COMM 290

2018W1 Midterm Review Solutions

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commerce
undergraduate
society

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Vocabulary Overview

Objective function – the function describing the problem's objective which you are attempting to maximize or minimize.

Optimal solution – the best set of decisions that maximizes the objective function while remaining within the constraints.

Target Cell – Contains the output of the objective function and is highlighted in green.

Constraint – A limitation of some sort posed with the problem. Always enclosed by a blue border.

Multiple optima – There are multiple sets of optimal solutions.

Feasible region – The region in which all solutions are valid and subject to the constraints.

Infeasible solution – There is no feasible region associated with your LP.

Unbounded solution – The feasible region is infinitely large, usually due to lack of a constraint, and the objective function behaves such that you are moving the isoprofit line outwards indefinitely.

Input Data – The data given to you as part of a problem. Usually highlighted in yellow.

Action Plan – The "action" you will take to solve the problem, which will be indicated inside red borders on excel.

Redundant constraint – A constraint which does not affect the feasible region.

Non-negativity constraint – A constraint which makes sure a "decision" cannot be a negative value.

RHS Allowable Increase/Decrease of a Binding Constraint – Range in which the right-hand-side of the constraint may move while keeping the constraint binding.

RHS Allowable Increase/Decrease of a Non-Binding Constraint – Range in which the right-hand-side of the constraint may move while keeping the constraint non-binding.

Allowable Increase/Decrease of an objective coefficient – Range in which the objective coefficient may move without disrupting the optimal solution.

Shadow Price – The increase in the value of the target cell for every one-unit increase of the RHS of a constraint.

Relative Reference – A reference in the form A1 that will change when auto-filled to other cells.

Absolute Reference – A reference in the form A1 that will not change when auto-filled to other cells.

Solving Algebraically + Graphically

Problem:

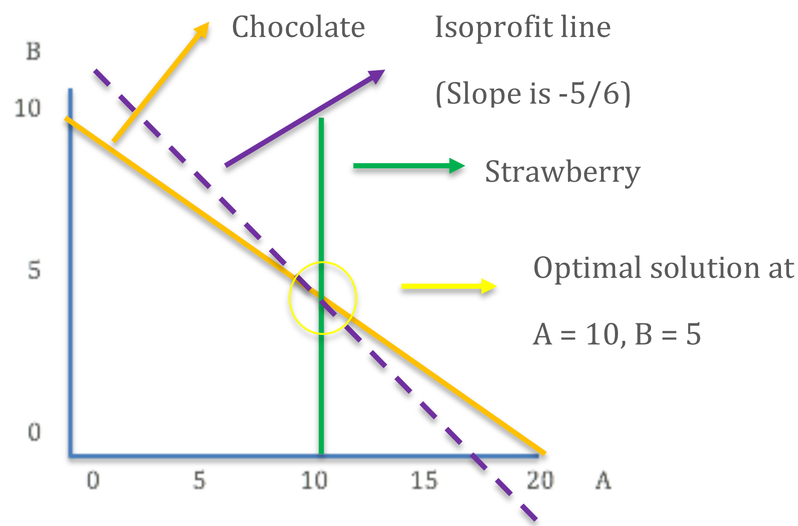
You are a student running a business selling two combos of Pocky: Combo A earns you $5, consisting of one chocolate and one strawberry, while Combo B earns you $6, consisting of two chocolates and no strawberry. You have 20 chocolates in stock and 10 strawberries in stock. Assume there are no costs associated with this model.

1. What is the objective function? Is this a maximizing or minimizing model?

   The objective function should represent the amount of profit we are making. Therefore, the objective function is max 5A + 6B.

2. List out all constraints in algebraic form. How many constraints are there?
   There are four constraints:
   Chocolate: A + 2B <= 20
   Strawberry: A <= 10
   A >= 0
   B >= 0

3. Draw out the constraints and label the optimal solution on the provided graph template. What is the optimal solution, and what is the profit at that optimal solution? What are the binding constraints?



Two binding constraints: strawberry, chocolate.

Since strawberry is binding, A = 10.

To find B, substitute A = 10 into chocolate constraint.

