commerce undergraduate society

COMMERCE MENTORSHIP PROGRAM

MIDTERM REVIEW SESSION

COMM 298

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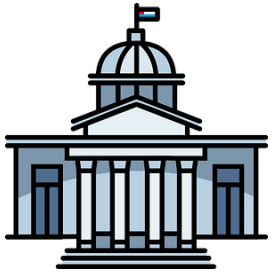
What is Finance?

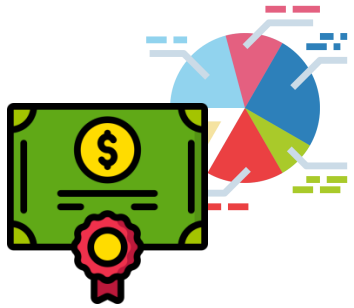
Individuals

Allocate Resources

Businesses



Institutions



Make Investments

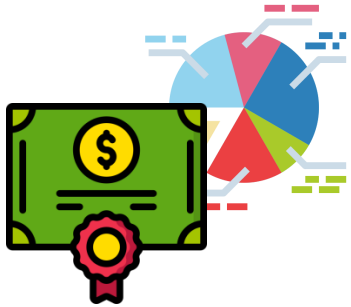
Manage Money

Maximize Value or Achieve Financial Objectives

Some Areas of Finance

Corporate Finance

- Corporate Decisions
- Capital Budgeting
- Capital Structure



Investments

- Bond & Stock Valuation
- Risk & Return
- Wealth Allocation

Personal Finance

- Individual/Home Decisions
- Retirement Planning
- Mortgage Loans

Some more areas: Financial institutions, markets, international finance, fintech (financial technology)

Firm Ownership

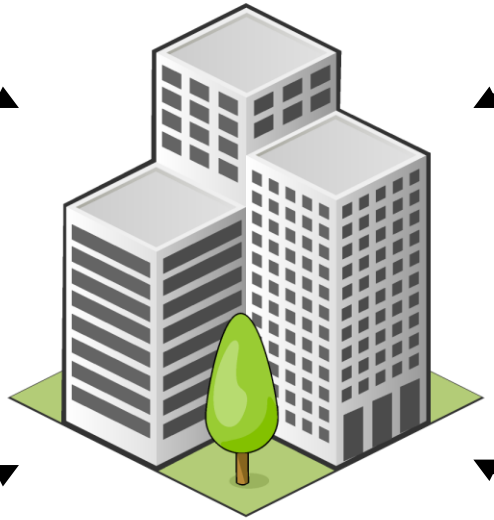
Sole Proprietorship

Partnership

Corporation

Characteristics of a Corporation

Many owners who each own a fraction of the company



Evolved from a sole proprietorship

Possesses Legal Powers

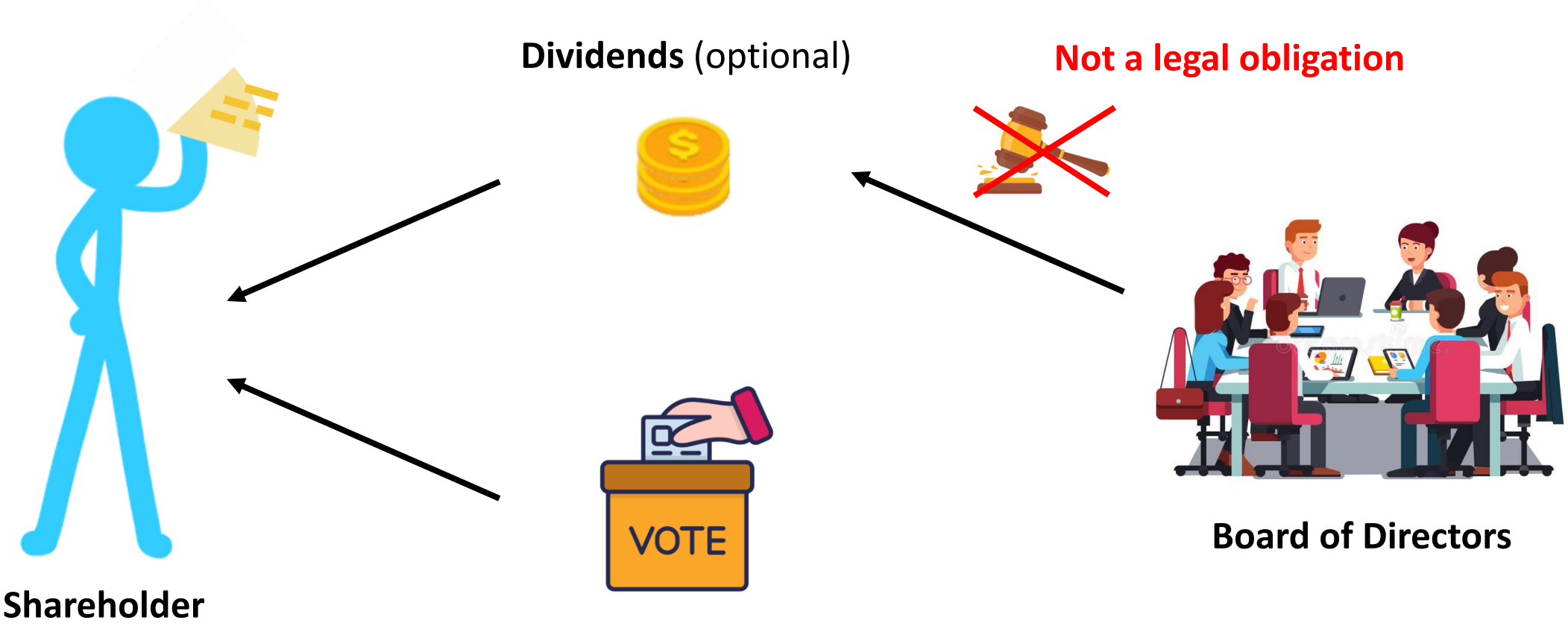
Contracts, Assets, Obligations

Separate entity from owners (shareholders)

Limited Liability

Equity & Shareholders

What Does a Shareholder Get?



Practice Question 1

Which of the following correctly describe a corporation? Select all that apply.

a) Shareholders under a corporation have unlimited liability for its obligations

b) Shareholders under a corporation are diverse and allow a substantial amount of capital to be raised

c) The financial manager of a corporation determines whether dividends will be paid or not

d) Usually, a shareholder who has more shares can cast more votes when electing the board of directors

e) There is a finite limit to how many owners a corporation can have as office space is never infinite

Practice Question 2

Which of the following is/are related to the field of corporate finance? Select all that apply.

a) A CEO wants to determine what capital structure (mix of debt & equity) to use next year

b) A CFO wants to determine how much to save each year for his daughter's RESP

c) A financial analyst wants to determine whether their firm should open a factory in USA given the costs

d) A financial manager wants to determine what dividend policy to use for their company's shareholders

e) A treasurer wants to determine the stocks he needs to purchase to obtain an efficient portfolio for his wife

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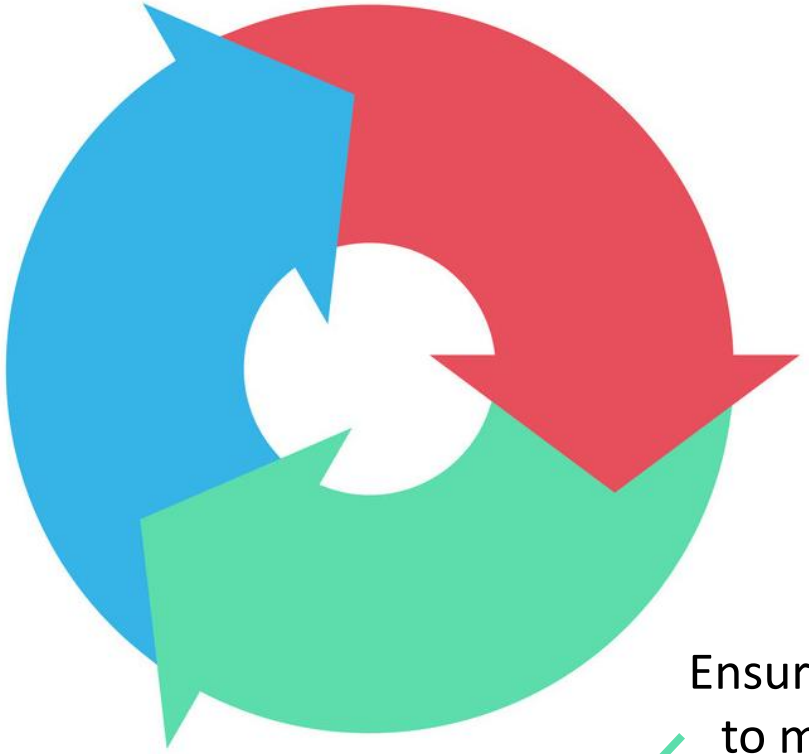
The Financial Manager's Duties

How do we pay for investments & projects?

Weighs cost & benefits of investments & projects



Financing

Investing

Do we raise money using equity or debt?

What investments & projects are worthwhile?

Ensure company has enough cash to meet day-to-day obligations

Managing Cash

What is the Primary Goal of a Corporation?

A firm's management should make decisions in the shareholders' interests. What should they do?

a) Maximize profit

b) Minimize costs

c) Maximize market share

d) Maximize the current value of the company's stock (maximize owners' equity)

Shareholders

We are better off if the value of our investment is maximized!

Should Companies Pursue ESG Instead?

Should we focus on shareholder welfare maximization (SWM) over shareholder value maximization (SVM)?

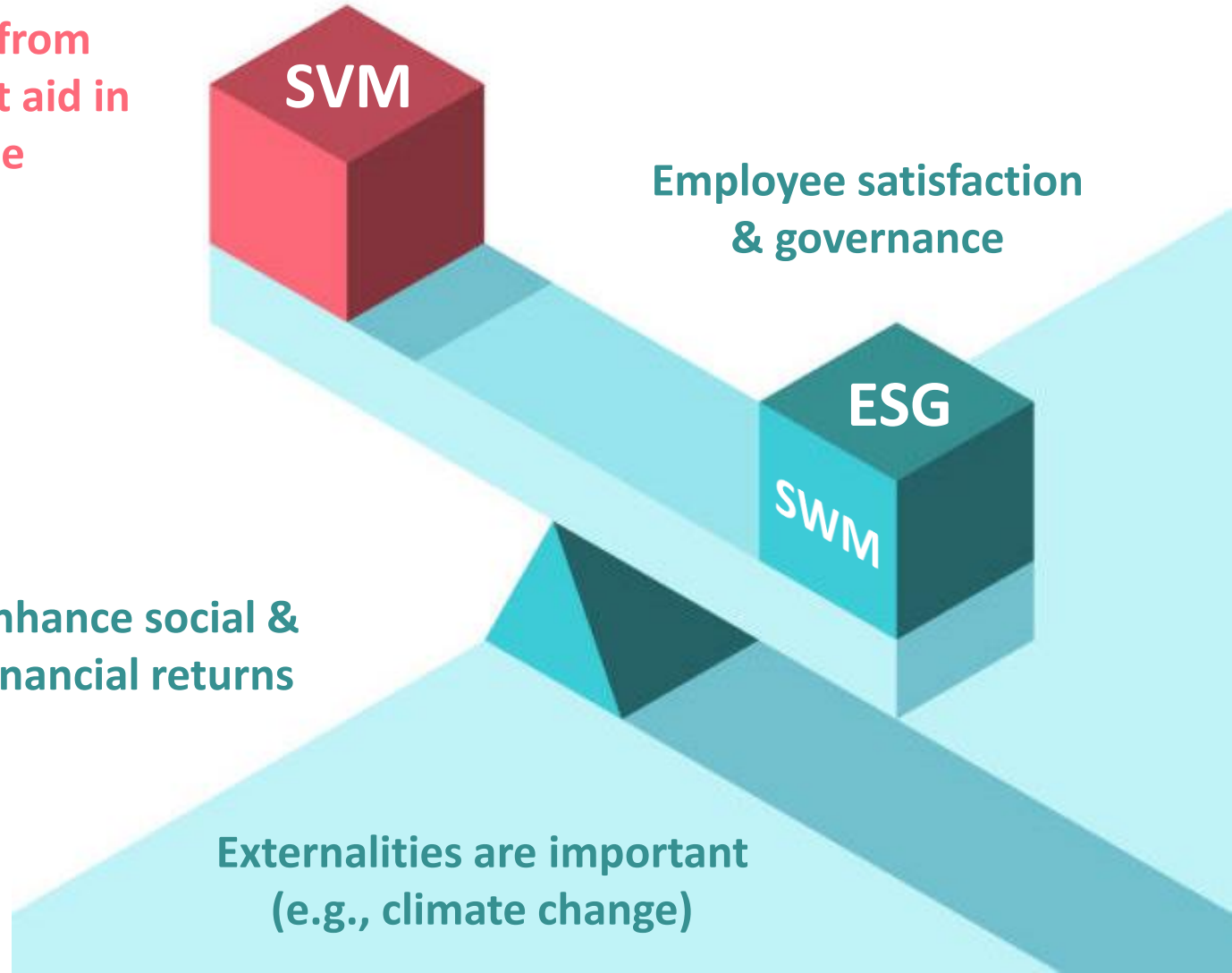
ESG distracts one from non-ESG factors that aid in long-term value

No scope for social goals

Dangerous to deviate from SVM

Focus on long-term value rather than if something falls under ESG

Some ESG factors have no link to performance



The Board of Directors

Shareholders

Shareholders Vote for Board of Directors

CEO

CEO Implements Rules Set by Board

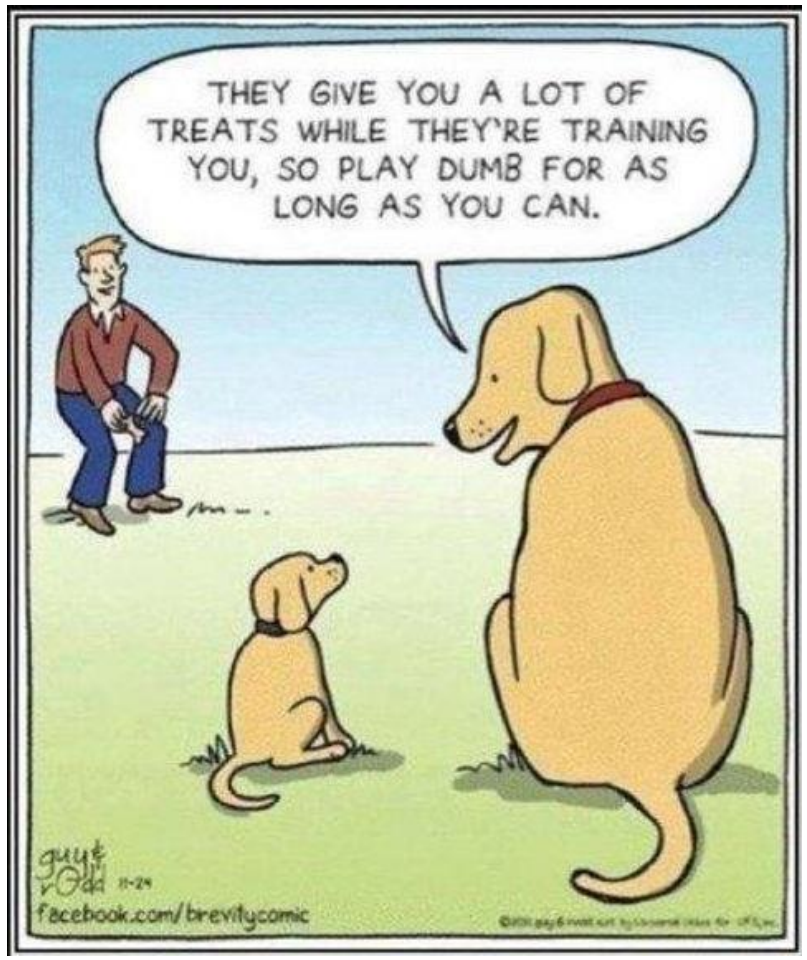
Board of Directors

Selects CFO

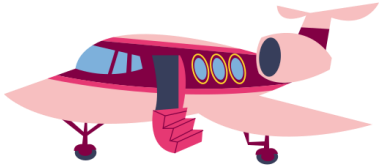
Board of Directors Hires Managers

The Principal-Agent Problem

Managers (agents) may put their own self-interest before their shareholders' (principals) interests



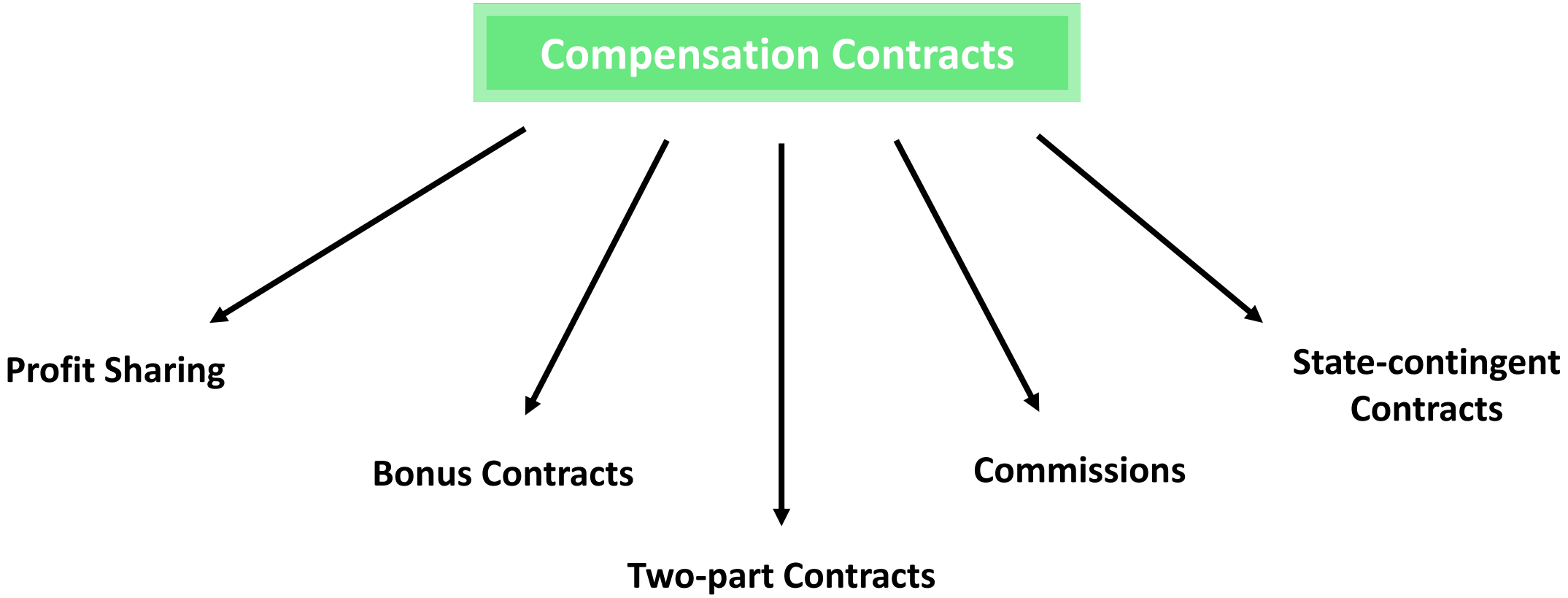
Shirking



Perquisites

Resolving the Principal-Agent Problem

We can use compensation contracts that relate corporate performance to manager compensation



Practice Question 1

Which of the following best illustrates a principal-agent problem in the corporate environment?

a) Managers taking an approved one-month vacation from work to go to Europe

b) Two employees arguing over the formatting and structure of an instruction manual, reducing productivity

c) Executives pursuing non-ESG activities that enhance profitability over the ESG activities shareholders want

d) Board of directors using their lunch breaks for personal matters (e.g., checking their phones)

e) Customer Engagement director travelling overseas to conduct business meetings with stakeholders

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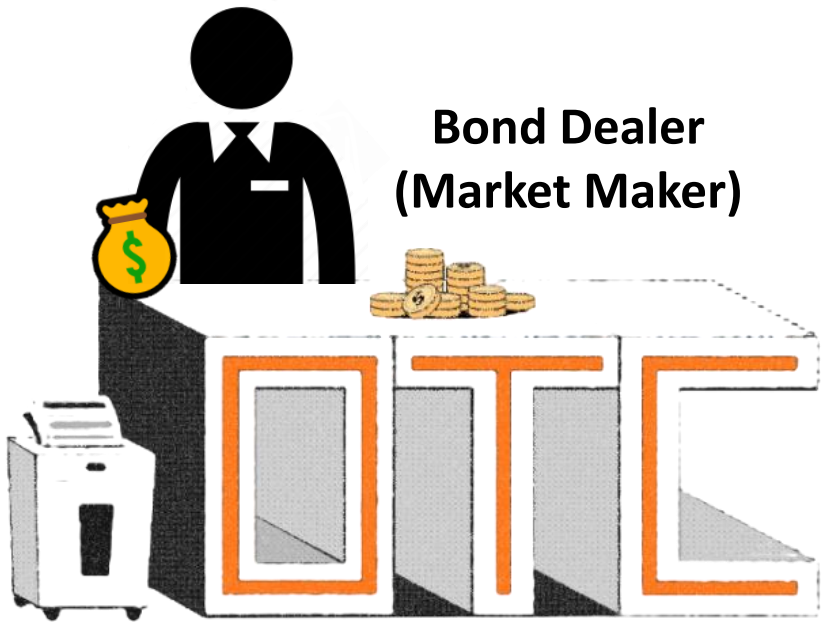
Financial Markets

Platforms or systems where financial assets (e.g., stocks, bonds, derivatives, etc.) are bought and sold



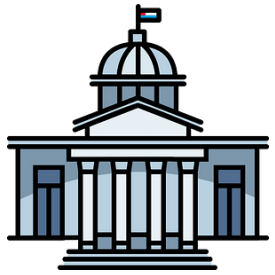
Financial Market Functions

(1) Allows individuals, governments, and businesses to efficiently raise capital through fund allocation

Who needs money?

Individuals

Businesses



Governments

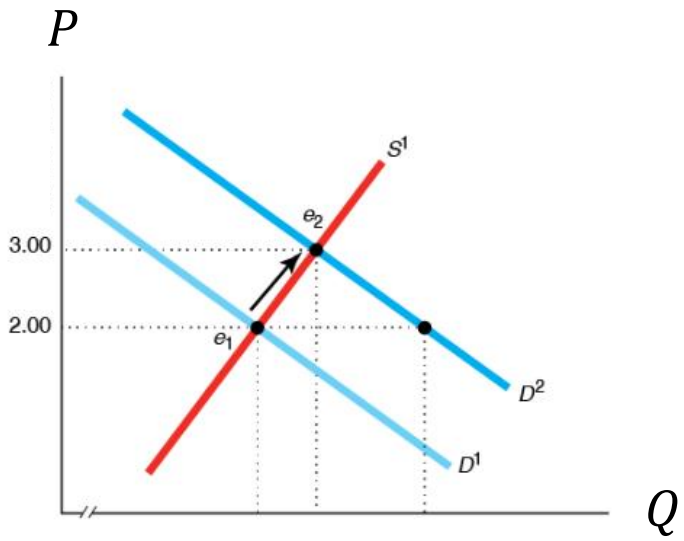
Who supplies money?

Investors

Businesses

Financial Market Functions (Continued)

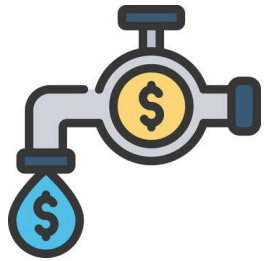
(2) A tool to help determine the fair value (market value) of financial assets (e.g., stocks)



Supply & demand help establish market prices

But is the price always right?

(3) Can sell assets for cash with ease without significantly impacting the asset's price (**liquidity**)



(4) Allows investors to manage financial risks (e.g., via derivatives – options, futures, etc.)

(5) Helps transmit information to investors as prices & trading volumes encode publicly available information

Financial Market Categorization

By Sale

Primary Market

Secondary Market

By Asset

Bonds

Stocks

Derivatives

By Maturity

Money Market

Capital Market

Categorization by Sale

Investor

Trades after original sale

Secondary Market

Investor

Original sale of assets

Primary Market

Corporation

Provides liquidity for investors (firm gets no money!)

