# **COMM** 204

# 2020W1 final review session

**Answer Key** 



**PREPARED BY** 

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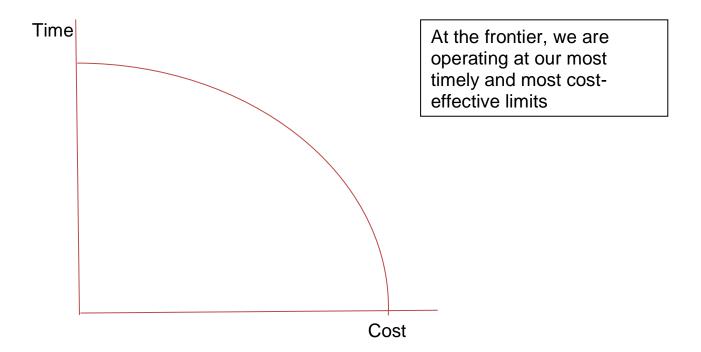
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# **The Operations Frontier**

Operations management finds the operations frontier, determines the optimal position for the firm to be on the frontier, and pushes the frontier out with innovation





# **Process Analysis**

### **Key Terms**

**Unit Flow:** Items that flow through the process

**Activities:** Transformation steps in the process

Resources: What performs activities

**Buffers:** Storage for flow units

**Decision Points:** Fork in the road

Theoretical Flow Time: Amount of time that a flow unit is in the process

(ex. 20 seconds)

Unit Load: Amount of time that a resource needs to process a flow unit

(ex. 5 seconds)

Capacity rate: Maximum possible output rate (ex. 3 bubbles teas/ minute)

Bottleneck: The slowest resource that determines the capacity rate for the

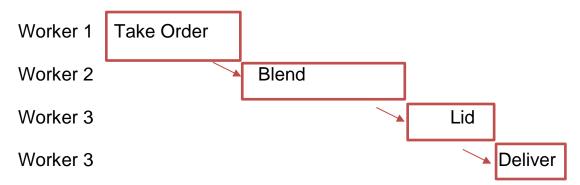
entire process \*there may be more than 1



# Linear Flow Diagram



### Swim Lane Flow Diagram



# **Gantt Diagram**

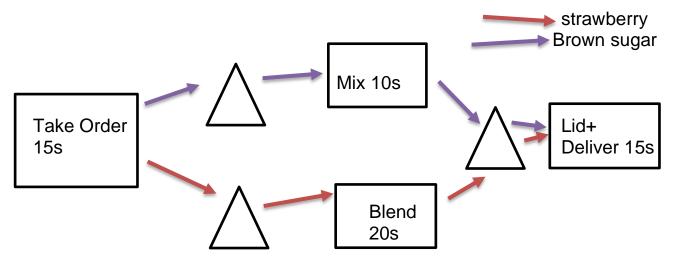
W1	Take	15s	,		15	S		15	S							
	Order															
W2	Blend			20	S		20:	S		20	S					
W3	Lid						10	S		10	S		10	S		
W3	Deliver								5			5			5	
									S			S			S	



- Q1. In the bubble tea example, what is the...
  - a) Theoretical flow time? 50 seconds
  - b) Unit load of worker 3? 15 seconds
  - c) Capacity rate of worker 3? 60/15= 4 bubble teas/minute
  - d) Capacity rate of worker 2? 60/20= 3 bubble teas/minute
  - e) Bottleneck? Worker 2 is the bottleneck because they have the HIGHEST UNIT LOAD, or in other words, the LOWEST CAPACITY RATE



Q2: A bubble tea shop offers two flavours: brown sugar milk tea and strawberry slush. Both teas require worker 1 to take the order, which takes 15 seconds, and worker 3 to put on the lid and deliver, which takes 15 seconds. The strawberry slush also requires worker 2 to blend for 20 seconds, while the brown sugar milk tea requires worker 2 to mix for 10 seconds.



 a) If customers only ordered brown sugar, what would the capacity rate be? (per minute)

Bottleneck: Workers 1 and 3

Capacity Rate=60/15=4 teas/minute

b) If there are 100 strawberry orders/hour and 60 brown sugar orders/hour, what is the bottleneck?

