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# Maximum Consensus based Localization and Protection Level Estimation using Synthetic LiDAR Range Images

Research  
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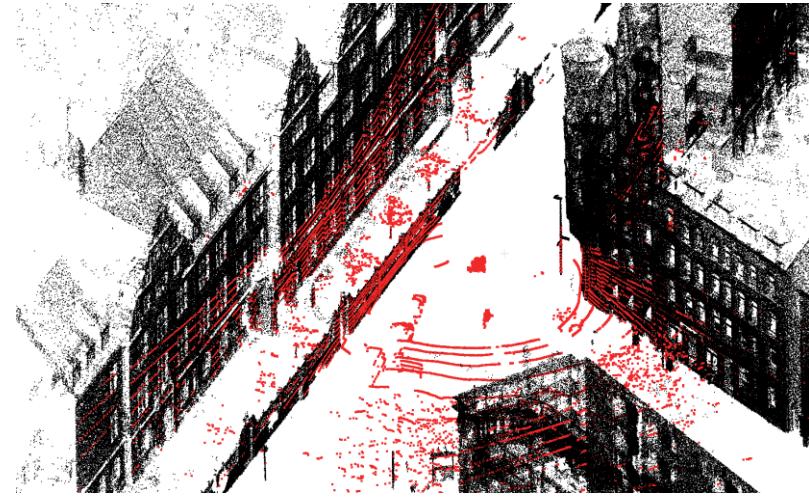


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# Motivation

- ▶ LiDAR based localization is typically based on two steps:
  - Establishing associations between a scan acquired by a LiDAR sensor mounted on the ego-vehicle and a map.
  - Minimizing the residuals based on a loss function, which matches the error distribution.
    - Quadratic loss/Least squares in case of normally distributed errors.
  
- ▶ Problem: Wrong associations (outliers) cause deviations from the distribution function
  - can't be modelled and result in a wrong estimate of the system state.
  
- ▶ New approach: Localization based on robust maximum consensus criterion.



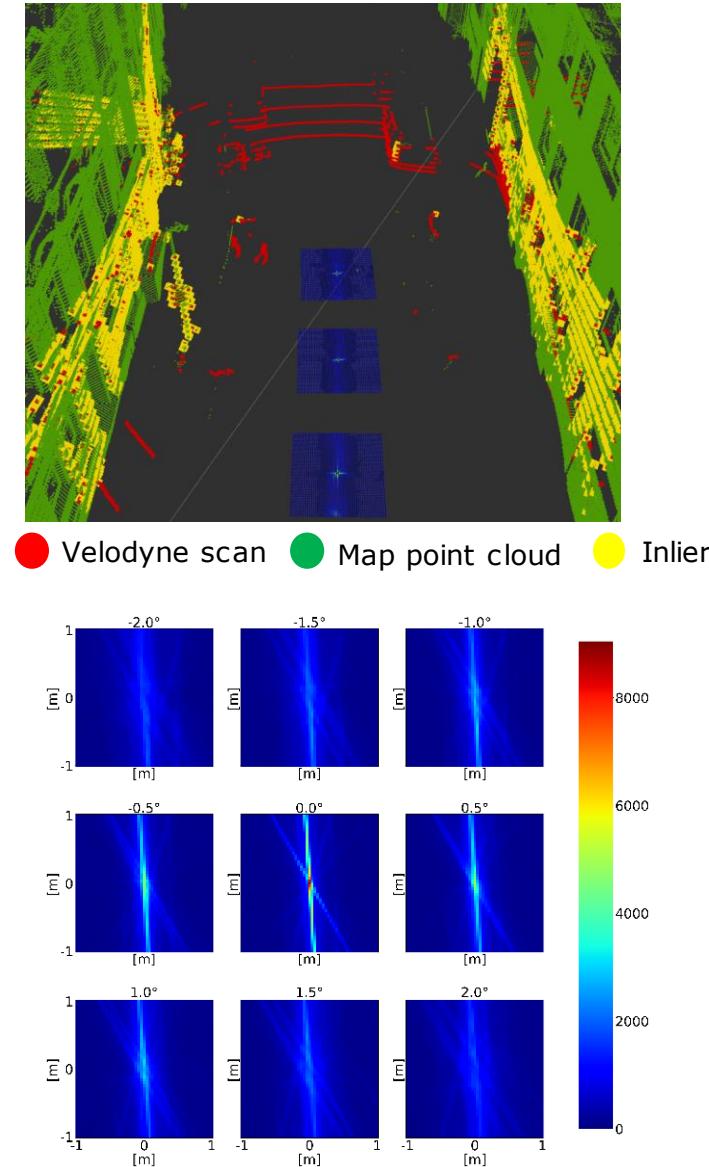


# Localization using Maximum Consensus: Basic principle

- ▶ Rough localization is given (e.g. by means of GNSS)
- ▶ Exact localization based on the registration of two point clouds using maximum consensus
  - Sparse 'car sensor' point cloud from a Velodyne VLP-16 scanner
  - Dense, high resolution 'map' point cloud obtained using a Riegl MMS
- ▶ 2D position: space of possible positions is discretized and the consensus set is computed for every cell
- ▶ Heading: car sensor scan is rotated around the up-axis in discrete angular steps and 2D 'position' accumulator is calculated

