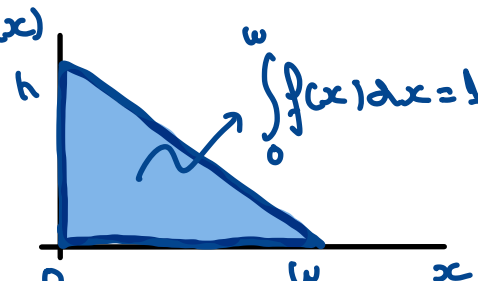$$f(x) = \begin{cases} \left(1 - x/w\right)h & 0 \le x \le w \\ 0 & , \text{ otherwise} \end{cases}$$

$$F(x) = \int_0^x \left(1 - \tfrac{t}{w}\right)h \, dt = ht \Big]_0^x - \frac{h}{2w} t^2 \Big]_0^x = hx - \frac{h}{2w}x^2 = hx\left(1 - \frac{x}{2w}\right)$$

Since the area must be 1 : $A = \dfrac{hw}{2} = 1 \implies \boxed{h = 2/w}$

$$F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{2x}{w}\left(1 - \dfrac{x}{2w}\right), & 0 \le x \le w \\ 1, & x > w \end{cases}$$

$$y = \frac{2x}{w}\left(1 - \frac{x}{2w}\right)$$
$$x^2 - 2wx + w^2 y = 0$$
$$x = w\left(1 \pm \sqrt{1-y}\right)$$

we require the root $\boxed{x = w(1 + \sqrt{1-y})}$ since $x \in [0, w]$