IDENTIFICATION IN MODELS FOR MATCHED PANEL DATA WITH TWO-SIDED RANDOM EFFECTS

Koen Jochmans*

Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France

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Abstract

This paper is concerned with models for matched worker-firm data in the presence of both worker and firm heterogeneity. We show that models with complementarity and sorting can be nonparametrically identified from short panel data while treating both worker and firm heterogeneity as discrete random effects. This paradigm is different from the framework of Bonhomme, Lamadon and Manresa (2019a), where identification results are derived under the assumption that worker effects are random but firm heterogeneity is observed. Our identification approach is constructive and our results appear to be the first of their kind in the context of matched panel data problems.

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*Address: Toulouse School of Economics, 1 esplanade de l'Université, 31080 Toulouse, France. E-mail: koen.jochmans@tse-fr.eu.

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Introduction

Matched panel data are often used to study the interaction between two types of units over a period of time. The importance of unobserved heterogeneity across the units in such data is well recognized and understanding its implications has received considerable attention. The seminal work of Abowd, Kramarz and Margolis (1999), for example, was concerned with matched worker-firm data. They regressed wages on worker and firm fixed effects to quantify the degree of heterogeneity in wages coming from, respectively, worker and firm heterogeneity and used the fixed-effect decomposition to investigate sorting patterns between workers and firms. Such decompositions have become a workhorse tool in the labor literature and have been widely adopted in other fields; teacher value-added studies (McCaffrey, Lockwood, Koretz, Louis, and Hamilton 2004, Chetty, Friedman and Rockoff 2014) are one example. To maintain focus, for the remainder of this paper we will use terminology from the worker-firm application; everything to follow naturally translates to other settings.

The regression approach of Abowd, Kramarz and Margolis (1999) has come under increased scrutiny. The linearity of the model does not permit any form of complementarity between workers and firms, which is at odds with theoretical models (see, e.g., Shimer and Smith 2000 or Eeckhout and Kircher 2011). It is also inherently static, as it rules out any dynamics in wages even within job spells (see Lentz, Piyapromdee and Robin 2023, pp. 2416-2417, for a discussion) and demands mobility decisions to be exogenous conditional on worker and firm heterogeneity. Furthermore, their fixed-effect decompositions produce highly unreliable results in the type of data to which they are usually applied (Jochmans and Weidner 2019).[1]

[1] The presence of bias in estimators of variance components is now well understood. It can be corrected for using any of a variety of estimators (see Andrews, Gill, Schank and Upward 2008, Kline, Saggio and Sølvsten 2020, Azkarate-Askasua and Zerecero 2024, and Babet, Godechot and Palladino 2025). However, even after bias correction, the estimators exhibit highly non-standard asymptotic behavior that, to date, is not fully understood. Kline, Saggio and Sølvsten (2020) present results for problems with a particular connectivity structure.

In influential work Bonhomme, Lamadon and Manresa (2019a) proposed an alternative framework that permits both complementarity and sorting. Their approach is based on the assumption that worker and firm heterogeneity is discrete and hinges on the presumption that firms can be consistently clustered by type from the cross-sectional distribution of wages in an initial step. Given such a consistent classification the firm types can be treated as observed in the data, so that the model effectively features only one-way heterogeneity (in the form of the worker effect). Arguments reminiscent of those used in the literature on multivariate finite mixtures (see. for example, Hu 2017 or Schennach 2020 for recent overviews of this literature) can then be used to identify certain aspects of the model from short panel data.

The theory underlying the consistent clustering of firms requires minimal firm size to diverge with the sample size. Such a paradigm does not align well with many available data sets, where many small firms are present (see also He and Robin 2025). A framework that allows for small firms to exist in large samples would be one where the number of workers and firms grow at the same rate. In such a case, average firm size is bounded, and so neither worker nor firm effects can be estimated consistently. Such a setting thus requires treating both the worker and firm heterogeneity as random effects. Several (parametric) attempts at estimating such models have been made (see Beffy, Kamionka, Kramarz and Robert 2003, Bonhomme, Lamadon and Manresa 2019b, and Abowd, McKinney and Schmutte 2019). However, to date, it is not known under what type of conditions models with two-sided heterogeneity can be (nonparametrically) identified, if at all. This paper attempts to make progress on this question.

