commerce undergraduate society

COMM 290 [SOLUTIONS]

FINAL EXAM REVIEW SESSION

Prepared by Tony Chen
Instructed by Simon Roberts

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Vocabulary Overview

First Half

Objective function – the function describing the problem's objective which you are attempting to maximize or minimize.

Optimal solution – the best set of decisions that maximizes the objective function while remaining within the constraints.

Target Cell – Contains the output of the objective function and is highlighted in green.

Constraint – A limitation of some sort posed with the problem. Always enclosed by a blue border.

Multiple optima – There are multiple sets of optimal solutions.

Feasible region – The region in which all solutions are valid and subject to the constraints.

Infeasible solution – There is no feasible region associated with your LP.

Unbounded solution – The feasible region is infinitely large, usually due to lack of a constraint, and the objective function behaves such that you are moving the isoprofit line outwards indefinitely.

Input Data – The data given to you as part of a problem. Usually highlighted in yellow.

Action Plan – The "action" you will take to solve the problem, which will be indicated inside red borders on excel.

Redundant constraint – A constraint which does not affect the feasible region.

Non-negativity constraint – A constraint which makes sure a "decision" cannot be a negative value.

RHS Allowable Increase/Decrease of a Binding Constraint – Range in which the right-hand-side of the constraint may move without changing the shadow price.

RHS Allowable Increase/Decrease of a Non-Binding Constraint – Range in which the right-hand-side of the constraint may move while keeping the constraint non-binding.

Allowable Increase/Decrease of an objective coefficient – Range in which the objective coefficient may move without changing the optimal solution.

Shadow Price – The increase in the value of the target cell for every one-unit increase of the RHS of a constraint.

Relative Reference – A reference in the form A1 that will change when auto-filled to other cells.

Absolute Reference – A reference in the form $A$1 that will not change when auto-filled to other cells.

Second Half

Probability Tree – Tree containing all possible outcomes of a probability problem.

Sample Space – All the possible outcomes.

Probability – The chance by which something will happen.

Independent – Knowing something about one outcome does not affect another.

Dependent – Knowing something about one outcome affects another.

Mutually Exclusive – Two outcomes are **mutually exclusive** if they cannot both occur at the same time.

Expected Monetary Value – The expected value of all monetary payoffs

Optimistic Decision Approach – Optimistic approach highlights the best payoff under any decision, and selects the decision with the maximum highest payout.

Maximin Conservative Approach – Conservative approach highlights the worst payoff under any decision, and selects the decision with the best worst-case payout.

Minimax Regret Approach – Regret approach calculates the difference between each outcome and the best outcome under each state. Then, select the decision that has the least worst-case regret.

Expected Value of Sample Information – The amount of profit gained by knowing another related state before making a decision.

Expected Value of Perfect Information – The amount of profit gained by knowing the state before making a decision.

Efficiency of Information – The % of EVPI extracted using sample information.

Expected Value – The average outcome of a random variable.

Variance – A measurement of how much a variable varies.

Standard Deviation – How much a variable will normally vary relative to the mean. Often used with the ± sign in combination with the mean. More about that in COMM 291…

Solving Algebraically + Graphically

Problem:

You are the manager of a fast-food restaurant, serving fries and burgers. Each burger results in \$2.50 of profit, while each order of fries results in \$1.50 of profit. Each burger requires 5 minutes of cooking and 2 minutes of packing, while each order of fries requires 3 minutes of cooking and 3 minutes of packing. You have 360 minutes for cooking and 220 minutes for packing. Due to demand, you must produce at least 30 of both burgers and fries.

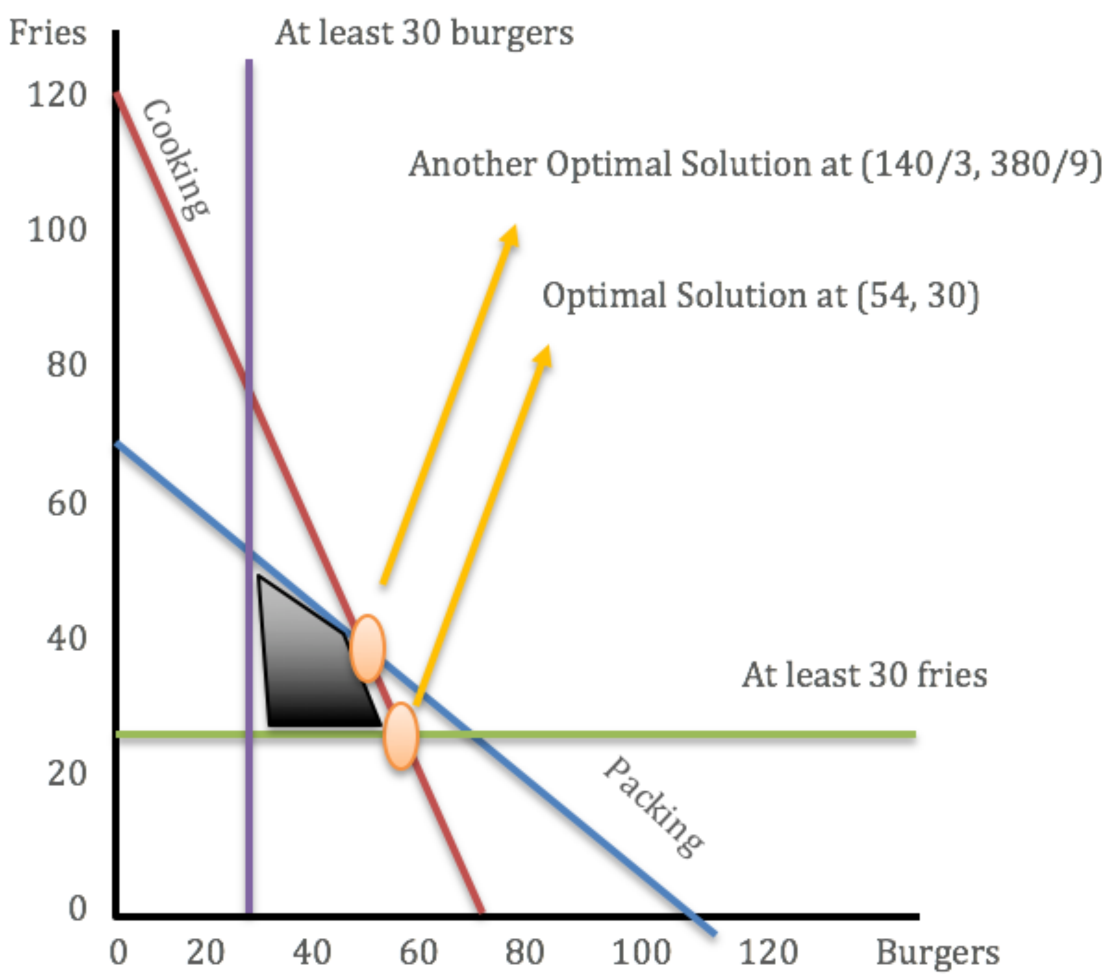
1. How many non-negativity constraints are there? What about total constraints?

