

# **COMMERCE MENTORSHIP PROGRAM**

# MIDTERM REVIEW SESSION MATH 104



**PREPARED BY** 

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### **Problems with Limits:**

1. Evaluate:

a) 
$$\lim_{x \to 11} \frac{2x^2 - 21x - 11}{x^2 - 6x - 55}$$

$$= \lim_{x \to 11} \frac{(2x+1)}{(x+5)} = \frac{(2(11)+1)}{(11+5)} = \frac{23}{16}$$

Answer:

Answer:

-(0)

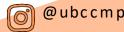
**b)**  $\lim_{x \to 0} \frac{x}{5 - \sqrt{x + 25}}$ 

$$= \lim_{x \to 0} \frac{x(5+\sqrt{x+25})}{-x} = \lim_{x \to 0} -5 - \sqrt{x+25} = -5 - \sqrt{0+25} = -5 - 5 = -10$$

c) 
$$\lim_{x\to 2} \frac{2x^2-7x-4}{x+4}$$

$$=\frac{1 \text{ im } 8-14-4}{6}$$











d) 
$$\lim_{x \to \sqrt{2}} \frac{x-2}{x(x-\sqrt{2})}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x+57)(x-57)}{x(x-57)}$$

$$= \lim_{X \to 52} (x+52) = (52+52) = 252 = 2$$

2. Evaluate the following limit **if** it exists. If it does not exist, state so.

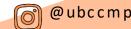
$$f(x) = \begin{cases} -2(x+1) + 2 & x < 0 \\ e^x + 1 & x \ge 0 \end{cases}$$

$$\lim_{x\to 0} f(x)$$
 exists if  $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} f(x)$ 

$$= \lim_{x\to 0} -2(x+1)+2$$

The limit does not exist











### **Continuity Problem:**

3. Given the restriction of a > 1, find the values for a and b such that f(x) is continuous for all real numbers

$$f(x) \begin{cases} (x-2)^2 + b & x > 2\\ \frac{1}{2}x + a & -2 \le x \le 2\\ -(x+a)^2 + 3 & x < -2 \end{cases}$$

Solving for a:

When 
$$x = -2$$

$$\frac{1}{2}(-2) + \alpha = -(-2+\alpha)^2 + 3$$

$$a^2 3a = 0$$

$$a(a-3)=0$$

$$a>1$$
:  $a=3$ 

$$\alpha = 3$$

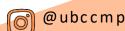
To be continuous, each piece has to connect to each other

Solving for b:  
when 
$$x = 2$$
  
 $2x+(3) = (x-2)^2+b$   
 $2(2)+3 = (2-2)^2+6$   
 $1+3 = b$   
 $b=4$ 

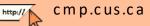
$$a = 3$$

$$b = 4$$









### **Using IVT:**

4. Prove that this equation has a root using IVT

$$f(x) = -2x^3 - 3x^2 + 4$$
  
The equation has a roof when  $f(x) = 0$   
First find two points, one above and one below  $f(x) = 0$ 

Example:

when 
$$x=0$$
  $f(0) = -2(0)^3 - 3(0)^2 + 4 = 4$   
when  $x=1$   $f(1) = -2(1)^3 - 3(1)^2 + 4 = -1$   
 $f(1) < 0 < f(0)$ 

Since f(x) is continuous, because of IVT we can say there exists an x value between 0 and 1 such that f(x) would be 0.

.. flx) has a root.



### **Problems involving Differentiation:**

5.

a) Carefully state the limit definition of the derivative of a function y = f(x)

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
, where the limit exists

b) Use the limit definition of the derivative to find f'(4) for the following function. No marks will be given for the use differentiation rules.

$$f(x) = x^{2} + 2$$

$$f(x+h) = (x+h)^{2} + \lambda$$

$$= x^{2} + \lambda x h + h^{2} + \lambda$$

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x^{2}+1xh+h^{2}+1) - (x^{2}+2)}{h}$$

$$= \lim_{h \to 0} \frac{2xh+h^{2}}{h}$$



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6. Let 
$$g(x) = [f(x)]^3 + 5e^{f(x)} + 2$$
,  $f(2) = 1$ ,  $f'(2) = 3$ 

