commerce undergraduate society

COMMERCE MENTORSHIP PROGRAM

FINAL REVIEW SESSION

COMM 290

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Definitions – 2nd Half of Course

Probability Tree – Tree containing all possible outcomes of a probability problem.

Sample Space – All the possible outcomes.

Probability – The chance by which something will happen.

Independent – Knowing something about one outcome does not affect another.

Dependent – Knowing something about one outcome affects another.

Mutually Exclusive – Two outcomes are mutually exclusive if they cannot both occur at the same time.

Expected Monetary Value (EMV) – The expected value of all monetary payoffs

Optimistic Decision Approach – Optimistic approach highlights the best payoff under any decision and selects the decision with the maximum highest payout.

Maximin Conservative Approach – Conservative approach highlights the worst payoff under any decision and selects the decision with the best worst-case payout.

Minimax Regret Approach – Regret approach calculates the difference between each outcome and the best outcome under each state. Then, select the decision that has the least worst- case regret.

Expected Value of Sample Information – The amount of profit gained by knowing another related state before making a decision.

Expected Value of Perfect Information – The amount of profit gained by knowing the state before making a decision.

Efficiency of Information – The % of EVPI extracted using sample information.

Expected Value – The average outcome of a random variable.

Variance – A measurement of how much a variable varies.

Standard Deviation – How much a variable will normally vary relative to the mean.

Probability Trees

Probability Tree – Tree containing all possible outcomes of a probability problem.

Sample Space – All the possible outcomes.

Probability – The chance by which something will happen.

Independent – Knowing something about one outcome does not affect another.

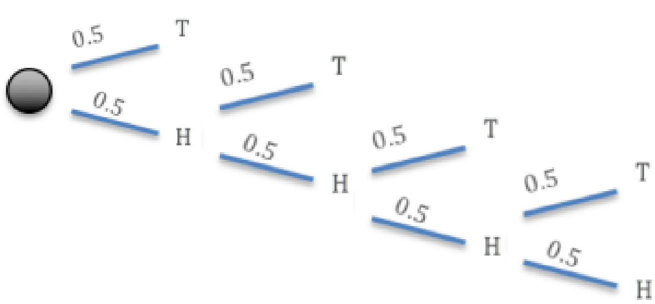
Dependent – Knowing something about one outcome affects another.

Mutually Exclusive – Two outcomes are mutually exclusive if they cannot both occur at the same time.

=RAND – Generates a random real number between 0 and 1.

Problem – Let's Play a Game: You meet a stranger at a sketchy subway station and agree to play a game. You flip 4 coins – 50% probability of heads. If heads, you get slapped by the stranger. If tails, you win $1,000. The game ends when you flip tails. Answer the following questions

1. Draw a probability tree for the given scenario. Be sure to include the sample space and probability of each outcome.



Sample Space	Probability
T	0.5
HT	0.25
HHT	0.125
HHHT	0.0625
HHHH	0.0625

2. What is the probability that you will get exactly two heads? What about exactly three heads?

   You will get two heads with 0.125 probability, and you will get three with 0.0625 probability.

3. After flipping one coin, which ended up being heads, what is the probability that your final outcome will be four heads?

   We are looking for P(HHHH | H) = P(HHHH) / P(H) = 0.0625/0.5 = 0.125

   (P(HHHH) = P(HHHH AND H) since you need H on first toss.

4. You play a game where you lose $500 if you get two or more heads from four flips. What is the probability that you will lose $500?

   You want P(HHT OR HHHT OR HHHH) which is represented by the sum of all three probabilities as the outcomes are mutually exclusive.

   Therefore, the answer is 0.0125 + 0.0625 + 0.0625 = 0.25.

Decisions & EMV

Expected Monetary Value (EMV) – The expected value of all monetary payoffs

Optimistic Decision Approach – Optimistic approach highlights the best payoff under any decision and selects the decision with the maximum highest payout.

Maximin Conservative Approach – Conservative approach highlights the worst payoff under any decision and selects the decision with the best worst-case payout.

Minimax Regret Approach – Regret approach calculates the difference between each outcome and the best outcome under each state. Then, select the decision that has the least worst- case regret.

