commerce undergraduate society

COMMERCE MENTORSHIP PROGRAM

MIDTERM REVIEW SESSION

COMM 290

CMP

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INTRODUCTION

Target Cell — Contains the output of the objective function - what are you trying to minimize or maximize?

Input Data — The data given to you as part of a problem

Constraint — A limitation presented in the problem

Action Plan — The “action” you will take to solve the problem

Relative Reference — Will change when auto-filled in other cells
(=A1)

Absolute Reference — Will not change when auto-filled in other cells
(=A1)

Mixed Reference — Will partially change when auto-filled in other cells
(=$A1 or =A$1)

=SUMPRODUCT — Returns the sum of the products of corresponding arrays
=SUMPRODUCT(array1, [array 2], [array 3]…)

SIMPLE LP

Problem – Pocky Dealer: You are a student running a business selling two combos of Pocky: Combo A earns you $5, consisting of one chocolate and one strawberry, while Combo B earns you $6, consisting of two chocolates and no strawberry. You have 20 chocolates in stock and 10 strawberries in stock. Assume there are no costs associated with this model.

What is the objective? Is this a maximizing or minimizing model?

The objective function is the profit that the company makes by selling the two products. We construct the objective function by multiplying the number of each products sold (represented by variables A & B) by their selling price.

Therefore, the objective function is max 5A + 6B.
It is a maximizing model because the business is trying to maximize their profits.

How many constraints are there? How would you write them algebraically?

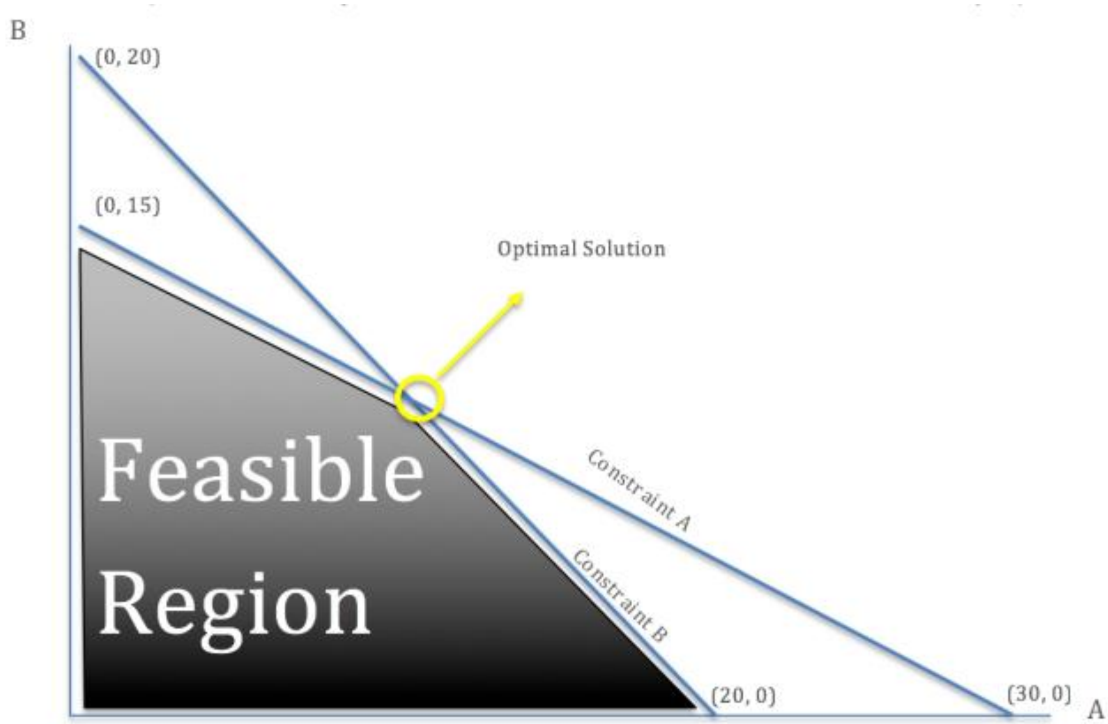
There are four constraints.
Aside from the two stated in the prompt in regards to the limited amount of each types of pokeys, there are also two non-negativity constraints for the two products.
Chocolate: A + 2B <= 20
Strawberry: A <= 10
A >= 0
B >= 0

