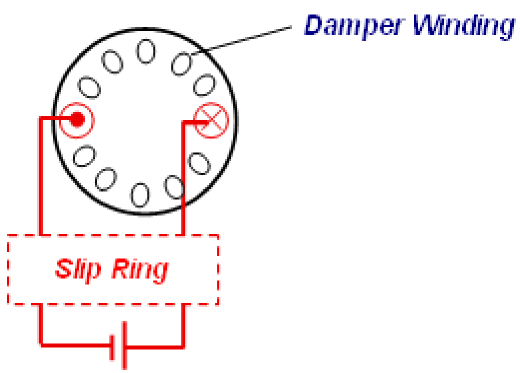
Chapter 4

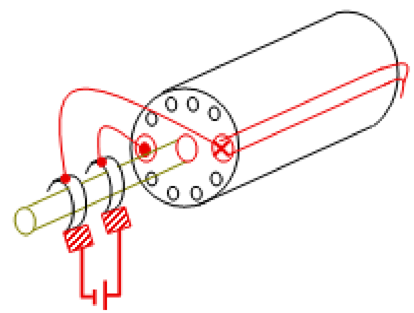
Modeling and Control of Surface Permanent Magnet Synchronous Motor; SPMSM

Synchronous Machines

- ชนิดของ Synchronous Motor (SM) … จำแนกตามโครงสร้างของ Rotor

  [1] Field Winding (+Damper Winding) … .ใช้กับมอเตอร์ขนาดมีพิกัดกำลังสูง (> kW)

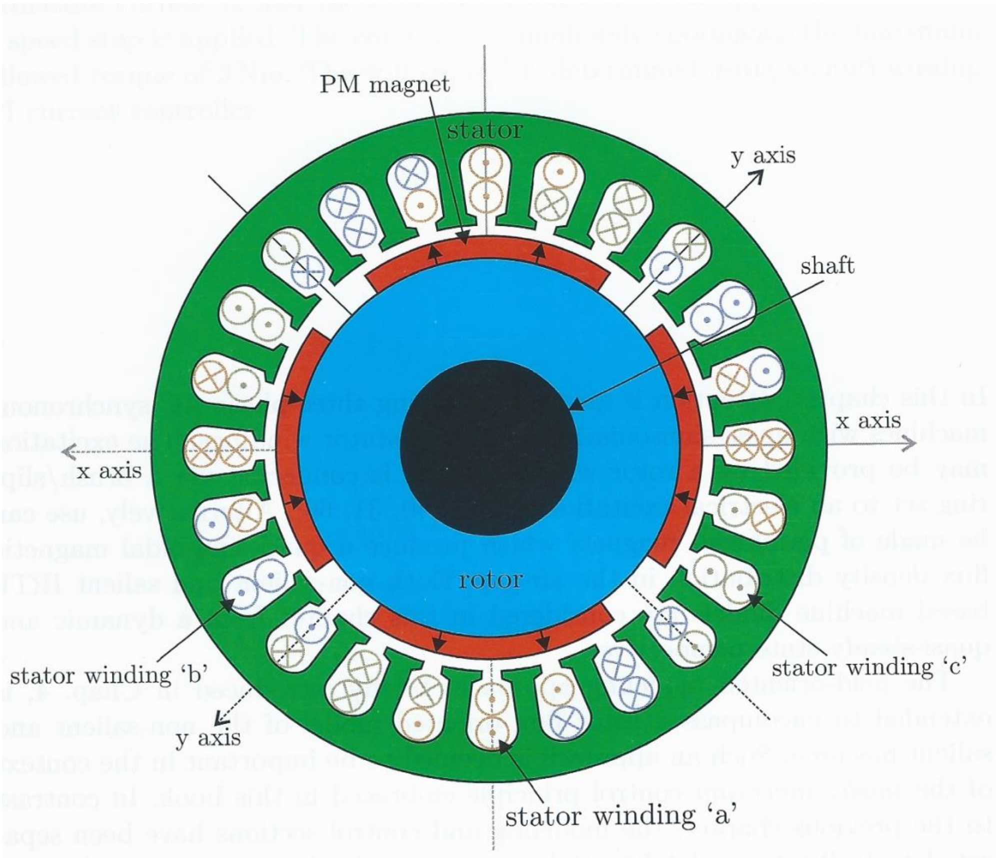




  [2] Permanent Magnet … พิกัด ~ kW….. ใช้ในงาน Servo, High Eff. Drives

  - Surface Permanent Magnet (SPM)
  - Interior Permanent Magnet (IPM)

Surface Permanent Magnet Synchronous Motor



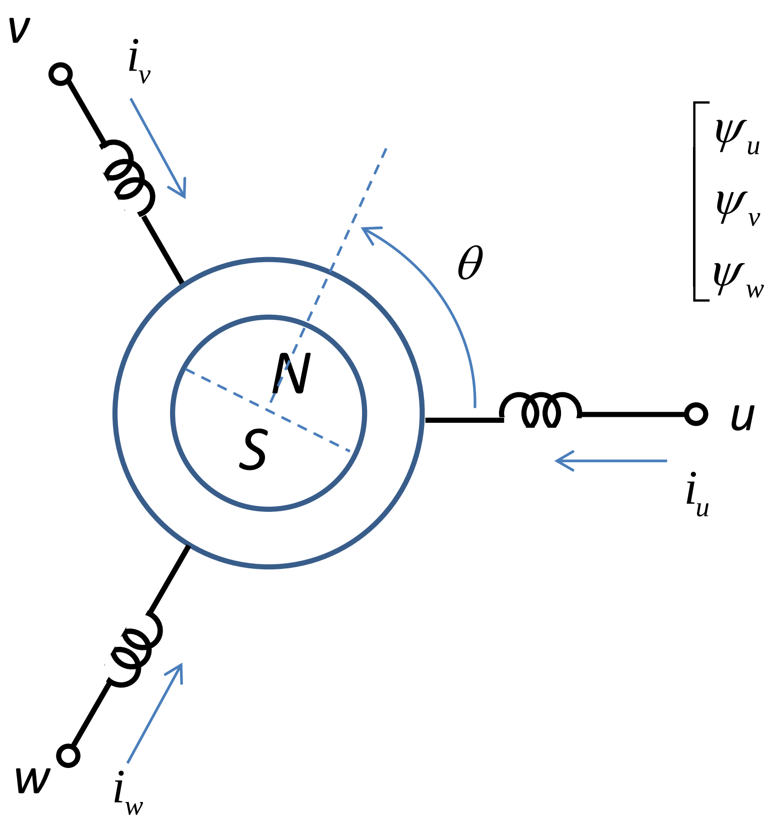
-Uniform Airgap : Non salient

☒ Large Airgap

☒ High Speed

- *Cross-sectional view of 4-pole SPMSM*

Dynamic Model of SPMSM



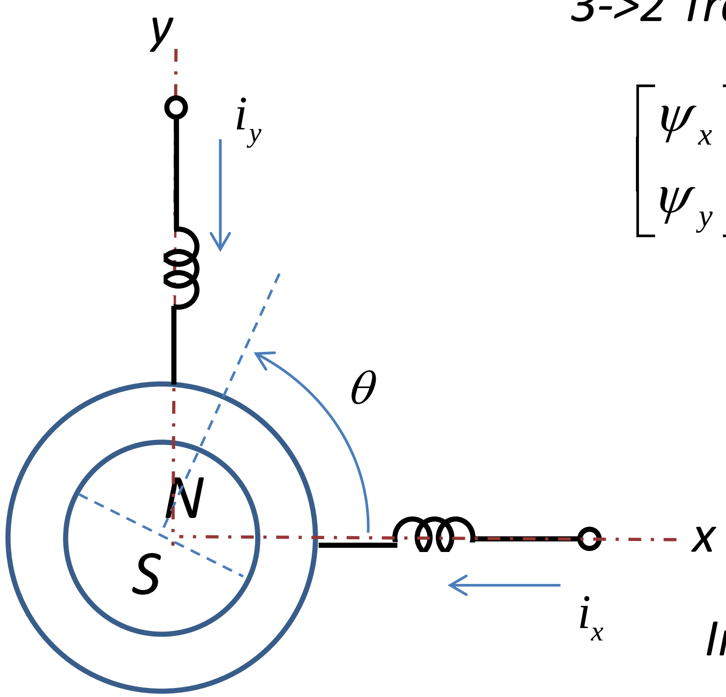
Flux Linkage on Windings :

$$\begin{bmatrix}\psi_u\\ \psi_v\\ \psi_w\end{bmatrix}=\begin{bmatrix} l+L & -\frac{1}{2}L & -\frac{1}{2}L\\ -\frac{1}{2}L & l+L & -\frac{1}{2}L\\ -\frac{1}{2}L & -\frac{1}{2}L & l+L\end{bmatrix}\begin{bmatrix}i_u\\ i_v\\ i_w\end{bmatrix}+\underbrace{\lambda'\begin{bmatrix}\cos\theta\\ \cos\left(\theta-120^o\right)\\ \cos\left(\theta-240^o\right)\end{bmatrix}}_{\text{Flux Linkage from permanent magnet}}$$

