

Basic DSP

Signal notation

signal as function

signals describe how physical quantity varies over time and/or space.

- Mathematically: a function of one or more independent variables
- continuous and discrete Indep Vbls
 - continuous: take any values
 - discrete: in limited set of values (typically integers)

Sampling

Often we obtain a discrete-time signal $x[n]$ by sampling a continuous time signal $x(t)$

$$x[n] = x(t) \Big|_{t=nT} = x(nT)$$

Sample at nT

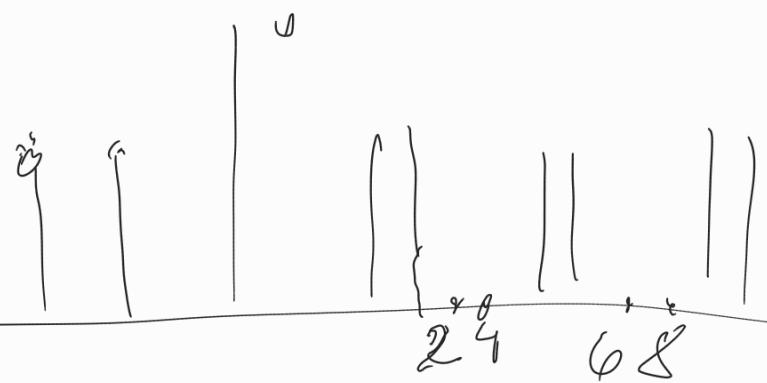


Displaying Signals

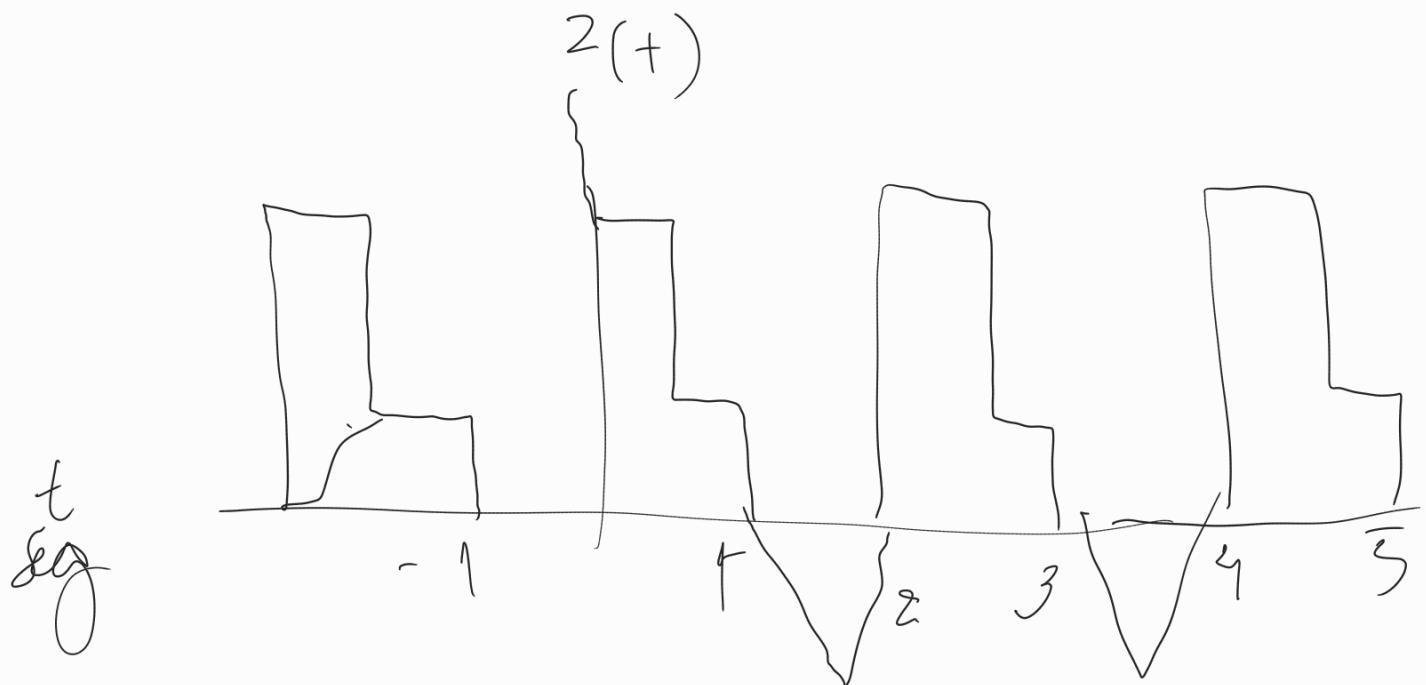
Signals stored in a computer must be discrete! Common to connect samples with straight lines, visual clarity with large number