Initial public offering (IPO)

Categorization by Asset Traded (Bonds – Fixed Income Market)

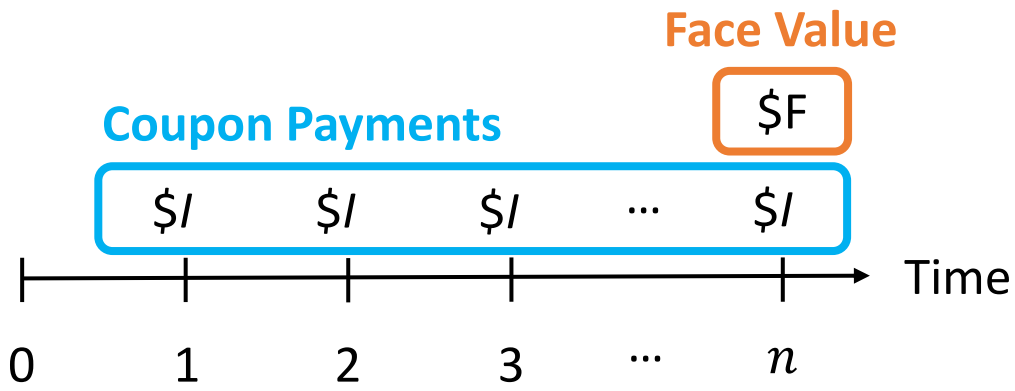
Investor

Secondary Market

Investor

Primary Market

Corporation



Categorization by Asset Traded (Stocks – Equity Market)

New York Stock Exchange (NYSE)

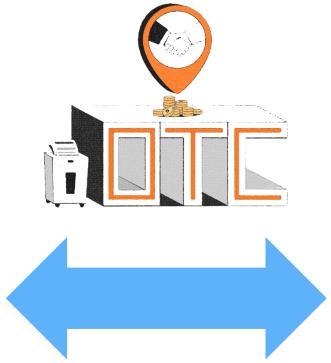
Toronto Stock Exchange (TSX)

National Association of Securities Dealers Automated Quotations (NASDAQ)

Categorization by Asset Traded (Derivatives Market)

Financial contracts whose value is derived from an underlying benchmark – used for risk management

Investor



Investor

Categorization by Maturity

Maturity is the date on which a financial asset reaches its end

Money Market

Short-term assets: Bonds (treasury bills), Certificates of Deposit, Commercial Paper

Capital Market

Long-term assets: Bonds (e.g., consol bond) & Stocks

Practice Question 1

The main roles of a financial market constitute which of the following? Select all that apply.

a) Provides liquidity to investors who want to sell securities

b) Solves principal-agent problems within corporations as public information is encoded in the prices of stocks

c) Enables investors to determine the fair value of assets traded

d) Allows corporations to issue debt (e.g., bonds) and equity (e.g., stocks) to fund their capital projects

e) Bridges the gap between shareholder value maximization and shareholder welfare maximization

Practice Question 2

Which of the following is/are true about financial assets? Select all that apply.

a) Investors who own a stock receive periodic payments (interest and principal) from the issuer

b) Financial assets with a maturity of more than one year are sold within the money market

c) The over-the-counter market is an example of a primary financial market where bonds are sold

d) Derivatives are used as a medium to achieve shareholder welfare maximization

e) None of the above

Practice Question 3

Which of the following is/are traded on the money market? Select all that apply.

a) A Government of Canada (GOC) bond issued on Jan. 7th, 2023 and matures on Dec. 16th, 2036

b) A zero-coupon treasury bill with 8 months until maturity

c) 100 shares from Apple Inc. that have no fixed maturity date

d) A commercial paper used to finance short-term liabilities

e) None of the above

Investors & Financial Institutions

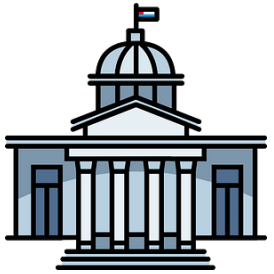
Investors

Corporations

- Net demander of financing
- Raise capital from markets
- $$ from investment paid back to investors

Households

- Net supplier of financing
- Purchase securities issued by corporations

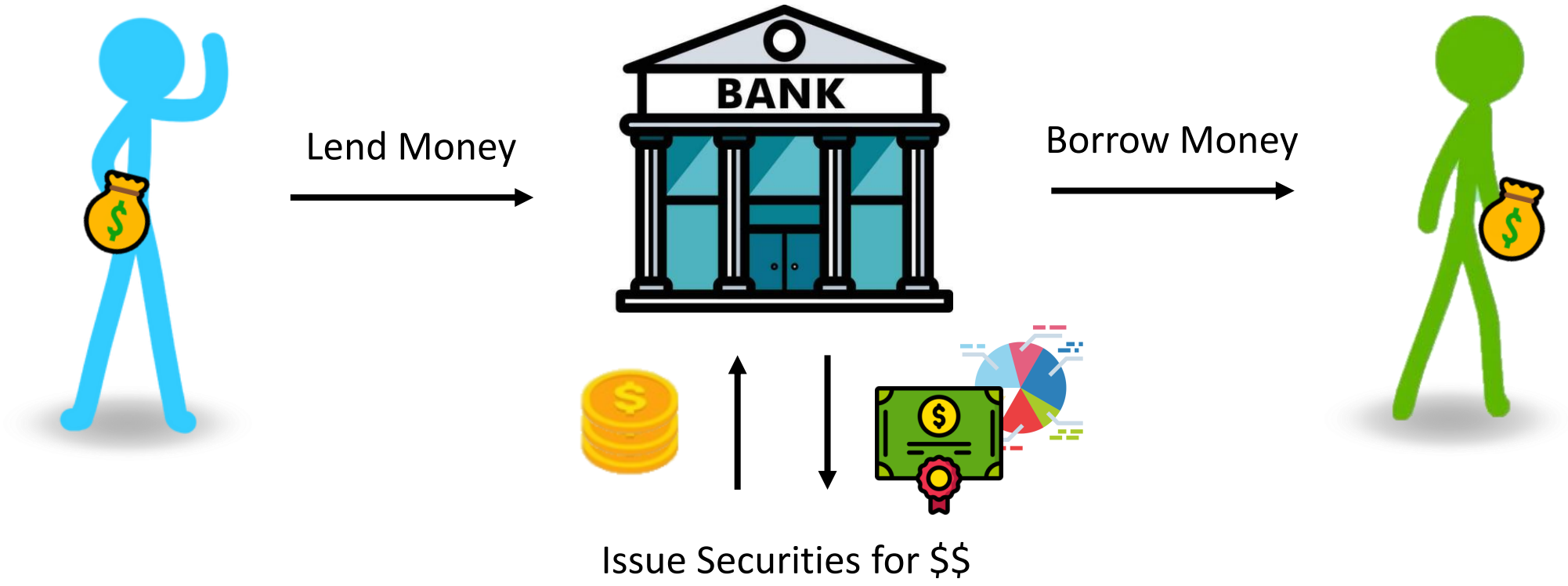


Governments

- Net supplier or demander
- Issues majority of debt instruments in bond market

Intermediation

"Matchmaking" between buyers (demand) and sellers (supply) is hard – we need intermediaries!



Do lenders and borrowers ever need to meet? NO!

Some Other Intermediaries

Investment Companies (e.g., mutual fund providers)

- Exploit economies of scale to pool and manage money from many investors
- Reduce access fees for investors as a large-scale portfolio with many financial assets is built

Investment Banks (e.g., Goldman Sachs)

- Help firms issue securities (via underwriting), go public, acquire other firms, or sell assets
- Advise the company on their asset's IPO price and market the asset

Venture Capital Funds & Private Equity Funds

- Equity investment in young, immature startups is called venture capital (VC)
- Private Equity funds are investments in private companies (one may convert a public firm to private)

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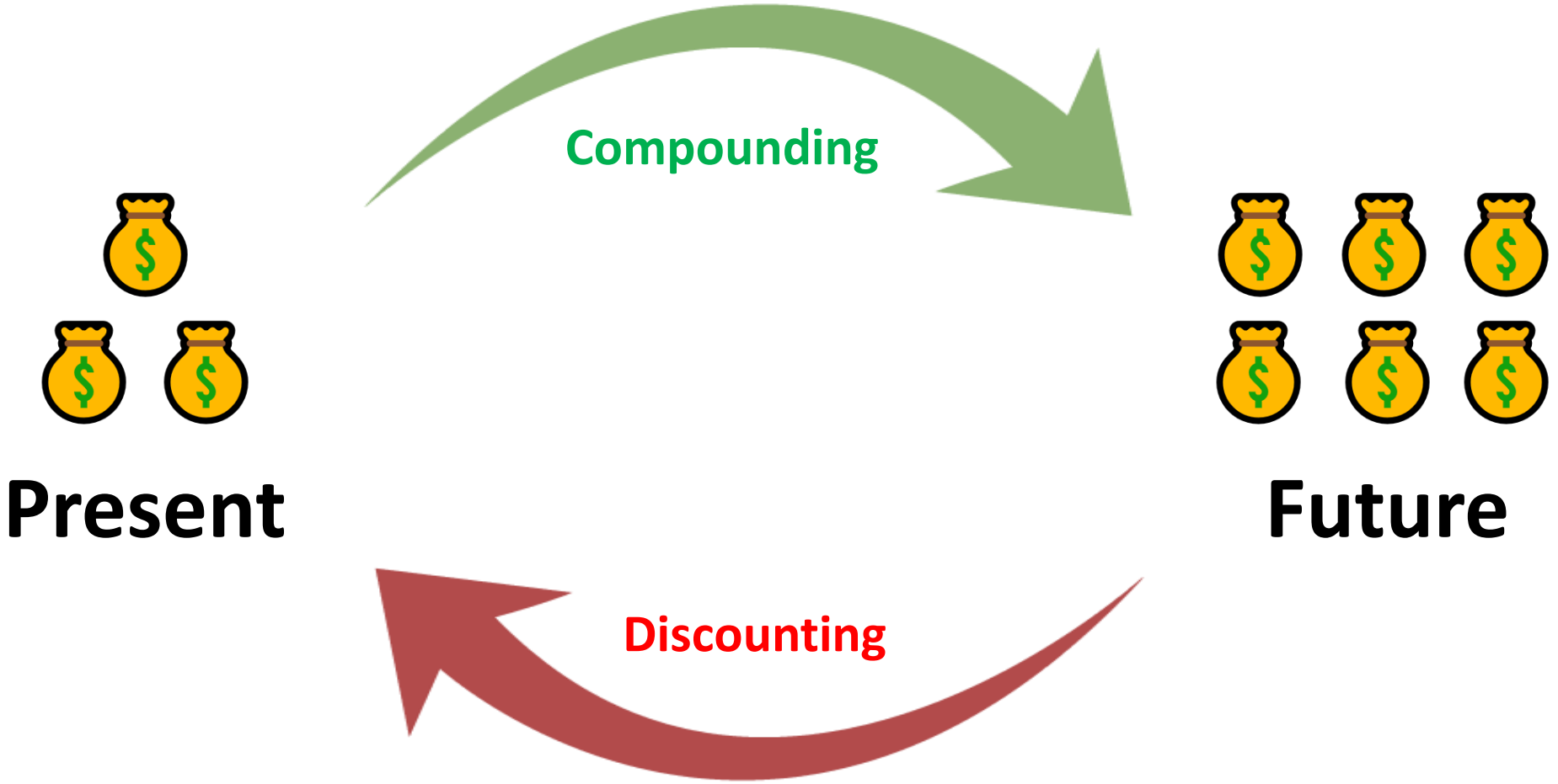
A dollar today is …

… worth more than a dollar tomorrow

Present & Future Value



Compounding & Future Value of One Period Cash Flows

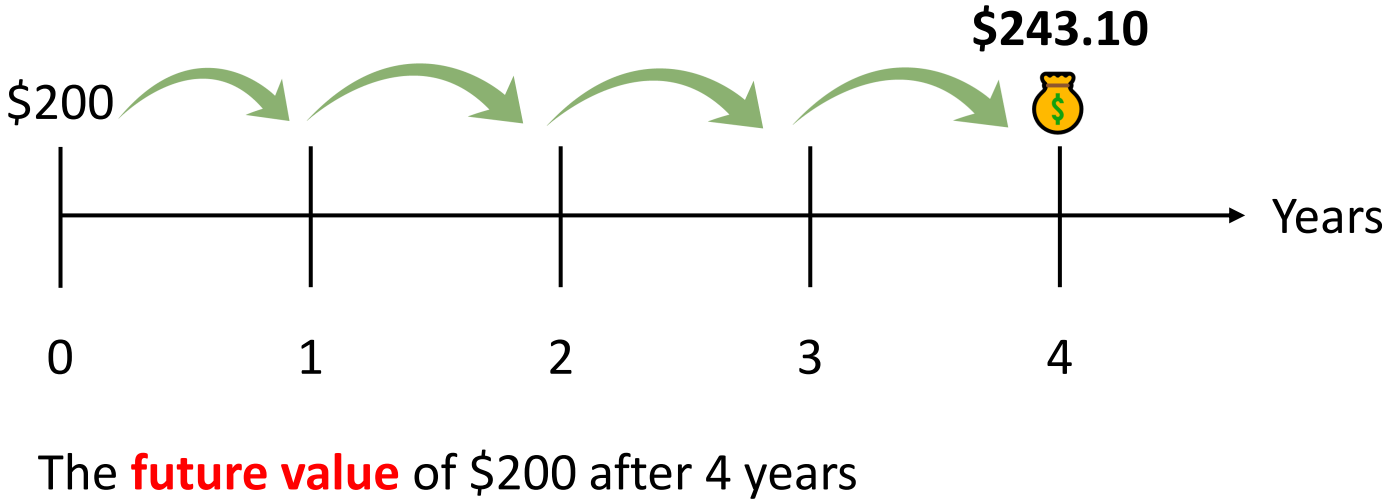
What is the value of a $200 investment compounded annually at 5% for 4 years?

After 1 Year: $200 \cdot (1.05) = \$210$

After 2 Years: $210 \cdot (1.05) = \$220.50$

After 3 Years: $220.50 \cdot (1.05) = \$231.53$

After 4 Years: $231.53 \cdot (1.05) = \$\mathbf{243.10}$



The **future value** of $200 after 4 years

Our calculations can be seen as $200 \cdot (1.05)^4 = \$243.10$

Future Value Formula

$$FV_n = C_0 \cdot (1 + r)^n$$

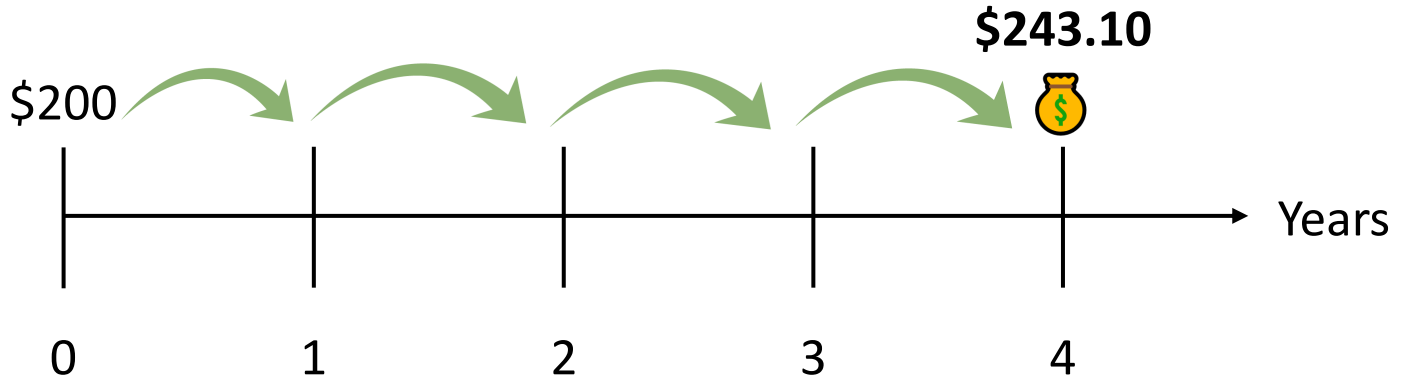
t stands for time here!

FV_n = Future value of cash flow at $t = n$

C_0 = Value of cash flow at $t = 0$

r = Effective interest rate

n = Number of periods



In our example: $n = 4$ $r = 0.05$ $C_0 = 200$ $FV_4 = 243.10$

Note: n and r must be in the same time units (e.g., if r is a yearly rate, n must be in years!)

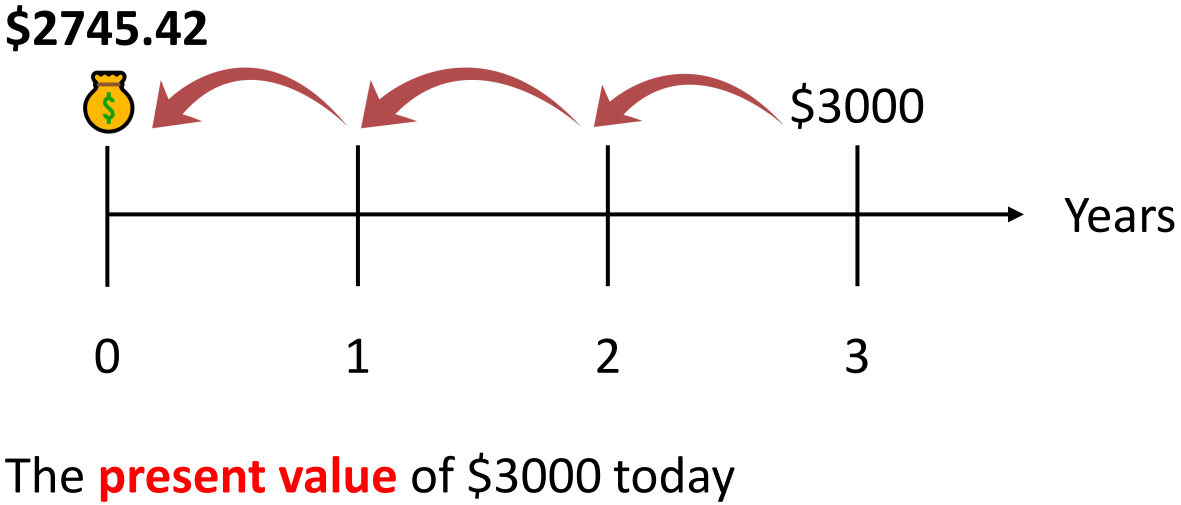
Discounting & Present Value of One Period Cash Flows

You will receive $3000 in lottery winnings 3 years from today. If your bank offers a 3% interest rate compounded annually, what is the value of your earnings today?

After 1 Year: $3000 \cdot (1.03)^{-1} = \$2912.62$

After 2 Years: $2912.62 \cdot (1.03)^{-1} = \$2827.79$

After 3 Years: $2827.79 \cdot (1.03)^{-1} = \$\mathbf{2745.42}$



The **present value** of $3000 today

Our calculations can be seen as $3000 \cdot (1.03)^{-3} = \$2745.42$

Present Value Formula

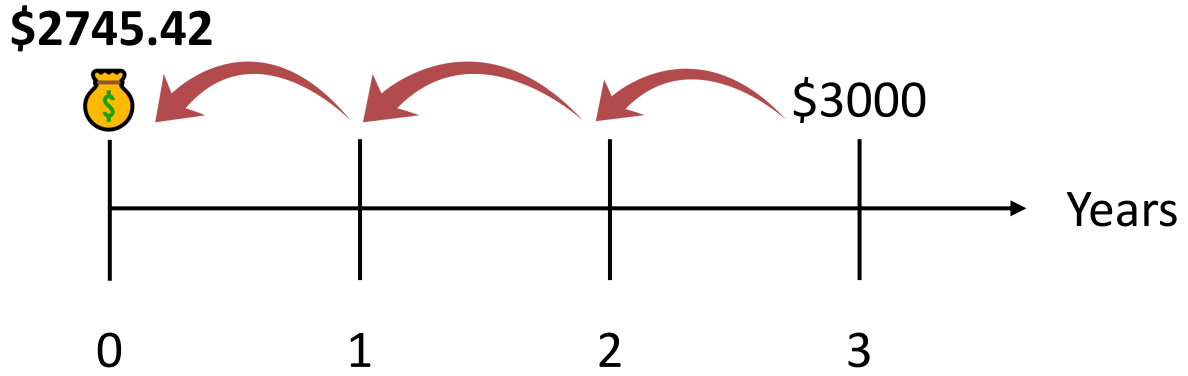
$$PV_0 = C_n \cdot (1 + r)^{-n}$$

C_n = Value of cash flow at $t = n$

PV_0 = Present value of cash flow at $t = 0$

r = Effective interest rate

n = Number of periods

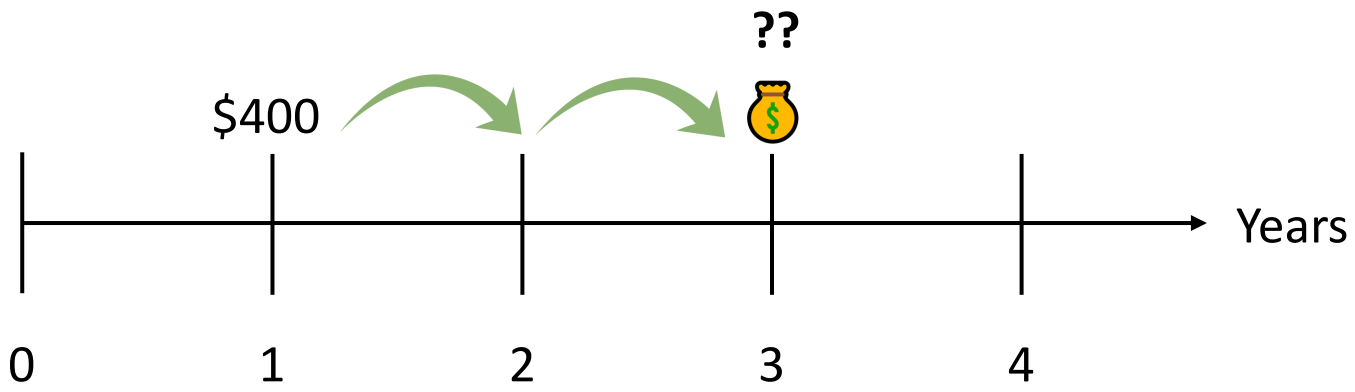


In our example: $n = 3$ $r = 0.03$ $PV_0 = 2745.42$ $C_3 = 3000$

Remark: As r increases, PV decreases – we are more impatient to receive our $$!

Valuation of One Period Cash Flows not at $t = 0$

I want to find the FV at $t = 3$ of a \$400 cash flow at $t = 1$ given a 5% yearly interest rate



We can modify the formula

$$FV_n = C_0 \cdot (1 + r)^n$$

How many periods are we moving this cash flow forward? Two!

$$FV_3 = C_1 \cdot (1 + r)^2$$

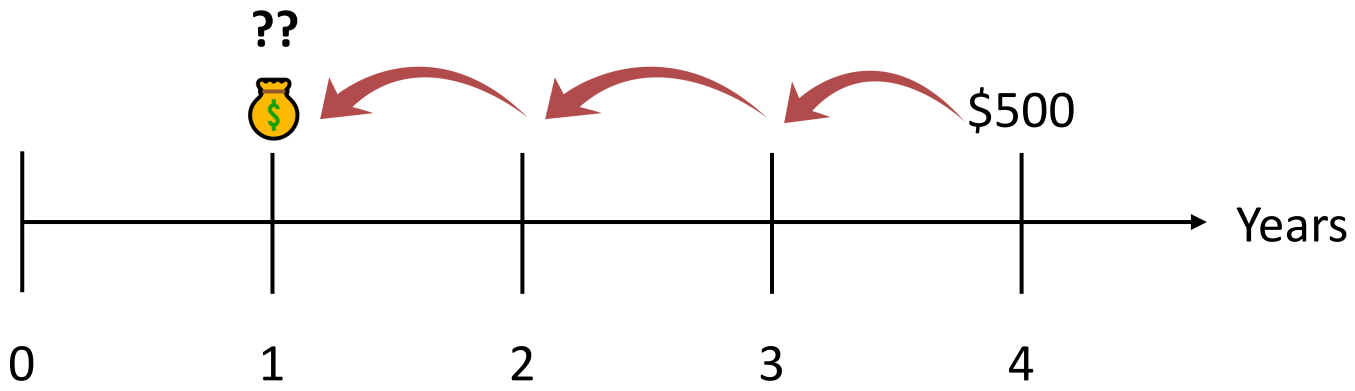
Want FV at $t = 3$

The cash flow (\$400) is at $t = 1$

Solution: $FV_3 = 400 \cdot (1 + 0.05)^2 = \$\mathbf{441}$

Valuation of One Period Cash Flows not at $t = 0$ (Continued)

I want to find the PV at $t = 1$ of a \$500 cash flow at $t = 4$ given a 2% yearly interest rate



We can modify the formula

$$PV_0 = C_n \cdot (1 + r)^{-n}$$

How many periods are we moving this cash flow back? Three!

$$PV_1 = C_4 \cdot (1 + r)^{-3}$$

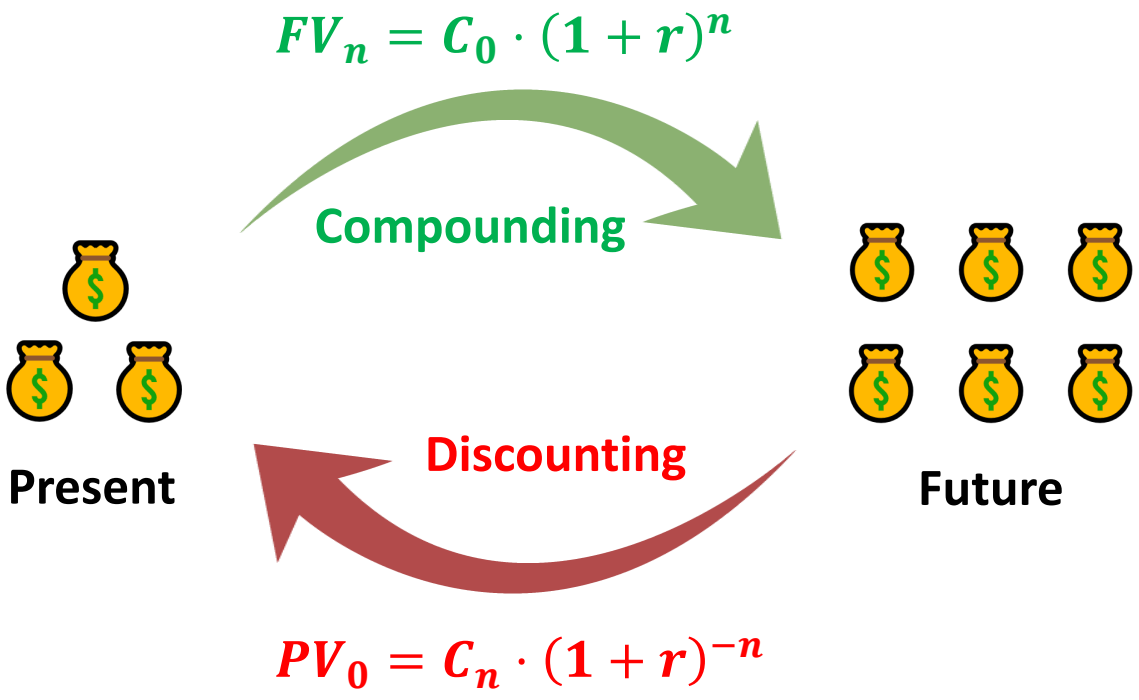
Want PV at $t = 1$

The cash flow (\$500) is at $t = 4$

Solution: $PV_1 = 500 \cdot (1 + 0.02)^{-3} = \$\mathbf{471.16}$

Summary of Discounting & Compounding One Period Cash Flows

Valuation of cash flows at $t = 0$



C_0 = Value of cash flow at $t = 0$

C_n = Value of cash flow at $t = n$

FV_n = Future value of cash flow at $t = n$

PV_0 = Present value of cash flow at $t = 0$

r = Effective interest rate

n = Number of periods

} **Same time units!**

Valuation of cash flows NOT at $t = 0$

- Consider the time period of the cash flow and where it needs to be compounded/discounted
- Look at the number of periods one has to compound/discount

Practice Question 1

You received a $250 stipend one year ago and immediately put it in a mutual fund that offers a yearly return of 1.7%. How much is your money worth 3 years from today?

