

$$\hat{\beta} = (x^T x)^{-1} x^T y$$

$$y = \beta x + e$$

* $E[\hat{\beta}]$:

$$\begin{aligned} E[\hat{\beta}] &= E[(x^T x)^{-1} x^T y] \\ &= \underbrace{(x^T x)^{-1} x^T x}_{\mathbf{I}} \beta + \underbrace{(x^T x)^{-1} x^T}_{\mathbf{0}} E[e] \end{aligned}$$

$$E[\hat{\beta}] = \mathbf{I} \beta$$

* $V[\hat{\beta}]$:

$$\begin{aligned} V[\hat{\beta}] &= V[\underbrace{(x^T x)^{-1}}_A x^T y] \\ &= \underbrace{(x^T x)^{-1}}_{\mathbf{I}} \underbrace{(x^T x)}_{\mathbf{I}} \underbrace{(x^T x)^{-1}}_{\mathbf{I}} V[y] \end{aligned}$$

$$V[\hat{\beta}] = (x^T x)^{-1} \sigma^2$$

$$\begin{aligned} V[\hat{Ax}] &= E[(y - \bar{y})^2] \\ &= E[y^2] - \bar{y}^2 \\ &= A^2 E[x^2] - A^2 \bar{x}^2 \\ &= A^2 (E[x^2] - \bar{x}^2) \\ &= A^2 V[x] \end{aligned}$$