (a) inductor current $i_L(t)$, (b) diode current $i_D(t)$.

- The diode current cannot be negative; therefore, the diode must become reverse-biased before the end of the switching period. There are now three subintervals during each switching period Ts.
- At the end of the second subinterval the diode current reaches zero, and for the remainder of the switching period neither the transistor nor the diode conduct. The converter operates in the discontinuous conduction mode.
- The condition for operation in the continuous and discontinuous conduction modes are:

$$I > \Delta i_L \quad for\ CCM$$

$$I < \Delta i_L \quad for\ DCM$$

Buck Converter in Discontinuous Conduction Mode

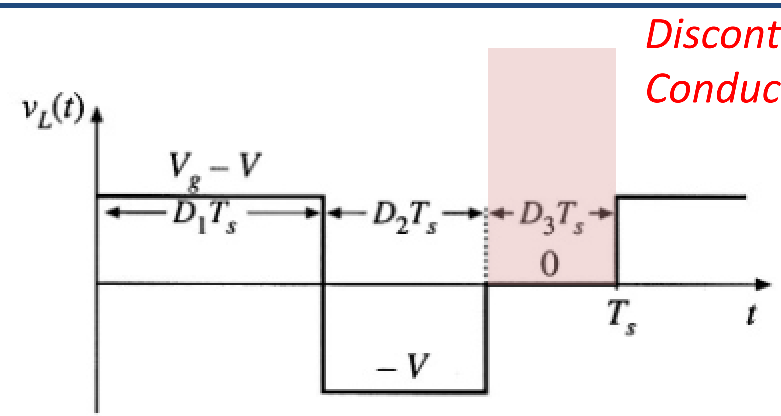


Fig. 5.8 Inductor voltage waveform $v_L(t)$, buck converter operating in discontinuous conduction mode.

- According to the principle of inductor volt-second balance, the dc component of this waveform must be zero. Since the waveform is rectangular, its dc component (or average value) is

$$\langle v_L(t) \rangle = D_1 (V_g - V) + D_2(-V) + D_3(0) = 0$$

- The output voltage yields

$$V = V_g \frac{D_1}{D_1 + D_2}$$

-D1 is the control input of converter, and can be considered know. But the subinterval 2 duty cycle D2 is unknown, and hence another equation is needed to eliminate D2 and solve for the output voltage V.

Buck Converter in Discontinuous Conduction Mode

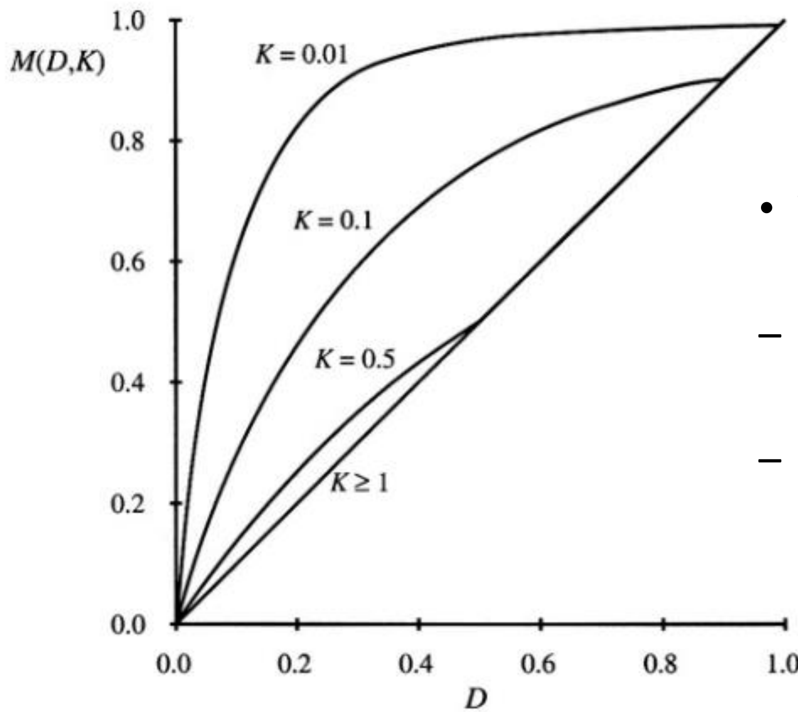


Fig. 5.11 Voltage conversion ratio $M(D, K)$, buck converter.

- The drawback of discontinuous mode:
  - It causes the output voltage to increase.
  - The output is dependent on Load R.

$$M(D_1, K) = V/V_g,$$

$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + \dfrac{4K}{D^2}}} & \text{for } K < K_{crit} \end{cases}$$

Boost Converter

From Buck Converter...

$$V_2 = DV_1$$

By interchanging the power source and load, the above equation must still hold; by solving for the load voltage V1, we obtain…

$$V_1 = \frac{1}{D} V_2$$

So the load voltage V1 is greater than the source voltage V2. It is a boost converter.

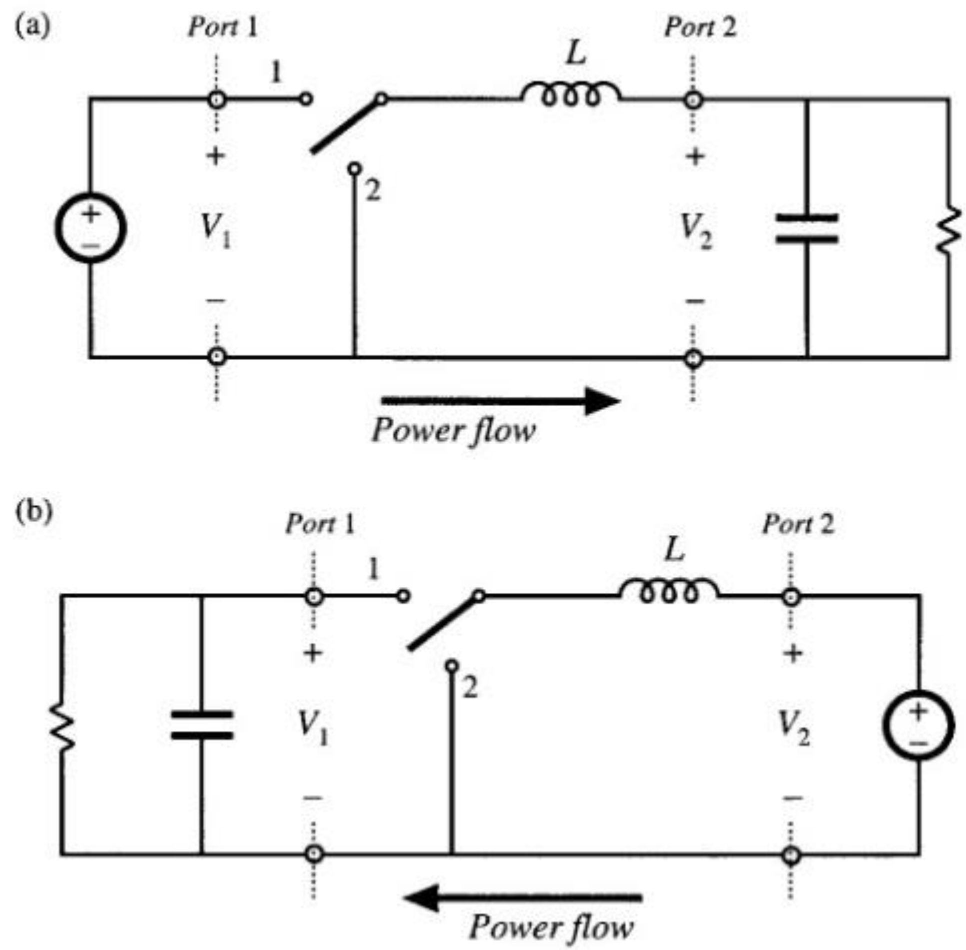
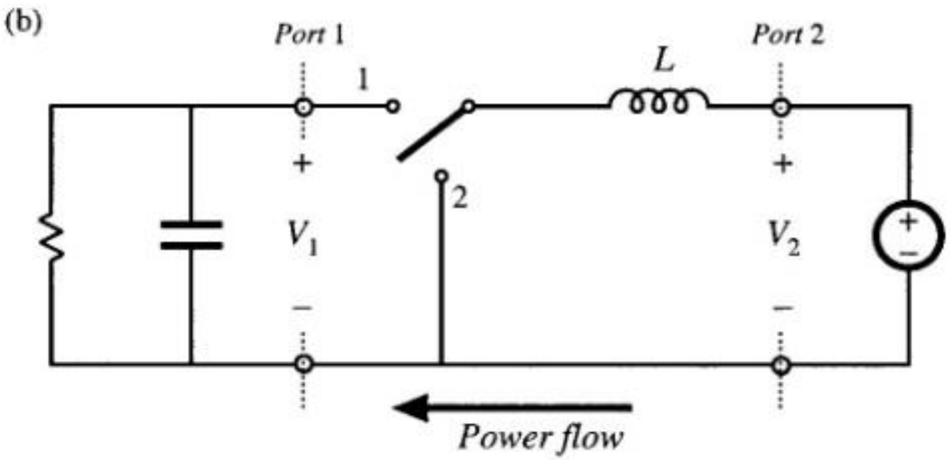
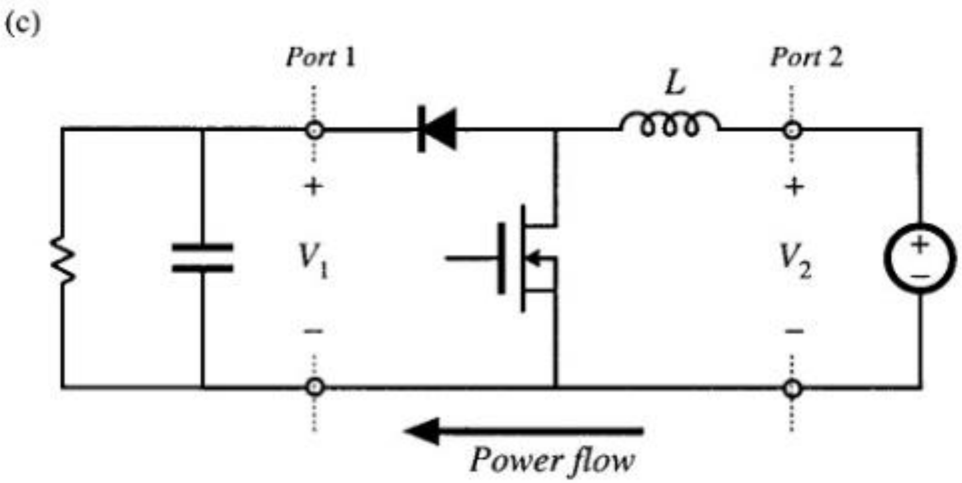


