COMM 290

2020W1 midterm review session

PREPARED BY

Tony Chen

Instructed By

Anna Feng

commerce undergraduate society

cus.cmp.ca
facebook.com/ubccmp
@ubccmp
twitter.com/ubccmp

TABLE OF CONTENT

Simple LP: Algebraic Solution

Definition & Key Points

Objective function – the function describing the problem's objective which you are attempting to maximize or minimize.
Optimal solution – the best set of decisions that maximizes the objective function while remaining within the constraints.
Constraint – A limitation of some sort posed with the problem. Enclosed by blue border.
Multiple optima – There are multiple sets of optimal solutions.
Non-negativity constraint – A constraint that makes sure a "decision" cannot be a negative value.

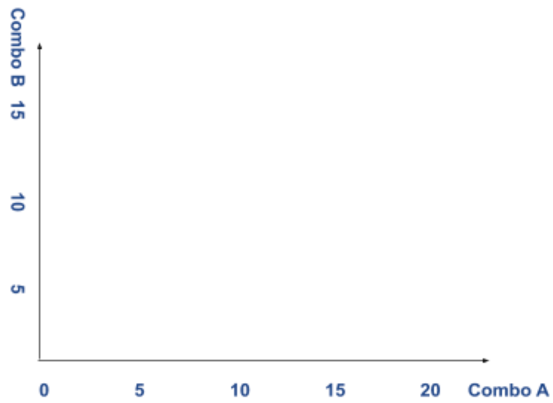
Practice Problems

You are a student running a business selling two combos of Pocky: Combo A earns you \$5, consisting of one chocolate and one strawberry, while Combo B earns you \$6, consisting of two chocolates and no strawberry. You have 20 chocolates in stock and 10 strawberries in stock. Assume there are no costs associated with this model.

1. What is the objective function? Is this a maximizing or minimizing model?

2. List out all constraints in algebraic form. How many constraints are there?

3. Draw out the constraints and label the optimal solution on the provided graph template. What is the optimal solution, and what is the profit at that optimal solution? What are the binding constraints?



4. Find the allowable increase and decrease of the coefficients for both Combo A and Combo B. (Do this by hand.)

5. Find the allowable increase and decrease of all constraints. (Do this in Excel).

Simple LP: Graphing

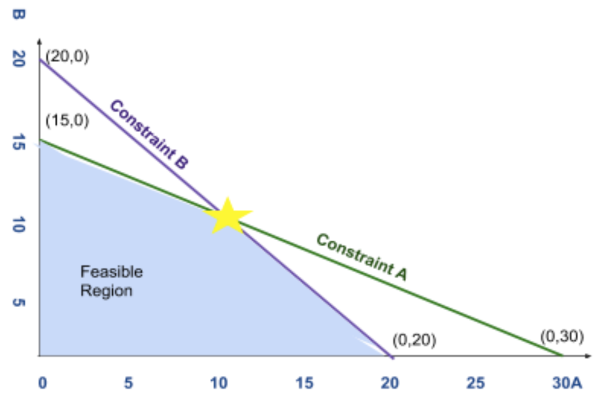
Definition & Key Points

Redundant constraint – A constraint which does not affect the feasible region.
Feasible region – The region in which all solutions are valid and subject to the constraints.
Infeasible solution – There is no feasible region associated with your LP.
Unbounded solution – The feasible region is infinitely large, usually due to lack of a constraint, and the objective function behaves such that you are moving the isoprofit line outwards indefinitely.

Practice Problems

Consider the following graph of an arbitrary linear programming model with the correct labelled optimal solution, feasible region. Assume this is a profit maximization model (That is, objective function in the form xA + yB).

