Frequency response: how system responds to sinusoids of differente frequencies

$$x[n] = e^{j\omega n} \longrightarrow \boxed{H} \longrightarrow y[n] = H(e^{j\omega})\, e^{j\omega n}$$

freq response - independent of n

Multiplication!

$$H(e^{j\omega}) = |H(e^{j\omega})|\, e^{j \measuredangle H(e^{j\omega})} \Rightarrow$$

$$y[n] = |H(e^{j\omega})|\, e^{j \measuredangle H(e^{j\omega})}\, e^{j\omega n}$$

$$= |H(e^{j\omega})|\, e^{j(\omega n + \measuredangle H(e^{j\omega}))}$$

- Amplitude of $e^{j\omega n}$ changed by $|H(e^{j\omega})|$ - magnitude response
- Phase of $e^{j\omega n}$ changed by $\measuredangle H(e^{j\omega})$ -

Phase response.

By linearity, if

$$x[n] = \alpha_1 e^{j\omega_1 n} + \alpha_2 e^{j\omega_2 n} \Rightarrow$$

$$y[n] = \alpha_1 H(e^{j\omega_1}) e^{j\omega_1 n} + \alpha_2 H(e^{j\omega_2}) e^{j\omega_2 n}$$

Relationship to impulse response

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] =$$

$$\sum_{k=-\infty}^{\infty} h[k] e^{j\omega(n-k)} = e^{j\omega} \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}$$