COMM 290

2020W1 midterm review session

Answer Key

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CMP

Simple LP: Algebraic Solution

You are a student running a business selling two combos of Pocky: Combo A earns you $5, consisting of one chocolate and one strawberry, while Combo B earns you $6, consisting of two chocolates and no strawberry. You have 20 chocolates in stock and 10 strawberries in stock. Assume there are no costs associated with this model.

1. What is the objective function? Is this a maximizing or minimizing model?

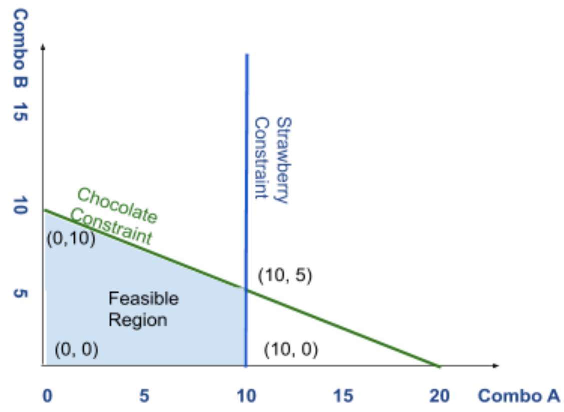
The objective function is the profit that the company makes by selling the two products. We construct the objective function by multiplying the number of each products sold (represented by variables A & B) by their selling price.
Therefore, the objective function is max 5A + 6B.
It is a maximizing model because the business is trying to maximize their profits.

2. List out all constraints in algebraic form. How many constraints are there?

There are four constraints.
Aside from the two stated in the prompt in regards to the limited amount of each types of pokeys, there are also two non-negativity constraints for the two products.
Chocolate: A + 2B <= 20 Strawberry: A <= 10
A >= 0
B >= 0

3. Draw out the constraints and label the optimal solution on the provided graph template. What is the optimal solution, and what is the profit at that optimal solution? What are the binding constraints?

After graphing the two constraints (with the non-negativity constraints being the two axis), we find the combination of A & B products at the interaction of each constraint. The number of each product and their profits are as follows:



# of Combo A	# of Combo B	Profit
0	0	0
0	10	60
10	5	80
10	0	50

The optimal solution is for the company to sell 10 combo A and 5 combo B, gaining a profit of $80. The two binding constraints are the amount of chocolate and strawberry.

4. Find the allowable increase and decrease of the coefficients for both Combo A and Combo B. (Do this by hand.)

1. Find the slope of the two binding constraints
    a. Chocolate: -1/2, strawberry: - infinity
2. Set -C1/C2 equal to the two slopes of the binding constraints
    a. C1 = objective coefficient of combo A = 5
    b. C2 = objective coefficient of combo B = 6
    c. Current slope = -C1/C2 = -5/6
3. Find allowable increase & decrease for C1 by keeping C2 constant

$$-\infty \leq -\frac{C1}{6} \leq -\frac{1}{2}$$

Multiply all sides by -6 (don't forget to change signs!)

    a. $$\infty \geq C1 \geq 3$$
    b. Since C1 is currently 5, and can be between 3 and infinity, the allowable increase and decrease for Combo A's objective coefficient are infinity (1E+30) and 2, respectively.
4. Find allowable increase & decrease for C2 by keeping C1 constant

$$-\infty \leq -\frac{5}{C2} \leq -\frac{1}{2}$$

Take the reciprocal of all sides (don't forget to change signs! Negative infinity is used or this does not make sense)

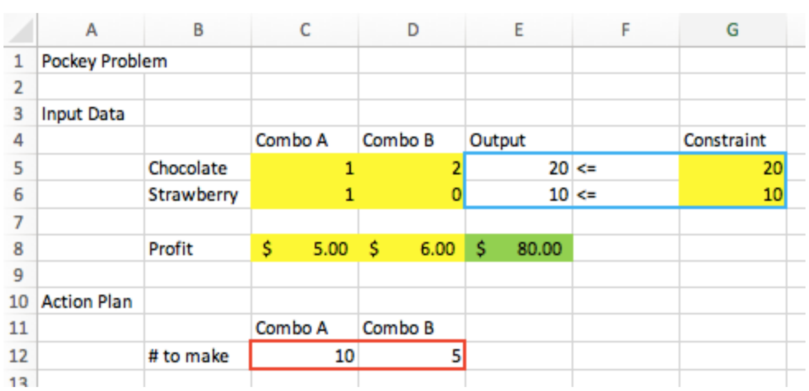
$$0 \geq -\frac{C2}{5} \geq -2$$

Multiply both sides by -5 (don't forget to change signs!)

    a. $$0 \leq C2 \leq 10$$
    b. Since C2 is currently 6, and can be between 0 and 10, the allowable increase and decrease for Combo B's objective coefficient are 4 and 6, respectively.

