

# Week 9 - Evaluation of Regression Models

## Introduction:

Welcome to this lecture on methods of evaluating regression models. Regression models are essential in mathematical modeling and data analysis, and their performance needs to be assessed rigorously. In this lecture, we will explore various evaluation methods and metrics used to assess the quality and accuracy of regression models.

## Importance of Model Evaluation:

Model evaluation is critical to determine how well a regression model fits the data and makes predictions. It helps in selecting the best model among competing models and guides model refinement. Accurate model evaluation is essential for making informed decisions in various fields, including economics, engineering, and the sciences.

## Common Evaluation Metrics:

Several metrics are commonly used to evaluate regression models: 1. Mean Squared Error (MSE):

MSE measures the average squared difference between predicted and actual values. It penalizes larger errors more than smaller ones, making it sensitive to outliers. The formula for MSE is:  $MSE = \Sigma(y - \bar{y})^2 / n$ , where  $y$  is the actual value,  $\bar{y}$  is the mean of actual values, and  $n$  is the number of data points. 2. Root Mean Squared Error (RMSE):

RMSE is the square root of MSE and is in the same units as the dependent variable. It provides a more interpretable measure of error.  $RMSE = \sqrt{MSE}$ . 3. Mean Absolute Error (MAE):

MAE measures the average absolute difference between predicted and actual values. It is less sensitive to outliers compared to MSE.  $MAE = \Sigma|y - \hat{y}| / n$ , where  $\hat{y}$  is the predicted value. 4. R-squared ( $R^2$ ):

$R^2$  represents the proportion of variance in the dependent variable explained by the model. It ranges from 0 to 1, where a higher value indicates a better fit.  $R^2 = 1 - (SSE / SST)$ , where

SSE is the sum of squared errors and SST is the total sum of squares. 5. Adjusted R-squared (Adjusted  $R^2$ ):

Adjusted  $R^2$  adjusts  $R^2$  for the number of predictors in the model. It penalizes the addition of unnecessary variables.  $\text{Adjusted } R^2 = 1 - [(1 - R^2) * ((n - 1) / (n - p - 1))]$ , where  $n$  is the number of data points and  $p$  is the number of predictors.

## **Model Validation Techniques:**

Various techniques are used to validate regression models:

### **1. Residual Analysis:**

Residuals are the differences between actual and predicted values. Residual plots are used to assess the distribution and pattern of residuals. Patterns in residuals may indicate violations of model assumptions.

### **2. Cross-Validation:**

Cross-validation techniques (e.g., k-fold cross-validation) divide the dataset into training and testing sets multiple times to assess model performance. It helps estimate how the model will generalize to new, unseen data.

### **3. Overfitting and Underfitting:**

Overfitting occurs when a model is too complex and fits the noise in the data. Underfitting happens when a model is too simple and fails to capture the underlying relationships. Model complexity should be carefully balanced.

### **4. Feature Importance:**

Assessing the importance of predictor variables helps in model interpretation. Techniques like feature selection or permutation importance can identify important predictors.

## **Conclusion:**

Evaluating regression models is essential to determine their accuracy, reliability, and suitability for a given problem. Metrics like MSE, RMSE, MAE,  $R^2$ , and adjusted  $R^2$  provide valuable insights into a model's performance. Model validation techniques and residual analysis help ensure robust and interpretable regression models.