COMM 290

FINAL REVIEW SESSION

by Anna Feng

commerce
undergraduate
society

Introduction + Agenda

- Survey Questions
- Final Exam Overview
- Final Exam Contents
  - Probability Tree
  - Value of Information
  - Decision and EMV
  - Random Variables
- Kahoot!
- Q & A

If we’ve met before… …

- New additions to the midterm review:
  - Vocabulary review
  - More time to attempt questions
  - More time to ask questions
- Review Package
  - Contains practice problems from before AND after midterm
  - Found on CMP Facebook event page, website or here
  - Answer key released after the review session

Survey Questions

How confident are you with regards to the upcoming final exam?

A. I can peer-tutor my friends

B. I can do most of the review questions but occasionally makes mistakes

C. I am still a little confused about the concepts of possibility

D. I know very little and have trouble understanding the questions

How prepared are you?

A. I already did all the practice problems I can find

B. I have a plan to finish the practice problems, but I haven’t done most of them yet

C. I will probably get to the practice questions at some point before the exam

D. I haven’t thought about reviewing for the exam yet

Final Exam General Tips

- Before the exam:
  - Prepare for technical problems and distractions
  - Create a review schedule for all classes
  - Use all your resources
- After the exam:
  - Save your notes on probabilities
  - Clarify any confusions
  - Celebrate :-)
- During the exam:
  - Stay calm and read the questions carefully
  - Use the canvas bookmark function
  - Be mindful of time
  - Don't panic!

Vocabulary

- **Probability Tree –** Tree containing all possible outcomes of a probability problem.
- **Sample Space –** All the possible outcomes.
- **Probability –** The chance by which something will happen.
- **Independent –** Knowing something about one outcome does not affect another.
- **Dependent –** Knowing something about one outcome affects another.
- **EMV –** The expected value of all monetary payoffs
- **Optimistic Decision Approach –** Optimistic approach highlights the best payoff under any decision, and selects the decision with the maximum highest payout.
- **Maximin Conservative Approach –** Conservative approach highlights the worst payoff under any decision, and selects the decision with the best worst-case payout.
- **Minimax Regret Approach –** Regret approach finds the difference compared to the best outcome under each state. Then, select the decision that has the least worst-case regret.

Vocabulary

- **Expected Value of Sample Information –** The amount of profit gained by knowing another related state before making a decision.
- **Expected Value of Perfect Information –** The amount of profit gained by knowing the state before making a decision.
- **Efficiency of Information –** The % of EVPI extracted using sample information.

- **Expected Value –** The average outcome of a random variable.
- **Variance –** A measurement of how much a variable varies.
- **Standard Deviation –** How much a variable will normally vary relative to the mean. Often used with the ± sign in combination with the mean. More about that in COMM 291…

Probability Tree

1. Think about the possible states
2. Draw out the tree
3. Find the probability of each branch: remember that these are joint probabilities!
4. Draw in sample space: that is, the possible outcomes
5. Label the probability associated with each sample space outcome

Practice: Coin Game

- Problem:
  - You are playing a game, where you will flip 4 coins. The game ends when you flip tails. Answer the following questions.
- Steps:

1. Think about the possible states
2. Draw out the tree
3. Find the probability of each branch: remember that these are joint probabilities!
4. Draw in sample space: that is, the possible outcomes
5. Label the probability associated with each sample space outcome

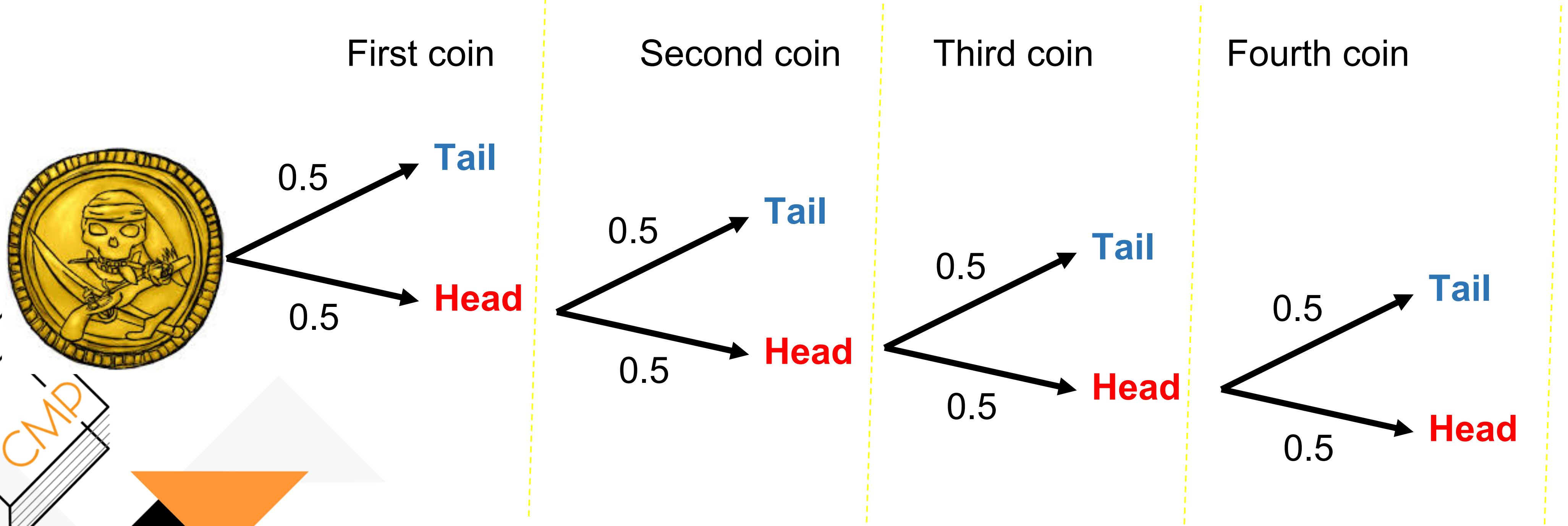
You are playing a game, where you will flip 4 coins. The game ends when you flip tails. Answer the following questions.

