

Question 3

Whenever we divide a number by 4, the possible results for remainders are from the set $\{0, 1, 2, 3\}$. The question is essentially asking us to select five elements out of a set containing only four elements. According to the pigeonhole principle, whenever there are more objects to be selected out of a set containing fewer buckets where these objects can fit, there must be at least two objects in the same bucket.

Question 4

Since each computer must connect to at least one other computer, we're in essence connecting 6 elements in the domain to 5 elements in the codomain, as for each computer there are only 5 choices to choose from. Similarly, in accordance with the pigeonhole principle, since we're putting more objects into buckets available, there must be at least two computers connected to the same number of other computers (1, 2, 3, 4, or 5).

Question 5

Think of the set as 50 distinct sets, each with 2 elements adding up to 101 (namely, $\{1, 100\}$, $\{2, 99\}$, $\{3, 98\}$, $\{4, 97\}$, ..., $\{50, 51\}$). We're being asked to select 51 numbers from these 50 sets, which means there are at least 2 numbers selected from the same set. As all of these 50 sets add up to 101, as soon as we have both numbers in one set selected, they must add up to 101.

Question 6

Think of the 100 addresses as 50 distinct sets, this time each containing 2 adjacent numbers ranging from 1000 to 1099 (namely, $\{1000, 1001\}$, $\{1002, 1003\}$, ..., $\{1098, 1099\}$). Since we're selecting 51 houses out of 50 sets, there are at least two houses picked from the same set, making them adjacent integers.

Question 7

Consider there being 2 types of integers, even and odd. Because we have a 2-axis space, there are $2 \times 2 = 4$ possibilities for a point, when its coordinates consist of even or odd integers (namely, (odd, odd), (even, even), (even, odd), or (odd, even)).

The midpoint of a line joining a pair of points whose coordinates are both integers has integer coordinates when and only when both points' x-axis values are even or odd, and both of their y-axis values are even or odd. Out of 5 points, we're essentially selecting 5 objects and fitting them into 4 possible buckets (4 possible combinations of x and y), so there must be at least one repetition. As soon as there's a pair that has both their x-axis values and y-axis values the same type (even or odd), we have a line whose midpoint consists of integer coordinates.