   There is a total of two non-negativity constraints (1 for each variable) and six total constraints.

2. Is this a maximizing or minimizing model?

   Since this model has to do with profit and not cost, it must be a maximizing model.

3. Complete the algebraic formulation of this model, and produce a graph labelling the correct optimal solution. Also label the feasible region.



Max 2.5B + 1.5F

Subject to:

B >= 0

F >= 0

5B + 3F <= 360 (Cooling)   2B + 3F <= 220 (Packing)

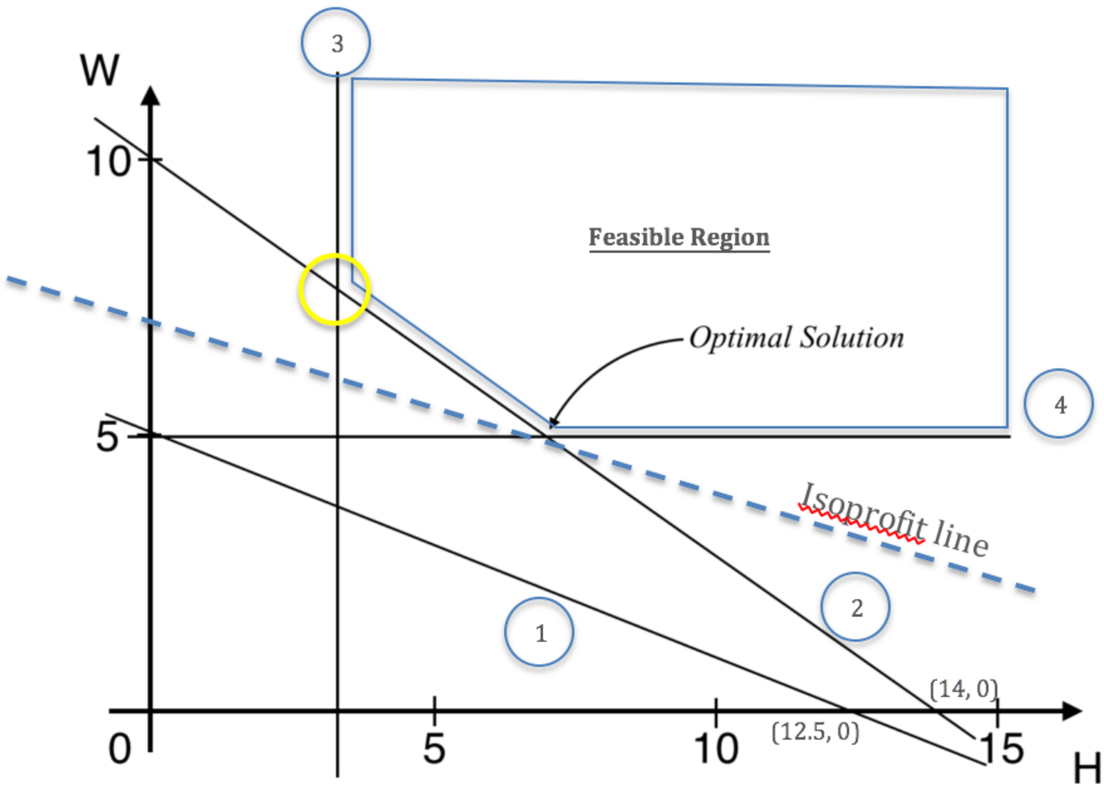
B >= 30 (minimum)   F >= 30 (minimum)

B >= 0 (non-negativity)   F >= 0 (non-negativity)

Understanding Graphs

Problem:

Consider the following modified graph for Pet Troodon found on the Sample Midterm Exam and answer the following questions.



1. Given the feasible region, and assuming both objective coefficients are positive, is this LP a maximizing or minimizing model?

   The LP must be a minimizing model because otherwise, the LP will be unbounded.

2. Which constraints are binding, and which are non-binding?

   Constraints 2 and 4 are binding, while the other ones are not.

3. What is one sample objective function that will lead to multiple optima at the current optimal solution **and** at the point labelled with the yellow circle?

   This objective function must have slope equal to that of constraint 2, or -5/7. Therefore, any objective function in the form of min xH + yW where –x/y = -5/7 would be correct.

4. Given an objective function of 6H + 10W, calculate by hand the shadow price of constraint 1's right-hand-side. Do the same for constraint 3.

   Both shadow prices are 0, since both constraints are non-binding.

5. Suppose due to an unexpected error, the sign of constraint 3 has been switched. Label the new optimal solution and the new feasible region.

   The yellow circle coincidentally labels the new optimal solution, while the purple polygon labels the new feasible region. The new optimal solution is at the intersection between constraint 2 and constraint 3.

6. Suppose that this model goes from a minimizing model to a maximizing model. What can you say about the optimal solution and the LP?

   As stated in Question 2, the LP will become unbounded, and there will be **no** optimal solution.

Interpreting an LP

Problem:

Consider this completed excel model for a modified version of Nationland Power and answer the following questions.

	A	B	C	D	E	F	G	H
1	**Nationland Power**							
2								
3	**Input Data**							
4			Solar	Wind	Diesel	Total		Constraint
5		CO2	50	7.5	735	34.82	<=	80
6		Domestic	0.95	0.25	1	0.70		
7		Social	0.05	0.01	0.4	0.04	>=	0.03
8						Total		
9		Profit	$ 0.08	$ 0.13	$ 0.05	$ 0.09786		
10								
11								
12	**Action Plan**							
13			Solar	Wind	Diesel	Total		Constraint
14		Proportion	0.6429	0.3571	0.0000	1	=	1
15								
16	**Additional Constraints**							
17				Output		Requirement		
18		At least 70% Domestic		70%	>=	70%		
19		At most 84% Domestic		70%	<=	84%		

1. What is the objective function? How many variables are there?

   The objective function is max 0.08S + 0.13W + 0.05D, the **three** variables being S W and D.

2. How many are binding constraints are there?

   There is a total of 3 binding constraints.