Periodicity

a signal that repeat a pattern
is said to be periodic



$$n = 4, 8, 12 \dots$$



Mathematically

$$T_0 = 2, 4, 6 \dots \text{sec}$$

$$x(t+T_0) = x(t) \text{ for all } t$$

$$x[n+N] = x[n] \text{ for all } n$$



Music, Saxophone, Sound

Summary:

treat signals as mathematical functions
independent variable can be continuous
or discrete values

Periodicity - does a signal repeat

Sinusoidal Signals

• Sinusoids
continuous and discrete time

frequencies

sine waves

occurs in nature

light, sound, etc.

light of a given wave

micro waves

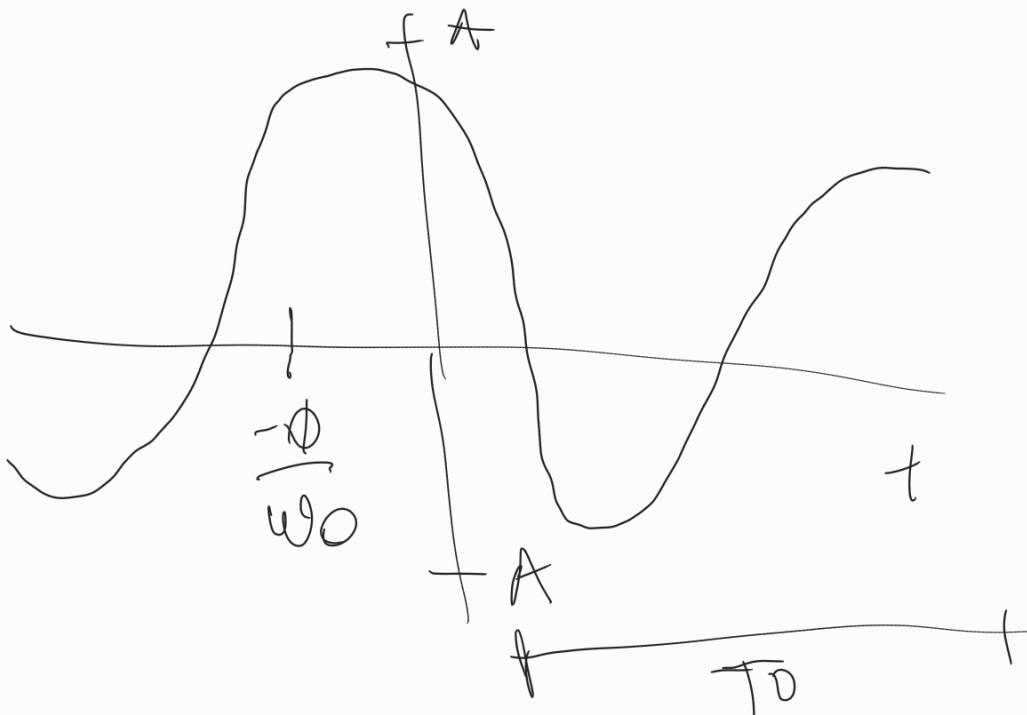
