Workout 3

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1. $$\begin{cases} -u'' = 10 \sin x, & 0<x<1 \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$

V_h

Define $\{\phi_i(x)\}_{i=1}^{n-1}$ by $\phi_i(x_j) = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$

and linear space $V_h = \{v \in C([0,1]) : v(0) = v(1) = 0$
$v(x)$ piecewise linear on $[x_i, x_{i+1}]$, $i = 0, \dots n-1\}$
$x_i := ih$ and $h = \frac{1}{n}$

Every function $v_h \in V_h$ can be written as a linear combination of the hat functions

$$u(x) \approx u_h = \sum_{i=1}^{n-1} c_i \phi_i(x) \quad (c_i \text{ coefficients})$$

Define FEM using basis functions, use the case where $n=4$.
I start with some derivation (actually part B)

Let $-u'' = 10 \sin x = f$

If we multiply $f = -u''$ by a function v and integrate by parts:

$$\Rightarrow \int_0^1 f v \, dx = \int_0^1 -u'' v \Leftrightarrow -[u'v]_0^{1} + \int_0^1 u'v' \, dx =$$

$$= -u'(1)v(1) + u'(0)v(0) + \int_0^1 u'v' \, dx$$