Calibration of partial factor design formats

Best practice and challenge

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Motivation for Calibration

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- Sustainable development of the build environment requires optimal balance between **safety** and **resource efficiency**.
- For **structural** design this balance can be identified using a **high** level design strategy - e.g. risk informed decision making.
- Daily life practical decisions require a simple and easy to use **low** level design strategy - e.g. partial factor design.

Levels of Structural Engineering Decision Making

	Commonly applied when:	Objective:
Risk-informed decision making: - decisions are taken with due consideration of the decision makers preferences.	Exceptional design situations in regard to uncertainties and consequences.	Maximize the expected utility of the decision maker.
Reliability-based design and assessment: - estimation of the probability of adverse events.	Unusual design situations in regard to uncertainties.	Satisfy reliability requirements.
Semi-probabilistic: - safety format prescribing design criteria in terms of the design equations and the analysis procedures to be used.	Usual design situations in regard to consequences and uncertainties. Default method of most design codes.	Satisfy deterministic design criteria.

Questions?

Reliability based design

Reliability requirement

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- the possible cause and /or **mode** of attaining a limit state;
- the possible **consequences** of failure in terms of risk to life, injury, potential economical losses;
- public **aversion** to failure;

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In the Eurocodes *appropriate* level of reliability is dependent on:

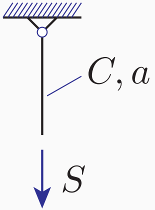
- the possible cause and /or **mode** of attaining a limit state;
- the possible **consequences** of failure in terms of risk to life, injury, potential economical losses;
- public **aversion** to failure;
- the **expense** and procedures necessary to **reduce** the risk of failure.

Reliability requirement

Reliability Class	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

Figure 1: Reliability requirements as stated in EN 1990:2002

Reliability based design - a simple example



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C,S Normal distributed.

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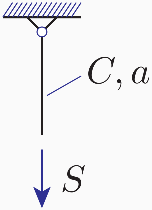

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- A design can be identified that **corresponds** to a specified reliability requirement.
- The most simple form of reliability problem was considered here, but in practice it is often much **more complex**.
- The achieved reliability is **conditional on utilised knowledge** - the reliability based design solution is also conditional on knowledge!
- Reliability is always dependent on specified **reference time**.

Questions?

Design Value Format

Derivation of design values

Based on the simple reliability problem:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (1)$$

And

$$\beta \overset{!}{=} \beta_{req}$$

Design values and characteristic values

The **design value** of a basic variable Y is defined as the multiplication or division of a corresponding **partial safety factor** γ_Y and the characteristic value y_k:

$$\frac{r_k}{\gamma_R} = r_d \geq e_d = \gamma_E e_k \tag{2}$$

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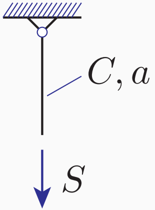
Note: Typical values for p are:

- resistance related variables: $p = 0.05$;
- permanent actions: $p = 0.5$;
- time-variable actions (yearly reference period): $p = 0.98$.

Design value format - a simple example



	μ	V
Capacity C $\left[kN/mm^2\right]$	1	0.10
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C,S Normal distributed.

Design value format - generalisation to other distributions

Normal:	$y_d = \mu_Y\left(1 + \alpha_Y \beta_t V_Y\right)$ $y_k = \mu_Y\left(1 + \Phi^{-1}(p) V_Y\right)$
Log-Normal:	$y_d = \mu_Y \exp\left(-\frac{1}{2}\ln\left(1+V_Y^2\right) + \alpha_Y \beta_t \sqrt{\ln\left(1+V_Y^2\right)}\right)$ $y_k = \mu_Y \exp\left(-\frac{1}{2}\ln\left(1+V_Y^2\right) + \Phi^{-1}(p) \sqrt{\ln\left(1+V_Y^2\right)}\right)$
Gumbel:	$y_d = \mu_Y\left(1 - V_Y \frac{\sqrt{6}}{\pi}\left(0.5772 + \ln\left(-\ln\left(\Phi(\alpha_Y \beta_t)\right)\right)\right)\right)$ $y_k = \mu_Y\left(1 - V_Y \frac{\sqrt{6}}{\pi}\left(0.5772 + \ln\left(-\ln\left(p\right)\right)\right)\right)$

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- … but only for **specific design cases**.
- The α values are case specific and their determination may be cumbersome.
- Both, α and the extreme value distribution representing the variable load have to relate to the same **time reference period** than the reliability target.