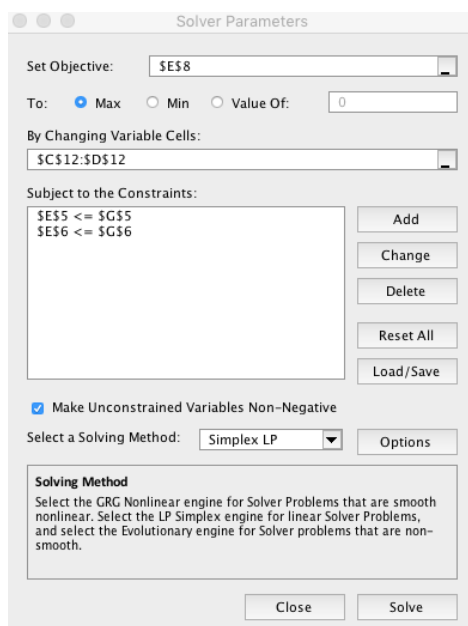
5. Find the allowable increase and decrease of all constraints. (Do this in Excel).

The excel model:



	A	B	C	D	E	F	G
1	Pockey Problem						
2							
3	Input Data						
4			Combo A	Combo B	Output		Constraint
5		Chocolate	1	2	20	<=	20
6		Strawberry	1	0	10	<=	10
7							
8		Profit	$ 5.00	$ 6.00	$ 80.00		
9							
10	Action Plan						
11			Combo A	Combo B			
12		# to make	10	5			
13							

The solver settings:



The sensitivity analysis:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	# to make Combo A	10	0	5	1E+30	2
D12	# to make Combo B	5	0	6	4	6

Constraints

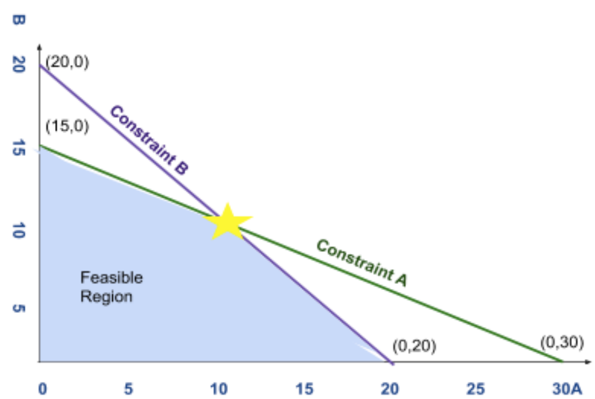
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E5	Chocolate Output	20	3	20	1E+30	10
E6	Strawberry Output	10	2	10	10	10

Simple LP: Graphing

Consider the following graph of an arbitrary linear programming model with the correct labelled optimal solution, feasible region. Assume this is a profit maximization model (That is, objective function in the form xA + yB).

1. Define the range of possible slopes for isoprofit lines that would lead to this optimal solution (marked by star). Give an example of a possible objective function.

The function of the two constraints are A + B <= 20 and 2B + A <= 30
The slopes of the two binding constraints are -1 and -1/2.
Therefore, -C1/C2 (slope of isoprofit line) of the objective function C1A + C2B must be between -1 and -1/2.
Therefore, a sample objective function would be max 2A + 3B.



2. Write two possible objective functions that would lead to multiple optima.

To have multiple optima, the slope of the isoprofit line must be equal to the slope of one of the constraints.
Therefore, any objective function C1A + C2B with –C1/C2 equal to either -1/2 or -1 would give you multiple optima.
Two example answers would be A + 2B and A + B.

3. Find the coordinates of the optimal solution. The two constraints are A + B = 20 and A + 2B = 30.

The coordinates of the optimal solution are the intersection of the two binding constraints. To find it, we set up a system of equation.

