

# Week 4 - Descriptive Statistics in Mathematical Modelling

## Introduction:

Welcome to this Module on Measures of Central Tendency and Measures of Variability in the context of mathematical modeling. These statistical concepts play a crucial role in summarizing and understanding data, which is essential for building accurate mathematical models.

## Measures of Central Tendency:

Measures of central tendency help us find the “center” or typical value of a dataset.

### 1. Mean (Average):

The mean is calculated by adding up all the values in a dataset and dividing by the number of observations. It is sensitive to outliers and can be skewed by extreme values.

The formula for the mean ( ) of a dataset with  $n$  observations is:

$$= (\sum x_i) / n$$

### 2. Median:

The median is the middle value when the data is arranged in ascending or descending order. It is not affected by extreme outliers and is a robust measure of central tendency. For datasets with an even number of observations, the median is the average of the two middle values.

### 3. Mode:

The mode is the value that appears most frequently in the dataset. Some datasets may have multiple modes (bimodal, trimodal, etc.), while others may have no mode (no value occurs more than once).

## Measures of Variability:

Measures of variability quantify how spread out or dispersed the data points are from the central tendency.

### 1. Range:

The range is the difference between the maximum and minimum values in the dataset. It is simple to calculate but sensitive to outliers.

### 2. Variance and Standard Deviation:

Variance measures the average squared deviation of each data point from the mean.

Standard deviation is the square root of the variance.

A smaller standard deviation indicates less variability in the data, while a larger one suggests greater variability.

The formulas for variance ( $\hat{\sigma}^2$ ) and standard deviation ( $\hat{\sigma}$ ) are:

$$\hat{\sigma}^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\hat{\sigma} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

### 3. Interquartile Range (IQR):

IQR is the range of the middle 50% of the data and is calculated as the difference between the third quartile (Q3) and the first quartile (Q1). It is less sensitive to outliers compared to the range.

## Applications in Mathematical Modeling:

Measures of central tendency and variability are vital in mathematical modeling for various reasons:

- They help summarize and understand the data used to build models.
- They provide insights into the distribution and characteristics of the data.
- They help identify outliers and understand their impact on the model.
- They guide the selection of appropriate probability distributions for modeling uncertainty.

**Conclusion:**

Measures of central tendency (mean, median, mode) and variability (range, variance, standard deviation, IQR) are essential statistical tools for summarizing and understanding data in the context of mathematical modeling.

These measures assist in characterizing the central behavior and spread of data, aiding model development, and decision-making.