Questions?

Generalising α

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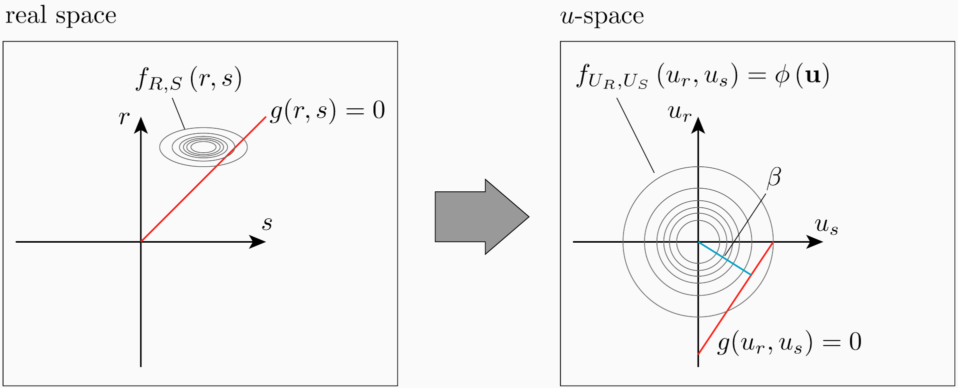
- The set of generalised α values shall lead to **safe** design solutions **for most of the cases**.
- **Alternative representation** of the reliability problem for an informed choice.

Hashofer-Lind representation of reliability problem



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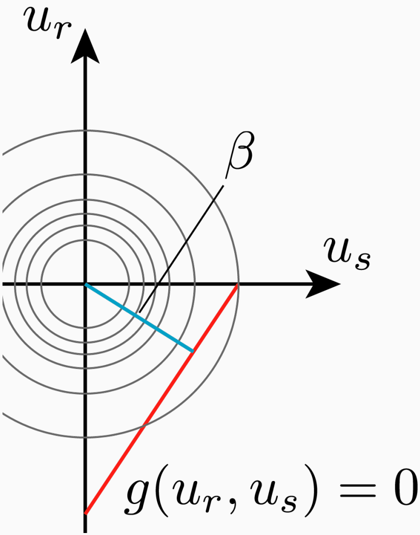

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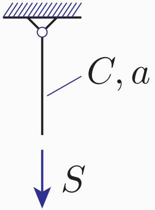

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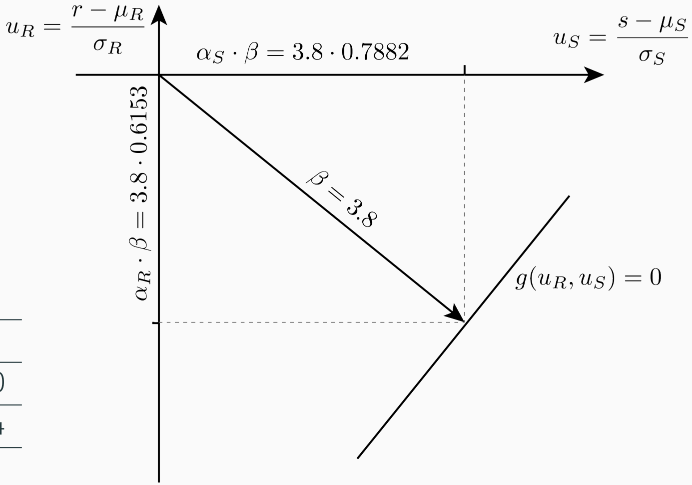
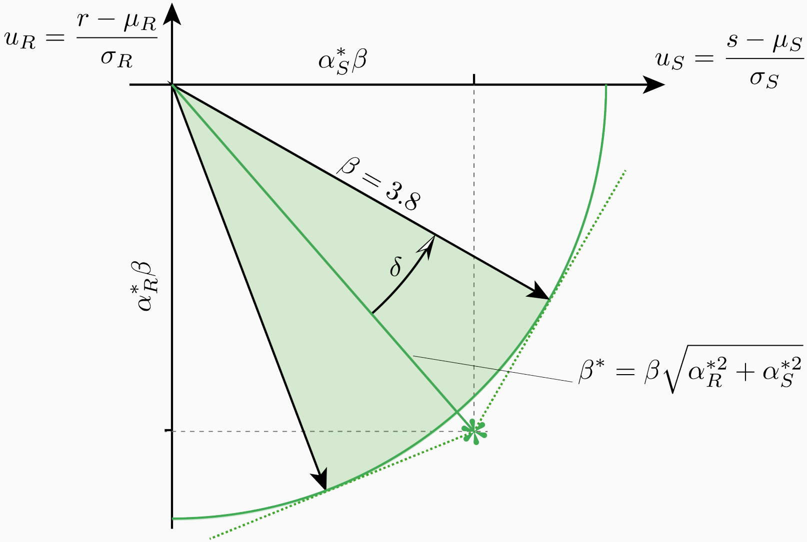


