The Dilution Rate of Government Debt

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Abstract

This paper introduces the dilution rate of government debt, which measures newly-issued short-term to previously-issued long-term government debt. The concept allows for dynamic maturity management analysis in linearized frameworks. I construct one such model and use it to motivate a structural vector autoregression (SVAR) which I estimate with U.S. post-war data. Debt dilution increases with negative shocks to the Fed's policy rate and U.S. tax rate. Positive shocks to debt dilution reduce inflation contemporaneously and increase long-run inflation. The results raise questions about deflationary effects from open market purchases and inflationary impacts of open market sales.

Keywords: Federal Reserve, U.S. Treasury, Debt maturity management, SVARs, U.S. inflation.

JEL Classification: E52, E58, E63, H63, C32.

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1 Introduction

The U.S. Federal Reserve (Fed) often deploys quantitative easing (QE) and interest rate cuts simultaneously during severe economic downturns, unleashing a powerful two-pronged approach to monetary stimulus. Slashing the federal funds rate reduces short-term borrowing costs for households, firms and the government, while purchasing long-term assets like U.S. Treasurys injects liquidity into financial markets.

Despite this policy marriage, a tractable theory describing the direct impact of short-term–for–long-term asset swaps on inflation remains absent, so it is difficult to answer questions like 'does QE contribute to low inflation during recessions?' and 'does quantitative tightening (QT) counteract contractionary monetary policy?' The main issue arises largely due to the interaction between DSGE linearization and the household's no-arbitrage condition on debt objects of differing maturities.[1] Without additional assumptions on this no-arbitrage condition (e.g., risk premia), these debt objects are approximately perfect substitutes.[2] I overcome the need for additional assumptions and bypass the perfect substitution issue in this paper, developing and outlining simple theoretical predictions relating maturity management to inflation outcomes that survive linearization.

Cochrane (2001) and Leeper and Leith (2017) develop a general theory of debt management in a constant endowment setting, allowing for an arbitrarily large number of debt maturities to be freely issued by the government. They characterize the equilibrium price level as a weighted sum of current and expected future discounted primary surpluses but with a weight structure difficult to penetrate intuitively. Cochrane (2023) writes, "These formulas likely hide additional interesting insights and special cases."

This paper investigates one such case, requiring only a resource constraint, household budget constraint, and household optimization over long- and short-term government debt. Limiting government issuance to two periods dramatically simplifies the general formulas in Cochrane's (2001) and Leeper and Leith's (2017) papers and allows for new, exciting perspectives on maturity

[1] Debt of various maturities can be used to complete markets in non-linear models like those in work by Angeletos (2002), Buera and Nicolini (2004) and Faraglia, Marcet, and Scott (2010). They are also useful insurance tools in non-linear, incomplete-markets models such as those in papers by Lustig, Sleet, and Yeltekin (2008), Debortoli, Nunes, and Yared (2017), and Faraglia et al. (2019)

[2] Woodford (2001) develops a second-best approach by examining a maturity structure consisting of perpetuities with exponentially-decaying coupons where the decay rate is held constant. Leeper and Zhou (2021) vary such a portfolio's rate of decay to compare welfare outcomes from policy across economies with different debt durations.

management as a policy tool.

The main object of interest in this paper is the dilution rate of government debt (hereby referred to as the dilution rate), which measures the amount of outstanding short-term government debt relative to the amount of outstanding previously-issued, unmatured long-term debt. The name, inspired by concepts discussed in Cochrane's (2023) book, describes how the government dilutes the real value of its unmatured debt when it issues new debt maturing on the same date. The concept is similar to that of a publically-traded company diluting its current market capitalization by issuing additional stock, reducing price per share.

Keeping expectations about future primary surpluses fixed, a theoretical government raises expected future inflation when it dilutes outstanding unmatured debt with short-term issuance. The increase in expected inflation reduces the market value of existing unmatured debt, lowering outstanding debt's market value. This reduction in government indebtedness relieves the aggregate price level from adjusting upward to revalue debt to equate the government's present value condition. Theoretical debt dilution reduces current prices at the expense of higher expected future prices.

The U.S.'s outstanding maturity structure is jointly determined by the U.S. Treasury and the Fed. The Treasury finances government deficits through new issuance while the Fed engages in open-market operations like QE and QT, exchanging ultra-short-term debt (reserves) with longer-term debt (Treasurys) at market prices. The residual, privately held structure represents consolidated government indebtedness. U.S. dilution is a function of fiscal and monetary debt management.

I divide the paper into three sections. First, I introduce the dilution rate. Second, I embed the dilution rate in a DSGE model with household optimization and government policy. Finally, I log-linearize the model to motivate a recursively-identified structural vector autoregression (SVAR) analysis of U.S. policy comprised of inflation and fiscal (tax rate), monetary (interest rate) and debt management (dilution rate) policy variables.

I find four main results from the SVAR analysis. First, a surprise decrease in tax rates causes a contemporaneous, transitory increase in debt dilution. Second, a surprise decrease in interest rates causes a contemporaneous and highly persistent increase in dilution. Third, in line with the model's theoretical predictions, an unexpected increase in dilution causes a contemporaneous, somewhat persistent decrease in inflation at the cost of a gradual, highly persistent long-run rise in inflation. Fourth, dilutive effects do not fully resolve the "price puzzle" originally described in Sims (1992) as an unexpected monetary tightening causes a contemporaneous, transitory increase in inflation.

In summary, I not only introduce a theoretical, linearizeable object that allows for intuitive analysis of dynamic maturity management policy, but I also support theoretical predictions of this object's relation to inflation with analogous causal findings in U.S. post-war data.