2B + 1(10) = 20 → B = 5

The optimal solution is indeed to make 10 of Combo A and 5 of Combo.

Profit = \$5(10) + \$6(5) = \$80

4. Find the allowable increase and decrease of the coefficients for both Combo A and Combo B. (Do this by hand.)

Given that slope of the two constraints are -1/2 (chocolate) and -infinity (strawberry, C2 = 0), we want to set the slope of the isoprofit line to be between -1/2 and -infinity. Given the objective function 5A + 6B (C1A + C2B), we want to set the two coefficients such that –C1/C2 is equal to -1/2 and -infinity. We will start by finding the allowable increase of C1 (by keeping C2 constant).

$$-\infty \leq -\frac{C1}{6} \leq -\frac{1}{2}$$

Multiply all sides by -6 (don't forget to change signs!)

$$\infty \geq C1 \geq 3$$

Since C1 is currently 5, and can be between 3 and infinity, the allowable increase and decrease for Combo A's objective coefficient are infinity (1E+30) and 2, respectively. To find allowable increase/decrease for C2:

$$-\infty \leq -\frac{5}{C2} \leq -\frac{1}{2}$$

Take the reciprocal of all sides (don't forget to change signs! Negative infinity is used or this does not make sense)

$$0 \geq -\frac{C2}{5} \geq -2$$

Multiply both sides by -5 (don't forget to change signs!)

$$0 \leq C2 \leq 10$$

Since C2 is currently 6, and can be between 0 and 10, the allowable increase and decrease for Combo B's objective coefficient are 4 and 6, respectively.

5. Find the allowable increase and decrease of all constraints. (Do this in Excel).

   To do this, you will have to construct a model. The model should look somewhat like this:

	A	B	C	D	E	F	G
1	Pocky Problem						
2							
3	Input Data						
4			Combo A	Combo B	Output		Constraint
5		Chocolate	1	2	20	<=	20
6		Strawberry	1	0	10	<=	10
7							
8		Profit	$ 7.00	$ 10.00	$ 120.00		
9							
10	Action Plan						
11			Combo A	Combo B			
12		# to make	10	5			

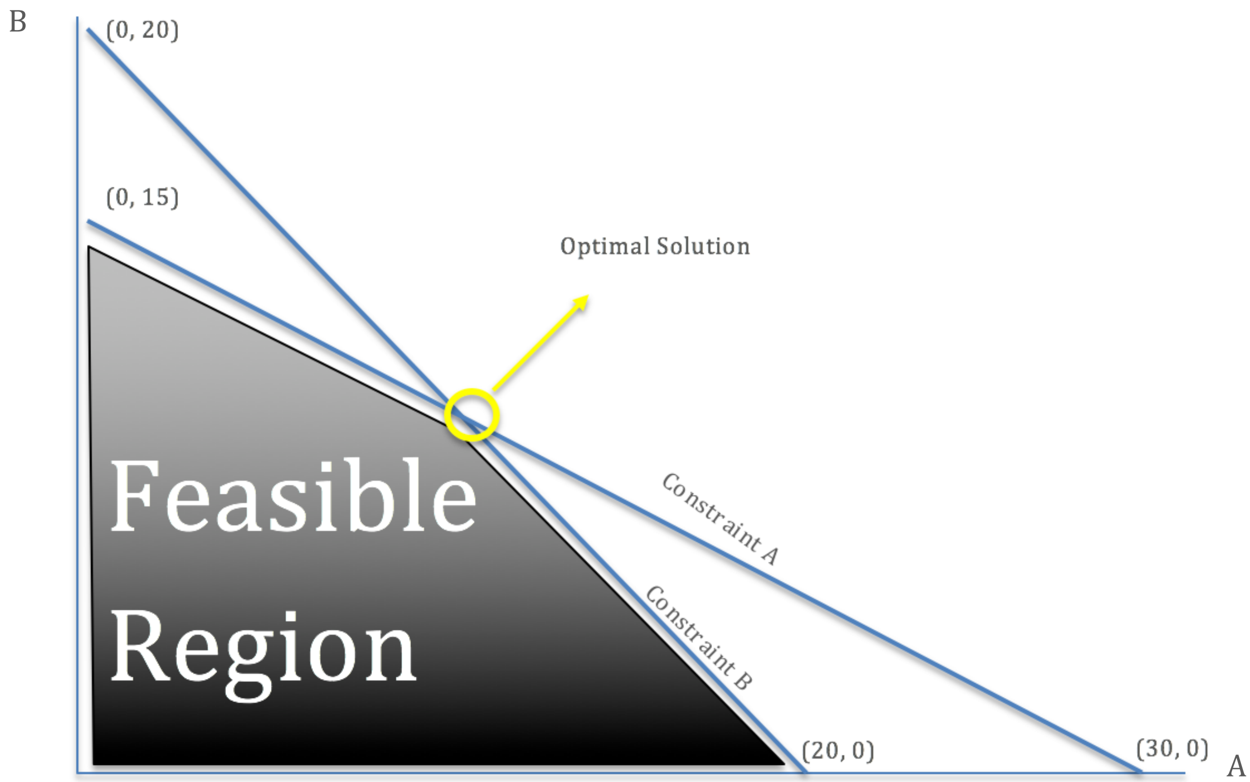
   When you produce a sensitivity report, you should see this:

	Allowable Increase	Allowable Decrease
Chocolate	1E+30	10
Strawberry	10	10

Understanding Graphs

Problem:

Consider the following graph of an arbitrary linear programming model with the correct labelled optimal solution, feasible region. Assume this is a profit maximization model (That is, objective function is in the form xA + yB).



1. Define the range of possible slopes for isoprofit lines that would lead to this optimal solution. Give an example of a possible objective function.

   The slopes of the two constraints are -1 and -1/2. Therefore, -C1/C2 (slope of isoprofit line) of the objective function C1A + C2B must be between -1 and -1/2.

Therefore, a sample objective function would be max 2A + 3B.

2. Write two possible objective functions which would lead to multiple optima.

   To have multiple optima, the slope of the isoprofit line must be equal to the slope of one of the constraints. Therefore, any objective function C1A + C2B with –C1/C2 equal to either -1/2 or -1 would give you multiple optima. Two example answers would be A + 2B and A + B.

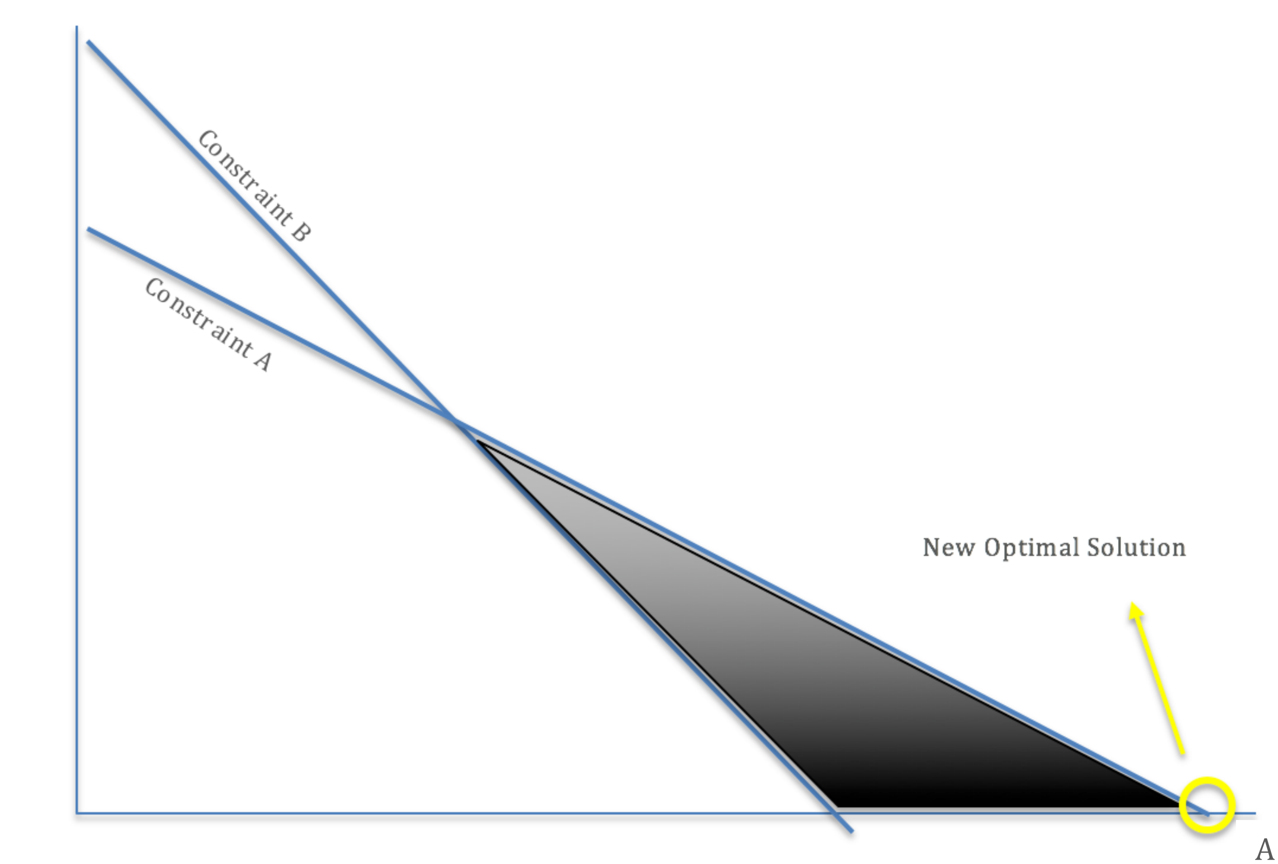
3. Find the coordinates of the optimal solution.

