commerce undergraduate society

COMM 290

FINAL EXAM REVIEW SESSION

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Vocabulary Overview

First Half

Objective function – the function describing the problem's objective which you are attempting to maximize or minimize.

Optimal solution – the best set of decisions that maximizes the objective function while remaining within the constraints.

Target Cell – Contains the output of the objective function and is highlighted in green.

Constraint – A limitation of some sort posed with the problem. Always enclosed by a blue border.

Multiple optima – There are multiple sets of optimal solutions.

Feasible region – The region in which all solutions are valid and subject to the constraints.

Infeasible solution – There is no feasible region associated with your LP.

Unbounded solution – The feasible region is infinitely large, usually due to lack of a constraint, and the objective function behaves such that you are moving the isoprofit line outwards indefinitely.

Input Data – The data given to you as part of a problem. Usually highlighted in yellow.

Action Plan – The "action" you will take to solve the problem, which will be indicated inside red borders on excel.

Redundant constraint – A constraint which does not affect the feasible region.

Non-negativity constraint – A constraint which makes sure a "decision" cannot be a negative value.

RHS Allowable Increase/Decrease of a Binding Constraint – Range in which the right-hand-side of the constraint may move without changing the shadow price.

RHS Allowable Increase/Decrease of a Non-Binding Constraint – Range in which the right-hand-side of the constraint may move while keeping the constraint non-binding.

Allowable Increase/Decrease of an objective coefficient – Range in which the objective coefficient may move without changing the optimal solution.

Shadow Price – The increase in the value of the target cell for every one-unit increase of the RHS of a constraint.

Relative Reference – A reference in the form A1 that will change when auto-filled to other cells.

Absolute Reference – A reference in the form A1 that will not change when auto-filled to other cells.

Second Half

Probability Tree – Tree containing all possible outcomes of a probability problem.

Sample Space – All the possible outcomes.

Probability – The chance by which something will happen.

Independent – Knowing something about one outcome does not affect another.

Dependent – Knowing something about one outcome affects another.

Mutually Exclusive – Two outcomes are **mutually exclusive** if they cannot both occur at the same time.

Expected Monetary Value – The expected value of all monetary payoffs

Optimistic Decision Approach – Optimistic approach highlights the best payoff under any decision, and selects the decision with the maximum highest payout.

Maximin Conservative Approach – Conservative approach highlights the worst payoff under any decision, and selects the decision with the best worst-case payout.

Minimax Regret Approach – Regret approach calculates the difference between each outcome and the best outcome under each state. Then, select the decision that has the least worst-case regret.

Expected Value of Sample Information – The amount of profit gained by knowing another related state before making a decision.

Expected Value of Perfect Information – The amount of profit gained by knowing the state before making a decision.

Efficiency of Information – The % of EVPI extracted using sample information.

Expected Value – The average outcome of a random variable.

Variance – A measurement of how much a variable varies.

Standard Deviation – How much a variable will normally vary relative to the mean. Often used with the ± sign in combination with the mean. More about that in COMM 291…

Solving Algebraically + Graphically

Problem:

You are the manager of a fast-food restaurant, serving fries and burgers. Each burger results in $2.50 of profit, while each order of fries results in $1.50 of profit. Each burger requires 5 minutes of cooking and 2 minutes of packing, while each order of fries requires 3 minutes of cooking and 3 minutes of packing. You have 360 minutes for cooking and 220 minutes for packing. Due to demand, you must produce at least 30 of both burgers and fries.

