

# COMM 204

2020W1 midterm review session

**Answer Key**



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undergraduate  
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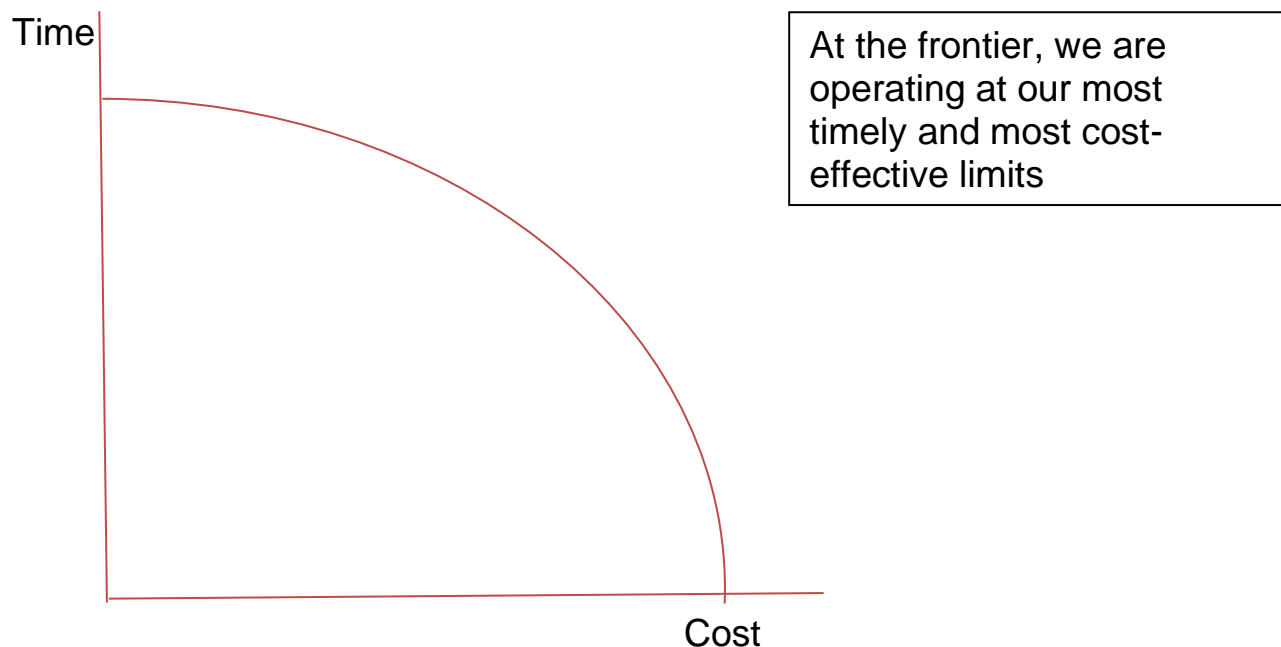
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# The Operations Frontier

How does operations management help the organization?

It provides tools that help identify and eliminate inefficiencies to ensure that the organization is as cost-efficient and timely as possible and innovates to improve operations. In other words, operations management finds the operations frontier, determines the optimal position for the firm to be on the frontier, and pushes the frontier out with innovation



# Process Analysis

## Key Terms

**Unit Flow:** Items that flow through the process

**Activities:** Transformation steps in the process



**Resources:** What performs activities

**Buffers:** Storage for flow units



**Decision Points:** Fork in the road



**Theoretical Flow Time:** Amount of time that a flow unit is in the process

(ex. 20 seconds)

**Unit Load:** Amount of time that a resource needs to process a flow unit

(ex. 5 seconds)