We know that 62.5% of orders are strawberry and 37.5% are brown sugar

Worker	Unit	Unit	Units load mix	Cap Rate mix
	load B	load S	(0.375B+0.625S)	-
1	15	15	15	60/15=4/min
2	10	20	16.25	60/16.26=3.7/min
3	15	15	15	60/15=4/m

Worker 2 is the bottleneck resource.

### More Key Terms

Throughput rate: Actual output rate (minimum of capacity rate and input rate) (ex. 3 bubble teas/minute)

Input Rate: Rate at which flow units arrive at the process (ex. 2 orders/minutes)

**Flow Time:** Average time for a unit to move through the system (ex. 4 minutes)

Flow Time Capacity Rate
Throughput rate

**Cycle Time:** Average time between completion of units (ex. 2 minutes)



# **Utilization**

**Utilization=Throughput Rate/Capacity Rate** 

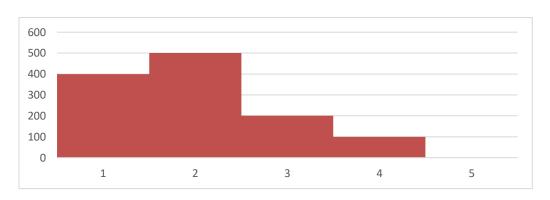
\*always less than or equal to 100%

Implied Utilization=Input Rate/Capacity Rate



# **Inventory Build-Up**

### **Discrete**



# Average Inventory=∑ *Inventory Build Up/*Total Slots

# Q3: Calculate Output and Inventory Build-up

Period	Input	Capacity	Output	Inventory
0				400
1	000	1000	1000	400+000 4000 200
I	900	1000	1000	400+900-1000=300
2	900	900	900	300+900-900=300
3	700	600	600	300+700-600=400
4	0	600	400	400+0-400=0



### **Continuous**

### Average Inventory=Area under the curve/total time

R<sub>i</sub>(t): input rate at time (t)

R<sub>o</sub>(t): output rate at time (t)

 $\Delta R(t)$ : instantaneous inventory accumulated at time (t) (slope)

$$\triangle R(t) = Ri(t) - Ro(t)$$

I(t): Number of units of inventory in process at time (t)

For a straight line:  $I(t_2)=I(t_1)+ \Delta R^*(t_2-t_1)$ 



Q4: a) Calculate average inventory

$$\frac{[(3*1/2)+(3*3+3*3/2)+(6*2)+(6*2/2)+(12*2/2)]}{8}=5.625$$

b) What is the instantaneous inventory accumulated at 2:00?

$$I(t_2)=I(t_1)+ \Delta R^*(t_2-t_1)$$

$$4=3+\Delta R^*(2-1)$$



 $\Delta R=1$ unit/hour

\*\*we could also calculate rise/run

# Little's Law

**Average Inventory (I):** The average number of units/customers in the system (ex. 5 bubble teas)

Average Throughput Rate (R): The average actual output rate (lower of capacity rate and input rate) (ex. 3 bubble teas/ minute)