1. How many non-negativity constraints are there? What about total constraints?

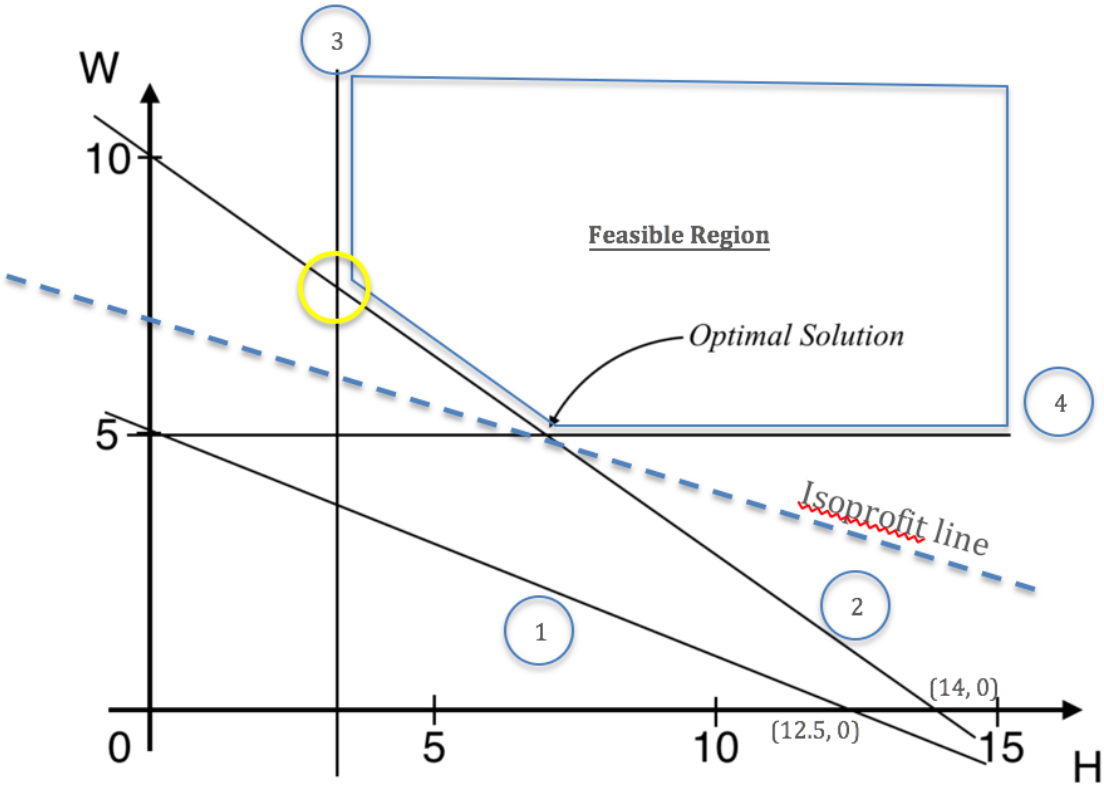
2. Is this a maximizing or minimizing model?

3. Complete the algebraic formulation of this model, and produce a graph labelling the correct optimal solution. Also label the feasible region.

Understanding Graphs

Problem:

Consider the following modified graph for Pet Troodon found on the Sample Midterm Exam and answer the following questions.



1. Given the feasible region, and assuming both objective coefficients are positive, is this LP a maximizing or minimizing model?

2. Which constraints are binding, and which are non-binding?

3. What is one sample objective function that will lead to multiple optima at the current optimal solution **and** at the point labelled with the yellow circle?

4. Given an objective function of 6H + 10W, calculate by hand the shadow price of constraint 1's right-hand-side. Do the same for constraint 3.

5. Suppose due to an unexpected error, the sign of constraint 3 has been switched. Label the new optimal solution and the new feasible region.

6. Suppose that this model goes from a minimizing model to a maximizing model. What can you say about the optimal solution and the LP?

Interpreting an LP

Problem:

Consider this completed excel model for a modified version of Nationland Power and answer the following questions.

	A	B	C	D	E	F	G	H
1	**Nationland Power**							
2								
3	**Input Data**							
4			Solar	Wind	Diesel	Total		Constraint
5		CO2	50	7.5	735	34.82	<=	80
6		Domestic	0.95	0.25	1	0.70		
7		Social	0.05	0.01	0.4	0.04	>=	0.03
8						Total		
9		Profit	$ 0.08	$ 0.13	$ 0.05	$ 0.09786		
10								
11								
12	**Action Plan**							
13			Solar	Wind	Diesel	Total		Constraint
14		Proportion	0.6429	0.3571	0.0000	1	=	1
15								
16	**Additional Constraints**							
17				Output		Requirement		
18		At least 70% Domestic		70%	>=	70%		
19		At most 84% Domestic		70%	<=	84%		

1. What is the objective function? How many variables are there?

2. How many are binding constraints are there?

3. What are the best equations for the cells F5, F14 and D18?

   F5:

   F14:

   D18:

4. What is the optimal solution? What is the value of the target cell?

5. Suppose the value in cell H5 decreased to 40. Will this change the optimal solution? Why or why not?

6. Suppose the value in cell H7 increased to 0.04. Will this change the optimal solution? Why or why not? What if it changed to 0.041?