**Capacity rate:** Maximum possible output rate (ex. 3 bubbles teas/ minute)

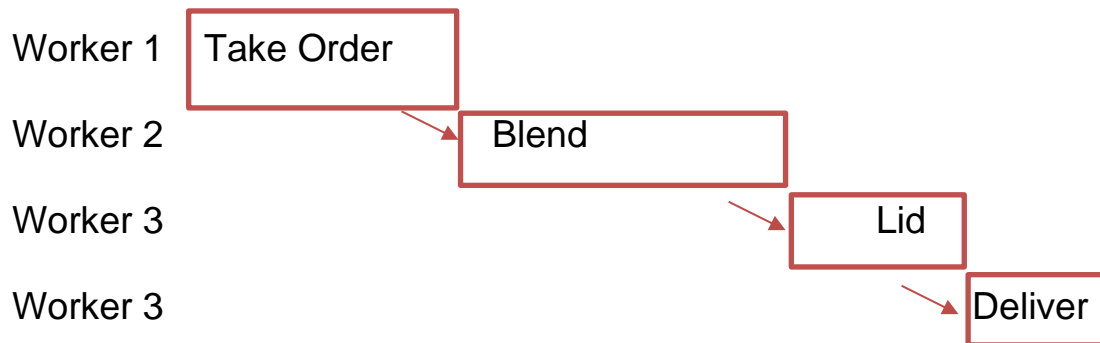
**Bottleneck:** The slowest resource that determines the capacity rate for the entire process \*there may be more than 1



### Linear Flow Diagram



### Swim Lane Flow Diagram



### Gantt Diagram

W1	Take Order	15s				15s				15s										
W2	Blend				20s				20s				20s							
W3	Lid								10s				10s				10s			
W3	Deliver									5s					5s				5s	



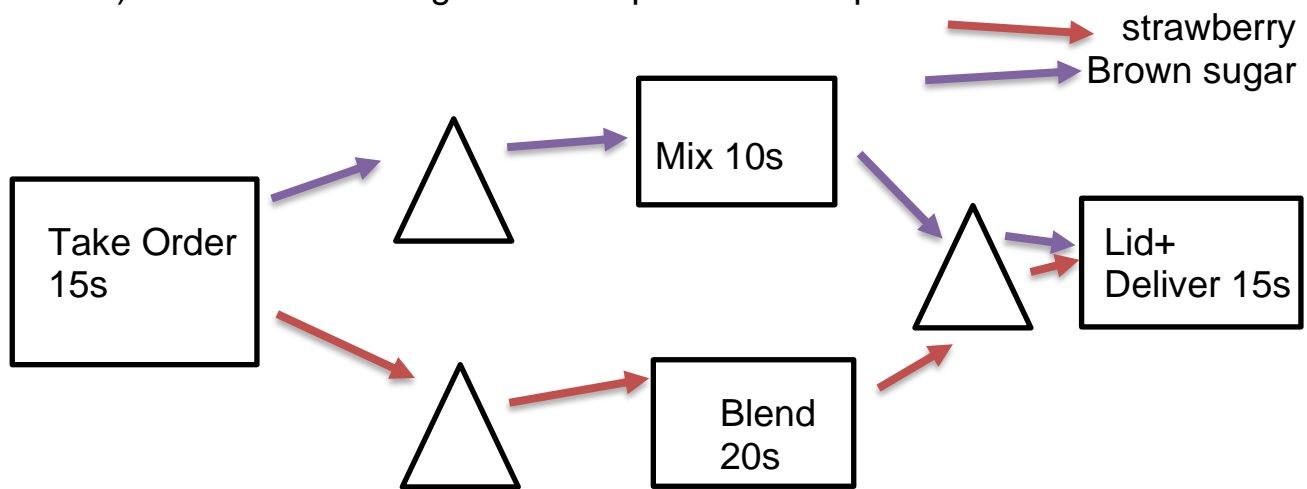
Q1: In the bubble tea example above, what is the...

- a) Theoretical flow time? **50 seconds**
- b) Unit load of worker 3? **15 seconds**
- c) Capacity rate of worker 3?  **$60/15 = 4$  bubble teas/minute**
- d) Capacity rate of worker 2?  **$60/20 = 3$  bubble teas/minute**
- e) Bottleneck? **Worker 2 is the bottleneck because they have the HIGHEST UNIT LOAD, or in other words, the LOWEST CAPACITY RATE**
- f) Capacity rate of the process? **3 bubble teas/minutes**
- g) For this question only: The owner of the bubble tea shop decides to buy a new, faster blender that takes 10 seconds to blend drinks.  
Who is the new bottleneck? **Workers 1 + 3**
- h) For this question only: The owner of the bubble tea shop decides to do takeout-only, so Worker 1's unit load is now 10 seconds.  
What is the capacity rate of the process? **3 bubble teas/minute**



