

COMMERCE MENTORSHIP PROGRAM

FINAL REVIEW SESSION

COMM 298



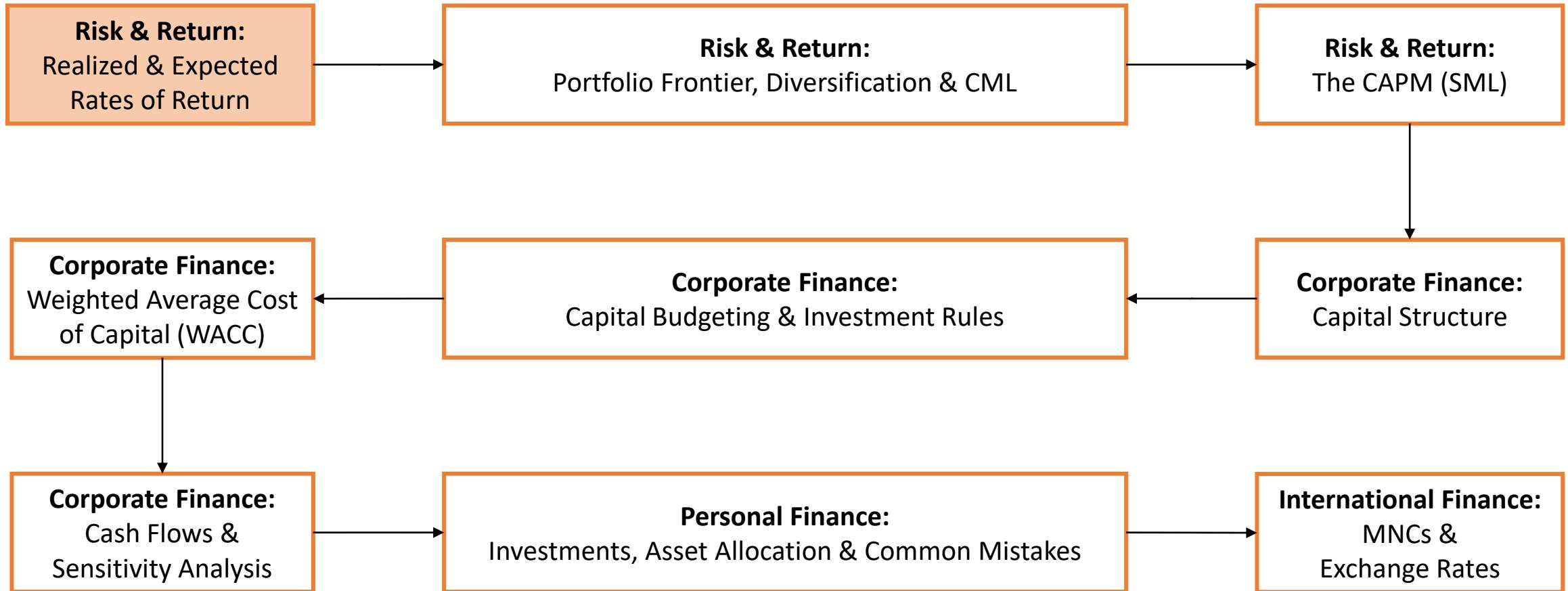
PREPARED BY

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**The 2023W2 C298
Final Exam is not
Cumulative !!**

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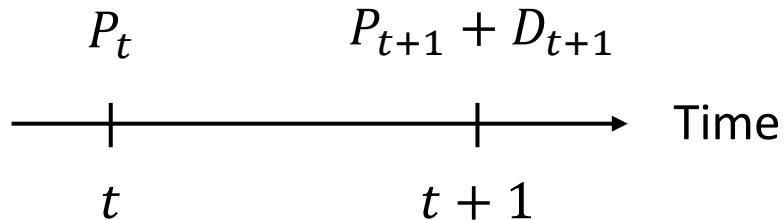




Returns on Individual Assets

One Period Realized Returns for Stocks

How much do the value of your assets (securities) change within a period relative to what you started off with?



Stocks

How did your asset value change?

(1) Received cash flow of dividends D_{t+1}



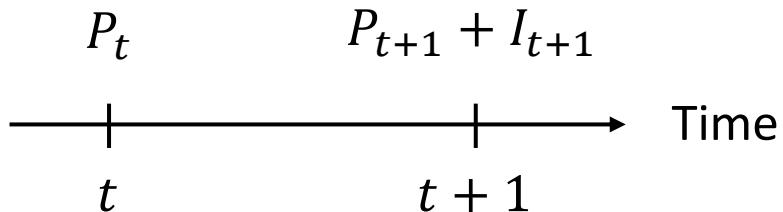
(2) Stock (asset) price changed from P_t to P_{t+1}



P_t = Stock price at time t
 D_{t+1} = Dividend received at time $t + 1$
 R_{t+1} = Realized returns from time t to $t + 1$

$$R_{t+1} = \frac{\text{Change in asset value}}{\text{What you started off with}}$$
$$R_{t+1} = \frac{D_{t+1} + (P_{t+1} - P_t)}{P_t}$$

One Period Realized Returns for Bonds



Bonds

How did your asset value change?

(1) Received cash flow of a coupon I_{t+1}



(2) Bond (asset) price changed from P_t to P_{t+1}



P_t = Bond price at time t

I_{t+1} = Coupon received at time $t + 1$

R_{t+1} = Realized returns from time t to $t + 1$

Change in asset value

$$R_{t+1} = \frac{I_{t+1} + (P_{t+1} - P_t)}{P_t}$$

What you started off with

Generalized One Period Realized Returns



Stocks

$$R_{t+1} = \frac{D_{t+1} + (P_{t+1} - P_t)}{P_t} = \underbrace{\frac{D_{t+1}}{P_t}}_{\text{Dividend Yield}} + \underbrace{\frac{P_{t+1} - P_t}{P_t}}_{\text{Capital Gain Yield}}$$



Bonds

$$R_{t+1} = \frac{I_{t+1} + (P_{t+1} - P_t)}{P_t} = \frac{I_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

In General:

$$R_{t+1} = \frac{\text{Cash Flows} + (P_{t+1} - P_t)}{P_t}$$

More Examples of Effective Rate Conversions

$r_{quarterly}$ r_{annual} (EAR)

$$r_{annual} = (1 + r_{quarterly})^4 - 1$$

There are **4** quarters in a year

r_{weekly} r_{annual} (EAR)

$$r_{annual} = (1 + r_{weekly})^{52} - 1$$

There are **52** weeks in a year

r_{2-year} $r_{quarterly}$

$$r_{quarterly} = (1 + r_{2-year})^{\frac{1}{8}} - 1$$

1/8 of a 2-year period is a quarter (3 months)

r_{3-week} r_{6-week}

$$r_{6-week} = (1 + r_{3-week})^2 - 1$$

Two 3-week periods make up a 6-week period

$r_{quarterly}$ $r_{monthly}$

$$r_{monthly} = (1 + r_{quarterly})^{\frac{1}{3}} - 1$$

1/3 of a quarter (3 months) is a single month

Multiple Period Realized Returns

What is the annualized return if the realized return is 3% every quarter?

$$\begin{aligned} R_{annual} &= (1 + R_{quarterly})^4 - 1 = (1 + 0.03)^4 - 1 \\ &= (1 + 0.03)(1 + 0.03)(1 + 0.03)(1 + 0.03) - 1 = 0.1255 \\ &\quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_1 \quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_2 \quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_3 \quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_4 \end{aligned}$$

What is the annualized return if the returns on each quarter are: Q1 = 3%, Q2 = 5%, Q3 = -1%, Q4 = 2%

$$\begin{aligned} R_{annual} &= (1 + 0.03)(1 + 0.05)(1 - 0.01)(1 + 0.02) - 1 = 0.0921 \\ &\quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_1 \quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_2 \quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_3 \quad \text{\scriptsize \textbf{\textcolor{blue}{R}}}_4 \end{aligned}$$

Annual Returns Given n Equal Length Periods in a Year:

$$R_{annual} = (1 + R_1)(1 + R_2)(1 + R_3) \dots (1 + R_{n-1})(1 + R_n) - 1$$

More on Realized Returns

Our calculation of realized returns over multiple periods assumes we reinvest all cash flows received into the same asset (e.g., reinvest dividends into the same stock) – this action allows us to compound returns

$$R_{annual} = (1 + R_1)(1 + R_2)(1 + R_3) \dots (1 + R_{n-1})(1 + R_n) - 1$$

Multiplication (compounding) – NOT addition

Realized vs Expected Returns:

Realized returns: What we *actually* get from an investment over a period that has already passed

Expected returns: An estimate of what we *hope* to get over a period that hasn't passed

Expected Returns & Statistics

Let R_t be the realized return for each period t (day, month, year, etc.). Then the mean, variance, and standard deviation across n periods of t length are as follows.

Mean (expected return): $E(R) = \bar{R} = \frac{R_1 + R_2 + \cdots + R_n}{n} = \sum_{t=1}^n \frac{R_t}{n}$ (doesn't reflect an asset's risk)

Variance (total risk): $Var(R) = s_R^2 = \frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \cdots + (R_n - \bar{R})^2}{n-1} = \sum_{t=1}^n \frac{(R_t - \bar{R})^2}{n-1}$

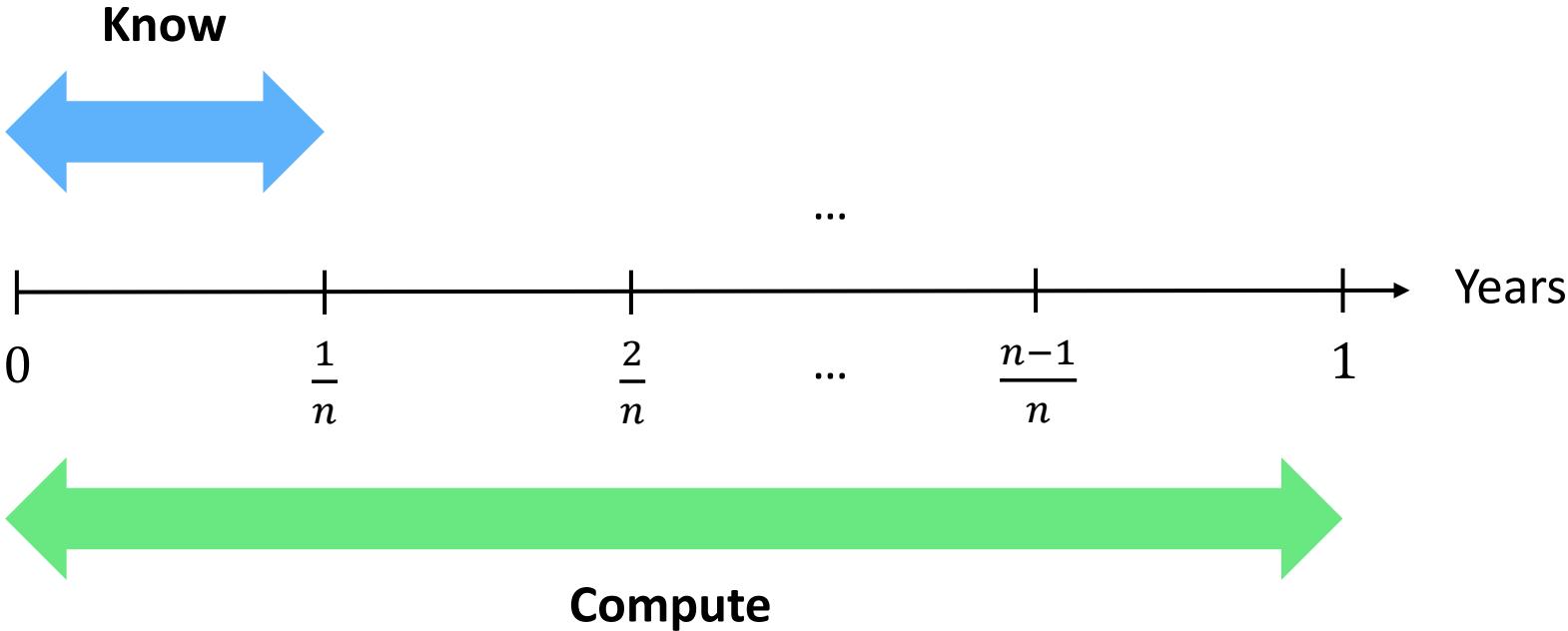
Standard Deviation (volatility): $SD(R) = s_R = \sqrt{Var(R)}$



Note: The time period of $Var(R)$ and $SD(R)$ should match the time period of \bar{R}

Annual Variance & Standard Deviation

If we divide a year into n equal periods and know the variance and standard deviation of returns for one period, we can *approximate* the variance and standard deviation of returns for the entire year



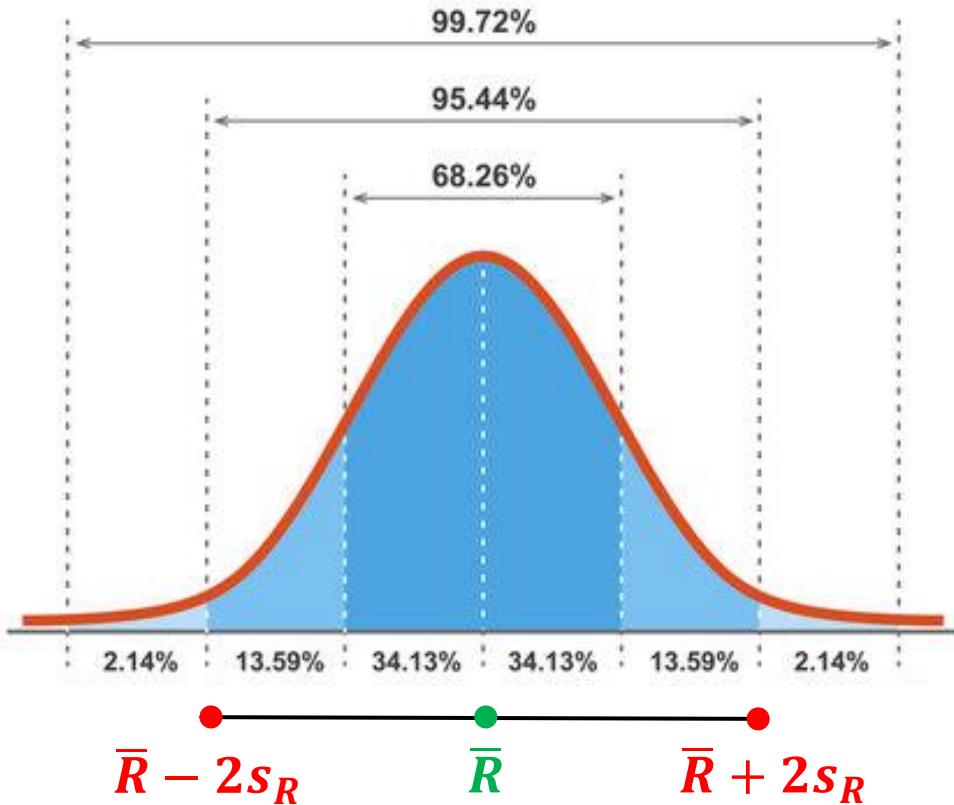
$$Var(R_{annual}) \approx n \cdot Var(R_{period})$$

(assuming independent returns between periods)

$$SD(R_{annual}) \approx \sqrt{n} \cdot SD(R_{period})$$

Normal Returns

If returns over a period are normally distributed, we can quantitatively estimate next period's return



$$\bar{R} = \text{Mean}$$

$$s_R = SD(R) = \text{Standard Deviation}$$

≈ 68% of data points lie within 1 SD of the mean

≈ 95% of data points lie within 2 SDs of the mean

≈ 99.7% of data points lie within 3 SDs of the mean

$$\bar{R} \pm 2 \cdot SD(R)$$

95% chance next period's return will be
within 2 SDs of the mean (\bar{R})

Normal Returns



Care Only About
MEAN and **VARIANCE**

Practice Question 1

You own a municipal bond that is priced at \$256 on Mar. 22nd, 2023. If it pays semi-annual coupons of \$20, what is your semi-annual realized return when you sell the bond on Sep. 22nd, 2023 for \$250?

Practice Question 2

A GIC you found on the internet website displayed realized returns of 3.4%, 2.8% and -2.4% for Q2, Q3, and Q4, respectively. If the annual realized return on the asset is 6%, what was the realized return for Q1?

Practice Question 3

Your parents bought some RBC stocks at the beginning of the year. The stocks yield a constant monthly return of 2.51% and provide quarterly dividend payments. If the capital gain yield over the first quarter was 3.75%, what was the dividend yield across this time?

Practice Question 4

Approximately 95% of yearly returns on Canadian common stocks from 1957 – 2017 lie between -22.6% and 43.2%. If the yearly returns are normally distributed, what is their mean and variance?

Returns on Portfolios



One Period Portfolio Realized Returns

The realized returns on a portfolio is a weighted average of each asset's return over a period t to $t + 1$

Two Assets (A & B): $R_{portfolio} = w_A R_A + w_B R_B$

$$w_A = \frac{V_A}{V_A + V_B}$$

$$w_B = \frac{V_B}{V_A + V_B}$$

R_A = Realized return on asset A from period t to $t + 1$

R_B = Realized return on asset B from period t to $t + 1$

V_A = Dollar amount invested into asset A at period t

V_B = Dollar amount invested into asset B at period t

n Assets: $R_{portfolio} = w_1 R_1 + w_2 R_2 + \dots + w_n R_n$

\$ invested into asset i

$$w_i = \frac{V_i}{\underbrace{V_1 + V_2 + \dots + V_n}_{\text{Total \$ invested in portfolio}}}$$

for $i \in \{1, 2, \dots, n\}$

Portfolio Expected Returns

Let w_A & w_B be weights on assets A & B and R_A & R_B be random variables. ρ_{AB} is the correlation coefficient between assets A & B .

Mean (expected value):

$$E(R_{portfolio}) = E(w_A R_A + w_B R_B) = w_A \cdot E(R_A) + w_B \cdot E(R_B)$$

Variance (total risk):

$$Var(R_{portfolio}) = Var(w_A R_A + w_B R_B) = w_A^2 \cdot Var(R_A) + w_B^2 \cdot Var(R_B) + 2w_A w_B \cdot \underbrace{\rho_{AB} \cdot SD(R_A) \cdot SD(R_B)}_{Cov(R_A, R_B) = \text{Covariance}}$$

Standard Deviation (volatility):

$$SD(R_{portfolio}) = \sqrt{Var(R_{portfolio})}$$

Correlation & Covariance

Covariance measures the degree to which two random variables move together

$$Cov(R_A, R_B) = \rho_{AB} \cdot SD(R_A) \cdot SD(R_B)$$

Correlation is a normalization of covariance and only takes on values from -1 to 1 (i.e., $-1 \leq \rho \leq 1$)

$$\rho_{AB} = \frac{Cov(R_A, R_B)}{SD(R_A) \cdot SD(R_B)}$$

Variance With Covariance:

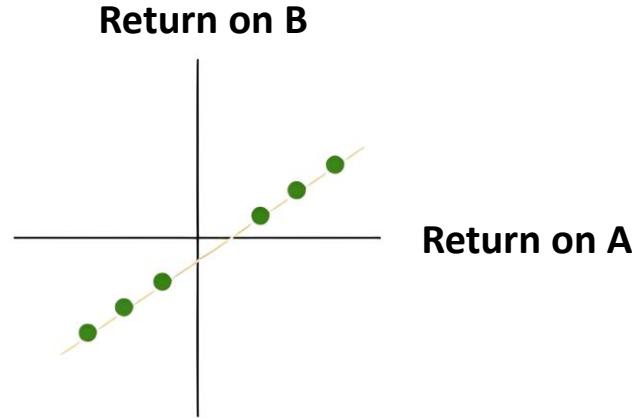
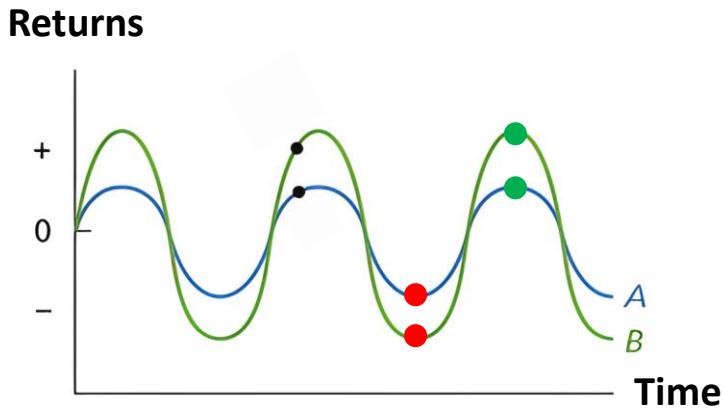
$$Var(R_{portfolio}) = w_A^2 \cdot Var(R_A) + w_B^2 \cdot Var(R_B) + 2w_Aw_B \cdot \textcolor{red}{Cov(R_A, R_B)}$$

Variance With Correlation:

$$Var(R_{portfolio}) = w_A^2 \cdot Var(R_A) + w_B^2 \cdot Var(R_B) + 2w_Aw_B \cdot \textcolor{red}{\rho_{AB}} \cdot SD(R_A) \cdot SD(R_B)$$

What Does Positive Correlation Tell us?

Perfect Positive Correlation ($\rho_{AB} = 1$):



When one asset takes on its highest return, the other always takes its highest return and vice versa

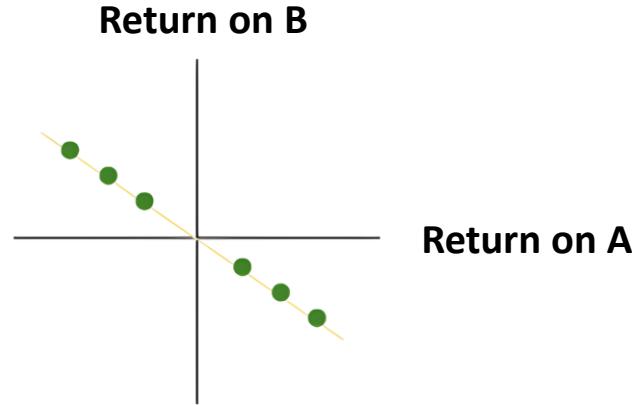
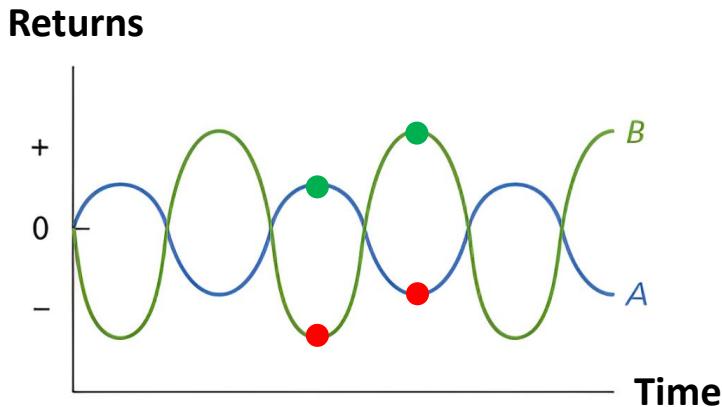
Positive Correlation ($0 < \rho_{AB} < 1$):

When the return on two assets tend to move in the same direction

The higher the number (i.e., closer to 1), the greater chance the above statement will be true at any given time

What Does Negative Correlation Tell us?

Perfect Negative Correlation ($\rho_{AB} = -1$):



When one asset takes on its highest return, the other always takes its lowest return and vice versa

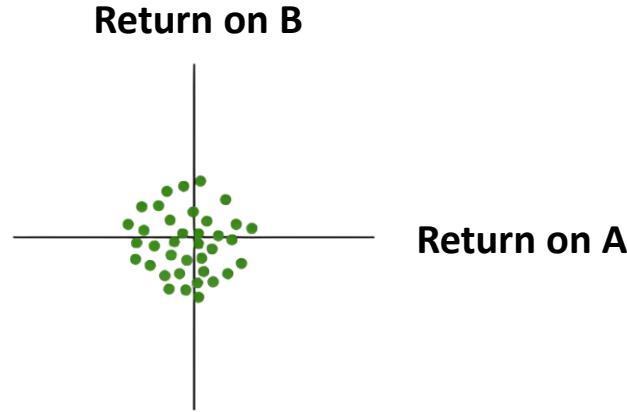
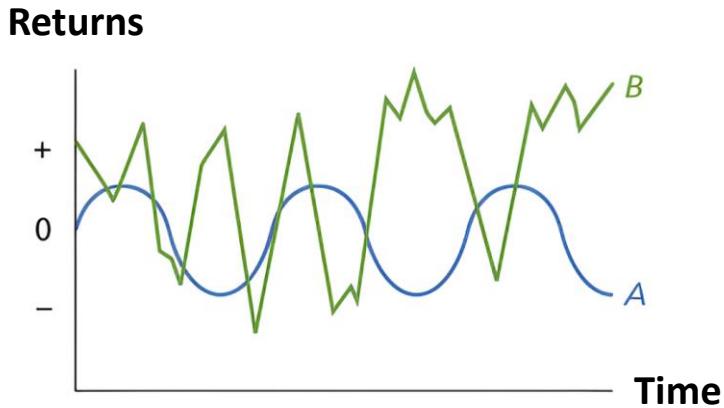
Negative Correlation ($-1 < \rho_{AB} < 0$):

When the return on two assets tend to move in the opposite direction

The lower the number (i.e., closer to -1), the greater chance the above statement will be true at any given time

What Does Zero Correlation Tell us?

Zero Correlation ($\rho_{AB} = 0$):



When the return on one asset is completely unrelated to the return of the other asset

The assets are independent!

Variance or Covariance?

What drives total risk in a portfolio – the variance of individual assets or the covariance between assets?

2-asset Portfolio Variance:

$$Var(R_{portfolio}) = w_A^2 \cdot Var(R_A) + w_B^2 \cdot Var(R_B) + 2w_Aw_B \cdot Cov(R_A, R_B)$$

	A	B
A	$w_A^2 \cdot Var(R_A)$	$w_Aw_B \cdot Cov(R_A, R_B)$
B	$w_Aw_B \cdot Cov(R_A, R_B)$	$w_B^2 \cdot Var(R_B)$

2 Variance Terms

2 Covariance Terms

Variance or Covariance? (Continued)

3-asset Portfolio Variance:

$$= w_A^2 \cdot \text{Var}(R_A) + w_B^2 \cdot \text{Var}(R_B) + w_C^2 \cdot \text{Var}(R_C) + 2w_Aw_B \cdot \text{Cov}(R_A, R_B) + 2w_Aw_C \cdot \text{Cov}(R_A, R_C) + 2w_Bw_C \cdot \text{Cov}(R_B, R_C)$$

A B C

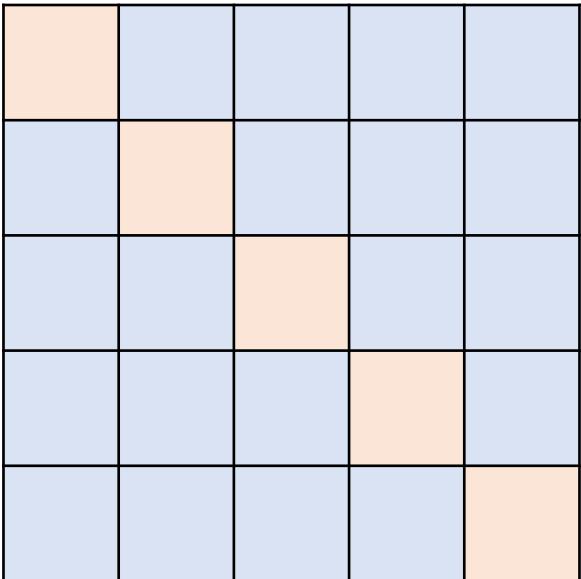
	A	B	C
A	$w_A^2 \cdot \text{Var}(R_A)$	$w_Aw_B \cdot \text{Cov}(R_A, R_B)$	$w_Aw_C \cdot \text{Cov}(R_A, R_C)$
B	$w_Aw_B \cdot \text{Cov}(R_A, R_B)$	$w_B^2 \cdot \text{Var}(R_B)$	$w_Bw_C \cdot \text{Cov}(R_B, R_C)$
C	$w_Aw_C \cdot \text{Cov}(R_A, R_C)$	$w_Bw_C \cdot \text{Cov}(R_B, R_C)$	$w_C^2 \cdot \text{Var}(R_C)$

3 Variance Terms

6 Covariance Terms

Variance or Covariance? (Continued)

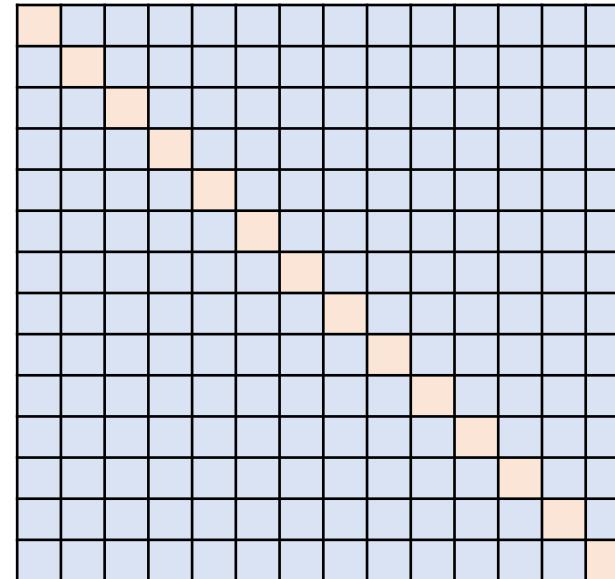
5 Assets



5 Variance Terms

20 Covariance Terms

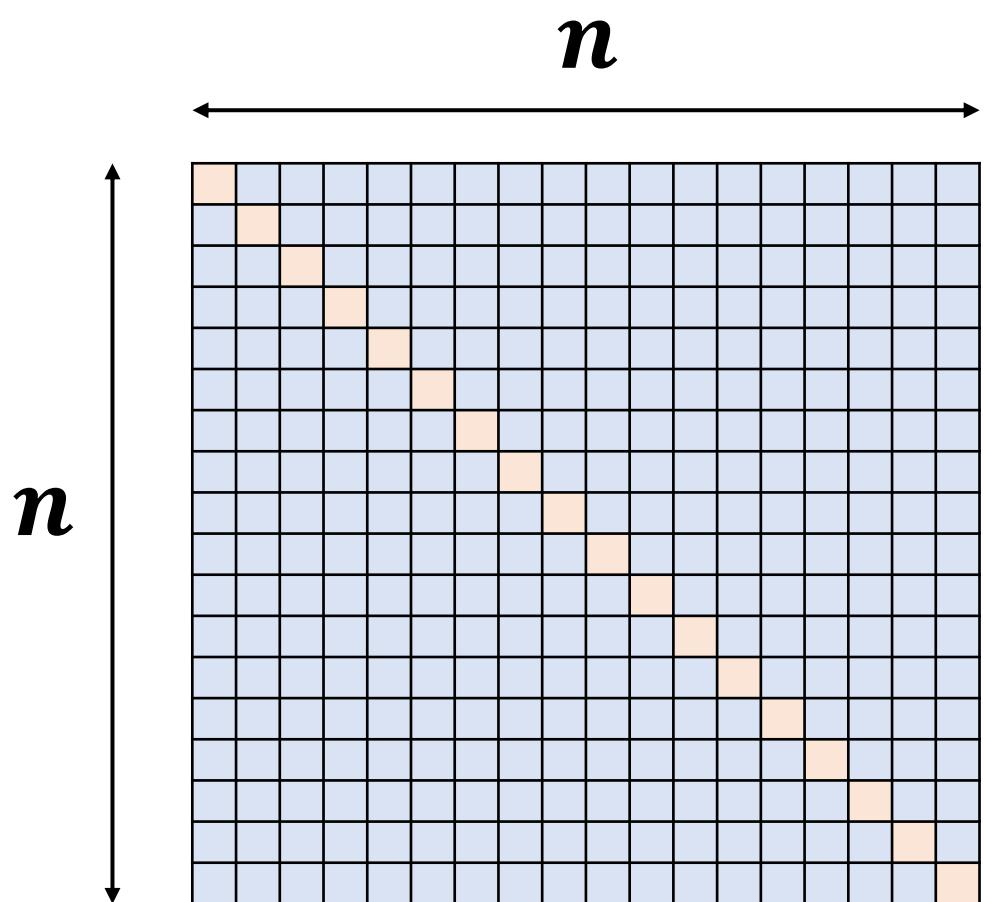
16 Assets



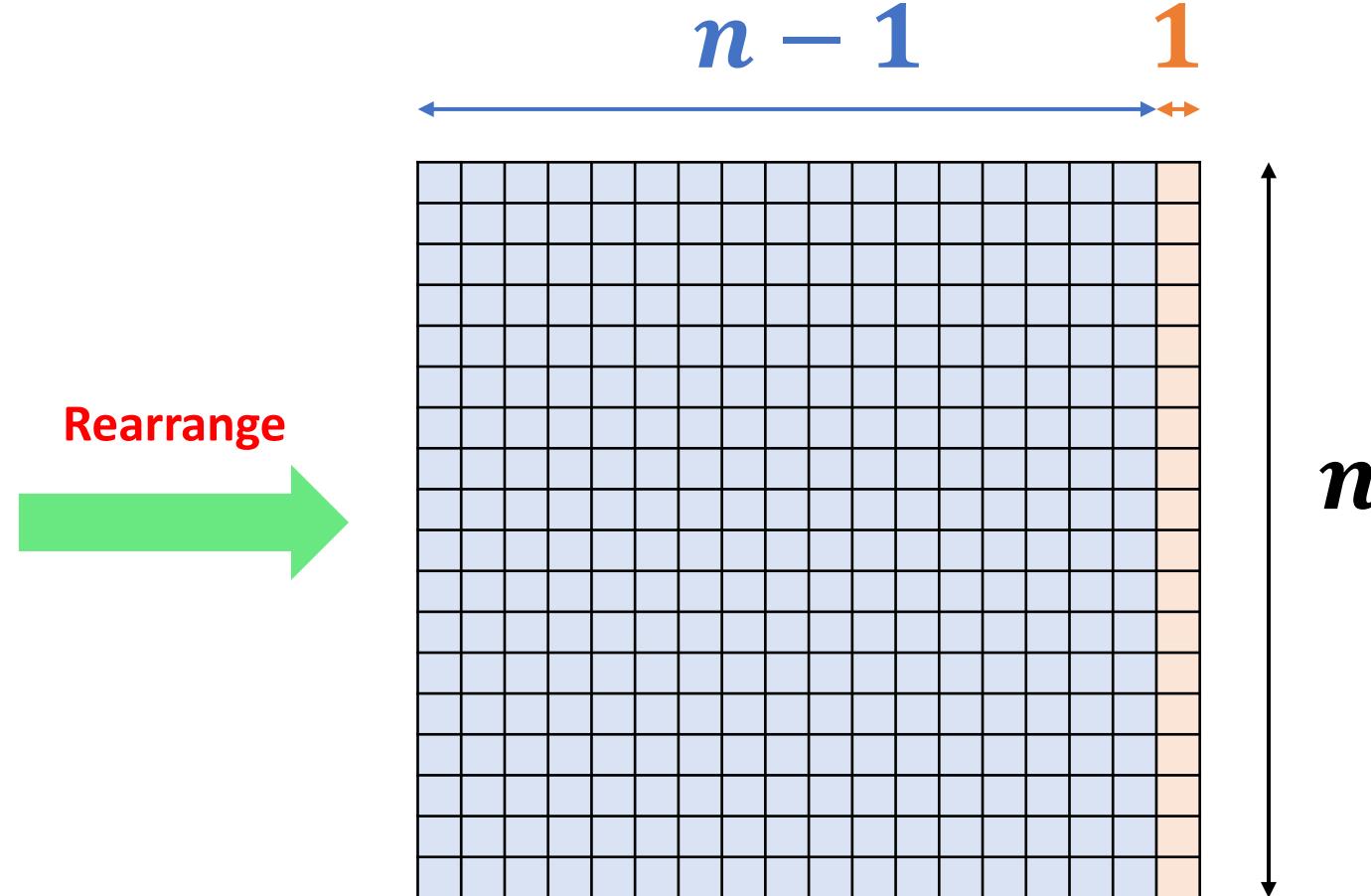
16 Variance Terms

240 Covariance Terms

General Formula for n Assets



n Variance Terms



$n(n - 1)$ Covariance Terms

Rearrange

Covariance Matters

Practice Question 5

Your finance professor invested \$8000 in two mystery stocks (A & B) at the start of the year. Their portfolio received an annual return of 7.8%. If they told you that $R_A = 2.3\%$ and $R_B = 8.7\%$, what percentage of their investment went towards stock A?

Practice Question 6

It is empirically known that stock A and B are perfectly negatively correlated with each other. Based on the sample data below, calculate the variance of a portfolio that contains 65% of stock A and 35% of stock B.