3. What are the best equations for the cells F5, F14 and D18?

   F5: =SUMPRODUCT(C5:E5, $C$14:$E$14)

   F14: =SUM(C14:E14)

   D18: =F6

4. What is the optimal solution? What is the value of the target cell?

   The optimal solution is to produce 0.64 KWh of Solar and 0.36 KWh of Wind per KWh produced. The value of the target cell is $0.10, which represents cost per KWh.

5. Suppose the value in cell H5 decreased to 40. Will this change the optimal solution? Why or why not?

   This can be done without using sensitivity analysis. Since the current output value is 34.82, it is still within the constraint even if the 80 changes to a 40. Therefore, the constraint remains non-binding and will not change the optimal solution.

6. Suppose the value in cell H7 increased to 0.04. Will this change the optimal solution? Why or why not? What if it changed to 0.041?

   If it changed to 0.04, it will not change the optimal solution since it follows the same explanation as the example in Question 5.

Sensitivity Analysis

Problem:

Given the attached sensitivity analysis of the modified version of Nationland Power with many cells blacked out, answer the following questions.

Microsoft Excel 15.15 Sensitivity Report
Worksheet: [Final Exam Review Spreadsheet.xlsx]Nationland Power
Report Created: 2016-12-03 5:03:50 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$14	Proportion Solar	0.642857143	0	[redacted]	0.05	0.024666667
$D$14	Proportion Wind	0.357142857	0	[redacted]	0.37	0.05
$E$14	Proportion Diesel	0	-0.026428571	[redacted]	0.026428571	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$18	At least 70% Domestic Output	[redacted]	-0.071428571	0.7	0.14	0.1
$D$19	At most 84% Domestic Output	0.7	[redacted]	0.84	[redacted]	[redacted]
$F$14	Proportion Total	1	0.147857143	1	1.333333333	0.263157895
$F$5	CO2 Total	34.82142857	[redacted]	80	[redacted]	[redacted]
$F$7	Social Total	0.035714286	[redacted]	0.03	[redacted]	[redacted]

1. What are the values within the cells F9, D16, E17, G17, H17, G19 and G20?

F9: 0.08 | D16: 0.7

E17: 0 | G17: 1E+30

G19: 1E+30 | G20: 0.005714

2. Suppose you must now get 75% of your electricity supply from domestic sources. By how much will this decrease your profit?

   The constraint that is affected is "At least 70% domestic," where the allowable increase is 0.14 and the shadow price is -0.071428. This will change your total profit by -0.071428 * 0.05, resulting in approximately 0.003 in lost profit.

3. Suppose you now must get at most 80% of your electricity domestically. By how much will this change the amount of wind energy used?

   This will not change any part of the solution, as the 0.04 decrease **is** within the allowable decrease and the constraint **remains** non-binding.

4. Is there evidence of multiple optima? Why or why not?

   There is no evidence of multiple optima, as there are no objective coefficients with an allowable change of 0.

5. What's the highest that the profit of wind power could go without influencing any change in the optimal solution? What about the profit of solar power?

   The allowable increase of that coefficient is 0.37, while it is currently 0.13. Therefore, the highest this number could go is 0.5. For solar power, the allowable increase is 0.05

and the value is currently 0.08. Therefore, the highest it could go is 0.13.

6. Suppose you must now dedicate $0.035 cents as opposed to $0.030 to social programs. Will this induce any change in the LP?

   This will not change the LP in any way as this change is within the allowable increase and the constraint is non-binding.

Blending Problem

Problem:

The following is the completed model and sensitivity analysis for the Lucky Strike model you discussed in class, and please use those to answer the following questions.

	A	B	C	D	E	F	G	H
1	**Lucky Strike**							
2								
3	**Input Data**							
4		Cost per litre ($)	Amount Available					
5	OilA	0.32	500000		Selling Price Regular	$ 0.42	litre	
6	OilB	0.38	275000		Selling Price Ultra	$ 0.51	litre	
7	OilC	0.34	425000					
8								
9	*Regular gas must be at least 45% OilA*				0.45			
10	*Regular gas must be at most 25% oilb*				0.25			
11	*Ultra Gas must be at most 35% OilC*				0.35			
12	*Ultra Gas must be at least 35% OilB*				0.35			
13								
14	**Action Plan**				Amount		Amount	
15			Reg	Ultra	Used		Available	
16		OilA	186428.6	313571.4	500000	<=	500000	*litres*
17		OilB	0	275000	275000	<=	275000	*litres*
18		OilC	227857.1	197142.9	425000	<=	425000	*litres*
19		Total	414285.7	785714.3				
20								
21	**Blending Constraints**				Model Output		Model Req	
22	***Regular*** *gas must be at* ***least*** *45% OilA*				186428.5714	>=	186428.57	
23	***Regular*** *gas must be at* ***most*** *25% oilb*				0	<=	103571.43	
24	***Ultra*** *Gas must be at* ***most*** *35% OilC*				197142.8571	<=	275000	
25	***Ultra*** *Gas must be at* ***least*** *35% OilB*				275000	>=	275000	
26								
27								
28								
29								
30	**Rev Cost Info**							
31			**Reg**	**Ultra**	**Total**			
32		**Rev**	$174,000.00	$400,714.29	$ 574,714.29			
33		**Costs**						
34		OilA	$ 59,657.14	$100,342.86	$ 160,000.00			
35		OilB	$ -	$104,500.00	$ 104,500.00			
36		OilC	$ 77,471.43	$ 67,028.57	$ 144,500.00			
37		**Profit**	$ 36,871.43	$128,842.86	$ 165,714.29			

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [Lucky.ST.2016W.rev.5B.L101.xlsx]Model
Report Created: 05/10/2016 9:34:20 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$16	OilA Reg	186428.5714	0	0.1	0	0.177777778
$D$16	OilA Ultra	313571.4286	0	0.19	0.177777778	0
$C$17	OilB Reg	0	-0.257142857	0.04	0.257142857	1E+30
$D$17	OilB Ultra	275000	0	0.13	1E+30	0.257142857
$C$18	OilC Reg	227857.1429	0	0.08	0.163636364	0
$D$18	OilC Ultra	197142.8571	0	0.17	0	0.163636364

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$19	OilA Used	500000	0.1	500000	358441.5584	141558.4416
$E$17	OilB Used	275000	0.297142857	275000	223076.9231	193006.993
$E$18	OilC Used	425000	0.08	425000	173015.873	414285.7143
$E$22	Regular gas must be at least 45% OilA Model Out	186428.5714	-5.82867E-16	0	77857.14286	186428.5714
$E$23	Regular gas must be at most 25% oilb Model Out	0	0	0	1E+30	103571.4286
$E$24	Ultra Gas must be at most 35% OilC Model Outp	197142.8571	0	0	1E+30	77857.14286
$E$25	Ultra Gas must be at least 35% OilB Model Outp	275000	-0.257142857	0	125454.5455	136250

1. What are the best formulas for cells E16, E22, G22 and E37?
   E16: =SUM(C16:D16)

   E22: =C16

   G22: =E9*C19

   E37: =SUM(C37:D37) OR E32 – SUM(E34:E36)
2. How many variables are in this problem? How many constraints are there? How many constraints are binding?