We are able to establish identification of all primitive parameters in two such models from three-wave panel data under mild assumptions. The primitives are the distributions of wages and mobility decisions conditional on the worker and firm effects, which are informative about heterogeneity and complementarity, and the joint distribution of worker and firm effects, which is informative about sorting. The first part of the paper focusses on a model that allows for Markovian dependence in wages within employment spells conditional on the worker and firm effect but requires exogenous mobility. We then give a corresponding result for a model that assumes away wage dynamics within employment spells but allows for endogenous mobility (see, e.g., Abowd, McKinney and Schmutte 2019) by permitting mobility decisions to depend on current wages even after conditioning on worker and firm heterogeneity.

In the models we look at we are able to retain most of the features of those considered in Bonhomme, Lamadon and Manresa (2019a). This is encouraging; the generalization from one-dimensional to two-dimensional heterogeneity is non-trivial. At the same time, the conditions needed to establish identification are weaker in certain important respects. Furthermore, our derivations further reveal a degree of overidentification. Hence, there appears to be scope for further generalization. Lastly, our arguments are constructive, permitting the construction of an estimator by replacing population quantities by sample counterparts, although we do not focus on this here for conciseness. This is useful as it implies that two-sided random-effect estimation can in fact be performed in a tractable manner.

1 The baseline model

We consider stationary panel data on workers followed over time. For each worker i and each time period t we observe the worker's wage, w_{it}, together with a binary indicator of job mobility, x_{it}, which captures whether the worker is switching employer between periods t and $t+1$ or not. We also known the identity of the firm where worker i was employed at time t, say $f(i,t)$.

Worker i and firm f are characterized by unobserved heterogeneity ϕ_i and ψ_f. We will follow Bonhomme, Lamadon and Manresa (2019a, pp. 703 and 708) and presume that both types of heterogeneity are discrete, with a known number of support points. The latter is presumed for brevity and can be relaxed.[2] Let ψ_{it} be shorthand notation for $\psi_{f(i,t)}$, that

[2]Under the assumptions spelled out below the smallest number of support points for the distribution of the worker effect and for the firm effect that can rationalize our model are in fact identified as the rank of the bivariate distribution of (w_{i1}, w_{i2}) for workers that change employment between periods one and two,

is, the effect of the firm where worker i is employed at time t. The joint distribution of the worker and firm heterogeneity then is

$$p(\phi, \psi) \coloneqq \mathbb{P}(\phi_i = \phi, \psi_{it} = \psi).$$

The wage and mobility processes are initialized in the following manner. First, workers independently draw their type with probability $p(\phi) \coloneqq \mathbb{P}(\phi_i = \phi) = \int p(\phi, \psi)\, d\psi$. Firms, in turn, draw their type independently according to $p(\psi) \coloneqq \mathbb{P}(\psi_f = \psi) = \int p(\phi, \psi)\, d\phi$. An initial allocation then follows from assigning a worker of type ϕ to a firm of type ψ with probability

$$p_\phi(\psi) \coloneqq \frac{p(\phi, \psi)}{p(\phi)}.$$

First period wages w_{i1} are drawn from the conditional distribution $Q_{\phi_i, \psi_{i1}}$, where we write

$$Q_{\phi,\psi}(w) \coloneqq \mathbb{P}(w_{it} \leq w | \phi_i = \phi, \psi_{it} = \psi),$$

independently for each worker. Next, the match quality between the worker and his current firm is evaluated. With probability $r_{\phi_i, \psi_{i1}}$, where

$$r_{\phi,\psi} \coloneqq \mathbb{P}(x_{it} = 1 | \phi_i = \phi, \psi_{it} = \psi),$$

