

# Non-Cooperation in Debt Maturity

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# Institutional Separation

Central bank separation from larger government can result in monetary policy shielded from political pressure

- Lower inflation levels, variances (Alesina & Summers, 1993)
- Higher growth rates (Cukierman, Kalaitzidakis, Summers & Webb, 1993)

Isolation comes with institutional differences

- Division of objectives (Stella & Lonnberg, 2008)
- Ability to commit to future policy (Gnocchi, 2013; Rogoff, 1985)

Though isolated, central banks and fiscal counterparts are tightly linked

- Both institutions leverage federal debt markets across maturities
  - FP: issues to finance spending, MP: conducts OMOs
- Government collects central bank remittances as income
  - Remittances = CB operating profits, paid lump-sum
- “Central bank isolation” only an approximation
  - Authority granted by lawmakers

# This paper

Institutional optimization implies strategic interaction

A coordination problem in joint-determination of maturity structure may emerge

I investigate theoretical consequences arising from this problem

1. Infinite-lived model, credible non-cooperative government institutions interact in federal debt markets across maturities

- Solve Ramsey planner's problem, coordinated government implementation
- Solve decentralized equilibrium with institutional non-cooperation

2. Analogous 3-period model

- Vary commitment power
- Compare outcomes to first-best

# The Model

## Households

Model: Lucas & Stokey (1983) with nominal debt

Infinitely-lived households choose consumption, labor supply and stocks of non-contingent government debt (across various maturities) to maximize utility, subject to a sequence of budget constraints

Utility function:

$$u(c_t, n_t \mid \pi_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi} - \frac{1}{2} \left( \frac{1}{\pi_t} - 1 \right)^2, \quad \pi_t = \frac{P_t}{P_{t-1}}$$

HHBC at  $t$ :

$$P_t c_t + \underbrace{\sum_{j=1}^J Q_t^{(t+j)} B_t^{(t+j)}}_{\text{MV(newly purchased debt)}} = (1 - \tau_t) P_t n_t + B_{t-1}^{(t)} + \underbrace{\sum_{j=1}^{J-1} Q_t^{(t+j)} B_{t-1}^{(t+j)}}_{\text{MV(unmatured } t-1 \text{ debt)}} \quad (1)$$

# Market-Clearing

Commodity, Asset Markets

Production technology owned by households, linear:  $y_t = n_t$

Goods market,  $J$  debt markets  $B_t = \left\{ B_t^{(t+j)} \right\}_{j=1}^J$

Goods market-clearing (aggregate resource constraint):

$$n_t = c_t + g_t$$

where  $g_t$  follows an  $S$ -state Markov process

Bond prices  $Q_t = \left\{ Q_t^{(t+j)} \right\}_{j=1}^J$  clear debt markets

- Pinned down by household optimization

# Ramsey Planner

## Efficient Benchmark

**Definition:** A welfare-maximizing Ramsey planner chooses  $\tau_t$ ,  $\pi_t$  and a **portfolio of complete-markets state contingent debt** in every period to maximize lifetime expected household welfare, subject to private optimization, the ARC and the household budget constraint.

Planner's solution called **Ramsey plan**

- Efficient benchmark of economy

If some issuance of  $B_t = B_t^*$  supports Ramsey plan, call  $B_t^*$  economy's **optimal maturity structure** at  $t$

Angeletos (2002) shows implementation with indexed debt possible when  $J \geq S$ , unique **optimal maturity structure** when  $J = S$

Focus on  $S = J = 2$  for today

# Consolidated, Benevolent Government

## Price Determination

$B_t$  in terms of nominal resources (dollars)

What determines value of dollars (in terms of real allocations)? Not so straightforward

Combine ARC, HHBC, HHFOCs  $\rightarrow$  forward-iterate on MV(government debt), HHTVC

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \beta \mathbb{E}_t \left[ \left( \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \right) \left( \frac{P_t}{P_{t+1}} \right) \right] \frac{B_{t-1}^{(t+1)}}{P_t}}_{\text{MV(outstanding government liabilities)/}P} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i})}_{\text{Exp. discounted current and future real primary surpluses}}$$

Want  $P_t$  only in terms of current and expected future allocations

Almost there, future expected  $P_{t+1}$  shows up on LHS

Household FOCs

# Consolidated, Benevolent Government

## Price Determination

Call the economy's **dilution rate**  $\frac{B_t^{(t+1)}}{B_{t-1}^{(t+1)}}$

When government sells ST debt while LT debt outstanding, it dilutes value of existing claims on  $t + 1$ 's surpluses. Expected future dilution affects current price of LT debt.

