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RESEARCH ARTICLE

Revenue-Maximizing Online Stable Task Assignment on Taxi-Dispatching Platforms

Jingwei Lv[1] **, Ze Zhao**[2]**, Shuzhen Yao**[1]**, Weifeng Lv**[1]

1 School of Computer Science and Engineering, Beihang University, Beijing 100191, China
2 School of Mathematical Science , Beihang University, Beijing 100191, China

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1 Introduction and contributions

With the rapid development of mobile Internet, taxi-dispatching platforms have become increasingly popular and important. For example, UBer and Didi are both famous O2O platforms covering the dominating portion of the taxi-dispatching market over the world. A central issue in taxi-dispatching platforms is task assignment.

The main purpose of the problem is matching drivers to passengers, on the premise of a balance between price and distance, to maximize platforms' revenue. In this letter, the baseline algorithm based on traditional solution [1] [2] for spatial assignment is performed. It fails to deliver satisfying results on revenue maximizing. Furthermore we propose a problem called *Revenue-Maximizing Online Stable Matching (RMOSM)* problem with a new algorithm *Equation-Substitutable Online Matching (ESOM)*. Compared to the baseline algorithm, the ESOM algorithm can cater to platforms' request for higher profits.

2 Revenue-Maximizing Online Stable Matching Problem

2.1 Stability [3]

Blocking Pair. *A blocking pair, denoted by* $< t, w > \in T \times W$*, satisfies the following condition:*

Table 1 Used notations

Notations	Elucidation
l_t/l_w	Location of t or w
s_t/s_w	Appearing time of t or w
d_t	Leaving time of t
p_t	Price of t
t_w	Threshold of w
$d(t,w)$	Distance between t and w
H_i	The $i_t h$ time window
T_i	The available tasks set during H_i
W_i	The available workers set during H_i
M_i	The matching set during H_i
M	The whole matching set

There is another pair $< t^*, w^* >$*, and*

$$\begin{aligned} &p_{t^*} < p_t \text{ or } w \text{ is unmatched} \\ &d(t,w^*) < d(t,w) \text{ or } t \text{ is unmatched} \end{aligned} \tag{1}$$

Stability. *A matching M is stable if* $\forall < t, w > \in T \times W$, $< t, w >$ *is not a blocking pair.*

2.2 RMOSM Problem

Revenue-Maximizing Online Stable Matching(RMOSM) Problem. *Given a set of tasks T, a set of workers W and time windows* $\{h_i\}_0^\infty$ *based on T and W, we define*

$$T_0 = \{t \in T | s_t \in H_0\} \tag{2}$$

$$W_0 = \{w \in W | s_w \in H_0\} \tag{3}$$

Our problem begins with T_0 *and* W_0*. We deal with* T_i *and* W_i *to find a stable matching* M_i*, then turn to next time window* H_{i+1} *for another stable matching problem about* T_{i+1} *and* W_{i+1}*, and*

$$T_{i+1} = \{t \in T | s_t < h_{i+1}, (s_t + d_t) > h_{i+1}, t \text{ is unmatched.}\} \tag{4}$$

$$W_{i+1} = \{w \in W | s_w < h_{i+1}, w \text{ is unmatched.}\} \tag{5}$$

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E-mail: $lvjingwei@buaa.edu.cn$

We get series of stable matching M_{i0}^{∞}. Then for each M_i we calculate the revenue R_i, and for certain fixed normal number n, define overall revenue R as $\Sigma_{i=0}^{n} R_i$.

The Revenue-Maximizing Online Stable Matching Problem is to find an algorithm that could give a stable matching with revenue as high as possible.

3 Baseline Algorithm

3.1 Baseline Algorithm

Algorithm 1 Baseline(T, W)

Input: Tasks:T, Workers:W
Output: Matching:M

$M \leftarrow \emptyset$
while $T \neq \emptyset$ **do**
  $t \leftarrow argmax_{t \in T} p_t$
  $W_t \leftarrow \{w \in W | d(t, w) \leq t_w\}$
  $w \leftarrow argmin_{w \in W_t} d(t, w)$
  Assert (t, w) into M
  Remove t from T
  Remove w from W
end while
return M

Complexity Analysis. For each time window, the time complexity of baseline is $O(|T_i||W_i|)$. As the largest number of time windows that one task covers is generally limited to a finite constant, with given T and W for the whole time period, the time complexity is $O(|T||W|)$.

3.2 Shortcomings of Baseline

In common situations of taxi-dispatching, distances between tasks and workers are often considered as their exact spatial Euclidean distance, which is for that available matching sets could be enlarged from the loose definition of distances, and the revenue usually improves. However, the baseline algorithm focuses little on the selection towards equal distances.

Thus, we ought to design a new algorithm with a new feature to deal with the equation situation properly.

4 ESOM Algorithm

4.1 Substitutable

Substitutable [4]. *Given a matching M and matched pair $(t, w) \in M$, worker w is substitutable if there exists an unmatched worker $w^{'}$ that $d(t, w) = d(t, w^{'})$.*

This concept is presented in [4] to deal with a pricing problem about matching.

4.2 Equation-Substitutable Online Matching Algorithm

The ESOM algorithm is based on traditional methods for stable matching like the Gale-Shapley algorithm or the Chain algorithm. The relaxed distance applied to our model, which divides exact Euclidean distance into some batches, creates more equal situations when tasks and workers compete to be assigned while matching. In this way, the substitutable concept offers workers more chances to be matched when distance's equation happens and helps to increase the number of matched pairs.