Sensitivity Analysis

Problem:

Given the attached sensitivity analysis of the modified version of Nationland Power with many cells blacked out, answer the following questions.

Microsoft Excel 15.15 Sensitivity Report
Worksheet: [Final Exam Review Spreadsheet.xlsx]Nationland Power
Report Created: 2016-12-03 5:03:50 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	Proportion Solar	0.642857143	0		0.05	0.024666667
D14	Proportion Wind	0.357142857	0		0.37	0.05
E14	Proportion Diesel	0	-0.026428571		0.026428571	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D18	At least 70% Domestic Output		-0.071428571	0.7	0.14	0.1
D19	At most 84% Domestic Output	0.7		0.84		
F14	Proportion Total	1	0.147857143	1	1.333333333	0.263157895
F5	CO2 Total	34.82142857		80		
F7	Social Total	0.035714286		0.03		

1. What are the values within the cells F9, D16, E17, G17, H17, G19 and G20?

F9: D16:

E17: G17:

G19: G20

2. Suppose you must now get 75% of your electricity supply from domestic sources. By how much will this decrease your cost?

3. Suppose you now must get at most 80% of your electricity domestically. By how much will this change the amount of wind energy used?

4. Is there evidence of multiple optima? Why or why not?

5. What's the highest that the profit of wind power could go without influencing any change in the optimal solution? What about the profit of solar power?

6. Suppose you must now dedicate $0.035 cents as opposed to $0.030 to social programs. Will this induce any change in the LP?

Blending Problem

Problem:

The following is the completed model and sensitivity analysis for the Lucky Strike model you discussed in class, and please use those to answer the following questions.

	A	B	C	D	E	F	G	H
1	**Lucky Strike**							
2								
3	**Input Data**							
4		Cost per litre ($)	Amount Available					
5	OilA	0.32	500000		Selling Price Regular	$ 0.42	litre	
6	OilB	0.38	275000		Selling Price Ultra	$ 0.51	litre	
7	OilC	0.34	425000					
8								
9	*Regular gas must be at least 45% OilA*				0.45			
10	*Regular gas must be at most 25% oilb*				0.25			
11	*Ultra Gas must be at most 35% OilC*				0.35			
12	*Ultra Gas must be at least 35% OilB*				0.35			
13								
14	**Action Plan**				Amount		Amount	
15			Reg	Ultra	Used		Available	
16		OilA	186428.6	313571.4	500000	<=	500000	*litres*
17		OilB	0	275000	275000	<=	275000	*litres*
18		OilC	227857.1	197142.9	425000	<=	425000	*litres*
19		Total	414285.7	785714.3				
20								
21	**Blending Constraints**				Model Output		Model Req	
22	***Regular*** *gas must be at* ***least*** *45% OilA*				186428.5714	>=	186428.57	
23	***Regular*** *gas must be at* ***most*** *25% oilb*				0	<=	103571.43	
24	***Ultra*** *Gas must be at* ***most*** *35% OilC*				197142.8571	<=	275000	
25	***Ultra*** *Gas must be at* ***least*** *35% OilB*				275000	>=	275000	
26								
27								
28								
29								
30	**Rev Cost Info**							
31			**Reg**	**Ultra**	**Total**			
32		**Rev**	$174,000.00	$400,714.29	$ 574,714.29			
33		**Costs**						
34		OilA	$ 59,657.14	$100,342.86	$ 160,000.00			
35		OilB	$ -	$104,500.00	$ 104,500.00			
36		OilC	$ 77,471.43	$ 67,028.57	$ 144,500.00			
37		**Profit**	$ 36,871.43	$128,842.86	$ 165,714.29			