$x_{i1} = 1$ and employment is terminated. In any subsequent period t there are then two possibilities, depending on the realization of x_{it-1}. If $x_{it-1} = 1$ the worker draws a new firm type ψ_{it} from the conditional distribution p_{ϕ_i}, followed by a new wage draw from the implied $Q_{\phi_i, \psi_{it}}$. If $x_{it-1} = 0$ the worker remains in the same firm, so that $f(i,t) = f(i, t-1)$ and, therefore, $\psi_{it} = \psi_{it-1}$. Within job spells wages are allowed to exhibit Markovian dependence, with transition kernel

$$Q_{\phi,\psi,w}(w') \coloneqq \mathbb{P}(w_{it} \leq w' | w_{it-1} = w, x_{it-1} = 0, \phi_i = \phi, \psi_{it} = \psi),$$

whose steady-state distribution is $Q_{\phi,\psi}$. When $x_{it-1} = 0$, w_{it} is thus drawn from $Q_{\phi_i, \psi_{it}, w_{it-1}}$. In either case, before moving on to the next period, the match quality between the worker

and as the rank of the bivariate distribution of $(w_{i_1 1}, w_{i_2 1})$ for pairs of distinct workers (i_1, i_2) employed in the same firm in the first time period.

and his current employer is again evaluated and they decide to separate with probability $r_{\phi_i,\psi_{it}}$.[3]

1.1 Assumptions and identification

Our aim is to nonparametrically identify the steady-state distributions $Q_{\phi,\psi}$, the transition kernels $Q_{\phi,\psi,w}$, and the separation probabilities $r_{\phi,\psi}$, as well as the joint distribution of worker and firm types $p(\phi,\psi)$. Of course, because the types are latent, it is understood that identification here will be up to an arbitrary relabelling of the types. In empirical applications one often given empirical content to the types (such as them being a measure of ability or efficiency, for example). In such cases, if desired, types can be ordered by a functional (such as the mean) of the wage distributions conditional on only worker or firm type, that is,

$$Q_\phi(w) \coloneqq \int Q_{\phi,\psi}(w)\, p_\phi(\psi)\, d\psi, \qquad \text{and} \qquad Q_\psi(w) \coloneqq \int Q_{\phi,\psi}(w)\, p_\psi(\phi)\, d\phi,$$

where we let $p_\psi(\phi) \coloneqq p(\phi,\psi)/p(\psi)$ in analogy to $p_\phi(\psi)$. For our purposes, however, such an ordering is not needed and, hence, is irrelevant.

We will work under two assumptions. To state them we let

$$P_\phi(w) \coloneqq \mathbb{P}(w_{it} \leq w, x_{it} = 1 | \phi_i = \phi) = \int r_{\phi,\psi}\, Q_{\phi,\psi}(w)\, p_\phi(\psi)\, d\psi.$$

The first assumption is a rank condition.

[3]Similar to Lentz, Piyapromdee and Robin (2023) it is possible to treat unemployment as a specific value of the firm effect, say ψ_0. If at the end of a period an employment spell terminates, a worker of type ϕ remains unemployed with probability $p_\phi(\psi_0)$. In this case the worker draws a wage (which can be unemployment benefits) from distribution Q_{ϕ,ψ_0} (which may be degenerate) and finds a job for the next period with probability $\int p_\phi(\psi)\, r_{\phi,\psi_0}\, d\psi - p_\phi(\psi_0)\, r_{\phi,\psi_0} = (1 - p_\phi(\psi_0))\, r_{\phi,\psi_0} =: c_\phi$, or remains unemployed with probability $p_\phi(\psi_0)\, r_{\phi,\psi_0} + (1 - r_{\phi,\psi_0}) = 1 - c_\phi$. This approach sees unemployment as an outside option. This is flexible and can be used in other contexts. In the specific setting of worker-firm data it ignores the fact that the unemployment status of an individual is observed in the data. There may be additional identifying content in observing individuals moving in and out of unemployment, but we do not explore this here.

Assumption 1. *The distributions P_ϕ and Q_ϕ, and Q_ψ are linearly independent in ϕ and ψ, respectively.*

Assumption 1 demands that changes in ϕ and ψ affect wages, which is intuitive. A simple sufficient condition is that the distributions $Q_{\phi,\psi}$ are linearly independent in (ϕ, ψ), but this is much stronger than needed. Assumption 1 permits, but does not require, the presence of complementarity; it can accommodate linear wage processes with additively-separable worker and firm heterogeneity, such as the model of Abowd, Kramarz and Margolis (1999). Conditions of this type are typical in the analysis of multivariate latent-variable models. In Bonhomme, Lamadon and Manresa (2019a) the corresponding restriction on the wage distribution conditional on the worker effect appears in Assumption 3(ii), where the $Q_{\phi,\psi}$ are required to be linearly independent in ϕ for each ψ. On the firm side, their Assumption B1, in turn, demands the conditional wage distributions Q_ψ to be well separated across the different ψ.