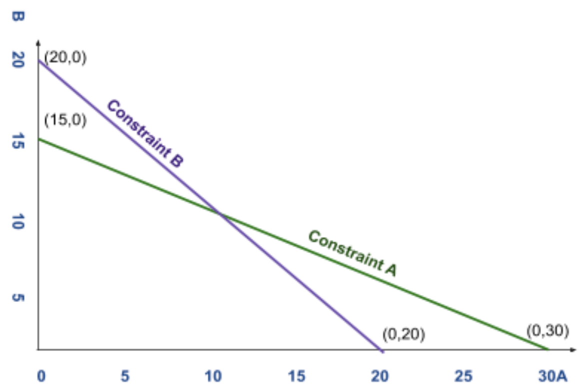
1. Define the range of possible slopes for isoprofit lines that would lead to this optimal solution (marked by a star). Give an example of a possible objective function.



2. Write two possible objective functions that would lead to multiple optima.

3. Find the coordinates of the optimal solution. The two constraints are A + B = 20 and A + 2B = 30.

4. Suppose the sign of Constraint B (<=) is changed to the >= sign. How will this change the feasible region? Assuming the objective function remains the same and the LP remains feasible, draw in the new feasible region and label the new optimal solution. If this LP becomes infeasible, explain why.



Solution Simple LP: Excel Solution

Definition & Key Points

Relative Reference – A reference in the form A1 that will change when auto-filled to other cells.
Absolute Reference – A reference in the form A1 that will not change when auto-filled to other cells.
Target Cell – Contains the output of the objective function and is highlighted in green.
Input Data – The data given to you as part of a problem. Usually highlighted in yellow.
Action Plan – The "action" you will take to solve the problem, which will be indicated inside red borders on excel.

Practice Problems

You are producing fruit juice producing four types of fruit juice: apple, citrus, pineapple and tropical. You have 20L of apple concentrate, 20L of orange concentrate and 10L of pineapple concentrate. You have 60L of water. The breakdown of each juice is as follows:

- **One bottle of apple juice requires 200mL of apple concentrate and 300mL water;**
- **One bottle of citrus juice requires 300mL of orange concentrate and 200mL water;**
- **One bottle of pineapple juice requires 250mL of pineapple concentrate and 250mL water;**
- **One bottle of tropical juice requires 150mL of pineapple, 100mL of orange, 50mL of apple and 200mL water.**

You profit $3 for every bottle of apple juice, $3 for every bottle of citrus juice, $6 for every bottle of pineapple juice and $5 for every bottle of tropical juice.

1. Complete the algebraic formulation for this problem. You do not have to solve for the optimal solution algebraically.

2. Solve for optimal solution in excel. What is the amount of profit made under the optimal solution? How many constraints are there? What about binding constraints?

3. Under the optimal solution, how many bottles of each fruit juice will you produce?'

4. Consider the following sensitivity analysis for Fruit Juice and answer the following questions.

a. Let's say a sudden decrease in demand for tropical fruit juice drops the profit of tropical juice to $3 per bottle. Will that change the optimal solution? Why or why not?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	Bottles to Make Apple	[redacted]	0	3	1E+30	1.4
D14	Bottles to Make Citrus	[redacted]	0	3	1E+30	1.05
E14	Bottles to Make Pineapple	[redacted]	0	6	1E+30	0.583333333
F14	Bottles to Make Tropical	[redacted]	-0.35	5	0.35	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G5	Apple Concentrate Total	20	15	20	4.444444444	20
G6	Orange Concentrate Total	20	10	20	10	20
G7	Pineapple Concentrate Total	10	24	10	6.666666667	10
G8	Water Total	53.33333333	0	60	1E+30	6.666666667

b. A drought has occurred and water supply has suddenly dropped from 60 litres to 45 litres. Will the optimal solution change? If yes, what is the new optimal solution, and how much profit will you make?

c. Suppose due to high demand, you must make at least 40 bottles of tropical juice. What is the new optimal solution, and how much profit will you make? Is this new constraint binding?

Sensitivity Analysis

Definition & Key Points

RHS Allowable Increase/Decrease of a Binding Constraint –Range where the right-hand side of the constraint may move while keeping the constraint binding.
RHS Allowable Increase/Decrease of a Non-Binding Constraint – Range where the right-hand side of the constraint may move while keeping the constraint non-binding.
Allowable Increase/Decrease of an objective coefficient – Range in which the objective coefficient may move without disrupting the optimal solution.
Shadow Price – The increase in the value of the target cell for every one-unit increase of the RHS of a constraint.