Think about the possible states

For each coin flip, the outcome can be either head or tail.

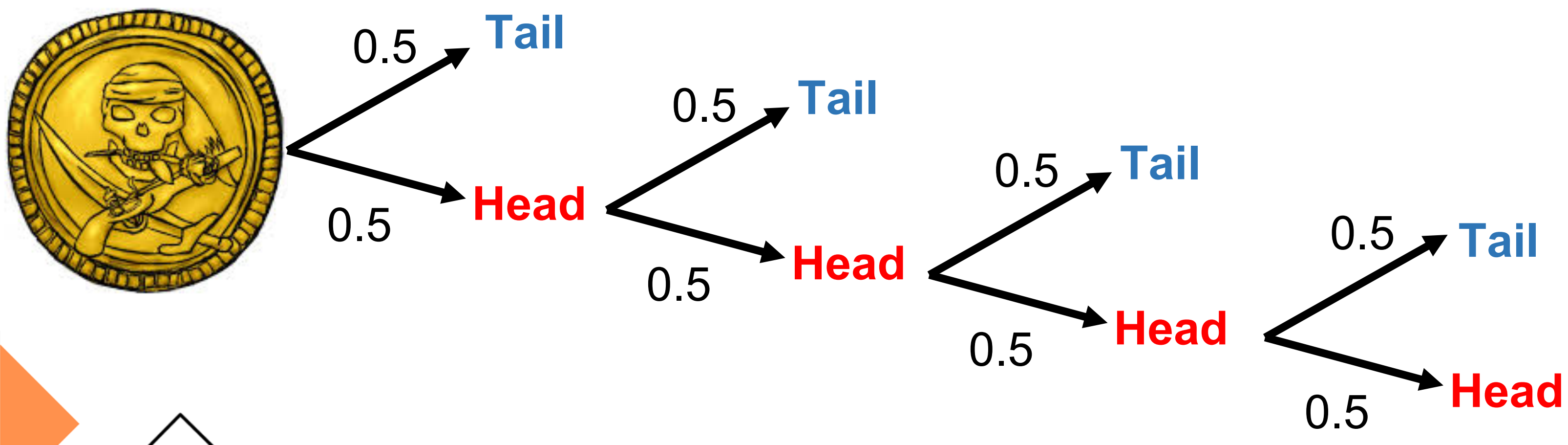
If the outcome is head, the next coin will be flipped

Draw out the tree, then fill in the probabilities

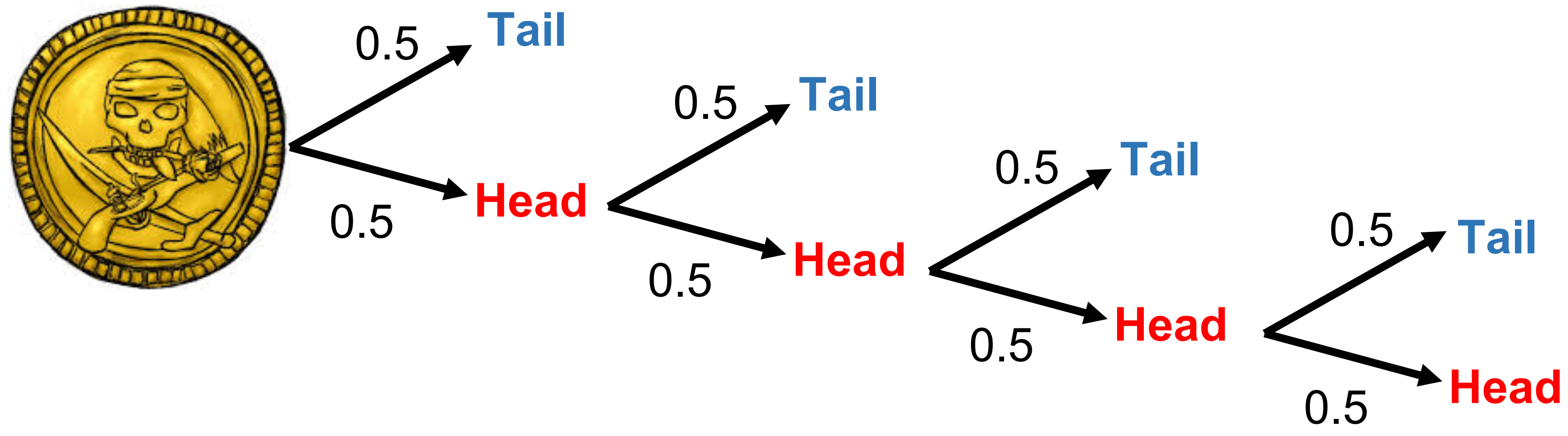


You are playing a game, where you will flip 4 coins. The game ends when you flip tails. Answer the following questions.

Draw in sample space: that is, the possible outcomes
Label the probability associated with each sample space outcome



Sample Space	Calculations	Probability
T	0.5	**0.5**
HT	0.5*0.5	**0.25**
HHT	0.5*0.5*0.5	**0.125**
HHHT	0.5*0.5*0.5*0.5	**0.0625**
HHHH	0.5*0.5*0.5*0.5	**0.0625**



Sample Space	Calculations	Probability
T	0.5	**0.5**
HT	0.5*0.5	**0.25**
HHT	0.5*0.5*0.5	**0.125**
HHHT	0.5*0.5*0.5*0.5	**0.0625**
HHHH	0.5*0.5*0.5*0.5	**0.0625**

2. What is the probability that you will get exactly two heads? What about exactly three heads?

- You will get two heads with 0.125 probability, and you will get three with 0.0625 probability.