   Assuming each unit of A and B require 1 unit of any constrained resource, the two constraints can be rewritten as A + B = 20, and A + 2B = 30; note that equal signs are allowed to be used due to them being binding constraints. The second constraint can be rewritten as A + 2B – 10 = 20, which we can set to be equal to A + B from the first constraint:

$$A + 2B - 10 = A + B$$
$$B - 10 = 0$$
$$B = 10$$
$$A + B = 20$$
$$A = 10$$
$$B = 10$$

4. Suppose the sign of Constraint B (<=) is changed to the >= sign. How will this change the feasible region? Assuming the objective function remains the same and the LP remains feasible, draw in the new feasible region and label the new optimal solution. If this LP becomes infeasible, explain why.

   The slope of the two constraints remain the same. The isoprofit line must also maintain the same slope. Therefore, if you take any isoprofit line with slope between -1/2 and -1, you will reach this optimal solution.



Fruit Juice - Excel

Problem (Solve in Excel):

You are a company producing fruit juice producing four types of fruit juice: apple, citrus, pineapple and tropical. You have 20L of apple concentrate, 20L of orange concentrate and 10L of pineapple concentrate. You have 60L of water. The breakdown of each juice is as follows:

- One bottle of apple juice requires 200mL of apple concentrate and 300mL water;
- One bottle of citrus juice requires 300mL of orange concentrate and 200mL water;
- One bottle of pineapple juice requires 250mL of pineapple concentrate and 250mL water;
- One bottle of tropical juice requires 150mL of pineapple, 100mL of orange, 50mL of apple and 200mL water.

You profit $3 for every bottle of apple juice, $3 for every bottle of citrus juice, $6 for every bottle of pineapple juice and $5 for every bottle of tropical juice.

1. Complete the algebraic formulation for this problem. You do not have to solve for optimal solution algebraically.

   Max. 3A + 3C + 6P + 5T
   Subject to
   ============================
   0.2A + 0.05T <= 20 (apple concentrate)
   0.3C + 0.1T <= 20 (orange concentrate)
   0.25P + 0.15T <= 10 (pineapple concentrate)
   0.3A + 0.2C + 0.25P + 0.2T <= 60 (water)
   A >= 0 (non-negativity)
   C >= 0 (non-negativity)
   P >= 0 (non-negativity)
   T >= 0 (non-negativity)

2. Solve for optimal solution in excel. What is the amount of profit made under the optimal solution? How many constraints are there? What about binding constraints?