$A + B = 20$

$A + 2B = 30$

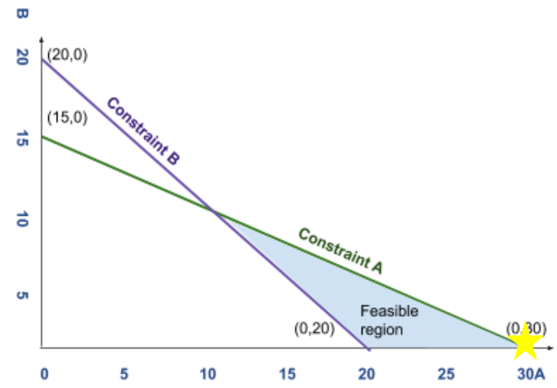
We can solve using substitution and rewrite the first equation as A = 20 – B

Substituting the expression of A into the second equation, we get 20 – B + 2B = 30.

Combining like term and simplifying: 20 + B = 30 , B = 10; A = 20 – 10 = 10.

4. Suppose the sign of Constraint B (<=) is changed to the >= sign. How will this change the feasible region? Assuming the objective function remains the same and the LP remains feasible, draw in the new feasible region and label the new optimal solution. If this LP becomes infeasible, explain why.

The slope of all constraints remains the same, but the feasible region had changed because the inequality of constraint B forces the feasible region to go outward (away from the origin). The original isoprofit line can now be pushed further away from the origin, making the new optimal solution (0,30) as marked by the star.



Solution Simple LP: Excel Solution

You are producing fruit juice producing four types of fruit juice: apple, citrus, pineapple and tropical. You have 20L of apple concentrate, 20L of orange concentrate and 10L of pineapple concentrate. You have 60L of water. The breakdown of each juice is as follows:

- **One bottle of apple juice requires 200mL of apple concentrate and 300mL water;**
- **One bottle of citrus juice requires 300mL of orange concentrate and 200mL water;**
- **One bottle of pineapple juice requires 250mL of pineapple concentrate and 250mL water;**
- **One bottle of tropical juice requires 150mL of pineapple, 100mL of orange, 50mL of apple and 200mL water.**

You profit $3 for every bottle of apple juice, $3 for every bottle of citrus juice, $6 for every bottle of pineapple juice and $5 for every bottle of tropical juice.

1. Complete the algebraic formulation for this problem. You do not have to solve for the optimal solution algebraically.

First, convert all unit into Litre.
Next define the variables: # of apple juice as A; citrus juice as C; pineapple juice as P and tropical juice as T.
Objective function: Max 3A + 3C + 6 P + 5T
Subjected to the following constraints:

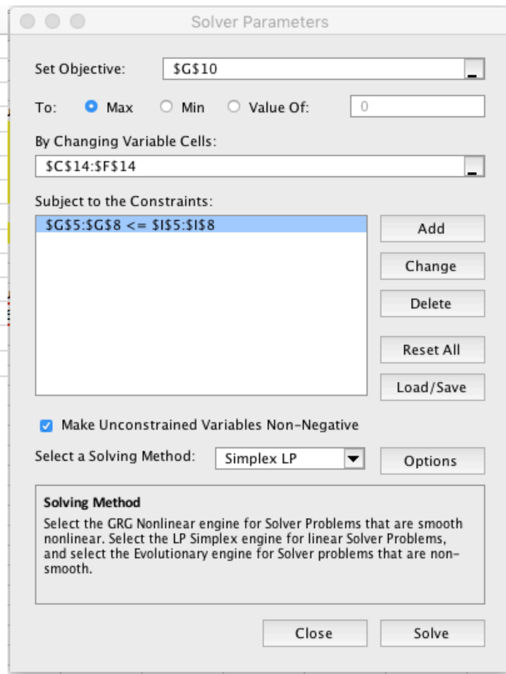
0.2A + 0.05T <= 20 (apple concentrate)
0.3C + 0.1T <= 20 (orange concentrate)
0.25P + 0.15T <= 10 (pineapple concentrate)
0.3A + 0.2C + 0.25P + 0.2T <= 60 (water)
A >= 0 (non-negativity)
C >= 0 (non-negativity)
P >= 0 (non-negativity)
T >= 0 (non-negativity)

2. Solve for optimal solution in excel. What is the amount of profit made under the optimal solution? How many constraints are there? What about binding constraints?