Generalisation



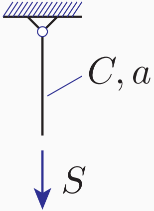
Generalisation chosen in the Eurocode

The following Eurocode standardized values can be used for a **50 years reference period**:

- If Y represents a strength related variable: $\alpha_Y = -0.8$
- If Y represents a load related variable: $\alpha_Y = 0.7$
- If Y is dominating the reliability problem: $\alpha_Y = (-)1$
- If Y represents a secondary strength or load related variable: $\alpha_Y = -0.8 \cdot 0.4$ or $\alpha_Y = 0.7 \cdot 0.4$ correspondingly.

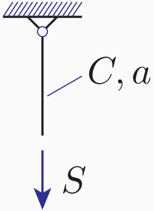
Reality check - extended examples

Initial Example continued



	Example 1			Example 2			Example 3a			Example 3b		
	Distr.	μ	V	Distr.	μ	V	Distr.	μ	V	Distr.	μ	V
Capacity C $[kN/mm^2]$	Normal	1	0.1	Normal	1	0.2	LogN	1	0.1	LogN	1	0.2
Load S $[kN]$	Normal	1	0.335	Normal	1	0.335	Gumbel	1	0.335	Gumbel	1	0.335

Initial Example continued



	Example 1			Example 2			Example 3a			Example 3b		
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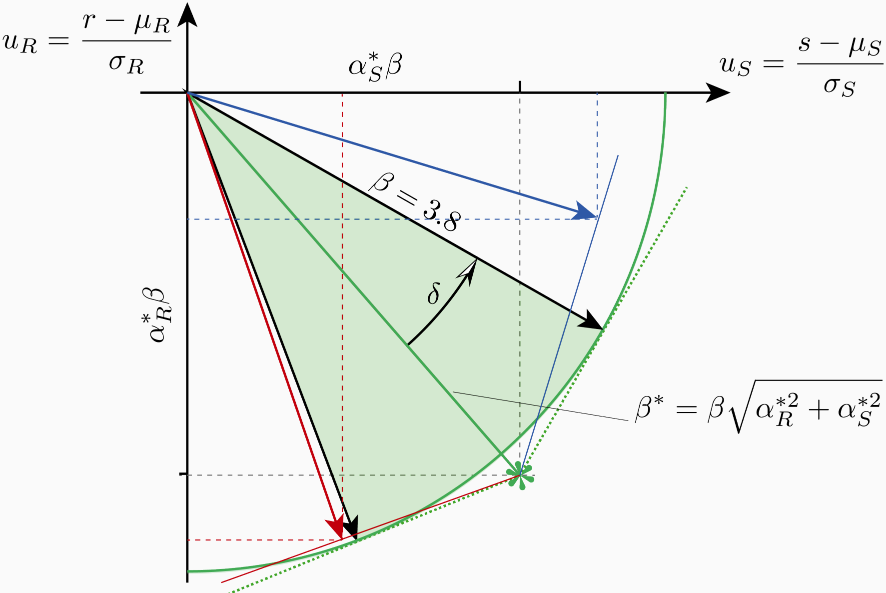
Capacity $C\ [kN/mm^2]$	Normal	1	0.1	Normal	1	0.2	LogN	1	0.1	LogN	1	0.2
Load $S\ [kN]$	Normal	1	0.335	Normal	1	0.335	Gumbel	1	0.335	Gumbel	1	0.335