Practice Problems

Consider the following sensitivity analysis for an unspecified maximization LP model. Some cells have had their numbers removed. Cells with a number attached and highlighted will be referred to in the questions in this section. Assume all questions are independent of one another.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C16	Decision 1	4500	0	0.33	0.01	0.016
D16	Decision 2	0	0	0.33	1E+30	1E+30
E16	Decision 3	62000	0	0.2	1E+30	0.017
F16	Decision 4	0	-0.062	-8.81E-10	0.03	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C25	Constraint 1	(1)	0.65	5400	2501	1008
C26	Constraint 2	[redacted]	0.55	21600	6500	5300
C27	Constraint 3	[redacted]	0.14	0	12000	0
C29	Constraint 4	[redacted]	0.62	7500	28000	7500
C28	Constraint 5	62000	0.062	(2)	22000	27000

1. What should be the value within the cell (1)? Explain.

2. What should be the value within the cell (2)? Explain.

3. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.01?

4. How much more profit would the firm earn if Decision 2's objective coefficient went up by 0.5?

5. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.01 and Decision 4's objective coefficient went up by 0.02?

6. How much more profit would the firm make if the RHS of Constraint 1 was increased by 2500?

7. How much more profit would the firm make if the RHS of Constraint 3 was increased by 1000?

Blending Problems

You are the owner of a store for junk computer parts. You have CPUs, RAMs, and SSDs, which you can supply at the cost of $14.4, $12 and $9 each, respectively. You offer two types of blends: basic and premium. For basic, you will charge $33 for each piece of hardware, while for premium you will charge $36 per piece of hardware. You have 200 CPUs, 300 RAMs and 400 SSDs in inventory. However, there are a few guidelines you must follow:

- **Basic must contain:**
  - **At least 30% SSDs;**

- ○ **At most 50% RAMs;**
- ○ **At least 30% CPUs;**
- ● **Premium must contain:**
  - ○ **At most 40% SSDs;**
  - ○ **At least 35% RAMs;**
  - ○ **At most 40% CPUs.**

You do not have to complete the excel model for this problem.

1. Consider the partially completed spreadsheet on the following page. There are some cells highlighted in blue which do not have their values filled in. What should be the best formula for each of the following cells labelled (a) to (g)?

(a) (b) (c) (d)
(e) (f) (g)

2. How much of each bulk should you produce to maximize profit? What is the breakdown of each part among each blend?

	A	B	C	D	E	F	G	H
1	Computer Parts							
2								
3	Input Data							
4			Cost($)			Basic	Premium	
5		CPUs	$ 14.40		Revenue	$ 33.00	$ 36.00	
6		RAMs	$ 12.00					
7		SSDs	$ 9.00					
8								
9								
10	Action Plan							
11			Basic	Premium	Total		Constraint	
12		CPUs			(b)	<=	200	
13		RAMs			0	<=	300	
14		SSDs			0	<=	400	
15		Output	(a)	0				
16								
17								
18								
19	Blending Constraints							
20						Output		Constraint
21		Basic must be at least		30%	SSD	(c)	>=	(d)
22		Basic must be at most		50%	RAM	0	<=	0
23		Basic must be at least		30%	CPU	0	>=	0
24		Premium must be at most		40%	SSD	0	<=	(e)
25		Premium must be at least		35%	RAM	0	>=	0
26		Premium must be at most		40%	CPU	0	<=	0
27								
28	Revenue/Cost							
29								
30			Basic	Ultra				
31		CPUs	$ -	$ -				
32		RAMs	(f)	$ -				
33		SSDs	$ -	$ -				
34			Revenue	(g)				
35			Profit	$ -				

3. Suppose you are doing an algebraic formulation for this blending problem. Write down all the blending constraints in algebraic form. Use the following labels: BC, BR, BS, PC, PR, PS, with the first letter representing the blend and the second letter representing the part.

