commerce undergraduate society

COMMERCE MENTORSHIP PROGRAM

FINAL REVIEW SESSION

MATH 104

CMP

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Page 12:

1) $2x^3 + 4y^2 = 5$

Differentiate y with respect to x

$$6x^2 + (4)(2y)\left(\frac{dy}{dx}\right) = 0$$

$$6x^2 + 8y\left(\frac{dy}{dx}\right) = 0$$

$$(8y)\frac{dy}{dx} = -6x^2$$

$$\boxed{\frac{dy}{dx} = -\frac{6x^2}{8y} = -\frac{3x^2}{4y}}$$

2) $x^2 + y^2 = 16$

Differentiate y with respect to x

$$2x + (2y)\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx} = -\frac{2x}{2y} = \frac{-x}{y}$$

Given $x = 3$

$$(3)^2 + y^2 = 16$$

$$9 + y^2 = 16$$

$$y^2 = 7$$

$$y = \pm\sqrt{7}$$

When $x = 3$, $y = \sqrt{7}$

$$\frac{dy}{dx} = -\frac{3}{\sqrt{7}}$$

$y = mx + b$ plug in $(3, \sqrt{7})$

$$\sqrt{7} = -\frac{3}{\sqrt{7}}(3) + b$$

$$b = \frac{9}{\sqrt{7}} + \sqrt{7}$$

$\therefore$

$$\boxed{y = -\frac{3}{\sqrt{7}}x + \frac{9}{\sqrt{7}} + \sqrt{7}}$$

When $x = 3$, $y = -\sqrt{7}$

$$\frac{dy}{dx} = \frac{3}{\sqrt{7}}$$

$y = mx + b$ plug in $(3, -\sqrt{7})$

$$-\sqrt{7} = \frac{3}{\sqrt{7}}(3) + b$$

$$b = -\frac{9}{\sqrt{7}} - \sqrt{7}$$

$\therefore$

$$\boxed{y = \frac{3}{\sqrt{7}}x - \frac{9}{\sqrt{7}} - \sqrt{7}}$$

Both answers must be given

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1) $$\lim_{x \to 1} \frac{x^3 - 3x^2 + 3x - 1}{x^3 + x^2 - x - 1}$$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Derive top and bottom function to get

$$\lim_{x \to 1} \frac{3x^2 - 6x + 3}{3x^2 + 2x - 1} = \frac{3(1)^2 - 6(1) + 3}{3(1)^2 + 2(1) - 1} = \boxed{0}$$

2) $y = 5x^{2x}$

$\ln y = \ln 5x^{2x} = \ln 5 + 2x \ln x$

Differentiate both sides

$$\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) = 0 + (2)(\ln x) + (2x)\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = (2\ln x + 2)(y)$$

$$\boxed{= (2\ln x + 2)(5x^{2x})}$$

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elasticity formula

i) $q = 100 - 10p$

$$e_p = \frac{dq}{dp} \cdot \frac{p}{q}$$

$$\frac{dq}{dp} = -10$$

At $p = 7$

$$q = 100 - 70 = 30$$

$$e_p = (-\cancel{10}) \cdot \frac{7}{\cancel{30}}$$

$$= \boxed{-\frac{7}{3}}$$

ii) $|e_p| = \frac{7}{3} > 1$

Therefore a decrease in price would increase revenues

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1)

$f(-2) = 4+2-6 = 0$

$f(3) = 9-3-6 = 0$

$$\frac{f(3)-f(2)}{3-2} = 0$$

To find c, find when

$f'(x) = 0$

$f'(x) = 2x-1$

$2x-1 = 0$

$x = \frac{1}{2} = c$

$$\boxed{c = \frac{1}{2}}$$

2) $f(3) = 6$ $\quad -3 \leq f'(x) \leq 5$ $\quad f(1) = ?$

Mean value theorem, $f'(x) = \dfrac{f(b)-f(a)}{b-a}$

Let $b = 3$ $\quad a = 1$

$$-3 \leq \frac{6-f(1)}{3-1} \leq 5$$

$$-3 \leq \frac{6-f(1)}{2} \leq 5$$

$$-6 \leq 6-f(1) \leq 10$$

$$-12 \leq -f(1) \leq 4$$

$$\boxed{-4 \leq f(1) \leq 12}$$

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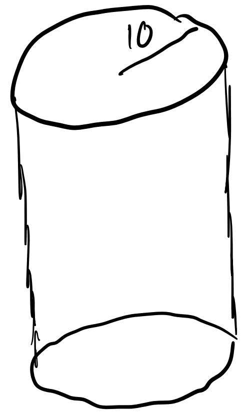
1) $V = \pi r^2 h$ radius (r) = 10