Fig. 6.2 Inversion of source and load transforms a buck converter into a boost converter: (a) buck converter, (b) inversion of source and load, (c) realization of switch.

Switch Realization





- Since power flows in the opposite direction, the standard buck converter unidirectional switch realization cannot be used for boost converter.

- By following the principle of switch realization, the switch can be realized by connecting a transistor between the inductor and ground, and a diode from the inductor to the load.

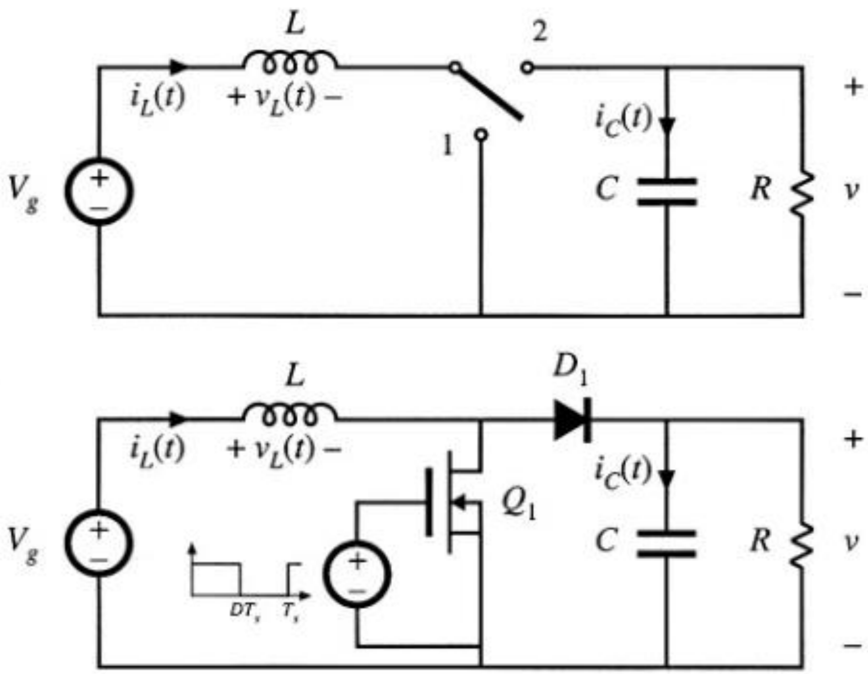
- In consequence, the transistor duty cycle D becomes the fraction of time which the SPDT switch spends in position 2, rather than in position 1.

- So we should interchange D with its compliment D’ and the conversion ratio of boost converter is

$$\boxed{V_1 = \frac{1}{D'} V_2}$$

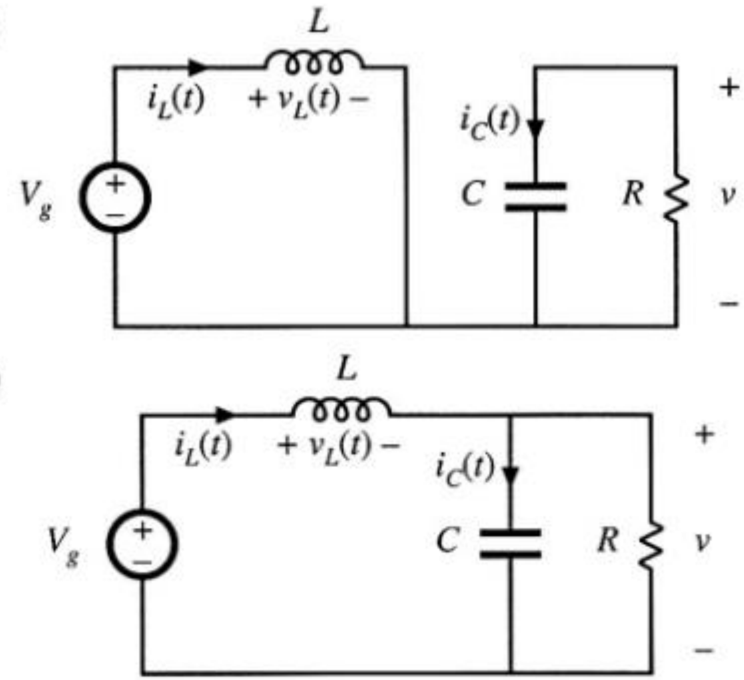
(a)

(b)



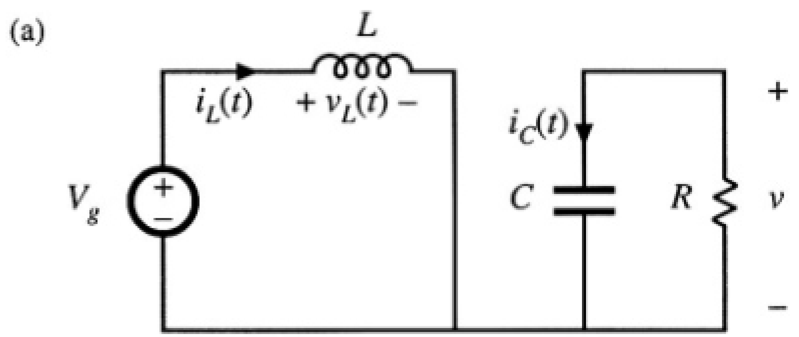
(a)

(b)



Steady-State Analysis with Linear Ripple Approximation

Q1 : on, (switch is in position 1)



$$v_L = V_g$$

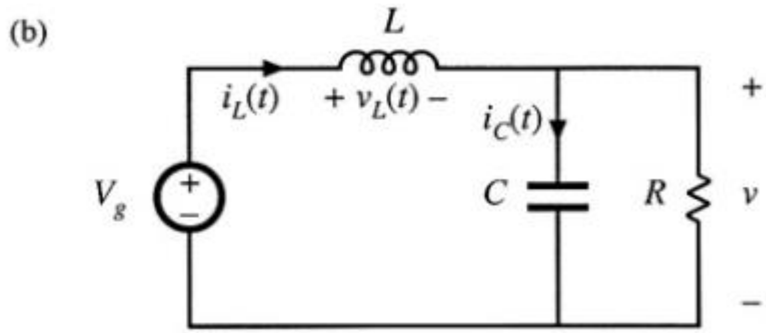
$$i_C = -\frac{v}{R}$$

Use of the linear ripple approximation, $v \approx V$, leads to

$$v_L = V_g$$

$$i_C = -\frac{V}{R}$$

Q1 : off, (switch is in position 2)



$$v_L = V_g - v$$

$$i_C = i_L - \frac{v}{R}$$

Use of the linear ripple approximation, $v \approx V$, and $i_L \approx I$, leads to

$$v_L = V_g - V$$

$$i_C = I - \frac{V}{R}$$

Steady-State Analysis with Linear Ripple Approximation

Q1 : on, (switch is in position 1)

Use of the linear ripple approximation, $v \approx V$, leads to

$$v_L = V_g$$

$$i_C = -\frac{V}{R}$$

Q1 : off, (switch is in position 2)

