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ABSTRACT

Ackermann steering geometry relates the steer angle of an inside tire to that of the outside tire. When turning the inside tire travels a shorter radius than the outside tire and thus must have a greater steer angle to avoid tire scrub. Classic Ackermann minimizes scrub by positioning both tires perpendicular to the turn center. It can have a significant impact on tire wear [1]. Ackermann analysis can also be used as a tuning tool in cases where classic Ackermann may not be the objective.

Ackermann has been around longer than the motor vehicle - over a century - but there is little rigorous analysis in the literature. There are two common measurements of Ackermann which give very different results. Both are used in texts and computer programs[8] Yet a literature search revealed only a couple sentences discussing the relationship between the two.

This paper presents a mathematical analysis of Ackermann which explains the two measurements and develops a formula relating the two metrics. It also derives additional analytical tools which provide better understanding of Ackermann steering geometry.

ACKERMANN ANALYSIS

The standard formula for classic Ackermann involves track and wheelbase. The formula can be improved by considering the effect of scrub radius. The proper formula involves the distance between kingpin axis's rather than track.

The relationship between steering input and steering angle is key to understanding Ackermann. The first and second derivatives of this function provide a formula for classic Ackermann. This formula is a metric for measuring Ackermann in any situation.

The angle between the steering arm and the tie-rod in plan view is more important than the angle of the steering arm alone. This can be shown by the formula described above.

The Jeantaud diagram projects the steering arms to an intersection. If those lines intersect near the rear axle it is assumed to represent classic Ackermann. But this convenient graphical representation is only an approximation. It neglects the important contribution of the steering tie-rod. The Jeantaud diagram can be enhanced to include the steering tie-rod and produce an accurate representation of classic Ackermann.

THE HISTORY OF ACKERMANN STEERING GEOMETRY

Munich carriage builder George Lankensperger invented a steering system for horse-drawn vehicles in 1817. When viewed from above this resembled a rectangle. His English agent, Rudolph Ackermann, took out a patent in his own name. Frenchman Charles Jeantaud improved the system in 1878 by inclining the steering arms so that lines drawn from the steering arms through the kingpin axis will converge near the rear axle. The system now resembles a trapezoid [2].

Dixon [3] refers to the difference in steer angles as the Langensperger (with a g rather than a k) angle. Dixon also refers to a Jeantaud diagram where the projections of the steering arms intersect near the rear axle. This drawing is common and can be a useful indicator, but it usually yields 50-70% Ackermann. Race Car Vehicle Dynamics [5] calls this a "reasonable approximation"

In his second edition Dixon [4] adds "With straight tie-rods, to obtain an Ackermann factor close to 1.0 may require the projected steering arm intersection point to be at about 60% of the distance to the rear axle."

Dixon reports that "the traditional London Taxi has almost perfect Ackermann over its full 60 degree of inner road-wheel steer angle." This statement highlights the fact that it is difficult to achieve classic Ackermann over a wide range of steering values. Miller [1] proposes a definition of Ackermann percentage based on a range of steering values.

Rather than debating "Ackerman" vs. "Ackermann" perhaps we should be arguing over "Lankensperger", "Langensperger" or "Jeantaud" steering geometry.

THE DEFINITION OF CLASSIC ACKERMANN

The terms "classic" and "perfect" Ackermann are often used interchangeably in the literature. This paper will use classic because "perfect" implies a desirable trait whereas classic is more neutral.

To avoid tire scrub when a vehicle is turning draw perpendicular lines from all four tires. If the perpendiculars from the front tires intersect along the lines drawn from the unsteered rear tires (Figure 1) then you have classic Ackermann for one steering angle. This is expressed mathematically as

$$\text{Netsteer} = -\text{Right}^2 * \text{Track} / \text{Wheelbase}$$

Where Right is the steer angle of the loaded tire.

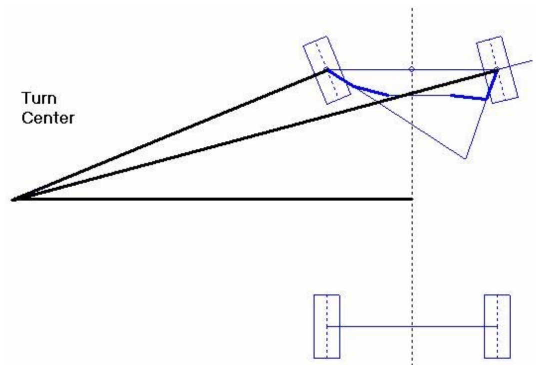

Figure 1: The Ackermann Turn Center

MEASURING ACKERMANN

The literature discusses 100% Ackermann without really discussing values other than 100% and 0% or parallel steer. Just what is 50% Ackermann? The simplest, and most natural, definition is shown below:

$$\text{Netsteer \%} = 100 * \frac{\text{Actual\_Netsteer}}{\text{Classic\_Netsteer}}$$

Formula 1 is the best measurement of Netsteer or Ackermann percentage.