oscillatory motion: pendulum

undamped spring-mass system

2. Sums of sinusoids can describe any signal

3. notion of frequency per wave or crepline

$$x(t) = A \cos(\omega_0 t + \phi)$$



Amplitude: A

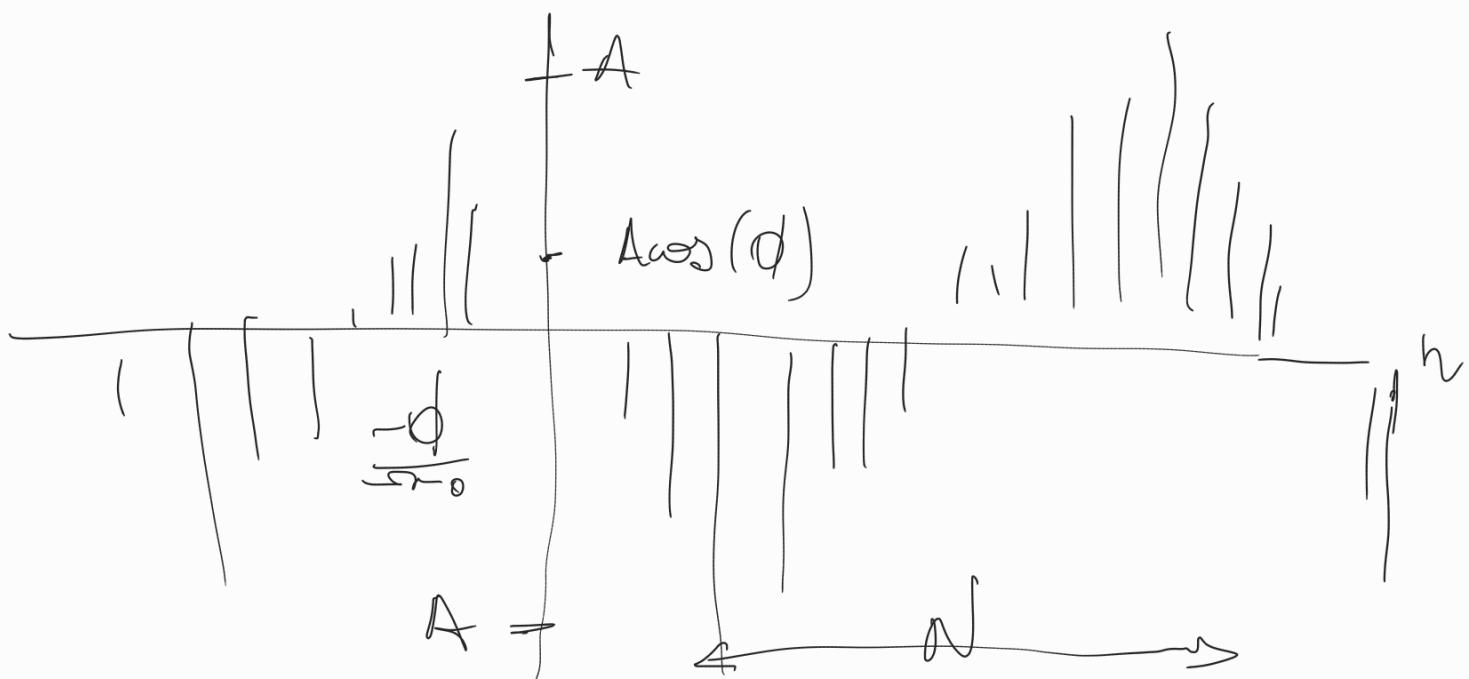
Phase Shift: ϕ

Frequency: ω_0

$$\text{Period } T_0 = \frac{2\pi}{\omega_0}$$

• Discrete-Time Sine waves

$$x[n] = A \cos(\Omega_0 n + \phi)$$



Amplitude: A

Phase Shift: ϕ

Frequency: Ω_0

$$\text{Period: } N = \frac{2\pi}{\Omega_0}$$

Frequency

\downarrow
period

(CT)

Period measured in seconds (CT)
or samples (DT)

