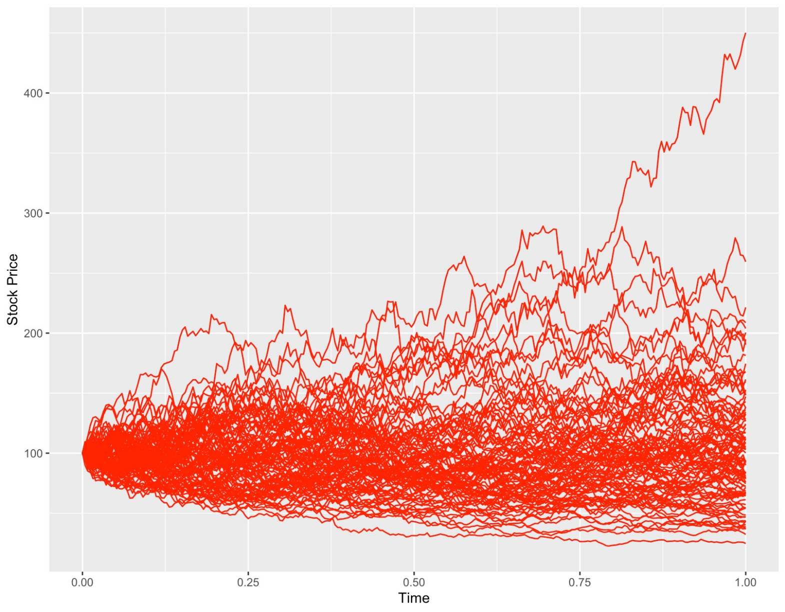
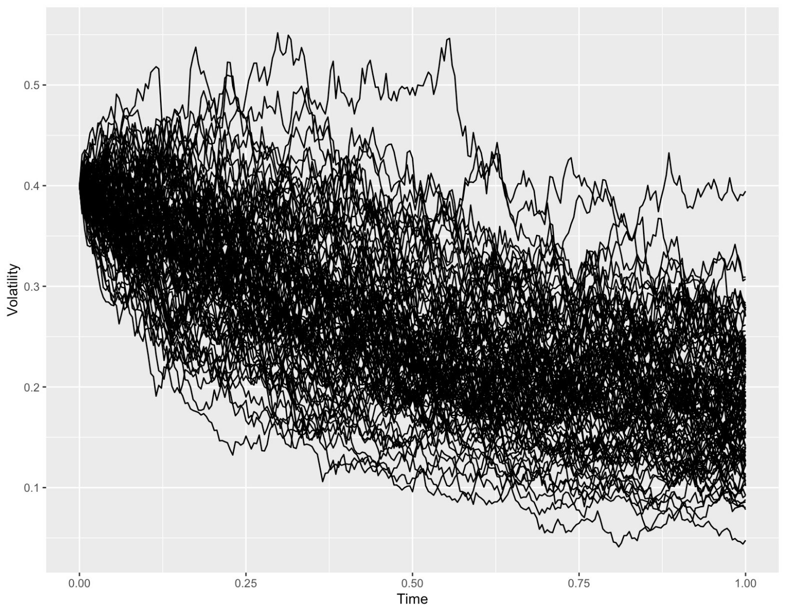
FINANCIAL SIMULATION TERM PROJECT

Hanqing Chen, Jocelyn McConnon, Marc Edward McDonald, Thomas Su

Market Simulation Method

- Heston Stochastic Volatility Model
  - $\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_{1t}$
  - $dV_t = \alpha(b - V_t)dt + \sigma\sqrt{V_t} dW_{2t}$
- We discretize Heston's Model into the following
  - $S_{t_{j+1}} = S_{t_j} + \mu S_{t_j}(t_{j+1} - t_j) + \sqrt{V_t} S_{t_j} \sqrt{(t_{j+1} - t_j)} Z_{1j}$
  - $V_{t_{j+1}} = V_{t_j} + \alpha\left(b - V_{t_j}\right)(t_{j+1} - t_j) + \sigma\sqrt{V_t}\sqrt{(t_{j+1} - t_j)} Z_{2j}$
- Heston's Model represents the real-world market better than a GBM model
  - GBM model assumes constant volatility
  - Real-world markets have cycle of volatility
- Values of μ, σ, and $volvol$ from last 20 years of S&P 500





Option Pricing Method

Our model was primary tested on European Options

- European Call: $\mathrm{P} = max(S - K, 0)$
- European Put: $\mathrm{P} = max(K - S, 0)$
- Price determined at expiration

We also implemented Up and Out Barrier Options

- Up and Out Barrier Call: $\mathrm{P} =$

$$\begin{cases} max(S - K, 0), & S_{\mathrm{max}} < B \\ 0, & S_{max} \geq B \end{cases}$$