Practice Question 2

Kita is a musician who mistakenly spent her money on a 6-string bass instead of a guitar. She now must save $1000 to obtain a guitar but only can put $300 in the bank today. If she can obtain a guitar in 15 years, what yearly interest rate does her bank offer?

Interest Rates & Conversions

Effective Rates

What is my **actual** return over a time period? An effective rate (denoted by r) will tell us that!

1) Suppose I start with \$100 and end with \$107 after 3 months (a quarter). This is a gain of 7%!

$\Rightarrow r_{quarterly} = 0.07$

2) Suppose I start with \$200 and end with \$250 after 1 month. This is a gain of 25%!

$\Rightarrow r_{monthly} = 0.25$

3) Suppose I start with \$50 and end with \$60 after 1 year. This is a gain of 20%!

$\Rightarrow r_{annual} = 0.20$ [There is a **special name** for r_{annual} – it is called the **effective annual rate (EAR)**]

Annual Percentage Rate

Shows the amount of yearly interest **without** considering the effects of compounding

How to determine if you are given an APR rate?

- You are explicitly told it is an APR rate
- It is annual (monthly, quarterly, semi-annual, etc. rates CANNOT be annual)
- You are told the compounding frequency

Example: Your bank offers you an interest rate of 10% compounded semi-annually. What is the APR?

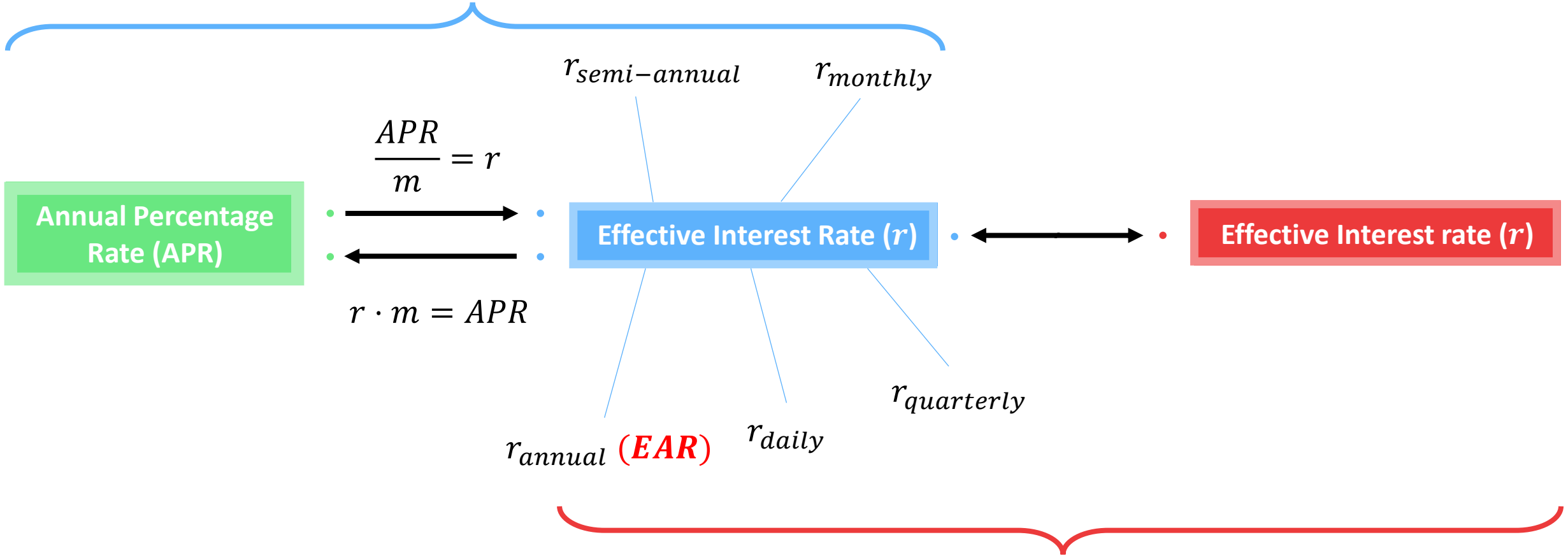
The APR is 10% (But the real return is 10.25% – more on how to compute this later)

Note: You MUST convert an APR rate to an effective rate before thinking of compounding/discounting!!!

How do I get an effective rate then?

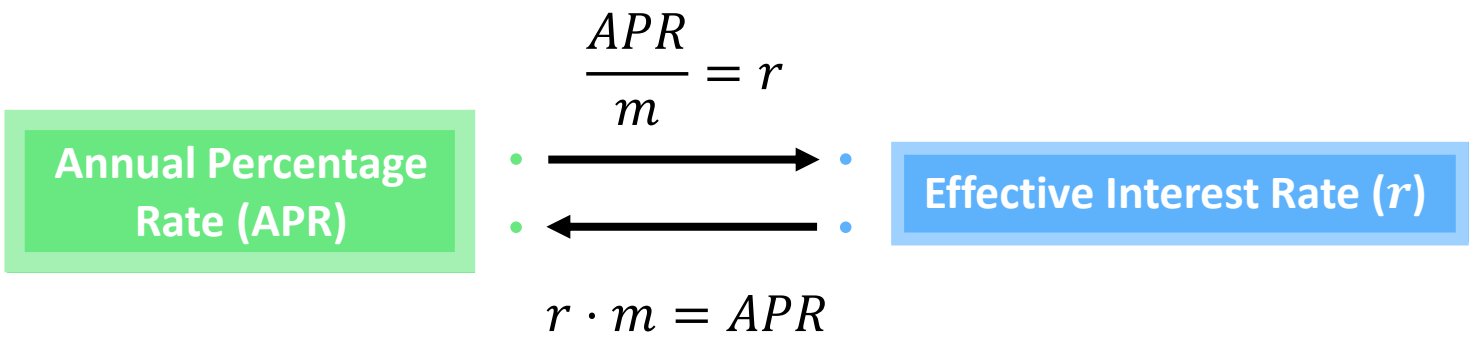
A Conversion Map for Interest Rates

How do I get from APR to an effective rate and vice versa?



How do I convert between effective rates?
(e.g., suppose I want to find $r_{monthly}$ given $r_{semi-annual}$)

Going Between APR & Effective Rates



m = # of compounding periods in ONE year

APR = Annual percentage rate

Remark: The type of effective rate (r) you get from APR entirely depends on the compounding frequency given

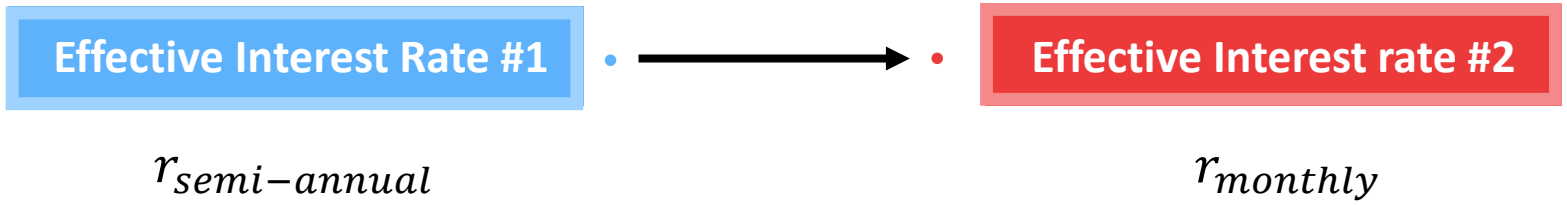
Compounding Frequency	(APR to Effective Rate)	Compounding Frequency	(APR to Effective Rate)
Daily ($m = 365$)	$\frac{APR}{365} = r_{daily}$	Quarterly ($m = 4$)	$\frac{APR}{4} = r_{quarterly}$
Weekly ($m = 52$)	$\frac{APR}{52} = r_{weekly}$	Semi-annually ($m = 2$)	$\frac{APR}{2} = r_{semi-annual}$
Monthly ($m = 12$)	$\frac{APR}{12} = r_{monthly}$	Annually ($m = 1$)	$\frac{APR}{1} = APR = r_{annual}$ **(EAR)**

Converting Between Effective Rates

Sometimes the occurrence of cash flows do not match up with the effective interest rate we get from APR.

If we wanted to discount/compound cash flows that occur monthly, we must use $r_{monthly}$. But what if we only can get $r_{semi-annual}$ from APR (compounded semi-annually)? We need to convert $r_{semi-annual}$ to $r_{monthly}$!

$$\boldsymbol{r_{effective\ \#2} = \left(1 + r_{effective\ \#1}\right)^{n} - 1}$$



Here, n is the number of times the time period of rate #1 goes into #2.

In our example we need to think, "how many half-years goes into one month?" – the answer is 1/6

Solution: $r_{monthly} = (1 + r_{semi-annual})^{\frac{1}{6}} - 1$

1/6 of half a year is a month

More Examples of Effective Rate Conversions

$r_{quarterly} \longrightarrow r_{annual}$ (EAR)	$r_{annual} = \left(1 + r_{quarterly}\right)^{\mathbf{4}} - 1$	There are **4** quarters in a year
$r_{weekly} \longrightarrow r_{annual}$ (EAR)	$r_{annual} = \left(1 + r_{weekly}\right)^{\mathbf{52}} - 1$	There are **52** weeks in a year
$r_{2-year} \longrightarrow r_{quarterly}$	$r_{quarterly} = \left(1 + r_{2-year}\right)^{\frac{\mathbf{1}}{\mathbf{8}}} - 1$	**1/8** of a 2-year period is a quarter (3 months)
$r_{3-week} \longrightarrow r_{6-week}$	$r_{6-week} = \left(1 + r_{3-week}\right)^{\mathbf{2}} - 1$	**Two** 3-week periods make up a 6-week period
$r_{quarterly} \longrightarrow r_{monthly}$	$r_{monthly} = \left(1 + r_{quarterly}\right)^{\frac{\mathbf{1}}{\mathbf{3}}} - 1$	**1/3** of a quarter (3 months) is a single month

Adjust the interest rate to match the cash flow frequency

The Effect of Compounding

How does r_{annual} (EAR) change when the number of compounding periods increases (holding APR fixed)?

Assume $APR = 0.1$ (10%) for the following scenarios

Compounding Frequency	APR → Effective	Effective → Effective [r_{annual} (EAR)]
Annually ($m = 1$)	$r_{annual} = \frac{APR}{1} = 0.1$	$r_{annual} = 0.1$
Semi-annually ($m = 2$)	$r_{semi-annual} = \frac{APR}{2} = 0.05$	$r_{annual} = (1 + r_{semi-annual})^2 - 1 = 0.1025$
Quarterly ($m = 4$)	$r_{quarterly} = \frac{APR}{4} = 0.025$	$r_{annual} = \left(1 + r_{quarterly}\right)^4 - 1 = 0.1038$
Monthly ($m = 12$)	$r_{monthly} = \frac{APR}{12} = 0.00833$	$r_{annual} = \left(1 + r_{monthly}\right)^{12} - 1 = 0.1047$
Daily ($m = 365$)	$r_{daily} = \frac{APR}{365} = 0.000274$	$r_{annual} = \left(1 + r_{daily}\right)^{365} - 1 = 0.1052$

Conclusion: As the compounding frequency (m) increases, the EAR **increases**

Practice Question 3

A $40000 investment you own today offers a 5% rate, compounded semi-annually. If you sell this investment 18 months later, how much money would you obtain?

Practice Question 4

A loan shark offers you $10000 at an APR rate that is compounded monthly. As a finance student, you realize this is illegal as you would have to pay him back $538972 after one year. If this is the case, what APR rate is the loan shark offering?

Practice Question 5

Which of the following is/are true about interest rates? Select all that apply.

a) Under annual compounding, the APR rate will always be equal to the EAR rate

b) Holding all else fixed, increasing the compounding frequency will decrease the EAR

c) The APR rate will have a greater chance at surpassing the EAR rate as we compound more frequently

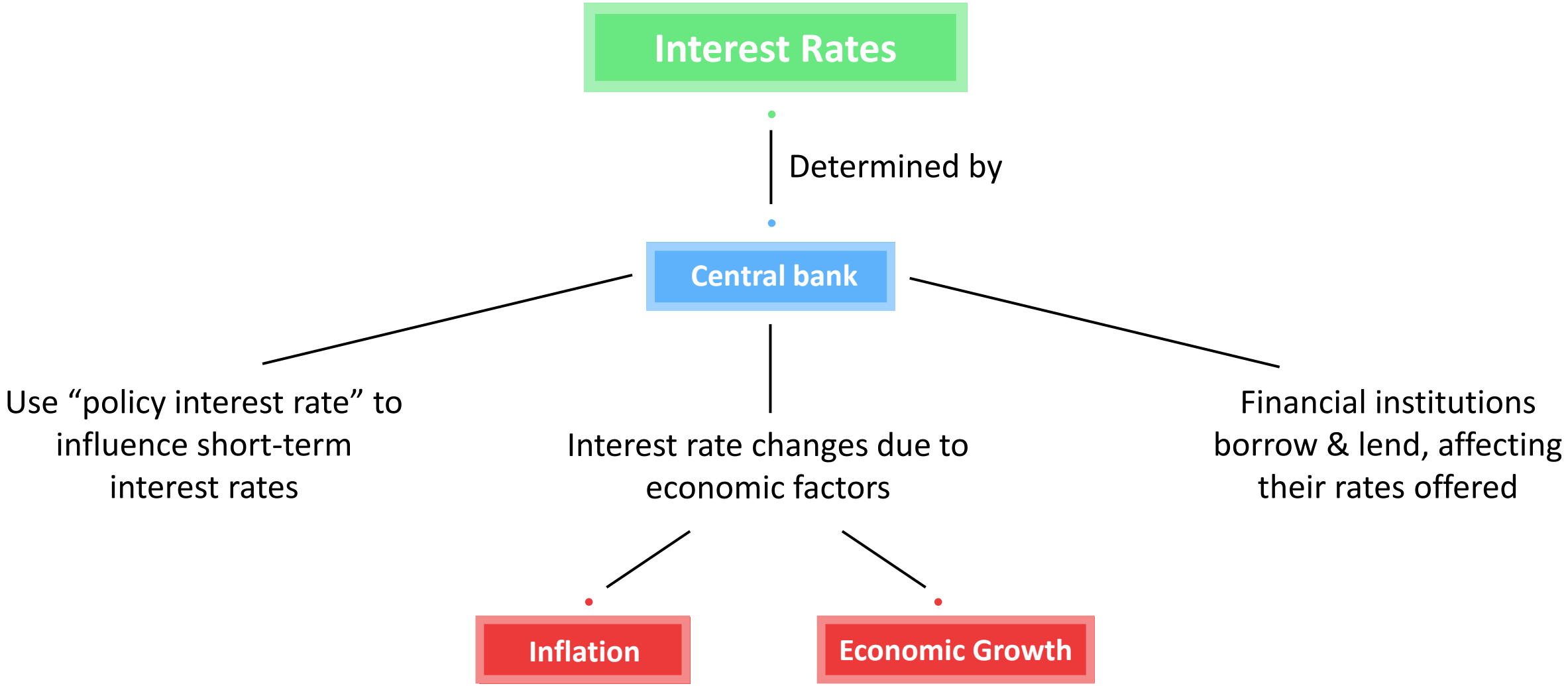
d) If a bank offers an APR rate of 5% compounded monthly, then $\frac{APR}{2}$ will give us $r_{semi-annual}$

e) None of the above

Determinants of Interest Rates

Interest Rates and the Central Bank

The market (nominal) interest rate (r) is determined by the **supply** of savings and **demand** for borrowing



Economic Determiner #1 - Inflation

Earning 5% yearly interest on your assets does not mean you can buy 5% more next year – there is inflation!

From ECON 102 (Fisher Equation): Real interest rate $\approx$ Market (nominal) interest rate (r) – Inflation

How much your buying power increases

What the central bank influences

Investors save less if inflation is high as the real interest rate will be low

Real interest rate $\approx$ Market (nominal) interest rate (r) – Inflation

To increase the real interest rate so people save more, the market interest rate (r) needs to increase

Real interest rate $\approx$ Market (nominal) interest rate (r) – Inflation

Economic Determiner #2 – Investments & Growth

The level of economic growth desired comes from investment – what determines our willingness to invest?

Answer: The present value of future benefits that come from the investment

Suppose interest rates are high (r ⬆)

$$PV_0 = C_n \cdot (1+r)^{-n} = \frac{C_n}{(1+r)^n}$$

(⬇ PV_0, ⬆ $(1+r)^n$)

Would you be more inclined to invest? NO!

Suppose interest rates are low (r ⬇)

$$PV_0 = C_n \cdot (1+r)^{-n} = \frac{C_n}{(1+r)^n}$$

(⬆ PV_0, ⬇ $(1+r)^n$)

Would you be more inclined to invest? YES!

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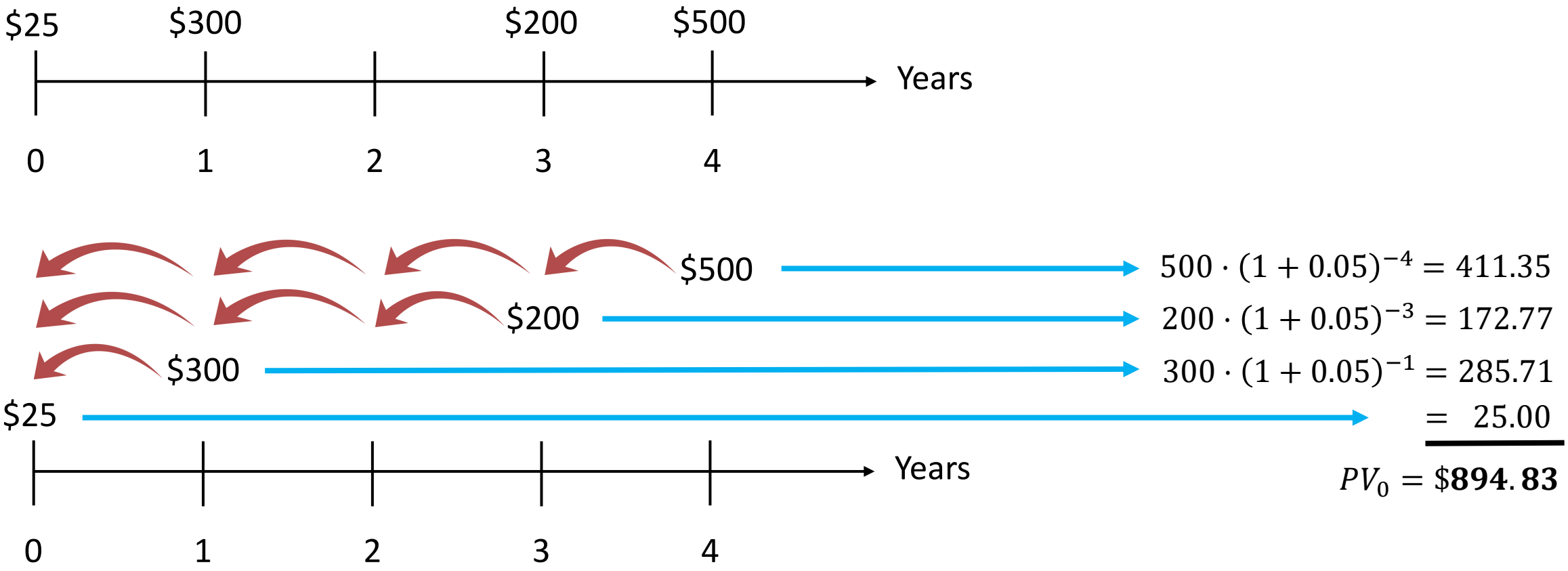
Valuing Multiple Cash Flows

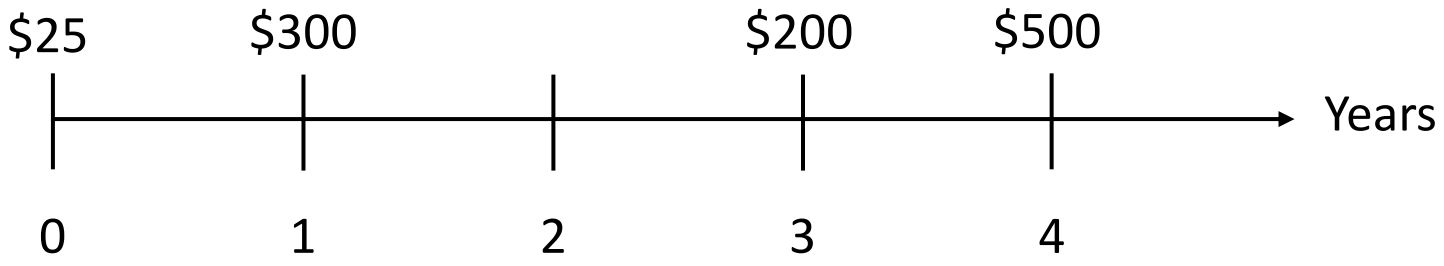
How do we value the following **streams** of cash flows? Value each single period's cash flow!

