Workout 2

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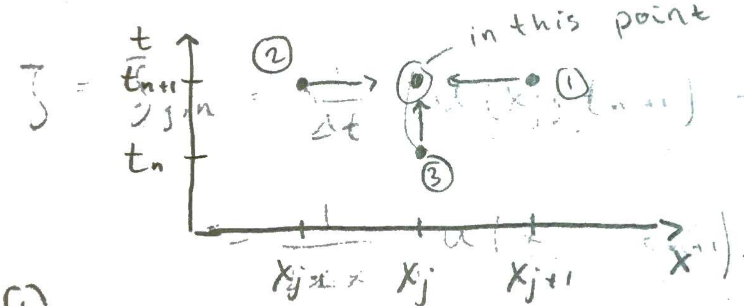
1. $$\begin{cases} u_t = u_x, & 0 < x < 1, \quad t > 0 \\ u(0,t) = u(1,t), & t > 0 \\ u(x,0) = f(x), & 0 < x < 1 \end{cases}$$

is approximated by:

$$\begin{cases} \dfrac{u_j^{n+1} - u_j^n}{\Delta t} = \dfrac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2\Delta x}, & j = 0,1,\dots, N-1, \quad n = 0,1,\dots \\ u_j^n = u_{j+N}^n, & \text{for all } j, \quad n = 0,1,\dots \\ u_j^0 = f(x_j), & j = 0,1,\dots, N-1 \end{cases}$$

a) Derive the LTE from the approximation

→ Taylors theorem around $u(x_j, t_{n+1})$



①

$$u(x_{j+1}, t_{n+1}) = u(x_j, t_{n+1}) + \Delta x\, u_x(x_j, t_{n+1}) + \frac{\Delta x^2}{2} u_{xx}(x_j, t_{n+1}) + \frac{\Delta x^3}{6} u_{xxx}(x_j, t_{n+1}) + O(\Delta x^4)$$

②

$$u(x_{j-1}, t_{n+1}) = u(x_j, t_{n+1}) - \Delta x\, u_x(x_j, t_{n+1}) + \frac{\Delta x^2}{2} u_{xx}(x_j, t_{n+1}) + \frac{\Delta x^3}{6} u_{xxx}(x_j, t_{n+1}) + O(\Delta x^4)$$

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