Basic DSP

Signal notation

signal as function

signals describes how physical quantity varies over time and/or space.

Mathematically: a function of one or more independent variable

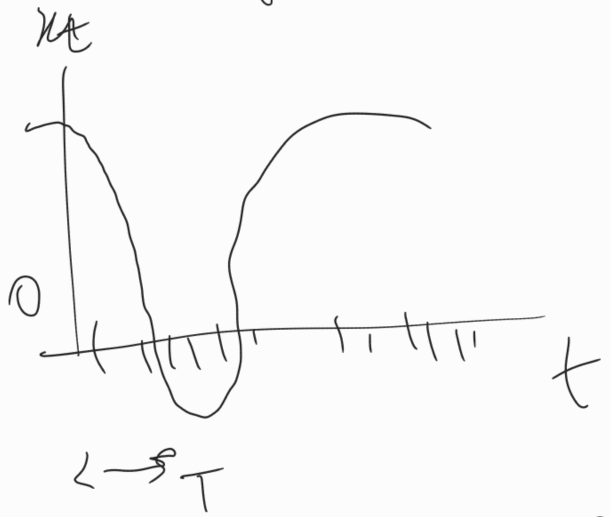
- Continuous and discrete Indep Vbls
  - continuous: take any values
  - discrete: in limited set of values (typically integers)

Sampling

Often we obtain a discrete-time signal $x[n]$ by sampling a continuous time signal $x(t)$

$$x[n] = x(t)\big|_{t=nT} = x(nT)$$

Sample at nT



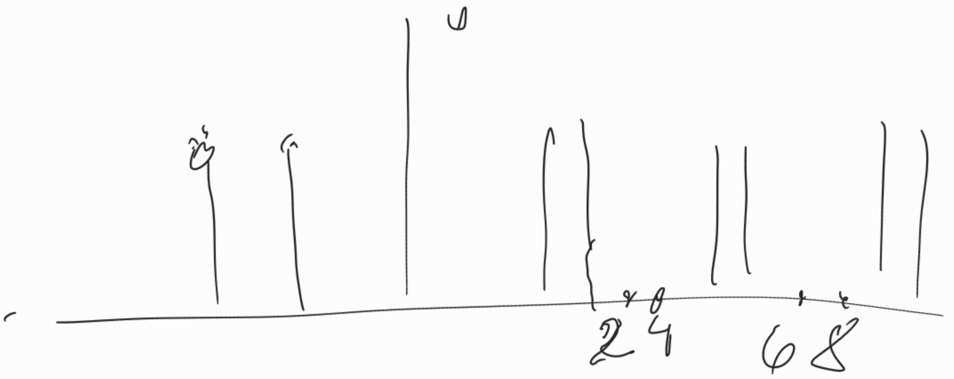
Displaying Signals

Signals stored in a computer must be discrete!

