HW4 Solution : Textbook P121-123, 4.1 4.2 4.3 4.5

(points)

4.1 (28) $\pi(LI) = P(Y=1) = \dfrac{\exp(\alpha+\beta\cdot LI)}{1+\exp(\alpha+\beta\cdot LI)}$

From Table 4.9, $\hat{\alpha} = -3.7771$ $\quad\hat{\beta} = 0.1449$

$\therefore\ \hat{\pi}(LI) = \dfrac{\exp(\hat{\alpha}+\hat{\beta}\cdot LI)}{1+\exp(\hat{\alpha}+\hat{\beta}\cdot LI)}$

a. (5) $\hat{\pi}(LI=8) = \dfrac{\exp(-3.7771+0.1449\times 8)}{1+\exp(-3.7771+0.1449\times 8)} \doteq \underline{0.068}$

b. (5) $\pi(LI) = 0.5 \Rightarrow LI = -\dfrac{\alpha}{\beta}$ the media effective level

$\therefore$ when $\hat{\pi} = 0.5$, $LI = -\dfrac{\hat{\alpha}}{\hat{\beta}} \doteq \underline{26.0}$

c. (6) the rate of change in π, $\dfrac{d\pi(LI)}{d\,LI} = \beta\cdot\pi(LI)[1-\pi(LI)]$

$\hat{\pi}(LI=8) = 0.068$ $\quad\therefore \left.\dfrac{d\pi}{d\,LI}\right|_{LI=8} = \hat{\beta}\cdot\hat{\pi}(LI=8)[1-\hat{\pi}(LI=8)]$

$= 0.1449\times 0.068\times(1-0.068) \doteq \underline{0.009}$

$\hat{\pi}(LI=26) = 0.5$ $\quad\therefore \left.\dfrac{d\pi}{dLI}\right|_{LI=26} = 0.1449\times 0.5^2 \doteq \underline{0.036}$

d. (6) $\hat{\pi}(LI=14) = \dfrac{\exp(-3.7771+0.1449\times 14)}{1+\exp(-3.7771+0.1449\times 14)} \doteq \underline{0.15}$

$\hat{\pi}(LI=28) \doteq \underline{0.57}$

$\hat{\pi}(LI=28) - \hat{\pi}(LI=14) = 0.57 - 0.15 = \underline{0.42}$

e. (6) When LI increases by 1, the odds multiply by e^{β}.

$\therefore$ the estimated odds of remission multiply by

$e^{\hat{\beta}} = \exp(0.1449) \doteq \underline{1.16}$.

4.2 (24) From Table 4.9, $\hat{\beta} = 0.1449$ $se(\hat{\beta}) = 0.0593$

a. (6) $H_0: \beta = 0$ vs. $H_a: \beta \neq 0$

Wald-type test: if $n \gg 1$ $Z = \frac{\hat{\beta} - 0}{se(\hat{\beta})} \overset{\text{under } H_0}{\sim} N(0,1)$

$Z_{obs} = \frac{0.1449}{0.0593} \doteq 2.44 > 1.96 = Z_{0.025}$

$\therefore$ reject H_0 under significant level 0.05.

This provides positive evidence of LI effect on cancer patient remission.

b. (6) 95% Wald CI of β: $\hat{\beta} \pm 1.96\, se(\hat{\beta}) \doteq (0.0287, 0.2611)$

$\therefore$ 95% Wald CI for e^{β} — OR corresponding to a 1-unit increase in LI:

$(e^{0.0287}, e^{0.2611}) \doteq (1.029, 1.298)$

The odds multiply by at least 1.029 and at most 1.298 for every 1-unit increase in LI with 95% confidence

c. (6) $H_0: \beta = 0$ vs. $H_a: \beta \neq 0$

LRT: from Table 4.9 LR Statistic

$\chi^2 = 8.30$ with DF = 1

p-value $(\chi^2 > 8.30) < 0.01$, reject H_0.

This provides strong evidence of LI effect.

d. (6) 95% LR CI for β: $(0.0425, 0.2846)$,

from Table 4.9 Likelihood Ratio 95% Conf. Limits.