2 The Dilution Rate of Government Debt

Consider the government's equilibrium flow government budget constraint (GBC) from papers by Cochrane (2001) and Leeper and Leith (2017) implied by the household budget constraint (HHBC), goods market clearing $C_t = Y_t$, and household optimization. Assume the special case where the government can freely borrow in $K = 2$ debt instruments and that its maximum available maturity is $J = 2$ periods to write the flow GBC as

$$\frac{B_{t-1}^{(t)}}{P_t} + \frac{Q_t^{(t+1)} B_{t-1}^{(t+1)}}{P_t} = \frac{Q_t^{(t+1)} B_t^{(t+1)}}{P_t} + \frac{Q_t^{(t+2)} B_t^{(t+2)}}{P_t} + s_t \tag{1}$$

where $B_t^{(t+j)}$ is outstanding j-period nominal debt with corresponding price $Q_t^{(t+j)}$, where P_t is the economy's aggregate price level, and where $s_t = \tau_t Y_t$ is government primary surplus.

Household optimization over $B_t^{(t+j)}$ determines the price of one-period debt and reveals the no-arbitrage condition $Q_t^{(t+2)} = Q_t^{(t+1)} \mathbb{E}_t Q_{t+1}^{(t+2)}$. Forward-iterate on the market value of debt to write the government's present value condition as

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}}_{MV(\text{Debt})/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[PV(\text{Prim. Spls.})]} \tag{2}$$

where the j-period real interest rate is defined to be $r_{t,t+j} = \beta^{-j} c_t^{-\sigma} / c_{t+j}^{-\sigma}$. The flow condition (2) says that the real market value of maturing and unmatured government debt must equal the expected discounted sum of current and future government primary surpluses.

The forward iteration outlined thus far is standard in models that examine fiscal policy's role in breaking Ricardian equivalence. Such forward-iteration is sufficient to express P_t alone as a function of current and expected future allocations when the debt takes the form of perpetuities with geometrically declining coupons, introduced in work by Woodford (2001).[3]

The government's present value condition (2) features an expectation including P_{t+1} within the price of unmatured long-term debt. Current and future debt variables $\left\{\left\{B_t^{(t+j)}, B_{t+1}^{(t+1+j)}, \cdots\right\}_j\right\}$ load into expectations about future price levels recursively; P_{t+1}'s determination includes expectations about P_{t+2} and so on. I express the government's present value condition so that P_t is exclusively a function of current and expected future allocations. When expectations about future debt policy are uncorrelated with those about primary surpluses, the present value condition (2) becomes

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}}_{(\text{Mat. Debt})/P_t} = \underbrace{s_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} \left(1 + \sum_{h=1}^{i} -\Psi_{t,t+h-1}^{-1}\right)}_{\mathbb{E}[PV(\text{Diluted Primary Surpluses})]} \tag{3}$$

where $\Psi_{t,t} = B_t^{(t+1)}/B_{t-1}^{(t+1)}$ is the economy's nominal dilution rate of government debt and where $\Psi_{t,t+i} = \left(B_t^{(t+1)} \cdots B_{t+i}^{(t+i+1)}\right) / \left(B_{t-1}^{(t+1)} \cdots B_{t+i-1}^{(t+i+1)}\right)$ is the i-period ahead product of dilution rates.[4]

The dilution rate relates the time t stock of outstanding short-term debt to the time $t-1$ stock of outstanding long-term debt. For some, it may be easier to think of the dilution rate as relating short-term to long-term debt outstanding, holding the date of maturity constant.

The rewritten present value condition (3) includes P_t as a function exclusively of current and expected future allocations. Writing the condition this way also transforms the government's time t debt supply decision from $\left(B_t^{(t+1)}, B_t^{(t+2)}\right)$ to $\Psi_{t,t}$. Though the algebraically relevant term in this rewritten condition is not the dilution rate, but the inverse dilution rate $\Psi_{t,t}^{-1}$. One can immediately see that the rewritten present value condition (3) collapses to the standard case when a government has no outstanding long-term debt $B_{t-1}^{(t+1)} = 0$. Additionally, it must be the case that the government is expected to issue a non-zero amount of short-term debt each period, or else $\Psi_{t,t+i}^{-1}$ becomes undefined for some i. Interpreting the inverse dilution rate directly is challenging, so I recommend thinking in terms of the (non-inverted) dilution rate and flipping the ratio when examining the algebra.

I name $\Psi_{t,t}$ the dilution rate because the government dilutes the real value of its unmatured long-term debt when it issues new short-term debt maturing at the same date, similar to a publicly

[3]This set of structures includes the 1-period-only case.

[4]Covariance terms make the rewritten present value condition (3) less straightforward while adding little expositional value. I relegate the general formula to its derivation in Appendix A.

traded company diluting its current market capitalization by issuing additional stock, reducing price per share.

To give a demonstration, a government enters time t with previously-issued long-term debt outstanding $B_{t-1}^{(t+1)} > 0$. The outstanding debt's market value is $Q_t^{(t+1)} B_{t-1}^{(t+1)}$, which includes expectations over P_{t+1}. Absent other changes to current or expected policy, a budget-neutral increase in $B_t^{(t+1)}$ and decrease in $B_t^{(t+2)}$ increases period $t+1$'s maturing debt and P_{t+1} as a result according to the present value condition (2). Household demand for debt maturing at $t+1$ falls with its real (deflated) payout, so that $Q_t^{(t+1)}$ falls. According to the present value condition (2), equilibrium prices are low when the market value of inherited debt is low. Short-term issuance dilutes long-term debt through $Q_t^{(t+1)}$, that dilution partially relieves government indebtedness at time t, and P_t is depressed. The mechanism here reflects a trade-off in the timing of inflation: surprise dilution reduces current prices at the cost of higher future prices.[5]

This exercise's effects are visible in the new present value condition (3): a surprise increase in $B_t^{(t+1)}$ and decrease in $B_t^{(t+2)}$ decreases $\Psi_{t,t}^{-1}$ and increases $\Psi_{t,t+1}^{-1}$ by less, increasing the effective discounted value of s_{t+1} and decreasing that of s_{t+2} also by less. The right-hand side increases and P_t falls.

