Debt Maturity Management Under Fiscal-Monetary Bargaining

Joe Anderson

University of Virginia

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Abstract

This paper develops a theoretical framework in which fiscal and monetary authorities interact strategically to determine the optimal choices of taxes, inflation, and the maturity structure of nominal debt to finance government spending. Fiscal and monetary policymakers are separate entities with potentially distinct objectives. Greater fiscal bargaining power leads to higher inflation and lower tax rates in the baseline case. This bargaining approach rationalizes large differences in U.S. inflation experiences, such as those observed post-GFC and post-COVID. I impute historical U.S. fiscal-monetary bargaining power, documenting substantial spikes in fiscal power in 1950, throughout the 1970s, and after COVID. Of the three, the recent post-COVID spike best approximated first-best outcomes because surprise inflation is more effective at financing highly-indebted governments.

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1 Introduction

The U.S. government combines inflation and explicit taxation to finance its debt portfolio. Explicit taxes directly raise revenue while inflation erodes nominal debt's real value. Figure 1 displays U.S. primary surplus/GDP and inflation rates over the last 110 years, highlighting the four largest fiscal expansions: WWI, WWII, the Great Financial Crisis (GFC) and COVID.

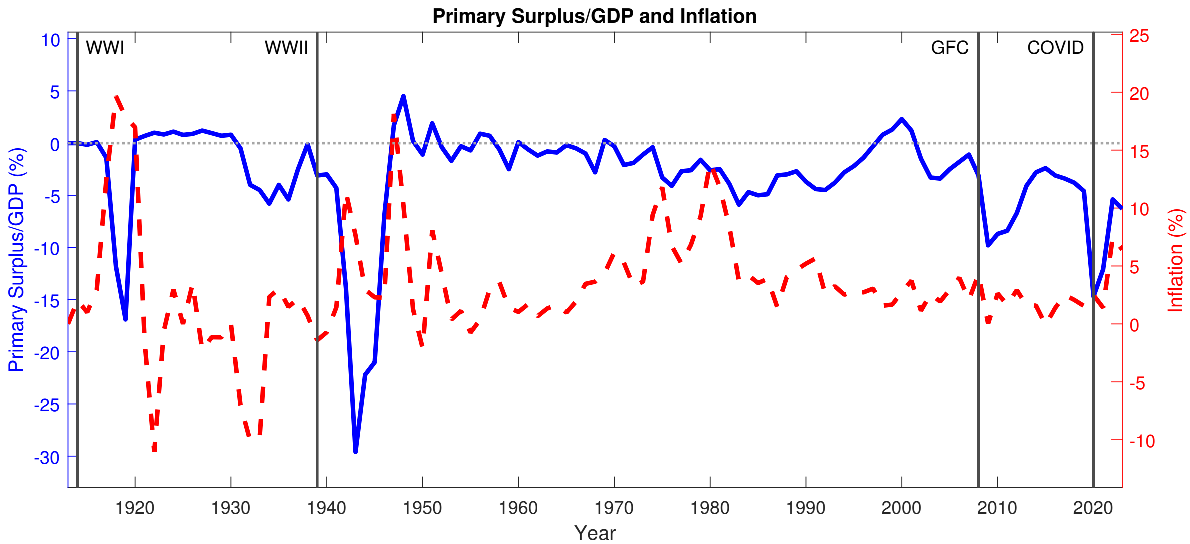

Figure 1: U.S. primary surplus/GDP and inflation rates (1912-2023)

The tax/inflation financing mix varied widely across these episodes. Average primary deficit/GDP was 2.1% in the years following WWI, 5.1% following the GFC, and much higher at 8.2% and 9.6% after WWII and COVID, respectively. Average inflation rates of around 5% eroded debt in the aftermath of WWI, WWII and COVID while inflation was below 2% after the GFC.[1] Figure 2 compares the cumulative change in prices following each event. Twelve-year cumulative price changes after WWI (77% increase) and WWII (65% increase) reach considerably higher levels than those after the GFC (24% increase). The four-year cumulative price change after COVID (19% increase) is close to surpassing the twelve-year one after the GFC.

When it comes to trading off debt financing between explicit taxes and surprise inflation, the U.S. employs a particular arrangement: fiscal policymakers manage tax policy while operationally independent central bankers focus on price stability. As a result, fiscal authorities (Congress/Treasury) and the Federal Reserve (Fed) jointly-determine the tax/inflation financing mix while disparately

[1] Following Hall and Sargent (2022), these averages are taken at twelve year horizons beginning from 1914, 1939 and 2008, with COVID's window shortened to a four year horizon starting from 2020.

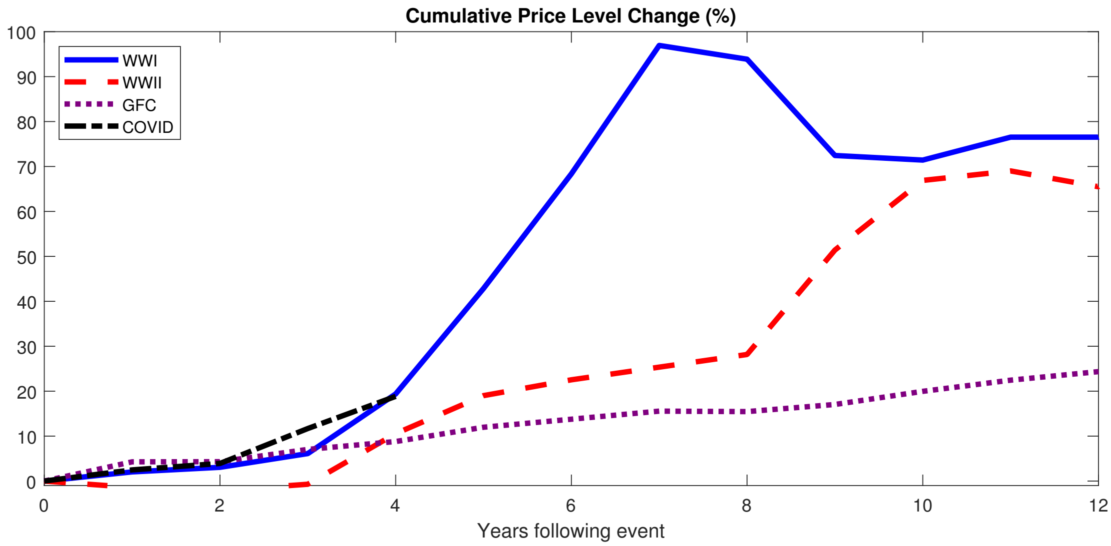

Figure 2: Cumulative price level changes. Year 0 represents 1913, 1938, 2007 and 2019 for WWI, WWII, the GFC and COVID, respectively.

internalizing the political costs associated with such a trade-off. How do operationally independent institutions arrive at a single mix? And what conditions lead such institutions to evaluate this trade-off in a manner consistent with that of the private sector? Taking the U.S.'s institutional structure as given, I propose a framework that explains how relative fiscal and monetary bargaining power shapes the economy's financing mix. Greater central bank power leads to lower inflation, while a stronger fiscal debt-manager results in lower taxes.

The government uses the maturity structure of public debt to intertemporally smooth the financing mix. The Treasury and Fed are the two biggest players in Treasury markets. The Treasury issues new debt at various maturities to finance deficits; the Fed executes open market purchases (sales) by issuing (redeeming) interest-bearing reserves to purchase (sell) long-term debt on secondary markets. Figure 3 illustrates how both policy authorities individually impact the maturity structure. The average maturity of privately-held debt (solid black line) moves in relation to the average maturities of outstanding Treasury-issued debt (dashed blue line), Fed-held debt (dotted red line) and Fed-issued reserves and currency (dot-dashed red line at 0 maturity). The privately-held maturity structure shortened when the Treasury shortened its issuance in both 2008-2009 and 2020. It also shortened from 2011-2014 when the Fed lengthened its portfolio via open market purchases as part of its QE3 program, despite mildly longer Treasury issuance.

Treasury officials often repeat two main debt management objectives when asked about issuance

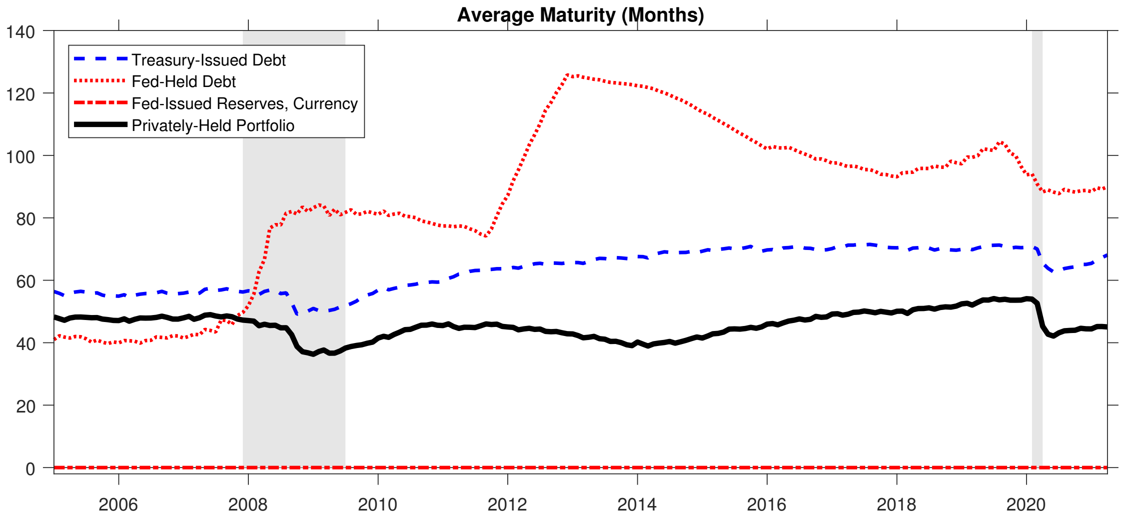

Figure 3: Treasury-issued, Fed-held, Fed-issued and net outstanding average maturities in the U.S. (2005-2022)

strategy: remaining 'regular and predictable' and 'minimizing borrowing costs.' 'Regular and predictable' refers to slow policy adjustments and advanced issuance notice given to primary market participants. 'Minimizing borrowing costs' includes facilitating demand for debt through communication with, and issuance strategies for, the Treasury Borrowing Advisory Committee (TBAC) - a consortium of private banking executives.

The Fed conducts monetary policy according to objectives outlined in the Federal Reserve Act: maximize employment, stabilize prices and maintain moderate long-term interest rates. There is no direct debt-management directive in its charter, which Fed Governor Christopher Waller reiterated in 2021, "Deficit financing and debt servicing issues play no role in our policy decisions and never will" [Waller (2021)]. This quote starkly contrasts with the Treasury's goal of 'minimizing borrowing costs,' highlighting differing institutional priorities on economic outcomes. The Treasury and Fed independently pursue individual objectives, manipulating the same maturity structure and financing mix.

Greenwood et al. (2015) argue the Treasury partially neutralized Fed QE efforts after the GFC through longer newly-issued debt. Miran and Roubini (2024) claim the Treasury offset the Fed's recent post-COVID quantitative tightening efforts by issuing large amounts of short-term debt. Both papers conclude that the institutions acted non-cooperatively during these periods. I push back on these claims: offsetting institutional debt positions do not necessarily imply non-cooperation. For many reasons, the Fed and Treasury might choose, cooperatively or not, to manage the structure in the way they did. The relevant question is: "How does non-cooperative institutional behavior influence macroeconomic outcomes under a rich maturity structure of public debt?" We are missing a theoretical framework that can address this question. This research develops such a framework and uses it to rationalize observed differences in historical financing mixes.

I add nominal debt to Lucas and Stokey (1983)'s economy, first considering a consolidated government before separating the government into two branches: a fiscal debt-manager that minimizes tax distortions and a central bank that minimizes inflation-driven welfare loss. Each version of the government needs to finance inherited obligations and random exogenous spending using a combination of costly inflation and labor taxes, intertemporally smoothing choices using a maturity structure of nominally non-contingent debt. The benevolent, committing, consolidated government implements complete markets outcomes when the maturity structure is sufficiently rich, as in studies with indexed debt like Angeletos (2002) and Buera and Nicolini (2004). Separate institutions simultaneously and non-cooperatively choose debt policy to minimize their individual financing burdens, potentially moving the economy away from first-best.

Regardless of government specification, current and expected future maturity management affects the economy's aggregate price level through the debt dilution rate of government debt, which measures the relative amount of newly-issued short-term debt maturing alongside previously-issued long-term debt. Unexpected debt dilution shifts current inflation into the future, altering its timing without changing its magnitude. An increase in expected future debt dilution increases contemporaneous inflation through the price of unmatured long-term debt. Contemporaneous, unmatured debt's value increases as a result of reduced inflation expectations, as nominal debt becomes more valuable when expected future aggregate prices fall. More valuable unmatured debt leads to a more indebted government, and higher current prices result.

The model reduces to a one-shot Nash game in state-contingent plans when both branches have the power to commit to future policy. Simultaneously moving players choose their tools subject to a single consolidated government budget constraint. Asymmetric bargaining pins down the baseline model's unique equilibrium and delivers a measure of an institution's relative strength.[2]

When relative monetary bargaining power is high, government debt is primarily financed through

[2]Equilibria when only one institution has commitment power are equivalent with those under two committing institutions where one institution holds unilateral bargaining power.

explicit taxes with surprise inflation playing a minor role. Conversely, when relative fiscal bargaining power is high, debt is primarily financed through inflation in order to keep tax rates low. In the baseline model, a powerful fiscal authority (high inflation) is considerably more costly than a powerful central bank (high taxes); first-best outcomes require a substantial amount of central bank bargaining power. More powerful fiscal policy is required to match first-best outcomes when social tolerance of inflation is high and when the government is highly indebted, relative to baseline. When households don't mind inflation, they are more willing to use it to keep distortionary taxes low. As inflation becomes costless (infinitely-costly), all-powerful fiscal (monetary) policy maximizes welfare. When the government inherits a large amount of debt, surprise inflation serves as a more powerful financing tool and is optimally used more. As the government inherits higher (lower) levels of debt, both tax and inflation rates rise (fall) across all bargaining power specifications. Ultimately, institutions' bargaining must align with household preferences to reach first-best.

Recent political proposals recommend that the president be personally consulted on all rate decisions, while others suggest strengthening Treasury oversight of the Fed, as reported by Restuccia, Timmons, and Leary (2024). Donald Trump's "Project 2025" outlines a proposal for elected officials to select the Fed's inflation target. Bargaining outcomes in the model allow for direct evaluation of these debates. Because interior bargaining outcomes rely on the central bank's operational independence from the fiscal policymaker, such a central bank provides a critical buffer against these unilaterally powerful fiscal outcomes.

I identify time-variation in intra-governmental bargaining power and use it to rationalize large observed differences in inflation outcomes from figures 1 and 2. For instance, to rationalize post-GFC and post-COVID inflation rates, given their respective debt structures, the model requires a post-COVID fiscal debt-manager with more than ten times the bargaining power compared to the post-GFC debt-manager.

Finally, I compare the bargaining time series to Drechsel (2024)'s data on interactions between the president and Fed officials. Fiscal power and president-Fed hours began to rise in the mid-1960s, spiked multiple times in the 70s, and remained low from the 90s until 2008, when president-Fed interactions ceased to be tracked by the White House. An additional spike in fiscal power occurred after the brief COVID recession. Compared to model-based first-best outcomes, the U.S. exhibited too much fiscal power in the 70s and too *little* fiscal power from 2009 onward (including after

COVID). The same amount of surprise inflation creates more government financing in high-debt economies relative to low-debt ones, so first-best outcomes require more fiscal power during periods of high government indebtedness.

2 Model

2.1 Model Environment

The model closely resembles Lucas and Stokey (1983). Consider an infinitely-lived flexible-price economy where discrete periods are indexed by $t \in \{0, 1, 2, \cdots\}$. Three types of agents inhabit the model: households, a debt-manager and a central bank.

A measure-1 continuum of identical price-taking households consume c_t and produce an aggregated good in every period equal to their labor supply n_t. Households own the economy's production technology, and their labor income is taxed at rate $\tau_t \in [0, 1)$. They lend (borrow) using a portfolio of nominal government bonds $B_t = \left\{B_t^{(t+j)}\right\}_j$, where $j \in \{j_1 = 1, j_2, \cdots, j_{K-1}, j_K = J\}$ represents a bond's term to maturity for $k \in \{1, \cdots, K\}$. K represents the number of debt instruments issued each period and J represents the maximum maturity of the portfolio. $K = J$ when each maturity is separated from the previous maturity by one time period, or $j_k - j_{k-1} = 1$ for all k. Otherwise, $J > K$.

Define $P_t > 0$ as the aggregate price level, which represents the exchange rate between nominal objects (hereafter referred to as 'dollars') and the numeraire good. Maturing bonds $B_t^{(t)}$ $\forall t$ each pay 1 dollar at maturity in all states of the economy. Finally, call $\pi_t = \frac{P_t}{P_{t-1}}$ the economy's gross inflation rate at time t.

A household's welfare is defined as the sum of discounted expected utility over its lifetime:

$$W_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{u\left(c_t\right) - v\left(n_t\right) - w\left(\pi_t\right)\right\} \quad \beta \in (0, 1) \tag{1}$$

where u, v and w are twice-differentiable, $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$ for $c_t, n_t > 0$, $\lim_{c_t \to \infty} u'\left(c_t\right) = 0$, $\lim_{n_t \to \infty} v'\left(n_t\right) = \infty$, $\lim_{c_t \to 0+} u'\left(c_t\right) = \infty$, $v'(0) = 0$, and where w is minimized at 1.

The government purchases g_t of the consumption good each period according to an exogenous

Markov process. The process is an S-state Markov chain with transition matrix $\mathcal{P}$. Call the vector of spending $g \equiv \{g(s)\}_s$ where $s \in \{1, \cdots, S\}$.

Total production is consumed by households and the government to yield the aggregate resource constraint (ARC):

$$n_t = c_t + g_t \quad \forall\ t \tag{2}$$

The government is split into two branches: the debt-manager and central bank. Each institution aims to maximize the sum of discounted expected household utility under a reweighting of utility components. The debt-manager chooses the issued supply of nominal government debt across maturities $\boldsymbol{B}_t^{dm} = \left\{\boldsymbol{B}_t^{(t+j),dm}\right\}_{j=1}^{J}$ and the distortionary labor income tax rate at each period to maximize its payout:

$$W_0^{dm} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\left(1-\rho^{dm}\right)\left[u\left(c_t\right) - v\left(n_t\right)\right] - \rho^{dm} w\left(\pi_t\right)\right\}\ ,\ \rho^{dm} \in [0,1] \tag{3}$$

and the central bank simultaneously chooses its own debt demand $\boldsymbol{B}_t^{cb} = \left\{\boldsymbol{B}_t^{(t+j),cb}\right\}_{j=1}^{J}$ as well as π_t each period to maximize its payout:

$$W_0^{cb} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\left(1-\rho^{cb}\right)\left[u\left(c_t\right) - v\left(n_t\right)\right] - \rho^{cb} w\left(\pi_t\right)\right\}\ ,\ \rho^{cb} \in [0,1] \tag{4}$$

where $\boldsymbol{B}_t^{dm} \in \mathbb{R}^K$ and $\boldsymbol{B}_t^{cb} \in \mathbb{R}^K$ are otherwise unrestricted. The maturity structure of outstanding government debt at time t is therefore $\boldsymbol{B}_t = \boldsymbol{B}_t^{dm} - \boldsymbol{B}_t^{cb}$.

Debt markets clear when debt jointly-issued by government institutions is held by households at each maturity:

$$\boldsymbol{B}_t = B_t \quad \forall\ t \tag{5}$$

2.2 Market Structure

K asset markets exist in every period: one for each circulating debt instrument. Debt is exchanged at nominal prices $Q_t \equiv \left\{Q_t^{(t+j)}\right\}_{j=1}^{J}$ where $Q_t^{(t+j)}$ is the time t price of a bond that matures at time $t+j$. Households lend (borrow) $Q_t^{(t+j)}$ dollars in period t to receive (pay) 1 dollar in period $t+j$, so that $Q_t^{(t)} = 1 \ \forall t$.

Each household chooses $\{c_t, n_t, B_t\}_{t=0}^{\infty}$ to maximize (1) subject to its flow budget constraint:

$$P_t c_t + \sum_{j=1}^{J} Q_t^{(t+j)} \left(B_t^{(t+j)} - B_{t-1}^{(t+j)} \right) \leq P_t \left(1 - \tau_t\right) n_t + B_{t-1}^{(t)} \tag{6}$$

where the sum on the left side of (6) represents new household borrowing across maturities.

Household debt holdings are subject to limits that eliminate Ponzi schemes:

$$B_t^{(t+j)} \in \left[\underline{B}, \overline{B} \right] \quad \forall \ t, j \tag{7}$$

where debt limits $\underline{B}$ and $\overline{B}$ are set to be sufficiently large so that (7) imposes no additional restrictions.

The debt-manager chooses fiscal policy $\left\{ \tau_t, \boldsymbol{B}_t^{dm} \right\}_{t=0}^{\infty}$ to maximize (3). The central bank chooses monetary policy $\left\{ \pi_t, \boldsymbol{B}_t^{cb} \right\}_{t=0}^{\infty}$ to maximize (4). Both institutions are constrained by household optimization and the other institution's simultaneous policy choice, which is taken as given at time t. Both institutions are additionally constrained by (2) and (6), combined here as the consolidated government's budget constraint (GBC):

$$B_{t-1}^{(t)} + P_t g_t \geq \sum_{j=1}^{J} Q_t^{(t+j)} \left(B_t^{(t+j)} - B_{t-1}^{(t+j)} \right) + P_t \tau_t n_t \tag{8}$$

3 Commitment-Flexible Markov Perfect Equilibrium

3.1 Equilibrium Definition

Define $x_t \equiv \{g_t, B_{t-1}\}$ as the economy's state space entering period t. Define $\mathcal{H}_T \equiv \{g_t, \{\eta_t^i\}_i\}_{t=0}^{T}$ as the history of exogenous states and history of play from period 0 to T, where η_t^i is institution i's time t policy choice. Define $\bar{\mathcal{H}}_t$ as the set of all feasible $\mathcal{H}_t$ at time t after starting the economy at a given set of initial conditions x_0.