		Statistical Measure	
Stock		$E(R)$	$SD(R)$
	A	1.7%	0.86%
	B	2.6%	0.32%

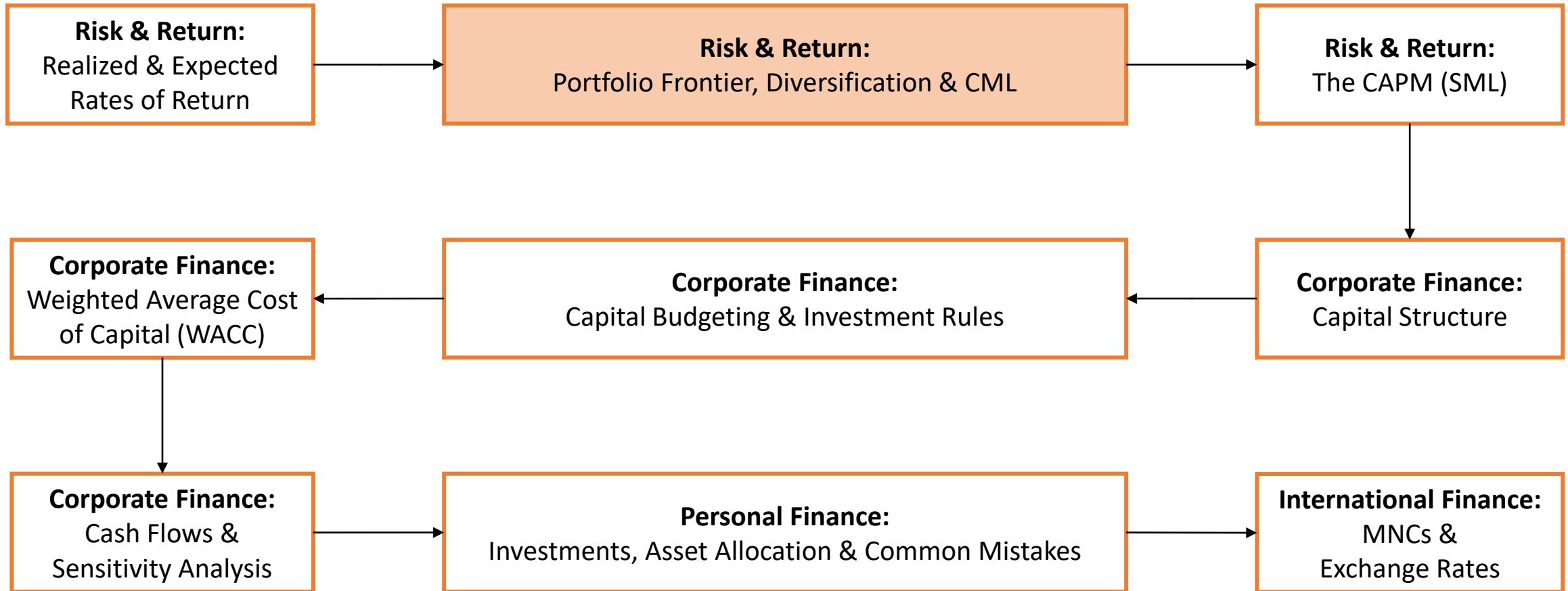
Practice Question 7

Which of the following is true about portfolio theory?

(Select all that apply)

- a) The covariance between assets has greater influence over a portfolio's variance as more assets are added
- b) The sign of the correlation coefficient and covariance between any two stocks will always be the same
- c) If the correlation coefficient between two stocks is not 0, the stocks are dependent
- d) If the variance of monthly returns on a portfolio is $Var(R)$, the annual variance is exactly $12 \cdot Var(R)$
- e) The correlation coefficient between two identical stocks is always 1

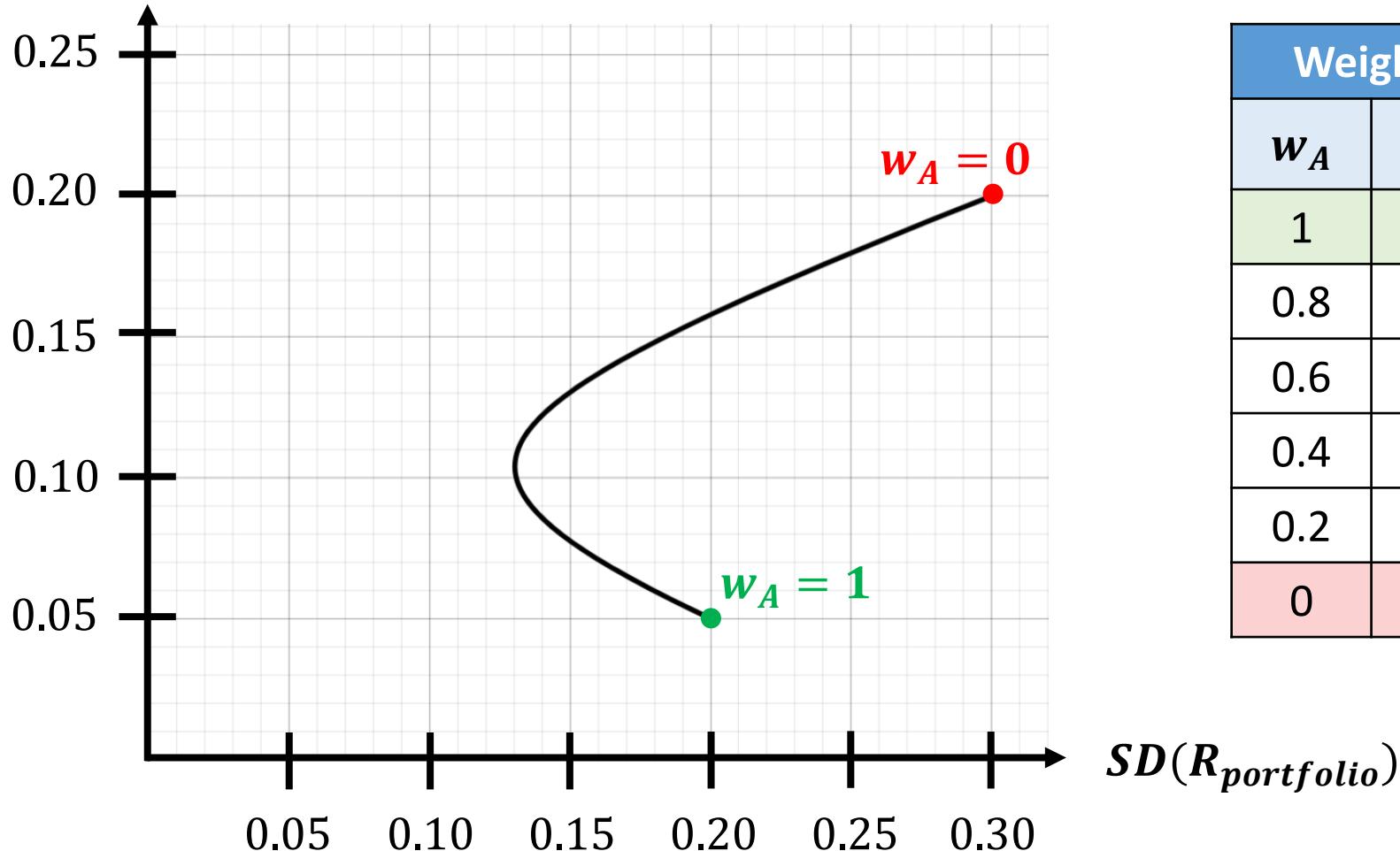
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The Portfolio Frontier

Let $E(R_A) = 0.05$, $E(R_B) = 0.20$, $SD(R_A) = 0.20$, $SD(R_B) = 0.30$, $\rho_{AB} = -0.40$. Now vary w_A & w_B

$E(R_{portfolio})$



Weights		Statistical Measure	
w_A	w_B	$E(R_{portfolio})$	$SD(R_{portfolio})$
1	0	0.05	0.20
0.8	0.2	0.08	0.15
0.6	0.4	0.11	0.13
0.4	0.6	0.14	0.17
0.2	0.8	0.17	0.23
0	1	0.20	0.30

Correlation & Diversification (Graphically)

Using the same data from the previous slide, let's now vary ρ_{AB} – what do we see given a fixed w_A ?

$$E(R_{\text{portfolio}})$$

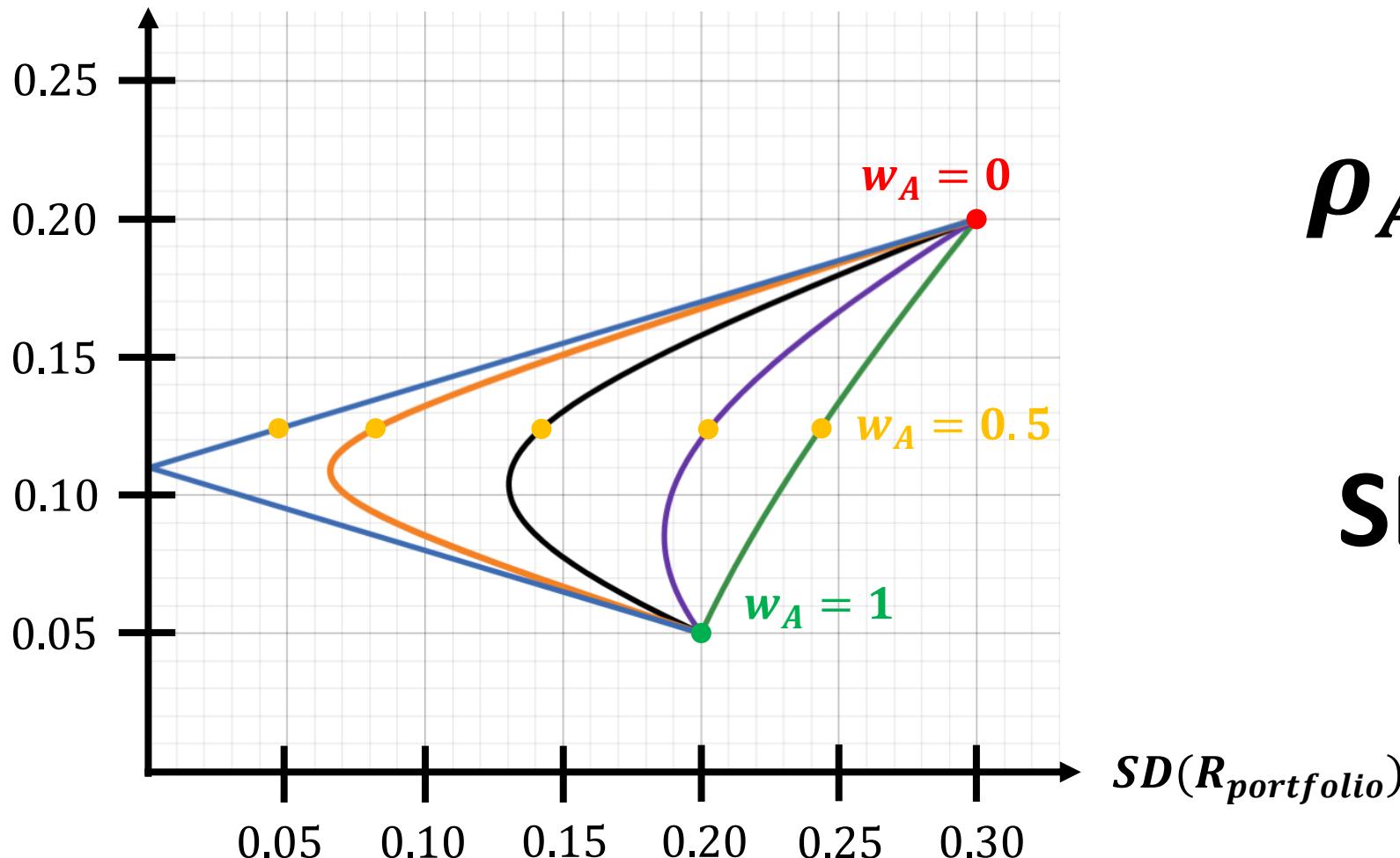
$$\rho_{AB} = -1$$

$$\rho_{AB} = -0.85$$

$$\rho_{AB} = -0.4$$

$$\rho_{AB} = 0.3$$

$$\rho_{AB} = 0.9$$



ρ_{AB} Decreases
↓
SD Decreases

Correlation & Diversification (Algebraically)

Why does decreasing ρ_{AB} decrease portfolio variance and standard deviation algebraically?

$$SD(R_{portfolio}) \downarrow = \sqrt{Var(R_{portfolio})} \downarrow = \sqrt{w_A^2 \cdot Var(R_A) + w_B^2 \cdot Var(R_B) + 2w_Aw_B \cdot \rho_{AB} \downarrow \cdot SD(R_A) \cdot SD(R_B)}$$

Diversification is the process of reducing risk by holding a number of different securities

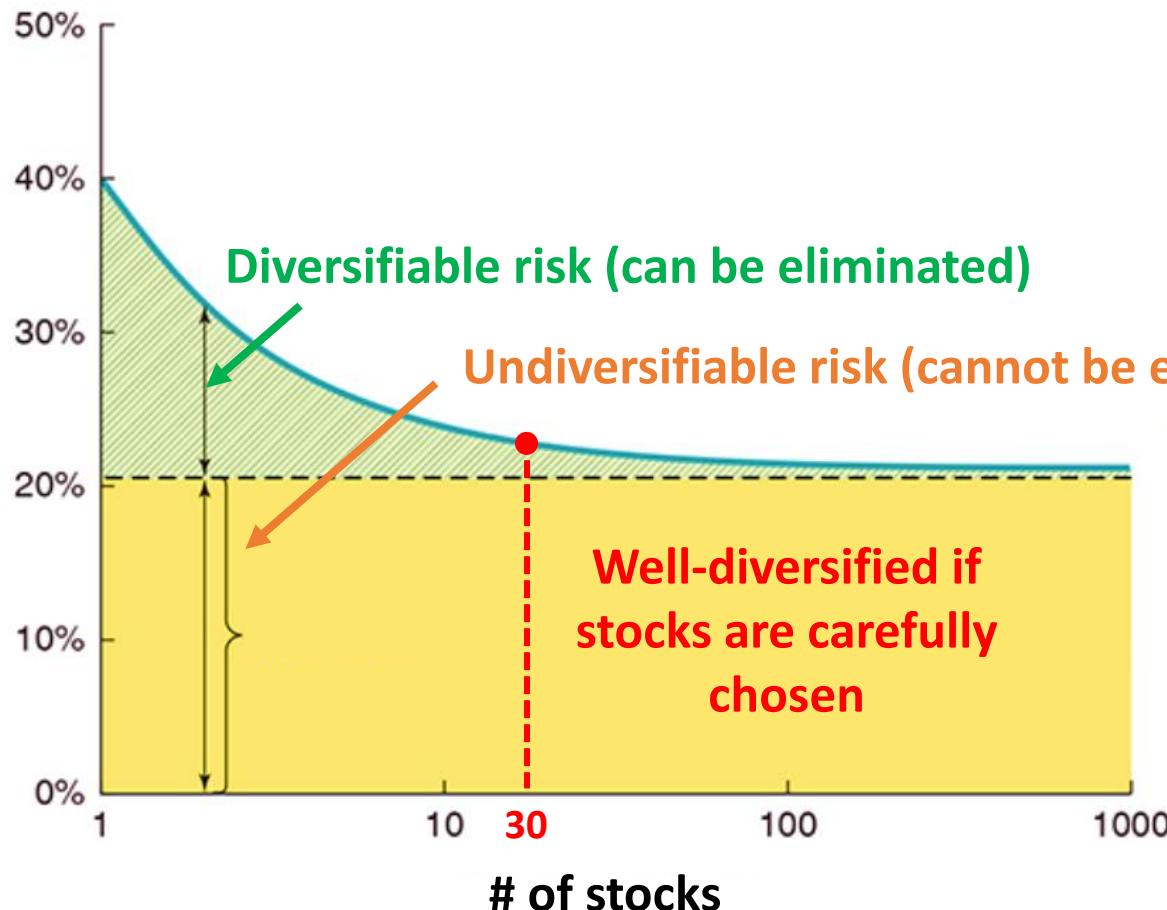
Facts About Diversification:

- (1) Adding an asset with a positive correlation into a portfolio doesn't make diversification useless
- (2) If an asset added to a portfolio isn't perfectly positively correlation ($\rho \neq 1$) with all existing assets in the portfolio, portfolio variance & SD will be reduced – otherwise the variance & SD will stay the same
- (3) Stocks from companies in the same industry tend to be affected by similar factors – bad for diversification
- (4) The lower an asset's ρ is with existing assets in the portfolio, the more effective diversification will be

Diversification in Stock Portfolios

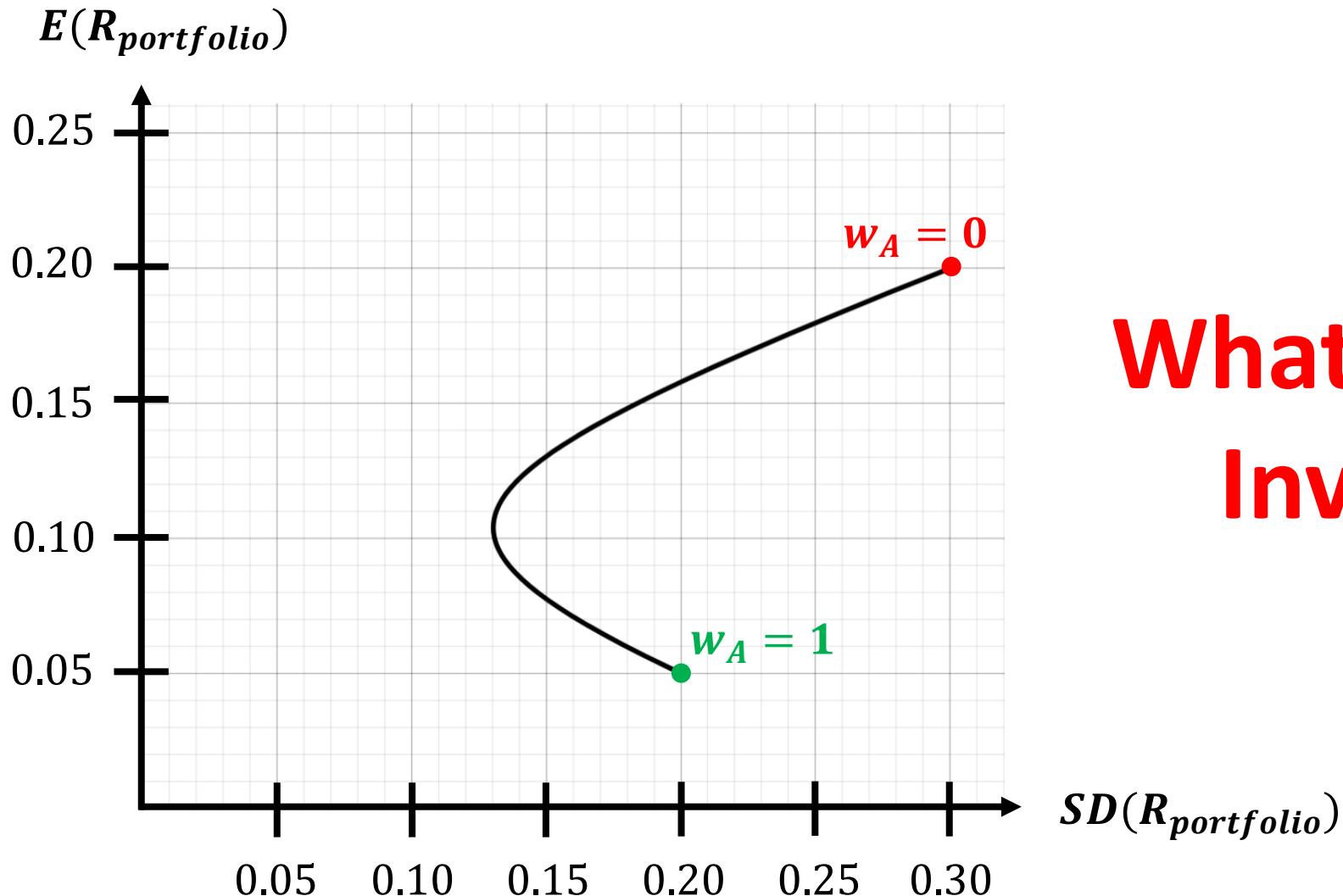
A portfolio is well-diversified when adding more assets will not reduce total portfolio variance/SD significantly

$SD(R_{portfolio})$



# of stocks	$SD(R_{portfolio})$
1	40.0%
2	32.0%
3	28.8%
4	27.1%
5	26.0%
10	23.7%
20	22.5%
30	22.1%
50	21.7%
100	21.5%
1000	21.2%

Motivating the Capital Market Line (CML)



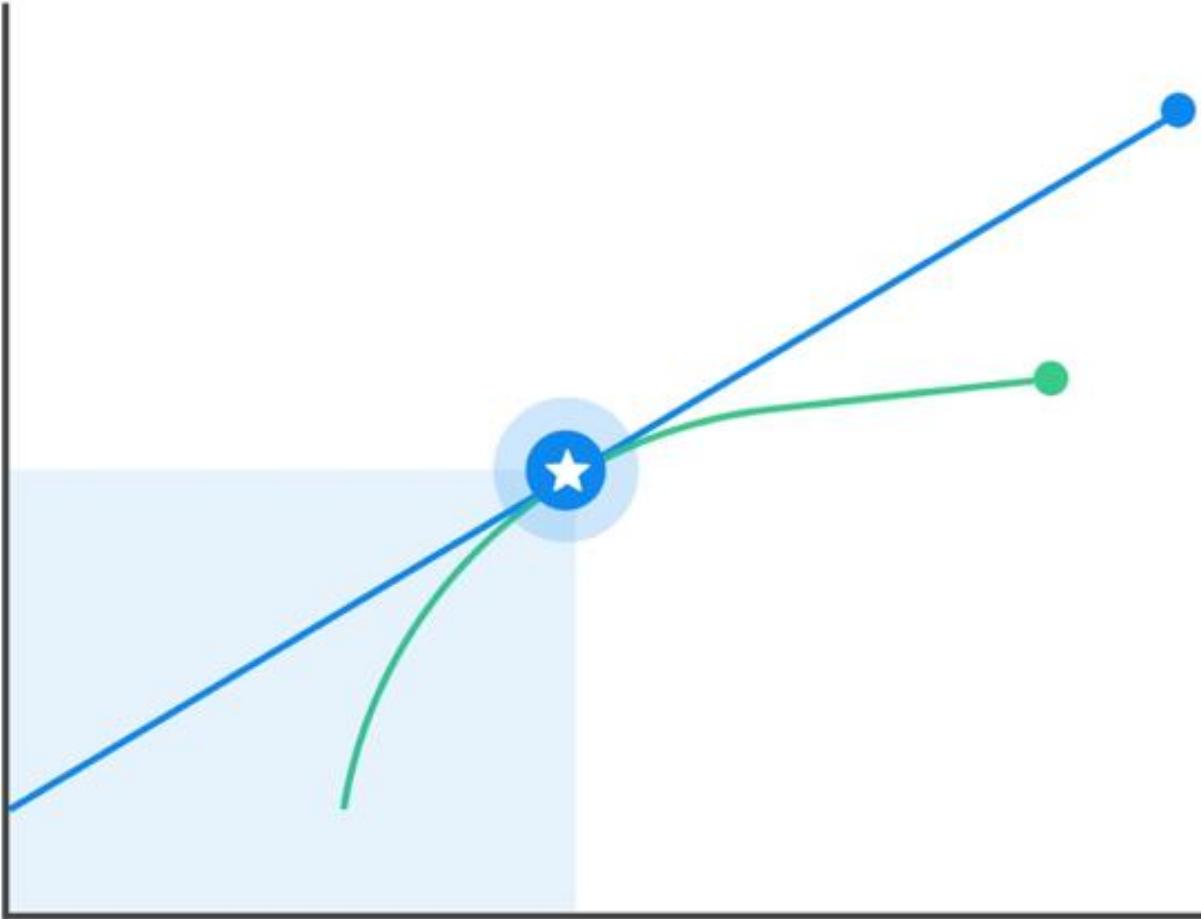
What Portfolios Will
Investors Hold?

Practice Question 1

Which of the following is true about risk and diversification?

(Select all that apply)

- a) It takes around 30 carefully chosen stocks to obtain a well-diversified portfolio
- b) Buying stocks from companies in the same industry will effectively diversify away risk
- c) Variance is a measure that accounts for both diversifiable and undiversifiable risk
- d) Adding an asset to a portfolio that has $0 < \rho < 1$ with all existing assets is useless for diversification
- e) The effectiveness of diversification depends on the strength of the correlation coefficient between securities



The Capital Market Line (CML)

The Mean-variance Criterion

Normal returns imply that investors only care about mean and variance (total risk) – but *how* do they care?

Mean-variance Criterion:

Investors like to maximize expected returns (mean) and minimize variance/standard deviation of returns

All investors are risk averse here!

Dominated & Efficient Portfolios:

A portfolio is *efficient* if there isn't a different portfolio that offers either of the following

- (1) Higher expected return for equal or lower variance/SD
- (2) Equal expected return for lower variance/SD

Otherwise, the portfolio is *dominated* as there is a different portfolio that offers either of the above

Which are Efficient Portfolios?

Portfolio	$E(R_{portfolio})$	$SD(R_{portfolio})$
A	1.5%	0.8%
B	2.0%	2.8%
C	3.5%	0.8%
D	3.5%	4.2%
E	4.0%	2.6%
F	4.5%	5.2%

Dominated by C

Dominated by C/E

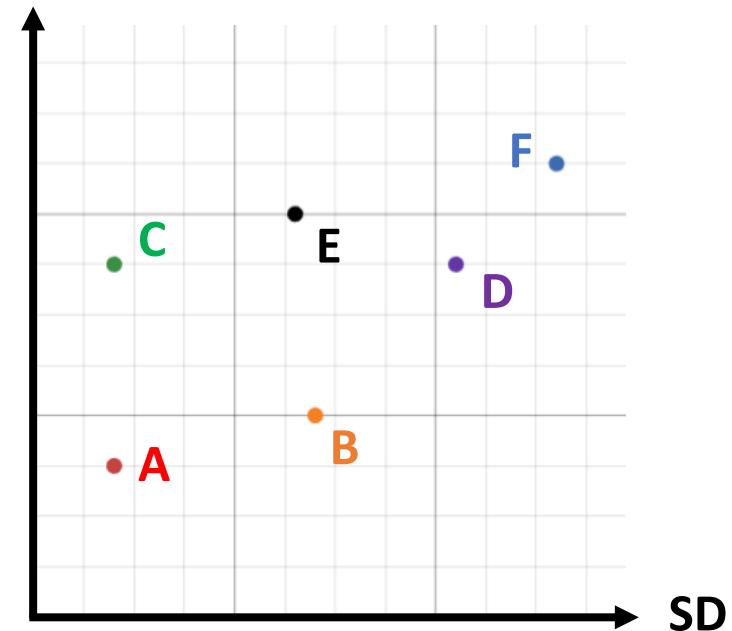
Efficient

Dominated by C/E

Efficient

Efficient

Expected Returns



Explanation:

A is dominated by C: Portfolio C has the same SD but higher expected return than portfolio A

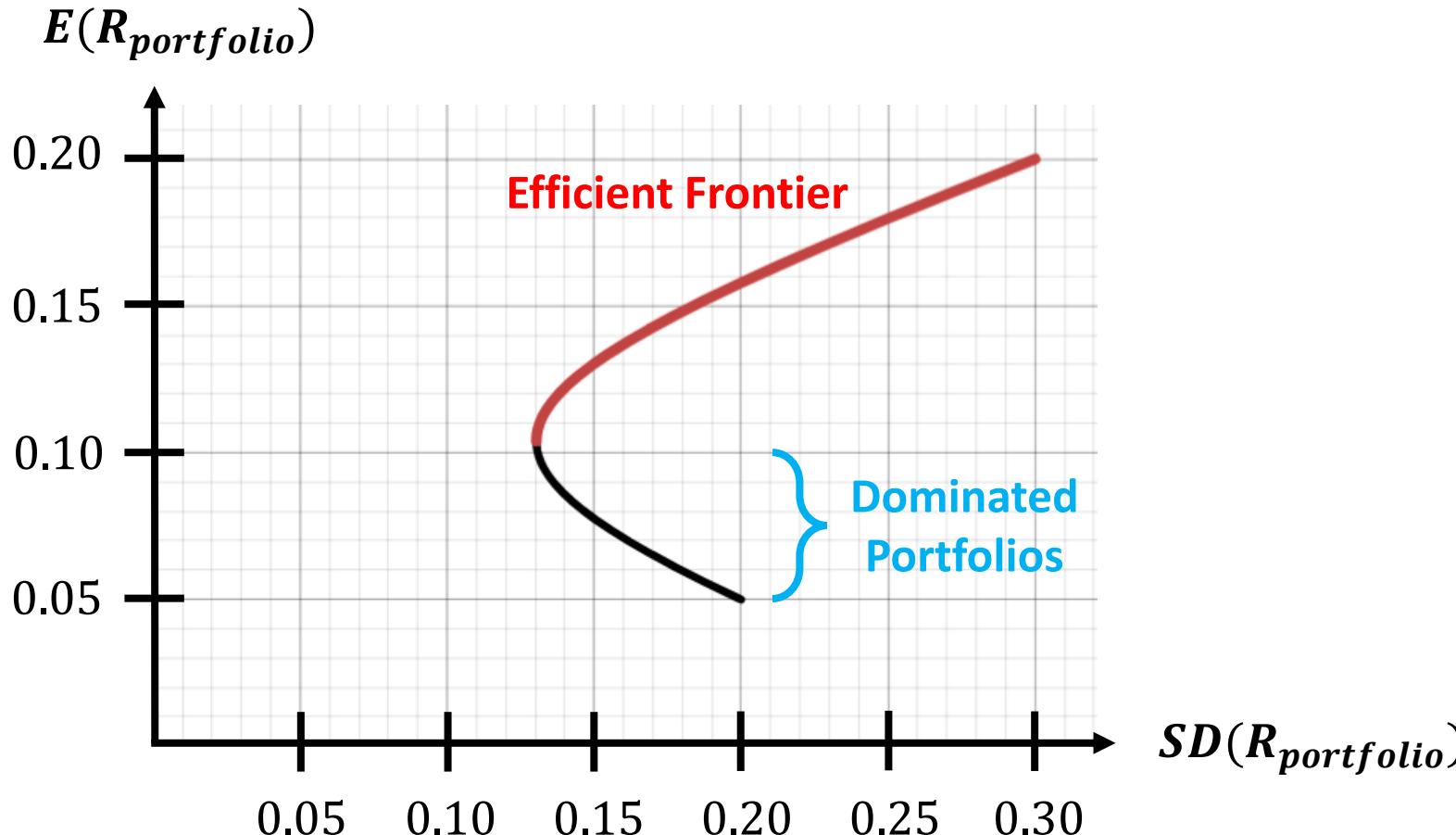
B is dominated by C & E: Portfolios C & E have higher expected returns and lower SD than portfolio B

D is dominated by C & E: Portfolios C & E have the same/higher expected returns but lower SD than portfolio D

The Efficient Frontier of Two Assets

Recall the example where $E(R_A) = 0.05$, $E(R_B) = 0.20$, $SD(R_A) = 0.20$, $SD(R_B) = 0.30$, $\rho_{AB} = -0.40$

⚠️ If two investments have the same variance/SD, investors will prefer the one with higher expected return

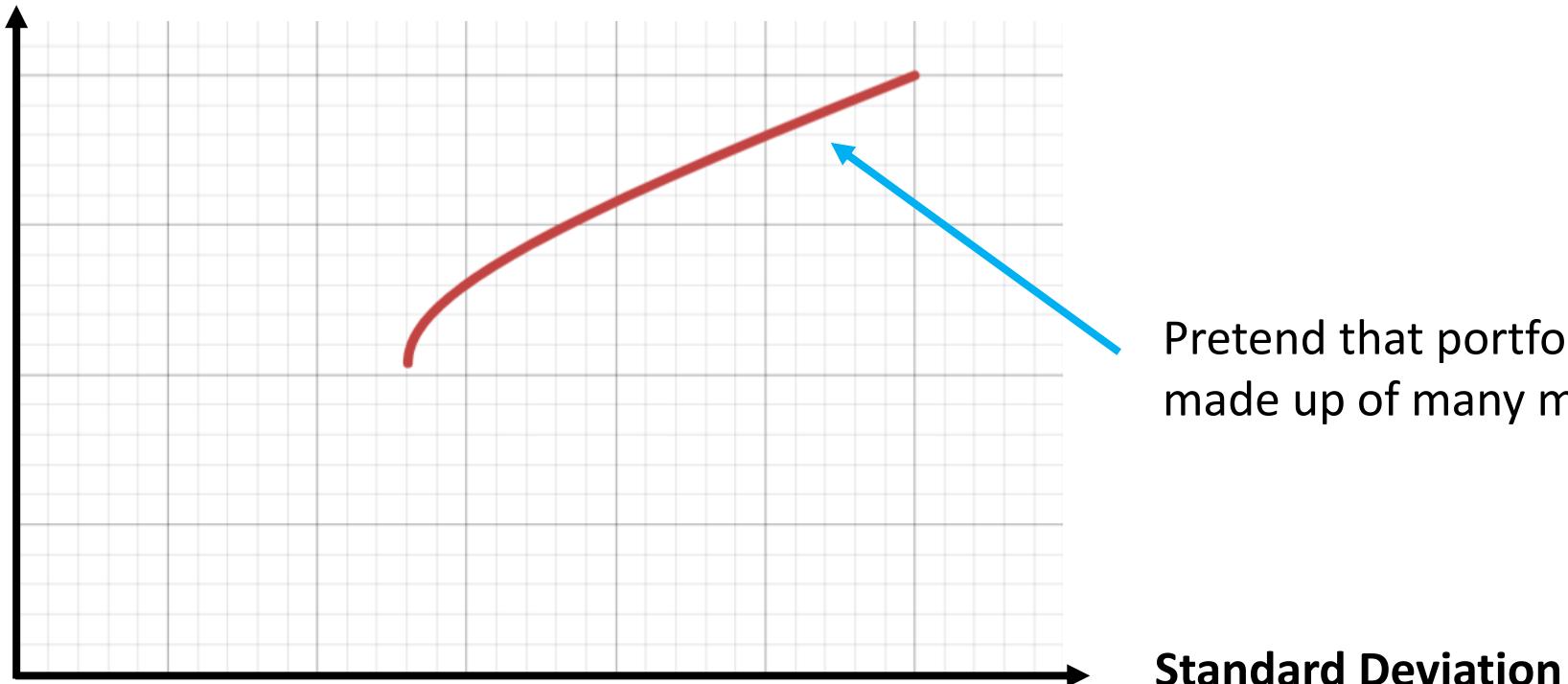


A Larger Scale Efficient Frontier

We can hypothetically consider an efficient frontier where our portfolios are made up of any combination of all risky assets in the world (e.g., stocks, bonds, options, gold, real estate, commodities, etc.)

We will consider this frontier moving forward!

Expected Returns



Pretend that portfolios on this frontier are made up of many more than 2 risky assets

Standard Deviation

Adding a Riskless Asset

What *complete* portfolios could be constructed if we combine a risk-free asset (e.g., GOC T-bill with 0 coupons) with a risky portfolio on the efficient frontier? Risk-free means no variance (expected = realized return)

I will use “ pf ” (portfolio) to denote an arbitrary portfolio on the efficient frontier constructed last slide

R_C = Return on complete portfolio (risk free asset + portfolio on efficient frontier)

R_f = Return on risk free asset (risk-free rate) $Var(R_f) = SD(R_f) = 0$ $E(R_f) = R_f$ $w_f = 1 - w_{pf}$

Mean:
$$\begin{aligned} E(R_C) &= w_{pf} \cdot E(R_{pf}) + w_f \cdot E(R_f) \\ &= w_{pf} \cdot E(R_{pf}) + (1 - w_{pf}) \cdot R_f \\ &= R_f + w_{pf} \cdot [E(R_{pf}) - R_f] \end{aligned}$$

Variance:
$$\begin{aligned} Var(R_C) &= w_{pf}^2 \cdot Var(R_{pf}) + w_f^2 \cdot Var(R_f) + 2w_{pf}w_f \cdot \rho \cdot SD(R_{pf}) \cdot SD(R_f) \\ &= w_{pf}^2 \cdot Var(R_{pf}) + w_f^2 \cdot 0 + 2w_{pf}w_f \cdot \rho \cdot SD(R_{pf}) \cdot 0 \\ &= w_{pf}^2 \cdot Var(R_{pf}) \end{aligned}$$

Complete Portfolio Combinations (Algebraically)

$$E(R_C) = R_f + w_{pf} \cdot [E(R_{pf}) - R_f] \quad (1)$$

$$Var(R_C) = w_{pf}^2 \cdot Var(R_{pf}) \quad (2)$$

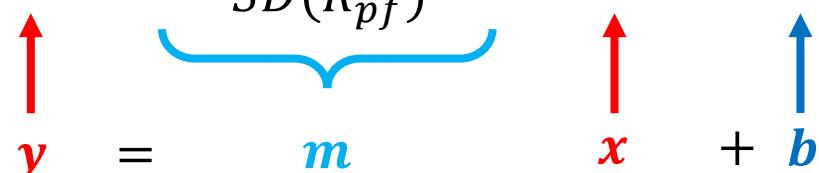
$$SD(R_C) = \sqrt{Var(R_C)} = \sqrt{w_{pf}^2 \cdot Var(R_{pf})} = \sqrt{w_{pf}^2} \cdot \sqrt{Var(R_{pf})} = w_{pf} \cdot SD(R_{pf}) \quad (3)$$

From equation (3) we can see that $SD(R_C) = w_{pf} \cdot SD(R_{pf}) \Leftrightarrow w_{pf} = \frac{SD(R_C)}{SD(R_{pf})}$ (4)

Substituting equation (4) into (1) we obtain $E(R_C) = R_f + \frac{SD(R_C)}{SD(R_{pf})} \cdot [E(R_{pf}) - R_f]$ (5)

Rearrange equation (5) we get

$$E(R_C) = \frac{[E(R_{pf}) - R_f]}{SD(R_{pf})} \cdot SD(R_C) + R_f$$



$$y = m \cdot x + b$$

It is a line!