Create the model and solve using Excel/Solver

GRAPHING

Feasible Region	The region in which all solutions are valid and subject to the constraints
Optimal Solution	The best set of decisions that maximizes/minimizes the objective function while remaining within the constraints
Multiple Optima	Multiple sets of optimal solutions
Infeasible Solution	There is no feasible region associated with your LP
Unbounded Solution	The feasible region is infinitely large, and the objective function behaves such that you are moving the isoprofit line outwards indefinitely
Redundant Constraint	A constraint that does not affect on the feasible region
Non-Negativity Constraint	A constraint which makes sure a “decision” cannot be a negative value

Problem: Consider the following graph of an arbitrary linear programming model with the correct labelled optimal solution, feasible region. Assume this is a profit maximization model (That is, objective function is in the form xA + yB).



Define the range of possible slopes for isoprofit lines that would lead to this optimal solution (marked by the yellow circle)
The slopes of the two binding constraints are -1 and -1/2.
Therefore, -C1/C2 (slope of isoprofit line) of the objective function C1A + C2B must be between -1 and -1/2.

Find the coordinates of the optimal solution. The two constraints are A + B = 20 and A + 2B = 30.
The coordinates of the optimal solution are the intersection of the two binding constraints. To find it, we set up a system of equations: A + B = 20; A + 2B = 30
Using substitution OR elimination, B= 10 and A= 10

Suppose the sign of Constraint B (<=) is changed to the >= sign. How will this change the feasible region?
Feasible region changes to the triangle with points (20,0), (30,0), (10,10)
In other words, the triangle to the right of the shaded feasible region above

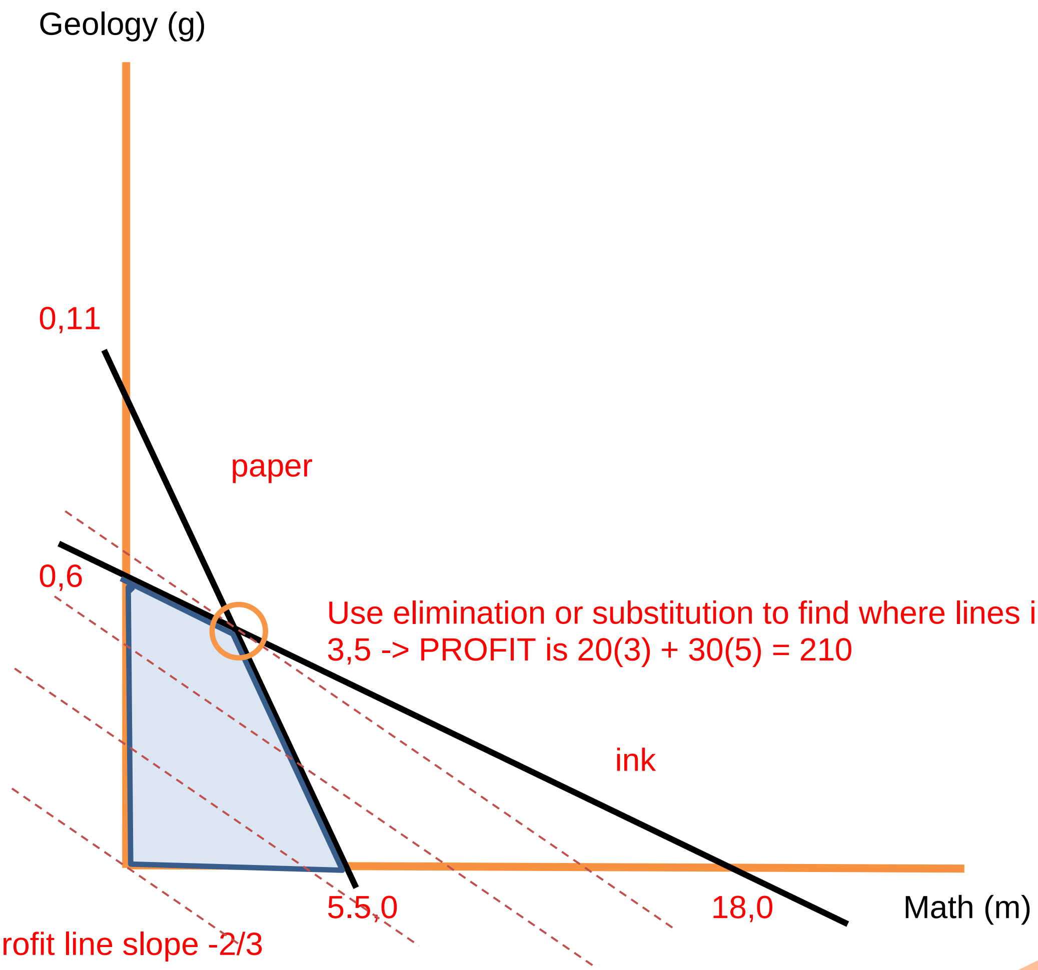
Problem – Rachel's Bootleg Textbooks: Rachel is furious with the high cost of virtual textbooks. Being the entrepreneurial Sauder snake she is, Rachel gets a PDF copy of the math textbook and geography textbook and decides to print copies to sell to students (don't actually do this).