Induced Voltage :

$$\begin{bmatrix}v_{un}\\ v_{vn}\\ v_{wn}\end{bmatrix}=R\begin{bmatrix}i_u\\ i_v\\ i_w\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\psi_u\\ \psi_v\\ \psi_w\end{bmatrix}$$

Dynamic Model of SPMSM



3->2 Transformation :

$$\begin{bmatrix}\psi_x\\ \psi_y\end{bmatrix} = T\begin{bmatrix}\psi_u\\ \psi_v\\ \psi_w\end{bmatrix} = \underbrace{\left(l+\frac{3}{2}L\right)}_{L_s}\cdot\begin{bmatrix}i_x\\ i_y\end{bmatrix} + \sqrt{\frac{3}{2}}\lambda'\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$$

$$= L_s\begin{bmatrix}i_x\\ i_y\end{bmatrix} + \underbrace{\lambda\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}}_{\vec{\lambda}} = L_s\,\vec{i}_s + \lambda\, e^{J\theta}$$

Induced Voltage :

$$\begin{bmatrix}v_x\\ v_y\end{bmatrix} = R\begin{bmatrix}i_x\\ i_y\end{bmatrix} + \frac{d}{dt}\begin{bmatrix}\psi_x\\ \psi_y\end{bmatrix}$$

$$\vec{v}_s = R\,\vec{i}_s + \frac{d}{dt}\left(L_s\,\vec{i}_s + \vec{\lambda}\right) = R\,\vec{i}_s + L_s\frac{d\vec{i}_s}{dt} + J\omega\vec{\lambda}$$

Power & Induced Torque

$$\vec{v}_s = R\,\vec{i}_s + L_s \frac{d\vec{i}_s}{dt} + J\omega\vec{\lambda}$$

SPMSM Model on Stator Reference Frame

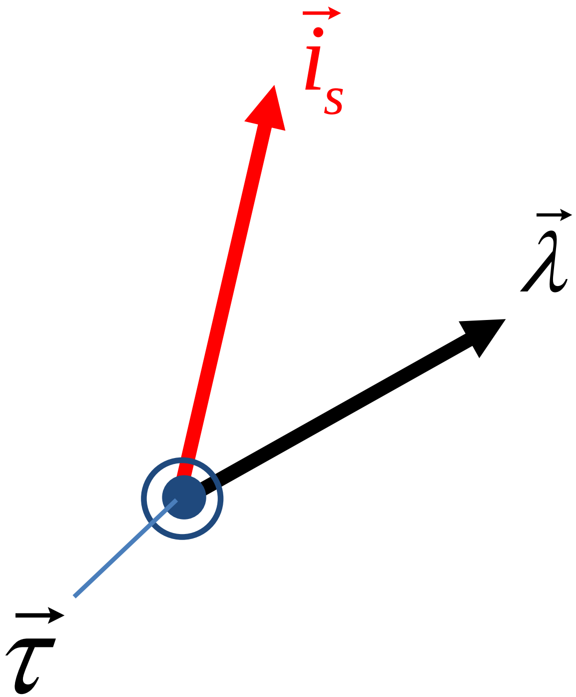
Power…

$$\vec{i}_s \bullet \vec{v}_s = \vec{i}_s \bullet \left\{ R\,\vec{i}_s + L_s \frac{d\vec{i}_s}{dt} + J\omega\vec{\lambda} \right\}$$

$$\underbrace{\vec{i}_s^{\,T}\vec{v}_s}_{\text{Input Power}} = \underbrace{\vec{i}_s^{\,T} R\,\vec{i}_s}_{\text{Copper Loss}} + \underbrace{\vec{i}_s^{\,T} L_s \frac{d\vec{i}_s}{dt}}_{\frac{d}{dt}\underbrace{\left(\frac{1}{2} L_s \vec{i}_s^{\,T}\vec{i}_s\right)}_{\text{Magnetic Energy}}} + \underbrace{\vec{i}_s^{\,T} J\omega\vec{\lambda}}_{\text{Mechanical Power}}$$

Power & Induced Torque

$$P_{mech} = \vec{i}_s^{\,T} J \omega \vec{\lambda} = \tau \cdot \omega_m = \tau \cdot \frac{\omega}{P}$$



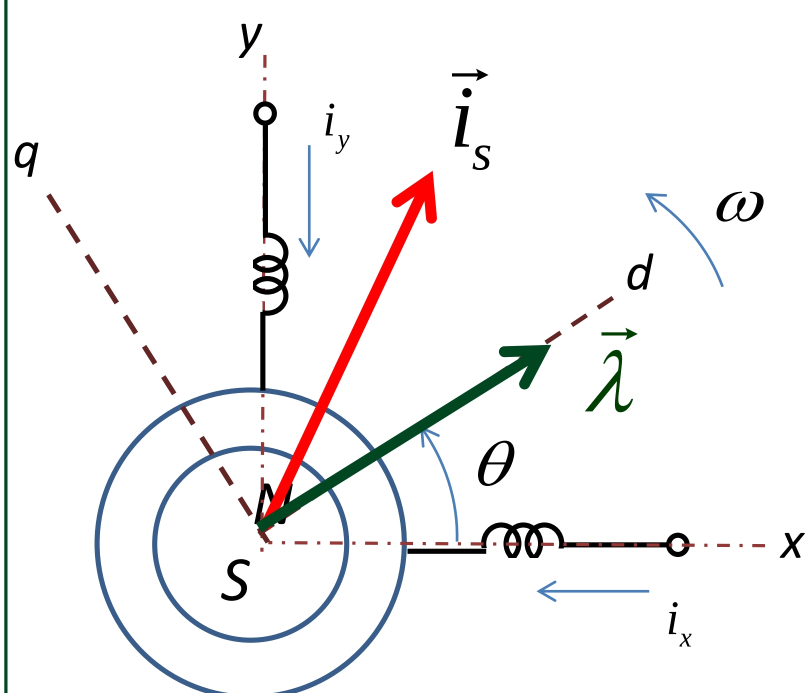
$$\tau = P\,\vec{i}_s^{\,T} J \vec{\lambda}$$

$$\vec{\tau} = P\,\vec{\lambda} \times \vec{i}_s$$

Dynamic Model of SPMSM on Rotating Reference Frame

$$T_1(\theta) \triangleq e^{-J\theta} \quad \times \quad \rightarrow$$

$$e^{-J\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Stator ref. frame

$$\vec{v}_s = R\vec{i}_s + L_s \frac{d\vec{i}_s}{dt} + J\omega\vec{\lambda}$$

Rotor ref. frame

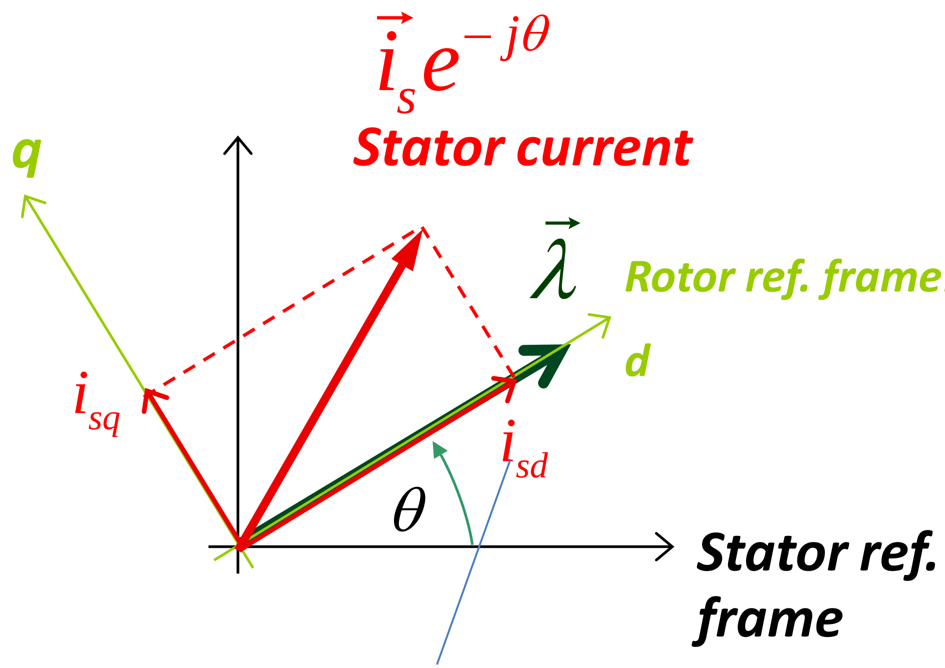
$$\vec{v}_s' = R\vec{i}_s{}' + L_s \underbrace{T_1(\theta)\frac{d\vec{i}_s}{dt}}_{\frac{d}{dt}\left(T_1(\theta)\vec{i}_s\right) = \frac{d\vec{i}_s{}'}{dt} - J\omega\vec{i}_s{}'} + J\omega\vec{\lambda}'$$