The Sharpe Ratio

$$E(R_C) = \frac{[E(R_{pf}) - R_f]}{SD(R_{pf})} \cdot SD(R_C) + R_f$$

$y = m x + b$

↑ ↑ ↑
 y m x $+$ b

***m* is the Sharpe Ratio**

Increase in expected return (reward)

$$S_{pf} = \frac{[E(R_{pf}) - R_f]}{SD(R_{pf})} = \text{Increase in expected return (reward) per unit of additional SD (variability)}$$

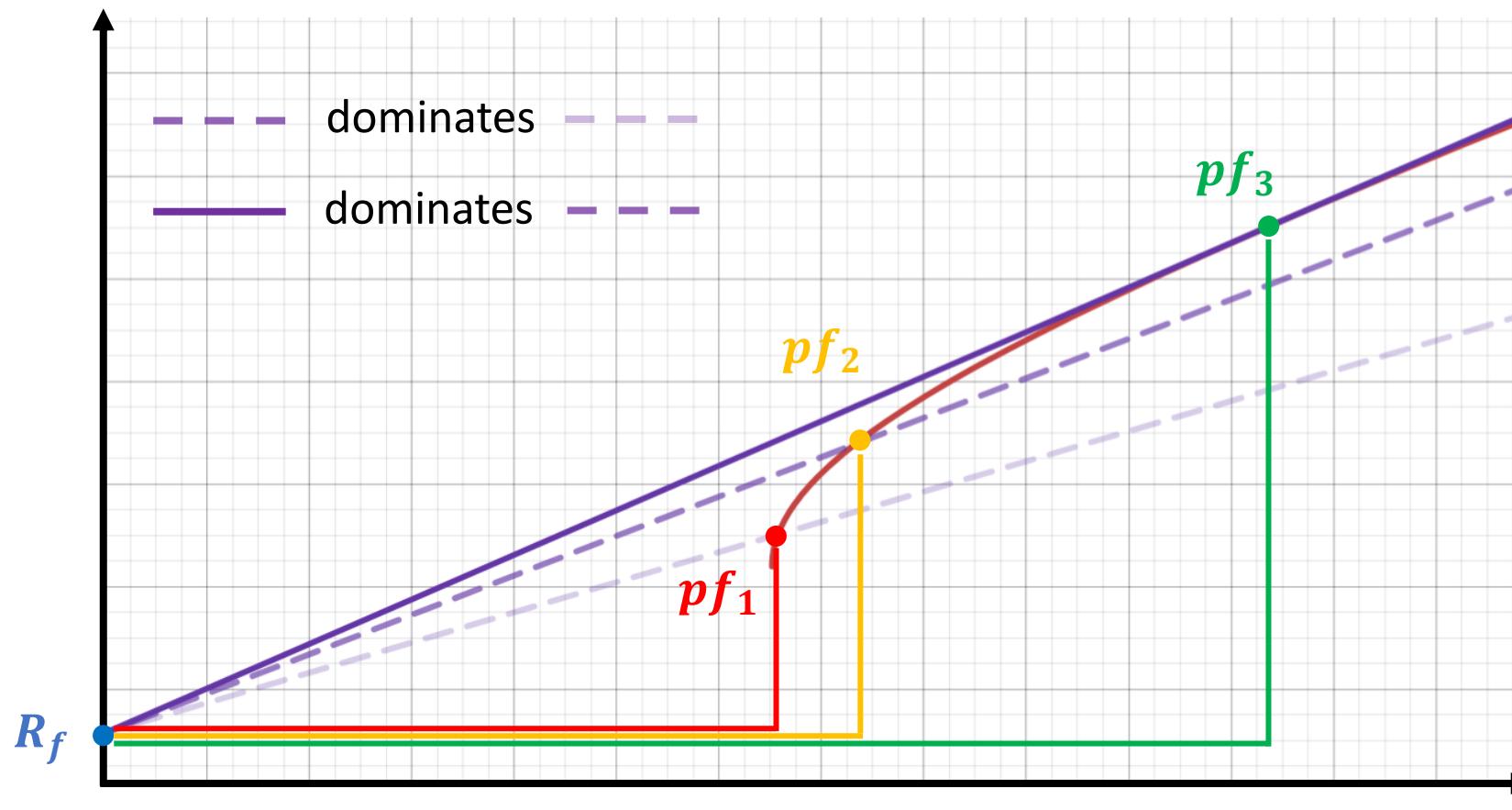
Increase in standard deviation (variability)



Complete Portfolio Combinations (Graphically)

The goal is to maximize the Sharpe ratio while holding a risk-free asset and a portfolio on the efficient frontier. In other words, we are maximizing our possible reward to variability ratio (i.e., the slope)!

$E(R_C)$ (y-variable)



$$E(R_C) = \frac{[E(R_{pf}) - R_f]}{SD(R_{pf})} \cdot SD(R_C) + R_f$$

$y = m x + b$

$$S_{pf} = \frac{[E(R_{pf}) - R_f]}{SD(R_{pf})}$$

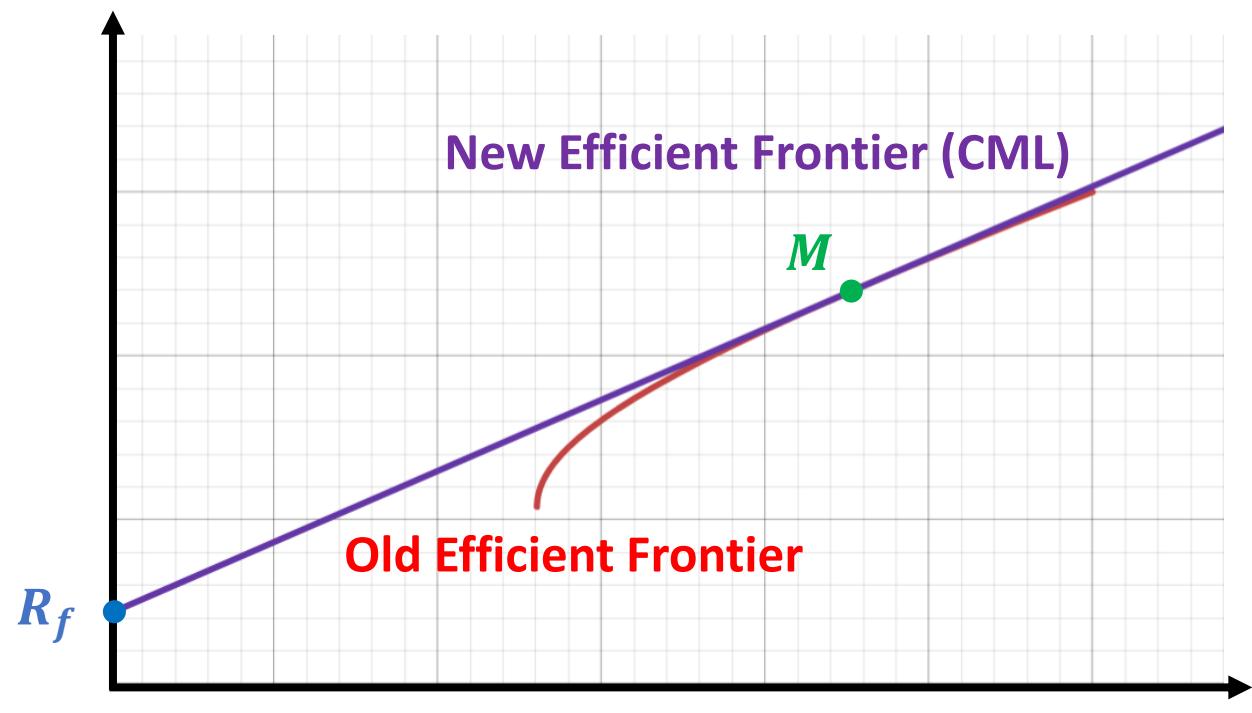
$$S_{pf_1} < S_{pf_2} < S_{pf_3}$$

The Capital Market Line (CML) & Market Portfolio

The capital market line maximizes the Sharpe ratio and is tangent to a portfolio on the old efficient frontier

Instead of calling the tangent portfolio “pf” (for portfolio) I will use “M” (for market portfolio)

$E(R_C)$ (y-variable)



$$E(R_C) = \frac{[E(R_M) - R_f]}{SD(R_M)} \cdot SD(R_C) + R_f$$

$y = m \cdot x + b$

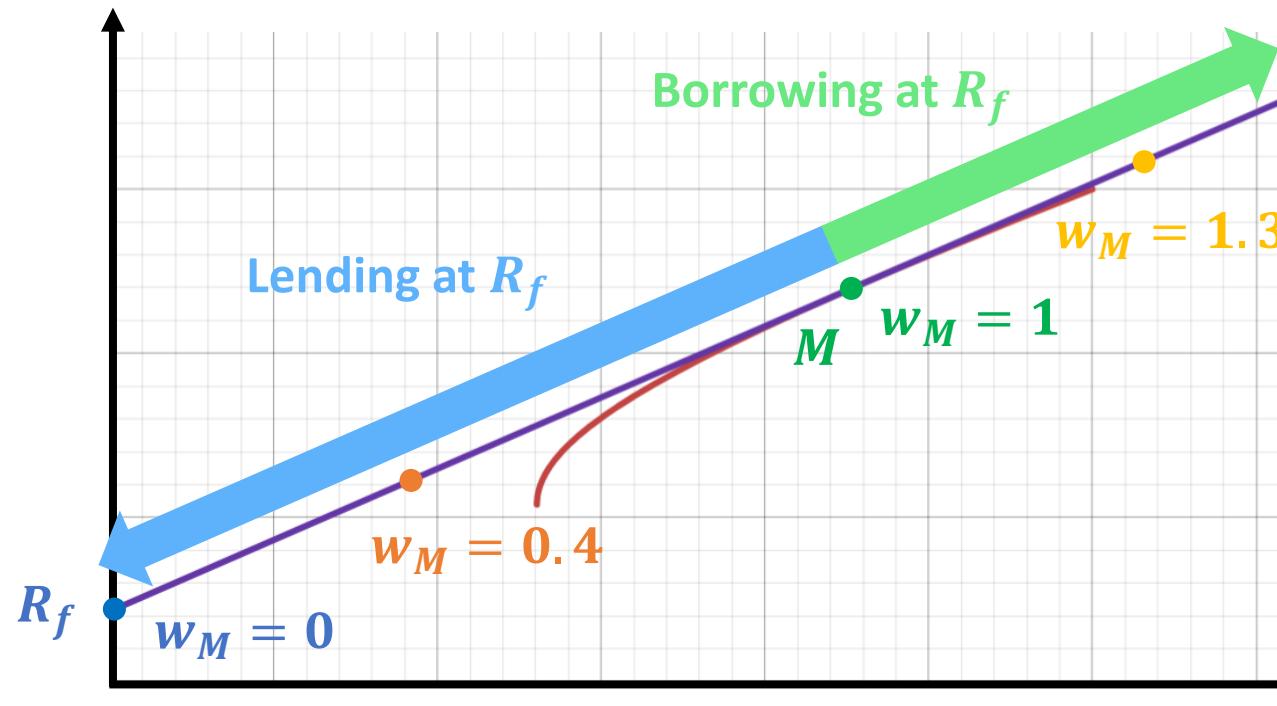
— dominates (except M)

Legend:
— dominates
(except M)

Lending & Borrowing at R_f – How Risk Adverse are you?

All rational investors (the entire *market*) will hold some proportion of the *market* portfolio and risk-free asset

$E(R_C)$



- Lend 100% of wealth in risk-free asset (e.g., GOC T-bills with 0 coupons)
- Lend 60% of wealth in risk-free asset and invest 40% of wealth in M
- Invest 100% of wealth in M
- Borrow amount equal to 30% of wealth and invest amount equal to 130% of wealth in M (here $w_f = -0.30$)

Moving further out on the CML does NOT make one risk preferring! The investor is just less risk adverse

Borrowing at R_f – Short Selling

When an investor borrows at R_f ($w_f < 0$), they are shorting selling (i.e., selling an asset they do not own)

How does short selling work?

- (1) Borrow the risk-free asset and sell it on the market
- (2) Buy the risk-free asset on the market and return it to the borrower

Why short sell?

Selling price > buying price \Rightarrow Profit

How can the buying price go below the selling price?

Stay tuned for our section on CAPM ...

Summary of Equations

R_C = Return on complete portfolio

R_f = Risk-free rate

p_f = Arbitrary portfolio on efficient frontier

M = Market portfolio

S = Sharpe ratio

w = Weight on asset/portfolio

Complete Portfolios not on the CML:

$$E(R_C) = R_f + w_{pf} \cdot [E(R_{pf}) - R_f] = \frac{[E(R_{pf}) - R_f]}{SD(R_{pf})} \cdot SD(R_C) + R_f = S_{pf} \cdot SD(R_C) + R_f$$

$$Var(R_C) = w_{pf}^2 \cdot Var(R_{pf})$$

$$SD(R_C) = w_{pf} \cdot SD(R_{pf})$$

Complete Portfolios on the CML:

$$E(R_C) = R_f + w_M \cdot [E(R_M) - R_f] = \frac{[E(R_M) - R_f]}{SD(R_M)} \cdot SD(R_C) + R_f = S_M \cdot SD(R_C) + R_f$$

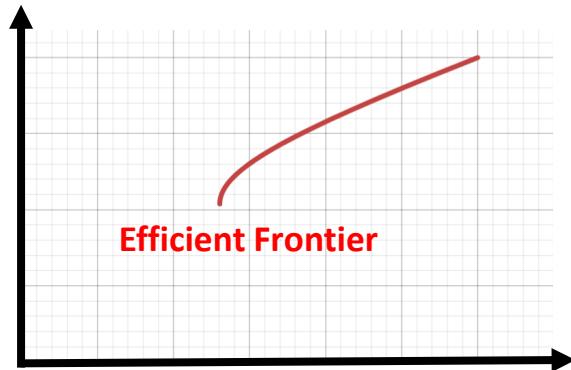
$$Var(R_C) = w_M^2 \cdot Var(R_M)$$

$$SD(R_C) = w_M \cdot SD(R_M)$$

Answering the Main Question

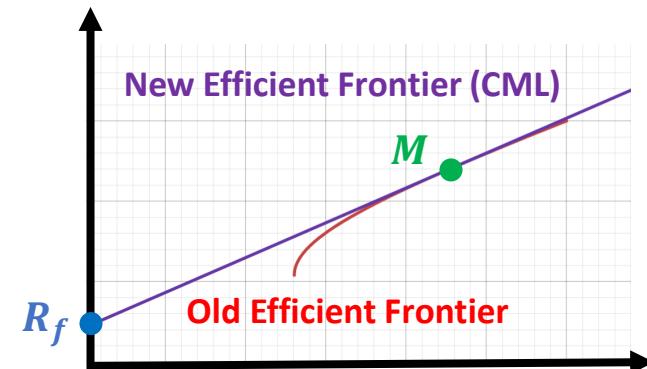
At the end of last section, we posed the question – what portfolios will all rational investors hold?

Case 1: Market has risky assets (no risk-free assets)



They will hold efficient portfolios on the efficient frontier

Case 2: Market has risky & risk-free assets (assume investors can lend/borrow at R_f)



They will hold some proportion of the market portfolio (M) and risk-free asset

Practice Question 2

If portfolio A has a higher expected return and standard deviation than portfolio B, a risk adverse investor will:

(Select all that apply)

- a) Choose to invest in portfolio A as it has a higher expected value
- b) Choose to invest in portfolio B because it has a lower standard deviation
- c) Choose to invest in portfolio B because investing in portfolio A makes them risk preferring
- d) Choose to invest in portfolio A because investing in portfolio B makes them risk preferring
- e) None of the above

Practice Question 3

Suppose portfolio A and B have the same Sharpe ratio. Which of the following is true?

(Select all that apply)

- a) Both portfolios must lie on the capital market line
- b) If portfolio A has a lower expected return than B, portfolio A will have a lower standard deviation than B
- c) At least one of the portfolios must lie on the old efficient frontier
- d) Both portfolios must be dominated by the market portfolio
- e) None of the above

Practice Question 4

Which of the following is true about the model of the portfolio frontier and capital market line?

(Select all that apply)

- a) In a 2-asset portfolio, $E(R_{portfolio})$ is the same for all $\rho_{AB} < 1$ holding all else constant
- b) A risk-free asset enables higher levels of expected return and standard deviation on an investor's portfolio
- c) Every portfolio on the old efficient frontier is dominated by a portfolio on the capital market line
- d) If an investor puts 100% of his wealth into the market portfolio, he is neither lending or borrowing at R_f
- e) There is no other portfolio that has a higher Sharpe ratio than the market portfolio

Practice Question 5

A complete portfolio on the CML has an expected return of 17%. If the market has an expected return of 15%:

(Select all that apply)

- a) The risk-free rate must be 2%
- b) The weight put on the risk-free asset in the complete portfolio must be negative
- c) The complete portfolio has greater total risk than the market portfolio
- d) An investor holding the complete portfolio is lending at the risk-free rate
- e) The complete portfolio has a higher expected return than any other portfolio on the old efficient frontier

Practice Question 6

State whether each portfolio is efficient or dominated. If it is dominated, state which portfolio(s) dominate it.

Portfolio	$E(R_{portfolio})$	$SD(R_{portfolio})$
A	1.36%	0.60%
B	2.72%	0.86%
C	3.40%	0.44%
D	5.04%	0.76%
E	6.39%	0.76%
F	10.20%	0.92%

Practice Question 7

You are given the following information about two complete portfolios on the CML

		Statistical Measure	
Portfolio		$E(R_C)$	$SD(R_C)$
	A	11.5%	10.0%
	B	22.3%	35.1%

- a) What is the Sharpe ratio of the market portfolio?

Practice Question 7

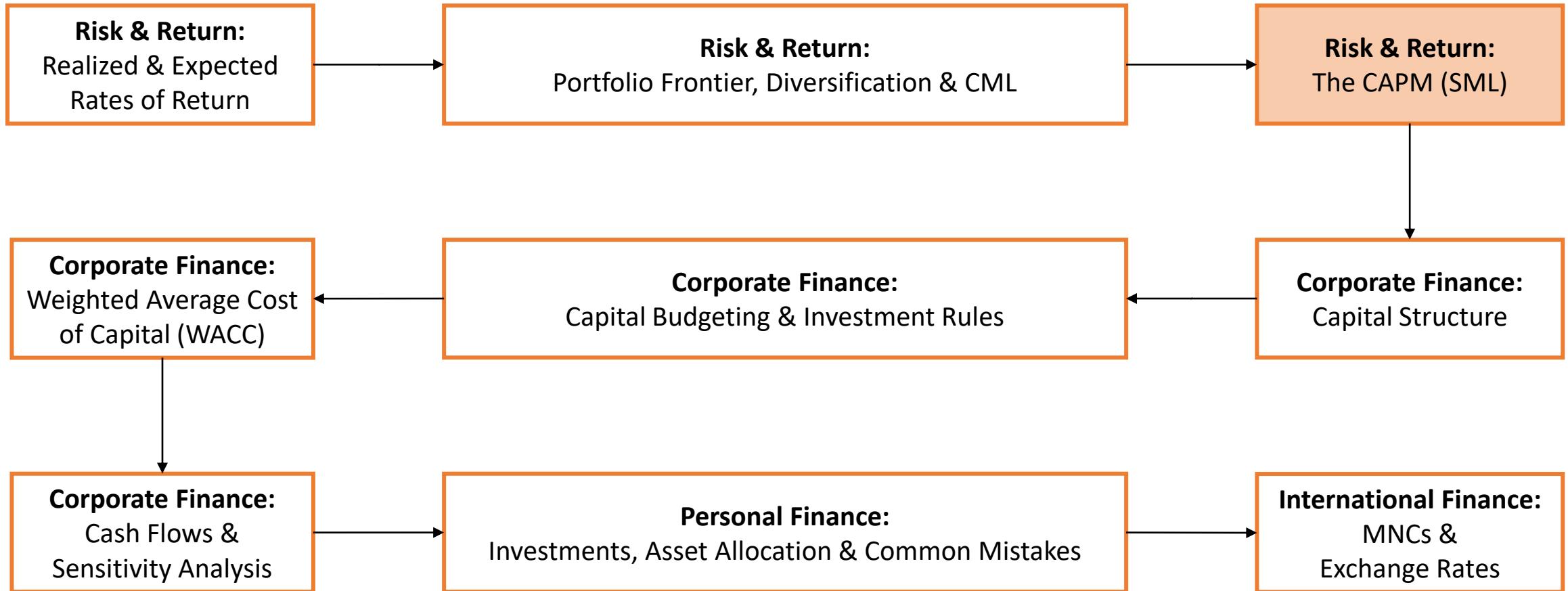
		Statistical Measure	
		$E(R_C)$	$SD(R_C)$
Portfolio	A	11.5%	10.0%
	B	22.3%	35.1%

b) What is the risk-free rate?

Practice Question 8

An investor constructed an efficient portfolio with an expected return of 10% and standard deviation of 20%. The investor was able to lend or borrow at the risk-free rate of 3%. If the expected return on the market is 8%, what is the standard deviation of the market portfolio?

TABLE OF CONTENTS



What Type of Risk Gets Rewards?

Investors should only be rewarded with a risk premium (higher expected returns) for risk they **MUST** take on

Systematic Risk (market/undiversifiable risk)



- There are macroeconomic events (e.g., severe world-wide recession) that affect all securities in the market (makes the return of almost every asset move in the same direction)
- Cannot be reduced through diversification (it's called *undiversifiable* risk – you can't avoid it!)

Unsystematic Risk (unique/independent/idiosyncratic/diversifiable risk)



- Risk that only affects an individual security (it's *unique* to that security)
- Comes from company specific news (e.g., factory burning down, CFO going to prison)
- Can be reduced through diversification

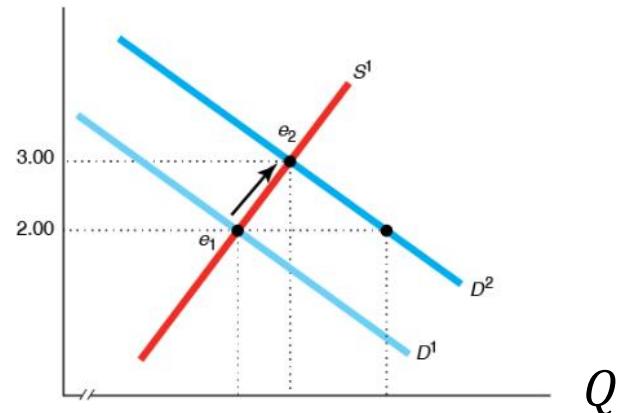
Why is Unsystematic Risk not Rewarded?

If we assume that the market for securities is competitive, any gains on unsystematic risk quickly disappear

Suppose an individual stock gave rewards for unsystematic risk



Current Share
Price (P_0)



Rewards for
Unsystematic Risk



Demand Increases



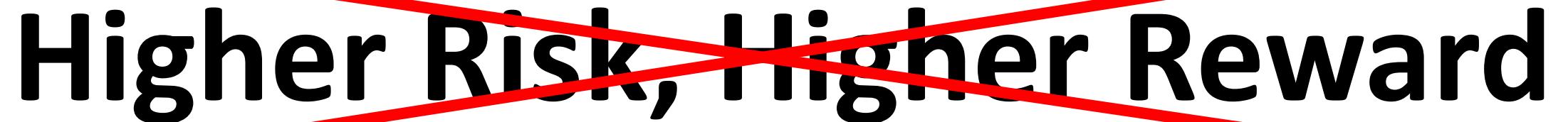
Expected Return Decreases

Assume $t = 1$ has not occurred yet,
so the returns are not realized

$$E(R) \downarrow = \frac{D_1 + (P_1 - P_0 \uparrow)}{P_0 \uparrow}$$



Conclusion: Unsystematic risk yields no additional expected returns (risk premium is 0) for investors



Higher Risk, Higher Reward

Higher Systematic Risk, Higher Reward

Historical Risks & Returns on Portfolios

How do the returns on large portfolios behave as their total risk increases?

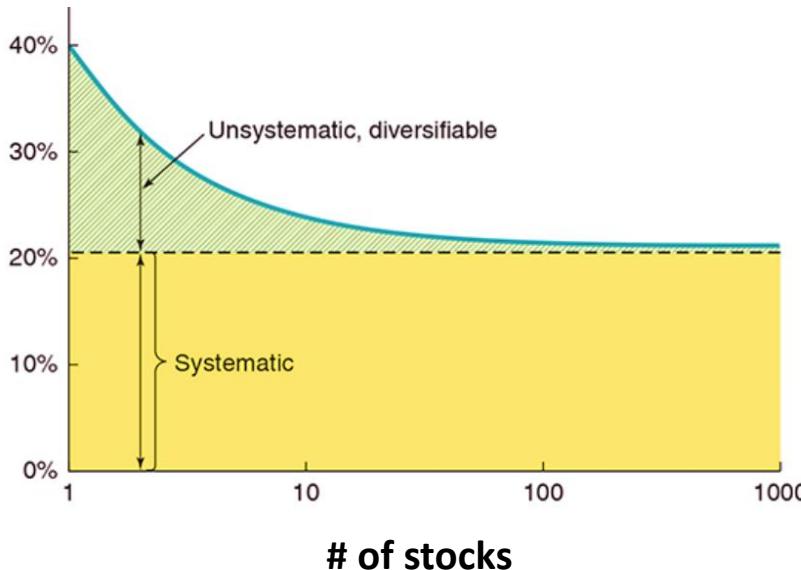
Large portfolios (e.g., S&P 500 Index – 500+ stocks)



As total risk (variance) increases, expected returns also increase

How is this consistent with the fact that only systematic risk is rewarded?

A large portfolio is *well-diversified* – there are negligible amounts of unsystematic risk



$$\text{Total risk} = \text{Systematic risk} + \text{Unsystematic risk}$$

$$\text{Total risk} \approx \text{Systematic risk} \quad (\text{unsystematic risk is negligible})$$

$$\text{Total risk} \uparrow \approx \text{Systematic risk} \uparrow$$

Expected return increases

Historical Risks & Returns on Individual Stocks

There is not much of a relationship between the historical return and total risk of individual stocks

Average Annual
Return

24%
20%
16%
12%
8%
4%
0%

Most ■ are here

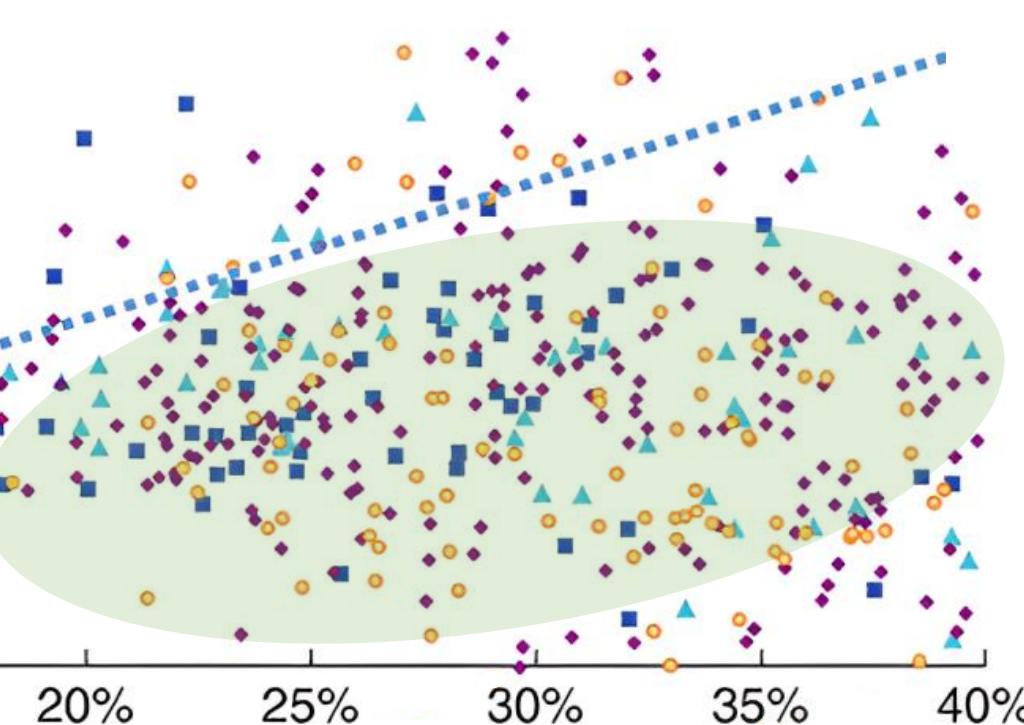
Portfolios

S&P 500 Index (in CAD)

Three-Month Canadian
Treasury Bills

S&P/TSX
Composite Index

Long-Term Government
of Canada Bonds



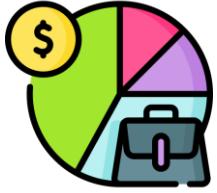
Lower returns
and/or higher risk
than portfolios

Volatility of
Annual Return

Market Capitalization: Low (smaller stocks) ■ High (larger stocks) ■

Key Takeaways – Motivating the CAPM (SML)

Total risk \approx Systematic risk (Unsystematic risk is negligible)



Large Portfolios
(well-diversified)



Total risk $\uparrow \Rightarrow$ Systematic risk $\uparrow \Rightarrow$ Expected return (Reward) \uparrow

Total risk = Systematic risk + Unsystematic risk



Individual Securities
(e.g., stocks)



Total risk $\uparrow \times$ Systematic risk $\uparrow \Rightarrow$ Expected return (Reward) \uparrow



Conclusion: Total risk is a good predictor of expected returns for large portfolios but not individual securities

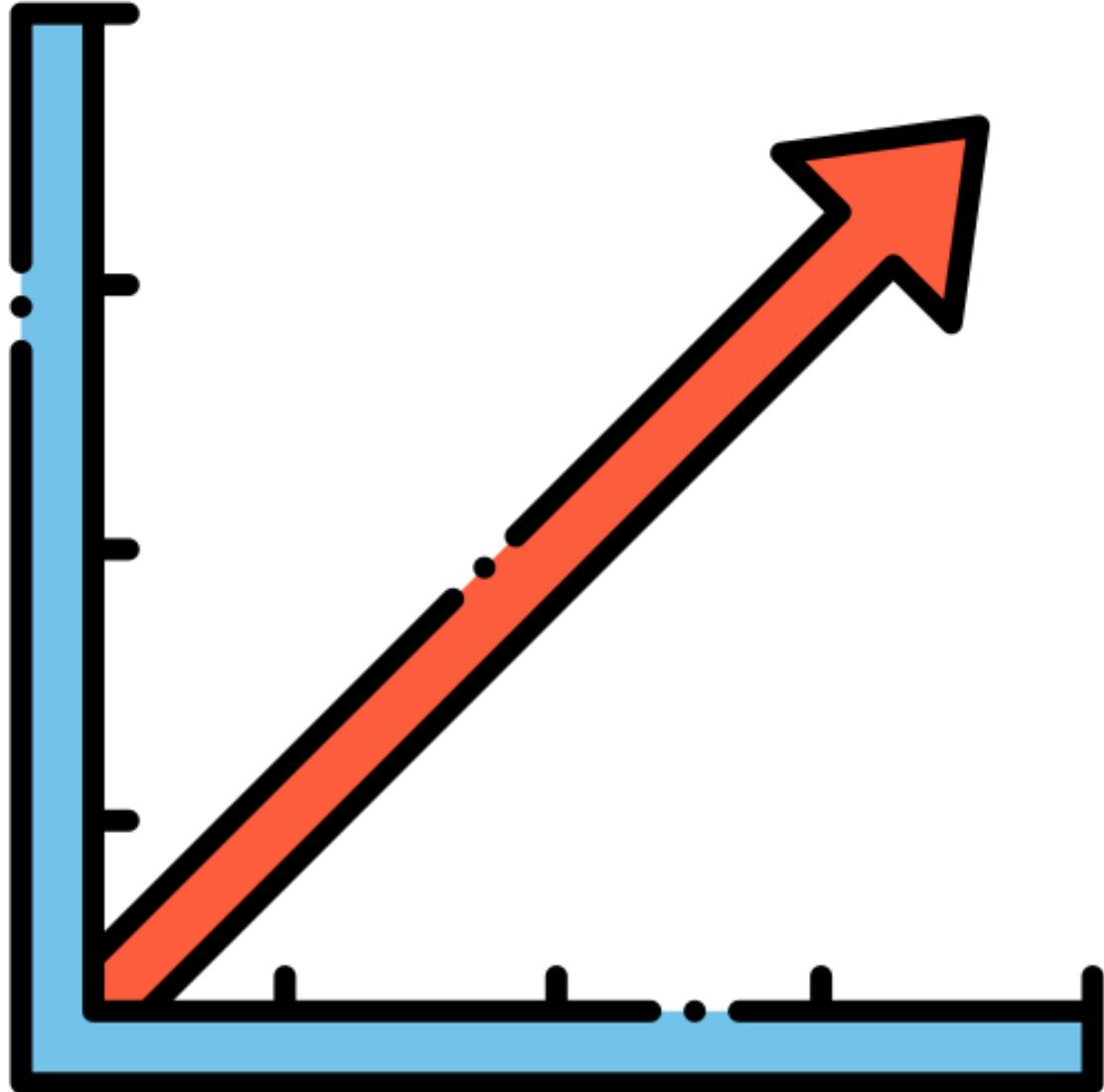
**How do we compute
expected returns on an
individual security?**

Practice Question 1

Which of the following statements about risk & return is true?

(Select all that apply)

- a) Most individual stocks have lower returns and higher total risk than large portfolios
- b) A company factory burning down is an example of systematic risk
- c) Investors are rewarded for the risk they bear regardless if they can diversify it away or not
- d) Larger stocks tend to be less volatile than smaller stocks
- e) Total risk is an appropriate measure of systematic risk for individual securities and diversified portfolios



The Capital Asset Pricing Model **(CAPM)**

Extracting Systematic Risk

The additional expected return on an individual asset is based on the amount of systematic risk it has

How can we only capture systematic risk?

The market portfolio (M) theoretically contains *every* risky asset in the world – it is fully diversified!

Fully diversified portfolio \Rightarrow Unsystematic risk = 0 \Rightarrow **Market portfolio only contains systematic risk**

How can we connect the market portfolio to individual assets?