$\frac{dV}{dt} = 15$ $\frac{dh}{dt} = ?$



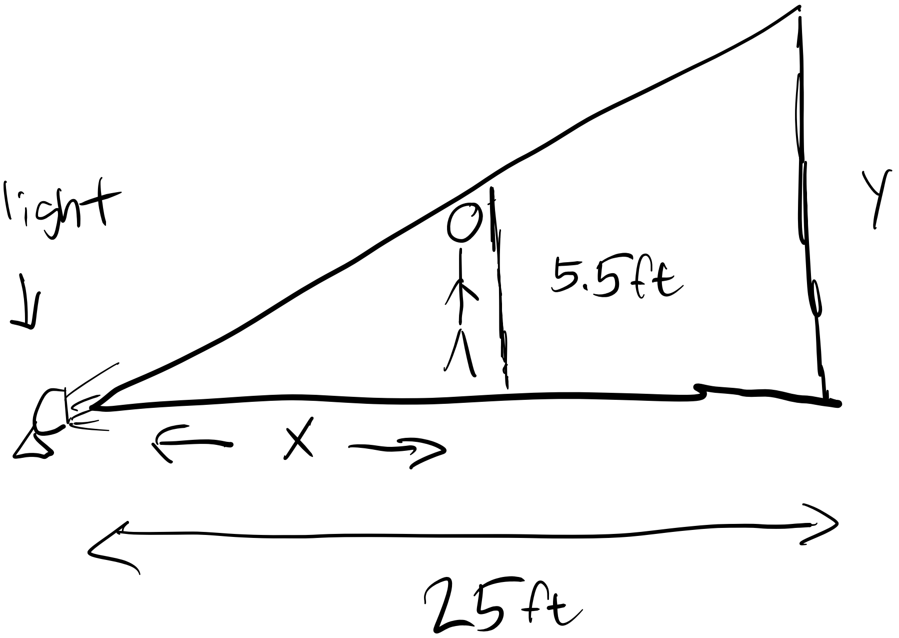
differentiate volume equation with respect to time

$$\frac{dV}{dt} = (\pi r^2)\frac{dh}{dt}$$

$$15 = (10^2)(\pi)\left(\frac{dh}{dt}\right)$$

$$\frac{dh}{dt} = \frac{15}{100\pi} = \boxed{\frac{3}{20\pi}}$$

2)



y = height of shadow

$x = 10$ ft, distance to wall

$\frac{dy}{dt} = ?$

$\frac{dx}{dt} = 2.5$ ft/second

$$\frac{y}{25} = \frac{5.5}{x}$$

$$y = \frac{137.5}{x}$$

$$\frac{dy}{dt} = -\frac{137.5}{x^2} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-137.5}{10^2} \cdot 2.5$$

$$\boxed{= (-1.375)(2.5)}$$

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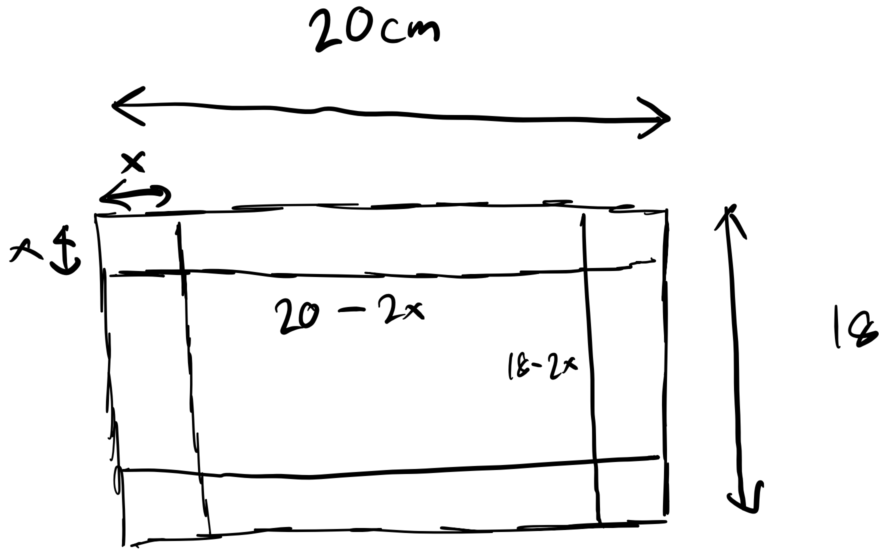
1) The height of the box is x.