Define economy's **inverse dilution rate** as  $a_t = \frac{B_{t-1}^{(t+1)}}{B_t^{(t+1)}}$

When future debt choices are known, GBC becomes

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}}_{(\text{Maturing debt})/P} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i})}_{\text{Exp. diluted discounted current and future real primary surpluses}} \left( 1 + \underbrace{\sum_{k=1}^i \prod_{h=1}^k -a_{t+h-1}}_{\text{maturity modifier}} \right) \quad (2)$$

Only  $P_t$  remaining on LHS  $\rightarrow$  (2) prices nominal resources at  $t$



# Inverse Dilution Rate

## Oscillating Signs

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i}) \left( 1 + \sum_{k=1}^i \prod_{h=1}^k -a_{t+h-1} \right) \quad (2)$$

Interacting (positive)  $a_t$  terms oscillate signs within sum going forward

For intuition, given exogenous future expected primary surpluses

$$a_t \uparrow \implies B_t^{(t+1)} \downarrow \rightarrow P_{t+1} \downarrow \rightarrow Q_t^{(t+1)} \uparrow \rightarrow P_t \uparrow$$

$$\mathbb{E}_t a_{t+1} \uparrow \implies B_{t+1}^{(t+2)} \downarrow \rightarrow P_{t+2} \downarrow \rightarrow Q_{t+1}^{(t+2)} \uparrow \rightarrow P_{t+1} \uparrow \rightarrow Q_t^{(t+1)} \downarrow \rightarrow P_t \downarrow$$

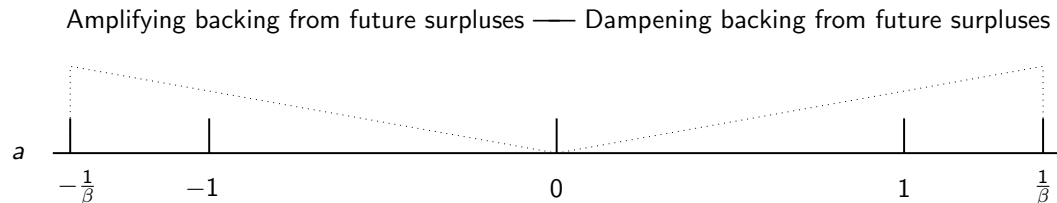
Positive future expected persistent dilution offsets within  $Q_t^{(t+1)}$

# Inverse Dilution Rate

## Dilution Ad Infinitum

What happens when a government dilutes every period forever?

Relative to ST-only case ( $a = 0$ )



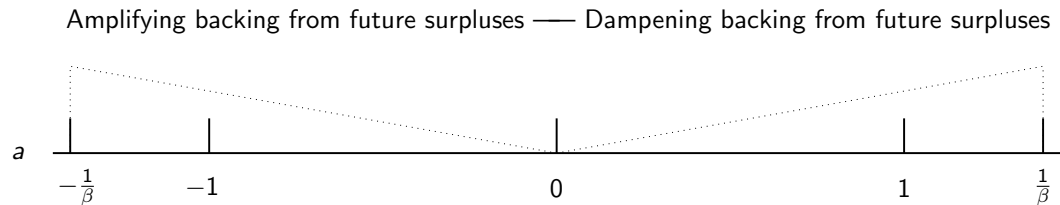
$a = 1$ : Government lets all LT debt run off every period

$a = -1$ : Government sells all LT assets, issues same magnitude of ST debt every period

RHS of (2) explodes when  $|a| > \frac{1}{\beta}$

# Inverse Dilution Rate

Dilution Ad Infinitum



Government delays final repayment of maturing debt when borrowing positive amounts in both ST and LT debt

Moves up final maturing debt payment when issuing ST debt but purchasing LT assets

Note: Cochrane (2001), Leeper & Leith (2017) provide more general formula ( $J > 2$ )

# Back to the Planner

## Ramsey Plan

Ramsey Planner constrained by  $t = 0$  pricing equation (2)

Define real debt allocations as  $b_t^{(t+j)} = \frac{B_t^{(t+j)}}{P_t}$

- (2) moves from  $B$  and  $P$  terms  $\rightarrow b$  and  $\pi$  terms

Pricing Equation Evolution

No inflation in constraint except  $\pi_0$  and  $\pi_1$

Committing Ramsey Planner:  $\pi_t = 1 \quad \forall t + i, i > 1$

Timeless perspective  $\implies \pi_t = 1 \quad \forall t$

Inflation never used to finance government spending  $\implies$  model reverts back to one with indexed debt  $\implies$  Ramsey plan: perfect smoothing of distortionary tax rate

Angeletos (2002), Buera & Nicolini (2004) explore this solution

# Consolidated, Benevolent Government

## Implementing the Ramsey Solution

Ramsey planner has access to a complete-markets portfolio of state-contingent assets, issues these assets to achieve zero net inflation and constant distortionary tax rate