- (1) All risky assets worldwide are in the market portfolio
- (2) By the CML, all investors will hold the market portfolio and risk-free asset in some proportion

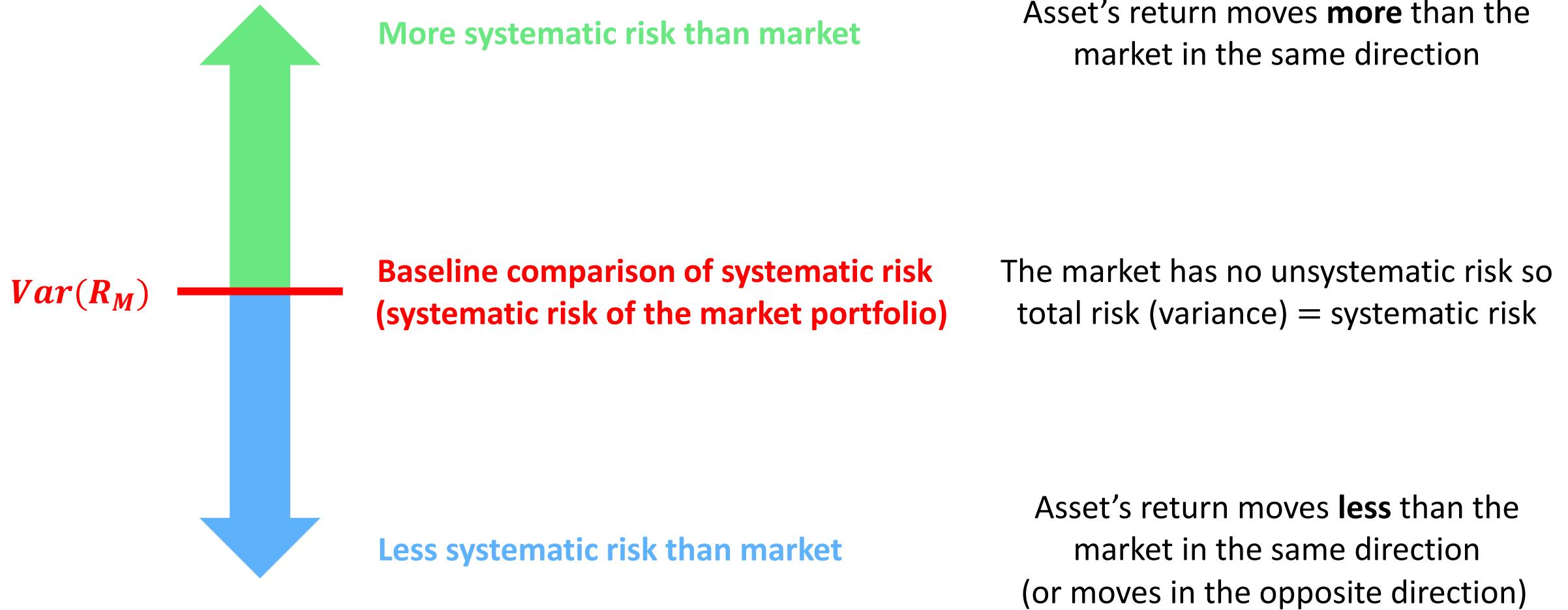
Combining (1) and (2), this means that any risky asset held by investors is in the market portfolio



Conclusion: The systematic risk of all individual assets averages out to the systematic risk of the market

Building a Quantitative Measure of Systematic Risk

An individual asset's systematic risk is computed relative to the systematic risk of the market (securities in M)



Beta – A Measure for Systematic Risk

Let's take our two biggest findings from the previous slide

- (1) The baseline comparison of systematic risk is the systematic risk of the market portfolio – $\text{Var}(R_M)$
- (2) The amount of systematic risk asset i has depends on how its return moves with the market – $\text{Cov}(R_i, R_M)$

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\text{Var}(R_M)}$$

 **Systematic risk
of asset i**

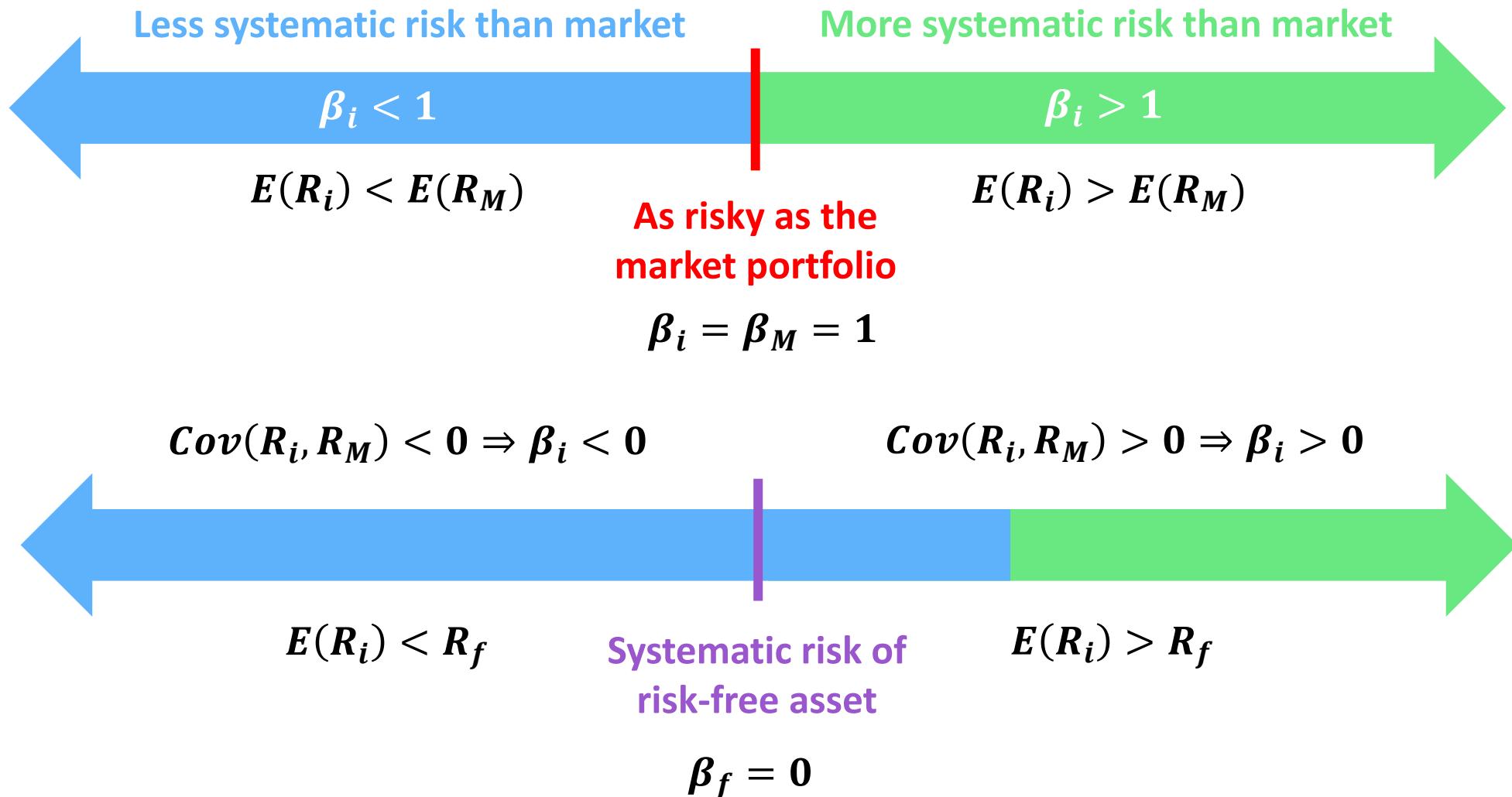
→ **Covariance of returns between asset i and the market portfolio**
→ **Variance of returns of the market portfolio**

Remark: If $\beta_i = x$, asset i has x times the systematic risk of the market

(e.g., $\beta_i = 0.5$ means that asset i has half as much systematic risk as the market)

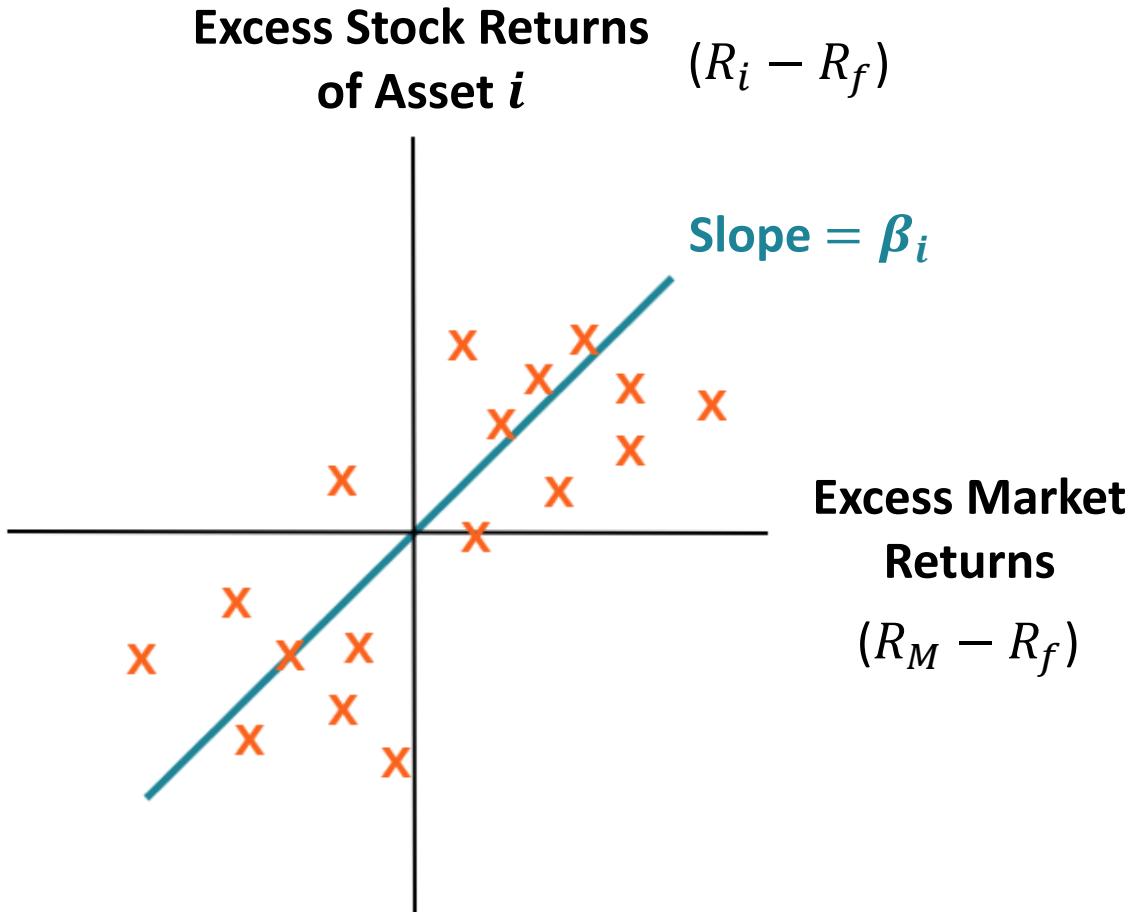
Interpretations of Beta

What can we say about a single asset i given its beta? Remember – higher systematic risk = higher returns!



How to Estimate Beta

Beta comes from the slope of the regression line of excess stock returns on excess market portfolio returns



Where does the data come from?

5 years of monthly returns are usually used

2 years of weekly returns can also be used

The more data points the more valid it is!

Additional Comments:

The market portfolio is a theoretical concept, so we use something that closely mimics it (a proxy) instead like the S&P 500 Index

We write R_i and R_M instead of $E(R_i)$ and $E(R_M)$ as historical data is used – no guessing here!

More Interpretations of Beta

The regression slope (β_i) tells us how much asset i 's return fluctuates relative to market return fluctuations

Assets With $\beta_i > 1$:

Particularly sensitive to market fluctuations (if $R_M \uparrow$ then $R_i \uparrow\uparrow$ and if $R_M \downarrow$ then $R_i \downarrow\downarrow$)

Assets that do really well in good times and very bad in bad times (e.g., luxury goods)

Have high expected returns and low prices (since $E(R_i)$ is high) to compensate for high systematic risk

Assets With $\beta_i < 1$:

Not so sensitive to market fluctuations (e.g., stocks from companies that produce necessities)

$\beta_i < 0$ assets (very rare) do well when the economy is bad and do bad when the economy is good (e.g., gold)

The CAPM & Security Market Line (Algebraically)

Systematic risk gives us the *additional* rewards for a single asset, but how do we compute the total rewards?

Two Components:

(1) The base “reward for waiting” : R_f

(2) The additional “reward for worrying” (risk premium of asset i) : $[E(R_M) - R_f] \cdot \beta_i$

$$E(R_i) = [E(R_M) - R_f] \cdot \beta_i + R_f$$

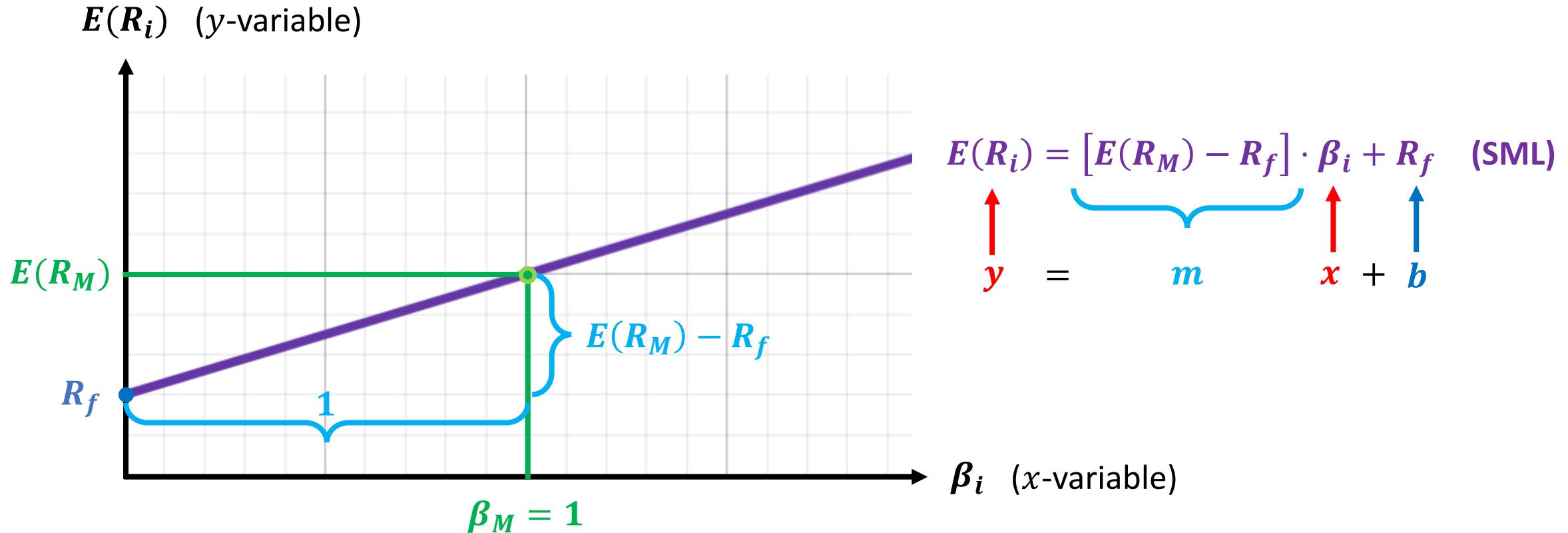
The equation $E(R_i) = [E(R_M) - R_f] \cdot \beta_i + R_f$ is shown with annotations. A red vertical arrow points up from the variable y to the term $E(R_M) - R_f$. A blue brace underneath the term $E(R_M) - R_f$ is labeled with the letter m . To the right of the brace, a red vertical arrow points up from the variable x to the term β_i . To the right of the term β_i , there is a plus sign followed by a blue vertical arrow pointing up from the variable b to the term R_f .

$E(R_i)$ = Expected return of asset i $E(R_M)$ = Expected return on the market

R_f = Risk-free rate β_i = Systematic risk of asset i

The CAPM & Security Market Line (Graphically)

The security market line (SML) is a graphical representation of the CAPM



Note: The SML extends to the left of the y -axis as beta can be negative (it isn't shown in the diagram)

Class Example Revisited

We saw that individual stocks often had higher total risk and lower returns than portfolios. Why?

The following information was given or computed about Teck (asset i) and the market portfolio (M)

$$E(R_M) = 0.10 \quad SD(R_M) = 0.15 \quad \beta_M = 1$$

$$E(R_i) = 0.096 \quad SD(R_i) = 0.50 \quad \beta_i = 0.9$$

Why was Teck's return lower than the market?

It has less systematic risk than the market – beta is lower!



An asset with high volatility (and thus high variance/total risk) but low beta is not risky because any risk unrelated to beta is unsystematic and can be diversified away

Beta of a Portfolio

We can also compute the expected return of portfolios using the CAPM (SML)

$$\beta_{portfolio} = w_1\beta_1 + w_2\beta_2 + \cdots + w_n\beta_n \quad (\text{for a portfolio with } n \text{ individual assets})$$

Using the above beta, this means that $E(R_{portfolio}) = [E(R_M) - R_f] \cdot \beta_{portfolio} + R_f$

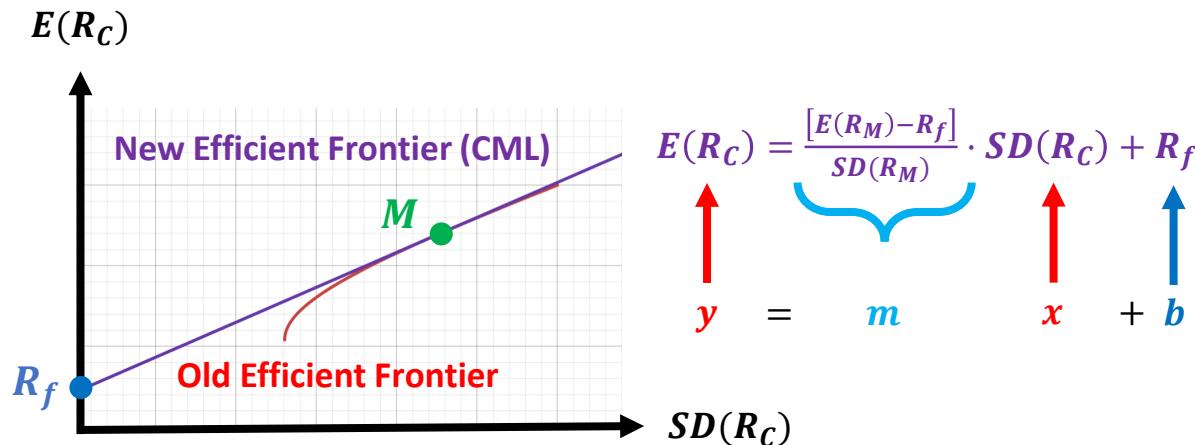
Why is the average β of all individual assets equal to 1?

By the formula, the systematic risk of the market portfolio is the average systematic risk of all assets in it

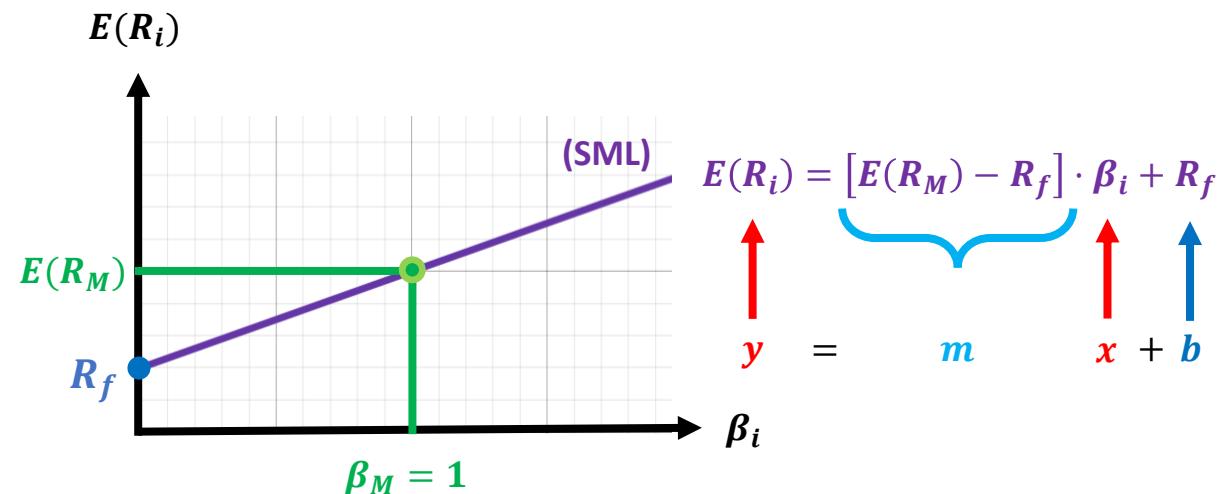
The market portfolio is comprised of all individual assets worldwide and has $\beta_M = 1$

Therefore, the average systematic risk (β) of all individual assets is 1

CML vs SML



- Shows what portfolios investors will hold (efficient portfolios – ones on the CML)
- Helps determine the location of the market portfolio (M)
- Does not help price individual assets



- Shows the relationship between an asset's systematic risk & expected return
- Does not help determine the location of the market portfolio (M)
- Helps us price individual assets via computation of expected returns

Practice Question 2

Which of the following is a correct interpretation of beta?

(Select all that apply)

- a) An asset with $\beta_i < 0$ will fluctuate opposite to the market on average
- b) Beta is the intercept of the regression line of excess stock returns on excess market returns
- c) Stocks from companies that produce necessities (e.g., toilet paper) tend to have lower values of beta
- d) An asset with $0 < \beta_i < 1$ exhibits higher expected returns than risk-free assets but not the market
- e) Assets with $\beta_i < 0$ will demand a negative risk premium

Practice Question 3

Which of the following is true when comparing the CML to the SML?

(Select all that apply)

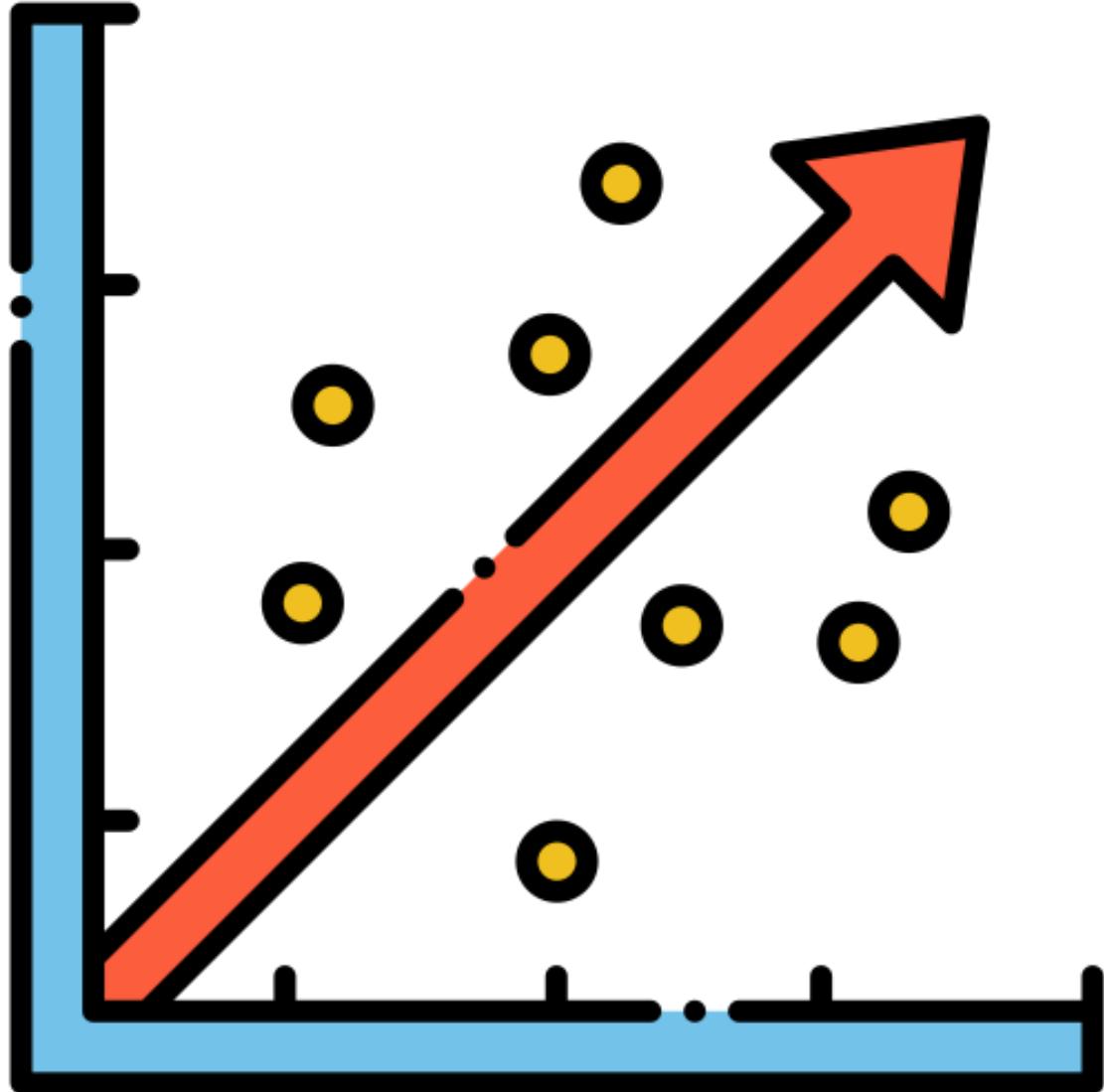
- a) Both have the same y -intercept
- b) The SML plots return against systematic risk while the CML plots return against unsystematic risk
- c) Both have the same slope
- d) The minimum y -value of both is the risk-free rate
- e) Only the SML is useful in determining the required return on individual securities

Practice Question 4

A stock has a standard deviation of 23% and a beta of 1.4. If the market standard deviation is 15%, what is the correlation coefficient between the stock and the market?

Practice Question 5

You own a portfolio that is equally split between two risky stocks (A & B) and a risk-free asset. The portfolio has an expected return of 27% and comes with the following information: $R_f = 5\%$, $E(R_M) = 20\%$. If the beta of stock A is 1.6, what is the beta of stock B?



CAPM & Mispriced Securities

Why Use the CAPM?

Our first attempt at computing expected returns for an asset used historical data to compute an average

$$E(R) = \bar{R} = \frac{R_1 + R_2 + \dots + R_n}{n} = \sum_{t=1}^n \frac{R_t}{n}$$

Why isn't this method preferred over the CAPM?

It is difficult to infer the average return of individual stocks from historical data (high volatility & near 0 returns)

Amount of data available for use changes for corporations

Why is the CAPM preferred?

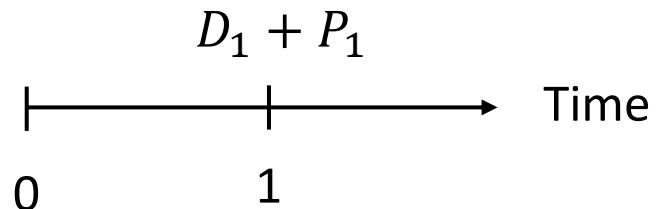
Regressions to estimate beta are quite accurate, so the CAPM estimates expected returns more accurately

Why is it Called the CAPM?

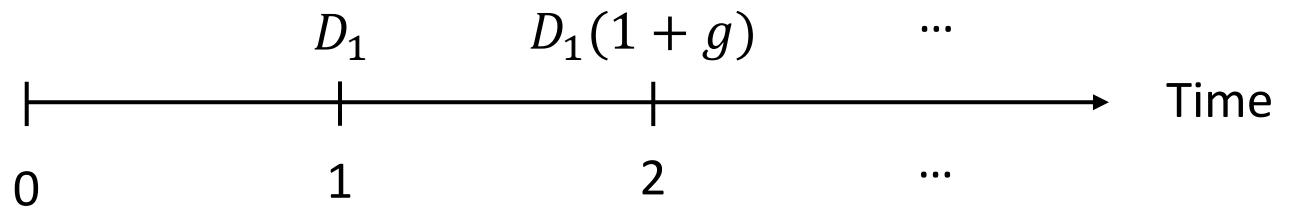
CAPM has the word “pricing” – what do the expected returns $E(R_i)$ from the SML have to do with pricing?

Recall the following formulas for the valuation for stocks

One-period Investor: $P_0 = \frac{D_1 + P_1}{1 + r}$



Constant dividend growth: $P_0 = \frac{D_1}{r - g}$



r was known as the *expected rate of return* – we now know to write this as $E(R_i)$

We were handed $E(R_i)$ before the midterm but can compute it now because of CAPM

Thus, we can discount future cash flows using the $E(R_i)$ from CAPM to determine the price of assets

Alpha of an Asset

CAPM allows us to compute a theoretical expected return and price for individual assets

What happens in reality?

An asset's expected return is determined by alternative returns on similar-risk securities. If this return doesn't match the CAPM expected return, the model concludes that the asset's market is not at equilibrium.

How do we capture this difference?

$$\alpha_i = E(R_i)^M - E(R_i)^{CAPM}$$

↑
**What investors
expect
(reality)**

↑
**What the
model says
(theoretical)**

α_i = Alpha of asset i (forward-looking)

$E(R_i)^M$ = Market expected return

$E(R_i)^{CAPM} = [E(R_M) - R_f] \cdot \beta_i + R_f$ = CAPM expected return

←
What we previously called $E(R_i)$

Alpha and Pricing Under CAPM

Under the CAPM framework, we can label an individual asset as properly priced, underpriced, or overpriced

Asset is Properly Priced ($\alpha_i = 0$):

Asset lies on the SML & the market for the asset is in equilibrium

Holds true for all assets if the market portfolio is efficient

Asset is Underpriced ($\alpha_i > 0$):

Asset lies above the SML & exhibits expected returns that are higher than what CAPM predicts

Market will tend to equilibrium in a competitive market where investors buy the asset

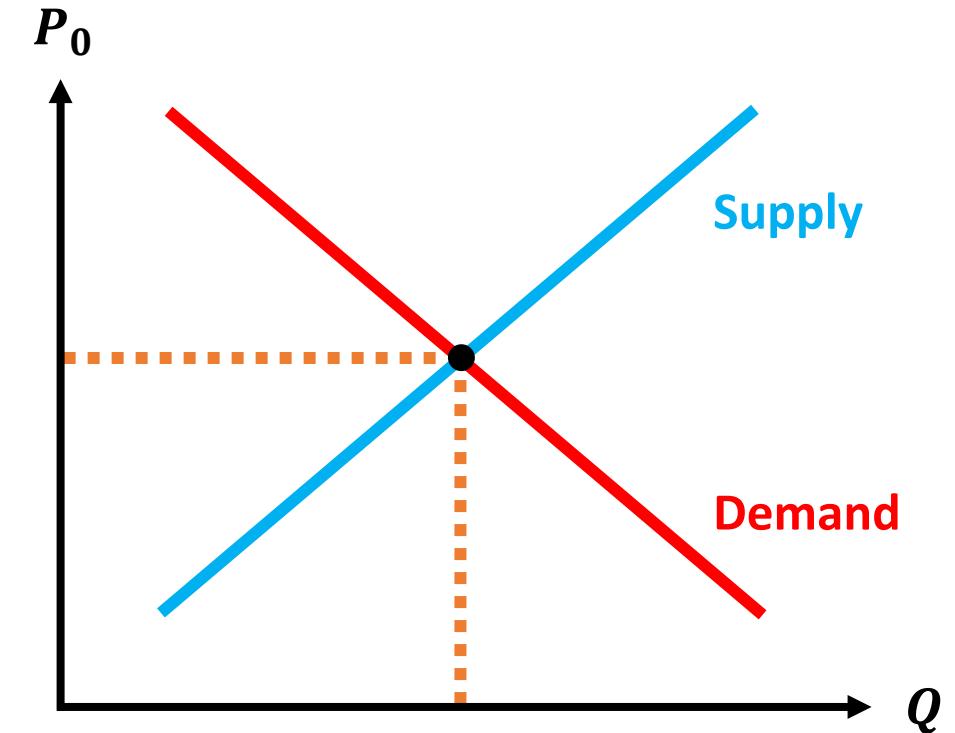
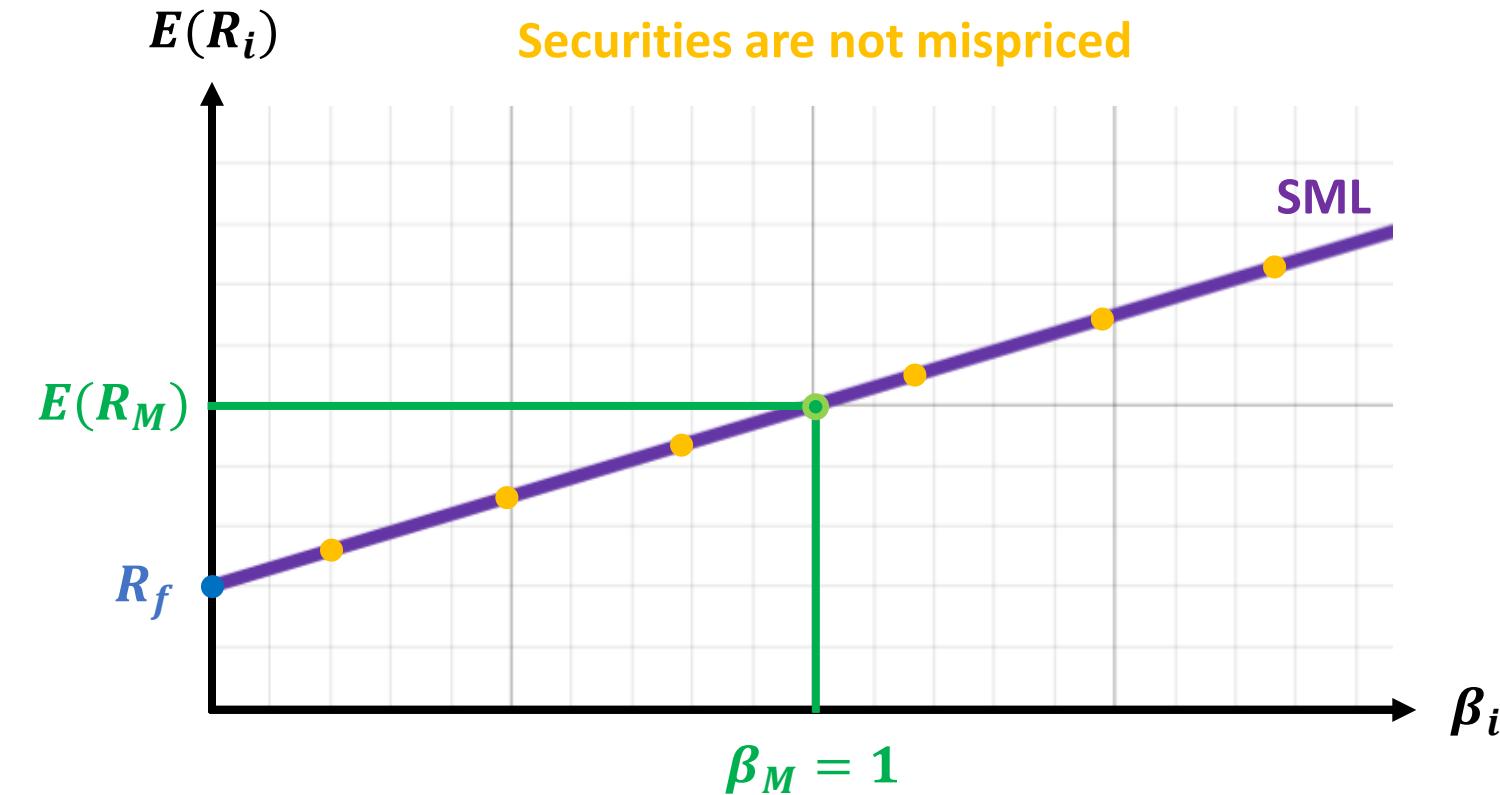
Asset is Overpriced ($\alpha_i < 0$):

Asset lies below the SML & exhibits expected returns that are lower than what CAPM predicts

Market will tend to equilibrium in a competitive market where investors sell/short-sell the asset

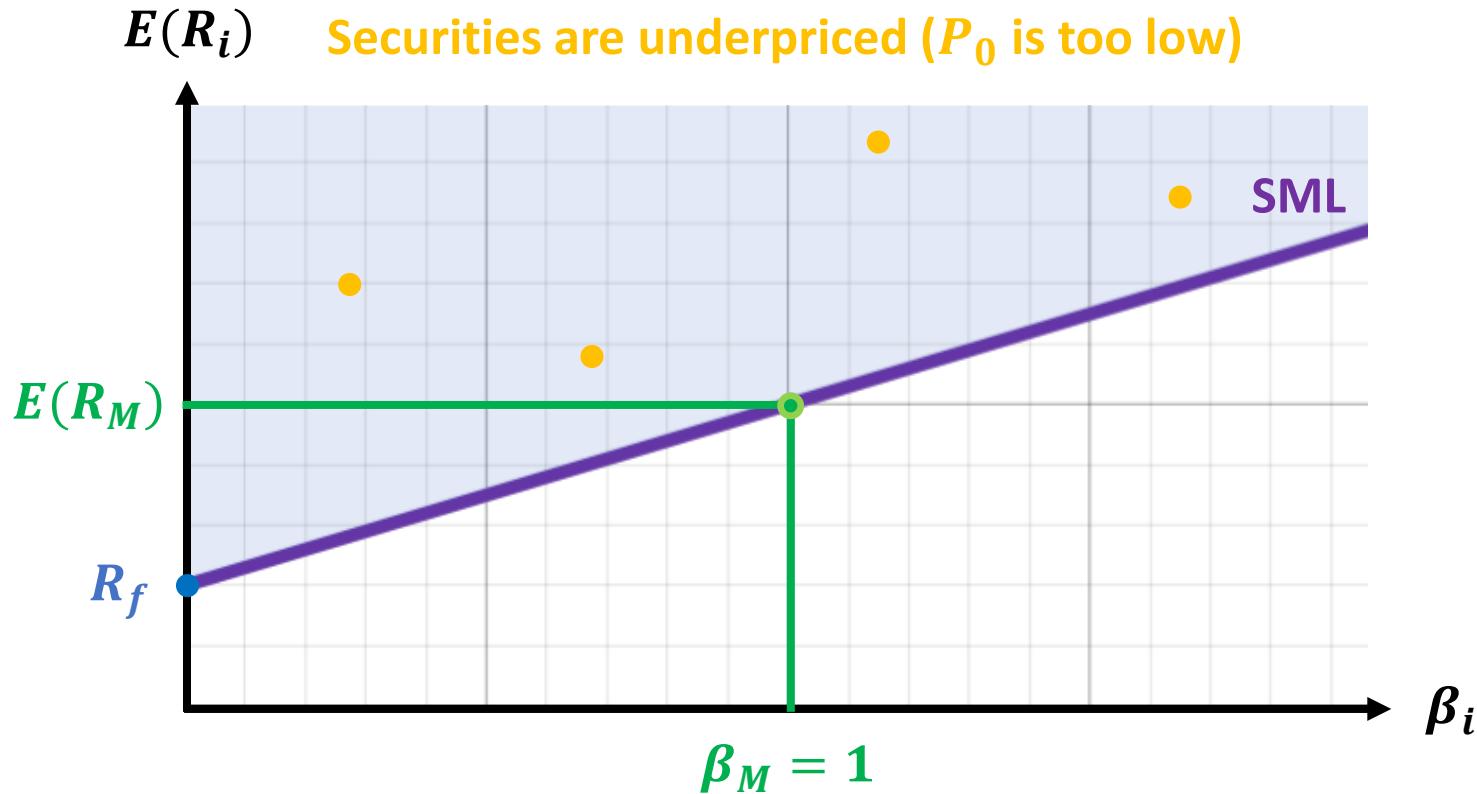
Zero Alpha

An asset with $\alpha_i = 0$ lies on the SML and has expected market returns such that $E(R_i)^M = E(R_i)^{CAPM}$



Positive Alpha

An asset with $\alpha_i > 0$ lies above the SML and has expected market returns such that $E(R_i)^M > E(R_i)^{CAPM}$



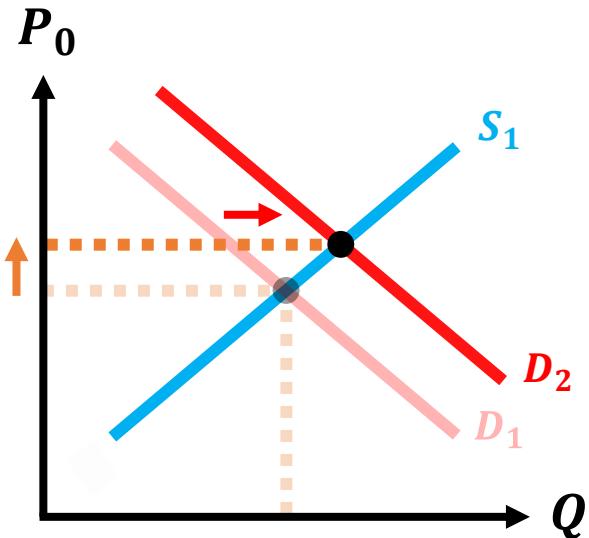
Let's focus on the case of stocks

$$E(R_i)^M \uparrow = \frac{D_1 + (P_1 - P_0)}{P_0}$$

Return to Equilibrium (Positive Alpha)

