$$V[\hat{\beta}] = V[Cy] = C\underbrace{V[y]}_{\sigma^2}C^T$$

$$V[\hat{\beta}] = CC^T\sigma^2$$

$$V[\hat{\beta}] = ((X^TX)^{-1}X^T + D)((X^TX)^{-1}X^T + D)^T\sigma^2$$

$$V[\hat{\beta}] = ((X^TX)^{-1}X^T + D)(X(X^TX)^{-1} + D^T)\sigma^2$$

$$V[\hat{\beta}] = \underbrace{(X^TX)^{-1}\sigma^2}_{V[\hat{\beta}_{LS}]} + DD^T\sigma^2$$

$$\boxed{V[\hat{\beta}] = V[\hat{\beta}_{LS}] + \sigma^2 DD^T}$$

$V[\hat{\beta}] > V[\hat{\beta}_{LS}]$

In other words, any other unbiased estimator must have a larger variance than the least square estimator $\hat{\beta}_{LS}$