Problem: You are an independent money investor, and you are looking at three options: buying stocks, buying bonds, and shorting stocks. The markets can be down, neutral, or up. The payoff matrix is provided and use it to answer the following questions

	Down (0.2)	Neutral (0.5)	Up (0.3)
Buy Stocks	-20	0	30
Buy Bonds	10	10	10
Short Stocks	30	0	-40

1. Which decision will you make if under the optimistic approach? What about conservative?

Optimistic: under the optimistic approach, the highest outcome of all three options are 30, 10 and 30, respectively. Therefore, you will choose to either buy stocks or short stocks.

Conservative: under the conservative approach, the worst outcome of all three options are -20, 10 and -40, respectively. Consequently, you will choose to buy bonds.

2. Construct a regret matrix. Which decision will you make if you are using the regret approach?

	Down (0.2)	Neutral (0.5)	Up (0.3)	Maximum
Buy Stocks	50	10	0	50
Buy Bonds	20	0	20	20
Short Stocks	0	10	70	70

You will choose to buy bonds because it holds the lowest maximum regret.

3. Which decision should you make if you are using the EMV (Expected Monetary Value) approach?

Buy Stocks: EMV = -20*0.2 + 0*0.5 + 30*0.3 = 5

Buy Bonds: 10 (constant payout)

Short Stocks: EMV = 30*0.2 + 0*0.5 – 40*0.3 = -6

You will choose to buy bonds.

Value of Information

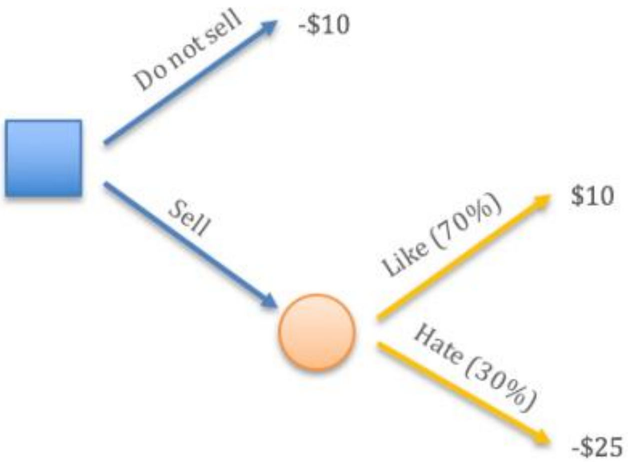
Expected Value of Sample Information – The amount of profit gained by knowing another related state before making a decision.

Expected Value of Perfect Information – The amount of profit gained by knowing the state before making a decision.

Efficiency of Information – The % of EVPI extracted using sample information.

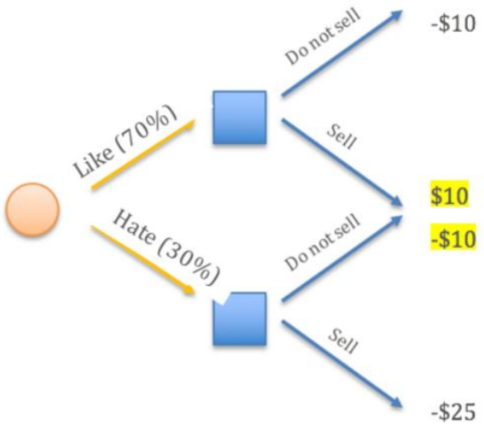
Problem – Makeshift Movie Theatre: You are selling movie tickets for the upcoming Marvel movie. You are unsure whether or not people will like the movie or not: forecasts say there is 0.7 chance that people will like it and 0.3 that they will not. If they like the movie, you will sell tickets for $10 of profit, and if they don't, you will sell the tickets but at a loss of $25. If you do not sell in either case, you will be at a loss of $10.

1. Draw a probability tree; under the EMV approach, what decision will you make?



The EMV for selling is 10*0.7 – 25*0.3 = -$0.5. Consequently, you will sell because the EMV is higher than the loss you take by not selling (-$10).

2. You can foresee the future at a cost. How much will you be willing to pay in order to foresee the future? Draw the probability tree associated with this scenario.