   The amount of profit made under the optimal solution is $740. There are a total of eight constraints, 4 of which are binding (non-negativity of tropical, apple concentrate, orange concentrate, pineapple concentrate).

3. Under the optimal solution, how many bottle of each fruit juice will you produce?’

   Under the optimal solution, you will produce 100 bottles of apple juice, 66.67 bottles of citrus juice, 40 bottles of pineapple juice and 0 bottles of tropical juice.

4. Consider the following sensitivity analysis for Fruit Juice and answer the following questions. If any behavior cannot be calculated, resolve the LP again with different parameters.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	Bottles to Make Apple	[redacted]	0	3	1E+30	1.4
D14	Bottles to Make Citrus	[redacted]	0	3	1E+30	1.05
E14	Bottles to Make Pineapple	[redacted]	0	6	1E+30	0.583333333
F14	Bottles to Make Tropical	[redacted]	-0.35	5	0.35	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G5	Apple Concentrate Total	20	15	20	4.444444444	20
G6	Orange Concentrate Total	20	10	20	10	20
G7	Pineapple Concentrate Total	10	24	10	6.666666667	10
G8	Water Total	53.33333333	0	60	1E+30	6.666666667

a. Let’s say a sudden decrease in demand for tropical fruit juice drops the profit of tropical juice to $3 per bottle. Will that change the optimal solution? Why or why not?

   This will not change the optimal solution as the allowable decrease for the objective coefficient of tropical fruit juice is 1E+30, and the decrease of $2 in selling price is within that allowable range.

b. A drought has occurred and the supply of water has suddenly dropped from 60 litres to 45 litres. Will the optimal solution change? If yes, what is the new optimal solution, and how much profit will you make?

The new optimal solution will change since the decrease by 15 is outside of the allowable decrease range (6.6666). Therefore, it will be unsolvable by our sensitivity report.

The new optimal solution is to make 75.92 bottles of apple juice, 44.44 bottles of citrus juice, 0 bottles of pineapple juice and 66.66 bottles of tropical juice, resulting in $694.44 of profit.

Intuitively, the drop from 60 to 53.33 will not affect profit. Only the drop from 53.33 to 45 will.

c. Suppose due to high demand, you must make at least 40 bottles of tropical juice. What is the new optimal solution, and how much profit will you make? Is this new constraint binding?

The optimal solution is to make 90 bottles of apple juice, 53.33 bottles of citrus juice, 16 bottles of pineapple juice and 40 bottles of tropical juice, which results in $726 of profit. This new constraint is binding.

Sensitivity Analysis

Problem:

Consider the following sensitivity analysis for an unspecified maximization LP model. Some cells have had their numbers removed. Cells with a number attached and highlighted will be referred to in the questions in this section. Assume all questions are independent of one another.

Variable Cells							
			Final	**Reduced**	**Objective**	**Allowable**	**Allowable**
	Cell	**Name**	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
	C16	Decision 1	4500	0	0.33	0.01	0.016
	D16	Decision 2	0	0	0.33	1E+30	1E+30
	E16	Decision 3	62000	0	0.2	1E+30	0.017
	F16	Decision 4	0	-0.062	-8.81E-10	0.03	1E+30
Constraints							
			Final	**Shadow**	**Constraint**	**Allowable**	**Allowable**
	Cell	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
	C25	Constraint 1	(1)	0.65	5400	2501	1008
	C26	Constraint 2	[redacted]	0.55	21600	6500	5300
	C27	Constraint 3	[redacted]	0.14	0	12000	0
	C29	Constraint 4	[redacted]	0.62	7500	28000	7500
	C28	Constraint 5	62000	0.062	(2)	22000	27000

1. What should be the value within the cell (1)? Explain.

   The value within cell (1) should be 5400 because the shadow price associated with the constraint is non-zero. Therefore, the constraint must be binding, resulting in a value of 5400 in cell (1).