Does so through optimal state-contingent payoffs  $z(g_t)$  from asset portfolio

Importantly, model lacks both lump sum transfers/taxes and state-contingent debt

Ramsey plan can only be implemented via MV of **previously-supplied** debt

$$z(g_t) = b_{t-1}^{(t)} + q_t^{(t+1)}(g_t) b_{t-1}^{(t+1)}$$

where  $q_t^{(t+1)}$  is price of unmatured real debt

$z(g)$

- $q_t^{(t+1)} = Q_t^{(t+1)}$  when  $\pi_t = \pi = 1$

Government understands nature of time- $t$  debt prices when supplying debt at  $t - 1$

- Uses this information to carefully issue

# Consolidated, Benevolent Government

## Implementing the Ramsey Solution

Optimal maturity structure: large ST asset position offset by large LT debt position

- Structure constant across states and time

LT debt revalues downward in high-spending (high-MU) states

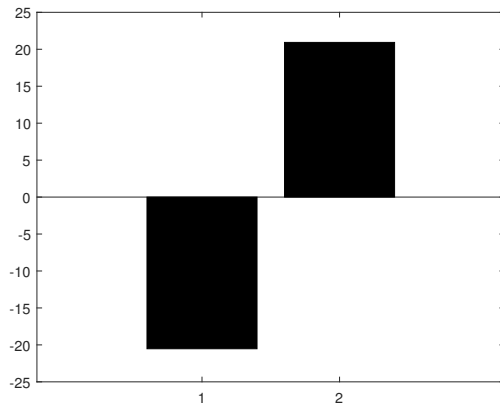
Numerical example to demonstrate another interpretation under nominal debt:

Variable	Calibration	Description
$(g_\ell, g_h)$	(.1751, .3570)	Spending amounts
$(p_{\ell\ell}, p_{\ell h})$ $(p_{h\ell}, p_{hh})$	(.985, .015) (.19, .81)	Transition matrix probabilities
$\beta$	.95	Time discount factor
$\sigma$	2	Inverse of intertemporal elasticity of substitution
$\varphi$	2	Inverse of Frisch elasticity of labor supply
$\theta$	3	Inflation coefficient
$(1, J)$	(1, 2)	Maturity lengths
$z(\ell)^*$	.05	MV of debt in low-spending state

# Consolidated, Benevolent Government

## Optimal Maturity Structure

Optimal maturity structure:



$$B_t^{(t+1)} = -18.55, B_t^{(t+2)} = 19.52 \implies a_t = a = -1.052 > -\frac{1}{\beta} = -1.053$$

# Consolidated, Benevolent Government

## New Interpretation of a Classic Result

Under constant  $a_t = a$ , (2) becomes

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i}) \left( 1 + \sum_{k=1}^i (-a)^k \right) \quad (3)$$

$B_{t-1}^{(t)} = B^{(1)}$  and  $P_t = P$  are constants

Constant tax rate  $\implies$  RHS is contingent only on exogenous state

$a$  chosen to equate RHS in low-spending state to RHS in high-spending state

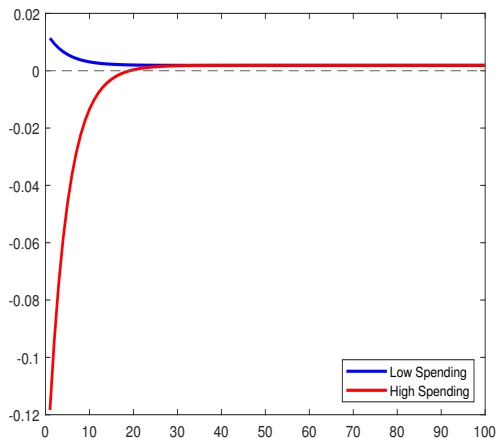
- Choice of  $a$  is unique

New interpretation: to avoid using ex-post inflation-financing, nominal debt issued strategically so that current and future expected dilution equates backing across states.