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [Lucky.ST.2016W.rev.5B.L101.xlsx]Model
Report Created: 05/10/2016 9:34:20 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C16	OilA Reg	186428.5714	0	0.1	0	0.177777778
D16	OilA Ultra	313571.4286	0	0.19	0.177777778	0
C17	OilB Reg	0	-0.257142857	0.04	0.257142857	1E+30
D17	OilB Ultra	275000	0	0.13	1E+30	0.257142857
C18	OilC Reg	227857.1429	0	0.08	0.163636364	0
D18	OilC Ultra	197142.8571	0	0.17	0	0.163636364

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E16	OilA Used	500000	0.1	500000	358441.5584	141558.4416
E17	OilB Used	275000	0.297142857	275000	223076.9231	193006.993
E18	OilC Used	425000	0.08	425000	173015.873	414285.7143
E22	Regular gas must be at least 45% OilA Model Out	186428.5714	-5.82867E-16	0	77857.14286	186428.5714
E23	Regular gas must be at most 25% oilb Model Out	0	0	0	1E+30	103571.4286
E24	Ultra Gas must be at most 35% OilC Model Outp	197142.8571	0	0	1E+30	77857.14286
E25	Ultra Gas must be at least 35% OilB Model Outp	275000	-0.257142857	0	125454.5455	136250

1. What are the best formulas for cells E16, E22, G22 and E37?

   E16:

   E22:

   G22:

   E37:

2. How many variables are in this problem? How many constraints are there? How many constraints are binding?

3. Why is the RHS for the blending constraints 0? Explain using algebraic formulation.

4. How much should you pay for each additional liter of OilC?

5. Is there evidence of multiple optima?

6. What is the total profit from OilC products?

7. Suppose the cost of OilA increased by $0.06. By how much will this change the target cell?

8. Suppose you can now sell regular oil for $0.05 more. By how much will this change the target cell?

9. Suppose you have access to 30,000 additional units of OilA. What will be the new value of target cell?

10. Suppose you have access to up to 200,000 additional units of OilC at the same cost of 0.34 per litre (In other words, the value in cell G18 is now 625,000). How can you best describe the change in the target cell?

Scheduling Problem

Problem:

Given the attached solution to a modified version of the Gotham City National Bank LP, answer the following questions.

	A	B	C	D	E	F	G	H	I	J
1	**Gotham City National Bank data**									
2	Source: Winston SMA problem 4.86									
3										
4	**Input Data**									
5										
6	**Time**	9am-10am	10am-11am	11am-Noon	Noon-1pm	1pm-2pm	2pm-3pm	3pm-4pm	4pm-5pm	#
7										**Employees**
8	Requirement									
9	FT1	4	4	4	4		4	4	4	4
10	FT2	2	2	2		2	2	2	2	2
11	PT1	0	0	0						0
12	PT2		2	2	2					2
13	PT3			1	1	1				1
14	PT4				0	0	0			0
15	PT5					0	0	0		0
16	PT6						1	1	1	1
17										
18	Supply	6	8	9	7	3	7	7	7	
19		>=	>=	>=	>=	>=	>=	>=	>=	
20	**Demand**	6	7	3	7	3	6	7	5	
21										
22	**Other constraints**						Model		Model	
23							Output		Requirement	
24	At most 4 part-time workers hired						4	<=	4	
25	# FT workers whose lunch is from 1:00 to 2:00 must = 4						4	=	4	
26										
27	**Cost Issues**									
28										
29	Total Cost Full Time		$ 672.00							
30	Total Cost Part Time		$ 108.00							
31	Total Cost		$ 780.00							

Microsoft Excel 15.15 Sensitivity Report
Worksheet: [Final Exam Review Spreadsheet.xlsx]Gotham
Report Created: 2016-12-03 6:09:27 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
J9	FT1 Employees	4	0	112	1E+30	1E+30
J10	FT2 Employees	2	0	112	1E+30	58
J11	PT1 Employees	0	0	27	58	112
J12	PT2 Employees	2	0	27	0	0
J13	PT3 Employees	1	0	27	0	0
J14	PT4 Employees	0	0	27	1E+30	0
J15	PT5 Employees	0	0	27	1E+30	0
J16	PT6 Employees	1	0	27	0	112