Q2: A bubble tea shop offers two flavours: brown sugar milk tea and strawberry slush. Both teas require worker 1 to take the order, which takes 15 seconds, and worker 3 to put on the lid and deliver, which takes 15 seconds. The strawberry slush also requires worker 2 to blend for 20 seconds, while the brown sugar milk tea requires worker 2 to mix for 10 seconds.

a) Draw the flow diagram that represents this process



b) If customers only ordered brown sugar, what would the capacity rate be? (per minute)

**Bottleneck: Workers 1 and 3**

**Capacity Rate =  $60/15 = 4$  teas/minute**



c) If there are 100 strawberry orders/hour and 60 brown sugar orders/hour, what is the bottleneck?

We know that 62.5% of orders are strawberry and 37.5% are brown sugar

Worker	Unit load B	Unit load S	Units load mix (0.375B+0.625S)	Cap Rate mix
1	15	15	15	$60/15=4/\text{min}$
2	10	20	16.25	$60/16.26=3.7/\text{min}$
3	15	15	15	$60/15=4/\text{m}$

Worker 2 is the bottleneck resource.

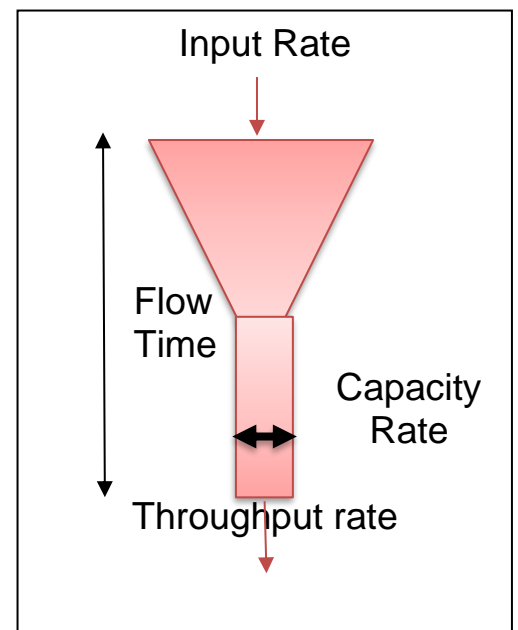
### More Key Terms

**Throughput rate:** Actual output rate (minimum of capacity rate and input rate) (ex. 3 bubble teas/minute)

**Input Rate:** Rate at which flow units arrive at the process (ex. 2 orders/minutes)

**Flow Time:** Average time for a unit to move through the system (ex. 4 minutes)

**Cycle Time:** Average time between completion of units (ex. 2 minutes)





# Utilization

$$\text{Utilization} = \text{Throughput Rate} / \text{Capacity Rate}$$

\*always less than or equal to 100%

$$\text{Implied Utilization} = \text{Input Rate} / \text{Capacity Rate}$$

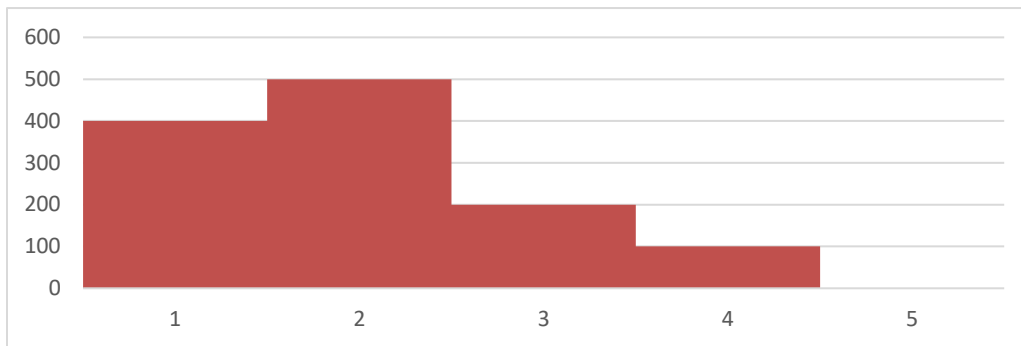
Q3: Complete this utilization profile

Resource	Capacity rate	Input Rate	Utilization
Worker 1	4 teas/minute	2 teas/minute	$2/4=50\%$
Worker 2	3 teas/minute	2 teas/minute	$2/3=67\%$
Worker 3	4 teas/minute	2 teas/minute	$2/4=50\%$



# Inventory Build-Up

## Discrete



$$\text{Average Inventory} = \frac{\sum \text{Inventory Build Up}}{\text{Total Slots}}$$

Q4a: Calculate average inventory

$$(400+500+200+100)/4 = 300$$

Q4b: Fill in the blank. Fewer, longer time slots (rather than the same data over more, shorter time slots) would mean that there is...