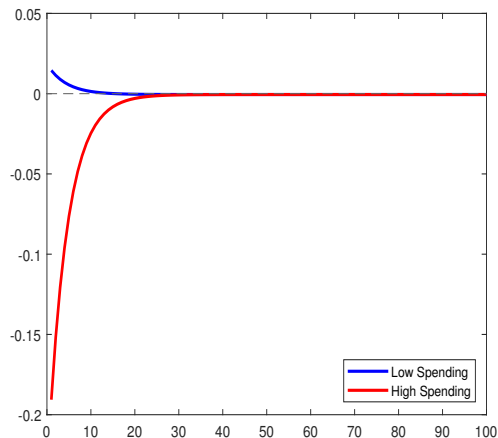


# Consolidated, Benevolent Government

## New Interpretation of a Classic Result



Exp. future primary surpluses

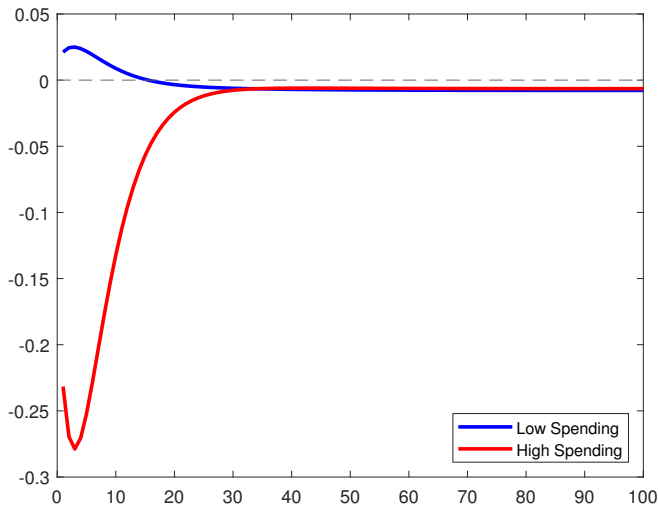


Exp. discounted future primary surpluses

Negative surplus states are high MU  $\rightarrow$  higher weights

# Consolidated, Benevolent Government

New Interpretation of a Classic Result



Zero-inflation  $a$  amplifies both curves equally until integrals equate

Exp. discounted, modified future surpluses

# Institutional Problem

## Two Government Institutions, Separate Objectives

Consider same model where two government institutions have separate objectives

Debt-manager implements fiscal policy, solves

$$\max_{\tau_t, b_t} \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi} \right) \right\}$$

Central bank implements monetary policy, solves

$$\max_{\pi_t, b_t} \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \theta \left( \frac{1}{\pi_t} - 1 \right)^2 \right\}$$

Subject to

$$\frac{1}{\pi_t} \left( b_{t-1}^{(t)} + \beta \mathbb{E}_t \left( \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma} \pi_{t+1}} \right) b_{t-1}^{(t+1)} \right) = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i}) \quad ,$$

household optimization and the ARC, taking other institution's choices as given

# Institutional Problem

## Solution Equivalency

Committing central bank sets future inflation to  $\pi_t = 1$  every period past  $t = 1$

Committing debt-manager sets constant tax rate  $\tau_t = \tau$  every period past  $t = 1$

Markov Perfect Nash Equilibrium matches Ramsey plan

Institutional non-cooperation not an issue

Many ways to move forward, I investigate how assumptions on fiscal and monetary commitment affect the problem

# Three Period Model

Consider the same model, but lasting for only three periods ( $t = 0, 1, 2$ )

Uncertainty about  $g_1$  and  $g_2$  resolved at  $t = 1$ ,  $g_1 = g_2 = g(s)$

- Allows for effectively complete markets (Ramsey solution feasible)

Variable	Calibration	Description
$g_0$	.2	Initial spending
$(g_\ell, g_h)$	(.15, .25)	Spending amounts
$(p_\ell, p_h)$	(.9, .1)	Transition probabilities
$\beta$	.95	Time discount factor
$\sigma$	2	Inverse of intertemporal elasticity of substitution
$\varphi$	2	Inverse of Frisch elasticity of labor supply
$\theta$	3	Inflation coefficient
$(1, J)$	(1, 2)	Maturity lengths
$(b_{-1}^{(0)}, b_{-1}^{(1)})$	(.7, .15)	Maturity structure entering $t = 0$

# Three Period Model

## Institutional Credibility

The exercise: vary commitment power of institutions and compare maturity structures, inflation paths, tax rate paths and welfare

Institutions solve same problems as before under three distinct endowments of commitment technology:

1. Both institutions able to commit to future policy
2. Central bank able to commit, fiscal policy unable to commit
3. Neither institution able to commit