An alternative measure is based on the turn center projection. If the turn center projects to the midpoint of the chassis, or 50% of the wheelbase, the inside wheel is turned **more** than the classic value. This suggests the Ackermann percentage should be 200% in Figure 1:

$$\text{Netsteer \%} = \frac{100 * \text{Wheelbase}}{\text{Turn Center Fore-aft}}$$

Formula 2 is an approximation of Netsteer or Ackermann percentage.

This method is less meaningful but it does illustrate the relationship between Ackermann and wheelbase. Trucks are often sold with different wheelbase depending upon application [1]. But unless the steering system is changed with each wheelbase the Ackermann steering will differ.

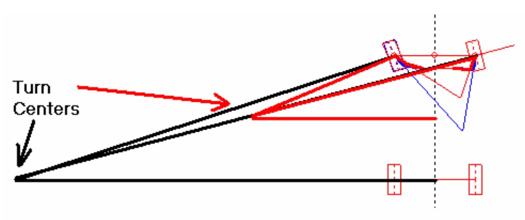

Figure 2: Ackermann Turn Centers

In Figure 2 above, the outside tire has an angle of 15 degrees. For classic Ackermann the inside tire has a steer angle of 18.031 degrees and a Netsteer of -3.031 degrees. For the other case the inside tire has a steer angle of 22.521 degrees and a Netsteer of -7.521 degrees. The TurnCenter projection is at 50% of the wheelbase yielding an Ackermann percentage of 200%. The ratio of Netsteer is -7.521 / -3.031 = 2.481 or 248.1%. The ratio of netsteer values is the best metric of partial Ackermann.

IMPROVING THE DEFINITION OF CLASSIC ACKERMANN

In the process of preparing this paper a case arose where WinGeo3 [8] and WinSteer [9] calculated 86% Ackermann yet the turn center was ahead of the rear axle, indicating more than 100% Ackermann. This is contradictory. The reason was the calculated Ackermann did not account for the fore-aft movement of the tires when they are steered.

With a vertical kingpin axis, the tire contact patch moves fore-aft a distance equivalent to the scrub radius multiplied by the sine of the steer angle. We can express this in algebra, with the details available from the primary author [9].

$$\text{Netsteer} = -\text{Right}^2 * \text{Tk} / \text{Wheelbase}$$

Where Right is the steer angle of the loaded tire and Tk is the distance between the kingpin axis's.

Having done the necessary algebra, the same conclusion can be drawn geometrically. The perpendicular to the tire drawn from the tire contact patch goes through the original axle line near the steering axis. It would go through the steering axis if the castor trail were zero. At this point we can adjust the classic formula by replacing the track T with the distance between the kingpin axis', which we call Tk.

Bird [10] uses kingpin location as an input to his computer programs which calculate Ackermann angles for heavy trucks.

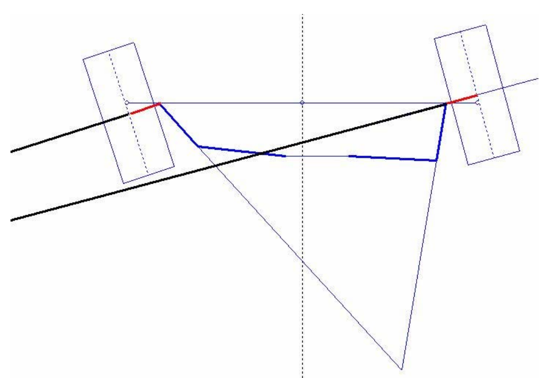

Figure 3: Ackermann with scrub radius

ASSUMPTIONS OF THIS PAPER

We will limit the analysis to symmetric suspensions with a steering input moving cross-car like a rack-and-pinion. We will temporarily ignore asymmetric suspensions because there are too many ways to achieve Ackermann. We will ignore other steering inputs such as the drag-link or the kart system with tabs attached to the hand wheel axis..

Trucks and oval track racing cars sometimes have a single tie-rod connecting both steering arms. This system can be modeled by placing the inner tie-rod points on the centerline of the car. This approximation is valid for small angles.

