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Lecture Notes

Calibration of partial factor design formats - best practice and challenges

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Synopsis:
These are the Notes that accompany the lecture given at the graduate course on "Reliability, risk, and decision analysis in structural engineering", Lund, 4.5.2022. As the time for the lecture is limited, it is suggested that students read the following text before the lecture. The text is structured into 4 chapters. **Chapter 1 and 2 to shall be read by all students participating in the course (reading time approx. 1 hour)**, chapters 3 and four are recommended.
Structural design standards are used for the daily design of structures. They comprise of simple rules that represent the current best practice and are generally applied to regular structures. Structural design standards do not account for risk, reliability and uncertainty in an explicit manner. However, the safety concept that is implemented is taking reference to uncertainty and reliability implicitly in terms of partial safety factors and characteristic values.
In this lecture, it is demonstrated that structural design standards correspond to the societal preference for safety and cost effectiveness. A direct correspondence between reliability and the choice of partial safety factors is developed for particular cases. However, it is also shown that the set up of structural design codes is merely a problem of proper generalisation and simplification.

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1. Concepts

1.1. Levels of design

The design of a structure is in principle a decision problem with the objective to identify a structural design with a performance that maximizes the expected utility (or equivalently minimizes the expected cost) for the corresponding decision maker (Rackwitz 2000). A common attribute of these decisions is that they have to be performed subject to uncertainties. In general, different levels of detail for the assessment of the structural performance are distinguished (ISO 2015); risk informed decision making, reliability based design, and semi-probabilistic design (see overview in Figure 1).

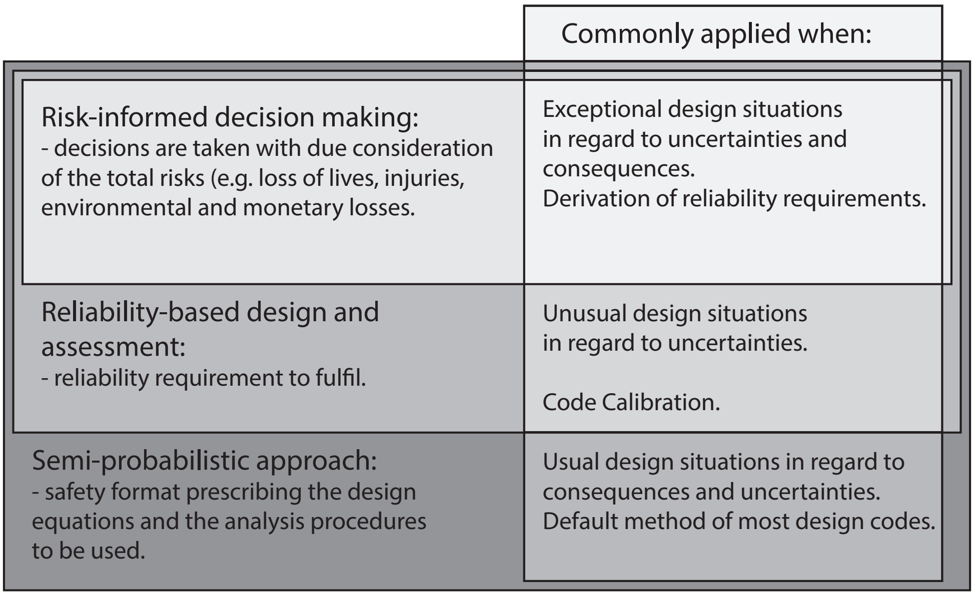

Figure 1: Different levels of detail for the assessment of the structural performance as arranged in ISO (2015)

The three different approaches are often associated to numbered levels: The semi-probabilistic approach corresponds to Level 1, the reliability based approach to Level 2 or 3 (dependent on the level of detail in uncertainty representation), and the risk informed approach corresponds to Level 4. The approaches on the different levels closely correspond to each other. The ability to account for the particular conditions of specific design situations and therefore identify a more optimal design solution is also increasing with increasing level. That is the reason why higher level approaches should be used to verify or calibrate lower level approaches. However, the complexity and difficulty is increasing with increasing level and from a regulative perspective, i.e. where a unified set of rules and assumptions for broad application shall be standardized, low level approaches are advantageous since their inherent low level of detail goes along with the allowance for broad generalization.

1.2. Risk informed decision making

Risk informed decision making allows for the highest level of detail and follows broadly the scheme of Bayesian Decision Analysis JCSS (2008). In the structural engineering context, decision alternatives may include structural measures, i.e. the dimension of cross sections, the choice of material grades, the design of a superstructure, repair and strengthening etc., and non-structural measures as e.g. the implementation of quality control and checking, inspection or the installation of structural health monitoring. Simplification or enhancement in the representation of the decision problem by models could also considered as a non-structural measure. It is assumed that executive decision making is not mechanically following the results of the formal decision analysis but that the well documented assessment informs the decision maker, such that he or she is able to identify his or her final decision.

Risk informed decision making is very flexible and can be applied for systems of different scale in space and time. Depending on the definition of the system boundaries, structural performance attributes as e.g. sustainability, resilience, robustness and reliability can be addressed within an analysis (Faber et al. 2018). The risk informed decision framework can also be applied for the calibration of lower level decision making methods.

Guidance and standardisation for risk based decision making can be found in ISO (2015). However, the flexibility and generality of the method requires a large amount of expertise and experience from the person or group of persons elaborating on a risk based decision analysis.

1.3. Reliability based design

In reliability based design, the design decision is chosen such that it complies to a predefined reliability requirement. The reliability requirement is defined based on past experience, i.e. specified as the inherent reliability of traditionally accepted design solutions (Baravalle et al. 2017), or it is based on formal calibration using risk informed methods. Following the latter, it may be differentiated between different types of decisions on structures in regard to consequences and the relative cost of implementing the decision as discussed in the keynote lecture from yesterday. In reliability based design, it is possible to assess the effect of a decision on the structural performance on a component level, i.e. only one possible failure mode is considered, or on a system level, i.e. the interaction of different failure modes in a structure are considered. However, as consequences are not represented explicitly in a reliability analysis, subordinate structural performance attributes as robustness, sustainability and resilience can not be considered explicitly by reliability based design.

Reliability based design is addressed in the international standard ISO (2015), more detailed guidance on structural reliability methods and uncertainty representation in regard to models, structural resistance and structural demands is found in the Probabilistic Model Code of the Joint Committee on Structural Safety, (JCSS 2001).

1.4. Semi-probabilistic design

The semi-probabilistic approach corresponds to the lowest level of detail. Here, a design decision is chosen such that it complies with the criterion that a design value of a resistance is larger than a design value of a corresponding load effect. Design values for the load bearing capacity r_d are chosen to have a sufficiently low non-exceedance probability and design values for loads e_d are chosen to have a sufficiently low exceedance probability such that the design criterion in the limit ($r_d = e_d$) corresponds to the required level of reliability.

In the so-called load and resistance factor design (LRFD) format (Ravindra et al. 1978) design values are estimated based on characteristic values and partial safety factors γ as e.g. $r_d = r_k/\gamma_M$ for resistance variables and $e_d = \gamma_E \cdot e_k$ for the effects of applied loads. Both, the definition of the characteristic value and the choice of partial safety factor, is made in order to meet the reliability requirements. However, the correspondence to reliability requirements is generally made for domains of structures for that generalised assumptions in regard to consequences and uncertainties are made. With semi-probabilistic design, structural performance on a component/failure mode level can be assessed. The explicit consideration of the interaction of failure modes in a structure, i.e. system effects, is not accommodated.

The principles of semi-probabilistic design are outlined in ISO (2015). It is the method of choice for most structural design decision problems and executive guidance and standardisation is found in several national and international design standards as, e.g. the Eurocodes (CEN 2002).

1.5. Relevance and correspondence

The different levels of engineering decision making are all relevant and support the superordinate objective of the safe and optimal development and maintenance of structures in the build environment. And the approaches on the different levels closely correspond to each other. The ability to account for the particular conditions of specific design situations and therefore identify a more optimal design solution is increasing with increasing level. That is the reason why higher level approaches are used to verify or calibrate lower level approaches. However, the complexity and difficulty is increasing with increasing level and from a regulative perspective, i.e. where a unified set of rules and assumptions for broad application shall be standardised, semi-probabilistic approaches are advantageous since their inherent low level of detail goes along with the allowance for broad generalisation. The design, calibration and layout of simplified decision making approaches, has to be based, directly or indirectly, on risk informed decision making, as this is the only level of detail that allows for the explicit consideration of the superordinate objectives in engineering decision making.