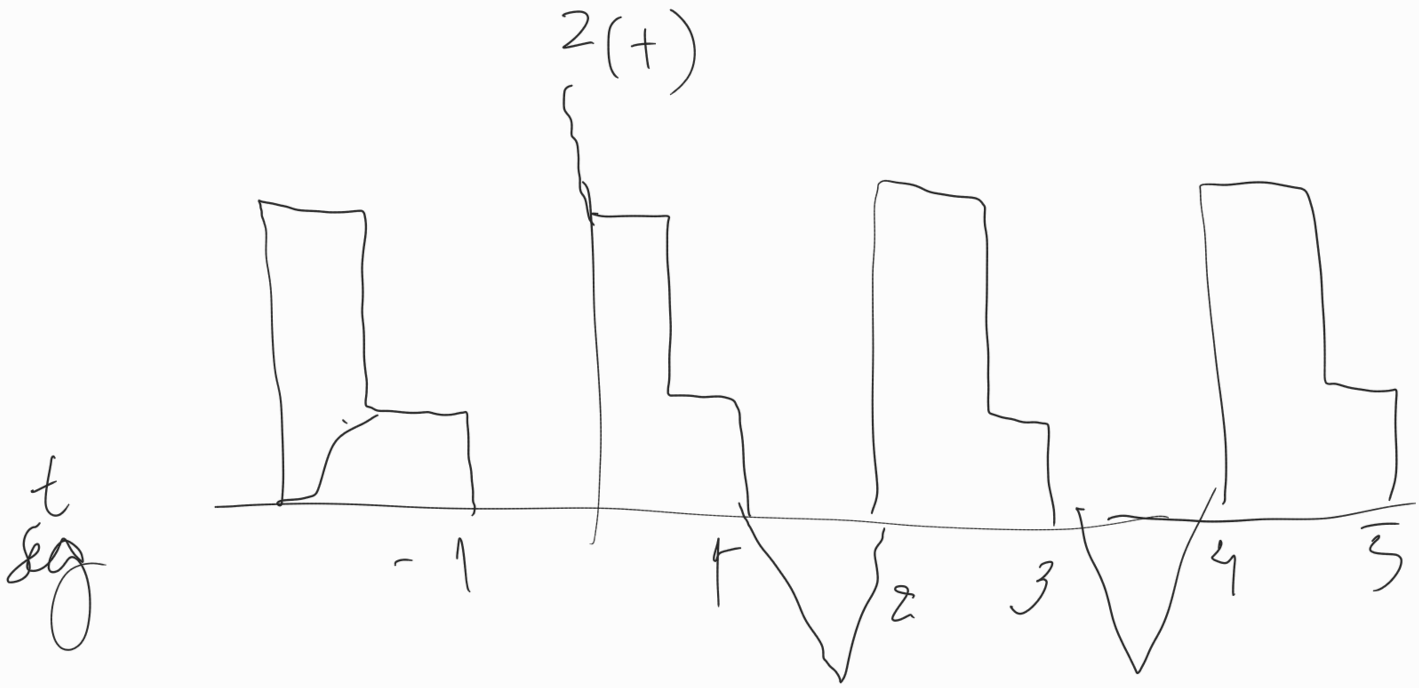
Common to connect samples with straight lines, visual clarity with large number

Periodicity

a signal that repeat a pattern is said to be periodic



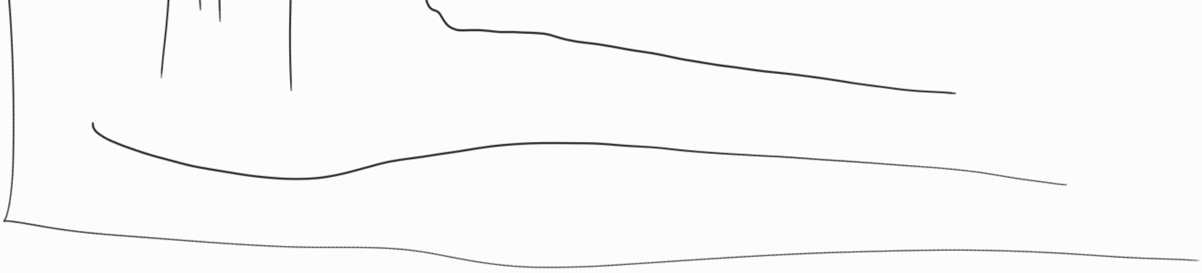
$N = 4, 8, 12 \dots$



Mathematically $\quad T_0 = 2, 4, 6 \dots$ sec

$$x(t+T_0) = x(t) \text{ for all } t$$

$$x[n+N] = x[n] \text{ for all } n$$



voice, saxophone, S sound

Summary:

treat signals as mathematical functions

independent variable can be continuous or discrete values.

Periodicity - does a signal repeat

Sinusoidal Signals

- sinusoids

continuous and discrete time

frequency

sinusoids are

occurs in nature:

light of a given color

microwaves

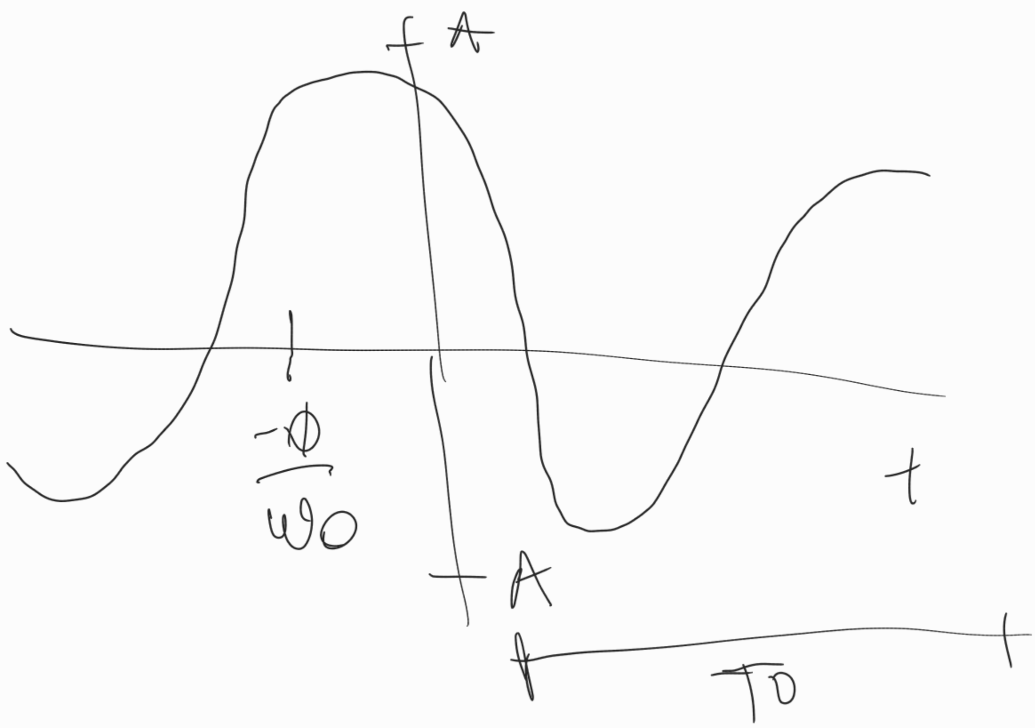
oscillatory motion: pendulum,

undamped spring-mass system

2. Sums of sinusoids can describe any signal

3. notion of frequency pervades our experience

$$x(t) = A\cos(\omega_0 t + \phi)$$



Amplitude: A

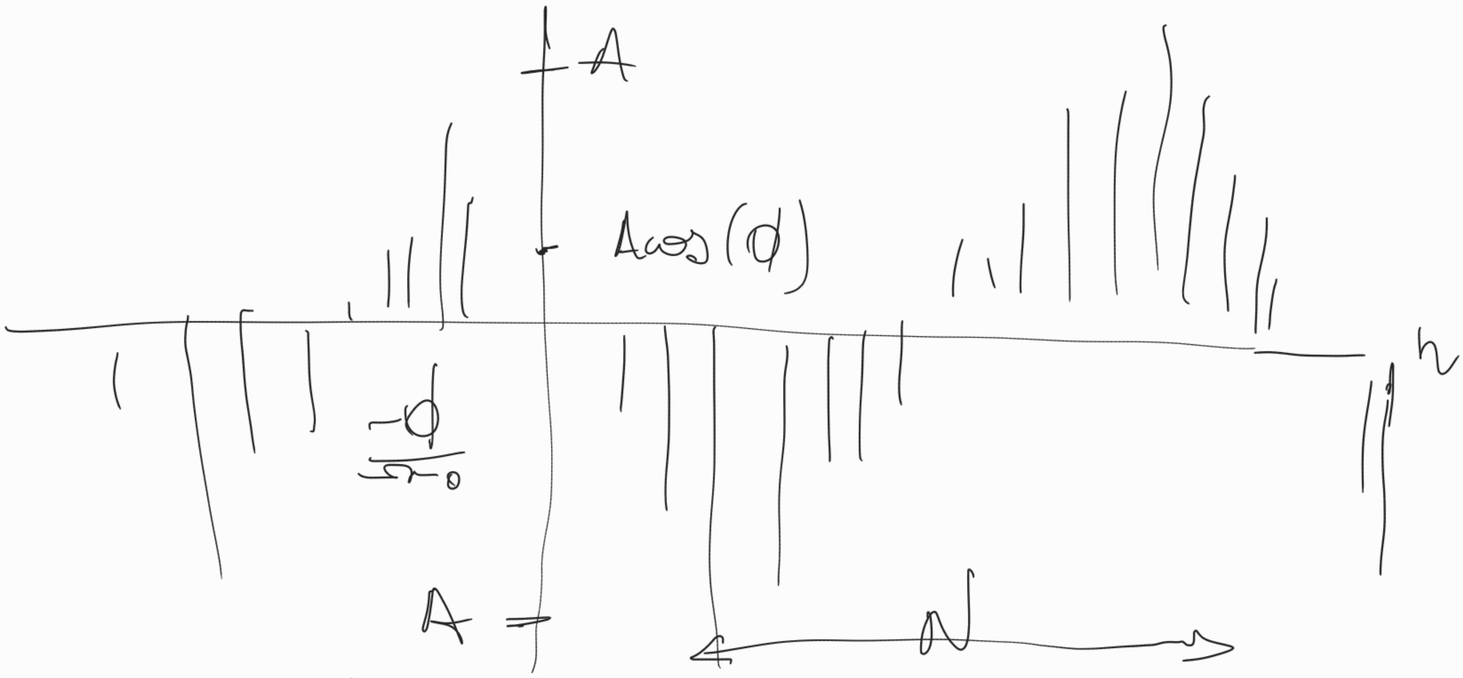
Phase Shift: ϕ

Frequency: ω_0

Period $T_0 = \frac{2\pi}{\omega_0}$

- Discrete-Time Sinusoids

$$x[n] = A\cos(\Omega_0 n + \phi)$$



Amplitude: A

Phase Shift: ϕ

Frequency: Ω_0

Period: $N = \frac{2\pi}{\Omega_0}$

Frequency

$\frac{1}{\text{period}}$

Period measured in seconds (CT)
or samples (DT)

Frequency measured in radians/time or cycles/time

CT Period: T_0 sec

$$f_0 = \frac{1}{T_0} \text{ cycles/sec or Hz}$$

DT Period: N samples

$$F_0 = \frac{1}{N} \text{ cycles/sample}$$

CT Period: T_0 sec | DT Period: N samples

$f_0 = \frac{1}{T_0}$ cycles/sec or Hz | $F_0 = \frac{1}{N}$ cycles/sample

$$w_0 = 2\pi f_0 = \frac{2\pi}{T_0} \text{ rads/sec}$$