In a steady state with $c_t = c$, $B_t^{(t+j)} = B^{(j)}$ and $s_t = s$, the rewritten present value condition (3) becomes

$$\frac{B^{(1)}}{P} = \left(\frac{1}{1-\beta}\right)\left(\frac{1}{1+\beta\Psi^{-1}}\right) s \tag{4}$$

The steady state present value condition (4) reveals a long-run restriction on the dilution rate that must hold to ensure equilibrium: $|B^{(1)}| > \beta |B^{(2)}|$. There must exist a sufficiently large amount of short-term debt (assets) to ensure equilibrium determinacy. Such a condition is easily met in U.S. data where most outstanding debt (Treasurys + reserves) matures within two years.

The Fed engages in open market operations when it implements rate policy, exchanging ultra-short-term debt (reserves) for long-term debt (U.S. Treasurys), among other assets. Additionally, the U.S. Treasury finances deficits by regularly auctioning off newly-issued debt to primary dealers, and it redeems debt from open markets periodically. The Treasury releases its auction schedule six months in advance, while the Fed frequently revises its open market operations strategies. Given

[5] A similar exercise is outlined in Cochrane (2023) in a three-period economy.

the nature of joint debt determination in the U.S., I hypothesize that dilutive policy effects largely derive from surprises to realized dilution rates rather than surprises to expected future dilution rates. As such, I determine whether innovations to the U.S.'s realized dilution rate affects macroeconomic outcomes and whether those effects match this section's theoretical predictions.

3 A Guiding Model

In this section, I describe a flexible-price economy augmented with rules governing monetary (interest rate) policy, fiscal (tax) policy and policy joint-implementation through the maturity structure of government debt. The model is meant to be a guide for the paper's empirical strategy.

Households choose consumption C_t and provide a fixed supply of labor to firms $N = 1$. Firms choose labor demand and face a tax rate τ_t on output, given a linear production function $Y_t = AN_t$. The resource constraint is $C_t = Y_t$, so consumption is constant at $C_t = A$ and the real interest is constant at β^{-1}. Monetary and fiscal rules are similar to those studied in the Fiscal Theory of the Price Level (FTPL) pioneered by Leeper (1991), Sims (1994) and Woodford (1995).

The government can borrow in short-term (1-period) and long-term (2-period) debt. To put the flow GBC (1) in terms of inflation rates rather than price levels, I define real (deflated) debt allocations $b_t^{(t+j)} = B_t^{(t+j)}/P_t$ and gross inflation $\pi_t = P_t/P_{t-1}$. Also, I define the one-period nominal interest rate i_t as the inverse of outstanding unmatured debt's price $Q_t^{(t+1)}$. The log-linearized present value condition (3) takes the form

$$0 = \hat{\pi}_t + (1-\beta)\,\hat{\tau}_t - \left(\frac{\beta}{1-\beta\psi^{-1}}\right)\left(\hat{\psi_t^{-1}} - \hat{i}_t\right) + \boldsymbol{E}_t + u_t^{GBC} \tag{5}$$

where $\hat{\pi}_t$, $\hat{\tau}_t$, and $\hat{i}_t$ are log-linearized inflation, nominal interest rate and tax rate, respectively, where $\psi_t = b_t^{(t+1)}/b_{t-1}^{(t+1)}$ is the economy's real dilution rate of government debt and $\hat{\psi_t^{-1}}$ is the log-linearized inverse dilution rate, and where $\boldsymbol{E}_t$ contains all expectations of variables dated time $t+1$ and beyond.[6] The shock u_t^{GBC} is mean-zero Gaussian. Inverse dilution rates without a t subscript ψ^{-1} relate to steady state values.

A tax rule determines the distortionary levy on household income as a linear function of model

[6] I derive this equilibrium condition in Appendix B.

variables, written as

$$\hat{\tau}_t = f_\tau\left(\hat{\pi}_t, \hat{\psi_t^{-1}}, \hat{i}_t\right) + u_t^{FP} \tag{6}$$

and a rate rule determines the nominal interest rate as a linear function of model variables, written as

$$\hat{i}_t = f_i\left(\hat{\pi}_t, \hat{\psi_t^{-1}}, \hat{\tau}_t\right) + u_t^{MP} \tag{7}$$

where u_t^{FP} and u_t^{MP} are Gaussian and mutually orthogonal with u_t^{IS}, u_t^{AS} and u_t^{GBC}.

Dilution is jointly determined by monetary and fiscal authorities who implement individual tax and interest rate policies. It is a linear function of model variables, written as

$$\hat{\psi_t^{-1}} = f_\psi\left(\hat{\pi}_t, \hat{i}_t, \hat{\tau}_t\right) + u_t^{\psi} \tag{8}$$

where u_t^{ψ} is mutually orthogonal with u_t^{GBC}, u_t^{FP} and u_t^{MP}.

Given tax policy (6), interest rate policy (7), and debt policy (8), inflation is determined by the present value condition (5).