	Example 1	Example 2	Example 3a	Example 3b
Section $[mm^2]$	2.62	5.03	3.56	4.21
α_R	0.615	0.949	0.298	0.516
α_S	0.788	0.316	0.955	0.856

Initial Example - Application of the generalized α- values

	Example 1	Example 2	Example 3a	Example 3b
Simplified Assumptions	$\alpha_R^* = -0.8;\ \alpha_S^* = 0.7;\ \beta_{req} = 3.8$			
Cross section $[mm^2]$	2.717	4.824	3.113	4.21
Real α_R	-0.630	-0.945	-0.291	-0.516
Real α_S	0.777	0.328	0.957	0.853
Real β	3.98	3.74	3.41	3.80

Ext. Example - Application of the generalized α- values



A simple calibration case study

$$
\begin{aligned}
H(R, G, Q, X_Q) &= zR_i - (1-a)G - aX_Q Q \quad \text{with} \\
z &= \gamma_{R_i} \frac{(1-a)\cdot\gamma_G\cdot g_k + a\cdot\gamma_Q\cdot q_k^*}{r_{k,i}}
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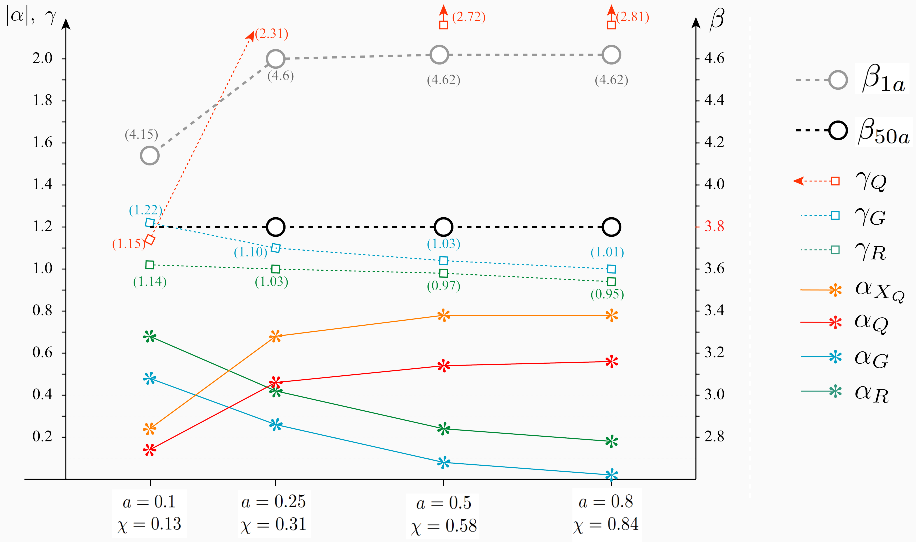
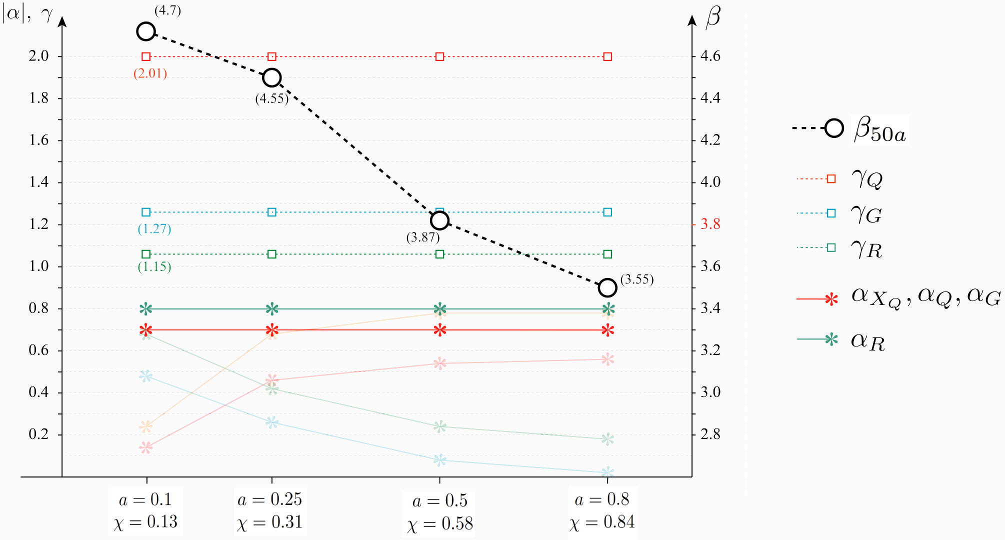
	Dist.	μ	V	p
Material 1	LN	1	0.1	0.05
Permanent	N	1	0.1	0.5
Variable (50a-max)	G	1	0.15	(see below)
Model Uncertainty	LN	1	0.3	

