

# Week 9 - Evaluation of Regression Models

## Introduction:

Welcome to this unit on methods of evaluating regression models. Regression models are essential in mathematical modeling and data analysis, and their performance needs to be assessed rigorously. Without proper evaluation, researchers may misinterpret the relationships between variables. They might erroneously conclude that certain predictors have a significant impact on the outcome when they do not or vice versa. Inadequately evaluated regression models may produce inaccurate predictions. If the model does not capture the underlying relationships between variables effectively, its predictive performance may be poor, leading to unreliable forecasts or estimates. In this unit, we will explore various evaluation methods and metrics used to assess the quality and accuracy of regression models.

## Importance of Model Evaluation:

Model evaluation is critical to determine how well a regression model fits the data and makes predictions. It helps in selecting the best model among competing models and guides model refinement. Accurate model evaluation is essential for making informed decisions in various fields, including economics, engineering, and the sciences.

## Common Evaluation Metrics:

Several metrics are commonly used to evaluate regression models:

1. Mean Squared Error (MSE):

MSE measures the average squared difference between predicted and actual values. It penalizes larger errors more than smaller ones, making it sensitive to outliers. The formula for MSE is:

$$MSE = \frac{\sum (y_i - \bar{y})^2}{n}$$

where  $y_i$  is the actual value,  $\bar{y}$  is the mean of actual values, and  $n$  is the number of data points.

## 2. Root Mean Squared Error (RMSE):

RMSE is the square root of MSE and is in the same units as the dependent variable. It provides a more interpretable measure of error.

$$RMSE = \sqrt{MSE}$$

## 3. Mean Absolute Error (MAE):

MAE measures the average absolute difference between predicted and actual values. It is less sensitive to outliers compared to MSE.

$$MAE = \frac{\sum |y_i - \hat{y}|}{n}$$

where  $\hat{y}$  is the predicted value.

## 4. Multiple R-squared ( $R^2$ ):

$R^2$  represents the proportion of variance in the dependent variable explained by the model. It ranges from 0 to 1, where a higher value indicates a better fit. For this we need to calculate two terms.

### *Sum of Squared Errors*

The sum of squares error (SSE) or residual sum of squares (RSS, where residual means remaining or unexplained) is the difference between the observed and predicted values.

$$SSE = \sum \epsilon_i^2$$

where  $\epsilon_i$  is the difference between the actual value of the dependent variable and the predicted value.

$$\epsilon_i = y_i - \hat{y}$$

*Sum of Squares Total* The sum of squares total (SST) or the total sum of squares (TSS) is the sum of squared differences between the observed dependent variables and the overall mean.

$$SST = \sum (y_i - \bar{y})^2$$

Where  $y_i$  is the observed dependent value and  $\bar{y}$  is the mean of the dependent variable.

From these two variables, you are able to calculate Multiple R-squared.

$$R^2 = 1 - (SSE/SST)$$

where SSE is the sum of squared errors and SST is the total sum of squares.

#### 5. Adjusted R-squared (Adjusted $R^2$ ):

Adjusted  $R^2$  adjusts  $R^2$  for the number of predictors in the model. It penalizes the addition of unnecessary variables.

$$AdjustedR^2 = 1 - [(1 - R^2) * ((n - 1)/(n - p - 1))]$$

where n is the number of data points and p is the number of predictors.