Use of the linear ripple approximation, $v \approx V$, and $i_L \approx I$, leads to

$$v_L = V_g - V$$

$$i_C = I - \frac{V}{R}$$

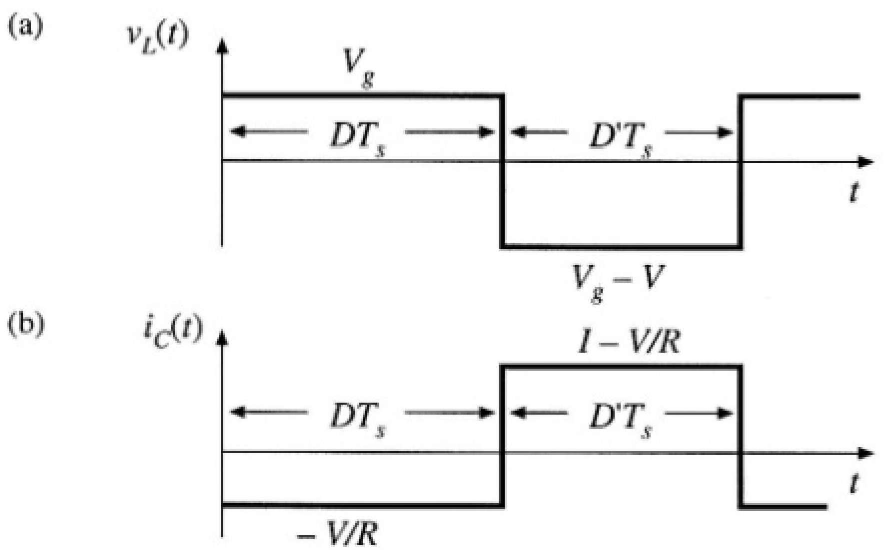
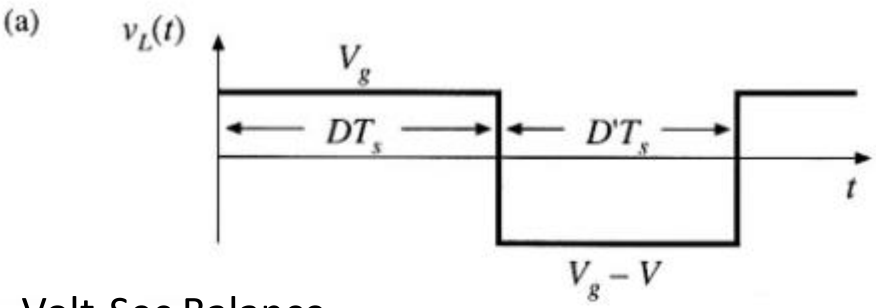


Fig. 2.15 Boost converter voltage and current waveforms.

Steady-State Analysis with Volt-Sec Balance



$$M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1-D}$$

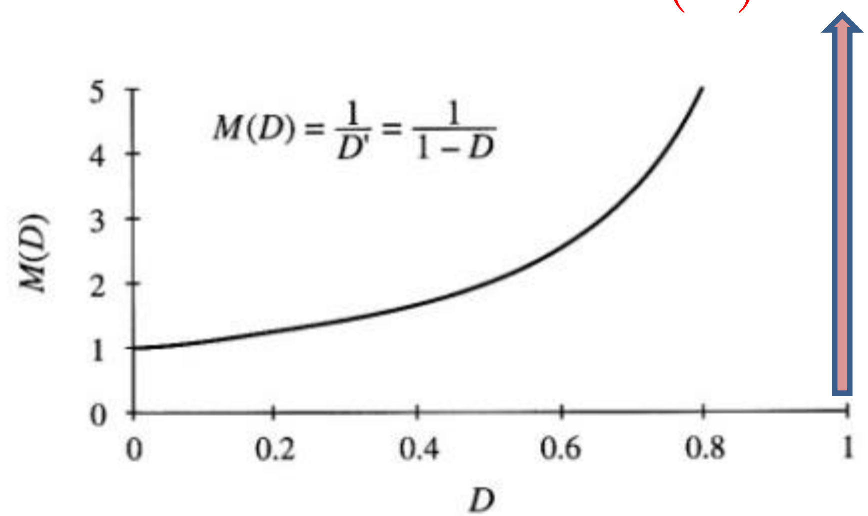
Volt-Sec Balance...

$$\int_0^{T_s} v_L(t)dt = (V_g)DT_s + (V_g - V)D'T_s$$

$$V_g(D + D') - VD' = 0$$

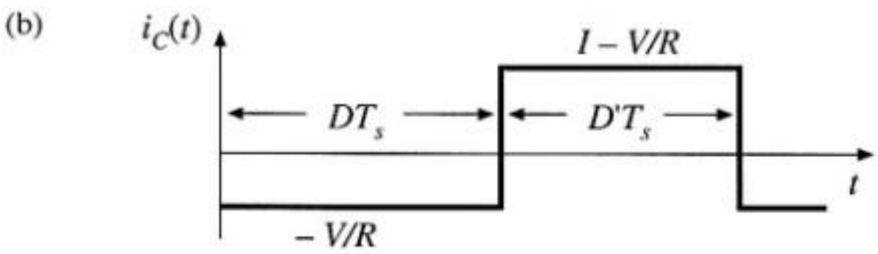
$$V = \frac{V_g}{D'}$$

$$M(D) \rightarrow \infty$$



- The output voltage increases as D increases, and in the ideal case tends to infinity as D tends to 1.
- *Maximum output voltage of a practical boost converter is indeed limited !!*

Steady-State Analysis with Amp-Sec Balance



- The inductor current DC component I becomes large as D approaches 1.
- *The inductor current is the input current, and it is greater than the load current. Why ???*

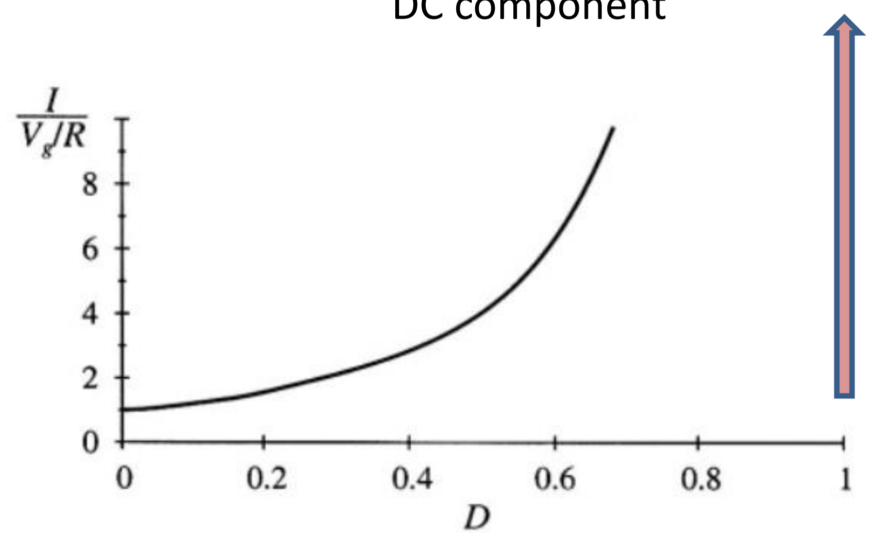
Amp-Sec Balance…

$$\int_0^{T_s} i_C(t)dt = \left(-\frac{V}{R}\right)DT_s + \left(I - \frac{V}{R}\right)D'T_s$$

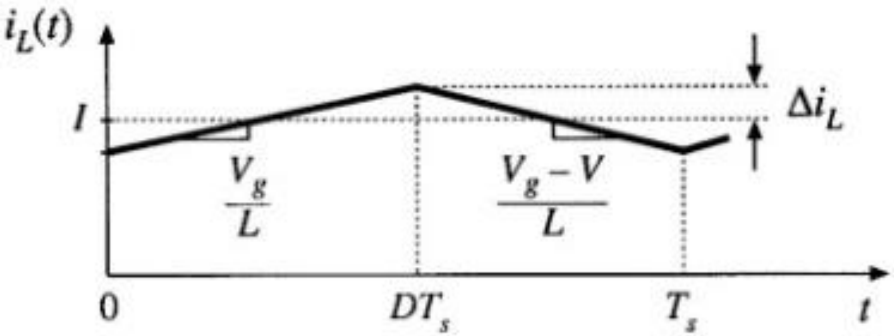
$$-\frac{V}{R}\left(D + D'\right) + ID' = 0$$

$$I = \frac{V}{D'R} \longrightarrow I = \frac{V_g}{D'^2R}$$

inductor current DC component $I \rightarrow \infty$



Steady-State Analysis: Inductor Current Waveform



$$2\Delta i_L = \frac{V_g}{L} DT_s$$

$$\boxed{\Delta i_L = \frac{V_g}{2L} DT_s}$$

- Switch in position 1

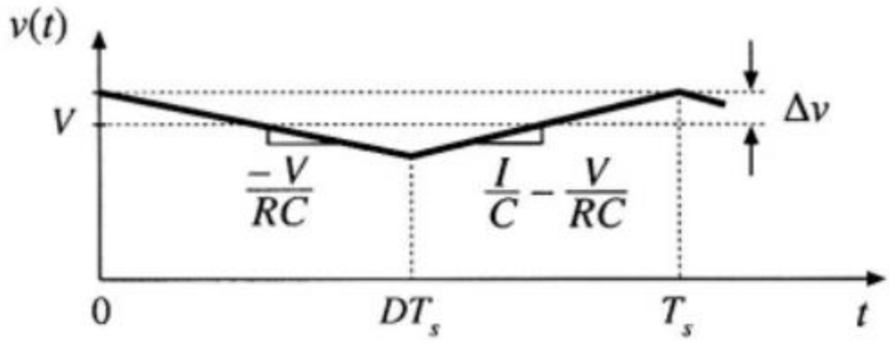
$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

- Switch in position 2

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$

This expression can be used to select the inductor value L such that a given value of diL is obtained.

Steady-State Analysis: Capacitor Voltage Waveform



$$-2\Delta v = \frac{-V}{RC} DT_s$$

$$\boxed{\Delta v = \frac{V}{2RC} DT_s}$$

- Switch in position 1

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$

- Switch in position 2

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

This expression can be used to select the capacitor value C to obtain a given output voltage ripple peak magnitude dv.

