Question 7

There are 6 vowels, each worth 3 points, and 20 consonants, each worth 1 point. The probability of picking each is 1/26.

a) Distribution of X is the set of all pairs (r, p(X=r)) for all r $\in$ X(S). The possible pair (3, 1/26) appears 6 times, while (1, 1/26) appears 20 times. Thus, the distribution is {(3, 6/26), (1, 20/26)}. ({p(X = 3) = 6/26, p(X = 1) = 20/26})
b) Expected value of X is $\sum$ p*r = ((1/26)*3)*6 + ((1/26)*1)*20 ~= 1.46.
c) Variance of X is $\sum$ p(s)*(X(s) – E(X))^2 = ((1/26)*(3-1.46)^2)*6 + ((1/26)*(1-1.46)^2)*20 ~= 0.71

Question 8

Total number of possible values of X is 26^2, from the set {2, 4, 6}. Possible scenarios are as follows:

1. 1 vowel + 1 consonant, X = 4
2. 2 vowels, X = 6
3. 2 consonants, X = 2

1. Probability of first scenario: 2*(6/26)*(20/26) = 240/26^2 ~= 0.355
2. Probability of second scenario: (6/26)^2 = 36/26^2 ~= 0.053
3. Probability of third scenario: (20/26)^2 = 400/26^2 ~= 0.592

a) Distribution of X = set of (value of scenario 1 * probability of scenario 1) + (value of scenario 2 + probability of scenario 2) + (value of scenario 3 * probability of scenario 3) = {(4, 0.355), (6, 0.053), (2, 0.592)}

b)
    a. Expected value is $\sum$ p*r = 0.355*4 + 0.053*6 + 0.592*2 ~= 2.92
    b. Expected value of the sum is 2*expected value of 1 deck. The latter is found in Part b) in the previous question, 1.46. So the expected value of the sum of ~= 2.92

Question 9

a) Suppose probability for 1, 2, 4, 5, 6 to come on each die is x, then probability for 3 to appear is 2x. 5x+2x = 7x = 1, so x = 1/7. The probability for 1 through 6 to come up on each die is therefore 1/7, 1/7, 2/7, 1/7, 1/7, and 1/7.

Expected value of the random variable is $\sum p*r$ =
1*1/7+2*1/7+3*2/7+4*1/7+5*1/7+6*1/7 = (1+2+6+4+5+6)/7 = 24/7 ~= 3.43

Expected value of the sum of two such dice is twice the expected value for either, or ~= 6.86.

b) Probability for each number to come up on a single fair die is 1/6. The expected value is thus 1/6*(1+2+3+4+5+6) = 3.5.

   The expected sum for 3 such dice is 3*3.5 = 10.5.