   There are six variables, 13 constraints (6 non-negativity), 6 of which are binding.

3. Why is the RHS for the blending constraints 0? Explain using algebraic formulation.

   When formulated algebraically, the constraint looks like this, which is in linear form:
   AR >= 0.45(AR + BR + CR)

   However, Excel's simplex LP will move all variables to the left hand side, resulting in the following. Please note that both forms are absolutely correct for algebraic formulation.
   0.55 AR – 0.45BR – 0.45CR >= 0

4. How much should you pay for each additional liter of OilC?

   Since the shadow price of the OilC used constraint is 0.08, you can expect to make an additional 0.08 profit for every unit of OilC available. Therefore, you should be willing to pay anything less than 0.08 per liter of OilC.

5. Is there evidence of multiple optima?

   There is evidence of multiple optima due to a variable cell (coefficient) allowable change of 0.

6. What is the total profit from OilC products?

   Total profit generated by OilC is 227857.14 * 0.08 + 197142.86 * 0.17 = 51,742.85 (see objective coefficients for OilC products).

7. Suppose the cost of OilA increased by $0.06. By how much will this change the target cell?

   We cannot predict the behavior of this change as this changes multiple objective coefficients: OilA Regular and OilA Ultra.

8. Suppose you can now sell regular oil for $0.05 more. By how much will this change the target cell?

   Similar to Question 7, this changes multiple objective coefficients (OilA Regular, OilB Regular and OilC Regular) and thus we do not know what will be the change in the target cell.

9. Suppose you have access to 30,000 additional units of OilA. What will be the new value of target cell?

   Every unit of OilA is worth an additional 0.1 of profit, and the increase of 30,000 is within allowable increase of 358,000. Therefore, the new value of the target cell will be 165,714 + 30,000*0.1 = 168,714.

10. Suppose you have access to up to 200,000 additional units of OilC at the same cost of 0.34 per litre (In other words, the value in cell G18 is now 625,000). How can you best describe the change in the target cell?

    The change in 200,000 is **outside** of the allowable increase of 173,015. However, since the more oil you have the more profit you are intuitively able to make, we can conclude that this will increase our profits by **at least** 173,015*0.08 = 13,841.

Scheduling Problem

Problem:

Given the attached solution to a modified version of the Gotham City National Bank LP, answer the following questions.

	A	B	C	D	E	F	G	H	I	J
1	**Gotham City National Bank data**									
2	Source: Winston SMA problem 4.86									
3										
4	**Input Data**									
5										
6	**Time**	9am-10am	10am-11am	11am-Noon	Noon-1pm	1pm-2pm	2pm-3pm	3pm-4pm	4pm-5pm	**#**
7										**Employees**
8	Requirement									
9	FT1	4	4	4	4		4	4	4	4
10	FT2	2	2	2		2	2	2	2	2
11	PT1	0	0	0						0
12	PT2		2	2	2					2
13	PT3			1	1	1				1
14	PT4				0	0	0			0
15	PT5					0	0	0		0
16	PT6						1	1	1	1
17										
18	Supply	6	8	9	7	3	7	7	7	
19		>=	>=	>=	>=	>=	>=	>=	>=	
20	**Demand**	6	7	3	7	3	6	7	5	
21										
22	**Other constraints**						Model		Model	
23							Output		Requirement	
24	At most 4 part-time workers hired						4	<=	4	
25	# FT workers whose lunch is from 1:00 to 2:00 must = 4						4	=	4	
26										
27	**Cost Issues**									
28										
29	Total Cost Full Time			$ 672.00						
30	Total Cost Part Time			$ 108.00						
31	Total Cost			$ 780.00						

	A	B	C	D	E	F	G	H
1	**Microsoft Excel 15.15 Sensitivity Report**							
2	**Worksheet: [Final Exam Review Spreadsheet.xlsx]Gotham**							
3	**Report Created: 2016-12-03 6:09:27 PM**							
4								
5								
6	Variable Cells							
7				**Final**	**Reduced**	**Objective**	**Allowable**	**Allowable**
8		**Cell**	**Name**	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
9		$J$9	FT1 Employees	4	0	112	1E+30	1E+30
10		$J$10	FT2 Employees	2	0	112	1E+30	58
11		$J$11	PT1 Employees	0	0	27	58	112
12		$J$12	PT2 Employees	2	0	27	0	0
13		$J$13	PT3 Employees	1	0	27	0	0
14		$J$14	PT4 Employees	0	0	27	1E+30	0
15		$J$15	PT5 Employees	0	0	27	1E+30	0
16		$J$16	PT6 Employees	1	0	27	0	112
17								
18	Constraints							
19				**Final**	**Shadow**	**Constraint**	**Allowable**	**Allowable**
20		**Cell**	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
21		$B$18	Supply 9am-10am	6	56	6	2	0
22		$C$18	Supply 10am-11am	8	0	7	1	1E+30
23		$D$18	Supply 11am-Noon	9	0	3	6	1E+30
24		$E$18	Supply Noon-1pm	7	56	7	0	0.666666667
25		$F$18	Supply 1pm-2pm	3	0	3	1	1
26		$G$18	Supply 2pm-3pm	7	0	6	1	1E+30
27		$H$18	Supply 3pm-4pm	7	56	7	0	1
28		$I$18	Supply 4pm-5pm	7	0	5	2	1E+30
29		$G$24	At most 4 part-time workers hired Output	4	-29	4	2	0
30		$G$25	# FT workers whose lunch is from 1:00 to 2:00 must = 4 Output	4	-56	4	0.4	0

1. How many variables are in this problem? How many constraints are binding?

   There are 8 variables 9 binding constraints.

2. What is the optimal solution, and what is the total cost at that optimal solution?

   The optimal solution is to hire 4 full time employees who take breaks at 1pm – 2pm, hire 2 full time employees who take breaks at noon, and 2, 1, 1 part-time employees starting at 10am, 11am and 2pm, respectively. Under the optimal solution, total cost is 780.

3. How much are full-time and part-time workers paid by the hour? There are two ways to solve this. (Hint: they have been modified from the original version!)