Volume of the box as a function of x.

$$V = (20-2x)(18-2x)(x)$$

length width height

$$= (360 - 40x - 36x + 4x^2)(x)$$

$$= (4x^2 - 76x + 360)(x)$$

$$= 4x^3 - 76x^2 + 360x$$

$$\frac{dv}{dx} = 12x^2 - 152x + 360$$

solve for x when $\frac{dv}{dx} = 0$

$$12x^2 - 152x + 360 = 0$$

$$3x^2 - 38x + 90 = 0$$

$$x = \frac{38 \pm \sqrt{38^2 - 4(90)(3)}}{6}$$

$$x = \frac{38 \pm \sqrt{364}}{6}$$

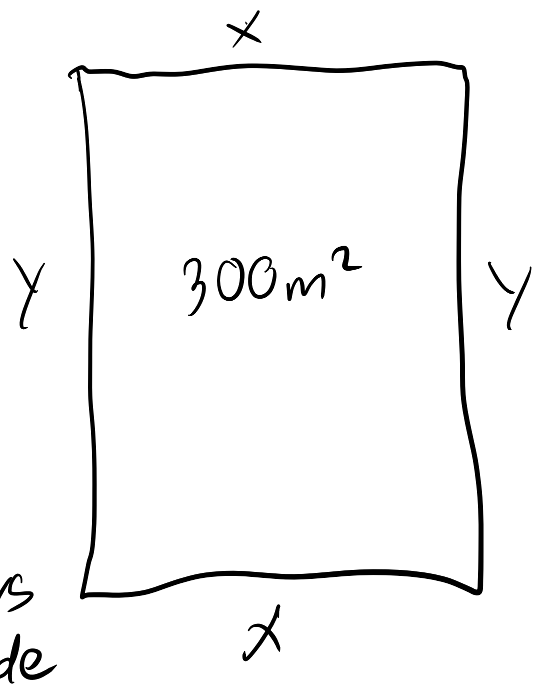
$$x = \frac{38 \pm 2\sqrt{91}}{6} \quad x = \frac{19 \pm \sqrt{91}}{3}$$

$x \neq \frac{19 + \sqrt{91}}{3}$ because then your length and width would be negative

$$\therefore \boxed{x = \frac{19 - \sqrt{91}}{3}}$$

2) Let x be the width,

let y be the length.



$$C(\text{cost}) = x + x + y + \left(\tfrac{1}{2}\right)y$$

$$= 2x + \tfrac{3}{2}y$$

neighbour pays half of one side

$$xy = 300$$

$$y = \frac{300}{x}$$

$$C = 2x + \frac{3}{2}\left(\frac{300}{x}\right)$$

$$= 2x + \frac{450}{x}$$

$$C' = 2 - \frac{450}{x^2}$$

Find when $C' = 0$

$$2 - \frac{450}{x^2} = 0$$

$$2x^2 = 450$$

$$x^2 = 225$$

$$x = \pm\sqrt{225}$$

$x \neq -\sqrt{225}$ cause negative side length

$$\boxed{x = \sqrt{225} \qquad y = \frac{300}{\sqrt{225}}}$$

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1) $f(x) = x^3 - 1$

Domain:

$-\infty \le x \le \infty$

Intercepts:

$f(0) = -1$

$0 = x^3 - 1$

$x^3 = 1$

$x = 1$

$(1,0)$

$(0,-1)$

Asymptotes:

none

First Derivative

$f'(x) = 3x^2$

$3x^2 = 0$

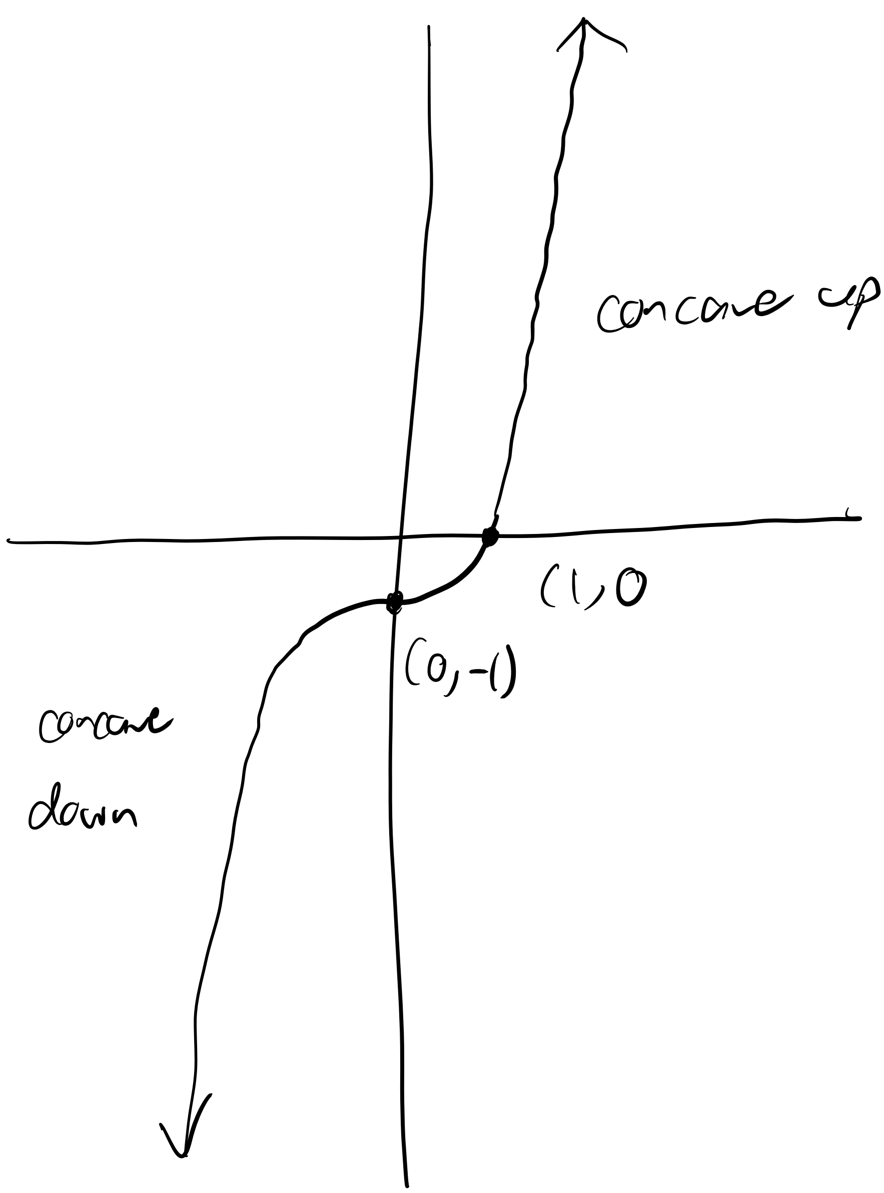
$x = 0$ critical point

Second derivative

$f''(x) = 6x$

$6x = 0$

$x = 0$ inflection point



2) $f(x) = x + \frac{1}{x}$

Domain:

$x \neq 0$

Intercepts:

no y intercept cause $x \neq 0$

$0 = x + \frac{1}{x}$ $\quad x^2 + 1 = 0$

no x intercept cause invalid solution

Asymptote:

$\lim_{x \to 0^+} x + \frac{1}{x} = \infty$

$\lim_{x \to 0^-} x + \frac{1}{x} = -\infty$

Vertical asymptote at $x = 0$

As x approaches $\pm\infty$, f(x) approaches x.

