Society of Actuaries

Canadian Institute of Actuaries

# Exam MLC
# Models for Life Contingencies

**Friday,**
**October 28, 2016**
**8:30 a.m. – 12:45 p.m.**

# MLC

## INSTRUCTIONS TO CANDIDATES

**General Instructions**

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

   Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

   Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Multiple-Choice Instructions**

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

   (a) Name
       (include last name, first name and middle initial)

   (b) Candidate Number
       (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
       (The supervisor will supply the number.)

   (d) Examination Part
       (Code the examination that you are taking by blackening the circle to the left of "Exam MLC")

   (e) Booklet Number
       (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

**CONTINUED ON INSIDE FRONT COVER**

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.

9. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.

6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

# Exam MLC

# SECTION A – Multiple-Choice

**1.** For the country of Bienna, you are given:

(i) Bienna publishes mortality rates in biennial form, that is, mortality rates are of the form:
$_2q_{2x}$, for $x = 0,1,2,...$

(ii) Deaths are assumed to be uniformly distributed between ages $2x$ and $2x+2$, for $x = 0,1,2,...$

(iii) $_2q_{50} = 0.02$

(iv) $_2q_{52} = 0.04$

Calculate the probability that (50) dies during the next 2.5 years.

(A) 0.02

(B) 0.03

(C) 0.04

(D) 0.05

(E) 0.06

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**2.** You are given the following survival function of a newborn:

$$S_0(x) = \begin{cases} 1 - \dfrac{x}{250}, & 0 \le x < 40 \\ 1 - \left(\dfrac{x}{100}\right)^2, & 40 \le x \le 100 \end{cases}$$

Calculate the probability that (30) dies within the next 20 years.

(A)   0.13

(B)   0.15

(C)   0.17

(D)   0.19

(E)   0.21

**USE THIS PAGE FOR YOUR SCRATCH WORK**

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**3.** In a population initially consisting of 75% females and 25% males, you are given:

    (i)    For a female, the force of mortality is constant and equals $\mu$.

    (ii)    For a male, the force of mortality is constant and equals $1.5\mu$.

    (iii)    At the end of 20 years, the population consists of 85% females and 15% males.

Calculate the probability that a female survives one year.

    (A)    0.89

    (B)    0.92

    (C)    0.94

    (D)    0.96

    (E)    0.99

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**4.** For a special whole life insurance on (x), you are given:

 (i) Death benefits are payable at the moment of death.

 (ii) The death benefit at time $t$ is $b_t = e^{0.02t}$, for $t \geq 0$.

 (iii) $\mu_{x+t} = 0.04$, for $t \geq 0$

 (iv) $\delta = 0.06$

 (v) $Z$ is the present value at issue random variable for this insurance.

Calculate $Var(Z)$.

 (A) 0.020

 (B) 0.036

 (C) 0.052

 (D) 0.068

 (E) 0.083

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**5.** You are given the following extract of ultimate mortality rates from a two-year select and ultimate mortality table:

| $x$ | $q_x$ |
|---|---|
| 50 | 0.045 |
| 51 | 0.050 |
| 52 | 0.055 |
| 53 | 0.060 |

The select mortality rates satisfy the following:

(i) $q_{[x]} = 0.7 q_x$

(ii) $q_{[x]+1} = 0.8 q_{x+1}$

You are also given that $i = 0.04$.

Calculate $A^{1}_{[50]:\overline{3}|}$ .

(A) 0.08

(B) 0.09

(C) 0.10

(D) 0.11

(E) 0.12

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**6.** For a special fully discrete whole life policy on (48), you are given:

    (i) The policy pays 5000 if the insured dies before his median curtate future lifetime at issue and 10,000 if he dies after his median curtate future lifetime at issue.

    (ii) Mortality follows the Illustrative Life Table.

    (iii) $i = 0.06$

Calculate the actuarial present value of benefits for this policy.

(A) 1420

(B) 1450

(C) 1480

(D) 1510

(E) 1540

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**7.** For a Type B Universal Life policy issued to (55) you are given:

     (i)     $AV_4 = 500$

     (ii)    The additional death benefit is 1,000,000.

     (iii)   The COI rate in year 5 is 20 per 1000 of insurance.