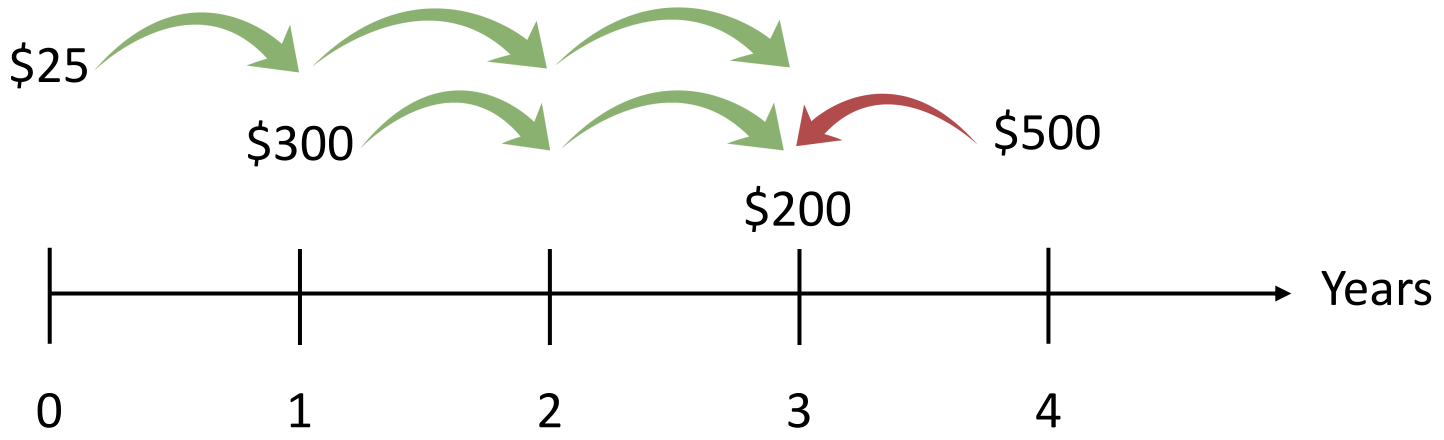
What is the present value of the following cash flows at $t = 0$? Assume an EAR of 5% (i.e., $r_{annual} = 0.05$)



Valuing Multiple Cash Flows (Continued)

What is the value of the following cash flows at $t = 3$? Assume an EAR of 5% (i.e., $r_{annual} = 0.05$)

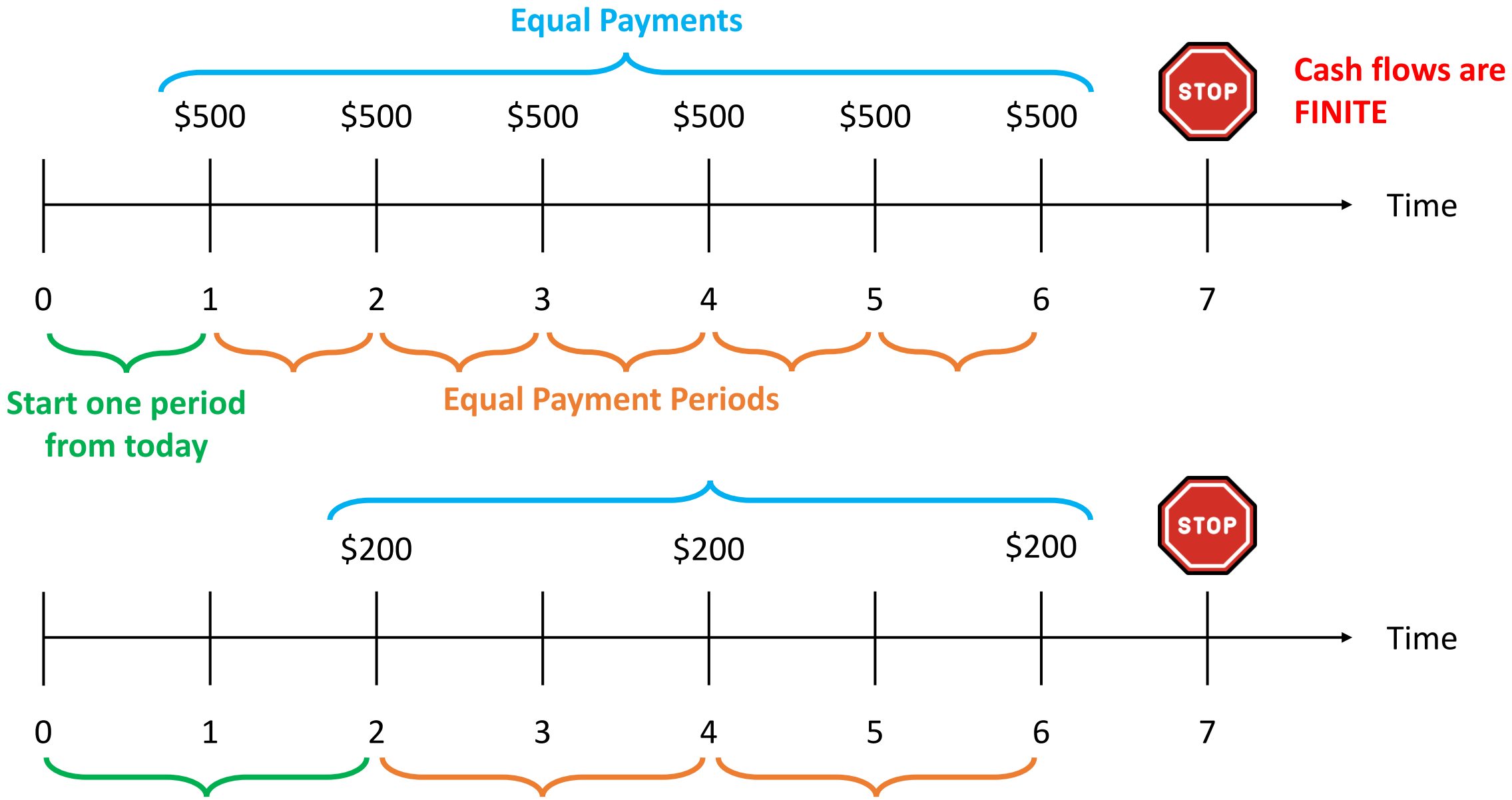




$$= 25 \cdot (1 + 0.05)^3 + 300 \cdot (1 + 0.05)^2 + 200 + 500 \cdot (1 + 0.05)^{-1} = \$\mathbf{1035.88}$$

Annuities and Perpetuities

Annuities


Equal Payments
$500
$500
$500
$500
$500
$500
STOP
Cash flows are FINITE
Time
0
1
2
3
4
5
6
7
Start one period from today
Equal Payment Periods
$200
$200
$200
STOP
Time
0
1
2
3
4
5
6
7

Annuity Formula for Present Value

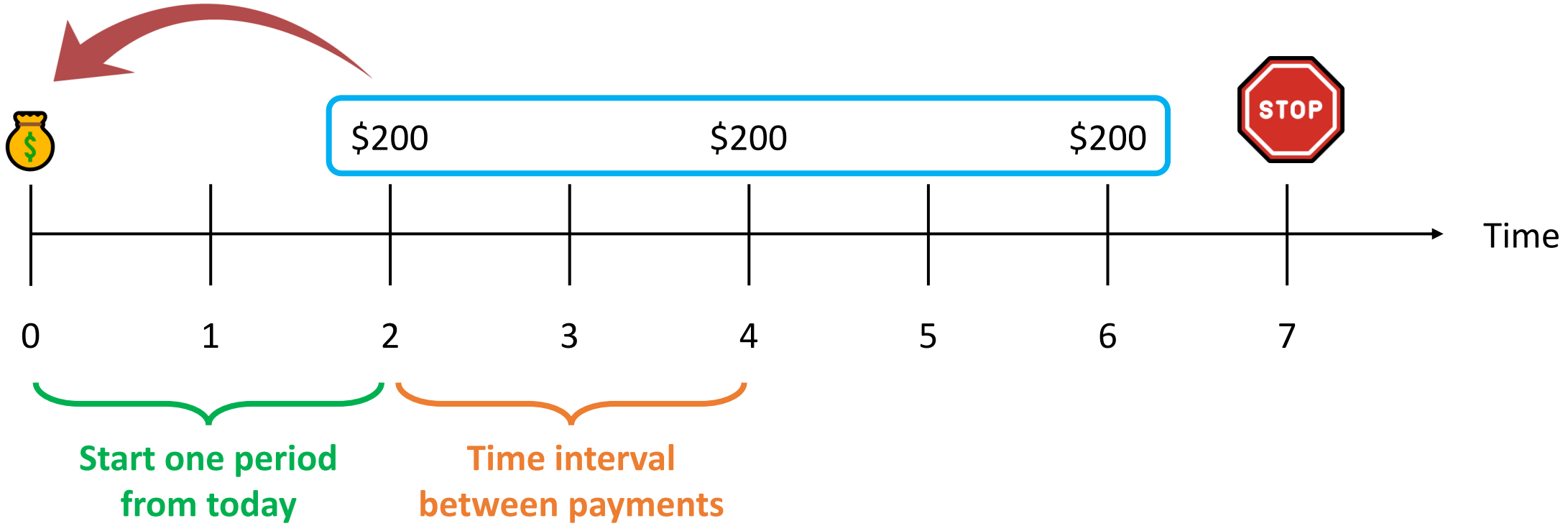
$$PV_0 = C\left[\frac{1-(1+r)^{-n}}{r}\right]$$

C = Level payment

r = Effective interest rate per period

n = Number of payments in annuity period

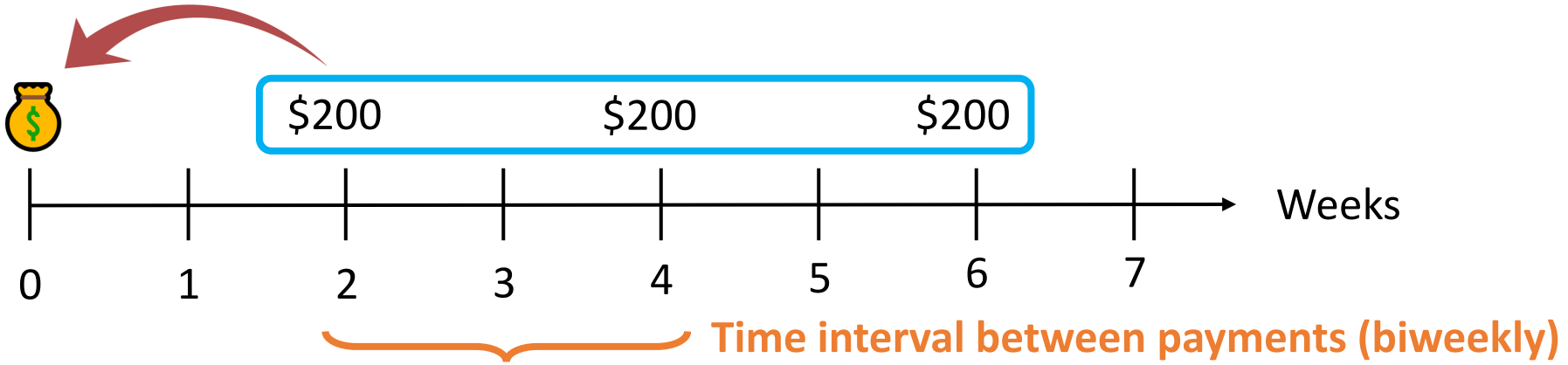
Same time units!



Using the PV Annuity Formula

If $r_{weekly} = 0.1$, what is the PV of these biweekly cash flows today?

$$PV_0 = C\left[\frac{1-(1+r)^{-n}}{r}\right]$$



$C = 200$ $n = 3$ biweekly payments

Note: Do we use $r_{weekly} = 0.1$ for the formula? NO! $n = 3$ **biweekly** payments, so we need $r_{\boldsymbol{biweekly}}$

$$r_{biweekly} = \left(1 + r_{weekly}\right)^2 - 1 = (1+0.1)^2 - 1 = 0.21$$

$$PV_0 = 200\left[\frac{1-(1+0.21)^{-3}}{0.21}\right] = \$\mathbf{414.79}$$

Annuity Formula for Future Value

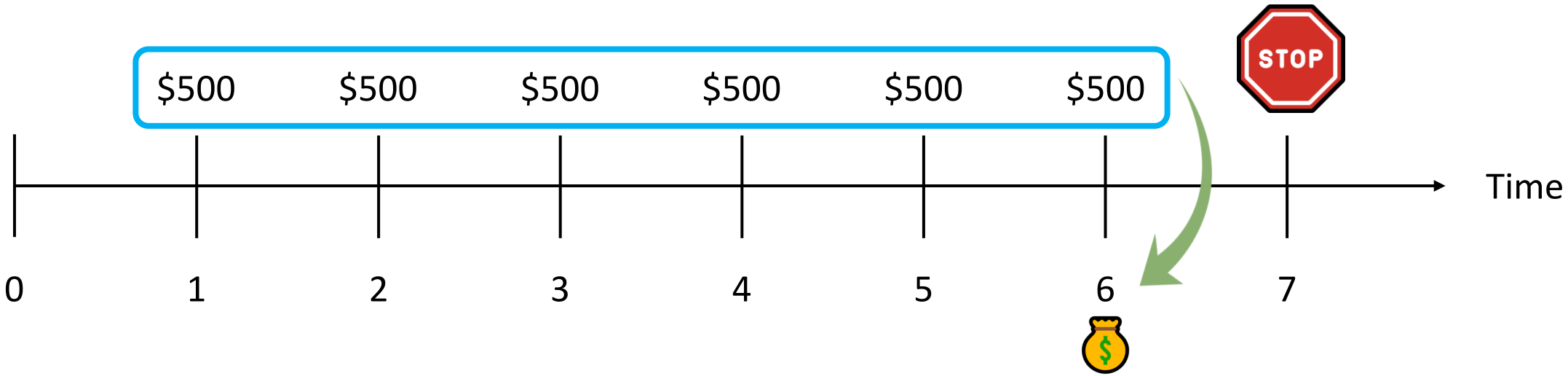
$$FV_n = C\left[\frac{(1+r)^n - 1}{r}\right]$$

C = Level payment

Same time units!

r = Effective interest rate per period

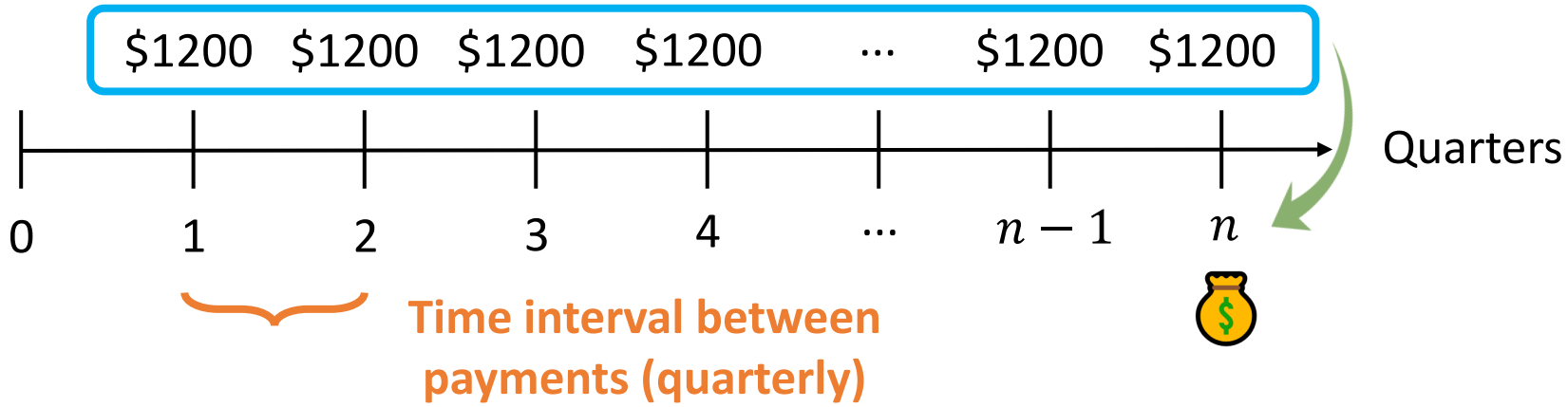
n = Number of payments in annuity period



Note: This gives you the FV on the date the last cash flow occurs, NOT one period after!

Using the FV Annuity Formula

George plans to deposit $1200 for his newborn daughter's RESP at end of each quarter. He wants $120000 saved up by the time he makes his last deposit. If his bank offers a yearly return of 4.8% APR compounded quarterly, how many years will it take to reach his goal if his first deposit is next quarter?



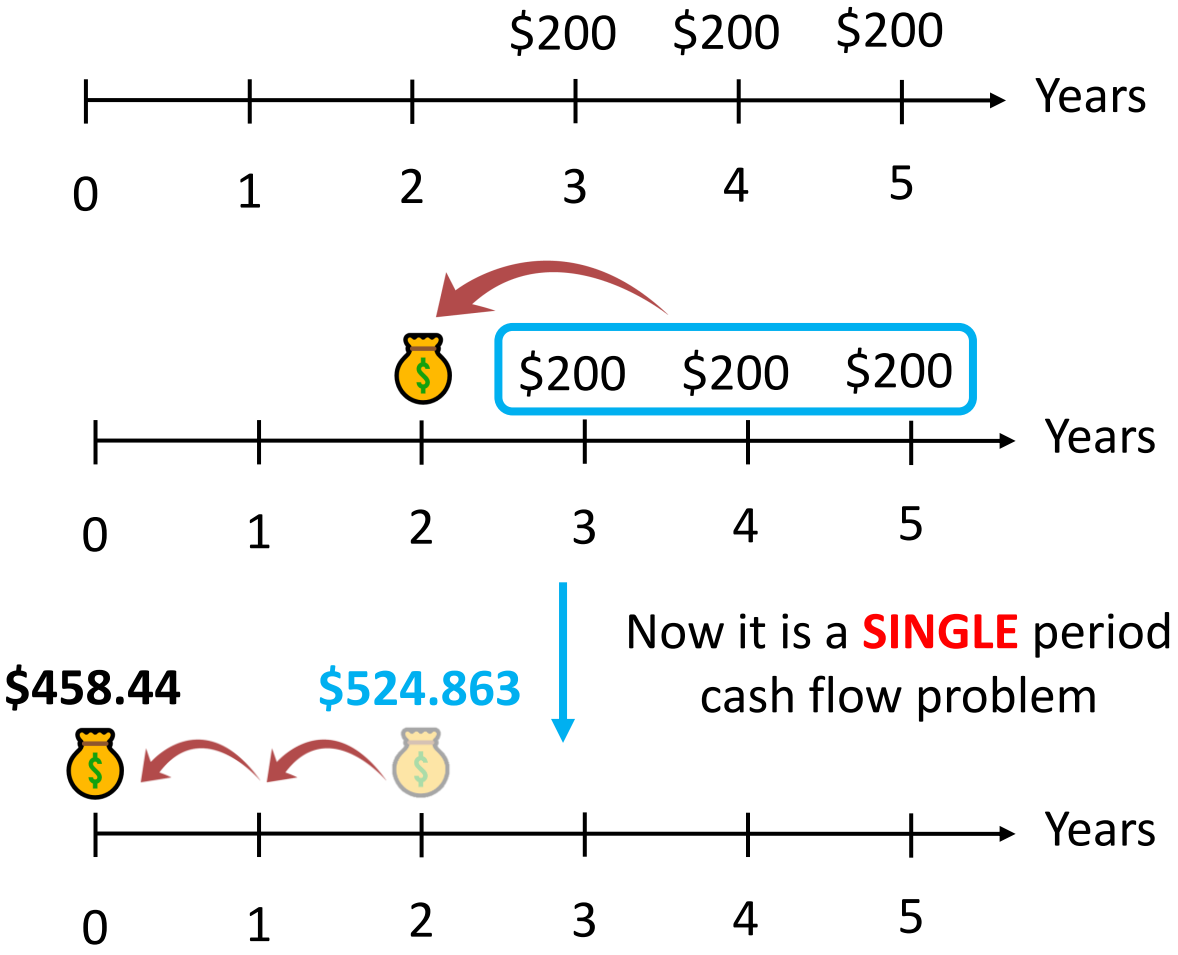
$$FV_n = C\left[\frac{(1+r)^n - 1}{r}\right]$$

$FV_n = 120000$ $C = 1200$ $m = 4$ $r_{quarterly} = \frac{APR}{m} = \frac{0.048}{4} = 0.012$ $n = ?$ **quarters**

$120000 = 1200\left[\frac{(1+0.012)^n - 1}{0.012}\right] \Rightarrow n = 66.1$ quarters (16.52 years)

Special Annuities – Moving Annuities

What is the value of the following cash flows at $t = 0$? Assume an EAR of 7%.



$$PV = C\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$C = 200$

$n = 3$ annual payments

$r_{annual} = 0.07$

$$PV_2 = 200\left[\frac{1-(1+0.07)^{-3}}{0.07}\right] = \$\mathbf{524.863}$$

$$PV_0 = PV_2 \cdot (1+r)^{-n}$$

$PV_2 = 524.863$

$n = 2$ years

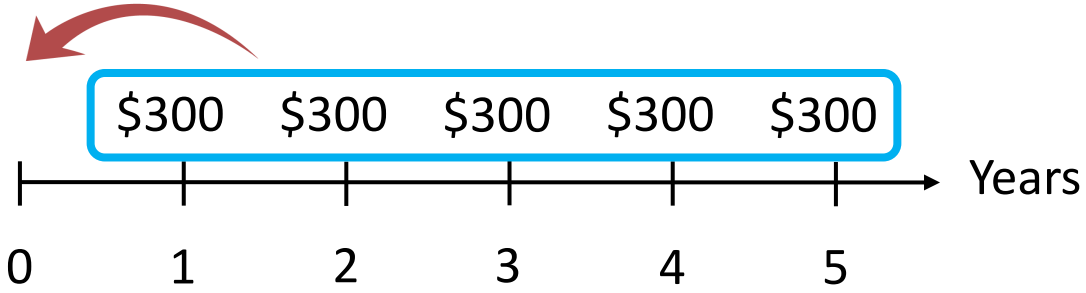
$r_{annual} = 0.07$

$$PV_0 = 524.863 \cdot (1+0.07)^{-2} = \$\mathbf{458.44}$$

Special Annuities – Annuity Due

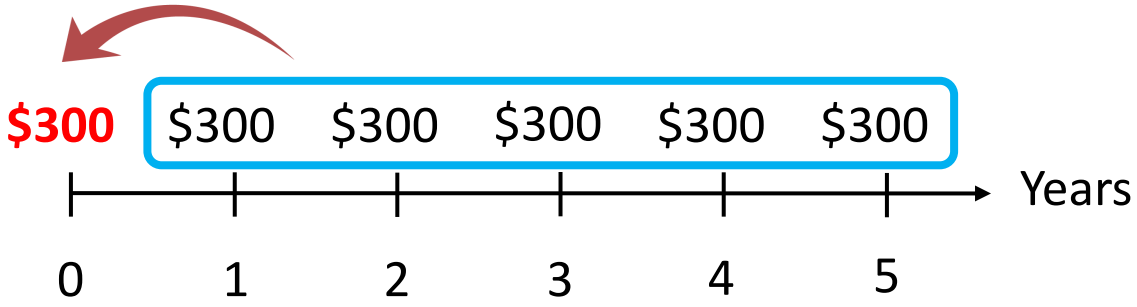
What is the value of the following streams cash flows at $t = 0$? Assume an EAR of 7%.

Regular Annuity $C = 300$ $r_{annual} = 0.07$ $n = 5$ annual payments



$$PV_0 = 300\left[\frac{1-(1+0.07)^{-5}}{0.07}\right] = \$1230.06$$

Annuity Due



$$PV_0 = 300\left[\frac{1-(1+0.07)^{-5}}{0.07}\right] + \mathbf{300} = \$1530.06$$

Perpetuities

Like an annuity, but the cash flows are *perpetual* – they never end (e.g., a consol bond)

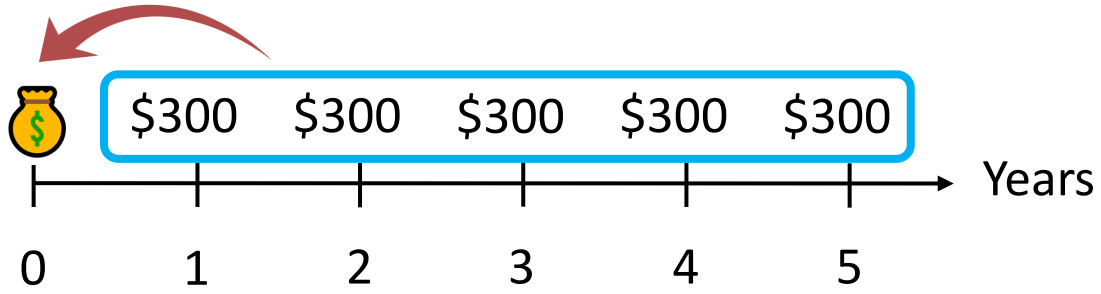
$$PV_0 = \frac{C}{r}$$

C = Level payment

r = Effective interest rate per period

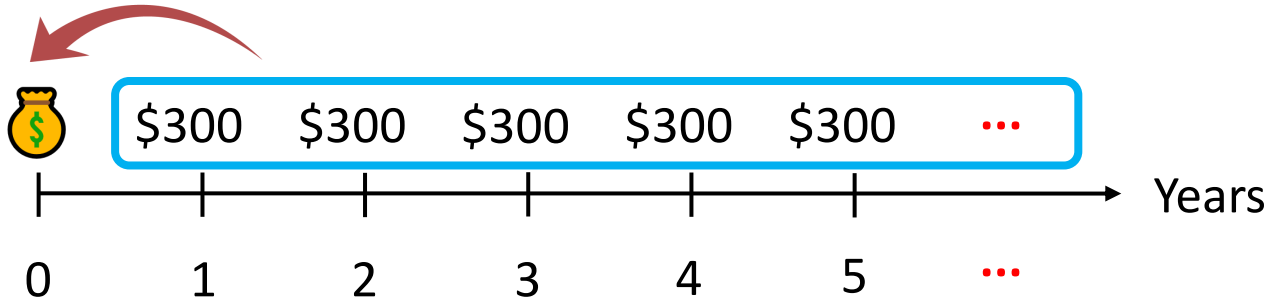
Still need to match the timing of cash flows (e.g., if payments occur every **month**, you must use $r_{monthly}$)

Regular Annuity



$$PV_0 = 300\left[\frac{1-(1+0.07)^{-5}}{0.07}\right] = \$1230.06$$

Perpetuity $C = 300$ $r_{annual} = 0.07$

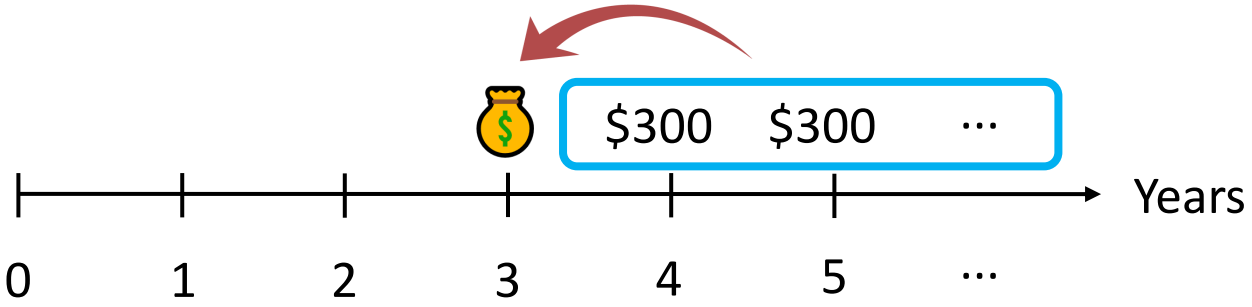


$$PV_0 = \frac{300}{0.07} = \$4285.71$$

Applying the Special Cases of Annuities to Perpetuities

In the future, you will own a consol bond that pays $300 every year for an indefinite amount of time. The first payment will start 4 years from today. If $r_{annual} = 0.07$, what is the PV of this perpetuity today?