3. After flipping one coin, which ended up being heads, what is the probability that your final outcome will be four heads?

- We are looking for P(HHHH | H) = P(HHHH) / P(H) = 0.0625/0.5 = 0.125
- (P(HHHH) = P(HHHH AND H) since you need H on first toss.)

4. You play a game where you win $1 if you get two or more heads from four flips. What is the probability that you will win $1?

- You want P(HHT OR HHHT OR HHHH) which is represented by the sum of all three probabilities as the outcomes are mutually exclusive.
- Therefore, the answer is 0.0125 + 0.0625 + 0.0625 = 0.25.

Break time!
Please come back at ________

Please fill out the feedback survey

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Decision Approaches

- Optimistic:
  - Label the best outcome for any decision. Select the decision that has the best labelled outcome.
- Conservative:
  - Label the worst outcome for any decision. Select the decision that has the best labelled out come (maximin).
- Regret:
  - Label the best outcome under each state. Then, find the difference in utility between each situation and the best outcome for **each state**. For each decision, label the highest regret. Select the decision with the lowest labelled value (minimax).

Decision and EMV

1. Think about which are your states, and which are decisions
2. Place the decisions (squares) and then states (circles)
3. Label each state with their respective probabilities
4. Find the outcome of each situation
5. Conduct EMV calculation on each decision branch

Practice: Money Investor

- Problem:
  - You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up.

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up.

Optimistic:?
Conservative:?
Regret:?

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up.

Optimistic:

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up.

Conservative:

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up.

Regret:

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	50 -20	10 0	0 30
Buy Bonds	20 10	0 10	20 10
Short Stocks	0 30	10 0	70 -40

You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up.

EMV:
Stocks: 5
(-20*0.2)+(0.5*0)+(0.3*30)
Bonds: 10
(10*0.2)+(0.5*10)+(0.3*10)
Short: -6
(30*0.2)+(0.5*0)+(0.3*-40)

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

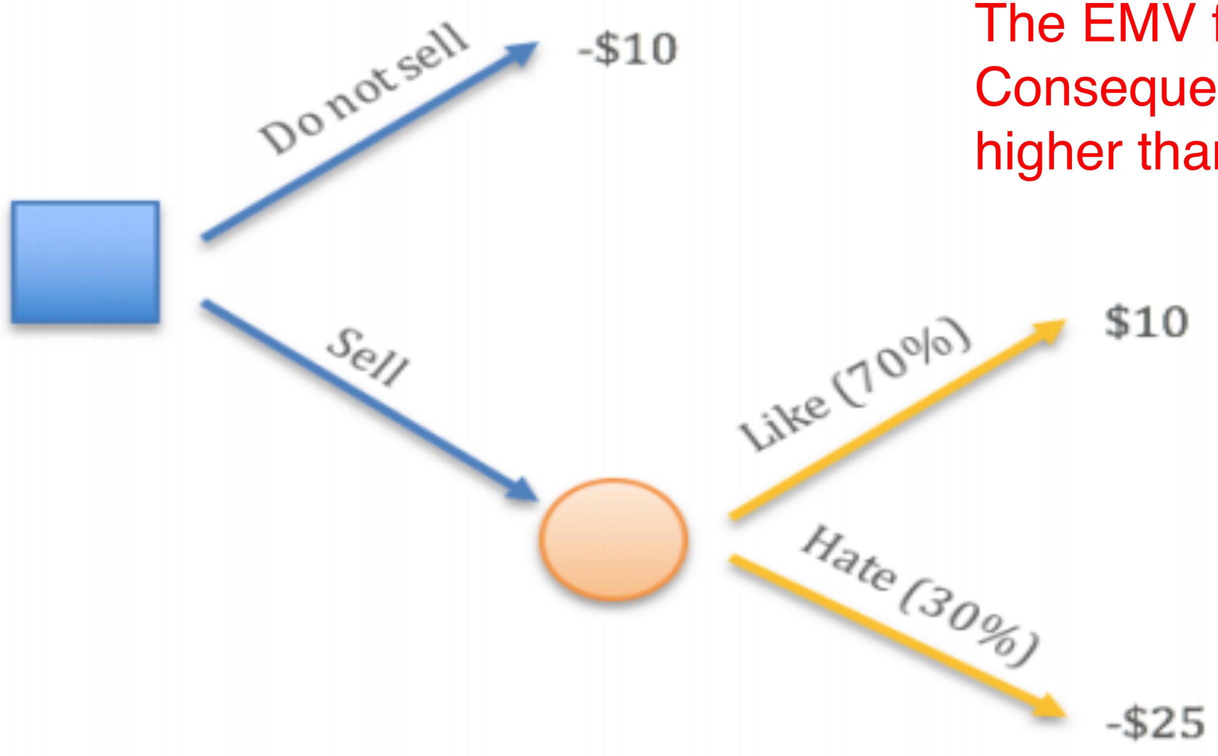
Value of Information

1. Draw the probability tree, with decisions and states
2. Remember to draw decisions first
3. Conduct EMV analysis on each branch
4. Select the best decision by EMV

Practice: Money Tickets

- Problem:
  - You're selling tickets for a movie. Forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.
- Steps:
  1. Draw the probability tree, with decisions and states
  2. Remember to draw decisions first
  3. Conduct EMV analysis on each branch
  4. Select the best decision by EMV

You're selling tickets for a movie. Forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.



