FT and DTFT Properties

A) Convolution - multiplication

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \xleftrightarrow{FT}$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \xleftrightarrow{DTFT}$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \xleftrightarrow{FT}$$

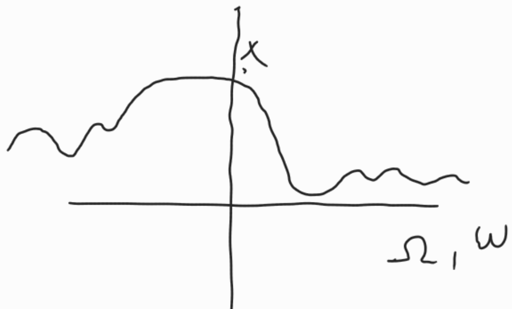
$$Y(\Omega) = X(\Omega)\, H(\Omega)$$

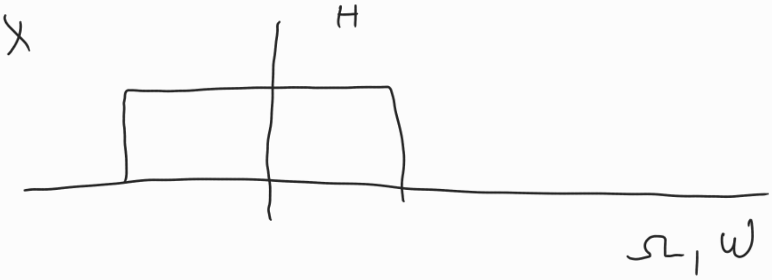
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \xleftrightarrow{DTFT}$$

$$Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})$$

$\xleftrightarrow{FT}$

$\xleftrightarrow{DTFT}$





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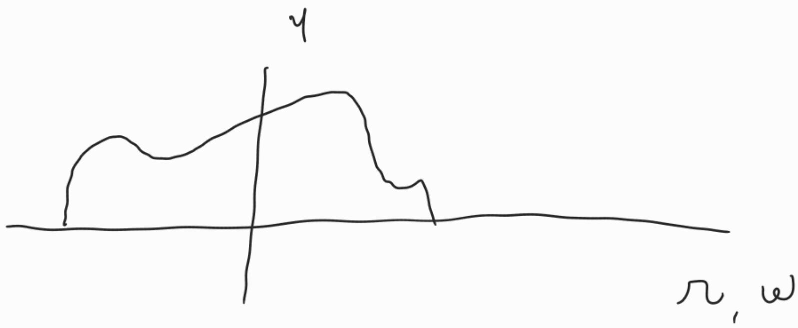


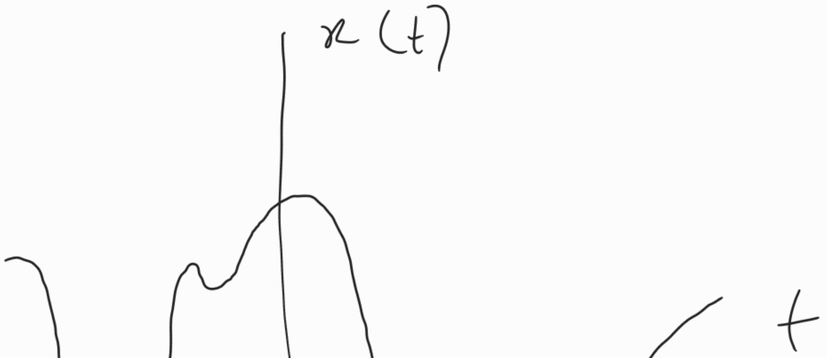
("Filtering)

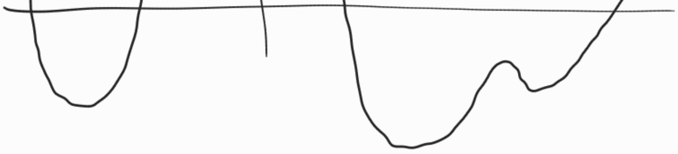
B) Multiplication - convolution (windowing)

$$z(t) = x(t)w(t) \xleftrightarrow{FT} Z(\Omega) = \frac{1}{2\pi} X(\Omega) * W(\Omega)$$

$$z[n] = x[n]w[n] \xleftrightarrow{DTFT} Z(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) * W(e^{j\omega})$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\gamma}) W(e^{j\omega - \gamma}) d\gamma$$