We will also ignore static toesteer. Theoretical discussion of Ackermann assumes a static steer angle of zero when the vehicle is not turning. In reality most vehicles run a static steer angle for stability, either toe-in or toe-out. We will assume the designer desires the same angles in a turn. This is equivalent to ignoring static toe-in or toe-out and assuming the vehicle has zero steer when running straight.

THE ACKERMANN FACTOR

The general formula f(angle) for steering input as a function of right tire steering angle can be differentiated to provide a fundamental formula for classic Ackermann steering geometry.

$$d^2N / dR^2 = d^2L / dR^2 = -2 f''(0) / f'(0)$$

We now have a formula for Netsteer in terms of f' and f''. This is a theoretical formula and is independent of the specific relationship. This formula applies to any steering system as the steer angle approaches zero. This is a metric for Netsteer at small angles. For classic Ackermann $-2 f''(0) / f'(0) = Tk / W$

The factor is based on angles measured in radians. It must be multiplied by Pi/180 for angles measured in degrees.

This can be verified with a curve fit. Take a suspension with classic Ackermann and generate a table of steering angle and steering input. As we limit the range of data the curve fit converges on the Ackermann factor. See [9] for an example.

SOURCES OF ACKERMANN

Ackermann comes from the difference between circular motion and linear motion. The steering input moves linearly while the steering arm/spindle/wheel/tire rotates about the steering axis, also known as the kingpin axis. The steering projection is a line from the kingpin axis through the steering arm at a constant height. If the steering projection is parallel to the centerline of the car then the left and right steer angles will be identical and there will be 0% Ackermann. But if the steering projection is at an angle then the inside and outside tires follow a different portion of the circle and asymmetric steer results. By carefully selecting angles we can manipulate the resulting Ackermann.

If the steering tie-rod were infinitely long (mathematical concept) or very long (engineering concept) this would be the only source of Ackermann. But since tie-rods are of finite length they represent a second source of circular motion. As the tire rotates the angle of the tie-rod, as seen in plan view, must change as the outer tie-rod point moves longitudinally. The primary motion is laterally, but there is a secondary movement longitudinally, and Ackermann is a second-order effect

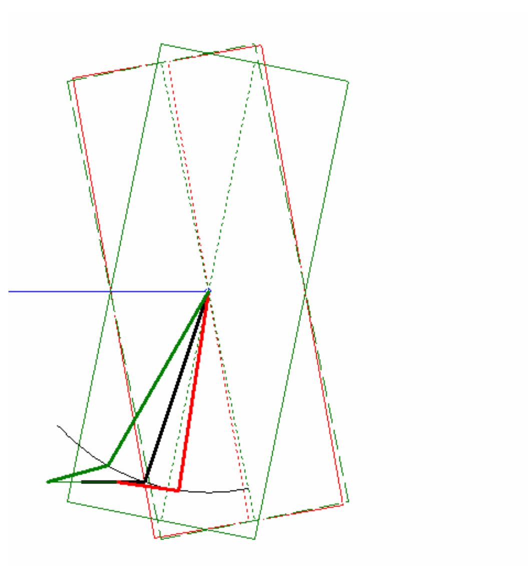

Figure 4: Rear Steer Steering Arm and Tie-Rod movement

TIE-ROD LENGTH

Tie-rod length must be considered in Ackermann analysis. When the tie-rod length is equal to the offset of the steering arm, as in Figure 4, and the steering arm is aimed at the center of the rear-axle, provides a case where both Ackermann calculations are at 100%.

The easiest way to understand the effect of tie-rod length is to consider a front steer case where the tie-rod length is equal to the steering arm offset, as in Figure 5.

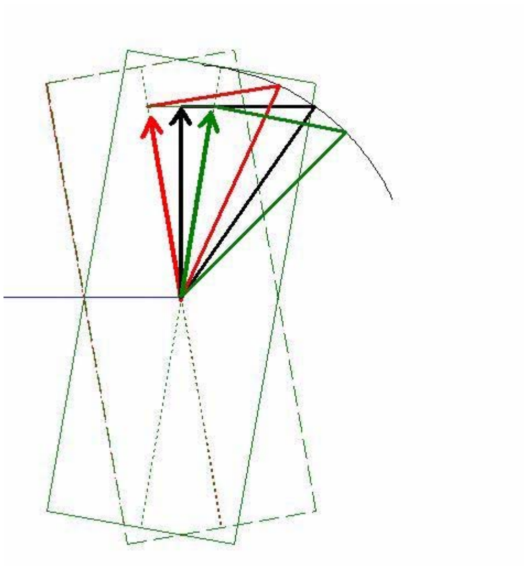

Figure 5: Front steer example where a short tie-rod produces 0% Ackermann

This example shows a virtual steering arm parallel to the vehicle centerline. The steering movement is symmetric and this would suggest zero Ackermann, and that is exactly what it produces.

