COMM 295

2018W1 Final Review Solutions

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Monopoly & Pricing with Market Power

Monopoly:

- As a single supplier, a monopolist faces the entire (downward sloping) market demand.
- However, MR is not equal to price as the monopolist must reduce price to sell more (due to downward sloping demand).

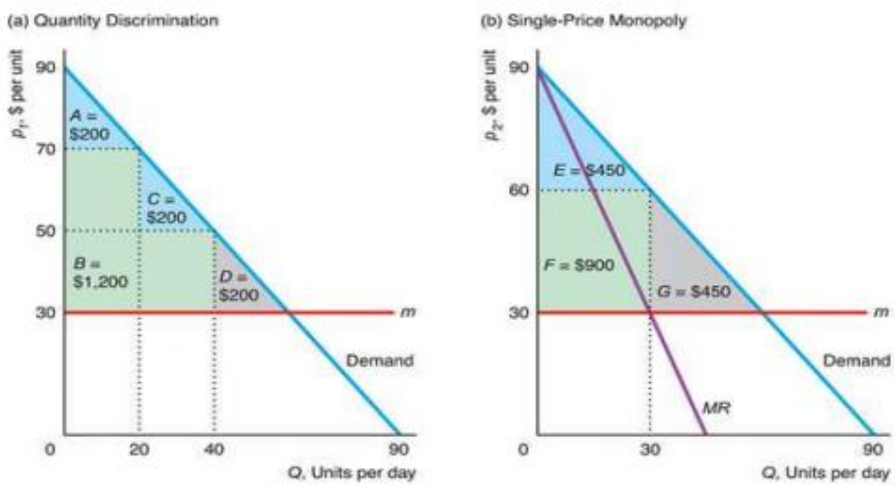
Pricing with Market Power

- Perfect price discrimination: Monopolist charges the maximum price that each consumer is willing to pay.
- Multi-Group price discrimination: Splitting consumers into two or more groups based on their demand curve and charging different prices to each group.
  $MR_1 = MR_2 = MC$
- Quantity-Based price discrimination (or non-linear price discrimination): Charging different prices based on the quantity
- Two-Part Tariff: Charging an entry fee and a usage fee
    - Identical consumer:
        - Single consumer:
            - usage fee of P = MC
            - entry fee = entire CS
        - Many consumers:
            - usage fee of P = MC
            - entry fee = entire CS/(# of consumers)
    - Different Consumers:
        - charge same entry fee:
            - usage fee P > MC
            - entry fee = CS of the consumer with lower demand
        - charge different entry fee:
            - usage fee P = MC
            - entry fee = CS of the consumer

A consumer has a demand curve given by p = 90 – Q. In the following pictures:

(a) shows the outcome of quantity-based price discrimination if the firm charges $70 each for the first 20 units and $50 each for additional units.

(b) Shows the outcome of profit-maximizing single price monopoly. The marginal cost is $30.



Which of the following statements is true:

A. The average price paid under situation (a) is the same as situation (b)
B. Revenue under situation (a) is double the revenue of situation (b)
C. Profits are higher in situation (b) than with situation (a)
D. None of the above

A) Average price paid in (a) is ($70 * 20 + $50 * 20)/40 = $60
Average price paid in (b) is $60

	Textbook	MyEconLab code	Bundle
Consumer 1	120	30	150
Consumer 2	110	90	200
Consumer 3	90	110	200
Consumer 4	30	120	150

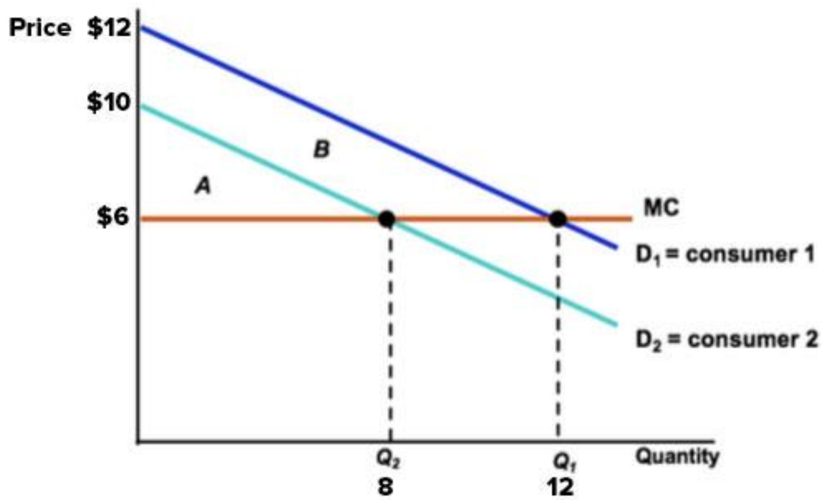
a) What is the revenue using stand-alone pricing textbook = $90, code = $90
b) What is the revenue using pure bundling pricing bundle = $150
c) What is the maximum revenue if you are able to use stand-alone pricing, pure bundling, or mixed bundling

a) 3*90 + 3*90 = 540
b) 4*150 = $600
c) Price to charge bundle = $200, textbook = $120, code = $120
Revenue = $640