## Model Validation Techniques in R:

### Summary Statistics:

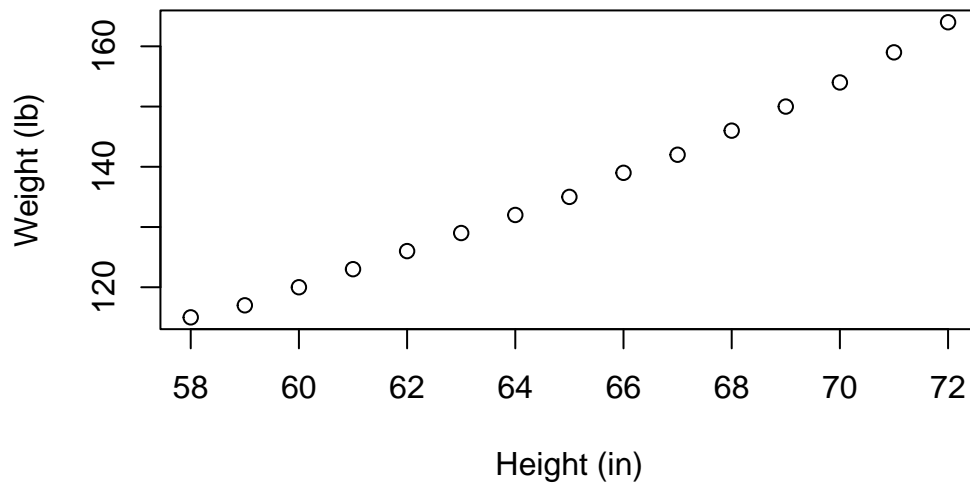
In this example, we will do an exploration of a dataset which includes American women's height and weight. The question we are asking, is can we predict a woman's height given her weight?

```
# You may need to install the datasets library (by using install.packages('datasets'))  
library(datasets)
```

In this example, it's worth plotting the data to get a sense of whether there may be a trend or not.

```
plot(women, xlab = "Height (in)", ylab = "Weight (lb)",  
      main = "women data: American women aged 30-39")
```

### women data: American women aged 30–39



The `summary()` function provides a summary of the regression model, including coefficients, standard errors, and metrics for evaluating the effectiveness of the model.

Example:

```
summary(lm_model)
```

Below we have created a linear regression model, where we use weight as the dependent variable and height as the independent variable.

```
lmHeight = lm(height~weight, data = women) #Create the linear regression
summary(lmHeight) #Review the results
```

Call:

```
lm(formula = height ~ weight, data = women)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.83233	-0.26249	0.08314	0.34353	0.49790

Coefficients:

```

              Estimate Std. Error t value Pr(>|t|)
(Intercept) 25.723456    1.043746   24.64 2.68e-12 ***
weight       0.287249    0.007588   37.85 1.09e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.44 on 13 degrees of freedom
Multiple R-squared:  0.991, Adjusted R-squared:  0.9903
F-statistic: 1433 on 1 and 13 DF,  p-value: 1.091e-14

```

The most important results are, our Multiple R-Squared and Adjusted R

### **Coefficient of Determination (R-squared):**

R-squared measures the proportion of the variance in the dependent variable that is explained by the independent variables in the model. Higher values indicate better fit.

If we just want to isolate this statistic, we can call the following function.

Example:

```
summary(lm_model)$r.squared
```

### **Adjusted R-squared:**

Adjusted R-squared penalizes the inclusion of unnecessary predictors in the model and provides a more conservative measure of model fit, especially for models with multiple predictors.

If we just want to isolate this statistic, we can call the following function.

Example:

```
summary(lm_model)$adj.r.squared
```

However, these statistics are included in the summary report given above.

## Root Mean Squared Error (RMSE) or Mean Absolute Error (MAE):

These metrics quantify the average difference between observed and predicted values. Lower values indicate better predictive accuracy.

Example

```
RMSE: sqrt(mean(residuals(lm_model)^2))
```

```
rmse = sqrt(mean(residuals(lmHeight)^2))
rmse
```

```
[1] 0.4096541
```

## Multiple Linear Regression

Generally speaking, modelling is going to be concerned with looking at more than one factor. In this case, what we can do is use multiple linear regression to assess whether multiple factors are more useful to predict a value.

In this case, we are looking at predicting a cars price based on some characteristics of the car.

```
automobile <- read.csv("~/automobile.csv")
```

For example, we could look at whether city.mpg is a good predictor of price but doing a simple linear regression model.