- i. Less excess demand
- ii. Less inventory build-up
- iii. Lower average inventory



Q5: a) Calculate Output and Inventory Build-up

Period	Input	Capacity	Output	Inventory
0				400
1	900	1000	1000	$400+900-1000=300$
2	900	900	900	$300+900-900=300$
3	700	600	600	$300+700-600=400$
4	0	600	400	$400+0-400=0$

b) What is average inventory?  $(300+300+400+0)/4=250$

### Continuous

**Average Inventory=Area under the curve/total time**

$R_i(t)$ : input rate at time (t)

$R_o(t)$ : output rate at time (t)

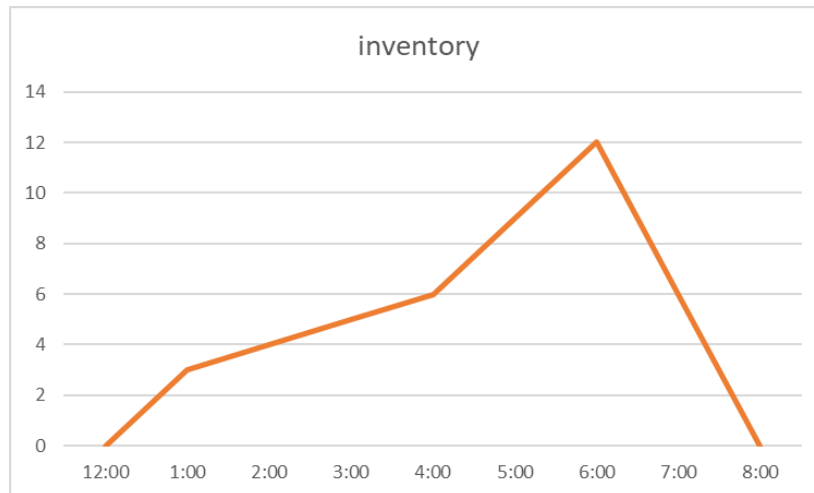
$\Delta R(t)$ : instantaneous inventory accumulated at time (t) (slope)

$$\Delta R(t) = R_i(t) - R_o(t)$$

$I(t)$ : Number of units of inventory in process at time (t)

**For a straight line:  $I(t_2) = I(t_1) + \Delta R^*(t_2 - t_1)$**





Q6: a) Calculate average inventory

$$\frac{[(3 \times 1/2) + (3 \times 3 + 3 \times 3/2) + (6 \times 2) + (6 \times 2/2) + (12 \times 2/2)]}{8} = 5.625$$

b) What is the instantaneous inventory accumulated at 2:00?

$$I(t_2) = I(t_1) + \Delta R \times (t_2 - t_1)$$

$$4 = 3 + \Delta R \times (2 - 1)$$

$$\Delta R = 1 \text{ unit/hour}$$

\*\*we could also calculate rise/run



# Little's Law

**Average Inventory (I):** The average number of units/customers in the system (ex. 5 bubble teas)

**Average Throughput Rate (R):** The average actual output rate (lower of capacity rate and input rate) (ex. 3 bubble teas/ minute)

**Average Flow Time (T):** The average time for a unit to move through the system (ex. Hours)

$$I = R * T$$

Q7: Imagine you are standing outside of Walmart for 5 hours. You notice that 50 people exit the store while you were standing there. You also notice that people spend, on average, 30 minutes inside Walmart. How many people would you expect to be inside Walmart at any given moment?