2. EUROCODE semi-probabilistic design

2.1. General framework and design values

In the Eurocodes a semi-probabilistic design method is introduced through partial factor design that is generally applied as default design method for common design situations. The partial factor design format is formulated such that it facilitates the identification of acceptable and feasible design solutions concerning new structures. The partial factor design format comprises:

- consequence class categorization (see EN1990 Annex B);

- design situations (see EN1990 and EN1991-EN1999);
- design equations (see EN1990 and EN1991-EN1999);
- design values (as determined according EN1990, section 6).

Design equations can in general be formulated for failure modes involving in principle both failure of individual cross sections of the structures, as well as for failure modes involving the failures of several cross sections of the structures. The principle form of the design equation is given as (compare EN1990, equation 6.1):

$$r_d - e_d \geq 0 \tag{1}$$

where the design values for the load bearing capacity r_d and the action effect e_d are obtained from:

$$r_d = r_d\left(\mathbf{x_d}; \mathbf{a_d}; \boldsymbol{\theta}_\mathbf{d}\right) \tag{2}$$

$$e_d = e_d\left(\mathbf{f_d}; \mathbf{a_d}; \boldsymbol{\theta}_\mathbf{d}\right) \tag{3}$$

where:

$\mathbf{x_d}$ is a vector of design values of material properties;

$\mathbf{f_d}$ is a vector of design values of actions;

$\mathbf{a_d}$ is a vector of design values of geometrical properties;

$\boldsymbol{\theta}_\mathbf{d}$ is a vector of design values of model uncertainties.

2.2. Partial factors

The design value of a basic variable Y is defined as the multiplication or division of a corresponding partial safety factor γ_Y and the characteristic value y_k. Accordingly, Eq. (1) can be rewritten in an extended form as:

$$\frac{r_k}{\gamma_R} = r_d \geq e_d = \gamma_E e_k \tag{4}$$

A characteristic value y_k is taken as a specified $p-$ fractile value from the statistical distribution $F_Y(y)$ that is chosen to represent the basic variable, as:

$$y_k = F_Y^{-1}(p) \tag{5}$$

where $F_Y(y)$ is the cumulative probability distribution function of the basic variable.
Note: Typical values for p are:

- resistance related variables: $p = 0.05$;
- permanent actions: $p = 0.5$;
- time-variable actions (yearly reference period): $p = 0.98$.

The partial safety factors for the various actions and materials characteristics entering the design equations are determined such as to account for the uncertainties associated with the loads and resistances which are of relevance for the given design situation in such a way that design solutions that fulfill the inequality in Eq. (4) are consistent with the reliability requirements of the Eurocodes.

2.3. Reliability requirements in the Eurocodes

A basic requirement stated in the Eurocode is that structures shall sustain its anticipated loads with appropriate level of reliability and in an economical way, see section 2.1(1) of EN 1990:2002. The term “appropriate level of reliability” is specified further in 2.2(3) where its choice is related to relevant factors, including:

(a) the possible cause and /or mode of attaining a limit state;

(b) the possible consequences of failure in terms of risk to life, injury, potential economical losses;

(c) public aversion to failure;

(d) the expense and procedures necessary to reduce the risk of failure.

A formal differentiation according to (a), (c) and (d) is not further considered in the present version of the Eurocode. According to (b), structures and components are classified, e.g., regarding the consequence of failure, whereas the reliability levels might be given either for classified structures as a whole or for classified components (see 2.2(4) of EN 1990:2002).

In EN 1990:2002, Annex B3.1, a classification of buildings and structures in terms of consequences is defined (3 classes). It is mentioned that importance of a failure mode for the consequences should be considered (see B3.1(2,3) of EN 1990:2002). This might lead to different classification of different failure modes in one structure. In EN 1990:2002, B3.2 minimum reliability levels for 3 consequence classes are recommended.

Table B2 - Recommended minimum values for reliability index β (ultimate limit states)

Reliability Class	**Minimum values for β**	
	1 year reference period	**50 years reference period**
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

Figure 2: Reliability requirements as stated in the EUROCODES.

2.4. Direct correspondence between reliability and design values - the design value approach

For specific cases, a direct correspondence between the design value and the reliability requirements can be established by the so called design value method. In order to demonstrate this, the simple reliability problem is revisited, where one normal distributed resistance variable R is compared with a normal distributed load variable S by a safety margin M that is defined correspondingly as normal distributed (for independent R and S), i.e. $M = R - S$. Failure is then defined as $M = R - S < 0$ and the failure probability as

$$\begin{aligned} p_F = \Pr\left(M = R - S < 0\right) &= \Phi\left(\frac{0-\mu_M}{\sigma_M}\right) \\ &= \Phi\left(\frac{-\mu_R+\mu_S}{\sqrt{\sigma_R^2+\sigma_S^2}}\right) \\ &= \Phi\left(-\beta\right) \end{aligned} \tag{6}$$

Thus, for this simple case, the reliability index β is defined as

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2+\sigma_S^2}} \tag{7}$$

In structural design e.g. according to the Eurocodes, the reliability requirement is given as a design requirement β_{req}. Therefore, it is useful to reformulate the above expression, i.e. to facilitate the identification of a combination of R and S that comply with, say, $\beta_{req} = 3.8$ such that $\beta \geq \beta_{req}$. The distance between the mean values that is in compliance with the required reliability is determined as $\mu_R - \mu_S \geq \beta_{req}\sigma_M$, whereas σ_M can be splitted into the contributions from R and S as

$$\mu_R - \mu_S \geq \beta_{req}\frac{\sigma_R}{\sqrt{\sigma_R^2+\sigma_S^2}}\sigma_R + \beta_{req}\frac{\sigma_S}{\sqrt{\sigma_R^2+\sigma_S^2}}\sigma_S$$

Introducing the weighting factors α_R and α_S as

$$\begin{aligned} \alpha_R &= \frac{\sigma_R}{\sqrt{\sigma_R^2+\sigma_S^2}} \\ \alpha_S &= \frac{\sigma_S}{\sqrt{\sigma_R^2+\sigma_S^2}} \end{aligned} \tag{8}$$

and separating the resistance from the load side, the following expression is obtained:

$$\begin{aligned} \mu_R - \alpha_R\beta_{req}\sigma_R &\geq \mu_S + \alpha_S\beta_{req}\sigma_S \\ \mu_R\left(1-\alpha_R\beta_{req}V_R\right) &\geq \mu_S\left(1+\alpha_S\beta_{req}V_S\right) \\ r_d &\geq s_d \end{aligned} \tag{9}$$

with the coefficients of variation V_R and V_S and so-called design values r_d and s_d.

Note: Equation (9) looks very convenient for structural design as it suggests the separation of load and resistance variables. But this is not really the case as the weighting factors α_R and

α_S contain the standard deviations of both variables. The implications of this are demonstrated by the following example.

Example 1. Consider the simple example, where a tension rod with material capacity C is exposed to a load S. Both variables are considered as independent and normal distributed with parameters given in Table 1. The cross section area a that complies with the reliability requirement $\beta_{req} = 3.8$ is to be specified by the application of Eq. (9).

Table 1: Mean value μ and coefficient of variation V of the capacity C and the load S.

	μ	V
Capacity $C\ [kN/mm^2]$	1	0.10
Load $S\ [kN]$	1	0.34

The required design value r_d for the resistance $R = a \cdot C$ can be expressed based on Equation (9):

$$r_d = a \cdot \mu_C \left(1 - \alpha_R \beta_{req} V_C\right) \geq \mu_S \left(1 + \alpha_S \beta_{req} V_S\right) = s_d$$

$$a \cdot \mu_C \left(1 - \frac{a\sigma_C}{\sqrt{a^2\sigma_C^2 + \sigma_S^2}} \beta_{req} V_C\right) \geq \mu_S \left(1 + \frac{\sigma_S}{\sqrt{a^2\sigma_C^2 + \sigma_S^2}} \beta_{req} V_S\right)$$

, i.e. the cross section area a is chosen such that the equation above is fulfilled.

Unfortunately, the identification of a is not as straight forward as it looks, since both, α_R and α_S are dependent on a. However, a solution can be found with $a = 2.615\ mm^2$, which corresponds to $r_d = s_d = 2\ kN$ and $\alpha_R = 0.6153$ and $\alpha_S = 0.7882$.

Obviously, the same result is obtained based on Equation (7), i.e. without the splitting into design values.

$$a \quad \text{such that} \quad \beta_{req} = 3.8 \leq \beta = \frac{a \cdot \mu_C - \mu_S}{\sqrt{a^2\sigma_C^2 + \sigma_S^2}}$$

$\triangle$

From the example it becomes obvious that the computation of the design value of the resistance, r_d, requires the evaluation of α_R and this in turn requires the full evaluation of the reliability problem. In order to circumvent this problem the EUROCODES introduce so-called *standardized α values*. In order to develop the background for the choice of these *standardized α values* it is useful to represent the considered simple reliability problem as it was first introduced by Hasshofer and Lind in 1974.