THE ACKERMANN FORMULA

This formulation assumes the relevant points are in one plane. This can mean a plane perpendicular to the kingpin axis. The simplest way to achieve this is to assume the kingpin axis is vertical and Caster=0 and Kingpin=0

Ackermann Factor = -2 f”(0) / f’(0)

Analysis of the steering arm and tie-rod relationship to rack travel produces derivatives which can be applied to the Ackermann Factor formula above.

```
f'(x) = Deriv1a + Deriv1b
f"(x) = Deriv2a + Deriv2b + Deriv2c + Deriv2d
Deriv1a =  ArmLength * cos( Alpha + x ) =  ArmX
Deriv2a = -ArmLength * sin( Alpha + x ) = -ArmY
These terms relate to the steering arm angle
and define the case with a VERY long tie rod
Deriv1b = -TieX * ArmY/TieY          =0 when TieX=0
Deriv2b =  TieX^2 * ArmY^2 / TieY^3  =0 when TieX=0
Deriv2c = -TieX  * ArmX  / TieY      =0 when TieX=0
Deriv2d = -ArmY^2 / TieY
```

Figure 6 The Ackermann Formula

The actual mathematical results and the derivation are available from the primary author.

RESULTS OF CALCULUS

Based on the geometric relationship [11] tan(x + y) = tan(x) + tan(y) / (1 - tan(x) * tan(y)). If we ignore the term TanArm (ArmY / TieY) then f””/f’ = tan(ArmAngle + TieAngle)

When TieX is small Deriv2b disappears and Deriv2c approaches $ArmY^2$ / TieY. When TieX=0 and TieY = ArmY Deriv2c is -$ArmY^2$ / TieY = -ArmY which is also Deriv1a. Thus the steering arm the tie rod each contribute 50% and we achieve 100% classic Ackermann.

THE STEERING ARM TO TIE-ROD ANGLE

Allan Staniforth raised the subject of Ackermann in January 2006. In his hillclimb car it was impossible to mount the steering rack in line with the steering arms. The rack had to be significantly forward of the steering arms for packaging reasons. Staniforth and Ian Scott devised a solution to this packaging problem. The revised steering improved the handling of the car.

Staniforth suggested the angle of the tie-rods could be offset by aiming the steering arms "at his rear" rather than at the rear end of the vehicle. This suggestion led to numeric confirmation with the computer program WinGeo3 [8] and then to the theoretical analysis presented here, and finally full implementation in the computer program WinSteer [9]

More important than the angle of the steering arm is the angle between the steering arm and the tie-rod. This is more significant than the arm angle. If the entire steering mechanism is rotated, maintaining the arm-to-tie-rod angle, there is little change to the Ackermann percentage.

During the review process Kevin Kwiatkowski uncovered a prior reference in Dixon's second edition [4]. "Moving the rack forward or backward to change the tie-rod angles can be a useful way to adjust the Ackermann factor, the most important angle being the angle between the tie-rod and the steering arm in plan view, the Ackermann factor being proportional to the deviation of this angle from 90 degrees." This sentence was published in 1966. Staniforth and Scott did their work in 1998/1999 and published in 2000 [12]. The authors of this paper had Dixon's 1991 English book [3] and were unaware of the new sentence in the second edition published by SAE [4].

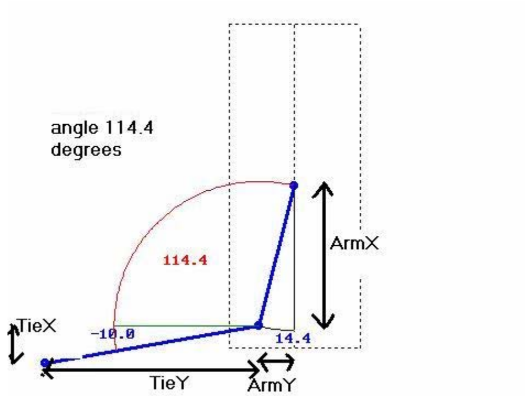

Figure 7: Steering Arm to Tie-Rod angle

The Steering Arm-Tie-rod angle is ArmAngle + TieAngle + 90 degrees. The cotangent of this angle represents the working of the steering system. We have already shown that, if we neglect a couple small terms, f''/f' is tan(ArmAngle + TieAngle) = cot(ArmAngle + TieAngle + 90) = cot(The Angle).