The second assumption is a support condition.

Assumption 2. *For all (ϕ, ψ) it holds that (i) $0 < p(\phi, \psi) < 1$ and that (ii) $0 < r_{\phi,\psi} < 1$.*

Assumption 2 has two parts. Part (i) is a full-support condition on the distribution of the latent types. It states that any worker type can match with any firm type with positive probability. Part (ii), in turn, states that any match between a worker and firm can terminate. Both Part (i) and Part (ii) are implied by Assumption 3(i) in Bonhomme, Lamadon and Manresa (2019a) applied to connecting cycles (as per their Definition 1) of length one. To see this note that their conditions are stated in terms of the conditional probabilities

$$p_{\psi_1,\psi_2}(\phi) := \mathbb{P}(\phi_i = \phi | \psi_{i1} = \psi_1, \psi_{i2} = \psi_2, x_{i1} = 1) \propto r_{\phi,\psi_1}\, p_\phi(\psi_2)\, p_\phi(\psi_1)\, p(\phi)$$

and then would demand that $p_{\psi_1,\psi_2}(\phi) > 0$ for all ψ_1, ψ_2 and ϕ, in which case Assumption 2 clearly holds. They further need these probabilities to vary sufficiently, in a specific sense. Among other things this requires that the p_{ψ_1,ψ_2} depend on both (ψ_1, ψ_2)—which, for example, demands the presence of sorting—and that $p_{\psi_1,\psi_2} \neq p_{\psi_2,\psi_1}$, which rules out job-mobility decisions that do not involve firm heterogeneity—i.e., $r_{\phi,\psi} = r_{\phi}$ for all ϕ—which covers the baseline setting where the termination probability of a job is common across workers and firms.

The following theorem states our main result.

Theorem 1. *Let Assumptions 1 and 2 hold. Then*

(i) the wage distributions $Q_{\phi,\psi}$ and transition kernels $Q_{\phi,\psi,w}$,

(ii) the mobility distributions $r_{\phi,\psi}$, and

(iii) the type distribution $p(\phi,\psi)$

are all nonparametrically identified up to relabelling of (ϕ,ψ) from panel data on wage trajectories and job transitions spanning three time periods.

Higher-order Markovian dependence in wages within employment spells can be allowed for, and can be recovered, if additional time periods are available. The proof, given below, extends naturally.

1.2 Connection to the literature

Our model is a stationary version of Bonhomme, Lamadon and Manresa (2019a, Section 2.1). Under the assumption that firm types are observed, their Theorem 1 gives conditions under which, from a two-wave panel, one may identify (i) the initial distribution of wages given worker and firm types, (ii) the same distribution in the subsequent period for workers that have changed employment between the two periods, (iii) the joint distribution of the worker and the firm types given that a switch in employer has taken place between the two periods, and, finally, (iv) the joint distribution of worker and firm types in the initial period. With the firm effects observed in the data they can permit (x_{it},ψ_{it}) to be Markovian, which we do not allow for. This is the chief difficulty encountered when treating both worker and firm effects as random.

The sets of theoretical models covered by Bonhomme, Lamadon and Manresa (2019a) and by our setup are not nested. The model of Shimer and Smith (2000), for example, fits our framework whereas it is ruled out by Assumption 3 of Bonhomme, Lamadon and Manresa (2019a) (as per their discussion on pp. 709); Hagedorn, Law and Manovskii (2017) features a related model. On the other hand it is more difficult to accommodate job-ladder models such as Burdett and Mortensen (1998) within the confines of our analysis. In either case, the manner in which wages and mobility decisions vary with worker and firm heterogeneity is not specified and, hence, could be nonlinear, accommodating general forms of complementarity between workers and firms. The assignment of workers to firms, in turn, is allowed to depend on their latent types. Hence, sorting is permitted (but, in our case, not required).

1.3 Proof of Theorem 1

To prove our main result we proceed in four steps. The first two of these serve to identify auxiliary parameters that will be used to identify the model parameters in the third and fourth step.