2.4.1. Alternative representation of the simple reliability problem

Hasofer et al. 1974 suggested an alternative representation of the reliability problem and a generalised definition of the reliability index that is also applicable to non-linear reliability problems with non-normal random variables. Applied to the simple reliability problem from above, i.e. one normal distributed resistance variable R is compared with a normal distributed load variable

S, a so-called limit state function is introduced, as

$$g(r, s) = r - s = 0 \tag{10}$$

The limit state function can be illustrated together with the joint probability density function of R and S, $f_{R,S}(r, s)$, compare Figure 3, where it can be seen that the limit state is dividing the possible domain of r and s into a failure domain, $g(r, s) \leq 0$ and a safe domain, $g(r, s) > 0$. The probability of failure can be computed by integrating the joint probability distribution function over the failure domain, i.e.

$$p_F = \int_{g(r,s)\leq 0} f_{R,S}(r, s) dr\, ds \tag{11}$$

By transforming the normal distributed random variables, R and S into standard normal variables $U_R = (R - \mu_R)/\sigma_R$ and $U_S = (S - \mu_S)/\sigma_S$, the limit state function can be written as a function of u_R and u_S, as

$$g(u_R, u_S) = (u_R\sigma_R + \mu_R) - (u_S\sigma_S + \mu_S) = 0 \tag{12}$$

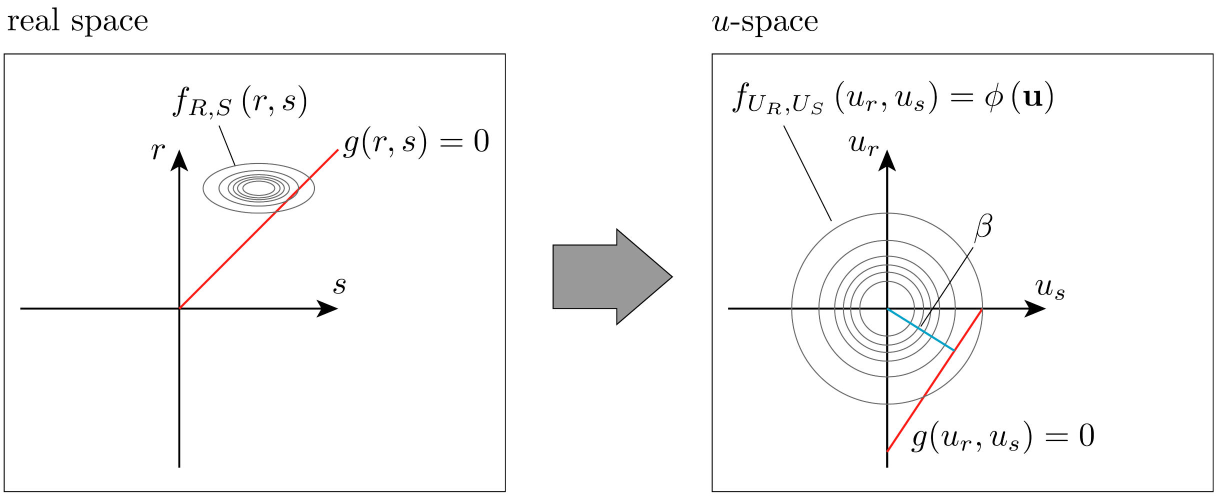

Figure 3: Left: The simple reliability problem with the resistance r and the load s. The topology of the joint probability density is indicated by the ellipses, the failure domain is below the limit state $g(r, s) = 0$, the safe domain is above.
Right: The same problem represented in the standard normal space and with the variables u_R and u_S. $f_{U_R,U_S}(u_R, u_S) = \phi(\mathbf{u})$ is the standard normal density.

Introducing the unit vector $\boldsymbol{\alpha}$, with the cosine of direction α_R and α_S, we find the reliability index β geometrically (compare Figure 3) as the shortest (orthogonal) distance between the limit state and the origin:

$$\begin{aligned} g(u_R, u_S) &= \beta - \alpha_R \cdot u_R - \alpha_S \cdot u_S = 0 \\ g(\mathbf{u}) &= \beta - \boldsymbol{\alpha}^T\mathbf{u} = 0 \end{aligned} \tag{13}$$

For the simple reliability problem the α-values and β are identical to Equation (7) and (8), the different geometrical interpretation, however, is much more general, that is

- The design point $\mathbf{u}^* = \boldsymbol{\alpha}\beta$ is the point on the limit state function $g(\mathbf{u}) = 0$ that is closest to the origin.
- $\boldsymbol{\alpha}$ is the normal vector on the limit state in the design point.
- β is the distance between the origin and the design point.
- In correspondence to Equation (7) $r_d = \alpha_R \beta \sigma_R + \mu_R$ and $s_d = \alpha_S \beta \sigma_S + \mu_S$ are defining the coordinates of the design point and are referred to as the design values of the resistance and the load.

Example 2. The alternative representation is applied to the example above and obviously, similar results are obtained. The corresponding graphical representation can be inspected in Figure 4.

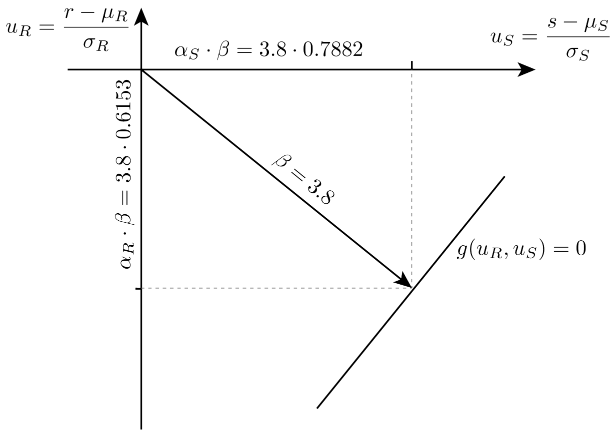

Figure 4: Illustration of Example 1 in the standard normal space.

△

2.5. *Derivation of generalized α-values*

In the following, the derivation of a set of generalised α-values that provide sufficiently accurate estimations of the design point for a relevant range of design situations is explored. Similar considerations have been done by König et al. 1981 and their results have been adapted in the current version of EN1990.

The idea is as follows: $\boldsymbol{\alpha}$ is a unit vector of length 1. If a set of generalised α-values is selected such that the corresponding vector sum is larger than 1, these generalised α-values could be applied to a range of cases and still lead to design solutions that comply with the required reliability. The idea is illustrated in Figure 5.

Accordingly and for linear limit state functions and normal distributed variables R and S, the range of cases for which the generalised α-values, if applied in combination both on the load and on the resistance side, result in design solutions that comply with the reliability requirement can be represented by the angle δ as

$$\cos \delta = \frac{\beta}{\beta^*} = \frac{1}{\sqrt{\alpha_R^{*2} + \alpha_S^{*2}}} \tag{14}$$

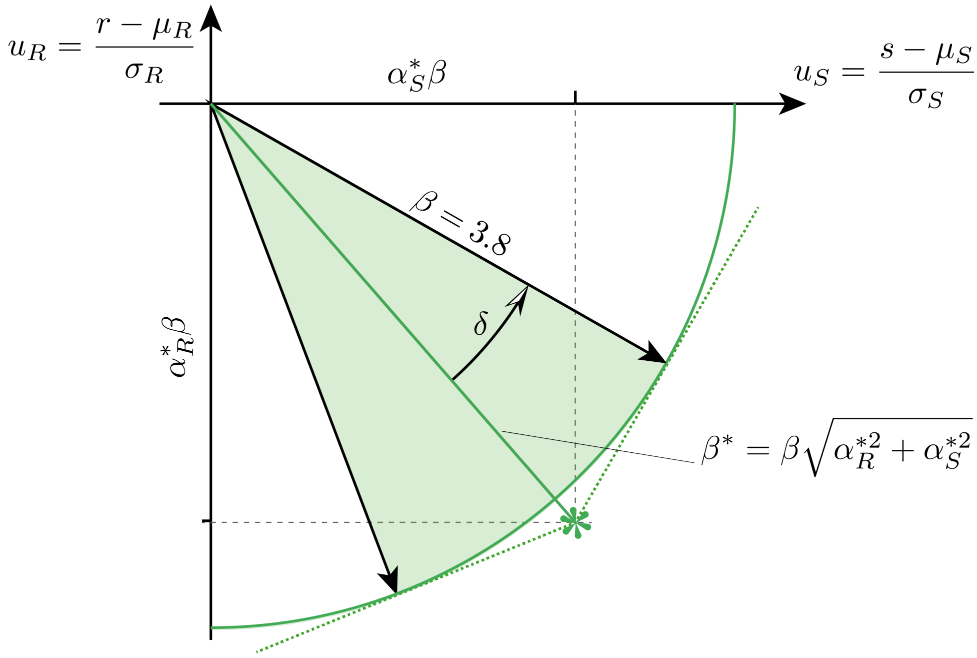

Figure 5: Illustration of the effect of the application of generalised α-values, α_R^* and α_S^* creating a range of design cases (indicated in green) for which compliance with the safety requirement can be achieved.