Given the following situation where a monopoly can charge different usage and different entry fees to the two different consumer types.

Consumer Type A has a demand curve: p = -0.5q + 12
Consumer Type B has a demand curve: p = -0.5q + 10
The firm has a marginal cost of $6 per unit

What is the profit maximizing usage fee?
What is the profit?



Usage fee is $6 (same as MC)

The profit in this situation is area of the smaller triangle + area of the larger triangle

Small triangle area = (10-6)*8/2

Large triangle area = (12-6)*12/2

$16 + $36 = $52

Oligopoly

Cournot Duopoly: two firms compete in choosing quantities (more realistic)
Bertrand Duopoly: two firms compete in choosing prices

Solving Cournot Model:

1. Find MR equation of each firm
- Remember to use the Q_A for Quantity of Firm A but the Price equation uses Q which is equal to $Q_A + Q_B$
2. Set MR = MC for both firms
3. Solve for Q_A and Q_B using the 2 questions.
4. Use Q to find price

Solving Cournot Model with first mover advantage:

1. Find MR equation of the following firm and set MR = MC
2. Find MR equation of the leader firm using the response function in step 1
3. Solve MR = MC for leader firm for the leader's Q
4. Solve for following firm's Q
5. Use Q to find the solution

Oligopoly

Two firms in a cournot duopoly have an inverse demand of P = 500 – 50Q and a cost function of C = 20Q. Find the equilibrium price, the quantity produced by each firm, and the profit of each firm.

1. Find MR equation.

$R_A = PQ_A = (500-50(Q_A+Q_B))Q_A$

$= 500Q_A - 50Q_A^2 + 50Q_AQ_B$

$MR_A = \partial R_A/\partial Q_A = 500 - 100Q_A - 50Q_B$

MR equation will be identical for Firm B

2. MR = MC

$500 - 100Q_A - 50Q_B = 20$

$100Q_A = 480 - 50Q_B$

$Q_A = 4.8 - 0.5Q_B$

By symmetry

$Q_B = 4.8 - 0.5Q_A$

3. Solve the system of equation

$Q_A = 4.8 - 0.5(4.8 - 0.5Q_A)$ by substitution

$Q_A = 2.4 + 0.25Q_A$

$Q_A = 3.2$

Therefore

$Q_B = 3.2$ as well and Q = 6.4

4. Use Q to find the solution

P = 500-50(6.4) = 180

Profit = 180*3.2 – 20*3.2 = 512

Price = 180

Quantity produced by each firm = 3.2

Profit of each firm = 512

Oligopoly

Assume that firm A is the first mover and firm B is the follower. Find the equilibrium price, the quantity produced by each firm, and the profit of each firm.

Firm B (follower) observes Q_A chosen by the leader and will choose Q_B such that $MR_B = MC_B$ to maximize $profit_B$. Their response will be: $Q_B = 4.8 – 0.5Q_A$ (derived in part a). $Firm_A$ will predict Firm B's response and therefore use Firm B's response function when making their first move.

$P = 500 – 50(Q_A+4.8 – 0.5Q_A) = 500 – 50Q_A – 240 + 25Q_A = 250 – 25Q_A$

$R_A = PQ_A = (250\text{-}25Q_A)Q_A = 250Q_A - 25Q_A^2$
$MR_A = 250 – 50Q_A$

MR = MC

$250 – 50Q_A = 20$
$Q_A = 4.6$

$Q_B = 4.8 – 0.5Q_A = 4.8 -0.5(4.6) = 2.5$ P
= 500-50(4.6+2.5) =145

$Q_A = 4.6$ Q_B
= 2.5

$Profit_A = 145(4.6)\text{-}20(4.6) = 575$
$Profit_B = 145(2.5)\text{-}20(2.5) = 312.5$

Game Theory

Static Game:

- each player acts once and at the same time

Dynamic Game:

- Stackelberg: Sequentially, one player goes first followed by the other
- Cournot: Repeatedly, multiple rounds of the game

When at least one player has a dominant strategy then the outcome is a unique Nash equilibrium.