```
model = lm(price ~ `city.mpg`, data = automobile) #Create the linear regression
summary(model) #Review the results
```

Call:

```
lm(formula = price ~ city.mpg, data = automobile)
```

Residuals:

Min	1Q	Median	3Q	Max
-7623	-3302	-1698	1632	22078

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
--	----------	------------	---------	----------

```
(Intercept) 35802.44    1690.14    21.18    <2e-16 ***
city.mpg     -891.42      64.79   -13.76    <2e-16 ***
```

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 5739 on 193 degrees of freedom
```

```
Multiple R-squared:  0.4951,    Adjusted R-squared:  0.4925
```

```
F-statistic: 189.3 on 1 and 193 DF,  p-value: < 2.2e-16
```

Not so great with a multiple R squared of 0.4993 and an adjusted R squared of 0.4967. Maybe we can use another variable and see if this helps the model. Lets add highway.mpg. Note, all we do to add another variable is use the + sign and then the next variable.

```
model = lm(price ~ `city.mpg` + `highway.mpg`, data = automobile) #Create the linear regression model
summary(model) #Review the results
```

Call:

```
lm(formula = price ~ city.mpg + highway.mpg, data = automobile)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-8441   -3476   -1111    1108   20397
```

Coefficients:

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  39172.6     2015.2   19.439  < 2e-16 ***
city.mpg      -129.8       266.7   -0.487  0.62687
highway.mpg   -735.3       250.0   -2.941  0.00368 **
```

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 5628 on 192 degrees of freedom
```

```
Multiple R-squared:  0.5169,    Adjusted R-squared:  0.5119
```

```
F-statistic: 102.7 on 2 and 192 DF,  p-value: < 2.2e-16
```

This has improved our multiple R squared to 0.5183 and adjusted R squared to 0.5132. Maybe we can add horsepower to our multiple linear regression model.

```
model = lm(price ~ `city.mpg` + `highway.mpg` + `horsepower`, data = automobile) #Create the linear regression model
summary(model) #Review the results
```