**Average Flow Time (T):** The average time for a unit to move through the system (ex. Hours)

I = R \* T

Days of Inventory: Average number of days that a unit of inventory is held

Days of Inventory= Cost of Inventory\*365/COGS

T= I/R

**Inventory Turnover**: How many times the inventory has been replaced in a year

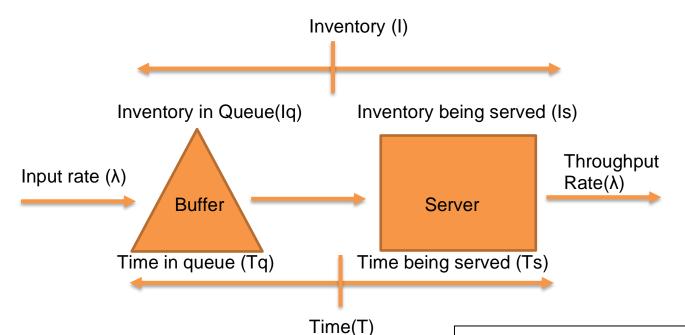
Inventory Turnover=COGS/Avg Inventory

**=Cost of Output/Cost of Input** 

= 1/T



# **Variability in Processes**



λ (units/time): long-run avg throughput rate

1/λ (time): inter-arrival time

 $\mu$  (units/time): long-run avg capacity rate of a

server

 $1/\mu$  (time): avg processing time of a server

c: number of servers

p: utilization

Ca: coefficient of variation for interarrival times

Cs: coefficient of variation for service times

T=Tq+Ts

I=Iq+Is

Iq=λ\*Tq

Is=λ\*Ts

**I**≡λT

$$q = \frac{p^{\sqrt{2(c+1)}}}{1-p} * \frac{ca^2 + cs^2}{2}$$

 $p = \lambda/c\mu$ 

Ca=SD( $1/\lambda$ )/mean( $1/\lambda$ )

 $Cs=SD(1/\mu)/mean(1/\mu)$ 



### **Queues**

G – "generally distributed" (must solve for it)

M – "exponentially distributed" (=1)

D – "deterministic" (=0)

### Interarrival time distribution/Service time distribution/# of servers

Queue Type	What's it mean?	PK Formula
G/G/1	"Interarrival time are generally distributed, service times are generally distributed, there is 1 server"	$Iq = \frac{p^2}{1-p} * \frac{C\alpha^2 + Cs^2}{2}$
M/M/1	"Interarrival times are exponentially distributed, service times are exponentially distributed, there is 1 server"	$Iq = \frac{p^2}{1-p}$
G/G/c	"Interarrival times are generally distributed, service times are generally distributed, there are c servers"	$q = \frac{p\sqrt{2(c+1)}}{1-p} * \frac{Ca^2 + Cs^2}{2}$
M/D/1	"Interarrival times are exponentially distributed, service times are deterministic, there is 1 server"	$Iq = \frac{p^2}{1-p} * \frac{1}{2}$



Q5: At Starbucks, Mary is the only server and can serve 45 customers per hour. On average, a new customer enters the store every 2 minutes. There are, on average, three customers in the store.

a) What is the utilization? How long do customers have to wait in line?

b) A second employee, Tim, joins Mary in the afternoon. Tim can serve customers just as quickly as Mary, and the store stays just as busy while they are together (inter-arrival time does not change). Service times and interarrival times are both exponentially distributed. Using PK formula, determine the average number of people waiting in line.

# **Project Management**

**Project:** Set of related tasks/activities that are directed towards some major output that require time to perform



Critical Activity: An activity which, if delayed, will delay entire project

**Critical Path**: The path on which all activities are critical activities. In other words, it is the path that takes the longest amount of time to complete.

**Crashing**: Reducing time to complete an activity

**Crash Time:** The minimum possible time that it will take to complete an activity if it is crashed as much as possible

Crash Cost: The cost of the activity if it is crashed as much as possible

**Crash Cost/day= (Crash Cost)-(Cost)/(Time)-(Crash Time)** 

**Crash Limit = Time - Crash Time** 

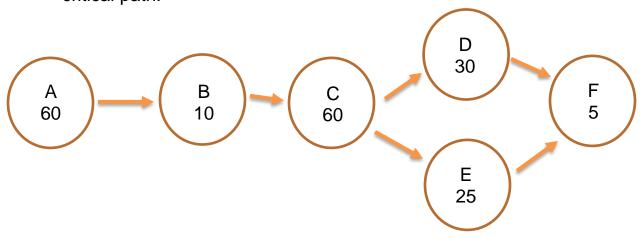


Q6: The plan to operate Lucy's Diner is below.

a) Calculate the crash limit and crash cost/day.