$\therefore$ 95% LR CI for e^{β}: $(e^{0.0425}, e^{0.2846}) \doteq (1.043, 1.329)$

Similar interpretation in b.

4.3 (18)

a. The percentage of complete games estimated to be decreasing
(6) by 0.0694 per decade.

b. $x=12$, $\hat{\pi} = 0.7578 - 0.0694 \times 12 = -0.075$

(6) $\because$ The percentage $\pi \in [0,1]$

$\therefore \hat{\pi} = -0.075$ is not plausible.

C. (6) $x=12$, $\hat{\pi} = \dfrac{\exp(1.148 - 0.315 \times 12)}{1 + \exp(1.148 - 0.315 \times 12)} \doteq 0.067$

This is more plausible than b, since $\hat{\pi} \in [0,1]$.

4.5 (30) X – Temperature $Y = \begin{cases} 1, & \text{if at least one primary O-ring suffered thermal distress (success)} \\ 0, & \text{otherwise} \end{cases}$

The R codes and outputs are provided on the last page.

From the R outputs, $\hat{\alpha} = 15.0429$

$\hat{\beta} = -0.2322$, $se(\hat{\beta}) = 0.1082$

a. (6) The estimated temperature effect : $\hat{\beta}$. $logit(\hat{\pi}) = 15.0429 - 0.2322x$

Interpretation of $\hat{\beta}$: When the temperature increases by 1 unit, the mean of log odds ratio of success, $\log \frac{\pi}{1-\pi}$, will decrease by 0.2322.

b. (6) $\hat{\pi}(x=31) = \dfrac{\exp(15.0429 - 0.2322 \times 31)}{1 + \exp(15.0429 - 0.2322 \times 31)} \doteq 0.997$

C. $\pi(x) = 0.50$, $x = -\dfrac{\hat{\alpha}}{\hat{\beta}} = -\dfrac{15.0429}{-0.2322} \doteq 64.78$
(6)

$$\left.\frac{d\pi(x)}{dx}\right|_{x=64.78} = \hat{\beta}\,\pi(x)(1-\pi(x)) = -0.2322 \times 0.5^2 \doteq -0.058$$

At $x = 64.78$, $\hat{\pi}$ decreases at the rate of 0.058.

d.(b) $e^{\hat{\beta}} \doteq 0.79$

Estimated odds of thermal distress multiply by 0.79 for each 1°F increase in temperature.

e.(b) $H_0: \beta = 0$ vs. $H_a: \beta \neq 0$

(i) Wald test : from R output $z_{obs} = -2.145$ P-value = $0.032 < 0.05$

$\therefore$ reject H_0, there is significant temperature effect on TD.

(ii) LRT : from R output, LRstat $\doteq 7.95$ P-value = $0.0048 < 0.05$

$\therefore$ reject H_0 with stronger evidence.

HW3
R-codes:

```
TD = c(0,1,0,0,0,0,0,0,1,1,1,0,0,1,0,0,0,0,0,0,1,0,1)
Temp = c(66,70,69,68,67,72,73,70,57,63,70,78,67,53,67,75,70,81,76,79,75,76,58)

logitfit = glm(TD~Temp, family=binomial(link=logit))
summary(logitfit)
(LR.stat = logitfit$null.deviance-logitfit$deviance)
(pvalue.LRT = 1-pchisq(logitfit$null.deviance-logitfit$deviance, logitfit$df.null-
logitfit$df.residual) )
```

outputs:

```
> summary(logitfit)

Call:
glm(formula = TD ~ Temp, family = binomial(link = logit))

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.0611  -0.7613  -0.3783   0.4524   2.2175

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  15.0429     7.3786   2.039   0.0415 *
Temp         -0.2322     0.1082  -2.145   0.0320 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 28.267  on 22  degrees of freedom
Residual deviance: 20.315  on 21  degrees of freedom
AIC: 24.315

Number of Fisher Scoring iterations: 5
> (LR.stat = logitfit$null.deviance-logitfit$deviance)
[1] 7.95196
> (pvalue.LRT = 1-pchisq(logitfit$null.deviance-logitfit$deviance, logitfit$df.null-
logitfit$df.residual) )
[1] 0.004803533
```