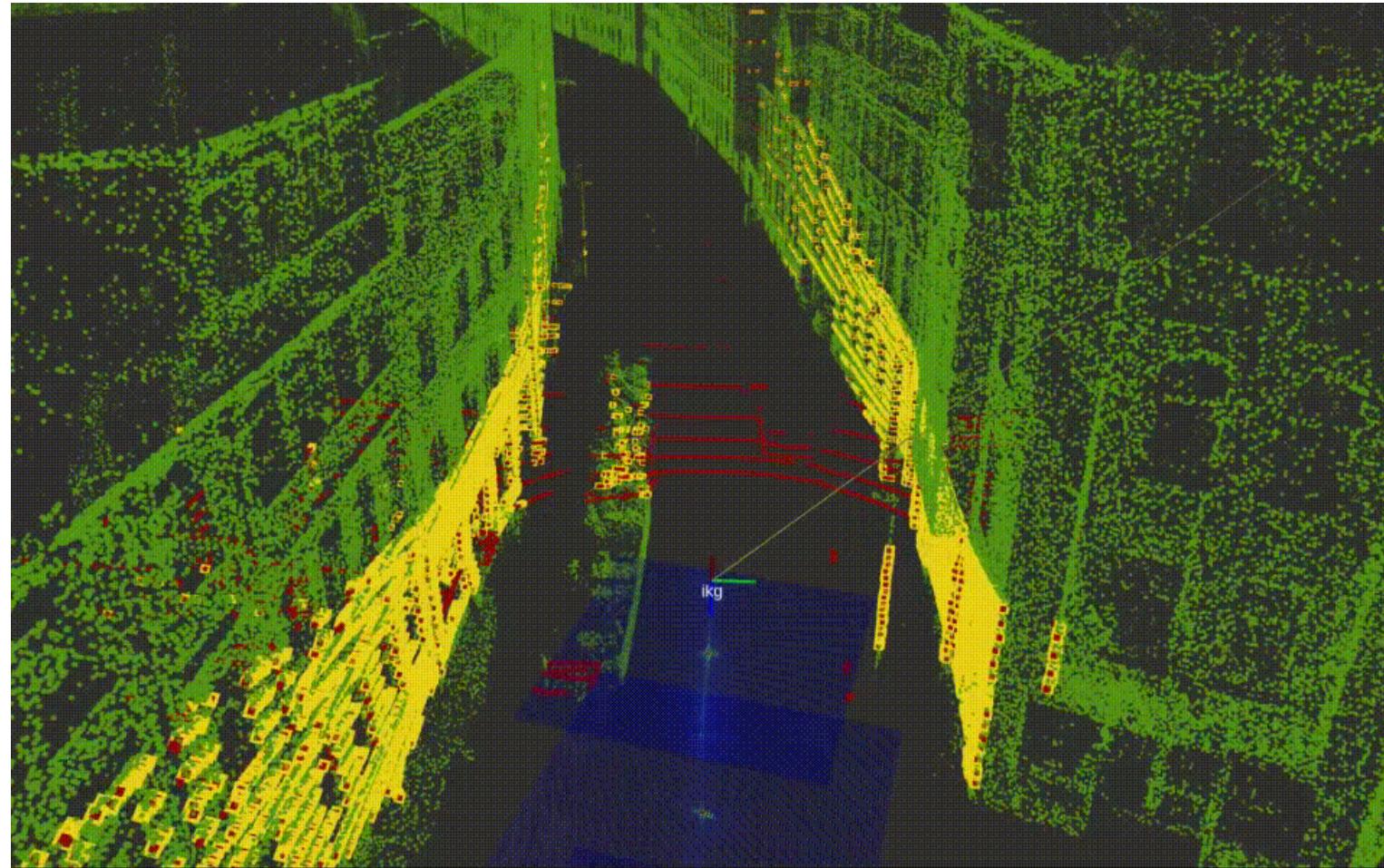
$$\theta^*, \mathbf{t}_{xy}^* = \arg \max_{\theta \in [-\pi, \pi), \mathbf{t}_{xy} \in \mathbb{R}^2} \Psi(\mathbf{R}_\theta, \mathbf{t})$$

$$\Psi(\mathbf{R}_\theta, \mathbf{t}) = \sum_{i=1}^n \sum_{j=1}^m \mathbb{I}(\|\mathbf{R}_\theta \mathbf{p}_i + \mathbf{t} - \mathbf{q}_j\|_\infty \leq \epsilon)$$





# Video: Localization using Maximum Consensus



● Velodyne scan

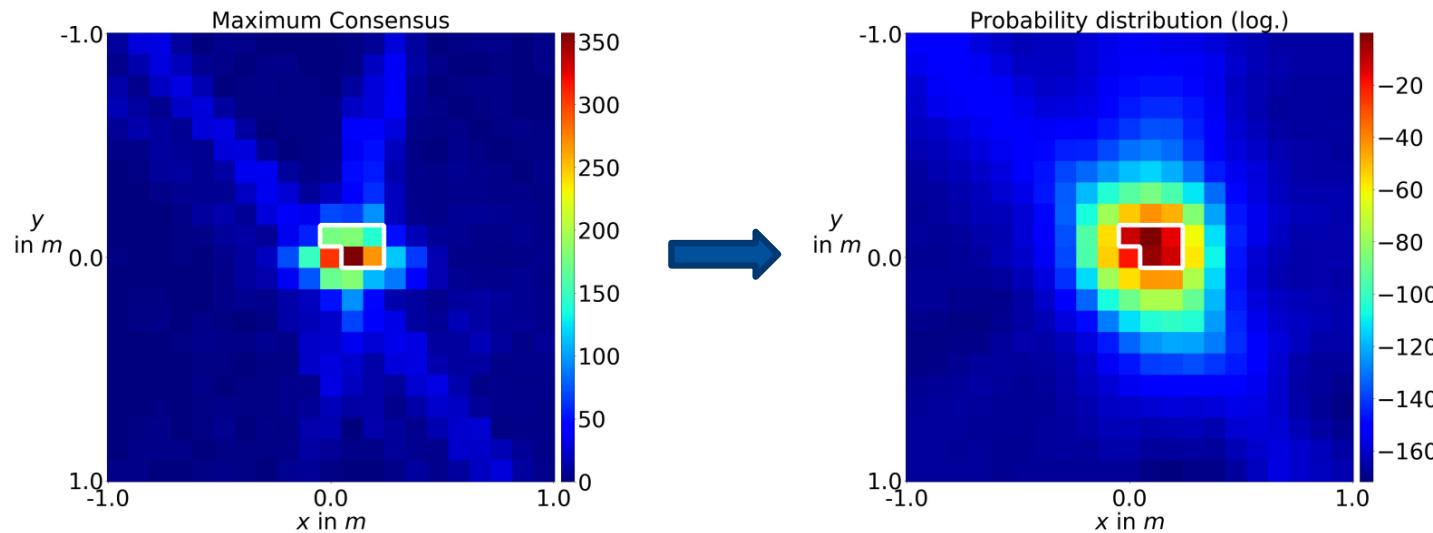
● Map point cloud

● Inlier

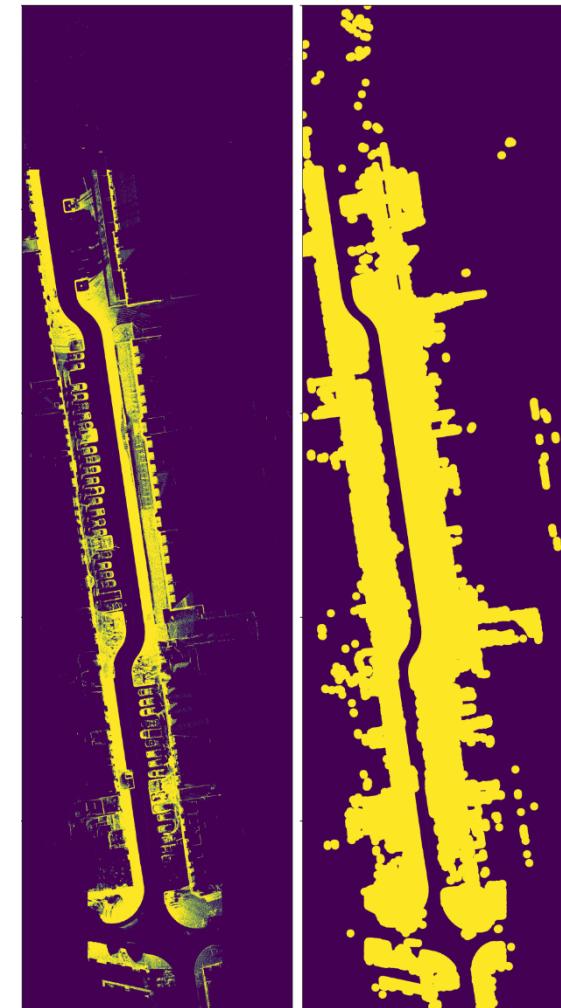


# Idea / Goal

- ▶ Goal: Transfer accumulator into a grid of probabilities



- ▶ Define confidence levels (Protection Level (PL))
- ▶ Vision: Integrity system raising alerts, if PL intersects with the Alert Limit (AL) given by a map and car sensor scans for dynamic objects.