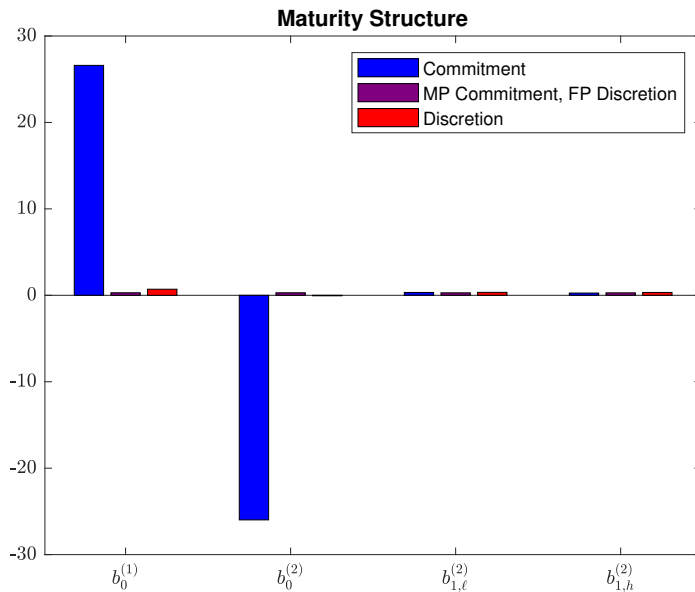
All Committing FOCs

Discretionary FP, Committing MP FOCs

All Discretionary FOCs

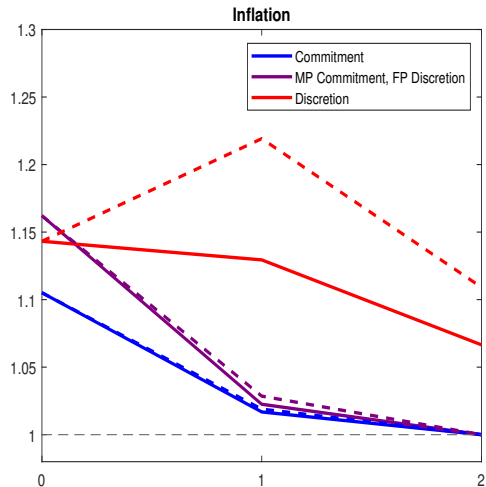
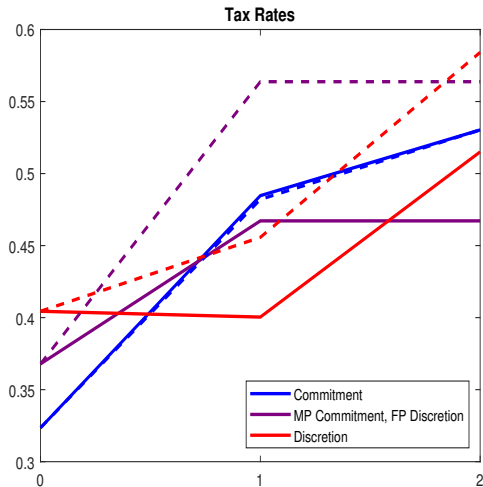
# Three Period Model

Institutional Interaction, Maturity Structure



# Three Period Model

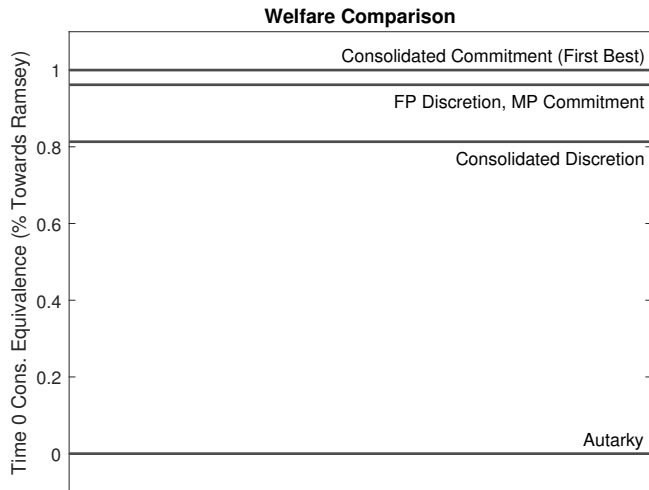
## Institutional Interaction, Inflation and Taxes





# Three Period Model

## Institutional Interaction, Welfare Comparison



# Takeaways

Federal economic institutions leverage maturity structure to achieve separate objectives

- Room for strategic interaction

In a model with:

- A maturity structure of non-contingent nominal debt
- A debt-manager using public debt to implement economy's distortionary tax rate
- A central bank using public debt to implement economy's inflation rate

Ramsey solution achieved in Markov Perfect Nash Equilibrium

- Inverse dilution rate a key player

Varying commitment power of institutions results in deviations from Ramsey plan

- Non-committing debt-manager results in realistic maturity structure, inflation paths, tax rates, etc.
- Welfare-improvement moving from a non-committing consolidated government to one with a central bank that commits to future inflation levels

# Moving Forward

Some facts:

- Financial frictions associated with fiscal and monetary policy debt transactions
- US Treasury highly averse to redeeming debt
- Federal reserve's balance sheet small in comparison to outstanding debt
- US Fed not able to issue LT liabilities
- Output responds to inflation in SR

Adding these into the infinite-lived version may create a non-degenerate role for inflation