The first step is concerned with identifying the distribution of wages conditional on the worker type alone, that is, the functions Q_ϕ, up to an arbitrary ordering of the ϕ. To do so we use the panel dimension of our setup. More precisely, we exploit the observation that in our model wages and mobility decisions are independent across job spells conditional on the worker effect. To see how this is helpful for our purposes consider the joint probability

$$\mathbb{P}(w_{i1} \leq w_1, x_{i1} = 1, w_{i2} \leq w_2, x_{i2} = 1, w_{i3} \leq w_3) \tag{1.1}$$

for chosen values (w_1, w_2, w_3). Here, workers switch employer between the first and second period, and again between the second and third period. The probability of this happening is non-zero under Assumption 2. By conditional independence, the joint probability in (1.1) factors as

$$\int P_\phi(w_1)\, P_\phi(w_2)\, Q_\phi(w_3)\, p(\phi)\, d\phi,$$

which is a tri-variate finite mixture. By Assumption 1 the distributions Q_ϕ and P_ϕ, seen as a function of ϕ, are linearly independent. By Assumption 2, $0 < p(\phi)$ for all ϕ. From

Allman, Matias and Rhodes (2009, Theorem 8) or Bonhomme, Jochmans and Robin (2016, Theorem 2) it then follows that the functions Q_ϕ are nonparametrically identified up to label swapping.

The second step of our proof, in turn, identifies the distribution of wages conditional on the firm type alone, that is, the functions Q_ψ, again up to an arbitrary ordering of the ψ. To do this we exploit the cross-sectional dimension of our problem and the fact that firm identities are known. Consider the cross-sectional distribution of wages of distinct workers (i_1, i_2, i_3) employed by the same firm f in time period t. It is helpful to make the dependence of wages on the firm explicit, by writing w_{ift} for the wage of worker i earned in firm f at time t. Then we can write the probability distribution in question, evaluated at (w_1, w_2, w_3), as

$$\mathbb{P}(w_{i_1ft} \leq w_1, w_{i_2ft} \leq w_2, w_{i_3ft} \leq w_3). \tag{1.2}$$

Wages of different workers employed at the same firm are independent conditional on the firm effect. Therefore, their joint probability in (1.2) factors as

$$\int Q_\psi(w_1)\, Q_\psi(w_2)\, Q_\psi(w_3)\, p(\psi)\, d\psi,$$

which is again a tri-variate finite mixture. In the same way as before, this representation implies that the Q_ψ are nonparametrically identified up to a relabelling of the firm types ψ.

In the third step of our proof we use the results obtained so far to recover the conditional wage distributions $Q_{\phi,\psi}$ for the labelling of worker and firm types from the previous two steps. This is done by looking at the joint distribution of wages for two distinct workers initially employed at the same firm, together with the next period's wage of one of them that switches employer at the end of the period. The distribution in question, as a function of (w_1, w, w_2) is

$$\mathbb{P}(w_{i_1f1} \leq w_1, w_{i_2f1} \leq w, x_{i_21} = 1, w_{i_22} \leq w_2).$$

Under the dynamics of our model this probability can be written in terms of the model primitives as

$$\iint Q_\phi(w_2)\, H(w, \phi, \psi)\, Q_\psi(w_1)\, d\phi\, d\psi, \tag{1.3}$$

where

$$H(w,\phi,\psi) \coloneqq \mathbb{P}(w_{it} \leq w, x_{it} = 1, \phi_i = \phi, \psi_{it} = \psi) = Q_{\phi,\psi}(w)\, r_{\phi,\psi}\, p(\phi,\psi).$$

The latter joint probability is identified because the Q_ϕ and Q_ψ are identified and are linearly independent. To see this take a collection of values for the wage, $w_1, \ldots, w_m$ for some finite integer m so that the matrices $(A)_{v,\phi} \coloneqq Q_\phi(w_v)$ and $(B)_{v,\psi} \coloneqq Q_\psi(w_v)$ have maximal column rank. By Assumption 1 such a set of values exists. Further, for any w, let $(C_w)_{v_1,v_2} \coloneqq \mathbb{P}(w_{i_1 f 1} \leq w_{v_1}, w_{i_2 f 1} \leq w, x_{i_2 1} = 1, w_{i_2 2} \leq w_{v_2})$ and $(D_w)_{\phi,\psi} \coloneqq H(w,\phi,\psi)$. Then, from (1.3), $C_w = A\, D_w B^\top$ so that $D_w = (A'A)^{-1} A' C_w B (B'B)^{-1}$, which contains the $H(w,\phi,\psi)$ for any w, is identified. A value w of particular interest is $w = +\infty$, for which

$$h(\phi,\psi) \coloneqq H(+\infty,\phi,\psi) = \mathbb{P}(x_{it} = 1, \phi_i = \phi, \psi_{it} = \psi) = r_{\phi,\psi}\, p(\phi,\psi).$$