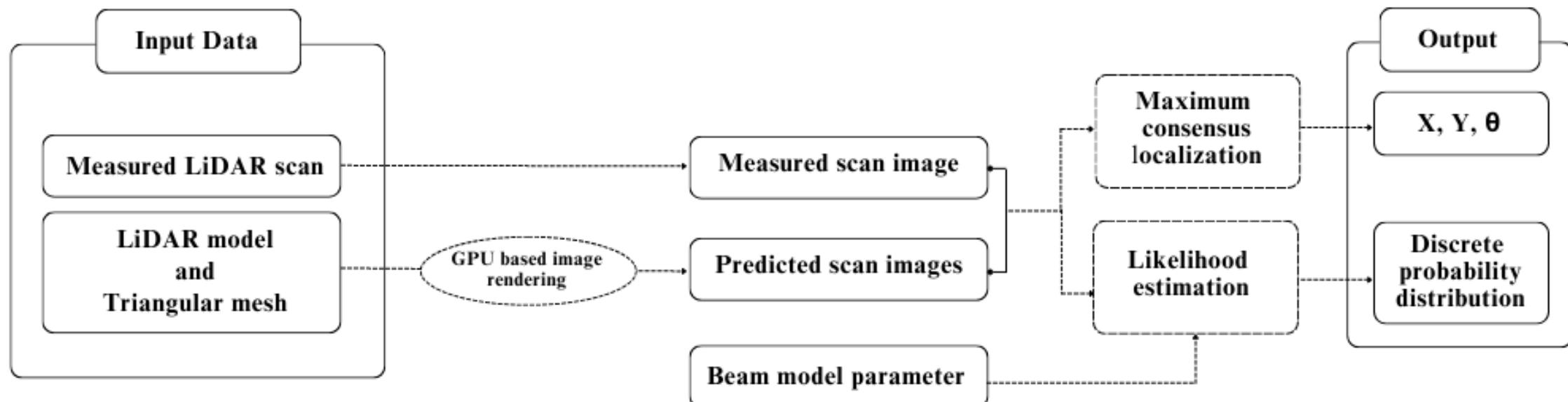
Map point cloud projected to the xy-plane (left) and buffered "Alert Limit map" (right).



# Overview Methodology

## ► Goals:

- Maximum consensus based localization and
- Protection level estimation using synthetic LiDAR range images

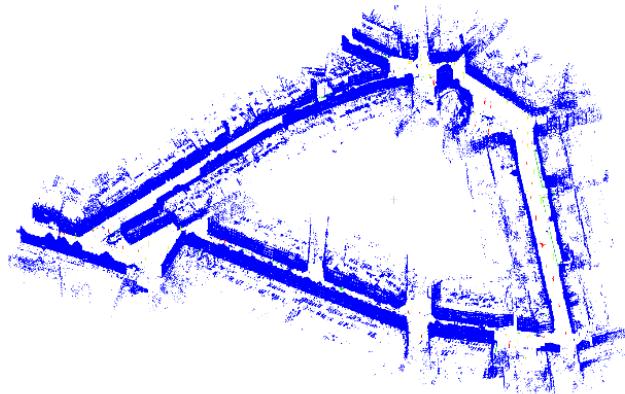




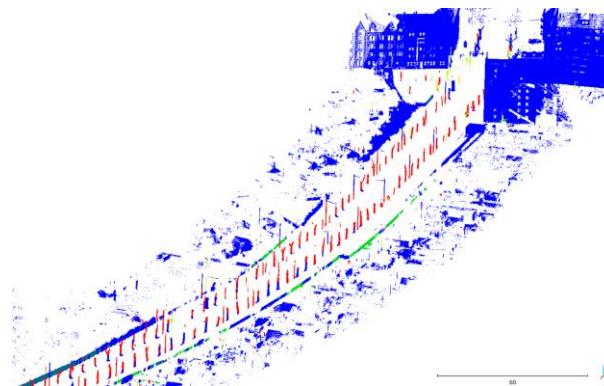
# Overview data sets

Environment	Area	LiDAR	Number of frames
Structured	Nordstadt	Velodyne VLP-16	300
Semi-structured	Haltenhoffstr.	PandarXT-32	1700
Unstructured	A2/B6	PandarXT-32	500

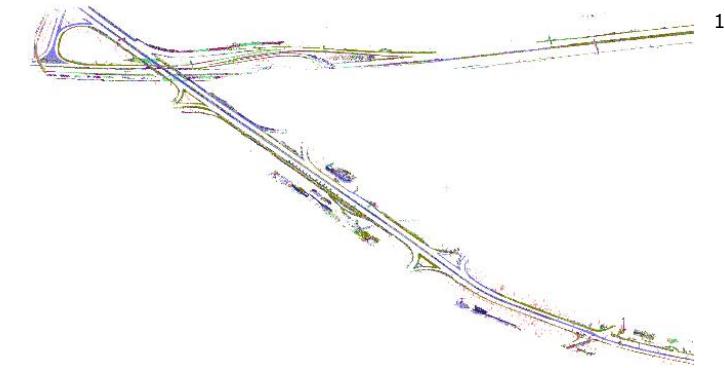
- ▶ Map point clouds of the three investigated areas with points of ground, parked cars and vegetation being removed:



Nordstadt



Haltenhoffstraße



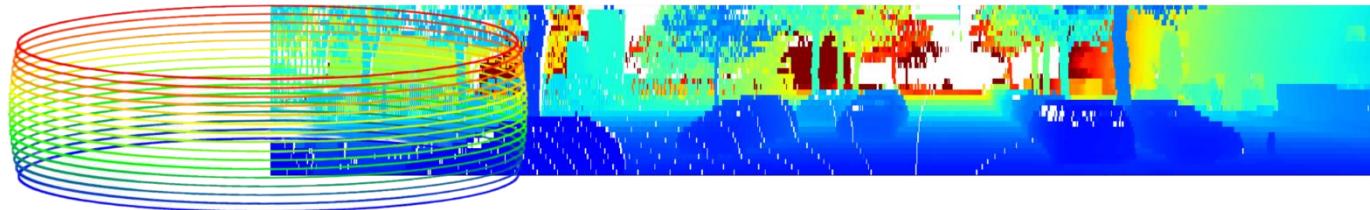
Highway (A2/B6)

- ▶ Data set of „Nordstadt“ area is publicly available: „LUCOOP: Leibniz University Cooperative Perception and Urban Navigation Dataset“ [2]



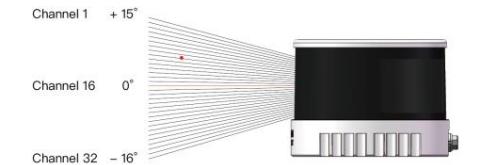
# Scan images

- ▶ Measured scan images:
  - Rows: Layer → Velodyne VLP-16: 16; Hesai PandarXT-32: 32
  - Columns: Azimuth (bins =  $360^\circ \times 1/\text{horizontal\_resolution}$ ) → Rotating LiDARs with a horizontal FOV of  $360^\circ$ 
    - Velodyne VLP-16:  $0.2^\circ$  → 1800 measurements per layer and rotation
    - Hesai PandarXT-32:  $0.18^\circ$  → 2000 measurements per layer and rotation
  - Pixel value = Measured distance



