



commerce  
undergraduate  
society

COMMERCE MENTORSHIP PROGRAM

# MIDTERM REVIEW SESSION

# COMM 290



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# TABLE OF CONTENTS



Introduction to LPs	3
Simple LP	4
Graphing	5
Sensitivity Analysis	9
Blending LP	12
Scheduling LP	14
Transportation LP	16

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# INTRODUCTION

## Target Cell

Contains the output of the objective function - what are you trying to minimize or maximize?

## Input Data

The data given to you as part of a problem

## Constraint

A limitation presented in the problem

## Action Plan

The “action” you will take to solve the problem



## Relative Reference

Will change when auto-filled in other cells  
(=A1)

## Absolute Reference

Will not change when auto-filled in other cells  
(=\$A\$1)

## Mixed Reference

Will partially change when auto-filled in other cells  
(=\$A1 or =A\$1)

## =SUMPRODUCT

Returns the sum of the products of corresponding arrays  
=SUMPRODUCT(array1, [array 2], [array 3]...)



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# SIMPLE LP

**Problem – Pocky Dealer:** You are a student running a business selling two combos of Pocky: Combo A earns you \$5, consisting of one chocolate and one strawberry, while Combo B earns you \$6, consisting of two chocolates and no strawberry. You have 20 chocolates in stock and 10 strawberries in stock. Assume there are no costs associated with this model.

What is the objective? Is this a maximizing or minimizing model?

How many constraints are there? How would you write them algebraically?

Create the model and solve using Excel/Solver



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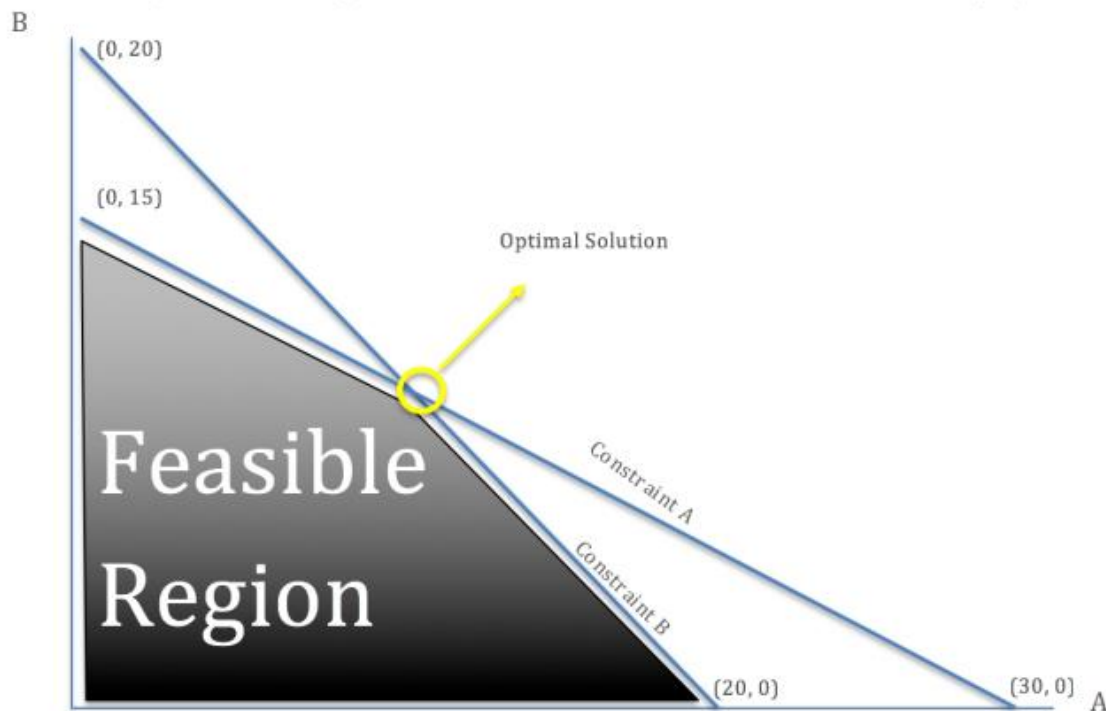
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# GRAPHING