   Method one: Looking at cost issues, full time workers cost Gotham 672. Divided among six full-time workers results in 112 per worker. Since each worker is paid for 8 hours, each worker is paid 14 per hour.

   Method two: The object coefficient for a full time employee is 112 (cost per employee). Each employee works for 8 hours, resulting in a 14 hourly wage.

4. Suppose Gotham may now hire six part-time workers. How much money will this change save? What if they can employ seven workers?

   The increase by two is within the allowable increase, and will save $58 for Gotham. If they can employ a seventh, the change in cost is unknown but intuitively, it should be no higher than the cost with six part-time workers due to part-time workers having a lower hourly wage..

5. Does this LP have evidence of multiple optima?

   Yes, there is evidence of multiple optima.

6. Suppose due to an influx in demand, you must now employ two additional workers for the 9am-10am shift. By how much will this change the target cell?

   The shadow price of that time slot is 56, and the change is within the allowable increase. Consequently, this will increase the target cell by 112.

7. Suppose due to an increase in demand, you now require 8 workers for the 11am-noon shift. By how much will this change the target cell?

   This will not change the target cell as the constraint remains non-binding.

Transportation Problem

Problem:

Given the attached solution to a modified version of the Powerco LP, answer the following questions.

	A	B	C	D	E	F	G	H	I
1	**Power Co**								
2									
3	**Input Data**								
4									
5		**Costs ($)**	City1	City2	City3	City4			
6		Plant1	$ 7.00	$ 6.00	$10.00	$ 7.00			
7		Plant2	$ 9.00	$10.00	$10.00	$ 4.00			
8		Plant3	$ 12.00	$ 9.00	$12.00	$ 5.00			
9									
10	**Action Plan**								
11					To		Amount		
12			City1	City2	City3	City4	Sent		**Supply**
13		Plant1	25	10	0	0	35	<=	35
14	From	Plant2	20	0	30	0	50	<=	50
15		Plant3	0	10	0	30	40	<=	40
16		Received	45	20	30	30			
17			>=	>=	>=	>=			
18		Demand	45	20	30	30			
19									
20	Total Cost		$ 955.00						

Microsoft Excel 15.15 Sensitivity Report
Worksheet: [Final Exam Review Spreadsheet.xlsx]Powerco
Report Created: 2016-12-03 6:36:53 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$13	Plant1 City1	25	0	7	1	0
$D$13	Plant1 City2	10	0	6	0	1
$E$13	Plant1 City3	0	2	10	1E+30	2
$F$13	Plant1 City4	0	5	7	1E+30	5
$C$14	Plant2 City1	20	0	9	0	1
$D$14	Plant2 City2	0	2	10	1E+30	2
$E$14	Plant2 City3	30	0	10	1	11
$F$14	Plant2 City4	0	0	4	1E+30	0
$C$15	Plant3 City1	0	2	12	1E+30	2
$D$15	Plant3 City2	10	0	9	1	0
$E$15	Plant3 City3	0	1	12	1E+30	1
$F$15	Plant3 City4	30	0	5	0	5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$16	Received City1	45	10	45	0	10
$D$16	Received City2	20	9	20	0	10
$E$16	Received City3	30	11	30	0	10
$F$16	Received City4	30	5	30	0	30
$G$13	Plant1 Sent	35	-3	35	10	0
$G$14	Plant2 Sent	50	-1	50	10	0
$G$15	Plant3 Sent	40	0	40	1E+30	0

1. How many variables are in this problem? How many constraints are there? How many constraints are binding?

   There are 12 variables, 19 constraints, 13 of which are binding.

2. What is the cost of electricity at the optimal solution?

   At the optimal solution, total electricity costs are $955.

3. Suppose due to unforeseen circumstances, City2 requires 5 additional units of electricity. By how much will the target cell change?

   The LP will become infeasible since it will make it impossible for supply to meet demand.

4. Suppose all shipments from Plant1 increased in price by $4. How will this change the target cell?

   We cannot determine the change, as it changes four objective coefficients while sensitivity analysis can only determine what happens when **one** variable is changed.

5. City1 has funded a new electricity plant, and now only requires 30 units to be shipped. How will this change the target cell?

The shadow price of City1 Received is 10, while the decrease of 15 is outside the allowable decrease of 10. However, intuitively the less units you ship, the cheaper it is. Consequently, the target cell will go down by **at least $100**.

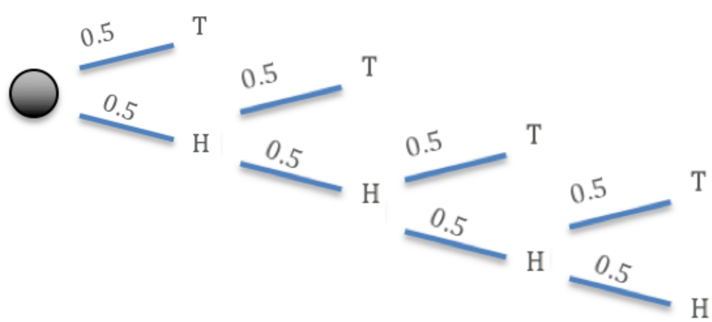
End of First Half – Break time!

Probability Trees

Problem:

You are playing a game, where you will flip 4 coins. The game ends when you flip tails. Answer the following questions.

1. Draw a probability tree for the given scenario. Be sure to include the sample space and probability of each outcome.



Sample Space	Probability
T	0.5
HT	0.25
HHT	0.125
HHHT	0.0625
HHHH	0.0625

2. What is the probability that you will get exactly two heads? What about exactly three heads?

   You will get two heads with 0.125 probability, and you will get three with 0.0625 probability.

3. After flipping one coin, which ended up being heads, what is the probability that your final outcome will be four heads?

   We are looking for P(HHHH | H) = P(HHHH) / P(H) = 0.0625/0.5 = 0.125
   (P(HHHH) = P(HHHH AND H) since you need H on first toss.)

4. You play a game where you win $1 if you get two or more heads from four flips. What is the probability that you will win $1?

   You want P(HHT OR HHHT OR HHHH) which is represented by the sum of all three probabilities as the outcomes are mutually exclusive.

   Therefore, the answer is 0.0125 + 0.0625 + 0.0625 = 0.25.

5. Fun bonus question: assume the game goes on indefinitely until you flip tails. Write an algebraic expression describing the probability of an x amount of heads over the course of a game (e.g. x = 3 if you flip 3 heads before the game ends). Check to see if your expression works for the possible outcomes in the game involving four flips (do not test with the HHHH case as there is no tails).