Having performed the necessary derivation, we can look at the geometry involved. If we rotate the entire co-ordinate system when we rotate the steering mechanism, then the links are not changed. The only items that move are the tire, and this makes no difference because we are dealing with changes in tire angle, and the steering input. Thus the only change is:

Tan(Arm) * (ArmY/TieT) / (1 - Tan(Arm)*Tan(Tie))

Since the ratio ArmY/TieY does not change when we rotate the mechanism, this term is proportional to Tan(Arm). Nothing else changes

CASTER ANGLE

The above formula was derived assuming the Caster and Kingpin angles are zero. This assumption allows the situation to be modeled in a two-dimensional plane. If we permit a non-zero caster angle we can construct a reference plane perpendicular to the kingpin axis and going through the steering arm tie-rod point. The steering tie-rod inner point still moves in this plane.

Then the above derivation applies except for the assumption that a steer angle of x degrees is represented by the same angle in the reference plane. This is a small correction. When we deal with larger steering angles this correction is dwarfed by other factors.

KINGPIN ANGLE

When the kingpin angle is not zero, we can still construct a reference plane perpendicular to the kingpin axis and going through the steering arm tie-rod point. But in this case the inner mount of the tie-rod no longer travels in the reference plane. This introduces other factors, but with the small kingpin angles usually used the correction should be small.

THE ANGLE IN THREE DIMENSIONS

When the kingpin axis is vertical it does not matter if the inboard end of the steering tie-rod is above or below the plane. The motions are the same: the length of the tie-rod is the diagonal distance between steering arm and the input device. As viewed from above, the motions in the reference plane are unchanged.

Even with a non-zero kingpin angle, vertical displacement of the steering system makes little difference.

ENHANCED JEANTAUD DIAGRAM

The Jeantaud diagram is quite graphic but it ignores the length of the steering tie-rod. An enhanced Jeantaud diagram can be drawn that does a good job of predicting the percentage of Ackermann. Let Base be the distance between the kingpin and steering arm perpendicular to the steering tie-rod. Create a length Offset equal to the Base squared divided by the length of the steering tie-rod. Then construct a point a distance Offset from the steering arm point along the axis of the tie-rod. In the example below the steering tie-rod is twice the length of the Base. Thus the Offset is half the Base.

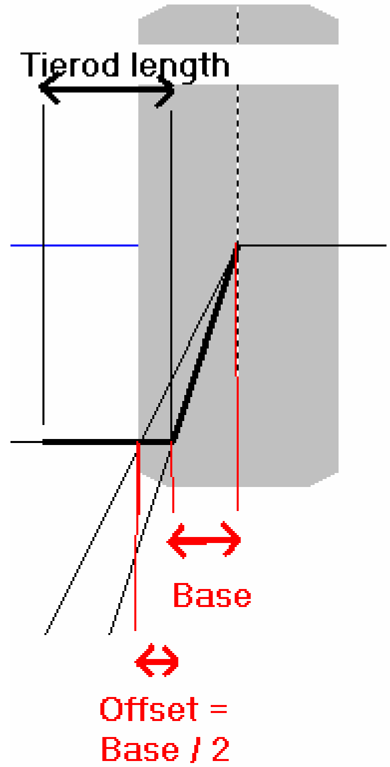

Figure 8 Geometric display of the Enhanced Jeantaud diagram

Then construct lines from the kingpin axis through this new point. The intersection of these lines should be near the center of the vehicle for classic Ackermann.

Rear steer example:

Enhanced Jeantaud - Front Steer, 1 degree

Offset 3.3 inches	Enhanced	Ackermann
Tierod 3.3 inches	100.081	100.112
6.6 inches	75.072	75.054
9.9 inches	66.724	66.705
13.2 inches	62.549	62.531
27.7 inches	55.989	55.978

Front steer example:

Enhanced Jeantaud - Rear Steer at 1 degree

Offset 3.3 inches	Enhanced	Ackermann
Tierod -3.3 inches	100.081	100.112
3.3 inches	0.115	-0.035
6.6 inches	24.934	24.984
9.9 inches	33.296	33.326
34.3 inches	45.191	45.195

That is very good agreement. If you reduce

That is very good agreement. If you reduce the steering angle to 0.1 degree the differences disappear.