Under Assumption 2 $h(\phi,\psi)$ is strictly positive. Therefore, for any chosen value w we have that

$$Q_{\phi,\psi}(w) = \frac{H(w,\phi,\psi)}{h(\phi,\psi)}$$

is nonparametrically identified up to the same labelling of worker and firm types as before.

In the fourth and final step of our proof we follow a similar approach as in the previous step to identify the remaining parameters, $Q_{\phi,\psi,w}$, $r_{\phi,\psi}$, and $p(\phi,\psi)$. Rather than looking at workers who switch employer after the first period we look at workers that switch in the second period. The relevant probability distribution, seen as a function of (w_1, w, w', w_2) is

$$\mathbb{P}(w_{i_1 f 1} \leq w_1, w_{i_2 f 1} \leq w, x_{i_2 1} = 0, w_{i_2 2} \leq w', x_{i_2 2} = 1, w_{i_2 3} \leq w_2).$$

This joint probability factors as

$$\textstyle\iint Q_\phi(w_2)\, G(w, w', \phi, \psi)\, Q_\psi(w_1)\, d\phi\, d\psi, \tag{1.4}$$

where

$$G(w, w', \phi, \psi) \coloneqq \mathbb{P}(w_{it-1} \leq w, w_{it} \leq w', x_{it-1} = 0, x_{it} = 1, \phi_i = \phi, \psi_{it} = \psi).$$

Observe that $G(w, w', \phi, \psi) = Q_{\phi,\psi}(w, w')\, r_{\phi,\psi}\, (1 - r_{\phi,\psi})\, p(\phi, \psi)$, where we use the shorthand

$$Q_{\phi,\psi}(w, w') \coloneqq \mathbb{P}(w_{it} \leq w, w_{it+1} \leq w' | x_{it} = 0, \phi_i = \phi, \psi_{it} = \psi)$$

for the joint distribution of two wage observations within a given employment spell. From this decomposition, by the same argument as used for the function H before, the function G is identified up to the same labelling of worker and firm types. From this we may then recover

$$g(\phi, \psi) \coloneqq G(+\infty, +\infty, \phi, \psi) = \mathbb{P}(x_{it-1} = 0, x_{it} = 1, \phi_i = \phi, \psi_{it} = \psi) = r_{\phi,\psi}(1 - r_{\phi,\psi})p(\phi, \psi),$$

given which we find

$$Q_{\phi,\psi}(w, w') = \frac{G(w, w', \phi, \psi)}{g(\phi, \psi)}$$

for any pair (w, w'). From this we then equally obtain the conditional distribution $Q_{\phi,\psi,w}(w')$. Furthermore, we also have

$$r_{\phi,\psi} = 1 - \frac{g(\phi, \psi)}{h(\phi, \psi)},$$

and with it,

$$p(\phi, \psi) = \frac{h(\phi, \psi)}{r_{\phi,\psi}} = \frac{h(\phi, \psi)^2}{h(\phi, \psi) - g(\phi, \psi)},$$

all again up to the same ordering of worker and firm types. All parameters of the model have thus been shown to be identified. □

2 Endogenous mobility

Extensions and variations of our model can be entertained. One alternative specification of interest allows for mobility decisions to depend on current wage, in addition to worker and firm effects. That is

$$r_{\phi,\psi,w} \coloneqq \mathbb{P}(x_{it} = 1 | w_{it} = w, \phi_i = \phi, \psi_{it} = \psi) \neq \mathbb{P}(x_{it} = 1 | \phi_i = \phi, \psi_{it} = \psi) = r_{\phi,\psi}.$$

Such dependence translates into what is usually referred to as endogenous mobility (see, for example, Abowd, McKinney and Schmutte 2019). Our identification approach can be modified to deal with this at the expense of ruling out Markovian dependence in wages within employment spells, i.e. $Q_{\phi,\psi,w} = Q_{\phi,\psi}$. Dealing with both at the same time appears to be more complicated.

