commerce undergraduate society

MATH 104/184

MIDTERM REVIEW SESSION

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Limits

The limit of a function is a value that the function **approaches** as the input **approaches** some value.

$$\lim_{x \to a} f(x) = c$$

This reads, "The limit of f(x) as x approaches a is equal to c"

We say that the limit exists as x -> a when our function approaches the same value from both sides, that is, when:

$$\lim_{x \to a+} f(x) \;=\; \lim_{x \to a-} f(x)$$

When evaluating a limit, your first step should be to plug in the value of x = a into your original function. If this works and gives you a numerical answer, no further work is required. Sometimes, you may end up plugging in x = a into a fraction that gives you the value of 0/0. This is called an **indeterminate form**. To get information from one of these indeterminate forms, some algebraic manipulation is usually required. (Covered in examples!)

1. Put your answer in the box provided and show your work. No credit will be given for an answer without the accompanying work.

(a) Evaluate $\lim_{x \to 7}\left(\frac{x^2 - 4x - 21}{3x^2 - 17x - 28}\right)$

Answer:

(b) Evaluate $\lim_{x \to -3}\left(\frac{\sqrt{2x + 22} - 4}{x + 3}\right)$

Answer:

(c) Evaluate $\lim_{x \to 1}\left(\frac{1}{x^2 - 1} - \frac{2}{x^4 - 1}\right)$

Answer:

Continuity

A function f is continuous at x = a when:

1) f(a) is defined
2) $\lim_{x \to a} f(x)$ exists
3) $\lim_{x \to a} f(x)$ = f(a)

In other words, the graph of the function is unbroken i.e. you could draw it from any point to another without lifting a pen from your paper.

It's important to note that many types of functions, such as polynomial functions, sin(x), cos(x) are continuous throughout their entire domains. Since proving the continuity of a function for a given domain is outside the scope of this course, you can state that these functions are continuous without providing proofs.

2. Find the values of a and b such that $f(x)$ is continuous for all real numbers.

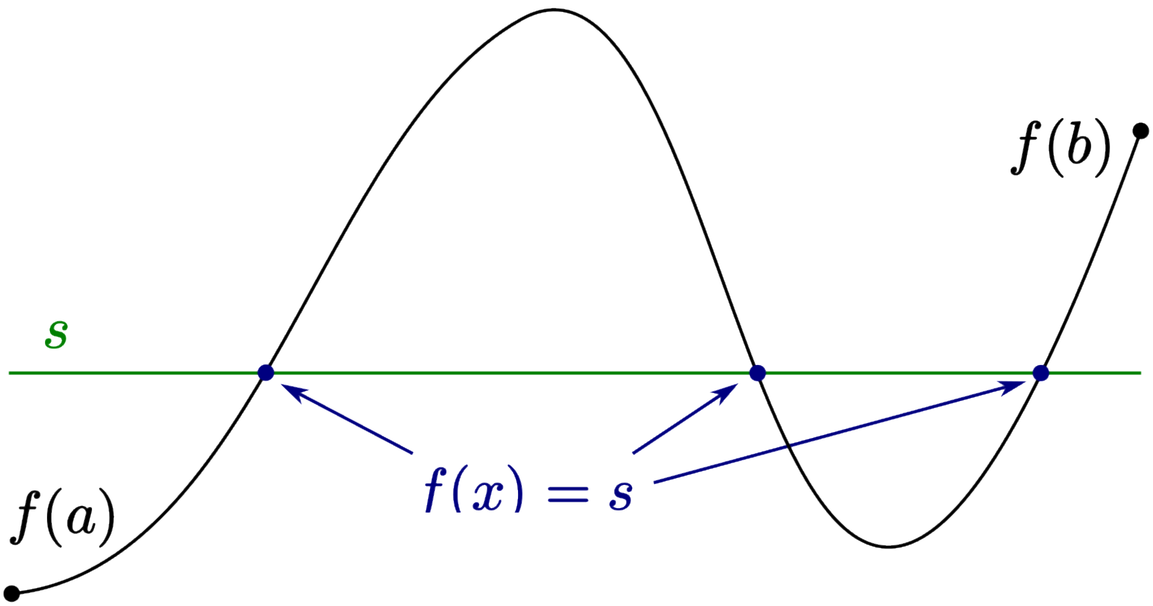
$$f(x)\begin{cases} e^x + a & \text{if} \quad x > 2 \\ bx^2 + 1 & \text{if} \quad 1 \le x \le 2 \\ 3x^3 - b & \text{if} \quad x < 1 \end{cases}$$

Intermediate Value Theorem (IVT)

Theorem: If f is a continuous function whose domain is in the interval [a,b], then there exists at least one c within [a,b] such that f(a) < f(c) < f(b).

