Question 5

a)

1. Base case: when n = 1, n^3+2n = 1+2 = 3, which is a multiplier of 3, which is tantamount to saying 3 dividing n^3+2n leaves no remainder. We see the statement holds true when n = 1.
2. Inductive step: we assume that (n-1)^3 + 2(n-1) is a multiplier of 3, for all n>=2. Toward this goal we see that n^3 + 2n = (n-1)^3 + 2(n-1) + (3n^2 – 3n +3), since the first part before the parentheses is n^3 – 3n^2 + 3n -1 + 2n – 2. The part inside the parentheses is a multiplier of 3, as it can be written in the form of 3(n^2-n+1). As n is a positive integer, n^2-n+1 is also an integer, making the entirety of the components making up n^3+2n a multiplier of 3.

b)

1. Base case: with the smallest positive integer >= 2, 2, it is a product of prime numbers by definition as its only divisors are 1 and itself.
2. Inductive hypothesis: every integer between 2 and n-1 is assumed to be a product of prime numbers. For any n, it must be either 1) a prime number itself, meaning it has no divisors other than 1 and itself, which makes it a product of prime numbers, or 2) a non-prime number, meaning it can be divided by two other integers, k1 and k2 (where neither k1 nor k2 is 1 or the non-prime number itself). Since the k1 and k2 must be smaller than n, and it is assumed all integers larger than 2 and smaller than n are products of prime numbers, n is also a product of prime numbers.