     (iv)   Expense charges are 500 per year plus 5% of premiums.

     (v)    $i^c = 0.045$

     (vi)   The policy does not have a no-lapse guarantee.

Calculate the minimum premium to be paid at the beginning of year 5 so that the policy does not lapse before the next premium is paid at the beginning of year 6.

(A)    20,150

(B)    20,650

(C)    21,150

(D)    21,650

(E)    22,150

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**8.** For a fully discrete whole life insurance policy of 1,000,000 on (50), you are given:

(i) The annual gross premium, calculated using the equivalence principle, is 19,526.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

Calculate the expense loading, $P^e$, for this policy.

(A) 550

(B) 650

(C) 750

(D) 850

(E) 950

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**9.** For a special fully discrete 2-year term insurance on $(x)$, you are given:

(i) $q_x = 0.01$

(ii) $q_{x+1} = 0.02$

(iii) $i = 0.05$

(iv) The death benefit in the first year is 100,000.

(v) Both the benefits and premiums increase by 1% in the second year.

Calculate the annual net premium in the first year.

(A) 1410

(B) 1417

(C) 1424

(D) 1431

(E) 1438

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For a fully discrete 3-year endowment insurance of 1000 on $(x)$, you are given:

(i)    $\mu_{x+t} = 0.06$, for $0 \le t \le 3$.

(ii)   $\delta = 0.06$

(iii)  The annual premium is 315.80.

(iv)   $L_0$ is the present value random variable for the loss at issue for this insurance.

Calculate $\Pr[L_0 > 0]$.

(A)    0.03

(B)    0.06

(C)    0.08

(D)    0.11

(E)    0.15

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**11.** For a fully discrete, 5-payment 10-year term insurance of 200,000 on (30), you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) The following expenses are incurred at the beginning of each respective year:

| | Year 1 | | Years 2 - 10 | |
|---|---|---|---|---|
| | Percent of Premium | Per Policy | Percent of Premium | Per Policy |
| Taxes | 5% | 0 | 5% | 0 |
| Commissions | 30% | 0 | 10% | 0 |
| Maintenance | 0% | 8 | 0% | 4 |

(iii) $i = 0.06$

(iv) $\ddot{a}_{30:\overline{5}|} = 4.4516$

(v) $\ddot{a}_{30:\overline{10}|} = 7.7465$

Calculate the annual gross premium using the equivalence principle.

(A) 750

(B) 780

(C) 800

(D) 820

(E) 850

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**12.** For a fully discrete whole life insurance policy of 100,000 on [55], a professional skydiver, you are given:

   (i) Premiums are paid annually.

   (ii) Mortality follows a 2-year select and ultimate table

   (iii) $i = 0.04$

   (iv) The following table of values for $A_{[x]+t}$ :

| $x$ | $A_{[x]}$ | $A_{[x]+1}$ | $A_{x+2}$ |
|-----|-----------|-------------|-----------|
| 55  | 0.23      | 0.24        | 0.25      |
| 56  | 0.25      | 0.26        | 0.27      |
| 57  | 0.27      | 0.28        | 0.29      |
| 58  | 0.29      | 0.30        | 0.31      |

Calculate the Full Preliminary Term reserve at time 3.

(A) 2700

(B) 3950

(C) 5200

(D) 6450

(E) 7800

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**13.** For a special fully discrete 2-year endowment insurance on $(x)$, you are given:

(i) The death benefit for year $k$ is $25{,}000k$ plus the net premium reserve at the end of year $k$, for $k = 1, 2$. For year 2, this net premium reserve is the net premium reserve just before the maturity benefit is paid.

(ii) The maturity benefit is 50,000.

(iii) $p_x = p_{x+1} = 0.85$

(iv) $i = 0.05$

(v) $P$ is the level annual net premium.

Calculate $P$.

(A) 27,650

(B) 27,960

(C) 28,200

(D) 28,540

(E) 28,730

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14. For a Type A universal life policy with a death benefit of 50,000, you are given:

(i)

| Policy Year | Annual Premium | Percent of Premium Charge | Annual Fixed Expense Charge | Annual Cost of Insurance Rate Per 1000 | Interest Credited |
|---|---|---|---|---|---|
| 2 | 3000 | 7% | 10 | 3.0 | 5% |

(ii) The account value at the end of year 1 is 2233.