A time 0 institution may have a technology that permits it to commit to its future $t > 0$ policies, which may be contingent on $\mathcal{H}_t$. When an institution has access to this technology, it is a 'committing' institution. When it does not posses such a technology, as in Kydland and Prescott (1977) and Rogoff (1985), it sets time t policy contingent only on x_t. Such an institution is 'non-committing.' All future policy chosen by a committing institution is taken as an additional constraint set by its opponent regardless of commitment status.

This research employs a commitment-flexible Markov perfect equilibrium (CFMPE) framework, focusing exclusively on CFMPE in pure strategies.[3] The CFMPE is 'commitment-flexible' due to its ability to nest two separate equilibrium concepts contingent on institutional commitment combinations. Under two committing institutions, a CFMPE is a Nash equilibrium where state contingent policies are all chosen at time 0, taking as given what the other player has chosen. Under two non-committing institutions, the CFMPE is a Markov perfect equilibrium.[4]

In each period t, the debt-manager's action is a selection of fiscal policy $\eta_t^{dm} \equiv \left\{\tau_t, \boldsymbol{B}_t^{dm}\right\}$ and the central bank's action is a selection of monetary policy $\eta_t^{cb} \equiv \left\{\pi_t, \boldsymbol{B}_t^{cb}\right\}$. Institutions act simultaneously and before households, who then choose allocations $\eta_t^{hh} \equiv \{c_t, n_t, B_t\}$ given $\{x_t, \tau_t, \pi_t, P_t, Q_t\}$.

Index institutions by $i \in \{dm, cb\}$ and denote i's opponent by $-i$. When i is a non-committing institution, define its time t subgame perfect strategy as $\gamma^i\left(x_t, \gamma^i(\cdot), \gamma^{-i}(\cdot)\right) \equiv \eta_t^i = \eta^i\left(x_t, \gamma^i(\cdot), \gamma^{-i}(\cdot)\right)\ \forall t$. When i is a committing institution, define its time 0 strategy as $\gamma_0^i\left(x_0, \gamma^i(\cdot), \gamma^{-i}(\cdot)\right) \equiv \left\{\{\eta_t^i\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty} = \left\{\{\eta^i\left(\mathcal{H}_t, \gamma^i(\cdot), \gamma^{-i}(\cdot)\right)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}$ and its time $t > 0$ strategy simply as its previously-chosen $\gamma_t^i\left(\mathcal{H}_t, \gamma^i(\cdot), \gamma^{-i}(\cdot)\right) \equiv \eta_t^i = \eta^i\left(\mathcal{H}_t, \gamma^{-i}(\cdot)\right)$. Combine these strategies into i's unified strategy profile $\gamma^i(\cdot) \equiv \{\gamma_0^i(\cdot), \gamma_t^i(\cdot)\}$.

A pure strategy CFMPE consists of a debt-manager strategy $\gamma^{dm}(\cdot)$, a central bank strategy $\gamma^{cb}(\cdot)$, a household strategy $\gamma^{hh}\left(x_t, \gamma^{dm}(\cdot), \gamma^{cb}(\cdot)\right) = \eta_t^{hh}\left(x_t, \gamma^{dm}(\cdot), \gamma^{cb}(\cdot)\right)$, a pricing function for the aggregate price level $P_t = \gamma^P\left(x_t, \gamma^{dm}(\cdot), \gamma^{cb}(\cdot)\right)$ and a pricing function for the vector of bond prices $Q_t = \gamma^Q\left(x_t, \gamma^{dm}(\cdot), \gamma^{cb}(\cdot)\right)$ such that in every period t:

1. The household strategy $\gamma^{hh}(\cdot)$ maximizes (1) given $\gamma^{dm}(\cdot)$, $\gamma^{cb}(\cdot)$, $\gamma^P(\cdot)$ and $\gamma^Q(\cdot)$ while satisfying (6) and (7),

2. The debt-manager's strategy $\gamma^{dm}(\cdot)$ maximizes (3) given $\gamma^{hh}(\cdot)$, $\gamma^{cb}(\cdot)$, $\gamma^P(\cdot)$ and $\gamma^Q(\cdot)$ while satisfying (8),

[3]For expositional convenience, all future mentions of CFMPE will indicate CFMPE in pure strategies.

[4]Because the policy game is played within the competitive equilibrium framework in Barro (1979) and Lucas and Stokey (1983), the nested Nash equilibrium and Markov perfect equilibrium concepts may also be described as 'Nash competitive equilibrium' and 'Markov perfect competitive equilibrium.'

3. The central bank's strategy $\gamma^{cb}(\cdot)$ maximizes (4) given $\gamma^{hh}(\cdot)$, $\gamma^{dm}(\cdot)$, $\gamma^{P}(\cdot)$ and $\gamma^{Q}(\cdot)$ while satisfying (8),

4. The set of pricing equations $\gamma^{P}(\cdot)$ and $\gamma^{Q}(\cdot)$ clear all markets, satisfying (2) and (5).

Households, the debt-manger and the central bank all have complete information about each other's problems and the underlying government spending process $\{g, \mathcal{P}\}$, and all entities are fully rational.

3.2 Household Optimization

Households' optimality conditions ensure (6) and (8) hold as strict equalities, and that

$$1 - \tau_t = \frac{v'(n_t)}{u'(c_t)} \quad \text{and} \quad Q_t^{(t+j)} = \beta^j \mathbb{E}_t \left[\frac{u'(c_{t+j}) P_t}{u'(c_t) P_{t+j}} \right] , \tag{9}$$

leading to a no-arbitrage result in government debt when all maturities are traded:

$$Q_t^{(t+j)} = Q_t^{(t+1)} \mathbb{E}_t \left[Q_{t+1}^{(t+2)} \cdots Q_{t+j-1}^{(t+j)} \right] . \tag{10}$$

The expected return on a long-term bond equals the expected return from rolling over short-term debt for the duration of the long bond's life.

Equation (9) and properties of $u(\cdot)$, $v(\cdot)$ and β imply the following transversality condition on the real market value of government debt:

$$\lim_{i \to \infty} \left(\frac{\sum_{j=0}^{J-1} \beta^{j+i} \mathbb{E}_t \left[\frac{u'(c_{t+j+i}) P_t}{u'(c_t) P_{t+j+i}} \right] B_{t-1+i}^{(t+j+i)}}{P_t} \right) = 0 \tag{11}$$

3.3 Price Level Determination

To determine the equilibrium price level P_t, combine (2), (6) and (9), forward-iterate on the market value of government debt and apply (11) to write:

$$\underbrace{\frac{\sum_{j=0}^{J-1}\beta^j\mathbb{E}_t\left[\frac{u'(c_{t+j})P_t}{u'(c_t)P_{t+j}}\right]B_{t-1}^{(t+j)}}{P_t}}_{\text{MV(outstanding government liabilities)}/P_t} = \underbrace{\frac{1}{u'(c_t)}\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[u'(c_{t+i})c_{t+i}-v'(n_{t+i})n_{t+i}\right]}_{\mathbb{E}_t[\text{PV(primary surpluses)}]} \tag{12}$$

so that the aggregate price level adjusts to ensure that the real value of outstanding nominal debt (left side) is fully backed by the expected present value of government primary surpluses (right side).[5] Equation (12) holds regardless of government set-up.

3.4 The Dilution Rate of Government Debt

How do expected and unexpected debt decisions affect government financing? Cochrane (2001) and Leeper and Leith (2017) address this question by deriving general formulas under $J = K \in \mathbb{N}$. Unfortunately, due to the complexity of these general results, any intuition behind the answer has remained largely impenetrable. An important special case has yet to be explored. For the remainder of the analysis, specify a maturity structure such that $J = K = 2$ so that (12) becomes:

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}+\beta\mathbb{E}_t\left[\frac{u'(c_{t+1})B_{t-1}^{(t+1)}}{u'(c_t)P_{t+1}}\right]}_{\text{MV(outstanding government liabilities)}/P_t} = \underbrace{\frac{1}{u'(c_t)}\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[u'(c_{t+i})c_{t+i}-v'(n_{t+i})n_{t+i}\right]}_{\mathbb{E}_t[\text{PV(primary surpluses)}]} \tag{13}$$

where the nominal price of maturing debt is constant at 1, but where the nominal price of unmatured two-period (long-term) debt $Q_t^{(t+1)}$ includes expectations about the period-ahead price level P_{t+1}. Current and future debt variables $\{B_t, B_{t+1}, \cdots\}$ are loaded into expectations about future prices levels recursively; P_{t+1}'s determination includes expectations about P_{t+2} and so on. I aim to express (13) so that P_t is exclusively a function of current and expected future allocations.

Given $B_{t-1} > 0$, debt's market value increases as expectations about P_{t+1} fall; households' valuations rise when debt is expected to pay out more numeraire upon maturity. Time $t-1$ households purchased claims on the government's discounted revenue stream at time $t+1$ when

[5] Notice that $u'(c_{t+i})c_{t+i} - v'(n_{t+i})n_{t+i} = u'(c_{t+i})(\tau_{t+i}n_{t+i} - g_{t+i})$ from (2) and (9).

they bought long-term debt. In the interim (at time t), the government may dilute these claims by issuing one-period (short-term) debt which is also set to mature at time $t+1$.[6] Holding the expected primary surplus path (resources available to exchange for dollars at time $t+1$) fixed, time t dilution lowers P_t at the expense of higher future price levels (due to larger future stocks of maturing debt).

Unexpected dilution represents a transfer of resources from long-term bondholders to taxpayers: bondholders expected more available real resources at time $t+1$, which justified a high time $t-1$ price for long-term debt, but taxpayers are not asked to supply those resources ex-post upon new time t short-term issuance.[7] Naturally, expected future dilution reduces the price of newly-issued long-term debt.

One can transform (13) so that all expectations of aggregate price levels within $Q_t^{(t+1)}$ are substituted for expectations about future debt issuance. It is useful to first define the economy's **dilution rate of government debt** as $\frac{B_t^{(t+1)}}{B_{t-1}^{(t+1)}}$, or the ratio of short-term debt to previously-issued long-term debt where both are set to mature on the same date.

Appendix B displays the dilution rate of U.S. government debt since WWII. Dilution has been variable throughout the sample, with large spikes during the mid 70s, the Great Recession and COVID. Relative to the rest of the time series, the 90s saw low and stable dilution.

It is convenient to define the economy's inverse dilution rate as $a_t \equiv \frac{B_{t-1}^{(t+1)}}{B_t^{(t+1)}}$, where a large, positive a_t indicates only slight levels of debt dilution. When expectations about future debt policy are uncorrelated with those about primary surpluses, (13) becomes:

$$\underbrace{\frac{B_{t-1}}{P_t}}_{\text{(Maturing debt)}/P_t} = \underbrace{\frac{1}{u'(c_t)}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[u'(c_{t+i})\, c_{t+i} - v'(n_{t+i})\, n_{t+i}\right] \left(1 + \underbrace{\sum_{k=1}^{i} \prod_{h=1}^{k} -a_{t+h-1}}_{\text{Dilution modifier}}\right)}_{\mathbb{E}_t[\text{PV(modified primary surpluses)}]} \tag{14}$$

so that P_t is a function of current and future expected allocations.[8] Equation (14)'s more general formula under $J \geq K = 2$ is derived in appendix C.

It is clear from (13) that long-term debt is required for current and expected dilution to affect

[6] This is similar to a publicly traded company issuing additional shares, which dilutes the value of shares held by existing owners.

[7] Cochrane (2023) briefly discusses these ideas in Chapter 7, Section 3.

[8] I write (14) in its current form for expositional purposes only. Covariance terms make (14) less straightforward when expectations about future debt policy are correlated with those of future primary surpluses.

P_t. When the dilution modifier is zero $\left(B_{t-1}^{(t+1)} = 0\right)$, (14) reverts to the standard pricing equation in models without a maturity structure.

Given $B_{t-1}^{(t+1)} > 0$ and absent changes to expected primary surpluses, additional expected future debt dilution may either raise or lower P_t (compared to the $B_{t-1}^{(t+1)} = 0$ case) depending on its timing. As a comparative statics exercise, (14) confirms that P_t falls as time t dilution increases ($a_t \downarrow$). Conversely, an isolated increase to expected time $t+1$ dilution ($a_{t+1} \downarrow$) decreases P_{t+1}, which raises $Q_t^{(t+1)}$. Since the time t government's unmatured debt becomes more expensive, P_t rises. Both unexpected and expected dilution affects time t financing through the price of unmatured debt, so maturity management policy is more effective under larger $B_{t-1}^{(t+1)}$ (in absolute value).

There are two ways to interpret the dilution modifier relative to the short-term-only ($K = J = 1$) case. First, as a modifier on the path of real interest rates: when households expect the government to dilute its long-term debt in the future, they require higher rates on their lending. Second, as a modifier on the effective timing of government primary surpluses: when the government utilizes both short-term and long-term borrowing, it delays the final repayment of maturing debt, thereby inducing more discounting of future primary surpluses.

Equation (14) reveals an interesting bit of interpretation. *Maturing debt* is ultimately still the driving debt force in price determination. Decisions about outstanding, current, and expected future long-term debt affect P_t only through effective discounting of future primary surpluses. Maturing debt, however, must be equalized to the sum of these expected discounted surpluses via P_t.

While (14) is a special case of the formulas outlined in Cochrane (2001) and Leeper and Leith (2017), it is easier to interpret. Once the number of maturities exceeds $K = 2$, the analog of the inverse dilution rate becomes an inverted matrix rather than an inverted ratio, which complicates any exposition about maturity management's effects on government financing.

3.4.1 A Constant Dilution Rate

Seminal papers on the maturity structure of government debt, such as Angeletos (2002) and Buera and Nicolini (2004), outline the optimality of a constant maturity structure in the first-order Markov case described above under $K = S = 2$. The result is a constant dilution rate of government debt. What are the implications of such a dilution process?

When the government chooses a single dilution rate every period $a_t = a_{t+1} = a$, the dilution modifier in (14) becomes $\sum_{k=1}^{i}(-a)^k$. The right side of (14) therefore is unbounded when $|a| > \frac{1}{\beta}$. The constant maturity structure cannot be too skewed towards either long-term debt or long-term asset positions when the bounded stock of real maturing debt $\frac{B_{t-1}^{(t)}}{P_t}$ is finite.

Figure 4 explores the spectrum of feasible constant inverse dilution rates and their effects on the present value of the stream of expected modified discounted surpluses ('fiscal backing' using the language of Cochrane (2011)) relative to the case where only short-term debt is used ($a = 0$).

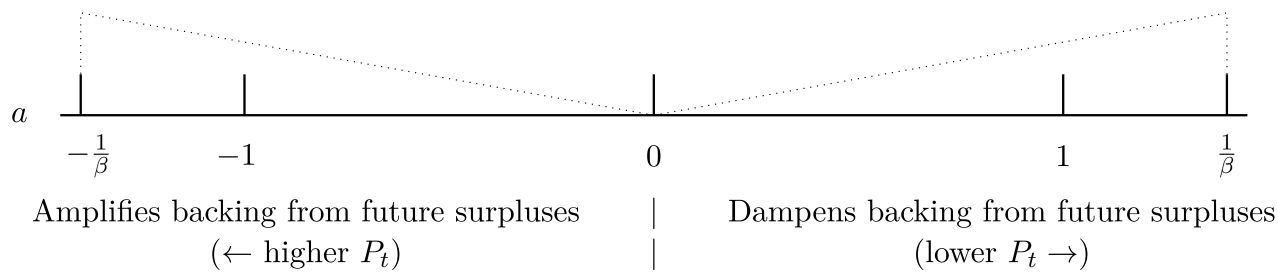

Figure 4: Dilutive effects on the price level relative to the $K = J = 1$ case

Price levels are higher (lower) when backing is amplified (dampened). Different inverse dilution rates imply different approaches to debt management. I review the debt-management approaches from examples marked in figure 4 under $B_{t-1}^{(t)} > 0$ in table 1.

Case	Description
$a = 1/\beta$	All new borrowing happens via long-term debt, using some of the borrowing to pay off a portion of last period's long-term debt prior to maturity.
$a = 1$	All new borrowing happens via long-term debt, where long-term debt becomes short-term debt and eventually runs off the balance sheet.
$a = 0$	Long-term debt is never issued and dilution is absent. This is a short-term-only ($K = J = 1$) economy.
$a = -1$	Government issues short-term debt and buys long-term assets in equal par values every period, financing spending, maturing debt and new long-term assets with long-term asset sales, short-term borrowing and taxes.
$a = -1/\beta$	Government buys more long-term assets than it borrows in short-term debt in par value terms every period, financing spending, maturing debt and new long-term assets with long-term asset sales, short-term borrowing and taxes.

Table 1: Special cases under a constant inverse dilution rate $a_t = a$ and $B_{t-1}^{(t)} > 0$

The result maps back to intuition from (13) under a constant maturity structure: given $B_t^{(t+1)} > 0$, a structure with positive constant dilution includes time $t-1$ long-term debt while one with negative constant dilution includes time $t-1$ long-term assets. When they share the same expected sum of discounted primary surpluses, price levels are lower for those with inherited long-term assets relative to those with inherited long-term debt.

I use the above results to reinterpret optimal maturity structure results described in Angeletos (2002) and Buera and Nicolini (2004) through the dilution lens in appendix H.3.

3.5 A Quasi-Primal Approach

Households derive utility from consumption, labor supply and inflation. Up until now, the analysis has focused on the relationships between nominal debt management, fiscal financing and the price level. To better align the model with agents' preferences, it is useful to convert equations from being in terms of aggregate price levels and nominal debt to instead being in terms of inflation rates and real debt allocations. Define a household's real (indexed) debt holdings maturing in period $t+j$ as $b_t^{(t+j)} \equiv \frac{B_t^{(t+j)}}{P_t}$ and the government's real debt supplied as $\boldsymbol{b}_t^{(t+j)} \equiv \frac{\boldsymbol{B}_t^{(t+j)}}{P_t}$, and define the vector of real debt allocations held by households as $b_t \equiv \left\{b_t^{(t+j)}\right\}_{j=1}^J$ and supplied by the government as $\boldsymbol{b}_t \equiv \left\{\boldsymbol{b}_t^{(t+j)}\right\}_{j=1}^J$. (13) can now be expressed as:

$$\frac{1}{\pi_t}\left\{u'\left(c_t\right)b_{t-1}^{(t)} + \beta\mathbb{E}_t\left[\frac{u'\left(c_{t+1}\right)}{\pi_{t+1}}\right]b_{t-1}^{(t+1)}\right\} = \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[u'\left(c_{t+i}\right)c_{t+i} - v'\left(n_{t+i}\right)n_{t+i}\right] \tag{15}$$

where (15) is called the economy's implementability constraint.

Any CFMPE must be a competitive equilibrium. Lucas and Stokey (1983) employ the primal approach to characterize a competitive equilibrium, which consists of substituting out all prices from the economy and writing the system in terms only of allocations. I employ a quasi-primal approach that follows Lucas and Stokey (1983) but includes π_t. I show that (2) and (15) are necessary and sufficient conditions for a competitive equilibrium.

Proposition 1 ***(competitive equilibrium)*** *A stochastic sequence* $\left\{\left\{c_t\left(\mathcal{H}_t\right), n_t\left(\mathcal{H}_t\right), g_t\left(\mathcal{H}_t\right),\right.\right.$ $\left.\left.\pi_t\left(\mathcal{H}_t\right)\right\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}$ *is a competitive equilibrium if and only if it satisfies* (2) $\forall\mathcal{H}_t$ *and* $\exists\left\{\left\{\left\{\boldsymbol{b}_t^{(t+j)}\left(\mathcal{H}_t\right)\right\}_{j=1}^J\right\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}$ *which satisfies* (15) $\forall\mathcal{H}_t$.

Proof: The proof can be found in appendix A.

Solving government institutions' problems subject only to (2) and (15) dramatically simplifies the analysis. Proposition 1 enables this simplification, showing that policy analysis subject to (2) and (15) is consistent with policy analysis subject to (2), (5), (6), (7), (8), (9) and (11).

Thanks to proposition 1, the implementability constraint (15) becomes the key to the game played between institutions. $\{c_t, n_t\}$ depends only on fiscal policy. $\{\pi_t\}$ depends only on monetary policy (inflation does not enter the ARC and is linearly separable in all agents' preferences). The implementability constraint (15) connects the two so that institutions' policies must be jointly consistent with (15). A debt-manager that commits to future tax policy constrains the central bank through (15). Symmetrically, a central bank that commits to future inflation policy constrains the debt-manager through (15). One can think of the institutional game being played *along* (15) so long as policy choices can be implemented using the maturity structure of debt.

3.6 Policy Feasibility

How does this model behave when pushed to extremes? Are there special cases that provide additional insight into potential outcomes from the institutional game? In which ways does this model connect to the large literature stemming from Lucas and Stokey (1983)? In which ways does it deviate?