Steady-State Equivalent Circuit Modeling, Losses and Efficiency

Steady-State Equivalent Circuit for DC-DC Converter

- The use of equivalent circuits is a physical and intuitive approach which allows the well-known techniques of circuit analysis to be employed.
- Only the important DC components are modeled; the switching ripple is ignored.
- The DC transformer is used to model the ideal functions performed by a dc-dc converter.
- This simply model correctly represents the relationships between the dc voltages and currents of the converter.
- The model can be refined by including losses, such as semiconductor forward voltage drops and on-resistances, inductor core and copper losses, etc.
- The resulting model can be directly solved to find the voltages, currents, losses and efficiency in the actual nonideal converter.

$$P_{in} = P_{out}$$

$$V_g I_g = VI$$

(100 % Efficiency)

This relation is valid only under equilibrium (dc) conditions !!

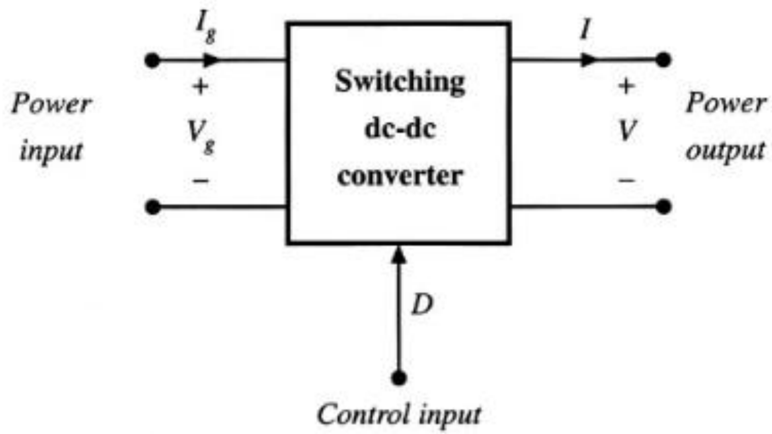


Fig. 3.1 Switching converter terminal quantities.

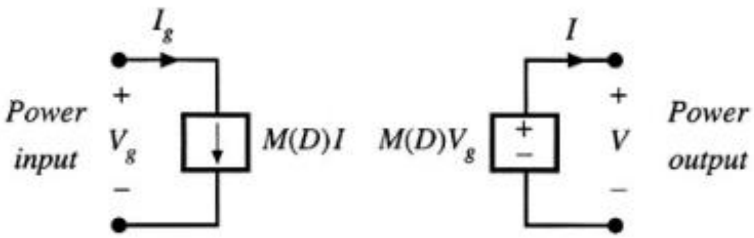
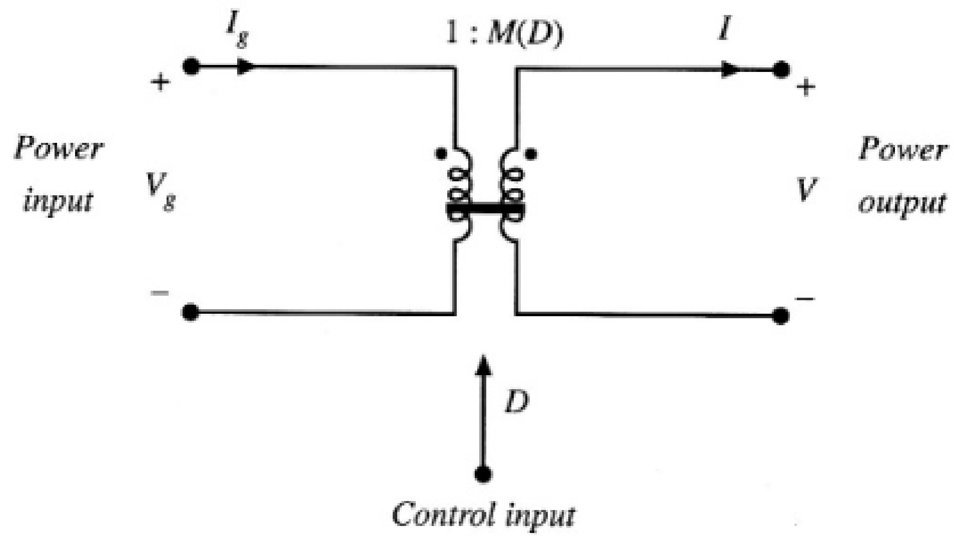


Fig. 3.2 A switching converter equivalent circuit using dependent sources, corresponding to Eqs. (3.3) and (3.4).

Ideal DC Transformer

Fig. 3.3 Ideal dc transformer model of a dc-dc converter operating in continuous conduction mode, corresponding to Eqs. (3.1) to (3.4).



$$V = M(D)V_g$$

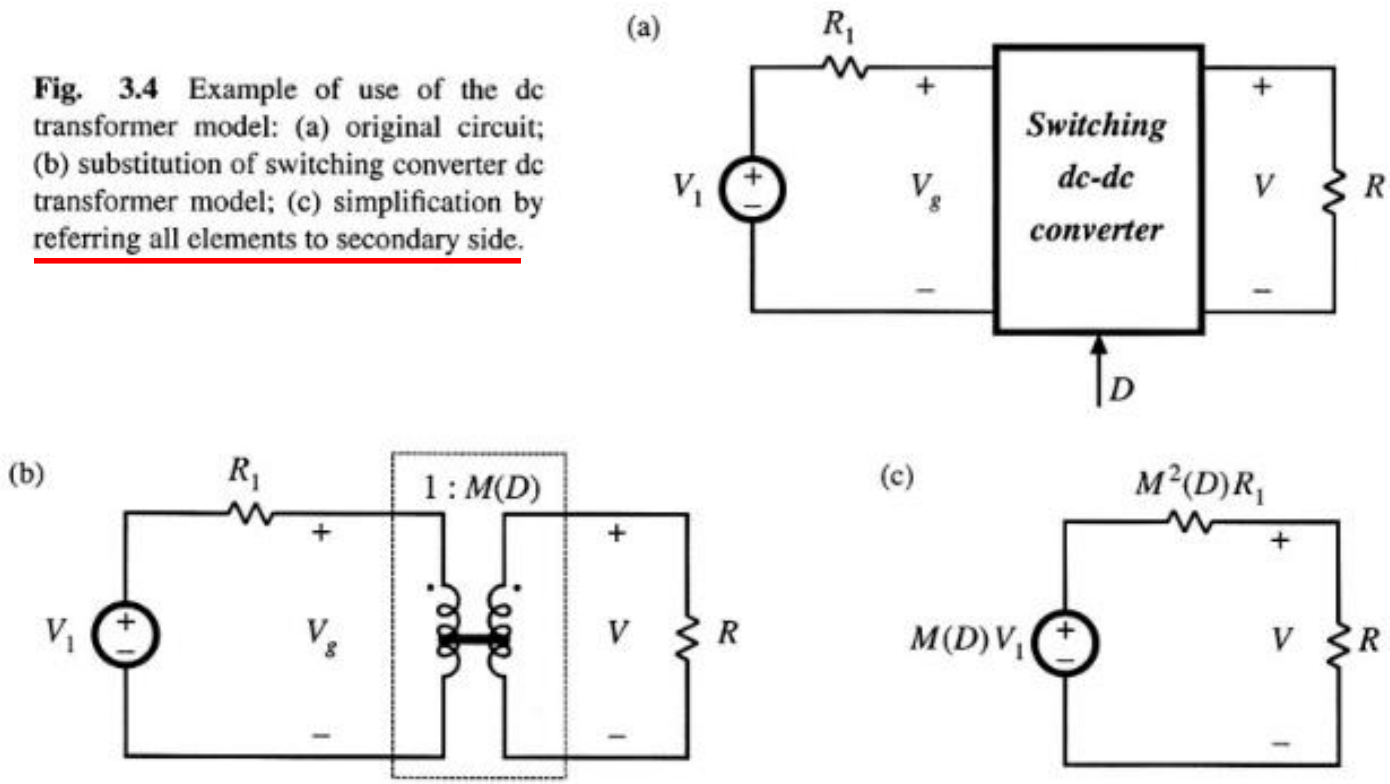
$$I_g = M(D)I$$

M(D) for buck converter ??

M(D) for boost converter ??

Use of DC Transformer Model

Fig. 3.4 Example of use of the dc transformer model: (a) original circuit; (b) substitution of switching converter dc transformer model; (c) simplification by referring all elements to secondary side.



The rules for manipulating and simplifying circuits containing transformers apply equally well to circuits containing dc-dc converters. (transformation of dc voltage and current levels)

$$V = M(D)V_1 \frac{R}{R + M^2(D)R_1}$$

Inclusion of Inductor Copper Loss

Fig. 3.5 Modeling inductor copper loss via series resistor R_L.

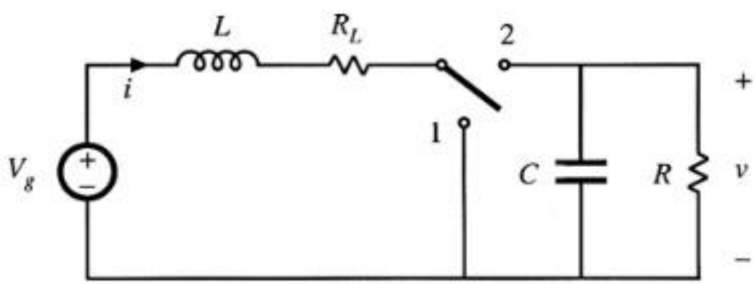


Fig. 3.6 Boost converter circuit, including inductor copper resistance R_L.

$$\boxed{\frac{V}{V_g} = \frac{1}{D'} \frac{1}{\left(1 + \frac{R_L}{D'^2 R}\right)}}$$

– The circuit with inclusion of inductor copper loss can be analyzed in the same manner as used for the ideal lossless converter, using the principles of inductor volt-sec balance, capacitor amp-sec balance, and the small-ripple approximation.