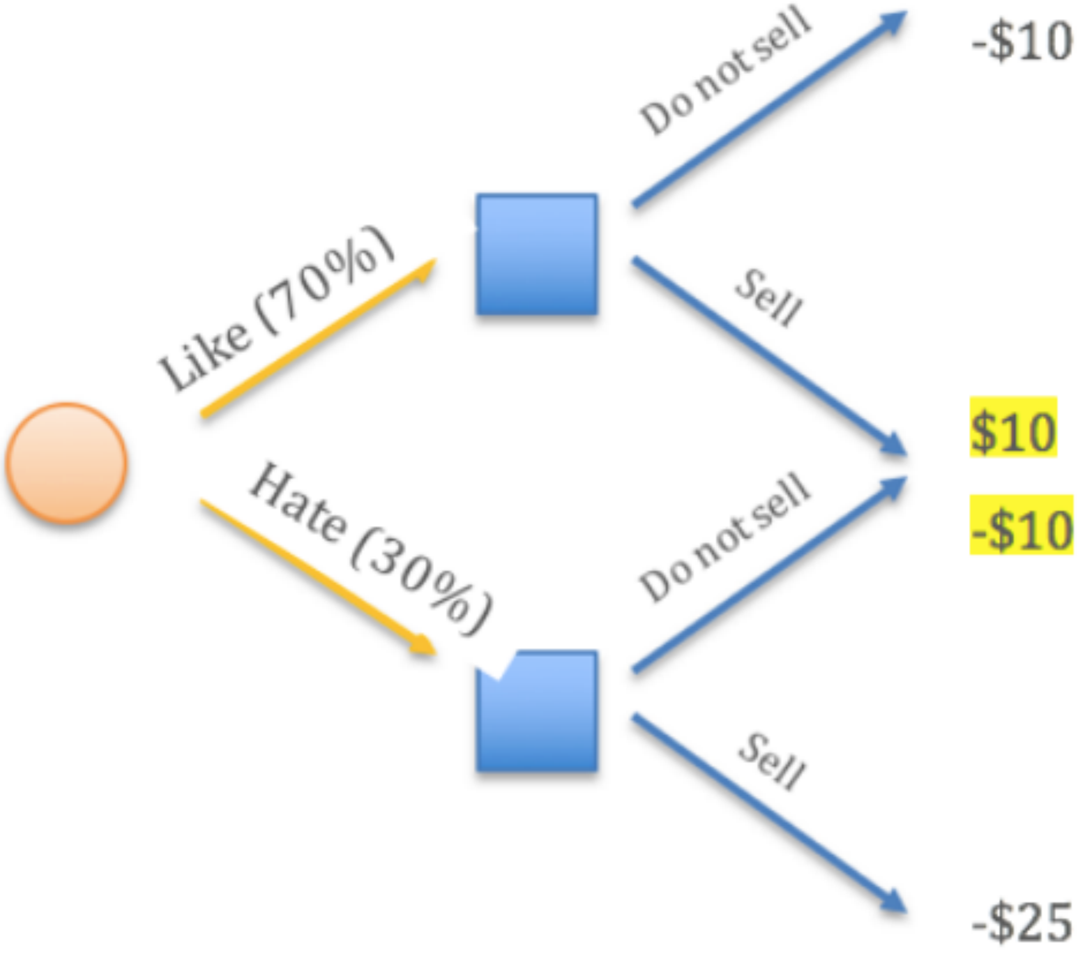
The EMV for selling is 10*0.7 – 25*0.3 = -$0.5. Consequently, you will sell because the EMV is higher than the loss you take by not selling (-$10).

Perfect Information

1. Draw the probability tree, with decisions and states
2. Remember to draw states first
3. For each state, indicate which decision you will make
4. Find the expected value by EMV approach

You’re selling tickets for a movie. Forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don’t, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10. **You can foresee the future at a cost. How much will you be willing to pay in order to foresee the future?**

You know that if people like the movie, you will sell, and if people do not like the movie you will not sell. Therefore, your EPPI (Expected Profit of Perfect Information) is $10*0.7 - $10*0.3 = $10*0.4 = $4. Compared to the old EMV of -$0.5, your value of perfect information is EPPI – EMV = $4.5. Therefore, you should be willing to pay up to $4.5 in order to tell the future.



Sample Information

1. Draw the probability tree, with states, decisions and states
2. Remember to draw additional information states first
3. Calculate the probability matrix
4. Find the probability at each branch, and the payoff
5. Select the decisions you will make using EMV
6. Calculate value of the information using EMV

Practice Movie Tickets(again)