Tit-for-Tat Strategy: A player responds in kind to an opponent's previous play, cooperating with cooperative rivals and retaliating against uncooperative ones.

- Infinitely games: Tit-for-Tat is rational
- Finite games: Defect from the start is rational

Game Theory

What is the nash equilibrium?

		Firm B		
		Large	Small	None
	Large	4, 4	12, 8	16, 9
	Small	8, 12	16, 16	20, 18
Firm A	None	9, 16	15, 20	18, 18

Eliminate the choice where the other 2 choices are better regardless of what the other firm chooses to narrow it down.

Firm B will never chose Large.

Firm A will never choose Large or None if Firm B will never choose Large.

(Firm A, Firm B)

(Small, None) for a payoff of (20,18)

Game Theory

		Player 2	
		Cheat	Cooperate
Player 1	Cheat	5 5	-20 50
	Cooperate	50 -20	20 20

1. What is the nash equilibrium if the game is played only once?
2. What is the nash equilibrium if the game is played 2 times?
3. What is the nash equilibrium if the game is played 10 times?

1. Both cheat
2. Both cheat
3. Both cheat

After the last game, there is no retaliation possible. But, in the last game before last game, knowing that the rival would defect in the last game, it will defect in the game before.

Going backwards, the only rational outcome is for both firms to defect every game.

Game Theory

Firm A and Firm B are competing in a game that can go on infinitiely.

(A payoff, B payoff)		**Firm B** Defect (Low Price)	Cooperate (High Price)
Firm A	Defect (Low Price)	50,50	100,25
	Cooperate (High Price)	25,100	75,75

Which of the following is true?

1. Tit-for-tat is a rational strategy
2. The sum of payoffs is maximized if they both cooperate
3. They will never defect

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1. T
2. T
3. F

Keyword in 3. is "never". That part makes the statement false. Although it is rational for both firms to cooperate we do not know what the discount factor is (interest rate). In other words, it is possible that when you take the present value of the cash flows that defecting results in a higher present value.

Math explanation if you like math (I don't think you need to know this)

Let $\sigma = \frac{1}{(1+r)}$

Profit(defect) = $100 + 50\sigma + 50\sigma^2 + 50\sigma^3 + \ldots = 100 + 50\frac{\sigma}{(1-\sigma)}$

Profit(cooperate) = $75 + 75\sigma + 75\sigma^2 + 75\sigma^3 + \ldots = \frac{75}{(1-\sigma)}$

Therefore, firms will never cheat ONLY if Profit(cooperate) > Profit(defect)

In other words, if $\frac{75}{(1-\sigma)} > 100 + 50\frac{\sigma}{(1-\sigma)}$

Uncertainty

Uncertainty
• Unlike the choice under certainty, choice under uncertainty is risky and so is difficult to make.
• How do we make choices when certain variables such as income and prices are uncertain?

• To measure risk, we must know:

1. All the possible outcomes.
2. The probability that each outcome will occur.

• Two measures to help describe and compare risky choices are:
1. Expected value.
2. Variability.

Expected Value E(X): The weighted average of the values resulting from all possible outcomes.

Assume n possible outcomes:
Values of possible outcomes: $X_1, X_2, \ldots, X_n$.
Probability of each outcome: $P_1, P_2, \ldots, P_n$.

$E(X) = P_1 X_1 + P_2 X_2 + \ldots + P_n X_n$

Describing Risk: Variability
• Variability: Variability comes from deviations in actual payoffs relative to the expected payoff.
• Greater variability of actual payoffs the expected value signals greater risk.
• Variance or Standard Deviation measures variability.

Variance: $\sigma^2 = P_1(X_1-E(X))^2 + P_2(X_2-E(X))^2 + \ldots + P_n(X_n-E(X))^2$

Standard Deviation: Take the square root of variance

Attitude Towards Risk
• You choose depending on your attitude towards risk as reflected in your utility function.
• If you dislike risk, then you may choose a riskier job only if it gives you sufficiently higher
expected value than the risky job.
• In other words, you choose the option that gives you the highest expected utility.

Uncertainty

Assume Galen has a utility function $U(X) = 3X^{0.5}$. They have the option between

job 1 which gives $65 with 0.6 and $25 with 0.4

job 2 which gives $48 with certainty

Which job does Galen take?