	Precedence	Time	Crash	Crash	Cost	Crash	Crash
			Time	Limit		Cost	cost/day
A	None	60	40	20	100	220	6
В	А	10	5	5	40	70	6
С	В	60	55	5	30	45	3
D	С	30	20	10	10	50	4
E	С	25	20	5	15	20	1
F	D, E	5	N/A	0	2	N/A	N/A

b) Draw the flow diagram that represents this process and determine the critical path.



Paths: ABCDF=165 days

ABCEF=160 days

so, path ABCDF is the critical path



c) Lucy wants to complete the project in 150 days. What activities should be crashed in order to minimize cost?

Paths: ABCDF=165 days // 155 // 150 Goal=165-150=15

ABCEF=160 days // 155 // 150 Limit: 35 // 30

Critical path shift:D(5) // none

Total Cost	# Days	Critical Path	Crashable	Best
197	165	ABCDF	A,B,C,D	C(5), D(5)
232	155	ABCDF/ABCEF	A,B,D+E	E(5), D(5)
257	150	ABCDF/ABCEF		

To minimize cost, Lucy should crash C and E for 5 days each, and D for 10 days.



# **Estimating**

**BCWS:** Budgeted cost of work scheduled to date.

**BCWC**: Budgeted cost of work completed to date

**ACWC:** Actual cost of work completed to date

If BCWC>BCWS, the project is ahead of schedule

If BCWC>ACWC, the project is underbudget

Q7: Complete the time-cost analysis table below. Is this project under, on, or over budget? Is it ahead, on, or behind of schedule?

Activity	Budget	Scheduled	Completed	ACWC	BCWC	BCWS
Α	\$100	80%	60%	\$80	60%*100	80%*100
					=\$60	=\$80
В	\$200	30%	40%	\$70	40%*200	30%*200
					=\$80	=\$60
Total				80+70	60+80	80+60
				=\$150	=\$140	=\$140

BCWS=BCWC, so the project is on schedule.

BCWC<ACWC, so the project is over budget.



# **Forecasting**

### **Time Series Analysis**

F<sub>t</sub>: Forecast at time (t)

n: number of periods

At: Actual data at time (t)

w: associated weight

 $\alpha$  = parameter - 1

Forecast Error =  $F_t - A_t$ 

Mean Absolute Deviation =  $[\Sigma(F_t-A_t)]/n$ 

Simple Moving Average:  $F_t = (A_{t-1} + ... + A_{t-n})/n$ 

Weighted Moving Average:  $F_t = w_1^*At_{-1}+...+w_{t-n}^*A_{t-n}$ 

Exponential Smoothing:  $F_{t}=F_{t-1}+\alpha(A_{t-1}-F_{t-1})$ 

Tips: When choosing "n" for a simple moving average, if there is

- a) HIGH variation WITHOUT a trend, choose a large n
- b) LOW variation WITH a trend, choose a small n



Q3: Franzene wants to forecast how much money she will spend at Shein in December. Forecast Franzene's spending for December using...

Month	Actual	Weight
	spending (\$)	
September	150	0.1
October	168	0.4
November	195	0.5

a) 3-month simple moving average

$$F_t = (A_{t-1} + ... + A_{t-n})/n$$
$$= (195 + 168 + 150)/3 = $171$$

b) 3-month weighted moving average

$$F_t = w_1^* A_{t-1} + ... + w_{t-n}^* A_{t-n}$$
$$= 0.5^* 195 + 0.4^* 168 + 0.1^* 150 = \$179.7$$

c) Exponential smoothing with  $\alpha$ =0.7. Hint: October's spending was forecasted to be 155.

$$F_{t}=F_{t-1}+\alpha(A_{t-1}-F_{t-1})$$

$$F_{November}$$
=155+0.7(168-155)=164.1

$$F_{December} = 164.1 + 0.7(195 - 164.1) = 185.73$$



# **Inventory Management**

### **Newsvendor Model**

**Newsvendor Model:** Uncertain demand, 1-time decision, perishable product.

Cu: Underage cost, or lost profit from not having enough inventory

**Co:** overage cost, or the wasted inventory

- 1) NV ratio = Cu/(Cu+Co)
- 2) Find Q such that Pr(D</=Q) = NV ratio
  - a. If DISCRETE, choose Q with the minimum CDF that is over the NV ratio
  - b. If CONTINUOUS,  $Q = \mu + \sigma (Z_{NV})$  OR  $Q = \mu + safety stock$ 
    - \*\* find  $Z_{NV}$  using a Z-table