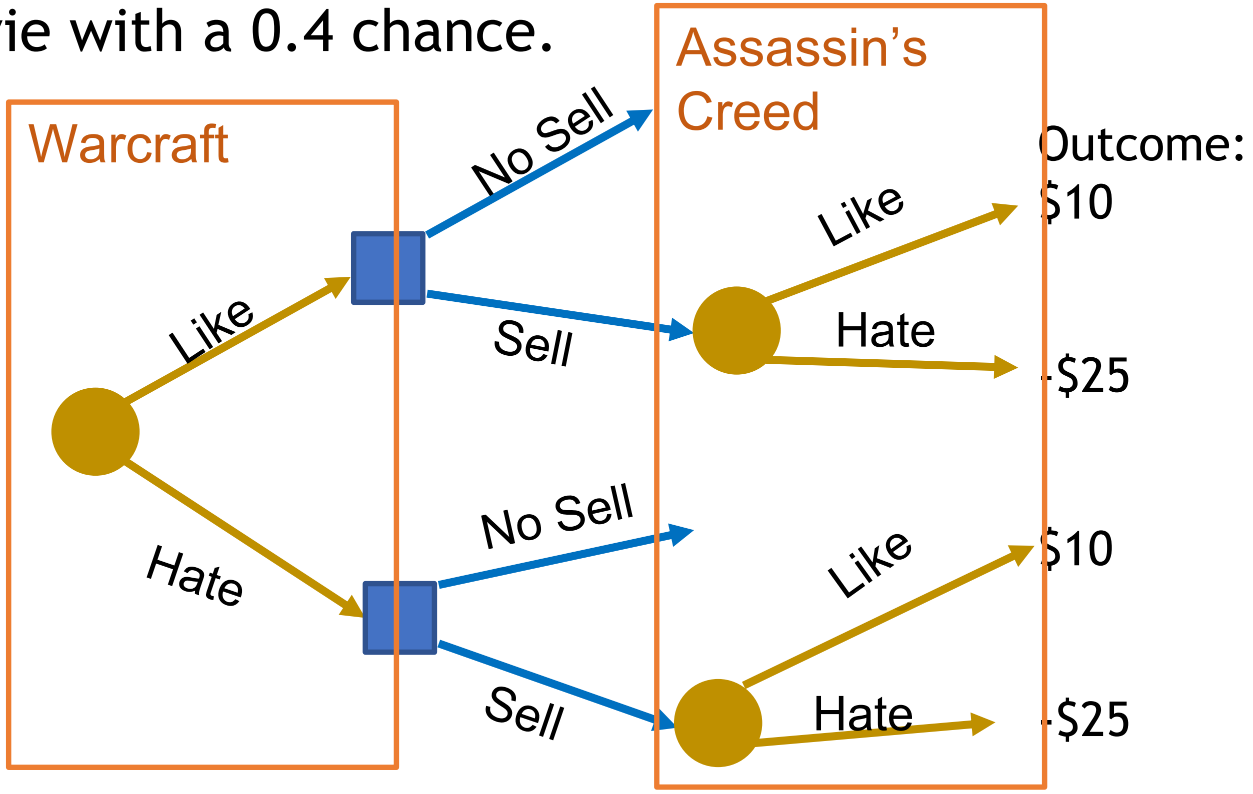
- Problem:
  - Suppose you get additional information that if people liked the Warcraft movie, there will be an 0.9 chance that people will like the Assassin’s Creed movie, while if people did not like the Warcraft movie, people will only like the Assassin’s Creed movie with a 0.4 chance.
- Steps:
  1. Draw tree
  2. Matrix
  3. Branch probabilities
  4. EMV

You're selling tickets for a movie. Forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.

Suppose you get additional information that if people liked the Warcraft movie, there will be an 0.9 chance that people will like the Assassin's Creed movie, while if people did not like the Warcraft movie, people will only like the Assassin's Creed movie with a 0.4 chance.

Steps:

1. Draw tree
2. Matrix
3. Branch probabilities
4. EMV



You're selling tickets for a movie. Forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.

Suppose you get additional information that if people liked the Warcraft movie, there will be an 0.9 chance that people will like the Assassin's Creed movie, while if people did not like the Warcraft movie, people will only like the Assassin's Creed movie with a 0.4 chance.

Steps:

1. Draw tree
2. Matrix
3. Branch probabilities
4. EMV

Let LW and HW denote if people like Warcraft or not, and LA and HA denote if people like assassins or not.

P(LA | LW) = 0.9 P(LA | HW) = 0.4 P(LA and LW) = 0.9 * P(LW) P(LA and HW) = 0.4 * P(HW)

Solve this system:

0.9 * P(LW) + 0.4 * P(HW) = 0.7 (1)

0.1 * P(LW) + 0.6 * P(HW) = 0.3 (2)

(2): P(LW) + 6 * P(HW) = 3, P(LW) = 3 – 6 * P(HW)