Calculate the account value at the end of year 2.

(A) 5100

(B) 5110

(C) 5120

(D) 5130

(E) 5140

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**15.** For a 5-year term insurance policy on $(x)$, you are given:

(i)    The profit signature is $\Pi = \{-550, 300, 275, 75, 150, 100\}$

(ii)   The risk discount rate is 12%.

Calculate the discounted payback period (DPP) for this policy.

(A)    1

(B)    2

(C)    3

(D)    4

(E)    5

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**16.** XYZ Life Company issues 5,000 fully discrete whole life insurance policies of 10,000 to lives each age 50, with independent future lifetimes.

You are given:

(i) The annual gross premium is 220 per policy.

(ii) Each policy is assumed to incur an expense of 30 at the beginning of each year.

(iii) Gross premiums and reserves are calculated using $q_{53} = 0.0068$ and $i = 5\%$.

(iv) At the end of the third policy year:
- The gross premium reserve per policy is 505
- There are 4,885 policies in force

(v) During the fourth policy year:
- The actual expense incurred per policy was 34
- There were a total of 42 actual deaths
- The earned interest rate was 6%

Calculate the profit for the fourth policy year.

(A) -97,000

(B) -83,000

(C) -69,000

(D) -55,000

(E) -41,000

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**17.** The gross annual premium, $G$, for a fully discrete 5-year endowment insurance of 1000 issued on $(x)$ is calculated using the equivalence principle.

You are given:

(i)  $1000P_{x:\overline{5|}} = 187.00$

(ii)  The expense reserve at the end of the first year, $_1V^e = -38.70$

(iii)  $q_x = 0.008$

(iv)  Expenses, payable at the beginning of the year, are:

| Year | Percent of Premium | Per Policy |
|------|--------------------|-----------|
| First | 25% | 10 |
| Renewal | 5% | 5 |

(v)  $i = 0.03$

Calculate $G$.

(A)  200

(B)  213

(C)  226

(D)  239

(E)  252

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**18.** For a member of a defined contribution plan who is age 65, you are given:

  (i) The member's current salary is 250,000, and there will be no future salary increases.

  (ii) The member's current retirement fund is 1,500,000 and there are no expected future contributions prior to retirement.

  (iii) The retirement fund grows at a rate of 8% annually.

  (iv) Upon retirement, the member will use her retirement fund as the single net premium for an annuity due that makes quarterly payments.

  (v) The single net premium is calculated using the Illustrative Life Table and $i = 0.06$.

Calculate, using the 2-term Woolhouse approximation, the member's replacement ratio upon retirement at age 66.

(A) 0.60

(B) 0.63

(C) 0.65

(D) 0.68

(E) 0.70

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**19.** Tim is a member of a defined benefit pension plan that provides its members, upon retirement at age 65, an annual pension payable monthly in advance equal to 2% of final salary per year of service. Final salary is defined to be the salary earned in the year immediately preceding retirement.

Tim's salary throughout 2015 was 150,000, and he will not receive a salary increase in 2016. However, his salary will increase by 2% every January 1, starting at January 1, 2017.

On the valuation date, January 1, 2016, Tim is age 45 with 10 years of past service.

You are given:

(i)  $i = 5\%$

(ii)  ${}_t p_{45}^{(\tau)} = 0.99^t$, for $t \le 20$

(iii)  $\ddot{a}_{65}^{(12)} = 13.75$

(iv)  There are no benefits other than retirement benefits.

Calculate the normal contribution for Tim on the valuation date assuming a traditional unit credit funding method.

(A)  12,700

(B)  14,000

(C)  16,500

(D)  18,900

(E)  20,800

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**20.** Kaitlyn entered a defined benefit plan on January 1, 1990 with a salary for 1990 of 50,000. You are given:

    (i) The annual retirement benefit is 2% of the final 5-year average salary for each year of service.

    (ii) Her normal retirement date is December 31, 2023.

    (iii) The reduction in benefit for early retirement is 7% for each year prior to her normal retirement date.