Each math textbook will sell for \$20 and each geography textbook will sell for \$30. Each math textbooks require 1 ounce of ink and 2 pounds of paper. Since the geology textbook has more photos, it requires 3 ounces of ink but only 1 pound of paper. Rachel only has 18 ounces of ink and 11 pounds of paper. What combination of textbooks should Rachel produce to make as much profit as possible?

Graph the problem and find optimal solution

Max 20m + 30g
s.t.

1m + 3g <= 18 ounces of ink
2m + 1g <= 11 pounds of paper
m >= 0, g >= 0



Questions – Based on the Rachel's Bootleg Textbooks problem:

What would happen to the feasible region if the inequality symbol for the ink constraint reversed? What if both constraints' inequality symbols flipped?

If ink constraint reversed inequality symbols, the new feasible region would become the small triangle above the currently shaded feasible region. If both flipped, the feasible region would become unbounded.

What would happen to the feasible region if we added another constraint $m+g<=20$?

Nothing – this constraint is redundant and does not shrink the feasible region.

What range of prices for math textbooks would give the same optimal solution?

Slope of isoprofit line = $-c_1/c_2 = -20/30$
Slope of binding constraint <= Slope of isoprofit line <= Slope of binding constraint

Isolate one variable in the isoprofit line – in this case, $-c_1$ represents math (x-axis)
$-2/1 <= -c_1/30 <= -1/3$

Compute each allowable increase/decrease:

$-2/1 <= -c_1/30$	$-c_1/30 <= -1/3$
$2/1 >= c_1/30$	$c_1/30 >= 1/3$
$60 >= c_1$	$c_1 >= 10$

Combining the two, we know that $10 <= c_1 <= 60$; the price for math textbooks has to be within \$10 and \$60; meaning the allowable increase is \$40 (\$60-\$20), and the allowable decrease is \$10 (\$20-\$10).

SENSITIVITY ANALYSIS

RHS Allowable Increase/Decrease of Binding Constraint	Range that the RH side of a binding constraint can move, keeping the shadow price the same
RHS Allowable Increase/Decrease of Non-Binding Constraint	Range that the RH side of the non-binding constraint can move, keeping the constraint non-binding and the shadow price the same
Allowable Increase/Decrease of Objective Coefficient	Range that an objective coefficient can move without changing the optimal solution
Shadow Price	The increase in the target cell's value for every unit Increase in the RH side of a constraint

Questions – Solve algebraically based on the Rachel's Bootleg Textbooks problem:

What if Rachel could get an extra pound of paper? How would this change the constraint and optimal solution?

Add 1 to RHS of constraint $\quad 2m + 1\,g <= 11 + 1$

Find new intersection between paper and ink constraint using substitution or elimination

$1m + 3g = 18$ // $2m + 1g <= 12$

$m=3.6$; $g=4.8$

If Rachel has the opportunity to buy the extra pound of paper for $8, should she?