The market will adjust to equilibrium in a competitive market (eliminates mispricing so market can be efficient)

$$E(R_i)^M \uparrow = \frac{D_1 + (P_1 - P_0) \downarrow}{P_0 \downarrow}$$



$$E(R_i)^M \downarrow = \frac{D_1 + (P_1 - P_0) \uparrow}{P_0 \uparrow}$$

Stock is underpriced/expected returns are too high relative to its β



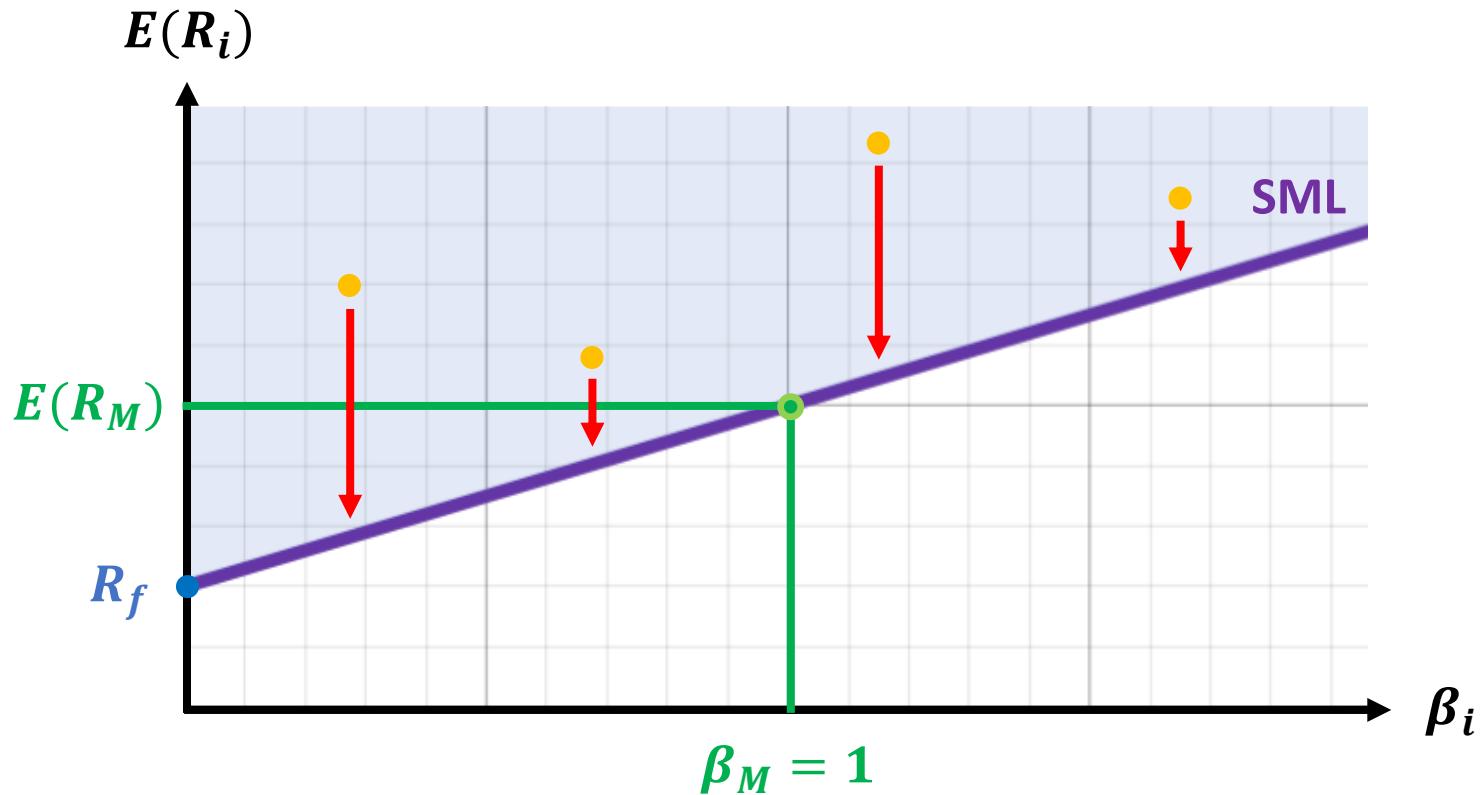
Investors buy stock and increase demand so price rises



Stock is properly priced/expected return decreases

Return to Equilibrium (Continued)

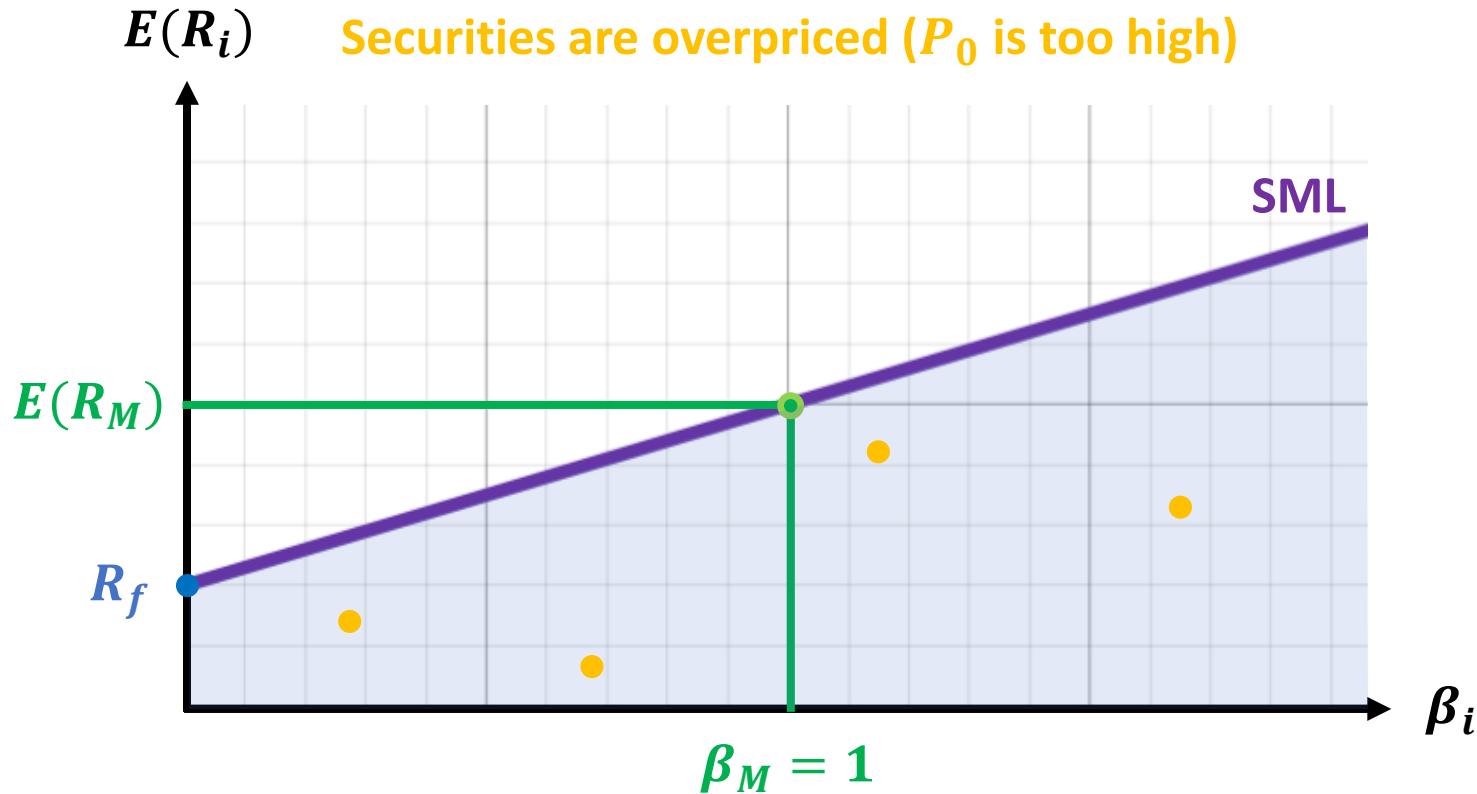
An underpriced asset will increase in price and fall down onto the SML as the market reaches equilibrium



$$E(R_i)^M \downarrow = \frac{D_1 + (P_1 - P_0 \uparrow)}{P_0 \uparrow}$$

Negative Alpha

An asset with $\alpha_i < 0$ lies below the SML and has expected market returns such that $E(R_i)^M < E(R_i)^{CAPM}$



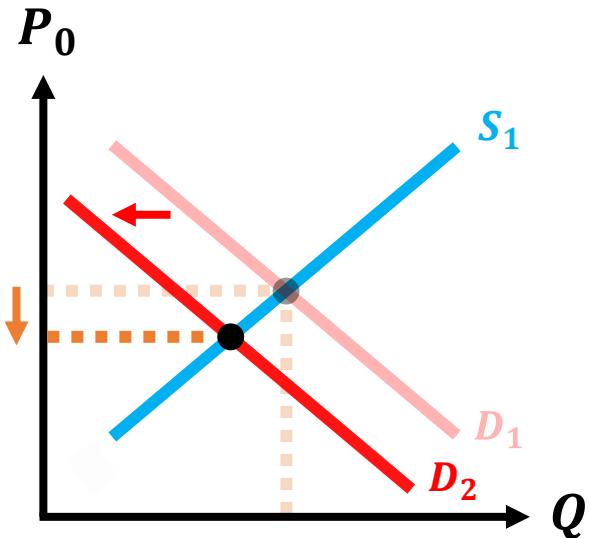
Let's focus on the case of stocks

$$E(R_i)^M \downarrow = \frac{D_1 + (P_1 - P_0 \uparrow)}{P_0 \uparrow}$$

Return to Equilibrium (Negative Alpha)

The market will adjust to equilibrium in a competitive market (eliminates mispricing so market can be efficient)

$$E(R_i)^M \downarrow = \frac{D_1 + (P_1 - P_0) \uparrow}{P_0 \uparrow}$$



Stock is overpriced/expected returns are too low relative to its β

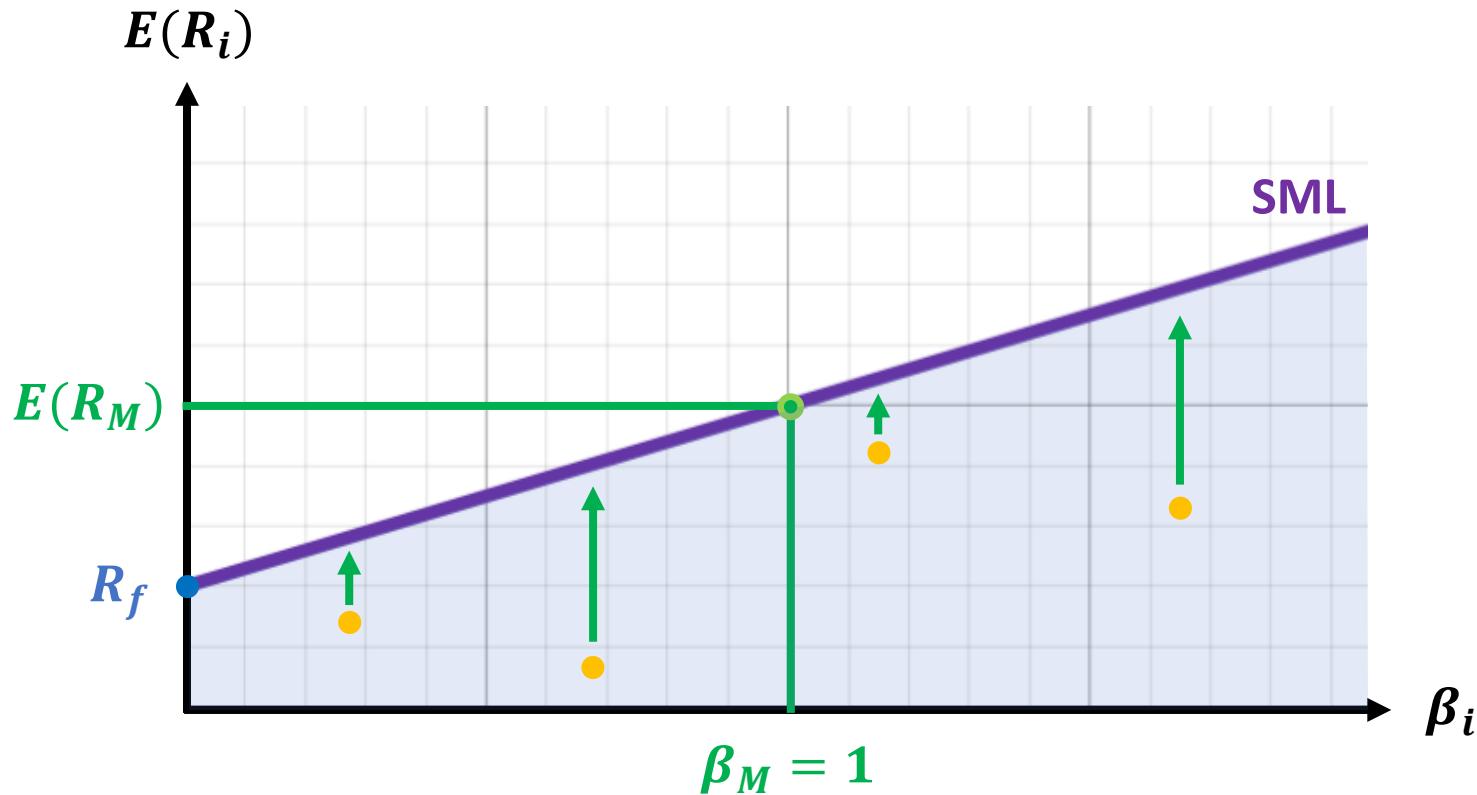
$$E(R_i)^M \uparrow = \frac{D_1 + (P_1 - P_0) \downarrow}{P_0 \downarrow}$$

Stock is properly priced/expected return increases



Return to Equilibrium (Continued)

An overpriced asset will decrease in price and rise onto the SML as the market reaches equilibrium



$$E(R_i)^M \uparrow = \frac{D_1 + (P_1 - P_0)}{P_0}$$

Short Selling (Revisited)

When an investor borrows an asset and sells it, they are shorting selling (i.e., selling an asset they do not own)

How does short selling work?

- (1) Borrow an asset (e.g., stock) and sell it on the market
- (2) Buy the asset on the market and return it to the borrower

Why short sell?

Selling price > buying price \Rightarrow Profit

How can the buying price go below the selling price?

~~Stay tuned for our section on CAPM ...~~

If an asset is overpriced when sold ($\alpha_i < 0$) we should expect its price to fall by the time we buy it

Problems With the CAPM

Under CAPM, a security that does not fall on the SML is labelled as mispriced (i.e., we think its price is wrong)

What if we consistently label an asset as mispriced?

Research has shown that small stocks have consistently earned higher returns than predicted by CAPM

Through the CAPM framework, we would consistently label these stocks as underpriced

However, are these assets truly mispriced or is there a flaw with predictions made by the CAPM?

Infrequent mispricing is OK!

The CAPM is a forward-looking model of expected return (i.e., it tries to predict the future)

There is no guarantee that the actual returns and price will match the theoretical numbers obtained by CAPM

If the actual returns differ from the expected returns infrequently, this does not invalidate the CAPM

Under diversification & Portfolio Biases

There is much evidence that individual investors fail to diversify their portfolios adequately. Why?

Familiarity Bias:

“I will buy assets in companies I am more familiar with!”

Relative Wealth Concerns:

“I only care about how my portfolio does relative to my peers!”

Investors will choose undiversified portfolios to match their peers (e.g., colleagues, friends, etc.)

Do these biases invalidate the CAPM?

Do Biases Invalidate CAPM?

Not always – it depends on how the bias affects the investor

Random Departures:

If individuals depart from the CAPM in random and unique ways, the combined effect cancels out

Society as a whole will hold the market portfolio, so market prices or returns will not be affected

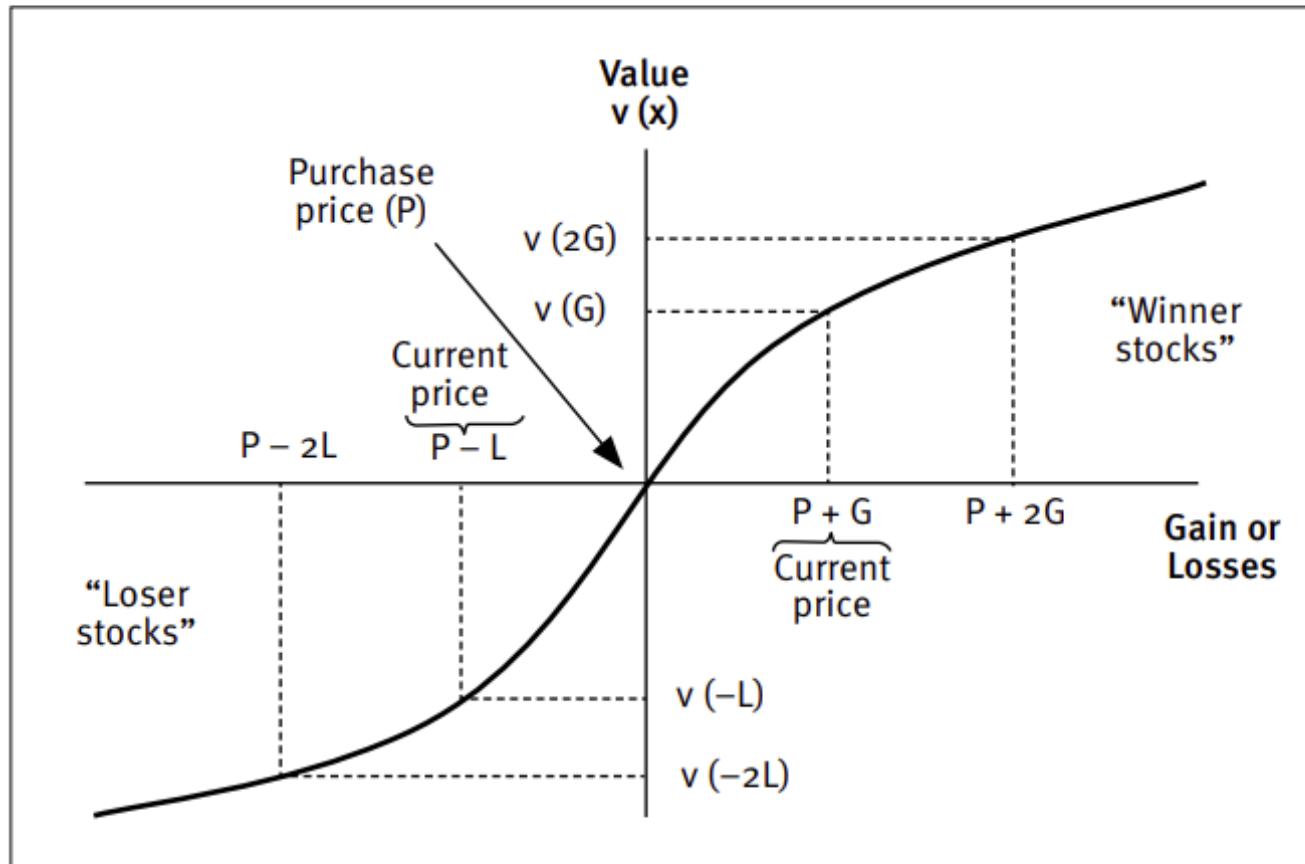
A Pattern in Departures:

If there are patterns in investors' departure from the CAPM due to bias, the market will be impacted

Disposition Effect: When an investor holds onto stocks that have lost value and sells stocks that have risen in value since the time of purchase

How the Disposition Effect Occurs (Optional)

Figure 1. Prospect theory and the disposition effect



What does Prospect theory say?

Risk averse in the domain of gains

Risk preferring in the domain of losses

Source: Weber and Camerer, 1998, p. 170.

Practice Question 6

Which of the following is a correct interpretation of alpha?

(Select all that apply)

- a) It is the slope of the regression line of excess market returns on excess stock returns
- b) It is the difference between the expected market return on an asset and CAPM expected return
- c) An asset with negative alpha will be bought by investors as the market tends to equilibrium
- d) An asset with positive alpha plots above the CML
- e) None of the above

Practice Question 7

Which of the following is true about the CAPM?

(Select all that apply)

- a) The model is invalidated whenever realized returns differ from the expected returns predicted by CAPM
- b) The familiarity bias and relative wealth concerns can make investors depart from the CAPM
- c) An investor who consistently holds onto stocks when they do well is exhibiting the disposition effect
- d) The CAPM is favoured over computations of historical average returns by a majority of corporations
- e) The CAPM isn't perfect in practice as small stocks often yield higher returns than predicted

Practice Question 8

The Interastral Peace Corporation's common stock currently sells at \$60. It pays no dividends and has an alpha of 2.3%. If the market expects a yearly return of 20% on this stock, what should its equilibrium year-end selling price be according to the CAPM?

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Capital Structure

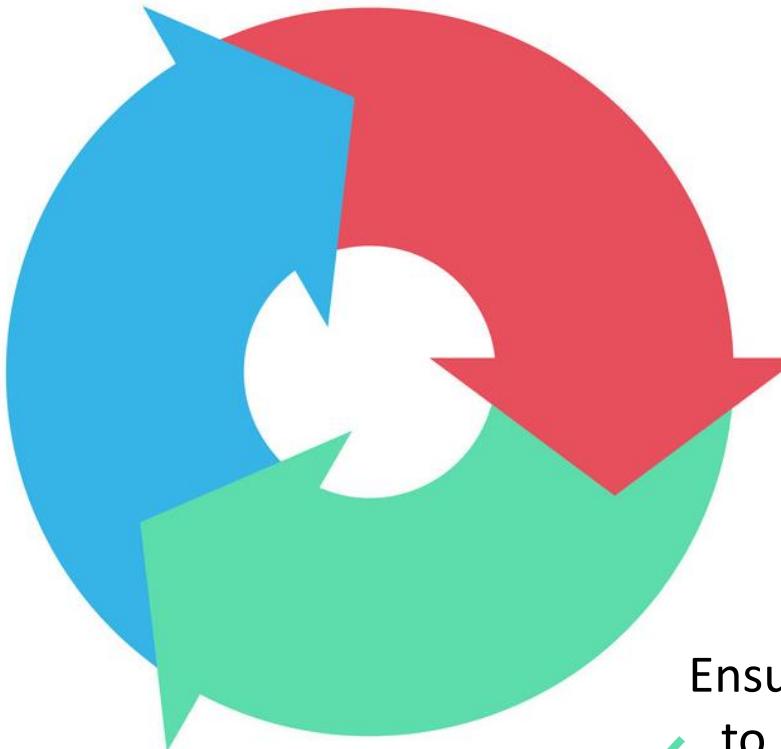


The Financial Manager's Duties

Investing

What investments & projects are worthwhile?

Weighs cost & benefits of investments & projects



Managing Cash

Ensure company has enough cash to meet day-to-day obligations

Financing

Do we raise money using equity or debt?

How do we pay for investments & projects?

What is the Primary Goal of a Corporation?

A firm's management should make decisions in the shareholders' interests. What should they do?

- a) Maximize profit
- b) Minimize costs
- c) Maximize market share
- d) **Maximize the current value of the company's stock (maximize owners' equity)**



Shareholders



We are better off if the value of our investment is maximized!

Financing Decisions

Step 1: What type of securities should be issued?



Step 2: In what proportion? (capital structure)



Step 3: How do we raise the capital?



Step 1: What Type of Securities are Issued?

The most common choices of financing are through equity alone or a mix of equity and debt

Useful Terminology Moving Forward:

D: Market value of debt (e.g., bonds, loans)

E: Market value of levered equity (e.g., common stock)

$V = D + E$: Market value of the firm

R_D = Required return on debt by investors (cost of capital on debt for company financing)

R_E = Required return on equity by investors (cost of capital on equity for company financing)

Step 2: In What Proportion? (Capital Structure)

How much should be funded from equity? How about debt?

Some Factors Companies Consider:

Investors expect different returns for different securities



R_D and R_E will likely be different – investors tend to require a greater return from equity than debt

Pricing, transaction costs, and the future



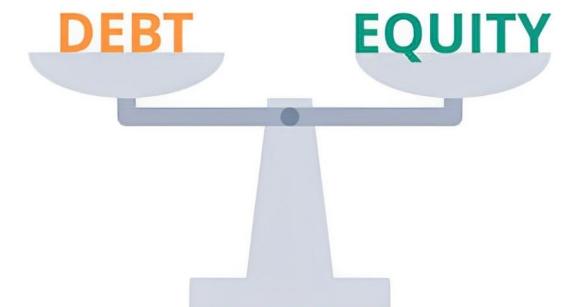
Will the securities issued be sold at a fair price and/or come with transaction costs (e.g., underwriting fees)?

Will the securities issued affect the company's future investing opportunities?

The company's use of cash



Does the company want to save cash, pay off debt, or pay dividends?



How Much Debt do Companies Take on?

The percentage weight put on debt financing is defined as D/V – the company's debt-to-value ratio

Utility companies have an average debt-to-value ratio of 50%. Why this high?

These companies have very stable cash flows – low chance to default on mandatory debt payments

They own many tangible assets that can be sold as collateral if they reach close to a state of bankruptcy

Software companies have an average debt-to-value ratio of 10%. Why this low?

Uncertain cash flows may lead them to default on mandatory debt payments

Equity financing is more attractive as there are no legal obligations to pay shareholders anything

Step 3: How do we Raise Capital?

Below are some examples (not an exhaustive list of possibilities)



Loans



Angel Investors



Stocks (IPO & SEO)



Venture Capital



Bonds



Private Equity Firms

Equity for Private Companies



Angel Investors: Individual investors who buy equity in small private firms (often startups)



Venture Capital: Private equity investment by businesses in young, immature startups



Private Equity Firms: Invests in the equity of existing, mature private firms or takes a public firm private by purchasing all its outstanding equity

Equity for Public Companies

Initial Public Offering (IPO): When a firm issues stocks to the public for the first time in the primary market

Advantages:

Provides the company with greater liquidity (higher trading volume) and better access to capital

Disadvantages:

Ownership of the firm is spread as more individuals own stocks of the company

Investors' ability to monitor the company's management decreases

Financial disclosure requires greater accountability, stringent requirements, and time

Seasoned Equity Offering (SEO): Issuance of shares that follow a company's IPO in the primary market

Similar to an IPO, but firms do not need to handle price-setting issues as a price for the stock exists

Private & Public Corporate Debt

Debt financing does not dilute the firm's ownership, but legally demands payments (e.g., interest, principal)

Private Corporate Debt:



Many young firms decide to undertake a bank loan

It is not publicly traded on a market, so issuance costs are avoided, but it is illiquid

Public Corporate Debt:



Firms provide a bond indenture (contract of "terms and conditions") to the market via a public bond issue

Bonds usually pay semi-annual coupons, have a face value of \$1000, and have maturities of < 30 years

Involves underwriting fees (e.g., a percentage commission on all money earned from bonds sold)

Secured Debt vs. Unsecured Debt



Secured

Borrowers put up an asset to serve as collateral for the loan

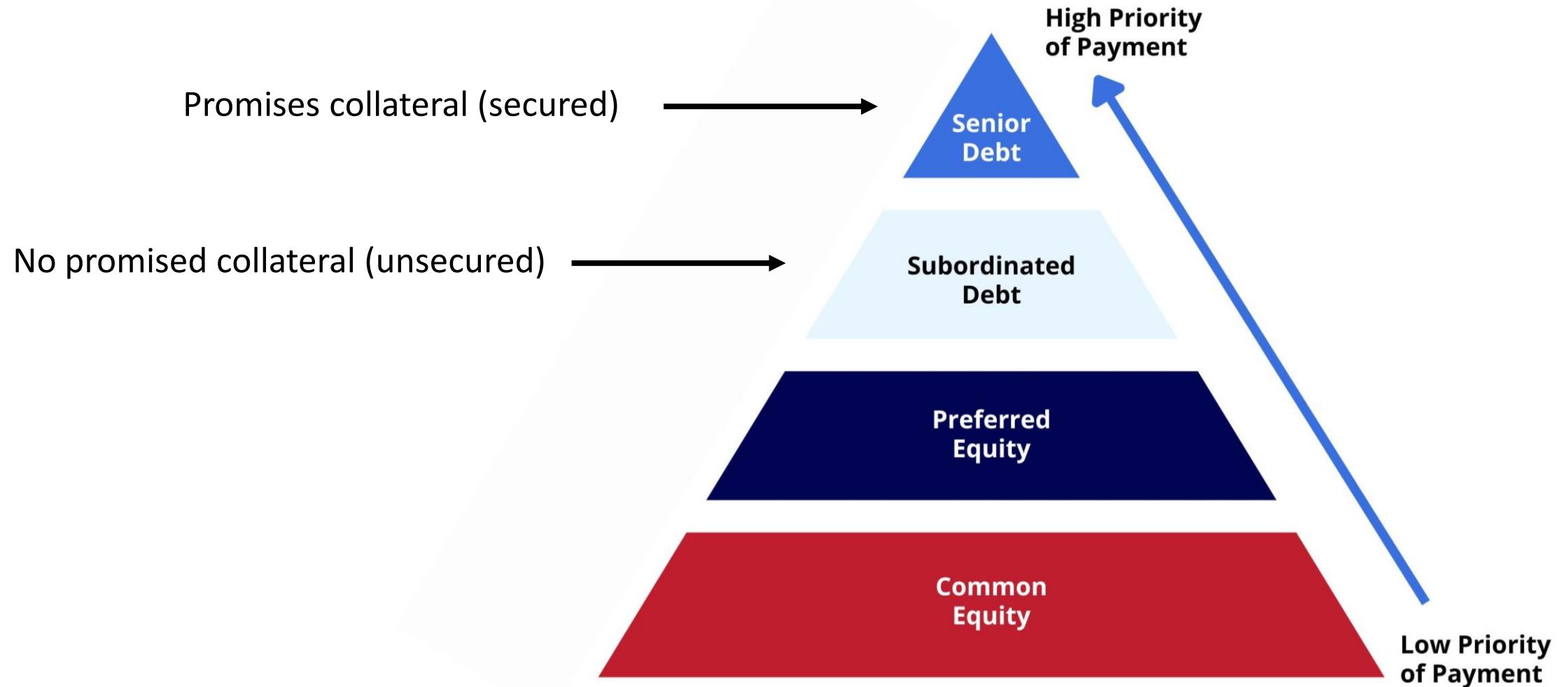


Unsecured

No collateral needed to secure the loan

Debt Seniority

Seniority: Definition of who gets paid 1st, 2nd, etc. if the firm doesn't have enough assets to pay everyone



Practice Question 1

Which of the following statements is true?

(Select all that apply)

- a) Most firms choose to finance with only debt alone or a mix of debt and equity
- b) Security prices and future investment opportunities can influence a firm's choice of capital structure
- c) Underwriting fees are a possible transaction cost firms may face when issuing securities
- d) A financial manager is responsible for evaluating the costs and benefits of projects
- e) The market value of a firm with unstable cash flows will likely be comprised of mostly equity

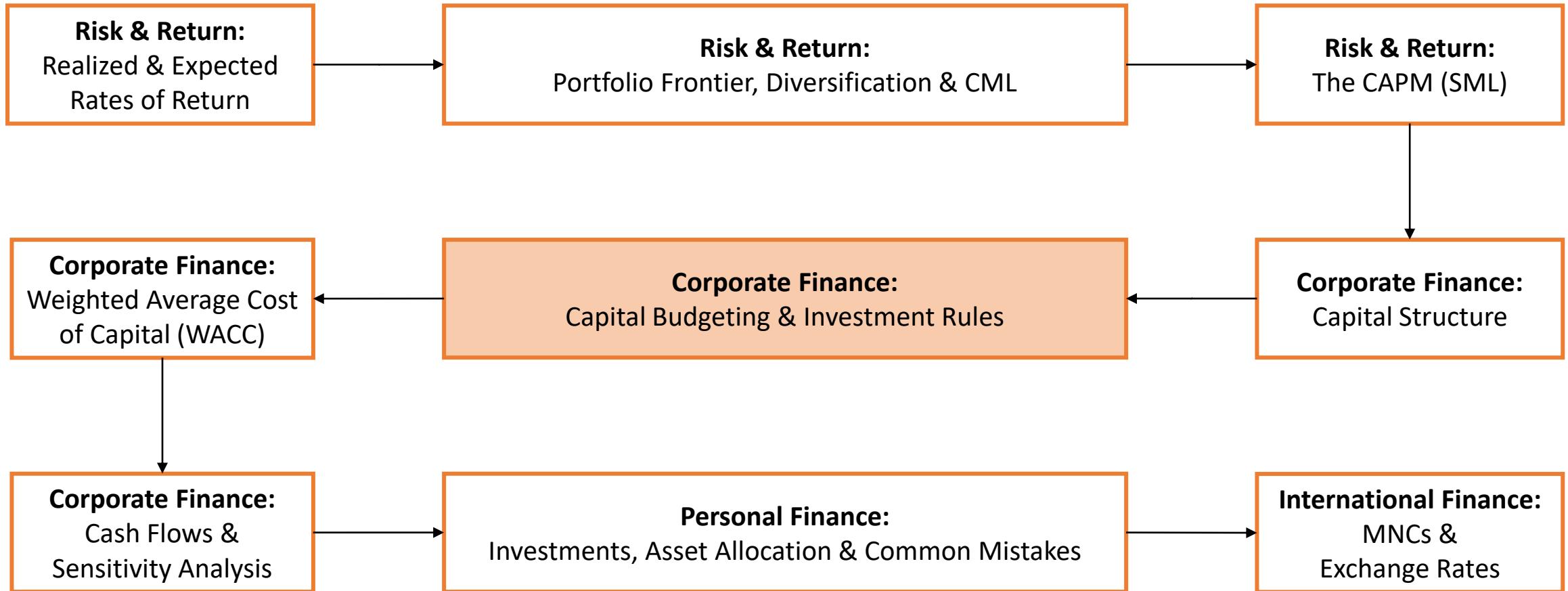
Practice Question 2

Which of the following statements is true?

(Select all that apply)

- a) Investors who own subordinate debt will be paid first when a company goes into bankruptcy
- b) Angel investors and venture capital are sources of equity financing for public companies
- c) Secured debt promises specific firm assets (e.g., land) that cannot be promised to anyone else
- d) A disadvantage of private corporate debt is that it is quite illiquid
- e) The IPO of a firm takes place in the primary market while the SEO takes place in the secondary market

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Risk & Return:
Realized & Expected
Rates of Return

Risk & Return:
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Corporate Finance:
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Corporate Finance:
Capital Structure

Corporate Finance:
Cash Flows &
Sensitivity Analysis

Personal Finance:
Investments, Asset Allocation & Common Mistakes

International Finance:
MNCs &
Exchange Rates

What is Capital Budgeting?

It is the process of making investments in assets (tangible or intangible) with long lives

Examples of Capital Budgeting Decisions:

- a) An NFL team considers replacing its stadium's natural grass turf with artificial turf
- b) A mining equipment company planning to build a new factory on vacant land
- c) UPS planning to invest in e-bikes for deliveries
- d) Fast food retail chain deciding whether to develop a new product line of burgers
- e) A supermarket company deciding whether to lease new space in a shopping mall

The list continues ...

Investment Rules for Capital Budgeting



Payback Period



Net Present Value



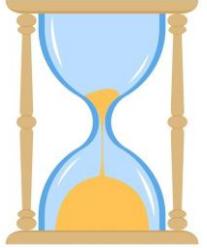
Internal Rate of Return



Profitability Index

What Makes a Good Investment Rule?

A good investment rule should meet the following criteria



Adjusts for the time value of money (we should see discounting here)



Adjusts for risk (higher risk should require a higher return)



Measures value created for the firm (shows the benefits/costs in dollar terms)

Payback Period Rule

Payback Period: The time until the cash flows (undiscounted) from a project offset the initial investment C_0

Payback Rule: Accept a project if the payback period is less than a prespecified length of time (cutoff period)

Cash Flow (Yearly)	Amount	Investment Offset
C_0	-1000	
C_1	300	300
C_2	500	800
C_3	-200	600
C_4	500	1100
C_5	1200	2300

1000 is between here

Year 4 pays \$500 but we only need \$400 to reach \$1000

$$\text{Payback Period} = 3 + \frac{400}{500} = 3.8 \text{ years}$$

(we would accept the project if the cutoff is more than 3.8 years)

Problems With the Payback Period Rule

Disadvantages:

- (1) It ignores time value of money (there is no r being used)
- (2) It does not consider cash flows after the payback period (e.g., C_5 on the previous slide)
- (3) It favours short-term projects
- (4) It does not adjust for risk explicitly (not possible to adjust in this method due to point #1)

Why do people still use this rule?

It is easy to use and calculate – very simple

Useful for small investments

Net Present Value (NPV) Rule

NPV: The value added today by taking on an investment (it is the net of the investment cost C_0)

$$NPV = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n}$$

Initial cost of undertaking project (negative)

Future cash flows (positive or negative)

C_n = Cash flow at time $t = n$
 r = Effective interest rate

NPV Rule: If projects aren't mutually exclusive and capital isn't scarce, accept all positive NPV projects

If projects are mutually exclusive (choosing one discards the rest): Choose the highest positive NPV project

If capital is scarce (rationed): Don't use the NPV rule – use the profitability index rule

NPV Example Calculation

If the firm's yearly cost of capital (r) is 12%, what is the NPV of the following set of cash flows? Assume this project is not mutually exclusive with any other project and that capital is not rationed.

Cash Flow (Yearly)	Amount
C_0	-1000
C_1	300
C_2	500
C_3	-200
C_4	500
C_5	1200

$$NPV = -1000 + \frac{300}{(1+0.12)} + \frac{500}{(1+0.12)^2} - \frac{200}{(1+0.12)^3} + \frac{500}{(1+0.12)^4} + \frac{1200}{(1+0.12)^5} = 522.77$$

$$NPV = 522.77 > 0 \quad (\text{take on the project})$$

Problems With the NPV Rule

Disadvantages:

The NPV rule can be hard to use in practice if estimations of cash flows & the cost of capital (r) are poor

Cash flows: Hard to estimate if the new project is irrelevant to what they are doing currently

Cost of capital: Hard to estimate if the new project is more or less risky than other projects undertaken

Why do people still use this rule?

