12.10.

$$\lambda^n + \sum_{k=1}^{n} c_k \lambda^{n-k} = 0 \quad \text{for any eigenvalue } \lambda.$$

If M is diagonalizable we can write

$$M = S^{-1} \Lambda S$$

for S unitary and Λ diagonal.

Now consider

$$M^n + \sum_{k=1}^{n} c_k M^{n-k} = S^{-1} \Lambda^n S + \sum_{k=1}^{n} c_k S^{-1} \Lambda^{n-k} S$$

$$= S^{-1} \left[\Lambda^n + \sum_{k=1}^{n} c_k \Lambda^{n-k} \right] S$$

Because Λ is diagonal, we can consider each element individually, in which case we recover the polynomial we started with. Since that polynomial vanishes, so does the expression in brackets for every element of the matrix. Hence

$$M^n + \sum_{k=1}^{n} c_k M^{n-k} = 0.$$