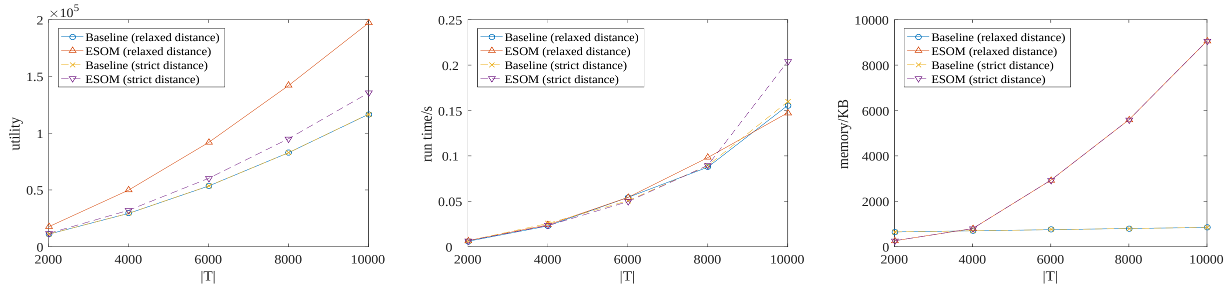
Algorithm 2 is the key part of the ESOM algorithm. in line 1, if W_i is not empty and t is not matched, the algorithm runs lines 2-17. In line 2, select w from W_t as who is nearest to t. In lines 3-5, if w is unmatched, add (t, w) to M_i and remove w from W_t; if not, we assume w was assigned to $t^{'}$ before. In line 8, we judge whether w is substitutable, and if so, rematch w with t rather than $t^{'}$ and replace t in T_i with $t^{'}$, then in line 11, if the price of $t^{'}$ and t is equal while $r_{t^{'}} < r_t$, we remove w from $W_{t^{'}}$; if not, we remove w from W_t. n line 17, we turn back to line 1 for the next round. When the cycle ends, return M_i,T_i and W_i as answers.

Algorithm 2 ESOM(M_i, T_i, W_i, t)

Input: Matching:M_i, Tasks:T_i, Workers:W_i, Available Workers:W_t,Task:t
Output: Matching:M_i, Tasks:T_i, Workers:W_i, Available Workers:W_t

1: **while** $W_t \neq \emptyset$ and t is unmatched **do**
2:   $w \leftarrow argmin_{w \in W_t} d(t, w)$
3:   **if** w is unmatched **then**
4:     Assert (t, w) into M_i
5:     Remove w from W_t
6:   **else**
7:     (Assume w was assigned to $t^{'}$)
8:     **if** w is substitutable **then**
9:       Replace $(t^{'}, w)$ in M_i with (t, w)
10:       Replace t in T_i with $t^{'}$
11:       **if** $p_{t^{'}} = p_t$ and $r_{t^{'}} < r_t$ **then**
12:         Remove w from $W_{t^{'}}$
13:       **end if**
14:     **else**
15:       Remove w form W_t
16:     **end if**
17:   **end if**
18: **end while**
19: **return** M_i, T_i, W_i, W_t

Complexity Analysis. For each time window, the time complexity of the ESOM algorithm is $O(|T_i||W_i|)$. With given T and W for the whole time period, the time complexity is $O(|T||W|)$.



(a) Utility, run time and memory of varying $|T|$ ($|W| = |T|$)

Fig. 1 Results on varying $|T|$

4.3 Competitive Ratio

In the RMOSM problem, we simplify the competitive ratio as $\frac{M}{M^*}$, where M is the result of the ESOM algorithm, and M^* is the optimal result.

Theorem. *For each time window, the profit R resulted from the ESOM algorithm satisfies that* $R \geq \frac{2}{3}R^*$, *where R is the optimal profit.*

As in ESOM algorithm tasks with higher prices have priority to be matched, the competitive ratio of ESOM is $\frac{2}{3}$.

5 Experimental Study

5.1 Experimental Setup

We use one real dataset, the New York City Taxi and Limousine Commission(NYC TLC) data set [5]. We also use synthetic datasets for evaluation. We generate the location, utility and appearing time following uniform distribution, too. Statistics of the synthetic datasets are shown in Table 2.

Table 2 Synthetic Dataset

Factor	Setting
$\lvert T\rvert$	1000, 2000, **3000**, 4000, 5000
$\lvert W\rvert$	1000, 2000, **3000**, 4000, 5000
$\lvert T\rvert$ ($\lvert T\rvert = \lvert W\rvert$)	2000, 4000, 6000, 8000, 10000
Threshold	50, 100, **200**, 300, 400
RelaxedDistance	1, 50, **100**, 150, 180
NumberOfTimePeriods	**50**
Bound	**3000**

5.2 Experiment Result

The experiment results on the synthetic datasets are shown in Fig.1.

6 Performance Analysis

In terms of total utility scores, the ESOM algorithm with relaxed distance always performs the best among four algorithms in both synthetic and real dataset. As for running time, with capacity's increment, the four algorithms are difference regarding to the number of matched pairs. And in the aspect of memory cost, although ESOM performs worst for the need of substitutable marks, it's efficient enough to be applied to real-time assignments with low memory cost.

7 Conclusion

We identify a problem about dynamic task assignment, called RMOSM Problem, then introduce the concept Substitutable and design a novel algorithm Equation-Substitutable Online Matching (ESOM). Finally we conduct experiments that verify the efficiency and effectiveness of the proposed approaches.

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