Call:

```
lm(formula = price ~ city.mpg + highway.mpg + horsepower, data = automobile)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-5480.2	-752.9	-13.0	387.1	7002.2

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	14649.69	2473.37	5.923	2.45e-08	***
city.mpg	-189.58	151.42	-1.252	0.212705	
highway.mpg	-22.40	132.05	-0.170	0.865567	
horsepower100	1060.15	1934.18	0.548	0.584511	
horsepower101	5849.27	1776.15	3.293	0.001262	**
horsepower102	-39.57	1673.98	-0.024	0.981178	
horsepower106	13354.16	2520.69	5.298	4.60e-07	***
horsepower110	4700.79	1540.68	3.051	0.002742	**
horsepower111	3399.03	1725.39	1.970	0.050870	.
horsepower112	346.95	2029.65	0.171	0.864524	
horsepower114	6650.92	1612.53	4.125	6.43e-05	***
horsepower115	6705.52	2426.64	2.763	0.006515	**
horsepower116	1287.99	1507.62	0.854	0.394429	
horsepower120	7837.08	2355.60	3.327	0.001129	**
horsepower121	12086.39	1764.20	6.851	2.32e-10	***
horsepower123	18475.04	1780.77	10.375	< 2e-16	***
horsepower134	10762.91	2400.95	4.483	1.55e-05	***
horsepower140	12896.14	2472.68	5.215	6.67e-07	***
horsepower142	7450.31	2382.32	3.127	0.002158	**
horsepower143	11575.08	2355.60	4.914	2.53e-06	***
horsepower145	3032.89	1673.70	1.812	0.072179	.
horsepower152	3013.72	1824.08	1.652	0.100803	
horsepower154	6034.69	2359.63	2.557	0.011640	*
horsepower155	23406.76	2137.49	10.951	< 2e-16	***
horsepower156	5304.68	1981.65	2.677	0.008345	**
horsepower160	7803.85	1639.14	4.761	4.87e-06	***
horsepower161	5862.68	1981.65	2.958	0.003647	**
horsepower162	7750.94	2003.05	3.870	0.000168	***
horsepower175	5992.89	2389.66	2.508	0.013323	*
horsepower176	22519.58	2074.71	10.854	< 2e-16	***
horsepower182	25116.56	1856.97	13.526	< 2e-16	***
horsepower184	31542.81	2182.14	14.455	< 2e-16	***
horsepower200	8787.33	2396.02	3.667	0.000350	***



horsepower207	23661.13	1800.60	13.141	< 2e-16	***
horsepower262	24195.62	2517.45	9.611	< 2e-16	***
horsepower48	598.70	2982.21	0.201	0.841189	
horsepower52	1280.10	2193.68	0.584	0.560495	
horsepower55	2100.34	2903.89	0.723	0.470747	
horsepower56	947.73	2174.11	0.436	0.663588	
horsepower58	2328.26	3091.47	0.753	0.452677	
horsepower60	-1105.91	2652.10	-0.417	0.677341	
horsepower62	-1131.42	1681.93	-0.673	0.502286	
horsepower68	-699.62	1607.03	-0.435	0.663997	
horsepower69	-915.30	1621.31	-0.565	0.573313	
horsepower70	-101.05	1659.60	-0.061	0.951537	
horsepower72	10444.83	2414.78	4.325	2.93e-05	***
horsepower73	-307.21	1835.50	-0.167	0.867325	
horsepower76	-1124.30	1744.26	-0.645	0.520290	
horsepower78	-2665.21	2369.89	-1.125	0.262733	
horsepower82	-910.19	1698.67	-0.536	0.592954	
horsepower84	881.14	1632.75	0.540	0.590309	
horsepower85	-547.82	1772.11	-0.309	0.757695	
horsepower86	240.62	1699.44	0.142	0.887615	
horsepower88	-307.13	1574.46	-0.195	0.845629	
horsepower90	1424.12	1786.63	0.797	0.426783	
horsepower92	1562.21	1731.51	0.902	0.368532	
horsepower94	969.15	1934.18	0.501	0.617134	
horsepower95	6533.21	1640.59	3.982	0.000111	***
horsepower97	2079.74	1623.08	1.281	0.202249	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1907 on 136 degrees of freedom

Multiple R-squared: 0.9607, Adjusted R-squared: 0.944

F-statistic: 57.35 on 58 and 136 DF, p-value: < 2.2e-16

We have improved this more dramatically this time, and our multiple R squared value rises to 0.6803 and adjusted R squared value rises to 0.6753.

We have dealt so far with just numerical data, lets see what happens if we use some categorical data by adding make to our linear regression.

```
model = lm(price ~ `city.mpg` + `highway.mpg` + `horsepower` + `make`, data = automobile)
summary(model) #Review the results
```

Call:

```
lm(formula = price ~ city.mpg + highway.mpg + horsepower + make,  
    data = automobile)
```

Residuals:

Min	1Q	Median	3Q	Max
-5504.8	-496.6	0.0	358.1	6123.4

Coefficients: (5 not defined because of singularities)

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	14007.84	2418.31	5.792	5.69e-08	***
city.mpg	-256.50	150.46	-1.705	0.090830	.
highway.mpg	47.96	125.02	0.384	0.701944	
horsepower100	-1286.45	3267.63	-0.394	0.694505	
horsepower101	1200.92	3999.71	0.300	0.764504	
horsepower102	651.57	2820.66	0.231	0.817709	
horsepower106	12088.96	3563.28	3.393	0.000938	***
horsepower110	2054.42	2810.73	0.731	0.466254	
horsepower111	5081.33	1670.00	3.043	0.002881	**
horsepower112	389.20	2882.23	0.135	0.892812	
horsepower114	5127.48	3216.33	1.594	0.113522	
horsepower115	2322.90	3329.43	0.698	0.486724	
horsepower116	1955.80	2707.41	0.722	0.471461	
horsepower120	10619.17	3358.39	3.162	0.001985	**
horsepower121	6234.17	4285.37	1.455	0.148348	
horsepower123	18830.25	1556.11	12.101	< 2e-16	***
horsepower134	9243.78	3609.56	2.561	0.011679	*
horsepower140	8587.31	3350.85	2.563	0.011622	*
horsepower142	4661.28	3391.26	1.374	0.171849	
horsepower143	11588.83	2039.23	5.683	9.45e-08	***
horsepower145	5539.82	2870.07	1.930	0.055941	.
horsepower152	3107.97	2803.65	1.109	0.269843	
horsepower154	6118.78	2042.77	2.995	0.003331	**
horsepower155	23852.94	1892.07	12.607	< 2e-16	***
horsepower156	5263.73	2964.68	1.775	0.078354	.
horsepower160	6964.37	2744.54	2.538	0.012445	*
horsepower161	5821.73	2964.68	1.964	0.051878	.
horsepower162	6235.24	3436.49	1.814	0.072111	.
horsepower175	6217.70	2072.58	3.000	0.003284	**
horsepower176	22828.48	1836.04	12.434	< 2e-16	***
horsepower182	19328.42	4338.77	4.455	1.90e-05	***
horsepower184	31995.85	1947.11	16.432	< 2e-16	***
horsepower200	8836.97	3124.00	2.829	0.005479	**

horsepower207	23681.74	1578.70	15.001	< 2e-16	***
horsepower262	24511.39	2227.27	11.005	< 2e-16	***
horsepower48	214.89	3538.71	0.061	0.951678	
horsepower52	-1246.34	3214.01	-0.388	0.698862	
horsepower55	2124.15	3370.47	0.630	0.529746	
horsepower56	779.74	2968.31	0.263	0.793242	
horsepower58	-99.44	4412.26	-0.023	0.982056	
horsepower60	-3425.47	4109.80	-0.833	0.406226	
horsepower62	-1291.67	2743.21	-0.471	0.638595	
horsepower68	1340.49	2823.16	0.475	0.635778	
horsepower69	-612.37	2524.82	-0.243	0.808775	
horsepower70	-330.04	2743.93	-0.120	0.904462	
horsepower72	13185.69	3381.24	3.900	0.000159	***
horsepower73	1723.99	2297.85	0.750	0.454566	
horsepower76	-3459.28	3731.64	-0.927	0.355780	
horsepower78	-5126.08	3836.87	-1.336	0.184075	
horsepower82	2175.20	2146.58	1.013	0.312941	
horsepower84	3779.89	2962.96	1.276	0.204521	
horsepower85	-2899.20	2994.19	-0.968	0.334855	
horsepower86	-2181.86	3747.71	-0.582	0.561534	
horsepower88	1293.11	2804.39	0.461	0.645561	
horsepower90	-863.08	3052.90	-0.283	0.777887	
horsepower92	1458.60	2768.79	0.527	0.599306	
horsepower94	3933.06	2331.64	1.687	0.094236	.
horsepower95	3892.47	3046.55	1.278	0.203834	
horsepower97	424.45	2796.43	0.152	0.879613	
makeaudi	4681.22	2618.14	1.788	0.076300	.
makebmw	5977.49	4012.84	1.490	0.138956	
makechevrolet	442.07	3005.76	0.147	0.883320	
makedodge	-2341.33	2586.02	-0.905	0.367078	
makehonda	2549.45	3510.03	0.726	0.469048	
makeisuzu	2668.50	3319.86	0.804	0.423104	
makejaguar	NA	NA	NA	NA	
makemazda	-2768.35	2657.86	-1.042	0.299705	
makemercedes-benz	NA	NA	NA	NA	
makemercury	NA	NA	NA	NA	
makemitsubishi	-2256.97	2513.42	-0.898	0.371001	
makenissan	111.68	2300.07	0.049	0.961354	
makepeugot	2946.91	2794.62	1.054	0.293775	
makeplymouth	-2298.36	2596.65	-0.885	0.377859	
makeporsche	NA	NA	NA	NA	
makerenault	NA	NA	NA	NA	
makesaab	1429.11	2515.69	0.568	0.571043	

makesubaru	-2831.79	1713.68	-1.652	0.101055
maketoyota	299.22	2475.04	0.121	0.903975
