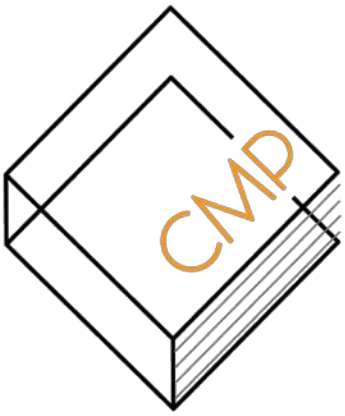


# MATH 104/184

# 2018W1 Midterm 1 Review Package

By N Morishige

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[15] **1. Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Evaluate $\lim_{x \to -1} \dfrac{\sqrt{4x+20}-4}{x+1}$.

Answer:

(b) Evaluate $\lim_{x \to 2} \dfrac{2x-4}{x^2+x-6}$.

Answer:

(c) Suppose $f(x)$ and $g(x)$ are continuous functions for all real numbers and $\lim_{x \to 2} f(x) = -2$, $\lim_{x \to -2} g(x) = 4$, and $\lim_{x \to 2} g(x) = 3$.

Evaluate $\lim_{x \to 2} \dfrac{f(x)}{2(g(x))^2+4}$.

Answer:

(d) Solve for $x$ if $\ln 2^{2x+4} = \ln 5$.

Answer:

(e) Find the inverse function for $f(x) = \dfrac{1}{4x+3}$ and state where it has an inverse. Explain your answer.

Answer:

**Long Problems**. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[10] **2.** Prove that the equation

$$x^3 - x^2 + 2x = 1 - 2\cos x$$

has a solution.

[10] **3.** Use the definition of the derivative as a limit to find $f'(4)$ for the following function. No marks will be given for the use of differentiation rules.

$$f(x) = \frac{x}{2x+5}.$$

[15] **4.** You manufacture chocolate teapots. The demand for your product as a function of price is given by the equation $q(p) = 200 - \sqrt{p}$.

(a) What is your revenue as a function of $p$?

(b) What is your revenue as a function of $q$?

(c) You have have fixed costs of 360000 KPW (North Korean won) and the variable cost of producing teapots is $q^3$. What is your profit as a function of quantity? What are your break-even points?

(d) At what price should you sell your chocolate teapots to make the maximum profit?

[10] **5.** Find the equation of the tangent line to the graph $f(x) = \dfrac{x}{\tan x + 1}$ at $x = 0$.

[10] **6.** Find numbers $a$ and $b$ that makes

$$f(x) = \begin{cases} \ln x + a & \text{if } x > 1 \\ x^2 + x - 2 & \text{if } 0 \leq x \leq 1 \\ 3x^3 - 4b\cos x & \text{if } x < 0 \end{cases}$$

$f(x)$ continuous for all real numbers. With these values, is $f(x)$ differentiable at $x = 0$? Is $f(x)$ differentiable at $x = 0$?