$$\Omega_0 = 2\pi F_0 = \frac{2\pi}{N} \text{ rads/samples or rad}$$

$$\longleftrightarrow$$

2π rads / cycles

Relating continuous and
Discrete time Frequency

Discrete Time Frequency

- Force samples of CT sinusoid to equal DT sinusoid

$$x[n] = x(t)\Big|_{t=nT}$$

$$A\cos(\Omega n + \phi) = A\cos(\omega nT + \phi)$$
$$= A\cos((\omega T)n + \phi)$$

$$\boxed{\Omega = \omega T}$$

$$\text{rads} = \frac{\text{rads}}{\text{sec}}\ \text{sec}$$

$$\Omega = 2\pi f T$$

$$\text{rads} = \frac{\text{rads}}{\text{cycles}}\ \frac{\text{cycles}}{\text{sec}}\ \text{sec}$$

Uniqueness

Continuous time: different frequencies

$\Downarrow$

different sinusoids

If $\omega_1 \neq \omega_2$, then $\sin(\omega_1 t) \neq \sin(\omega_2 t)$

Discrete-time: shift frequency by $m2\pi \Rightarrow$ same sinusoid

If $\Omega_2 = \Omega_1 + m2\pi$

then $\sin(\Omega_1 n) = \sin(\Omega_2 n)$

$\sin((\Omega_1 + m\pi)n) = \sin(\Omega_1 n)\cos(m2\pi n) +$
$\cos(\Omega_1 n)\sin(m2\pi n)$