In plain English, this theorem essentially states that for every continuous function, for any points a and b, that there's a point in between a and b where the function's value at that point takes on any value in between f(a) and f(b).

MEMORIZE THIS THEOREM



In this illustration, you can see that s is a value between f(a) and f(b). IVT would be able to tell us, without looking at the graph, that there is at least one x value such that f(x) = s. Many practical applications of the IVT will have you setting up an equation algebraically to prove that a solution to a given function exists.

3. Prove that the equation below has a root.

$$f(x) = 4x^4 - 16x^3 + 2x^2 - x + 9$$

Derivatives

The derivative is the **instantaneous rate of change** of a function at a specific point.

The derivative of f(x) is given by

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

Provided that the limit exists. We say that f(x) is **differentiable** where this limit exists.

f′(a), the derivative of f(x) at x = a can also be described by

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

A useful property that the derivative has is that its value at x = a is equal to the slope of the line that is tangent to f(x) at x = a.

In other words,

$f'(a)$ = the slope of the line tangent to f(x) at x = a

4.

(a) Carefully state the limit definition of the derivative of the function $y = f(x)$

(b) Use the limit definition of the derivative to find $f'(9)$ for the following function. No marks will be given for the use of differentiation rules.

$$f(x) = -6\sqrt{x}$$

5. Let: $h(x) = e^{3f(x)} + [f(x)]^2 \qquad f(1) = 2 \qquad f'(1) = 5$

Find $h'(1)$

Answer:

6. Find the equations of the lines parallel to $2y - 2x = 2019^{2020}$ and tangent to the graph of $y = 2x + 3x^{-1}$

7. Professor Fenceman, an economics professor at the University of Building Connections sells 200 of his award-winning review packages at $110 each. He rents an industrial-grade printer at $50 per term and each review pack costs $70 to produce. Dr. Fenceman's students are price-sensitive, and some will decide not to buy the review package if the price is too high. He estimates that for every $20 increase in his review package price, 20 students will choose not to purchase it.

(a) Find the linear demand function for the review package as a function of q (quantity).

(b) Find the Revenue function as a function of q (quantity)

(c) What should Dr. Fenceman charge to maximize profit?

CHALLENGE

Find the equation of the line that passes through the origin and is tangent to the graph of $f(x) = \ln x$

Note: It's unlikely that you'll have a question like this on a 50-minute midterm, but it's good practice to familiarize yourself with derivatives. Draw a diagram and try to visualize what's going on.

Appendix: Formulae

$$\lim_{x \to a} \left[f\left(x\right) + g\left(x\right) \right] = \lim_{x \to a} f\left(x\right) + \lim_{x \to a} g\left(x\right).$$

$$\lim_{x \to a} kf\left(x\right) = k \lim_{x \to a} f\left(x\right).$$

$$\lim_{x \to a} \left[f\left(x\right) + g\left(x\right) \right] = \lim_{x \to a} f\left(x\right) + \lim_{x \to a} g\left(x\right).$$

$$\lim_{x \to a} \frac{f\left(x\right)}{g\left(x\right)} = \frac{\lim\limits_{x \to a} f\left(x\right)}{\lim\limits_{x \to a} g\left(x\right)}, \quad \text{if} \quad \lim_{x \to a} g\left(x\right) \neq 0.$$

$$\lim_{x \to a} b^{f(x)} = b^{\lim\limits_{x \to a} f(x)},$$

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

Business Terminology and notation

Quantity **q**

Price **p**

Revenue = **pq** (Refers to the money earned by selling **q** items at **p** price)

Costs = **C(q)** (Refers to the total cost of producing **q** items)

- Fixed Cost (Stays constant regardless of production
- Variable Cost (Varies depending on the **q** value)

Break-Even Point (**Where R(q) = C(q) -> Profit = 0**)

Profit(Loss) = **Revenue – Cost**

Marginal Cost **= C'(q)** (The cost to produce an additional unit)

Marginal Revenue = **R'(q)** (The additional revenue gained from selling an additional unit)

"Marginal" means that you should take the derivative of that function.

Profit is maximized where:

- P'(q) = 0, or
- MR = MC, or
- Marginal Profit = 0.