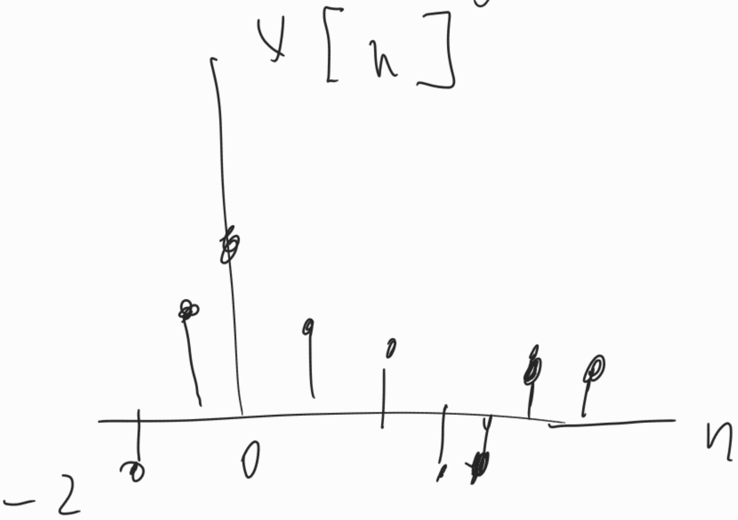
LTI Systems

System: maps an input signal to an output signal

$x(t)$ or $x[n]$ $\longrightarrow$ $\boxed{H}$ $\longrightarrow$ $y(t)$ or $y[n]$

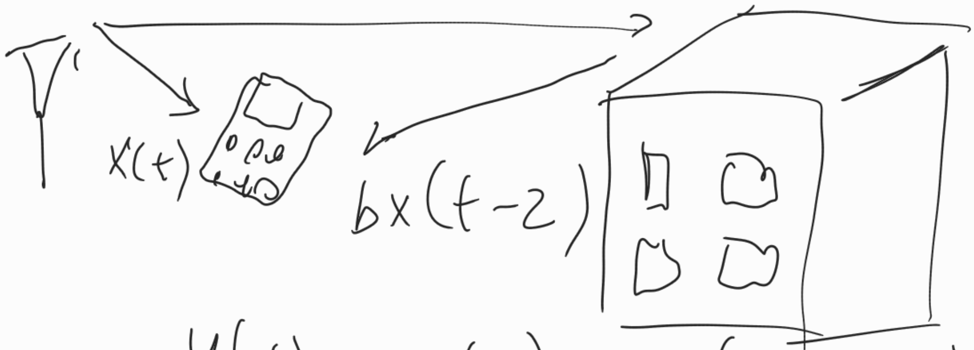
continuous signal

Discrete time signal



system 1) Model a physical phenomenon

2) implement desired characteristic



$y(t) = x(t) + bx(t-\tau)$

$$z(t) = y(t) - by(t-\tau) + b^2 y(t-2\tau) - b^3 y(t-3\tau) + b^4 y(t-4\tau)$$

$\vdots$

$x(t^4) + \cancel{bx(t-\tau)}$

$- \cancel{bx(t-\tau)} - b^2 x(\cancel{t-2\tau})$

$$+ b^2 \cancel{x(t-2\tau)} + b^3 \cancel{x(t-3\tau)}$$
$$- \cancel{b^3 x(t-3\tau)} - b^4 \cancel{x(t-4\tau)}$$

$$\vdots$$

Linear System

Superposition holds : Sum of inputs
$\Rightarrow$ sum of outputs

$$x_1[n] \rightarrow \boxed{H} \rightarrow y_1[n]$$
$$x_2[n] \rightarrow \boxed{H} \rightarrow y_2[n]$$
$$a x_1[n] + b x_2[n] \longrightarrow \boxed{H} \longrightarrow a y_1[n] + b y_2[n]$$

Time Invariance

System responds the same now it does later

$$x[n] \longrightarrow \boxed{H} \longrightarrow y[n]$$

$$\Downarrow$$

$x[n-n_0] \longrightarrow \boxed{H} \longrightarrow y[n-n_0]$

Limit LTI

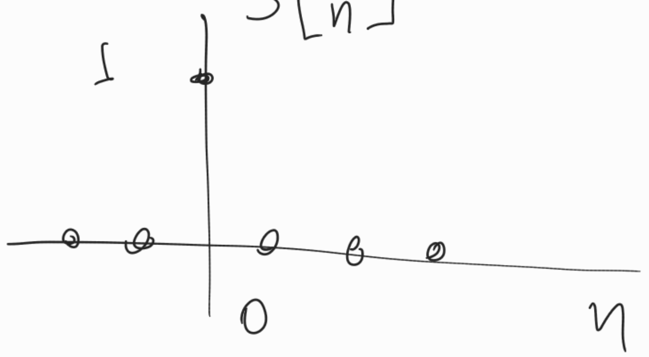
Satisfies both

Intere variant system

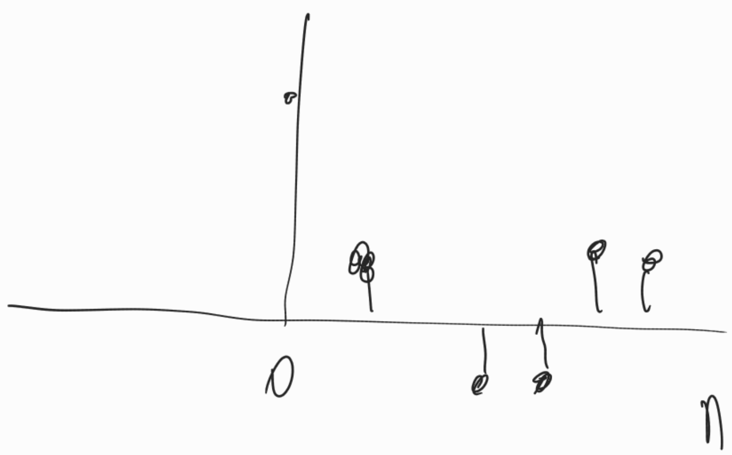
IVO for LTI systems

impulses response tells all

$\delta[n]$



$\delta[n] \to \boxed{H} \to h[n]$



$x[n]$ input $\Rightarrow$

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

∞

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$= h[n] * x[n]$$

causal before input starts

$h[n] < 0 \quad n < 0$

Stable

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

Difference Equations

Important class of LTI systems

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x(n-k)$$

1) model physical systems
2) design filters
3) implement (compute) filter

$y[n] - 1/2\, y[n-1] = x[n]$

$\rightarrow \quad y[n] = \frac{1}{2} y[n-1] + x[n]$

impulse response $\quad x[n] = \delta[n]$

$y[-1] = 0$

$y[0] = \frac{1}{2} \cdot 0 + 1 = 1$

$y[1] = \frac{1}{2} \cdot 1 + 0 = \frac{1}{2}$

$y[2] = \frac{1}{2} \cdot \frac{1}{2} + 0 = \frac{1}{4}$

$\vdots$