

$$P_m = \rho^m P_0$$

$$\sum_{m=0}^{\infty} P_m = 1$$

$$\rho < 1$$

P_0 in terms of λ and μ :

$E[N]$:

$$\sum_{m=0}^{\infty} P_m = 1$$

$$\sum_{m=0}^{\infty} \rho^m P_0 = 1$$

$$P_0 = \left(\sum_{m=0}^{\infty} \rho^m \right)^{-1}$$

$$P_0 = \left(1 + \underbrace{\rho + \rho^2 + \dots + \rho^m}_{GP} \right)^{-1}$$

$$P_0 = \left(\frac{1}{1-\rho} \right)^{-1} = 1 - \rho$$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$N = \sum_{m=0}^{\infty} m P_m = \sum_{m=0}^{\infty} m \rho^m P_0$$

$$N = \underbrace{(1-\rho)}_{P_0} \sum_{m=0}^{\infty} m \rho^{m-1}$$

$$N = (1-\rho) \rho \sum_{m=0}^{\infty} m \rho^{m-1}$$

$$N = (1-\rho) \rho \frac{d}{d\rho} \underbrace{\sum_{m=0}^{\infty} \rho^m}_{GP}$$

$$N = (1-\rho) \rho \frac{d}{d\rho} \left(\frac{1}{1-\rho} \right)$$

$$N = \cancel{(1-\rho)} \rho \frac{1}{(\cancel{1-\rho})^2} = \frac{\rho}{(1-\rho)}$$

$$N = \frac{\lambda}{\mu - \lambda}$$