Rotor ref. frame

$$\vec{v}_s' = R\vec{i}_s{}' + L_s \frac{d\vec{i}_s'}{dt} + J\omega L_s\vec{i}_s' + J\omega\vec{\lambda}'$$

Vector Control of SPMSM

สมการแรงบิด



$$\vec{\tau} = P\vec{\lambda} \times \vec{i}_s$$

$$\tau = \mathrm{Im}\left[P|\lambda|e^{-j\theta} \cdot \vec{i}_s\right]$$

$$= P|\lambda|\,\mathrm{Im}\left[\vec{i}_s e^{-j\theta}\right]$$

$$= P|\lambda|\,\mathrm{Im}\left[i_{sd} + j i_{sq}\right]$$

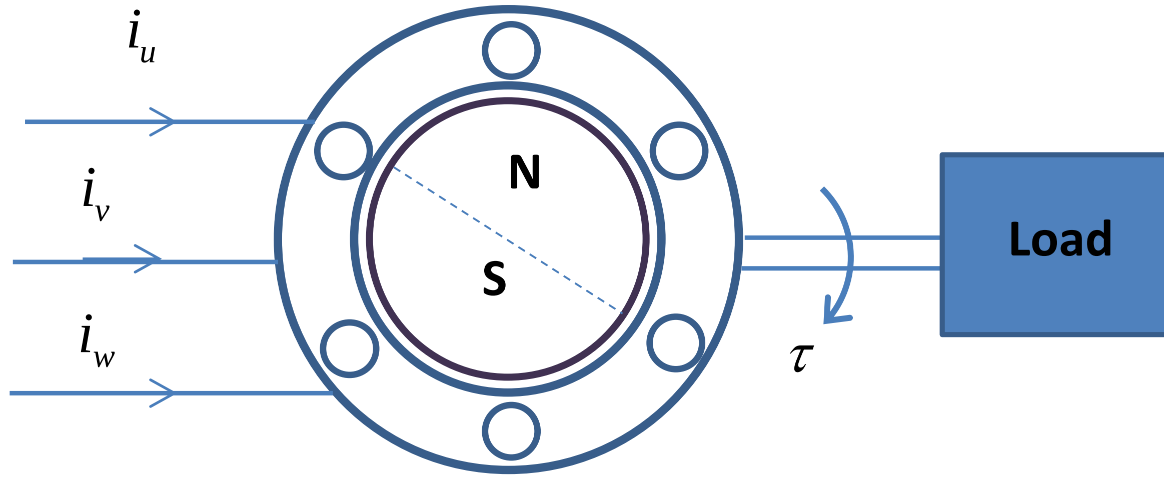
$$= P|\lambda| i_{sq}$$

i_{sd} ไม่เกี่ยวกับการสร้างแรงบิด

Vector Control of SPMSM

$$\tau = \underbrace{P\left|\lambda\right|}_{\text{Permanent Magnet}} i_{sq}$$

ต้องการควบคุมแรงบิด -> จะต้องจ่ายกระแสสเตเตอร์อย่างไร ?



ควบคุมแรงบิด ผ่าน...

- กระแส i_{sq}
- ให้กระแส $i_{sd} = 0$ **(maximum torque per amp.)**

Vector Control of SPMSM

Ex P = 2, λ = 0.1 [Wb], ต้องการ $\tau(t)$ = 1 [N.m]

$$\theta(t) = \frac{\pi}{6} + 100\pi t$$ (มุมของโรเตอร์ทางไฟฟ้า)

$$\tau(t) = P|\lambda| i_{sq} \quad \Rightarrow \quad 1 = 2 \times 0.1 \times i_{sq} \quad \rightarrow \quad i_{sq} = 5\,[A]$$

อยากได้

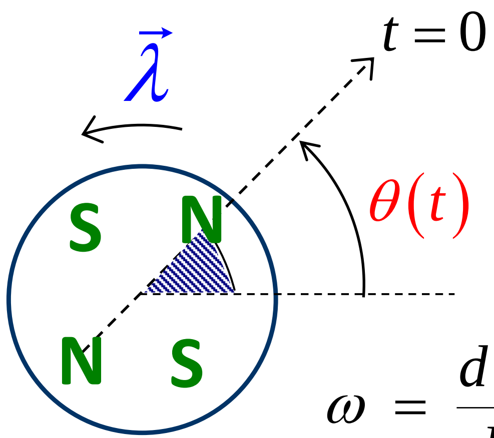
$$\left.\begin{aligned} i_{s1}(t) &= \\ i_{s2}(t) &= \\ i_{s3}(t) &= \end{aligned}\right\} \Leftrightarrow \quad \vec{i}_s \quad \Leftrightarrow \quad \vec{i}_s e^{-j\varepsilon} = i_{sd} + j i_{sq}$$

$$= 0 + j5$$

$$\theta(t) = \frac{\pi}{6} + 100\pi t$$

ถ้าต้องการ **Maximum Torque/Amp.** $\rightarrow$ $i_{sd} = 0$

Vector Control of SPMSM



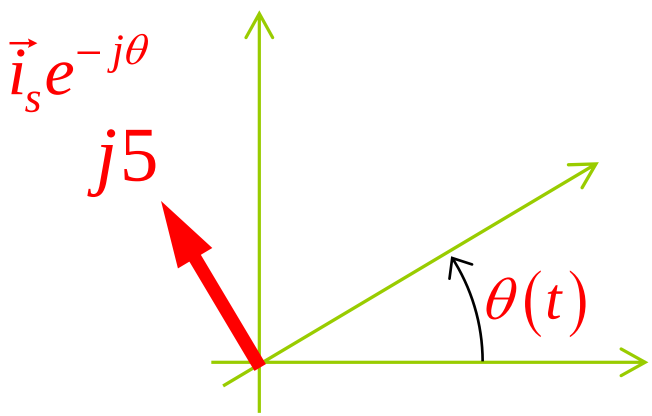
$$\vec{i}_s(t) = \left(i_{sd} + j i_{sq}\right) \cdot e^{j\theta}$$