Q8: Suppose Club Kiddos, a children's indoor playground business, resells pizzas. At the beginning of the day, all the pizza must be purchased, and they cannot sell day-old pizza. Each pizza costs \$5, and they are sold for \$20.

a) What are the overage and underage costs? What is NV ratio?Cu=20-5=\$15 Co=\$5NV ratio=15/(15+5)=0.75

b) Given the probabilities of pizza demand shown below, how many packs of hotdogs should Club Kiddos buy?

d	Prob	Prob(D =d)</th
	(D=d)	aka CDF
1 pizza	0.5	0.5
2 pizzas	0.3	0.8
3 pizzas	0.2	1.0

We choose the # of pizzas with
the smallest CDF that is equal to,
or greater than, the NV ratio.

0.8>0.75, so Club Kiddos should
purchase 2 pizzas.



Q4: Suppose that Candy Craze, a small candy retailer, purchases its candy every month, and it cannot sell month-old lollipops. Monthly demand for lollipops is approximately normally distributed, with an average of 5lbs, and a standard deviation of 1lbs. Each pound of lollipops costs the business \$40, and the store sells lollipops for \$70 per pound. Unsold lollipops are sold on clearance for \$20 per pound.

a) What is the NV ratio?

=0.6

\*\* I have contacted a few profs, and they agree that the correct Cu is \$30

b) How many pounds of lollipops should Candy Craze purchase? What is the safety stock?

$$Z_{0.6}=0.255$$

Safety Stock= 
$$\sigma^*(Z_{NV})$$
 Q=  $\mu$  + Safety stock  
=1(0.255) =5+0.255



### **EOQ Model**

EOQ Model: Fixed order quantity, order is triggered by stock-level

C=cost per unit

D=annual demand

S=ordering cost per order

Q=batch size

H=annual holding cost per unit of average inventory

i=%carrying cost (interest)

LT=lead time

ROP=Reorder Point (demand during lead time)

SS=Safety stock (extra inventory kept in case of uncertain demand)

H=i\*C

Frequency=D/Q

**Cycle Time=Q/D** 

ROP(reorder point)=D<sub>LT</sub>+SS

Annual Setup Cost=(D/Q)\*S

IF the question asks for this

Annual Holding Cost=[(Q/2)+ROP]\*H

Annual Total Cost= (D/Q)\*S + [(Q/2)+ROP]\*H (+ C\*

Pay upon shipment: Retailer owns pipeline inventor

Cash on delivery: Retailer DOES NOT own pipeline inventory



	Type A	Type B	Type C
	Certain Demand, No	Certain Demand,	Uncertain
	lead time	Lead Time	Demand, Lead
			Time
Q <sub>opt</sub>	$\sqrt{\frac{2*S*D}{H}}$	$\sqrt{\frac{2*S*D}{H}}$	$\sqrt{\frac{2*S*D}{H}}$
ROP	0	μ <sub>LT</sub>	μ <sub>LT</sub> +SS
Average	Q/2	Shipment:	Shipment:
Inventory		Q/2+µ <sub>LT</sub>	Q/2+ µ <sub>LT</sub> +SS
		Delivery: Q/2	Delivery:
			Q/2+SS
Safety	0	0	σ <sub>LT</sub> *Z <sub>Service Level</sub>
Stock			OR
			$=\sqrt{LT}*\sigma*Z_{SL}$



Q10: Mr. Roberts provides his students with pencils, and he purchases whenever his pencil inventory hits a certain level. He knows that his students need 200 pencils per school year. It takes Mr. Roberts 18 days to receive a shipment of pencils (assume 360 days in a year). Pencils cost \$2/pencil and shipping costs \$20 per order. Pencils may be stored at a cost of \$0.50 per pencil per year. Assume that Mr. Roberts owns the pencils while they are being shipped.

a) How many pencils should Mr. Roberts buy per order?