makevolkswagen	2408.00	2640.16	0.912	0.363564
makevolvo	1747.37	2946.61	0.593	0.554289

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1643 on 120 degrees of freedom

Multiple R-squared: 0.9743, Adjusted R-squared: 0.9584

F-statistic: 61.38 on 74 and 120 DF, p-value: < 2.2e-16

So some things to note with this output. Firstly, our multiple R squared value rises again to 0.9179, and our adjusted R squared value rises to 0.908. But our output looks different. We can see that each make of car is represented, and that some have some \* next to their entries, which denotes significance (we can see that these models are a lot better (significant) than others). Before we go any further, let's look at what p values are.

As we know, in a linear regression model, coefficients represent the relationship between the independent variable(s) and the dependent variable. For example, in a simple linear regression  $y = mx + b$ , 'm' is the coefficient (slope) of the independent variable 'x'.

*Null Hypothesis (H0):* The null hypothesis states that there is no relationship between the independent variable(s) and the dependent variable, i.e., the coefficient(s) equal zero.

*Alternative Hypothesis (H1):* The alternative hypothesis complements the null hypothesis by proposing that there is indeed a relationship between the independent variable(s) and the dependent variable, contrary to the null hypothesis. It suggests that the coefficient(s) in the regression equation are not equal to zero.

*P-value:* The p-value associated with each coefficient indicates the probability of observing the coefficient's value, or more extreme, if the null hypothesis is true. In other words, it tells us the likelihood of the observed relationship occurring by random chance. A significant p-value leads to rejecting the null hypothesis in favor of the alternative hypothesis, indicating that there is evidence supporting a relationship between the independent variable(s) and the dependent variable. Conversely, a non-significant p-value suggests that there is insufficient evidence to reject the null hypothesis, indicating that the relationship between the variables is not statistically significant.

*Interpretation:* If the p-value is small (typically less than a chosen significance level, often 0.05), it suggests that the coefficient is statistically significant. In other words, we reject the null hypothesis and conclude that there is evidence of a significant relationship between the independent variable and the dependent variable. If the p-value is large, it suggests that the coefficient is not statistically significant. In this case, we fail to reject the null hypothesis, indicating that there is insufficient evidence to conclude a significant relationship.

*Decision:* Based on the p-value, you decide whether to retain or reject the null hypothesis. If you reject the null hypothesis, it implies that the variable associated with the coefficient in question is indeed contributing significantly to the model's predictive power.

Overall, p-values help researchers determine the validity of their regression models and the significance of the variables included. However, it's essential to interpret p-values in conjunction with other metrics and considerations to make informed decisions about model selection and variable significance.

Now we know what a p value is, we can see that there are certain values which are more useful than others, as indicated by the asterisks (\*). Lets check to see if this is right. Remember, we can create a subset of values by using the which command. Lets put all mercedes cars into a smaller dataframe. To compare, lets also put all hondas into a smaller dataframe.