$$= j5 \cdot e^{j\left(\frac{\pi}{6} + 200\pi t\right)}$$

$$= 5 \cdot e^{j\left(\frac{2\pi}{3} + 200\pi t\right)}$$

$$\omega = \frac{d\theta}{dt} = 100\pi \left[rad/s\right] \text{ (elec.)}$$

$$N = \frac{3000}{2} = 1500 \left[rpm\right] \text{ (mech.)}$$

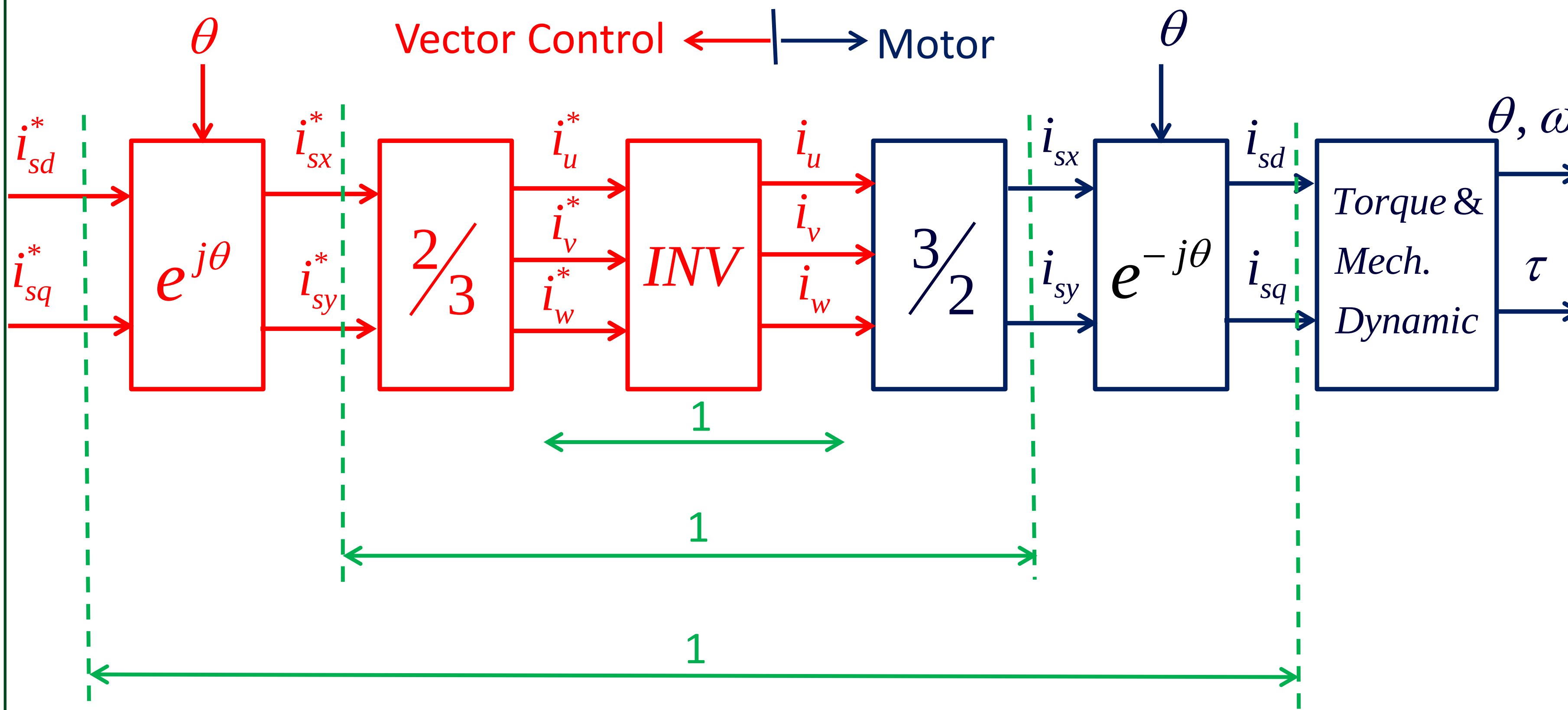


$$i_u(t) = 5\sqrt{\frac{2}{3}} \cos\left(100\pi t + \frac{2\pi}{3}\right)$$

$$i_v(t) = 5\sqrt{\frac{2}{3}} \cos\left(100\pi t\right)$$

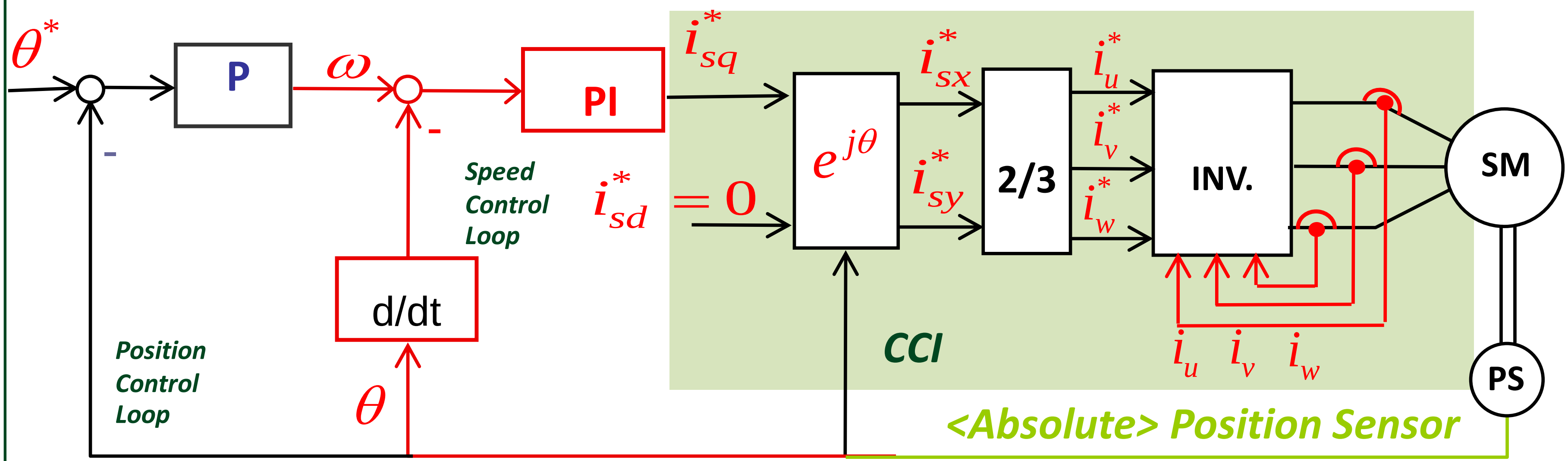
$$i_w(t) = 5\sqrt{\frac{2}{3}} \cos\left(100\pi t - \frac{2\pi}{3}\right)$$

Overall Block Diagram



θ สามารถตรวจวัดได้ด้วย Position Sensor; Encoder, Resolver

Vector Control with Current-Controlled Inverter

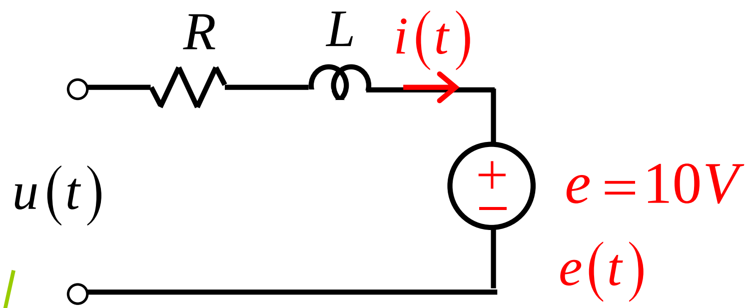


☑ ***Response*** *ไว*

☑ *ไม่ต้องการค่าพารามิเตอร์มอเตอร์* R, L_s

☒ *ใช้* ***Gain*** *สูงในการควบคุมกระแส*

Principle of Vector Control by Voltage Source Inverter



- Voltage equation :

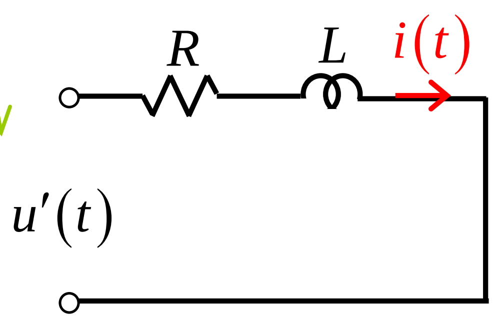
$$R i(t) + L\frac{d i(t)}{dt} + e(t) = u(t)$$

- Commanded voltage with induced voltage compensation:

กำหนด $u(t) = \underset{e(t)}{10} + u'(t)$

ควบคุมกระแสผ่านแหล่งจ่ายแรงดัน

$$R i(t) + L\frac{d i(t)}{dt} + \underset{10}{\cancel{e(t)}} = u'(t) + \underset{e(t)}{\cancel{10}}$$



- Decoupled current dynamic :

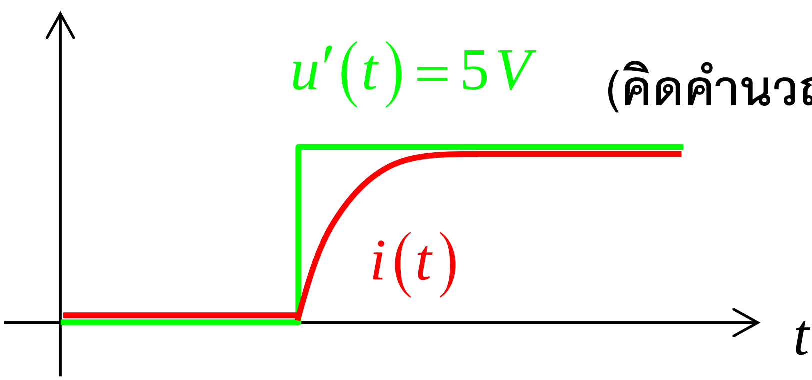
ง่าย $R i(t) + L\frac{d i(t)}{dt} = u'(t)$

Principle of Vector Control by Voltage Source Inverter

สมมุติว่า $R = 1\Omega, \quad L = 1\,mH$

ต้องการให้ได้ $i(t) = 5A$

$\Rightarrow$ จ่ายแรงดัน $u'(t) = 5\times 1 = 5V$ (คงที่)



(คิดคำนวณ)

$$\therefore u(t) = 5 + e(t)$$
$$= 5 + 10 = \boxed{15V}$$

จ่ายจริง

Principle of Vector Control by Decoupling Control

$$\vec{v}_s' = R\vec{i}_s{}' + L_s \frac{d\vec{i}_s'}{dt} + J\omega L_s \vec{i}_s' + J\omega\vec{\lambda}'$$

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = \begin{bmatrix} R & -\omega L_s \\ \omega L_s & R \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + L_s \frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega\lambda \end{bmatrix}$$

มีการเชื่อมโยงแรง
เคลื่อนเหนี่ยวนำ
ระหว่าง ***d-q axes !***

d-axis: $$v_{sd} = R\,i_{sd} + L_s \frac{di_{sd}}{dt} \boxed{- \omega L_s\, i_{sq}}$$

q-axis: $$v_{sq} = R\,i_{sq} + L_s \frac{di_{sq}}{dt} \boxed{+ \omega L_s\, i_{sd} + \omega\,\lambda}$$