4 A Structural Vector Autoregression

I use the model described in Section 3 to motivate a structural VAR at a quarterly frequency to ask three questions: 'How do monetary and tax policy affect dilution?,' 'How does dilution affect inflation?,' and 'Does the addition of dilution eliminate the 'price puzzle' effect from a monetary policy tightening?' As in work by Sims (1992), Bernanke and Blinder (1992), and Blanchard and Perotti (2002), policy decisions are made conditional on lagged information. Specifically, I disallow fiscal and monetary policy rules from being affected by contemporaneous shocks with the exception that the Fed may react to contemporaneous tax policy innovations. Debt policy responds to tax and interest rate shocks contemporaneously to implement monetary and fiscal policy.

Finally, I apply the present value condition (5) to allow inflation $\hat{\pi}_t$ to be affected by contemporaneous shocks to each variable. This leaves me with a recursively identified impact matrix. The full SVAR is written as

$$\boldsymbol{z}_t = \boldsymbol{B}_0^{-1}\boldsymbol{C} + \underbrace{\boldsymbol{B}_0^{-1}\boldsymbol{B}\left(L\right)}_{\boldsymbol{A}(L)}\boldsymbol{y}_{t-\rho} + \underbrace{\boldsymbol{B}_0^{-1}\boldsymbol{u}_t}_{\boldsymbol{v}_t} \tag{9}$$

where $\boldsymbol{z}_t = \left[\hat{\tau}_t \; \hat{i}_t \; \hat{\psi_t^{-1}} \; \hat{\pi}_t\right]'$ and where $\rho = 4$ lags. I list the identification scheme in Table 1.

	u_t^{FP}	u_t^{MP}	u_t^{ψ}	u_t^{GBC}
$\hat{\tau}$	X			
$\hat{i}$	X	X		
$\hat{\psi^{-1}}$	X	X	X	
$\hat{\pi}$	X	X	X	X

Table 1: VAR identification strategy. An X indicates where a shock from the top row is allowed to contemporaneously affect a variable from the left column.

where an X indicates which shocks from the top row are allowed to contemporaneously affect variables from the left column.

Data from 1949Q1–2022Q4 is quarterly. I calculate tax rates as U.S. receipts over U.S. GDP. Interest rates are average NY Fed discount rates from 1949Q1–1954Q2 and average Fed Funds rates from 1954Q3–2022Q4. Inflation is annualized growth rates in NIPA's GDP deflator. Finally, the inverse dilution rate is calculated as previous-quarter long-term debt over short-term debt, where short-term debt is defined as (total reserves) + (outstanding Treasurys maturing within 1 quarter) and long-term debt is defined as (Treasurys maturing after 1 quarter). All data is seasonally adjusted and in terms of percentage log deviations from their mean. As in the theory, tax rates are net rates, and interest, dilution and inflation rates are in gross terms. A full data description is available in Appendix D.

5 Results

I apply the Minnesota prior from Sims's (1980) work to plot IRFs using the Matlab code made available by Ferroni and Canova (2025) using Ferroni and Canova's (2025) default hyperparameters. The full set of IRFs can be found in Appendix C.

I answer the first question, 'How do monetary and tax policy affect dilution?' using IRFs from fiscal u_t^{FP} (left panel) and monetary u_t^{MP} (right panel) shocks on inverse dilution in Figure 1. An unexpected one percent deviation from the mean in tax rates causes a contemporaneous and transitory jump of less than one percent deviation from the mean in inverse dilution in the same direction at the 68% credible level. Conversely, an unexpected one percent deviation from the mean in interest rates causes a contemporaneous and highly persistent jump of almost a 4 percent deviation from the mean in inverse dilution in the same direction, lasting for 32 quarters at the 90%

credible level.

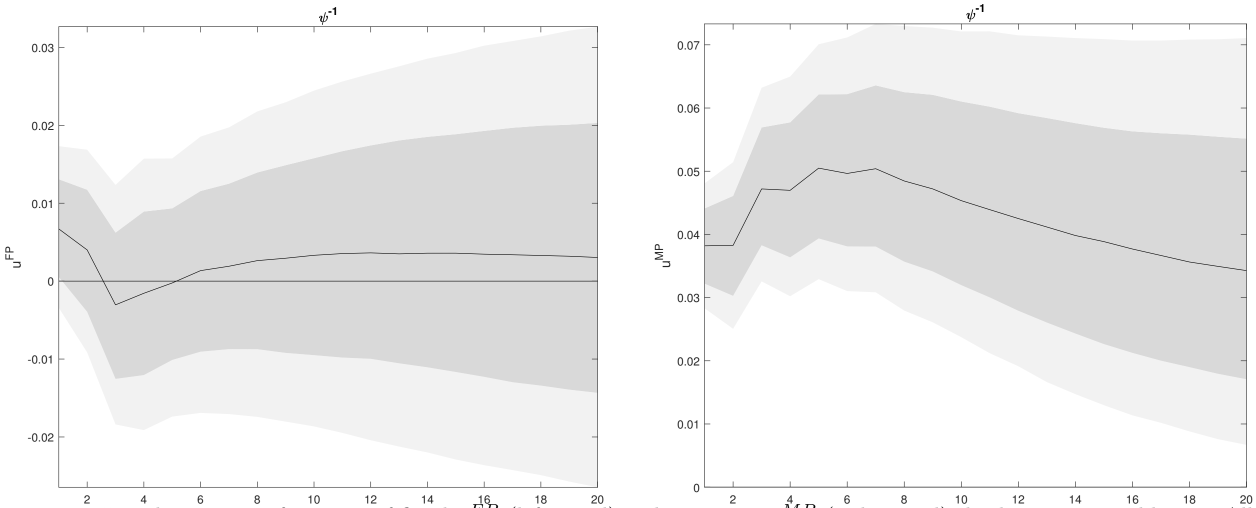

Figure 1: Impulse response functions of fiscal u_t^{FP} (left panel) and monetary u_t^{MP} (right panel) shocks on inverse dilution. All data in percentage deviations from mean. Dark and light bands represent 68% and 90% credible intervals, respectively. The response horizon is 20 quarters.