    (iv) Every January 1, each employee receives a 2.5% increase in salary.
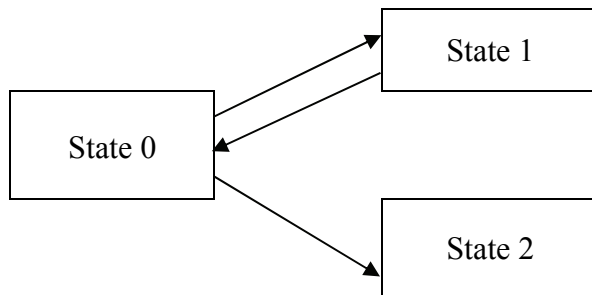
    (v) Kaitlyn retires on December 31, 2020.

Calculate Kaitlyn's annual retirement benefit.

(A) 45,200

(B) 46,100

(C) 47,000

(D) 48,900

(E) 50,000

# Exam MLC

# SECTION B – Written-Answer

**1.** (*7 points*) You are given the following 3-state Markov model:



For all states $i$ and $j$, and for all ages $x \geq 0$, ${}_{t}p_{x}^{ij}$ is a differentiable function of $t$, and for $i \neq j$:

$$\mu_{x}^{ij} = \lim_{h \to 0+} \frac{{}_{h}p_{x}^{ij}}{h}$$

(a) (*1 point*) Define the symbols ${}_{t}p_{x}^{00}$ and ${}_{t}p_{x}^{\overline{00}}$, and explain why these probabilities are not equal for this model.

(b) (*4 points*) The probability ${}_{t+h}p_{x}^{00}$ can be expressed as

${}_{t+h}p_{x}^{00} = {}_{t}p_{x}^{00} {}_{h}p_{x+t}^{00} + {}_{t}p_{x}^{01} {}_{h}p_{x+t}^{10}$. Use this equation to derive the Kolmogorov forward differential equation for ${}_{t}p_{x}^{00}$.

(c) (*2 points*) You are also given:

- $\mu_{x+t}^{01} = 0.5$, for all $t$
- $\mu_{x+t}^{02} = kt$, for all $t$
- ${}_{2}p_{x}^{\overline{00}} = 0.165$

Calculate $k$.

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**2.** (*11 points*) SoA Life simultaneously sells $N$ two-year joint life term insurance policies, each with a death benefit of 10,000 payable at the end of the year of death of the first death. The ages at issue are 65 and 70. The premiums are payable annually during the term of the policy.

You are given:

(i) The future lifetimes are independent.

(ii) $q_{65+t} = 0.05 + 0.01t$, for $t = 0, 1, 2, ..., 10$

(iii) The spot rates are stochastic with the following distribution:

| Scenario Number | Probability | One Year Spot Rate | Two Year Spot Rate |
|---|---|---|---|
| 1 | 0.40 | 0.05 | 0.06 |
| 2 | 0.60 | 0.05 | 0.07 |

(iv) The annual premium per policy, $P$, is calculated using the equivalence principle.

(a) (*3 points*) Show that $P$ is 1400 to the nearest 100. You should calculate $P$ to the nearest 1.

$L_{0,j}$ is the loss-at-issue random variable for the $j^{th}$ insurance policy.

(b) (*4 points*)

(i) Calculate the expected value and variance of $L_{0,j}$ given that the spot rates follow scenario 1.

(ii) Calculate the expected value and variance of $L_{0,j}$ given that the spot rates follow scenario 2.

**2.** **Continued**

Let $L = \displaystyle\sum_{j=1}^{N} L_{0,j}$

(c)  (*2 points*)  Express $Var[L]$ in terms of $N$.

(d)  (*1 point*)  Demonstrate mathematically that the interest rate risk in these policies is not diversifiable.

(e)  (*1 point*)  Briefly explain why the interest rate risk in these policies is not diversifiable.

**3.** (*9 points*) You are calculating the expected present value (EPV) of group insurance benefits for Gary, age 35, an active employee of BMK. You are given:

- (i) The benefit for death of an active employee is 100,000. The benefit for disability of an active employee is 50,000. There are no other benefits.

- (ii) Benefits are payable as a lump sum at the end of the year of the event.

- (iii) Decrements follow the Illustrative Service Table, with $d$, $w$, and $i$ representing death, withdrawal, and disability, respectively.