```
# This code takes all rows where make is mercedes-benz and puts it into a new dataframe called mercedes
mercedes <- automobile[which(automobile$make == 'mercedes-benz'),]
# This code takes all rows where make is honda and puts it into a new dataframe called honda
honda <- automobile[which(automobile$make == 'honda'),]
```

Now lets run our linear regression model, just for all mercedes cars and all honda cars.

```
model = lm(price ~ `city.mpg` + `highway.mpg` + horsepower, data = mercedes) #Create the model
summary(model) #Review the results
```

Call:

```
lm(formula = price ~ city.mpg + highway.mpg + horsepower, data = mercedes)
```

Residuals:

```
    59    60    61    62    63    64    65    66
-2842  -146  -218  3206  -436   436 -2220  2220
```

Coefficients: (2 not defined because of singularities)

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	64192	4479	14.331	2.98e-05 ***
city.mpg	-23734	9725	-2.440	0.0586 .
highway.mpg	19454	8549	2.276	0.0719 .
horsepower155	NA	NA	NA	NA
horsepower184	NA	NA	NA	NA

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2394 on 5 degrees of freedom

Multiple R-squared: 0.9112, Adjusted R-squared: 0.8757  
F-statistic: 25.65 on 2 and 5 DF, p-value: 0.002351

```
model = lm(price ~ `city.mpg` + `highway.mpg` + horsepower, data = honda) #Create the linear model  
summary(model) #Review the results
```

Call:

```
lm(formula = price ~ city.mpg + highway.mpg + horsepower, data = honda)
```

Residuals:

Min	1Q	Median	3Q	Max
-1138	0	0	67	1262

Coefficients: (1 not defined because of singularities)

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	25852.7	10375.5	2.492	0.0471 *
city.mpg	-806.0	620.3	-1.299	0.2415
highway.mpg	149.8	310.1	0.483	0.6463
horsepower101	2243.3	1272.7	1.763	0.1284
horsepower58	12033.7	9498.1	1.267	0.2521
horsepower60	3884.7	5116.7	0.759	0.4765
horsepower76	297.8	1798.8	0.166	0.8740
horsepower86	NA	NA	NA	NA

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 744.3 on 6 degrees of freedom

Multiple R-squared: 0.9348, Adjusted R-squared: 0.8697

F-statistic: 14.34 on 6 and 6 DF, p-value: 0.002504

We can see that the model is better for mercedes cars 0.9112 and 0.8757, than for honda cars 0.8722 and 0.8297.

## Practical Questions

The main purpose of this week, is to see how we evaluate linear regression models. In our dataset, we have a number of different variables described below.

Variable	Data
make:	alfa-romero, audi, bmw, chevrolet, dodge, honda, isuzu, jaguar, mazda, mercedes-benz, mercury, mitsubishi, nissan, peugot, plymouth, porsche, renault, saab, subaru, toyota, volkswagen, volvo
fuel-type:	diesel, gas.
aspiration:	std, turbo.
num-of-doors:	four, two.
body-style:	hardtop, wagon, sedan, hatchback, convertible.
drive-wheels:	4wd, fwd, rwd.
engine-location:	front, rear.
wheel-base:	continuous from 86.6 120.9.
length:	continuous from 141.1 to 208.1.
width:	continuous from 60.3 to 72.3.
height:	continuous from 47.8 to 59.8.
curb-weight:	continuous from 1488 to 4066.
engine-type:	dohc, dohc, l, ohc, ohcf, ohcv, rotor.
num-of-cylinders:	eight, five, four, six, three, twelve, two.
engine-size:	continuous from 61 to 326.
fuel-system:	1bbl, 2bbl, 4bbl, idi, mfi, mpfi, spdi, spfi.
bore:	continuous from 2.54 to 3.94.
stroke:	continuous from 2.07 to 4.17.
compression-ratio:	continuous from 7 to 23.
horsepower:	continuous from 48 to 288.
peak-rpm:	continuous from 4150 to 6600.
city-mpg:	continuous from 13 to 49.
highway-mpg:	continuous from 16 to 54.
price:	continuous from 5118 to 45400.

Create models to answer the following questions in a R Markdown document.

1. What is the best single variable for predicting price?
2. Rank, in order, each variable according to Adjusted R squared and also Root Mean Squared. Are the orders different?
3. Is there a combination of variables that improves our model? HINT: Combine the single variables that scored best in various ways to see if you can improve the model.

4. Now that you have found your best combination, answer the following questions:
5. Does the model perform better for certain makes of car?
6. Does the model work better for standard or turbo cars?
7. Does the model work for cars with two or four doors, or is there no difference?

### **Conclusion:**

Evaluating regression models is essential to determine their accuracy, reliability, and suitability for a given problem. Metrics like MSE, RMSE, MAE,  $R^2$ , and adjusted  $R^2$  provide valuable insights into a model's performance. Model validation techniques and residual analysis help ensure robust and interpretable regression models.