Calculate shadow price of paper

Using the solution above, the new profit with extra paper is $216 instead of $210

$20(3.6) + 30\,(4.8) = \$216$

Therefore, the shadow price is $6. Rachel's profit increases by $6 with one extra pound of paper. Thus, Rachel should not pay $8 for an additional pound of paper.

Sensitivity Report – for the Rachel's Bootleg Textbooks problem:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Math textbook	3	0	20	40	10
D12	Geography textbook	5	0	30	30	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E5	Ink	18	8	18	15	12.5
E6	Paper	11	6	11	25	5

Based on the sensitivity report above:

What is the objective formula?
Look at objective coefficients – 20m + 30g

What is the optimal solution and total profit?
Look at final value – 3 math, 5 geography
Put into objective coefficient for total profit – 20(3) + 30(5) = 210

How much could the price of a math textbook increase, keeping the same optimal solution?
Look at allowable increase – 40, meaning math price could increase up to 60

Are the constraints binding?
Look at constraints final value and constraint R.H. side – yes, they are both binding because the L.H. side = R.H. side

How much would profit change with 1 more ounce of ink? How about 1 less ounce of ink?
Look at ink shadow price – profit would increase by 8.
With 1 less ounce, profit decreases by the shadow price of 8

Sensitivity Report – Additional Practice Problem

Assume this is the sensitivity report for a profit maximizing LP model

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C16	Decision 1	4500	0	0.33	0.01	0.016
D16	Decision 2	0	0	0.33	1E+30	1E+30
E16	Decision 3	62000	0	0.2	1E+30	0.017
F16	Decision 4	0	-0.062	-8.81E-10	0.03	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C25	Constraint 1	(1)	0.65	5400	2501	1008
C26	Constraint 2	[redacted]	0.55	21600	6500	5300
C27	Constraint 3	[redacted]	0.14	0	12000	0
C29	Constraint 4	[redacted]	0.62	7500	28000	7500
C28	Constraint 5	62000	0.062	(2)	22000	27000

What are the missing values (1) and (2)?

(1) 5400 – since the shadow price is non-zero, we know the constraint is binding, meaning the LHS must equal the RHS

(2) 62000 – for similar reasoning, the LHS must equal the RHS

How much more profit would the firm make if the RHS of Constraint 1 was increased by 2500?

The increase of 2500 is within Constraint 1's allowable increase, and a shadow price of 0.65 means for every additional constrained unit, the profit goes up by 0.65. Therefore, the company earns 0.65 * 2500 = 1625 additional profit.

How much more profit would the firm make if the RHS of Constraint 3 was increased by 1000?

Similar to the previous question, the 1000 increase is within Constraint 3's allowable increase, and Constraint 3 has a non-zero shadow price of 0.14. Therefore, the firm makes 1000 * 0.14 = 140 additional profit

BLENDING LP

Problem – Kostco Hot Dogs: You are in charge of procurement at Kostco (not Costco), a company that sells hot dogs.

Kostco sells two types of hot dogs: premium and classic. The hot dogs are made with a combination of 3 different ingredients: beef, pork, and mystery meat. Each type of meat has its own cost per pound and maximum amount you can order from the meat supplier (as shown in the table below).

In order to ensure quality, the premium hot dog must be at least 50% beef and cannot contain more than 5% mystery meat. The classic hot dog can have mystery meat, but no more than 25%.

How much of each type of meat should Kostco order?

Input Data						
		Cost per pound	Pounds Available		Selling Price	
	Beef	$ 3.00	1,000		Classic	$ 3.00
	Pork	$ 2.00	3,000		Premiun	$ 5.00
	Mystery meat	$ 1.50	2,000			

Fill in the Blanks – Based on the Kostco Hot Dogs problem:

	A	B	C	D	E	F	G	H
1	**Kostco Hot Dogs**							
2								
3	**Input Data**							
4			Cost per pound	Pounds Available		Selling Price		
5		Beef	$ 3.00	1,000		Classic	$ 3.00	
6		Pork	$ 2.00	3,000		Premiun	$ 5.00	
7		Mystery meat	$ 1.50	2,000				
8								
9		Premium must be at least 50% beef				50%		
10		Premium must not contain more than 5% mystery meat				5%		
11		Classic can not be more than 25% mystery meat				25%		
12								
13	**Action Plan**				Model		Model	
14			Classic	Premium	Output		Requirement	*units*
15		Beef	0	1000	1000	<=	A	*pounds*
16		Pork	2100	900	3000	<=	3,000	*pounds*
17		Mystery meat	700	100	B	<=	2,000	*pounds*
18		Total	2800	2000				
19								
20	**Blending**				Model		Model	
21	**Constraints**				Output		Requirement	
22		Premium must be at least 50% beef			1000	>=	1000	
23		Premium must not contain more than 5% mystery meat			100	<=	C	
24		Classic can not be more than 25% mystery meat			700	<=	700	
25								
26	**Revenue and Cost Info**							
27			Classic	Premium	Total			
28		**Revenue**	D	$ 10,000.00	$ 18,400.00			
29		**Costs**						
30		Beef	$ -	$ 3,000.00	$ 3,000.00			
31		Pork	$ 4,200.00	$ 1,800.00	$ 6,000.00			
32		Mystery meat	$ 1,050.00	$ 150.00	$ 1,200.00			
33		**Profit**	$ 3,150.00	$ 5,050.00	$ 8,200.00			

Note: Write both the formula and the numerical output

A] =D5 , 1000 B] =SUM(C17:D17), 800 C] =D18*F10, 100 D] G5*C18, 8400

How would you write the blending constraints algebraically?
Premium Beef >= 0.5 x (Premium Beef + Premium Pork + Premium Mystery)
Premium Mystery <= 0.05 x (Premium Beef + Premium Pork + Premium Mystery)
Classic Mystery <= 0.25 x (Classic Beef + Classic Pork + Classic Mystery)

SCHEDULING LP

Problem – CUS 24-hr Fast Food: After enough Sauder students protest their high student fees, the CUS promises to open up a budget 24-hr fast-food restaurant in Henry Angus.

Your restaurant has six labour shifts per 24-hour period, starting at 12am, 4am, 8am, 12pm, 4pm, 8pm, and 12pm. You have access to workers who work two consecutive shifts a day. Due to fluctuations in demand, your required labour at different time periods is as follows:

Shift Time	12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am
Workers Required	3	4	7	8	6	5

Find the scheduling method that will use the minimum amount of workers. The completed LP has been provided to you.

CUS 24-hr Fast Food								
		Time Periods Covered						
		12am	4am	8am	12pm	4pm	8pm	Labour
	12am	4	4					4
	4am		0	0				0
	8am			7	7			7
	12pm				1	1		1
	4pm					5	5	5
Start	8pm	0					0	0
Time	Supply	4	4	7	8	6	5	17
		>=	>=	>=	>=	>=	>=	
	Required	3	4	7	8	6	5	

Questions – Based on the CUS 24-hr Fast Food problem:

Is this a maximizing or minimizing model? What are you trying to maximize or minimize?

Minimize the total amount of volunteers needed for the day

How many constraints are there – fill in the table:

	Binding	Non-Binding
Regular	5	1
Non-Negativity	2	4

Answer below questions based on sensitivity report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
I5	12am Labour	4	0	1	0	1
I6	4am Labour	0	0	1	1	0
I7	8am Labour	7	0	1	0	1
I8	12pm Labour	1	0	1	1	0
I9	4pm Labour	5	0	1	0	1
I10	8pm Labour	0	0	1	1E+30	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C11	Shift 12am	4	0	3	1	1E+30
D11	Shift 4am	4	1	4	1E+30	1
E11	Shift 8am	7	0	7	1	0
F11	Shift 12pm	8	1	8	0	1
G11	Shift 4pm	6	0	6	1	0
H11	Shift 8pm	5	1	5	0	1

Due to an overnight frat party, your demand for workers at 12am-4am goes up to four. Will this affect your optimal solution?
There is an allowable increase of 1 for the 12am-4am labour demand. Since the demand at that time is non-binding, it will remain non-binding after the increase to 4 and thus there is no change in the optimal solution.

Suppose the demand for workers at 4am-8am increases by one. How will this affect the optimal solution?
Consider that the labour demand at 4am-8am has a shadow price of 1 and an allowable increase of infinity. Therefore, if the demand goes up by 1,
you will require 1 additional worker

TRANSPORTATION LP

Problem – SkipTheTuition: In order to pay your super high tuition, which seems to increase every year, you take a side gig as a delivery person for SkipTheDishes.