The excel model:

	A	B	C	D	E	F	G	H	I
1	Fruit Juice								
2									
3	Input Data								
4			Apple	Citrus	Pineapple	Tropical	Output		
5		Apple	0.2	0	0	0.05	20	<=	20
6		Orange	0	0.3	0	0.1	20	<=	20
7		Pineapple	0	0	0.25	0.15	10	<=	10
8		Water	0.3	0.2	0.25	0.2	53.3333333	<=	60
9									
10		Profit	$ 3.00	$ 3.00	$ 6.00	$ 5.00	$ 740.00		
11									
12	Action Plan								
13			Apple	Citrus	Pineapple	Tropical			
14		# to make	100	66.6666667	40	0			
15									

The solver setting:



The sensitivity analysis:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	# to make Apple	100	0	3	1E+30	1.4
D14	# to make Citrus	66.66666667	0	3	1E+30	1.05
E14	# to make Pineapple	40	0	6	1E+30	0.583333333
F14	# to make Tropical	0	-0.35	5	0.35	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G5	Apple Output	20	15	20	4.444444444	20
G6	Orange Output	20	10	20	10	20
G7	Pineapple Output	10	24	10	6.666666667	10
G8	Water Output	53.33333333	0	60	1E+30	6.666666667

The amount of profit made under the optimal solution is $740 (found under cell G10). There are a total of eight constraints, 4 of which are limited amount of concentration, and the other four are non-negativity constraints.
Four of those constraints are binding (non-negativity of tropical, apple concentrate, orange concentrate, pineapple concentrate).
We can find the binding constraints by observing the final amount of each ingredients used and the final number of products.

3. Under the optimal solution, how many bottles of each fruit juice will you produce?’

Under the optimal solution, you will produce 100 bottles of apple juice, 66.67 bottles of citrus juice, 40 bottles of pineapple juice and 0 bottles of tropical juice.
We can find this information under the action plan cells.

4. Consider the following sensitivity analysis for Fruit Juice and answer the following questions.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	Bottles to Make Apple		0	3	1E+30	1.4
D14	Bottles to Make Citrus		0	3	1E+30	1.05
E14	Bottles to Make Pineapple		0	6	1E+30	0.583333333
F14	Bottles to Make Tropical		-0.35	5	0.35	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G5	Apple Concentrate Total	20	15	20	4.444444444	20
G6	Orange Concentrate Total	20	10	20	10	20
G7	Pineapple Concentrate Total	10	24	10	6.666666667	10
G8	Water Total	53.33333333	0	60	1E+30	6.666666667

a. Let’s say a sudden decrease in demand for tropical fruit juice drops the profit of tropical juice to $3 per bottle. Will that change the optimal solution? Why or why not?

This will not change the optimal solution as the allowable decrease for the objective coefficient of tropical fruit juice is 1E+30, and the decrease of $2 in selling price is within that allowable range.

b. A drought has occurred and water supply has suddenly dropped from 60 litres to 45 litres. Will the optimal solution change? If yes, what is the new optimal solution, and how much profit will you make?

The new optimal solution will change since the decrease by 15 is outside of the allowable decrease range (6.6666). Therefore, it will be unsolvable by our sensitivity report.
The new optimal solution is to make 75.92 bottles of apple juice, 44.44 bottles of citrus juice, 0 bottles of pineapple juice and 66.66 bottles of tropical juice, resulting in $694.44 of profit.
Intuitively, the drop from 60 to 53.33 will not affect profit. Only the drop from 53.33 to 45 will.

c. Suppose due to high demand, you must make at least 40 bottles of tropical juice. What is the new optimal solution, and how much profit will you make? Is this new constraint binding?

The optimal solution is to make 90 bottles of apple juice, 53.33 bottles of citrus juice, 16 bottles of pineapple juice and 40 bottles of tropical juice, which results in $726 of profit. This new constraint is binding. (the effect of an additional constraint cannot be determined by the original sensitivity report)

Sensitivity Analysis

Consider the following sensitivity analysis for an unspecified maximization LP model. Some cells have had their numbers removed. Cells with a number attached and highlighted will be referred to in the questions in this section. Assume all questions are independent of one another.