$$T = 1/2 \text{ hour} \quad R = 50 \text{ people} / 5 \text{ hours} = 10 \text{ people/hour}$$

$$I = R * T$$

$$I = 10 * 1/2$$

$$I = 5 \text{ people}$$



**Days of Inventory:** Average number of days that a unit of inventory is held

$$\text{Days of Inventory} = \text{Cost of Inventory} \times 365 / \text{COGS}$$

$$T = I/R$$

**Inventory Turnover:** How many times the inventory has been replaced in a year

$$\text{Inventory Turnover} = \text{COGS} / \text{Avg Inventory}$$

$$= \text{Cost of Output} / \text{Cost of Input}$$

$$= 1/T$$

Q8: Given COGS=\$20 000, Inventory turns=8, solve for average inventory.

$$\text{Inventory Turnover} = \text{COGS} / \text{Inv}$$

$$\text{Inv} = 20\,000 / 8$$

$$\text{Inv} = \$2500$$

Q9: Given COGS=\$20 000, Average inventory=\$5000, what is days of inventory?

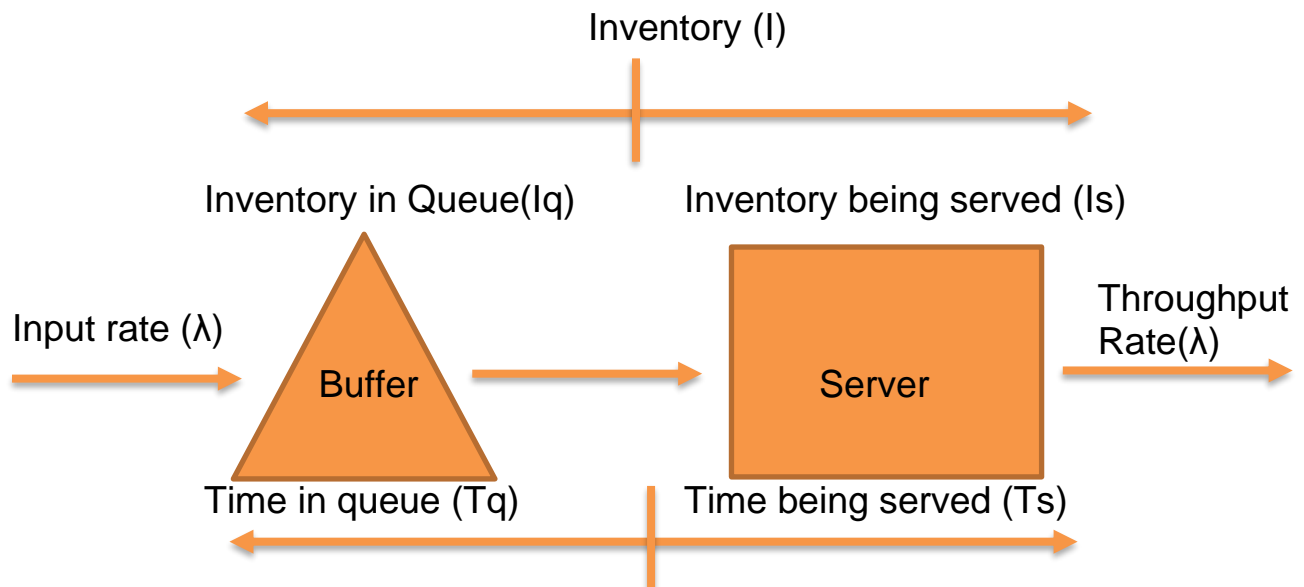
$$\text{Days of Inventory} = 365(I) / \text{COGS}$$

$$= 365(5000) / 20000$$

$$= 91.25 \text{ days}$$



# Variability in Processes



$\lambda$  (units/time): long-run avg throughput rate

$1/\lambda$  (time): inter-arrival time

$\mu$  (units/time): long-run avg capacity rate of a server

$1/\mu$  (time): avg processing time of a server

$c$ : number of servers

$p$ : utilization

$Ca$ : coefficient of variation for interarrival times

$Cs$ : coefficient of variation for service times

$$T = Tq + Ts$$

$$I = Iq + Is$$

$$Iq = \lambda * Tq$$

$$Is = \lambda * Ts$$

$$I = \lambda T$$

$$Iq = \frac{p\sqrt{2(c+1)}}{1-p} * \frac{Ca^2 + Cs^2}{2}$$

$$p = \lambda / c\mu$$

$$Ca = SD(1/\lambda) / \text{mean}(1/\lambda)$$

$$Cs = SD(1/\mu) / \text{mean}(1/\mu)$$



## Queues

G – “generally distributed” (must solve for it)

