

GM assumptions: $E[e] = 0$, $V[e] = \sigma^2$ and $\text{Cov}(e_1, e_2) = 0$

Let $\hat{\beta} = Cy$ where $C = (x^T x)^{-1} x^T + D$

$$E[\hat{\beta}] = E[Cy] = E[(x^T x)^{-1} x^T + D](\beta x + e)$$

$$E[\hat{\beta}] = ((x^T x)^{-1} x^T + D) \times \beta + ((x^T x)^{-1} x^T + D) E[e]$$

$$E[\hat{\beta}] = ((x^T x)^{-1} x^T + D) \times \beta$$

$$E[\hat{\beta}] = \underbrace{(x^T x)^{-1} x^T x}_I \beta + D \times \beta$$

$$E[\hat{\beta}] = (I + Dx) \beta$$

$\hat{\beta}$ is only an unbiased estimate of β if

$$Dx = 0_{//}$$