

Week 5 - Visualisations in Mathematical Modelling

Introduction:

Welcome to this lecture on the role of visualizations in mathematical modeling.

Tables and visualizations are indispensable tools in the context of mathematical modeling, offering a powerful means to analyze, interpret, and communicate complex data and model results. Their utility spans across various stages of the mathematical modeling process, from initial data exploration to the presentation of final model outcomes. Here's why using tables and visualizations is particularly beneficial in this context:

Enhanced Data Understanding: Tables organize data in a structured format, making it easier to identify patterns, trends, and anomalies. Visualizations such as graphs, charts, and heat maps provide an immediate visual context that can reveal insights which might not be obvious from raw data alone. This enhanced understanding is crucial for selecting the appropriate model and for feature engineering.

Effective Communication: Mathematical models often involve complex computations and concepts that can be challenging to explain. Tables and visualizations simplify the communication of these complexities, making the models more accessible to non-expert stakeholders. They can effectively convey key findings, model performance metrics, and comparison of model predictions with actual data.

Assumptions and Relationships Exploration: Before developing a model, it's important to explore the relationships between variables. Scatter plots, correlation matrices, and other visual tools help in understanding these relationships and in verifying the assumptions of the model, such as linearity or distributional assumptions.

Model Validation and Diagnosis: After a model is built, visualizations are crucial for validating the model and diagnosing issues. Residual plots, for example, can help identify patterns of errors in predictions, indicating potential problems with the model, such as non-linearity or heteroscedasticity. Comparison of actual vs. predicted outcomes through visual plots can also highlight the model's accuracy and areas for improvement.

Comparative Analysis: Tables and visualizations facilitate the comparison of different models or different versions of a model. Through side-by-side comparisons or overlaying plots, one can easily assess which model performs better based on various metrics, leading to more informed decision-making.

Interactivity and Exploration: Interactive visualizations allow users to explore data and model results in more depth. For instance, adjusting parameters to see real-time effects on outcomes can provide insights into the sensitivity of the model and help in understanding how different variables influence predictions.

Decision Making and Strategic Insights: Ultimately, the goal of mathematical modeling is often to inform decision-making. Tables and visualizations distill complex model outputs into actionable insights, highlighting key takeaways and guiding strategic decisions. For example, a well-designed dashboard can provide a comprehensive overview of model results, trends over time, and predictive forecasts, all of which are critical for making informed decisions.

In summary, tables and visualizations bridge the gap between complex mathematical models and practical application, making them essential tools in the mathematical modeling process. They enhance understanding, facilitate communication, aid in model validation and comparison, enable deeper exploration, and support decision-making, thereby maximizing the value derived from models.

Tables

Plots

There are various types of plots used in mathematical modeling:

1. Scatter Plots:

Scatter plots display individual data points as points on a graph. They are useful for exploring relationships between two continuous variables. Scatter plots can reveal trends, clusters, and outliers.

2. Line Plots:

Line plots show the change in one or more variables over a continuous domain (e.g., time). They are often used for time series data or to visualize functions.

3. Histograms and Frequency Distributions:

Histograms display the distribution of a single variable by dividing it into intervals (bins) and counting the number of observations in each bin. They provide insights into the data's central tendency, variability, and shape.

4. Box Plots (Box-and-Whisker Plots):

Box plots summarize the distribution of a dataset by displaying its quartiles, median, and potential outliers. They help visualize the spread and skewness of data.

5. Heatmaps:

Heatmaps use colors to represent data values in a two-dimensional matrix. They are useful for visualizing relationships in large datasets, such as correlation matrices.

6. Contour Plots:

Contour plots show the level curves of a two-dimensional function. They are often used in mathematical modeling to represent surfaces or contour maps.

7. 3D Plots:

3D plots visualize three-dimensional functions or datasets. They provide a spatial view of the data or model.

8. Geographic Maps:

Geographic maps display data geospatially. They are used in modeling applications related to geography, such as climate modeling or disease spread.

9. Network Graphs:

Network graphs represent relationships between entities (nodes) and their connections (edges). They are employed in modeling complex networks, such as social networks or transportation systems.

10. Animated Visualizations:

Animated visualizations show how data or model results change over time. They are valuable for dynamic modeling scenarios, such as simulations.

Benefits of Visualizations in Mathematical Modeling:

Enhance Understanding: Visualizations make complex data and models more accessible and interpretable. **Facilitate Communication:** Visualizations help convey findings and insights to a broad audience. **Detect Patterns:** Visualizations reveal patterns, trends, and outliers in the data. **Model Validation:** Visualizations aid in comparing model predictions to observed data. **Decision Support:** Visualizations assist in decision-making by providing a clear view of model outcomes.

Conclusion:

Visualizations are indispensable in mathematical modeling, helping researchers and analysts explore data, communicate findings, and validate models. The choice of visualization depends on the type of data and the modeling goals. Effective visualizations enhance model understanding and support informed decision-making.