Discrete time analog of differential equation "ring of computation" for system outputs

$$x[n] \rightarrow \boxed{H} \rightarrow y[n]$$

$$y[n] + a_1 y[n-1] + a_2 y[n-2] + \dots + a_N y[n-N] =$$

$$b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \qquad (a_0 = 1)$$

N^{th} order

$N + M + 1$ parameters $\{a_k, b_k\}$ completely define the system

$\boxed{\text{Computation}}$

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

present past input

output   outputs

Example

$$y[n] - \frac{1}{2} y[n-1] = \frac{1}{4} x[n] + \frac{1}{4} x[n-1]$$

Rewrite

$$y[n] = \frac{1}{2} y[n-1] + \frac{1}{4} x[n] + \frac{1}{4} x[n-1]$$

Assume start with $n=0$

$$y[0] = \frac{1}{2} y[-1] + \frac{1}{4} x[0] + \frac{1}{4} x[-1]$$

initial condition

Matlab "FILTER"

Suppose $\quad x[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

$$y[-1] = 0$$

$$y[0] = \frac{1}{2} \cdot 0 + \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 0 = \frac{1}{4}$$

$y[1] =$

$$y[1] = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 1 = \frac{5}{8}$$

$$y[2] = \frac{1}{2}\cdot\frac{5}{8} + \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 1 = \frac{13}{16}$$

$$y[3] = \frac{1}{2}\cdot\frac{13}{16} + \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 1 = \frac{29}{32}$$

$$\vdots$$

Examples:

1) 6 points averaging

$$y[n] = \frac{1}{6}\left\{x[n] + x[n-1] + \dots + x[n-5]\right\}$$

2) 6 point differencing

$$y[n] = \frac{1}{6}\left\{x[n] - x[n-1] + x[n-2] - \dots - x[n-5]\right\}$$

3) Recursive low pass

$$y[n] = 0.95\,y[n-1] + 0.05\,x[n]$$

4) Recursive High Pass

$$y[n] = 0,95\, y[n-1] + 0.05\, x[n]$$

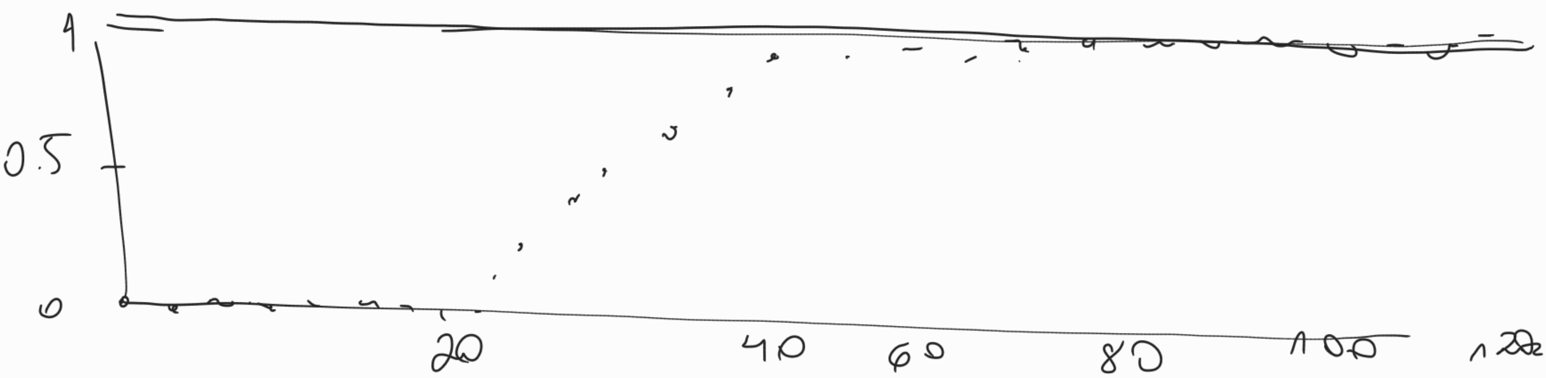
Inputs:

a) $x[n] = \begin{cases} 0 & n < 20 \\ 1 & n \geq 20 \end{cases}$

b) $x[n] = \begin{cases} 0 & n < 0 \\ \cos\left(\pi \frac{n}{8}\right) & n \geq 0 \end{cases}$

c) $x[n] = \begin{cases} 0 & n < 0 \\ \cos\left(7\pi \frac{n}{8}\right) & n \geq 0 \end{cases}$