Assume the number of exogenous spending states $S = 2$ for the rest of the analysis (and recall that we have already assumed $K = J = 2$) so that, in the spirit of Buera and Nicolini (2004)'s first order Markov case, the economy's time t nominal payout matrix A_t and wealth transfer vector Z_t are defined as:

$$\begin{aligned} A_t &\equiv \begin{bmatrix} \left(\frac{1}{\pi_t}|g_t = g(1)\right) & \beta \mathbb{E}_t \left(\dfrac{u'(c_{t+1})}{u'(c_t)\,\pi_t \pi_{t+1}}|g_t = g(1)\right) \\ \left(\frac{1}{\pi_t}|g_t = g(2)\right) & \beta \mathbb{E}_t \left(\dfrac{u'(c_{t+1})}{u'(c_t)\,\pi_t \pi_{t+1}}|g_t = g(2)\right) \end{bmatrix} \\ Z_t &\equiv \begin{bmatrix} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\frac{u'(c_{t+i})c_{t+i} - v'(n_{t+i})n_{t+i}}{u'(c_t)}|g_t = g(1)\right) \\ \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\frac{u'(c_{t+i})c_{t+i} - v'(n_{t+i})n_{t+i}}{u'(c_t)}|g_t = g(2)\right) \end{bmatrix} \end{aligned} \tag{16}$$

where A_t includes ex-post payouts from saving in debt of maturity j (column) upon entering state s (row). Equation (16) allows (15) in all exogenous states to be written as $A_t b_{t-1} = Z_t$, given b_{t-1}.

Angeletos (2002) and Buera and Nicolini (2004) prove that any path of taxes resulting in an non-singular A_t under $\pi_t = \pi = 1$ can be implemented using a sufficiently rich maturity structure, $K \geq S$, and that a unique maturity structure exists when $K = S$.

Consider a central bank which commits to zero net inflation (a constant price level) every period: $\left\{\{\pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}=\{1,1,\cdots\}$. Let $\mathcal{T}$ then be defined as:

$$\mathcal{T}\equiv\left\{\left\{\{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}:\det(A_t)\neq 0\ \forall t,\forall\mathcal{H}_t\in\bar{\mathcal{H}}_t \text{ and } \tau_t(\mathcal{H}_t)\in[0,1)\ \forall t,\forall\mathcal{H}_t\in\bar{\mathcal{H}}_t\right\}$$

but where tax paths in $\mathcal{T}$ do not necessarily need to satisfy (15) under $\pi_t=1\ \forall t$.

The central bank's ability to inflate or deflate inherited nominal debt (assets) allows for many feasible joint inflation/tax rate paths. The definition of $\mathcal{T}$ is sufficient to establish Proposition 2.

Proposition 2 ***(monetary revaluation, fiscal freedom)*** *Under committing monetary and fiscal policy and either $b_{-1}^{(0)}>0,b_{-1}^{(1)}<0$ or $b_{-1}^{(0)}<0,b_{-1}^{(1)}>0$, for any tax path $\left\{\{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}\in\mathcal{T}$, $\exists\left\{\{\pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}=\{\pi_0(x_0),\pi_1(x_0),1,1,\cdots\}$ and $\exists\left\{\left\{\left\{\boldsymbol{b}_t^{(t+j)}(\mathcal{H}_t)\right\}_{j=1}^{J}\right\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}$ such that (2) and (15) hold.*

Proof: The proof can be found in appendix A.

If the government inherits short-term debt maturing at time 0 and long-term nominal assets maturing at time 1, it is feasible to inflate the economy at time 0 (reducing the real burden of maturing debt) and deflate the economy at time 1 (increasing the lump-sum real payout from households to the government) so that the left side of (15) equals any real number.[9]

This type of revaluation can support any feasible path of tax rates: where $\tau_t\in[0,1)$ in every period and, to ensure a well-defined maturity structure, where consumption is never equated across spending states, $c_t(1)\neq c_t(2)$ in every period.[10]

To solve for the π_0 and π_1 that achieve this result, given a tax path in $\mathcal{T}$, rewrite (15) at time 0 simply as $\frac{1}{\pi_0}\left(\mathfrak{m}+\frac{1}{\pi_1}\mathfrak{n}\right)=\mathfrak{D}$. Time 0 inflation always satisfies $\pi_0=\frac{\mathfrak{m}}{\mathfrak{D}}+\frac{1}{\pi_1}\frac{\mathfrak{n}}{\mathfrak{D}}$. When the government

[9] This is also true if it inherits short-term assets and long-term debt.

[10] A consolidated, committing, benevolent government chooses such a path whenever $g(1)\neq g(2)$.

inherits short-term debt ($\mathfrak{m} > 0$) and long-term assets ($\mathfrak{n} < 0$), time 1 inflation needs to satisfy

$$\begin{cases} \pi_1 > -\frac{\mathfrak{n}}{\mathfrak{m}} \text{ if } \mathfrak{D} > 0 \\ \pi_1 = -\frac{\mathfrak{n}}{\mathfrak{m}} \text{ if } \mathfrak{D} = 0 \\ \pi_1 < -\frac{\mathfrak{n}}{\mathfrak{m}} \text{ if } \mathfrak{D} < 0 \end{cases}$$

where reversing the signs of $\mathfrak{m}$ and $\mathfrak{n}$ reverses the required inequalities for π_1.

When the promised tax path implies a positive expected present value of future primary surpluses and the government inherits short-term debt and long-term assets, period 1 inflation (that which applies only to long-term assets through $Q_0^{(1)}$) needs to be sufficiently high. When the government instead inherits short-term assets and long-term debt, period 1 inflation (that which applies now only to long-term debt) needs to be sufficiently low. These thresholds are reversed under a tax path that implies a negative expected present value of current and future primary surpluses. Corollary 1 addresses one such case.

Corollary 1 ***(zero tax path feasibility)*** *Under committing monetary and fiscal policy, either* $b_{-1}^{(0)} < 0$ *or* $b_{-1}^{(1)} < 0$ *and the tax path* $\left\{\{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty} = \{0, 0, \cdots\}$, $\exists \left\{\{\pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty} = \{\pi_0(x_0), \pi_1(x_0), 1, 1, \cdots\}$ *and* $\exists \left\{\left\{\left\{\boldsymbol{b}_t^{(t+j)}(\mathcal{H}_t)\right\}_{j=1}^{J}\right\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty}$ *such that* (2) *and* (15) *hold.*

Proof: The proof can be found in appendix A.

The central bank can support a debt-manager that commits to taxes equal to zero in every period so long as the government inherits either short-term or long-term assets. In the case of short-term inherited assets and long-term inherited debt (under $\mathfrak{m} + \mathfrak{n} = 0$), the central bank promises to inflate the economy at time 1 and deflates the economy at time 0. In the case of short-term debt and long-term assets (again under $\mathfrak{m} + \mathfrak{n} = 0$), the central bank promises deflation at time 1 and inflates at period 0. More generally, a perfectly-accommodating central bank trades off time 0 inflation's effects on $\mathfrak{m}$ with those on $\mathfrak{n}$ through the interaction of π_0 and π_1, which shows up in the price of unmatured debt.

When the government enters period 0 with both short-term and long-term assets, a perfectly-accommodating central bank has more freedom with respect to π_0 and π_1 adjustments to arrive at an asset valuation to support zero tax path. In all cases, the portfolio's resulting real value is used to finance all future spending without the need for tax revenue under a well-defined maturity implementation.

4 Three Period Model

In order to address the question, 'How does non-cooperative institutional behavior influence macroeconomic outcomes under a rich maturity structure of public debt?' in a tractable way, consider an analogous three-period model to the one described above. Time is indexed $t \in \{0,1,2\}$ with $g_1, g_2 \in \{g_\ell, g_h\}$ denoting low-spending and high-spending exogenous states. The economy encounters a low-spending regime in period 1 with probability p. It remains in the low- (high-)spending state in period 2 with certainty when it experiences the low- (high-)spending regime in period 1. This ensures the complete markets solution is feasible and implementable under a consolidated, committing, benevolent government with access to 2 debt objects.[11]

Indexing in the three period versions of (1), (3) and (4) matches $t \in \{0,1,2\}$ so that the implementability constraint (15) at time 0 becomes:

$$\frac{1}{\pi_0}\left\{u'\left(c_0\right) b_{-1}^{(0)} + \beta \mathbb{E}_0\left[\frac{u'\left(c_1\right)}{\pi_1}\right] b_{-1}^{(1)}\right\}$$
$$= \left[u'\left(c_0\right) c_0 - v'\left(n_0\right) n_0\right] + \beta \mathbb{E}_0\left[u'\left(c_1\right) c_1 - v'\left(n_1\right) n_1\right] + \beta^2 \mathbb{E}_0\left[u'\left(c_2\right) c_2 - v'\left(n_2\right) n_2\right] \tag{17}$$

4.1 First-Best: A Consolidated, Committing, Benevolent Government

A consolidated, committing, benevolent government chooses $\left\{\left\{\tau_t\left(\mathcal{H}_t\right), \pi_t\left(\mathcal{H}_t\right), \boldsymbol{b}_t\left(\mathcal{H}_t\right)\right\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ to maximize (1) subject to (2) and (17). Applying the quasi-primal approach described in section

[11] The three period model is exclusively considered for the remainder of the paper's main body. Infinite-period analysis can be found in appendix H.

3.5 transforms the problem so the government equivalently chooses $\left\{\left\{c_t\left(\mathcal{H}_t\right), n_t\left(\mathcal{H}_t\right), \pi_t\left(\mathcal{H}_t\right),\right.\right.$ $\left.\left.\boldsymbol{b}_t\left(\mathcal{H}_t\right)\right\}_{\mathcal{H}_t\in\bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$. Its FOCs imply:

$$c_t, n_t : u'\left(c_t\right)+\lambda_0\left[u'\left(c_t\right)+u''\left(c_t\right)\left(c_t-\frac{b_{-1}^{(t)}}{\pi_{t-1}\pi_t}\right)\right]=v'\left(n_t\right)+\lambda_0\left[v'\left(n_t\right)+v''\left(n_t\right)n_t\right]\ \ \forall\, t\in\{0,1,2\} \tag{18}$$

$$\pi_t : w'\left(\pi_t\right)\pi_t^2-\lambda_0\left\{\left[\frac{u'\left(c_t\right)}{\pi_{t-1}}\right]b_{-1}^{(t)}+\beta\mathbb{E}_t\left[\frac{u'\left(c_{t+1}\right)}{\pi_{t+1}}\right]b_{-1}^{(t+1)}\right\}=0\ \ \forall\, t\in\{0,1,2\} \tag{19}$$

where λ_0 is the Lagrange multiplier on (17) and where $\pi_{-1}=1$, $b_{-1}^{(2)}=0$.

The consolidated, committing, benevolent government's optimal choice of $\{\tau_t, \pi_t\}_{t=0}^{2}$ simultaneously balances three goals. The first is an intratemporal motive to align marginal welfare losses at time t between τ_t and π_t use. The second is an intertemporal motive to smooth expected welfare losses across time. Both of these motives take the amount of financing needed to satisfy (17) as given. The third goal is embedded in (18)'s and (19)'s terms which include b_{-1}: the government uses τ_t and π_t to lessen its financing burden by devaluing (appreciating) inherited debt (assets).

A straightforward way to demonstrate the third goal is to set $b_{t-1}=0$. The resulting FOCs read $\lambda_0\left[u'\left(c_t\right)+u''\left(c_t\right)c_t-v'\left(n_t\right)-v''\left(n_t\right)n_t\right]=v'\left(n_t\right)-u'\left(c_t\right)\ \forall t$ and $\pi_t=1\ \forall t$. Neither taxes nor inflation can relax (17) if there is no initial debt.

Using arguments from section 2.2.2 in Buera and Nicolini (2004), it is straightforward to see that the consolidated, committing, benevolent government implements the complete markets Ramsey (first-best) plan with a $J=2$ portfolio of non-contingent debt so long as $c_{1,\ell}\neq c_{1,h}$ along such a plan. This is because, so long as the ex-post price of unmatured debt is not identical across time 1 states, there exists a linear combination of non-contingent debt that exactly replicates the state-contingent household transfers required to achieve such a plan.

Debortoli, Nunes, and Yared (2017) show that this condition is not met (so that first-best is not implementable) when the consolidated, committing, benevolent government inherits $b_{-1}=0$.[12] They do so under indexed debt in a model where τ_t and n_t are exogenously fixed and where utility with respect to consumption is $\log\left(c_t\right)$. I extend their proof to include the class of models described until now where u and v are both standard CES. Lemma 1 is the result.

[12]This is the 'full persistence limit' limiting case in proposition 2.

Lemma 1 ***(non-implementation under $b_{-1} = 0$)*** *Under a consolidated, committing, benevolent government where $b_{-1} = 0$, where u satisfies $\frac{u''(c)c}{u'(c)} = \sigma \geq 0$, where v satisfies $\frac{v''(n)n}{v'(n)} = \varphi \geq 0$, and where either $\sigma > 0$ or $\varphi > 0$, no finite maturity structure b_0 can implement the government's welfare-maximizing choices $\{c_t, n_t, \pi_t\}_{t=0}^{2}$.*

Proof: This proof can be found in appendix A.

CES utility implies a constant tax rate $\tau_t = \tau \ \forall t$. $b_{-1} = 0$ implies $\pi_t = 1 \ \forall t$. Because $g_1 = g_2 \in \{g_s\}_s$, $c_1 = c_2$ with certainty so that $\left(Q_1^{(2)} | g_1 = g_\ell\right) = \left(Q_1^{(2)} | g_1 = g_h\right) = \beta$. No finite maturity structure exists to perfectly insure the economy as unmatured debt's price does not vary across time 1 spending regimes along the consolidated, committing, benevolent government's plan.

Debortoli, Nunes, and Yared (2017) avoids an undefined time 0 maturity structure by assuming spending states are non-absorbing. This choice disallows the consolidated, committing, benevolent government from implementing first-best because only one maturity is demanded at time 1 and cannot fully insure the government against aggregate time 2 uncertainty. This paper instead sets $b_{-1} \neq 0$ to avoid both non-implementation and an undefined time 0 maturity structure.

4.2 Baseline: A Committing Debt-Manager and Committing Central Bank

A committing debt-manager chooses $\left\{\{\tau_t(\mathcal{H}_t), \boldsymbol{b}_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ to maximize (3) and a committing central bank chooses $\left\{\{\pi_t(\mathcal{H}_t), \boldsymbol{b}_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ to maximize (4). Identical to the quasi-primal approach described in section 3.5, use (2) and (9) to convert the committing debt-manager's problem so that it instead chooses $\left\{\{c_t(\mathcal{H}_t), n_t(\mathcal{H}_t), \boldsymbol{b}_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ to maximize (3). Both are constrained by (2) and (17). Institutions' plans are chosen and committed to simultaneously at time 0 to produce a one-shot Nash game in state-contingent plans. To fix ideas, assume a committing debt-manager can individually satisfy (2) and (17) under a constant $\pi_t = \pi = 1$.

Set $\rho^{cb} = 1$ and $\rho^{dm} = 0$ so that the central bank is responsible for smoothing inflation while the debt-manager is in charge of smoothing tax distortions. Institutions need to finance the consolidated government and do so non-cooperatively, through the jointly-determined maturity structure must support the resulting financing plan.

The committing debt-manager's FOCs imply:

$$u'(c_t) + \lambda_0^{dm}\left[u'(c_t) + u''(c_t)\left(c_t - \frac{b_{-1}^{(t)}}{\pi_{t-1}\pi_t}\right)\right] = v'(n_t) + \lambda_0^{dm}\left[v'(n_t) + v''(n_t)\, n_t\right] \quad \forall\, t \in \{0,1,2\} \tag{20}$$

while the committing central bank's FOCs imply:

$$w'(\pi_t)\,\pi_t^2 - \lambda_0^{cb}\left\{\left[\frac{u'(c_t)}{\pi_{t-1}}\right] b_{-1}^{(t)} + \beta \mathbb{E}_t\left[\frac{u'(c_{t+1})}{\pi_{t+1}}\right] b_{-1}^{(t+1)}\right\} = 0 \;\; \forall\, t \in \{0,1,2\} \tag{21}$$

Expressions (20) and (21) are identical to (18) and (19) except for the replacement of λ_0 with λ_0^{dm} in (20) and that of λ_0 with λ_0^{cb} in (21).

Splitting the government into two optimizing institutions results in a second multiplier λ_0^{dm}, λ_0^{cb}. With no additional restrictions on the model, there are multiple equilibria: many policy mixes consistent with (2) and (17) as is stated more formally in the following proposition.

Proposition 3 ***(CFMPE sufficiency)*** *A CFMPE exists under two committing institutions if* $\exists \lambda_0^{dm} \notin \{-\infty, \infty\}$*,* $\exists \lambda_0^{cb} \notin \{-\infty, \infty\}$ *and* $\exists b_0(x_0) \notin \{-\infty, \infty\}$ *such that* (2)*,* (17) *and* $\tau_t \in [0,1)\ \forall t$ *are satisfied when the debt-manager follows* (20)*, the central bank follows* (21) *and such that* $\left\{b_0(x_0), \left\{b_1^{(2)}(\mathcal{H}_1)\right\}_{\mathcal{H}_1 \in \bar{\mathcal{H}}_1}\right\}$ *implements* $\left\{\{c_t(\mathcal{H}_t), n_t(\mathcal{H}_t), \pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$.

Proof: This proof can be found in appendix A.

So long as the debt-manager's and central bank's plans follow (20) and (21) while the ARC (2), the implementability constraint (17) and all $\tau_t \in [0,1)$ are satisfied under a finite maturity structure, the outcome is a CFMPE.[13] In such a scenario, given its opponent's state-contingent plan, no institution finds it desirable to deviate from its own state-contingent plan. Given its opponent's policy, an institution needs to ensure that optimizing households have enough resources to finance themselves while the government remains solvent. When institutions are on their best response functions along (2), a deviation either implies violating (17) or optimal smoothing of individual payout losses.

[13] There exists additional CFMPE in high-debt or high asset economies where institutional FOCs imply violation of $\tau_t = [0,1)$ in at least one period and state while satisfying (2) and (17). Institutions in such CFMPE choose policy conditional on a binding $\tau_t = [0,1)$ in that period and state. These CFMPE are not considered in this analysis, yet are the reason why proposition 3 is merely a sufficiency statement.

For instance, in the case when the debt-manager sets taxes lower (higher) than they would need to be to finance both inherited debt and current and expected future spending along a given inflation plan $\left\{\{\pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$, the central bank optimally responds by further inflating (deflating) the inherited debt stock so as to ensure the ARC (2) and the implementability constraint (17) hold while following its optimality condition (21). Similarly, in the case when the central bank inflates (deflates) the inherited debt stock more than needed along a given tax plan $\left\{\{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$, the debt-manager optimally responds by decreasing (increasing) the tax path so as to ensure (2) and (17) hold while following its optimality condition (20).

Either institution can act in an unconstrained manner, but only so long as the other institution 'picks up the slack.' When $b_{-1} > 0$, an unconstrained debt-manager sets taxes only to finance new government spending without financing any inherited debt, relying on the central bank to hyperinflate all debt away at time 0 $\left(\lambda_0^{cb} \to \infty\right)$. Regardless of b_{-1}, an unconstrained central bank sets inflation to 1 in every period leaving the debt-manager to set taxes to finance both the entire stock of inherited debt (assets) as well as current and future spending with explicit taxes under at a constant price level $\left(\lambda_0^{cb} = 0\right)$. Additionally, by the same logic as that in lemma 1, $b_{-1} \neq 0$ implies implementation is feasible with $b_0(x_0) \notin \{-\infty, \infty\}$. The continuum between these extremes all satisfy CFMPE.

Assume $b_{-1} > 0$ holds for the remainder of the analysis. Under this condition, $\pi_0 > 1 \implies \pi_1 > 1$ through (21). Define a set of inherited debt structures $\hat{b}_{-1}$ as:

$$\hat{b}_{-1} \equiv \left\{b_{-1} \in \mathbb{R}^2_{++} : \forall b'_{-1} > b_{-1}, \pi_0 > 1 \text{ is required for CFMPE}\right\}$$

to characterize another subset of CFMPE $\left(\lambda_0^{cb} < 0\right)$ in lemma 2.

Lemma 2 ***(Payoff Dominated CFMPE)*** *Under two committing institutions,* $\forall b_{-1} \in \left(0, \hat{b}_{-1}\right)$, $\exists \lambda_0^{dm} > 0$ *and* $\exists \lambda_0^{cb} < 0$ *which satisfies a CFMPE.*

Proof: This proof can be found in appendix A.

As long as solvency is feasible under deflation, there exists CFMPE beyond the continuum observed between unconstrained institutions (that characterized by $\lambda_0^{cb} = 0$ to $\lambda_0^{cb} \to \infty$). Notice that, because the central bank does not have a constraint that prevents it from always following (21) (in the way the debt-manager has with $\tau_t \in [0, 1)$), the continuum of CFMPE can be indexed by $\lambda_0^{cb} \in [-N, \infty)$ where $N \geq 0$ is a constant. The set of CFMPE described in lemma 2 (where $\lambda_0^{cb} < 0$) is named 'payoff dominated' due to the fact that both institutions could individually be made better off by moving the economy towards the CFMPE with $\lambda_0^{cb} = 0$. The next section further refines the CFMPE concept to eliminate these equilibria.

4.2.1 Payoff Dominance: A Refinement

Payoff dominance, as proposed by Harsanyi and Selten (1988), is a refinement criterion for selecting among multiple Nash equilibria. It asserts that, when faced with multiple equilibria, rational, non-cooperative players will coordinate to eliminate equilibria where both players can be individually made better-off.

Section 4.2 identifies all CFMPE of the Nash game in state-contingent plans played by two committing institutions and indexes them from $\lambda_0^{cb} \in [-N, \infty)$. Because $b_{-1} > 0$, all CFMPE satisfying $\lambda_0^{cb} < 0$ are strictly Pareto-dominated by the CFMPE at $\lambda_0^{cb} = 0$ as the central bank is made better off by increasing inflation towards 1 while the debt-manager is made better off by decreasing tax rates, all while satisfying (2) and (17).

