# Generalized quantum-inspired evolutionary algorithm for combinatorial optimization prob

Julio Alegría Reyner, Yván Jesús Túpac Valdivia
San Pablo Catholic University
{julio,ytupac}@ucsp.pe

**Abstract:** This work proposes the Generalized Quantum-Inspired Evolutionary Algorithm (GQIEA). Like QIEA, GQIEA is also based on the quantum computing principle of superposition of states, but extends it not only to be used for binary values $(0, 1)$, but for any finite set of values $(1, 2, \ldots, n)$. GQIEA also defines a generalized Q-gate operator, which is a variation operator to drive the individuals toward better solutions. To verify GQIEA effectiveness and applicability, this is applied to the Assignment and Knapsack Problems. Results show that GQIEA has a good performance, even with a small population.

## QIEA

QIEA [Han, 2002], is an Evolutionary Algorithm composed by individuals composed by Q-bits, an evaluation function, and a population dynamic. Instead classical evolutionary algorithm, this model uses:

- An updating operator called Q-gate.
- An observation process.

**Representation:**
A Q-bit is the smallest unit of information in QIEA, which is defined using a pair of numbers ($\alpha$, $\beta$) as shown in equation 1.

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{1}$$

where the following identity is satisfied:

$$|\alpha|^2 + |\beta|^2 = 1$$

A Q-bit individual consists of a Q-bits string defined as Eq. 2.

$$\left[\begin{array}{c|c|c|c} \alpha_1 & \alpha_2 & \ldots & \alpha_m \\ \beta_1 & \beta_2 & \ldots & \beta_m \end{array}\right] \tag{2}$$

The population updating is performed by **Q-gate**. This operator is defined as a rotation matrix of Eq. 3.

$$\mathbf{U}(\Delta\theta_i) = \begin{bmatrix} \cos(\Delta\theta_i) & -\sin(\Delta\theta_i) \\ \sin(\Delta\theta_i) & \cos(\Delta\theta_i) \end{bmatrix} \tag{3}$$

applied to every column of the quantum individual $\mathbf{q}_i$ and ensuring compliance of the identity $|\alpha_k'|^2 + |\beta_k'|^2 = 1$

## QQIEA

The proposed Quantum-Inspired Evolutionary Algorithm instead a Q-bit uses a generalized Q-bit (GQ-bit):

**Representation:** A generalized Q-bit is defined as a $n$-uple, where $n$ is the number of possible observed states $|S|$ of the GQ-bit, as shown in Eq. 4.

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \tag{4}$$

and a GQ individual is a GQ-bits string of the Eq. 5.

$$\left[\begin{array}{c|c|c|c} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,m} \\ \alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n,1} & \alpha_{n,2} & \cdots & \alpha_{n,m} \end{array}\right] \tag{5}$$

**GQ-gate:** GQIEA also defines a GQ-gate operator.
A $j$-th GQ-bit of a GQ-bit represents the superposition of $n$ states $S = \{s_1, s_2, \ldots, s_n\}$. Let the state $s_\phi$ be the observed state on best individual, then the new $\alpha$'s of the GQ-bit will be determined as follows:

**Arithmetic GQ-gate:**

$$\alpha_i = \begin{cases} \alpha_i + \Delta_t & \text{if } i = \phi \\ \alpha_i & \text{if } i \neq \phi \end{cases} \quad , \quad i = 1, 2, \ldots, n \tag{6}$$

**Geometric GQ-gate:**

$$\alpha_i = \begin{cases} \alpha_i \cdot \Delta_t & \text{if } i = \phi \\ \alpha_i & \text{if } i \neq \phi \end{cases} \quad , \quad i = 1, 2, \ldots, n \tag{7}$$

A normalization is done after $\alpha_i$ updating, to ensure $\sum_i \alpha_i = 1.0$

## Case Studies

**Bounded Knapsack Problem [Kereller, 2004]:** Tests performed over the KP generalized version: Bounded Knapsack problem, like KP but allowing several copies of the same $k$ object, limited by maximum $m_k$, as Eq. 8:

$$\max \sum_{\forall k \in K} v_k x_k, \quad x_k \in \{0, 1, \ldots, m_k\}, \quad \text{such that} \sum_{\forall k \in K} w_k x_k \leq W \tag{8}$$