There are three restaurants you deliver identical ramen from: Kokoro, Danbo, and Kinton. You must deliver these meals to UBC residences, Totem, Orchard, and Vanier, one unit at a time. The supply and demand are as follows:

- Kokoro has 7 bowls, Danbo has 16, and Kinton has 13
- Totem requires 17 bowls, Orchard requires 5, Vanier requires 14

The time it takes to deliver to and from each location are as follows:

		Deliver to		
	Time (mins)	Totem	Orchard	Vanier
Deliver From	Kokoro	4	4.5	2.5
	Danbo	3	4	2.5
	Kinton	3.5	3	2

How can you complete all your deliveries in the least amount of time possible?

The action plan has been completed for you. Based on the information in the prompt, fill in all the remaining unpopulated cells. Include both the formula and the numerical output for each cell using a formula.

	A	B	C	D	E	F	G	H	I
1	**SkipTheTuition**								
2									
3	**Input data**								
4				**Deliver to**					
5		**Time (mins)**	Totem	Orchard	Vanier				
6	**Deliver From**	Kokoro	4	4.5	2.5				
7		Danbo	3	4	2.5				
8		Kinton	3.5	3	2				
9									
10	**Action Plan**								
11				**Deliver to**					
12		**Shipments**	Totem	Orchard	Vanier	Shipped		Supply	*units*
13	**From**	Kokoro	7	0	0	7*	<=	7	*bowls*
14		Danbo	2	0	14	16	<=	16	*bowls*
15		Kinton	8	5	0	13	<=	13	*bowls*
16		Received	17^	5	14	*units*			
17			>=	>=	>=				* =SUM(C13:E13)
18		Demand	17	5	14	*bowls*			^ =SUM(C13:C15)
19									+ =SUMPRODUCT(
20	**Total Time**	98+							C6:E8,C13:E15)

How many binding constraints are there in total?

10; 6 regular constraints and 4 non-negativity constraints are binding

What will happen to the LP if suddenly, an explosion happens at Kokoro and three of the seven ramen bowls in stock are destroyed?

The LP would become infeasible as there is not enough supply to meet demand

What are the objective coefficients in this model?

The travel time associated with each shipping route

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C13	Kokoro Totem	0	0	4	1E+30	0
D13	Kokoro Orchard	0	1	4.5	1E+30	1
E13	Kokoro Vanier	7	0	2.5	0	0.5
C14	Danbo Totem	16	0	3	1	1E+30
D14	Danbo Orchard	0	1.5	4	1E+30	1.5
E14	Danbo Vanier	0	1	2.5	1E+30	1
C15	Kinton Totem	1	0	3.5	0	1
D15	Kinton Orchard	5	0	3	1	3.5
E15	Kinton Vanier	7	0	2	0.5	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C16	Received Totem	17	4	17	0	1
D16	Received Orchard	5	3.5	5	0	5
E16	Received Vanier	14	2.5	14	0	7
F13	Kokoro Shipped	7	0	7	1E+30	0
F14	Danbo Shipped	16	-1	16	1	0
F15	Kinton Shipped	13	-0.5	13	7	0

Questions – based on sensitivity analysis:

Is there evidence of multiple optima in this LP?
Since there are many objective coefficients with a 0 allowable increase/decrease, there is indeed evidence of multiple optima

It now takes only 2.5 minutes to deliver from Kinton to Orchard. Will this change the optimal solution? What will be the new value in the target cell?
The decrease of 0.5 is within the allowable decrease of 3.5. The optimal solution does not change, and the new value of the target cell decreases from 98 to 95.5

Suppose Danbo has increased its supply by one. How will this affect the target cell under the optimal solution?
An increase of 1 is within the allowable increase for the supply at Danbo. The shadow price is a non-zero value of -1; therefore, if the constraint goes up by 1, the target cell will go down by 1.

Due to construction around UBC (what's new?), all delivery times to Vanier have been increased by 3. What will be the new optimal solution?
We don't know for sure, since this will change three objective coefficients at the same time