Principle of Vector Control by Decoupling Control

กำหนดให้ Decoupling Control :

$$v_{sd} = v'_{sd} \boxed{- \omega L_s i_{sq}}$$

$$v_{sq} = v'_{sq} \boxed{+ \omega L_s i_{sd} + \omega \lambda}$$

ชดเชยแรงเคลื่อนเหนี่ยวนำระหว่าง d-q axes

Decoupled Stator Dynamic :

d-axis:

$$v'_{sd} = R i_{sd} + L_s \frac{di_{sd}}{dt}$$

q-axis:

$$v'_{sq} = R i_{sq} + L_s \frac{di_{sq}}{dt}$$

Principle of Vector Control by Decoupling Control

กำหนดให้ Decoupling Control :

$$v'_{sd} = R i^{*}_{sd} \quad \longrightarrow \quad R i^{*}_{sd} = R i_{sd} + L_s \frac{di_{sd}}{dt}$$

$$v'_{sq} = R i^{*}_{sq} \quad \longrightarrow \quad R i^{*}_{sq} = R i_{sq} + L_s \frac{di_{sq}}{dt}$$

Decoupled Dynamic

First-Order Response;

Time constant : L_s / R

$$v'_{sd} \longrightarrow i_{sd} \longrightarrow \lambda_s$$ **Stator Flux**

$$v'_{sq} \longrightarrow i_{sq} \longrightarrow \tau$$

$$\frac{L_s}{R}$$

First-order lag

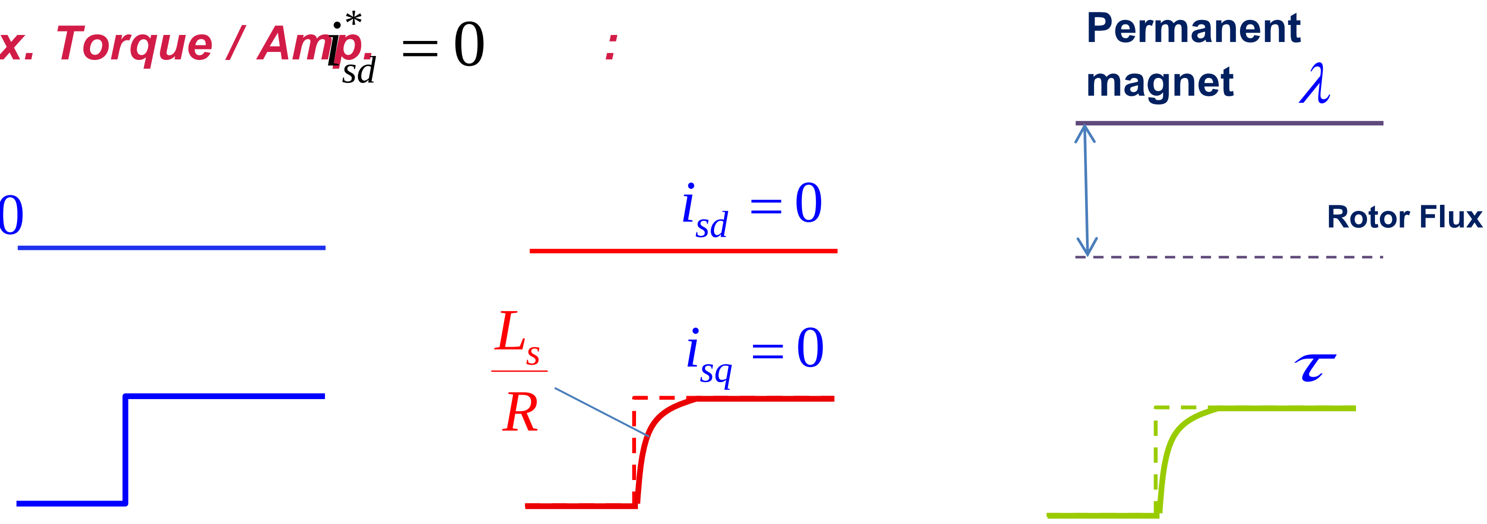
ไม่มี lag

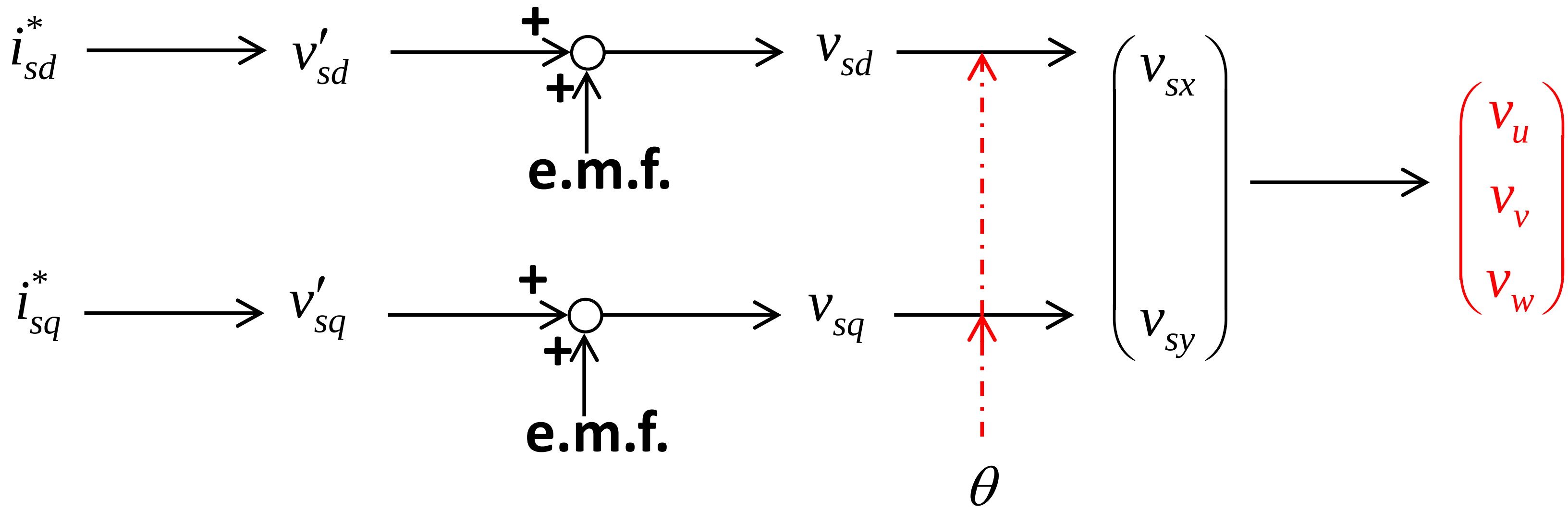
Principle of Vector Control by Decoupling Control

ในกรณี ***Max. Torque / Amp.*** $i^*_{sd} = 0$ **:**

$v'_{sd} = Ri^*_{sd} = 0$

$v'_{sq} = Ri^*_{sq}$





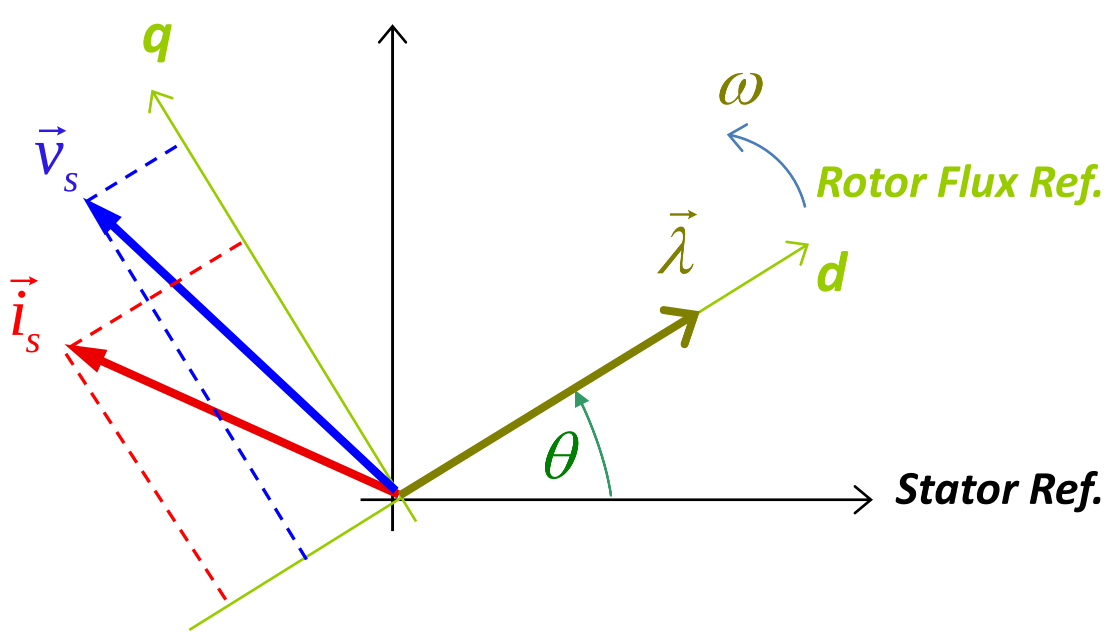
Principle of Vector Control by Decoupling Control