Government debt is used to intertemporally smooth taxes and inflation. Figure 1 uncovers a distinction in the way fiscal and monetary policy implements their policy through debt management.

When fiscal surpluses surprisingly fall, the joint government opts to temporarily dilute long-term debt holders by shifting the U.S.'s debt position short.

The Fed sells Treasurys (decreasing dilution) with surprise contractionary policy and buys Treasurys (increasing dilution) with expansionary policy. The joint government appears to follow suit because these relationships shine through in Figure 1's right panel.

I answer the second and third questions, 'How does dilution affect inflation?' and 'Does the addition of dilution eliminate the 'price puzzle' effect from a monetary policy tightening?' using IRFs from inverse dilution u_t^{ψ} (left panel) and monetary policy u_t^{MP} (right panel) shocks on inflation in Figure 2.

An unexpected one percent deviation from the mean in the inverse dilution rate causes between a 0.1 and 0.2 percent contemporaneous deviation from the mean in inflation in the same direction, lasting for two quarters at the 90% credible level. The same unexpected shock increases long-run inflation at the 68% credible level, doing so over quarters 22–112. Additionally, an unexpected one percent deviation from the mean in the Fed's policy rate causes between a 0.1 and 0.25 percent contemporaneous and transitory deviation from the mean in inflation in the same direction, also at the 90% credible level.

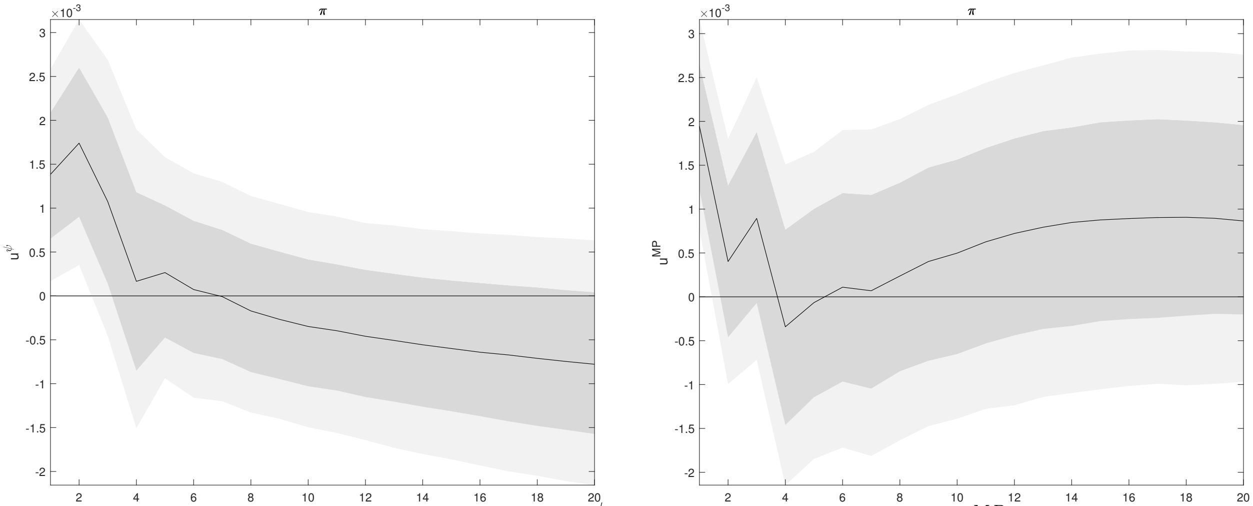

Figure 2: Impulse response functions of inverse dilution u_t^{ψ} (left panel) and monetary policy u_t^{MP} (right panel) shocks on inflation. All data in percentage deviations from mean. Dark and light bands represent 68% and 90% credible intervals, respectively. The response horizon is 20 quarters.

An increase in dilution (decrease in inverse dilution) reduces inflation in the short run and increases inflation in the long run – a result in line with Section 2's theoretical predictions. The exact timing of these effects suggests that surprise debt dilution may be an effective tool to fight inflation: the positive effects are immediate, while the negative effects only begin to surface in five years.

Despite this dilution result, the pesky 'price puzzle' plaguing modern macroeconomics remains: contractionary monetary policy has immediate and transitory inflationary consequences which are 90% credible in quarter 1. The selection of directional responses is listed in Table 2 in more detail.

Variable	Direction	Shock	Credible IRF Response Range (qtrs)	
			68%	90%
$\hat{\psi^{-1}}$	↑	u_t^{FP}	1	None
$\hat{\psi^{-1}}$	↑	u_t^{MP}	1–99	1–32
$\hat{\pi}$	↑	u_t^{MP}	1	1
$\hat{\pi}$	↑	u_t^{ψ}	1–3	1–2
$\hat{\pi}$	↓	u_t^{ψ}	22–112	None

Table 2: Statistically credible IRF ranges. This is the range during which a response of given variable in given direction is credible when responding to given shock. Shock occurs at quarter 1.

6 Conclusion

This paper introduces a new concept to the literature on public debt management. The dilution rate of government debt measures the amount of currently outstanding short-term debt to previously-issued, unmatured long-term debt. Such a concept is useful for three reasons. First, it can be widely applied to economic models with nominal debt. Second, it survives log-linear approximations common in much of modern macroeconomics. Third, it comes with clean interpretations and tractable algebra. Such an innovation improves on Woodford's (2001) constantly geometrically-declining structure because it allows for analysis of dynamic maturity responses to unexpected economic shocks.