   The probability of an x amount heads is $0.5 * 0.5^x$, or $0.5^{(x+1)}$, since there is a 0.5 chance of getting heads on every flip. However, you want to know how many heads you get over the course of a game, and recall that the ending condition is flipping tails. Thus, you must flip tails at the end of the sequence, which happens with 0.5 chance.

Decisions and EMV

Problem:

You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up The payoff matrix is provided and use it to answer the following questions.

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

1. Which decision will you make if under the optimistic approach? What about conservative?

   Optimistic: under the optimistic approach, the highest outcome of all three options are 30, 10 and 30, respectively. Therefore, you will choose to either buy stocks or short stocks.

   Conservative: under the conservative approach, the worst outcome of all three options are -20, 10 and -40, respectively. Consequently, you will choose to buy bonds.

2. Construct a regret matrix. Which decision will you make if you are using the regret approach?

	Down (0.2)	Neutral (0.5)	Up (0.3)	Maximum
Buy Stocks	50	10	0	50
Buy Bonds	20	0	20	20
Short Stocks	0	10	70	70

You will choose to **buy bonds** because it holds the lowest highest possible regret.

3. Which decision should you make if you are using the EMV (Expected Monetary Value) approach?

Buy Stocks: EMV = -20*0.2 + 0*0.5 + 30*0.3 = 5

Buy Bonds: 10 (constant payout)

Short Stocks: EMV = 30*0.2 + 0*0.5 – 40*0.3 = -6

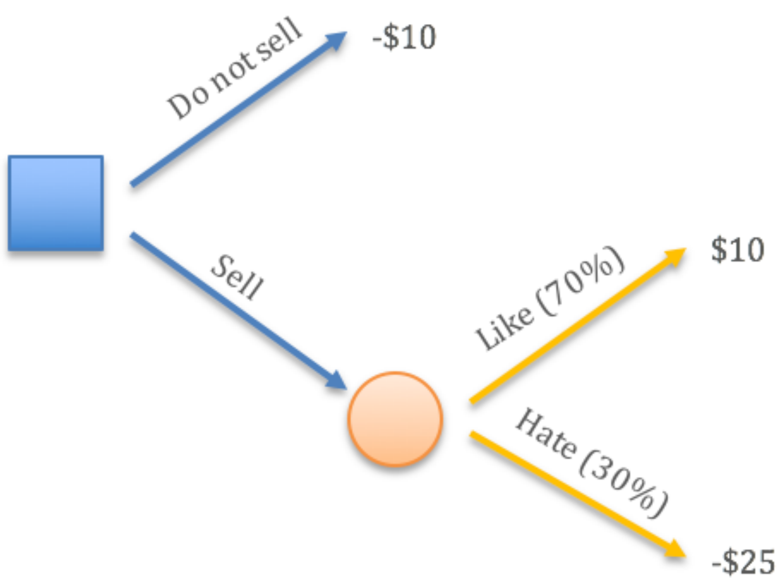
You will choose to **buy bonds**.

Value of Information

Problem:

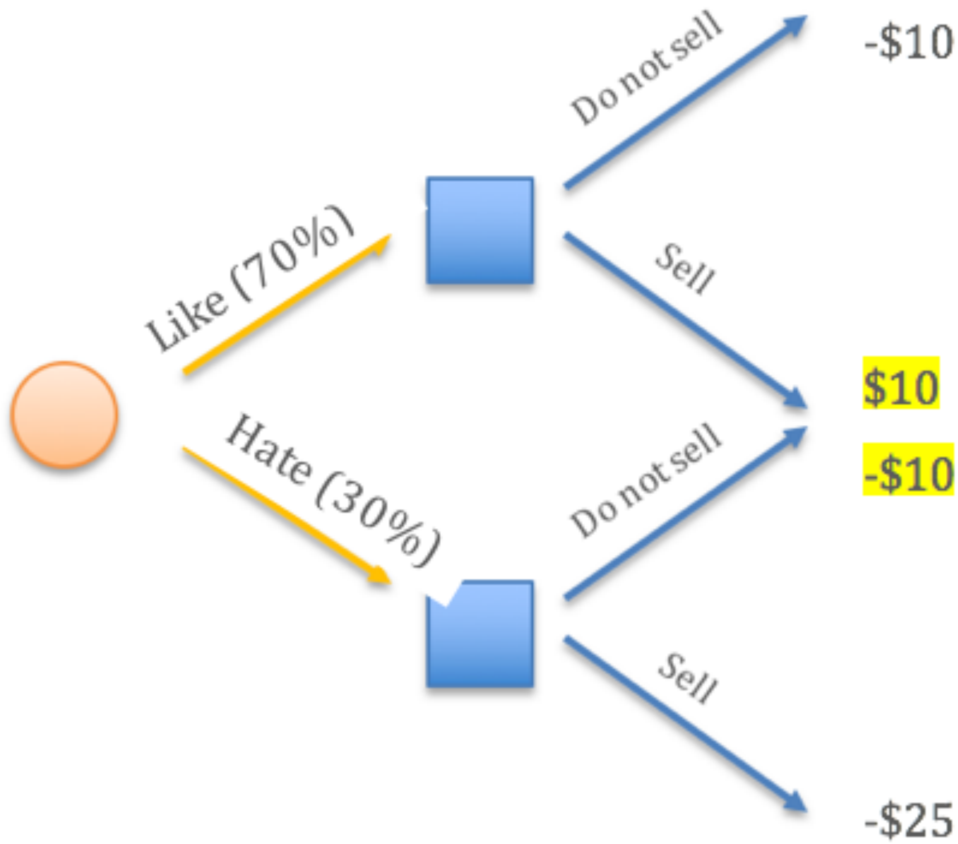
You are selling movie tickets for the upcoming Assassin's Creed movie. You are unsure whether or not people will like the movie or not: forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.

1. Draw a probability tree; under the EMV approach, what decision will you make?



The EMV for selling is 10*0.7 – 25*0.3 = -$0.5. Consequently, you will sell because the EMV is higher than the loss you take by not selling (-$10).

2. You can foresee the future at a cost. How much will you be willing to pay in order to foresee the future? Draw the probability tree associated with this scenario.



You know that if people like the movie, you will sell, and if people do not like the movie you will not sell. Therefore, your EPPI (Expected Profit of Perfect Information) is \$10*0.7 - \$10*0.3 = \$10*0.4 = \$4. Compared to the old EMV of -\$0.5, your value of perfect information is EPPI – EMV = \$4.5. Therefore, you should be willing to pay up to \$4.5 in order to tell the future.