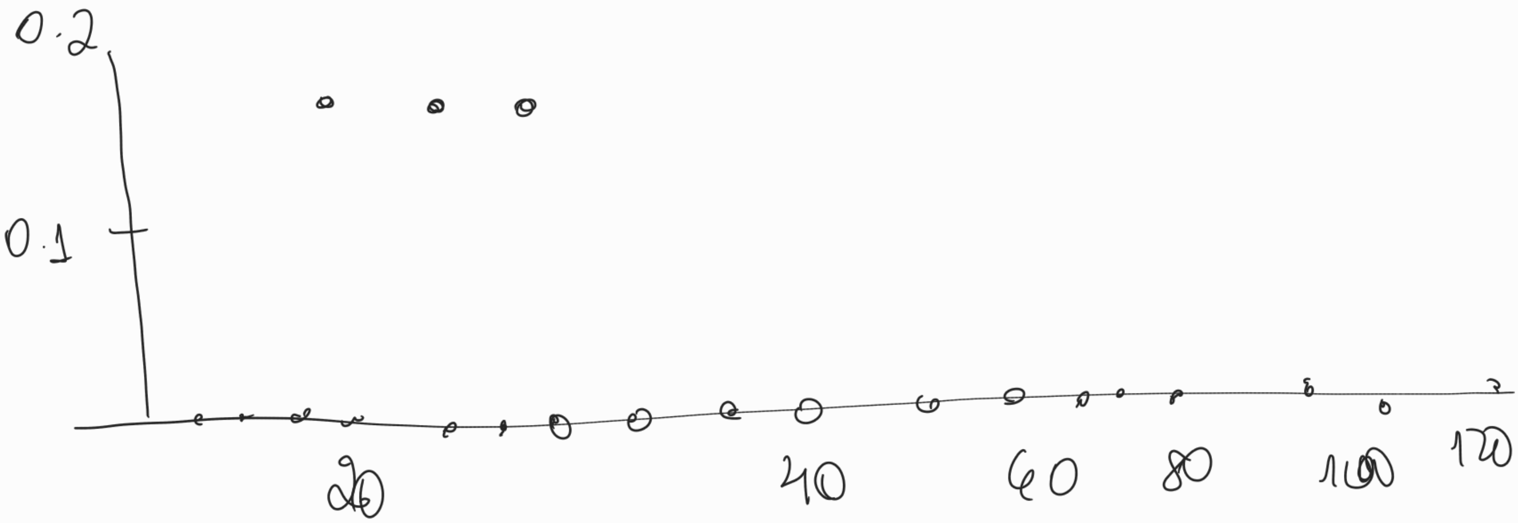
$$y[n] = \frac{1}{6} \{ x[n] + x[n-1] + \ldots + [n-5] \}$$



$$y[n] = \frac{1}{6} \{ x[n] - x[n-1] + x[n-2] - \ldots$$

$$x[\;\;\;-\;\;\;] \}$$

$= x[n-5]\}$



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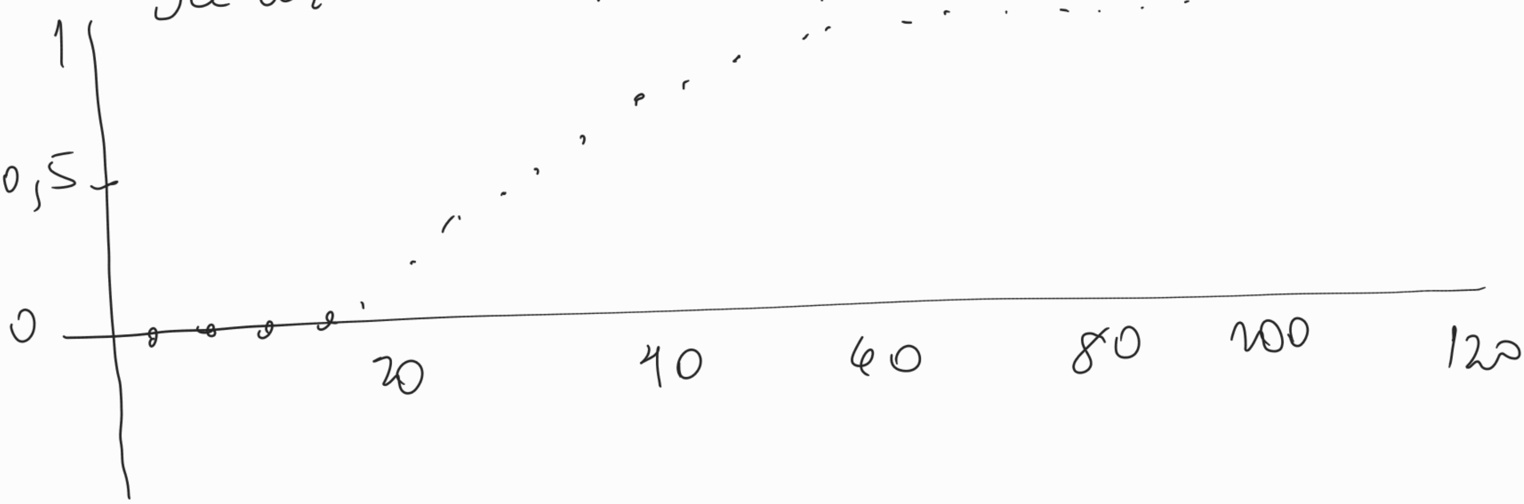
recursive low pass : step response



$$y[n] = -0.95\, y[n-1] + 0.05\, x[n]$$

recursive high pass : step response