

(1)

$$q_{ii}(t) = - \sum_{j \in S} q_{ij}(t)$$

(4)

$$p_{ij}(t, t+\Delta t) = q_{ij}(t) \Delta t$$

from 3:

$$q_{ii}(t) + \sum_j \frac{p_{ij}(t, t+\Delta t)}{\Delta t} = 0$$

$$\sum_j p_{ij}(t, t+\Delta t) = -q_{ii}(t) \Delta t$$

3 → 2

$$p_{ii}(t, t+\Delta t) - q_{ii}(t) \Delta t = 1$$

$$p_{ii}(t, t+\Delta t) = 1 + q_{ii}(t) \Delta t$$

(5)

(2)

$$p_{ii}(t) + \sum p_{ij}(t) = 1 \quad q_{ii}(t) + \sum q_{ij}(t) = 0$$

$$P = \begin{bmatrix} \textcircled{*} & * & * & \dots & * \\ p_{ii} & \sum p_{ij} & & & \end{bmatrix}$$

(3)

$$Q = \begin{bmatrix} \textcircled{*} & * & * & \dots & * \\ q_{ii} & \sum q_{ij} & & & \end{bmatrix}$$

$$Q = \begin{bmatrix} \frac{p_{ii}(t, t+\Delta t) - 1}{\Delta t} & \dots & \frac{p_{ij}(t, t+\Delta t)}{\Delta t} \\ \vdots & \ddots & \vdots \\ * & * & * \end{bmatrix}$$

$q_{ii}$                        $q_{ij}$

$$Q(t) = \lim_{\Delta t \rightarrow 0} \left\{ \frac{P(t, t+\Delta t) - I}{\Delta t} \right\}$$