This is a EOQ Type B model.

$$Q = \sqrt{\frac{2*S*D}{H}} = \sqrt{\frac{2*20*200}{0.5}} = 40\sqrt{10} = 126.5$$

b) What is the ROP?

$$ROP = \mu_{LT} = 0.05*200 = 10$$



c) On average, how many pencils does Mr. Roberts have?

Average Inventory= 
$$Q/2+\mu_{LT}$$
  
=126.5/2 + 10  
=73.25

d) How much money does Mr. Roberts spend, in total, for storing/ordering pencils?

e) If demand for pencils was uncertain, with a standard deviation of 50 pencils per year, and Mr. Roberts wanted to ensure a service level of 90%, how much safety stock should Mr. Roberts hold?

SS=
$$\sqrt{LT*\sigma*Z_{SL}}$$
  
= $\sqrt{0.05*50*1.285}$   
=14.37



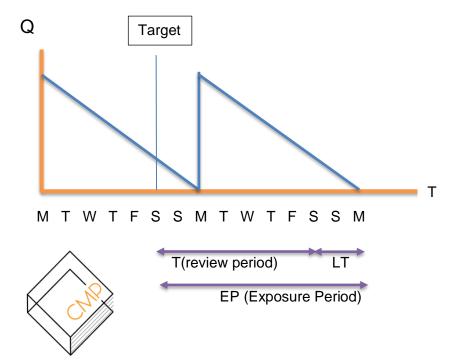
# **Periodic Review Model**

# Periodic Review Model: Fixed cycle time

# **I**=Current Inventory

$$EP = LT + T$$

	No Uncertainty	Uncertainty
Target	EP*D	EP*µ+SS
SS	0	$\sigma_{\text{EP}}^*Z_{\text{SL}}$
		$=\sqrt{EP}^* \sigma^* Z_{SL}$
Q	Target-I	Target-I
Average Inventory	Shipment: Q/2 + D*LT	Shipment:
	Delivery: Q/2	Q/2+SS+D*LT
		Delivery: Q/2+SS



Q11: Given  $\mu$ =40 units/day, T=20 days, LT=4 days, SL=99%, I=200 units, and  $\sigma$ =2units/day, calculate the a) order quantity

Q=Target – I Target= EP\*µ+SS

EP=LT+T =4+20=24 days

SS= $\sqrt{EP}^* \sigma^* Z_{SL}$ = $\sqrt{24}^*2^*2.325$ =22.78

Target=24\*40+22.78 =982.78

Q=982.78-200 =782.78

783 units should be ordered

b) average inventory (assuming cash on delivery) average Inventory = Q/2+SS =782.78/2 + 22.78 =414.17 Average Inventory is 414.17



# **Extra Questions**

 A manager at Starbucks wants to decrease queue time. Give 3 examples of ways to do this.

Hire another worker, decrease variability in service/interarrival time, innovate, etc.

2. In an M/M/2 queue, utilization is 0.8. What is Iq?



- 3. On Sundays, Julia is the only nail tech at Nailz Express Salon. From experience, she knows that customers entering the salon arrive in a poisson distribution, and her service time follows an exponential distribution. Customers typically come in at a rate of 3 per hour, and it takes Julia 12 minutes to do someone's nails.
  - a) What is the average utilization of Julia's time?

M/M/1 
$$\lambda$$
=3/hour Ts=1/ $\mu$ =12m=0.2hours p=?  $\mu$ =5/hour p=  $\lambda$ /c $\mu$ 
p=3/(1\*5)=0.6

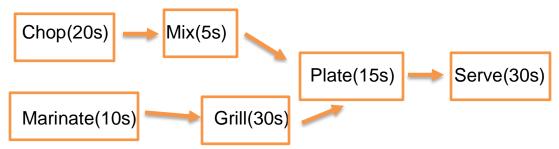
b) How long, on average, must customers wait to be served?