$= \sin(\Omega_1 n)$

- Compare two continuous and discrete time define

Summary - sinusoids

Sinusoidal signals are common and useful

frequency is inversely related to the period

Discrete and continuous time frequency are related through sampling

$\Omega = \omega T$

Discrete-time sinusoids are only unique over $0 < \Omega < 2\pi$

Exponential, Step and Impulse signals

1. Decay or growth : Exponentials
2. Steps - sudden change ; switch
3. Impulses - perturbation ; "kick the tires"

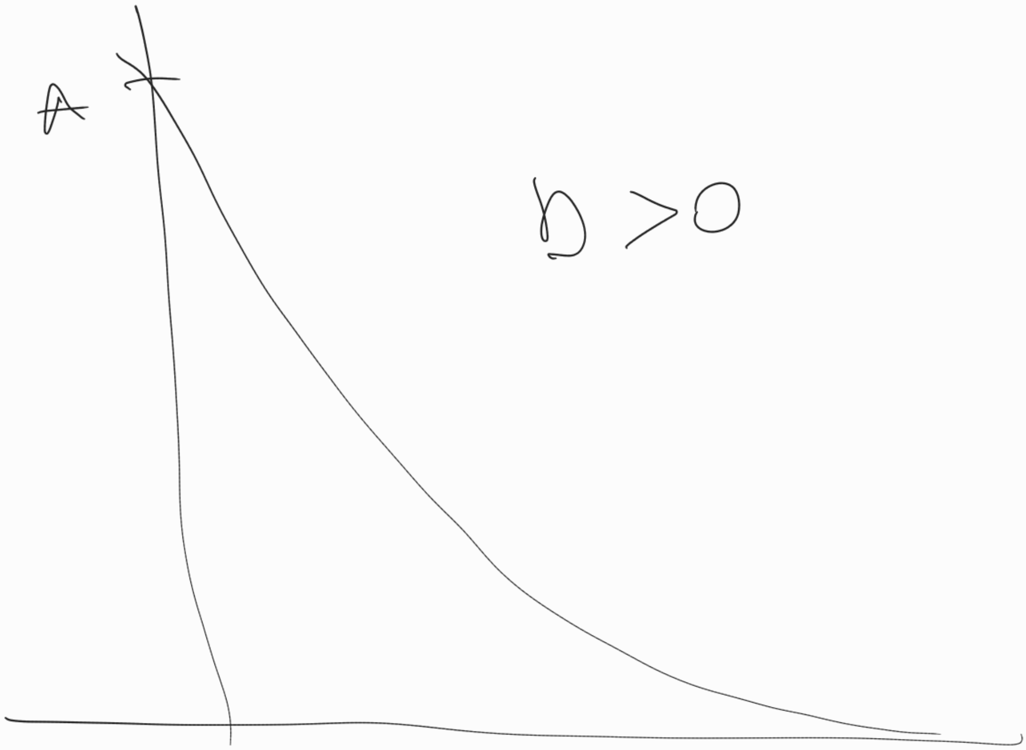
Contents

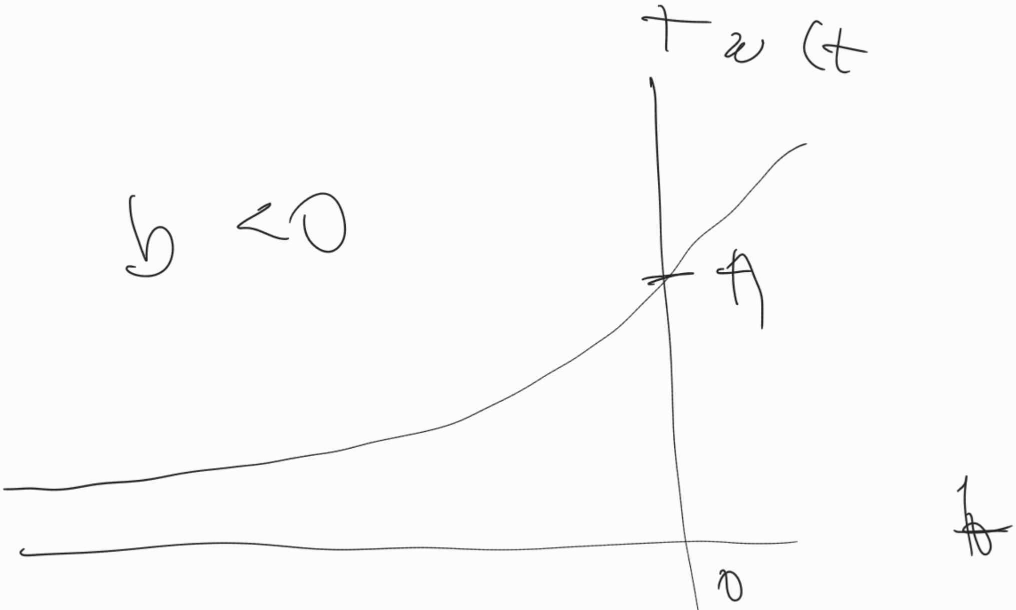
- exponentials
- exponentially damped sinusoids
- examples
- steps
- impulses