- ▶ Predicted scan images:
  - Requirements to predict scan images (for arbitrary poses): Representation of the environment → here: triangular mesh, and the LiDAR model
  - Relevant technical specifications of the LiDAR model: Horizontal and vertical field of view (FOV) and resolutions

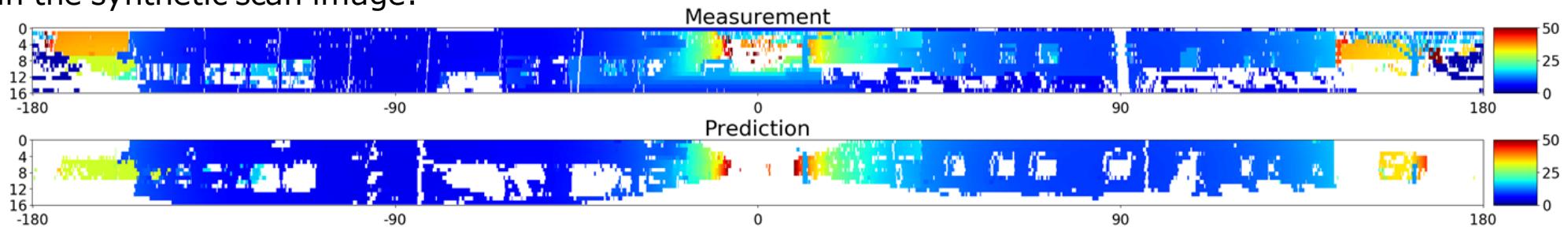
LiDAR	Rings	FOV (V.)	Res. (V.)	Res. (H.)
VLP-16	16	$30^\circ$ (-15° to +15°)	$2^\circ$	$0.2^\circ$
PandarXT-32	32	$31^\circ$ (-16° to +15°)	$1^\circ$	$0.18^\circ$





# Rendering of scan images

- ▶ Generation of triangular meshes, which enable fast image rendering using GPUs
  - Generation from 3D LiDAR point clouds using VDBFusion library [4]
    - Does not require any assumptions about the size of the area → well suited for the large RIEGL MMS point clouds
    - Combines TSDF with the VDB data structure [5] → practical meshing approach, which is fast and flexible with respect to the input point clouds
  - Parameters: Voxel size: 0.05m-0.1m; Truncation distance: 0.12m-0.25m
- ▶ GPU based rendering of predicted scan images using OpenGL
  - The rendering requires: Triangular mesh, LiDAR model specifications, LiDAR pose consisting of  $(x, y, z, \varphi, \theta, \psi)$
- ▶ Points of ground, vegetation and parked cars are removed in the 3D map point cloud using labels from manual annotation or labels from a classification [1]
  - Consequently, these classes are also not represented in the triangular mesh and in the synthetic scan image.



Am kleinen Felde (Nordstadt)

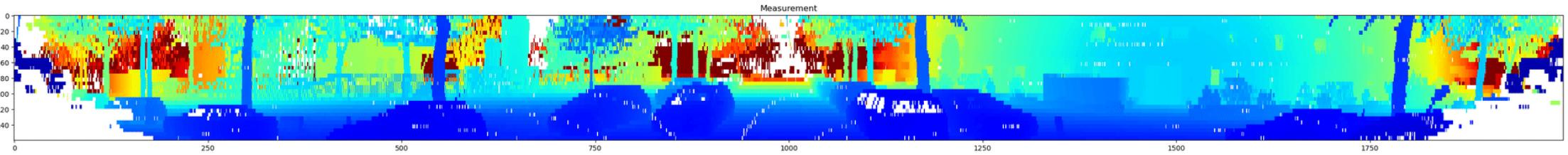


Haltenhoffstraße

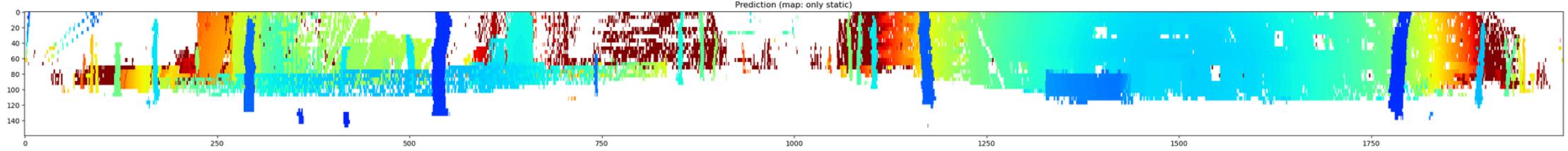


# Haltenhoffstraße (Hesai PandarXT-32)

Measurement



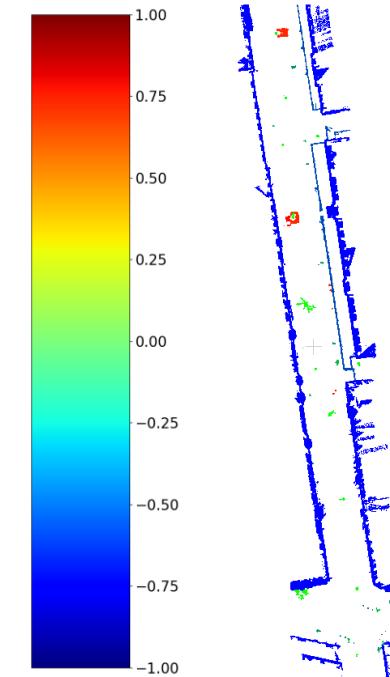
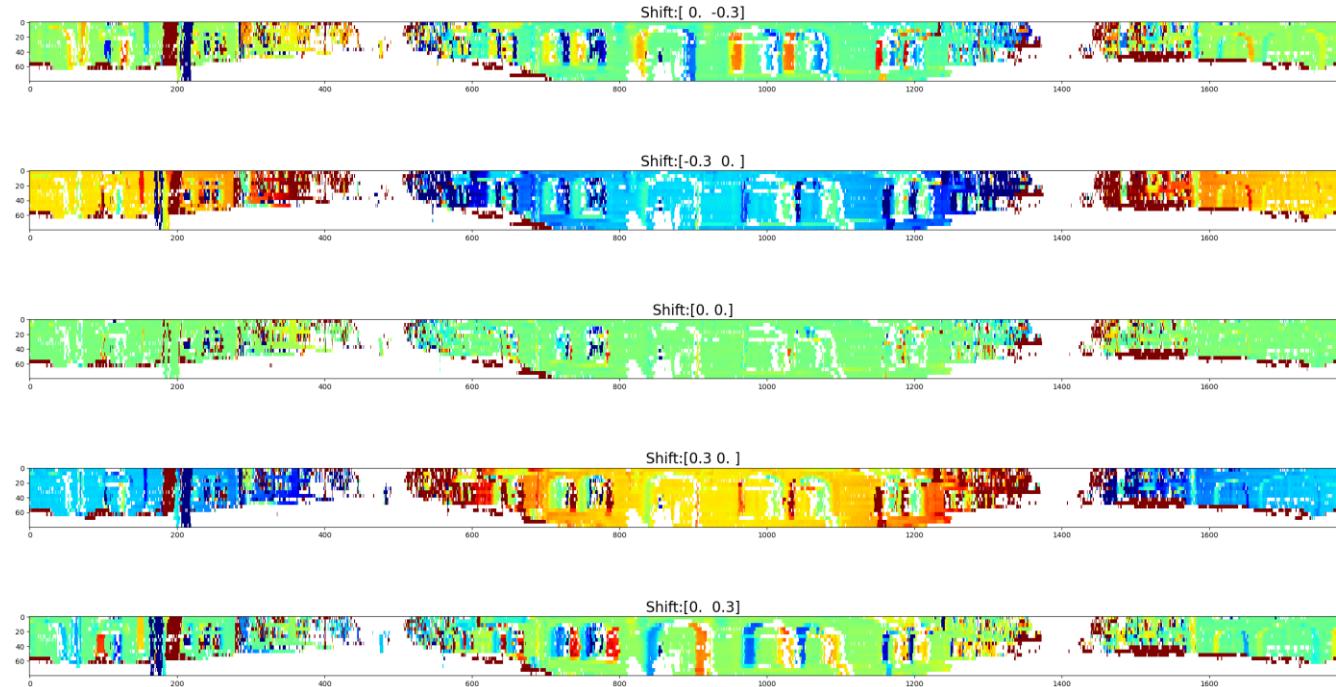
Prediction