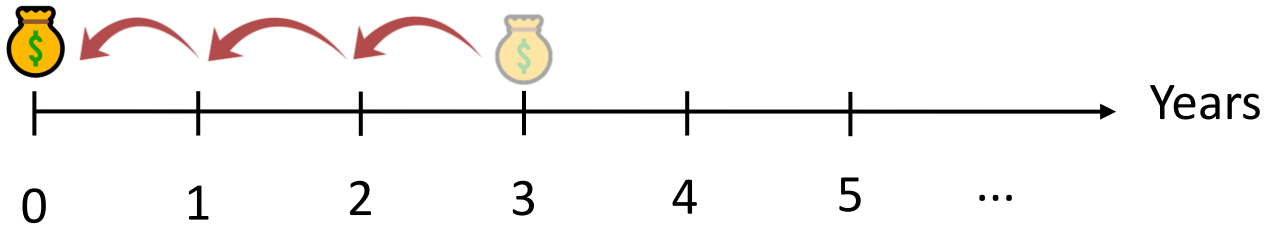
"Moving Perpetuities"



$$PV_3 = \frac{C}{r} = \frac{300}{0.07} = \$4285.71$$

Now it is a **SINGLE** period cash flow problem

$\$3498.42$ $\$4285.71$



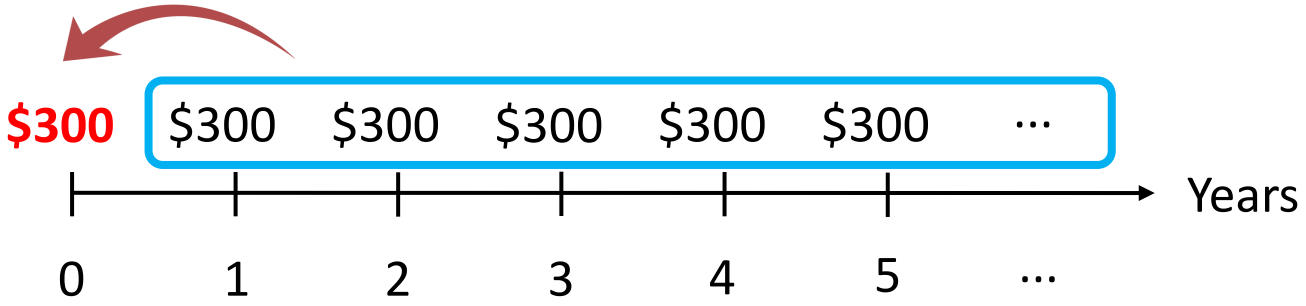
$$PV_0 = PV_3 \cdot (1+r)^{-n}$$

$$PV_0 = 4285.71 \cdot (1+0.07)^{-3} = \$3498.42$$

Applying the Special Cases of Annuities to Perpetuities (Continued)

A consol bond you own pays $300 every year for an indefinite amount of time. Somehow, the issuer has mistakenly given you the first payment today. If $r_{annual} = 0.07$, what is the PV of this perpetuity today?

"Perpetuity Due"



$$PV_0 = \frac{300}{0.07} + \mathbf{300} = \$4585.71$$

Practice Question 1

Today your parents have $10000 in the bank gaining interest at 10% APR, compounded monthly. They recently won a lottery that rewards fixed, monthly payments at the end of each month. The payments will start this month and occur for 10 years. All winnings will be immediately deposited into the bank when received.

a) If your parents can exactly afford a $80000 car in 10 years, what is the monthly payout won from the lottery?

Practice Question 1

Assume your answer to the last question is $275

b) Suppose the lottery changes their payout scheme and awards your parents a lump-sum of cash equal to the present value of their winnings instead. What is the price of the most expensive car your parents can buy today?

Practice Question 2

Your enrollment in a pension fund has landed you with an annuity of $900 every 2 months! The first payment will be a month from today on Jan. 31st, 2025 and the last payment will be on Nov. 30th, 2032. If you can reinvest all your money into a stock that gives a return of 1.48% every 2 months, how much would you have gained by the end of the 2032 (i.e., 8 years from today)?

Practice Question 3

How much is a perpetuity of $75 per year starting today if the interest rate is 22% APR compounded weekly?

Practice Question 4

Which of the following is/are true assuming all cash flows are positive? Select all that apply.

a) Holding all else constant, the PV of a perpetuity can never be less than the PV of an annuity

b) An example of an annuity is a consol bond

c) Holding all else constant, one would prefer a perpetuity of $300 every 2 months starting today over a perpetuity of $150 every month starting today

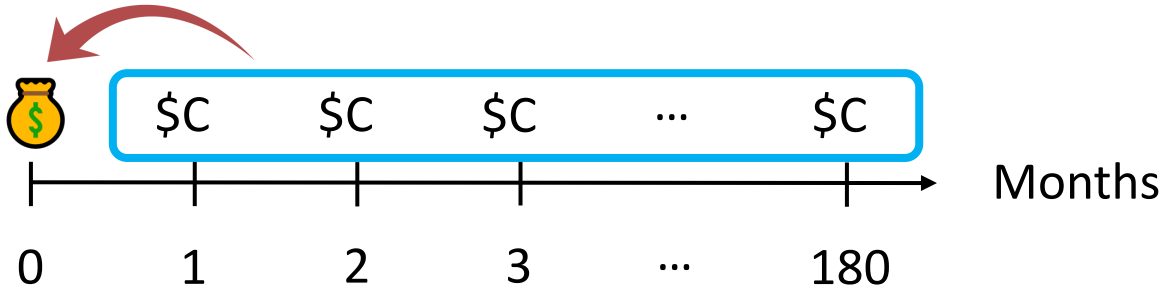
d) The fixed interval payments in a mortgage loan is an example of a perpetuity

e) One would prefer an annuity of $200 per year starting today under a 5% APR rate compounded semi-annually over the same annuity with annual compounding instead

Applications of Annuities: Mortgage Loans

Mortgage Loans

At the beginning of the month, Noble took out a $200000 mortgage to buy a townhouse. The 15-year loan requires him to make monthly payments under an APR of 7.54% compounded quarterly. How much does he need to pay each month if his first payment is due at the end of this month?



$n = 180$ **monthly** payments $\quad PV_0 = 200000 \quad r_{quarterly} = \frac{APR}{m} = \frac{0.0754}{4} = 0.01885$

$r_{monthly} = \left(1 + r_{quarterly}\right)^{\frac{1}{3}} - 1 = 0.00624426$ (Want $r_{monthly}$ as payments are **monthly**)

$PV_0 = C\left[\frac{1-(1+r)^{-n}}{r}\right] \Rightarrow 200000 = C\left[\frac{1-(1+0.00624426)^{-180}}{0.00624426}\right]$ $\quad C = \$1853.24$

What is Behind the Scenes?

$$C = \text{Period Interest } (I) + \text{Period Principal } (p)$$

PV at time t

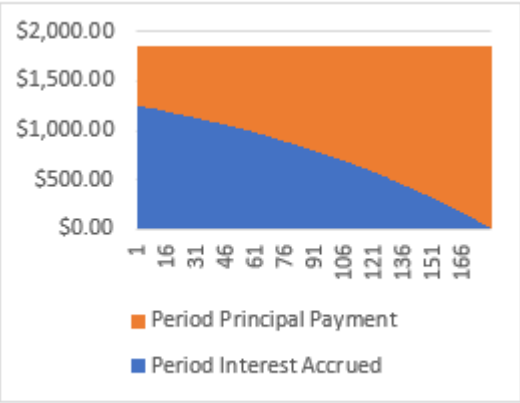
Months	Beginning Period Mortgage Principal	Period Payment	Period Interest Accrued	Period Principal Payment	End Period Account Balance
1	\$200,000 $= PV_0$	\$1,853.24	\$1,248.85	\$604.39	\$199,395.61
2	\$199,395.61	\$1,853.24	\$1,245.08	\$608.16	\$198,787.45
3	\$198,787.45	\$1,853.24	\$1,241.28	\$611.96	\$198,175.48
4	\$198,175.48	\$1,853.24	\$1,237.46	\$615.78	\$197,559.70
5	\$197,559.70	\$1,853.24	\$1,233.61	\$619.63	\$196,940.07
6	\$196,940.07	\$1,853.24	\$1,229.75	\$623.50	\$196,316.58
7	\$196,316.58	\$1,853.24	\$1,225.85	\$627.39	\$195,689.19
8	\$195,689.19	\$1,853.24	\$1,221.93	\$631.31	\$195,057.88
9	\$195,057.88	\$1,853.24	\$1,217.99	\$635.25	\$194,422.63
⋮	⋮	⋮	⋮	⋮	⋮
175	\$10,880.43	\$1,853.24	\$67.94	\$1,785.30	\$9,095.13
176	\$9,095.13	\$1,853.24	\$56.79	\$1,796.45	\$7,298.68
177	\$7,298.68	\$1,853.24	\$45.57	\$1,807.67	\$5,491.01
178	\$5,491.01	\$1,853.24	\$34.29	\$1,818.95	\$3,672.05
179	\$3,672.05	\$1,853.24	\$22.93	\$1,830.31	\$1,841.74
180	\$1,841.74	\$1,853.24	\$11.50	\$1,841.74	\$0.00

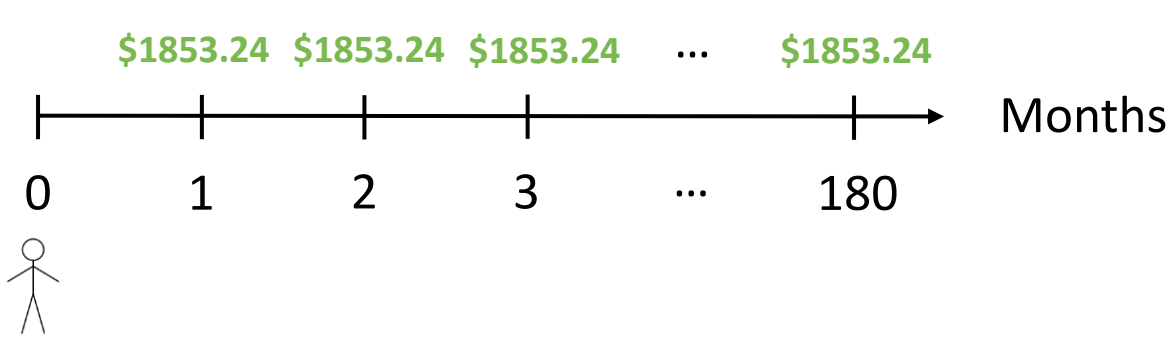
PV_1

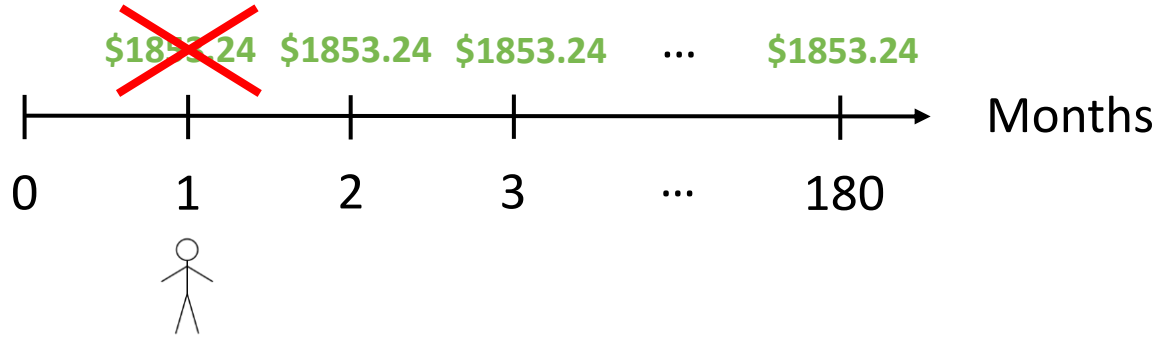
PV_2

As time passes …

p ⬆ and I ⬇







Practice Question 5

a) Michael's 40-year mortgage of $650000 has a contract interest rate of 4.24% APR compounded semi-annually. He is required to make quarterly payments, with the first starting a quarter from today. Under these terms, how much will he need to pay each quarter?

Practice Question 5

For the rest of the question, assume that the quarterly payments are $8500 each

b) How much does Michael owe after he makes his 37^{th} payment?

Practice Question 5

For the rest of the question, assume that the quarterly payments are $8500 each

c) If $2663.18 of the 50^{th} payment went to pay the interest, how much went to pay the principal?

Practice Question 5

For the rest of the question, assume that the quarterly payments are $8500 each

d) Let's rewind time. In an alternative world, Michael realized he had trouble paying off his mortgage after he made his 37th payment. His friend offered to pay off the remaining payments if Michael gave him ownership of the house. If the house is worth $590000 today, which of the following is true? Select the best option.

i) Michael should take the offer as the PV of his house is less than the PV of his remaining payments

ii) Michael should take the offer as the PV of his house is more than the PV of his remaining payments

iii) Michael should not take the offer as the PV of his house is less than the PV of his remaining payments

iv) Michael should not take the offer as the PV of his house is more than the PV of his remaining payments

v) Michael is indifferent between the choices as the PV of both options is the same

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What is a Bond?

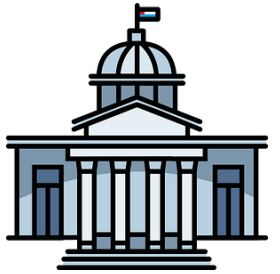
Debt instrument issued to investors for funds in exchange for periodic payments back to the investors

Investors

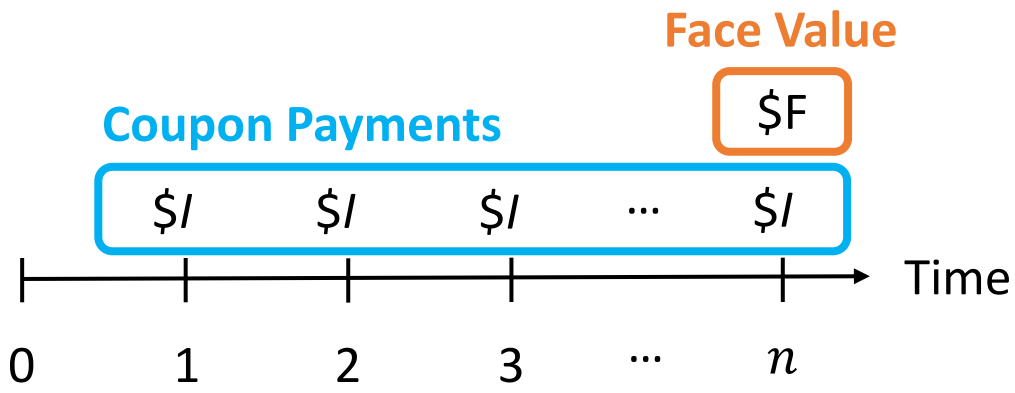
Bond

Today

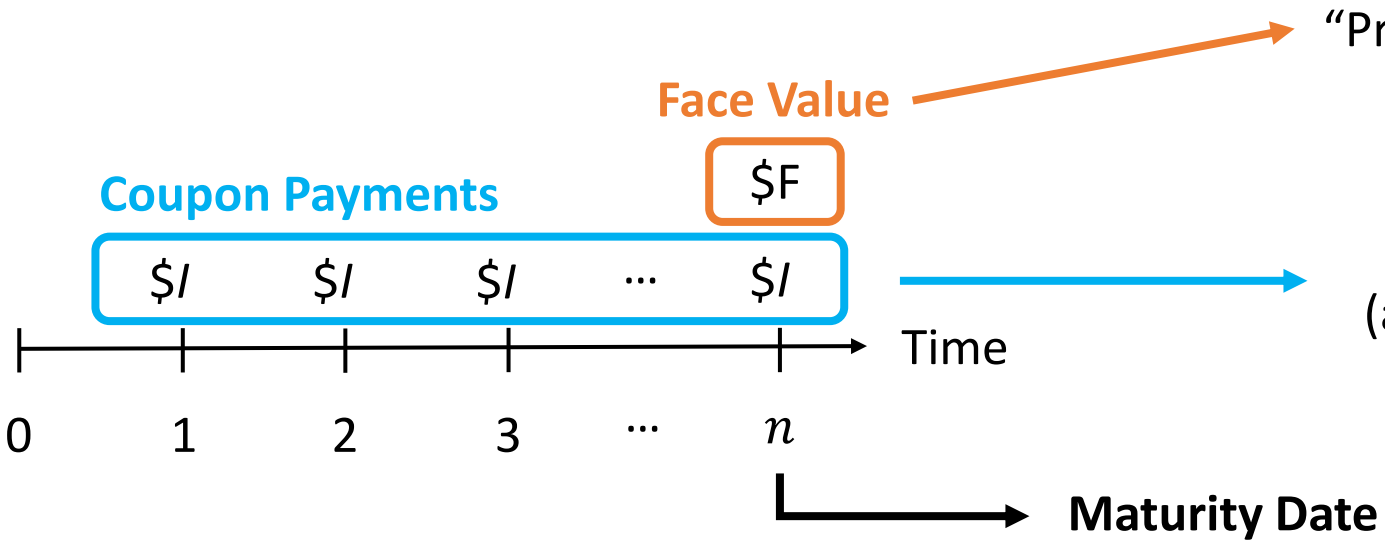
Corporations



Governments



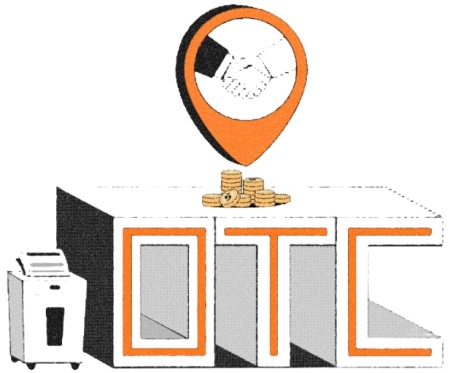
What is a Bond? (Continued)



Most bonds are interest only loans
(an exception is a zero-coupon bond)

"Fixed income" securities – strictly defined payments (interest payments don't change)

Usually bought and sold "over-the-counter" between investors (secondary market)



Zero-coupon Bonds

A bond without coupons (e.g., treasury bills) where the investor only receives the face value at maturity

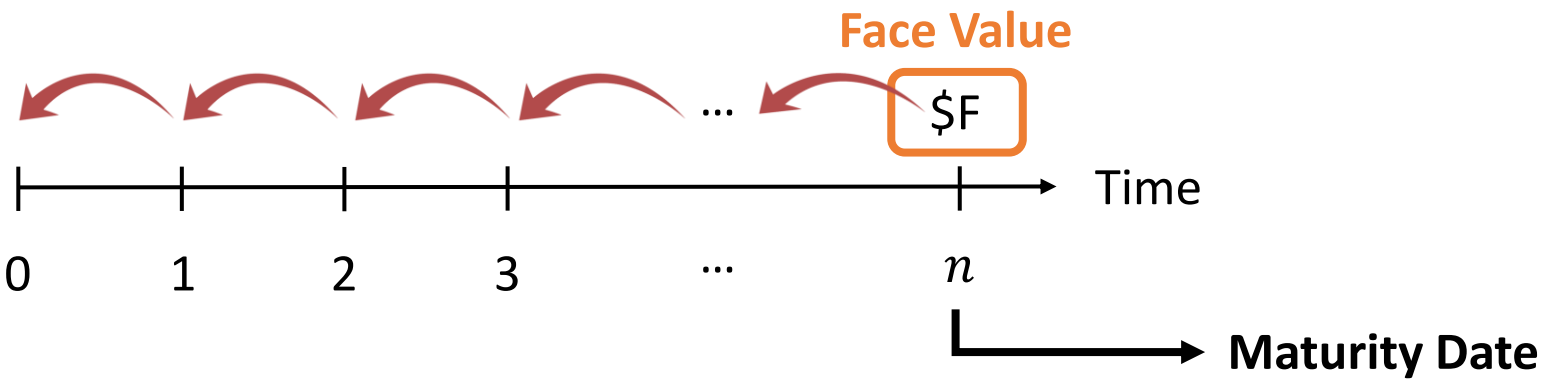
$$P_0 = F \cdot (1 + r)^{-n}$$

P_0 = Price of Bond

r = Effective interest rate

n = # of periods

Same time units!



Remarks:

- Zero-coupon bonds always sell at a discount (i.e., bond price $<$ face value)
- r is the rate of return (i.e., the reward for taking on risk – the higher the risk, the higher the r)
- Bond prices and r are inversely proportional (i.e., as r increases, bond price decreases)

How to get the Effective Rate r for Zero-coupon Bonds?