สรุป Decoupling Control :

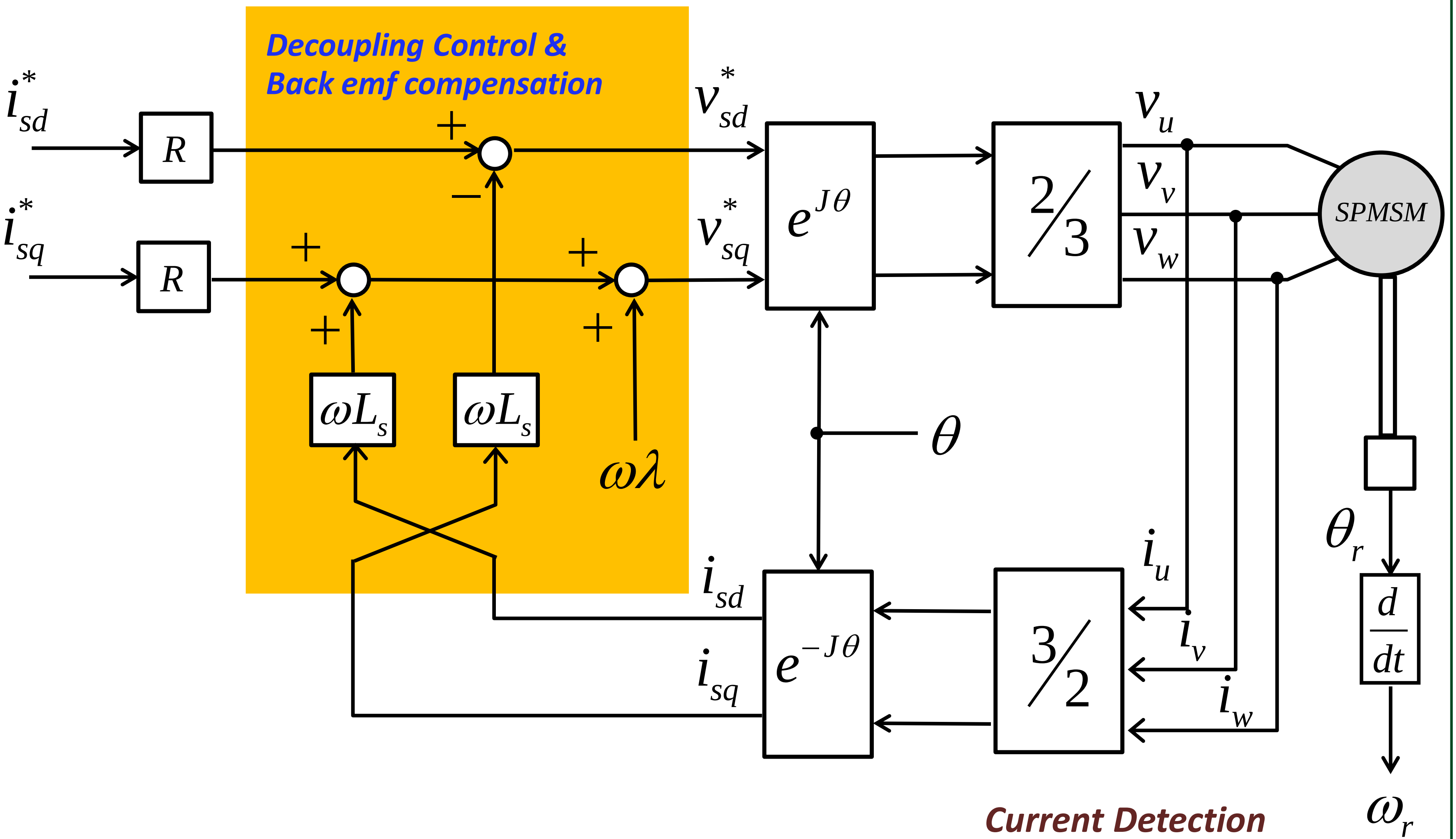
$$v_{sd}^{*} = R\,i_{sd}^{*} - \omega L_s\, i_{sq}$$

$$v_{sq}^{*} = R i_{sq}^{*} + \omega L_s\, i_{sd} + \omega\lambda$$

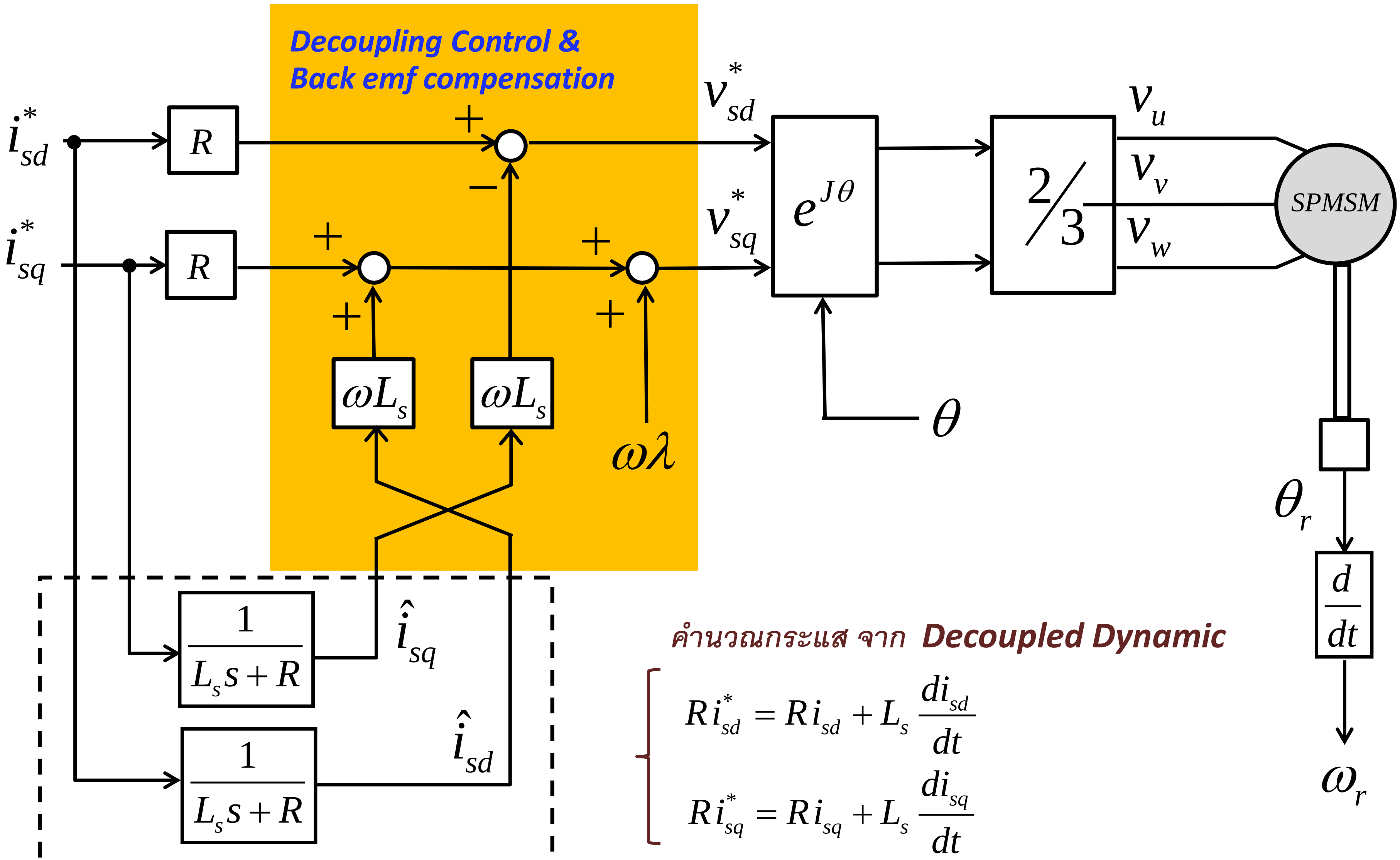
- ☑ ***Feed-Forward Control :*** *ไม่มีปัญหาเรื่องเสถียรภาพ*
- ☑ *ต้องการค่าพารามิเตอร์มอเตอร์ของมอเตอร์* ***R, Ls***



Decoupling Control Scheme #1(with current detection)