- (iv) $i = 0.08$

(a) (*2 points*) Show that the EPV of benefits for Gary for the first three years is 500 to the nearest 100. You should calculate the EPV to the nearest 1.

BMK amends its benefit plan so that the lump sum is payable at the moment of death or disability.

You assume decrements are uniformly distributed over each year of age in the multiple decrement table.

(b) (*1 point*) Calculate the EPV of benefits for Gary for the first three years.

You revise your assumption about the timing of decrements to assume that the force of decrement for each decrement is constant over each year of age.

(c) (*2 points*)

- (i) Show that $\mu_{35}^{(\tau)} = 0.06$ to the nearest 0.01. You should calculate $\mu_{35}^{(\tau)}$ to the nearest 0.0001.

- (ii) Calculate $\mu_{35}^{(d)}$ and $\mu_{35}^{(i)}$.

(d) (*2 points*) Calculate the EPV of benefits for Gary in the first year.

**3.** **Continued**

(e) (*2 points*) BMK decides to offer a death benefit of an additional 50,000 that is payable at the moment of death if death occurs within two years of becoming disabled for active lives who become disabled before age 36.

You are given:

(i) The force of mortality for disabled lives, $\mu_x$, is 0.5, for all attained ages $x$.

(ii) Mortality is the only decrement for disabled lives.

Calculate the EPV of this additional death benefit for an active life currently age 35.

**4.** (*11 points*) For a special fully discrete 3-year term insurance issued to [77], you are given:

(i)

| Policy Year | Death Benefit |
|---|---|
| 1 | 500,000 |
| 2 | 300,000 |
| 3 | 100,000 |

(ii) The following one-year select and ultimate mortality table:

| $x$ | $q_{[x]}$ | $q_{x+1}$ | $x+1$ |
|---|---|---|---|
| 77 | 0.030 | 0.060 | 78 |
| 78 | 0.036 | 0.067 | 79 |
| 79 | 0.042 | 0.075 | 80 |

(iii) $v = 0.95$

(iv) The level annual premium, $P$, is determined using the equivalence principle.

(v) $L_0$ is the loss-at-issue random variable.

(a) (*2 points*) Show that $P$ equals 12,840 to the nearest 10. You should calculate $P$ to the nearest 1.

(b) (*1 point*) Calculate the probability that $L_0 > 0$.

(c) (*3 points*) Show that the standard deviation of $L_0$ is 105,000 to the nearest 5000. You should calculate the standard deviation to the nearest 1000.

(d) (*2 points*) The MRH Life Insurance Company (MRH) sells 8000 of these policies to lives, all select lives age 77, who are assumed to have independent future lifetimes. $L$ is the aggregate loss-at-issue random variable for these 8000 policies.

Using the normal approximation, calculate the $90^{th}$ percentile of $L$.

**4.** **Continued**

(e)   (*2 points*)  MRH discovers that 7200 individuals purchased the 8000 policies, where 200 policyholders have each purchased 5 policies, while the remaining 7000 policyholders have each purchased a single policy.  $L^*$ is the aggregate loss-at-issue random variable for these 8000 policies.

Using the normal approximation, calculate the 90$^{\text{th}}$ percentile of $L^*$.

(f)   (*1 point*)  Briefly explain why the values of the 90$^{\text{th}}$ percentiles of $L$ and $L^*$ differ.

**5.** (*10 points*) You are using profit testing to determine the premium for a fully discrete 3-year term insurance policy of 100,000 on (70). You are given:

(i) Commissions are 52% of premium in year 1, 10% of premium in year 2, and 0% in year 3.

(ii) The only other expenses are pre-contract expenses equal to 30% of first year premium.

(iii) There are no withdrawal benefits.

(iv) Profit testing is based on Full Preliminary Term reserves.

(v) Full Preliminary Term reserve assumptions:

- $q_{70} = 0.017$; $q_{71} = 0.019$; $q_{72} = 0.021$
- $i = 0.055$

(vi) Profit testing assumptions:

- $p_{70}^{(\tau)} = 0.82$; $q_{71}^{(death)} = 0.018$; $q_{71}^{(withdrawal)} = 0.10$
- $i = 0.06$

(vii) The hurdle rate is 0.10.