# Maximum consensus localization

- ▶ Localization w.r.t three degrees of freedom → xy-position in the plane and heading angle  $\theta$
- ▶ Count of inliers as objective function → w.r.t measured and expected ranges in observation space (before: point matches defined in terms of Euclidean distance in 3D object space)
- ▶ 2D search range of xy-positions is discretized into a regular grid and for each position within the grid:
  - Predicted scan image is rendered
  - Calculate the deviations between measured and predicted scan images → image of equal size containing the differences as the pixel values.



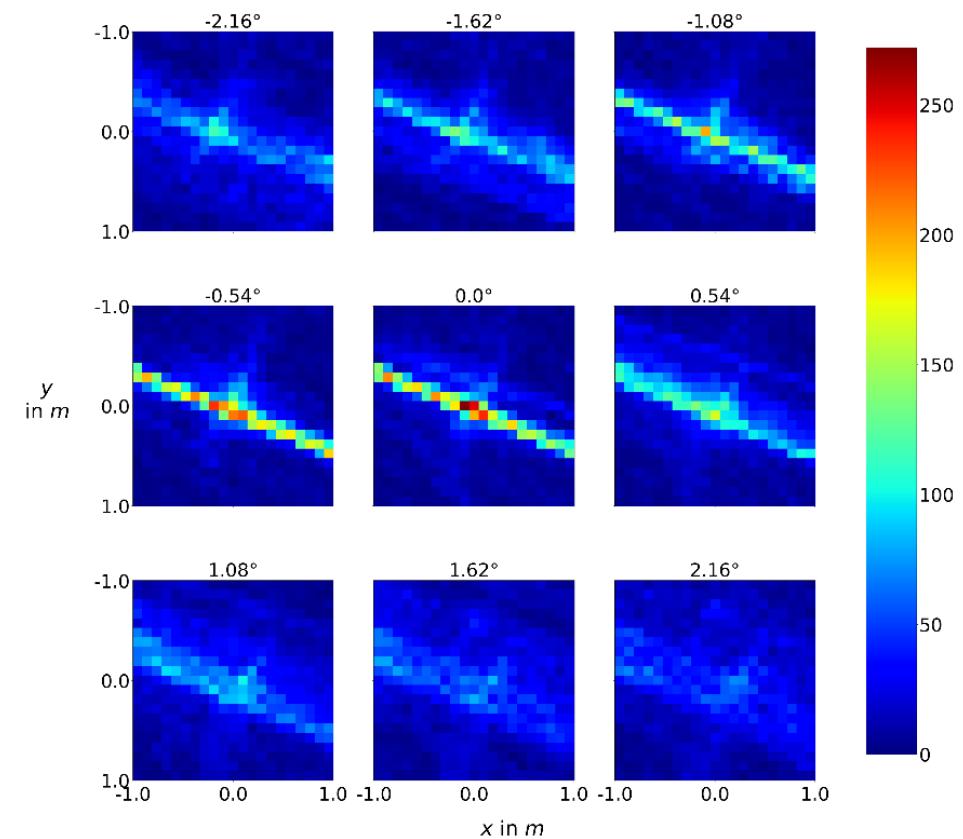


# Maximum consensus localization

- ▶ To include the heading angle  $\theta$  in the search:
  - Predicted scan image is shifted horizontally → difference image is calculated for each shift
  - Discretization of the heading search range depending on the horizontal resolution of the LiDAR sensor (here:  $= 0.18^\circ / 0.2^\circ$ ) → can not reach a step size below the horizontal resolution (= shift by one column)
  - Here: smallest step size set to three columns → angular resolution of  $0.54^\circ$  (Hesai PandarXT-32) and  $0.6^\circ$  (Velodyne VLP-16)
- ▶ Resulting objective function:

$$\theta^*, \mathbf{t}_{xy}^* = \arg \max_{\theta \in [-\pi, \pi), \mathbf{t}_{xy} \in \mathbb{R}^2} \Psi(\mathbf{R}_\theta, \mathbf{t})$$

$$\Psi(\mathbf{R}_\theta, \mathbf{t}) = \sum_{c=1}^n \sum_{r=1}^m \mathbb{I}(|u(\mathbf{R}_\theta, \mathbf{t}, c, r) - v(c, r)| \leq \epsilon)$$

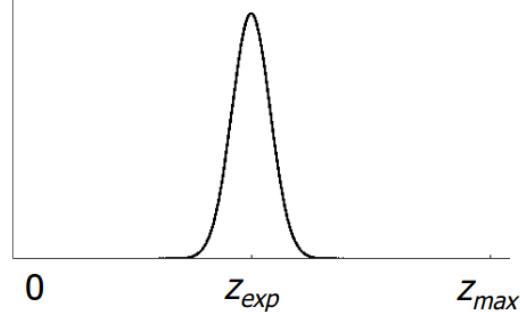


Search spaces in the xy-plane for the nine heading angles within the heading search range from  $-2.16^\circ$  to  $+2.16^\circ$ .