– Or the circuit analysis technique of dc transformer model !!

– V/Vg contains two terms:
  - The first term 1/D' is the ideal conversion ratio, with RL=0.
  - The second term, describes the effect of the inductor winding resistance.

Output Voltage vs Duty Cycle: Boost Converter with Inductor Copper Loss

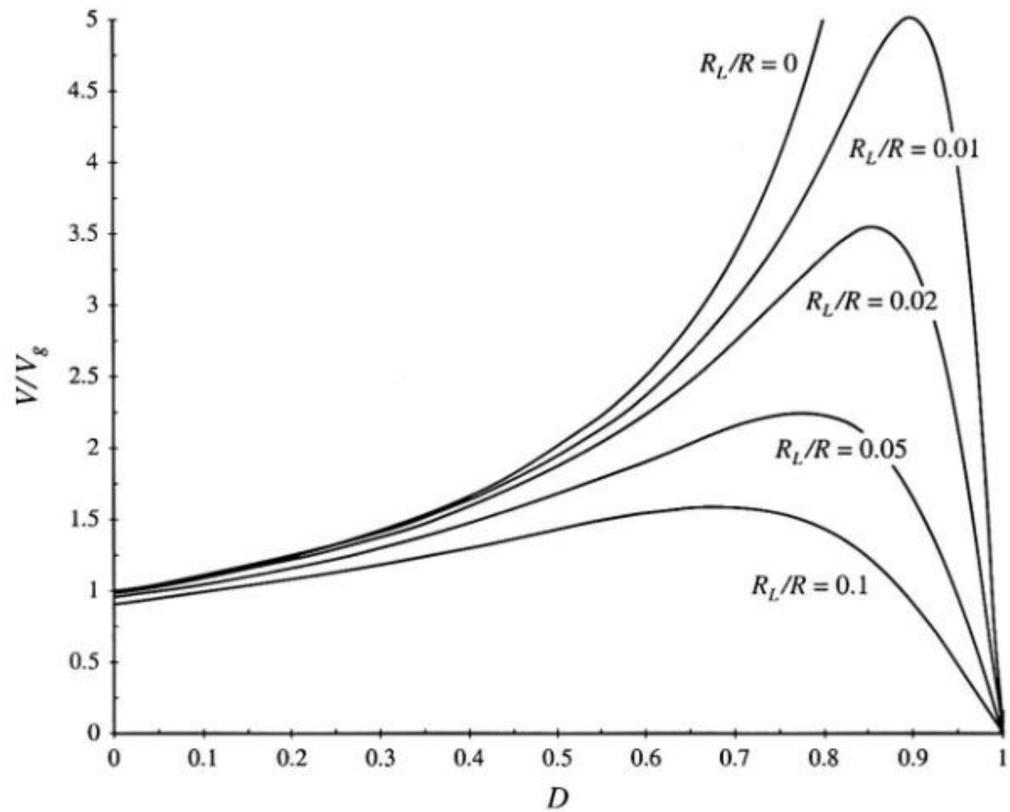


Fig. 3.9 Output voltage vs. duty cycle, boost converter with inductor copper loss.

$$\frac{V}{V_g} = \frac{1}{D'} \frac{1}{\left(1 + \frac{R_L}{D'^2 R}\right)}$$

- RL causes a major qualitative change in the V/Vg curve.
- Rather than approaching infinity at D=1, the curve tends to zero.
- *What happens at D=1 (the switch is always in position 1.) ???*
- RL limits the maximum voltage that the converter can produce.

Efficiency

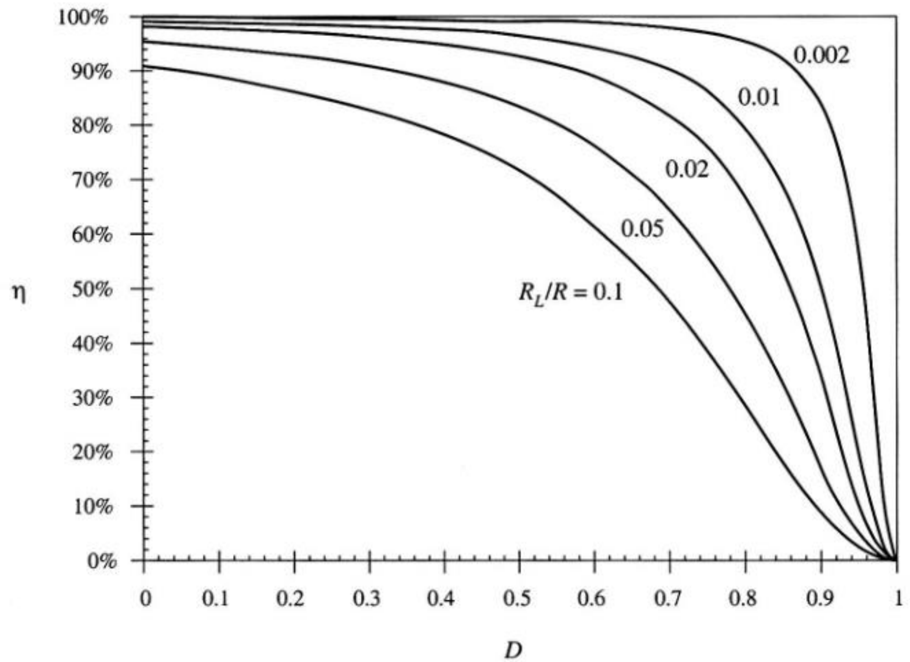


Fig. 3.15 Efficiency vs. duty cycle, boost converter with inductor copper loss.

$$P_{in} = (V_g)(I) \qquad P_{out} = (V)(D'I)$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{(V)(D'I)}{(V_g)(I)} = \frac{V}{V_g} D'$$

$$\boxed{\eta = \frac{1}{1 + \frac{R_L}{D'^2 R}}}$$

- To obtain high efficiency, the inductor winding resistance RL should be much smaller that D’2 R, the load resistance referred to the primary side of the ideal dc transformer.

- The efficiency is typically high at low duty cycles, but decreases rapidly to zero near D=1.

Buck-Boost Converter

Cascade Connection of Converters

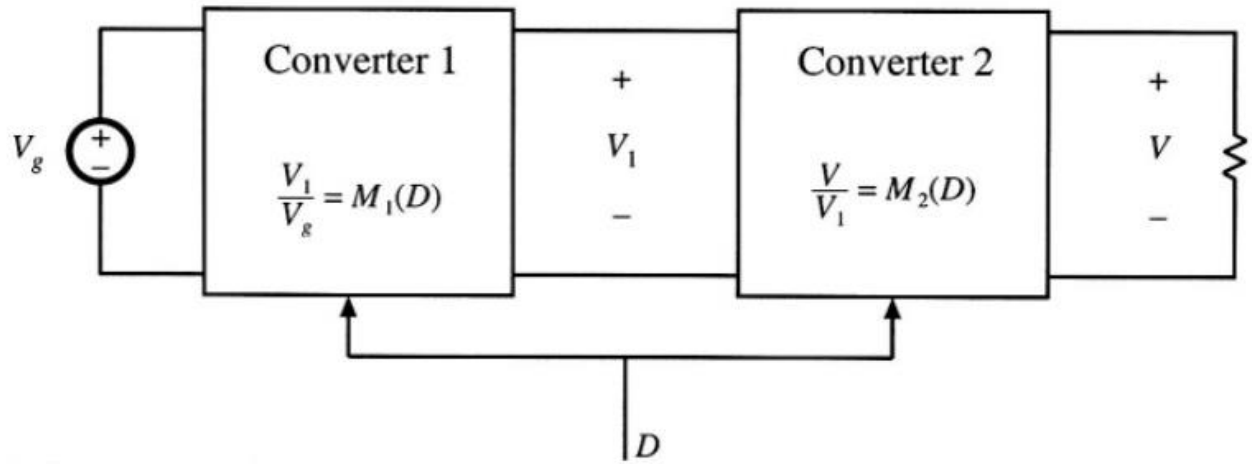


Fig. 6.3 Cascade connection of converters.