(a) (*2 points*) Show that the Full Preliminary Term reserve at the end of year 2 is 100 to the nearest 10. You should calculate the reserve to the nearest 1.

You started with a tentative premium of 2200. Your resulting profit vector was $(-660, 115, \text{Pr}_2, 340)$.

(b) (*2 points*) Show that $\text{Pr}_2$, the expected profit in year 2 per policy inforce at the start of year 2, is 210 to the nearest 10. You should calculate the expected profit to the nearest 1.

(c) (*1 point*) Calculate the net present value (NPV) of this insurance policy.

**5.** **Continued**

$\text{Pr}_1^*$ is the expected profit in year 1 per policy in force at the start of year 1, based on a gross premium of $G$.

It is possible to express $\text{Pr}_1^*$ as $k(G-2200)+\text{Pr}_1$ where:

        (i)      $k$ is a constant

        (ii)     $G$ is the gross premium for the policy

        (iii)    $\text{Pr}_1$ is the expected profit in year 1 per policy in force at the start of year 1, based on a gross premium of 2200

(d)    (*2 points*) Show that $k$ is 0.5 to the nearest 0.1. You should calculate $k$ to the nearest 0.001.

(e)    (*2 points*) Calculate the gross premium, $G$, such that the NPV is 100.

(f)    (*1 point*) If the pre-contract expenses increase by 10% of the premium and the commissions at the start of year 1 decrease by 10% of the premium, state whether the NPV will increase, decrease, or remain unchanged. Justify your answer. A numerical calculation of the change is not required.

**6.** (*8 points*) You are given the following information for a final average earnings defined benefit pension plan sponsored by company ABC:

    (i)     There are only two members:
- Tom was born January 1, 1976 and entered the plan on January 1, 2014. Tom's salary for the year 2016 is 65,000.
- Ken was born on January 1, 1966 and entered the plan on January 1, 2011. Ken's salary for the year 2016 is 80,000.

    (ii)    Benefits take the form of a life annuity, payable monthly in advance.

    (iii)   The final average earnings are defined as the salary in the final year of service.

    (iv)   The accrual rate is 1.3%.

    (v)    On termination the benefit is deferred to age 65.

    (vi)   You are also provided with the following actuarial assumptions and methods:

| | |
|---|---|
| Interest Rate: | 6.5% |
| Salary increases: | 3.0% on January 1 of each year |
| Retirement rates: | 100% at age 65 |
| Termination rates: | 6% per year at the end of each of the first 3 years of service and 0% thereafter |
| Other pre-retirement decrements: | None |
| Funding method: | Projected Unit Credit |

$$\ddot{a}_{65}^{(12)} = 9.6$$

(a)    (*2 points*) Briefly describe the difference between the traditional unit credit and projected unit credit funding methods. Include in your description an example (numerical or verbal) of how each method treats salaries at valuation time $t$ for a final average defined benefit plan for determining the value of the liabilities at time $t$.

(b)    (*3 points*) Calculate the total normal contributions for both Tom and Ken at January 1, 2016.

**6.** **Continued**

(c)   (*3 points*)  Following Tom's termination on December 31, 2016 the ABC Company announces that effective January 1, 2017 the following new flexible retirement benefit provisions will be put in place:

- Ken will be able to take retirement at age 62 or age 65.

- If Ken takes his retirement benefit at age 62 there will be an additional "bridge" benefit payable to the earlier of the date of death and age 65 equal to 15 per month for each year of service.

- If Ken takes his retirement benefit at age 62 there will be an actuarial reduction of 21% that will apply to both his normal and additional "bridge" benefits.  No reduction applies if Ken retires at age 65.

- It is assumed that Ken has a 50% probability of taking his retirement at age 62 and a 50% probability that he will take it at age 65.

You are also given:

$$\ddot{a}_{62}^{(12)} = 10.3$$
$$\ddot{a}_{62:\overline{3|}}^{(12)} = 2.7$$

Calculate the total actuarial liability and normal contribution at January 1, 2017 for Ken based on the new plan provisions.

**\*\*END OF EXAMINATION\*\***

**USE THIS PAGE FOR YOUR SCRATCH WORK**