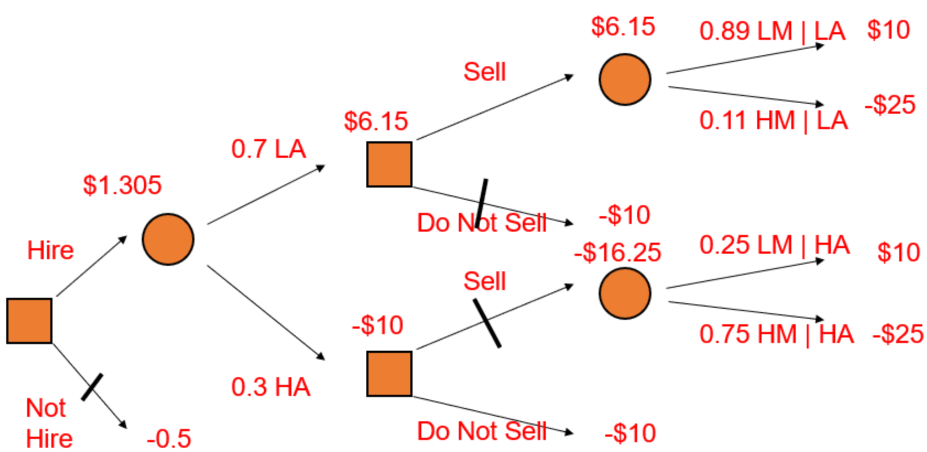
You know that if people like the movie, you will sell, and if people do not like the movie you will not sell. Therefore, your EPPI (Expected Profit of Perfect Information) is $10*0.7 - $10*0.3 = $10*0.4 = $4. Compared to the old EMV of -$0.5, your value of perfect information is EPPI – EMV = $4.5. Therefore, you should be willing to pay up to $4.5 in order to tell the future.

3. Suppose you can hire a movie expert to improve probability estimates. In the past, the expert predicted audiences were excited for action movies when they liked Marvel movies with a 90% probability. While audiences hated action movies, when they hated Marvel with a 75% chance. Construct a new tree, and find the EVSI (Expected Value of Sample Information).

P(Like Action | Like Marvel) = 0.9   P(Hate Action | Hate Marvel) = 0.75

	LM	HM	
LA	0.625	0.075	0.7
HA	0.075	0.225	0.3
	0.7	0.3	1

$P(LA \mid LM) = \frac{P(LA \,\&\, LM)}{P(LM)}$ $0.9 = \frac{P(LA \,\&\, LM)}{0.7}$ $P(HA \mid HM) = \frac{P(HA \,\&\, HM)}{P(HM)}$ $0.75 = \frac{P(HA \,\&\, HM)}{0.3}$



Note:
P (LM | LA) =
(LM&LA) / P(LA) =
0.625 / 0.7 = 0.89
(OR 0.9 DEPENDING ON ROUNDING)

Note:
P (LM | HA) =
(LM&HA) / P(HA) =
0.075 / 0.3 = 0.25
(OR 0.23 DEPENDING ON ROUNDING)

PROFIT = $1.305
VALUE = $1.805

*NOTE that numbers may vary slightly depending on how you rounded intermediate calculations

Random Variables

Expected Value – The average outcome of a random variable.

Variance – A measurement of how much a variable varies.

Standard Deviation – How much a variable will normally vary relative to the mean

Problem: Consider the following probability distribution and answer the following questions

X	0	1	2	3	4
P(X)	0.3	0.15	0.25	0.2	0.1

1. Find expected value, variance, and standard deviation.

   E(X) = 0.3*0 + 0.15*1 + 0.25*2 + 0.2*3 + 0.1*4 = 1.65

   VAR(X) = 0.3*$(0 – 1.65)^2$ + 0.15*$(1 – 1.65)^2$ + 0.25*$(2 – 1.65)^2$ + 0.2*$(3 – 1.65)^2$ + 0.1*$(4 – 1.65)^2$ = 1.8275

   STDEV(X) = sqrt(VAR(X)) = 1.351

2. Find P(2 OR 3) and P(0 OR 2).

   P(2 OR 3) = P(2) + P(3) = 0.25 + 0.2 = 0.45

   P(0 OR 2) = P(0) + P(2) = 0.3 + 0.25 = 0.55

3. You pick two numbers at random with replacement. Find P(1st = 0 AND 2nd = 4) and P(2nd = 1 | 1st = 4).

   P(1st = 0 AND 2nd = 4) and P(2nd = 1 | 1st = 4).