## Beam model definition and parameter estimation [6]

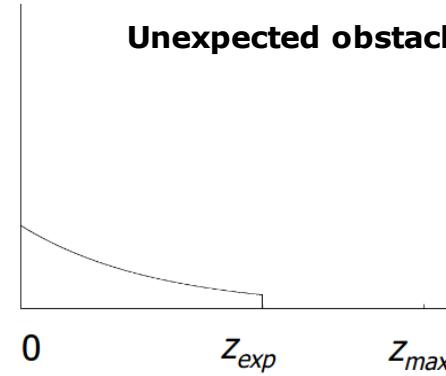
**Measurement noise**



$$p_{hit}(z^k | z^{k*}) = \begin{cases} \eta \mathcal{N}(z^k, z^{k*}, \sigma_{hit}^2) & \text{if } 0 \leq z^k \leq z_{max} \\ 0 & \text{otherwise} \end{cases}$$

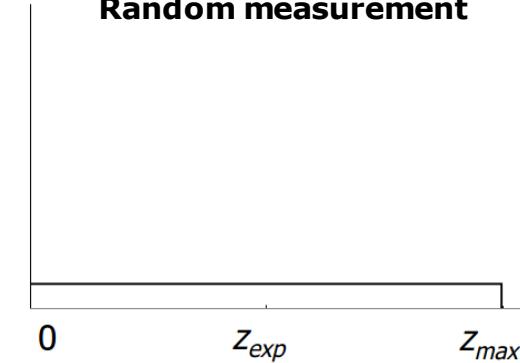
$$\text{with } \mathcal{N}(z^k, z^{k*}, \sigma_{hit}^2) = \frac{1}{\sqrt{2\pi\sigma_{hit}^2}} e^{-\frac{(z^k - z^{k*})^2}{2\sigma_{hit}^2}},$$

**Unexpected obstacles**



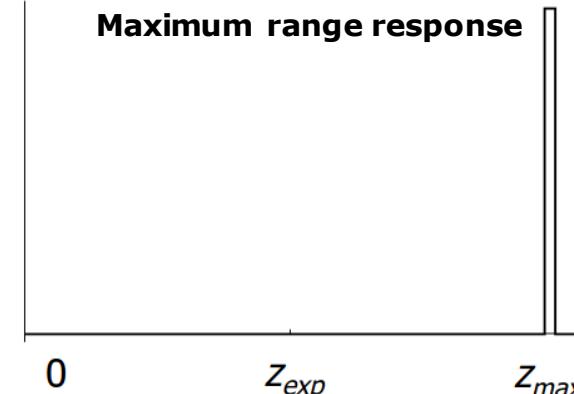
$$p_{short}(z^k | z^{k*}) = \begin{cases} \eta \lambda_{short} e^{-\lambda_{short} z^k} & \text{if } 0 \leq z^k \leq z^{k*} \\ 0 & \text{otherwise,} \end{cases}$$

**Random measurement**



$$p_{rand}(z^k | z^{k*}) = \begin{cases} \frac{1}{z_{max}} & \text{if } 0 \leq z^k < z_{max} \\ 0 & \text{otherwise.} \end{cases}$$

**Maximum range response**

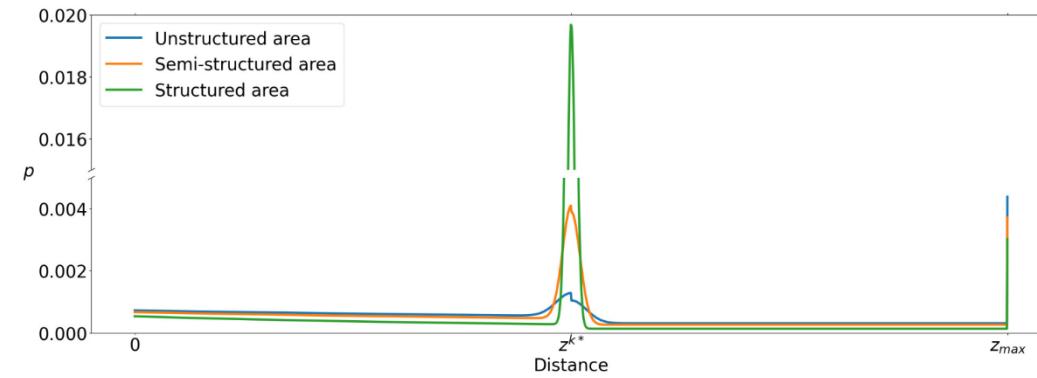
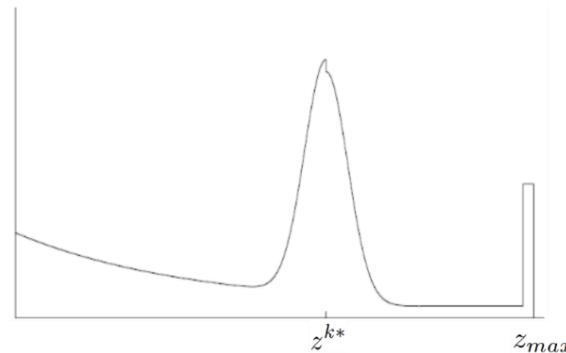


$$p_{max}(z^k | z^{k*}) = \begin{cases} 1 & \text{if } z = z_{max} \\ 0 & \text{otherwise,} \end{cases}$$



# Beam model definition and parameter estimation

- ▶ Beam model parameters  $\Theta$  ( $\sigma_{hit}$ ,  $\lambda_{short}$ ,  $w_{hit}$ ,  $w_{short}$ ,  $w_{random}$ ,  $w_{max}$ ) are estimated from the measured scan data [6]
  - Goal of the adjustment process: Identify a set of beam model parameters  $\Theta$  that maximizes the likelihood  $p(Z|X, m, \Theta)$  between actual and expected measurements → iterative process  
Z: set of real measurements, X: set of ground truth poses, m: map (→ here: mesh)
  - Expected distances are determined using ray-tracing
- ▶ In this work, for each type of environment, a set of intrinsic beam model parameters is estimated using the ground truth trajectory obtained by the RIEGL MMS and all available measurement epochs



$$p(z^k | z^{k*}) = \begin{pmatrix} w_{hit} \\ w_{short} \\ w_{max} \\ w_{rand} \end{pmatrix}^T \cdot \begin{pmatrix} p_{hit}(z^k | z^{k*}) \\ p_{short}(z^k | z^{k*}) \\ p_{max}(z^k | z^{k*}) \\ p_{rand}(z^k | z^{k*}) \end{pmatrix}$$

Name	$\sigma_{hit}$	$\lambda_{short}$	$w_{hit}$	$w_{short}$	$w_{max}$	$w_{rand}$
Structured	0.1	0.11	0.47	0.15	0.2	0.18
Semi-struct.	0.23	0.07	0.13	0.34	0.21	0.32
Unstructured	0.34	0.06	0.03	0.41	0.22	0.34