Variable Cells							
	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
	C16	Decision 1	4500	0	0.33	0.01	0.016
	D16	Decision 2	0	0	0.33	1E+30	1E+30
	E16	Decision 3	62000	0	0.2	1E+30	0.017
	F16	Decision 4	0	-0.062	-8.81E-10	0.03	1E+30
Constraints							
	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
	C25	Constraint 1	(1)	0.65	5400	2501	1008
	C26	Constraint 2	[redacted]	0.55	21600	6500	5300
	C27	Constraint 3	[redacted]	0.14	0	12000	0
	C29	Constraint 4	[redacted]	0.62	7500	28000	7500
	C28	Constraint 5	62000	0.062	(2)	22000	27000

1. What should be the value within the cell (1)? Explain.

The value within cell (1) should be 5400 because the shadow price associated with the constraint is non-zero. Therefore, the constraint must be binding, resulting in a value of 5400 in cell (1) which is equivalent to the RHS value.

2. What should be the value within the cell (2)? Explain.

The value within cell (2) should be 62000 because similar to question 1, due to the presence of a shadow price the constraint must be binding. Therefore, the RHS must be equal to the final value which is 62000.

3. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.01?

An increase in 0.01 of the objective coefficient is within the allowable increase, so the optimal values remain the same. Since the objective coefficient has gone up by 0.01, the company now makes 0.01 more of profit for each decision unit, resulting in 45 of increased profit.

4. How much more profit would the firm earn if Decision 2's objective coefficient went up by 0.5?

Since Decision 2 is not used at all, and the increase of 0.5 is within its allowable increase, the company earns 0 extra profit.

5. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.01 and Decision 4's objective coefficient went up by 0.02?

Sensitivity analysis cannot predict the behavior of themodel when multiple variables are changed at the same time. Thus, it cannot be determined from sensitivity analysis alone.

6. How much more profit would the firm make if the RHS of Constraint 1 was increased by 2500?

The increase of 2500 is within Constraint 1's allowable increase, and a shadow price of 0.65 means for every additional constrained unit, the profit goes up by 0.65. Therefore, the company earns 0.65 * 2500 = 1625 additional profit.

7. How much more profit would the firm make if the RHS of Constraint 3 was increased by 1000?

Similar to Question 6, the 1000 increase is within Constraint 3's allowable increase, and Constraint 3 has a non-zero shadow price of 0.14. Therefore, the firm makes 1000 * 0.14 = 140 additional profit.

Blending Problems

You are the owner of a store for junk computer parts. You have CPUs, RAMs, and SSDs, which you can supply at the cost of $14.4, $12 and $9 each, respectively. You offer two types of blends: basic and premium. For basic, you will charge $33 for each piece of hardware, while for premium you will charge $36 per piece of hardware. You have 200 CPUs, 300 RAMs and 400 SSDs in inventory. However, there are a few guidelines you must follow:

- **Basic must contain:**
  - **At least 30% SSDs;**
  - **At most 50% RAMs;**
  - **At least 30% CPUs;**
- **Premium must contain:**
  - **At most 40% SSDs;**
  - **At least 35% RAMs;**
  - **At most 40% CPUs.**

You do not have to complete the excel model for this problem.

1. Consider the partially completed spreadsheet on the following page. There are some cells highlighted in blue which do not have their values filled in. What should be the best formula for each of the following cells labelled (a) to (g)?

	A	B	C	D	E	F	G	H
1	Computer Parts							
2								
3	Input Data							
4			Cost($)			Basic	Premium	
5		CPUs	$ 14.40		Revenue	$ 33.00	$ 36.00	
6		RAMs	$ 12.00					
7		SSDs	$ 9.00					
8								
9								
10	Action Plan							
11			Basic	Premium	Total		Constraint	
12		CPUs			(b)	<=	200	
13		RAMs			0	<=	300	
14		SSDs			0	<=	400	
15		Output	(a)	0				
16								
17								
18								
19	Blending Constraints							
20						Output		Constraint
21		Basic must be at least		30%	SSD	(c)	>=	(d)
22		Basic must be at most		50%	RAM	0	<=	0
23		Basic must be at least		30%	CPU	0	>=	0
24		Premium must be at most		40%	SSD	0	<=	(e)
25		Premium must be at least		35%	RAM	0	>=	0
26		Premium must be at most		40%	CPU	0	<=	0
27								
28	Revenue/Cost							
29								
30			Basic	Ultra				
31		CPUs	$ -	$ -				
32		RAMs	(f)	$ -				
33		SSDs	$ -	$ -				
34			Revenue	(g)				
35			Profit	$ -				