Apply Harsanyi and Selten (1988)'s payoff dominance refinement to the CFMPE definition so as to eliminate all CFMPE indexed as $\lambda_0^{cb} \in [-N, 0)$, leaving the set of payoff-dominant CFMPE to only include equilibria indexed as $\lambda_0^{cb} \in [0, \infty)$.[14]

4.2.2 Quantitative Exercise

To further understand of the nature of the CFMPE multiplicity, specify the per period household payoff for the rest of the analysis as:

$$u(c_t) - v(n_t) - w(\pi_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n^{1+\varphi}}{1+\varphi} - \frac{1}{2}\theta\left(\frac{1}{\pi} - 1\right)^2 \tag{22}$$

with $\sigma = \varphi = 2$. Each period is a year, so $\beta = .9875^4$ is the annualized time discount factor from Angeletos (2002) and Buera and Nicolini (2004). Government spending $\{g_\ell, g_h\} = \{0.224, 0.464\}$

[14]For expositional convenience, all future mentions of CFMPE will indicate payoff-dominant CFMPE in pure strategies.

and low-spending probability $p = 75/76$ match U.S. spending data from 1942-2021, setting $g_0 = g_\ell$. $b_{-1} = \{0.221, 0.271\}$ matches the average U.S. maturity structure from 1947-2022. The functional form of $w(\pi_t)$ is taken from Sims (2013), and $\theta = 3.87$ so that the consolidated, committing, benevolent government chooses $\pi_0 = 1.032$, the average inflation rate over 1947-2022.[15]

The relationship between institutional Lagrange multipliers $\left\{\lambda_0^{dm}, \lambda_0^{cb}\right\}$ becomes important in measuring individual contributions to government financing. To evaluate this relationship, index equilibria by $\omega \in [0, 1)$ according to:

$$\omega \lambda_0^{dm} = (1 - \omega) \lambda_0^{cb} \tag{23}$$

where $\omega = 0$ represents an unconstrained central bank and $\omega \to 1$ represents an unconstrained debt-manager.

Figure 5's top-left panel displays the equilibrium time 0 tax (solid blue line) and inflation (dotted red line) rates at each CFMPE, and its top-right panel plots lifetime welfare (in terms of time 0 consumption equivalence, normalized to the consolidated, committing, benevolent government outcome) along all equilibria indexed by $\omega \in [0, 1)$. Its bottom-left panel illustrates how the jointly-issued time 0 maturity structure varies with the degree to which institutions are constrained, and its bottom-right panel displays surprise (solid blue line) and expected future (dotted red line) dilution rates. Appendix F reports variables as CFMPE, now indexed by ω, vary from one corner to the other.

The economy matches the consolidated, committing, benevolent government when $\omega = 0.5$ so that $\lambda_0^{dm} = \lambda_0^{cb}$, which necessarily occurs when $\lambda_0^{dm} = \lambda_0^{cb} = \lambda_0$. Institutions non-cooperatively reach first-best when they receive the same individual payoff from marginally relaxing (17). Welfare losses under an unconstrained central bank $\omega = 0$ are considerably less than those from an unconstrained debt-manager $\omega \to 1$ relative to first-best.

Four backing components adjust in offsetting ways as the central bank acts more constrained ($\omega \to 1$). First, times 1 and 2 taxes fall leading to lower expected primary surpluses. Second, future taxes rise *relative to* time 0 taxes, leading to an increase in discount rates (less discounting from real rates). Third, increased use of $B_0^{(1)}$ relative to the (fixed) level of $B_{-1}^{(1)}$ increases surprise debt

[15] These choices are discussed in greater detail in appendix D.

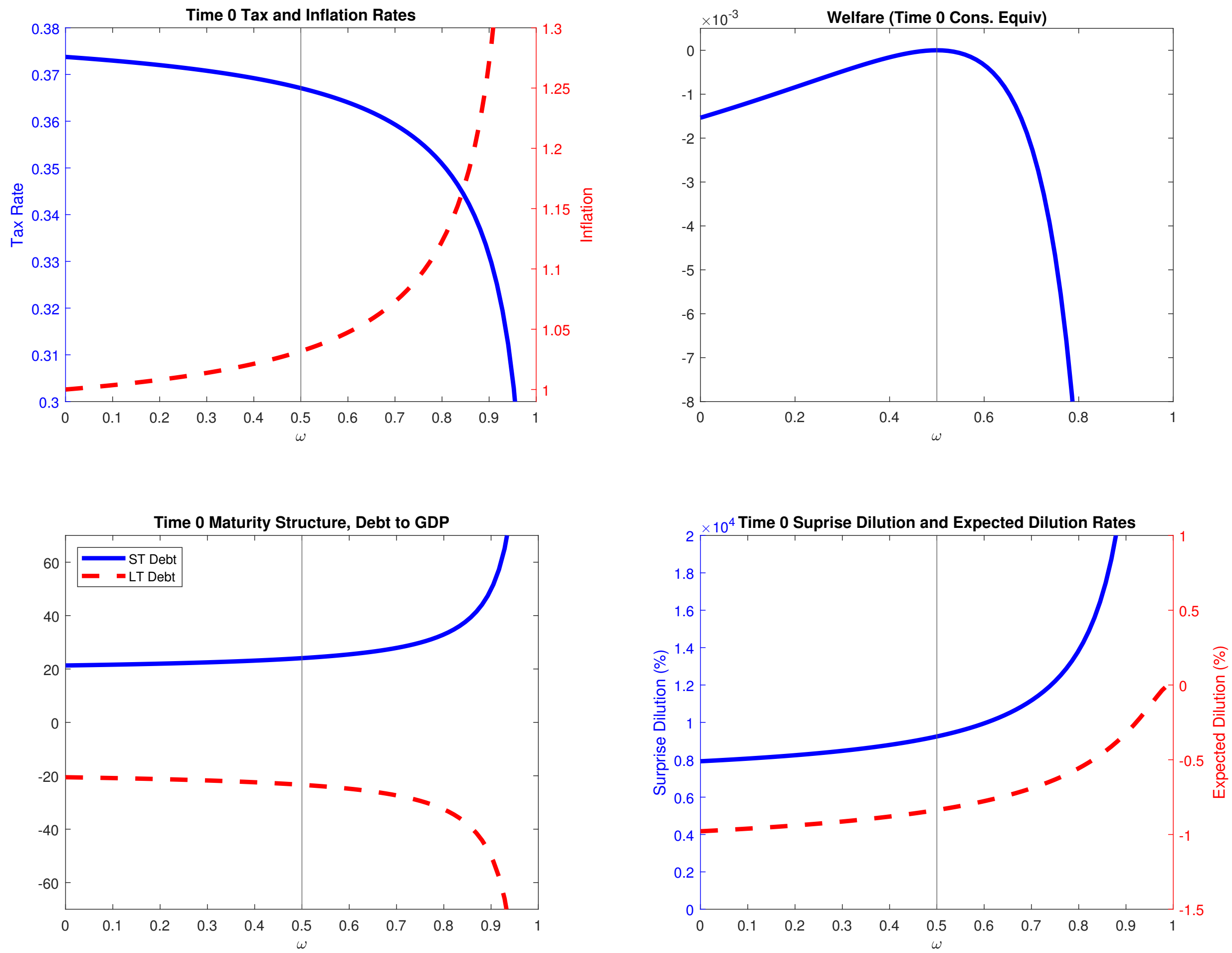

Figure 5: Time 0 financing, welfare, debt maturity structures, dilution under CFMPE indexed by $\omega \in [0, 1)$

dilution leading to less discounting from the dilution modifier. Fourth, increased use of (negative) $B_0^{(2)}$ relative to future expected (positive) $B_1^{(2)}$ increases dilution of time $t+2$ surpluses leading to more discounting from the dilution modifier. Falling primary surpluses and increased expected future dilution reduces backing. Rising real interest rates and surprise dilution partially offset this reduction's effects on time 0 prices. The result is seen in the top-left panel: $\pi_0 \to \infty$ as $\omega \to 1$.

All structures along the continuum feature positions with large amounts of short-term debt and offsetting long-term assets.[16] Beginning with Angeletos (2002) and Buera and Nicolini (2004), skewed non-contingent, indexed maturity structures have consistently shown up in optimal maturity structure analyses under committing governments. It continues to be true here when both institutions can commit. The ability to set expectations allows institutions to perfectly, yet non-cooperatively,

[16]This is unlike conventional short-term debt/long-term asset positions found in the optimal maturity literature with a consolidated, committing, benevolent government. The difference arises from the absorbing high-spending state.

target future allocations, as the maturity structure is set to exactly transfer (tax) resources to (from) households in a lump-sum way through state-contingent changes in its market value.

Lemma 1 explains the increasing skewness of maturity structures as $\omega \to 1$. When income taxes are used less to relax the implementability constraint (17) (as inflation is doing more of that job) and more to smooth tax distortions, the entire tax path approaches a constant distortion-minimizing rate. Time 1 debt prices approach $Q_{1,\ell} = Q_{1,h}$ as the economy exhibits smoother taxes. In the limit, the lack of debt price variation across states renders the market value of the maturity structure worthless in insuring the economy.

4.3 Unique CFMPE Under Bargaining

Arguments for reducing the Fed's operational independence have emerged as part of the U.S.'s 2024 presidential race. Some support a fiscal seat on the FMOC, some call for the president to be personally consulted on rate decisions, and others suggest stronger Fed oversight by the Treasury. Closing this paper's model provides an opportunity to directly address these debates.

I select unique equilibria using asymmetric bargaining solutions from the literature on oligopolistic collusion and surplus sharing. Fiscal and monetary institutions bargain over surpluses gained from agreeing on a single equilibrium out of the above multiplicity. I begin with a Nash bargaining solution originally introduced in Nash (1950) and continue with a Kalai-Smorodinksy bargaining solution originally introduced in Kalai and Smorodinsky (1975).

Each institution's disagreement payout is its worst-case feasible CFMPE (corner) described in section 4.2.2 so that variations in bargaining power span the entire continuum of CFMPE.[17] A central bank with extreme bargaining power controls all government tools to ensure $\pi_t = \pi = 1$ every period. A debt-manager with extreme bargaining power disregards previously issued government debt to minimize taxes. The intermediate space serves as an opportunity to study inflation and tax rate determination as outcomes of intra-governmental bargaining, which can be interpreted as explicit negotiating or as an outcome from a larger political economy equilibrium.

[17] One can view these disagreement payouts as outcomes from two pessimistic institutions. Both believe that the other takes over the entire government should negotiations break down.

4.3.1 Nash Bargaining

To pin down a unique CFMPE under Nash bargaining, replace (23) with the asymmetric bargaining surplus proposed in Harsanyi and Selten (1972) constructed by combining institution-specific surpluses as:

$$\left(W_0^{dm} - d^{dm}\right)^{\alpha} \left(W_0^{cb} - d^{cb}\right)^{1-\alpha} \ , \ \alpha \in [0,1) \tag{24}$$

where d^i is institution i's payout under its worst-case feasible CFMPE described in section 4.2.2 and where α captures the debt-manager's bargaining power relative to the central bank. Maximize (24) for a given α to arrive at the model's solution.

Unlike ω in (23), α in (24) has economic interpretation. A higher α indicates greater bargaining strength for the debt-manager relative to the central bank. Such a debt-manager has more power to influence negotiations its favor.

The CFMPE under $\alpha = 0$ corresponds with that under $\omega = 0$ and the the CFMPE under $\alpha \to 1$ corresponds with that under $\omega \to 1$. Denote α^* as the amount of debt-manager bargaining power needed to match first-best allocations. First-best ($\omega = 0.5$) requires $\alpha = \alpha^* = 0.0025$: a large amount of central bank bargaining power.

Unilateral central bank bargaining power ($\alpha = 0$) approximates first-best. Under this arrangement, welfare can be improved by ceding only a sliver of bargaining power to the debt-manager. Welfare under $\alpha = 0$ is approximately equal to that under $\alpha = .01$ so that households prefer a unilaterally powerful central bank to a government with $\alpha > .01$. A large fiscal-monetary asymmetry in downside risk drives this result: bad central bank outcomes are disproportionately costly to welfare compared to bad debt-manager outcomes.

4.3.2 Kalai-Smorodinsky Bargaining

I also consider the asymmetric Kalai-Smorodinsky (KS) bargaining solution proposed in Dubra (2001), which, rather than (24), closes the model through:

$$\frac{W_0^{dm} - d^{dm}}{m^{dm} - d^{dm}} = \left(\frac{\kappa}{1-\kappa}\right) \frac{W_0^{cb} - d^{cb}}{m^{cb} - d^{cb}} \ , \ \kappa \in [0,1) \tag{25}$$

where m^i is institution i's maximum possible (corner) payout and where κ is now the debt-manager's relative bargaining power parameter.

Nash bargaining maximizes the product of utility gains while KS bargaining equalizes the ratios of institutional maximal gains, both satisfying basic axioms of to two-player bargaining problems.[18] Nash is 'fair' in the sense that it balances relative payoff improvements while KS is 'fair' in ensuring that when one player has more to gain, it gains more in absolute terms.

The CFMPE under $\kappa = 0$ corresponds with that under $\alpha = 0$ and $\omega = 0$ and the the CFMPE under $\kappa \to 1$ corresponds with that under $\alpha \to 1$ and $\omega \to 1$. Denote κ^* as the amount of debt-manager bargaining power needed to achieve first-best allocations. First-best ($\alpha^* = 0.0025$, $\omega = 0.5$) requires $\kappa = \kappa^* = 0.059$. While still large, KS suggests the central bank requires less bargaining power than Nash to achieve welfare-maximization due to the normalization inherent in (25).

The findings suggest that fiscal-monetary bargaining power can play a major role in tax and inflation determination. And, while cooperation matching a consolidated, committing, benevolent government is always weakly welfare-improving relative to non-cooperative outcomes, cooperation under fiscal control is far worse than that under monetary control. An independent central bank is a requirement for outcomes far away from unilateral fiscal bargaining power, so maintaining such a central bank provides a critical buffer against fiscally-driven, potentially harmful financing decisions.

4.3.3 Equivalence Between Bargaining and Differences in Commitment

Many consider the U.S.'s institutional arrangement to include Fed credibility and fiscal non-credibility. Fiscal policy faces direct political pressures that incentivize trading off long-term economic performance for short-term gains while the Fed is more insulated from such pressures. For this reason, choosing a consolidated government's commitment specification is not always straightforward. Fortunately, this paper's framework and equilibrium definition allows for a committing central bank and non-committing debt-manager and for a committing debt-manager and non-committing central bank.[19]

When the debt-manager has commitment power while the central bank does not, the resulting

[18]KS satisfies the monotonicity axiom while Nash does not, which is discussed in both Kalai and Smorodinsky (1975) and Dubra (2001).

[19]Other work that includes varying levels of fiscal and monetary commitment within the same government includes Gnocchi (2013) and Gnocchi and Lambertini (2016).

allocations match those in the $\alpha, \kappa, \omega \to 1$ case under committing institutions.[20] When the central bank has commitment power while the debt-manager does not, the CFMPE matches that in the $\alpha, \kappa, \omega = 0$ case with committing institutions. Equilibria featuring unilateral bargaining power with a committing opponent and those featuring commitment power with a non-committing opponent are equivalent. Appendix E reviews and analyzes these cases in greater detail.

4.4 Numerical Results

This section explores numerical choices made in section 4.2.2, investigating how results vary with θ and b_{-1} across bargaining solutions. It then outlines bargaining power endowments needed to match inflation outcomes post-Great Financial Crisis (GFC) and post-COVID.

4.4.1 Inflation's Welfare Costs

How much does the baseline $\theta = 3.87$ drive the paper's results, specifically regarding the requirement for substantial central bank bargaining power to match a consolidated, committing, benevolent government's allocations? To address these reservations, table 2 lists α^* and κ^* under household sensitivities $\theta \in \{1, 3.87, 10\}$. Figure 6 displays tax, inflation, welfare and debt maturity outcomes in each case. Predictably, higher debt-manager bargaining power matches first-best allocations as households' dislike for inflation wanes. This finding is consistent across Nash and KS solutions.

Case	θ	α^*	κ^*	π_0	τ_0	Dil_0	$\mathbb{E}_0[Dil_1]$
Tolerant	1	0.031	0.168	1.119	.352	136	$-.006$
Baseline	3.87	0.0025	0.059	1.032	.367	92.5	$-.008$
Intolerant	10	$3.77e^{-4}$	0.025	1.012	.371	84.2	$-.009$

Table 2: Debt-manager bargaining power needed to match first-best under Nash and KS solutions, varying HH inflation sensitivity

A consolidated, committing, benevolent government sends $\pi_0 \to \infty$ as $\theta \to 0$, and opts for $\pi_t = \pi \to 1$ in every period as $\theta \to \infty$. As inflation's welfare costs approach zero, an all-powerful debt-manager maximizes welfare by ignoring (hyperinflating) inherited debt while financing current and future government spending with taxes. Under the latter, an omnipotent central bank maximizes

[20]The debt-manager commitment/central bank non-commitment case is not a CFMPE due to time 0's undefined inflation rate and maturity structure, but all other variables (tax rates, consumption, labor supply, time 1 and 2 inflation and debt choices) are identical to those under $\alpha, \kappa, \omega \to 1$.

welfare by ensuring prices are constant so that taxes finance both inherited debt and spending.

Figure 6 displays tax, inflation, welfare, maturity structure and dilution outcomes across the cases. The $\alpha, \kappa = 0$ economy across θ have identical taxes, inflation rates and maturity structures. The $\alpha, \kappa \to 1$ economy across θ are identical along these dimensions also. In both cases, extreme institutional bargaining power dominates θ's value. In all other cases, inflation-tolerant economies use more inflation financing and less explicit tax financing conditional on a single bargaining power arrangement.

The maturity structure remains less skewed as debt-manager bargaining power increases when θ is high. Lower inflation increases tax financing, and less-smooth taxes increase across-state variation in future debt prices. A less-skewed maturity structure results.

Surprise time 0 dilution and expected future dilution fall as θ rises. Lower surprise dilution is associated with higher time 0 inflation while more (in absolute terms) expected (negative) future dilution increases the amplification of expected future surpluses. The two effects largely offset, but the change to expected future dilution dominates. This effect aids higher taxes to achieve a lower P_0 and a lower π_0.

Naturally, first-best welfare declines as θ increases across economies. Compared to a single economy's first-best outcome, 'too much' central bank bargaining power (e.g. $\kappa < \kappa^*$) is relatively less costly in economies with a high θ. Welfare costs are low when a powerful central bank skews debt financing toward explicit taxes in economies with high inflation intolerance. Near first-best, 'too much' debt-manager bargaining power (e.g. $\kappa > \kappa^*$) also proves less costly in high-θ economies. This occurs because high-θ economies have low absolute welfare, making small deviations around first-best less expensive. This effect doesn't last: debt-managers become increasingly detrimental as $\alpha \to 1$ in high-θ economies, aligning with the intuition that more inflation financing worsens welfare in economies when inflation is more costly.

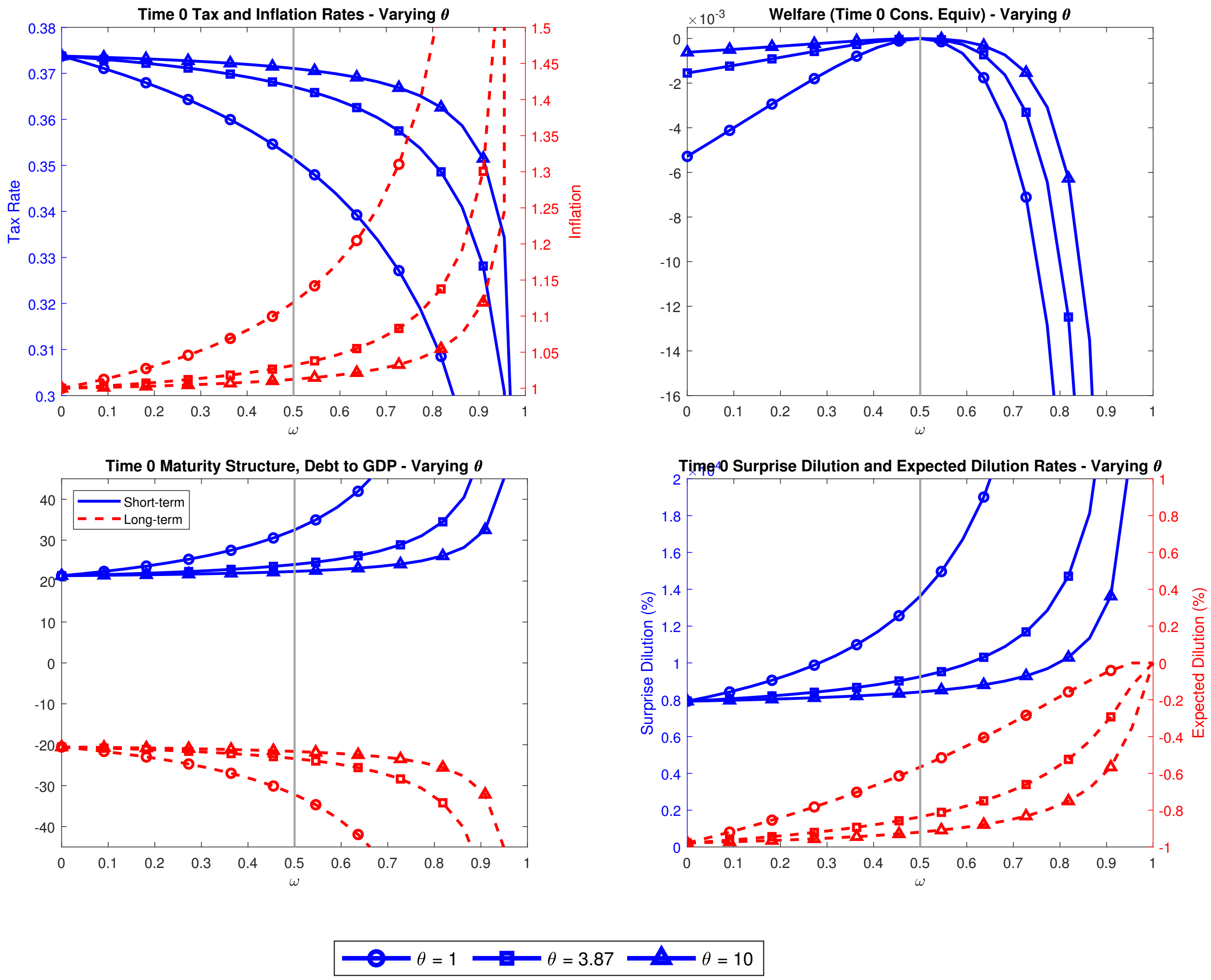

Figure 6: Time 0 financing, welfare, debt maturity structures, dilution under CFMPE indexed by $\omega \in [0, 1)$, varying HH inflation sensitivity

4.4.2 Inherited Debt

Consider two alternate initial maturity structures to the one described in 4.2.2. The first is the U.S.'s debt-to-GDP maturity structure at the end of 2009: $\{0.3131, 0.3001\}$, and the second is that at the end of 2020: $\{0.5766, 0.4635\}$. Different post-recession versions of the U.S. inherited these maturity structures in 2010 and 2021. Inflation after the GFC remained at or below the Fed's 1.02 target for a decade while inflation after the U.S.'s COVID episode quickly grew to 1.06, its highest level in 40 years. Figure 7 plots the U.S. par value debt-to-GDP in short- and long-term debt from 1948-2022. This section focuses on the U.S.'s debt structures entering 2009 and 2020 marked as "Post-GFC" and "Post-COVID".