# Pricing Equation Evolution

[Back](#)

$$\frac{B_{t-1}^{(t)}}{P_t} + \beta \mathbb{E}_t \left[ \left( \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \right) \left( \frac{P_t}{P_{t+1}} \right) \right] \frac{B_{t-1}^{(t+1)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i}) \quad (2)$$

$$\frac{b_{t-1}^{(t)}}{P_t} P_{t-1} + \beta \mathbb{E}_t \left[ \left( \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \right) \left( \frac{P_t}{P_{t+1}} \right) \right] \frac{b_{t-1}^{(t+1)}}{P_t} P_{t-1} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i})$$

$$\frac{1}{\pi_t} \left( b_{t-1}^{(t)} + \beta \mathbb{E}_t \left( \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma} \pi_{t+1}} \right) b_{t-1}^{(t+1)} \right) = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{c_{t+i}^{-\sigma}}{c_t^{-\sigma}} \right) (\tau_{t+i} n_{t+i} - g_{t+i})$$

# Optimal Ramsey Transfers

Back

$z(g_t)$  is expressed using RHS of implementability constraint *along the Ramsey plan*.  
Because the plan is  $\tau_t = \tau$ , RHS is contingent only on random draw of  $g_t$  every period:

$$z(g_t) = \frac{\mathbb{E}_g \sum_{i=0}^{\infty} \beta^i (\tau n_{t+i} - g_{t+i})}{c_t^{-\sigma}}$$

conditional on exogenous state of economy being  $g_t$ .

# Household FOCs

Back

Households choose  $c_t$ ,  $n_t$  and  $\{B_t^{(t+j)}\}_{j=1}^J$  to maximize expected lifetime utility:

$$\tau_t = 1 - \frac{n_t^\varphi}{c_t^{-\sigma}} \quad , \quad Q_t^{(t+j)} = \beta^j \mathbb{E}_t \left[ \frac{c_{t+j}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+j}} \right] \quad \forall j$$

Government budget constraint can be written as:

$$\underbrace{B_{t-1}^{(t)} + \sum_{j=1}^{J-1} Q_t^{(t+j)} B_{t-1}^{(t+j)}}_{\text{MV(outstanding government liabilities)}} = \underbrace{P_t (\tau_t n_t - g_t)}_{\text{primary surplus}} + \underbrace{Q_t^{(t+1)} \left( B_t^{(t+1)} + \sum_{j=1}^{J-1} Q_{t+1}^{(t+1+j)} B_t^{(t+1+j)} \right)}_{\text{MV(new liability position)}}$$

# FOCs for Committing Consolidated Government

$\lambda_0 : \text{Imp}, \phi_t : \text{ARC}$  [Back](#)

$$c_0^{-\sigma} + \lambda_0 \left[ (1 - \sigma) c_0^{-\sigma} + \sigma \left( \frac{c_0^{-1-\sigma}}{\pi_0} \right) b_{-1}^{(0)} \right] = \mu_0$$

$$c_1^{-\sigma} + \lambda_0 \left[ (1 - \sigma) c_1^{-\sigma} + \sigma \left( \frac{c_1^{-1-\sigma}}{\pi_0 \pi_1} \right) b_{-1}^{(1)} \right] = \mu_1$$

$$c_2^{-\sigma} + \lambda_0 (1 - \sigma) c_2^{-\sigma} = \mu_2$$

$$n_0^\varphi + \lambda_0 (1 + \varphi) n_0^\varphi = \mu_0$$

$$n_1^\varphi + \lambda_0 (1 + \varphi) n_1^\varphi = \mu_1$$

$$n_2^\varphi + \lambda_0 (1 + \varphi) n_2^\varphi = \mu_2$$

$$\theta \left( \frac{1}{\pi_0} - 1 \right) = -\lambda_0 \left[ c_0^{-\sigma} b_{-1}^{(0)} + \beta \mathbb{E}_0 \left( \frac{c_1^{-\sigma}}{\pi_1} \right) b_{-1}^{(1)} \right]$$

$$\theta \left( \frac{1}{\pi_1} - 1 \right) = -\lambda_0 \left( \frac{c_1^{-\sigma}}{\pi_0} \right) b_{-1}^{(1)}$$

$$\theta \left( \frac{1}{\pi_2} - 1 \right) = 0$$

# FOCs for Discretionary FP, Committing MP

$\lambda_0$  : Imp,  $\mu_t$  : GBC,  $\phi_t$  : ARC (Solve through repeated looping through individual institutional problems)