(a)= SUM(C12:C14) (b) = SUM(C12:D12)
(c) = C14 (d) = D21 * C1
(e) = D24 * C16 (f) = C13 * $C6
(g) = SUMPRODUCT(C15:D15)

2. How much of each bulk should you produce to maximize profit? What is the breakdown of each part among each blend?

The basic blend should contain 40 CPUs and 93.33 SSDs. The Premium blend should contain 160 CPUs, 300 RAMS and 306.67 SSDs.
The completed excel model is shown on the following page.

3. Suppose you are doing an algebraic formulation for this blending problem. Write down all the blending constraints in algebraic form. Use the following labels: BC, BR, BS, PC, PR, PS, with the first letter representing the blend and the second letter representing the part.

0.7BS – 0.3BR – 0.3BC >= 0 (Basic must be at least 30% SSD)
0.5BR – 0.5BS – 0.5BC <= 0 (Basic must be at most 50% RAM)
0.7BC – 0.3BR – 0.3BS >= 0 (Basic must be at least 30% CPU)
0.6PS – 0.4PC – 0.4PR <= 0 (Premium must be at most 40% SSD)
0.65PR – 0.35PC – 0.35PS >= 0 (Premium must be at least 35% RAM)
0.6PC – 0.4PS – 0.4PR <= 0 (Premium must be at most 40% CPU)

The completed excel model:

	A	B	C	D	E	F	G	H
1	Computer Parts							
2								
3	Input Data							
4			Cost			Basic	Premium	
5		CPUs	$ 14.40		Revenue	$ 33.00	$ 36.00	
6		RAMs	$ 12.00					
7		SSDs	$ 9.00					
8								
9	Action Plan							
10			Basic	Premium	Total		Constraint	
11		CPUs	40	160	200	<=	200	
12		RAMS	0	300	300	<=	300	
13		SSDs	93.3333333	306.666667	400	<=	400	
14		Output	133.333333	766.666667				
15								
16	Blending Constraints							
17						Output		
18		Basic must be at least		0.3	SSD	93.333333	>=	40
19		Basic must be at most		0.5	RAM	0	<=	66.666667
20		Basic must be at least		0.3	CPU	40	>=	40
21		Premium must be at most		0.4	SSD	306.66667	<=	306.66667
22		Premium must be at least		0.35	RAM	300	>=	268.33333
23		Premium must be at most		0.4	CPU	160	<=	306.66667
24								
25	Revenue/Cost							
26			Basic	Ultra				
27		CPUs	$ 576.00	$ 2,304.00				
28		RAMS	$ -	$ 3,600.00				
29		SSDs	$ 840.00	$ 2,760.00				
30			Revenue	32000				
31			Profit	$ 21,920.00				
32								

4. Consider the sensitivity analysis on the following page for Computer Parts with the optimal solution blacked out and answer the following questions.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	CPUs Basic	[redacted]	0	18.6	3	1E+30
D12	CPUs Premium	[redacted]	0	21.6	1E+30	3
C13	RAMs Basic	[redacted]	-3	21	3	1E+30
D13	RAMs Premium	[redacted]	0	24	1E+30	3
C14	SSDs Basic	[redacted]	0	24	3	24
D14	SSDs Premium	[redacted]	0	27	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E12	CPUs Total	[redacted]	21.6	200	146.6666667	160
E13	RAMs Total	[redacted]	24	300	1E+30	31.66666667
E14	SSDs Total	[redacted]	24	400	1E+30	53.33333333
F21	SSD Output	[redacted]	0	40	53.33333333	1E+30
F22	RAM Output	[redacted]	0	66.66666667	1E+30	66.66666667
F23	CPU Output	[redacted]	-3	40	160	40
F24	SSD Output	[redacted]	3	306.6666667	53.33333333	306.6666667
F25	RAM Output	[redacted]	0	268.3333333	31.66666667	1E+30
F26	CPU Output	[redacted]	0	306.6666667	1E+30	146.6666667

a. One non-negativity constraint is binding. How many of the other constraints are binding?