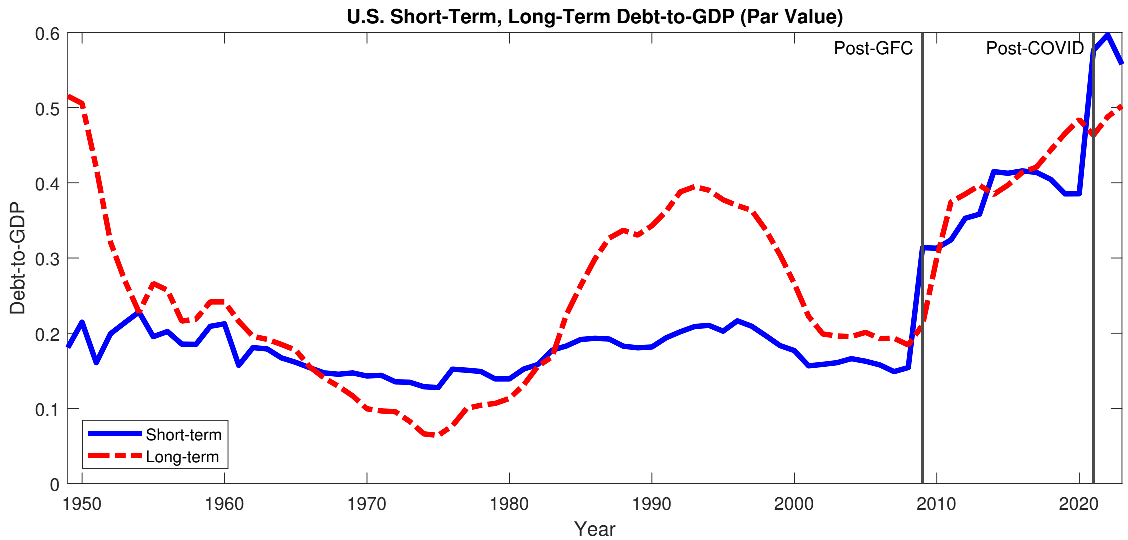

Figure 7: U.S. debt/GDP (1948-2022), calculated where $B_t^{(t+1)}$ = (outstanding Fed liabilities + outstanding Treasury debt set to mature in ≤ 1 year) and $B_t^{(t+1)}$ = (outstanding Treasury debt set to mature in > 1 year)

Table 3 displays α^* and κ^* under the the two maturity structures described above and compares them to the baseline case. First-best outcomes align with increased fiscal bargaining power in economies with more inherited debt. The consolidated, committing, benevolent government uses more inflation financing as the amount of inherited nominal debt rises. This happens for two reasons. First, welfare loss from tax distortions become increasingly detrimental (convex) as taxes increase relative to welfare loss from inflation. Second, a high-debt economy gets more financing from the same inflation rate relative to a low-debt economy even though inflation hurts households equally across the two.

Case	b_{t-1} (data)	α^*	κ^*	π_0	τ_0	Dil_0	$\mathbb{E}_0[Dil_1]$
Baseline	$\{0.221, 0.271\}$	0.0025	0.059	1.032	.367	92.4	$-.008$
$b_{t-1} = b_{2009}$	$\{0.313, 0.300\}$	0.0058	0.090	1.050	.389	62.5	$-.015$
$b_{t-1} = b_{2020}$	$\{0.577, 0.464\}$	0.056	0.255	1.167	.439	19.8	$-.062$

Table 3: Debt-manager bargaining power needed to match first-best under Nash and KS solutions, varying inherited debt stock

Figure 5 plots tax, inflation, welfare and maturity structure outcomes across economies with varied inherited structures. Like the exercise from section 4.4.1, allocations across all economies are identical under an all-powerful debt-manager ($\alpha, \kappa \to 1$) as hyperinflation fully finances the initial debt position. Otherwise, tax and inflation rates are higher across all bargaining power specifications in more indebted economies.

In the extreme case when $b_{-1} \to 0$, taxes are fully smooth under all bargaining power endowments so that the economy is as well-off under any α, κ as it is under a consolidated, committing, benevolent government. Welfare in high-debt economies is more sensitive to deviations of α, κ from α^*, κ^* than that in low-debt economies, as can be seen in the top-right panel of figure 8.

Economies with higher initial indebtedness feature less-skewed maturity structures because high initial debt induces large differences in state-contingent tax rates. Time 0 dilution is less positive and time 1 dilution is more negative in high-debt economies, approaching more realistic inverse dilution rates than those from the benchmark economy listed in appendix F. High-debt economies discount time 1 surpluses more through the maturity structure than do low-debt economies precisely because inflation financing is used more in these economies.

Finally, surprise time 0 dilution and expected future dilution fall as the level of inherited debt rises. Lower surprise dilution is associated with higher time 0 inflation while more (in absolute terms) expected (negative) future dilution increases the amplification of expected future surpluses. Just like when varying θ, the two effects offset, and again, the change to expected future dilution dominates. As higher debt uses more inflation and taxes (tighter government budget constraint), dilution is used more heavily to finance the government.

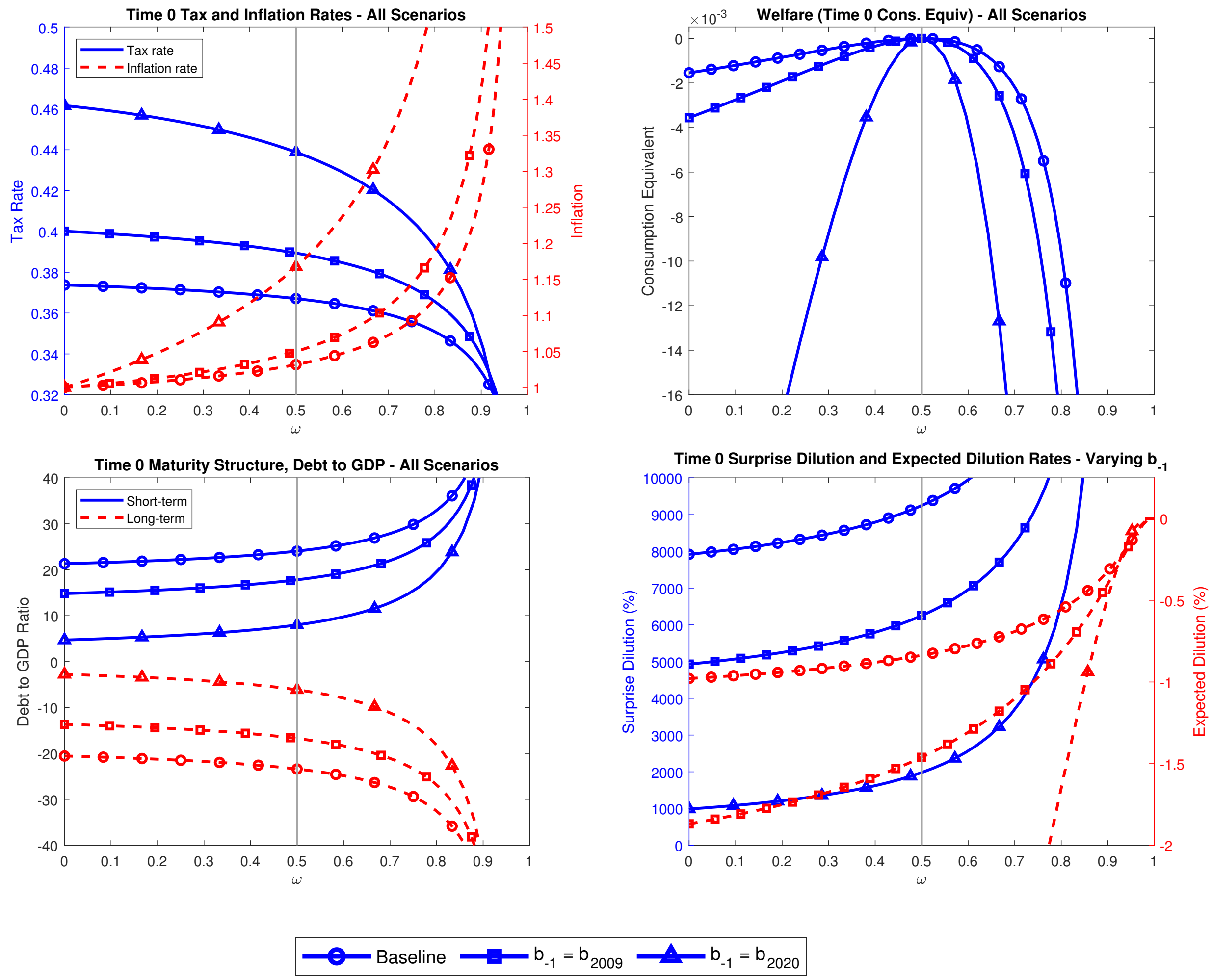

Figure 8: Time 0 financing, welfare, debt maturity structures, dilution under CFMPE indexed by $\omega \in [0, 1)$, varying initial debt stock

4.5 A History of American Bargaining

I use the model to impute the history of the U.S.'s relative bargaining power over the Fed from 1948-2021.[21] For external validation I compare this series (blue solid line) to Drechsel (2024)'s annual series on the number of hours presidents met with Fed officials (red dotted line). Figure 9 plots this comparison.

[21] This consists of solving the three-period model every year, given $\{b_{-1}, g_0\}$ from the data, and selecting the value of α,κ that sets π_0 equal to observed inflation.

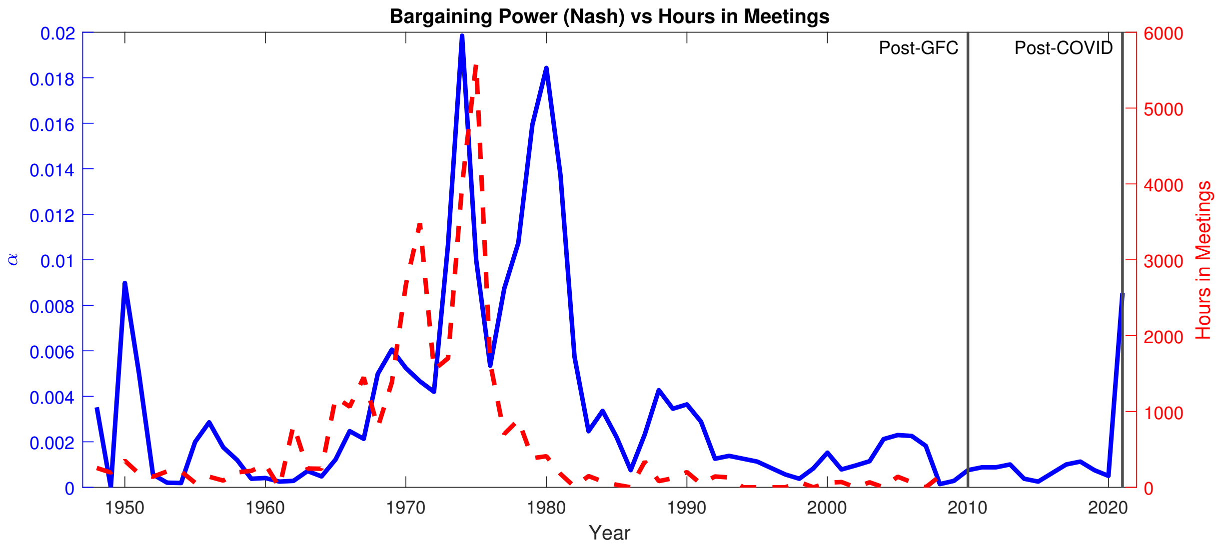

Figure 9: Comparing calculated fiscal bargaining power with hours spent between presidents and Fed officials annually (1948-2021)

Both series exhibit two large spikes between 1970-1980, with bargaining power spiking in 1974 and 1980 and hours spiking in 1971 and 1975.[22] Aside from a large increase in bargaining power in 1950 – the year before the Treasury-Fed Accord, which relieved the Fed from an obligation to support bond prices – the two tell similar stories. After 1980, both fiscal power and president-Fed interactions remain small until 2021, the last year in the sample. Drechsel (2024)'s series ends in 2008.

In 1964, President Lyndon B. Johnson pushed Fed Chair William McChesney Martin against a wall and exclaimed, "Martin, my boys are dying in Vietnam, and you won't print the money I need."

In 1971, while under pressure from President Richard Nixon to cut rates, Fed Chair Arthur Burns wrote in his personal journal, "I am convinced that the President will do anything to be reelected."

In 1980, Fed Chair Paul Volcker implored Congress to align fiscal tools with monetary goals, stating, "Monetary policy cannot – without peril – be relied on alone to halt inflation. The other major tools of public policy must also be brought to bear on the problem, with fiscal policy playing a central role."

In 2009, President Barack Obama expressed his intent to run fiscal surpluses, announcing, "Today I'm pledging to cut the deficit we inherited in half by the end of my first term in office."

[22]Drechsel (2024) explains the 1971 spike as part of a larger tactic from Nixon to put pressure on his longtime friend Arthur Burns to help Nixon get reelected.

Sentiments from these quotes are visible in figure 9. 1964 saw the beginning of a large rise in fiscal power, lasting well through 1971. 1980 saw the beginning of a large decline. Fiscal bargaining hit a 40-year low in 2009.

Tables 4 and 5 report average calculated fiscal bargaining power across presidents and Fed chairs. Jimmy Carter (1977-1981) and G. William Miller (1978-1979) top their respective lists. Carter was in office for the entire 1980 bargaining spike, while 1974's spike is split between the Nixon (1969-1974) and Ford (1974-1977) administrations. Miller's tenure, while brief, occurred during a drastic rise in fiscal power. He and Volcker (1979-1987) split the 1980 spike, with Volcker remaining in office long enough to witness low fiscal power's return, which persisted until the end of his final term. Arthur Burns (1970-1978) is second among central bankers because his tenure began when fiscal power was low (relative to the rest of the 70s), despite being in office for the entirety of the largest spike in fiscal bargaining power on record.

President	α	κ	Term
Carter (D)	0.014	0.100	02/1977–02/1981
Ford (R)	0.010	0.080	08/1974–02/1977
Biden (D)	0.009	0.132	02/2021–11/2024
Nixon (R)	0.008	0.074	02/1969–08/1974
Reagan (R)	0.005	0.065	02/1981–02/1989
Truman (D)	0.004	0.064	01/1948–02/1953
Bush, H. W. (R)	0.003	0.064	02/1989–02/1993
Johnson (D)	0.003	0.046	12/1963–02/1969
Bush, W. (R)	0.002	0.039	02/2001–02/2009
Trump (R)	0.002	0.049	02/2017–02/2021
Eisenhower (R)	0.002	0.035	02/1953–02/1961
Clinton (D)	0.001	0.040	02/1993–02/2001
Obama (D)	0.001	0.035	02/2009–02/2017
Kennedy (D)	0.001	0.023	02/1961–12/1963

Table 4: Comparing average fiscal bargaining across presidents (sorted by α)

Fed Chair	α	κ	Term
Miller	0.014	0.099	03/1978–08/1979
Burns	0.009	0.078	02/1970–03/1978
Volcker	0.007	0.072	08/1979–08/1987
McCabe	0.005	0.071	01/1948–04/1951
Powell	0.003	0.067	02/2018–11/2024
Martin	0.002	0.040	04/1951–02/1970
Greenspan	0.002	0.048	08/1987–02/2006
Bernanke	0.001	0.037	02/2006–02/2014
Yellen	0.001	0.036	02/2014–02/2018

Table 5: Comparing average fiscal bargaining across Fed chairs (sorted by α)

Sargent (1986) credits Neil Wallace for describing fiscal-monetary coordination during the Reagan and Volcker years as a 'game of chicken,' where monetary and fiscal policymakers pledged to pursue conflicting plans (given the U.S.'s GBC) with the expectation that the other would 'chicken out.' Importantly, in a game of chicken, only one player wins; neither ties nor partial victories occur. This paper's bargaining approach quantifies and refines Wallace's 1980s game of chicken. During Reagan's first term (1981-1985), which coincided entirely with Volcker's tenure as Fed Chair, fiscal bargaining power collapsed to around its 1965 level, where it remained until Volcker's final term ended in 1987. Time-variation in U.S. fiscal power reinforces the notion that Volcker and the Fed forced Reagan and fiscal policy to flinch first, and does so in a measurable way.

Figure 10 displays the history of calculated Nash bargaining power measures shown above (solid blue line) alongside those associated with first-best outcomes (dotted red line).

Two periods stick out: 1965-1985 and 2009-2021. Fiscal power in the 20 year span between 1965 and 1985 was consistently orders of magnitude above that required for first-best. The economy had a small debt burden, yet high (welfare-reducing) inflation. Fiscal power from 2009-2021 has been about an order of magnitude *lower* than optimal. Even the large inflationary period post-COVID is only about 15% of what the model suggests.

Government indebtedness is the culprit. Surprise inflation is considerably more valuable financing high-debt economies than low-debt ones. Households experience the same welfare loss from the same amount of inflation regardless of the country's debt position, but high-debt economies receive a larger transfer from bondholders when inflating a larger outstanding debt stock. The model's all-powerful central bank sets inflation to zero. Strong fiscal policy drives optimally high inflation when inherited debt is large.

Figure 7 shows that the debt-to-GDP ratio spiked during the GFC, steadily increased afterward, and spiked again during COVID. The same 1% of surprise inflation, and the fiscal bargaining power needed to drive this inflation, becomes more valuable throughout the period. Table 6 shows the fiscal power needed to rationalize post-GFC and post-COVID inflation rates marked in figures 9 and 10. The model requires an extremely powerful central bank ($\alpha, \kappa < .05$) to achieve 1.8% inflation when the government inherits 2009's debt structure. A ten-fold (four-fold) increase in fiscal bargaining power under Nash (KS) bargaining justifies the sharp post-COVID rise in inflation. Despite the increase in fiscal bargaining power leading up to 2021 and resulting in 6.2% inflation, substantially more was required to reach first-best.

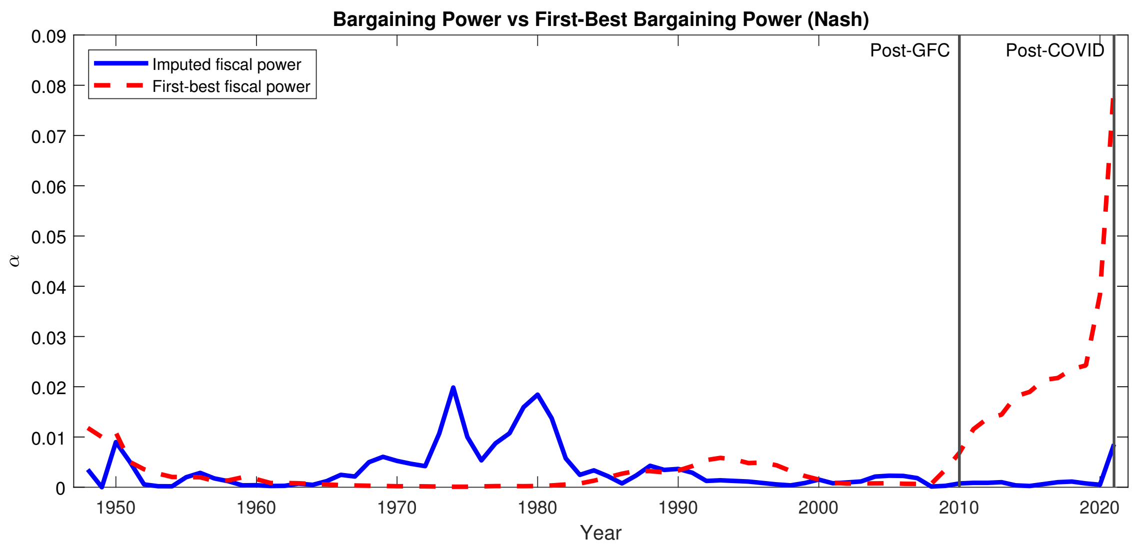

Figure 10: Comparing imputed fiscal bargaining power with that which matches first-best (1948-2021)

Year	b_{t-1} (data)	g_0 (data)	π_t (data)	α	κ	α^*	κ^*	π_0^*
2010	$\{0.313, 0.300\}$	.315	1.018	$7.54e^{-4}$	0.036	0.007	0.097	1.055
2021	$\{0.577, 0.464\}$	.263	1.062	0.009	0.132	0.08	0.285	1.207

Table 6: Debt-manager bargaining power required to match inflation data compared with debt-manager bargaining power needed to match first-best, given inherited maturity structure and real spending (2010 and 2021)

High levels of inherited debt in both 2010 and 2021 lead to a more aggressive use of first-best inflation financing (5.5% post-GFC, 20.7% post-COVID) than what the U.S. employed (1.8% post-GFC, 6.2% post-COVID). As inflation becomes a more powerful financing tool, more of it is optimally used. After the GFC, first-best required close to the level of fiscal bargaining power experienced after COVID (under Nash bargaining).