2. What should be the value within the cell (2)? Explain.

   The value within cell (2) should be 62000 because similar to question 1, due to the presence of a shadow price the constraint must be binding. Therefore, the RHS must be equal to the final value which is 62000.

3. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.01?

   An increase in 0.01 of the objective coefficient is within the allowable increase, so the optimal values remain the same. Since the objective coefficient has gone up by 0.01, the company now makes 0.01 more of profit for each decision unit, resulting in 45 of increased profit.

4. How much more profit would the firm earn if Decision 2's objective coefficient went up by 0.5?

   Since Decision 2 is not used at all, and the increase of 0.5 is within its allowable increase, the company earns 0 extra profit.

5. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.01 and Decision 4's objective coefficient went up by 0.02?

   Sensitivity analysis cannot predict the behavior of the model when multiple variables are changed at the same time. Thus, it cannot be determined from sensitivity analysis alone.

6. How much more profit would the firm make if the RHS of Constraint 1 was increased by 2500?

   The increase of 2500 is within Constraint 1's allowable increase, and a shadow price of 0.65 means for every additional constrained unit, the profit goes up by 0.65. Therefore, the company earns 0.65 * 2500 = 1625 additional profit.

7. How much more profit would the firm make if the RHS of Constraint 3 was increased by 1000?

   Similar to Question 6, the 1000 increase is within Constraint 3's allowable increase, and Constraint 3 has a non-zero shadow price of 0.14. Therefore, the firm makes 1000 * 0.14 = 140 additional profit.

Computer Parts – Blending Problem

Problem:

You are the owner of a store for junk computer parts. You have CPUs, RAMs, and SSDs, which you can supply at the cost of $14.4, $12 and $9 each, respectively. You offer two types of blends: basic and premium. For basic, you will charge $33 for each piece of hardware, while for premium you will charge $36 per piece of hardware. You have 200 CPUs, 300 RAMs and 400 SSDs in inventory. However, there are a few guidelines you must follow:

- Basic must contain:
    - At least 30% SSDs;
    - At most 50% RAMs;
    - At least 30% CPUs;
- Premium must contain:
    - At most 40% SSDs;
    - At least 35% RAMs;
    - At most 40% CPUs.

You do not have to complete the excel model for this problem. All questions will be on the next page.

1. Consider the following partially completed spreadsheet. There are some cells highlighted in blue which do not have their values filled in. What should be the best formula for each of the following cells labelled (a) to (g)?

	A	B	C	D	E	F	G	H
1	Computer Parts							
2								
3	Input Data							
4			Cost($)			Basic	Premium	
5		CPUs	$ 14.40		Revenue	$ 33.00	$ 36.00	
6		RAMs	$ 12.00					
7		SSDs	$ 9.00					
8								
9								
10	Action Plan							
11			Basic	Premium	Total		Constraint	
12		CPUs			(b)	<=	200	
13		RAMs			0	<=	300	
14		SSDs			0	<=	400	
15		Output	(a)	0				
16								
17								
18								
19	Blending Constraints							
20						Output		Constraint
21		Basic must be at least		30%	SSD	(c)	>=	(d)
22		Basic must be at most		50%	RAM	0	<=	0
23		Basic must be at least		30%	CPU	0	>=	0
24		Premium must be at most		40%	SSD	0	<=	(e)
25		Premium must be at least		35%	RAM	0	>=	0
26		Premium must be at most		40%	CPU	0	<=	0
27								
28	Revenue/Cost							
29								
30			Basic	Ultra				
31		CPUs	$ -	$ -				
32		RAMs	(f)	$ -				
33		SSDs	$ -	$ -				
34			Revenue	(g)				
35			Profit	$ -				

(a) = SUM(C12:C14)

(b) = SUM(C12:D12)

(c) = C14

(d) = D21 * C15
(e) = D24 * C16
(f) = C13 * $C6
(g) = SUMPRODUCT(C15:D15, F5:F6)

2. How much of each bulk should you produce to maximize profit? What is the breakdown of each part among each blend

   Here is the action plan under the optimal solution.

