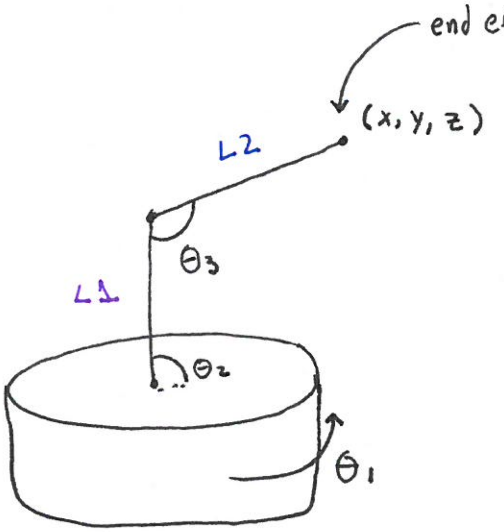
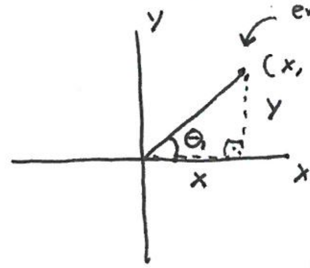
INVERSE KINEMATICS

① • start with a simplified version of the robotic arm that is missing the hand



★ solving for θ_1



- take slice of x - y plane

$$\boxed{\begin{aligned}\theta_1 &= \arctan\left(\tfrac{y}{x}\right)\\ &= \text{atan2}(y, x)^{*}\end{aligned}}$$

* atan2 is an Arduino function that computes the arctangent in the range $(-\pi, \pi]$ so if you want the range $[0, 2\pi]$ just add π

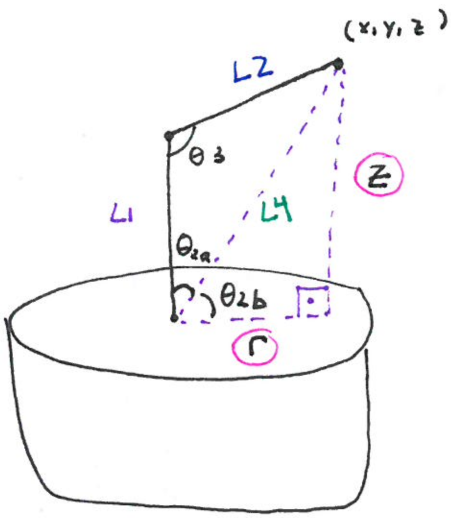
• Problem: How do we find θ_2 given just (x, y, z) of end effector

• Solution: draw an imaginary right triangle so we can write equations to solve for $\theta_2 + \theta_3$ using

LAW OF COSINES !!!

redraw

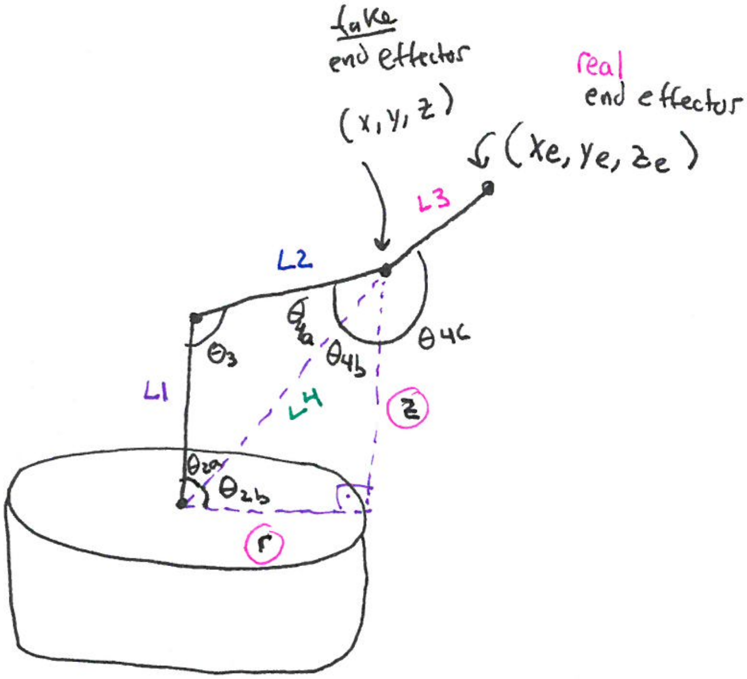
②



$$\boxed{\begin{aligned}\theta_{2a} &= \arccos\left(\frac{L_1^2 + L_4^2 - L_2^2}{2L_1L_4}\right)\\ \theta_{2b} &= \arctan\left(\frac{z}{r}\right) = \text{atan2}(z, r)\\ \theta_3 &= \arccos\left(\frac{L_2^2 + L_1^2 - L_4^2}{2L_2L_1}\right)\end{aligned}}$$

Hooray! Done? No, now we have to add back in the hand part, introducing a new angle, θ_4

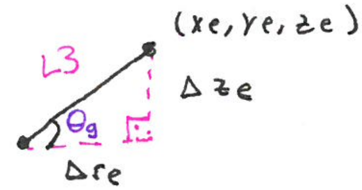
③



Problem:

- now we need to find z and r based on xe, ye, and ze since we don't actually know the position (x, y, z) of the fake end effector
- Solution: draw another imaginary right Δ

closeup



θ_g is a newly defined angle with respect to the ground

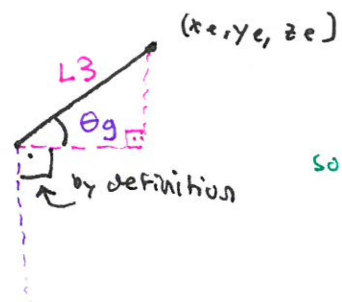
$$\boxed{\begin{aligned} \Delta z_e &= L3 \sin(\theta_g) \\ \Delta r_e &= L3 \cos(\theta_g) \end{aligned}}$$

we need θ_g to solve for Δz_e and Δr_e

so
$$\boxed{\begin{aligned} z &= z_e - \Delta z_e \\ r &= -\Delta r_e + r_e \quad \text{(sorry)} \end{aligned}}$$

solve for θ_4

$$\boxed{\begin{aligned} \theta_{4a} &= 180^\circ - \theta_3 - \theta_{2a} \\ \theta_{4b} &= 90^\circ - \theta_{2b} \\ \theta_{4c} &= 90^\circ + \theta_g \\ \theta_4 &= \theta_{4a} + \theta_{4b} + \theta_{4c} \end{aligned}}$$



so $90^\circ + \theta_g = \theta_{4c}$:)