Continuous time Exponentials

$$x(t) = Ae^{-bt} \quad \text{or}$$

$$x(t) = A\exp\{-bt\}$$

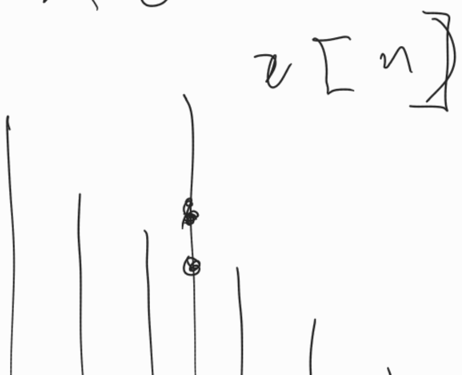




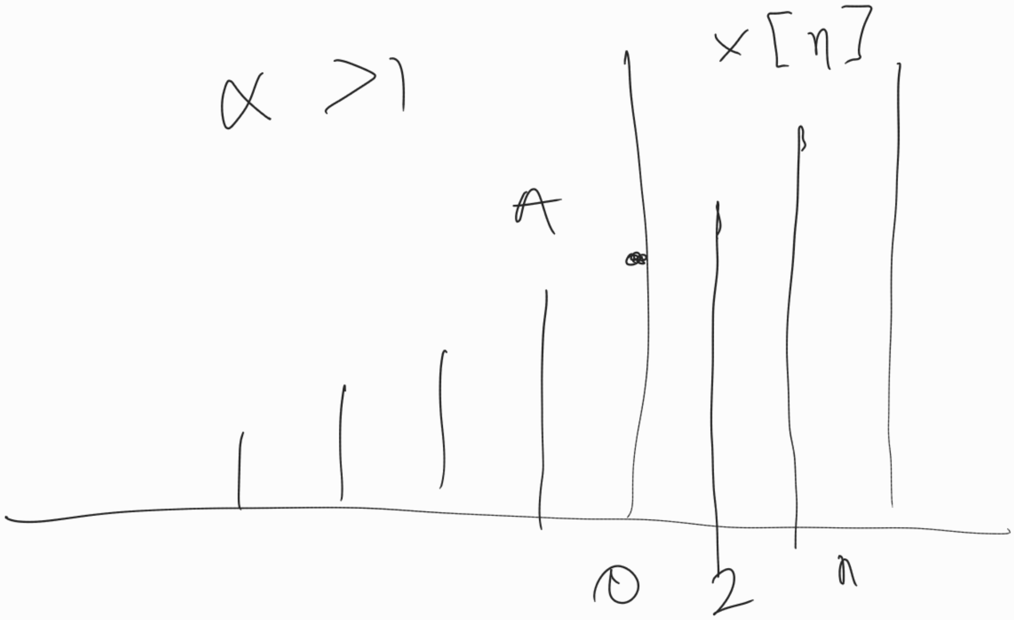
$$x[n] = Ae^{-bn} \text{ ; let}$$

$$\alpha = e^{-b}$$

$$= A\alpha^{n}$$

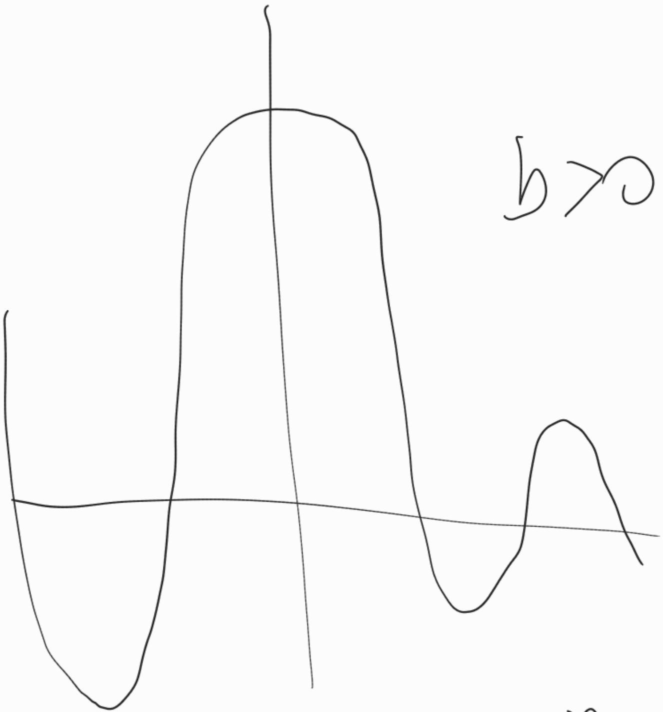


$0 < \alpha < 1$

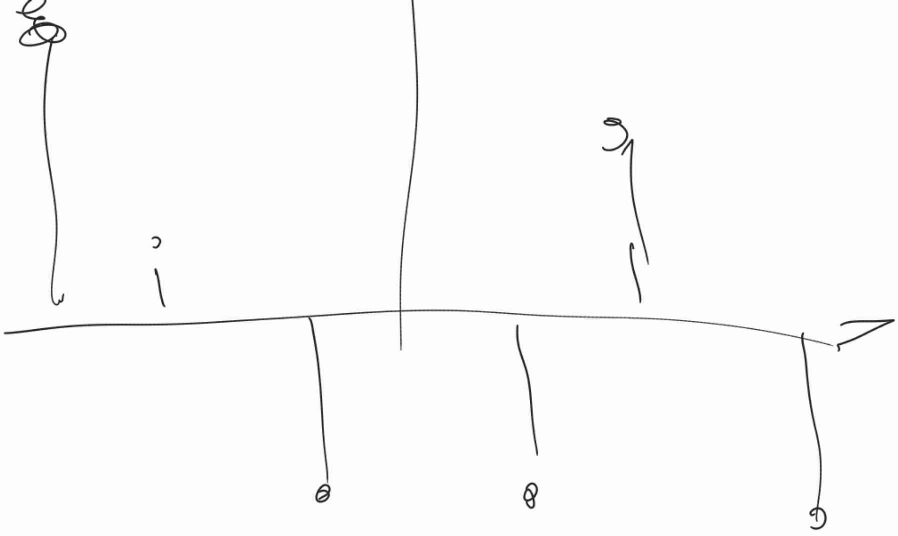