5 Conclusion

> *"I feel that the president should have at least [a] say in [making interest rate decisions], yeah, I feel that strongly."* – Donald Trump, 2024

What are the implications of a weak central bank? What about a strong one? This paper examines how non-cooperative behavior between fiscal and monetary authorities affects macroeconomic outcomes under a rich maturity structure of public debt. Modeling the debt-manager and central bank as separate entities with distinct objectives provides insight into the interactions that shape inflation, taxation, and debt management policies.

The analysis reveals several key findings. First, when both institutions have commitment power, there exists a multiplicity of equilibria ranging from a zero-inflation economy to one with hyperinflation. The welfare-maximizing outcome in the baseline model requires the central bank to have substantially more bargaining power than the debt-manager, outlining how central bank independence plays a crucial role in achieving optimal policy outcomes.

Second, the relative bargaining power between fiscal and monetary authorities can significantly impact inflation outcomes. For instance, drastically different inflation experiences following the GFC and COVID can be rationalized by changes in the relative bargaining power of these institutions.

Third, imputed bargaining power measurements match president-Fed meeting data and features notable spikes in 1950, the 1970s and 2021. Two periods stand out in the analysis: 1965-1985 and 2009-2021. From 1965 to 1985, the U.S. government consistently exhibited levels of fiscal power orders of magnitude more than that needed for first-best outcomes. From 2009 to 2021 fiscal power was considerably lower than the model's recommendations. Larger levels of government debt increase the effectiveness of inflation as a financing tool - higher debt makes surprise inflation more valuable for financing, yet costs households the same as surprise inflation in low-debt economies.

Fourth, the government's maturity structure serves as a powerful tool for implementing optimal policies. A highly skewed maturity profile allows the government to hedge against future spending uncertainty through state-contingent changes in debt prices. This finding aligns with and extends the classic maturity results from Angeletos (2002) and Buera and Nicolini (2004).

Fifth, expected and unexpected movements in the economy's debt dilution rate play key roles in price level determination. By strategically issuing debt to equate fiscal backing across spending states, the government avoids relying on ex-post inflation financing. This provides the optimal maturity structure literature with a new set of economic interpretations.

This work opens several avenues for future research. Extending the model to include additional realistic features, such as nominal rigidities or financial frictions, could provide further insights into the transmission of strategic fiscal-monetary interactions. Adding the U.S.'s debt dilution rate to well-identified empirical studies may uncover evidence of previously-unexplored optimal balance sheet policy.

In conclusion, this paper demonstrates that the interplay between fiscal and monetary policy, mediated through the maturity structure of government debt, has profound implications for macroeconomic outcomes. Understanding and managing these complex interactions is crucial for designing effective policy frameworks in an era of high government debt and constantly-evolving fiscal and monetary policy.

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A Appendix: Proofs

Proof for Proposition 1

Formally, let $\left\{ \{c_t(\mathcal{H}_t), n_t(\mathcal{H}_t), g_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t} \right\}_{t=0}^{\infty}$ represent a stochastic sequence. Substituting (2), (9), and the definitions $b_t^{(t+j)} = \frac{B_t^{(t+j)}}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}} > 0$ into (6), while considering (7), forward-iterating, and applying (11), results in (15). If a stochastic sequence $\left\{ \{c_t(\mathcal{H}_t), n_t(\mathcal{H}_t), g_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t} \right\}_{t=0}^{\infty}$ is generated by a competitive equilibrium, then it necessarily satisfies (2) and (15).

Let government institutions jointly choose the associated level of debt $\left\{ \left\{ \left\{ \boldsymbol{b}_t^{(t+j)}(\mathcal{H}_t) \right\}_{j=1}^{J} \right\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t} \right\}_{t=0}^{\infty}$, let the debt-manager choose a tax sequence $\left\{ \{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t} \right\}_{t=0}^{\infty}$ and the central bank choose a sequence of inflation rates $\left\{ \{\pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t} \right\}_{t=0}^{\infty}$ such that (9) is satisfied. (15) and (2) imply (6) and (8) as well as (5) are satisfied, given definitions $\boldsymbol{b}_t^{(t+j)} = \frac{\boldsymbol{B}_t^{(t+j)}}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}} > 0$. All optimality conditions, dynamic budget constraints, and market clearing criteria are satisfied, so the equilibrium is competitive.

Proof for Proposition 2

Proposition 2 can be shown in three steps. First, that $c_t > 0$ adjusts such that (2) holds under all tax rates from $\tau_t \in [0, 1)$. Second, that A_t is non-singular under the central bank's path of inflation, given $\left\{ \{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t} \right\}_{t=0}^{\infty} \in \mathcal{T}$. Third, that there exists π_0 and π_1 such that (15) holds under the initial debt positions outlined in the proposition.

Combine (2) and (9) to write $1 - \tau_t = \frac{v'(c_t+g_t)}{u'(c_t)}$ and evaluate it over $\tau_t \in [0, 1)$. Properties of the utility function imply $\lim_{c_t \to \infty} \frac{v'(c_t+g_t)}{u'(c_t)} = \infty$ and $\lim_{c_t \to 0+} \frac{v'(c_t+g_t)}{u'(c_t)} = 0$ so that, for any $g_s \in g$ and $\tau_t \in [0, 1)$, there exists some $c_t > 0$ such that $1 - \tau_t = \frac{v'(c_t+g_t)}{u'(c_t)}$ holds.

Angeletos (2002) and Buera and Nicolini (2004) prove that any complete markets outcome can be replicated by a unique maturity structure when $K = S$, given non-singular A_t and the utility function and Markov properties featured in this model. All tax paths in $\mathcal{T}$ satisfy this property when $\pi_t = 1 \; \forall t$. We need to demonstrate that any inflation path satisfying $\{\pi_t\}_{t=0}^{\infty} = \{\pi_0(x_0), \pi_1(x_0), 1, 1, \cdots\}$ also yield non-singular $A_t \; \forall t$. This holds trivially for $t > 1$. For $t \in \{0, 1\}$, inflation depends solely on the initial state, so $t = 1$ state revelations don't affect the central bank's promised π_1. If A_0 and A_1 are non-singular when $\pi_0 = \pi_1 = 1$, then they remain non-singular along the central bank's inflation path, given constant inflation across *states* at $t = 0$ and $t = 1$. Finally, because

$\tau_t \in [0,1)$ is bounded $\forall t$, there always exists some combination of inflation rates that satisfy (15), given $\left\{\{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{\infty} \in \mathcal{T}$.

The third step is shown in the text.

Proof for Corollary 1

To prove corollary 1 one needs to show that the zero tax rate path is in $\mathcal{T}$ and that the utility-weighted time 0 RHS of (15) $\frac{1}{c_0}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^i \left[u'(c_t)c_t - v'(n_t)n_t\right]$ is negative.

To see that the zero tax rate path is in $\mathcal{T}$, notice two things. First, that under $\tau_t = 0\ \forall t$, $(u'(c_t)|g_t = g(1)) \neq (u'(c_t)|g_t = g(2))\ \forall t$ from (2) and (9). Second, inflation depends solely on the initial state, so $t = 1$ state revelations don't affect the central bank's promised π_1 (and all inflation falls out of the determinant of $A_t\ \forall t$). Using arguments found towards the end of Buera and Nicolini (2004) section 2.2.2, these facts are necessary and sufficient for A_t to be non-singular $\forall t$. Thus the zero tax rate path is in $\mathcal{T}$.

To see that $\frac{1}{c_0}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^i \left[u'(c_t)c_t - v'(n_t)n_t\right]$, simply notice that $u'(c_t)c_t - v'(n_t)n_t = u'(c_t)(\tau_t n_t - g_t)$, which is necessarily negative $\forall t$.

Proof for Lemma 1

First notice that the consolidated, committing, benevolent government's time 1 implementability constraint is:

$$\frac{1}{\pi_1}\left\{u'(c_1)b_0^{(1)} + \beta\left[\frac{u'(c_2)}{\pi_2}\right]b_0^{(2)}\right\} = \left[u'(c_1)c_1 - v'(n_1)n_1\right] + \beta\left[u'(c_2)c_2 - v'(n_2)n_2\right] \tag{26}$$

where expectations are not needed at time 1. Recall that the government chooses $\pi_t = 1\ \forall t$ under $b_{-1} = 0$. (26) can be written in matrix notation in both states $\{g_{1,\ell}, g_{1,h}\}$ as $A_1 b_0 = Z_1$, where A_1 and Z_1 are defined as in (16):

$$A_1 \equiv \begin{bmatrix} 1 & \beta\left(\dfrac{u'(c_2)}{u'(c_1)}|g_1 = g_\ell\right) \\ 1 & \beta\left(\dfrac{u'(c_2)}{u'(c_1)}|g_1 = g_h\right) \end{bmatrix} \quad , \quad Z_1 \equiv \begin{bmatrix} \sum_{t=1}^{2} \beta^{t-1}\left(\frac{u'(c_t)c_t - v'(n_t)n_t}{u'(c_1)}|g_1 = g_\ell\right) \\ \sum_{t=1}^{2} \beta^{t-1}\left(\frac{u'(c_t)c_t - v'(n_t)n_t}{u'(c_1)}|g_1 = g_h\right) \end{bmatrix} \tag{27}$$

and where the time 0 optimal maturity structure is given by $b_0 = A_1^{-1}Z_1$ along the government's welfare-maximizing plan, so long as A_1 is not rank deficient. I will prove that, under lemma 1's maintained assumptions, a finite optimal structure b_0 cannot exist because $\det(A_1) = 0$. By the assumed CES properties of u and v, (18) becomes:

$$\frac{v'(n_t)}{u'(c_t)} = \frac{1 + \lambda_0 (1 - \sigma)}{1 + \lambda_0 (1 - \varphi)} \;\; \forall \; t \tag{28}$$

so that (9) implies that the tax rate is held constant over time and states $\{\{\tau_t\}_s\}_{t=0}^{2} = \{\tau, \{\tau, \tau\}, \{\tau, \tau\}\}$. Substitute (9) for n_t into (2) to write:

$$(1 - \tau) u'(c_t) = v'(c_t + g_\ell) \;\; \forall t \in \{1, 2\} \tag{29}$$

$$(1 - \tau) u'(c_t) = v'(c_t + g_h) \;\; \forall t \in \{1, 2\} \tag{30}$$

where the standard CES properties of u and v guarantee the existence of such a c_t for $g_s > 0$ and $\tau \in [0, 1)$. (29) and (30) determine c_t at both $t = 1$ and $t = 2$, so $c_1 = c_2$ across all state realizations resulting in $\det(A_1) = 0$.

Proof for Proposition 3

The proof here is straightforward. Under two committing institutions, a CFMPE reduces to a one-shot, simultaneous Nash game in state-contingent plans within a competitive equilibrium framework. By way of proposition 1, satisfaction of (2) and (17) implies the existence of a competitive equilibrium, including household optimization, market clearing and well defined pricing functions. Satisfaction of (20) implies debt-manager optimization, taking $\left\{\{\pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ as given. Satisfaction of (21) implies central bank optimization, taking taking $\left\{\{c_t(\mathcal{H}_t), n_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ as given. The definition of a CFMPE is satisfied when all simultaneously hold, which requires well-defined institutional Lagrange multipliers $\lambda_0^{dm}, \lambda_0^{cb} \notin \{-\infty, \infty\}$ and a well-defined an implementation scheme $\left\{b_0(x_0), \left\{b_1^{(2)}(\mathcal{H}_1)\right\}_{\mathcal{H}_1 \in \bar{\mathcal{H}}_1}\right\}$.

Proof for Lemma 2

It has already been assumed that both a committing debt-manager can feasibly individually satisfy (2) and (17) under $\pi_t = 1 \; \forall t$ and that choices resulting in equilibrium non-existence are individually worse for each institution than the lowest-payout outcome when existence is achieved.

$0 < b_{t-1} < \hat{b}_{t-1} \implies \exists \varepsilon \in \mathbb{R}^2_{++}$ for which $b'_{t-1} = b_{t-1} + \varepsilon < \hat{b}_{t-1}$. Due to the properties of u, v and w and the definition of $\hat{b}_{t-1}$, $\exists \left\{ \{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t} \right\}_{t=0}^{\infty} \in \mathcal{T}$ for which an economy inheriting b'_{t-1} requires either $\pi_0 < 1$ or $\pi_1 < 1$ so that (2) and (17) hold. Finally, by (21), $\pi_0 < 1 \iff \pi_1 < 1 \implies \lambda_0^{cb} < 0$.

B Appendix: A Time Series of American Dilution

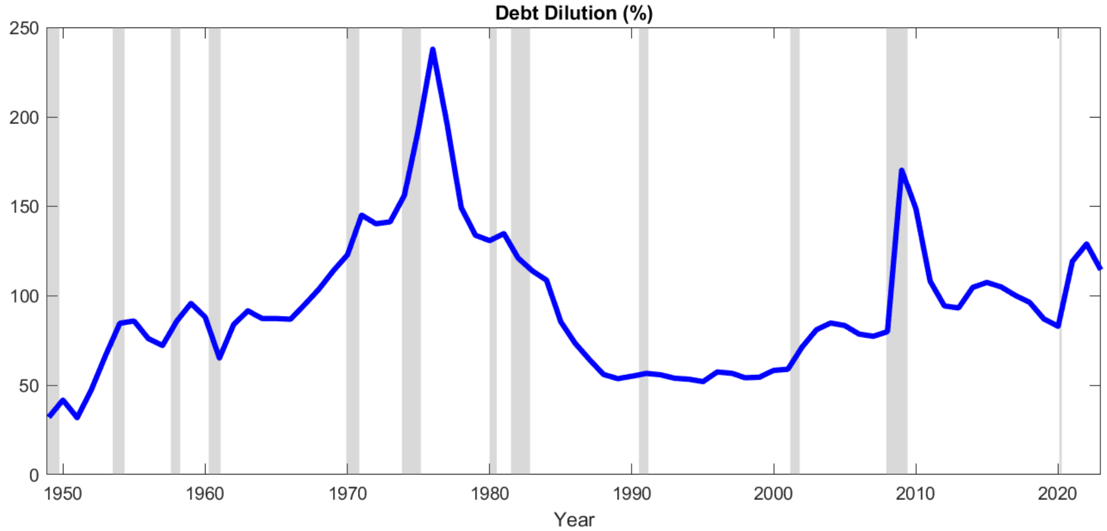

Figure 11: U.S. debt/GDP dilution rate (1948-2022), dilution calculated where $B_t^{(t+1)}$ = (outstanding Fed liabilities + outstanding Treasury debt set to mature in ≤ 1 year)$_t$ and $B_{t-1}^{(t+1)}$ = (outstanding Treasury debt set to mature in > 1 year)$_{t-1}$.

C Appendix: Deriving the Dilution Modifier when $K = 2$

Begin with (12) when $K = 2$ and $J \geq 2$:

$$\frac{B_{t-1}^{(t)}}{P_t} + \frac{\beta^{J-1}\mathbb{E}_t\left[\frac{u'\left(c_{t+J-1}\right)P_t}{u'\left(c_t\right)P_{t+J-1}}\right]B_{t-1}^{(t+J-1)}}{P_t} = \frac{1}{u'\left(c_t\right)}\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[u'\left(c_{t+i}\right)c_{t+i} - v'\left(n_{t+i}\right)n_{t+i}\right] \tag{31}$$

Substituting in for $Q_t^{(t+J-1)}$ using (9), multiplying by 1 and rearranging gives:

$$\frac{u'\left(c_t\right)B_{t-1}^{(t)}}{P_t} + \beta^{J-1}\mathbb{E}_t\left[\left(\frac{B_{t-1}^{(t+J-1)}}{B_{t-1+J-1}^{(t+J-1)}}\right)\frac{u'\left(c_{t+J-1}\right)B_{t-1+J-1}^{(t+J-1)}}{P_{t+J-1}}\right] = \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[u'\left(c_{t+i}\right)c_{t+i} - v'\left(n_{t+i}\right)n_{t+i}\right] \tag{32}$$

(32) sets up a forward-iteration in the $\frac{u'(c_{t+J-1})B_{t-1+J-1}^{(t)}}{P_{t+J-1}}$ term within the market value of long-term debt.

$\frac{B_t^{(t+1)}}{B_{t-J+1}^{(t+1)}}$ is the economy's dilution rate at time t. Define $a_t \equiv \frac{B_{t-J+1}^{(t+1)}}{B_t^{(t+1)}}$ as the economy's inverse dilution rate to write (32) as:

$$\frac{u'\left(c_t\right)B_{t-1}^{(t)}}{P_t} = \left\{\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[u'\left(c_{t+i}\right)c_{t+i} - v'\left(n_{t+i}\right)n_{t+i}\right]\right\} - \beta^{J-1}\mathbb{E}_t\left[\frac{a_{t-1+J-1}u'\left(c_{t+J-1}\right)B_{t-1+J-1}^{(t+J-1)}}{P_{t+J-1}}\right] \tag{33}$$

Then, using the law of iterated expectations after repeated substitutions of the $\frac{u'(c_{t+J-1})B_{t-1+J-1}^{(t)}}{P_{t+J-1}}$ term, the price level can be written only as a function of current and future expected allocations. When households have perfect foresight of debt management policy, express this relation as:

$$\frac{B_{t-1}^{(t)}}{P_t} = \frac{1}{u'\left(c_t\right)}\sum_{i=0}^{\infty}\beta^i\left[u'\left(c_{t+i}\right)c_{t+i} - v'\left(n_{t+i}\right)n_{t+i}\right]\left(1 + \underbrace{\sum_{k=1}^{\left\lfloor\frac{i}{J-1}\right\rfloor}\prod_{h=1}^{k} -a_{t+h(J-1)-1}}_{\text{Dilution modifier}}\right) \tag{34}$$

where $\lfloor\cdot\rfloor$ represents the floor function, which rounds all decimals down to the integer. And where a constant $a_t = a \;\forall t$ must satisfy $a < \left|(1/\beta)^{J-1}\right|$.

D Appendix: Numerical Exercise Parameterization

The numerical specification given in section 4.2 is rewritten here:

Parameter	Value	Description
β	0.9875^4	Time discount rate
σ	2	Inverse of IES
φ	2	Inverse of Frisch elasticity of labor
θ	3.87	Relative sensitivity to inflation
g_0	0.224	Initial government spending
(g_ℓ, g_h)	$(0.224, 0.464)$	Low and high-spending amounts
$\left(b_{-1}^{(0)}, b_{-1}^{(1)}\right)$	$(0.221, 0.271)$	Inherited maturity structure
p	75/76	Probability of entering low-spending state

Table 7: Numerical specifications

β is the quarterly time discount rate found in Angeletos (2002) and Buera and Nicolini (2004) converted to an annual rate and σ and φ are consistent with common estimates of IES and Frisch elasticities.

The government spending states are taken from the BLS's National Income and Product Accounts (NIPA) tables. In table 30200-A, for each year between 1942 and 2021, take the sum of consumption expenditures (line 25), gross government investment (line 45) and net purchases of nonproduced assets (line 27). Then net out consumption of fixed capital (line 48). Divide the resulting sum by GDP found in table 10105-A for the corresponding year. The result is a time series of (government spending)/GDP. Call this series $\{x_t\}_{t=1942}^{2021}$

Given this time series, choose g_ℓ and g_h to minimize the following expression:

$$\min_{g_\ell; g_h} \left\{ \sum_{t=1942}^{2021} \min \left[(g_\ell - x_t)^2 , (g_h - x_t)^2 \right] \right\} \tag{35}$$

The result of this minimization is $(g_\ell \ , \ g_h) = (0.224 \ , \ 0.464)$.

Identifying high-spending regimes and low-spending regimes in the data show that the first four years (1942-1945) were high-spending regimes while the following seventy-six years (1946-2021) were low-spending. The transition probability from low-spending to high-spending in the data is zero. To generate model dynamics, the probability of entering in the high state from the low-spending state is calibrated to be $p_{\ell,h} = 1/76$.

The inherited maturity structure is chosen to match the average U.S. maturity structure from

1947-2022 where short-term debt is defined as [(outstanding Treasury debt with 1 year or less until maturity)$_t$ + (Fed liabilities)$_t$]/GDP$_t$ and where long-term debt is defined as (outstanding Treasury debt with time-to-mature of greater than 1 year)$_t$/GDP$_t$. Fed liabilities is the sum of reserves outstanding, currency outstanding, reserve repurchase agreements and other deposit liabilities. θ is chosen so that the consolidated, committing, benevolent government chooses $\pi_0 = 1.032$, the average inflation rate over 1947-2022 as defined by the GDP deflator.

The infinite-lived model uses the same paramaterization as the three-period model for consistency, but, in addition to the transition probability from the low state in time t to the low state in time $t+1$, $p_{\ell\ell} = p = 75/76$, it also includes the transition probability $p_{hh} = 3/4$, as, out of four high-spending states since 1942 in the U.S., only one of them transitioned to a low-spending state.

E Appendix: Unique CFMPE Under a Commitment Discrepancy

As one committing institution gains bargaining power over its committing opponent, the economy moves toward a corner. How does such an outcome compare with those where the government is comprised of one committing institution and one non-committing institution?