It is the most accurate & reliable rule which precedes when other rules (e.g., IRR) lead to conflicting results

In other words, always base your decision on the NPV when rules conflict

Internal Rate of Return (IRR) Rule

IRR: The interest rate that makes the NPV of an investment equal to zero

The IRR is a rate of return number that summarizes the merits of an investment project

IRR Rule: If projects are not mutually exclusive, accept a project if the IRR exceeds the cost of capital

An Important Assumption:

The NPV of a project should decline smoothly as the cost of capital increases (i.e., $NPV \downarrow$ as $r \uparrow$)

Graphically, the NPV profile is “nicely-behaved” (monotonic decreasing, differentiable)

Advantages:

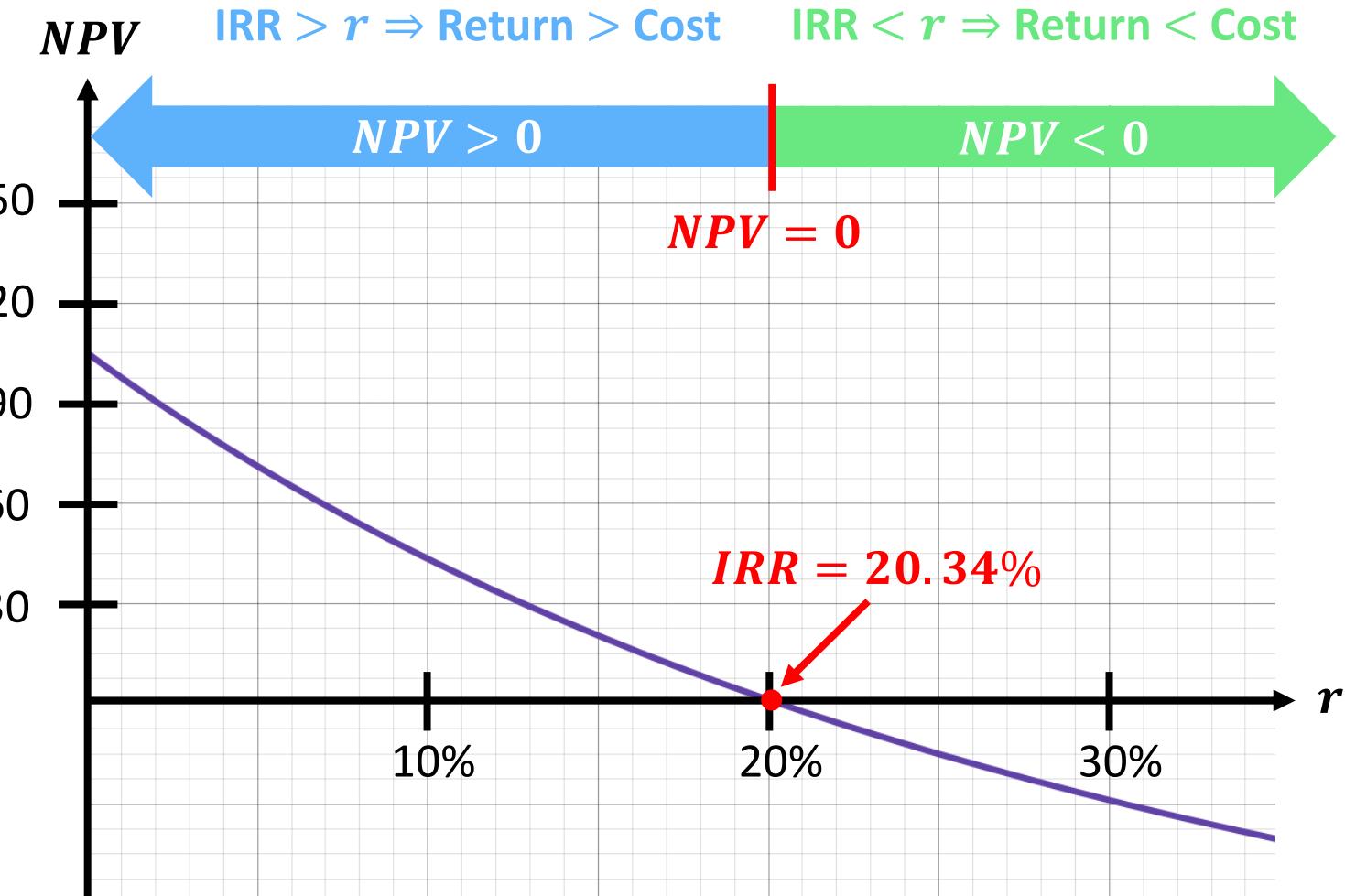
IRR generally gives the same answer as NPV about project acceptability

If IRR is big enough, you may not need to estimate the cost of capital accurately

A Smooth NPV Profile

Cash Flow	Amount
C_0	-105
C_1	20
C_2	45
C_3	50
C_4	60

r	NPV
0%	70.00
15%	13.60
20%	0.79
21%	-1.52
22%	-3.75



IRR is like a “return” (what you get) while r is like a “cost”

A Problem With the IRR Rule

If projects are mutually exclusive, our intuition would tell us to accept the one with the highest IRR holding all else constant since IRR is a measure of “return” (isn’t a higher return better?)

But this is NOT always correct!

Cash Flow	Amount
C_0	-105
C_1	20
C_2	45
C_3	50
C_4	60

Project A
 $(IRR = 20.34\%)$

Cash Flow	Amount
C_0	-80
C_1	45
C_2	35
C_3	30
C_4	25

Project B
 $(IRR = 28.11\%)$

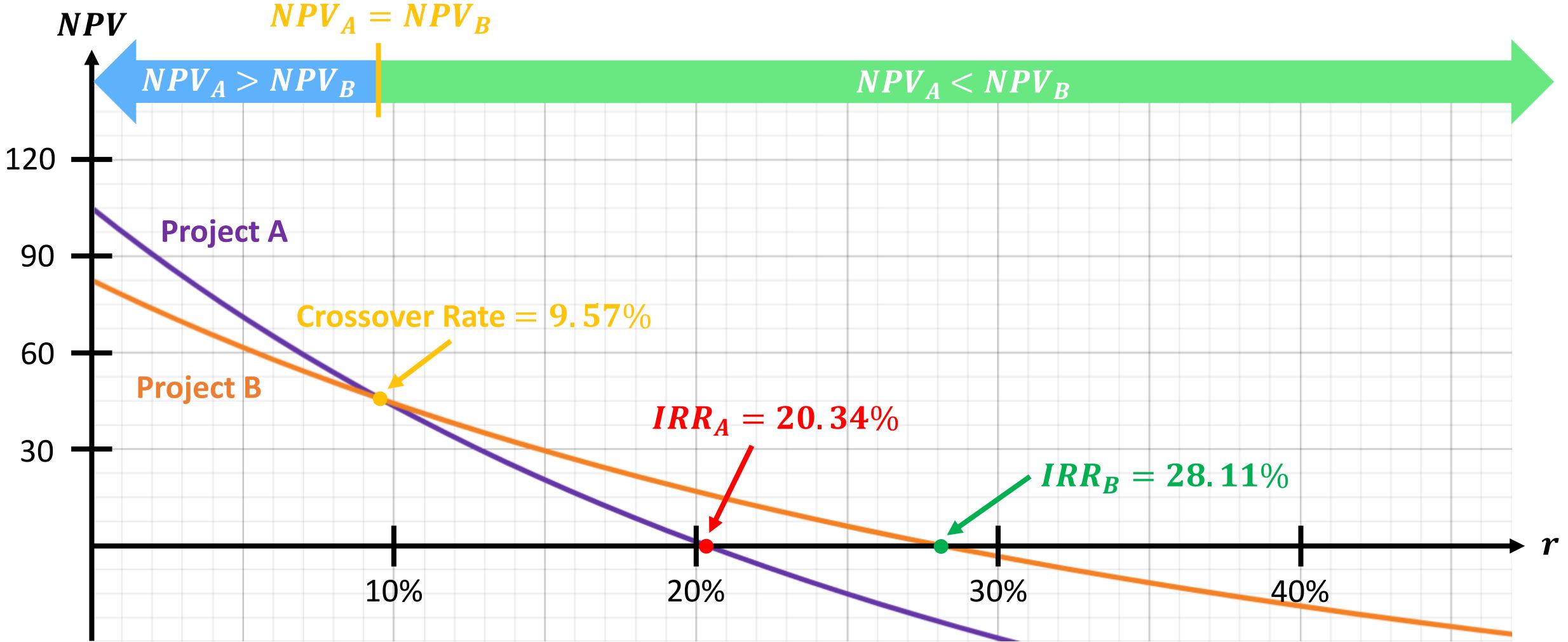
r	NPV_A	NPV_B
0%	70.00	55.00
5%	47.42	41.09
10%	28.92	29.45
15%	13.60	19.62
20%	0.79	11.22

Comparing NPVs

By the IRR rule we would always choose project B, but by the NPV rule, it depends on the cost of capital (r)

The Crossover Point

The crossover point is where the NPV of two mutually exclusive projects take the same value



More on the IRR Rule Assumption (Optional)

When using the IRR rule, we assumed that the NPV profile is “nicely-behaved”

What causes it to be nicely behaved?

Conventional cash flows – when a project has a negative cash flow (C_0) then all positive cash flows

In other words, the sign between cash flows when ordered from C_0 to C_n only change once

Example:

Cash Flow	Amount
C_0	-105
C_1	20
C_2	45
C_3	50
C_4	60

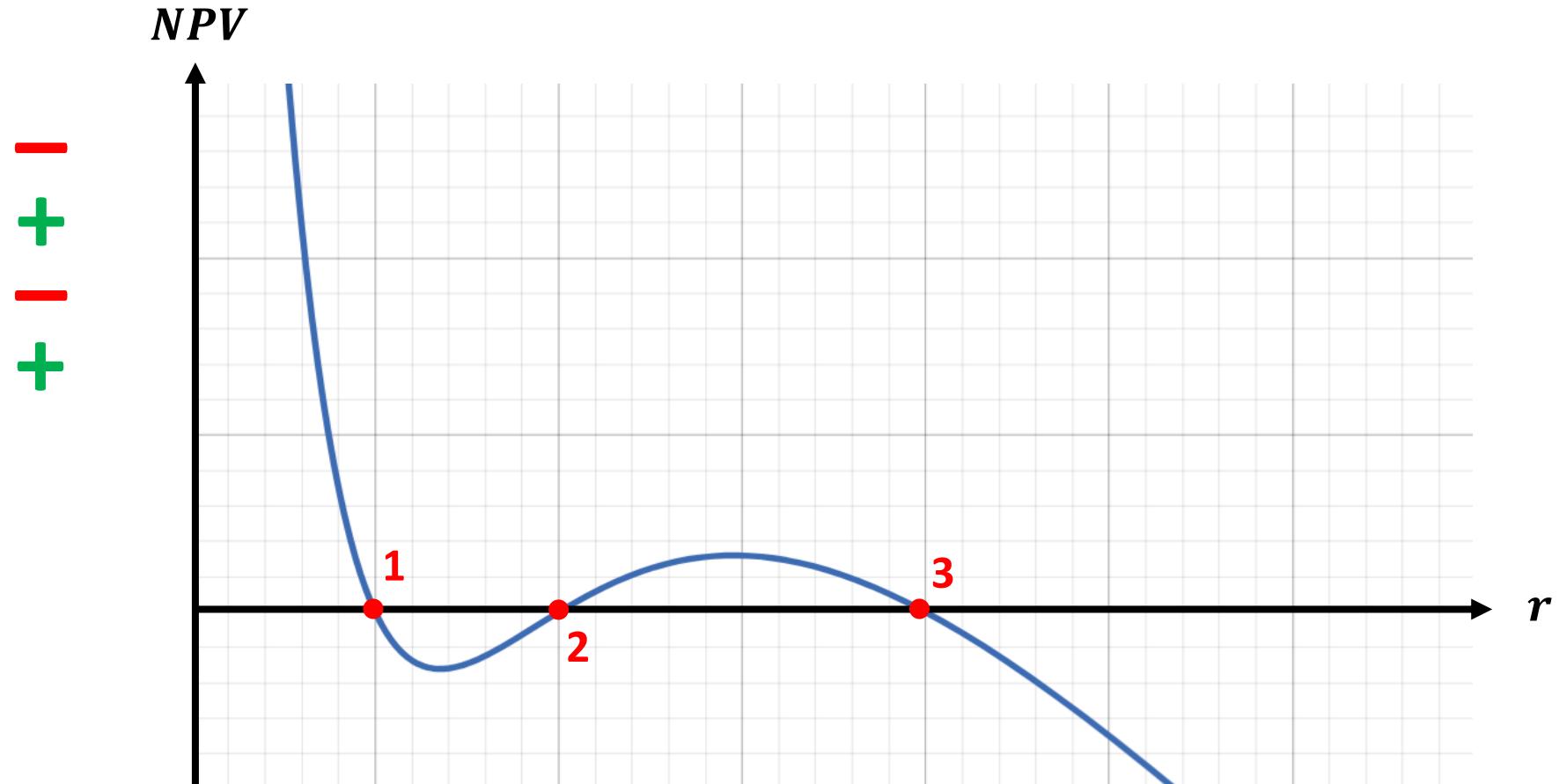


Sign Change

Another Problem With the IRR Rule (Optional)

If cash flows are unconventional (more than one sign change), there could be more than one IRR

Cash Flow	Amount
C_0	-10
C_1	65
C_2	-135
C_3	90



Which IRR is useful?

None of them!

When there are k sign changes, the maximum number of IRRs is k (i.e., there are k or less IRRs)

Descartes' Rule of Signs & Multiple IRRs (Really Optional)

Let $p(x)$ be a polynomial with real coefficients and terms arranged from highest to lowest degree

Theorem (Descartes' rule of signs): The number of positive real roots (multiplicity included) of a polynomial $p(x)$ is at most the number of sign changes in the coefficients of $p(x)$

We now tie this back to multiple IRRs

Let $R(r) = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n}$ (assume there are k sign changes between $\{C_0, C_1, \dots, C_n\}$)

Then $R(r) = R(r) \cdot \frac{(1+r)^n}{(1+r)^n} = \frac{C_0(1+r)^n + C_1(1+r)^{n-1} + \cdots + C_n}{(1+r)^n}$, so it is sufficient to find the roots of the numerator

We define $q(r) = C_0(1+r)^n + C_1(1+r)^{n-1} + \cdots + C_n$

Expanding $q(r)$ we obtain $q(r) = a_0r^n + a_1r^{n-1} + \cdots + a_n$ (has $\leq k$ sign changes)

By Descartes' rule of signs, $q(r)$ has at most k positive real roots, so $R(r)$ must too (implying $\leq k$ IRRs)

Profitability Index Rule

Profitability Index: The NPV of an investment's future cash flow divided by its initial cost (C_0)

$$PI = \frac{\text{Value Created}}{\text{Resource Used}} = \frac{NPV}{C_0}$$

← Initial cost in absolute (positive) terms

Shows how much NPV one gets for each dollar invested in the project

Profitability Index Rule: Accept the projects with the highest profitability indices first

Only use this rule when capital is scarce (rationed)!

Profitability Index Calculation

Suppose a firm has a cost of capital of 6% and a budget of \$10 million. Which of projects should it take?

Project	Cash Flows			Measures	
	C_0	C_1	C_2	NPV	PI
A	-10m	35m	10m	31.92m	3.19
B	-5m	5m	20m	17.52m	3.50
C	-5m	10m	15m	17.78m	3.56

$$PI_A = \frac{31.92}{10} = 3.19 \quad \text{← We get \$3.19 of NPV for every \$1 we invest in the project}$$

$$PI_B = \frac{17.52}{5} = 3.50 \quad \text{We should take projects B & C}$$

$$PI_C = \frac{17.78}{5} = 3.56 \quad \text{Notice how } NPV_B + NPV_C > NPV_A$$

What do Managers Use in Reality?

Capital Budgeting Technique

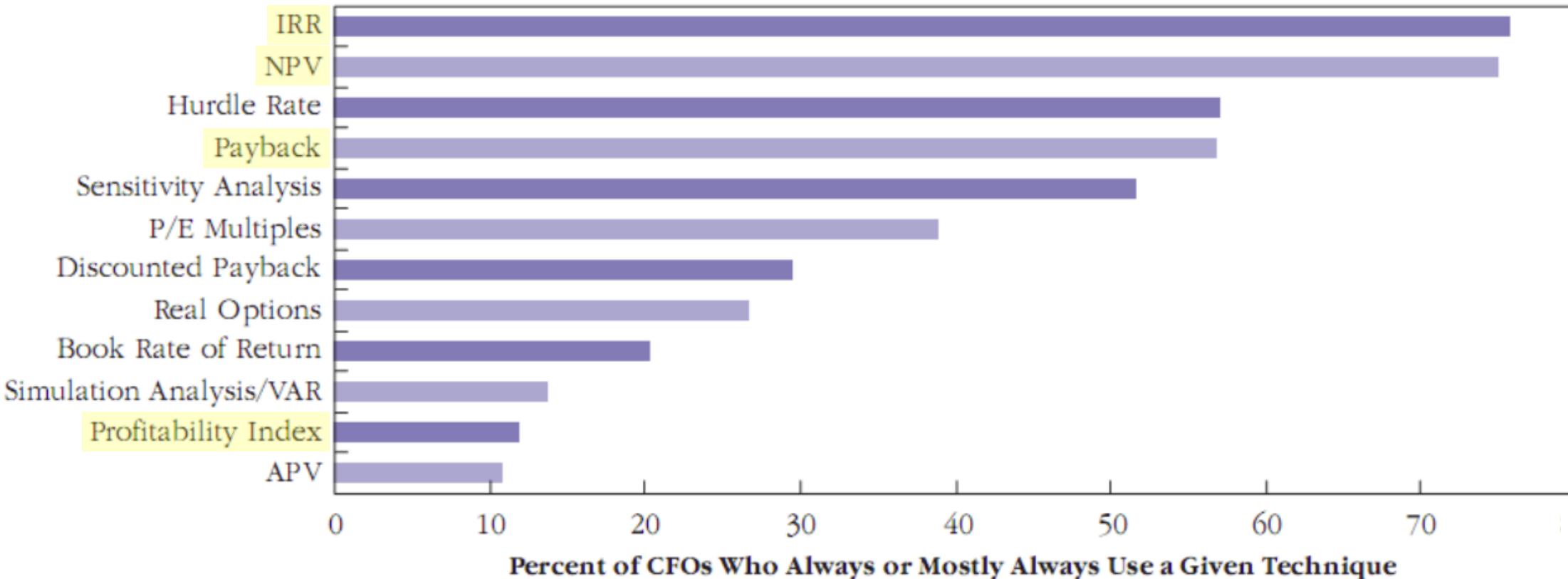


Figure 2.1 Popularity of capital budgeting techniques (Graham & Harvey; 2001)

Where to Next?

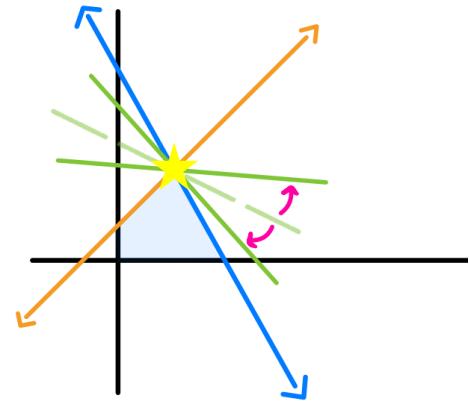
So far, we have handed you the cost of capital (r) and cash flows in a project. What are we doing next?



**How do we find
the cost of capital?**



**What cash flows
do we consider?**



**How do changes
affect NPV?**

Practice Question 1

If two projects are not mutually exclusive and provide poor estimations of their cost of capital:

(Select all that apply)

- a) The payback period rule can be used to evaluate both projects
- b) The NPV rule can be used to evaluate both projects
- c) The IRR rule can be used to evaluation both projects
- d) The profitability index rule can be used to evaluate both projects
- e) None of the rules are appropriate to evaluate both projects

Practice Question 2

Two mutually exclusive projects (A & B) have smooth declining NPV profiles. Project A has an IRR rate 12% while project B has an IRR of 10%. If their cross-over rate is 6%, which of the following is true?

(Select all that apply)

- a) If the cost of capital is 5%, a firm will invest in project A and not in project B
- b) If the cost of capital is 4%, a firm will invest in project B and not in project A
- c) If the cost of capital is 6%, a firm will be indifferent between project A and B
- d) If the cost of capital is 8%, a firm will invest in project B and not in project A
- e) None of the above

Practice Question 3

You have been asked to analyze the following four projects for a client. Each project provides non-zero cash flows in every year for the next three years. Answer the questions posed in the following slides.

Project	Cash Flows				Measures			
	C_0	C_1	C_2	C_3	Payback	IRR	NPV	PI
A	-240	25	100	375	2.3 years	31.9%	(a)	
B	-75	5	(b)		1.7 years	31.4%	46.38	0.62
C	-100	40	30	50	(c)	9.3%	8.50	0.09
D	-300		50	400	2.4 years	26.5%	176.60	(d)

Practice Question 3

Project	Cash Flows				Measures			
	C_0	C_1	C_2	C_3	Payback	IRR	NPV	PI
A	-240	25	100	375	2.3 years	31.9%	(a)	
B	-75	5	(b)		1.7 years	31.4%	46.38	0.62
C	-100	40	30	50	(c)	9.3%	8.50	0.09
D	-300		50	400	2.4 years	26.5%	176.60	(d)

a) If the client's cost of capital is 5%, what is the NPV of project A?

Practice Question 3

Project	Cash Flows				Measures			
	C_0	C_1	C_2	C_3	Payback	IRR	NPV	PI
A	-240	25	100	375	2.3 years	31.9%	(a)	
B	-75	5	(b)		1.7 years	31.4%	46.38	0.62
C	-100	40	30	50	(c)	9.3%	8.50	0.09
D	-300		50	400	2.4 years	26.5%	176.60	(d)

b) If the client's cost of capital is 5%, what cash flow is received in the 2nd year of project B?

Practice Question 3

Project	Cash Flows				Measures			
	C_0	C_1	C_2	C_3	Payback	IRR	NPV	PI
A	-240	25	100	375	2.3 years	31.9%	(a)	
B	-75	5	(b)		1.7 years	31.4%	46.38	0.62
C	-100	40	30	50	(c)	9.3%	8.50	0.09
D	-300		50	400	2.4 years	26.5%	176.60	(d)

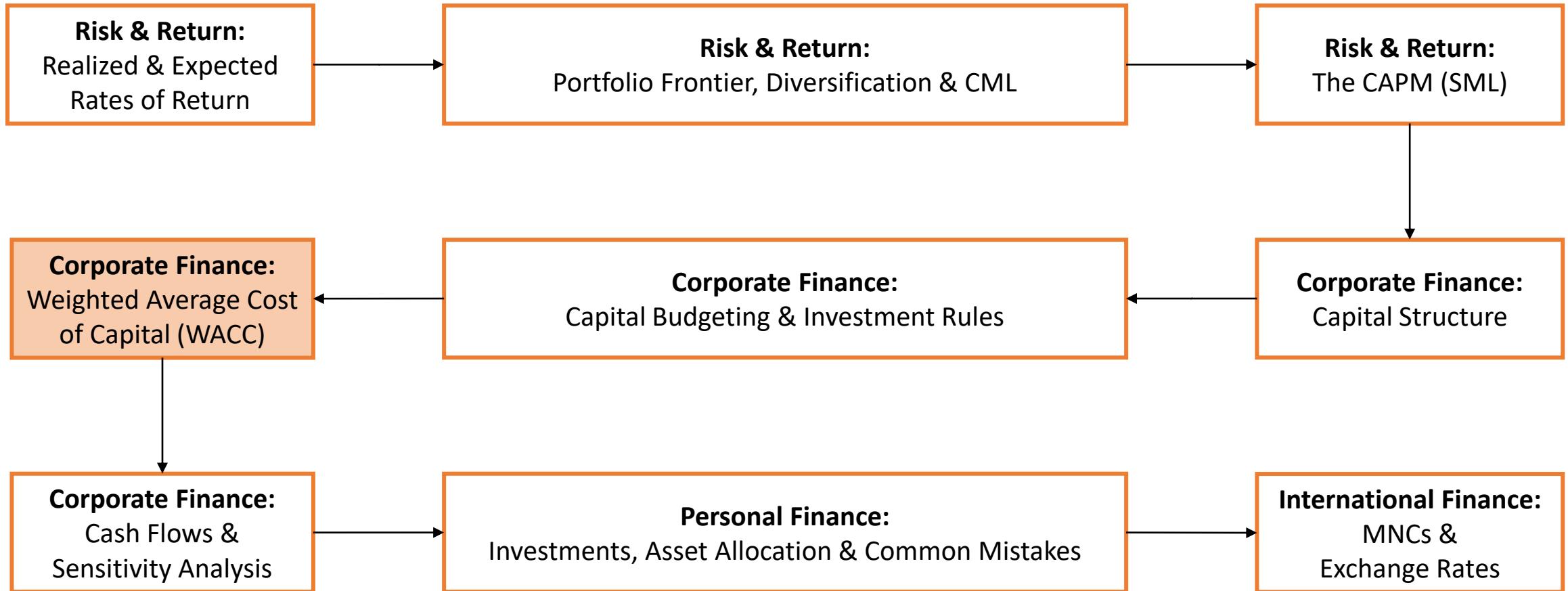
c) If the client's cost of capital is 5%, what is the payback period of project C?

Practice Question 3

Project	Cash Flows				Measures			
	C_0	C_1	C_2	C_3	Payback	IRR	NPV	PI
A	-240	25	100	375	2.3 years	31.9%	(a)	
B	-75	5	(b)		1.7 years	31.4%	46.38	0.62
C	-100	40	30	50	(c)	9.3%	8.50	0.09
D	-300		50	400	2.4 years	26.5%	176.60	(d)

d) If the client's cost of capital is 5%, what is the profitability index of project D?

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Cost of Capital

The cost of capital (r) in an investment project has many equivalent names – discount rate, required return

Why so many names?

Discount rate: From a TVM perspective, we need to properly value a project's cash flows by discounting

Required return: Investors require compensation for the money put towards a firm's project when they buy securities (e.g., stocks, bonds) – this compensation comes from the return on the firm's assets

Cost of capital: A firm needs to compensate their investors who lend money for their projects as there is uncertainty to whether the project will succeed or not (there is risk!)

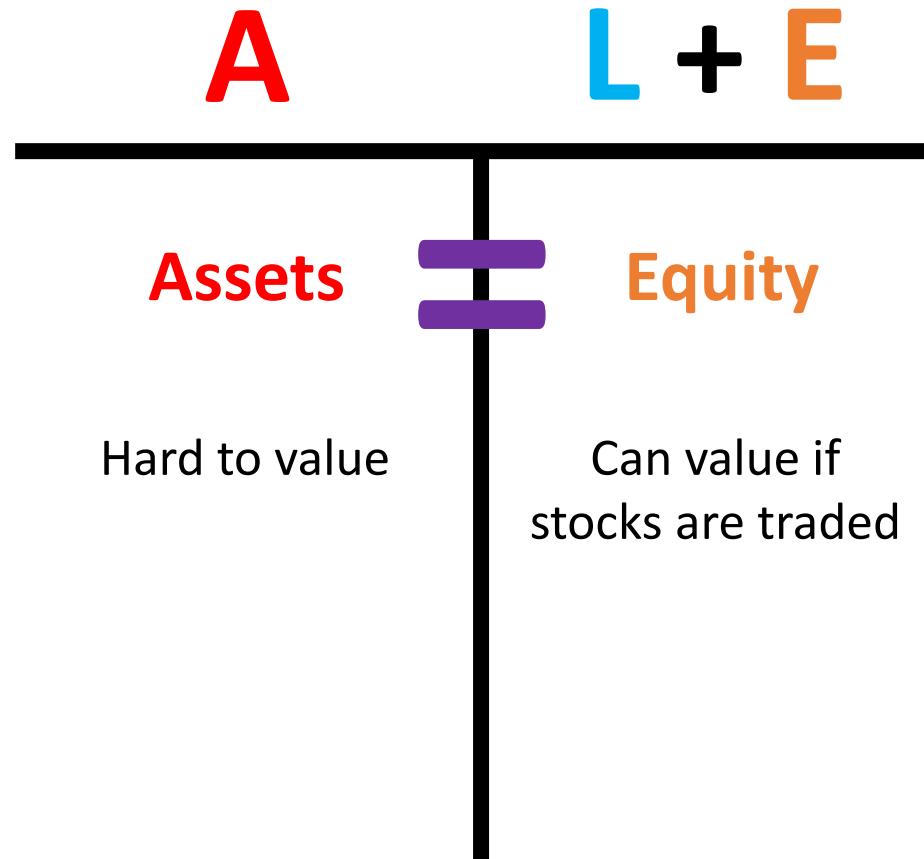
What will we consider?

- (1) The cost of capital of a firm only with equity
- (2) The cost of capital of a firm with debt and equity (i.e., a leveraged firm)

**Most firms finance with
only equity or a mix of
equity and debt**

A Firm With Only Equity

A firm's cost of capital reflects the return assets should be providing on average – but this is hard to value



Owning all the firm's assets = Owning all the equity

Value of firm's assets = Value of equity $(V = E)$

Risk of firm's assets = Risk of equity

Rate of return from firm's assets = Rate of return on equity

Firm's cost of capital = Investors' required return on equity

$R_{WACC} = R_E$ (WACC = Weight average cost of capital)

Estimating the Cost of Equity

Use the CAPM (SML equation) to estimate the cost of equity

$$R_E = [E(R_M) - R_f] \cdot \beta_E + R_f$$

Advantages:

Explicitly adjusts for risk (through β_E)

It applies no matter what the firm's dividend policy is – doesn't depend on D_1 unlike in the DDM & GGM

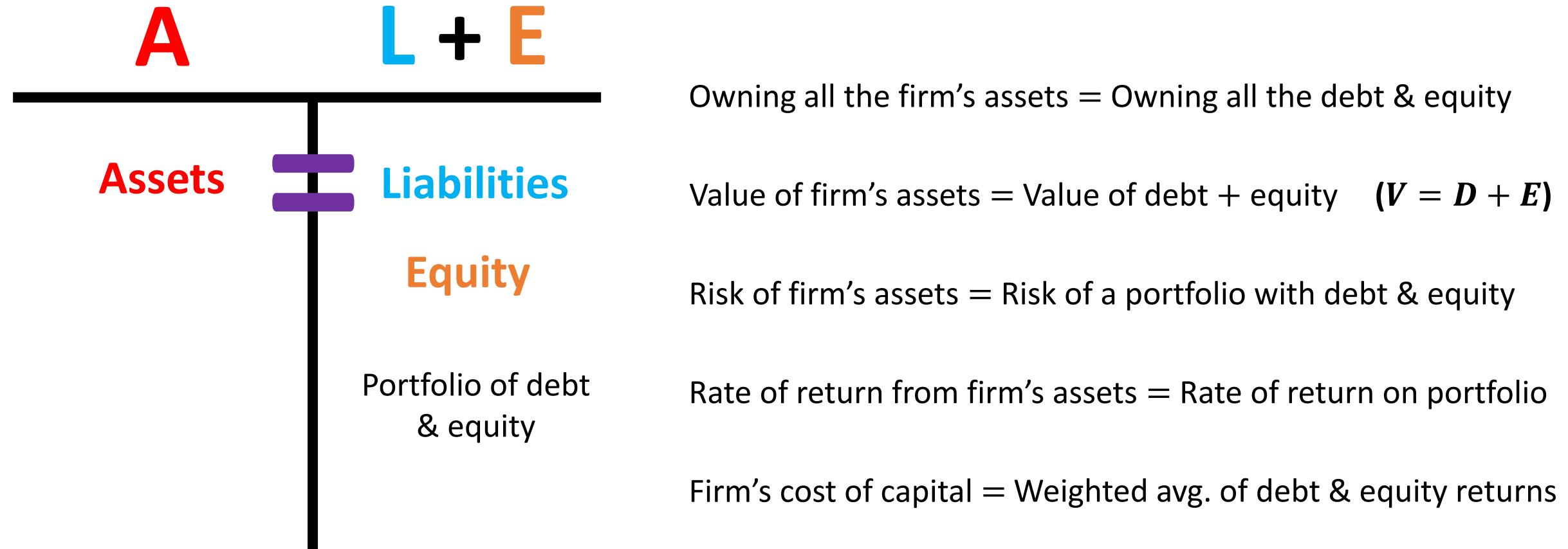
Disadvantages:

R_E can be inaccurate if the estimate of the market risk premium $[E(R_m) - R_f]$ and beta (β_E) are poor

It uses historical data (in the estimate of β_E) to predict future returns (CAPM is a forward-looking model)

A Firm With Debt & Equity (Levered Firm)

Recall that the expected return on a portfolio is a weighted average of the returns on the assets in it



$$R_{WACC} = w_D \cdot R_D + w_E \cdot R_E = \left(\frac{D}{V}\right) \cdot R_D + \left(\frac{E}{V}\right) \cdot R_E$$

Estimating the Cost of Equity & Debt

Cost of Equity:

$$R_E = [E(R_M) - R_f] \cdot \beta_E + R_f$$

Cost of Debt:

It is the expected return demanded by investors on a firm's debt – use the *YTM* on their outstanding bonds

$$R_D = YTM$$

If a bond is not publicly traded (i.e., we don't know its *YTM*), use the *YTM* from a different bond (a proxy) that has the same rating and a similar maturity time as the bond of interest

Why are we using an APR rate (YTM) and not an effective rate?

It has been like this in practice for centuries – arguably it's better to use an effective rate, but let's not touch it

The Value of Debt & Equity

The construction of R_{WACC} relied on owning (purchasing) all the firm's equity and/or debt

The price paid for each security class is the market price, so we use market values to compute D and E

Market Value of Equity:

$E = \# \text{ of shares} \times \text{Market price per share} = \text{Market capitalization}$

Market Value of Debt:

$D = \# \text{ of bonds outstanding} \times \text{Market price per bond}$

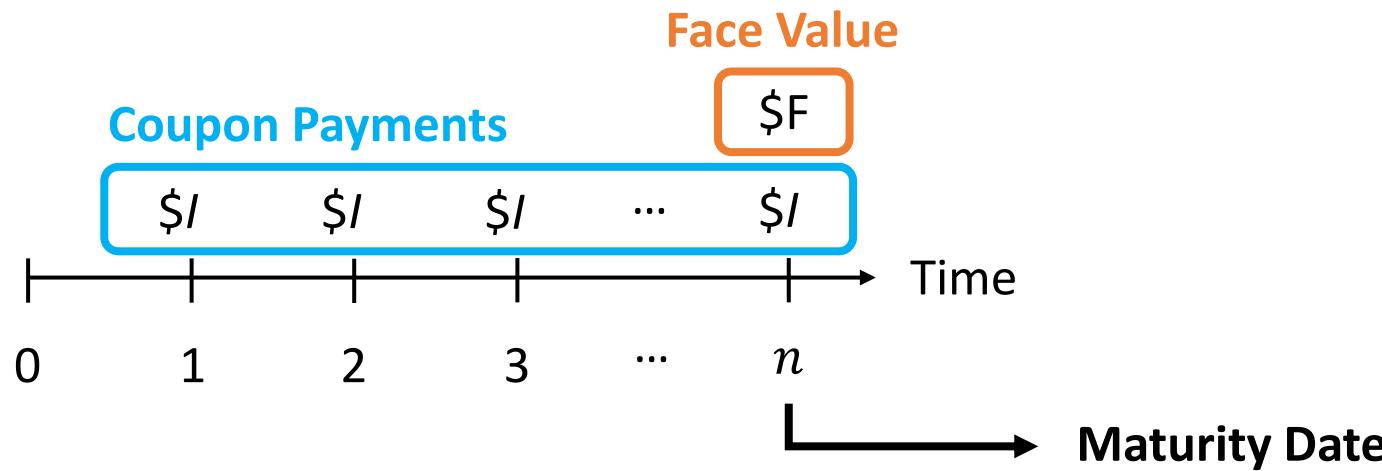
If a bond is not publicly traded (i.e., we don't know its YTM), use the YTM from a different bond (a proxy) that has the same rating and a similar maturity time as the bond of interest

Coupon Bonds

A bond with coupons where the investor receives an annuity of coupons + principal at maturity

$$P_0 = I \left[\frac{1 - (1+r)^{-n}}{r} \right] + F \cdot (1 + r)^{-n}$$

PV of Coupon Payments
PV of Face Value

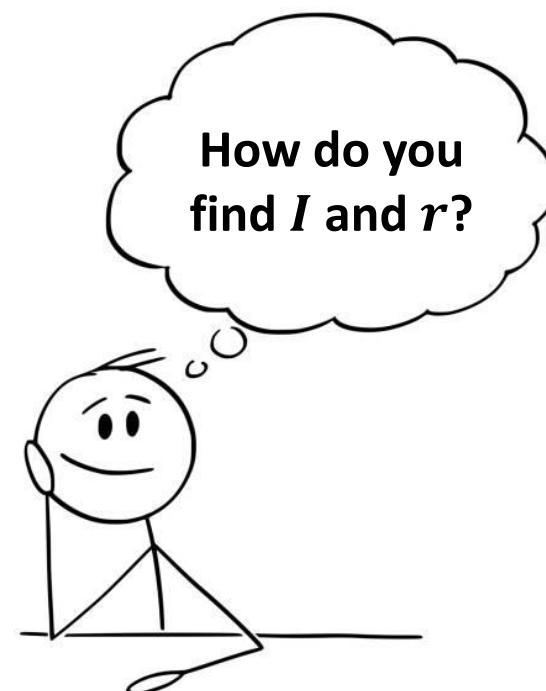


P_0 = Price of Bond

r = Effective interest rate

n = # of payments before bond matures

Same time units!



How to get the Coupon Payments (I)

Unlike yield to maturity (YTM), the coupon rate (CR) is a **FIXED** APR rate and never changes across time

$$I = F \cdot \frac{CR}{k}$$

I = Coupon (Interest) Payment F = Face Value CR = Coupon Rate
 k = # of coupon payments in ONE year

If we rearrange to solve for the coupon rate (CR), we can define it as the total coupon payment you get in one year expressed as a percentage of the bond's face value

$$CR = \frac{I \cdot k}{F} = \frac{\text{Total Coupon Payment in One year}}{\text{Face Value}}$$



Note: The number of coupon payments in a year (k) equals the frequency of compounding (m)

Summary of WACC

Formulas:

$$R_{WACC} = \left(\frac{D}{V}\right) \cdot R_D + \left(\frac{E}{V}\right) \cdot R_E$$

$$R_D = YTM \quad R_E = [E(R_M) - R_f] \cdot \beta_E + R_f$$

E = # of shares \times Market price per share

D = # of bonds outstanding \times Market price per bond

When can R_{WACC} be used as a discount rate when evaluating investments?

It is when the investment has risk characteristics similar to the firm's existing assets and projects

Practice Question 1

Which of the following factors increases a levered company's WACC assuming $R_E > R_D$?

(Select all that apply)

- a) An increase in the expected return of the market portfolio
- b) A decrease in the number of outstanding shares
- c) An increase in the number of outstanding bonds
- d) A decrease in the beta of the company's equity
- e) A decrease in the coupon rate of outstanding bonds

Practice Question 2

a) SoulGlad Company has 1258 outstanding shares that demand a yearly return of 6%. Each share will pay a dividend of \$40 next year. If dividends are expected to grow at a rate of 3% each year, what is the company's market value of equity?

Practice Question 2

b) SoulGlad Company also has 5000 outstanding bonds that mature in 27 years. Each bond has a face value of \$1000, a coupon rate of 8%, and YTM of 10%. If coupon payments occur yearly, what is the company's market value of debt?

Practice Question 3

a) The Divination Corporation's equity has a beta of 1.5. Investors can currently borrow and lend at the risk-free rate of 4%. If the expected return of the market portfolio is 8%, what is the company's cost of equity?

Practice Question 3

Assume that Divination Corporation's cost of equity is 12%

- b) Suppose the Divination Corporation has a market value of \$180000. Its outstanding bonds pay semi-annual coupons and demand a semi-annual return of 4.5%. If the bonds have an aggregate market value of \$70000, what is the company's weighted average cost of capital?

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Investment Projects

Investment projects often fall into one of the following six categories (not all inclusive!)



