Question 5

a) $5n^3+2n^2+3n \le 5n^3+2n^3$ when:

$$2n^2+3n \le 2n^3$$

$$n(2n^2-2n-3) \ge 0$$

when $n>0$,

one of the solutions to the quadratic formula

$2n^2-2n-3$ is $n=1.8288$

we can thus use 2 as n_0 and 7 as C_1

$5n^3+2n^2+3n \ge 5n^3$ also when $n \ge n_0$

thus C_2 is 5

Because $\exists\, n_0, C_1, C_2$ such that $C_2 g(n) \le f(n) \le C_1 g(n)$

where $g(n)=n^3$ and $f(n)=5n^3+2n^2+3n$,

we have proved $5n^3+2n^2+3n = \theta(n^3)$

b) $\sqrt{7n^2+2n-8} \leq \sqrt{9n^2} = 3n$

because $7n^2+2n-8 \leq 9n^2$ always holds, as

proven by : $2n^2-2n+8 = 2(n^2-n+4)$

$$= 2\left[(n-\tfrac{1}{2})^2+\tfrac{15}{4}\right] \geq 0$$

Thus C_1 can be 3

$\sqrt{7n^2+2n-8} \geq \sqrt{4n^2} = 2n$ when

$3n^2+2n-8 = (3n-4)(n+2) \geq 0$

namely, when $n \geq \frac{4}{3}$ or when $n \leq -2$

We can thus set n_0 to 2 and C_2 to 2

Because $\exists\, n_0, C_1, C_2 \mid C_2 g(n) \leq f(n) \leq C_1 g(n)$

where $C_1 = 3$, $C_2 = 2$, $n_0 = 2$, $f(n) = \sqrt{7n^2+2n-8}$

and $g(n) = n$,

$f(n) = \theta(n)$