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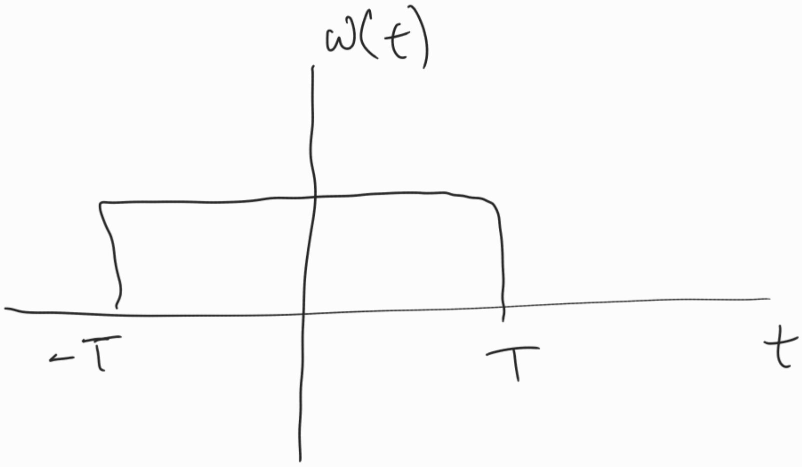
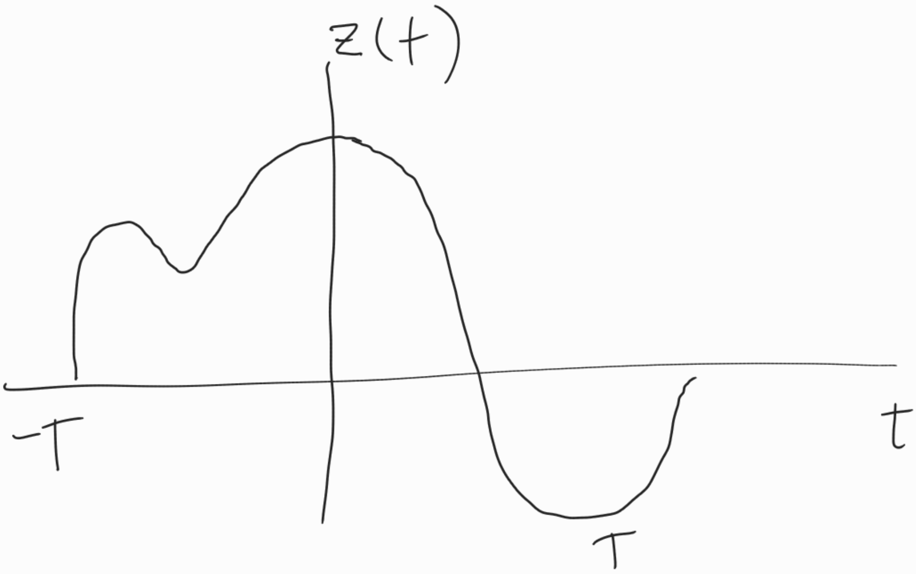
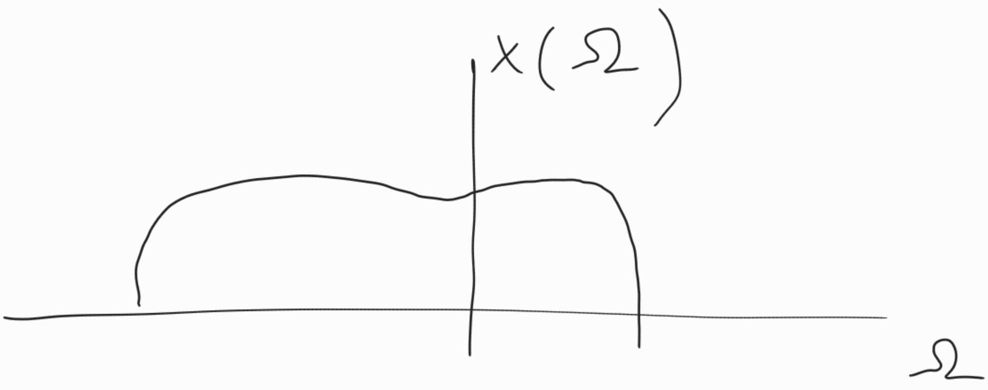
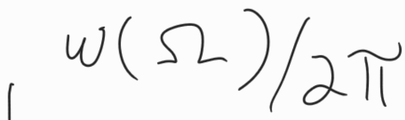