E.1 A Committing Debt-Manager and Non-Committing Central Bank

First evaluate the case where the debt-manager has commitment power and the central bank does not. The debt-manager takes time 0 central bank policy $\{\pi_0(x_0), \boldsymbol{b}_0(x_0)\}$ as given and simultaneously chooses its entire state-contingent plan $\left\{\{\tau_t(\mathcal{H}_t), \boldsymbol{b}_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ to maximize (3), taking future central bank best response functions as additional constraints. The time 0 central bank takes the debt-manager's state contingent plan as given and chooses time 0 policy to maximize (4), understanding how future central banks behave. Times 1 and 2 central banks take the debt-manager's promised policy as constraints in their problems, choosing $\{\pi_1(x_1), \boldsymbol{b}_1(x_1)\}$ and $\{\pi_2(x_2)\}$, respectively. A CFMPE exists when none of the four players (time 0 debt-manager, times 0, 1 and 2 central banks) wish to deviate from their plans, given the information set and choices of the other three players. There are two main cases to consider. The first is $\frac{1}{\pi_0} > 0$ and the second is $\frac{1}{\pi_0} = 0$, with the second case being the $\alpha, \kappa \to 1$ limiting case from section 4.3.

Under $\frac{1}{\pi_0} > 0$, the real market value of $b_{-1}^{(1)}$ at time 0 is strictly positive when $\frac{1}{\pi_1} > 0$. The time 1 central bank takes promised debt-manager policy as a constraint, so the debt-manager regrets any tax plan that doesn't imply $\frac{1}{\pi_1} = 0$ in order to minimize its smoothed tax path implied by (20). Time 1 hyperinflation implies undefined debt prices $Q_0^{(1)}$ according to (9) and thus violates the definition of a CFMPE.

Under $\frac{1}{\pi_0} = 0$, the real market value of all inherited debt b_{-1} at time 0 is zero. As discussed in lemma 1, there is no finite maturity structure that implements the tax distortion-minimizing plan under zero inherited debt. (7) and $P_0 < \infty$ are violated and, as such, so is the definition of a CFMPE. This reasoning is the same for why no CFMPE in the $\alpha, \kappa \to 1$ corner exists.

There exists no CFMPE when the debt-manager has access to commitment technology while the central bank does not. All potential $\pi_0 = \infty$ CFMPE are eliminated under a committing debt-manager and non-committing central bank using the same logic which eliminates these potential

CFMPE under two committing institutions.

E.2 A Committing Central Bank and Non-Committing Debt-Manager

When the central bank has commitment power while the debt-manager does not, the central bank takes time 0 fiscal policy $\{\tau_0(x_0), \boldsymbol{b}_0(x_0)\}$ as given and simultaneously chooses its state-contingent plan $\left\{\{\pi_t(\mathcal{H}_t), \boldsymbol{b}_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ to maximize (4), taking future debt-manager best response functions as additional constraints. The time 0 debt-manager takes the central bank's state contingent plan as given and chooses time 0 policy to maximize (3), understanding how future debt-managers behave. Times 1 and 2 debt-managers take promised monetary policy as constraints in their problems, choosing $\{\tau_1(x_1), \boldsymbol{b}_1(x_1)\}$ and $\{\tau_2(x_2)\}$, respectively. A CFMPE exists when none of the four players (time 0 central bank, times 0, 1 and 2 debt-managers) wish to deviate from their plans, given the information set and choices of the other three players.

Begin this section's analysis by noting that the economy described in section 4.2.2 can be entirely financed through taxes at times 1 and 2. That is, there exists some tax schedule $\left\{0, \left\{\{\tau_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=1}^{2}\right\}$ such that (2) and (17) hold under $\pi_t = 1\ \forall t$. Consider two cases under this commitment technology endowment. The first is $\exists \pi_t' \in \left\{\{\pi_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2} : \pi_t' \neq 1$ and the second is $\pi_t = 1\ \forall t$.

Under the first case, the central bank always regrets not setting $\pi_t = 1\ \forall t$. Even in the extreme case when the time 0 debt-manager sets $\tau_0 = 0$, there still exists a central bank policy plan $\left\{\{\pi_t(\mathcal{H}_t), \boldsymbol{b}_t(\mathcal{H}_t)\}_{\mathcal{H}_t \in \bar{\mathcal{H}}_t}\right\}_{t=0}^{2}$ such that times 1 and 2 debt-managers are constrained to fully finance the government under $\pi_t = 1$. Thus, the central bank always chooses to deviate from these outcomes which violates the CFMPE definition.

Under the second case, a CFMPE exists so long as neither the central bank nor the time 0 debt-manager wishes to deviate from $\boldsymbol{b}_0$. When $\pi_t = 1\ \forall t$, the central bank is indifferent about the path of issued debt. One can imagine two CFMPE of interest under this arrangement: one where the central bank 'allows' debt-managers to select debt under $\pi_t = 1\ \forall t$ and one where the time 0 debt-manager and the central bank agree on the debt plan implied by two committing institutions under $\alpha, \kappa = 0$. The first would have the central bank commit to debt policy in line with what time-consistent debt-managers would choose under $\pi_t = 1\ \forall t$. The second would have the central bank commit to debt policy in line with maximizing the debt-manager's payout also conditional on $\pi_t = 1\ \forall t$. The following refinement selects among the entire set of CFMPE under this commitment discrepancy.

E.3 The $\boldsymbol{\rho}$ Limit: A Refinement

Consider the economy when institution $i \in \{dm, cb\}$ has commitment power and when institution $-i$ does not have such technology under $\rho^{dm} = 1$ and $\rho^{cb} = 0$. When faced with multiple equilibria, pick the subset of equilibria to which are approached as the committing institution's weight on the non-committing institution's payout falls to zero. That is, keeping ρ^{-i} fixed, refine the CFMPE definition to eliminate all CFMPE to which are not approached as $\rho^i \to \mathbb{1}_{dm}(i)$. This refinement selects on-equilibrium paths that maximize player $-i$'s payout conditional on player i's payout being maximized.

Under section E.2's central bank commitment, debt-manager non-commitment, $\rho^{dm} = 1$ case, all CFMPE for $\rho^{cb} \in (0, 1)$ match that in the $\alpha, \kappa = 0$ full (dual) commitment case. This same CFMPE is trivially the limit of CFMPE as $\rho^{cb} \to 0$. Apply the ρ limit refinement to the CFMPE definition so as to eliminate all CFMPE other than that which matches $\alpha, \kappa = 0$.[23]

The unique (refined) CFMPE under a non-committing debt-manager and committing central bank is equivalent to the CFMPE under two committing institutions when the central bank has unilateral bargaining power.

[23]For expositional convenience, all future mentions of CFMPE will indicate payoff-dominant, indifferent-altruistic CFMPE in pure strategies.

F Appendix: Numerical Exercise Outcomes

Table 8 displays how endogenous variables move as the three-period baseline economy varies from $\omega = 0$ to $\omega \to 1$ under two committing institutions. Call the time t dilution modifier on time h surpluses in state s as $\mathrm{M}_{t,h,s}$.

Variable	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 0.9$	$\omega = 0.95$	$\omega \to 1$
c_0	0.78	0.79	0.79	0.79	0.80	0.81	0.84
$(c_{1,\ell}, c_{1,h})$	(0.79,0.70)	(0.79,0.70)	(0.79,0.70)	(0.79,0.70)	(0.80,0.71)	(0.81,0.71)	(0.84,0.74)
$(c_{2,\ell}, c_{2,h})$	(0.76,0.67)	(0.76,0.67)	(0.77,0.67)	(0.77,0.68)	(0.79,0.69)	(0.80,0.71)	(0.84,0.74)
n_0	1.01	1.01	1.01	1.02	1.02	1.03	1.06
$(n_{1,\ell}, n_{1,h})$	(1.01,1.16)	(1.01,1.16)	(1.01,1.16)	(1.02,1.16)	(1.03,1.17)	(1.04,1.18)	(1.06,1.20)
$(n_{2,\ell}, n_{2,h})$	(0.99,1.13)	(0.99,1.13)	(0.99,1.13)	(1.00,1.14)	(1.01,1.15)	(1.03,1.17)	(1.06,1.20)
τ_0	0.37	0.37	0.37	0.36	0.33	0.30	0.21
$(\tau_{1,\ell}, \tau_{1,h})$	(0.36,0.35)	(0.36,0.35)	(0.36,0.35)	(0.35,0.34)	(0.32,0.32)	(0.29,0.29)	(0.21,0.21)
$(\tau_{2,\ell}, \tau_{2,h})$	(0.44,0.44)	(0.43,0.43)	(0.43,0.43)	(0.41,0.41)	(0.37,0.37)	(0.32,0.32)	(0.21,0.21)
π_0	1.00	1.01	1.03	1.10	1.30	1.76	∞
$(\pi_{1,\ell}, \pi_{1,h})$	(1.00,1.00)	(1.01,1.01)	(1.02,1.02)	(1.05,1.06)	(1.12,1.16)	(1.18,1.24)	(1.00,1.00)
$(\pi_{2,\ell}, \pi_{2,h})$	(1.00,1.00)	(1.00,1.00)	(1.00,1.00)	(1.00,1.00)	(1.00,1.00)	(1.00,1.00)	(1.00,1.00)
$b_0^{(1)}$	21.5	22.4	24.3	30.6	54.3	115	∞
$b_0^{(2)}$	-20.7	-21.7	-23.6	-30.1	-54.8	-118	$-\infty$
$(b_{1,\ell}^{(2)}, b_{1,h}^{(2)})$	(0.21,0.03)	(0.20,0.02)	(0.20,0.02)	(0.18,0.00)	(0.15,-0.04)	(0.11,-0.09)	(0.00,-0.21)
$Q_0^{(1)}$	0.94	0.94	0.93	0.90	0.85	0.81	0.00
$Q_0^{(2)}$	0.96	0.96	0.94	0.91	0.84	0.79	0.00
$(Q_{1,\ell}^{(2)}, Q_{1,h}^{(2)})$	(1.02,1.04)	(1.02,1.04)	(1.01,1.03)	(1.00,1.02)	(0.99,0.99)	(0.97,0.98)	(0.95,0.95)
$q_0^{(1)}$	0.94	0.94	0.94	0.95	0.95	0.95	0.95
$q_0^{(2)}$	0.96	0.96	0.96	0.95	0.94	0.93	0.91
$(q_{1,\ell}^{(2)}, q_{1,h}^{(2)})$	(1.02,1.04)	(1.02,1.04)	(1.01,1.03)	(1.00,1.02)	(0.99,0.99)	(0.97,0.98)	(0.95,0.95)
a_0	0.01	0.01	0.01	0.01	0.00	0.00	0.00
$(a_{1,\ell}, a_{1,h})$	(-101,-746)	(-106,-879)	(-118,-1291)	(-158,-51530)	(-332,1195)	(-948,1071)	$(-\infty,\infty)$
$\mathbb{E}_0 a_1$	-110	-117	-134	-834	-312	-922	∞
$\mathrm{M}_{0,1}$	-0.01	-0.01	-0.01	-0.01	-0.00	-0.00	-0.00
$\mathbb{E}_0 \mathrm{M}_{0,2}$	-1.40	-1.41	-1.46	-6.77	-1.20	-1.24	undef
$(\mathrm{M}_{1,2,\ell}, \mathrm{M}_{1,2,h})$	(101,746)	(106,879)	(118,1291)	(158,51530)	(332,-1195)	(948,-1071)	$(\infty,-\infty)$
λ_0^{dm}	0.16	0.16	0.16	0.15	0.13	0.11	0.06
λ_0^{cb}	0.00	0.05	0.16	0.45	1.26	2.48	∞
W_0^{dm}	-4.64	-4.63	-4.63	-4.62	-4.61	-4.59	-4.56
W_0^{cb}	0.00	0.00	0.00	-0.02	-0.12	-0.40	-1.94
W_0	-4.64	-4.63	-4.63	-4.64	-4.73	-4.99	-6.49

Table 8: Baseline outcomes from CFMPE along $\omega \in [0, 1)$

G Appendix: Bargaining History, Kalai-Smorodinsky Solution

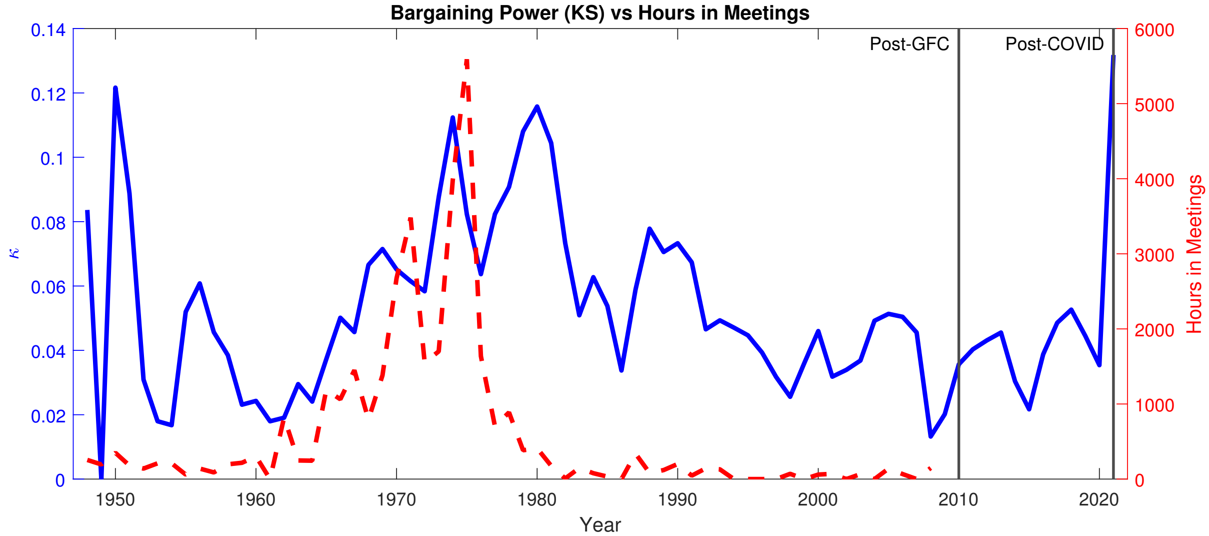

Figure 12: Comparing imputed fiscal bargaining power with hours spent between presidents and Fed officials annually (1948-2021)

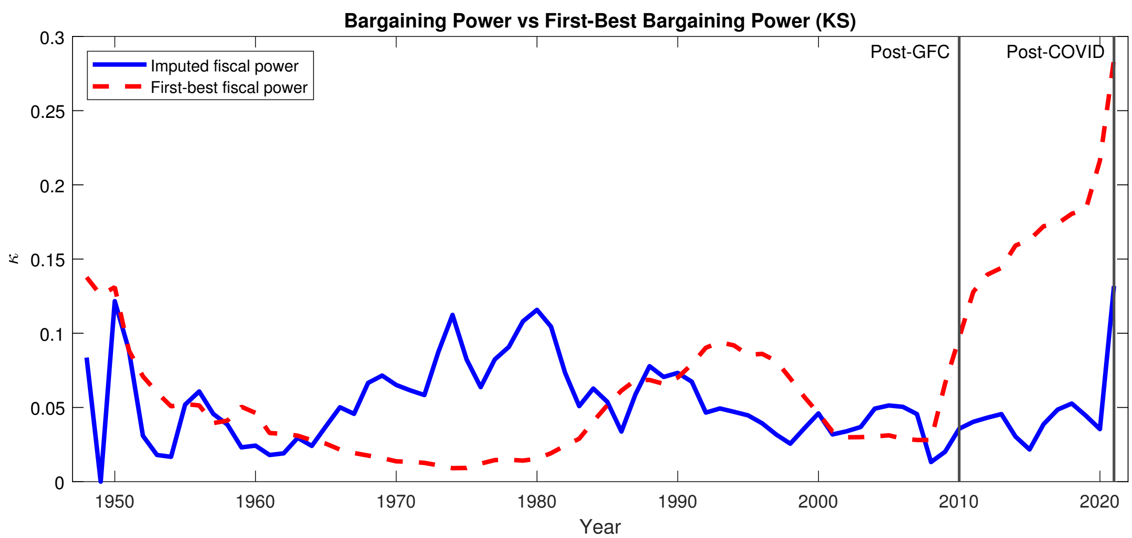

Figure 13: Comparing imputed fiscal bargaining power with that which matches first-best (1948-2021)

H Appendix: Infinite-Lived Model

Now turn back to the infinitely lived model under the timeless perspecitve proposed in Woodford (1999) described in sections 2 and 3. Section H.2 analyzes the model under various commitment endowments using methods from Sargent and Velde (1999). When both institutions lack the ability to commit to future policy, the two only agree on a single maturity structure when the bargaining power parameter is both endogenous and time-varying. The following analysis does not consider this case.

H.1 First-Best: A Consolidated, Committing, Benevolent Government

Appendix I derives the economy's Ramsey planner's FOCs:

$$c_t, n_t: \qquad u'(c_t) + \lambda_0 \left[u'(c_t) + u''(c_t)\, c_t\right] = v'(n_t) + \lambda_0 \left[v'(n_t) + v''(n_t)\, n_t\right] \tag{36}$$

$$\pi_t: \qquad \pi_t = 1 \tag{37}$$

so that $\{c_t, n_t, \pi_t\}_t$ and λ_0 are outcomes of (2), (15), (36), (37), given time-t expectations over the exogenous path of $\{g_t\}$. Call the resulting allocations (plus inflation) the economy's 'first-best' outcome for the remainder of the analysis. (37) reduces the model (via optimal policy) back to one with only real debt as in Lucas and Stokey (1983).

Inflation is optimally not used by the timeless committing Ramsey planner because it imposes costs on households without yielding any budgetary benefit. The lack of b_{-1} removes the need for initial debt devaluation. Rational adjustments of Q_t to inflation keeps it from relaxing (15).

Apply the same utility specification from (22) so that tax rates are optimally held constant $\tau_t = \tau^* \ \forall t$. A perfectly smooth tax rate finances all current and future government spending while minimizing distortions over households' lives. Appendix I.1 outlines how the maturity structure implements the consolidated, committing, benevolent government's optimal plan. Applying the same numerical values to utility parameters and $\{g_s\}_s$ as in section 4.2.2, and setting $p_{\ell\ell} = \frac{75}{76}$ and $p_{hh} = \frac{3}{4}$ fully specifies the model.[24]

The time-invariant, state-invariant maturity structure that supports 'first-best' features a large

[24]Appendix D discusses these decisions.

short-term asset (negative debt) position and a large, offsetting long-term debt position. Figure 14 displays the the unique, welfare-maximizing maturity structure's debt-to-GDP ratio in the low-spending state $\left\{\frac{b_t^{(t+1)}}{n_\ell}, \frac{b_t^{(t+2)}}{n_\ell}\right\}$.

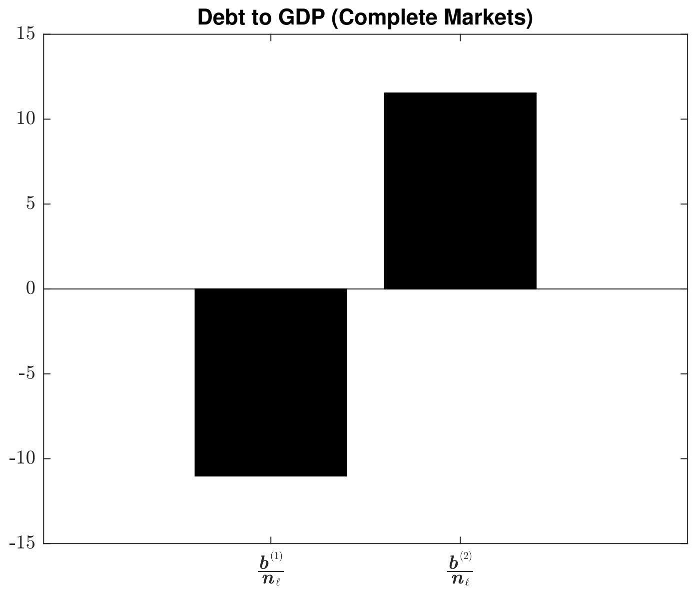

Figure 14: Debt/GDP under a consolidated, committing, benevolent government in the low-spending state

Like in the three-period model, carefully selected, highly skewed positions perfectly hedge against future spending uncertainty through future (time $t+1$) debt price movements. Targeting specific $t+1$ debt payouts with access only to debt instruments that have similar return profiles (as is the case with 1- and 2-period non-contingent debt) generates the high skewness of the position.

Debt prices fall relative to expectations when the economy enters a high-spending (high marginal utility) regime. The government's long-term debt position becomes less burdensome in the states in which it needs to finance higher spending. Appendix F includes the model's Ramsey solution and consolidated, committing, benevolent government implementation.

Table 9 displays the solutions to the infinite-lived baseline model.

H.2 Under Various Commitment Endowments

H.2.1 Baseline: A Committing Debt-Manager and Committing Central Bank

A committing debt-manager maximizes (3) and a committing central bank maximizes (4), both doing so subject to (2) and (15). Continue to apply Woodford (1999)'s timeless perspective.