   P(1st = 0 AND 2nd = 4) = P(0) * P(4) = 0.3 * 0.1 = 0.03

   P(2nd = 1 | 1st = 4) = P(1) = 0.15

4. You pick two numbers at random without replacement. Find P(1st = 0 AND 2nd = 4).

P(1st = 0 AND 2nd = 4) = P(0) * P(4 | 0) = 0.3 * 0.1/(1 – P(0) =
0.3 * 0.1/0.7 = 0.042

5. Suppose you will take whatever number you pick out, and construct a square with that given number as the side length. What is the expected value, variance, and standard deviation of the area of your square?

Y	0	1	4	9	16
P(Y)	0.3	0.15	0.25	0.2	0.1

E(Y) = 0.3*0 + 0.15*1 + 0.25*4 + 0.2*9 + 0.1*16 = 4.55

VAR(Y) = $0.3*(0 – 4.55)^2 + 0.15*(1 – 4.55)^2 + 0.25*(4 – 4.55)^2 + 0.2*(9 – 4.55)^2 + 0.1*(16 – 4.55)^2 = 25.2475$

STDEV(Y) = sqrt(VAR(X)) = 5.0247

6. This question is unrelated to the previous scenario. Given $E(X^2) = 20$ and E(X) = 5, find VAR(X) and STDEV(X).

Note that VAR(X) = $E(X^2) – E(X)^2 = 20 – 25 = 5$ (abs)
STDEV(X) = sqrt(5) = 2.236

Independent Bi-Variable Problem

Problem: Given the following probability distribution for a closed auction, answer the following questions. Assume the two players’ strategies are independent.

		Player 1 (A)			
		Bid $1	Bid $3	Bid $5	
Player 2 (B)	Bid $2	0.1	0.18	0.12	0.4
	Bid $4	0.1	0.18	0.12	0.4
	Bid $6	0.05	0.09	0.06	0.2
		0.25	0.45	0.3	

1. What is the probability that Player 1 wins? What is the probability that Player 2 wins?

The probability that Player 1 wins is equal to the sum of all probabilities in the cells where player 1’s bid s which adds up to 0.42. Since all the outcomes are mutually exclusive, you may add them to find the probability that one of them happens.

The probability that Player 2 wins is 1 – P(Player 1 wins) = 1 – 0.42 = 0.58. Since there will never be a situation where two bids are equal, one player must win

2. Find the expected value of (A + (-B)). What does this mean?

$(A) = 0.25*1 + 0.45*3 + 0.3*5 = 3.1$

$E(-B) = E(B) * -1 = (0.4*2 + 0.4*4 + 0.2*6) * -1 = -3.6$

$E(A + (-B)) = E(A) + E(-B) = -0.5$. This shows that on average, Player B has a bid that is 0.5 larger than that of Player A, and on average Player B will win.

3. Find the expected value, variance, and standard deviation of the the winning bid.

$E(X) = 0.1*2 + 0.18*3 + 0.28*4 + 0.24*5 + 0.2*6 = 4.26$

$VAR(X) = 0.1*(2 – 4.26)^2 + 0.18*(3 – 4.26)^2 + 0.28*(4 – 4.26)^2 + 0.24*(5 – 4.26)^2 + 0.2*(6 – 4.26)^2 = 1.5524$

$STDEV(X) = 1.2460$

Dependent Bi-variable Problem

Problem: Given the following probability distribution, answer the following questions.

	Y = 0	Y = 1	Y = 2	Total
X = 0	0.1	0.2	0.1	0.4
X = 1	0.15	0.1	0.05	0.3
X = 2	0.05	0.1	0.15	0.3
Total	0.3	0.4	0.3	1.0

1. How can you tell that this probability distribution is dependent (besides that it's given)? Explain.

   Looking at (0, 0), the joint probability does not equal to P(X = 0) * P(Y = 0).

2. Find P(X = 2 | Y = 1).

   P (X = 2 | Y = 1) = P (X = 2 AND Y = 1) / P(Y = 1) = 0.1/0.4 = 0.25

3. Find P(X = 1 AND Y = 2).

   P(1 AND 2) = 0.05 (taken directly from table)

4. Find P(X = 1 OR (Y = 1 OR 2)).

   P(Y = 1 OR Y = 2) = P(Y = 1) + P(Y = 2) = 0.7

   P (X = 1 OR (Y = 1 OR 2)) = P(X = 1) + P(Y = 1 OR 2) – (P(X = 1) AND (Y = 1 OR 2)) = 0.3 + 0.7 – 0.15 = 0.85

   (ALTERNATIVE: 1 – P(X = 0 AND Y = 0) – P(X = 2 AND Y = 0))