$Q^* = X_Q Q_{1a}$ and q_k^* such that $F_{Q^*}\left(q_k^*\right) = 0.98$

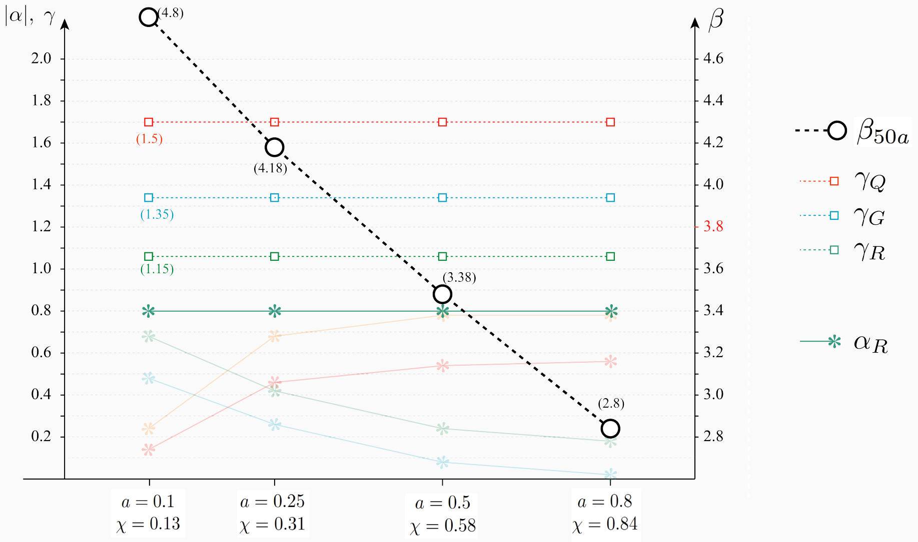
A simple calibration case study - real alpha values



A simple calibration case study - generalized alpha values



A simple calibration case study - generalized alpha values applied on material



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- Especially the application of the generalised α-value **on single variables in isolation** is not effective and, as demonstrated in this note, the obtained safety levels are **partly not acceptable**.
- It is recommended to **reconsider the recommendation of the design value approach** with its generalised α-values in the revision of the Eurocodes.

Questions?

Alternative approach to calibration

Calibration as an optimisation problem

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- We search for the *best compromise*.

$$\min_{\gamma_R^*,\gamma_G^*,\gamma_Q^*} \left\{ \sum\nolimits_{i=1}^{n} \left(\beta_t - \beta_i(\gamma_R, \gamma_G, \gamma_Q, \mathcal{D}_i) \right)^2 \right\} \tag{5}$$

Calibration as an optimisation problem

- Partial factors to be applied for a domain of design situations.
- We search for the *best compromise*.
- The best compromise to be identified by simple least square difference to the target, as

$$\min_{\gamma_R^*,\gamma_G^*,\gamma_Q^*} \left\{ \sum_{i=1}^{n} \left(\beta_t - \beta_i(\gamma_R, \gamma_G, \gamma_Q, \mathcal{D}_i) \right)^2 \right\} \tag{5}$$

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