2.6. The generalised α-values in the Eurocodes

In the Eurocodes (prEN 1990:**E2020** and EN1990:2002) the generalised α-values for the dominant resistance and load variable are

$$\begin{aligned} \alpha^*_{R,EC} &= -0.8 \\ \alpha^*_{S,EC} &= 0.7 \end{aligned} \tag{15}$$

With $\beta_{req} = 3.8$ and following up the illustration in Figure 5, this corresponds to $\beta^* = 4.04$ and $\delta = 19.8°$. When Equation (9) is applied for evaluating the design values r_d and s_d the design solutions identified based on the criterion $r_d = s_d$ have a reliability index larger than 3.8 for normal distributed variables and real α-values in the range of $\alpha_R > -0.956$ and $\alpha_S < 0.9$.

2.6.1. Generalisation of the Design Value Approach to other than Normal distributions

With proper transformation, the variables design value can be approximated.

Accordingly, the design value is defined as

$$y_d = F_Y^{-1}\left(\Phi(\alpha_Y \beta_t)\right) \tag{16}$$

where y_d is the design value of a variable Y, F_Y is its cumulative distribution function, α_Y (with $|\alpha_Y| \leq 1$) is a sensitivity factor indicating the importance of Y in the reliability estimation, and β_t is the target value for the reliability index specifying the reliability requirement.

Design value y_d and characteristic value y_k for some common distributions are determined according:

Normal:	$y_d = \mu_Y \left(1 + \alpha_Y \beta_t V_Y\right)$ $y_k = \mu_Y \left(1 + \Phi^{-1}(p) V_Y\right)$
Log-Normal:	$y_d = \mu_Y \exp\left(-\frac{1}{2}\ln\left(1+V_Y^2\right) + \alpha_Y \beta_t \sqrt{\ln\left(1+V_Y^2\right)}\right)$ $y_k = \mu_Y \exp\left(-\frac{1}{2}\ln\left(1+V_Y^2\right) + \Phi^{-1}(p) \sqrt{\ln\left(1+V_Y^2\right)}\right)$
Gumbel:	$y_d = \mu_Y \left(1 - V_Y \frac{\sqrt{6}}{\pi}\left(0.5772 + \ln\left(-\ln\left(\Phi(\alpha_Y \beta_t)\right)\right)\right)\right)$ $y_k = \mu_Y \left(1 - V_Y \frac{\sqrt{6}}{\pi}\left(0.5772 + \ln\left(-\ln\left(p\right)\right)\right)\right)$

The FORM sensitivity factors α_Y are resulting from the reliability analysis of the the design situation at hand. If no reliability analysis is performed, the following Eurocode standardized values can be used for a 50 years reference period:

- If Y represents a strength related variable: $\alpha_Y = -0.8$
- If Y represents a load related variable: $\alpha_Y = 0.7$
- If Y is dominating the reliability problem: $\alpha_Y = (-)1$
- If Y represents a secondary strength or load related variable: $\alpha_Y = -0.8 \cdot 0.4$ or $\alpha_Y = 0.7 \cdot 0.4$ correspondingly.

Self-weight is usually represented by a Normal distribution; Resistance variables are often represented by a Log-Normal distribution; the extreme values per reference period of time-variable actions are represented by the Gumbel distribution.

3. Application examples

E-1.1. Code calibration

NOTE: A similar example is discussed alongside a Jupyter Notebook during the lecture. Note that the input variables are not entirely identical (and therefore neither the results).

Geometry

The simply supported beam in Figure 6 is to be designed with a given target reliability. The basic random variables that represent the situation are given in Table 3. The resulting beam span and cross-section modulus are dependent on the design philosophy that is followed. The following formats are to be considered: reliability based design, load and resistance factor design and global partial safety factor.

The limit state function for bending moment at mid-span is:

$$g(f_m, g, q) = W \cdot f_m - g\frac{l^2}{8} - q\frac{l}{4} \leq 0 \tag{17}$$

Table 3: Basic random variables (* $\equiv$ yearly maxima).

	G	Q^*	F_m
Distribution	Normal	Normal	Normal
Mean (μ_X)	2 N/mm	10000 N	200 MPa
Standard dev. (σ_X)	0.30 N/mm	4000 N	20 MPa
Coeff. of var. (COV_X)	15%	40%	10%
Characteristic fractile (k_X)	0.50	0.95	0.05

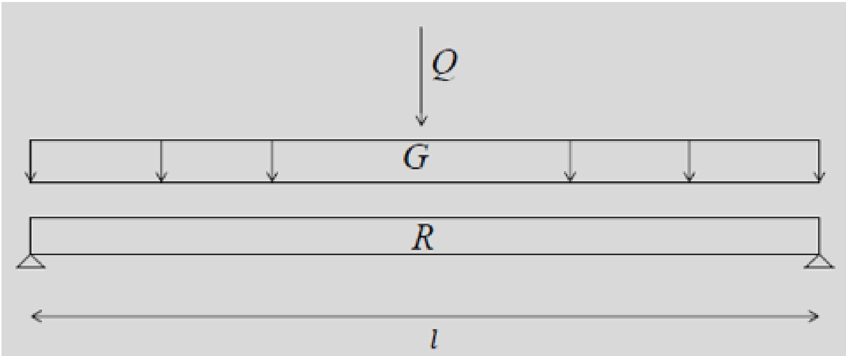

Figure 6: Beam geometry and loads.

The corresponding reliability index is:

$$\beta = \frac{W\mu_{F_m} - \mu_G \frac{l^2}{8} - \mu_Q \frac{l}{4}}{\sqrt{(W\sigma_{F_m})^2 + \left(\sigma_G \frac{l^2}{8}\right)^2 + \left(\sigma_Q \frac{l}{4}\right)^2}} \tag{18}$$

E-1.2. Reliability based design

The beam cross-section W can be chosen such that the reliability index of the beam corresponds to the target reliability, i.e. $\beta = \beta_t$. This corresponds to the so-called **reliability based design.** E.g. for $l = 6$ m, the section plastic modulus giving $\beta = \beta_t$ is found to be $W = 3.01 \cdot 10^5$. The cross-section plastic modulus is plotted against the beam length in Figure 7.

E-1.2.1. Load and resistance factor design format

Although modern structural design codes (like Eurocodes) allow reliability based design, they propose simpler design procedures in the **semi-probabilistic design format**, where design equations are written in the **load and resistance factor design format** (LRFD). This format is referred to as **semi-probabilistic**, since random variables (represented by distribution functions) are reduced to design values, which are obtained from characteristic values as $r_d = r_k/\gamma_R$ (for resistance) and $s_d = s_k/\gamma_S$ (for actions). The format is simpler, since the designer can check the design just by comparing design values, without the necessity of using reliability analyses.

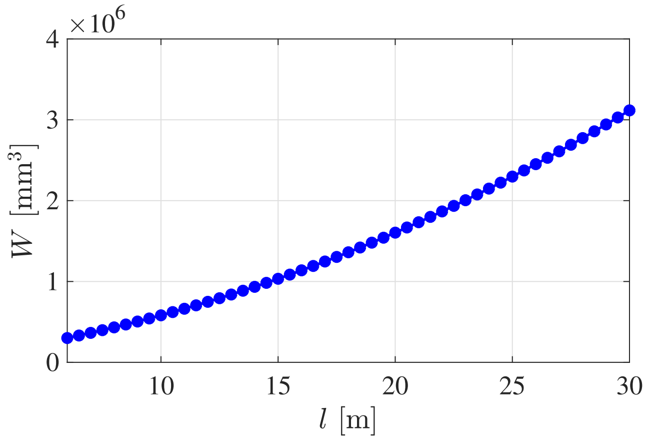

Figure 7: Reliability based design for different beam lengths for $\beta = \beta_t = 4.2$.

The design equation of the beam above is:

$$W \cdot f_{m,d} - g_d \frac{l^2}{8} - q_d \frac{l}{4} \leq 0 \tag{19}$$

where:

- $f_{m,d} = \dfrac{f_{m,k}}{\gamma_R}$; $g_d = g_k \gamma_G$ and $q_d = q_k \gamma_Q$ are the **design values**;
- $f_{m,k} = F_{F_m}^{-1}(k_{F_m})$; $g_k = F_G^{-1}(k_G)$ and $q_k = F_Q^{-1}(k_Q)$ are the **characteristic values** corresponding to the characteristic fractiles k_{F_m}, k_G and k_Q, which are defined arbitrarily and
- γ_G; γ_R and γ_Q are the **partial safety factors (p.s.f.)**.