The model is thus the same as before with the exception that, now, in every period, workers wages w_{it} and mobility decisions x_{it} are determined jointly according to distribution

$$Q_{\phi,\psi}(w,x) \coloneqq \mathbb{P}(w_{it} \leq w, x_{it} = x | \phi_i = \phi, \psi_{it} = \psi) = \{x\, r_{\phi,\psi,w} + (1-x)(1-r_{\phi,\psi,w})\}\, Q_{\phi,\psi}(w),$$

which no longer factors.

The following theorem concerns identification in the model with endogenous mobility.

Theorem 2. *Let Assumptions 1 and 2 hold. Then*

(i) the joint distributions $Q_{\phi,\psi}(w,x)$,

(ii) all implied marginal and conditional distributions, and

(iii) the type distribution $p(\phi,\psi)$

are all nonparametrically identified up to relabelling of (ϕ,ψ) from panel data on wage trajectories and job transitions spanning three time periods.

The proof of Theorem 2 is similar in spirit to the proof of Theorem 1. The first and second second step of the proof require no modification as the factorizations in (1.1) and (1.2) continue to go through. Therefore, the Q_ϕ and Q_ψ are identified up to relabelling of, respectively, the worker and firm types. The third and fourth step of the proof change. While the decompositions in (1.3) and (1.4) still hold, the terms that can be recovered from them,

$$H(w,\phi,\psi) = \mathbb{P}(w_{it} \leq w, x_{it} = 1, \phi_i = \phi, \psi_{it} = \psi)$$

and

$$G(w,w',\phi,\psi) = \mathbb{P}(w_{it-1} \leq w, w_{it} \leq w', x_{it-1} = 0, x_{it} = 1, \phi_i = \phi, \psi_{it} = \psi),$$

now factor differently. First, by conditional independence of the (w_{it}, x_{it}) we observe that

$$H(w,\phi,\psi) = Q_{\phi,\psi}(w,1)\, p(\phi,\psi), \qquad G(w,w',\phi,\psi) = Q(w',1)\, Q(w,0)\, p(\phi,\psi),$$

from which we identify

$$Q_{\phi,\psi}(w,0) = \frac{G(w,w',\phi,\psi)}{H(w',\phi,\psi)}.$$

Next, because we still have that $h(\phi,\psi) = H(+\infty,\phi,\psi) = r_{\phi,\psi}\, p(\phi,\psi)$ and also that $g(\phi,\psi) = G(+\infty,+\infty,\phi,\psi) = (1-r_{\phi,\psi})\, r_{\phi,\psi}\, p(\phi,\psi)$ we recover, in the same way as before,

$$r_{\phi,\psi} = 1 - \frac{g(\phi,\psi)}{h(\phi,\psi)},$$

from which

$$Q_{\phi,\psi}(w,1) = \frac{H(w,\phi,\psi)}{h(\phi,\psi)}\, r_{\phi,\psi} = \frac{H(w,\phi,\psi)\,(h(\phi,\psi)-g(\phi,\psi))}{h(\phi,\psi)^2}$$

follows. Therefore $Q_{\phi,\psi}(w,x)$ is identified for all (w,x) up to ordering of worker and firm types. Identification of the various implied marginal and conditional distributions is an immediate consequence. The type distribution, for the same ordering, then again follows as

$$p(\phi,\psi) = \frac{h(\phi,\psi)}{r_{\phi,\psi}}.$$

This completes the proof. □

Conclusion

In this paper we have given identification results for models for matched panel data with discrete two-sided unobserved heterogeneity. Our approach differs from the one followed in Bonhomme, Lamadon and Manresa (2019a) and Lentz, Piyapromdee and Robin (2023) in that we treat the heterogeneity on both sides as random effects. This by-passes the need to consistently estimate the heterogeneity on (at least) one side. The latter is fundamental to the identification results available to date but may be difficult to do in many situations of interest. Our approach is nonparametric and constructive, permitting the construction of an estimator by replacing population quantities by sample counterparts. Our derivations reveal that the models we consider are overidentified. Hence, there appears to be scope for further generalization.

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