- Converter can also be connected in cascade.
- Converter 1 has conversion ratio M1(D), $V_1 = M_1(D)V_g$ which is applied to thee input of the second converter.
- Assume that the converter 2 is driven with the same duty D applied to converter 1. If converter 2 has conversion ratio M2(D), $V = M_2(D)V_1$
- The conversion ration of the composite converter is the product of the individual conversion ratios…

$$\boxed{\frac{V}{V_g} = M(D) = M_1(D)M_2(D)}$$

Cascade Connection of Buck Converter and Boost Converter

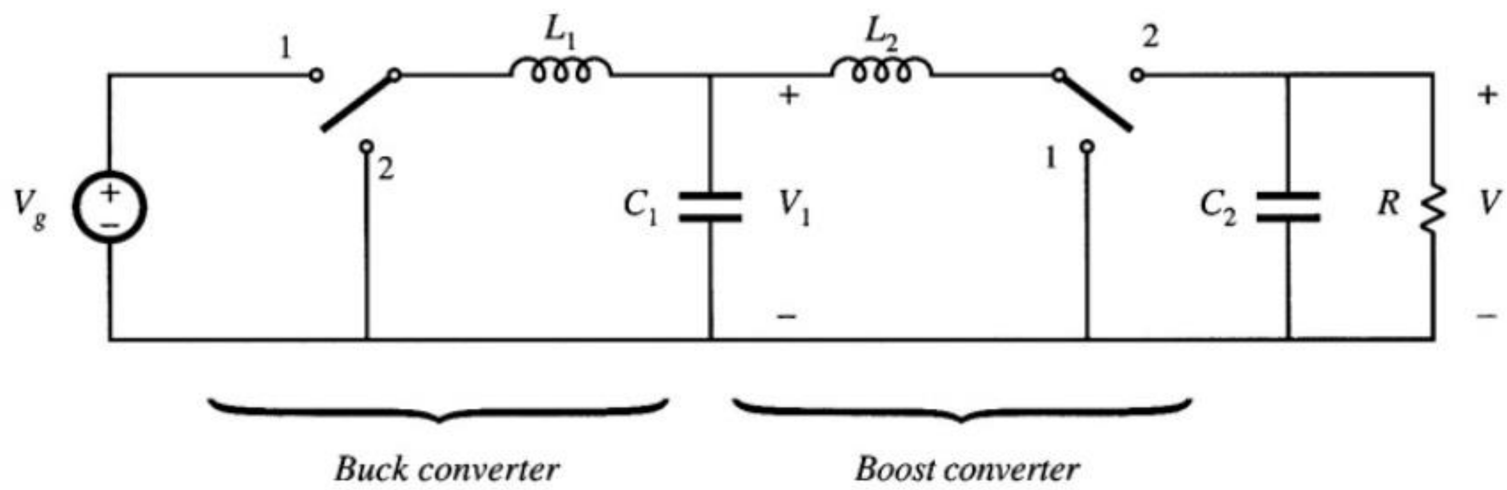
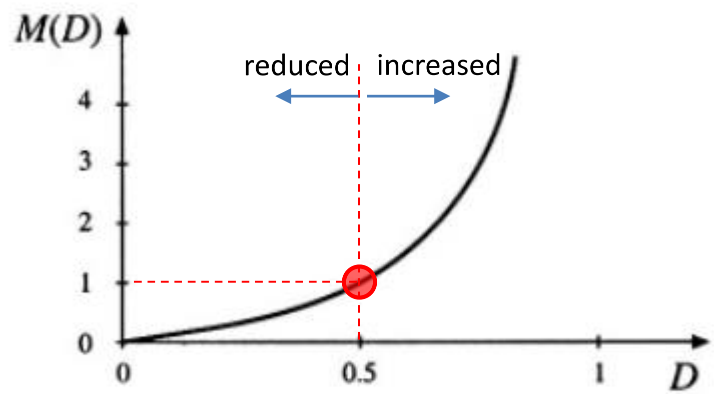


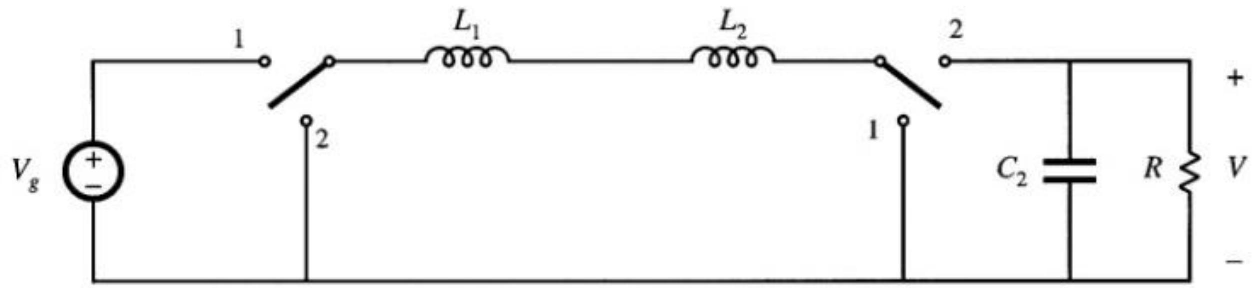
Fig. 6.4 Cascade connection of buck converter and boost converter.

Noninverting buck-boost conversion ratio

- Buck converter has conversion ratio… $\dfrac{V_1}{V_g} = D$
- Boost converter has conversion ratio… $\dfrac{V}{V_1} = \dfrac{1}{1-D}$
- So the composite conversion ratio is $\boxed{\dfrac{V}{V_g} = \dfrac{D}{1-D}}$



Simplification of Cascaded Buck and Boost Converter



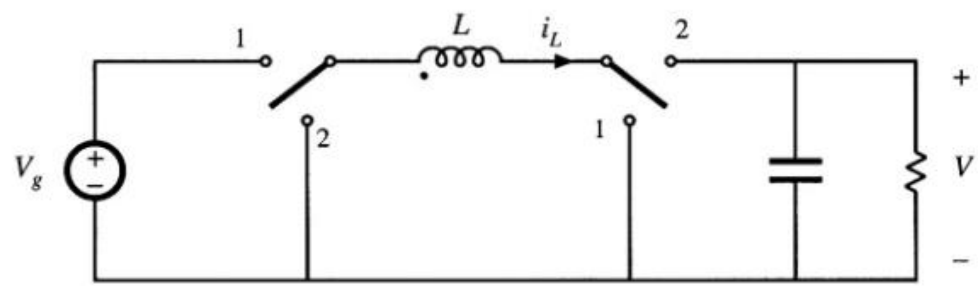
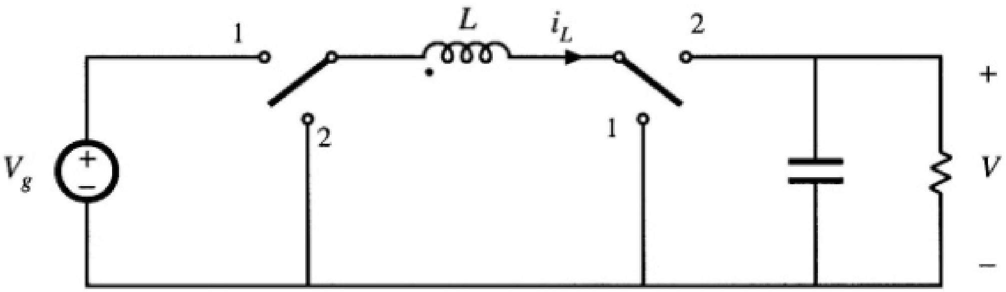


Fig. 6.5 Simplification of the cascaded buck and boost converter circuit of Fig. 6.4: (a) removal of capacitor C_1. (b) combining of inductors L_1 and L_2.

- **Removal of capacitor C1** : inductors L1 and L2, along with capacitor C1, form a three-pole low-pass filter. The *conversion ratio does not depend on the number of poles* present in the low-pass filter, and so the same steady-state output voltage should be obtained when a simpler low-pass filter is used. So capacitor C1 is removed.
- **Combining of L1 and L2** : Inductor L1 and L2 are now in series, and can be combined into a single inductor.

Simplification of Cascaded Buck and Boost Converter



- *The switches of the converter can also be simplified, leading to a negative output voltage.*
- When the switches are in position 1, the inductor is connected to the input source Vg, and energy is transferred from the source to the inductor.
- When the switches are in position 2, the inductor is then connected to the load, and energy is transferred from the inductor to the load.

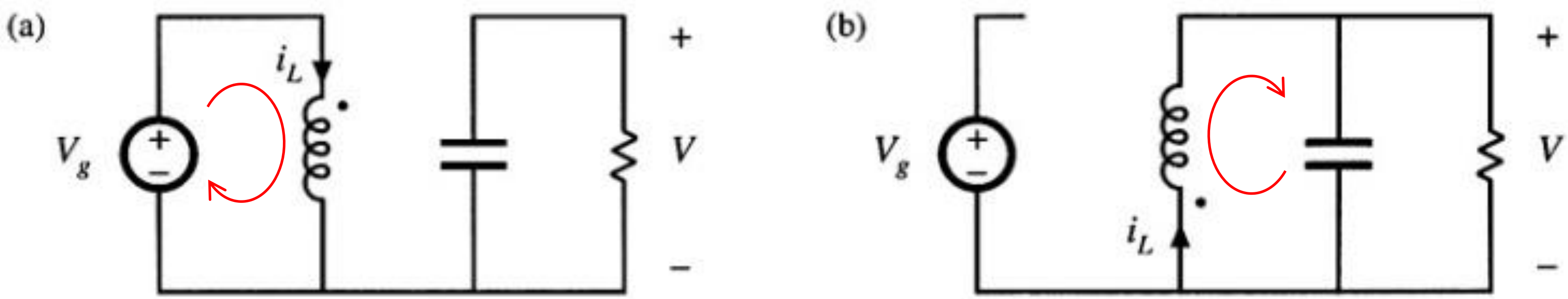


Fig. 6.6 Connections of the circuit of Fig. 6.5(b): (a) while the switches are in position 1, (b) while the switches are in position 2.