3. Suppose you get additional information that if people liked the Warcraft movie, there will be an 0.9 chance that people will like the Assassin's Creed movie, while if people did not like the Warcraft movie, people will only like the Assassin's Creed movie with a 0.4 chance. Construct a new probability tree, and find the EVSI (Expected Value of Sample Information).

   Let LW and HW denote if people like Warcraft or not, and LA and HA denote if people like assassins or not.

   P(LA | LW) = 0.9 P(LA | HW) = 0.4

   P(LA and LW) = 0.9 * P(LW) P(LA and HW) = 0.4 * P(HW)

	LA	HA	Total
LW	0.9 * P(LW)	0.1 * P(LW)	P(LW)
HW	0.4 * P(HW)	0.6 * P(HW)	P(HW)
Total	0.7	0.3	1

   Solve this system:

   0.9 * P(LW) + 0.4 * P(HW) = 0.7 (1)

   0.1 * P(LW) + 0.6 * P(HW) = 0.3 (2)

   From (2): P(LW) + 6 * P(HW) = 3

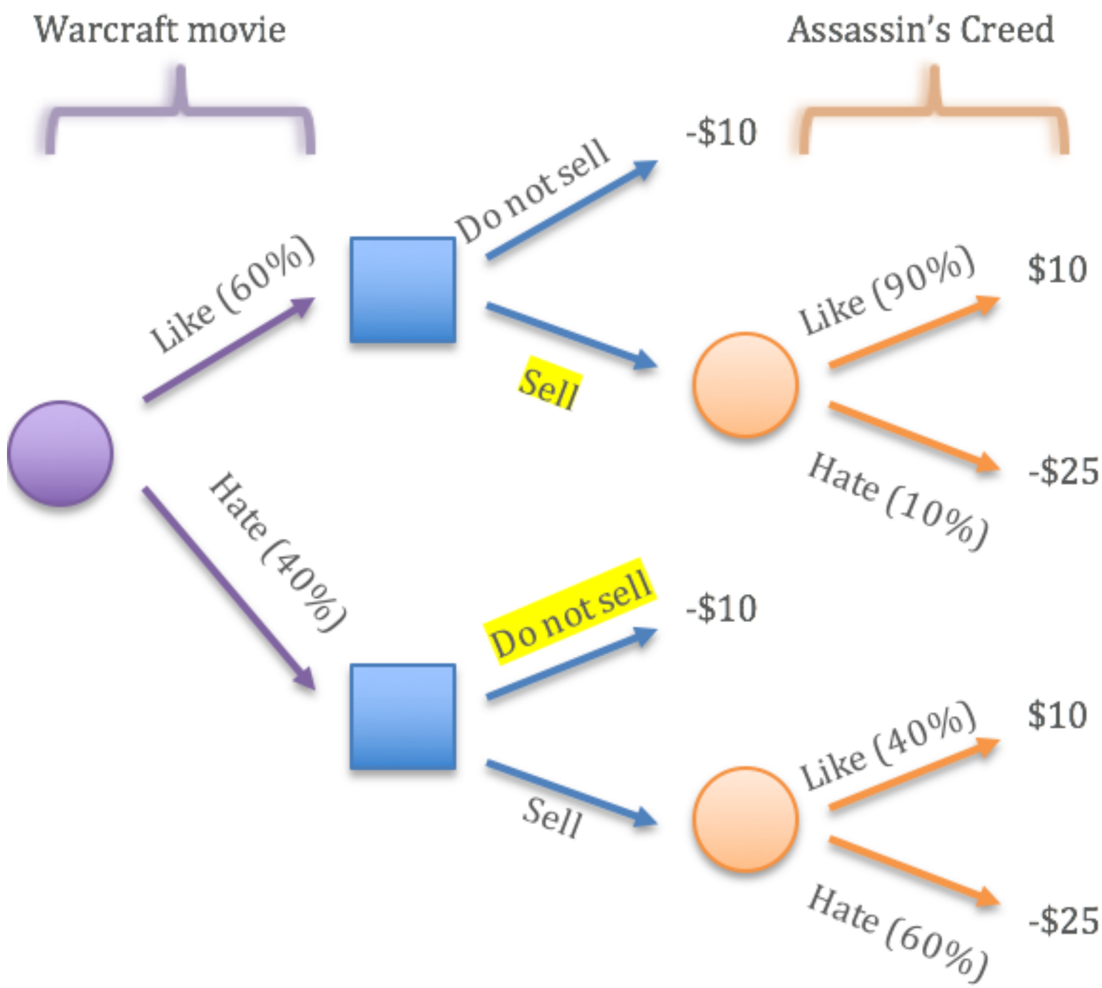
   P(LW) = 3 – 6 * P(HW)

   Plug into (1): 0.9 * (3 – 6 * P(HW)) + 0.4 * P(HW) = 0.7

   2.7 – 5.4 * P(HW) + 0.4 * P(HW) = 0.7

   5 * P(HW) = 2

   P(HW) = 0.4 P(LW) = 0.6



First, calculate the EMV of the two outcomes. If people like the Warcraft movie, the EMV for Assassin's Creed ticket sales is $10*0.9 - $25*0.1 which is equal to $6.5, and you will buy rather than not sell and get -$10. If people hate the Warcraft movie, the EMV for Assassin's Creed ticket sales is $10*0.4 - $25*0.6 which is equal to -$11, and you will not sell because you would rather get -$10. Your total EMV for the entire scenario will therefore be $6.5*0.6 - $10*0.4 = -$0.1, which is also your EPSI (Expected Profit with Sample Information). EVSI = EPSI – EMV = -$0.1 - -$0.5 = -$0.1 + 0.5 = $0.4.

Random Variables

Problem:

Consider the following probability distribution and answer the following questions.

X	0	1	2	3	4
P(X)	0.3	0.15	0.25	0.2	0.1

1. Find expected value, variance, and standard deviation.

   E(X) = 0.3*0 + 0.15*1 + 0.25*2 + 0.2*3 + 0.1*4 = 1.65

   VAR(X) = 0.3*(0 – 1.65)2 + 0.15*(1 – 1.65)2 + 0.25*(2 – 1.65)2 + 0.2*(3 – 1.65)2 + 0.1*(4 – 1.65)2 = 1.8275

   STDEV(X) = sqrt(VAR(X)) = 1.351

2. Find P(2 OR 3) and P(0 OR 2).

   P(2 OR 3) = P(2) + P(3) = 0.25 + 0.2 = 0.45

   P(0 OR 2) = P(0) + P(2) = 0.3 + 0.25 = 0.55

3. You pick two numbers at random **with** replacement. Find $P(1^{st} = 0 \text{ AND } 2^{nd} = 4)$ and $P(2^{nd} = 1 \mid 1^{st} = 4)$.