Markets quote the return on bonds using an APR rate called the "Yield to Maturity" (YTM)

Yield to Maturity (YTM)

$$\frac{YTM}{m} = r$$

$$r \cdot m = YTM$$

Effective Interest Rate (r)

m = # of compounding periods in ONE year

YTM = Yield to maturity

Compounding Frequency	(YTM to Effective Rate)	Compounding Frequency	(YTM to Effective Rate)
Daily ($m = 365$)	$\frac{YTM}{365} = r_{daily}$	Quarterly ($m = 4$)	$\frac{YTM}{4} = r_{quarterly}$
Weekly ($m = 52$)	$\frac{YTM}{52} = r_{weekly}$	Semi-annually ($m = 2$)	$\frac{YTM}{2} = r_{semi-annual}$
Monthly ($m = 12$)	$\frac{YTM}{12} = r_{monthly}$	Annually ($m = 1$)	$\frac{YTM}{1} = YTM = r_{annual}$ **(EAR)**

Most common compounding period for bonds

Coupon Bonds

A bond with coupons where the investor receives an annuity of coupons + principal at maturity

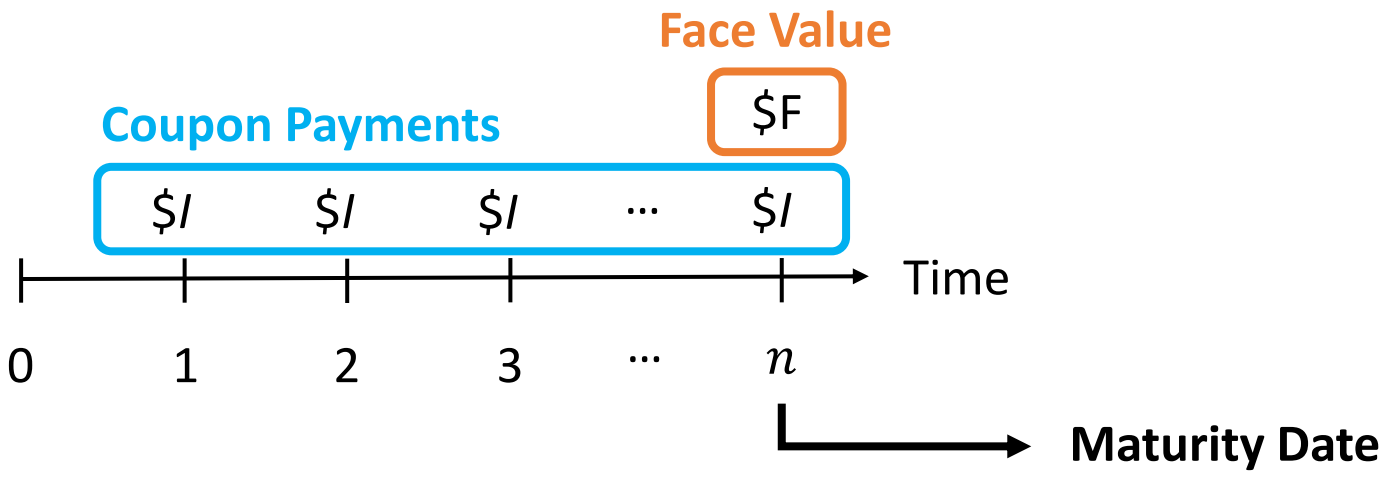
$$P_0 = \underbrace{I\left[\frac{1-(1+r)^{-n}}{r}\right]}_{\text{PV of Coupon Payments}} + \underbrace{F \cdot (1+r)^{-n}}_{\text{PV of Face Value}}$$

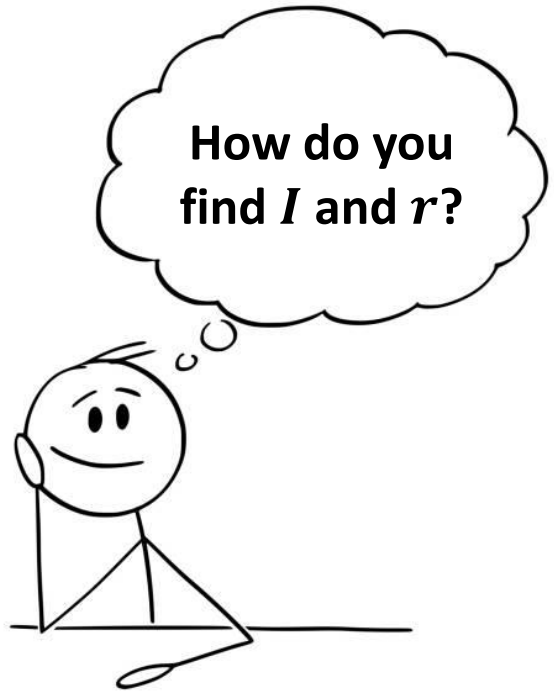
P_0 = Price of Bond

r = Effective interest rate

n = # of payments before bond matures

Same time units!





How to get the Coupon Payments (I)

Unlike yield to maturity (YTM), the coupon rate (CR) is a **FIXED** APR rate and never changes across time

$$I = F \cdot \frac{CR}{k}$$

I = Coupon (Interest) Payment F = Face Value CR = Coupon Rate

k = # of coupon payments in ONE year

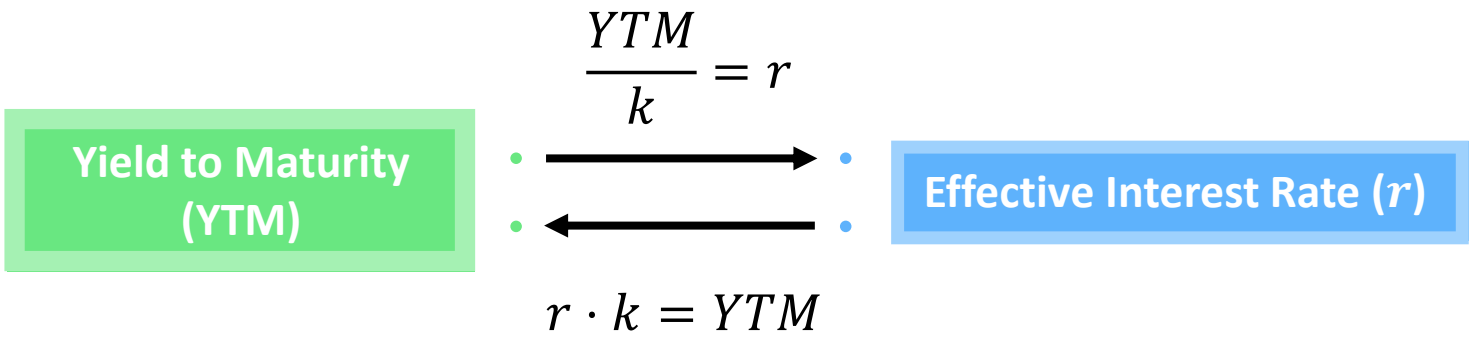
If we rearrange to solve for the coupon rate (CR), we can define it as the total coupon payment you get in one year expressed as a percentage of the bond's face value

$$CR = \frac{I \cdot k}{F} = \frac{Total\ Coupon\ Payment\ in\ One\ year}{Face\ Value}$$

Note: The number of coupon payments in a year (k) equals the frequency of compounding (m)

How to get the Effective Rate r for Coupon Bonds?

Recall that $m = k$ for a coupon bond so we can find r using the following



k = # of coupon payments in ONE year

YTM = Yield to maturity

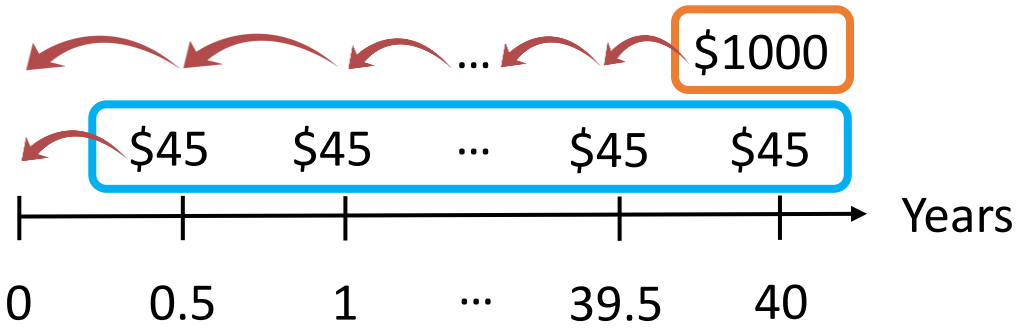
Coupon Payment Frequency	(YTM to Effective Rate)	Coupon Payment Frequency	(YTM to Effective Rate)
Daily ($k = 365$)	$\frac{YTM}{365} = r_{daily}$	Quarterly ($k = 4$)	$\frac{YTM}{4} = r_{quarterly}$
Weekly ($k = 52$)	$\frac{YTM}{52} = r_{weekly}$	Semi-annually ($k = 2$)	$\frac{YTM}{2} = r_{semi-annual}$
Monthly ($k = 12$)	$\frac{YTM}{12} = r_{monthly}$	Annually ($k = 1$)	$\frac{YTM}{1} = YTM = r_{annual}$ **(EAR)**

Most common frequency of coupon payments

Example Calculation of Coupon Bond

Today, you bought a Government of Canada (GOC) bond that has a coupon rate of 9%, a face value of $1000, and a YTM of 6%. It pays semi-annual coupons and matures in 40 years. How much did the bond cost you?

$I = F \cdot \frac{CR}{k}$ $F = 1000$ $CR = 0.09$ $k = 2$ coupons per year $I = 1000 \cdot \frac{0.09}{2} = \$45$



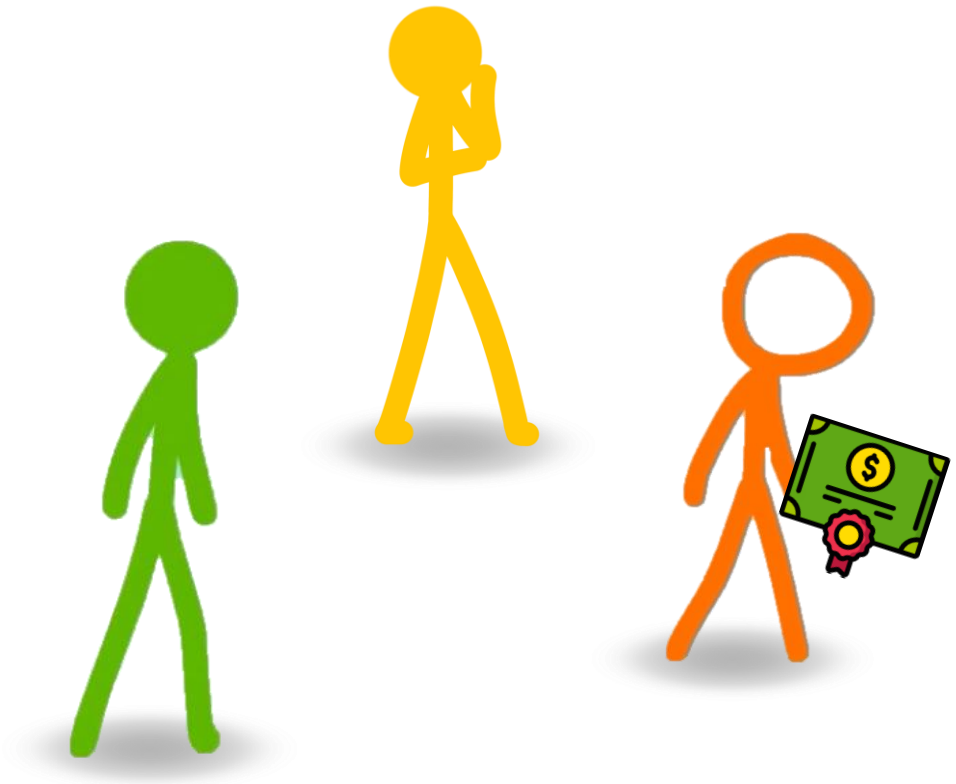
$n = 80$ **semi-annual** payments

$r_{semi-annual} = \frac{YTM}{k} = \frac{0.06}{2} = 0.03$

$$P_0 = I\left[\frac{1-(1+r)^{-n}}{r}\right] + F \cdot (1+r)^{-n} = \underbrace{45\left[\frac{1-(1+0.03)^{-80}}{0.03}\right]}_{\text{PV of Coupon Payments}} + \underbrace{1000(1+0.03)^{-80}}_{\text{PV of Face Value}} = \$\mathbf{1453.01}$$

Bond Price Quotations

Bond prices are quoted as a percent of its face value – a $1000 face value bond trading at $1052 would be quoted as 105.20 (105.20% of the face value)

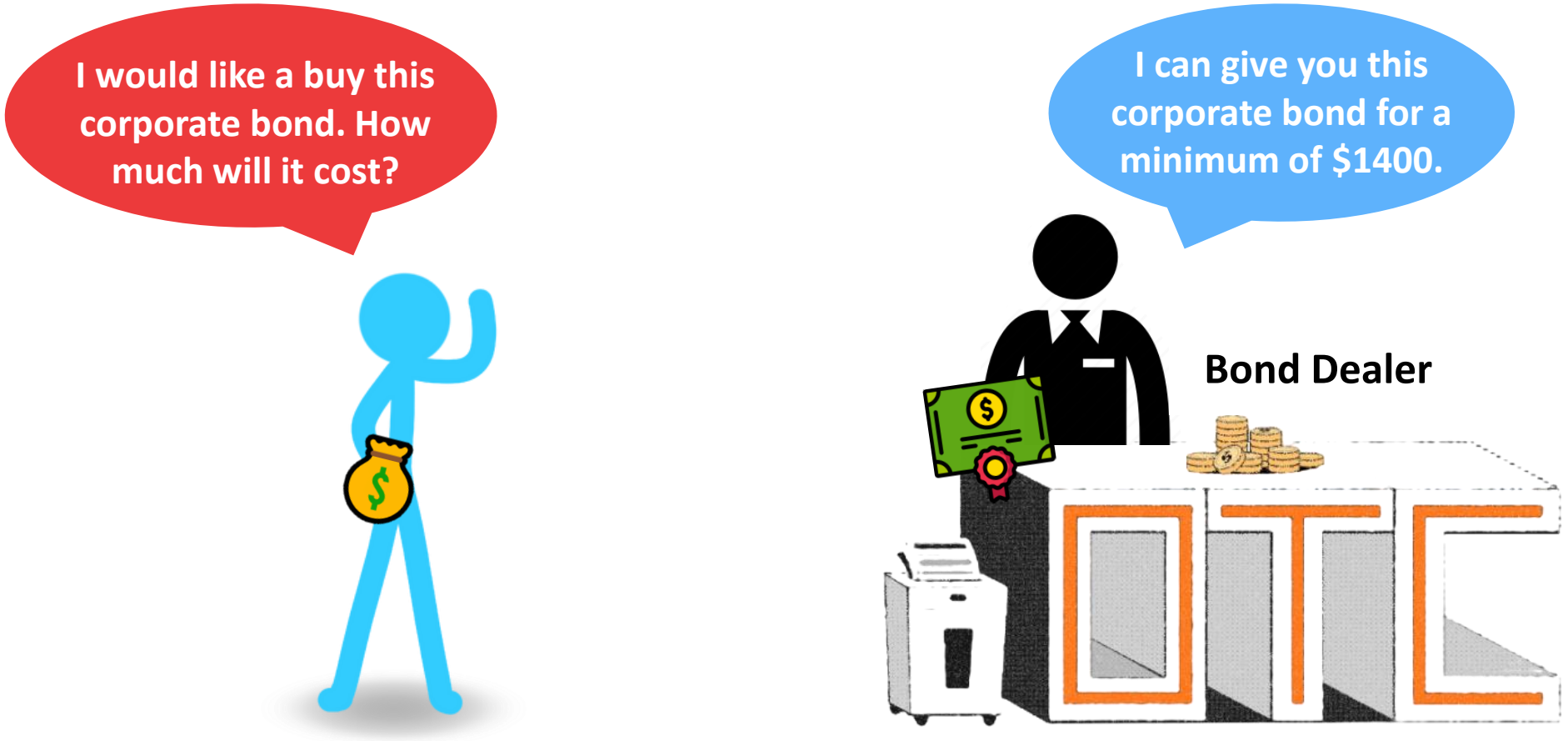


Bid Price

In an OTC bond market, the bid price is the **maximum** price a bond dealer is willing to pay to obtain a bond

Ask Price

In an OTC bond market, the ask price is the **minimum** price a bond dealer is willing to sell a bond for



Practice Question 1

What is the YTM of a one year, $1000 face value zero-coupon bond if it currently sells for $987.65? Assume the frequency of compounding is semi-annual.

Practice Question 2

a) A 30-year corporate bond has a YTM of 12%, a coupon rate of 9.6%, and a face value of $5000. How much money do you receive per coupon if the bond's coupon payments are yearly?

Practice Question 2

For the rest of the question, assume that the coupon payments are $500 each

b) If you bought the corporate bond today, how much would you have to pay?

Practice Question 2

For the rest of the question, assume that the coupon payments are $500 each

c) It has been a few years and you found out that your brother bought the same corporate bond 8 years ago and received his most recent coupon today. If the YTM is now 8%, how much money would he get from selling his bond right now?

Practice Question 3

Which of the following is/are true about bonds? Select all that apply.

a) Holding all else constant, a zero-coupon bond would always be priced lower than a coupon-bond

b) Bonds can be traded in a primary market but not in a secondary market

c) If a bond is sold at double its face value, the price would be quoted as 100 because the selling price is 100% more than its face value

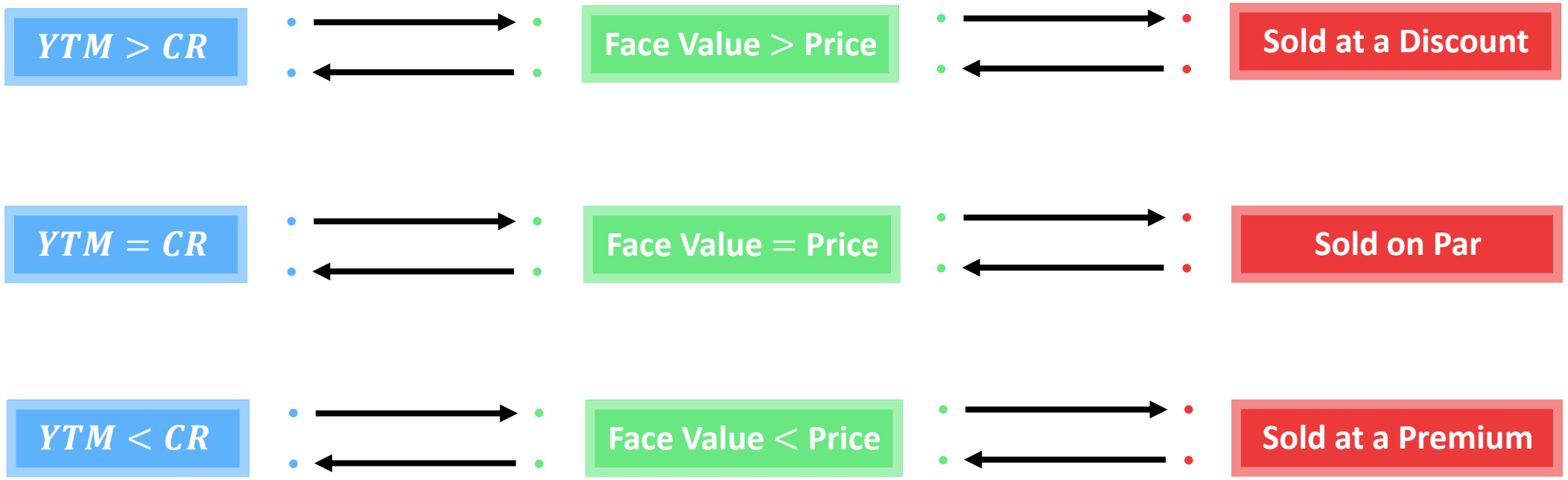
d) Treasury bills are an example of a zero-coupon bond

e) Both the coupon rate (CR) and YTM are APR rates that can fluctuate as time passes

More on YTM & Coupon Rates

YTM & Coupon Rates

The YTM and coupon rate are both APR rates, so we can do a comparison to understand bond prices



Conclusion: The better deal you get (i.e., higher CR relative to YTM), the more you must pay

More on YTM

Unlike the coupon rate (which stays constant across time), a bond's YTM can fluctuate across time

How do we interpret the YTM? What does it mean?

Answer: YTM is the *expected* average annual rate of return in APR terms when the following conditions are met

(1) You buy the bond today and hold it until maturity (hence it is called yield to maturity)

(2) You reinvest the coupons earned at that same YTM rate

When may your expectation deviate? (i.e., you get something different than your YTM)

(1) When you sell off the bond before it matures

(2) You reinvest the coupons earned at a different YTM rate or don't reinvest them at all

Practice Question 4

State whether the following bonds are traded at a premium, at a discount, or at par

a) A $1234.56 corporate bond that has a face value of $1000

b) An Apple Inc. bond that offers semi-annual coupons of $44.50, has a face value of $2000, and a YTM of 4.9%

c) A 7-year John Deere bond that pays semi-annual coupons, has a coupon rate of 4.7%, and offers an effective semi-annual return of 2.5%

Practice Question 5

a) List the following bonds' prices from smallest to largest. Each bond has the same face value and pays annual coupons.

i) 10-year bond with a coupon rate of 5% and YTM of 8%

ii) 12-year bond with a coupon rate of 9% and YTM of 8%

iii) 15-year bond with a coupon rate of 9% and YTM of 8%

iv) 8-year bond with a coupon rate of 5% and YTM of 8%

b) Select the correct option listed in brackets

i) Fixing all else, a premium bond will (increase/decrease) in price as its maturity period becomes longer

ii) Fixing all else, a discount bond will (increase/decrease) in price as its maturity period becomes longer

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What Determines a Bond's Yield?

The following contribute to a bond's YTM

(1) Nominal interest rate (inflation & purchasing power : the real interest rate)

- Real Interest Rate $\approx$ Nominal Interest Rate $-$ Inflation

(2) Interest rate risk premium

- How sensitive is the bond's price when r changes?

(3) Default (credit) risk premium

- How likely will we never be paid back for the bond we purchased?

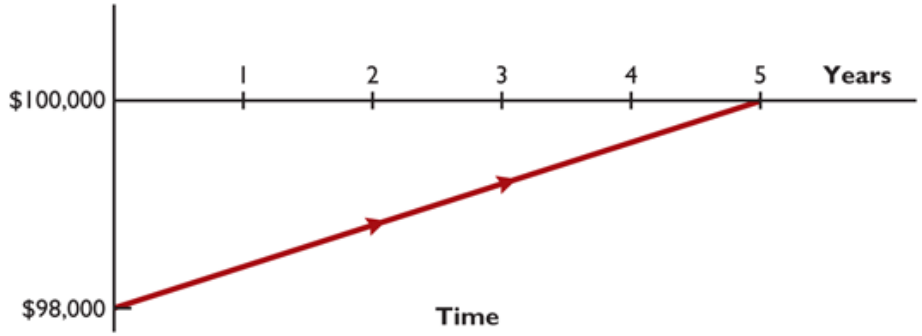
(4) Liquidity premium

- How easy is it to sell off the bond for cash (i.e., liquidate it)?

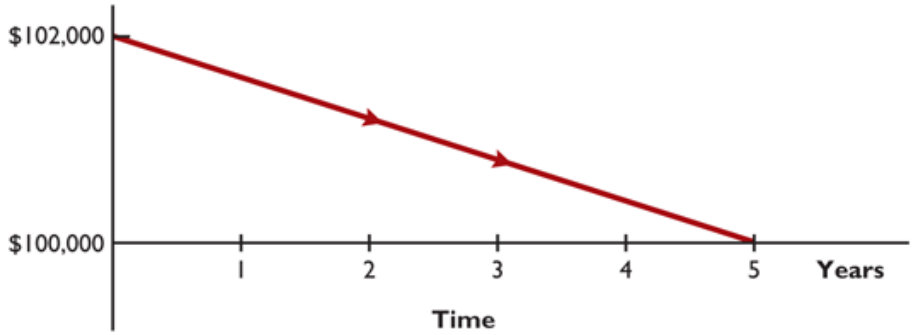
The focus for the next few slides

Why do Bond Prices Change?