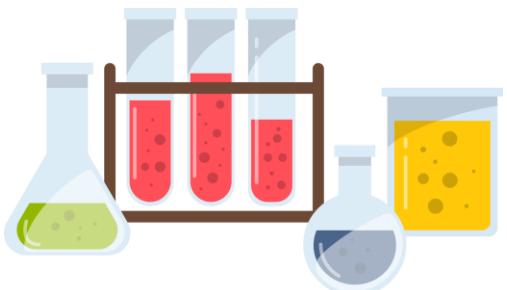
New Products



Expansion of
Existing Products



Replace Equipment
or Building



Research & Development



Exploration



Safety or Pollution
Control Devices

Cash Flow vs Profits

An investment project uses a piece of equipment that generates operating cash flows of \$6000 starting one year from today. The cash flows will last for 10 years. The equipment costed \$40000 and can be salvaged for \$4000 when the cash flows end. What is the NPV of this project if the firm's cost of capital is 6%?

Source of Cash Flow	Cash Flows			
	c_0	c_1	...	c_{10}
Operating	0	6000	...	6000
Equipment	-40000	0	...	4000

$$NPV = 6000 \left[\frac{1 - (1 + 0.06)^{-10}}{0.06} \right] - 40000 + \frac{4000}{(1 + 0.06)^{10}}$$

$$NPV = 6394.10 \quad (\text{correct way})$$

Cash Flow vs Profits (Continued)

An investment project uses a piece of equipment that generates operating cash flows of \$6000 starting one year from today. The cash flows will last for 10 years. The equipment costed \$40000 and can be salvaged for \$4000 when the cash flows end. What is the NPV of this project if the firm's cost of capital is 6%?

Source of Profit	Profit			
	c_0	c_1	...	c_{10}
Operating	0	6000	...	6000
Depreciation	0	-3600	...	-3600

$$\text{Depreciation} = \frac{\text{Cost} - \text{Salvage Value}}{\text{Useful Life}}$$

$$\text{Depreciation} = \frac{40000 - 4000}{10} = 3600$$

$$NPV = 2400 \left[\frac{1 - (1 + 0.06)^{-10}}{0.06} \right] = 17664.21 \quad (\text{incorrect way})$$

What is the Difference?

Source of Cash Flow	Cash Flows			
	c_0	c_1	...	c_{10}
Equipment	-40000	0	...	4000
Operating	0	6000	...	6000

$NPV = 6394.10$

Source of Profit	Profit			
	c_0	c_1	...	c_{10}
Depreciation	0	-3600	...	-3600
Operating	0	6000	...	6000

$NPV = 17664.21$

Why can the NPV from cash flows differ from the NPV of profits?

- (1) Depreciation (accounting doesn't expense right away, but the cash is paid right away)
- (2) Timing of revenue/expense recognition (revenue/expenses may not match with cash received/spent)

Discount Cash Flows
NOT Profits

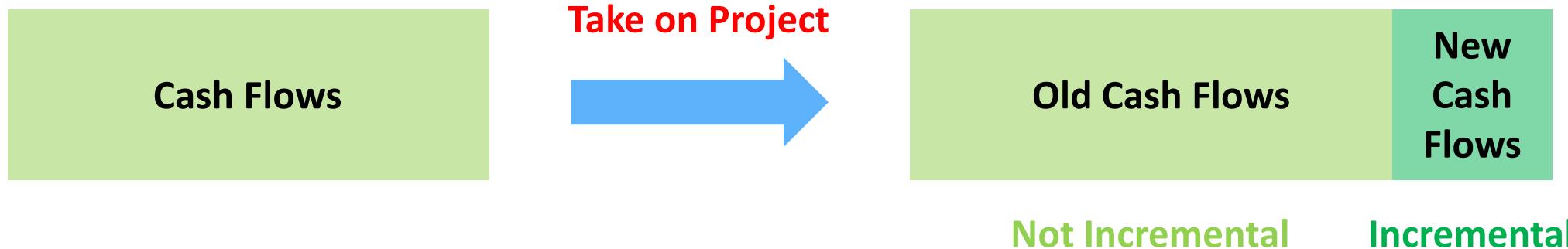
Incremental Cash Flows

Stand-alone Principle: Evaluation of a project based on its incremental cash flows

Incremental Cash Flows: Any cash flows (even incidental) that change by taking on a project/investment

Will this cash flow occur only if we accept the project?

- (1) Yes: Include in analysis because it is an incremental cash flow
- (2) No: Do not include because it will occur anyways (not incremental)
- (3) Partially: Only include the part that occurs as a direct result of taking on the project



What to Include – Incremental Cash Flows

A firm's product line will generate a net cash flow of \$150000 every year for 15 years. This product line will shut down afterwards. The firm can choose to expand this line for a cost of \$50000 today. In doing so, the firm will earn \$10000 more each year starting next year. If the firm's R_{WACC} is 4%, what is the NPV of this expansion?

“Net cash flow of \$150000 ...” **(not incremental)**

“Cost of \$50000 today” **(incremental)**

“Earn \$10000 more ...” **(incremental)**

$$NPV = -50000 + 10000 \left[\frac{1 - (1 + 0.04)^{-15}}{0.04} \right] = 61183.87$$

Cost of
\$50000 today

Earn \$10000 more each
year starting next year

What to Include – Opportunity Costs

Opportunity Cost: The most valuable alternative that is given up if a particular investment is undertaken

Arises when existing assets used by the project can be used elsewhere

A farming company is considering building a new barn on a vacant plot of land they own. If they do not use the land, they may sell it at a market price of \$500000. The materials to build the barn will cost \$80000. By building the barn, the farm will have enough storage to handle the number of purchase orders it receives each year. The barn will take 1 year to build, and operating cash flows are expected to increase by \$90000 per year for the next 17 years upon completion.

Market price of land **(opportunity cost)**

Materials to build the barn **(not an opportunity cost)**

Market price of land **(not an opportunity cost)**

What to Include – Changes in Net Working Capital

The *changes* in net working capital refer to net amount of current assets and liabilities added

$$\Delta \text{ Net Working Capital (NWC)} = \text{Added current assets} - \text{Added current liabilities}$$



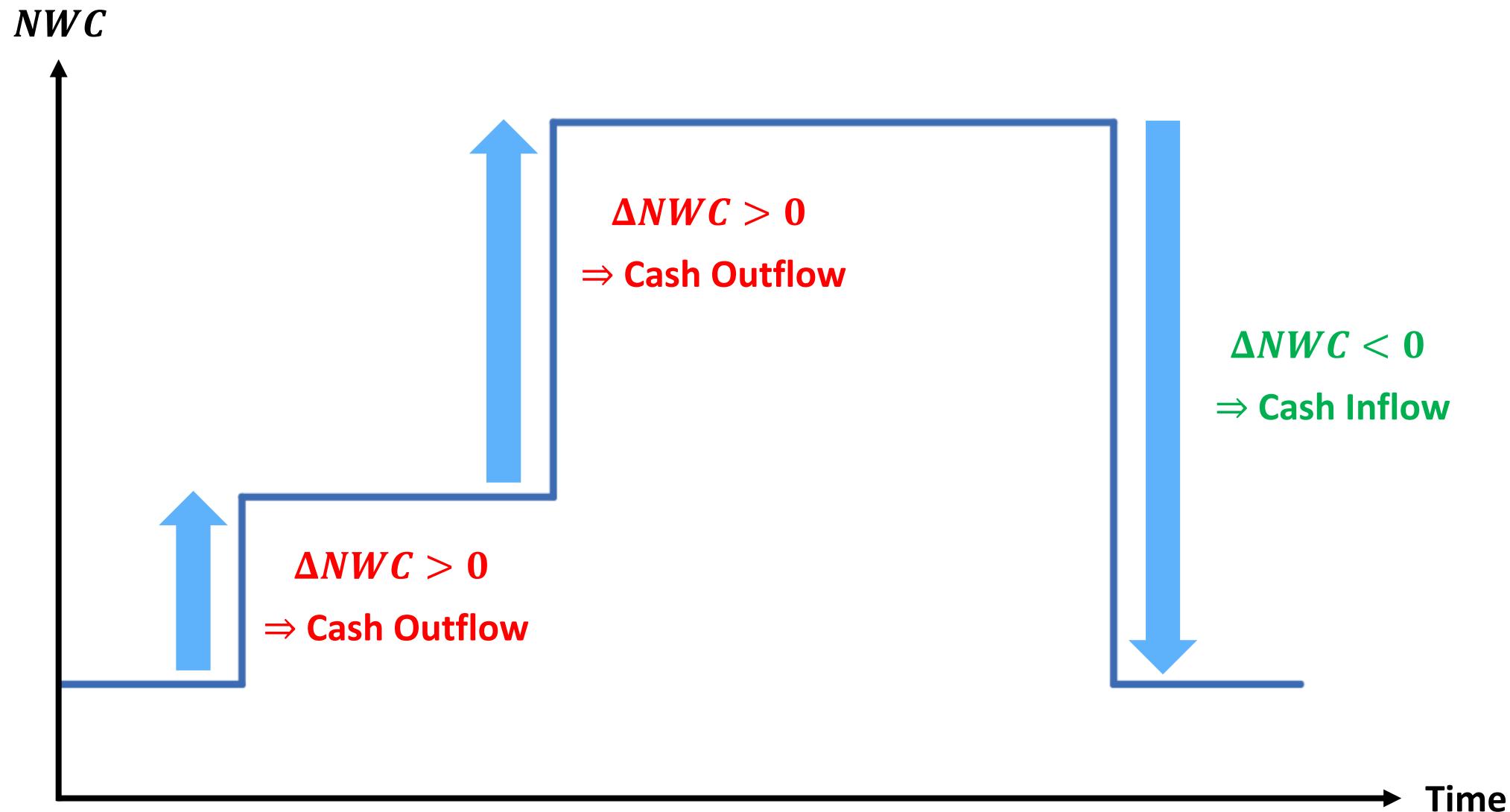
Obtaining
assets cause a
cash outflow



Obtaining a liability
(e.g., loan) causes
cash inflows

- (1) $\Delta \text{ Net Working Capital (NWC)} > 0$ represents a net cash outflow \Rightarrow deduct this net cash flow in analysis
- (2) $\Delta \text{ Net Working Capital (NWC)} < 0$ represents a net cash inflow \Rightarrow add this net cash flow in analysis

Changes in Net Working Capital Graph



What to Not Include – Sunk Costs & Financing Costs

Sunk Cost: A cost that has already been incurred regardless the project investment is made

A mining company wants to relocate their operations to a different ore deposit. To find the ore deposit, they spent \$40000 in exploration costs. If the company relocates, fixed costs will increase by \$100000. Once all the equipment is set up one year from now, operating cash flows will increase by \$30000 per year for 8 years.

Exploration costs **(sunk cost)**

Fixed costs **(not a sunk cost)**

Operating cash flows **(not a sunk cost)**

When a company finances a project, they will make interest, principal, and/or dividend payments

These actions involve cash outflows – why are they not included?

These payments are already accounted for by their cost of capital (R_{WACC})

Summary of Investment Decisions

What should we consider when we make investment decisions?

Consider:

Incremental cash flows

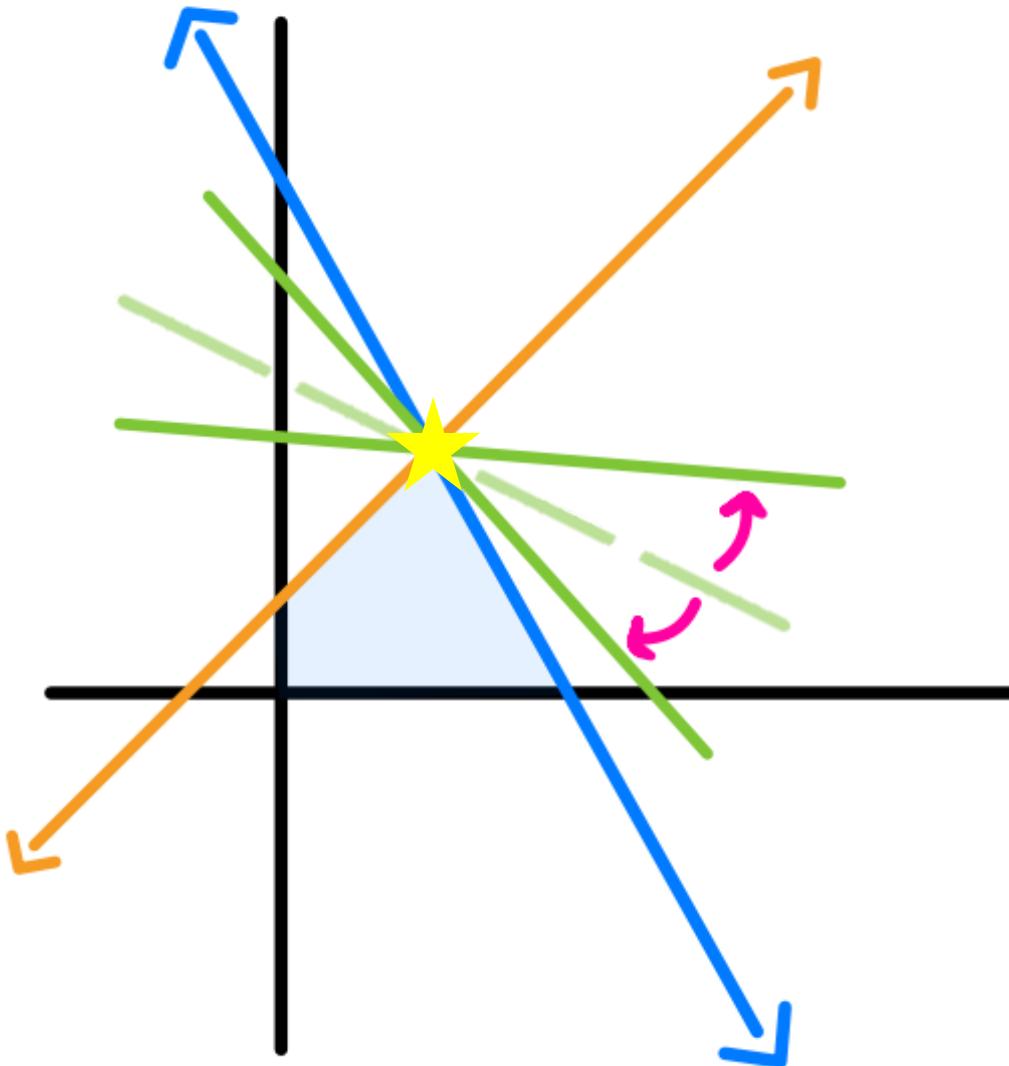
Opportunity costs

Cash flows due to changes in net working capital

Do Not Consider:

Sunk costs

Financing costs



Sensitivity & Scenario Analysis

Sensitivity vs Scenario Analysis

Sensitivity Analysis: (What we will work with in this course)

Assumes that the variables are *independent* of each other

Examines what happens to NPV when *one* variable is changed

Some variables of interest include variable costs (VC), fixed costs (FC), sales quantity (Q), cost of capital (r)

Reveals the variables that are driving cash flows (i.e., the variables that affect NPV the most when changed)

Scenario Analysis:

Assumes that the variables are *interdependent* of each other (e.g., price can affect quantity sold)

Each scenario examined has a different set of values for key variables (makes us redo the entire NPV analysis)

NPV Break-even

A pharmaceutical firm is considering a project that costs \$900000 today. The project has a six-year life and requires a return of 15%. If the project proceeds, a new type of medicine can be sold next year. The price per unit is \$2000 while the variable cost per unit is \$1000. Fixed costs are \$250000 each year. If the project's NPV is \$46120.67 when 500 units are sold each year, what is the NPV break-even quantity of units sold?

Source of Cash Flow	Cash Flows				
	c_0	c_1	...	c_6	
Initial Cost	-900000	0	...	0	
Operating (New)	0	A_{new}	...	A_{new}	

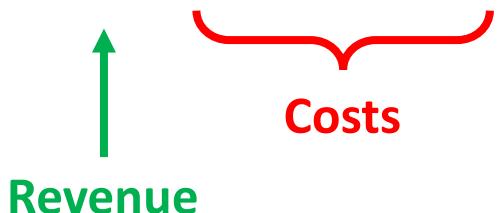
$$P = 2000 \quad VC = 1000 \quad FC = 250000$$

$$Q_{old} = 500 \quad A = Q(P - VC) - FC$$

$$Q_{new} = ? \quad n = 6 \quad R_{WACC} = 0.15$$

$$\text{Remark: } A = PQ - VC \cdot Q - FC = Q(P - VC) - FC$$

(all revenue & costs are cash based)



NPV Break-even (Continued)

Source of Cash Flow	Cash Flows			
	c_0	c_1	...	c_6
Initial Cost	-900000	0	...	0
Operating (New)	0	A_{new}	...	A_{new}

$$P = 2000 \quad VC = 1000 \quad FC = 250000$$

$$Q_{old} = 500 \quad A = Q(P - VC) - FC$$

$$Q_{new} = ? \quad n = 6 \quad R_{WACC} = 0.15$$

$$NPV_{operating-old} = [Q_{old} \cdot (P - VC) - FC] \cdot \left[\frac{1 - (1 + R_{WACC})^{-n}}{R_{WACC}} \right]$$

$$= [500 \cdot (2000 - 1000) - 250000] \cdot \left[\frac{1 - (1 + 0.15)^{-6}}{0.15} \right] = 946120.67$$

Want NPV to drop by \$46120.67 to get to 0 NPV



$$NPV_{operating-new} = [Q_{new} \cdot (P - VC) - FC] \cdot \left[\frac{1 - (1 + R_{WACC})^{-n}}{R_{WACC}} \right]$$

$$= [Q_{new} \cdot (2000 - 1000) - 250000] \cdot \left[\frac{1 - (1 + 0.15)^{-6}}{0.15} \right] = 946120.67 - 46120.67 = 900000$$

$$= [1000 \cdot Q_{new} - 250000] \cdot [3.7845] = 900000 \quad Q_{new} = 487.81$$

NPV Sensitivity Analysis

A pharmaceutical firm is considering a project that costs \$900000 today. The project has a six-year life and requires a return of 15%. If the project proceeds, a new type of medicine can be sold next year. The price per unit is \$2000 while the variable cost per unit is \$1000. Fixed costs are \$250000 each year. If the project's NPV is \$46120.67 when 500 units are sold each year, what is change in NPV if sales drop by 100 units each year?

$$P = 2000 \quad VC = 1000 \quad FC = 250000 \quad n = 6 \quad R_{WACC} = 0.15 \quad \Delta Q = -100$$

$$A = \underbrace{Q(P - VC)}_{\text{Revenue per unit}} - FC$$

When Q changes, only this part is impacted

$$\Delta NPV = \Delta Q(P - VC) \cdot \left[\frac{1 - (1 + R_{WACC})^{-n}}{R_{WACC}} \right] = -100 \cdot (2000 - 1000) \cdot \left[\frac{1 - (1 + 0.15)^{-6}}{0.15} \right] = -378448.27$$

Practice Problem 1

Dreamjolt Hostelry has invested \$1000000 to date on a new drink. The drink is now ready to be put on their menu. Their marketing department estimates that 50000 drinks will be sold each year for the next four years. The price of each drink will be \$7, and the variable cost of each drink will be \$4. Yearly fixed costs will be \$300000, which is comprised of \$60000 new fixed costs and \$240000 existing fixed costs.

To prepare the drinks, the Hostelry requires some special equipment. The equipment will be purchased immediately and costs \$250000. At the end of the 4th year, the equipment can be salvaged for \$20000. Furthermore, Dreamjolt Hostelry's cost of capital is 10%. Answer the questions in the following slides.

Practice Problem 1

a) What is the NPV of all non-operating cash flows?

Practice Problem 1

b) What is the NPV of all operating cash flows?

Practice Problem 1

Assume the NPV of the entire project is \$50000 and the NPV of all operating cash flows is \$300000

- c) What is the NPV break-even price?

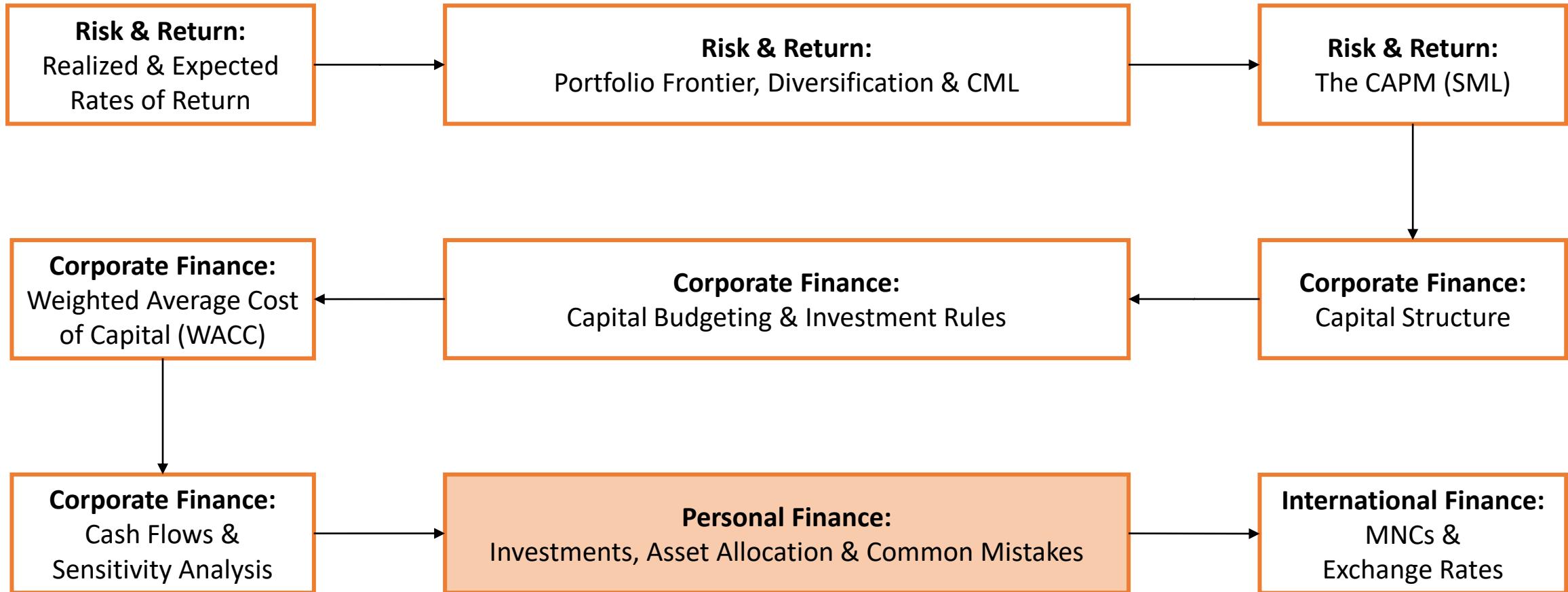
Practice Problem 1

d) If incremental fixed costs decrease by \$20000, how will this affect NPV?

Practice Problem 1

e) If the salvage value of the equipment increases by \$4250, how will this affect NPV?

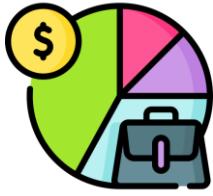
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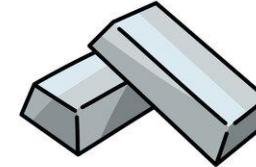
Some Investment Instruments



Individual Securities



Portfolios



Metals



Real Estate



Private Equity

Facts About Mutual Funds

What are mutual funds?

Portfolios of investments (e.g., stocks, bonds, etc.) that are consistent with portfolio theory

How are they managed?

Actively-managed: Managers pick the most “promising” stocks (less common today < 50%)

Passively-managed: Fund replicates the returns of a benchmark

What are some additional characteristics?

Open-ended: The fund issues new shares as more people invest – can sell stocks back to the fund at the current market value

Mutual funds can specialize in specific investments types (e.g., government bonds, large-cap stocks, etc.)

Specialization in Investments

Specific Country:

RBF1952	RBC China Equity Fund	F	International Equity Funds	High
RBF2322	RBC China Equity Fund US\$	F	International Equity Funds	High
RBF676	RBC Japanese Equity Fund	F	International Equity Funds	Medium

Specific Market Capitalization Size:

RBF5542	RBC U.S. Mid-Cap Value Equity Fund US\$	F	U.S. Equity Funds	Medium
RBF693	RBC U.S. Small-Cap Core Equity Fund	F	U.S. Equity Funds	Medium to high

Specific Security:

RBF1639	RBC \$U.S. Short-Term Government Bond Fund	F	Fixed Income Funds	Low
RBF1654	RBC \$U.S. Short-Term Corporate Bond Fund ³²	F	Fixed Income Funds	Low

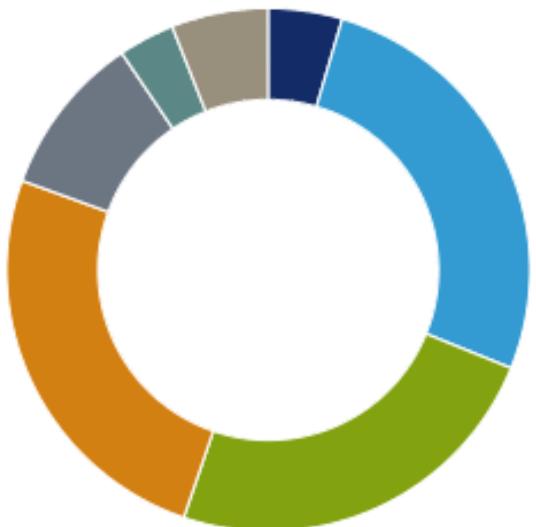
More Facts About Mutual Funds

Some mutual funds are less specialized and contain many asset classes (e.g., stocks, bonds, derivatives, cash)

Portfolio analysis

Asset type mix

As of 3/31/2024



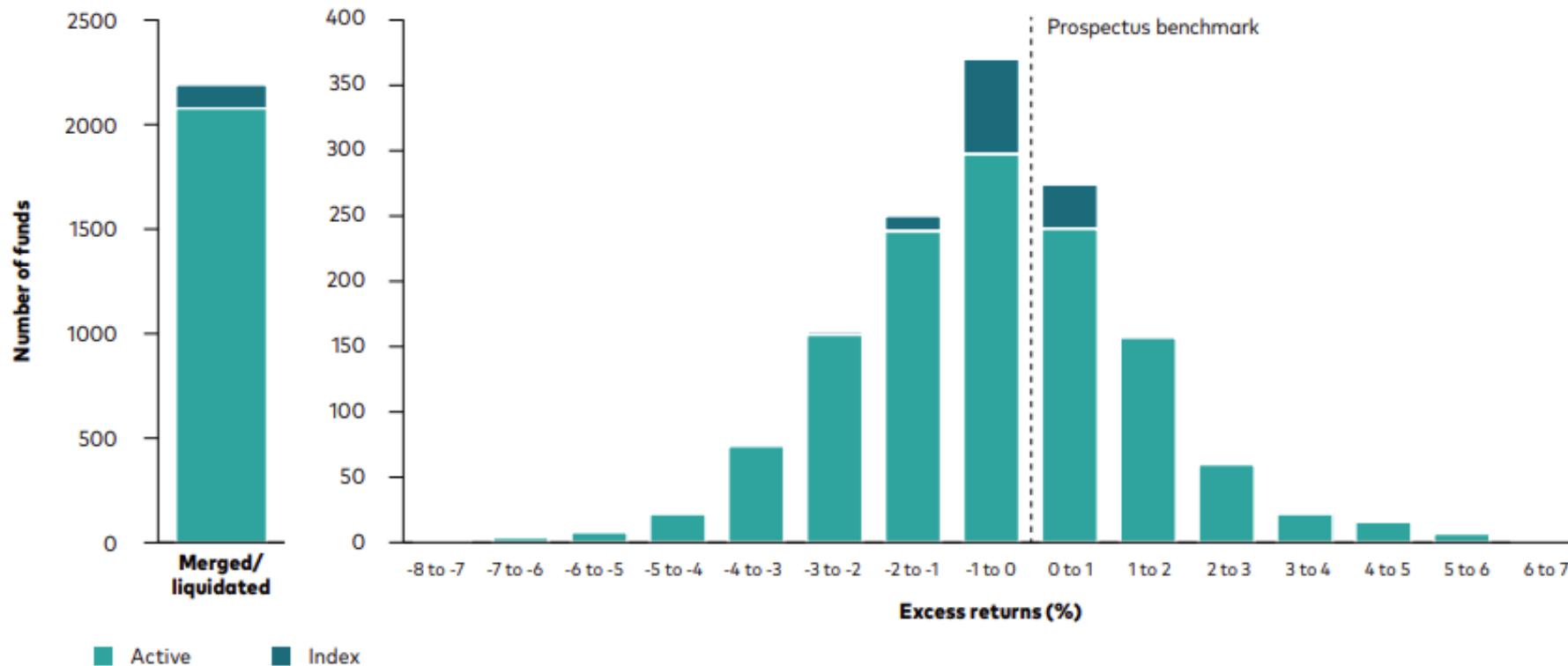
Cash	4.6%
Fixed income	26.6%
Canadian equities	24.1%
U.S. equities	25.3%
European equities	10.0%
Asia-Pacific equities	3.5%
Emerging markets equities	6.0%

Even More Facts About Mutual Funds

The return on an actively managed mutual fund depends on the fund manager's skill at picking investments

But there is no evidence that managers can persistently beat the market

a. Distribution of equity funds' excess return



Facts About Exchanged Traded Funds (ETFs)

What are ETFs?

Like mutual funds but are continuously traded on an exchange – provides more liquidity

Specific types?

Index ETFs (most common): Tries to replicate some index based on stocks, bonds, commodities

Example: SPDR S&P 500 tracks the S&P 500 Index

Other characteristics?

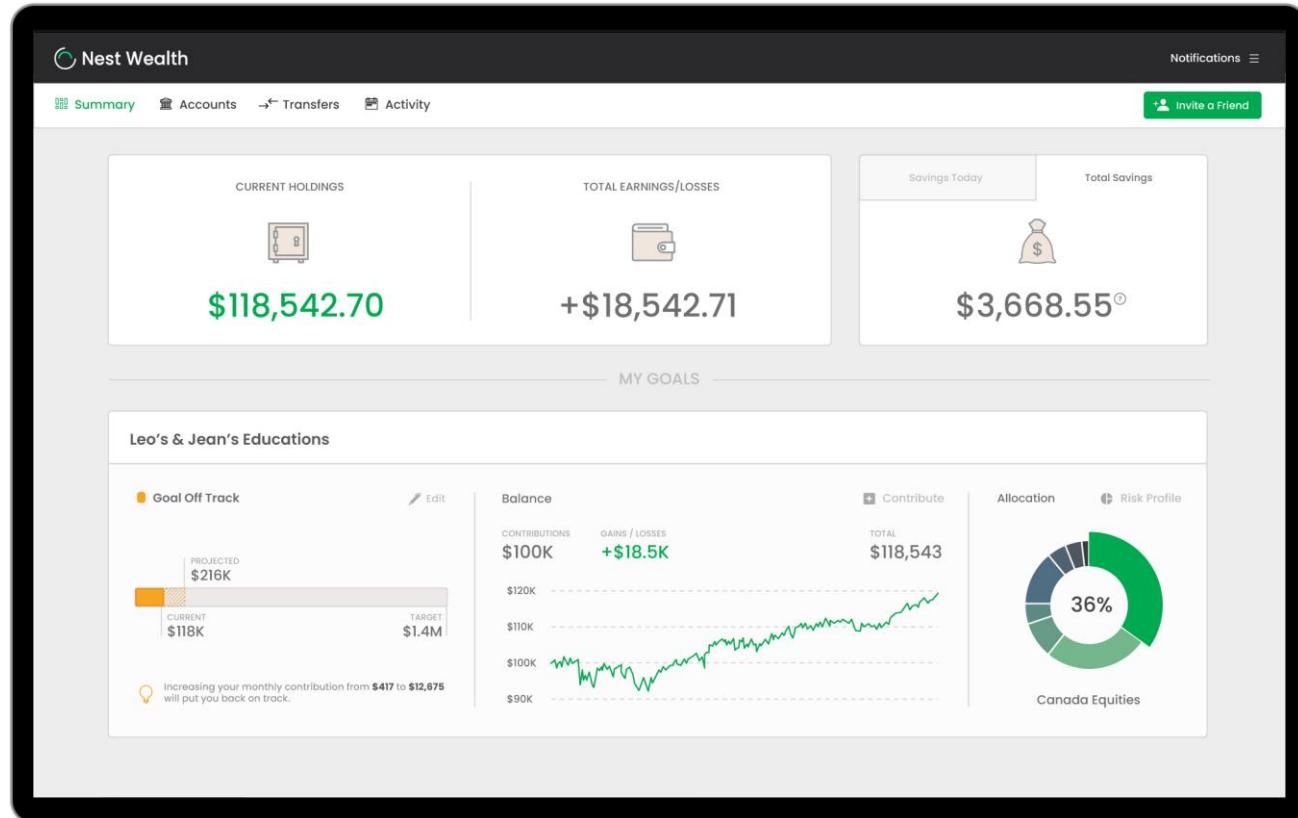
Typically have lower fees than mutual funds

Effectively replicates the target index, so the tracking error is close to 0

Robo-advisors

What are Robo-advisors?

Automated, algorithm-driven financial services with little to no human supervision (portfolio on “auto-pilot”)



Costs of Investment

Management Fees:

Investing in a mutual fund or ETF makes you pay a fee of 0.10% – 3%

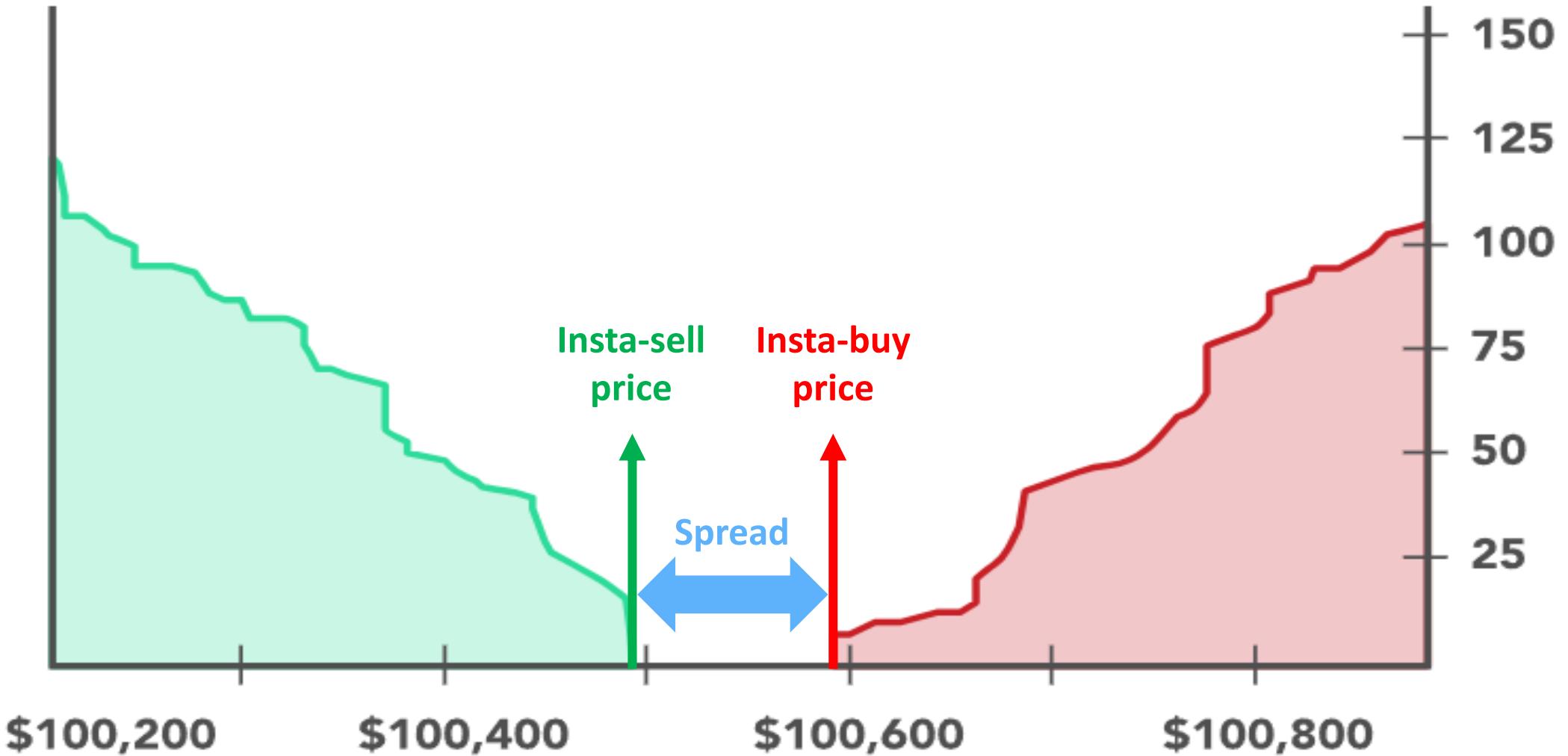
Brokerage Fee:

Fee to open & maintain an account ($\approx \$100$ per year + a small fee of $\approx \$5 – \10 per transaction)

Bid-ask Spread:

The price difference at which you can instantly buy an asset and instantly sell the same asset simultaneously

Bid-ask Spread



Can we Pick Stocks?

Do stocks even perform well?

Most individual stocks do worse than treasury bills, but diversified stock portfolios do better

Only a few stocks generate high returns which drive the whole stock market performance upwards

Stock-picking is hard!

How do we know our stock is performing well?

Compare the stock you own to an index (benchmark) with similar investment scope and risk

Investing Across the Lifespan

How easy is it to access my funds?