Frequency measured in radians/
Time or cycles/time

CT Period : T_0 sec

$$f_0 = \frac{1}{T_0} \text{ cycles/sec or Hz}$$

DT Period : N samples

$$F_0 = \frac{1}{N} \text{ cycles/sample}$$

CT Period : T_0 sec

$$f_0 = \frac{1}{T_0} \text{ cycles/sec or Hz}$$

DT Period : N samples

$$F_0 = \frac{1}{N} \text{ cycles/sample}$$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \text{ rads/sec}$$

$$\Omega_0 = 2\pi F_0 = \frac{2\pi}{N} \frac{\text{rads}}{\text{sample}}$$

or
rad

$$\overbrace{\quad}^{2\pi \text{ rads/cycles}}$$

Relating continuous and

Digital to a \sqrt{N} factor

Discrete Time Frequency

For samples of CT sinusoid to equal DT sinusoid

$$x[n] = x(t) \Big| t=nT$$

$$\begin{aligned} A \cos(\Omega_n + \phi) &= A \cos(\omega_n T + \phi) \\ &= A \cos((\omega T)_n + \phi) \end{aligned}$$

$$\boxed{\Omega_n = \omega T}$$

$$\text{rads} = \frac{\text{rad s}}{\text{sec}}$$

$$\Omega = 2\pi f T$$

$$\text{rad s} = \frac{\text{rads}}{\text{cycles}} \frac{\text{cy des}}{\text{sec}}$$

Uniqueness

Continuous time: different frequencies



Different sinusoids

If $\omega_1 \neq \omega_2$, then $\sin(\omega_1 t) \neq \sin(\omega_2 t)$

Discrete-time: shift frequency
by $m 2\pi \Rightarrow$ same sinusoid

$$\text{if } \Omega_2 = \Omega_1 + m 2\pi$$

$$\text{then } \sin(\Omega_2 n) = \sin(\Omega_1 n)$$

$$\sin((\Omega_1 + m\pi)n) = \sin(\Omega_1 n) \cos(m 2\pi) + \\ \cos(\Omega_1 n) \sin(m 2\pi n)$$

$$= \sin(\Omega_1 n)$$

o Compare two continuous and discrete time sine from

Summary - sinusoids

Sinusoidal signals are common and useful

frequency is inversely related to the period

Discrete and continuous time frequency are related through sampling

$$\Omega = \omega T$$

Discrete-time sinusoids are only unique over $0 < \Omega < 2\pi$

Exponential, Step and Impul~~s~~ signals

- 1 - Decay or growth : Exponentials
2. Steps - sudden change ; switch
3. Impulses - perturbation, "kick the tires"