Holding all else constant, a bond's price tends towards its face value as it gets closer to maturity



Bond sold at discount



Bond sold at premium

The market interest rate (r) can fluctuate

$$P_0 = I\left[\frac{1-(1+r)^{-n}}{r}\right] + F \cdot (1+r)^{-n}$$

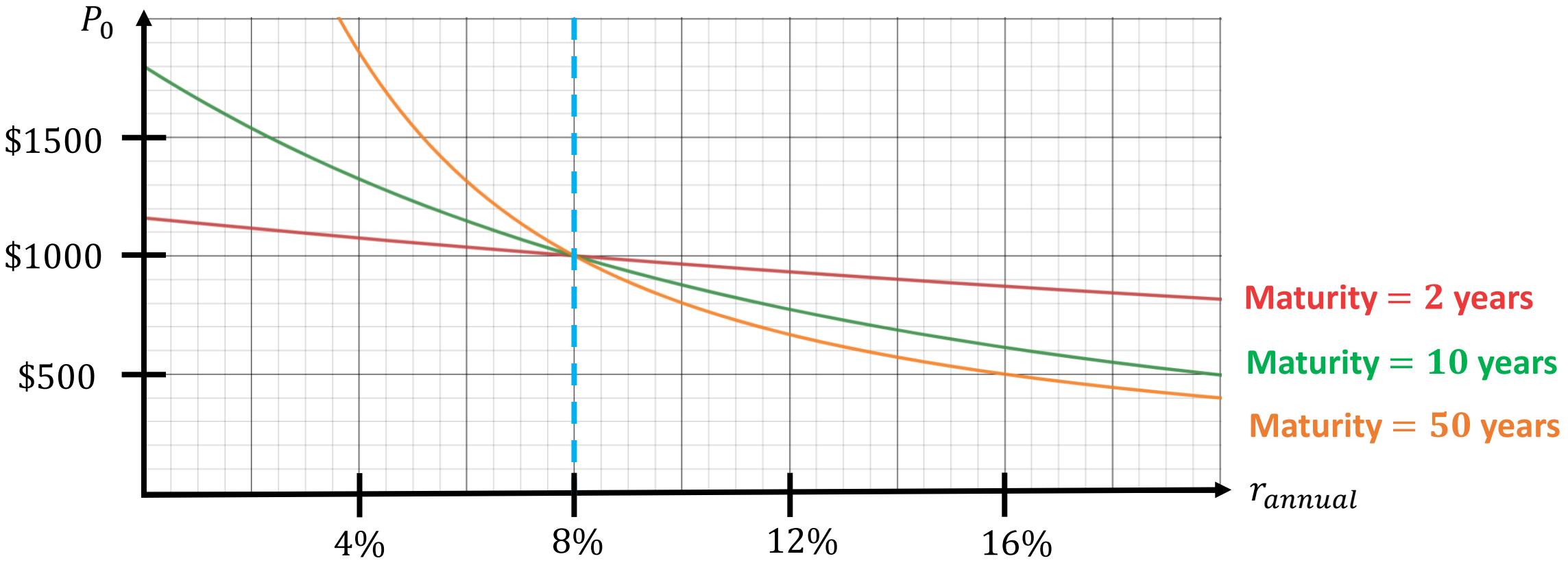
As the market rate (r) ⬆ the price (P) will ⬇

As the market rate (r) ⬇ the price (P) will ⬆

Interest Rate Risk (Maturity Length)

Prices fluctuate as r changes, but depending on a bond's characteristics, they will fluctuate more or less

The following bonds have : $CR = 8\%$ Yearly coupon payments Face Value: $\$1000$

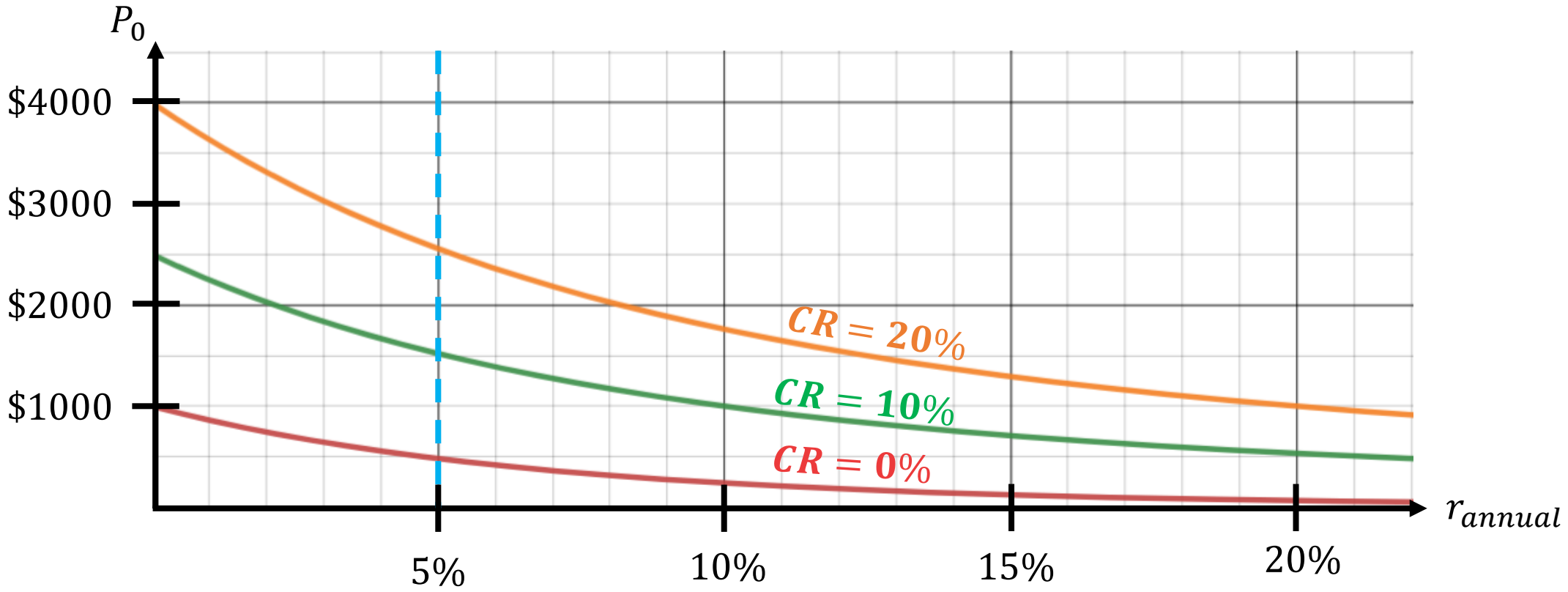


Conclusion: Longer maturity bonds have greater **percentual** price fluctuation (more interest rate risk!)

Interest Rate Risk (Coupon Rate)

How does the coupon rate (CR) affect bond prices as r changes?

The following bonds have : 15 years until maturity Yearly coupon payments Face Value: $1000



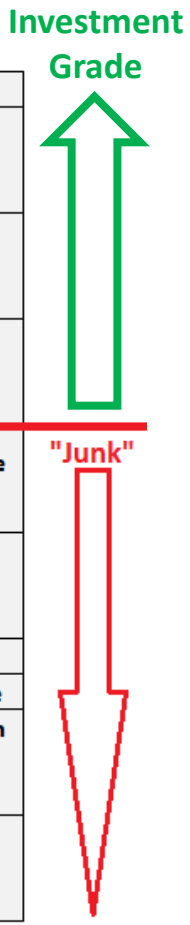
Conclusion: Bonds with lower coupon rates have greater **percentual** price fluctuation (more interest rate risk!)

Default (Credit) Risk & Bond Ratings

How likely will a bond issuer default on your payments? We can compare this likelihood with bond ratings!

Credit Rating Scales by Agency, Long-Term

Moody's	S&P	Fitch	
Aaa	AAA	AAA	Prime
Aa1	AA+	AA+	High grade
Aa2	AA	AA	
Aa3	AA-	AA-	
A1	A+	A+	Upper medium grade
A2	A	A	
A3	A-	A-	
Baa1	BBB+	BBB+	Lower medium grade
Baa2	BBB	BBB	
Baa3	BBB-	BBB-	
Ba1	BB+	BB+	Non-investment grade speculative
Ba2	BB	BB	
Ba3	BB-	BB-	
B1	B+	B+	Highly speculative
B2	B	B	
B3	B-	B-	
Caa1	CCC+	CCC	Substantial risk
Caa2	CCC		Extremely speculative
Caa3	CCC-		Default imminent with little prospect for recovery
Ca	CC	CC	
	C	C	
C	D	D	In default
/			
/			



Very likely to default

Remarks:

- The **default premium** is the difference between the YTM on a risky bond vs a government bond of *similar* maturity and coupon
- Assume government bonds do not have default risk
- The lower the grade, the higher chance for default, so investors need to be compensated with a greater reward (r)

Why does Greater Risk Result in a Higher YTM?

Investors need to be compensated for the risk they take – greater risk means greater return (r)

$$\frac{YTM}{k} = r \Leftrightarrow YTM = r \cdot k$$

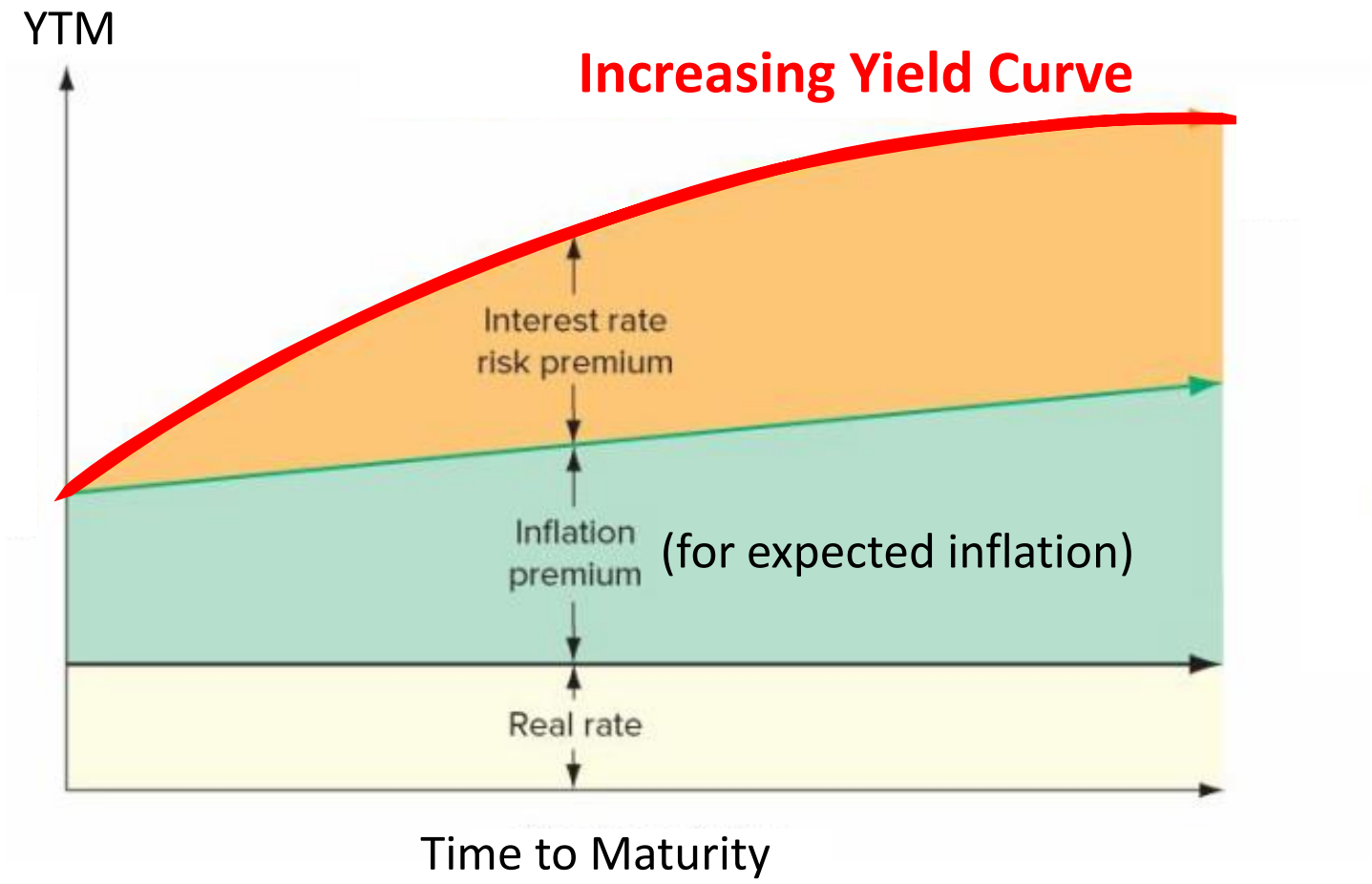
Holding k fixed, we see that YTM moves in the same direction as r

$YTM \uparrow = \uparrow r \cdot k$ As r **increases**, the YTM **increases**

$YTM \downarrow = \downarrow r \cdot k$ As r **decreases**, the YTM **decreases**

Yield Curve Example

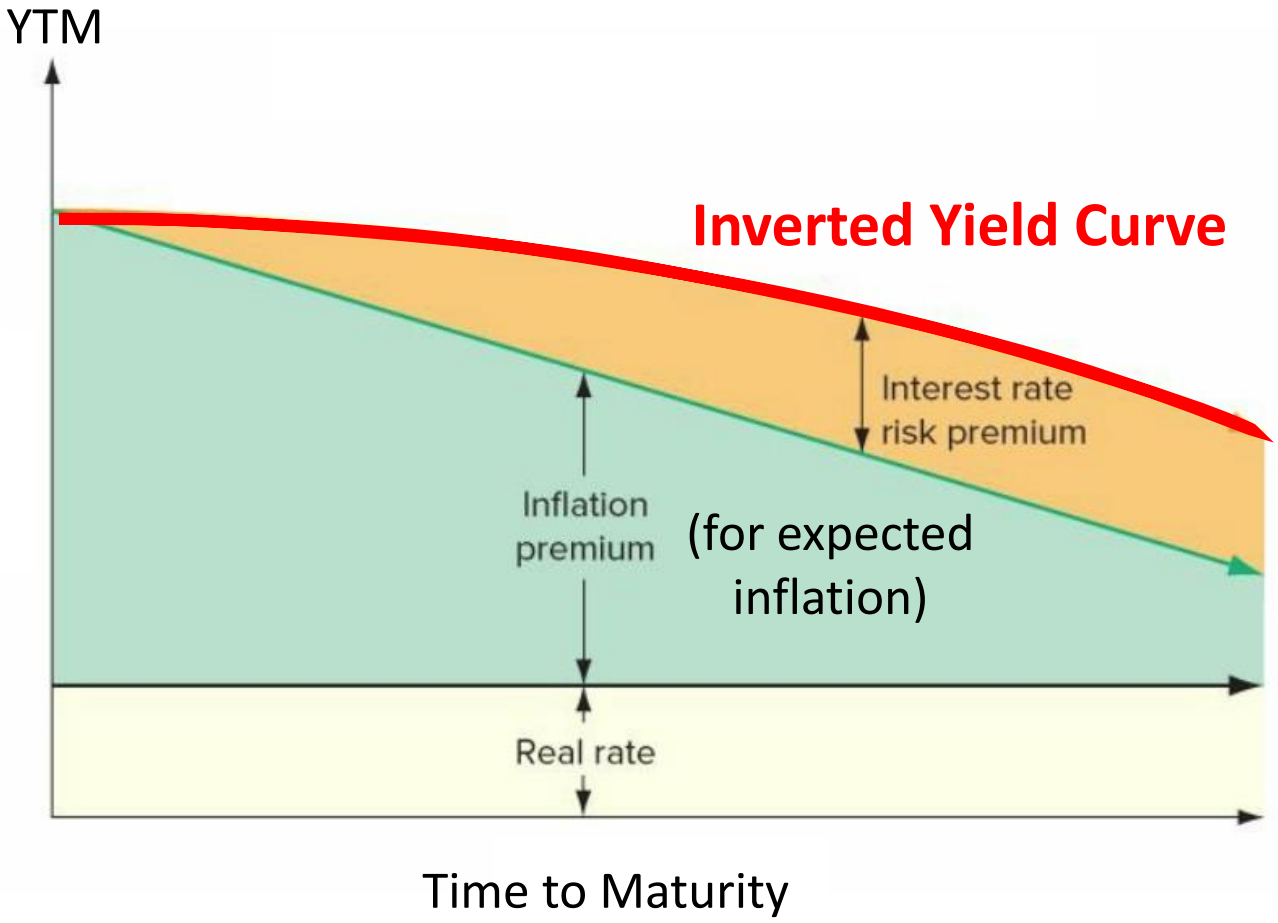
An increasing (steep) yield curve signals economic growth and rising interest rates in the future



Disclaimer: For 2023W2, you do not need to know this diagram! It's just a visual of some concepts mentioned

Yield Curve Example (Continued)

A decreasing (inverted) yield curve signals economic recession and declining interest rates in the future



Disclaimer: For 2023W2, you do not need to know this diagram! It's just a visual of some concepts mentioned

Practice Question 1

Suppose you are giving the following three bonds – assume all of them have a face value of $1000

i) A- grade John Deere bond with 8 years until maturity, coupon rate of 3.05%, and semi-annual coupons

ii) BBB grade WSP Global bond with 12 years until maturity, coupon rate of 2.74%, and semi-annual coupons

iii) BB grade Tesla bond with 10 year until maturity, coupon rate of 3.42%, and semi-annual coupons

a) Which bond has the highest credit risk?

b) Which bond has the highest interest rate risk?

c) You found a comparable Government of Canada bond with a YTM of 7% to the Tesla bond. If the Tesla bond has a YTM of 12%, what is the default premium of the Tesla bond?

Practice Question 2

Which of the following is/are true about an inverse sloping yield curve for bonds? Select all that apply.

a) A smaller interest rate risk premium will be expected in longer maturity bonds

b) The economy is forecasted to undergo a recession

c) Interest rates are expected to rise in the future

d) Longer maturity bonds will have a smaller YTM

e) A larger interest rate risk premium will be expected in shorter maturity bonds

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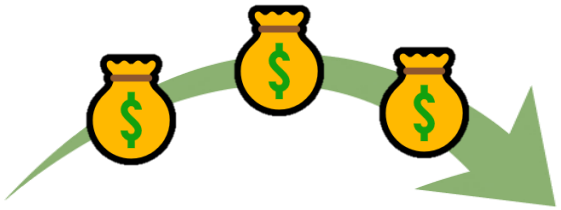
Information About Stocks

Equity instrument (e.g., common stock, preferred stock, etc.) issued by corporations in efforts to raise funds

Stockholders

Stocks



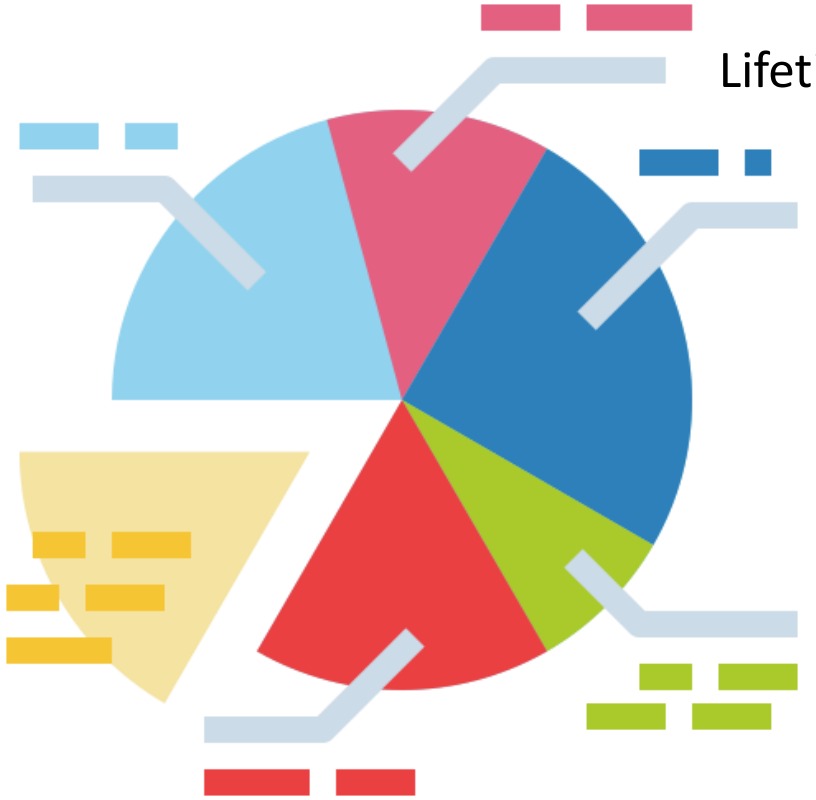
Dividends (optional)

Corporations

Voting rights
(usually 1 vote)

Not a liability until declared by board of directors

Common Stock Characteristics



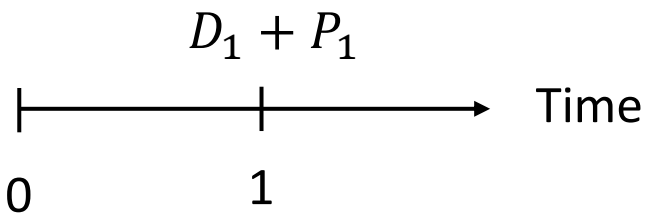
Lifetime of a corporation could be infinite (unlike a bond's maturity date)

Dividends are uncertain (unlike a bond's coupon payments)

There is no easy way to observe the return that investors require on common shares (unlike a bond's YTM)

Common Stock Valuation (No Growth)

One-period Investor:



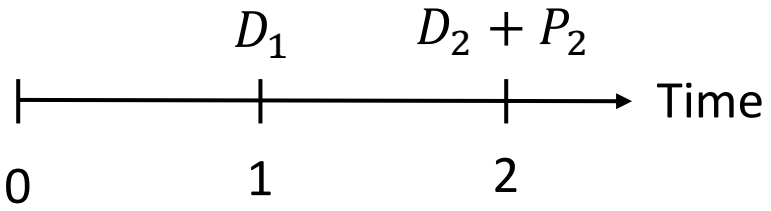
$$P_0 = \frac{D_1+P_1}{1+r} \quad \Leftrightarrow \quad r = \frac{D_1}{P_0} + \frac{P_1-P_0}{P_0}$$

Expected Return

Dividend Yield

Capital Gain Yield

Two-period Investor:



$$P_0 = \frac{D_1}{1+r} + \frac{D_2+P_2}{(1+r)^2}$$

In general (Dividend Discount Model – DDM):

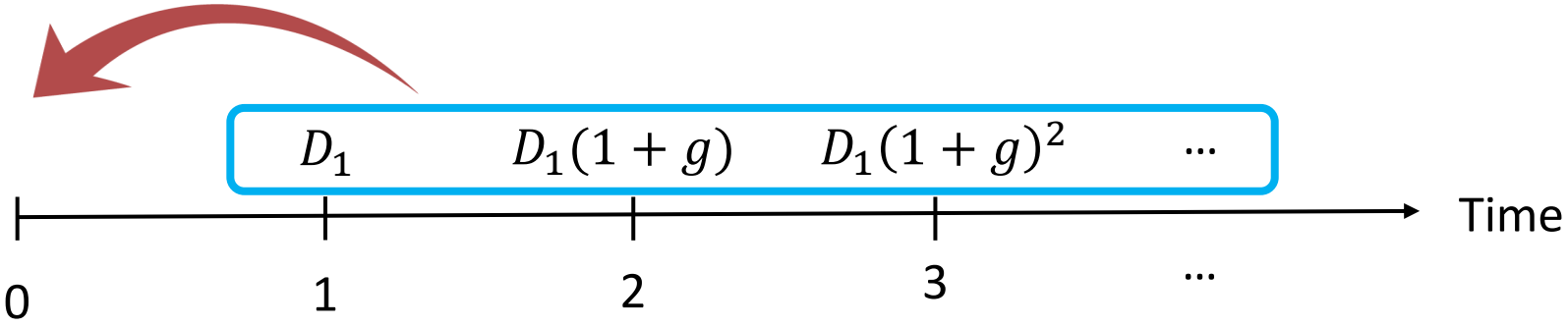
$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \frac{D_4}{(1+r)^4} + \cdots$$

The stock price equals the present value of ALL expected future dividends

Common Stock Valuation (Constant Growth)

It is difficulty to forecast dividend payments in the far future, so we need future dividend projections

What if the dividends acted like a perpetuity but changed at a fixed percentage (g) each period?



Where $D_1 = D_0(1+g)$

Gordon Growth Model (GGM):

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}$$

Under the condition of $g < r$

If $g \geq r$ the sum of our discounted dividends would diverge to infinity as the PV of each subsequent period's dividend would never decrease

Note: The dividend today (D_0) is not part of the PV of future cash flows for P_0 but is used to find D_1

Constant Growth Valuation Example

It is far in the future and Reddit has finally done it's initial public offering (IPO). They just paid a dividend of $5 per share today. Yearly dividends are expected to increase at a rate of 2% per year forever. If the yearly market return is 3% on securities with the same risk as Reddit's stock, what is the price of their stock today?

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}$$

We are not given D_1 so we need to obtain it from D_0

$g = 0.02$ $r = 0.03$ $D_1 = D_0(1+g) = 5 \cdot (1+0.02) = 5.10$

$$P_0 = \frac{5.10}{0.03 - 0.02} = \mathbf{\$510}$$

Interpreting the GGM Formula & Growth

We can write the GGM formula in terms of r to get a glimpse of what it is made up and determined by?

$$P_0 = \frac{D_1}{r-g} \quad \Leftrightarrow \quad r = \frac{D_1}{P_0} + g$$

$\frac{D_1}{P_0}$ is the **dividend yield** and is "*forward looking*" – looks into the *future* as it uses next period's dividend (D_1)

g is the **capital gain yield** and is also "*forward looking*" – looks at the *expected future* growth in dividends

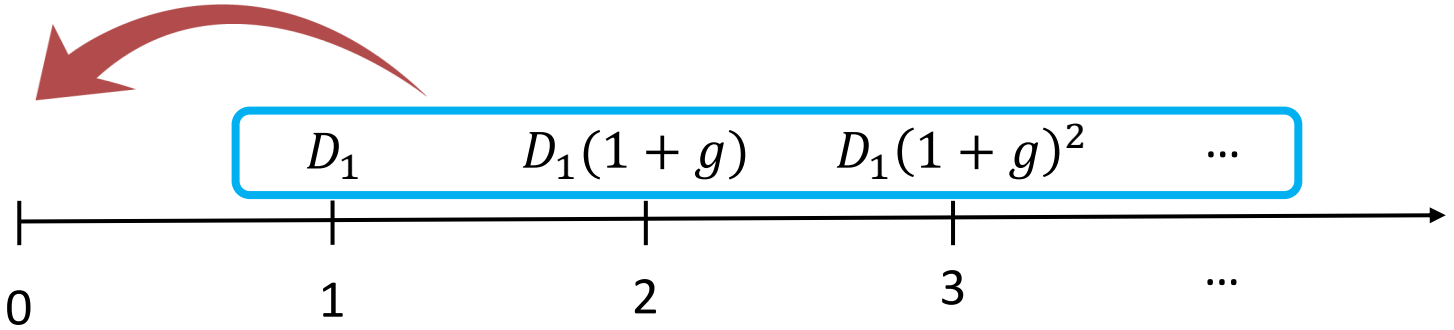
Is r determined by expected dividends (D_1) and the growth of the firm (g)?

NO! The *expected* rate of return (r) is market determined and driven by returns on similar risk securities

If a company changes their dividend (D_1) or growth rate (g), P_0 will change – a company cannot control r

What About Including D_0?