The minimum cross-section plastic modulus satisfying Eq. (19) is:

$$W_{min} = \frac{g_K \gamma_G \dfrac{l^2}{8} + q_K \gamma_Q \dfrac{l}{4}}{\dfrac{f_{m,k}}{\gamma_R}} \tag{20}$$

3.0.1. Calibration of partial safety factors for a given span

The p.s.f. need to be **calibrated** so that the design given in Eq. (20) fulfills a target safety. This corresponds to answer the question *which values of the p.s.f. in Eq. (20) provides a design (i.e. W) associated with the level of risk $\beta = \beta_t$?*

The limit state function in Eq. (17) is rewritten with $W = W_{min}$:

$$g(f_m, g, q) = \frac{g_K \gamma_G \dfrac{l^2}{8} + q_K \gamma_Q \dfrac{l}{4}}{\dfrac{f_{m,k}}{\gamma_R}} f_m - g \frac{l^2}{8} - q \frac{l}{4} \leq 0 \tag{21}$$

Consequently Eq. (18) becomes:

$$\beta = \frac{\left(\dfrac{g_K\gamma_G\dfrac{l^2}{8}+q_K\gamma_Q\dfrac{l}{4}}{\dfrac{f_{m,k}}{\gamma_R}}\right)\mu_{F_m}-\mu_G\dfrac{l^2}{8}-\mu_Q\dfrac{l}{4}}{\sqrt{\left(\left(\dfrac{g_K\gamma_G\dfrac{l^2}{8}+q_K\gamma_Q\dfrac{l}{4}}{\dfrac{f_{m,k}}{\gamma_R}}\right)\sigma_{F_m}\right)^2+\left(\sigma_G\dfrac{l^2}{8}\right)^2+\left(\sigma_Q\dfrac{l}{4}\right)^2}} \tag{22}$$

where it is clear that the reliability of the design depends on the partial safety factors, i.e. $\beta = \beta(\gamma_R, \gamma_G, \gamma_Q)$.

The calibrate partial safety factors $(\gamma_R^*, \gamma_G^*, \gamma_Q^*)$ are found by setting $\beta(\gamma_R^*, \gamma_G^*, \gamma_Q^*) \equiv \beta_t$. Note that there are infinite sets of $(\gamma_R^*, \gamma_G^*, \gamma_Q^*)$ which satisfy the equality. By fixing one p.s.f. (e.g. $\gamma_R^* = 1.05$) a unique set is obtained.

By way of example, setting $l = 6$ m, the calibrated partial safety factors result in $(\gamma_R^* = 1.05, \gamma_G^* = 1.344, \gamma_Q^* = 1.437)$, yielding $W = 300610$ mm^3. Numerical minimization in Matlab© has been used.

Design value method - FORM sensitivity factors method

An alternative method for p.s.f. calibration makes use of the FORM sensitivity factors, which are:

$$\alpha_{F_m} = \frac{-W\sigma_{F_m}}{k} \ ; \ \alpha_G = \frac{\left(\sigma_G\dfrac{l^2}{8}\right)}{k} \ ; \ \alpha_Q = \frac{\left(\sigma_Q\dfrac{l}{4}\right)}{k} \tag{23}$$

with:

$$k = \sqrt{(W\sigma_{F_m})^2+\left(\sigma_G\frac{l^2}{8}\right)^2+\left(\sigma_Q\frac{l}{4}\right)^2} \tag{24}$$

Introducing $W = 3.01\cdot 10^5$ mm^3 that was found for $l = 6$ m, the sensitivity factors are $(\alpha_{F_m} = -0.699, \alpha_Q = 0.698, \alpha_G = 0.157)$. The corresponding partial safety factors are:

$$\begin{aligned}
\gamma_R^* &= \frac{f_{m,k}}{f_{m,d}} = \frac{1+\Phi^{-1}(k_{F_m})\cdot COV_{F_m}}{1+\alpha_{F_m}\beta_t COV_{F_m}} = 1.18\\
\gamma_Q^* &= \frac{q_d}{q_k} = \frac{1+\alpha_Q\beta_t\cdot COV_Q}{1+\Phi^{-1}(k_Q)COV_Q} = 1.31\\
\gamma_G^* &= \frac{g_d}{g_k} = \frac{1+\alpha_G\beta_t\cdot COV_G}{1+\Phi^{-1}(k_G)COV_G} = 1.10
\end{aligned} \tag{25}$$

It is highlighted that the two sets of optimized p.s.f. $(\gamma_R^* = 1.05, \gamma_G^* = 1.344, \gamma_Q^* = 1.437)$ and $(\gamma_R^* = 1.18, \gamma_G^* = 1.31, \gamma_Q^* = 1.10)$ are equivalent in the sense that they lead to the same design (i.e. same W) and the same reliability index (i.e. $\beta = \beta_t = 4.2$) for $l = 6m$.

Calibration of partial safety factors for different scenarios

In general, the optimized p.s.f. obtained above using the FORM sensitivity factor method result in a reliability $\beta \neq \beta_t$, for a beam span $l \neq 6$ m, cf. Figure 8. This is due to the fact that the fraction of the external moment induced by G (or Q) changes with the beam span. The reliability for $l = 6$ m is equal to the target, since that was the condition imposed for calibration. The reliability index is therefore dependent on the beam span (or equivalently the ratio between the moments induced by G and Q), i.e. $\beta = \beta(\gamma_R, \gamma_G, \gamma_Q, l)$.

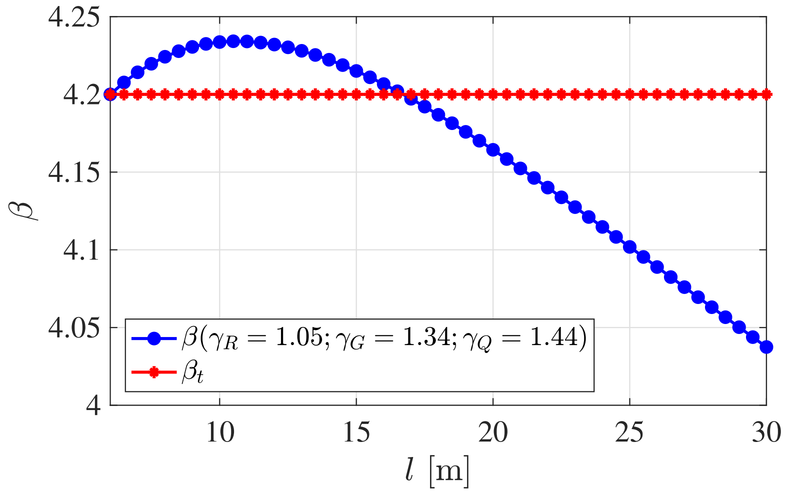

Figure 8: Reliability index of the beam designed with p.s.f optimized for $l = 6$ m.

In order to get a constant reliability, i.e. independent of the geometry, a new set of p.s.f should be calibrated for each beam length, see Figure 9. This is not practical and not implemented in design codes. The problem is solved by finding the set of p.s.f which ensures the reliability to be as homogeneous as possible over a range of possible scenarios. The set can be obtained by solving the minimization problem in Eq. (26).

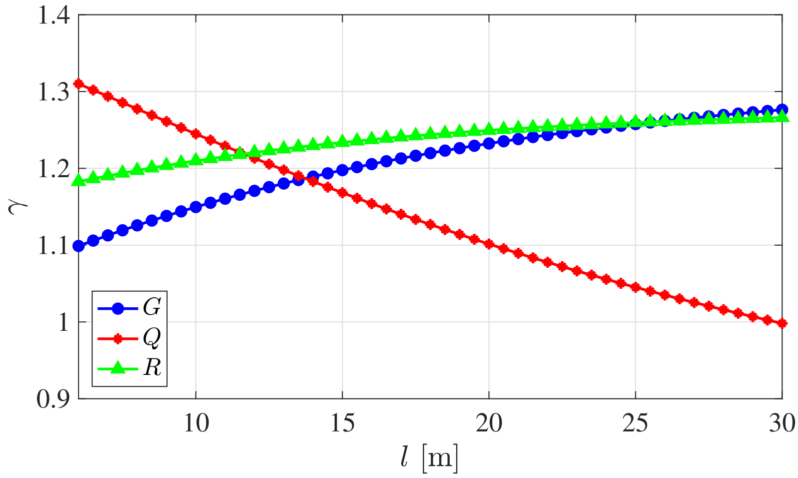

Figure 9: Partial safety factors giving constant reliability, calibrated with FORM α-method giving $\beta = \beta_t = 4.2$.

$$\min_{\gamma_R^*,\gamma_G^*,\gamma_Q^*} \left\{ \sum_{i=1}^{n} \left(\beta_t - \beta_i(\gamma_R, \gamma_G, \gamma_Q, l_i)\right)^2 \right\} \tag{26}$$

The set of p.s.f obtained is ($\gamma_R^* = 1.08, \gamma_G^* = 1.38, \gamma_Q^* = 1.35$). The corresponding reliability is plotted in Figure 10.