Variable	Description	Result
τ^*	Constant tax rate	0.222
$\{z_\ell, z_h\}$	Ramsey transfers	$\{0, -0.687\}$
$\{c_\ell, c_h\}$	Consumption	$\{0.834, 0.736\}$
$\{n_\ell, n_h\}$	Labor supply	$\{1.06, 1.20\}$
$\boldsymbol{b} = \boldsymbol{b}(\ell\} = \boldsymbol{b}(h)$	Outstanding maturity structure	$\{-11.65, 12.21\}$
a	Constant inverse dilution rate	-1.048
$\left\{q_\ell^{(1)}, q_\ell^{(2)}\right\}$	Asset prices in the low-spending state	$\{0.955, 0.91\}$
$\left\{q_h^{(1)}, q_\ell^{(2)}\right\}$	Asset prices in the high-spending state	$\{0.898, 0.817\}$

Table 9: Results under a consolidated, committing government

The central bank follows (37) while the debt-manager follows (36), replacing λ_0 with λ_0^{dm}. Like the three-period model under $b_{-1} = 0$, the debt-manager gains no additional capacity to lower taxes from additional inflation. Concurrently, the central bank is not constrained by either (2) or (15) so it is free to achieve its global optimum, setting inflation $\pi_t = 1\ \forall t$. Then, the debt-manager solves the consolidated, committing, benevolent government's problem, setting $\tau = \tau^*\ \forall t$. The unique CFMPE under two committing institutions is identical to that under a single committing, benevolent government.

H.2.2 A Committing Debt-Manager and Non-Committing Central Bank

A committing debt-manager maximizes (3) and a non-committing central bank maximizes (4), both doing so subject to (2), (15) and future central bank optimization. Continue to apply Woodford (1999)'s timeless perspective.

It continues to be the case that inflation has no budgetary effect on (15). The debt-manager commits to $\tau = \tau^*\ \forall t$, indifferent about future inflation policy. Given this current and promised future policy, the time t central bank understands that ex-post inflation ($\pi_{t+1} \neq 1$) financing will be required in one or both future exogenous states should the maturity structure be chosen differently than that described in appendix I.1. The non-committing central bank thus optimally sets the maturity structure according to that described in appendix I.1 in order to keep its future self unconstrained by (2) and (15).

The unique CFMPE under a committing debt-manager and non-committing central bank is identical to that from sections H.1 and H.2.1. The Ramsey plan is achieved and implemented so long as fiscal policy has commitment power.

H.2.3 A Non-Committing Debt-Manager and Committing Central Bank

A non-committing debt-manager maximizes (3) and a committing central bank maximizes (4), both doing so subject to (2), (15) and future debt-manager optimization. Continue to apply Woodford (1999)'s timeless perspective.

Like in section H.2.1, the committing central bank sets $\pi_t = 1 \ \forall t$. Using the ρ limit refinement outlined in section E.3, the central bank also commits to future maturity structures consistent with debt-manager's payout maximization as in section H.2.1. Given the central bank's committed future inflation and debt policies, the time t debt-manager understands that the economy can be financed using a single, perfectly-smooth tax rate $\tau_t = \tau^* \ \forall t$. The committing central bank maximizes household welfare by removing all discretion from the non-committing debt-manager.

The unique CFMPE under a non-committing debt-manager and committing central bank is identical to that from sections H.1, H.2.1 and H.2.2. The Ramsey plan is achieved and implemented so long as either institution has commitment power.

H.3 Reinterpreting an Optimal Result

Maintaining a constant, highly-skewed maturity structure over time requires an active government. It issues new long-term debt, buys back all un-matured debt from the previous period and purchases short-term assets from households at each t, a debt management scheme found in section 3.4.1's table 1.

Committing fiscal policy leads to an economy with constant short-term debt $B_{t-1}^{(t)} = B^{(1)} \ \forall t$, constant long-term debt $B_{t-1}^{(t+1)} = B^{(2)} \ \forall t$ and a constant price level $P_t = P \ \forall t$. The RHS of (14) is contingent only on the exogenous state because the tax rate is constant $\tau_t = \tau^* \ \forall t$. (14) allows for a new economic interpretation of the classic, constant, skewed optimal maturity structure originally described in Angeletos (2002) and Buera and Nicolini (2004): to avoid using ex-post inflation financing, nominal debt is issued strategically so that current and future dilution equates fiscal backing across spending states.

To get a better picture of this idea visually, consider the expected paths of future government primary surpluses upon entering each of the two spending states, displayed in Figure 15.

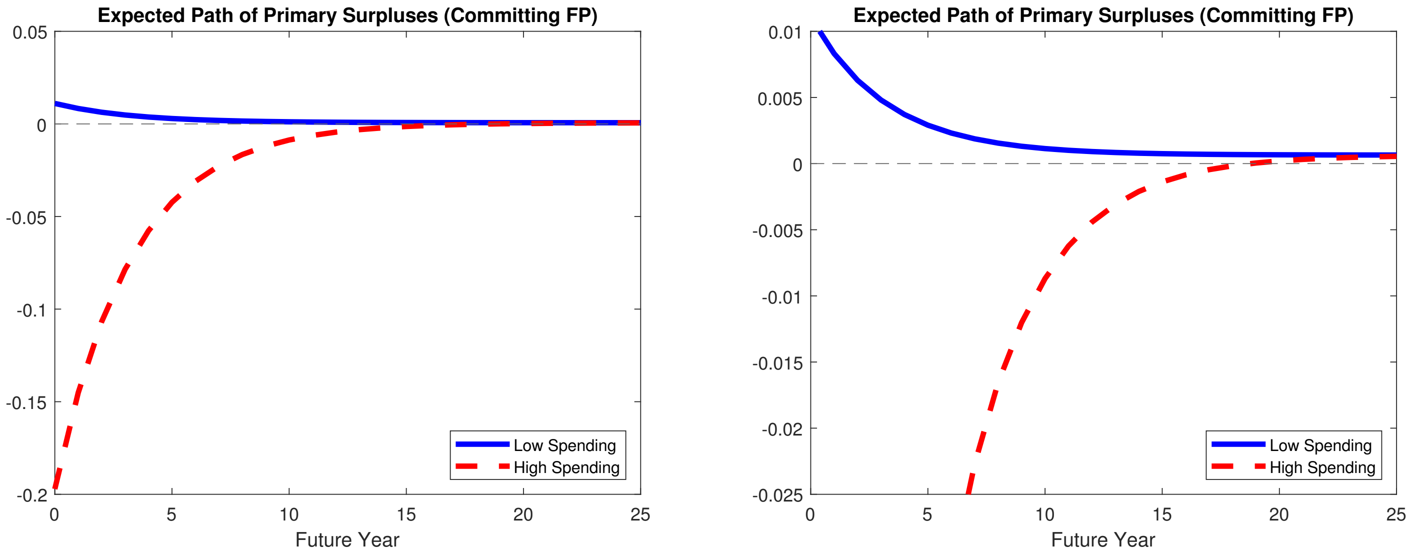

Figure 15: Present and expected future primary surpluses under a committing debt-manager. Right panel is the left panel re-scaled.

Committing fiscal policy runs a primary surplus in the low-spending state ($s = \ell$), and households rationally expect positive primary surpluses for all time thereafter. The government runs a primary deficit in the high-spending state ($s = h$). In this case, households don't expect the government to run another primary surplus until time $t + 19$ at which positive primary surpluses are expected ad infinitum. The path of $t + i$ expected primary surpluses converges to a positive number as $i \to \infty$ regardless of the time t state.

Figure 16 displays the corresponding paths of expected *discounted* future government primary surpluses in each time t state. Because marginal utility is greater in high-spending states relative to that in low-spending states, discount rates weigh deficits more (discount deficits less) than surpluses. The result is that, while expected primary surpluses converge to a positive number, expected discounted primary surpluses converge to zero from below (the compounding β^i naturally dominates as $i \to \infty$). Finally, and importantly, all expected discounted deficits starting at time $t + 20$ and beyond are less-negative under the high-spending-regime than they are under the low-spending-one.

The integral of the solid (dotted) line to zero is the RHS of (13) when the economy experiences a low-spending (high-spending) time t regime. When inflation is costly, the optimal maturity structure features no long-term debt only when these integrals are equal. Otherwise, with only short-term debt, ex-post adjustments to the price level P_t would need adjust to satisfy (13).

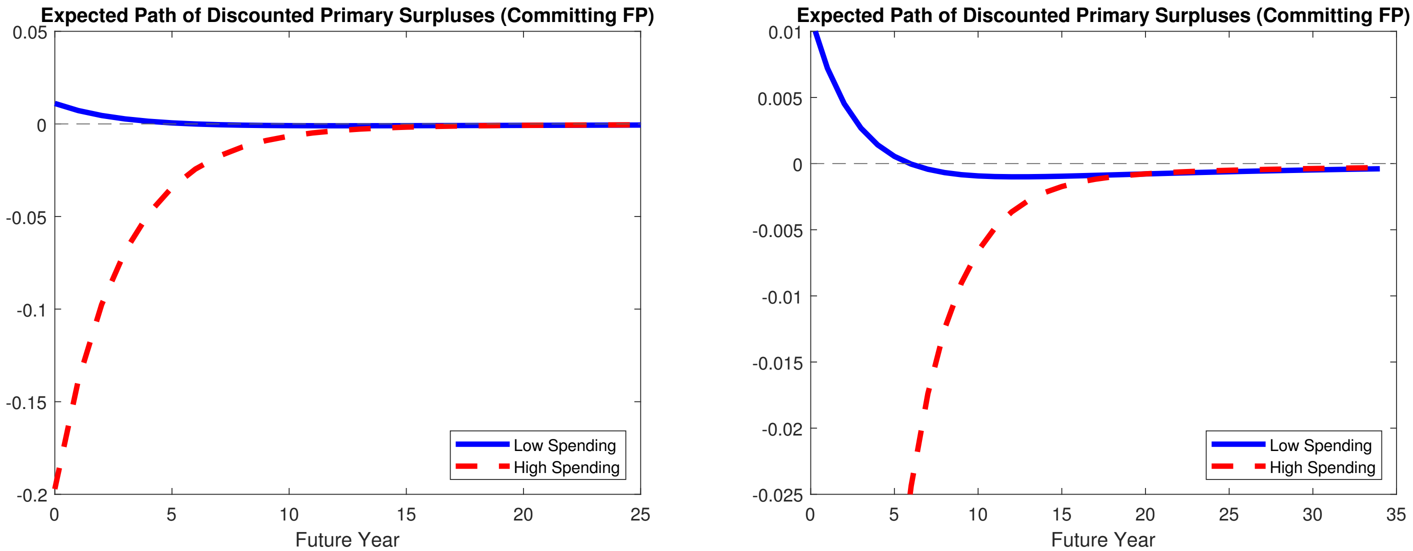

Figure 16: Present and expected future discounted primary surpluses under a committing debt-manager. Right panel is the left panel re-scaled.

Figure 16 clearly shows that the solid and dotted integrals are not equivalent. In order to equate them, the government adjusts the maturity structure of its debt supply such that current and expected future debt dilution affects expected surpluses' discounting until the integrals of expected discounted, *modified* primary surpluses are equated, as shown in figure 17.

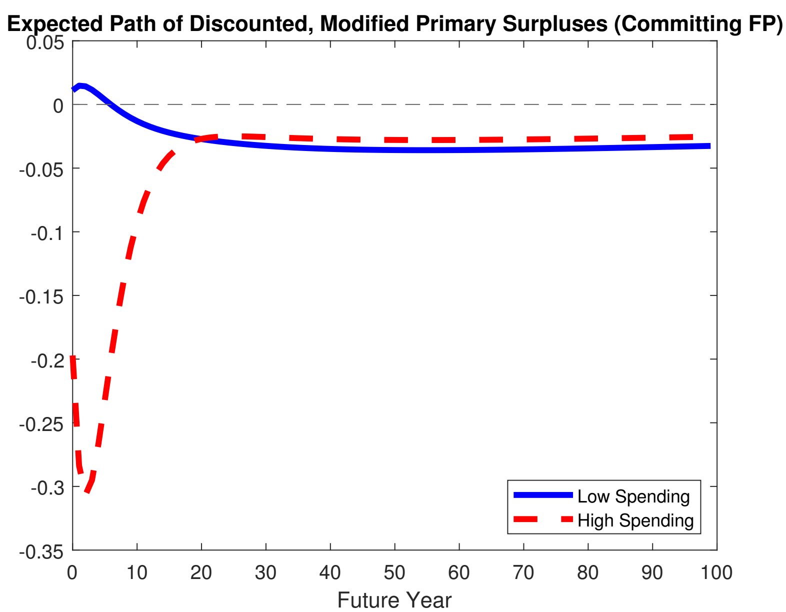

Figure 17: Present and expected future discounted, modified primary surpluses under a committing debt-manager

It is clear from comparing the scale of figures 16 and 17 that the optimal constant (inflation-minimizing) dilution rate amplifies fiscal backing. The large amplification 'fights' β^i so that the space between low- and high-spending discounted, modified primary surpluses extends from time $t + 20$ into the far future to make up for the large initial discrepancy in fiscal expectations. The economy's inverse dilution rate is highly negative at $a = -1.048 > -\frac{1}{\beta} = -1.052$, which comes close to maximizing the feasible amplification of future surpluses. As a comparison between how future surpluses are discounted under a high-spending time t regime in a short-term only ($a = 0$) economy and an economy with optimal maturity management ($a = -1.048$), the expected discounted, modified $t + i$ primary surplus rises to -0.005 at $i = 12$ in the former, and rises to -0.005 at $i = 1070$ in the latter. The dilutive effect delays discounting by over a thousand years.

To avoid using ex-post inflation financing, the committing debt-manager issues nominal debt strategically so that current and future dilution equates fiscal backing across spending states. This strategic issuance drastically changes the nature of how expected future government revenues are discounted back to the present.

I Appendix: Complete Markets Solution and Implementation

To develop an efficient benchmark, consider the infinite-lived consolidated, committing, benevolent government tasked with choosing allocations that maximize household welfare using its policy tools $\{\tau_t, \pi_t, \boldsymbol{b}_t\}$.

Use the quasi-primal approach described in section 3.5 so that the government chooses allocations and inflation $\{c_t, , n_t, \pi_t, \boldsymbol{b}_t\}$ to maximize (1) subject to (2), (5), (7) and (15). Its FOCs are:

$$c_t: \qquad u'(c_t) + \lambda_0 \left[u'(c_t) + u''(c_t) c_t - \beta^t u''(c_t) \left(\prod_{k=0}^{t} \frac{1}{\pi_k} \right) b_{-1}^{(t)} \right] = \phi_t \tag{38}$$

$$n_t: \qquad v'(n_t) + \lambda_0 \left[v'(n_t) + v''(n_t) n_t \right] = \phi_t \tag{39}$$

$$\pi_t: \qquad w'(\pi_t) - \lambda_0 \left(\prod_{i=0}^{t-1} \frac{1}{\pi_i} \right) \sum_{j=t}^{J-1} \beta^j \left[u'(c_j) \pi_t \left(\prod_{k=t}^{j} \frac{1}{\pi_k} \right) \right] b_{-1}^{(j)} = 0 \tag{40}$$

where λ_0 and ϕ_t are the Lagrange multipliers on (15) and (2), respectively. And where $b_t^{(t+i)} = 0$ when $i > J$.

Using the timeless perspective proposed in Woodford (1999), these FOCs become:

$$c_t, n_t: \qquad u'(c_t) + \lambda_0 \left[u'(c_t) + u''(c_t) c_t \right] = v'(n_t) + \lambda_0 \left[v'(n_t) + v''(n_t) n_t \right] \tag{36}$$

$$\pi_t: \qquad \pi_t = 1 \tag{37}$$

so that $\{c_t, n_t, \pi_t\}_t$ and λ_0 are outcomes of (2), (15), (36), (37), given time-t expectations over the exogenous path of $\{g_t\}$. (37) reduces the model back to one with only real debt as is the case in Lucas and Stokey (1983) via optimal policy.

I.1 Implementation Using the Maturity Structure

Despite the lack of access to a full portfolio of state-contingent debt, the committing, benevolent, consolidated government may still implement the complete markets solution if it has access to enough debt maturities. Angeletos (2002) shows first-best is implementable so long as $I \geq S$, and that b_t is uniquely determined when $I = S$. In these cases, the consolidated, committing, benevolent government replicates the complete markets portfolio using linear combinations of available debt maturities.

Assume $I = S$ for the rest of the analysis so that the complete markets solution will always be feasible should any specified government set-up be replaced with a consolidated, committing, benevolent one.

Use the utility specification from (22) so that (36) and (37) become:

$$c_t, n_t: \quad [1 + \lambda_0 (1 - \sigma)] c_t^{-\sigma} = [1 + \lambda_0 (1 + \varphi)] n_t^{\varphi} \tag{41}$$

$$\pi_t: \quad \pi_t = 1 \tag{42}$$

where (41) can be rewritten as:

$$\frac{n_t^{\varphi}}{c_t^{-\sigma}} = \frac{1 + \lambda_0 (1 - \sigma)}{1 + \lambda_0 (1 + \varphi)} \tag{43}$$

λ_0 is a constant and household intratemporal tradeoffs yield $1 - \tau_t = \frac{n_t^{\varphi}}{c_t^{-\sigma}}$ from (9). Both $\tau_t = \tau^*$ and $\pi_t = 1$ are optimally held constant by the consolidated government for all time. Call this optimal policy the government's 'Ramsey plan' which is achievable using a state-contingent portfolio of nominal debt. The goal is to find a non-contingent debt scheme that implements these optimal choices at every period.

The consolidated, committing, benevolent government would like to tax and transfer from households in a lump-sum, state-contingent manner using the maturity structure. Angeletos (2002) and Buera and Nicolini (2004) prove that such a maturity structure exists and is unique when $I = S$. To characterize this implementation, define z_t^* as the welfare-maximizing lump-sum transfer (lump-sum tax) the government sends to households along the Ramsey plan. Define c_t^* and n_t^* as consumption and labor supply choices along the Ramsey plan to rewrite (15) when the government is optimizing as:

$$z_t^* = \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[c_{t+i}^{*\,1-\sigma} - n_{t+i}^{*\,1+\varphi} \right]}{c_t^{*\,-\sigma}} \tag{44}$$

and notice that the RHS of (15) is conditional only on the exogenous state of the economy at time t, given that inflation and tax rates are constant over time and states.

$z_t^* = z(s)$ can be thought of as either the time t market value of state-contingent debt issued by the government at time $t-1$ or the total amount of transfer payments the government makes to the public at time t along the Ramsey tax plan. Because of the lack of state-contingent debt and lump-sum tax technology, z_t^* can only be achieved by the government through manipulation of the maturity structure.

Lucas and Stokey (1983) show that a non-committing central government can construct a debt issuance plan that supports the plan τ^* and induce its successor to do the same. Buera and Nicolini (2004) abstracts from the time inconsistency problem but develops a method for backing out the time-invariant, welfare-maximizing maturity structure in the economy assuming perfectly credible government policy. This paper follows Buera and Nicolini (2004)'s lead for the complete-market specification. The key to optimal management: the central government issues (redeems) a specific debt portfolio so that price changes in non-matured debt brings the market price of the entire portfolio at time t in line with z_t^*.

Given (15), (42), (43) and the stochastic process for g, the welfare-maximizing $z(s)$ is solved for in every state. Define $Z_t^* = Z^*$ $\forall t$ as an $S \times 1$ vector of state-contingent wealth transfers from the government to households along the Ramsey plan:

$$Z_t^* = Z^* = \begin{bmatrix} z(1) & z(2) & \ldots & z(S) \end{bmatrix}'$$

and define A_t^* as an $S \times J$ payout matrix, where each element is the ex-post market value of previously-issued j-period debt along the Ramsey plan:

$$A_t^* = \begin{bmatrix} 1 & \beta \frac{\mathbb{E}_t\left[c_{t+j_2-1}^{*-\sigma} | s=1\right]}{c_t^{*-\sigma}} & \ldots & \beta^{J-1} \frac{\mathbb{E}_t\left[c_{t+J-1}^{*-\sigma} | s=1\right]}{c_t^{*-\sigma}} \\ 1 & \beta \frac{\mathbb{E}_t\left[c_{t+j_2-1}^{*-\sigma} | s=2\right]}{c_t^{*-\sigma}} & \ldots & \beta^{J-1} \frac{\mathbb{E}_t\left[c_{t+J-1}^{*-\sigma} | s=2\right]}{c_t^{*-\sigma}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \beta \frac{\mathbb{E}_t\left[c_{t+j_2-1}^{*-\sigma} | s=S\right]}{c_t^{*-\sigma}} & \ldots & \beta^{J-1} \frac{\mathbb{E}_t\left[c_{t+J-1}^{*-\sigma} | s=S\right]}{c_t^{*-\sigma}} \end{bmatrix} = \begin{bmatrix} 1 & q_{t,1}^{(t+j_2-1)^*} & \ldots & q_{t,1}^{(t+J-1)^*} \\ 1 & q_{t,2}^{(t+j_2-1)^*} & \ldots & q_{t,2}^{(t+J-1)^*} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & q_{t,S}^{(t+j_2-1)^*} & \ldots & q_{t,S}^{(t+J-1)^*} \end{bmatrix}$$

As shown in Buera and Nicolini (2004), the nature of the stochastic process is first-order Markov, so A_t^* is independent of history and, therefore, invariant with respect to the time period and the state. A_t^* can be written as $A_t^* = A^*$. Given Z^*, A^* and the LHS of (15), the following identity holds for government transfers to the public in each period along the Ramsey tax plan:

$$Z^{*} = A^{*}\boldsymbol{b}_{t-1}(s)^{*} \tag{45}$$

Premultiplying both sides of this equation by ${A^{*}}^{-1}$ yields the optimal maturity structure of non-contingent debt:

$$\boldsymbol{b}_{t-1}(s)^{*} = {A^{*}}^{-1} Z^{*} \tag{46}$$

Because both ${A^{*}}^{-1}$ and Z^{*} are constant matrices, the welfare-maximizing debt structure is both state-invariant and thus time-invariant:

$$\boldsymbol{b}_{t-1}(s)^{*} = \begin{bmatrix} \boldsymbol{b}_{t-1}^{(t)}(s)^{*} & \boldsymbol{b}_{t-1}^{(t+1)}(s)^{*} & \dots & \boldsymbol{b}_{t-1}^{(t+J-1)}(s)^{*} \end{bmatrix}' = \boldsymbol{b}^{*} \quad \forall t, s \tag{47}$$