<b>Feasible Region</b>	The region in which all solutions are valid and subject to the constraints
<b>Optimal Solution</b>	The best set of decisions that maximizes/minimizes the objective function while remaining within the constraints
<b>Multiple Optima</b>	Multiple sets of optimal solutions
<b>Infeasible Solution</b>	There is no feasible region associated with your LP
<b>Unbounded Solution</b>	The feasible region is infinitely large, and the objective function behaves such that you are moving the isoprofit line outwards indefinitely
<b>Redundant Constraint</b>	A constraint that does not affect on the feasible region
<b>Non-Negativity Constraint</b>	A constraint which makes sure a “decision” cannot be a negative value



**Problem:** Consider the following graph of an arbitrary linear programming model with the correct labelled optimal solution, feasible region. Assume this is a profit maximization model (That is, objective function is in the form  $x_A + y_B$ ).



Define the range of possible slopes for isoprofit lines that would lead to this optimal solution (marked by the yellow circle)

Find the coordinates of the optimal solution. The two constraints are  $A + B = 20$  and  $A + 2B = 30$ .

Suppose the sign of Constraint B ( $\leq$ ) is changed to the  $\geq$  sign. How will this change the feasible region?

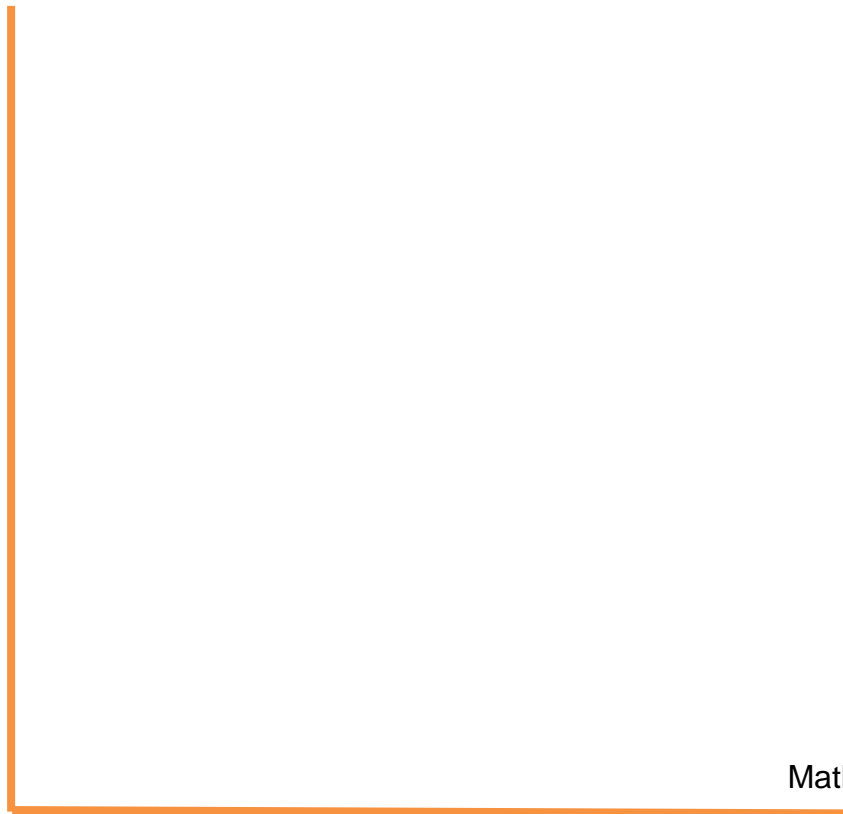


**Problem – Rachel’s Bootleg Textbooks:** Rachel is furious with the high cost of virtual textbooks. Being the entrepreneurial Sauder snake she is, Rachel gets a PDF copy of the math textbook and geography textbook and decides to print copies to sell to students (don’t actually do this).