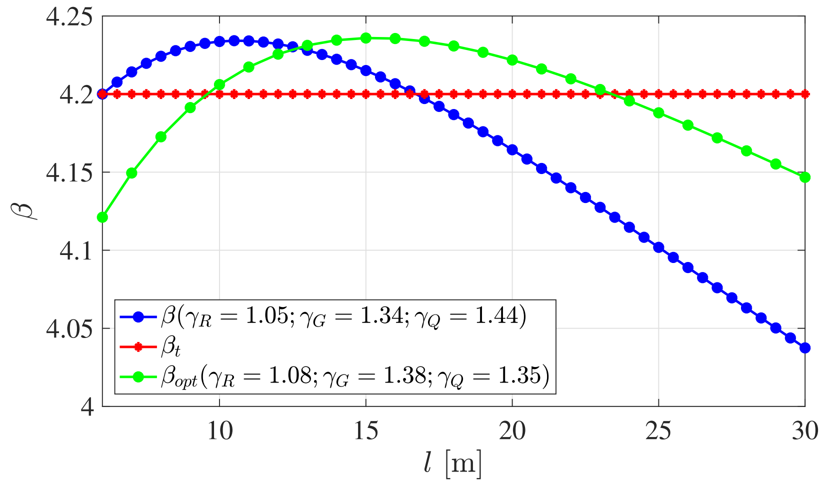

Figure 10: Comparison of the reliability index for p.s.f. calibrated for $l = 6$ m and for different scenarios simultaneously (β_{opt}).

E-1.2.2. Global partial safety factor

An alternative (and older) format of semi-probabilistic design equations makes use of the global safety factor (SF). The design equation with the global safety factor applied on the mean values reads:

$$\frac{W\mu_{F_m}}{\mu_G \frac{l^2}{8} + \mu_Q \frac{l}{4}} > SF \tag{27}$$

where the numerator is the mean value of the moment resistance and the denominator is the mean value of the action induced moment.

The minimum cross-section plastic modulus satisfying Eq. (27) is:

$$W_{min} = \frac{SF}{\mu_{F_m}} \left(\mu_G \frac{l^2}{8} + \mu_Q \frac{l}{4} \right) \tag{28}$$

Calibration of the global safety factor for a given span

As before, SF can be calibrated so that Eq. (28) provides a design with the target level of risk. The limit state function in Eq. (17) is rewritten with $W = W_{min}$:

$$g(f_m, g, q) = \frac{SF}{\mu_{F_m}} \left(\mu_G \frac{l^2}{8} + \mu_Q \frac{l}{4} \right) \cdot f_m - g\frac{l^2}{8} - q\frac{l}{4} \leq 0 \tag{29}$$

Consequently Eq. (18) becomes:

$$\beta = \frac{\dfrac{SF}{\mu_{F_m}}\left(\mu_G\dfrac{l^2}{8}+\mu_Q\dfrac{l}{4}\right)\mu_{F_m}-\mu_G\dfrac{l^2}{8}-\mu_Q\dfrac{l}{4}}{\sqrt{\left(\dfrac{SF}{\mu_{F_m}}\left(\mu_G\dfrac{l^2}{8}+\mu_Q\dfrac{l}{4}\right)\sigma_{F_m}\right)^2+\left(\sigma_G\dfrac{l^2}{8}\right)^2+\left(\sigma_Q\dfrac{l}{4}\right)^2}} \tag{30}$$

where it is clear that the reliability of the design is depending on the global safety factor, i.e. $\beta = \beta(SF)$.

The global safety factor is calibrated by setting $\beta(SF) \equiv \beta_t$. As an example, considering $l = 6$ m, the calibrated global safety factor is found with numerical minimization in Matlab© to be $SF^* = 2.51$, giving $W = 300610$ mm^3.

Calibration of the global safety factor for different scenarios

As before, the design performed with the calibrated SF for $l = 6$ m results in a different reliability for different spans. The SF can be calibrated so to give a reliability as homogeneous as possible over the different design scenarios. Therefore, as previously, the SF can be calibrated by solving a minimization problem, cf. Eq. (31). The optimized SF obtained is $SF^* = 2.20$, see Figure 11.

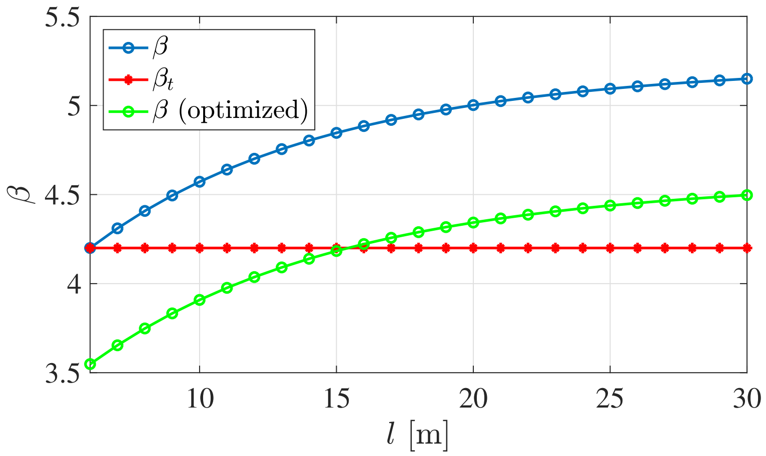

Figure 11: Reliability index for SF calibrated for $l = 6$ m (in blue) and for different scenarios simultaneously (in green).

$$\min_{SF^*}\left\{\sum_{i=1}^{n}\left(\beta_t-\beta_i(SF^*,l_i)\right)^2\right\} \tag{31}$$

Comments

It can be observed from Figure 10 and Figure 11 that, by changing the SF the curve is only shifted up and down while the three p.s.f. shift and distort the curve. This leads to higher levels of optimization (i.e. a flatter curve) with p.s.f.. The values of $\sum_{i=1}^{n}(\beta_t-\beta_i)^2$ show clearly this

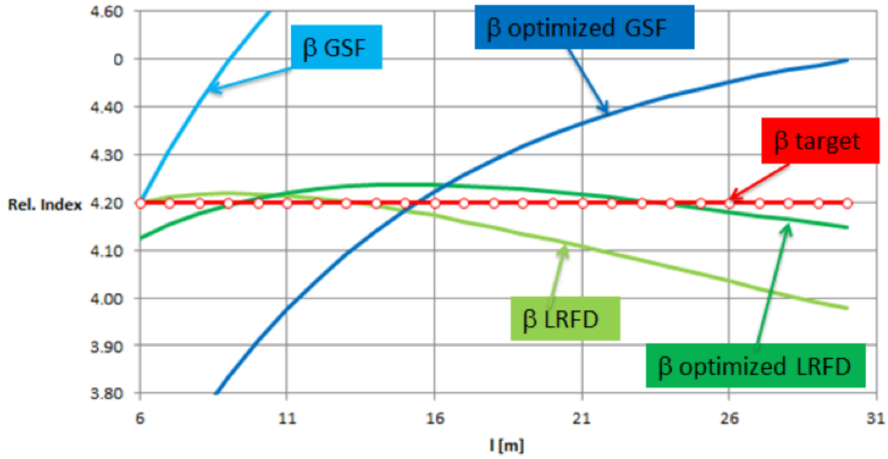

Figure 12: Comparison of β values for SF and $LRFD$ formats before and after optimization.

differences in the two formats. In fact, after optimization $\sum_{i=1}^{n}(\beta_t - \beta_i)^2 = 0.026$ for the p.s.f. format and $\sum_{i=1}^{n}(\beta_t - \beta_i)^2 = 1.88$ for the global safety factor format. The load and resistance factor design method offers therefore a more homogenized level of safety.

4. Real Example

In the context of the revision of the Eurocode a calibration exercises was considered (in CEN TC250/SC10/WG1) where the objective was to calibrate the load partial factors for wind load, snow load and live load in buildings. In the following, some results of this exercise are presented. The study is also published as Köhler et al. 2019.