Job 1: $E(U(X)) = 0.6*(3*65^{0.5}) + 0.4*(3*25^{0.5}) = 20.51$

Job 2: $E(U(X)) = 1*(3*48^{0.5}) = 20.78$

He will take job 2

How much does a certain job need to pay to offer him the same utility as job 1? What is his risk premium?

Job 1 had $E(U(X)) = 20.51$

$E(U(X)) = 20.51 = 3X^{0.5}$

$X = 46.74$

Which tells us job that pays 46.74 with certainty would have an $E(U(X)) = 20.51$ the same as Job 1 and has $E(X) = 46.74$ (because it is with certainty)

$E(X)$ of job 1 = $0.6*65 + 0.4*25 = 49$

This means that even though the expected value of Job 1 is 49. When put through Galen's utility function Job 1 and a job that pays 46.74 with certainty are equally attractive to him.

Risk premium = 49 – 46.74 = 2.26

Asymmetric Information

• Asymmetric Information: a situation where one party knows more than others.

• Many markets such as insurance, financial credit and employment are characterized by asymmetric information about product quality.

• Such asymmetric information leads to opportunistic behavior:
  - Adverse selection or lemons problem.
  - Moral hazard.

Asymmetric Information

In a market of used gaming consoles, both good used gaming consoles and inferior used gaming consoles are available. Owners of those gaming consoles have information of the actual qualities of the gaming consoles, whereas the buyers do not. In the market, 50% of all used 3DS are good, and 50% are inferior. All buyers are risk neutral and are willing to pay $130 for a good used 3DS, but only $80 for an inferior used 3DS. The owners of the good used 3DS are willing to sell them at a price no lower than $120. The owners of the inferior used 3DS are willing to sell them at a price no lower than $60.
1. What is the equilibrium price?
2. For what relative fractions of good used 3DS and inferior used 3DS will adverse selection not occur?

1. Buyers are only willing to pay a price that is the expected value of a used 3DS
   Expected value of used 3DS=0.5*80+0.5*130=$105
   $105 is the price that a buyer is willing to pay for a used 3DS regardless of whether the used 3DS is good or inferior because the buyer cannot know the actual qualities of the 3DS
   Sellers of inferior used 3DS are willing to accept a price no lower than $60, so they are willing to sell at the price of $105 ($105>$60)

   Sellers of good used 3DS are willing to accept a price no lower than $120, so they are not willing to accept buyers' offer of $105. As a result, no good used 3DS gets sold.

   Only inferior used 3DS will be sold. Realizing that there are only inferior 3DS left in the market, the equilibrium price will be $80, which is the price buyers are willing to pay for an inferior 3DS.

2. Adverse selection won't occur if buyer's maximum willingness to pay (expected value) is at least or more than the price of the good used 3DS seller is willing to sell for. Set x to be fraction of good used 3DS available in the market and (1-x) to be the fraction of inferior used 3DS available in the market.
   x*130+(1-x)*80=120
   130x+80-80x=120
   40=50x
   x=0.8 If there are at least 80% of good used 3DS in the gaming console market, then buyers will be willing to pay $120 for a used 3DS of ambiguous quality, meeting a good used 3DS seller's reservation price of $120.

Asymmetric Information

Suppose that half the population is healthy and the other half is unhealthy. If a healthy gets sick, the medical cost is \$1,000; for an unhealthy this cost is \$10,000. In each year, the probability that anyone gets sick is 0.4. Although each person knows whether he or she is healthy, the insurance company does not. Both the insurance company and the people are risk neutral.

If the insurance company offers complete actuarially fair (i.e. 0 profits for the company) insurance, what is the premium?
At the price that you determined in part a, do healthy people purchase the insurance?
If only unhealthy people purchase insurance, what is the price of insurance?

EV of medical coverage for both sick and healthy people combined:
EV = 0.4*(0.5*10,000 + 0.5*1,000) = 2,200.
Thus, actuarially fair premium (that yields zero profits to the insurance company)
= 2,200 per head/year.

Each healthy person's expected willingness to pay = 0.4*1,000 = 400/year.
This is less than the actuarially fair premium of 2,200/year.
Therefore, healthy people will not buy the medical coverage–adverse selection problem.

Each unhealthy person's willingness to pay for medical coverage
= 0.4*10,000 = 4,000/year.
This is higher than the premium charge.
Thus, they will buy the insurance.
Given that the insurance company knows only unhealthy people buy insurance, at equilibrium,
it will charge 4,000 per head/year.