HEAT-BATH RANDOM WALKS WITH MARKOV BASES

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ABSTRACT. Graphs on lattice points are studied whose edges come from a finite set of allowed moves of arbitrary length. We show that the diameter of these graphs on fibers of a fixed integer matrix can be bounded from above by a constant. We then study the mixing behaviour of heat-bath random walks on these graphs. We also state explicit conditions on the set of moves so that the heat-bath random walk, a generalization of the Glauber dynamics, is an expander in fixed dimension.

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1. INTRODUCTION

A *fiber graph* is a graph on the finitely many lattice points $\mathcal{F} \subset \mathbb{Z}^d$ of a polytope where two lattice points are connected by an edge if their difference lies in a finite set of allowed moves $\mathcal{M} \subset \mathbb{Z}^d$. The implicit structure of these graphs makes them a useful tool to explore the set of lattice points randomly: At the current lattice point $u \in \mathcal{F}$, an element $m \in \pm\mathcal{M}$ is sampled and the random walk moves along m if $u + m \in \mathcal{F}$ and stays at u otherwise. The corresponding Markov chain is irreducible if the underlying fiber graph is connected and the set $\mathcal{M}$ is called a *Markov basis* for $\mathcal{F}$ in this case. This paper investigates the *heat-bath* version of this random walk: At the current lattice point $u \in \mathcal{F}$, we sample $m \in \mathcal{M}$ and move to a random element in the integer ray $(u + \mathbb{Z} \cdot m) \cap \mathcal{F}$. The authors of [6] discovered that this random walk can be seen as a discrete version of the *hit-and-run* algorithm [15, 26, 16] that has been used frequently to sample from all the points of a polytope – not only from its lattice points. The popularity of the continuous version of the hit-and-run algorithm has not spread to its discrete analog, and not much is known about its mixing behaviour. One reason is that it is already challenging to guarantee that all points in the underlying set $\mathcal{F}$ can be

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