4. Consider the sensitivity analysis on the following page for Computer Parts with the optimal solution blacked out and answer the following questions.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	CPUs Basic		0	18.6	3	1E+30
D12	CPUs Premium		0	21.6	1E+30	3
C13	RAMs Basic		-3	21	3	1E+30
D13	RAMs Premium		0	24	1E+30	3
C14	SSDs Basic		0	24	3	24
D14	SSDs Premium		0	27	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E12	CPUs Total		21.6	200	146.6666667	160
E13	RAMs Total		24	300	1E+30	31.66666667
E14	SSDs Total		24	400	1E+30	53.33333333
F21	SSD Output		0	40	53.33333333	1E+30
F22	RAM Output		0	66.66666667	1E+30	66.66666667
F23	CPU Output		-3	40	160	40
F24	SSD Output		3	306.6666667	53.33333333	306.6666667
F25	RAM Output		0	268.3333333	31.66666667	1E+30
F26	CPU Output		0	306.6666667	1E+30	146.6666667

a. One non-negativity constraint is binding. How many of the other constraints are binding?

b. Due to an increase in demand, the price of the basic blend has increased from $33 to $36. Will this change the optimal solution? If yes, by how much will this increase the value in the target cell? If not, why not?

Scheduling Problems

You are the manager of a 24-hour fast-food restaurant on campus. Your restaurant offers six labour shifts per 24-hour period, starting at 12am, 4am, 8am, 12pm, 4pm, 8pm, and 12pm. You have access to workers who work two consecutive shifts a day. Due to fluctuations in demand, your required labour at different time periods is as follows:

- **12am-4am: 3 workers**
- **4am-8am: 4 workers**
- **8am-12pm: 7 workers**
- **12pm-4pm: 8 workers**
- **4pm-8pm: 6 workers**
- **8pm-12am: 5 workers**

Find the scheduling method that will use the minimum amount of workers. Produce a sensitivity analysis. Assume all sensitivity report questions are fully independent.

1. Is this a maximizing or minimizing model? What are you trying to maximize or minimize?

2. What is the optimal solution? How many binding constraints are there? What about non-binding?

3. Due to an overnight frat party, your demand for workers at 12am-4am goes up to four. Will this affect your optimal solution? Why or why not? If yes, what is the new optimal solution? Do not modify your LP.

4. Suppose the demand for workers at 4am-8am increases by one. How will this affect the optimal solution?

Transportation Problems

You are the manager of a few Canada post branches in Vancouver. You have three branches under your control: Robson, Pine and Oak, and you must deliver units of identical supplies to UBC, YVR Airport and Oakridge center. The supply and demand at each location is as follows:

- **Robson holds 7 units, Pine holds 16 and Oak holds 13.**
- **UBC requires 17, YVR Airport requires 5 and Oakridge requires 14.**

The shipping costs are as follows:

		Deliver To:		
		UBC	YVR Airport	Oakridge
Deliver From:	Robson	$4.00	$4.50	$2.50
	Pine	$3.00	$4.00	$2.50
	Oak	$3.50	$3.00	$2.00

Solve for the optimal solution, and produce a sensitivity analysis.

1. What is the optimal solution, and how many constraints are binding?

2. What will happen to the LP if suddenly, an explosion happens at the Robson branch and three of the seven units in stock are destroyed?

3. What are the objective coefficients in this model?

4. Refer to your produced sensitivity analysis for the next questions. If you cannot answer the question without running Solver again, please answer “we don’t know for sure.”

   a. Is there evidence of multiple optima in this LP?

   b. Suppose due to the acquisition of a new truck, it now costs $2.50 to deliver from Oak to YVR airport. Will this change the optimal solution? What will be the new value in the target cell?

   c. Suppose Pine has increased its supply by one. How will this affect the target cell under the optimal solution?

d. Suppose Pine increases its supply by five to a total of 21. What will be the new amount of parcels shipped from Pine?

e. Suppose Oak increases its supply by five to a total of 18. What will be the new target cell value under the new optimal solution?

f. Due to new hires, the cost of all shipments from Oak have been reduced by $0.10. What will be the new optimal solution?

g. Shipments from Pine to UBC are now free. What are the new optimal solution and target cell value?