I apply this concept to U.S. post-war data using a SVAR motivated by a flexible-price economy with inflation, fiscal policy, monetary policy and debt management policy. Four main findings arise. First, a surprise tax cut causes a contemporaneous, transitory increase in debt dilution. Second, a surprise interest rate cut causes a contemporaneous and highly persistent increase in dilution. Third, in line with the model's theoretical predictions, an unexpected increase in dilution causes a contemporaneous, somewhat persistent decrease in inflation at the cost of a gradual, highly persistent long-run rise in inflation. Fourth, dilutive effects do not fully resolve the "price puzzle" originally described in Sims (1992) – an unexpected monetary tightening causes a contemporaneous, transitory increase in inflation.

These results may spark new debate in government policy because they imply that open market purchases decrease inflation while open market sales increase inflation: QE contributes to suppressed prices during recessions wile QT counteracts contractionary monetary policy by increasing the price level. Could it have been the case that inflation was kept below the Fed's 2% target during the 2010s, at least in part, as a result of QE? Or that the Treasury suppressed inflation when it issued short-term debt in 2024 during QT? The results from this paper suggest so.

Moving forward, it may be useful to apply the dilution rate of government debt to existing theories of optimal debt management, such as those with complete markets economies like Buera and Nicolini (2004) and incomplete markets economics like Lustig, Sleet, and Yeltekin (2008), Debortoli, Nunes, and Yared (2017) and Faraglia et al. (2019). Such a re-interpretation may connect those theories to yet-to-come linearized theories of optimal debt responses to shocks.

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A Appendix: Deriving the Rewritten Present Value Condition

This section derives the rewritten present value condition (3) from the text, as in the paper by Cochrane (2001). I begin with the well-known present value condition (2):

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1}P_{t+1}}}_{MV(\text{Debt})/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[PV(\text{Prim. Spls.})]}$$

and subtract both sides by the market value of unmatured debt:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1}P_{t+1}}$$

then multiply the right-most term by $1 = B_t^{(t+1)}/B_t^{(t+1)}$ to write:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{B_{t-1}^{(t+1)}}{r_{t,t+1}B_t^{(t+1)}} \right) \frac{B_t^{(t+1)}}{P_{t+1}}$$

noticing that $B_t^{(t+1)}/P_{t+1}$ is $B_{t-1}^{(t)}/P_t$ iterated one period into the future.

Apply Section 2's definition of the nominal dilution rate to write:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{\Psi_{t,t}^{-1}}{r_{t,t+1}} \right) \frac{B_t^{(t+1)}}{P_{t+1}}$$

and forward-iterate once on this condition:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{\Psi_{t,t}^{-1}}{r_{t,t+1}} \right) \left[\sum_{i=0}^{\infty} \frac{s_{t+1+i}}{r_{t+1,t+1+i}} - \left(\frac{\Psi_{t+1,t+1}^{-1}}{r_{t+1,t+2}} \right) \frac{B_{t+1}^{(t+2)}}{P_{t+2}} \right]$$

which simplifies to:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \Psi_{t,t}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+1+i}}{r_{t,t+1+i}} + \mathbb{E}_t \left(\frac{\Psi_{t,t+1}^{-1}}{r_{t,t+2}} \right) \frac{B_{t+1}^{(t+2)}}{P_{t+2}}$$

and continuing this forward-iteration reveals:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \Psi_{t,t}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+1+i}}{r_{t,t+1+i}} + \mathbb{E}_t \Psi_{t,t+1}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+2+i}}{r_{t,t+2+i}} - \mathbb{E}_t \Psi_{t,t+2}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+3+i}}{r_{t,t+3+i}} + \cdots - \cdots$$

so that I can write:

$$\frac{B_{t-1}^{(t)}}{P_t} = s_t + \left(\mathbb{E}_t \frac{s_{t+1}}{r_{t,t+1}} - \Psi_{t,t}^{-1}\mathbb{E}_t \frac{s_{t+1}}{r_{t,t+1}}\right) + \left(\mathbb{E}_t \frac{s_{t+2}}{r_{t,t+2}} - \Psi_{t,t}^{-1}\mathbb{E}_t \frac{s_{t+2}}{r_{t,t+2}} + \mathbb{E}_t \Psi_{t,t+1}^{-1} \frac{s_{t+2}}{r_{t,t+2}}\right) + \cdots$$

where, starting with the term $\mathbb{E}_t \Psi_{t,t+1}^{-1} \frac{s_{t+2}}{r_{t,t+2}}$, expectations of future debt policy may be correlated with that of future primary surpluses. If we allow this covariance to be zero, we can freely factor $\Psi_{t,t+i}^{-1}$ out of every $\mathbb{E}_t \Psi_{t,t+i-1}^{-1} \frac{s_{t+i}}{r_{t,t+i}}$ term and group terms by s_{t+i} to write equation (3):

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}}_{\text{(Mat. Debt)}/P_t} = \underbrace{s_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} \left(1 + \sum_{h=1}^{i} -\Psi_{t,t+h-1}^{-1}\right)}_{\mathbb{E}[PV(\text{Diluted Primary Surpluses})]} \tag{3}$$

such that the household TVC holds and that the long-run condition $|B_t^{(t+1)}| > \beta |B_t^{(t+2)}|$ as $t \to \infty$ holds.