[Back](#)

$$c_0^{-\sigma} + \mu_0 \left[ (1 - \sigma) c_0^{-\sigma} + \sigma \left( \frac{c_0^{-1-\sigma}}{\pi_0} \right) b_{-1}^{(0)} \right] = \phi_0$$

$$c_1^{-\sigma} + \mu_1 \left[ (1 - \sigma) c_1^{-\sigma} + \sigma \left( \frac{c_1^{-1-\sigma}}{\pi_1} \right) b_0^{(1)} \right] = \phi_1$$

$$c_2^{-\sigma} + \mu_2 \left[ (1 - \sigma) c_2^{-\sigma} + \sigma \left( \frac{c_2^{-1-\sigma}}{\pi_2} \right) b_1^{(2)} \right] = \phi_2$$

$$n_0^\varphi + \mu_0 (1 + \varphi) n_0^\varphi = \phi_0$$

$$n_1^\varphi + \mu_1 (1 + \varphi) n_1^\varphi = \phi_1$$

$$n_2^\varphi + \mu_2 (1 + \varphi) n_2^\varphi = \phi_2$$

$$\theta \left( \frac{1}{\pi_0} - 1 \right) = -\lambda_0 \left[ c_0^{-\sigma} b_{-1}^{(0)} + \beta \mathbb{E}_0 \left( \frac{c_1^{-\sigma}}{\pi_1} \right) b_{-1}^{(1)} \right]$$

$$\theta \left( \frac{1}{\pi_1} - 1 \right) = -\lambda_0 \left( \frac{c_1^{-\sigma}}{\pi_0} \right) b_{-1}^{(1)}$$

$$\theta \left( \frac{1}{\pi_2} - 1 \right) = 0$$

$$\mu_0 \mathbb{E}_0 \left( \frac{c_1^{-\sigma}}{\pi_1} \right) = \mathbb{E}_0 \left( \frac{\mu_1 c_1^{-\sigma}}{\pi_1} \right)$$

$$\mu_0 \mathbb{E}_0 \left( \frac{c_2^{-\sigma}}{\pi_2} \right) = \mathbb{E}_0 \left( \frac{\mu_1 c_2^{-\sigma}}{\pi_2} \right)$$

$$\mu_1 = \mu_2$$



# FOCs for Discretionary Consolidated Government

$\mu_t$  : GBC,  $\phi_t$  : ARC [Back](#)

$$c_0^{-\sigma} + \mu_0 \left[ (1 - \sigma) c_0^{-\sigma} + \sigma \left( \frac{c_0^{-1-\sigma}}{\pi_0} \right) b_{-1}^{(0)} \right] = \phi_0$$

$$c_1^{-\sigma} + \mu_1 \left[ (1 - \sigma) c_1^{-\sigma} + \sigma \left( \frac{c_1^{-1-\sigma}}{\pi_1} \right) b_0^{(1)} \right] = \phi_1$$

$$c_2^{-\sigma} + \mu_2 \left[ (1 - \sigma) c_2^{-\sigma} + \sigma \left( \frac{c_2^{-1-\sigma}}{\pi_2} \right) b_1^{(2)} \right] = \phi_2$$

$$n_0^\varphi + \mu_0 (1 + \varphi) n_0^\varphi = \phi_0$$

$$n_1^\varphi + \mu_1 (1 + \varphi) n_1^\varphi = \phi_1$$

$$n_2^\varphi + \mu_2 (1 + \varphi) n_2^\varphi = \phi_2$$

$$\theta \left( \frac{1}{\pi_0} - 1 \right) = -\mu_0 c_0^{-\sigma} b_{-1}^{(0)}$$

$$\theta \left( \frac{1}{\pi_1} - 1 \right) = -\mu_1 c_1^{-\sigma} b_0^{(1)}$$

$$\theta \left( \frac{1}{\pi_2} - 1 \right) = -\mu_2 c_2^{-\sigma} b_1^{(2)}$$

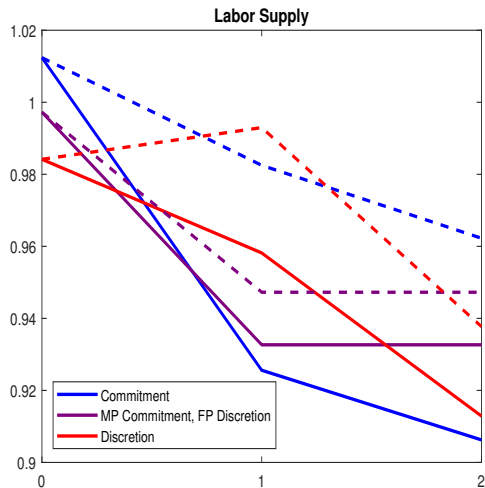
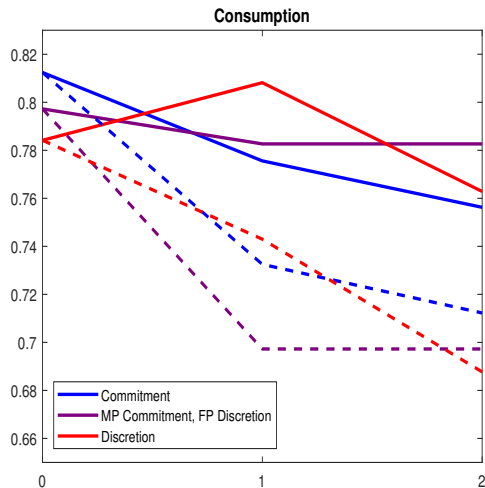
$$\mu_0 \mathbb{E}_0 \left( \frac{c_1^{-\sigma}}{\pi_1} \right) = \mathbb{E}_0 \left( \frac{\mu_1 c_1^{-\sigma}}{\pi_1} \right)$$