Each math textbook will sell for \$20 and each geography textbook will sell for \$30. Each math textbooks require 1 ounce of ink and 2 pounds of paper. Since the geology textbook has more photos, it requires 3 ounces of ink but only 1 pound of paper. Rachel only has 18 ounces of ink and 11 pounds of paper. What combination of textbooks should Rachel produce to make as much profit as possible?

**Graph the problem and find optimal solution**

Geology (g)



Math (m)



**Questions** – Based on the Rachel's Bootleg Textbooks problem:

What would happen to the feasible region if the inequality symbol for the ink constraint reversed? What if both constraints' inequality symbols flipped?

What would happen to the feasible region if we added another constraint  $m+g \leq 20$ ?

What range of prices for math textbooks would give the same optimal solution?



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# SENSITIVITY ANALYSIS

## **RHS Allowable Increase/Decrease of Binding Constraint**

Range that the RH side of a binding constraint can move, keeping the shadow price the same

## **RHS Allowable Increase/Decrease of Non-Binding Constraint**

Range that the RH side of the non-binding constraint can move, keeping the constraint non-binding and the shadow price the same

## **Allowable Increase/Decrease of Objective Coefficient**

Range that an objective coefficient can move without changing the optimal solution

## **Shadow Price**

The increase in the target cell's value for every unit Increase in the RH side of a constraint

**Questions** – Solve algebraically based on the Rachel's Bootleg Textbooks problem:

What if Rachel could get an extra pound of paper? How would this change the constraint and optimal solution?

If Rachel has the opportunity to buy the extra pound of paper for \$8, should she?



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## Sensitivity Report – for the Rachel's Bootleg Textbooks problem:

### Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$12	Math textbook	3	0	20	40	10
\$D\$12	Geography textbook	5	0	30	30	20

### Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$5	Ink	18	8	18	15	12.5
\$E\$6	Paper	11	6	11	25	5

Based on the sensitivity report above:

What is the objective formula?

What is the optimal solution and total profit?

How much could the price of a math textbook increase, keeping the same optimal solution?

Are the constraints binding?

How much would profit change with 1 more ounce of ink? How about 1 less ounce of ink?



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## Sensitivity Report – Additional Practice Problem

Assume this is the sensitivity report for a profit maximizing LP model

Variable Cells							
	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
	\$C\$16	Decision 1	4500	0	0.33	0.01	0.016
	\$D\$16	Decision 2	0	0	0.33	1E+30	1E+30
	\$E\$16	Decision 3	62000	0	0.2	1E+30	0.017
	\$F\$16	Decision 4	0	-0.062	-8.81E-10	0.03	1E+30
Constraints							
	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
	\$C\$25	Constraint 1	( 1 )	0.65	5400	2501	1008
	\$C\$26	Constraint 2		0.55	21600	6500	5300
	\$C\$27	Constraint 3		0.14	0	12000	0
	\$C\$29	Constraint 4		0.62	7500	28000	7500
	\$C\$28	Constraint 5	62000	0.062	( 2 )	22000	27000

What are the missing values (1) and (2)?

How much more profit would the firm make if the RHS of Constraint 1 was increased by 2500?

How much more profit would the firm make if the RHS of Constraint 3 was increased by 1000?



# BLENDING LP

**Problem – Costco Hot Dogs:** You are in charge of procurement at Costco (not Costco), a company that sells hot dogs.

Costco sells two types of hot dogs: premium and classic. The hot dogs are made with a combination of 3 different ingredients: beef, pork, and mystery meat. Each type of meat has its own cost per pound and maximum amount you can order from the meat supplier (as shown in the table below).

In order to ensure quality, the premium hot dog must be at least 50% beef and cannot contain more than 5% mystery meat. The classic hot dog can have mystery meat, but no more than 25%.

