

## Workout 1

### Linear systems of Equations

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1. 
$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \\ 0 \\ \beta \end{pmatrix} \quad \text{where } \alpha = -1 \\ \beta = 1$$

$A$                        $b$

$$U = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \begin{matrix} \left(\frac{1}{2}\right) \\ \left(\frac{1}{2}\right) \\ \left(\frac{1}{2}\right) \end{matrix} \sim \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -3/2 & 1 & 0 \\ 0 & 0 & -4/3 & 1 \\ 0 & 0 & 0 & -15/4 \end{pmatrix}$$

remove those three

$$L = \begin{pmatrix} -2 & \times & 0 & 0 \\ 1 & -2 & \times & 0 \\ 0 & 1 & -2 & \times \\ 0 & 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 \end{pmatrix}$$

Now we solve  $u$  when  $b = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$  and  $A = LU$

First:  $Ly = b \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow$

$$\Rightarrow \begin{cases} y_1 = -1 \rightarrow y_1 = -1 \\ -\frac{1}{2}y_1 + y_2 = 0 \rightarrow y_2 = -1/2 \\ -\frac{2}{3}y_2 + y_3 = 0 \rightarrow y_3 = \frac{2}{3}(-\frac{1}{2}) = -\frac{2}{6} \\ -\frac{3}{4}y_3 + y_4 = 1 \rightarrow y_4 = \frac{3}{4}(-\frac{2}{6}) + 1 = -\frac{6}{24} + 1 = \frac{3}{4} \end{cases}$$

$$y = \begin{pmatrix} -1 \\ -1/2 \\ -2/6 \\ 3/4 \end{pmatrix}$$

So now we have to do  $Ux = y$   
and solve it  $\longrightarrow$