$\xleftrightarrow{FT}$







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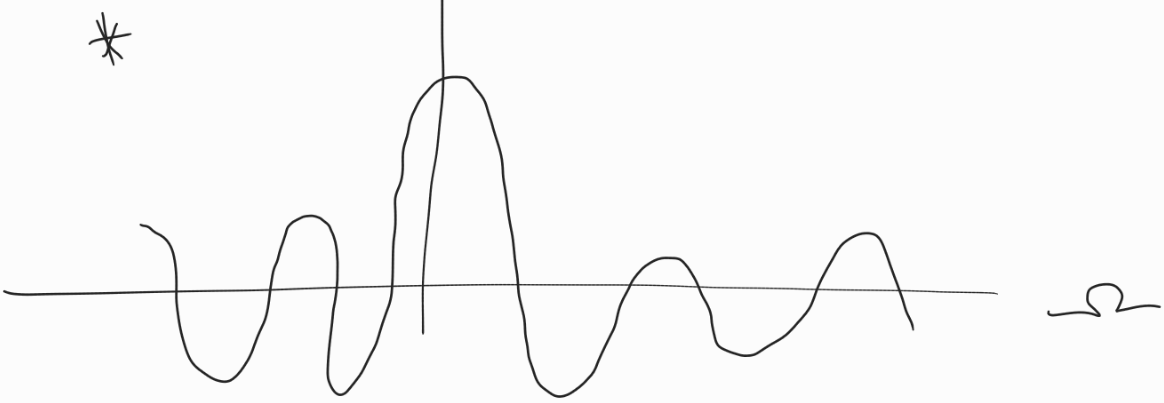
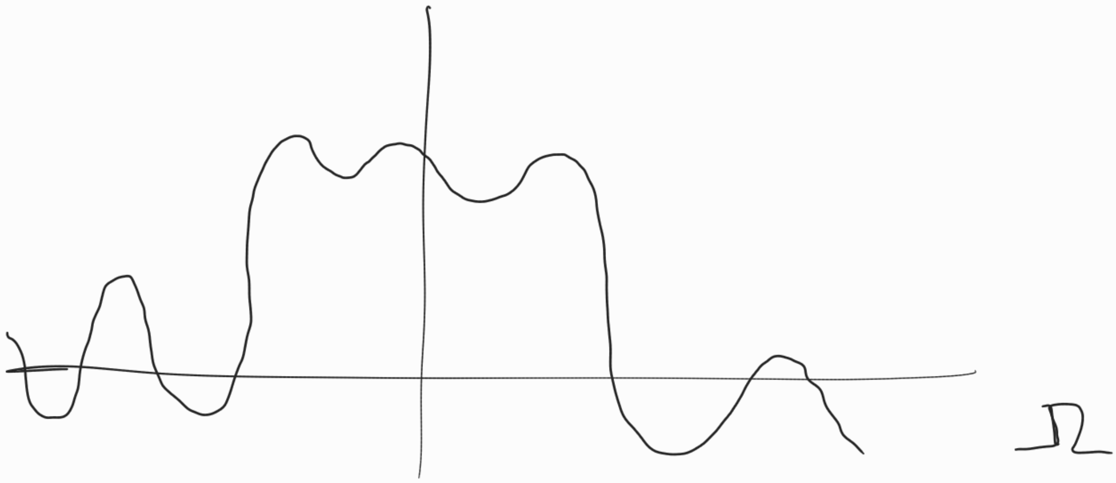