Action Plan						
		Basic	Premium	Total		Constraint
	CPUs	40	160	200	<=	200
	RAMs	0	300	300	<=	300
	SSDs	93.33	306.67	400	<=	400
	Output	133.33	766.67			

3. Suppose you are doing an algebraic formulation for this blending problem. Write down all the blending constraints in algebraic form. Use the following labels: BC, BR, BS, PC, PR, PS, with the first letter representing the blend and the second letter representing the part.

   0.7BS – 0.3BR – 0.3BC >= 0 (Basic must be at least 30% SSD)
   0.5BR – 0.5BS – 0.5BC <= 0 (Basic must be at most 50% RAM)
   0.7BC – 0.3BR – 0.3BS >= 0 (Basic must be at least 30% CPU)
   0.6PS – 0.4PC – 0.4PR <= 0 (Premium must be at most 40% SSD)
   0.65PR – 0.35PC – 0.35PS >= 0 (Premium must be at least 35% RAM)

0.6PC – 0.4PS – 0.4PR <= 0 (Premium must be at most 40% CPU)

4. Consider the following sensitivity analysis for Computer Parts with the optimal solution blacked out and answer the following questions.

Microsoft Excel 15.15 Sensitivity Report
Worksheet: [COMM 290 CMP Computation Spreadsheets.xlsx]Computer Parts - Solved
Report Created: 2017-10-16 9:35:57 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	CPUs Basic	[redacted]	0	18.6	3	1E+30
D12	CPUs Premium	[redacted]	0	21.6	1E+30	3
C13	RAMs Basic	[redacted]	-3	21	3	1E+30
D13	RAMs Premium	[redacted]	0	24	1E+30	3
C14	SSDs Basic	[redacted]	0	24	3	24
D14	SSDs Premium	[redacted]	0	27	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E12	CPUs Total	[redacted]	21.6	200	146.6666667	160
E13	RAMs Total	[redacted]	24	300	1E+30	31.66666667
E14	SSDs Total	[redacted]	24	400	1E+30	53.33333333
F21	SSD Output	[redacted]	0	40	53.33333333	1E+30
F22	RAM Output	[redacted]	0	66.66666667	1E+30	66.66666667
F23	CPU Output	[redacted]	-3	40	160	40
F24	SSD Output	[redacted]	3	306.6666667	53.33333333	306.6666667
F25	RAM Output	[redacted]	0	268.3333333	31.66666667	1E+30
F26	CPU Output	[redacted]	0	306.6666667	1E+30	146.6666667

a. One non-negativity constraint is binding. How many of the other constraints are binding?

Recall a binding constraint will always have an associated shadow price. Therefore, there must be 5 binding constraints.

b. Due to an increase in demand, the price of the basic blend has increased from \$33 to \$36. Will this change the optimal solution? If yes, by how much will this increase the value in the target cell? If not, why not?

   Cannot be evaluated as this changes multiple coefficients.

Food Services - Scheduling

Problem:

You are the manager of a 24-hour fast-food restaurant on campus. Your restaurant offers six labour shifts per 24-hour period, starting at 12am, 4am, 8am, 12pm, 4pm, 8pm, and 12pm. You have access to workers who work two consecutive shifts a day. Due to fluctuations in demand, your required labor at different time periods is as follows:

- **12am-4am: 3 workers**
- **4am-8am: 4 workers**
- **8am-12pm: 7 workers**
- **12pm-4pm: 8 workers**
- **4pm-8pm: 6 workers**
- **8pm-12am: 5 workers**

Find the scheduling method that will use the minimum amount of workers. Produce a sensitivity analysis. Assume all sensitivity report questions are fully independent.

1. Is this a maximizing or minimizing model? What are you trying to maximize or minimize?

   This is a minimizing model, and you are trying to minimize the amount of workers used.

2. What is the optimal solution? How many binding constraints are there? What about non-binding?

   Here is the action plan under the optimal solution.