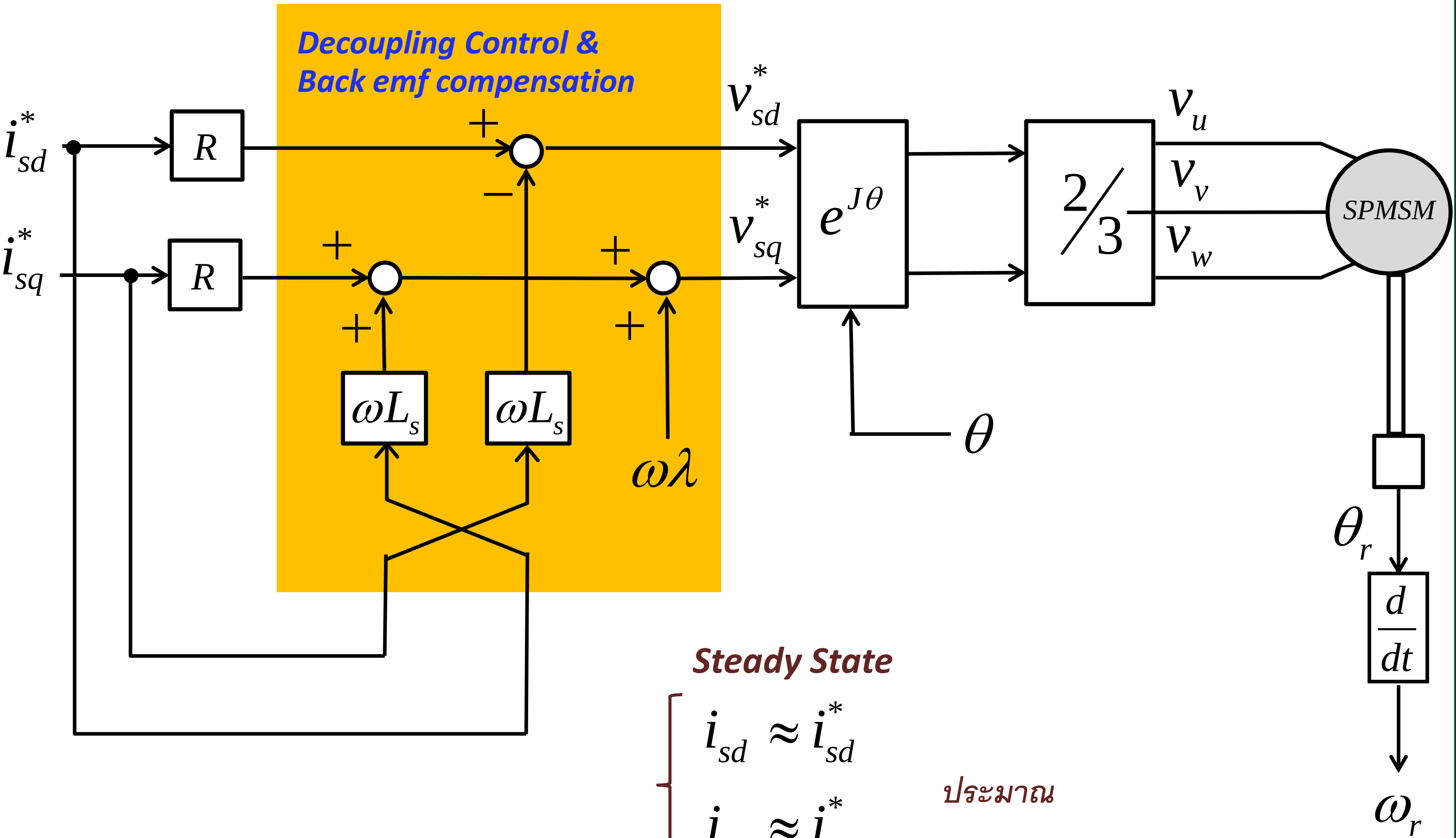
Decoupling Control Scheme #2 (with current estimation)



คำนวณกระแส จาก **Decoupled Dynamic**

$$
\begin{cases}
R i_{sd}^* = R i_{sd} + L_s \dfrac{di_{sd}}{dt} \\
R i_{sq}^* = R i_{sq} + L_s \dfrac{di_{sq}}{dt}
\end{cases}
$$

Decoupling Control Scheme #3 (simplified)