Recall a binding constraint will always have an associated shadow price. Therefore, there must be 5 binding constraints. We can also infer this based on the fact that an there are six total decisions, so there must be six total constraints.

b. Due to an increase in demand, the price of the basic blend has increased from $33 to $36. Will this change the optimal solution? If yes, by how much will this increase the value in the target cell? If not, why not?

Cannot be evaluated as this changes multiple coefficients.

Scheduling Problems

You are the manager of a 24-hour fast-food restaurant on campus. Your restaurant offers six labour shifts per 24-hour period, starting at 12am, 4am, 8am, 12pm, 4pm, 8pm, and 12pm. You have access to workers who work two consecutive shifts a day. Due to fluctuations in demand, your required labour at different time periods is as follows:

- **12am-4am: 3 workers**
- **4am-8am: 4 workers**
- **8am-12pm: 7 workers**
- **12pm-4pm: 8 workers**
- **4pm-8pm: 6 workers**
- **8pm-12am: 5 workers**

Find the scheduling method that will use the minimum amount of workers. Produce a sensitivity analysis. Assume all sensitivity report questions are fully independent.

1. Is this a maximizing or minimizing model? What are you trying to maximize or minimize?

This is a minimizing model, and you are trying to minimize the amount of workers used.

2. What is the optimal solution? How many binding constraints are there? What about non-binding?

The excel model:

	A	B	C	D	E	F	G	H	I
1	Food Service								
2									
3					Time Periods Covered				
4			12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12pm	Labour
5		12am	4	4					4
6		4am		0	0				0
7		8am			7	7			7
8		12pm				1	1		1
9		4pm					5	5	5
10		8pm	0					0	0
11	Start Time	Supply	4	4	7	8	6	5	17
12			>=	>=	>=	>=	>=	>=	
13		Demand	3	4	7	8	6	5	

The optimal solution is to have 4 people come in at 12am, 7 people come in at 8am, 1 person comes in at 12pm, and 5 people come in at 4pm.

There are 12 constraints in total, the six four-hour demand and six non-negativity constraints.

The six binding constraints are: staff requirements at 4am-8am, 8am-12pm, 12pm-4pm, 4pm-8pm, 8pm-12pm, and the non-negativity constraints at 4am and 8pm.

3. Due to an overnight frat party, your demand for workers at 12am-4am goes up to four. Will this affect your optimal solution? Why or why not? If yes, what is the new optimal solution? Do not modify your LP.

The sensitivity analysis is shown on the second page. Looking at the sensitivity analysis, there is an allowable increase of 1 for the 12am-4am labour demand. Since the demand at that time is non-binding, it will remain non-binding after the increase to 4 and thus there is no change in the optimal solution. Since this constraint becomes binding, any further changes will affect your solution.

4. Suppose the demand for workers at 4am-8am increases by one. How will this affect the optimal solution?

Sensitivity Report:
Consider that the labour demand at 4am-8am has a shadow price of 1 and an allowable increase of infinity. Therefore, if the demand goes up by 1, you will require 1 additional worker.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
I5	12am Labour	4	0	1	0	1
I6	4am Labour	0	0	1	1	0
I7	8am Labour	7	0	1	0	1
I8	12pm Labour	1	0	1	1	0
I9	4pm Labour	5	0	1	0	1
I10	8pm Labour	0	0	1	1E+30	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C11	Supply 12am-4am	4	0	3	1	1E+30
D11	Supply 4am-8am	4	1	4	1E+30	1
E11	Supply 8am-12pm	7	0	7	1	0
F11	Supply 12pm-4pm	8	1	8	0	1
G11	Supply 4pm-8pm	6	0	6	1	0
H11	Supply 8pm-12pm	5	1	5	0	1

Transportation Problems

You are the manager of a few Canada post branches in Vancouver. You have three branches under your control: Robson, Pine and Oak, and you must deliver units of identical supplies to UBC, YVR Airport and Oakridge center. The supply and demand at each location is as follows:

- **Robson holds 7 units, Pine holds 16 and Oak holds 13.**
- **UBC requires 17, YVR Airport requires 5 and Oakridge requires 14.**

The shipping costs are as follows:

		Deliver To:		
		UBC	YVR Airport	Oakridge
Deliver From:	Robson	$4.00	$4.50	$2.50
	Pine	$3.00	$4.00	$2.50
	Oak	$3.50	$3.00	$2.00

Solve for the optimal solution, and produce a sensitivity analysis.