Food Service								
		Time Periods Covered						
		12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am	Labour
Start Time	12am	4	4					4
	4am		0	0				0
	8am			7	7			7
	12pm				1	1		1
	4pm					5	5	5
	8pm	0					0	0
	Supply	4	4	7	8	6	5	17
		>=	>=	>=	>=	>=	>=	
	Required	3	4	7	8	6	5	

3. Due to an overnight frat party, your demand for workers at 12am-4am goes up to four. Will this affect your optimal solution? Why or why not? If yes, what is the new optimal solution? Do not modify your LP.

   Looking at the sensitivity analysis, there is an allowable increase of 1 for the 12am-4am labour demand. Since the demand at that time is non-binding, it will remain non-binding after the increase to 4 and thus there is no change in the optimal solution. Since this constraint becomes binding, any further changes will affect your solution.

4. Suppose the demand for workers at 4am-8am increases by one. How will this affect the optimal solution?

   Consider that the labour demand at 4am-8am has a shadow price of 1. Therefore, if the demand goes up by 1, you will require 1 additional worker.

Canada Post – Transportation

Problem:

You are the manager of a few Canada post branches in Vancouver. You have three branches under your control: Robson, Pine and Oak, and you must deliver units of identical supplies to UBC, YVR Airport and Oakridge center. The supply and demand at each location is as follows:

- Robson holds 7 units, Pine holds 16 and Oak holds 13.
- UBC requires 17, YVR Airport requires 5 and Oakridge requires 14.
- The shipping costs are as follows:

		Deliver To:		
		UBC	YVR Airport	Oakridge
Deliver From:	Robson	$4.00	$4.50	$2.50
	Pine	$3.00	$4.00	$2.50
	Oak	$3.50	$3.00	$2.00

Solve for the optimal solution, and produce a sensitivity analysis.

1. What is the optimal solution, and how many constraints are binding?

   The optimal solution is attached below (10 binding constraints), and total cost is $98:

Shipments	UBC	YVR	Oakridge
Robson	0	0	7
Pine	16	0	0
Oak	1	5	7

2. What will happen to the LP if suddenly, an explosion happens at the Robson branch and three of the seven units in stock are destroyed?

   This will make the LP infeasible, as the amount of units in stock is equal to demand at this point in the model. If three parcels in stock are removed, then there will be a shortage in supply and thus making the LP infeasible.

3. What are the objective coefficients in this model?

   The objective coefficients in this model are the nine costs associated with each shipping route.

4. Refer to your produced sensitivity analysis for the next questions. If you cannot answer the question without running Solver again, please answer "we don't know for sure."
    a. Is there evidence of multiple optima in this LP?

       Since there are many objective coefficients with a 0 allowable increase/decrease, there is indeed evidence of multiple optima.

    b. Suppose due to the acquisition of a new truck, it now costs $2.50 to deliver from Oak to YVR airport. Will this change the optimal solution? What will be the new value in the target cell?

       The decrease in $0.50 is within the allowable decrease of 3.5. The optimal solution does not change, but the new value of the target cell decreases from $98 to $95.50 (0.5 * 5 decrease).

    c. Suppose Pine has increased their supply by one. How will this affect the target cell under the optimal solution?

       An increase of 1 is within the allowable increase for the supply at Pine. The shadow price is a non-zero value of -1; therefore, if the constraint goes up by 1, the target cell will go down by $1.

d. Suppose Pine increases their supply by five to a total of 21. What will be the new amount of parcels shipped from Pine?

We don't know for sure, as the increase of 5 is beyond the allowable increase.

e. Suppose Oak increases their supply by five to a total of 18. What will be the new target cell value under the new optimal solution?

Since Oak's supply has an allowable increase of 5 and a shadow price of -0.5, the new target cell value will decrease by 5 * 0.5 to a new value of $95.50.

f. Due to new hires, the cost of all shipments from Oak have been reduced by $0.10. What will be the new optimal solution?

We don't know for sure, since this will change three objective coefficients at the same time.

g. Shipments from Pine to UBC are now free. What is the new optimal solution and target cell value?

The decrease from 16 to 0 is within the allowable decrease of 1E+30, so the optimal solution remains the same. Since shipments from Pine to UBC would cost $48 in the original model, that amount will be subtracted from the target cell since these shipments are now free. The new target cell value will be $50.