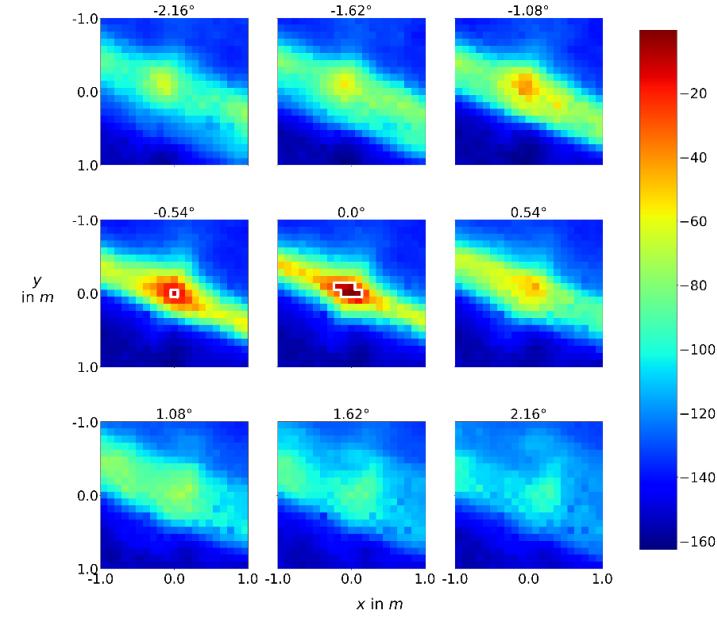
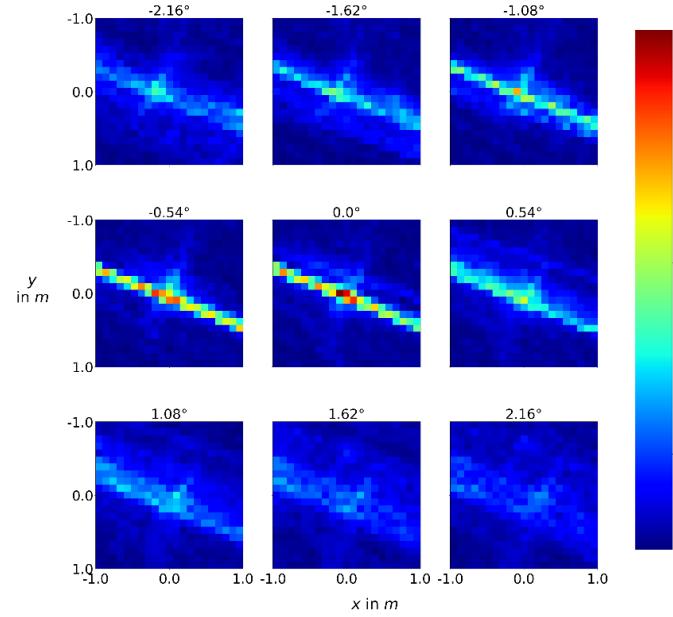
# Estimation of discrete probability distribution and definition of a protection level



- ▶ For every pixel of each predicted scan image  $p(z|x,m)$  is drawn from the beam model with  $z$ : measured range,  $x$ : candidate poses,  $m$ : map
- ▶ The discrete probability distribution over all candidate poses  $p(x|Z,m)$  is estimated as:

$$p(x|Z,m) = \frac{p(Z|x,m) \cdot p(x|m)}{\sum_{\tilde{x}} p(Z|\tilde{x},m)p(\tilde{x}|m)} \quad \text{with} \quad p(Z|x,m) = \prod_i p(z_i|x,m)$$

- ▶ The protection level is defined by the grid cells with a probability  $p > 1 \cdot 10^{-7}$ .





## Experiments – Maximum consensus based localization

### ▶ Robustness:

- Maximum consensus localization: exhaustive search → supposed to provide globally optimal localization solutions
- Sought: Portion of epochs, in which the localization fundamentally fails → deviation of solution from the ground truth beyond a margin explainable by inaccuracies in the pipeline: margin = 0.2 m
- For comparison: point-to-plane ICPs initialized in a similar grid from -1 m to +1 m around the ground truth position  
→ sought: portion of epochs, in which not all of the ICPs converge to the ground truth

### ▶ Accuracy:

- Maximum consensus localization using a grid with an increased resolution of 2 cm
- Calculation of mean absolute deviation (MAD) between the pose of the highest count and the ground truth

Environment	Failures ICP	Failures Max. Cons.	MAD $xy$ Max. Cons.	MAD $\theta$ Max. Cons.
Structured	0%	0	0.032 m	0.023°
Semi-structured	1.8%	0.3%	0.049 m	0.068°
Unstructured	9.2%	3.8%	0.101 m	0.378°

# Experiments - Estimation of discrete probability distribution and protection level



- ▶ Pose with the highest probability within the search range is considered as localization solution → similar evaluation as maximum consensus localization w.r.t. the ground truth
- ▶ Test of estimated protection level w.r.t the requirement that it contains the true pose

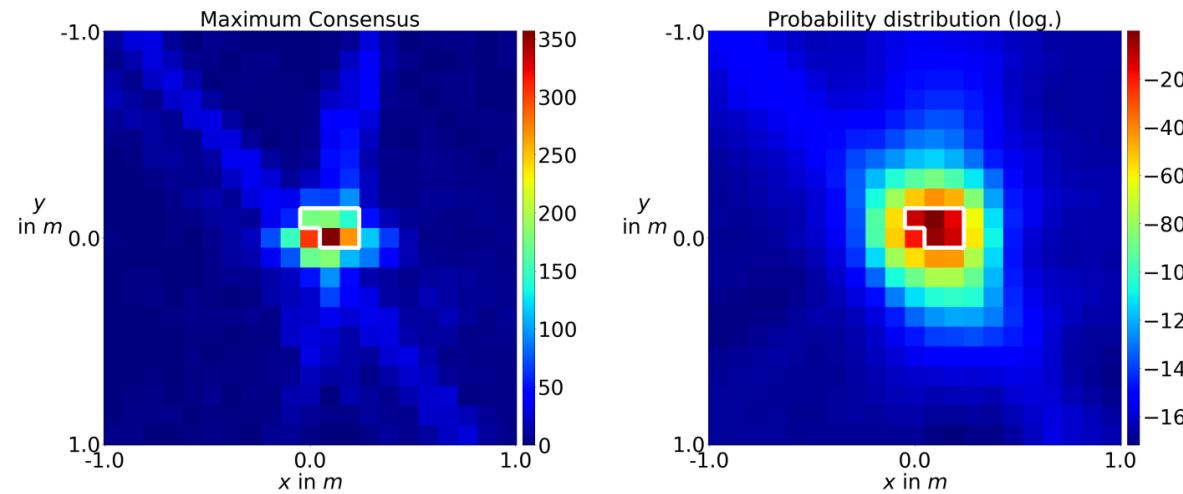
Environment	Failures Prob. distr.	MAD $xy$ Prob. distr.	MAD $\theta$ Prob. distr.	Portion PL check ✓
Structured	0%	0.034 m	0°	92.7%
Semi-structured	0%	0.046 m	0.032°	88.2%
Unstructured	0.6%	0.089 m	0.008°	86.4%

- ▶ Protection level test:
  - Requirement that the PL contains the true pose is not fulfilled for a significant portion in all three test areas
  - PL does not cover the true pose: state of misleading information (in terms of Stanford diagram)
  - Driving scenarios are not extremely challenging and no exceptional events occur  
→ expectation: portion of successful PL checks at or close to 100% (no states of misleading information)

# Experiments - Estimation of discrete probability distribution and protection level



- ▶ Maximum consensus localization only shows a little deviation from the true pose → no fundamental fail, but PL does not cover the ground truth pose.