1. What is the optimal solution, and how many constraints are binding?

The finished excel model:
The optimal solution is for UBC to receive 16 units from Pine and 1 from Oak; YVR to receive all 5 units from Oak and for Oakrighe to receive 6 units from Robson and 7 from Oak.
There are 10 binding constraints, 4 of which are non-negativity constraints.

	A	B	C	D	E	F	G	H	I
1	Transportation								
2	Input Data			Deliver To:					
3		Delvier from:		UBC	YVR	oakridge			
4			Robson	$ 4.00	$ 4.50	$ 2.50			
5			Pine	$ 3.00	$ 4.00	$ 2.50			
6			Oak	$ 3.50	$ 3.00	$ 2.00			
7									
8	Action Plan								
9				Deliver To:					
10		Delvier from:		UBC	YVR	oakridge	Total		Supply
11			Robson	-	-	7.00	7.00	<=	7
12			Pine	16.00	-	-	16.00	<=	16
13			Oak	1.00	5.00	7.00	13.00	<=	13
14			Total	17.00	5.00	14.00	98		
15				>=	>=	>=			
16			Demand	17	5	14			

2. What will happen to the LP if suddenly, an explosion happens at the Robson branch and three of the seven units in stock are destroyed?

This will make the LP infeasible, as the number of units in stock is equal to demand at this point in the model. If three parcels in stock are removed, then there will be a shortage in supply and thus making the LP infeasible.

3. What are the objective coefficients in this model?
The objective coefficients in this model are the nine costsassociated with each shipping route.

4. Refer to your produced sensitivity analysis for the next questions. If you cannot answer the question without running Solver again, please answer “we don’t know for sure.”

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D11	Robson UBC	0	0	4	1E+30	0
E11	Robson YVR	0	1	4.5	1E+30	1
F11	Robson oakridge	7	0	2.5	0	0.5
D12	Pine UBC	16	0	3	1	1E+30
E12	Pine YVR	0	1.5	4	1E+30	1.5
F12	Pine oakridge	0	1	2.5	1E+30	1
D13	Oak UBC	1	0	3.5	0	1
E13	Oak YVR	5	0	3	1	3.5
F13	Oak oakridge	7	0	2	0.5	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D14	Total UBC	17	4	17	0	1
E14	Total YVR	5	3.5	5	0	5
F14	Total oakridge	14	2.5	14	0	7
G11	Robson Total	7	0	7	1E+30	0
G12	Pine Total	16	-1	16	1	0
G13	Oak Total	13	-0.5	13	7	0

a. Is there evidence of multiple optima in this LP?
Since there are many objective coefficients with a 0 allowable increase/decrease, there is indeed evidence of multiple optima.

b. Suppose due to the acquisition of a new truck, it now costs $2.50 to deliver from Oak to YVR airport. Will this change the optimal solution? What will be the new value in the target cell?
The decrease in $0.50 is within the allowable decrease of 3.5. The optimal solution does not change, and the new value of the target cell decreases from $98 to $95.5

c. Suppose Pine has increased its supply by one. How will this affect the target cell under the optimal solution?
An increase of 1 is within the allowable increase for the supply at Pine. The shadow price is a non-zero value of -1; therefore, if the constraint goes up by 1, the target cell will go down by $1.

d. Suppose Pine increases its supply by five to a total of 21. What will be the new amount of parcels shipped from Pine?
We don’t know for sure, as the increase of 5 is beyond the allowable increase.

e. Suppose Oak increases its supply by five to a total of 18. What will be the new target cell value under the new optimal solution?
Since Oak’s supply has an allowable increase of 5 and a shadow price of -0.5, the new target cell value will decrease by 5 * 0.5 to a new value of $95.50.

f. Due to new hires, the cost of all shipments from Oak have been reduced by $0.10. What will be the new optimal solution?
We don’t know for sure, since this will change three objective coefficients at the same time.

g. Shipments from Pine to UBC are now free. What are the new optimal solution and target cell value?
The decrease from 16 to 0 is within the allowable decrease of 1E+30, so the optimal solution remains the same. Since shipments from Pine to UBC would cost $48 in the original model, that amount will be subtracted from the target cell since these shipments are now free. The new target cell value will be $50.