Contents

- exponentials
- exponentially damped sinusoid

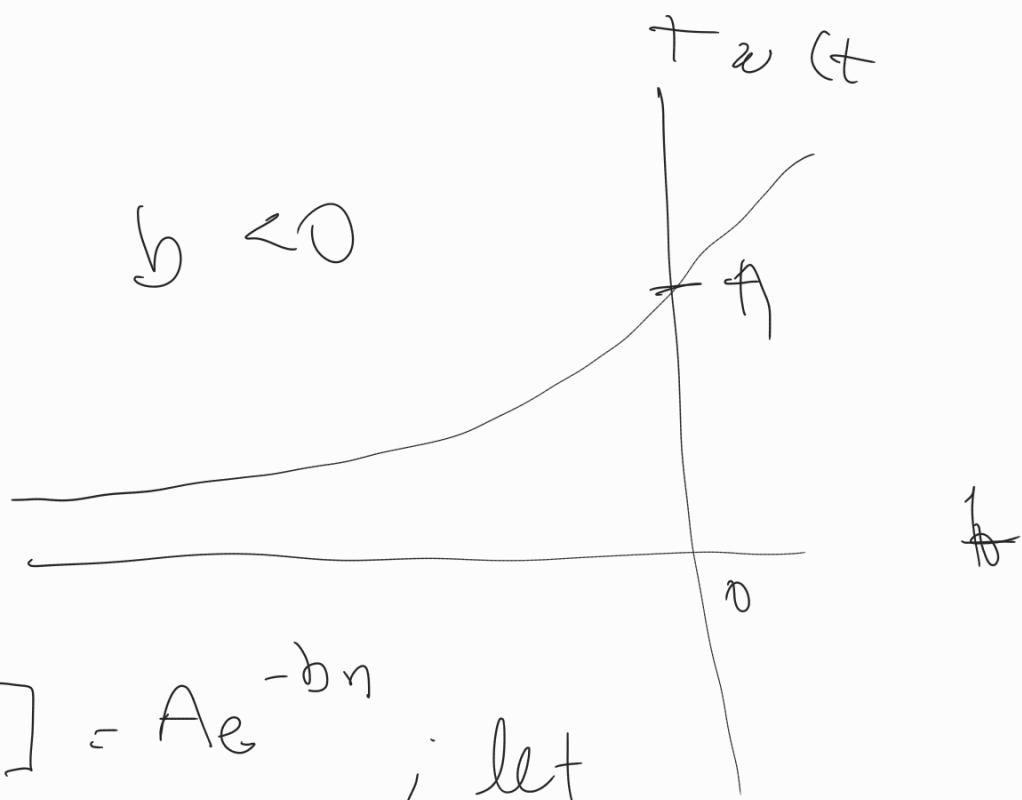
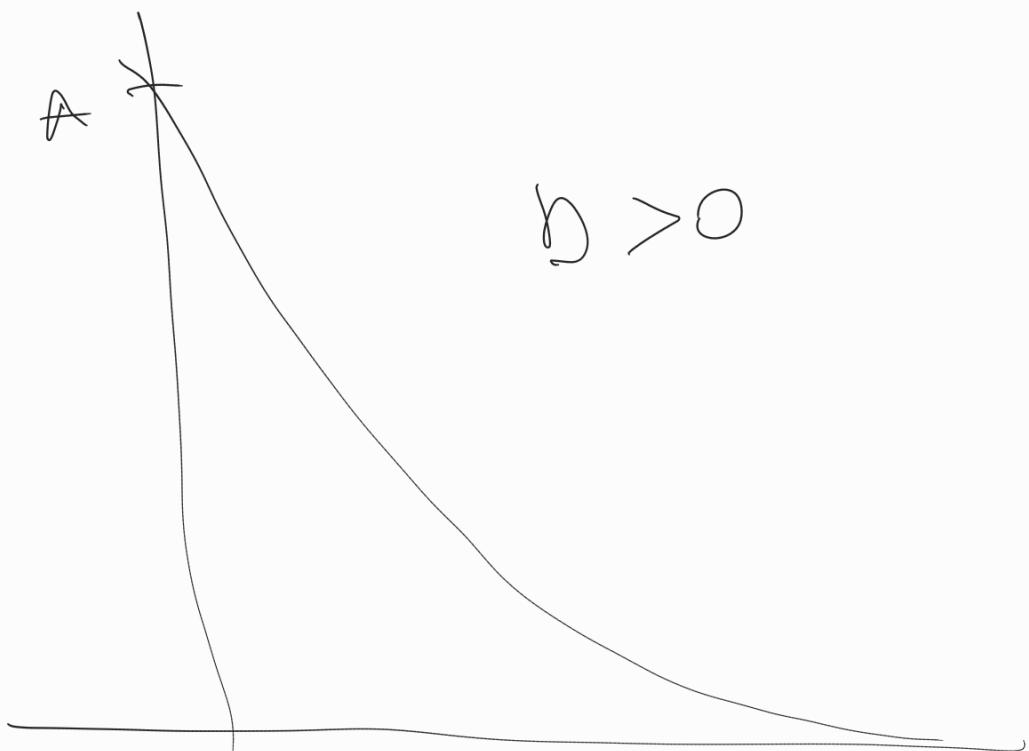
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- examples
- steps
- impulses