- ▶ Reasons for probability distributions with a too high confidence on a certain localization solution:
  - Estimation of the probability grids assumes independent measurements and does not consider existing correlations.
  - Beam model does not fit measurements → only one beam model is estimated for a drive over multiple kilometers.



# Conclusion and outlook

- ▶ We proposed a novel localization pipeline based on synthetic LiDAR range images combining
  - a robust localization approach using the maximum consensus criterion and
  - an estimation of probability distributions using a beam model for range sensors.
- ▶ The maximum consensus based localization approach provides robust and accurate results, however, still with some space for improvement
  - Future research: more robust and accurate approach based on point-to-plane adjustment.
- ▶ Estimated discrete probability distributions are too optimistic
  - Challenge: Finding a beam model (or in general loss function), which is in accordance to the error distribution of our measurements to correctly estimate the probability distribution for the vehicle pose (in highly dynamic setting even more difficult)
  - Future research: Application of the adaptive loss function introduced by Barron [7,8] for the estimation of probability distributions → a scaling parameter  $\alpha$ , which determines the shape of the loss function, is constantly estimated and updated. Depending on the outlier ratio a more robust shape is selected.



# References

- ▶ <sup>1</sup>Leichter, A., Internal Work.
- ▶ <sup>2</sup>J. Axmann et al., "LUCOOP: Leibniz University Cooperative Perception and Urban Navigation Dataset," 2023 IEEE Intelligent Vehicles Symposium (IV), Anchorage, AK, USA, 2023, pp. 1-8, doi: 10.1109/IV55152.2023.10186693.
- ▶ <sup>3</sup>Hesai Technology Co., Ltd., PandarXT-32 32-Channel Medium-Range Mechanical LiDAR User Manual, 2021.
- ▶ <sup>4</sup>I. Vizzo, T. Guadagnino, J. Behley, and C. Stachniss, "Vdbfusion: Flexible and efficient tsdf integration of range sensor data," Sensors, vol. 22, no. 3, 2022. [Online]. Available: <https://www.mdpi.com/1424-8220/22/3/1296>
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- ▶ <sup>7</sup>J. T. Barron. A General and Adaptive Robust Loss Function. In Proc. of the IEEE Conf. on Computer Vision and Pattern Recognition (CVPR), 2019.
- ▶ <sup>8</sup>N. Chebrolu, T. Läbe, O. Vysotska, J. Behley, and C. Stachniss, "Adaptive Robust Kernels for Non-Linear Least Squares Problems," IEEE Robotics and Automation Letters (RA-L), vol. 6, pp. 2240-2247, 2021.
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- ▶ R. W. Wolcott and R. M. Eustice, "Visual localization within LIDAR maps for automated urban driving," in 2014 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2014, pp. 176-183.
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Thank you for your attention.  
Questions? Comments?



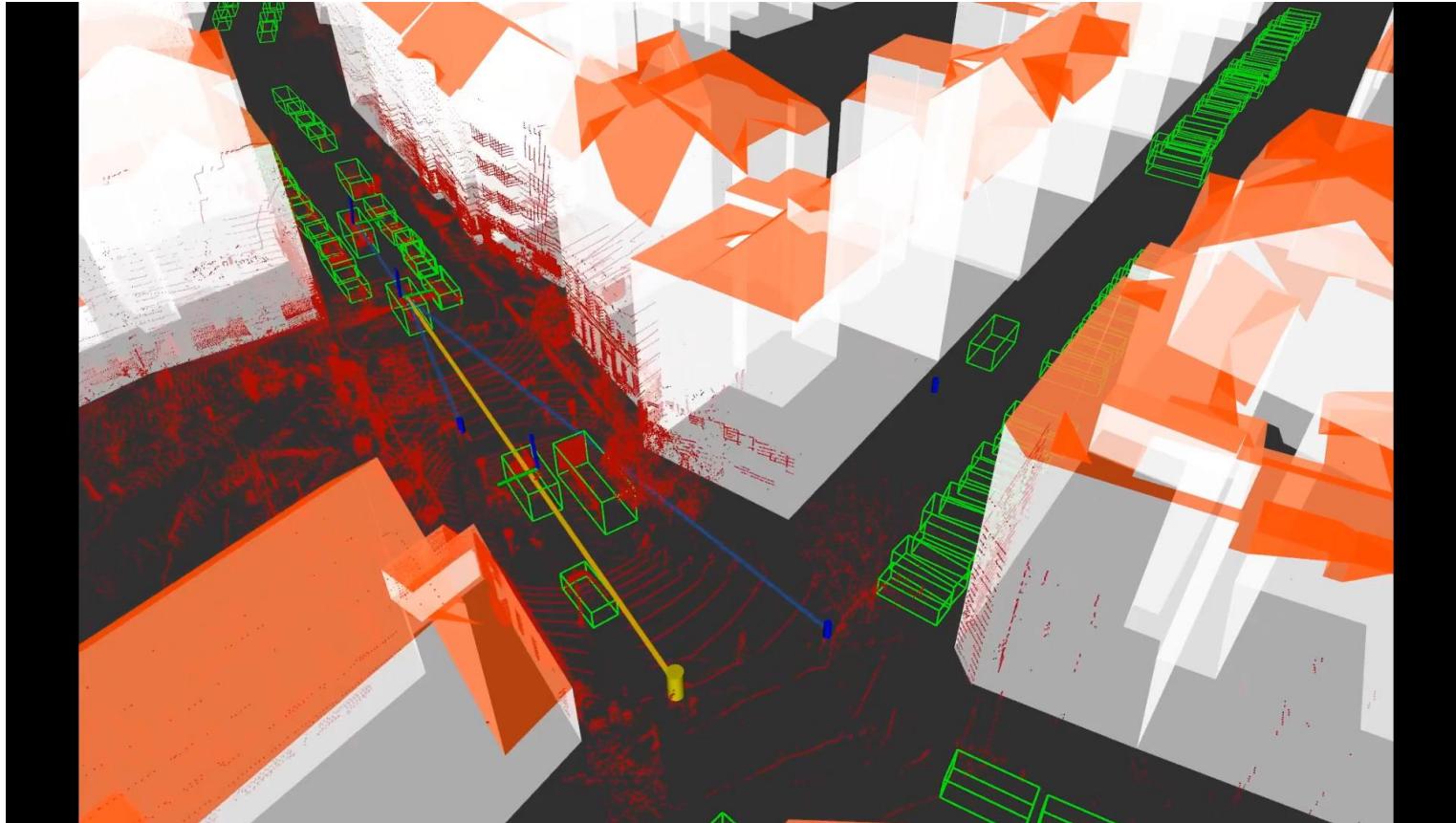
# Backup

# LUCOOP: Leibniz University Cooperative Perception and Urban Navigation Dataset

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- ▶ <https://data.uni-hannover.de/dataset/lucoop-leibniz-university-cooperative-perception-and-urban-navigation-dataset>



- ▶ LUCOOP contributors: Yunshuang Yuan, Qianqian Zou, Dominik Ernst, Benjamin Tennstedt, Jingyao Su, Rozhin Moftizadeh, Jeldrik Axmann, Hamza Alkhatib, Claus Brenner, Steffen Schön