$$\mu_0 \mathbb{E}_0 \left( \frac{c_2^{-\sigma}}{\pi_2} \right) = \mathbb{E}_0 \left( \frac{\mu_1 c_2^{-\sigma}}{\pi_2} \right)$$

$$\mu_1 = \mu_2$$

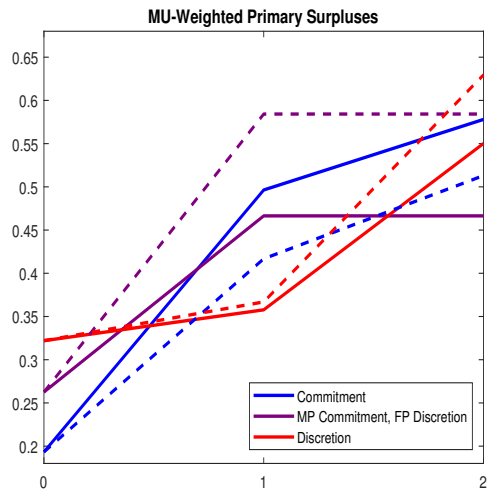
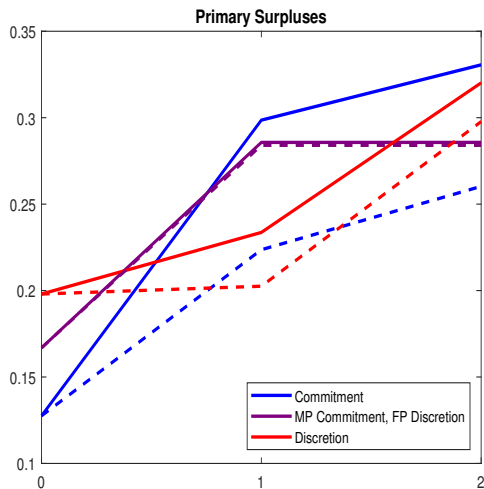
# Paths of Consumption and Labor Supply

[Back](#)



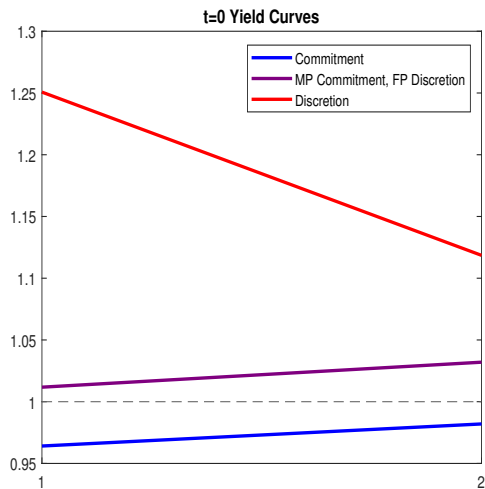
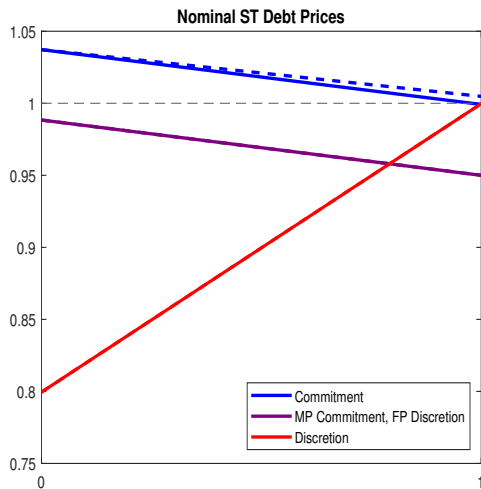
# Paths of Primary Surpluses

[Back](#)



# Debt Prices and the Yield Curve

[Back](#)



# Welfare Comparison in Levels

[Back](#)