**Obtained Results:**

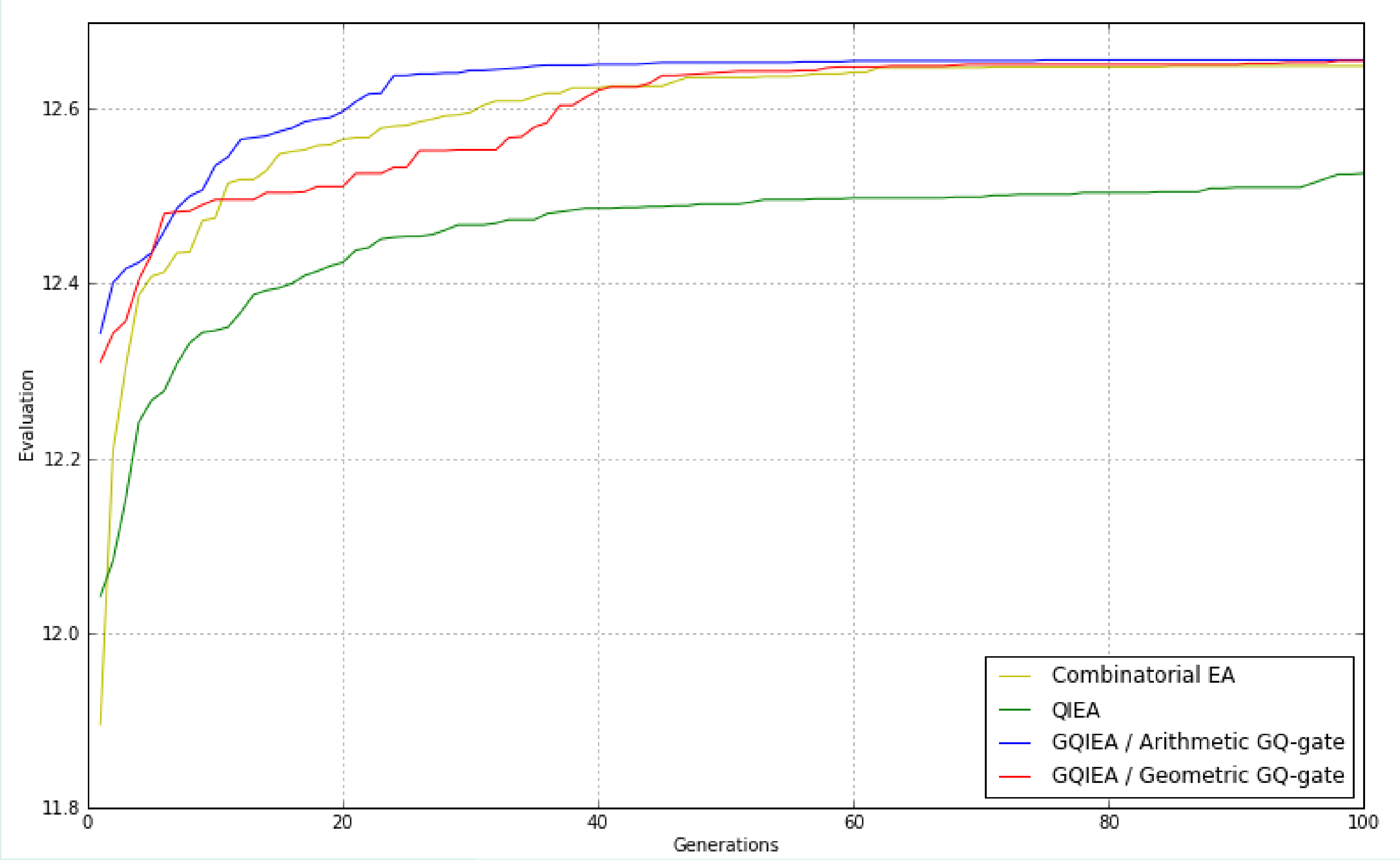


Figure 1: Comparison of an average of 10 executions of the Combinatorial EA, the QIEA and the GQIEA, applied to the Bounded Knapsack Problem.

| Algorithm | Result | Runtime |
|---|---|---|
| Combinatorial EA | 12.65 | 145.962 s |
| QIEA | 12.51 | 49.924 s |
| GQIEA (Arithmetic QGate) | 12.65 | 31.970 s |
| GQIEA (Geometric QGate) | 12.65 | 36.970 s |

Table 1: Results of the algorithms applied to the Bounded Knapsack Problem

- GQIEA (both GQ-gates) and CEA reach the same final result, QIEA was left behind.
- GQIEA was faster than combinatorial EA (slowest runtime) and QIEA, by dimensionality reduction.

## Conclusions

- A new more generally evolutionary model was proposed. The individuals representation allows superposition of solutions in any set of finite states, to improve the performance of QIEA in combinatorial optimization problems.
- Two updating operators were proposed: arithmetic GQ-gate and geometric GQgate.
- The GQIEA was compared against the Combinatorial EA and original QIEA. Outcomes show that the GQIEA outperforms other algorithms in results quality and convergence.

## References

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