Find g'(2)

$$g(x) = [f(x)]^{3} + 5e^{f(x)} + 2$$

$$g'(x) = 3f'(x) [f(x)]^{2} + 5f'(x)e^{f(x)}$$

$$g'(2) = 3f'(2) [f(2)]^{2} + 5f'(2)e^{f(2)}$$

$$= 3(3)(1)^{2} + 5(3)e^{f(2)}$$

$$= 29 + |5e^{f(2)}|$$



\$15

## **Lengthier Problems:**

7. Mr. Smith sells crates of watermelons. Currently he sells 70 crates of watermelons for \$20. He purchases a watermelon packager that costs \$100. Each crate \$20 to produce. His customers are price sensitive. As the price increases, less people will buy. He estimates that every 2\$ increase in price results in 4 less customers purchasing watermelons.

a) Find the linear demand function for the crates as a function of quantity (q)

$$\frac{\Delta p}{\Delta q} = \frac{2}{-4} = -\frac{1}{2}$$

$$p = \frac{\Delta p}{\Delta q} = \frac{4}{5}$$

$$p = \frac{4}{5}$$

Solving for 
$$b$$
:
$$(70,20)$$

$$20 = -\frac{1}{2}(70) + b$$

$$20 = -35 + b$$

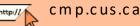
$$b = 55$$

b) Find the Revenue function as a function of quantity (q)  $R(a) = p \cdot a \quad (price + fines a confly)$   $P = -\frac{1}{2}a + \frac{55}{3}a$   $R(a) = \left(-\frac{1}{2}a + \frac{55}{3}\right)a$ 









c) How much should Mr. Smith charge for each crate of watermelons to maximize his profit?

Profit = 
$$P(a) = R(a) - C(a)$$
 (revenue - cost)

 $R(a) = -\frac{1}{2}a^{2} + 55a$  (from part b)

 $C(a) = 15a + 100$  (10 per crote, 100 for machine)

•  $P(a) = -\frac{1}{2}a^{2} + 55a - 15a - 100$ 
 $= -\frac{1}{2}a^{2} + 40a - 100$ 
 $P'(a) = (-\frac{1}{2})(2)(a) + 40$   $P'(a) = 0$ 
 $= -a + 40$   $-a + 40$ 
 $a = 40$  (optimal quartity)

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 $a = -\frac{1}{2}a + 55$  (price furtion from a)

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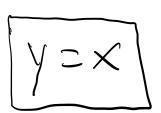
8. Find the equation of the tangent line to 
$$f(x) = 2x^2 + 5x + 2$$
 when  $x = -1$ 

$$f'(x) = 4x + 5$$

$$f(-1) = 2(-1)^2 + 5(-1) + 2 = 2 - 5 + 2 = -1$$

When 
$$x = -(f(x) = -1)$$

$$... y = mx + b (-1, -1) m = 1$$











### **Challenge Question:**

9. Find the equation of the line that passes through (0, -3) and is tangent to the graph of

Let 
$$(a, f(a))$$
 be the point the tangent  
line intersects  $f(x)$   

$$\frac{\Delta y}{\Delta x} = \frac{f(a) - (-3)}{a - 0} = \frac{f(a) + 3}{a} = \frac{a^3 + 2a^2 + 44}{a}$$

$$f'(a) = 3a^2 + 4a$$

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$$f'(a) = 3a^2 + 4a$$

$$\frac{a^3 + 2a^2 + 4}{a} = 3a^3 + 4a$$

$$\frac{a^3 + 2a^2 + 4}{a} = 3a^3 + 4a$$

$$2a^3 + 2a^2 - 4 = 0$$

$$a^3 + a^2 - 2 = 0$$

$$(a - 1)(a^4 + 2a + 2) = 0$$

$$a = 1$$

$$f'(1) = 3(1)^{2} + 4(1)$$

$$= 7$$

$$y = mx + b \quad (0, -3)$$

$$-3 = 7(0) + b$$

$$10 = -3$$

$$12 = 7x - 3$$

$$13 = 3(1)^{2} + 4(1)$$

$$-3 = 7(0) + b$$

$$10 = -3$$