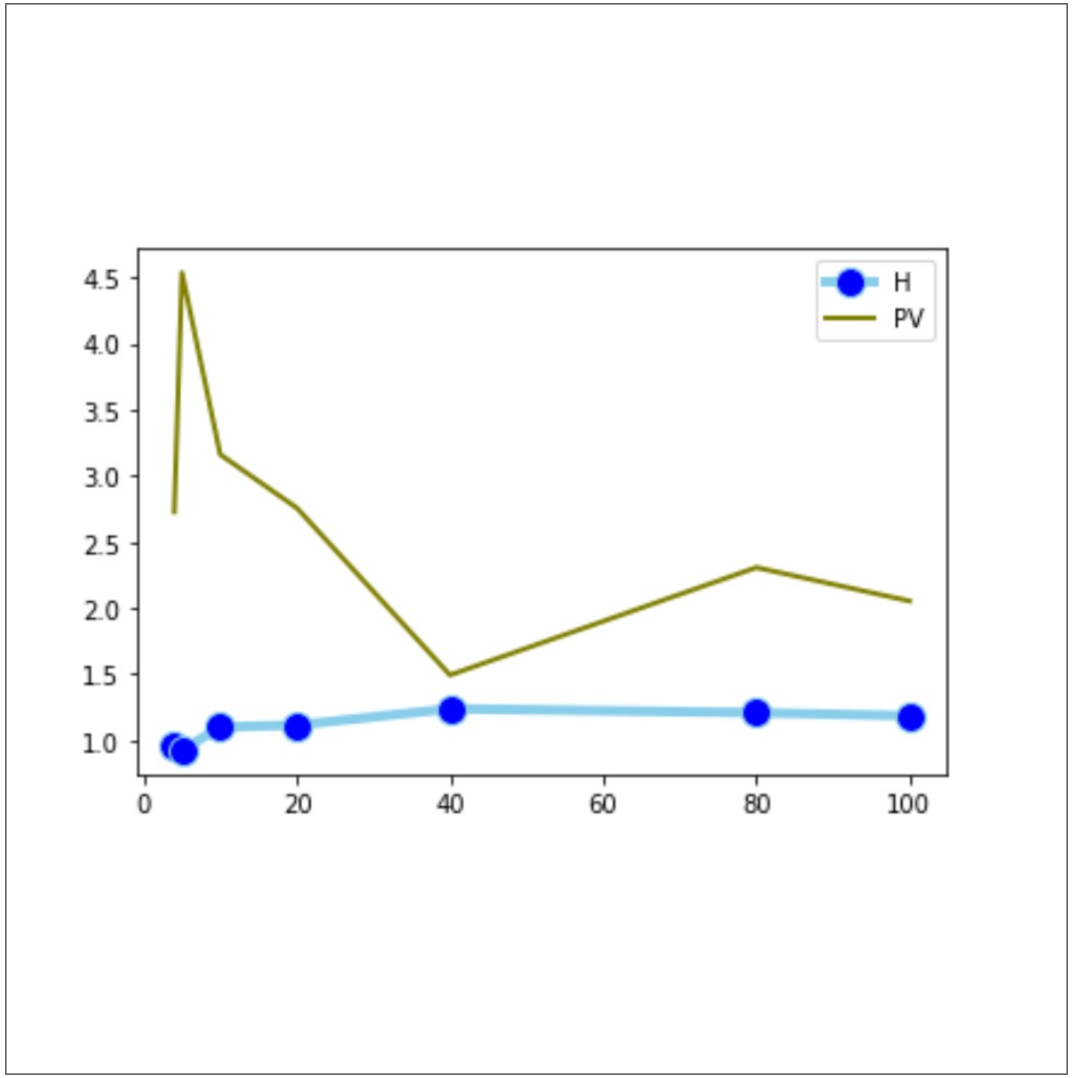
- Up and Out Barrier Put: $\mathrm{P} =$

$$\begin{cases} max(K - S, 0), & S_{\mathrm{min}} > B \\ 0, & S_{max} \leq B \end{cases}$$

- Price can reach \$0 at any point in option's life

Delta Estimation Method

- Since we implemented the European and Barrier option pricing models, we needed to calculate delta for the prices simulated in both situations.
- The delta of an option is the slope of the stock price vs. option value at a given point in time.
- To do this we used a finite difference method to approximate the derivative.
- Common Random Numbers is a variance reduction technique which we used ensure the distance between the observations remained consistent throughout our simulation.
  - This was done using the same seed to generate the normal distribution.
  - Heston's model was used to calculate the stock value.



Hedging Method

- Our hedging strategy is to be delta-neutral
- This means that as the path dependent deltas fluctuate according to the changes in stock prices, our portfolio is rebalanced such that the overall delta of the portfolio is zero.
- Rebalancing 10 times was the optimal tradeoff between performance and value
- The portfolio value includes an assumed 1% cost of trading

Results

- Overall hedge performance for European options over 12 iterations is 0.985
  - This metric is calculated by taking the standard deviation of the portfolio value / average option price
- Sharpe Ratio tells you how well the returns are given the level of volatility
  - We estimated the Sharpe ratio to be 1.015 using 1/H.perf
  - A Sharpe Ratio above 1 are considered a good return on volatility

Future Improvements

ADD GAMMA HEDGING

ADD SUPPORT FOR OTHER OPTIONS

CALCULATE SHARPE RATIO EXACTLY

ADD LATTICE FOR BARRIER OPTION PREMIUM?

ADD RISK MANAGEMENT FOR CREDIT VAR

Thank you!
Questions?

Constants Used:

- S0 <- 49
- K <- 50
- r <- 0.05
- t <- 20/52
- M <- 500
- N <- 10
- h <- 0.1
- alpha <- 2
- b <- 0.16
- V0 <- 0.4