

Question 7

- a) $26^8 = 208827064576$
- b) $P(26,8) = 26!/(26-8)! = 62990928000$
- c) $26^7 = 8031810176$
- d) Since X is already chosen in the first digit and the letters can't be repeated, the problem is akin to selecting and arranging 7 elements out of a set of 25, or $P(25,7) = 25!/(25-7)! = 3315312000$

Question 8

Number of permutations between the domain with n elements and codomain m elements is $P(m, n)$, where $m \geq n$. In our case, $n = 5$, thus:

- a) 0 since a one-to-one function is not possible as there are fewer elements in the codomain than in the domain.
- b) $P(5,5) = 5! = 120$
- c) $P(6,5) = 6!/(6-5)! = 720$
- d) $P(7,5) = 7!/(7-5)! = 2520$

Question 9

Step 1: Find total number of possible subsets with 100 elements: 2^{100}

Step 2: Find number of possible subsets that contain only 1 element: $C(100,1) = 100!/1!(100-1)! = 100$

Step 3: Remember to also subtract the case of empty set (1) from the total, as the empty set is a subset of all sets

Step 4: Total possible subsets containing more than 1 element = $2^{100} - 100 - 1 = 1267650600228229401496703205275$

Question 10

- a) Step 1: Find set of possible individuals selected, besides the bride: $C(9,5) = 9!/5!(9-5)! = 126$
Step 2: Find number of permutations with 6 people: $6! = 720$
Step 3: Total possible permutations: Step 1 * Step 2 = 90720
- b) Step 1: Find set of possible individuals selected, besides the couple: $C(8,4) = 8!/4!(8-4)! = 70$
Step 2: Find number of permutations with 6 people: $6! = 720$
Step 3: Total possible permutations: Step 1 * Step 2 = 50400
- c)
 - i. Method 1:
Step 1: Find set of possible individuals selected, if neither of the couple is selected:
 $C(8,6) = (8!/6!(8-6)!) = 28$

Step 2: Total possible permutations = total possible set if no restrictions are placed - # of permutations if both of the couple are selected - # of permutations if neither of them is selected = $P(10,6) - 50400 - P(8,6) = 151200 - 50400 - 20160 = 80640$

ii. Method 2 (just because I like doing it for fun):

Step 1: Find set of possible individuals selected, if the bride is selected and the groom is not: $C(8,5) = 8!/(5!(8-5)!) = 56$

Step 2: Find number of permutations with 6 people: $6! = 720$

Step 3: Total possible permutations if bride is selected and groom is not = Step 1 * Step 2 = 40320

Step 4: Possible permutations if exactly one of the couple is selected = number of permutations if only the bride is selected + number of permutations if only the groom is selected = $2 * \text{number of permutations if only the bride is selected} = 2 * \text{Step 3} = 80640$

Question 11

- a) Combinations of exactly 3 ones: $C(12,3) = 12!/(3!(12-3)!) = 220$
- b) # of combinations of no one, 1 one, 2 ones, and 3 ones = $C(12,0) + C(12,1) + C(12,2) + C(12,3) = 1 + 12 + 66 + 220 = 299$
- c) Number of total possible combinations – number of combinations containing 2 ones – number of combinations containing 1 one - number of combinations containing no one = $2^{12} - C(12,2) - C(12,1) - C(12,0) = 4017$

Question 12

- a) $7! = 5040$
- b) Total number of possible permutations – number of permutations when E and D are adjacent (meaning, when either “ED” or “DE” appears) = $8! - 2*7! = 40320 - 2*5040 = 30240$
- c) Add numbers of permutations of “CD” and “DE” and subtract out that of “CDE” to avoid double counting: $2*7! - 6! = 10080 - 720 = 9360$