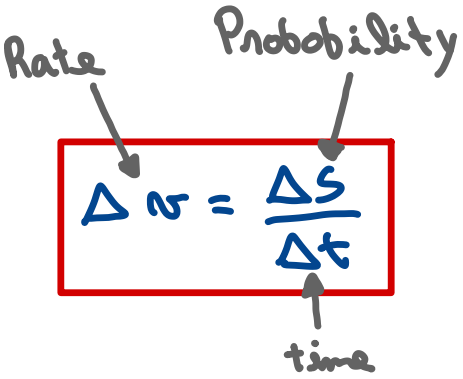
$$\begin{cases} p_{ij}(t, t+\Delta t) = q_{ij}(t)\,\Delta t \\ 1 - p_{ii}(t, t+\Delta t) = \sum_{j \in S} p_{ij}(t, t+\Delta t) \end{cases}$$



$$1 - p_{ii}(t, t+\Delta t) = \sum_{j \in S} p_{ij}(t, t+\Delta t)$$

$$1 - q_{ii}(t)\Delta t = \sum_{j \in S} q_{ij}(t)\Delta t$$

take the limit $\Delta t \to 0$ and divide by Δt

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t}^{0} - \lim_{\Delta t \to 0} \frac{q_{ii}(t)\Delta t}{\Delta t} = \lim_{\Delta t \to 0} \sum_{j \in S} \frac{q_{ij}(t)\Delta t}{\Delta t}$$

$$\boxed{q_{ii}(t) = -\sum_{j \in S} q_{ij}(t)}$$