M – “exponentially distributed” (=1)

D – “deterministic” (=0)

### Interarrival time distribution/Service time distribution/# of servers

Queue Type	What's it mean?	PK Formula
G/G/1	“Interarrival time are generally distributed, service times are generally distributed, there is 1 server”	$lq = \frac{p^2}{1-p} * \frac{Ca^2 + Cs^2}{2}$
M/M/1	“Interarrival times are exponentially distributed, service times are exponentially distributed, there is 1 server”	$lq = \frac{p^2}{1-p}$
G/G/c	“Interarrival times are generally distributed, service times are generally distributed, there are c servers”	$lq = \frac{p^{\sqrt{2(c+1)}}}{1-p} * \frac{Ca^2 + Cs^2}{2}$
M/D/1	“Interarrival times are exponentially distributed, service times are deterministic, there is 1 server”	$lq = \frac{p^2}{1-p} * \frac{1}{2}$





Q10: At Starbucks, Mary is the only server and can serve 45 customers per hour. On average, a new customer enters the store every 2 minutes. There are, on average, three customers in the store.

a) What is the utilization? How long do customers have to wait in line?

$$c=1 \quad 1/\lambda=2 \text{ mins} \quad \mu=45/\text{hr}=0.75/\text{min} \quad l=3 \quad p=? \quad Tq=?$$

$$p = \lambda / c\mu$$

$$Tq = T - Ts$$

$$= 0.5 / 1(0.75)$$

$$= (l/\lambda) - (1/\mu)$$

$$= 67\%$$

$$= (3/2) - (1/0.75)$$

$$= 0.17 \text{ minutes}$$

b) A second employee, Tim, joins Mary in the afternoon. Tim can serve customers just as quickly as Mary, and the store stays just as busy while they are together (inter-arrival time does not change). Service times and interarrival times are both exponentially distributed. Using PK formula, determine the average number of people waiting in line.

$$c=2 \quad 1/\lambda=2 \text{ mins} \quad \mu=45/\text{hr}=0.75/\text{min} \quad l=3 \quad lq=? \quad M/M/2$$

$$lq = \frac{p\sqrt{2(c+1)}}{1-p}$$

$$p = \lambda / c\mu$$

$$= \frac{0.33\sqrt{2(2+1)}}{1-0.33}$$

$$= 0.5 / 2(0.75)$$

$$= 0.099$$

$$= 0.33$$



## Extra Questions

1. A manager at Starbucks wants to decrease queue time. Give 3 examples of ways to do this.

Hire another worker, decrease variability in service/interarrival time, innovate, etc.

2. In an M/M/2 queue, utilization is 0.8. What is  $l_q$ ?

$$p=0.8$$

$$c=2$$

$$l_q=?$$

$$l_q = \frac{p\sqrt{2(c+1)}}{1-p}$$

$$l_q = \frac{0.8\sqrt{2(2+1)}}{1-0.8}$$

$$l_q = \frac{0.8\sqrt{6}}{0.2} = 2.895$$



3. On Sundays, Julia is the only nail tech at Nailz Express Salon. From experience, she knows that customers entering the salon arrive in a poisson distribution, and her service time follows an exponential distribution. Customers typically come in at a rate of 3 per hour, and it takes Julia 12 minutes to do someone's nails.

a) What is the average utilization of Julia's time?

$$M/M/1 \quad \lambda=3/\text{hour} \quad T_s=1/\mu=12\text{m}=0.2\text{hours} \quad p=?$$

$$\mu=5/\text{hour} \quad p=\lambda/\mu$$

$$p=3/(1*5)=0.6$$

b) How long, on average, must customers wait to be served?