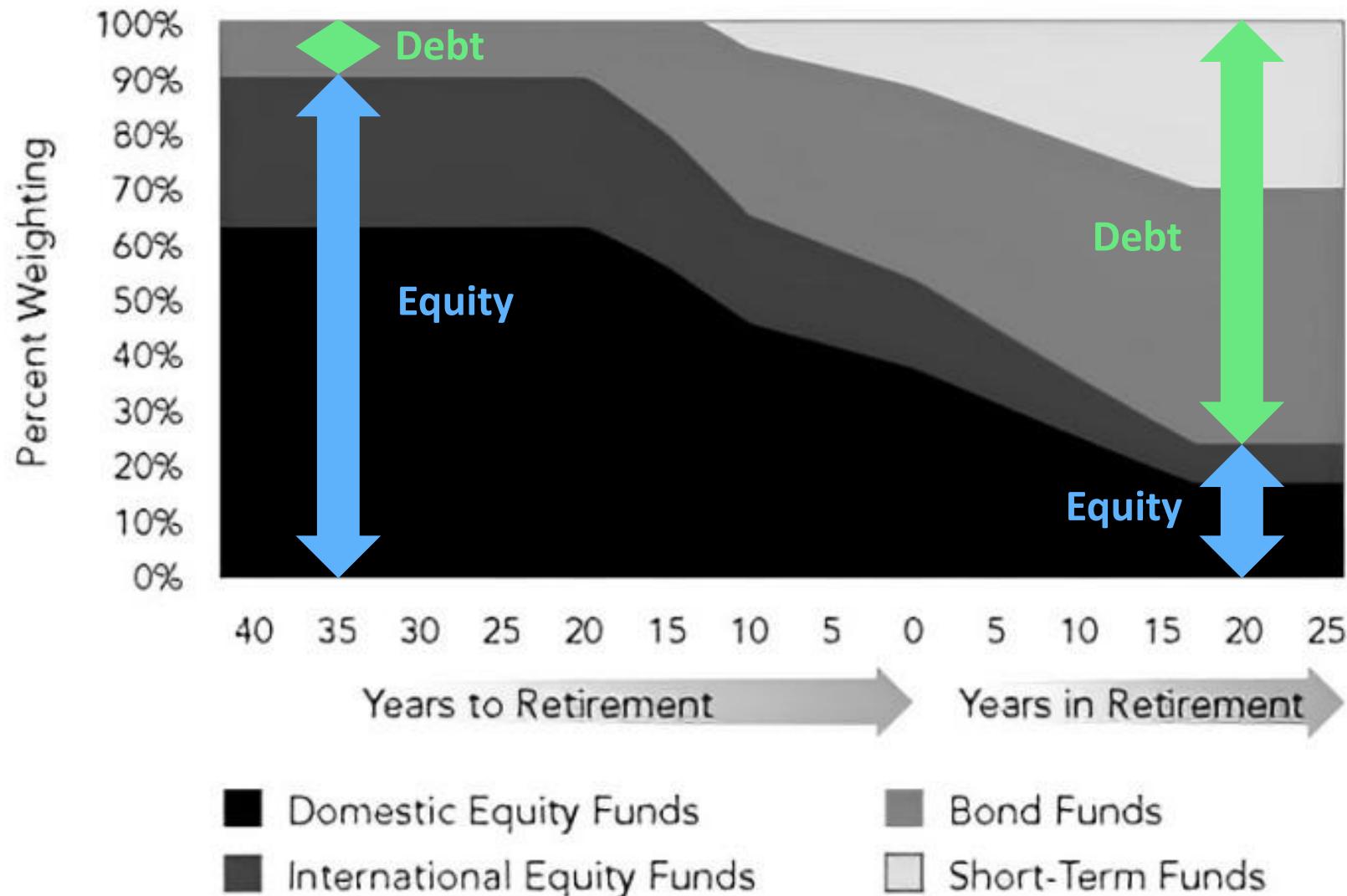
Investors in early stages of life should consider funds that are easy to access

If you don't need easy access to funds, one may want to invest in small-capitalization growth funds

How much risk do I bear?

As you grow older, consider shifting your portfolio to less risky assets (e.g., go from equity to debt)

Risk Across the Lifespan



What has History Said?

Set Realistic Goals:

Aiming for unrealistic goals may cause you to take unnecessary amounts of risk

Should you borrow to invest?

Expected returns go up but this action amplifies risk

From portfolio theory we know that $SD(R_{portfolio})$ increases as we borrow more at R_f

Should you take more risk to recover from losses?

This could lead to future losses – think of how gambling works!

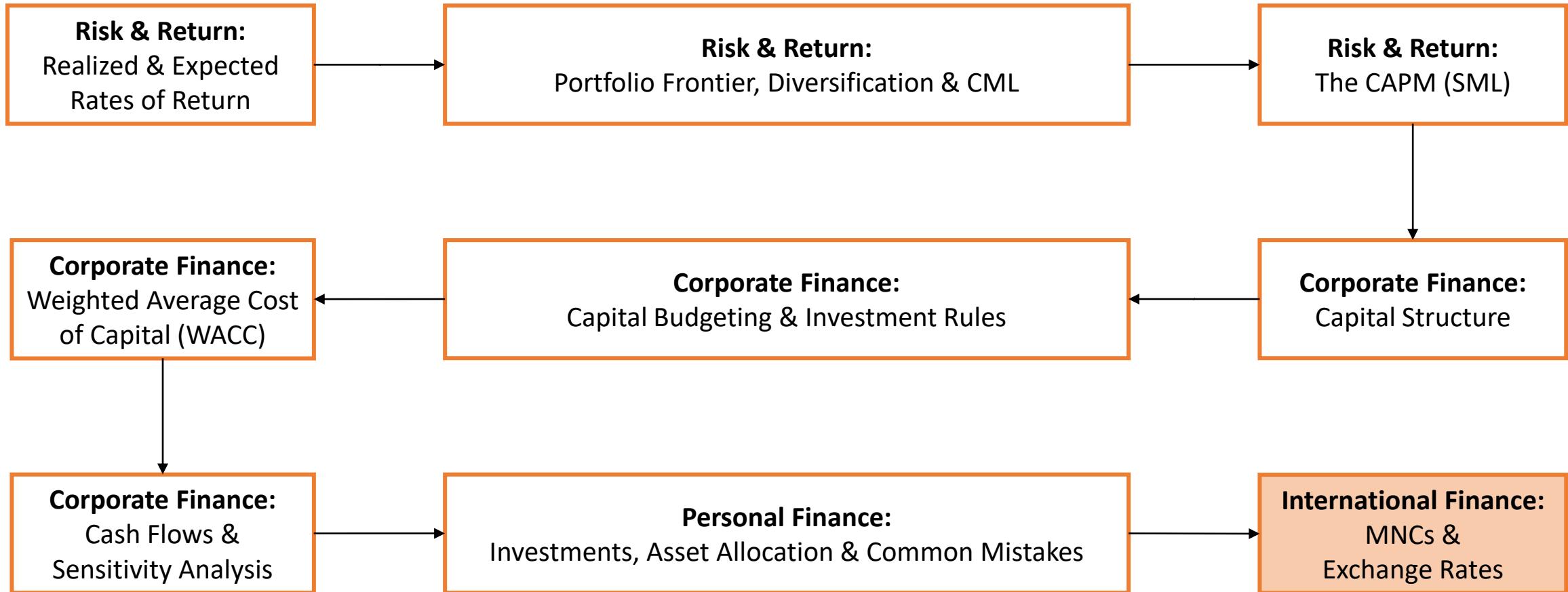
Practice Question 1

Which of the following is true about investments & personal finance?

(Select all that apply)

- a) The return on individual stocks are worse than the return on treasury bills on average
- b) Most mutual funds today are actively-managed
- c) One should always borrow to invest as it yields a higher expected return
- d) A university student's portfolio will likely bear more risk than a retired professor's portfolio
- e) ETFs replicate some indices like the S&P 500 quite successfully as their tracking error is near 0

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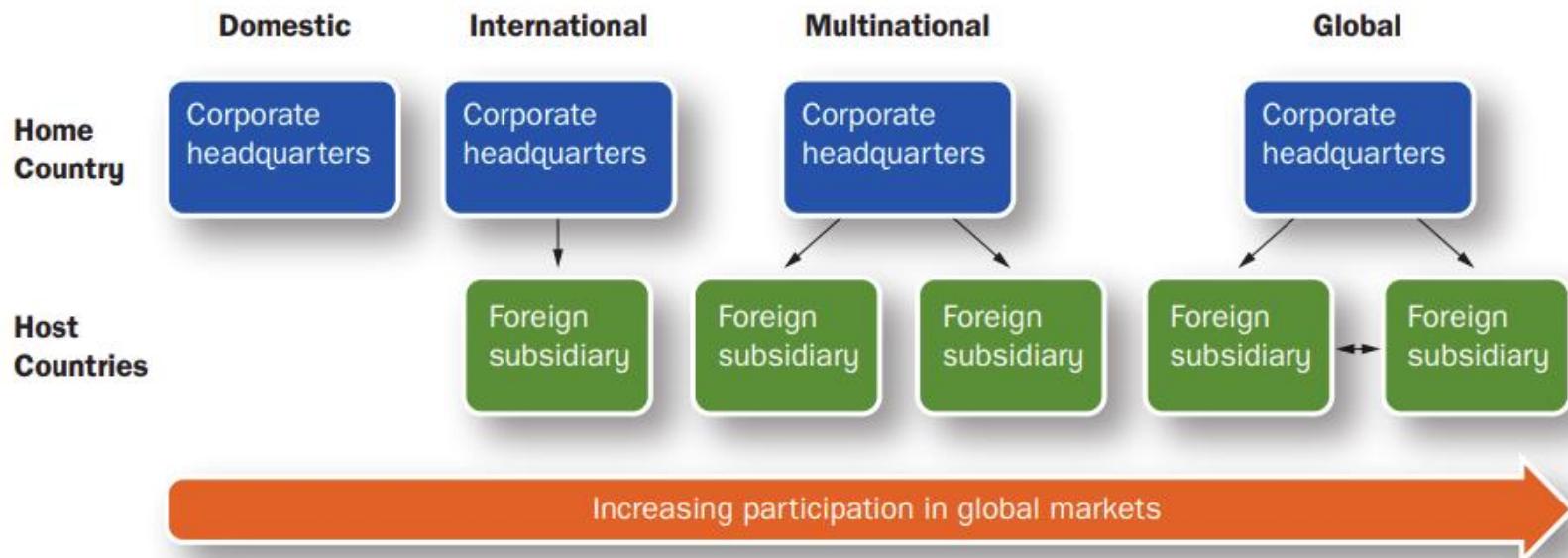
What is a Multinational Corporation?

A multinational corporation (MNC) is involved in selling goods or services in more than one country

Possess Centralized Management:

Parent company in home country & subsidiaries in foreign countries

Can raise capital in several markets and coordinate resource allocation (investment/financing decisions)

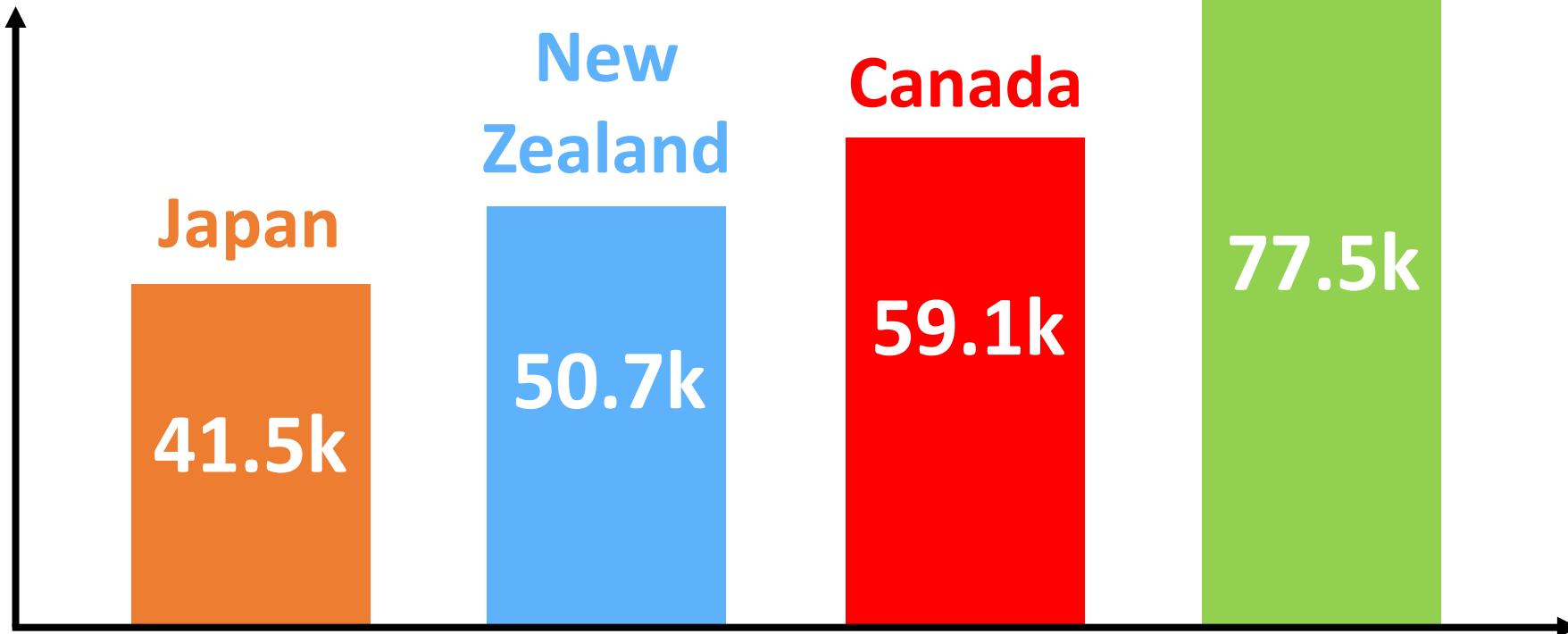


Factors of Production in MNCs

MNCs rely on the international mobility of factors of production (land, labour, capital)

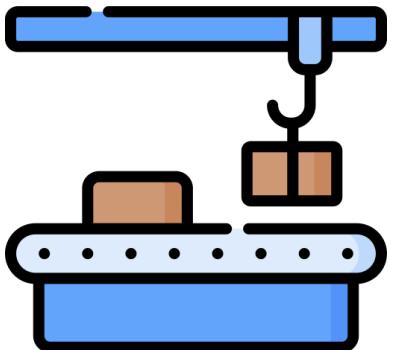
Differences in Labour Costs:

Mean Wage
2022 (USD)

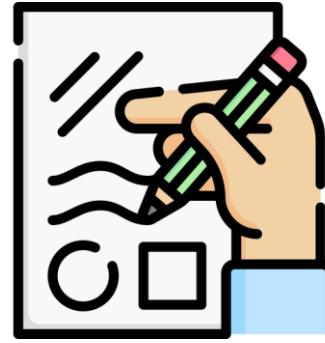


Factors of Production in MNCs (Continued)

Unique National Attributes & Skills:



Assembly (China)



Designing (USA)



Mining (Mongolia)

Factors of Production in MNCs (Continued)

Natural Resources & Geography:



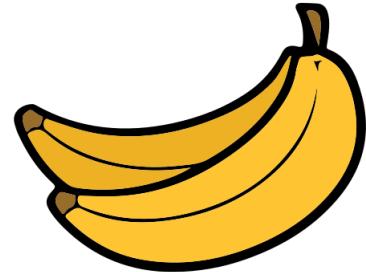
Coal (USA)



Oil (Saudi Arabia)



Silk (China)



Bananas (India)



Desert



Sea/Ocean



Rainforest

Why Become a Multinational Corporation?

Search for Raw Materials:

Foreign investments were made to seek out oil and ore

Market Seeking:

Can sell your service or good overseas in foreign markets

Cost Minimization:

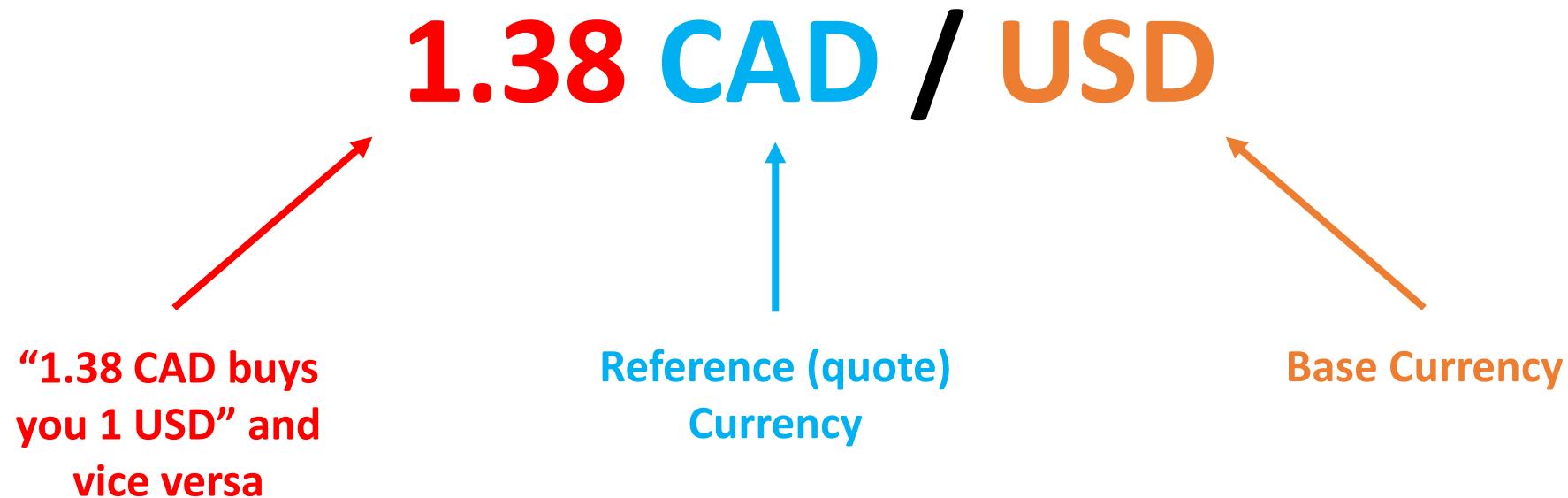
The cost of producing overseas could be cheaper

Exploring Financial-market Imperfections:

Firm can achieve diversification from operating in countries

What Makes International Finance Special?

Exchange Rate: The price of one nation's currency in terms of another



Foreign Exchange Risk: Changes in exchange rates that affect the firm's profitability and value

Types of Exchange Rates

Spot Rate:

Exchange rate used when currencies are traded for “immediate” delivery

Delivery arrives 2 days later

Forward Rate:

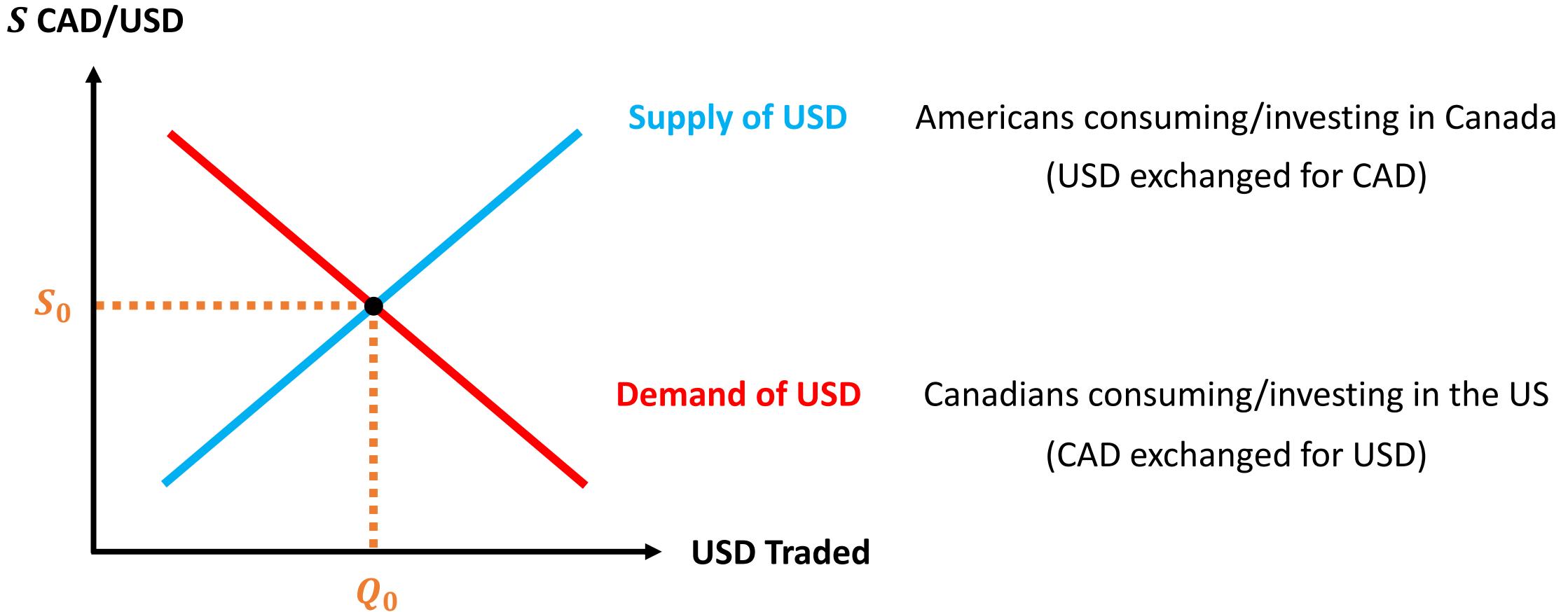
Exchange rate used for foreign exchange in the future

It is a quoted, fixed rate that cannot be changed once you enter a forward contract

You “lock in” the future rate at which you will exchange currencies today – reduces foreign exchange risk

How are Exchange Rates Set?

Exchange rates are set by market-clearing (equilibrium) prices from foreign supply and demand



Currency Appreciation & Depreciation

Currencies will appreciate and depreciate as the equilibrium exchange rate moves around

Currency Appreciation:

Increase in the value of the *base currency* with respect to the reference currency

Example: 1.38 CAD/USD rises to 1.42 CAD/USD (USD is appreciating with respect to CAD)

Think of “getting more” out of the same USD

Currency Depreciation:

Decrease in the value of the *base currency* with respect to the reference currency

Example: 1.38 CAD/USD drops to 1.33 CAD/USD (USD is depreciating with respect to CAD)

Think of “getting less” out of the same USD

The Duality of Appreciation & Depreciation

 **1.38 CAD / USD** \leftrightarrow  **0.72 USD / CAD**

USD Appreciates

(more attractive to buy/invest in US)

CAD Depreciates

(less attractive to buy/invest in Canada)

 **1.38 CAD / USD** \leftrightarrow  **0.72 USD / CAD**

USD Depreciates

(less attractive to buy/invest in US)

CAD Appreciates

(more attractive to buy/invest in Canada)

Calculating Changes in Exchange Rates

The following formula calculates the appreciation or depreciation of the *base currency* as a percent change

$$\Delta S = \frac{S_1 - S_0}{S_0}$$

S_0 = Old value of the exchange rate

S_1 = New value of the exchange rate

S_0 CAD/USD  S_1 CAD/USD (Measures appreciation/depreciation of USD)

S_0 USD/CAD  S_1 USD/CAD (Measures appreciation/depreciation of CAD)

$\Delta S > 0 \Leftrightarrow$ Base currency appreciates (reference currency depreciates)

$\Delta S < 0 \Leftrightarrow$ Base currency depreciates (reference currency appreciates)

What Affects the Equilibrium Exchange Rate?



Inflation



Economic Growth



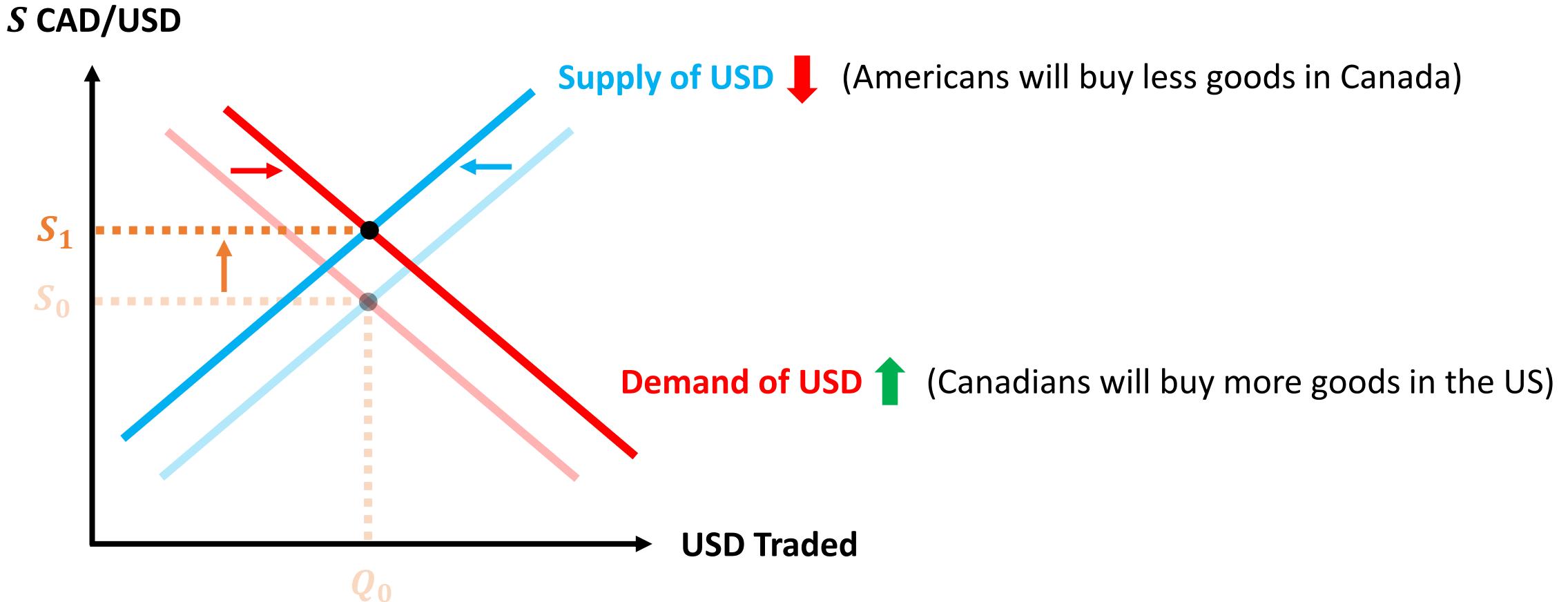
Interest Rates



Political & Economic Risks

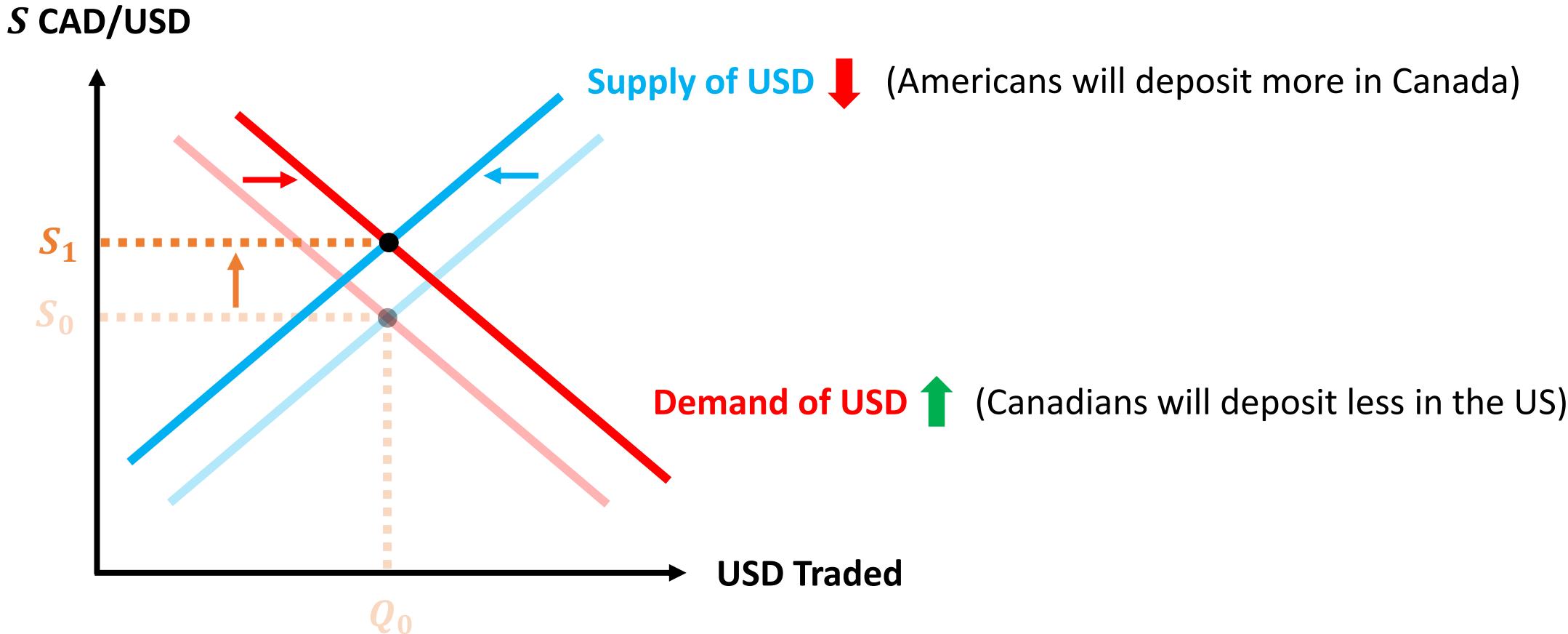
Factor #1 – Inflation Rates

What if Canada's prices went up? USD appreciates & CAD depreciates!



Factor #2 – Relative Interest Rates

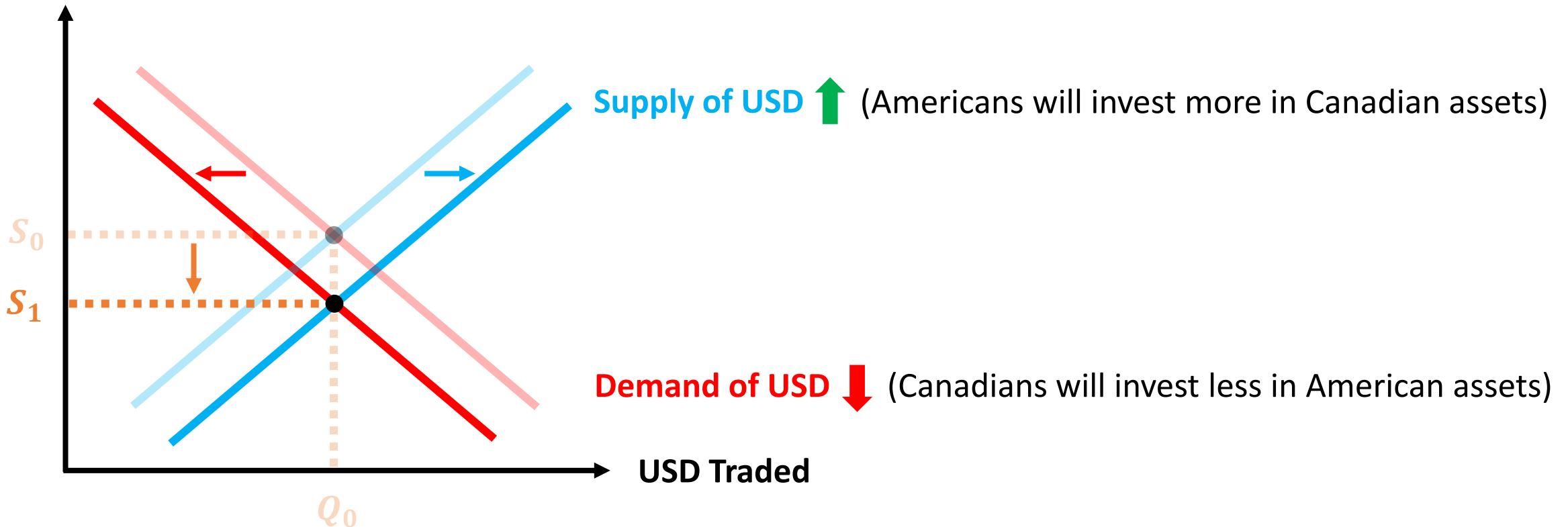
What if US's real interest rate went up? USD appreciates & CAD depreciates!



Factor #3 – Relative Economic Growth

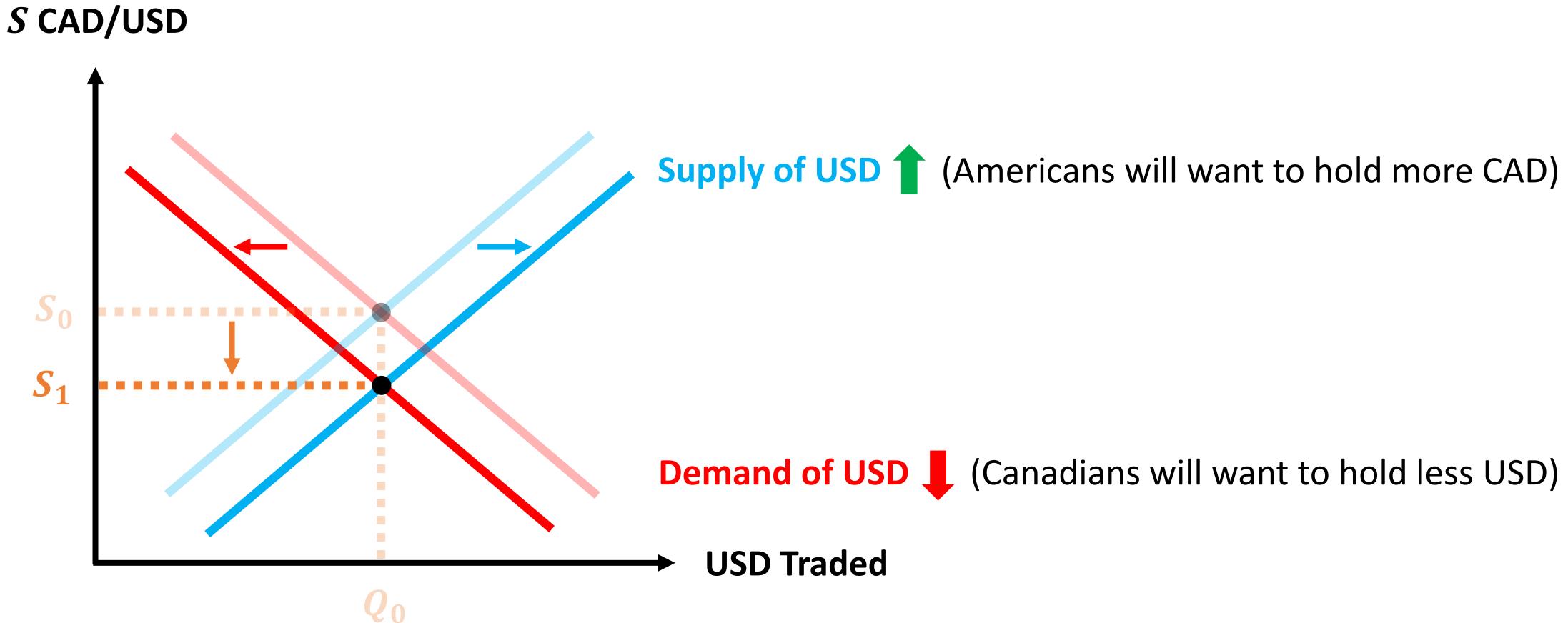
What if Canada's GDP is expected to grow more than the US's GDP? USD depreciates & CAD appreciates!

S CAD/USD



Factor #4 – Political & Economic Risks

What if US has higher political risk? USD depreciates & CAD appreciates! Investors prefer low-risk currencies



What Does the Central Bank do?

It can use monetary policy to affect equilibrium exchange rates by controlling economic growth and stability

Bank of Canada Objectives:

- (1) Allow Canadians to spend & invest with more confidence
- (2) Encourages long-term investment in Canada's economy
- (3) Sustain job creation and induce greater productivity

What can Monetary Policy do?

There are two major things the Bank of Canada (BoC) can do with monetary policy

Inflation-control Target:

Changes the policy interest rate to change market interest rates (r)

Lower interest rates encourage borrowing (PV of investments go up) and spending (higher inflation)

Higher interest rates slows down borrowing (PV of investments go down) and spending (lower inflation)

Flexible Exchange Rate:

Can intervene in the currency market when the value of CAD is too volatile or there is no market liquidity

Takes around 6-8 quarters to work – aims to protect the value of the CAD

Pros & Cons of a Strong Currency

If we compare Canada to the US, a strong CAD will mean a high USD/CAD or low CAD/USD ratio

Practically we can think of it as: “If I go from Canada to another country for vacation, I will feel rich”

Advantages:

Priced of imported goods, services, and raw materials are lower (one unit of CAD gives more foreign currency)

Foreign investment costs are lower

Disadvantages:

Exports become less competitive in foreign markets (one unit of foreign currency gives less CAD)

Firms face more domestic competition from lower-priced imports

Foreign firms have higher operating costs in Canada, so there will be less direct foreign investment

Pros & Cons of a Weak Currency

If we compare Canada to the US, a weak CAD will mean a low USD/CAD or high CAD/USD ratio

Practically we can think of it as: “If I go from Canada to another country for vacation, I will feel poor”

Advantages:

Exports become more competitive in foreign markets (one unit of foreign currency gives more CAD)

Firms are more competitive domestically against higher-priced imports

Foreign firms have lower operating costs in Canada, so there will be more direct foreign investment

Disadvantages:

Prices of imported goods, services, and raw materials are higher (one unit of CAD gives less foreign currency)

Practice Question 1

Which of the following is true about a multinational corporation?

(Select all that apply)

- a) It is comprised of a parent company in a home country and subsidiaries in foreign countries
- b) It often adapts to and exploits differences in costs, labour, and skills across countries
- c) Its goods or services are produced and sold in more than one country
- d) Its management is decentralized to allow global coordination when allocating resources
- e) It may form because of market seeking, market imperfections, and cost minimization

Practice Question 2

Which of the following will make the Canadian dollar appreciate with respect to the American dollar?

(Select all that apply)

- a) An increase in the real interest rate of the United States
- b) When investors expect the GDP of Canada to fall more than the GDP of America next year
- c) A public announcement that states that Canada's economic risk will fall
- d) The price of US goods increase by 10%
- e) The political risk of Canada increases due to an upcoming election

Practice Question 3

At the beginning of 2022, one British pound could buy €1.19 worth of German goods. At the end of 2022, the same British pound could only buy €1.13 worth of German goods. How much did the British pound change with respect to the Euro (€)?