Plug into (1): 0.9 * (3 – 6 * P(HW)) + 0.4 * P(HW) = 0.7

2.7 – 5.4 * P(HW) + 0.4 * P(HW) = 0.7 5 * P(HW) = 2

P(HW) = 0.4 P(LW) = 0.6

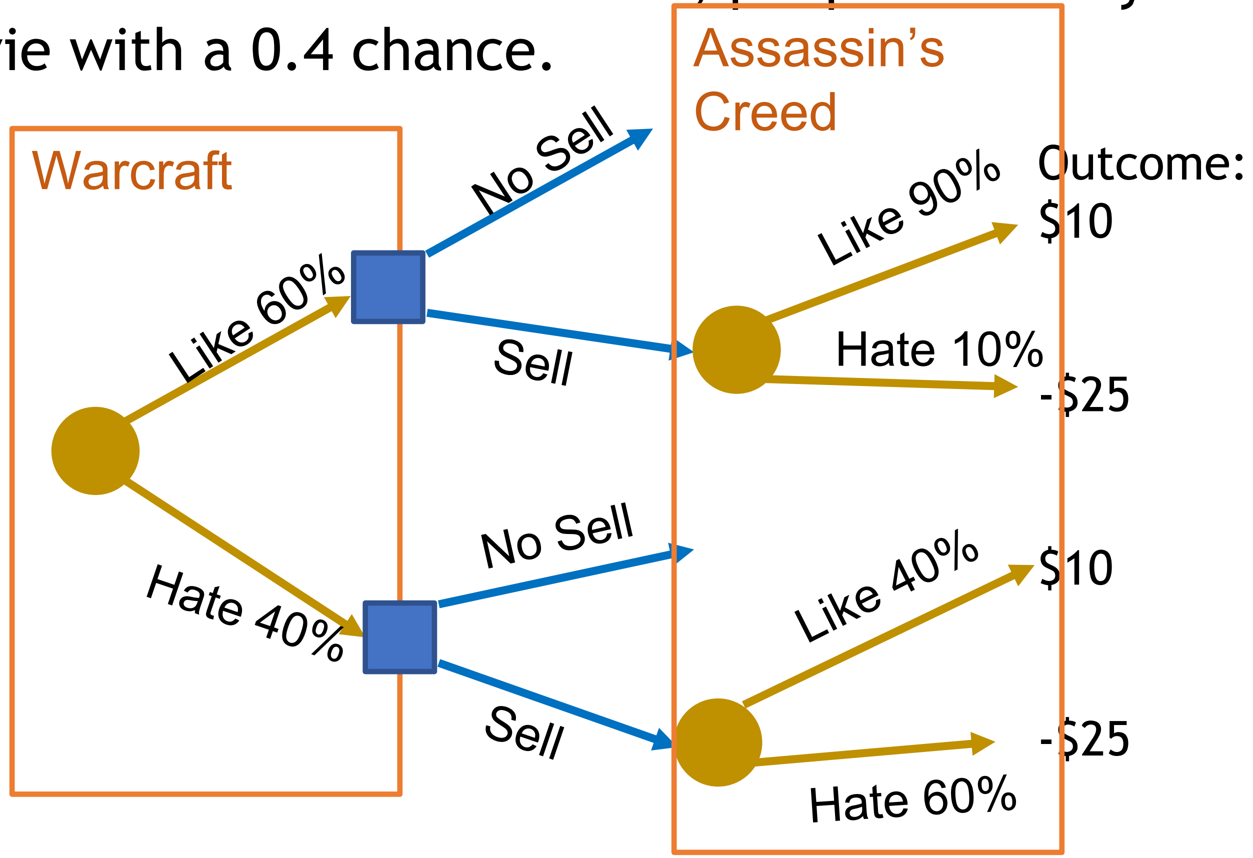
	LA	HA	Total
LW	0.9 * P(LW)	0.1 * P(LW)	P(LW)
HW	0.4 * P(HW)	0.6 * P(HW)	P(HW)
Total	0.7	0.3	1

You're selling tickets for a movie. Forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.

Suppose you get additional information that if people liked the Warcraft movie, there will be an 0.9 chance that people will like the Assassin's Creed movie, while if people did not like the Warcraft movie, people will only like the Assassin's Creed movie with a 0.4 chance.

Steps:

1. Draw tree
2. Matrix
3. Branch probabilities
4. EMV



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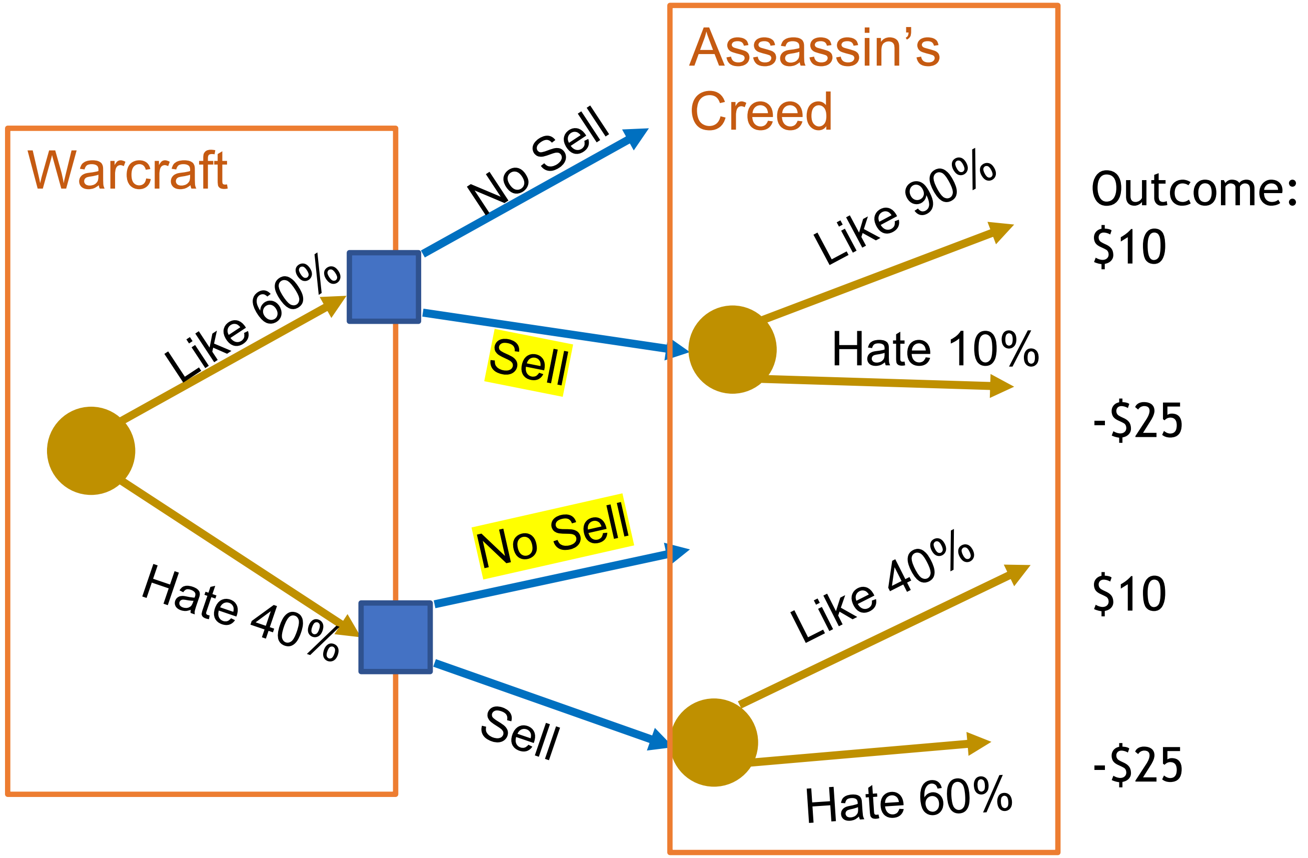