Exponentially Damped Sinusoids

$$x(t) = Ae^{-bt} \cos(\omega_0 t$$

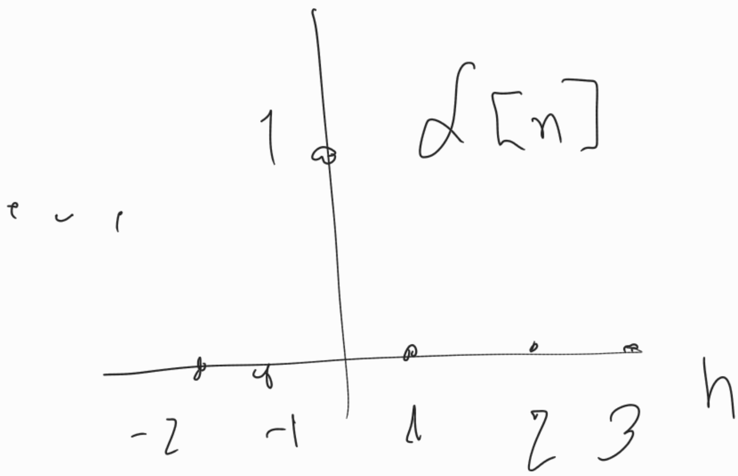


$$x[n] = A\alpha^n \cos(\Omega_0 n)$$



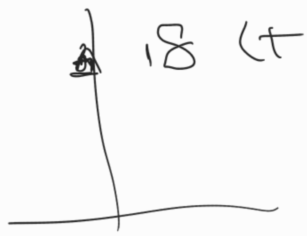
Impulses

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1 & n = 0 \end{cases}$$



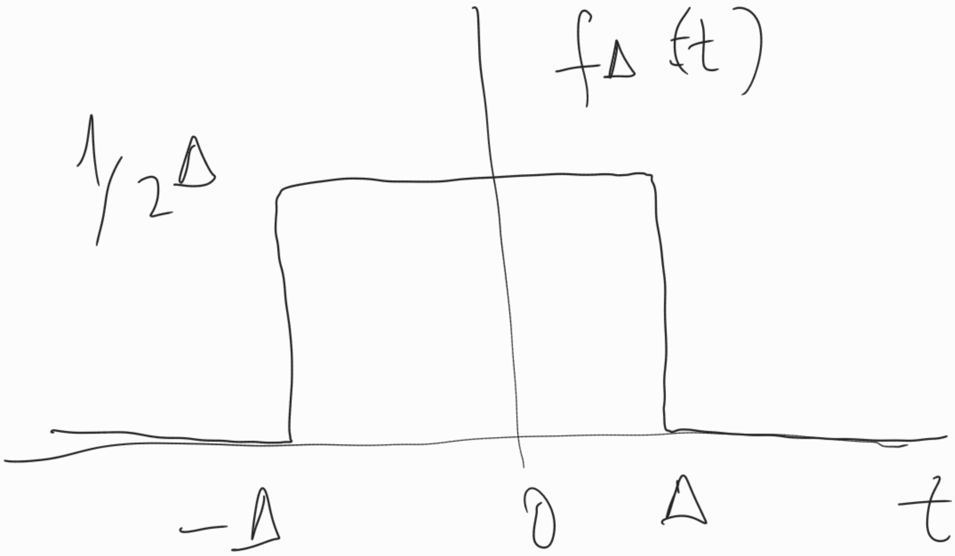
$$\delta[t] = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