B Appendix: Deriving the Log-Linearized Present Value Condition

This section is dedicated to deriving the log-linearized present value condition (5) found in Section 3. I begin with the well-known present value condition (2):

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}}_{MV(\text{Debt})/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[PV(\text{Prim. Spls.})]}$$

and define real (deflated) debt as $b_t^{(t+j)} = B_t^{(t+j)}/P_t$ and gross inflation as $\pi_t = P_t/P_{t-1}$ to write this equation as:

$$\frac{1}{\pi_t}\left[b_{t-1}^{(t)} + \frac{b_{t-1}^{(t+1)}}{i_t}\right] = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}}$$

where the short-term nominal rate is $i_t = \mathbb{E}_t r_{t,t+1} \pi_{t+1}$.[7]

Then I use Section 3's definition of the real dilution rate ψ_t and the fact that real rates are constant $r_{t,t+1} = \beta^{-1}$ to write:

$$b_{t-1}^{(t)} = \pi_t \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i s_{t+i} - \frac{\psi_t^{-1}}{i_t} b_t^{(t+1)}$$

and continually forward-iterate on b_t^{t+1} to get:

$$b_{t-1}^{(t)} = \pi_t \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i s_{t+i} - \frac{\psi_t^{-1}}{i_t} \mathbb{E}_t \pi_{t+1} \sum_{i=0}^{\infty} \beta^{1+i} s_{t+1+i} + \frac{\psi_t^{-1}}{i_t} \mathbb{E}_t \frac{\psi_{t+1}^{-1}}{i_{t+1}} \pi_{t+2} \sum_{i=0}^{\infty} \beta^{2+i} s_{t+2+i}$$
$$- \frac{\psi_t^{-1}}{i_t} \mathbb{E}_t \frac{\psi_{t+1}^{-1} \psi_{t+2}^{-1}}{i_{t+1} i_{t+2}} \pi_{t+3} \sum_{i=0}^{\infty} \beta^{3+i} s_{t+3+i} + \cdots - \cdots \tag{10}$$

I examine a constant inflation, surplus and dilution steady state where $\pi_t = \pi = 1$, $s_t = s$, and $\psi_t = \psi$ to write:

$$b_{t-1}^{(t)} = \left(\frac{1}{1-\beta}\right) s - \beta \psi^{-1} \left(\frac{1}{1-\beta}\right) s + \beta^2 \psi^{-2} \left(\frac{1}{1-\beta}\right) s - \beta^3 \psi^{-3} \left(\frac{1}{1-\beta}\right) s + \cdots - \cdots$$

[7] Solving the model will reveal a tight connection between the nominal interest rate and debt maturity management. Both must be consistent with the same expectation $\mathbb{E}_t[\pi_{t+1}]$ according to the Fisher equation and the time $t+1$ intertemporal condition. In a richer model with real and nominal rigidities, such a restriction loosens.

which simplifies to the real-valued version of (4):

$$b_{t-1}^{(t)} = \left(\frac{1}{1-\beta}\right)\left(\frac{1}{1+\beta\psi^{-1}}\right)s$$

so long as $|\psi| < \beta^{-1}$ holds.

From here, notice that ψ_t^{-1}/i_t can be factored out of all but the first term on the RHS of (10), and that π_t only affects the first term on the RHS of (10). I write (10) in terms of log-deviations, treating $b_{t-1}^{(t)}$ as a constant:

$$0 = (1+\hat{\pi}_t)\left(\frac{1}{1-\beta}\right)s + (1+\hat{s})\,s - \left(1+\hat{\psi_t^{-1}}\right)\left(1-\hat{i}_t\right)\beta\left(\frac{1}{1-\beta}\right)\left(\frac{1}{1-\beta\psi^{-1}}\right)s + \mathbb{E}_t$$

where $\mathbb{E}_t$ includes only terms dated from $t+1$ onward.

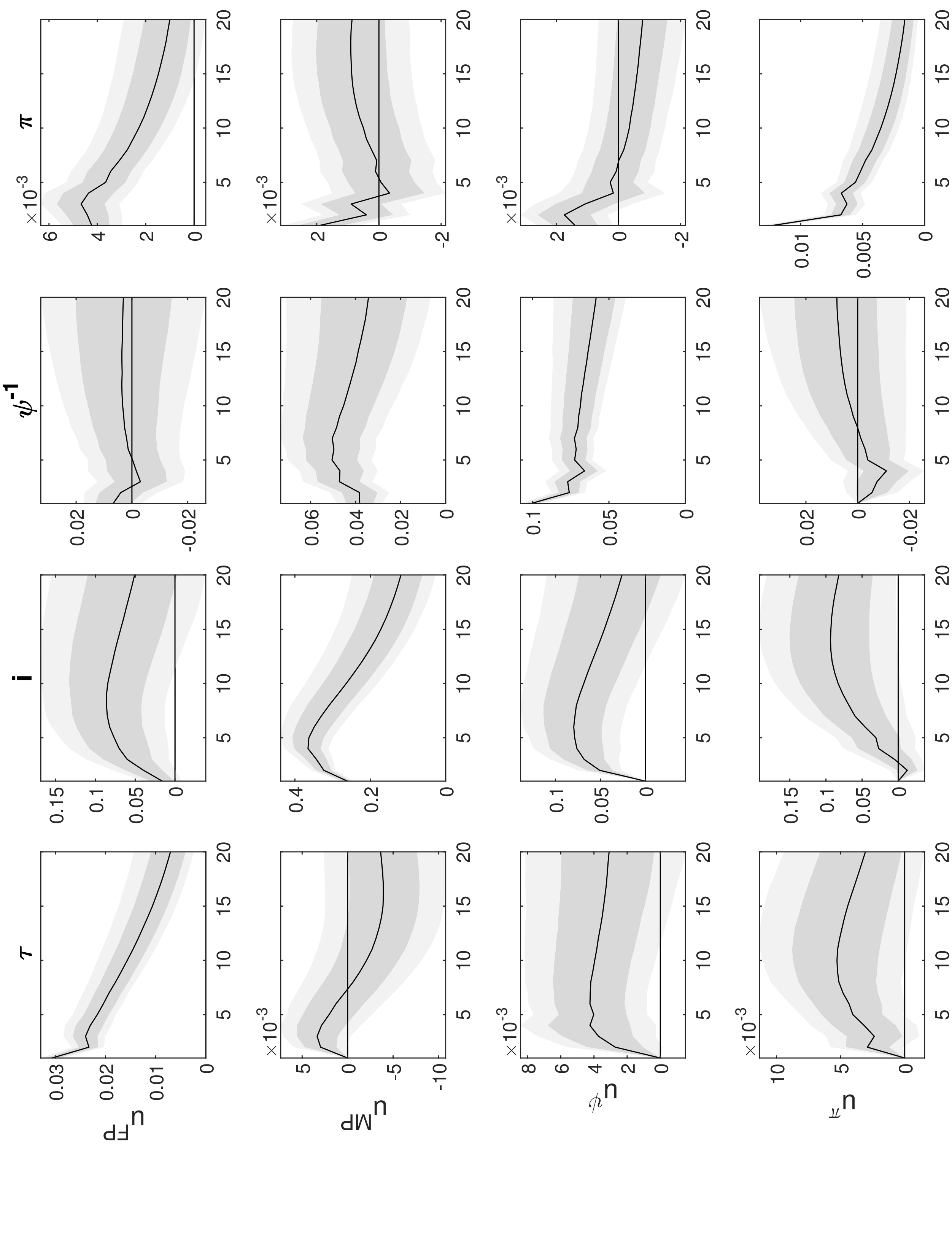
Approximating second order terms to zero and applying the steady state condition (4) yields:

$$0 = \left(\frac{1}{1-\beta}\right)s\hat{\pi}_t + s\hat{s} - \beta\left(\frac{1}{1-\beta}\right)\left(\frac{1}{1-\beta\psi^{-1}}\right)s\left(\hat{\psi_t^{-1}} - \hat{i}_t\right) + \mathbb{E}_t$$

where dividing everywhere by s, multiplying everywhere by $1-\beta$, substituting $\hat{\tau}_t$ for $\hat{s}_t$ and adding the shock u_t^{GBC} yields log-linearized equation (5) in Section 3:

$$0 = \hat{\pi}_t + (1-\beta)\,\hat{\tau}_t - \left(\frac{\beta}{1-\beta\psi^{-1}}\right)\left(\hat{\psi_t^{-1}} - \hat{i}_t\right) + \boldsymbol{E}_t + u_t^{GBC} \tag{5}$$

C Appendix: Complete IRF Listing



D Appendix: Data Sources and Construction

This paper uses four main time series from 1949Q1–2022Q4: U.S. tax rate, Fed's policy rate, U.S. real debt dilution rate, and U.S. price levels. Their construction and data sources are below:

- U.S. Tax Rate, constructed as total U.S. receipts over U.S. GDP:
  - Receipts: Section 3, table T30200-Q, line 40 in the National Income and Product Accounts (NIPA) from the BEA and found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`.
  - GDP: Gross domestic product. Section 1, table T10105-Q, line 1 in the National Income and Product Accounts (NIPA) from the BEA. Found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`.
- Fed's Policy Rate, average NY Fed discount rates from 1949Q1–1954Q2 and average Fed Funds rates from 1954Q3–2022Q4:
  - NY Fed discount rates: National Bureau of Economic Research, Release: NBER Macrohistory Database. `https://fred.stlouisfed.org/series/M13009USM156NNBR`
  - Fed Funds effective rate: Source: Board of Governors of the Federal Reserve System (US), Release: H.15 Selected Interest Rates. `https://fred.stlouisfed.org/series/FEDFUNDS`
- U.S. Inflation (1943–2023)
  - Growth rate in annual GDP deflator. Section 1, table T10109-A, line 1 in the NIPA from the BEA. Found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`
- U.S. Real Dilution Rate
  - Constructed as previous-period [(long-term debt outstanding)/(price level)] divided by [(short-term debt outstanding)/(price level)].
  - Long-term debt is Privately-Held Treasurys maturing in more than one quarter.

- Short-term debt calculated as (Reserves Outstanding) + (Privately-Held Treasurys maturing in less than one quarter).
    * Privately-Held Debt (1948Q4–2022Q4). Center for Research in Security Prices (CRSP) U.S. Treasury Database
    * Reserves Outstanding (1949Q1–2002Q4). Calculated as the average reserves outstanding/GDP from 2002–2022 multiplied by GDP from 1942–2001. Member Bank Reserve Account. Source: Center for Financial Stability Release: The Federal Reserve System's Weekly Balance Sheet Since 1914. `https://fred.stlouisfed.org/series/LDMB`
    * Reserves Outstanding (2003Q1–2022Q4). Calculated as (Federal Reserve Notes, Net of F.R. Bank Holdings) + (Deposits with F.R. Banks, Other Than Reserve Balances) + (Other Deposits at the Fed) + (Term Deposits Held by Depository Institutions) - (U.S. Treasury, Supplementary Financing Account) - (Treasury balance in TGA) + (Reverse Repurchase Agreements).
        · Federal Reserve Notes, Net of F.R. Bank Holdings. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLFN`.
        · Deposits with F.R. Banks, Other Than Reserve Balances. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WDFOL`.
        · Other Deposits at the Fed. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLODL`.
        · Term Deposits Held by Depository Institutions. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/TERMT`.
        · U.S. Treasury, Supplementary Financing Account. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLSFAL`.
        · Treasury balance in TGA. Daily Treasury Statements, found at `https://fsapps.`

fiscal.treasury.gov/dts/issues.

- Reverse Repurchase Agreements. Table H.4.1.T5 on the Fed's weekly balance sheet and found at https://fred.stlouisfed.org/series/WLRRAL.