EMV of the two outcomes:
If people like the Warcraft movie, the EMV for Assassin's Creed ticket sales is $10*0.9 - $25*0.1 = $6.5 (prefer sell over not sell).
If people hate the Warcraft movie, the EMV for Assassin's Creed ticket sales is $10*0.4 - $25*0.6 =-$11 (will not sell and rather get -$10).
EPSI (Expected Profit with Sample Information) is $6.5*0.6 - $10*0.4 = - $0.1
EVSI = EPSI – EMV = -$0.1 - -$0.5 = -$0.1 + 0.5 = $0.4.

Steps:
1. Draw tree
2. Matrix
3. Branch probabilities
4. EMV

Random Variable

1. You will get a distribution table
2. Recall these formulas:
3. P(A OR B) = P(A) + P(B)
4. P(A AND B) = P(A) * P(B)
5. P(A | B) = P(A and B) / P(B)
6. P(B | A) = P(A | B) * P(B) / P(A)
7. E(x): $\sum x * p(x)$
8. VAR(x) = STDEV^2: $\sum\left(x - E(x)\right)^2 * p(x)$

Variable Mutations

1. You may end up mutating your variables with a constant:
2. So, we have these formulas:
    1. $E(aX) = aE(X)$
    2. $VAR(aX) = a^2 * VAR(X)$
    3. $E(X + Y) = E(X) + E(Y)$
    4. $VAR(X + Y) = VAR(X) + VAR(Y)$

Practice

X	0	1	2	3	4
P(X)	0.3	0.15	0.25	0.2	0.1

1. Find expected value, variance, and standard deviation.
2. Find P(2 OR 3) and P(0 OR 2).
3. You pick two numbers at random with replacement. Find P(1st = 0 AND 2nd = 4) and P(2nd =1 | 1st = 4).
4. You pick two numbers at random without replacement. Find P(1st = 0 AND 2nd = 4) and P(2nd = 1 | 1st = 4).
5. Suppose you will take whatever number you pick out and construct a square with that given number as the side length. What is the expected value, variance, and standard deviation of the area of your square?

Practice

X	0	1	2	3	4
P(X)	0.3	0.15	0.25	0.2	0.1

1. Find expected value, variance, and standard deviation.

- E(X) = 0.3*0 + 0.15*1 + 0.25*2 + 0.2*3 + 0.1*4 = 1.65
- VAR(X) = $0.3*(0 – 1.65)^2 + 0.15*(1 – 1.65)^2 + 0.25*(2 – 1.65)^2 + 0.2*(3 – 1.65)^2 + 0.1*(4 – 1.65)^2$ = 1.8275
- STDEV(X) = sqrt(VAR(X)) = 1.351

2. Find P(2 OR 3) and P(0 OR 2).

- P(2 OR 3) = P(2) + P(3) = 0.25 + 0.2 = 0.45
- P(0 OR 2) = P(0) + P(2) = 0.3 + 0.25 = 0.55

Practice

X	0	1	2	3	4
P(X)	0.3	0.15	0.25	0.2	0.1

3. You pick two numbers at random with replacement. Find P(1st = 0 AND 2nd = 4) and P(2nd =1 | 1st = 4).

- P(1st = 0 AND 2nd = 4) and P(2nd = 1 | 1st = 4).
- P(1st = 0 AND 2nd = 4) = P(0) * P(4) = 0.3 * 0.1 = 0.03
- P(2nd = 1 | 1st = 4) = P(1) = 0.15

4. You pick two numbers at random without replacement. Find P(1st = 0 AND 2nd = 4) and P(2nd = 1 | 1st = 4).

- P(1st = 0 AND 2nd = 4) = P(0) * P(4 | 0) = 0.3 * 0.1/(1 – P(0) = 0.3 * 0.1/0.7 = 0.042
- P(2nd = 1 | 1st = 4) = 0.15 / (1 – P(4)) = 0.15 / 0.9 = 0.167

Practice

X	0	1	2	3	4
P(X)	0.3	0.15	0.25	0.2	0.1

5. Suppose you will take whatever number you pick out and construct a square with that given number as the side length. What is the expected value, variance, and standard deviation of the area of your square?

E(Y) = 0.3*0 + 0.15*1 + 0.25*4 + 0.2*9 + 0.1*16 = 4.55

VAR(Y) = $0.3*(0 - 4.55)^2 + 0.15*(1 - 4.55)^2 + 0.25*(4 - 4.55)^2 + 0.2*(9 - 4.55)^2 + 0.1*(16 - 4.55)^2$ = 25.2475

STDEV(Y) = sqrt(VAR(X)) = 5.0247

Y	0	1	4	9	16
P(Y)	0.3	0.15	0.25	0.2	0.1

Bivariable Problem

1. Define dependency
2. Similar Formulas as random variable section

Given the following probability distribution for a closed auction, answer the following questions. Assume the two players' strategies are independent. Complete the table before answering any questions.

1. What is the probability that Player 1 wins? What is the probability that Player 2 wins?
2. Find the expected value of (A + (-B)). What does this mean?