Oblique asymptote at $f(x) = x$

First Derivative

$f'(x) = 1 - \frac{1}{x^2}$

$1 - \frac{1}{x^2} = 0$

$\frac{1}{x^2} = 1$

$x = \pm 1$

$x = 1, f(x) = 2$

$x = -1, f(x) = -2$

critical points

$(1, 2)$

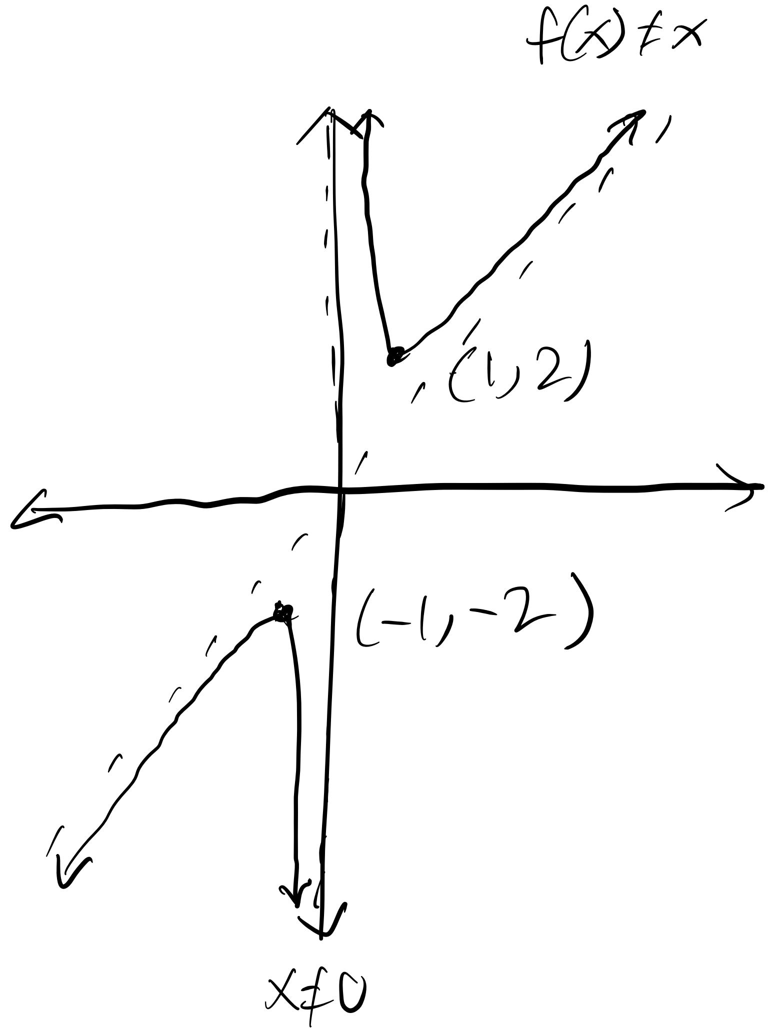
$(-1, -2)$

Second Derivative

$f''(x) = \frac{2}{x^3}$

$\frac{2}{x^3} = 0$

no inflection points



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1) $f(x) = xe^x$

$f(a) = f(1) = (1)e^1 = e$

$f'(x) = (1)(e^x) + (x)e^x$
$= (1+x)(e^x)$

$f'(1) = (1+1)(e^1) = 2e$

$f''(x) = (1)(e^x) + (1+x)e^x$
$= (2+x)(e^x)$

$f''(1) = (2+1)(e^1) = 3e$

$f'''(x) = (1)(e^x) + (2+x)e^x$
$= (3+x)(e^x)$

$f'''(1) = (3+1)(e^1) = 4e$

Third-Degree Taylor Polynomial

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{6}(x-a)^3$$

$$\boxed{f(x) \approx e + 2e(x-1) + \frac{3e}{2}(x-1)^2 + \frac{2e}{3}(x-1)^3}$$

2) $f(x) = \tan(x)$ $\quad f(a) = f(\frac{\pi}{4}) = 1$

$$f'(x) = \sec^2(x)$$
$$= \frac{1}{\cos^2 x}$$

$$f'(\frac{\pi}{4}) = \frac{1}{\cos^2(\frac{\pi}{4})} = \frac{1}{(\frac{\sqrt{2}}{2})^2} = \frac{1}{(\frac{2}{4})} = 2$$

$$f(x) \approx 1 + 2(x - \frac{\pi}{4})$$

$$\boxed{\approx 1 + 2x - \frac{\pi}{2}}$$