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B18	Supply 9am-10am	6	56	6	2	0
C18	Supply 10am-11am	8	0	7	1	1E+30
D18	Supply 11am-Noon	9	0	3	6	1E+30
E18	Supply Noon-1pm	7	56	7	0	0.666666667
F18	Supply 1pm-2pm	3	0	3	1	1
G18	Supply 2pm-3pm	7	0	6	1	1E+30
H18	Supply 3pm-4pm	7	56	7	0	1
I18	Supply 4pm-5pm	7	0	5	2	1E+30
G24	At most 4 part-time workers hired Output	4	-29	4	2	0
G25	# FT workers whose lunch is from 1:00 to 2:00 must = 4 Output	4	-56	4	0.4	0

1. How many variables are in this problem? How many constraints are binding?

2. What is the optimal solution, and what is the total cost at that optimal solution?

3. How much are full-time and part-time workers paid by the hour? There are two ways to solve this. (Hint: they have been modified from the original version!)

4. Suppose Gotham may now hire six part-time workers. How much money will this change save? What if they can employ seven workers?

5. Does this LP have evidence of multiple optima?

6. Suppose due to an influx in demand, you must now employ two additional workers for the 9am-10am shift. By how much will this change the target cell?

7. Suppose due to a decrease in demand, you now require 8 workers for the 11am-noon shift. By how much will this change the target cell?

Transportation Problem

Problem:

Given the attached solution to a modified version of the Powerco LP, answer the following questions.

	A	B	C	D	E	F	G	H	I
1	**Power Co**								
2									
3	**Input Data**								
4									
5		**Costs ($)**	City1	City2	City3	City4			
6		Plant1	$ 7.00	$ 6.00	$10.00	$ 7.00			
7		Plant2	$ 9.00	$10.00	$10.00	$ 4.00			
8		Plant3	$ 12.00	$ 9.00	$12.00	$ 5.00			
9									
10	**Action Plan**								
11					To		Amount		
12			City1	City2	City3	City4	Sent		Supply
13		Plant1	25	10	0	0	35	<=	35
14	From	Plant2	20	0	30	0	50	<=	50
15		Plant3	0	10	0	30	40	<=	40
16		Received	45	20	30	30			
17			>=	>=	>=	>=			
18		Demand	45	20	30	30			
19									
20	Total Cost		$ 955.00						

Microsoft Excel 15.15 Sensitivity Report
Worksheet: [Final Exam Review Spreadsheet.xlsx]Powerco
Report Created: 2016-12-03 6:36:53 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C13	Plant1 City1	25	0	7	1	0
D13	Plant1 City2	10	0	6	0	1
E13	Plant1 City3	0	2	10	1E+30	2
F13	Plant1 City4	0	5	7	1E+30	5
C14	Plant2 City1	20	0	9	0	1
D14	Plant2 City2	0	2	10	1E+30	2
E14	Plant2 City3	30	0	10	1	11
F14	Plant2 City4	0	0	4	1E+30	0
C15	Plant3 City1	0	2	12	1E+30	2
D15	Plant3 City2	10	0	9	1	0
E15	Plant3 City3	0	1	12	1E+30	1
F15	Plant3 City4	30	0	5	0	5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C16	Received City1	45	10	45	0	10
D16	Received City2	20	9	20	0	10
E16	Received City3	30	11	30	0	10
F16	Received City4	30	5	30	0	30
G13	Plant1 Sent	35	-3	35	10	0
G14	Plant2 Sent	50	-1	50	10	0
G15	Plant3 Sent	40	0	40	1E+30	0

1. How many variables are in this problem? How many constraints are there? How many constraints are binding?

2. What is the cost of electricity at the optimal solution?

3. Suppose due to unforeseen circumstances, City2 requires 5 additional units of electricity. By how much will the target cell change?

4. Suppose all shipments from Plant1 increased in price by $4. How will this change the target cell?

5. City1 has funded a new electricity plant, and now only requires 30 units to be shipped. How will this change the target cell?