4.1. Framework

The objective of the assessment of the safety factors of existing design codes is to confirm their absolute magnitude and to evaluate whether a change of the safety factors would lead to a better correspondence with the reliability requirements stated in the code. The key points of the followed framework are:

- The assessment is formulated as a calibration problem, where the load partial factors γ_S are the calibration variables.
- The objective of the calibration is the minimisation of variability of reliability levels in the considered domain of design equations D. The corresponding objective function (to be minimised) is
$$M(\gamma_S) = \sum_{k \in D} w_k \left(\beta_k(\gamma_\Gamma 30803) - \beta_{target}\right)^2 \tag{32}$$
- The domain of design equations D is defined as all design equations for which the load partial factors apply.
- The domain of design equations is represented by a reduced set of design equations representing the main structural materials and loads induced by wind and snow, and permanent, sustained and intermittent actions. The importance of the different design equations is

represented by the weighting factor w_k Assumptions are made for the formulation of this representative set.

- Design equations are represented in a partial factor design format as specified in the Eurocodes and with the corresponding limit states. All load and resistance related variables and model uncertainties are represented as random variables. Assumptions are made for the specification of the random variables, whereas the Probabilistic Model Code of the Joint committee on Structural Safety (**JCSS2001**) is used as the main reference.
- The target reliability level β_{target} is defined as the average reliability level of the considered set of design equations with the partial factors of the present Eurocode (EN1990:2002) (CEN 2002).

4.2. Case Study Eurocodes

Published in 2002, the Eurocodes consist of 10 European Standards, EN 1990 - EN 1999, providing a common basis for the design of buildings and other civil engineering structures (CEN 2002). In 2012 the European Commission issued a mandate (M/515 EN) for a revision of the Eurocodes in order to amend and extend the scope. The revision is currently ongoing and part of it is the assessment of the load related partial factors that are recommended by the Eurocodes by means of reliability based code calibration.

4.2.1. Method

Limit state function and design situations

$$g(\mathbf{X}, p_{ij}) = p_{ij}\Theta_{R,i}R_i - (1-a_Q)(a_G G_S + (1-a_G)G_P) - a_Q\Theta_{Q,j}Q_j \tag{33}$$

$$p_{ij} = max \left\{ \begin{array}{c} \frac{\gamma_{M,i}}{\theta_{Ri,k}r_{i,k}} \left\{(1-a_Q)\left[a_G g_{S,k}\gamma_{GS} + (1-a_G)g_{P,k}\gamma_{GP}\right] + a_Q\psi_{0,j}\gamma_Q\theta_{Qj,k}q_{j,k}\right\} \\ \frac{\gamma_{M,i}}{\theta_{Ri,k}r_{i,k}} \left\{(1-a_Q)\left[a_G g_{S,k}\zeta\gamma_{GS} + (1-a_G)g_{P,k}\zeta\gamma_{GP}\right] + a_Q\gamma_Q\theta_{Qj,k}q_{j,k}\right\} \end{array} \right\} \tag{34}$$

$$p_{ij} = \frac{\gamma_{M,i}}{\theta_{Ri,k}r_{i,k}} \left\{(1-a_Q)\left[a_G g_{S,k}\gamma_{GS} + (1-a_G)g_{P,k}\gamma_{GP}\right] + a_Q\gamma_Q\theta_{Qj,k}q_{j,k}\right\} \tag{35}$$

A generic linear limit state function is formulated for assessing the load partial safety factors. With the limit state function different structural materials and different variables can be considered, i.e. in Eq. (33) the failure mode is dominated by a material property R_i and the loads are the effects of the self-weight (G_S), the permanent load (G_P) and one variable load (Q_j).

The design variable in Eq. (33) is determined by the design equations of CEN 2002 for the material property i and the variable load j in Eq.(34) for "6.10ab" and in Eq. (35)

Table 4: Stochastic models based on **JCSS2001** *(*yearly maxima).*

Random variable		Distr. type	Mean(μ)	COV	Ch. Fract.(value)
Resistance model unc. (steel)	$\Theta_{R,1}$	Logn.	1.00	0.05	(μ)
Resistance model unc. (concrete)	$\Theta_{R,2}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (rebar)	$\Theta_{R,3}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (glulam)	$\Theta_{R,4}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (solid timber)	$\Theta_{R,5}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (masonry)	$\Theta_{R,6}$	Logn.	1.16	0.175	(μ)
Mat. property (steel yielding strength)	R_1	Logn.	1.00	0.07	($\mu - 2\sigma$)
Mat. property (concrete compr. capacity)	R_2	Logn.	1.00	0.15	0.05
Mat. Property (rebar yielding strength)	R_3	Logn.	1.00	0.07	0.05
Mat. property (glulam bending strength)	R_4	Logn.	1.00	0.15	0.05
Mat. property (solid timber bending strength)	R_5	Logn.	1.00	0.20	0.05
Mat. property (masonry compr. strength)	R_6	Logn.	1.00	0.16	0.05
Self-weight (steel)	$G_{S,1}$	Norm.	1.00	0.04	0.50
Self-weight (concrete)	$G_{S,2}$	Norm.	1.00	0.05	0.50
Self-weight (rebar)	$G_{S,3}$	Norm.	1.00	0.05	0.50
Self-weight (glulam)	$G_{S,4}$	Norm.	1.00	0.10	0.50
Self-weight (solid timber)	$G_{S,5}$	Norm.	1.00	0.10	0.50
Self-weight (masonry)	$G_{S,6}$	Norm.	1.00	0.065	0.50
Permanent load	G_P	Norm.	1.00	0.10	0.50
Permanent load (large COV)	$G_{P,v}$	Norm.	1.00	0.20	0.95
Wind time-invariant part	$\Theta_{Q,1}$	Logn.	0.79	0.24	(1.095)
Snow time-invariant part	$\Theta_{Q,2}$	Logn.	1.00	0.30	($\mu + \sigma$)
Imposed load model uncertainty	$\Theta_{Q,3}$	Logn.	1.00	0.10	(1.00)
Wind mean reference velocity pressure*	Q_1	Gumb.	1.00	0.25	0.98
Snow load on roof*	Q_2	Gumb.	1.00	0.40	0.98
Imposed load*	Q_3	Gumb.	1.00	0.53	0.98

Table 5: Material properties, weights and ranges of variations of a_G and a_Q.

i	Mat. property	$w_{R,i}$ (weight)	a_G range	a_Q range	$\gamma_{M,i}$ recommended in current Eurocodes
1	Structural steel yield strength	40%	6*[0.6; 1.0]	[0.2; 0.8]	1.00
2	Concrete compressive strength	15%		[0.1; 0.7]	1.50
3	Re-bar yield strength	25%		[0.1; 0.7]	1.15
4	Glulam timber bending strength	7.5%		[0.2; 0.8]	1.25
5	solid timber bending strength	2.5%		[0.2; 0.8]	1.30
6	Masonry compression strength	10%		[0.1; 0.7]	1.50

for “6.10”. Six material properties listed in Table 5 are considered. Wind ($j = 1$), snow ($j = 2$) and imposed ($j = 3$) loads are considered.

The notation, the random variables and the probabilistic models are reported in Table 4 and Table 5. a_Q is a parameter representing different proportions between variable and permanent loads ($a_Q = 1$ for variable load only). Ten equally spaced and equally weighted

Table 6: Results of the calibration.

	Eq. (34) / (6.10a&b in Eurocodes)		Eq. (35) / (6.10 in Eurocodes)	
	Eurocode values	Calibrated values	Eurocode values	Calibrated values
$\gamma_{R,EC}$	$\gamma_{R,EC} = [1.00, 1.50, 1.15, 1.25, 1.30, 1.50]$			
γ_{GS}	1.35	1.18	1.35	1.15
γ_{GP}	1.35	1.23	1.35	1.22
ζ	0.85	0.85	/	/
γ_{Q1} (wind)	1.50	1.62	1.50	1.63
γ_{Q2} (snow)	1.50	1.59	1.50	1.64
γ_{Q3} (imposed)	1.50	1.62	1.50	1.65

values in the ranges reported in Table 5 are considered. a_G is a parameter representing different proportions between permanent load and self-weight ($a_G = 1$ for self-weight only). Three equally spaced and equally weighted values in the ranges reported in Table 5 are considered.

4.2.2. Assessment Strategy

Safety level of the existing EN1990 The assessment of the reliability level of the present Eurocodes design equations, i.e. format and partial factors according to CEN 2002 is assessed first. The weighted average reliability index of a domain D of design equations representing 6 material resistances, 3 different dominant variable loads, 3 different permanent load proportions and 10 different relative proportions of variable load relative to the total load, i.e. the weighted average from 6 x 3 x 3 x 10 = 540 design equations is computed in Eq.

$$E\left[\beta_{EC}\right] = C \sum_{k \in D} w_k \beta_k \left(\boldsymbol{\gamma_{EC}}\right) \tag{36}$$

C is a normalisation constant equal to one divided of the sum of all weights w_k. Independent from the detailed assumptions taken, the pattern as shown in Figure 13 is observed when assessing the safety level of the existing Eurocodes. The following aspects are indicated by Figure 13 :

- The scatter of the reliability level considering all design equations is large.
- There is higher variability of the reliability level between materials than within a material for different loads.
- The computed average reliability levels are significantly lower than the reliability target in EN 1990:2002 that is $\beta_{target,EC} = 4.7$ for the yearly reference period.