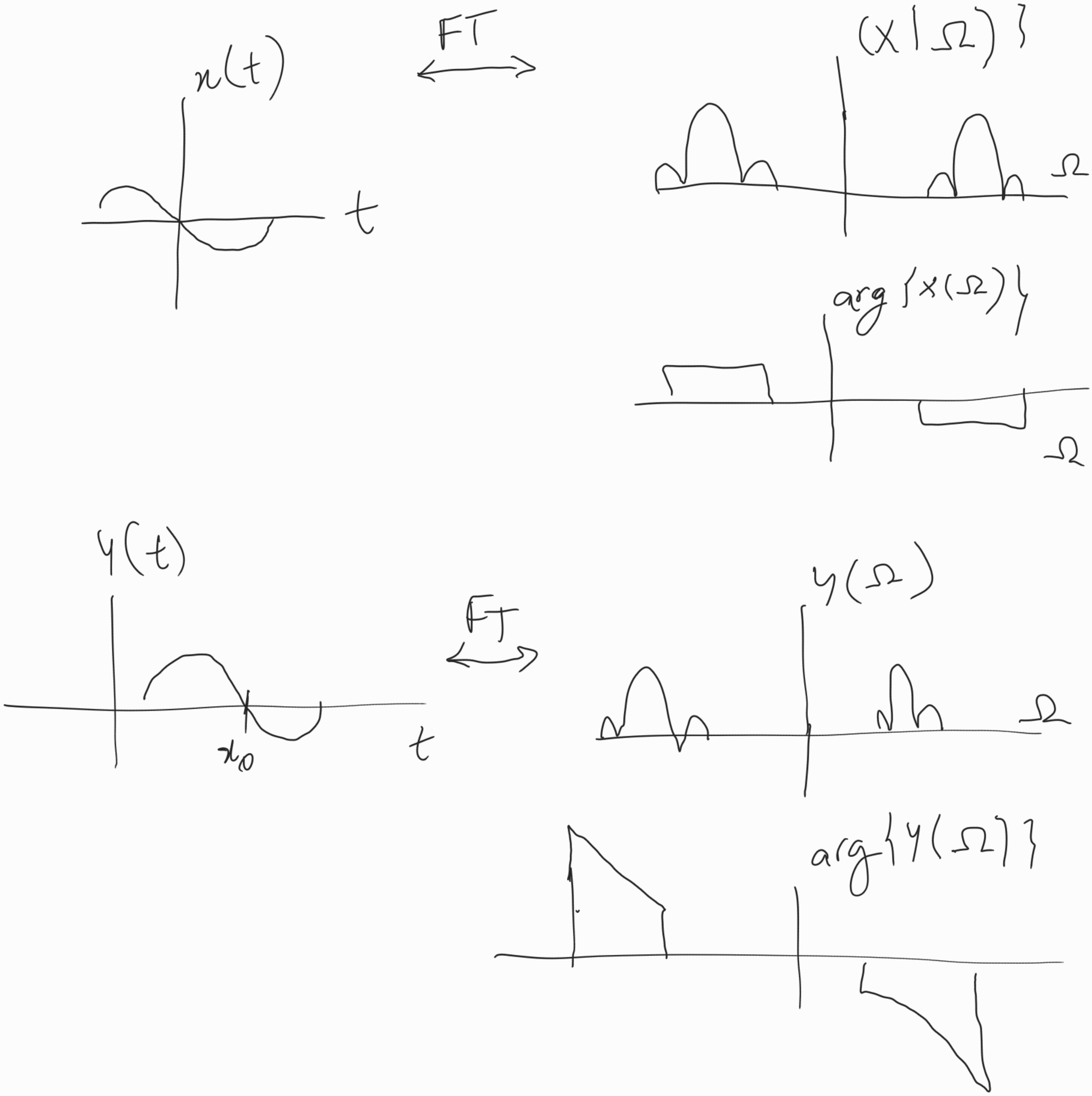
$z(\Omega)$



c) Time Shift

$$y(t) = x(t-t_0) \xleftrightarrow{FT} Y(\Omega) = e^{-j\Omega t_0} X(\Omega)$$

$$y[n] = x[n-n_0] \xleftrightarrow{DTFT} Y(e^{jw}) = e^{-jwn_0} X(e^{jw})$$



b) FT Representation for Periodic Signals

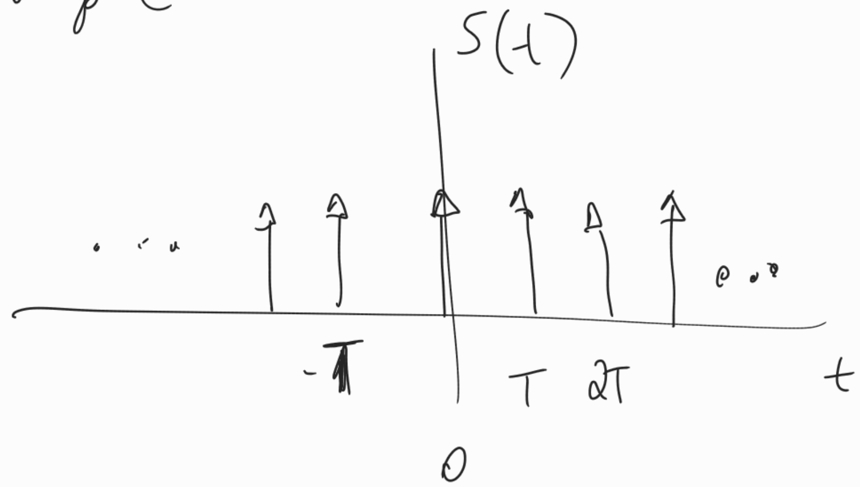
$X(t)$ has fund period T:

$$x(t) \xleftrightarrow{FS;\ \Omega_0 = \frac{2\pi}{T}} X[k] = \frac{1}{T}\int_0^T x(t)\, e^{-jk\Omega_0 t}\, dt$$

then

$$x(t) \xleftrightarrow{FT} X(\Omega) = 2\pi \sum_{k=-\infty}^{\infty} x[k]\,\delta(\Omega - k_0\Omega_0)$$

Example



$$S(t) = \sum_{l=\infty}^{\infty} \delta(t - lT)$$

① Find FS

$$S[k] = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{T} \cdot 1 = \frac{1}{T}$$

② $S(\Omega) = \sum^{\infty} 1\,\delta(\Omega - k\frac{2\pi}{\ })$

$$(2\pi) S(\Omega) = 2\pi \sum_{k=-\infty} \frac{1}{T} \delta\left(\Omega - k\frac{2\pi}{T}\right)$$