Continuous Time Exponentials

$$x(t) = A e^{-bt} \quad \text{or}$$

$$x(t) = A \exp\{-bt\}$$

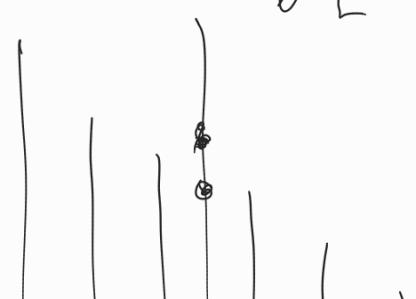


$$x[n] = Ae^{-bn} \quad ; \text{ let}$$

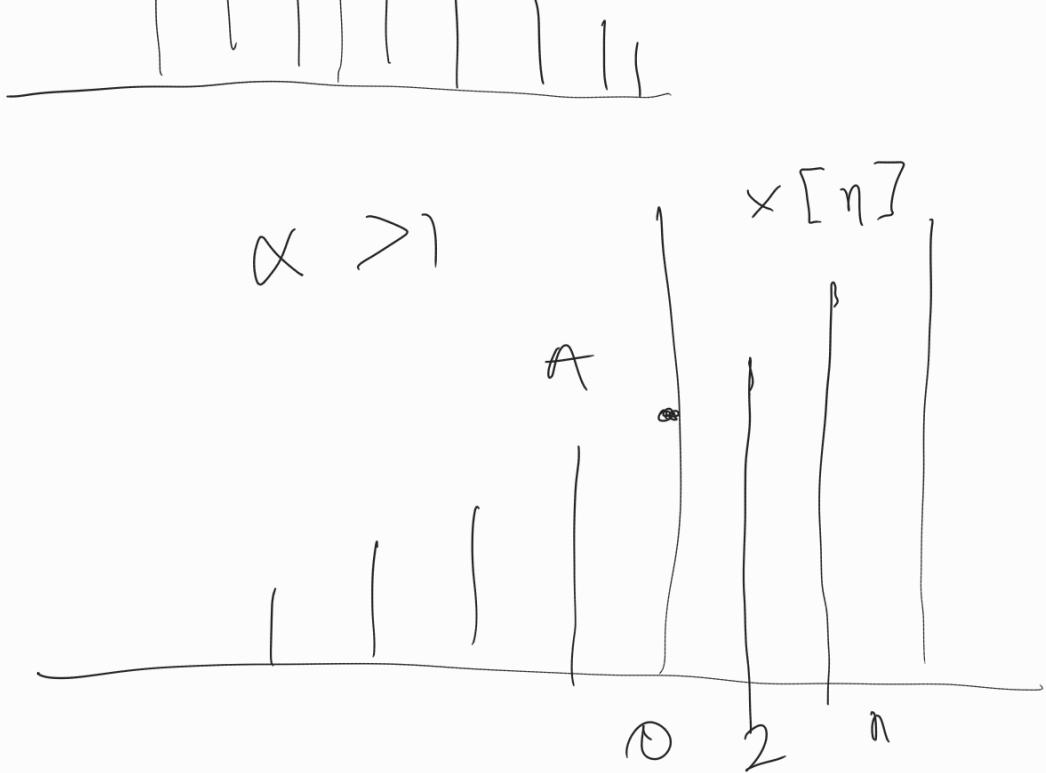
$$\alpha = e^{-b}$$

$$= A\alpha^n$$

$$x[n]$$

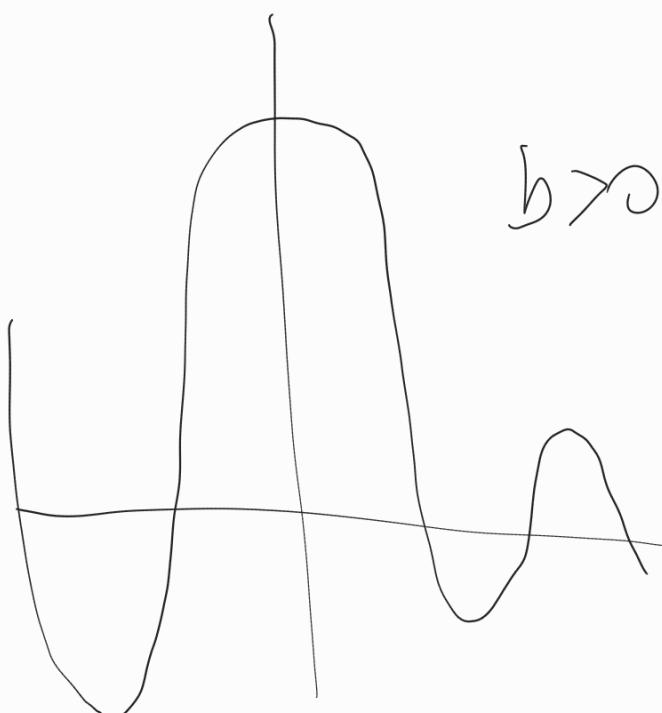


$$0 < \alpha < 1$$



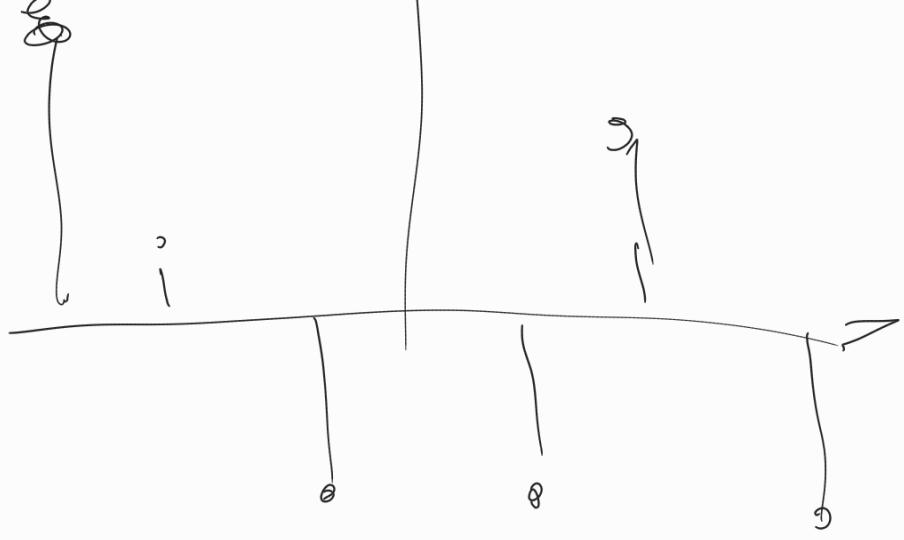