Tq=? 
$$Iq = \frac{p^2}{1-p}$$
  $Iq = \lambda^* Tq$   $Iq = (0.6^2)/(1-0.6)$   $0.9 = 4^* Tq$   $Iq = 0.9$  customers  $Iq = 0.3$  hours

c) How much time, on average, are customers spending in the



4. The flow diagram below represents the production line for a meal at a restaurant. Each step has 1 worker, and orders for both types of meals come in at the same rate.



a) What is the capacity rate of the process?

Since the bottleneck is the server, the capacity rate is 60/30=2 meals/minute

b) If the head chef can hire one more worker, what step would this worker be assigned to, and what would the new bottleneck and capacity rate of the process be?

The new worker should be assigned to the bottleneck step, so they would be a server. The new bottlenecks would now be the griller (60/(30\*0.5)=4/min), the plater (60/15=4/min), and the servers (60/30/2=4/min), making the new capacity rate of the process 4 meals per minute.



5. A call center receives 100 calls per hour from noon until 5pm. The center can process 90 calls per hour from noon until 3pm, and 60 calls per hour from then onwards. How many calls will be on hold at

1pm: 10

2pm: 20

3pm: 30

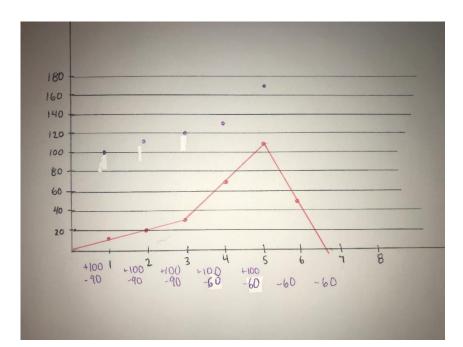
4pm: 70

5pm: 110

6pm: 50

7pm:0

8pm: 0



- 6. A) What is the name of the theory that explains why small changes in demand for a retailer will cause drastic consequences for producers?
  Bullwhip effect
  - b) How can these drastic consequences be reduced?
     Providing different levels of the supply chain with as much information as possible



- 7. The number of new COVID cases in BC is tracked.
- a) Prepare a 3-period moving average forecast
- b) Prepare a 3-period weighted moving average forecast using w1=0.7, w2=0.2, w3=0.1
- c) Which forecast is better? (Hint: Use MAD)

Date	# Cases	3-day	Absolute	3-day	Absolute
		simple	forecast	weighted	forecast
			error		error
Nov 22	586				
Nov 23	947				
Nov 24	945				
Nov 25	924	826	98	909.5	14.5
Nov 26	902	938.67	36.67	930.5	28.5
MAD:			67.33		21.5

The weighted moving average has a smaller MAD, so in this case it is

better



- 8. You own a flower company, where you sell, on average, 3000 roses per year, with an order cost of \$30 per order, a carrying cost of 15% per year, a unit cost of \$2 per rose, a shipping time of 3 days, and a standard deviation of daily demand of 50 roses. You plan to operate at a 95% service level (assume 360 days/year)
- a) Find EOQ

This is a Type C EOQ problem.

b) Calculate the reorder point

c) What is the cycle time? (in years)

Cycle Time=
$$Q/D = 774.6/3000=0.258$$



A plane has 100 seats. Each ticket is priced at \$400. On average, there are 10 no-shows with a standard deviation of 5 passengers. To maximize your profits, you want to overbook. If you overbook too many passengers, you must give a full refund and pay any extra passengers \$850 to find another flight. How many tickets should be sold for the flight?

This is a newsvendor problem. Cu=\$400Co=\$850

NVratio = Cu/(Cu+Co) = 400/(400+850) = 0.32

Using a Ztable, we know that Z0.32 =-0.465

Q=10+5(-0.465)=7.675 extra tickets, so sell a maximum of 107 tickets.



# Mia's Top 5 Tips

Understand the question before finding the answer!

If you can't figure out the answer... move on!

Don't forget to pay attention to units!

Practice makes perfect!

Fuel your brain!

Please submit your feedback to http://bit.ly/FeedbackCMP