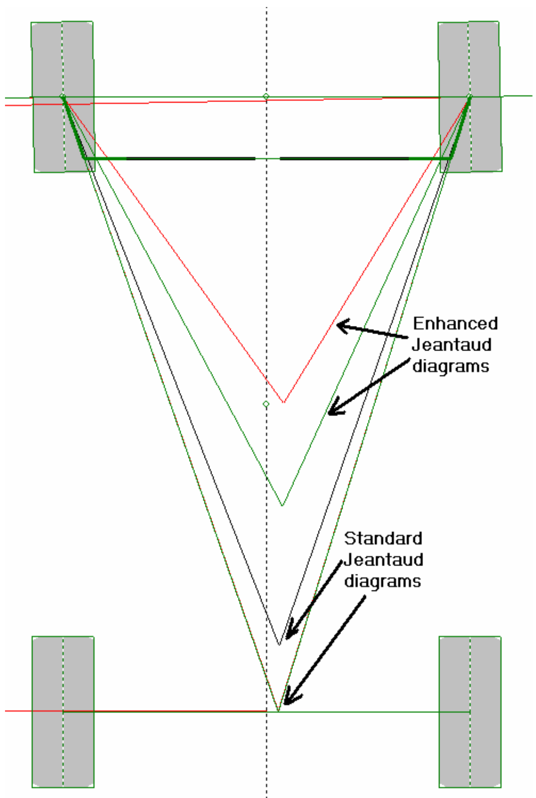

Figure 9 Enhanced Jeantaud diagrams

ASYMMETRIC SUSPENSIONS

Oval track racing involves asymmetric suspensions. The analysis here refers to symmetric suspensions because the asymmetric case offers too many opportunities to create Netsteer. One of the easiest is to shorten the length of the inside steering arm. This works just fine as long as the car only turns one direction. It the car must turn the opposite direction, perhaps to avoid an accident, it yields anti Ackermann.

But the Ackermann formula provides a metric for Ackermann. If we calculate the metric for each side and then average the values, the result is a useful indicator of combined Ackermann. A combination of 150% Ackermann on the right and 50% on the left yields 99.28% Ackermann. One example is hardly proof but it suggests this idea merits further investigation.

CONCLUSIONS

This paper should enhance understanding of Ackermann steering geometry.

1. It creates a way to measure Ackermann. The ratio of Actual to Classic netsteer is the most natural metric.

2. The derivation of a Ackermann Formula presents a useful analytical tool with each derivative term having a simple geometrical representation.

3. The angle of the steering arm is the primary factor, but the tie-rod angle and length play a significant role in Ackermann. However, the Arm-to-Tie-rod angle is a better predictor of Ackermann than the angle of the steering arm.

4. The Enhanced Jeantaud diagram is accurate when the tie-rods run cross-car. This is an accurate metric with geometric understanding but it must be aimed at the middle of the car, not the rear axle.

WHAT THE PAPER DOES NOT DO

This is a thorough analysis yet there are several things it does not do:

A. Should you use 100% classic Ackermann on your race car or street car? This paper provides no answer. Pneumatic tires are complex objects and their optimum performance has little to do with minimizing scrub. Zapletal [13], [14], [15] presents a discussion of these factors.

B. Ackermann is most crucial at large steer angles. With heavy trucks away from Interstate highways full-lock steering is common and has significant effect on tire wear. The calculus in this paper deals with small steer angles.

C. Racing tires operate at significant slip angles. This analysis ignores slip angles. See Zapletal.

D. This analysis assumes everything is rigid. In the real world everything bends, as demonstrated on a Kinematics and Compliance machine.

Ackermann is significant at large steer angles, for instance inner-city trucks at full lock. But large steer angles are rare in motor sports. If we consider a 6-degree steer angle, the difference between parallel steer and 100% classic Ackermann is 0.4 degrees. Measured as a length this is 5 mm or 3/8 of an inch. The difference between 90% Ackermann and 100% (0.5 mm or 1/16 inch) is much less than the precision available on setup.

PROCESS

Most of the mathematical relationships in this paper were discovered numerically and then verified with paper and pencil. Scientific papers usually begin with theory and proceed to numeric verification. The actual intellectual process may not follow the traditional pattern.

Several of the most significant developments were made while preparing the SAE paper. The rigor required for publication can lead to additional breakthroughs and understanding. The Enhanced Jeantaud diagram was discovered after the initial draft had been reviewed.

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