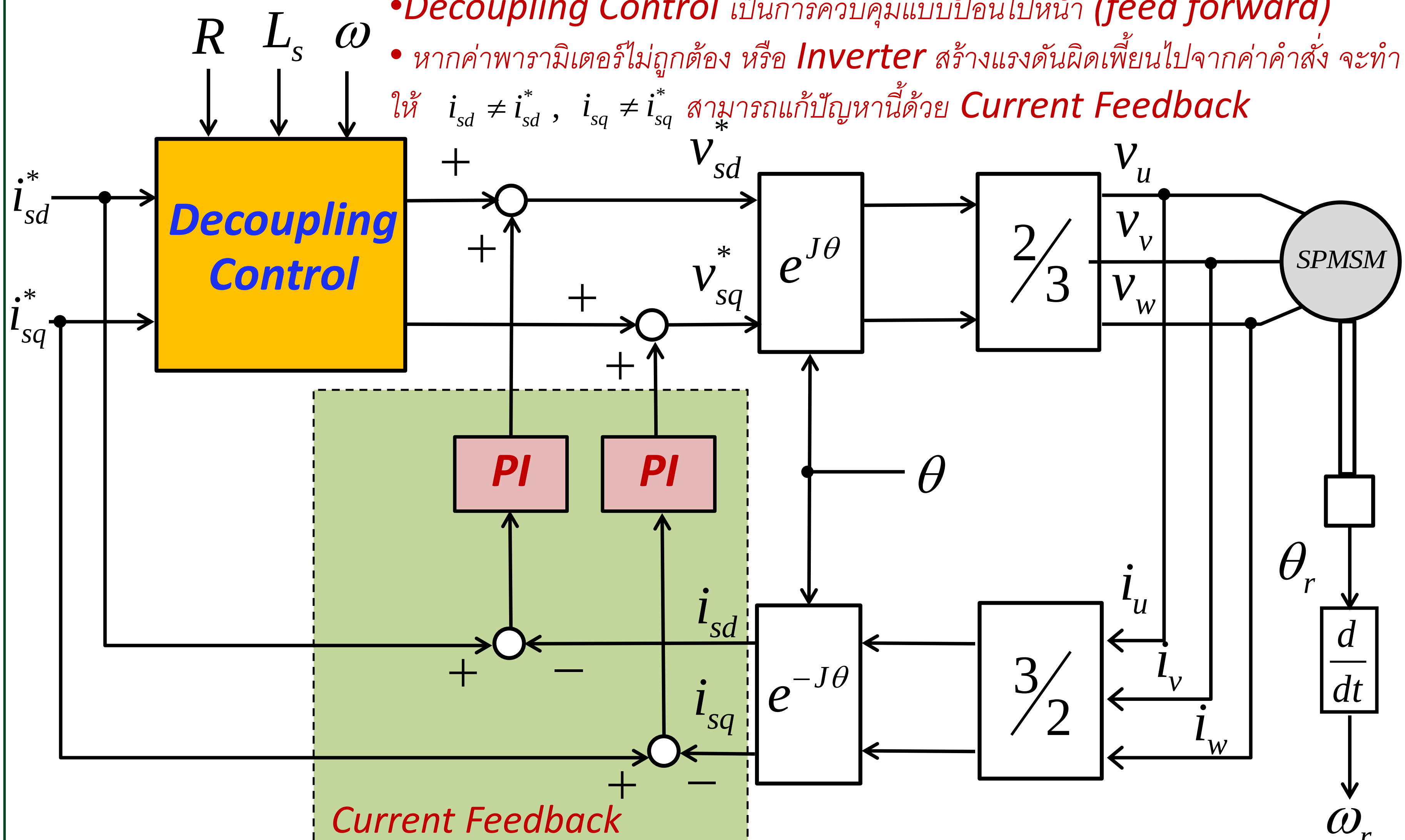
Steady State

$$\begin{cases} i_{sd} \approx i_{sd}^* \\ i_{sq} \approx i_{sq}^* \end{cases}$$

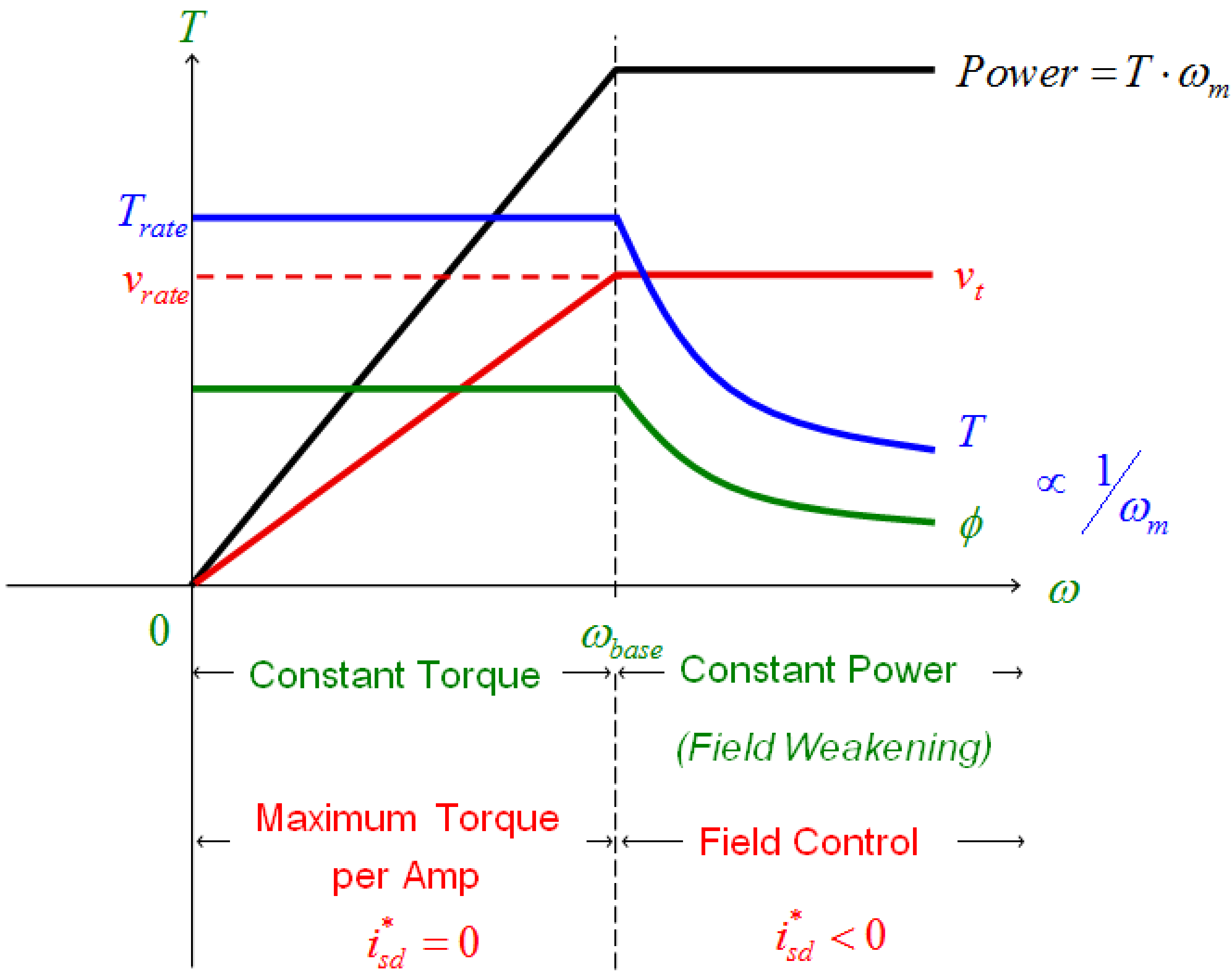
ประมาณ

Decoupling Control with Current Feedback

- *Decoupling Control เป็นการควบคุมแบบป้อนไปหน้า (feed forward)*
- *หากค่าพารามิเตอร์ไม่ถูกต้อง หรือ Inverter สร้างแรงดันผิดเพี้ยนไปจากค่าคำสั่ง จะทำให้* $i_{sd} \neq i_{sd}^*$, $i_{sq} \neq i_{sq}^*$ *สามารถแก้ปัญหานี้ด้วย Current Feedback*



Field-Weakening Operation



Field-Weakening Operation for SPMSM Drives

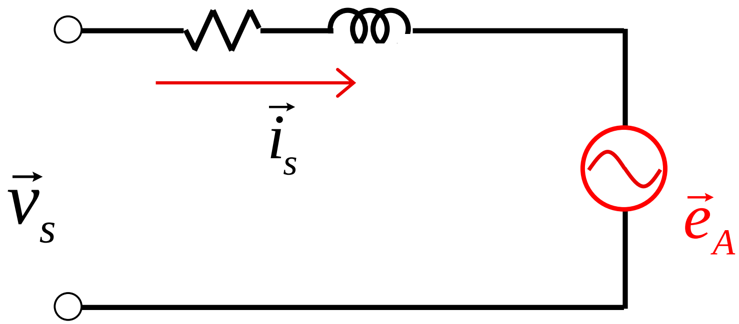
- *ในกรณีที่มอเตอร์ทำงานที่ความเร็วสูงกว่าพิกัด* $\omega_r > \omega_{rated}$
- *แรงเคลื่อนเหนี่ยวนำจะมีค่าสูง* $\Rightarrow$ *แรงดันสเตเตอร์ สูงกว่าค่าพิกัด* $|\vec{v}_s| > V_{rated}$

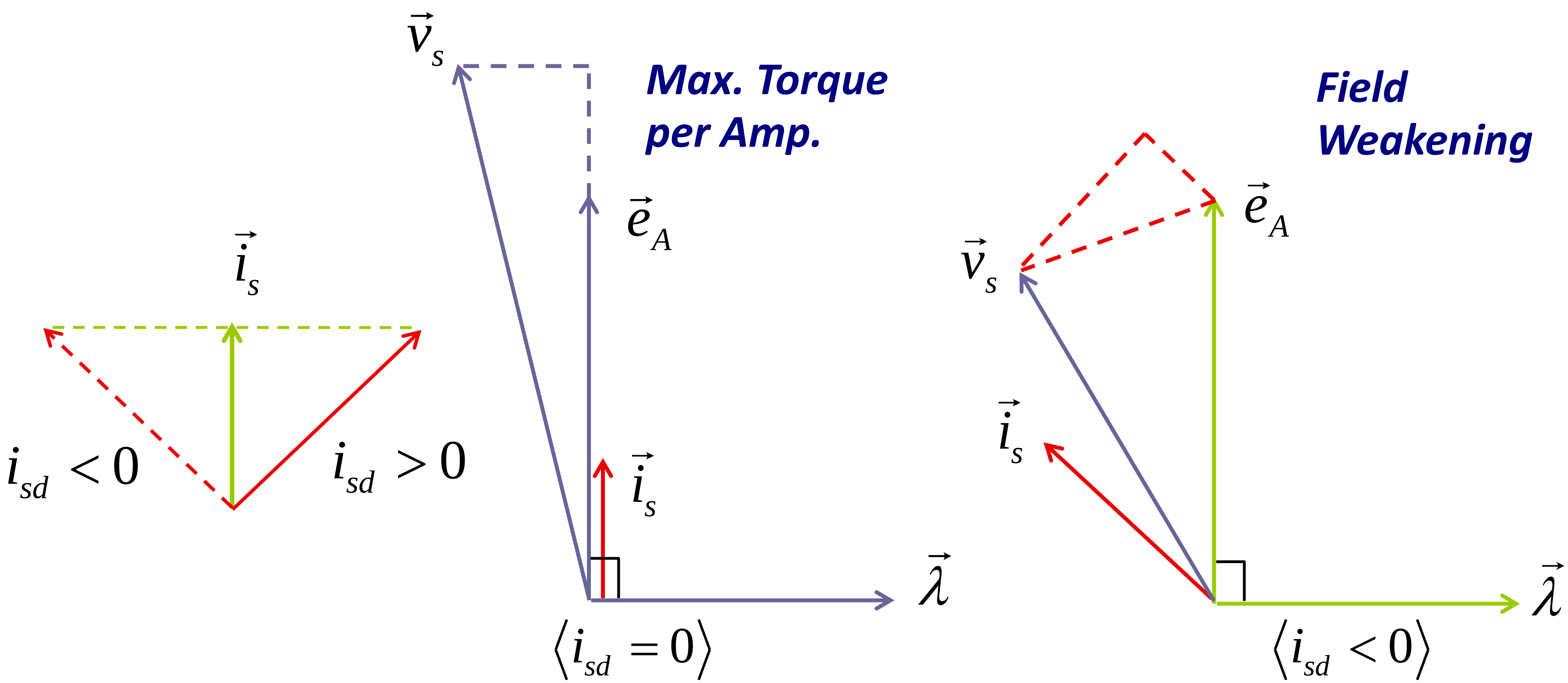
$$\vec{v}_s' = R\vec{i}_s' + L_s \frac{d\vec{i}_s'}{dt} + \underbrace{J\omega L_s \vec{i}_s'}_{\text{แรงเคลื่อนเหนี่ยวนำจากสเตเตอร์ฟลักซ์}} + \underbrace{J\omega\vec{\lambda}'}_{\text{แรงเคลื่อนเหนี่ยวนำจาก PM } \vec{e}_A}$$

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + L_s \frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -\omega L_s \\ \omega L_s & 0 \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}} + \begin{bmatrix} 0 \\ \omega\lambda \end{bmatrix}$$

- *ใช้กระแสในแกน **d** ลดขนาดแรงเคลื่อนเหนี่ยวนำได้* $(i_{sd} < 0)$

Field-Weakening Operation for SPMSM Drives





Field-Weakening Operation for SPMSM Drives