End of First Half - Break time!

Probability Trees

Problem:

You are playing a game, where you will flip 4 coins. The game ends when you flip tails. Answer the following questions.

1. Draw a probability tree for the given scenario. Be sure to include the sample space and probability of each outcome.

2. What is the probability that you will get exactly two heads? What about exactly three heads?

3. After flipping one coin, which ended up being heads, what is the probability that your final outcome will be four heads?

4. You play a game where you win $1 if you get two or more heads from four flips. What is the probability that you will win $1?

5. Fun bonus question: assume the game goes on indefinitely until you flip tails. Write an algebraic expression describing the probability of an x amount of heads over the course of a game (e.g. x = 3 if you flip 3 heads before the game ends). Check to see if your expression works for the possible outcomes in the game involving four flips (do not test with the HHHH case as there is no tails).

Decisions and EMV

Problem:

You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up The payoff matrix is provided and use it to answer the following questions.

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

1. Which decision will you make if under the optimistic approach? What about conservative?

2. Construct a regret matrix. Which decision will you make if you are using the regret approach?

3. Which decision should you make if you are using the EMV (Expected Monetary Value) approach?

Value of Information

Problem:

You are selling movie tickets for the upcoming Assassin's Creed movie. You are unsure whether or not people will like the movie or not: forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.

1. Draw a probability tree; under the EMV approach, what decision will you make?

2. You can foresee the future at a cost. How much will you be willing to pay in order to foresee the future? Draw the probability tree associated with this scenario.

3. Suppose you get additional information that if people liked the Warcraft movie, there will be an 0.9 chance that people will like the Assassin's Creed movie, while if people did not like the Warcraft movie, people will only like the Assassin's Creed movie with a 0.4 chance. Construct a new probability tree, and find the EVSI (Expected Value of Sample Information).

Random Variables

Problem:

Consider the following probability distribution and answer the following questions.

X	0	1	2	3	4
P(X)	0.3	0.15	0.25	0.2	0.1

1. Find expected value, variance, and standard deviation.

2. Find P(2 OR 3) and P(0 OR 2).

3. You pick two numbers at random **with** replacement. Find $P(1^{st} = 0 \text{ AND } 2^{nd} = 4)$ and $P(2^{nd} = 1 \mid 1^{st} = 4)$.

4. You pick two numbers at random **without** replacement. Find $P(1^{st} = 0 \text{ AND } 2^{nd} = 4)$ and $P(2^{nd} = 1 \mid 1^{st} = 4)$.

5. Suppose you will take whatever number you pick out, and construct a square with that given number as the side length. What is the expected value, variance, and standard deviation of the area of your square? (Hint: draw a new probability distribution)

6. **This question is unrelated to the previous scenario.** Given $E(X^2) = 20$ and $E(X) = 5$, find VAR(X) and STDEV(X).

Independent Bi-Variable Problem

Problem:

Given the following probability distribution for a closed auction, answer the following questions. Assume the two players' strategies are independent. Complete the table before answering any questions.

		Player 1 (A)			
		Bid $1	Bid $3	Bid $5	
Player 2 (B)	Bid $2				0.4
	Bid $4				0.4
	Bid $6				0.2
		0.25	0.45	0.3	

1. What is the probability that Player 1 wins? What is the probability that Player 2 wins?

2. Find the expected value of (A + (-B)). What does this mean?

3. Find the expected value, variance, and standard deviation of the following:
    a. The winning bid

    b. The total amount bid by both players (don't make a new distribution table)

Dependent Bi-Variable Problem

Problem:

Given the following probability distribution, answer the following questions.

	Y = 0	Y = 1	Y = 2	Total
X = 0	0.1	0.2	0.1	0.4
X = 1	0.15	0.1	0.05	0.3
X = 2	0.05	0.1	0.15	0.3
Total	0.3	0.4	0.3	1.0

1. How can you tell that this probability distribution is dependent (besides that it's given)? Explain.

2. Find P(X = 2 | Y = 1).

3. Find P(X = 1 AND Y = 2).

4. Find P(X = 1 OR (Y = 1 OR 2)).