$$\boxed{\hat{\beta} = (X^TX)^{-1}X^TY} \qquad \boxed{Y = \beta X + e}$$

* $E[\hat{\beta}]$:

$$E[\hat{\beta}] = E[(X^TX)^{-1}X^TY]$$
$$= \underbrace{(X^TX)^{-1}X^TX}_{I}\beta + (X^TX)^{-1}X^T\cancelto{0}{E[e]}$$

$$\boxed{E[\hat{\beta}] = I\beta}$$

* $V[\hat{\beta}]$:

$$V[\hat{\beta}] = V[\overbrace{(X^TX)^{-1}X^T}^{A}Y]$$
$$= \underbrace{\cancel{(X^TX)^{-1}}\,\cancel{(X^TX)}}_{I}(X^TX)^{-1}V[\cancelto{\sigma^2}{Y}]$$

$$\boxed{V[\hat{\beta}] = (X^TX)^{-1}\sigma^2}$$

$$V[\overbrace{AX}^{Y}] = E[(Y-\bar{Y})^2]$$
$$= E[Y^2] - \bar{Y}^2$$
$$= A^2E[X^2] - A^2\bar{X}^2$$
$$= A^2(E[X^2] - \bar{X}^2)$$
$$= A^2V[X]$$