How much of each type of meat should Costco order?

Input Data						
		Cost per pound		Pounds Available		Selling Price
	Beef	\$	3.00		1,000	Classic \$ 3.00
	Pork	\$	2.00		3,000	Premium \$ 5.00
	Mystery meat	\$	1.50		2,000	



## Fill in the Blanks – Based on the Costco Hot Dogs problem:

	A	B	C	D	E	F	G	H
1	<b>Kostco Hot Dogs</b>							
2								
3	<b>Input Data</b>							
4			Cost per pound	Pounds Available		Selling Price		
5		Beef	\$ 3.00	1,000		Classic	\$ 3.00	
6		Pork	\$ 2.00	3,000		Premium	\$ 5.00	
7		Mystery meat	\$ 1.50	2,000				
8								
9		Premium must be at least 50% beef				50%		
10		Premium must not contain more than 5% mystery meat				5%		
11		Classic can not be more than 25% mystery meat				25%		
12								
13	<b>Action Plan</b>				Model		Model	
14			Classic	Premium	Output		Requirement	units
15		Beef	0	1000	1000	<=	A	pounds
16		Pork	2100	900	3000	<=	3,000	pounds
17		Mystery meat	700	100	B	<=	2,000	pounds
18		Total	2800	2000				
19								
20	<b>Blending</b>				Model		Model	
21	<b>Constraints</b>				Output		Requirement	
22		Premium must be at least 50% beef				1000	>=	1000
23		Premium must not contain more than 5% mystery meat				100	<=	C
24		Classic can not be more than 25% mystery meat				700	<=	700
25								
26	<b>Revenue and Cost Info</b>							
27			Classic	Premium	Total			
28		Revenue	D	\$ 10,000.00	\$ 18,400.00			
29		Costs						
30		Beef	\$ -	\$ 3,000.00	\$ 3,000.00			
31		Pork	\$ 4,200.00	\$ 1,800.00	\$ 6,000.00			
32		Mystery meat	\$ 1,050.00	\$ 150.00	\$ 1,200.00			
33		Profit	\$ 3,150.00	\$ 5,050.00	\$ 8,200.00			

Note: Write both the formula and the numerical output

A)                      B)                      C)                      D)

How would you write the blending constraints algebraically?



# SCHEDULING LP

**Problem – CUS 24-hr Fast Food:** After enough Sauder students protest their high student fees, the CUS promises to open up a budget 24-hr fast-food restaurant in Henry Angus.

Your restaurant has six labour shifts per 24-hour period, starting at 12am, 4am, 8am, 12pm, 4pm, 8pm, and 12pm. You have access to workers who work two consecutive shifts a day. Due to fluctuations in demand, your required labour at different time periods is as follows:

Shift Time	12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am
Workers Required	3	4	7	8	6	5

Find the scheduling method that will use the minimum amount of workers. The completed LP has been provided to you.

CUS 24-hr Fast Food								
		Time Periods Covered						
		12am	4am	8am	12pm	4pm	8pm	Labour
Start Time	12am	4	4					4
	4am		0	0				0
	8am			7	7			7
	12pm				1	1		1
	4pm					5	5	5
	8pm	0					0	0
	Supply	4	4	7	8	6	5	17
		>=	>=	>=	>=	>=	>=	
Required		3	4	7	8	6	5	



## Questions – Based on the CUS 24-hr Fast Food problem:

Is this a maximizing or minimizing model? What are you trying to maximize or minimize?