Simplification of Cascaded Buck and Boost Converter

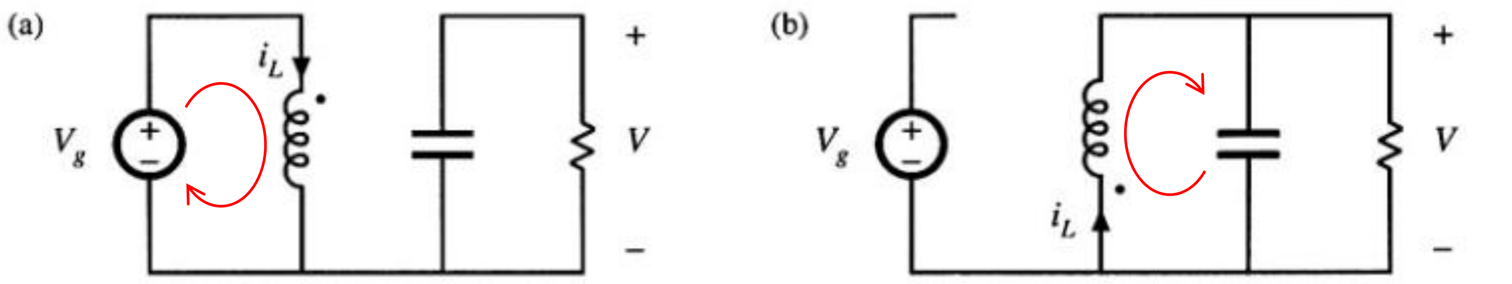


Fig. 6.6 Connections of the circuit of Fig. 6.5(b): (a) while the switches are in position 1, (b) while the switches are in position 2.

Noninverting buck-boost converter,

V > 0

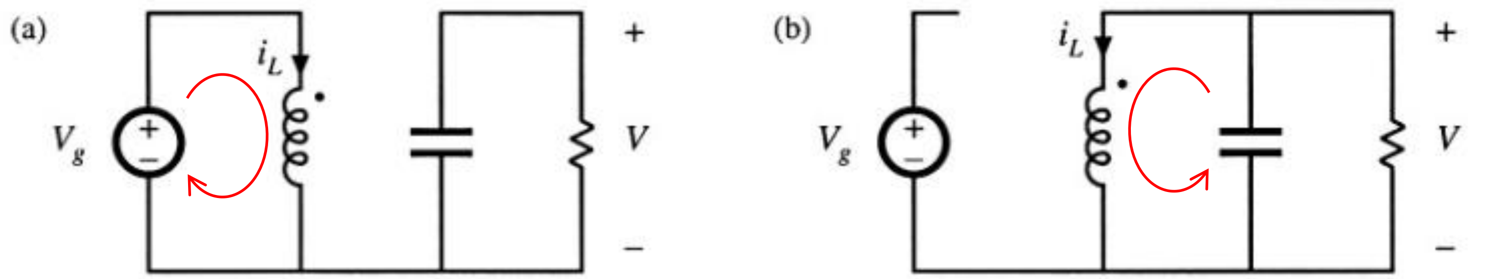


Fig. 6.7 Reversal of the output voltage polarity, by reversing the inductor connections while the switches are in position 2: (a) connections with the switches in position 1, (b) connections with the switches in position 2.

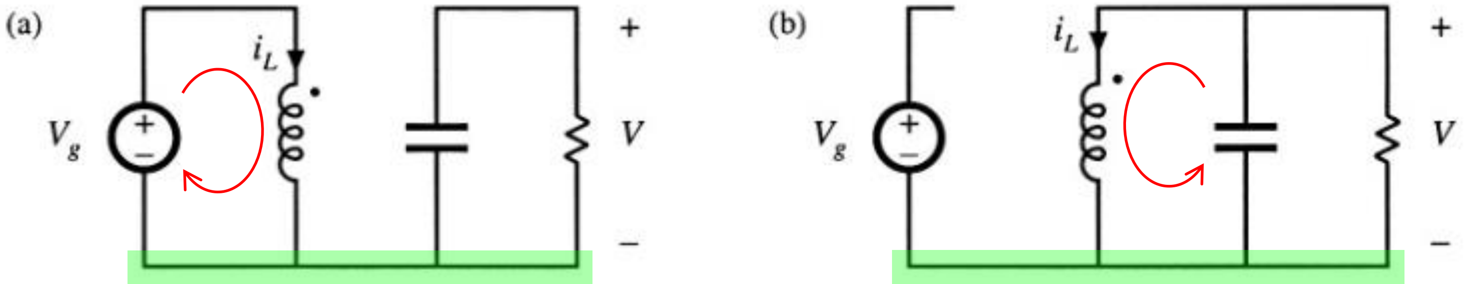
Inverting buck-boost converter,

V < 0.

- To obtain a negative output (V < 0), we can simply reverse the polarity of the inductor during one of the subintervals (say, while the switches are in position 2)
- So the composite conversion ratio becomes …

$$\boxed{\frac{V}{V_g} = -\frac{D}{1-D}}$$

Simplification of Inverting Buck-Boost Converter with One SPDT

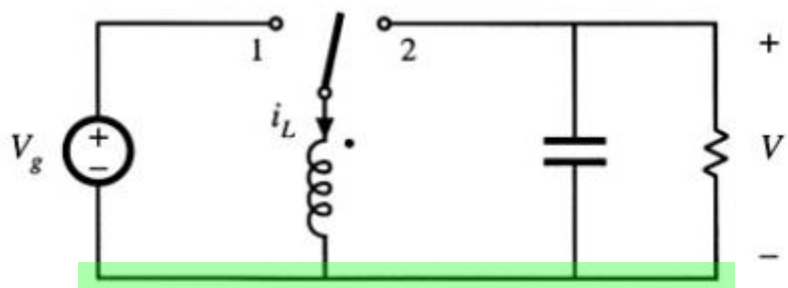


Inverting buck-boost converter,

$V < 0$.

Fig. 6.7 Reversal of the output voltage polarity, by reversing the inductor connections while the switches are in position 2: (a) connections with the switches in position 1, (b) connections with the switches in position 2.

Fig. 6.8 Converter circuit obtained from the subcircuits of Fig. 6.7.

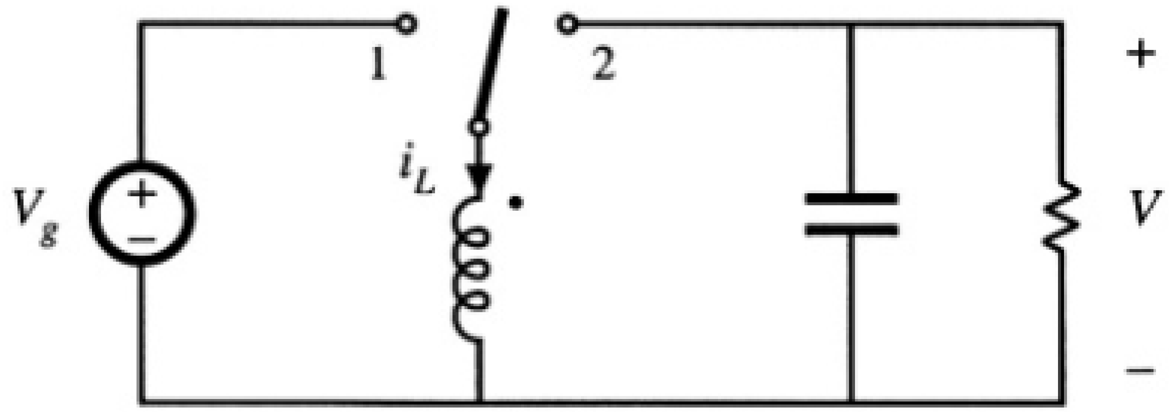


Inverting buck-boost converter,

$V < 0$.

- Note that one side of the inductor is now always connected to ground, while the other side is switched between the input source and the load.
- Hence only one SPDT switch is needed, and the converter circuit of Fig. 6.8 is obtained, this circuit is recognized as the conventional buck-boost converter.

