Question 5.

1. a) Yes. Because if $f(n_1)=f(n_2)$, $n_1-1=n_2-1$, $n_1=n_2$, by definition $f(n)$ is a one-to-one function.

b) no. -2 and 2 have the same image, 5.

c) Yes. If $f(n_1)=f(n_2)$, $n_1^3=n_2^3$, $n_1=n_2$, by definition $f(n)$ is one-to-one.

d) NO. ~~$f(2)=f(1)=1$, for example~~

~~$\cancel{\#}$~~ $f(2)=f(1)=1$, for example

II. a) & d) are onto

For any arbitrary n, $f(n)=n-1$ has a corresponding preimage; so does $f(n)=\lceil \frac{n}{2} \rceil$, as we can find $f(n)=\frac{n}{2}$ if n is even, or $f(n)=\lceil \frac{n}{2} \rceil$ if n odd. There are always preimages for $f(n) \in \mathbb{Z}$ in $f(n)=n^2+1$

b) & c) are not onto. For instance, there's no preimage of 7, as 6 is not the square of an integer. There's also no preimage of 10 for $f(n)=n^3$, as 10 is not the cube of an integer.

Question 6

a) Yes $f(x)$ is one-to-one because when $f(x_1)=f(x_2)$, $x_1=x_2$, as proved by $-3x_1+4=-3x_2+4$

$$x_1=x_2$$

$f(x)$ is also onto as $\forall y \in R, \exists x \in R$ $x=\frac{4-y}{3}$

$y=-3x+4 \rightarrow x=\frac{4-y}{3}$, $f(x)=f(\frac{4-y}{3})=-3\times\frac{4-y}{3}+4$
$=y-4+4=y$

b) No. $f(-2)=f(2)$ so $f(x)$ is not one-to-one, let alone bijection.

c) No. $f(x)$ is not defined at -2 so it's not a function.

d) Yes. If $f(x_1)=f(x_2)$, $x_1=x_2$, so $f(x)$ is one-to-one. It's also surjective because we can always find a preimage of any arbitrary $f(x)$, $\sqrt[5]{x-1}$

Question 7:

~~a) x^3~~

a) $\begin{cases} f(x)=2x+3 \text{ if } x\geq 0 \\ f(x)=2|x| \text{ if } x\leq 0 \end{cases}$

b) $|x|+1$

c) $\begin{cases} f(x)=2x+1 \text{ if } x\geq 0 \\ f(x)=2|x| \text{ if } x<0 \end{cases}$

d) $\frac{x-3}{x-4}$

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Question 8

a) $f \circ g = 3x(2x-2)+5 = 6x - 6 + 5$
$= 6x - 1$

$g \circ f = 2x(3x+5) - 2 = 6x + 10 - 2 = 6x + 8$

b) $a(cx+d)+b = c(ax+b)+d$

$acx + ad + b = acx + bc + d$

$(a-1)d = (c-1)b$

$$\frac{d}{b} = \frac{c-1}{a-1}$$

Question 9

a) No. $f \circ g$ only gurantees $\forall f(g(x)), \exists g(x) \mid$

$\forall c \in C, \exists b \in B \mid b = g(x) \wedge c = f(g(x))$

It only means all elements in C have preimages in B, but it's insufficient to prove all elements in B have preimages in A

b) Yes. If f and g are both onto,

$\forall b \in B, \exists a \in A$ and $\forall c \in C, \exists b \in B$.

g being onto guarantees every image in B has a preimage in A, or the codomain B = range of $g(x)$

f being onto guarantees every image in C has a preimage in B, thus guaranteeing every element in B has a corresponding image in C, and codomain C = range