Exponentially Damped Sinusoids

$$x(t) = A e^{-bt} \cos(\omega_0 t)$$



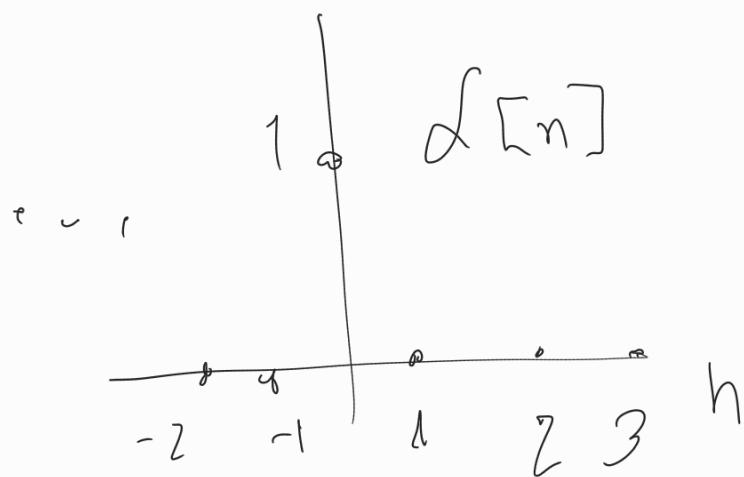
$$x[n] = A \alpha^n \cos(\Omega_0 n)$$

↑



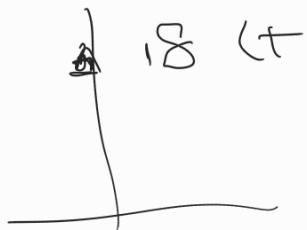
Impulses

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

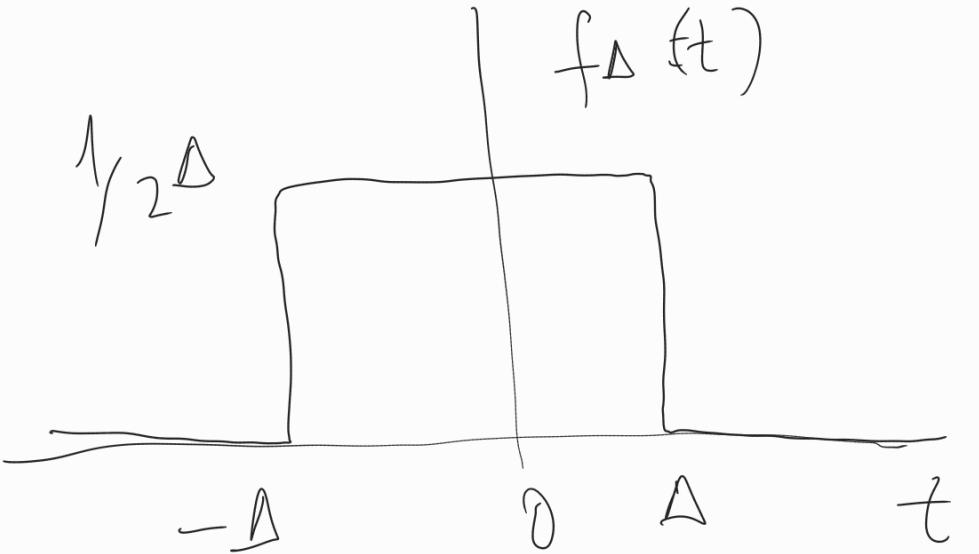


$$\delta[t] = 0 \quad t \neq 0$$

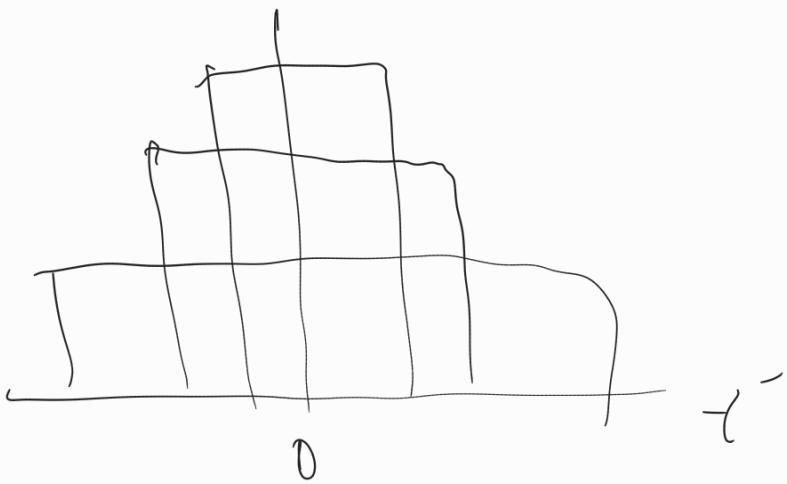
$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$



Response at time t_0 is $\int_{-\infty}^{\infty} h(t) \delta(t - t_0) dt$



$$S(t) \sim \lim_{\Delta \rightarrow 0} f_\Delta(+)$$



- generalized functions

$$\int_{-\infty}^{\infty} f(t) S(t-t_0) dt = f(t_0)$$

only meaningful math within
integral
filtering property

natural growth /

exponentes (de cosy')

el X po