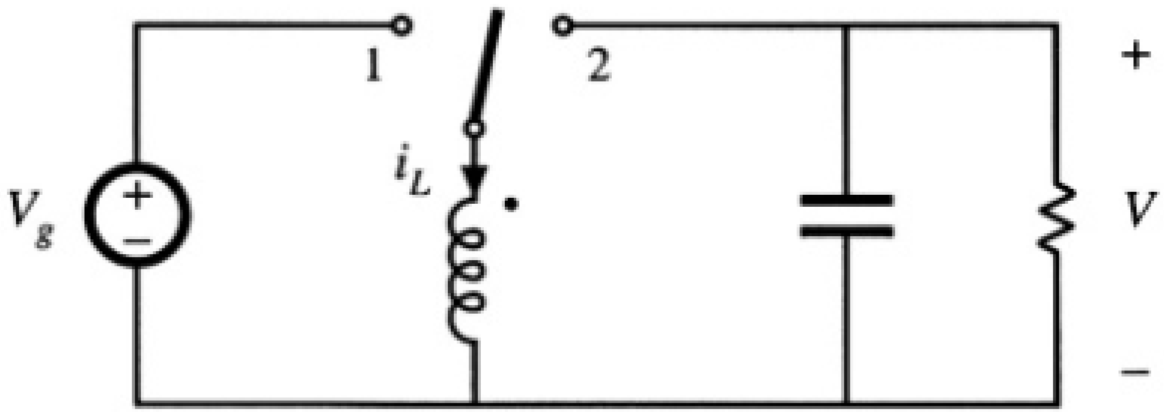
(Inverting) Buck-Boost Converter



$V < 0.$

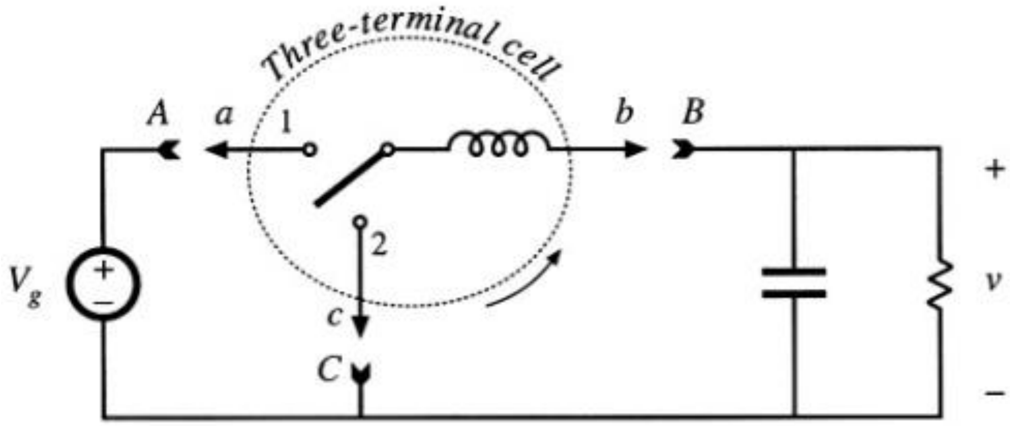
- Buck-boost converter can be viewed as a cascade connection of buck and boost converters
- Equivalent circuit model of the buck-boost converter contains a 1:D (buck) DC transformer, followed by a D’:1 (boost) DC transformer.
- It inherits the pulsating input current of the buck converter, and the pulsating output current of the boost converter.

Switches Realization for Buck-Boost Converter



$V < 0$.

Rotation of Three-Terminal Cell



Connections	Converter
a-A b-B c-C	Buck
a-C b-A c-B	Boost
a-A b-C c-B	Buck-Boost

- The buck, boost and buck-boost converters each contain an inductor that is connected to a SPDT switch.
- The inductor-switch network can be viewed as a basic cell having of the three terminals labeled *a, b*, and *c*.
- There are three distinct ways to connect this cell between the source and load.

List of DC-DC Converters

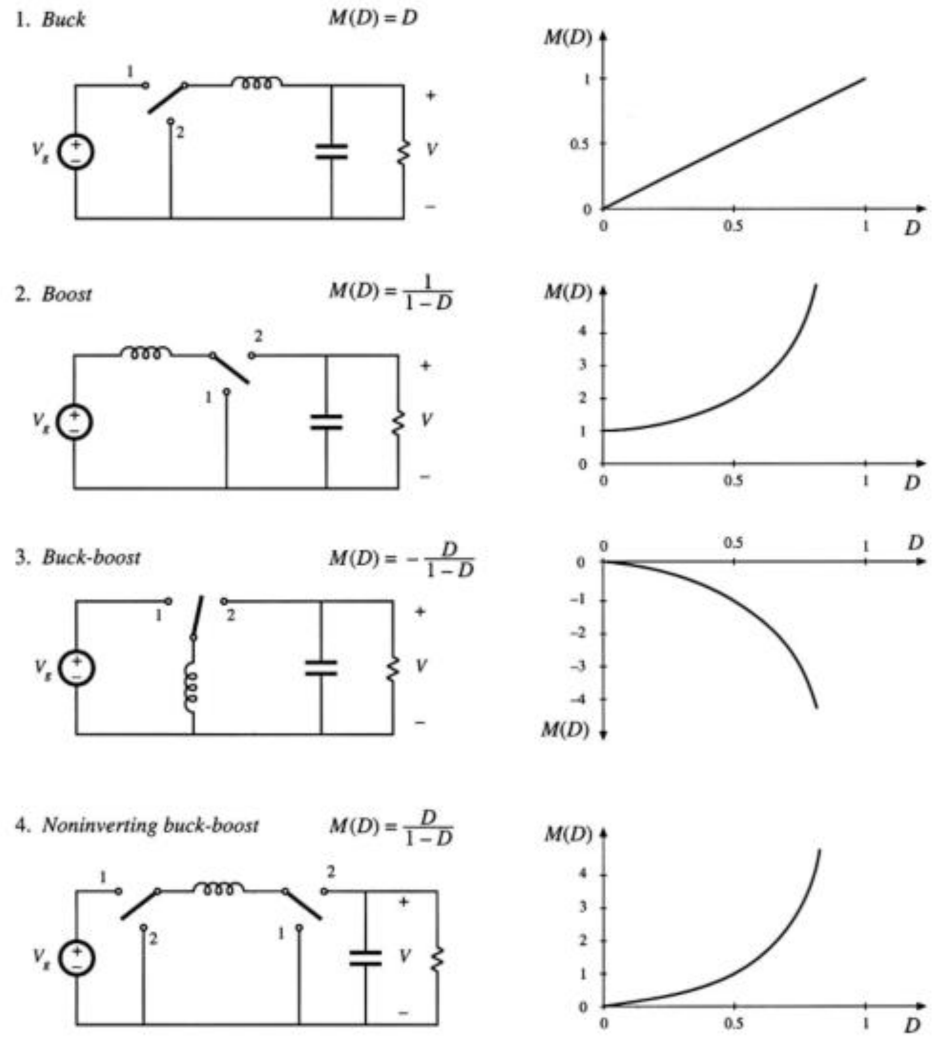
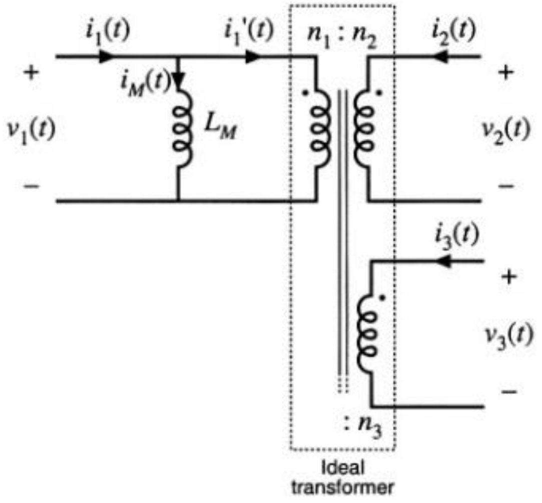


Fig. 6.14 Eight members of the basic class of single-input single-output converters containing a single inductor.

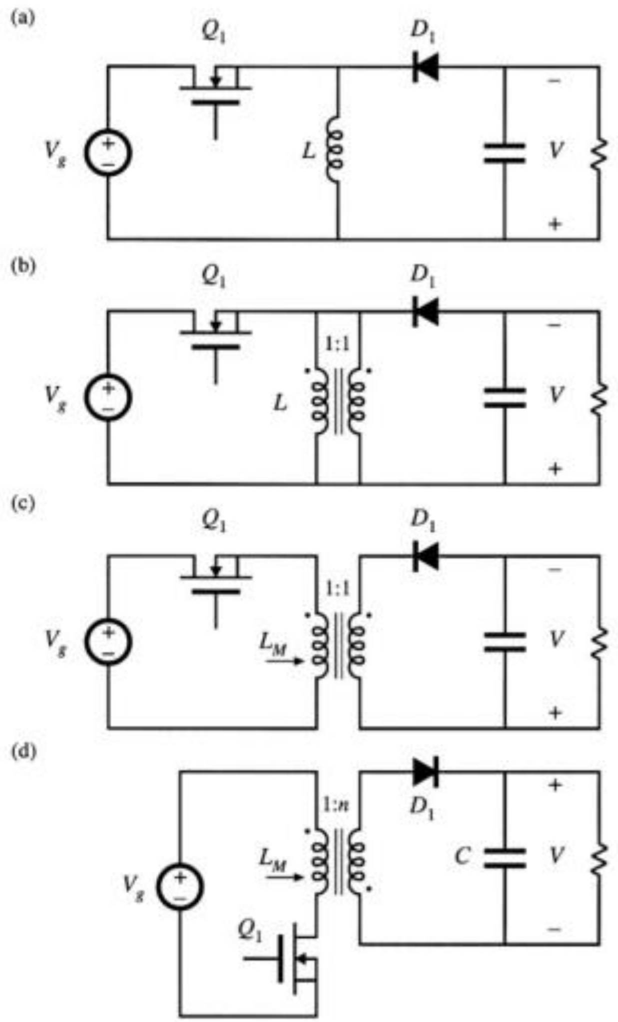
Transformer Isolation

- Large number of applications desire to *incorporate a transformer* into a switching converter, to obtain *dc isolation* between the converter input and output.
- Transformer size and weight vary inversely with frequency, uses of 50 Hz or 60 Hz make converter bulky,
- Incorporating the *transformer* into the converter which operates at switching frequency of tens or hundreds of kilohertz, can significantly *reduce the transformer size and weight*.
- When a *large step-up or step-down conversion ratio* is required, the use of transformer can allow better optimization.
- *Multiple dc outputs* can also be obtained in an inexpensive manner, by adding multiple secondary windings.



Flyback Converter

Fig. 6.30 Derivation of the flyback converter: (a) buck–boost converter; (b) inductor L is wound with two parallel wires; (c) inductor windings are isolated, leading to the flyback converter; (d) with a $1{:}n$ turns ratio and positive output.



- The *flyback* converter is based on the *buck-boost* converter.
- The *magnetizing inductance LM functions in the same manner as inductor L* of the original buck-boost converter.
  - When Q1 conducts, energy from the dc source Vg is stored in LM.
  - When diode D1 conducts, this store energy is transferred to the load, with *the inductor voltage and current scaled according to the 1:n turns ratio*.