$$T_q=? \quad l_q=\frac{p^2}{1-p} \quad l_q=\lambda*T_q$$

$$l_q=(0.6^2)/(1-0.6) \quad 0.9=4*T_q$$

$$l_q=0.9 \text{ customers} \quad T_q=0.225 \text{ hours}$$

c) How much time, on average, are customers spending in the

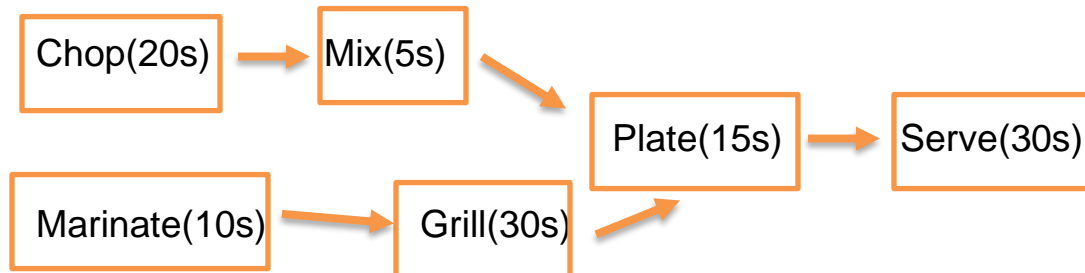
salon?  $T=?$

$$T=T_s+T_q$$

$$=0.2+0.225=0.425 \text{ hours}$$



4. The flow diagram below represents the production line for a meal at a restaurant. Each step has 1 worker, and orders for both types of meals come in at the same rate.



- a) What is the capacity rate of the process?

Since the bottleneck is the server, the capacity rate is  
 $60/30=2$  meals/minute

- b) If the head chef can hire one more worker, what step would this worker be assigned to, and what would the new bottleneck and capacity rate of the process be?

The new worker should be assigned to the bottleneck step, so they would be a server. The new bottlenecks would now be the griller ( $60/(30 \times 0.5)=4/\text{min}$ ), the plater ( $60/15=4/\text{min}$ ), and the servers ( $60/30/2=4/\text{min}$ ), making the new capacity rate of the process 4 meals per minute.



5. A call center receives 100 calls per hour from noon until 5pm. The center can process 90 calls per hour from noon until 3pm, and 60 calls per hour from then onwards. How many calls will be on hold at

1pm : 10

2pm : 20

3pm : 30

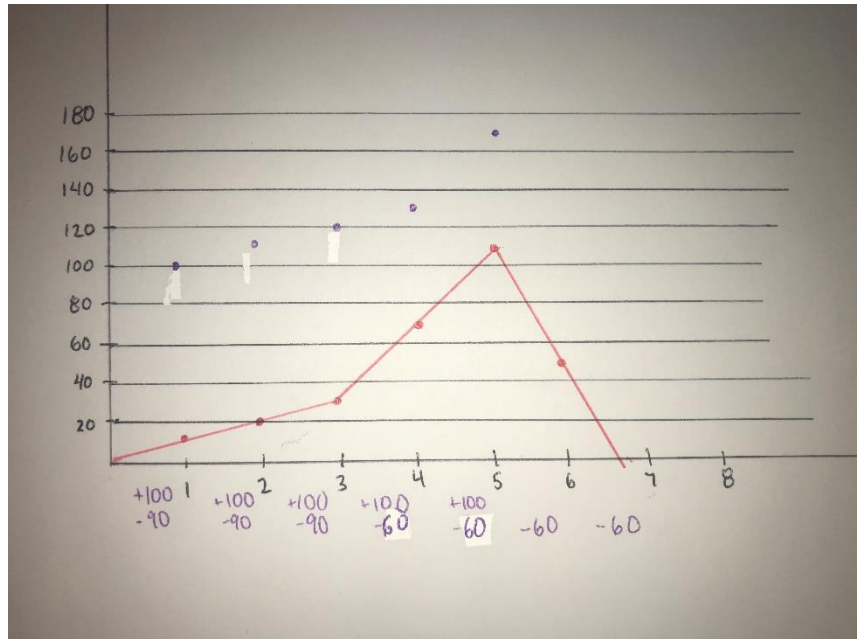
4pm : 70

5pm : 110

6pm : 50

7pm : 0

8pm : 0



# Mia's Top 5 Tips

**Understand the question before finding the answer!**

**If you can't figure out the answer... move on!**

**Don't forget to pay attention to units!**

**Practice makes perfect!**

**Fuel your brain!**

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