		Player 1 (A)			
		Bid $1	Bid $3	Bid $5	
Player 2 (B)	Bid $2	0.1	0.18	0.12	0.4
	Bid $4	0.1	0.18	0.12	0.4
	Bid $6	0.05	0.09	0.06	0.2
		0.25	0.45	0.3	

Given the following probability distribution for a closed auction, answer the following questions. Assume the two players' strategies are independent. Complete the table before answering any questions.

1. What is the probability that Player 1 wins? What is the probability that Player 2 wins?

Sum of all probabilities

A wins: yellow

B wins: green

		Player 1 (A)			
		Bid $1	Bid $3	Bid $5	
Player 2 (B)	Bid $2	0.1	0.18	0.12	0.4
	Bid $4	0.1	0.18	0.12	0.4
	Bid $6	0.05	0.09	0.06	0.2
		0.25	0.45	0.3	

Given the following probability distribution for a closed auction, answer the following questions. Assume the two players’ strategies are independent. Complete the table before answering any questions.

2. Find the expected value of (A + (B)). What does this mean?

(A) = 0.25*1 + 0.45*3 + 0.3*5 = 3.1

E(-B) = E(B) * -1 = (0.4*2 + 0.4*4 + 0.2*6) * -1 = -3.6

E(A + (-B)) = E(A) + E(-B) = -0.5.

This shows that on average, Player B has a bid that is 0.5 larger than that of Player A, and on average Player B will win.

		Player 1 (A)			
		Bid $1	Bid $3	Bid $5	
Player 2 (B)	Bid $2	0.1	0.18	0.12	0.4
	Bid $4	0.1	0.18	0.12	0.4
	Bid $6	0.05	0.09	0.06	0.2
		0.25	0.45	0.3	

Given the following probability distribution for a closed auction, answer the following questions. Assume the two players' strategies are independent. Complete the table before answering any questions.

Extra Problems:

Find the expected value, variance, and standard deviation of the following:

a. The winning bid

b. The total amount bid by both players

		Player 1 (A)			
		Bid \$1	Bid \$3	Bid \$5	
Player 2 (B)	Bid \$2	0.1	0.18	0.12	0.4
	Bid \$4	0.1	0.18	0.12	0.4
	Bid \$6	0.05	0.09	0.06	0.2
		0.25	0.45	0.3	

Problem:
Given the following probability distribution, answer the following questions.

1. Dependency?
2. Find P(X = 2 | Y = 1).
3. Find P(X = 1 AND Y = 2).
4. Find P(X = 1 OR (Y = 1 OR 2)).

	Y = 0	Y = 1	Y = 2	Total
X = 0	0.1	0.2	0.1	0.4
X = 1	0.15	0.1	0.05	0.3
X = 2	0.05	0.1	0.15	0.3
Total	0.3	0.4	0.3	1.0

Problem:
Given the following probability distribution, answer the following questions.

Dependency?

Looking at (0, 0), the joint probability does not equal to P(X = 0) * P(Y = 0).

Find P(X = 2 | Y = 1).

P (X = 2 | Y = 1) = P (X = 2 AND Y = 1) / P(Y = 1) = 0.1/0.4 = 0.25

	Y = 0	Y = 1	Y = 2	Total
X = 0	0.1	0.2	0.1	0.4
X = 1	0.15	0.1	0.05	0.3
X = 2	0.05	0.1	0.15	0.3
Total	0.3	0.4	0.3	1.0

Problem:
Given the following probability distribution, answer the following questions.

	Y = 0	Y = 1	Y = 2	Total
X = 0	0.1	0.2	0.1	0.4
X = 1	0.15	0.1	0.05	0.3
X = 2	0.05	0.1	0.15	0.3
Total	0.3	0.4	0.3	1.0

Find P(X = 1 AND Y = 2).

P(1 AND 2) = 0.05 (taken directly from table)

Find P(X = 1 OR (Y = 1 OR 2)).

P(Y = 1 OR Y = 2) = P(Y = 1) + P(Y = 2) = 0.7

P (X = 1 OR (Y = 1 OR 2)) = P(X = 1) + P(Y = 1 OR 2) – (P(X = 1) AND (Y = 1 OR 2)) = 0.3 + 0.7 – 0.15 = 0.85 (ALTERNATIVE: 1 – P(X = 0 AND Y = 0) – P(X = 2 AND Y = 0))

Break time!
Please come back at

____

Please fill out the feedback survey

[Ask for link]

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Bonus Section – Sensitivity Analysis

- The first section contains variable cells: these will have to do with your **decisions** and **objective coefficients.** The cell "address" contains the location of your decision cell.
  - Do not worry about reduced cost.
- **Rules of Allowables:**
  - Increasing / Decreasing the **objective coefficient** within the allowable range **DOES NOT CHANGE** the optimal solution, only the **TARGET CELL.**
  - Let's say a coefficient changes within the allowable range. You will still make the same amount of any decision: you will just make a different amount of profit.

Bonus Section – Sensitivity Analysis

- The second section contains constraints, and we are most concerned about the R.H. side.
- **Rules of Allowables:**
  - Increasing / Decreasing the **RHS of a binding constraint** within the allowable range **DOES** the optimal solution **AND** the **TARGET CELL**.
    - Change is equal to **change * shadow price**. In other words, **all binding constraints will have a shadow price.**
  - Increasing / Decreasing the **RHS of a non-binding constraint** within the allowable range **DOES NOT CHANGE** the optimal solution **NOR** the target cell.
    - One change will always be 1E+30; the other change would be the difference between amount used and the constraint limit.

Question time! Ask me anything

If you must go, best of luck on your exam!
You’ll all do great

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