   $P(1^{st} = 0 \text{ AND } 2^{nd} = 4) = P(0) * P(4) = 0.3 * 0.1 = 0.03$

   $P(2^{nd} = 1 \mid 1^{st} = 4) = P(1) = 0.15$

4. You pick two numbers at random **without** replacement. Find $P(1^{st} = 0 \text{ AND } 2^{nd} = 4)$ and $P(2^{nd} = 1 \mid 1^{st} = 4)$.

   $P(1^{st} = 0 \text{ AND } 2^{nd} = 4) = P(0) * P(4 \mid 0) = 0.3 * 0.1/(1 – P(0) = 0.3 * 0.1/0.7 = 0.042$

   $P(2^{nd} = 1 \mid 1^{st} = 4) = 0.15 / (1 – P(4)) = 0.15 / 0.9 = 0.167$

5. Suppose you will take whatever number you pick out, and construct a square with that given number as the side length. What is the expected value, variance, and standard deviation of the area of your square? (Hint: draw a new probability distribution)

Y	0	1	4	9	16
P(Y)	0.3	0.15	0.25	0.2	0.1

$E(Y) = 0.3*0 + 0.15*1 + 0.25*4 + 0.2*9 + 0.1*16 = 4.55$

VAR(Y) = $0.3*(0 - 4.55)^2 + 0.15*(1 - 4.55)^2 + 0.25*(4 - 4.55)^2 + 0.2*(9 - 4.55)^2 + 0.1*(16 - 4.55)^2 = 25.2475$

STDEV(Y) = sqrt(VAR(X)) = 5.0247

6. **This question is unrelated to the previous scenario.**
   Given $E(X^2) = 20$ and $E(X) = 5$, find VAR(X) and STDEV(X).

   Note that VAR(X) = $E(X^2) - E(X)^2 = 20 - 25 = 5$ (abs)

   STDEV(X) = sqrt(5) = 2.236

Independent Bi-Variable Problem

Problem:

Given the following probability distribution for a closed auction, answer the following questions. Assume the two players' strategies are independent. Complete the table before answering any questions.

		Player 1 (A)			
		Bid $1	Bid $3	Bid $5	
Player 2 (B)	Bid $2	0.1	0.18	0.12	0.4
	Bid $4	0.1	0.18	0.12	0.4
	Bid $6	0.05	0.09	0.06	0.2
		0.25	0.45	0.3	

1. What is the probability that Player 1 wins? What is the probability that Player 2 wins?

   The probability that Player 1 wins is equal to the sum of all probabilities in the cells highlighted in yellow, which adds up to 0.42. Since all the outcomes are mutually exclusive, you may add them to find the probability that one of them happens.

The probability that Player 2 wins is 1 – P(Player 1 wins) = 1 – 0.42 = 0.58. Since there will never be a situation where two bids are equal, one player must win.

2. Find the expected value of (A + (-B)). What does this mean?

(A) = 0.25*1 + 0.45*3 + 0.3*5 = 3.1

E(-B) = E(B) * -1 = (0.4*2 + 0.4*4 + 0.2*6) * -1 = -3.6

E(A + (-B)) = E(A) + E(-B) = -0.5. This shows that on average, Player B has a bid that is 0.5 larger than that of Player A, and on average Player B will win.

3. Find the expected value, variance, and standard deviation of the following:
    a. The winning bid

		Player 1 (A)			
		Bid $1	**Bid $3**	**Bid $5**	
Player 2 (B)	**Bid $2**	$2 wins 0.1	$3 wins 0.18	$5 wins 0.12	0.4
	Bid $4	$4 wins 0.1	$4 wins 0.18	$5 wins 0.12	0.4
	Bid $6	$6 wins 0.05	$6 wins 0.09	$6 wins 0.06	0.2
		0.25	0.45	0.30	

$E(X) = 0.1*2 + 0.18*3 + 0.28*4 + 0.24*5 + 0.2*6 = 4.26$

$VAR(X) = 0.1*(2 - 4.26)^2 + 0.18*(3 - 4.26)^2 + 0.28*(4 - 4.26)^2 + 0.24*(5 - 4.26)^2 + 0.2*(6 - 4.26)^2 = 1.5524$

$STDEV(X) = 1.2460$

b. The total amount bid by both players (don't make a new distribution table)

Here, you don't want to calculate this by summing everything up and making a probability distribution. Instead, we are looking for E(A + B), VAR(A + B) and STDEV(A + B).

$E(A + B) = E(A) + E(B) = 6.7$

$VAR(A) = 0.25*(1 - 3.1)^2 + 0.45(3 - 3.1)^2 + 0.3(5 - 3.1)^2 = 2.19$

$VAR(B) = 0.4*(2 - 3.6)^2 + 0.4(4 - 3.6)^2 + 0.2(6 - 3.6)^2 = 2.24$

$VAR(A + B) = VAR(A) + VAR(B) = 4.43$

$STDEV(A + B) = 2.1048$

Dependent Bi-Variable Problem

Problem:

Given the following probability distribution, answer the following questions.

	Y = 0	Y = 1	Y = 2	Total
X = 0	0.1	0.2	0.1	0.4
X = 1	0.15	0.1	0.05	0.3
X = 2	0.05	0.1	0.15	0.3
Total	0.3	0.4	0.3	1.0

1. How can you tell that this probability distribution is dependent (besides that it's given)? Explain.

   Looking at (0, 0), the joint probability does not equal to P(X = 0) * P(Y = 0).

2. Find P(X = 2 | Y = 1).

   P (X = 2 | Y = 1) = P (X = 2 AND Y = 1) / P(Y = 1) = 0.1/0.4 = 0.25

3. Find $P(X = 1 \text{ AND } Y = 2)$.

   $P(1 \text{ AND } 2) = 0.05$ (taken directly from table)

4. Find $P(X = 1 \text{ OR } (Y = 1 \text{ OR } 2))$.

   $P(Y = 1 \text{ OR } Y = 2) = P(Y = 1) + P(Y = 2) = 0.7$

   $P(X = 1 \text{ OR } (Y = 1 \text{ OR } 2)) = P(X = 1) + P(Y = 1 \text{ OR } 2) - (P(X = 1) \text{ AND } (Y = 1 \text{ OR } 2)) = 0.3 + 0.7 - 0.15 = 0.85$
   (ALTERNATIVE: $1 - P(X = 0 \text{ AND } Y = 0) - P(X = 2 \text{ AND } Y = 0)$)