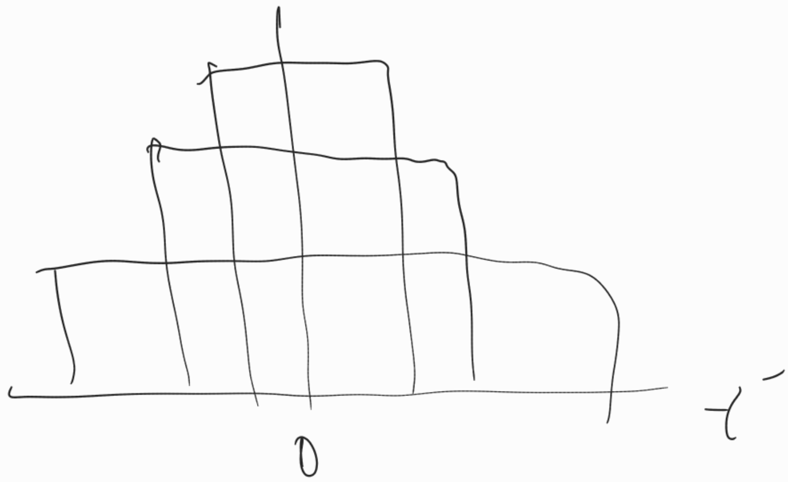


Continuous time Impulses

(continuous from jump)



$$\delta(t) \sim \lim_{\Delta \to 0} f_\Delta(t)$$



- generalized functions

$$\int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt = f(t_0$$

only meaningful math within integral

sifting property

...entals natural growth/(

exponentielles (de cong)

e x po