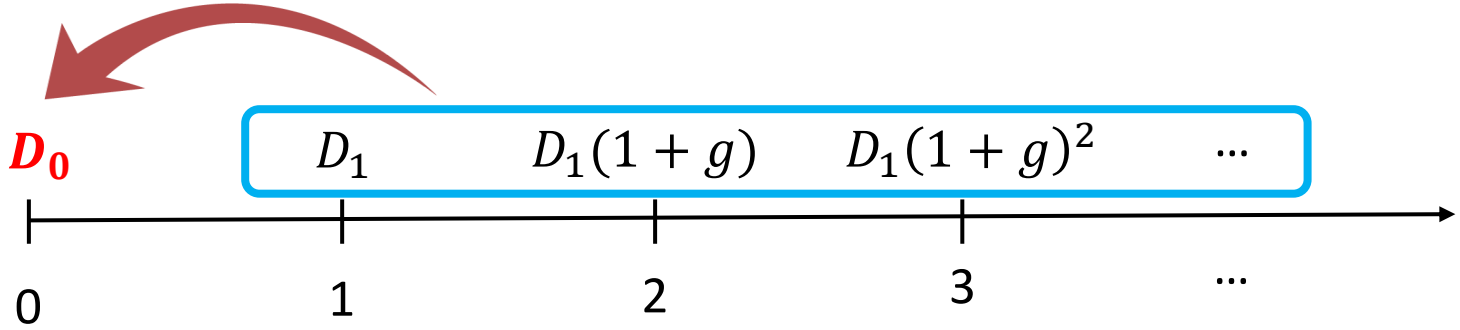
In the GGM, our calculation excludes D_0 – in this case we are calculating the ex-dividend price (P_0^{ex})



Just a fancier way to call P_0

$$P_0^{ex} = \frac{D_1}{r-g}$$

What if we wanted to include D_0 into our stock's price? We can do that as well!



$$P_0^{cum} = \frac{D_1}{r-g} + D_0$$

Means "with" in Latin – it is *with* the extra dividend, D_0

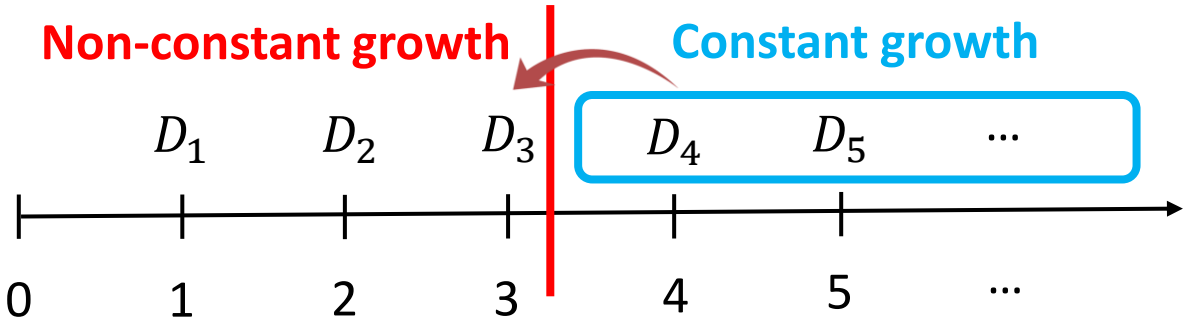
Common Stock Valuation (Non-constant Growth)

What if our growth rate g is not constant for a finite period but stables off at some period in the future?

Step 1: Find the numerical value of all dividends up to and including the period where growth first stabilizes

Step 2: Discount the constant growth dividends using the GGM to get its terminal value (TV)

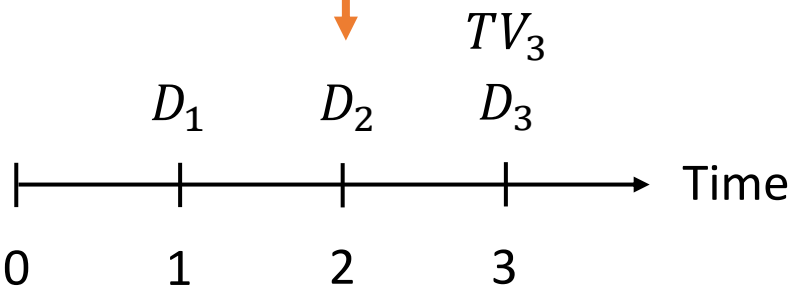
Step 3: Discount the terminal value and each individual dividend in the non-constant growth period to $t = 0$



$$TV_3 = \frac{D_4}{r - g}$$ **(use GGM)**

Stable growth rate

Now discount each individual value (use DDM)



$$P_0 = D_1(1+r)^{-1} + D_2(1+r)^{-2} + D_3(1+r)^{-3} + TV_3(1+r)^{-3}$$

Non-constant Growth Valuation Example

A firm just paid a dividend of $2 today. While the expected annual growth rate is 12% over the next year, it will be -3% the following year. After that, annual dividend growth is expected to be 8% each year forever. If the required return is 10% per year, what is the price of the firm's stock today if it pays yearly dividends?

$D_0 = \$2$ (not included in P_0 as divided was *just paid*)

$D_1 = D_0 \cdot (1 + 0.12) = 2 \cdot (1 + 0.12) = \$2.24$

$D_2 = D_1 \cdot (1 - 0.03) = 2.24 \cdot (1 - 0.03) = \$2.1728$

$D_3 = D_2 \cdot (1 + 0.08) = 2.1728 \cdot (1 + 0.08) = \$2.3466$ (first stable growth dividend)

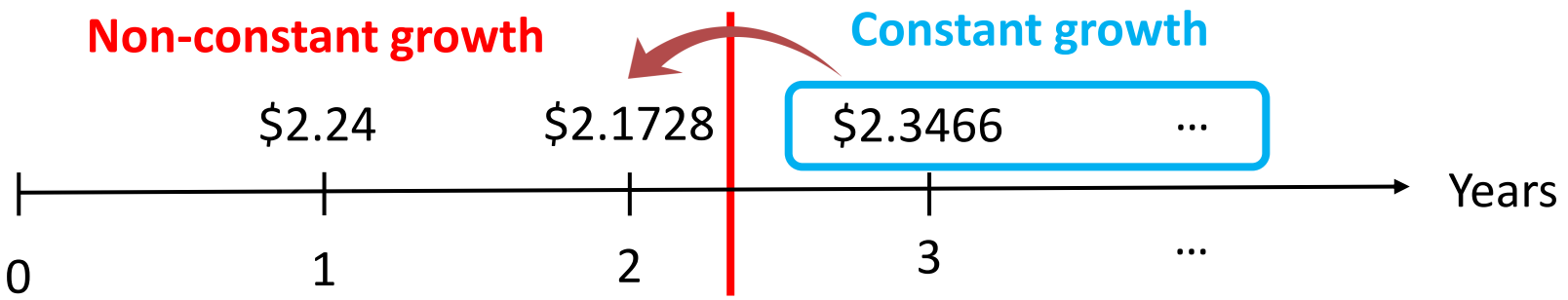
Stable growth rate

During year 1, the growth rate (g) is 12% but r is 10% – is this a violation of $g < r$?

NO! The condition $g < r$ only applies to when growth has stabilized (i.e., when we apply the GGM formula)!

The stable growth rate for the rest of time is 8% – and this is *less* than our r of 10%

Non-constant Growth Valuation Example (Continued)

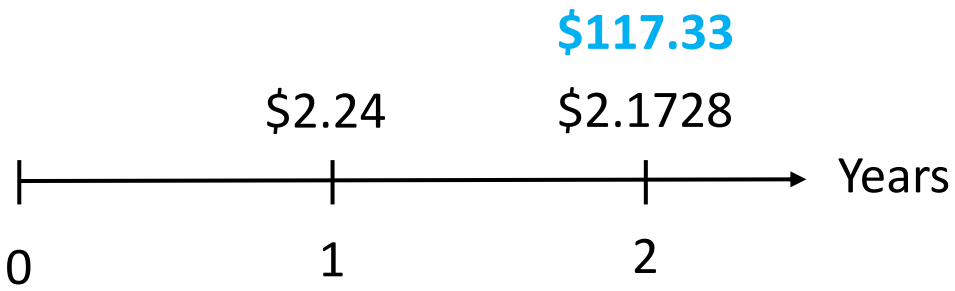


$D_1 = \$2.24$ $r = 0.10$

$D_2 = \$2.1728$ $g = 0.08$

$D_3 = \$2.3466$

$$TV_2 = \frac{D_3}{r-g} = \frac{2.3466}{0.10-0.08} = \mathbf{\$117.33}$$



$$P_0 = 2.24 \cdot (1+0.10)^{-1} + 2.1728 \cdot (1+0.10)^{-2} + 117.33 \cdot (1+0.10)^{-2} = \mathbf{\$100.98}$$

Limitations of the DDM

The DDM (and GGM) relies on a firm's *expected* future dividends, which holds much uncertainty

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \frac{D_4}{(1+r)^4} + \cdots$$

Dividend Discount Model (DDM)

$$P_0 = \frac{D_1}{r-g}$$

Gordon Growth Model (GGM)

Why are expected dividends uncertain?

(1) Future dividends are affected by company policies, market prices, and a firm's expected growth rate (g)

(2) Future dividends are hard to predict with historical data

(3) Some companies don't pay dividends currently

Comparable Firms and the P/E Ratio

We can estimate the stock price of a firm using the price-earnings (P/E) ratio of a comparable firm

$$P/E = \frac{P_0}{EPS_1} = \frac{Payout\ Ratio}{r-g} \Leftrightarrow P_0 = P/E \cdot EPS_1$$

P_0 = Price of firm's stock today

EPS_1 (Forecasted earnings per share) $= \frac{Net\ Income}{Total\ Outstanding\ Shares}$

Expected net income over the next twelve months (NTM)

Assumptions about the comparable firm:

(1) Generates similar future cash flows

(2) Has a similar payout ratio, growth rate, and expected return (similar risk)

Practice Question 1

Which of the following is/are true about stocks? Select all that apply.

a) As r increases, stock price increases, and as expected growth rate (g) decreases, stock price decreases

b) Companies are forced to pay dividends and can undergo legal action if they are not paid

c) The expected return on a stock with growing dividends consists of a capital gain yield and a dividend yield

d) In general, the more stocks one has, the more votes they can cast when electing the board of directors

e) The P/E valuation method focuses on using a company's historical earnings in the last 12 months

f) Companies control their expected return by adjusting the dividends they pay and future dividend growth rate

g) The ex-dividend price of a stock will never be higher than the cum-dividend price of the same stock

Practice Question 2

a) This morning, Frieren & Fern paid a yearly dividend of $10 to their shareholders. If investors require an annual return of 13% for their stock, what is their stock price today under the following yearly growth rates?

i) $g = 8\%$

ii) $g = -5\%$

iii) $g = 14\%$

b) If $g = 8\%$ what is the company's cum-dividend price today?

Practice Question 3

Himmel Corporation offers yearly dividends to their shareholders. They just paid their shareholders $5 and are looking at a 6% annual growth in dividends for the next 2 years. Afterwards, their annual growth rate will drop to 3% forever. Answer the following if investors require an annual return of 8%.

a) What is the terminal value at $t = 2$?

Practice Question 3

For the rest of the question, assume that the terminal value is $120

b) What is the price of Himmel Corporation's stock today?

Practice Question 4

Stark Co. does not pay dividends but is considered similar to Eisen & Sons. Eisen & Sons estimates that their P/E ratio will be 8.7x next year.

a) If Stark Co.’s EPS is projected to be $30, what is the price of their stock today?

b) Select the correct option listed in brackets

If the price of the stock is $300 today, an investor would (buy/not buy) it because it’s (overvalued/undervalued)

TABLE OF CONTENTS

Information & Valuation

I'm going to buy a few units of this stock for $100 each!

I'm going to sell this stock once the price hits $120!

People value the same security differently – they internalize this information and revise their valuation

Not Everyone has the Same Information

Our valuation of stocks focused on *expected* dividends and *expected* growth rate of dividends

How might our valuation of a stock be different than the market price?

(1) The quality of your information from your estimates might be different

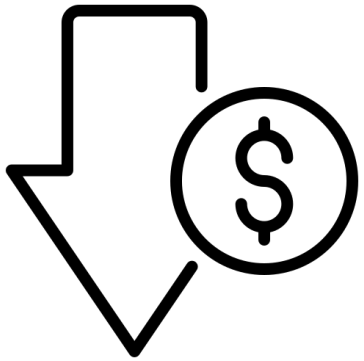
(2) You may have superior information compared to other investors (e.g., insider information)

Publicly traded stocks provide accurate information, so the market price likely reflects the true stock value

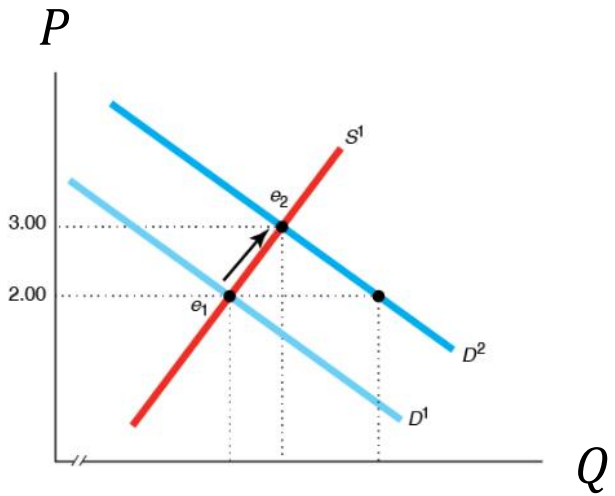
What factors may cause a stock's price to change?

Investor Competition & Price Adjustment

Investor competition adjusts the price of securities when new information is announced



$$P_0 \downarrow = \frac{D_1}{\uparrow r - g}$$



Stock is Undervalued → Expected Return r is too High → Demand Increases

Conclusion: If investors now know that a stock is undervalued, prices will adjust and increase

EMH: Competition Among Investors Works to Eliminate Mispricing

What Else Causes Prices to Adjust?

Degree of Investor Competition

Nature of New Information

Private

Public

Stock Price

Types of Information

(1) Public, Easily Interpretable Information (e.g., news reports, public data sources)

- Information available to all investors (hence, public)
- Competition among investors is fierce so prices reflect new information very quickly

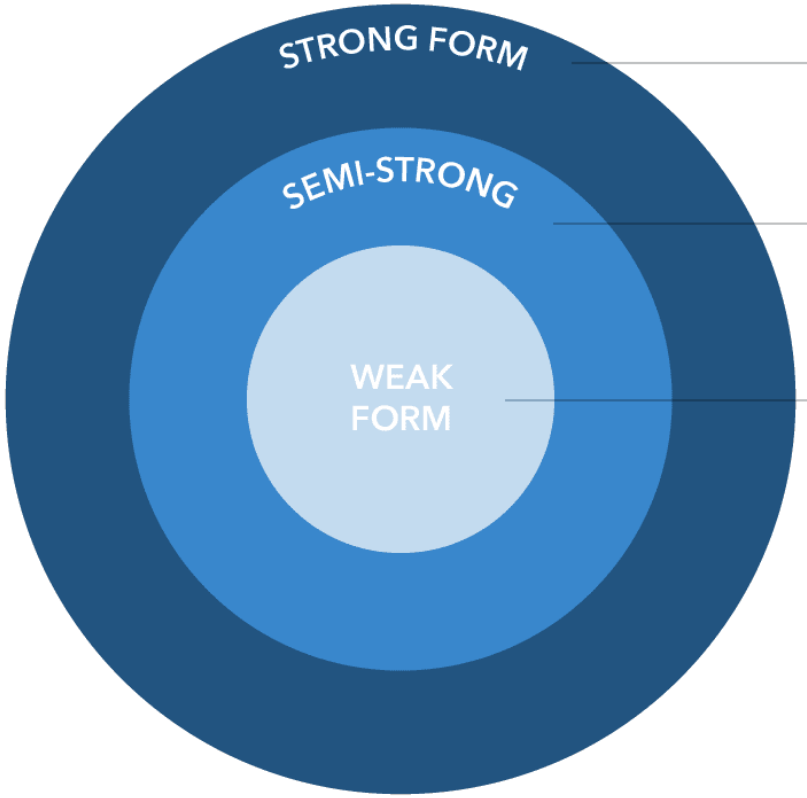
(2) Private or Hard-to-Interpret Information

- Confidential or insider information (material, non-public, price sensitive information)
- Requires expertise and efforts to understand the consequences
- Prices move more slowly as only a select few investors will trade and potentially gain a profit
- Competition to exploit this information will increase over time, so the only barrier to reducing inefficiency is the cost of obtaining information

When is a Market Efficient?

Recall the EMH: Competition among investors works to *eliminate mispricing*

If EMH is true, an efficient market should *quickly* and *accurately* reflect new info (past, public, and/or private)



Strong Form Efficient: Prices reflect all information, public or private

Semi-strong Form Efficient: Prices reflect all public and historical info

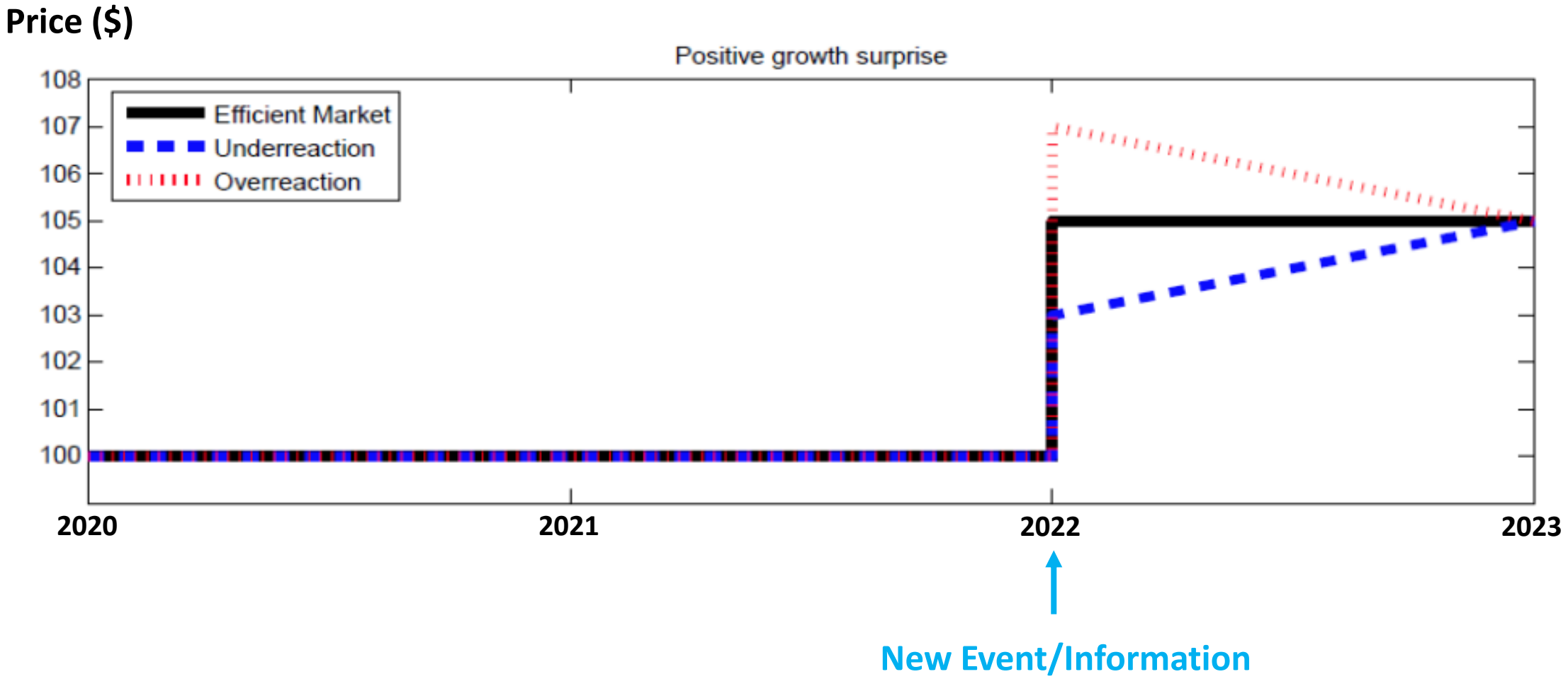
Weak Form Efficient: Prices only reflect historical information

Strong Form ⇒ Semi-strong Form ⇒ Weak Form

Weak Form ⇒ Semi-strong Form ⇒ Strong Form

Inefficient Markets

If EMH is true, an efficient market should *quickly* and *accurately* reflect new info (past, public, and/or private)



Are Markets Strong Form Efficient?

If a market is strong form efficient, we should NOT be able to gain anything with insider information as prices fully reflect all information. There are no secrets being held!

Evidence Against Strong Form Efficiency

(1) There *is* value in obtaining private information – there are also laws preventing insider trading (it is illegal, unethical, and unfair to uninformed investors)

(2) Analysts/researchers (who deal with private information) conduct research as they are compensated by earning trading profits – if there were no profits can be made, why conduct research?

(3) Prices follow a random walk (are unpredictable), but in a strong form efficient market, they would have to be predictable – this is a contradiction!

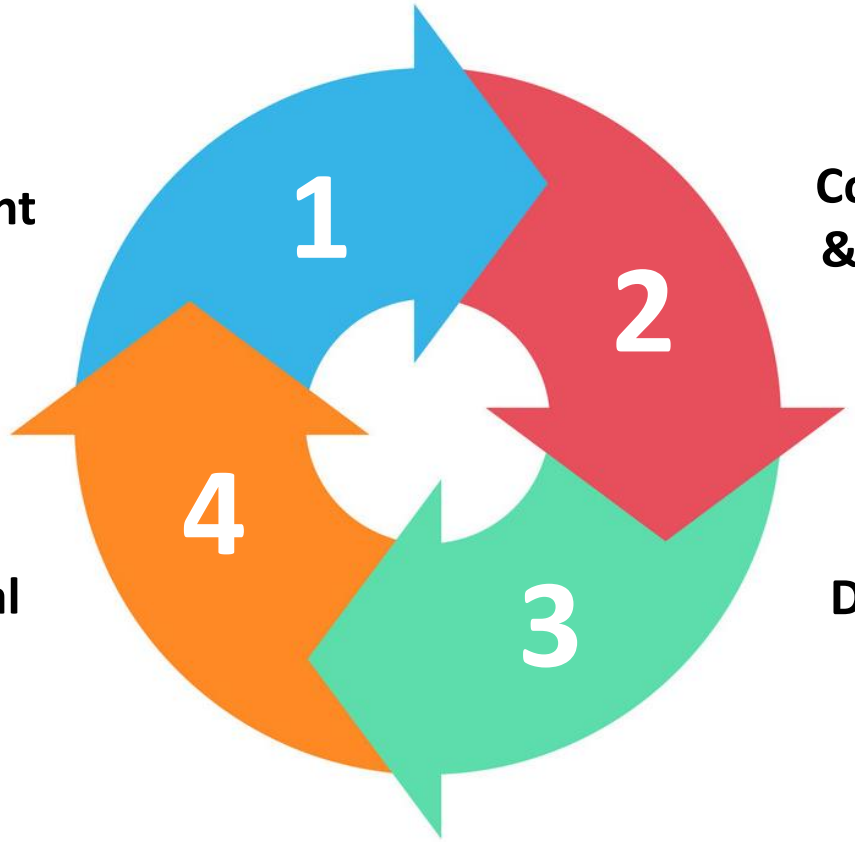
One Caveat About (2)

Grossman-Stiglitz Paradox: If prices reflect all information, no one will conduct research. But if no one conducts research, how will prices reflect all information?

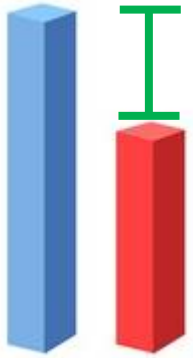
Testing Market Efficiency

Event studies allow us to analyze the impact of an event on an asset's price

Identify an Event



Collect Data Before & After Event Date



Calculate Abnormal Return

Define Benchmark Return

Practice Question 1

Which of the following statements is/are true? Select all that apply.

a) The stock return can be defined as the sum of the benchmark return and abnormal return

b) The prices of securities in a semi-strong form efficient market reflect both historical and public information

c) There is strong evidence against markets being semi-strong form efficient today

d) A market may be weak form efficient regardless of whether it is semi-strong form efficient or not

e) Data quality and superior information can cause differences in prices obtained from valuation methods

f) Information held back from a company's legal policies is an example of insider information

g) Since investor competition is less fierce over private information, prices will never adjust as time passes

Practice Question 2

Assume a semi-strong form efficient market. Your friend is a technical analyst (one who investigates past price patterns) and claims that he can predict future prices to “beat the market”. Select the option that best explains the validity of his claim.

a) Your friend is correct as a semi-form efficient market implies that one can make gains from public data

b) Your friend is incorrect as the market must be strong form efficient

c) Your friend is correct as a semi-form efficient market implies that one can make gains from historical data

d) Your friend is incorrect as prices already reflect all historical information in a semi-form efficient market

e) None of the above