Calibration of the load partial factors The calibration is performed with the objective to reduce the variability of reliability indexes. As only the load factors are subject to calibration, it is suggested to consider the weighted average yearly reliability associated to the Eurocode recommended reliability elements as a reliability target, i.e. as determined with Eq. (36) as $\beta_{target} = E\left[\beta_{EC}\right]$.

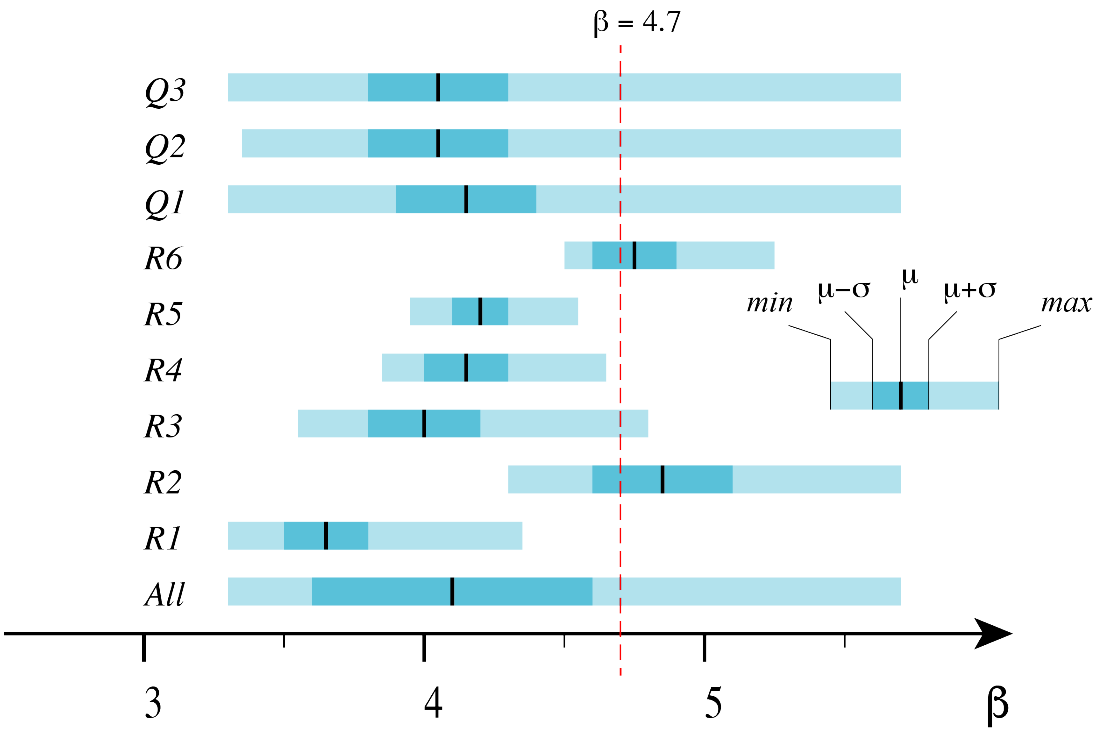

Figure 13: Magnitude and variability of the estimated yearly reliability indexes: situation with recommended safety factors given in EN1990 (CEN 2002)

The calibrated load partial safety factors are identified by solving the following minimisation problem over the domain of considered design equations D:

$$\gamma_{S,opt} = \arg\min_{\gamma_S} \sum_{k \in D} w_k (\beta_k(\gamma_S, \gamma_{R,EC}) - \beta_{target})^2 \tag{37}$$

where $\gamma_{S,opt} = [\gamma_{GS}, \gamma_{GP}, \gamma_{Q1}, \gamma_{Q2}, \gamma_{Q3}]$ are the load partial factors that are calibrated; $\gamma_{R,EC}$ are the material partial factors that are not calibrated (they are fixed to the values reported in Table 5).
The results of the calibration for the design equations corresponding to Eq. (34) and Eq. (35) (equations 6.10a&b and 6.10 in the Eurocodes correspondingly) are listed in Table 6.

The results show that the safety factors for permanent actions are lower than the present value but relative similar for self-weight and permanent load and also similar for Eurocode equations 6.10 and 6.10 ab. For variable load it can be seen that all safety factors have similar values for different loads and also for Eurocode equations Eq. 6.10 and 6.10 ab. The resulting effect on the reliability indexes is illustrated in Figure 14.

The following observations can be made:

- By calibration of the load factors the variability of reliability indexes can be reduced.
- Also the variability within the different materials can be reduced considerably.
- The variability in between the materials is unaffected by the calibration of the load factors. This confirms that the resulting partial factors are insensitive to the simplistic

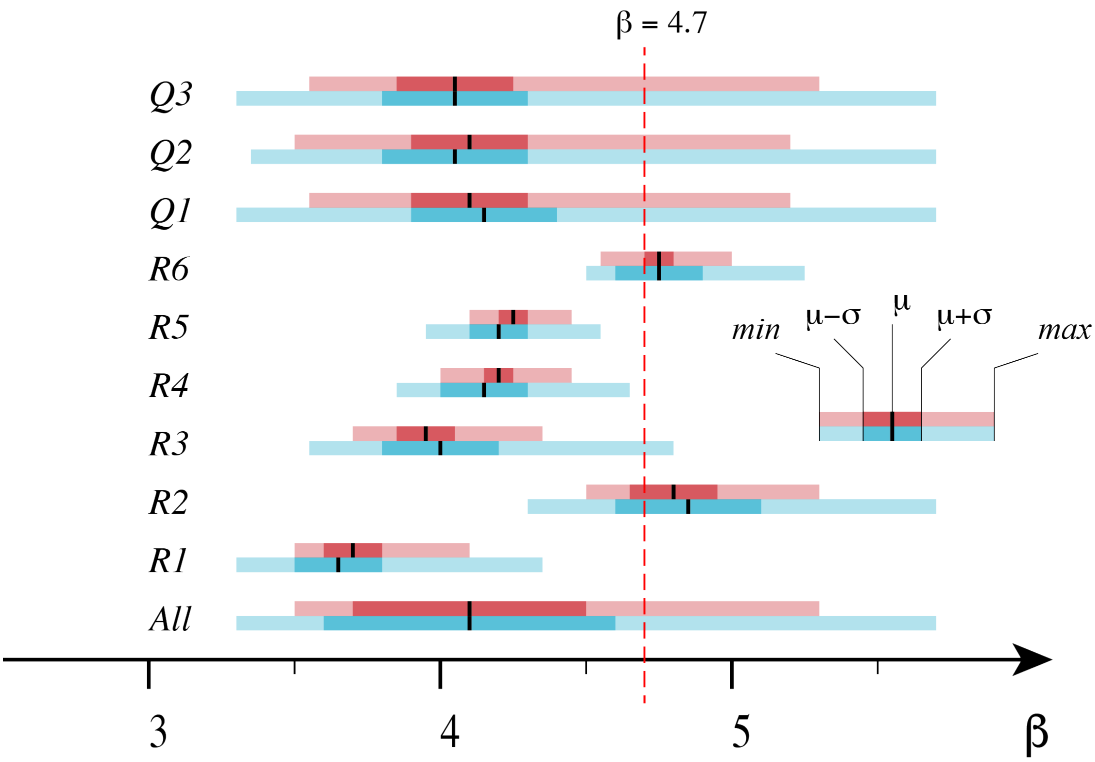

Figure 14: Magnitude and variability of the estimated yearly reliability indexes: comparison of the safety factors of EN1990 (blue) with the case where the load factors are calibrated (red). (Analysis based on Eq. 6.10 of EN 1990:2002 (CEN 2002).

representation of material resistance.

- This variability can only be reduced by calibration of the material factors.

4.3. Conclusion

The presented framework for the assessment and calibration of existing semi-probabilistic design codes. A case study was introduced for the calibration of the load safety factors of the Eurocodes. Conditional on the assumptions, the study indicates that:

- it is not necessary to distinguish two types of permanent loads in the safety concept;
- it is reasonable to apply the same safety factor to all variable loads considered in this study;
- the existing partial factors seem too high for permanent loads;
- the existing partial factors seem too low for variable loads;
- the reliability target in the existing Eurocodes is higher than the average reliability level implied by the current safety factors.

For thew continuation of the study, the load partial factors should be included in the calibration. This would be done by fixing the partial load factors to the ones obtained in the presented study and calibrate the partial resistance factors per relevant material. This implies a representation of load bearing capacity on a higher level of detail and with due consideration of the existing expertise and literature that is available for the corresponding materials.

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