How many constraints are there – fill in the table:

	Binding	Non-Binding
Regular		
Non-Negativity		

Answer below questions based on sensitivity report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$I\$5	12am Labour	4	0	1	1	0
\$I\$6	4am Labour	0	0	1	1	0
\$I\$7	8am Labour	7	0	1	0	1
\$I\$8	12pm Labour	1	0	1	1	0
\$I\$9	4pm Labour	5	0	1	0	1
\$I\$10	8pm Labour	0	0	1	1E+30	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$C\$11	Shift 12am	4	0	3	1	1E+30
\$D\$11	Shift 4am	4	1	4	1E+30	1
\$E\$11	Shift 8am	7	0	7	1	0
\$F\$11	Shift 12pm	8	1	8	0	1
\$G\$11	Shift 4pm	6	0	6	1	0
\$H\$11	Shift 8pm	5	1	5	0	1

Due to an overnight frat party, your demand for workers at 12am-4am goes up to four. Will this affect your optimal solution?

Suppose the demand for workers at 4am-8am increases by one. How will this affect the optimal solution?



# TRANSPORTATION LP

**Problem – SkipTheTuition:** In order to pay your super high tuition, which seems to increase every year, you take a side gig as a delivery person for SkipTheDishes.

There are three restaurants you deliver identical ramen from: Kokoro, Danbo, and Kinton. You must deliver these meals to UBC residences, Totem, Orchard, and Vanier, one unit at a time. The supply and demand are as follows:

- Kokoro has 7 bowls, Danbo has 16, and Kinton has 13
- Totem requires 17 bowls, Orchard requires 5, Vanier requires 14

The time it takes to deliver to and from each location are as follows:

	Time (mins)	Deliver to		
		Totem	Orchard	Vanier
Deliver From	Kokoro	4	4.5	2.5
	Danbo	3	4	2.5
	Kinton	3.5	3	2

How can you complete all your deliveries in the least amount of time possible?





The action plan has been completed for you. Based on the information in the prompt, fill in all the remaining unpopulated cells. Include both the formula and the numerical output for each cell using a formula.

	A	B	C	D	E	F	G	H	I
1	SkipTheTuition								
2									
3	Input data								
4			Deliver to						
5	Deliver From	Time (mins)	Totem	Orchard	Vanier				
6		Kokoro							
7		Danbo							
8		Kinton							
9									
10	Action Plan								
11			Deliver to						
12	From	Shipments	Totem	Orchard	Vanier	Shipped	Supply	units	
13		Kokoro	7	0	0				
14		Danbo	2	0	14				
15		Kinton	8	5	0				
16		Received					units		
17		Demand							
18									
19									
20	Total Time								

How many binding constraints are there in total?

What will happen to the LP if suddenly, an explosion happens at Kokoro and three of the seven ramen bowls in stock are destroyed?

What are the objective coefficients in this model?



### Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$13	Kokoro Totem	0	0	4	1E+30	0
\$D\$13	Kokoro Orchard	0	1	4.5	1E+30	1
\$E\$13	Kokoro Vanier	7	0	2.5	0	0.5
\$C\$14	Danbo Totem	16	0	3	1	1E+30
\$D\$14	Danbo Orchard	0	1.5	4	1E+30	1.5
\$E\$14	Danbo Vanier	0	1	2.5	1E+30	1
\$C\$15	Kinton Totem	1	0	3.5	0	1
\$D\$15	Kinton Orchard	5	0	3	1	3.5
\$E\$15	Kinton Vanier	7	0	2	0.5	0

### Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$C\$16	Received Totem	17	4	17	0	1
\$D\$16	Received Orchard	5	3.5	5	0	5
\$E\$16	Received Vanier	14	2.5	14	0	7
\$F\$13	Kokoro Shipped	7	0	7	1E+30	0
\$F\$14	Danbo Shipped	16	-1	16	1	0
\$F\$15	Kinton Shipped	13	-0.5	13	7	0

### Questions – based on sensitivity analysis:

Is there evidence of multiple optima in this LP?

It now takes only 2.5 minutes to deliver from Kinton to Orchard. Will this change the optimal solution? What will be the new value in the target cell?

Suppose Danbo has increased its supply by one. How will this affect the target cell under the optimal solution?

Due to construction around UBC (what's new?), all delivery times to Vanier have been increased by 3. What will be the new optimal solution?

